PROPOSED METHODS FOR DETERMINING THE EFFECT OF U-236 AND Np-237 ON THE VALUE OF URANIUM AS FEED FOR PRESSURIZED WATER POWER REACTORS

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PROPOSED METHODS FOR DETERMINING THE EFFECT OF U-236 AND Np-237 ON THE VALUE OF URANIUM AS FEED FOR PRESSURIZED WATER POWER REACTORS

A. Introduction

The procedure presently used to determine power reactor fuel cycle economics treats the price of uranium as a function only of its U-235 content and as independent of the amount of U-236 present. This is an acceptable simplification so long as the U-236 content of uranium purchased or sold by reactor operators remains below a few tenths of a percent. Under government fuel ownership conditions, there was no alternative pricing procedure for uranium which accounted for U-236 content, so indeed, reactor operators had no choice but to value their uranium on the existing AEC price scale.

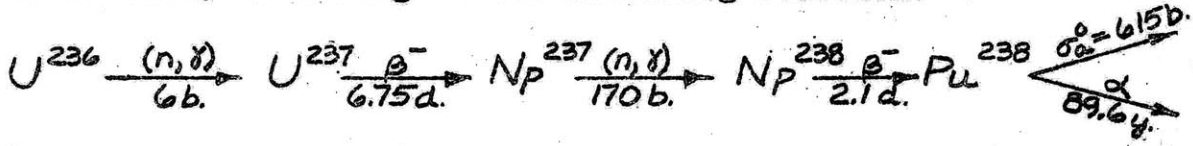
However, the onset of private fuel ownership makes it unnecessary for the operator to be bound to a fixed price scale for uranium and prices involved in uranium transactions between two parties (neither of which need be the AEC) will be mutually agreed upon, based on considerations of fuel value to the purchaser and fuel cost to the seller. The reactor operator, of course, retains the option to purchase natural uranium on the open market and pay the AEC for toll enrichment, thereby acquiring a product free of U-236, at a price consistent with the existing AEC scale for mixtures of U-235 and U-238. The bulk of the uranium of other than natural enrichment sold on the open market is likely to be in the form of UO_3, resulting from the processing of irradiated fuel material and the subsequent conversion of UNH to UO_3 form, which is suitable for shipping. As the burnup of fuel in power reactors is increased, the U-236 content of this UO_3 will likewise increase. Also, the general level of U-236 in feed to reactors, hence in fuel discharged, will increase as a larger proportion of diffusion plant feed is made up of fuel from power reactors.

In a word, large amounts of uranium will be available for sale to reactor operators, and this fuel can be expected to contain significant amounts of U-236. The reactor operator will not be bound to a price schedule for this material, but can offer to buy it at a price which reflects the effect of the U-236 content on his reactor economics. The operator would not be expected to pay the same price as he would if the U-236 were replaced by U-238 (the current procedure) since increased U-236 content will affect fuel cycle costs in the following ways:

1. It will reduce the reactivity lifetime of fuel of a given U-235 content in power reactors, because U-236 is a thermal poison whereas the U-238 it replaces is a fertile material. This will tend to increase fuel cycle costs.

2. It will increase the amount of separative work expended in a diffusion plant to produce uranium of a specified U-235 content, and thus will increase fuel cycle costs.

3. It will increase the amount of Np-237 produced in power reactors. Np-237 has value as a target material for the production of Pu-238, a valuable heat source for thermoelectric converters, according to the following reaction:



Present day cost procedures do not consider the sale of Np-237, but as appreciable quantities of Np-237 are produced, this will undoubtedly be done. The credit received for Np-237 will tend to reduce fuel cycle costs and is thus a positive characteristic of increasing U-236 content.

Obviously, the effect on fuel cycle economics of increasing U-236 content in uranium will be different for each reactor type, and in fact, for each method of fuel management for a particular reactor type. Thus, it will be necessary for each reactor operator to determine what the value of uranium of any U-236 composition is when used in his reactor so that he will know what price he can afford without penalizing his fuel economics. On the other

hand, the fuel seller must choose between selling his irradiated fuel or recycling it in his own reactor. If the price he must receive to avoid penalizing his fuel economics is more than the value of the material to anyone else, he would keep his fuel and recycle it; however, if his fuel is worth more to someone else than the break-even price he requires, he would sell it and use fresh feed material.

The purpose of this study is to establish the value of uranium over a range of U-235 and U-236 isotopic compositions when the uranium is used as feed to a typical large PWR. The importance of the PWR in power generation makes it a logical choice for the study. Also, the effect of Np-237 sale on fuel cycle economics in general and how the value of uranium would vary with the price received for Np-237, will be determined. The resulting uranium values will provide a means of estimating the effect of U-236 and Np-237 on fuel cycle economics for a representative PWR design, but the procedures developed could be extended to the estimation of feed fuel value for a wide range of other reactor types with only slight revision.

B. General Procedure

As mentioned above, reactor operators will have a choice between obtaining fresh, U-236-free UF_6 as their makeup material, priced on the existing AEC scale, or purchasing irradiated, U-236-bearing UO_3 as makeup, valued according to its effect on reactor economics. The principle to be followed in determining the value of uranium of a given composition, when used in a particular reactor, for a specified fuel flow model, and for a specified sale price for Np-237, is that the over-all fuel cycle cost with makeup uranium of this composition shall equal the overall fuel cycle cost for the same fuel cycle with makeup uranium containing no U-236, priced on the existing AEC scale and operated at the feed enrichment which gives minimum fuel cycle cost. If the price of uranium is set equal to its value determined in this way, it will be a matter of indifference to the reactor operator whether he obtains fuel of optimum enrichment containing no U-236 at the

current AEC price scale, or fuel containing U-236 priced according to this principle. Thus, the reactor operator knows what the minimum fuel cycle cost is when he obtains U-236-free feed on a known price scale and, since he would refuse to operate at a higher fuel cycle cost, he sets the value for feed containing U-236 such that the resulting fuel cycle cost remains at this same minimum.

For Np-237 prices between zero and a figure to be determined, the presence of U-236 in the feed would cause the uranium value to be less than the value of uranium with zero U-236 content; however, above this Np-237 price, the presence of sufficient U-236 in the feed could actually increase the value of uranium over the value of feed with zero U-236 content. By carrying out the analysis with a range of Np-237 prices, this effect can be examined. Further discussion of the Np-237 price effect is postponed to Section E, where a procedure for estimating the cost of producing Np-237 in the reference PWR is outlined. This cost would be one of the points considered in the range of Np-237 prices.

The reference PWR chosen for the study is the 450 MWe (1346 MWt) San Onofre Nuclear Generating Station designed by Westinghouse for Southern California Edison Company and San Diego Gas and Electric Company (1). Zircaloy-4 will be selected as the reference cladding material; however, since the level of U-236 buildup depends upon feed enrichment, portions of the study might be repeated using stainless steel cladding to gauge the effect of the different enrichment requirements. Due to its apparent advantages in large cores, modified multibatch scatter refueling will be used as the fuel management scheme (2). This procedure (3) differs from straight scatter refueling in that fresh fuel is first added to an outer annular ring, from which it is then used as feed to the remainder of the core, which is managed by straight scatter (see Figure 1). In effect, we then have a region refueled scatter-wise and surrounded by an annular region which feeds the central region with assemblies that have been irradiated for one period (between reloadings). This modification tends to provide

a flatter power distribution than does straight scatter for the reference design core size, since the outer core power is increased by the fresh fuel there. For larger cores, the neutronic coupling between portions of the core is poorer and straight scatter is used since the modified scheme would result in excessive power peaking in the fresh-fuel outer ring (2). A 4-batch refueling scheme will be used, as this apparently corresponds to a (roughly) one-year refueling interval preferred by power companies.

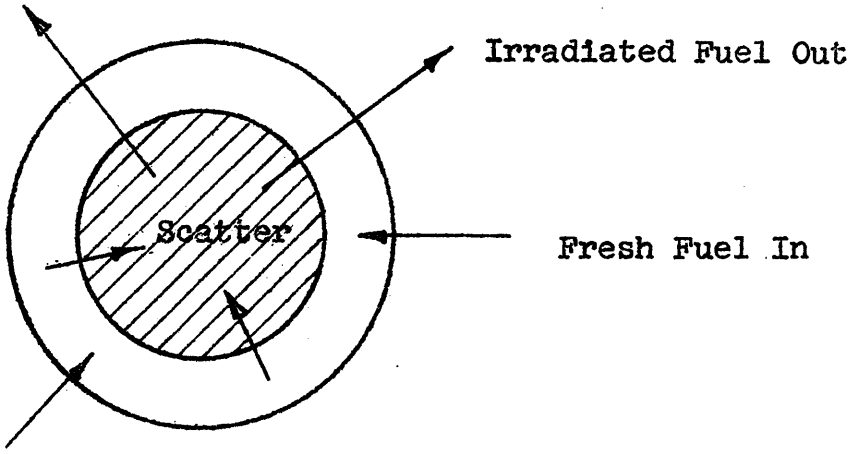


Figure 1

Two fuel cycle flowsheets will be examined for the reference PWR. The first (Figure 2) involves the recycle of processed fuel directly to the fuel fabricator, where it is blended with feed of high enrichment to form the reactor charge. The second (Figure 3) considers the recycle of fuel back through a gaseous diffusion plant where it is re-enriched and then mixed with feed of moderate enrichment. In Section D, a procedure is described leading to a variation of Figure 3 which enables one to consider feed material of very low enrichment which is first fed to a diffusion plant for upgrading prior to being mixed with the re-enriched recycle stream. By examining these flowsheets, it is possible to determine the effect of U-236 on the value for much of the fuel likely to be discharged from existing reactors and sold for reactor feed. Makeup for Figure 2 would most likely be processed fuel discharged

from research, testing or submarine reactors, while Figure 3 could utilize fuel discharged from a wide range of reactors as feed.

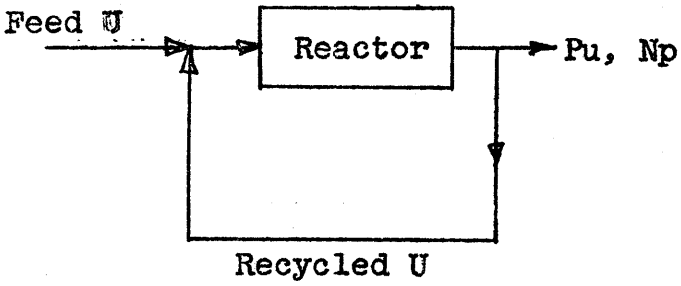


Figure 2

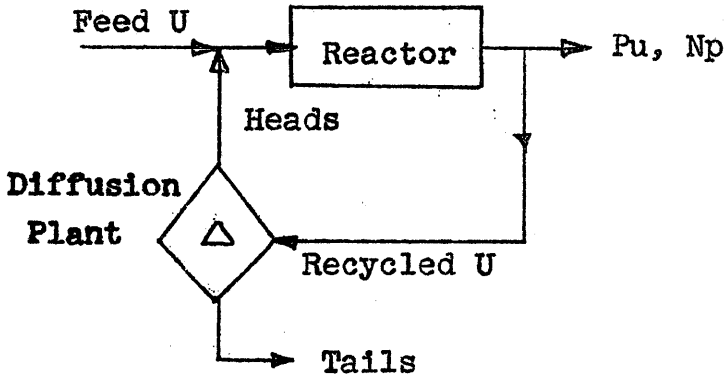


Figure 3

Note that both fuel cycles involve recycle of processed uranium and the immediate sale of fissile Pu and Np-237 after processing. The Np-237 is sold since it is economically desirable to remove this potentially valuable product from the system as quickly as possible after it has been formed. It was decided to sell Pu rather than recycle it in the PWR for this study, although the case where Pu is recycled back to the fabricator (bypassing the diffusion plant in the second case) after reprocessing could be the subject of another study leading to uranium feed values. However, the presence of Pu isotopes in the various fuel streams throughout the fuel cycle would considerably increase the complexity of the analysis and would destroy the basic simplicity of the procedures described in Section F. The recycle of the processed uranium, rather than selling it, is necessary to avoid having two streams of unknown value in the analysis. It would be impossible to set distinct values for two fuel materials at once. In Figure 3, the tails stream from the diffusion plant has zero value if its composition is properly fixed.

Economic analyses will be performed only for the steady-state fuel cycle flowsheets since this gives a common basis upon

which to compare values for uranium feed of various compositions and leads to a considerably simplified analysis.

In addition to the effects of U-236 content, Np-237 price, and clad material upon the value of uranium feed, the effect of natural uranium market price will be examined, also. Not only will this price affect the price of uranium, free of U-236, purchased from the AEC, which is needed for both recycle schemes, but the optimum tails composition in Figure 3 will be directly affected, thus influencing the amount of U-236 which leaves the diffusion plant in the tails stream. Hence, feed values will be determined for more than one natural uranium price.

A basic assumption which is unavoidable in performing the study is that the product obtained from any diffusion plant operation (each of which represents a toll enrichment transaction between the reactor operator and the AEC) is the only product stream from the plant and has a U-236 content which would result from the use of a feed stream having a composition which we have specified. In actual toll enrichment transactions (4), the reactor operator presents material of known composition to the AEC (e.g. natural uranium or uranium discharged from a reactor) and requests material having a higher U-235 content. Instead of using the operator's feed material to produce the desired product (we assume that it is used!), the AEC may actually furnish product material which was enriched from a different feed material. Thus, lack of control over the U-236 content in the product uranium could result, since the composition of feed streams and other product streams of the diffusion plant will be relatively unpredictable. The AEC will charge the operator the cost of separative work involved in producing the enriched material from the feed he furnishes.

Due to the impossibility of predicting the composition of all possible feed and product streams of the diffusion plant at some future date, it is necessary to make the above assumption; again, our feed stream is the only one present, our product stream is the only one present, and our product stream results from the re-enriching of our feed stream. It is further assumed that two

kinds of diffusion plants are operating. One accepts uranium feed streams containing U-236 and performs toll enrichment in the manner described above. The second accepts natural uranium feed only and provides the enriched product, free of U-236, which is used as feed to the flowsheets in Figures 2 and 3 when feed costs are based on the AEC price scale. In the cost analysis, it is assumed that the latter material can be purchased directly from the AEC without actually supplying natural uranium for toll enrichment; hence, the delay in receiving enriched product from toll enrichment is not involved for this feed uranium. The same cost per unit of separative work is assumed for all diffusion plant operations.

Another assumption made concerning diffusion plant operation is that the tails composition is always optimum for the price which the PWR operator pays for his natural uranium. If the market price for natural uranium is reasonably stable, this is a realistic assumption.

A final assumption, which leads to considerable simplification of the analysis at a slight loss in flexibility, is that the cost of converting UO_3 to UO_2, per kg of uranium, is the same as the cost of converting UF_6 to UO_2. Slight differences which might actually exist do not warrant the inclusion of numerous additional items in the cost equations, particularly since this is not a major contributor to the overall fuel cycle cost. In many situations indicated by Figures 2 and 3, the fabricator receives a stream of UF_6 and a stream of UO_3 for conversion to UO_2 and subsequent fabrication, and it is convenient to assign a single, overall cost of fabrication (including conversion) for each kg of uranium shipped to the reactor. In order to secure a homogeneous mixture of any two streams, regardless of their chemical form, it is likely that they will both be put into solution for mixing, after which the homogeneous solution will be converted to UO_2. Thus, since neither stream would be converted directly to UO_2, the assumption of a single cost of conversion per kg of reactor feed is not unreasonable.

The effect of U-236 on the value of uranium feed can be seen by comparing fuel values determined for material of zero U-236 content with those for material having increased U-236 content. It is also of interest to compare these results with both the existing AEC price scale (with U-236 considered as U-238) and the scale developed by de la Garza, et.al. (5), for mixtures of U-235, U-236, and U-238. The de la Garza scale is based only on diffusion plant considerations and neglects any effects of U-236 on the reactor being fed by the plant. The final evaluation of results is discussed in detail in Section H, step 15.

C. Recycle to Fabricator

The flowsheet used for the first recycle scheme is shown in Figure 4. Processed uranium is recycled to the fabricator, where it is blended with more-highly-enriched makeup feed material before being returned to the reactor. In this scheme, the only means available for re-enriching the spent fuel is by blending; thus, the makeup feed enrichments considered for this flowsheet will be much higher than those required for recycle through the diffusion plant (discussed in Section D). Since Figure 4 shows that U-236 is not removed from the cycle (except for small losses of fuel which are unavoidable during fabrication and processing), the steady-state concentration of U-236 in recycled uranium will be considerably greater than for the recycle to the diffusion plant, since in that case, an appreciable amount of U-236 is discharged from the cycle in the tails stream.

The nomenclature used is also given in Figure 4. The full-power output of the reactor is P MW(e). Flow rates F_i are steady-state, time-averaged values for total uranium at various points in the cycle, based on reactor operation at a load factor L. The ratio of U-235-to-U-238 in each stream is denoted by R_i, while y_i represents the weight fraction of U-236 in the uranium.

The first step is the determination of all F_i, R_i and y_i values for steady-state recycling conditions over a range of feed compositions, i.e., for various combinations of R and y. Steady-

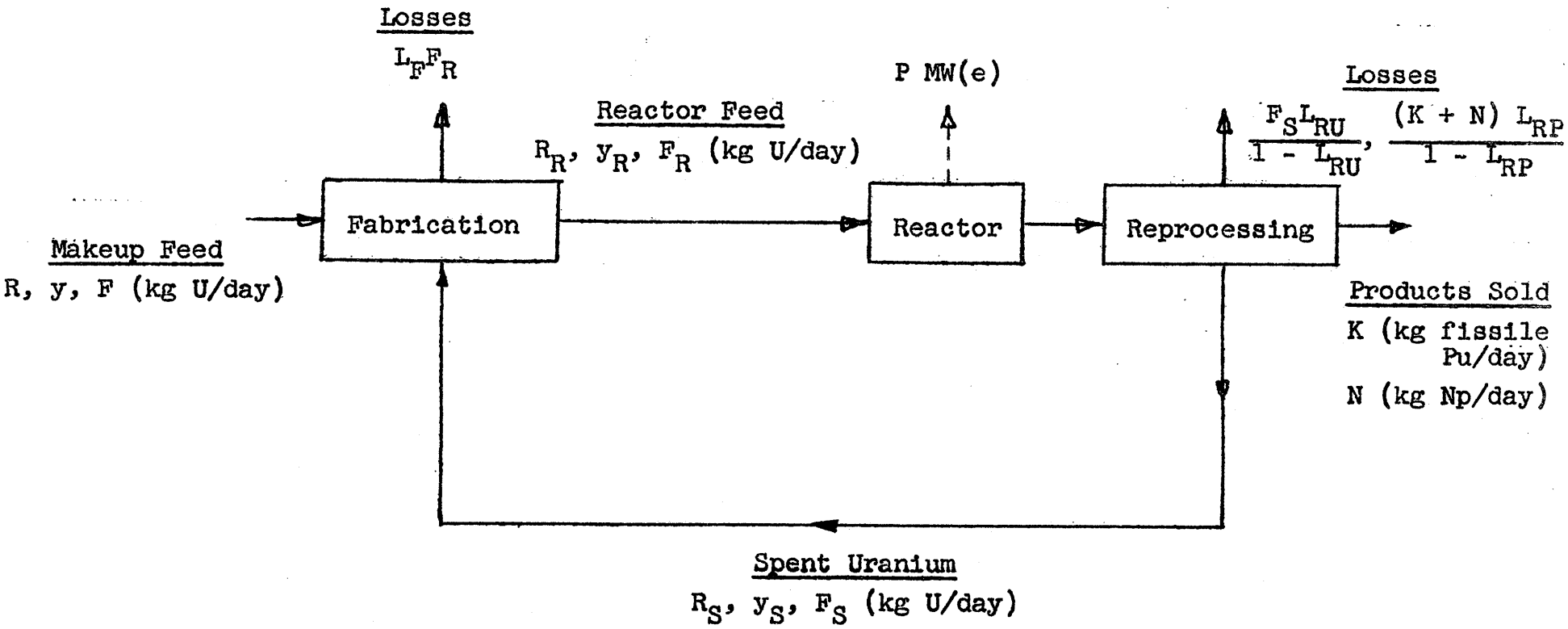


Figure 4. Recycle of Uranium to Fuel Fabricator

state conditions for y = 0 and various R values are particularly important. The procedure to be used for determining the steady-state cycle corresponding to a given feed composition is discussed in Section F.

It is then possible to perform the fuel value calculation for each of the feed compositions, provided assumptions are made about the unit price of the following materials:

1) fissile plutonium, C_K
2) neptunium, C_N
3) reactor feed uranium, C_R
4) spent uranium, C_S
5) makeup feed uranium, C_1

We propose to make the following assumptions:

1. C_K is to be specified as 10/12 times the unit price of U-235 contained in 90% enriched uranium, based on the existing AEC price scale. For the present AEC scale this procedure gives C_K = \$10,000/kg of fissile Pu, which is the value currently in effect.

2. C_N is a parameter to be varied from zero to some arbitrary upper limit. The variation of C_N will be discussed further in Section E. For the present, it can be assumed that C_N has a single specified value.

3. and 4. Prices of reactor feed, C_R, and reactor tails, C_S, are needed only to compute inventory charges. For this purpose, as a reasonable approximation, these may be assigned the same value they would have as UF_6 on the existing AEC price scale for mixtures of U-235 and U-238, with U-236 treated as U-238.

5. When the net feed contains no U-236 and is purchased from the AEC, its price is given by the existing AEC price scale for mixtures of U-235 and U-238. The minimum fuel cycle cost, C_P^*, realizable with such feed will occur at the optimum abundance ratio, R*. When the net feed is purchased as UO_3 from another source, and may contain U-236, its unit value, C_1, is to be determined from the condition that the net fuel cycle cost is to be the same as the minimum fuel cycle cost, C_P^*.

It should be noted that for 1., 3., 4., and 5., the existing AEC price scale will depend upon the market price for natural uranium. Hence, C_p^* and R* will also vary with this price. For the present discussion, it is assumed that natural uranium has a single specified price, although the effect on the results from varying this price will be investigated.

The net fuel cycle cost, C_p, in mills/kwhr, is given by:

$24 PLC_P$ (cost of electricity, \$/day) =

$$FC_1(R,y)$$ cost of net feed

$$+F_R C_F$$ cost of fabrication

$$+\left(\frac{F_S}{1-L_{RU}}+\frac{N+K}{1-L_{RP}}\right)(C_A+C_T)$$ cost of reprocessing and shipping

$$-KC_K$$ credit for plutonium

$$-NC_N$$ credit for neptunium

$$+it_F\left(\frac{C_R}{1-L_F}+C_F\right)F_R$$ interest on inventory during fuel fabrication

$$+\frac{iI}{2\times 365}\left[\frac{C_R}{1-L_F}+C_F+\frac{KC_K+NC_N+F_S(C_S-C_C)}{F_R}-\left(\frac{F_S}{1-L_{RU}}+\frac{N+K}{1-L_{RP}}\right)\frac{(C_A+C_T)}{F_R}\right]$$

interest on mean value of reactor inventory

$$+it_{RU}F_S\left[C_S-C_C-\frac{(C_A+C_T)}{1-L_{RU}}\right]$$ interest on uranium inventory during reprocessing

$$+it_{RP}\left[KC_K+NC_N-\frac{(N+K)}{1-L_{RP}}(C_A+C_T)\right]$$ interest on Pu + Np inventory during reprocessing

(1)

In this equation, C_F is the unit fabrication cost per kg of uranium leaving fabrication and includes the cost of converting UO_3 or UF_6 to UO_2; C_A is the cost of reprocessing per kg of fuel fed to the reprocessing plant and is assumed to include charges for converting UNH to UO_3, which is a convenient form for shipping uranium to the fabricator; t_F is the average pre-reactor holdup time; t_{RU} and t_{RP} are the average post-reactor holdup times for uranium and for Pu + Np, respectively; i is the annual interest rate; and I is the total initial uranium loading of the reactor. Interest charges on net makeup material are assumed to be included in the interest charge during fuel fabrication. The fraction of uranium lost during fabrication is L_F (based on material leaving fabrication), while L_{RP} and L_{RU} represent the fractional losses of Pu + Np and of uranium, respectively, during processing (based on material fed to processing). C_T is the unit shipping cost for irradiated fuel material. The unit cost of converting UO_3 to UF_6 is given by C_C.

To determine the unit value of makeup uranium of any composition R,y, the procedure is as follows. Fuel cycle costs, C_P, are evaluated using Equation (1) for a series of net feeds containing no U-236, with different R values, and priced on the AEC scale. The minimum fuel cycle cost C_P^* corresponding to the optimum net feed abundance ratio R* is determined. The unit value $C_1(R,y)$ of net feed with specified R and y, purchased as UO_3, can be evaluated by setting $C_P = C_P^*$ and solving (1) for $C_1(R,y)$. Note that all steady-state fuel flow rates now correspond to the use of the R,y material as net feed material. $C_1(R,y)$ is found to be:

$$C_1(R,y) = \frac{1}{F}\left\{24PLC_P^* - F_R C_F - \left(\frac{F_S}{1-L_{RU}} + \frac{N+K}{1-L_{RP}}\right)(C_A+C_T)\right.$$
$$+ KC_K + NC_N - it_F\left(\frac{C_R}{1-L_F} + C_F\right)F_R$$
$$- \frac{iI}{730}\left[\frac{C_R}{1-L_F} + C_F + \frac{KC_K + NC_N + F_S(C_S - C_C)}{F_R} - \left(\frac{F_S}{1-L_{RU}} + \frac{N+K}{1-L_{RP}}\right)\frac{(C_A+C_T)}{F_R}\right]$$
$$\left. - it_{RU}F_S\left[C_S - C_C - \frac{(C_A+C_T)}{1-L_{RU}}\right] - it_{RP}\left[KC_K + NC_N - \frac{(N+K)}{1-L_{RP}}(C_A+C_T)\right]\right\} \qquad (2)$$

For a fixed ratio of U-235 to U-238 in the net feed, as y increases (i.e. as U-236 is assumed to replace U-235 + U-238 to a greater extent), F will increase and K/F will decrease. Other flow ratios will remain about the same, except for N/F, which could increase somewhat. Thus, Equation (2) will lead to a lowering of net feed value as y increases, unless the unit price for neptunium, C_N, is large. It is expected that the results for net feed value as a function of R and y, for a fixed C_N and a fixed price for natural uranium, will resemble Figure 5.

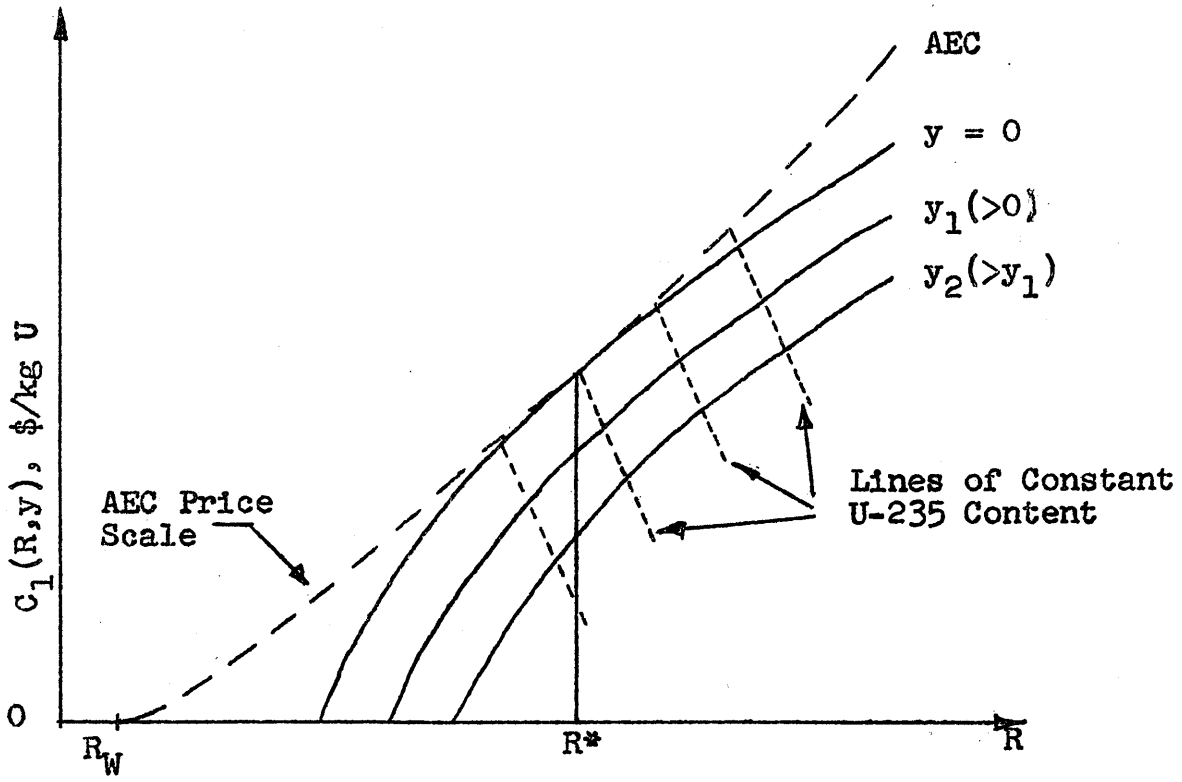


Figure 5

It should be noted that the fuel value calculated for feed equals the price on the AEC scale at the optimum abundance ratio R* for y = 0, because of the principle used in calculating fuel value. Further, it can be shown that the curve of calculated fuel value versus abundance ratio at y = 0 is tangent to the curve

representing the AEC price scale at $R = R^*$ and lies below the AEC price scale at all other values of R. If the calculated value was not less than the AEC price at $R \neq R^*$, some value of R other than R^* would have led to a fuel cycle cost lower than C_P^*.

The fact that the calculated fuel value line at $y = 0$ is tangent to the line representing the AEC price scale is a consequence of the assumption that the cost of converting UF_6 (as purchased from the AEC) to UO_2 is the same as the cost of converting UO_3 (as purchased from a reprocessing plant) to UO_2.

The reasoning which leads to the appearance of the $y = 0$ curve and its position relative to the AEC price scale as shown in Figure 5 is discussed further in Appendix I.

Note that the fuel value drops to zero at some $R > R_W$ along each line of constant y, reflecting the fact that either the minimum R for maintaining reactor operation has been reached or that the operator can no longer pay for feed and still maintain his fuel cycle cost at C_P^*. As y increases, zero feed value is reached at increasing R values. These points are dealt with in detail in Appendix I.

For this method of recycle operation, the points of zero fuel value will occur for high R, since the net feed must provide a substantial amount of U-235 but only a relatively small amount of U-238.

At first glance, it appears inconsistent to have $y = 0$ fuel values lower than the existing AEC prices, but this is a logical result of the fuel value definition. It would appear generally more profitable to the possessor of $y = 0$ material to return his uranium to the AEC for credit in subsequent toll enrichment rather than sell it to the PWR operator. However, this conclusion cannot be reached in a general sense, but will depend upon the chemical form of the uranium. Return to the AEC would require a conversion step to UF_6, while sale to the PWR operator would not, if the uranium is in UO_3 form. In this case, it would be more profitable to sell to the PWR operator for all U-235/U-238 ratios such that the price on the AEC scale minus $C_1(R,0)$ is less than the UO_3-to-UF_6 conversion cost.

As the price of Np-237 increases, the fuel values at constant $y > 0$ in Figure 5 will generally increase until, at some high Np-237 price, the $y > 0$ lines will begin to exceed the $y = 0$ line, reflecting an increase in fuel value due to the presence of U-236.

D. Recycle Through Diffusion Plant

A second possible procedure for recycling spent uranium is to first re-enrich the uranium by gaseous diffusion before returning it to the fuel fabricator, where it is mixed with makeup feed uranium to form the reactor feed stream. The basic flowsheet and nomenclature for this recycle scheme are given in Figure 6. The diffusion plant is assumed to be so operated that, at each point where two streams are mixed, both have the same U-235-to-U-238 abundance ratio, R. The same condition is applied at the point where makeup feed and diffusion plant product are mixed, i.e.

$$R = R_P \quad . \qquad (3)$$

De la Garza, Garrett, and Murphy (5) call a diffusion cascade operated in this way a "matched-R cascade". These authors show that the distribution of U-236 between product and waste in such a matched-R cascade is given by

$$\frac{y_P F_P}{(R_P)^{1/3}} + \frac{y_W F_W}{(R_W)^{1/3}} = \frac{y_S F_S}{(1+L_C)(R_S)^{1/3}} \quad , \qquad (4)$$

where L_C is the fractional loss of uranium during the $UO_3 \longrightarrow UF_6$ conversion (based on the product from conversion). The separative work expended per day, on the average, in such a matched-R cascade is

$$\Delta = F_P \phi_P + F_W \phi_W - \frac{F_S}{1+L_C} \phi_S \quad , \qquad (5)$$

where ϕ_i, the separation potential, is

$$\phi_i = (2x_i + 4y_i - 1) \ln R_i \quad , \qquad (6)$$

in which x_i is the weight fraction of U-235,

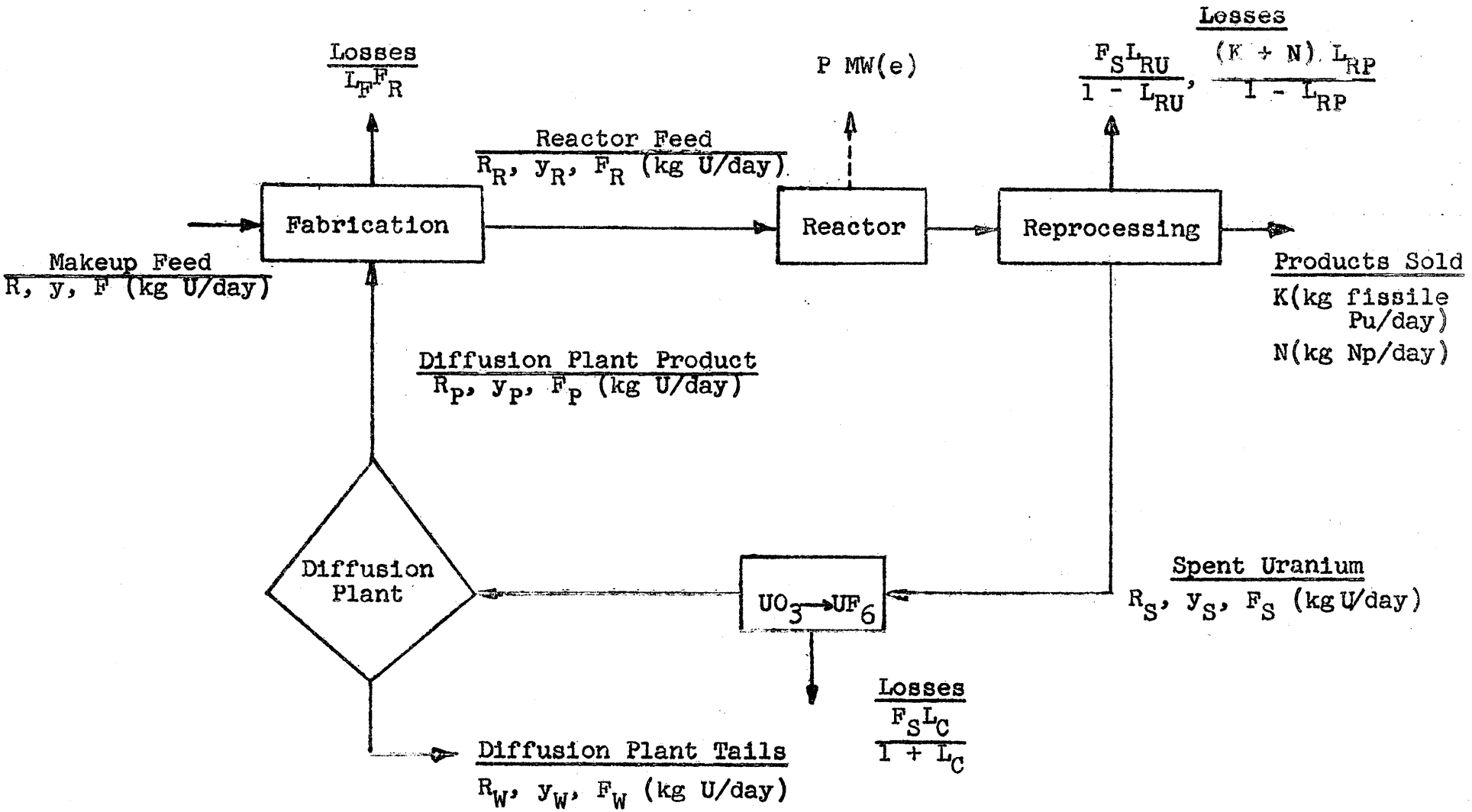


Figure 6. Recycle of Uranium Through Diffusion Plant - Makeup Feed Fed Directly to Fabricator

$$x_i = \frac{R_i(1-y_i)}{1+R_i} \qquad (7)$$

and y_i is the weight fraction of U-236.

With a known unit cost of separative work, C_Δ, e.g. \$30/kg U, the cost of re-enriching spent fuel from the reactor in the diffusion plant is $C_\Delta \Delta$, \$/day.

Since spent fuel re-enrichment is carried out prior to refabrication in this scheme, the makeup feed enrichments considered will be much lower than those required for recycle directly to the fuel fabricator. In addition, since a significant amount of U-236 is removed from the cycle in the diffusion plant tails, the steady-state concentration of U-236 in recycled uranium will be generally lower than in Figure 4.

The first step is again the calculation of all F_i, R_i, and y_i values for the steady-state cycle over a range of feed compositions, i.e. for various R,y points. The procedure used to accomplish this is described in Section F.

The principle used to determine the value of makeup feed for this method of recycle is identical to that described in Section C. For a specified Np-237 price and a fixed price for natural uranium, the value, $C_f(R,y)$, of makeup feed as UO_3 having composition R,y is such that the fuel cycle cost which results from its use is equal to the minimum fuel cycle cost, $C_P^\$$, attainable when makeup feed containing no U-236 is purchased from the AEC and is priced on the existing AEC scale. The various assumptions made in Section C also apply to this case. Diffusion plant cost considerations must now be included in the equation for makeup feed value. The separative work term, ΔC_Δ; the credit for tails, $-F_W C_W$; the cost of converting recycled UO_3 to UF_6, $F_S C_C$; and the inventory charge on the product from toll enrichment, $it_E F_P C_D$, must all be included in the right side of Equation (1). This leads to the following equation for $C_f(R,y)$:

$$C_f(R,y) = \frac{1}{F}\left\{24PLC_P^* - F_R C_F - \left(\frac{F_S}{1-L_{RU}} + \frac{N+K}{1-L_{RP}}\right)(C_A+C_T) + KC_K + NC_N \right.$$
$$- F_S C_C - \Delta C_\Delta + F_W C_W - it_F\left(\frac{C_R}{1-L_F} + C_F\right)F_R - it_E F_P C_D$$
$$- it_{RU} F_S\left[C_S - C_C - \frac{(C_A+C_T)}{1-L_{RU}}\right] - it_{RP}\left[KC_K + NC_N - \frac{(N+K)}{1-L_{RP}}(C_A+C_T)\right]$$
$$\left. - \frac{iI}{730}\left[\frac{C_R}{1-L_F} + C_F + \frac{KC_K + NC_N + F_S(C_S-C_C)}{F_R} - \left(\frac{F_S}{1-L_{RU}} + \frac{N+K}{1-L_{RP}}\right)\frac{(C_A+C_T)}{F_R}\right]\right\} \quad (8)$$

In the above, t_E is the time interval between the delivery of uranium to the AEC for toll enrichment and the receipt of product uranium. C_D is the price of the product from toll enrichment and is approximated here by the price on the existing AEC scale for mixtures of U-235 and U-238, with U-236 treated as U-238. The choice of C_W is discussed in the following paragraphs.

If the PWR operator could specify the operating conditions for the gaseous diffusion plant in Figure 6, he would operate with the R_W which gives him minimum fuel cycle cost for the overall flowsheet. This optimum R_W would differ from the R_W obtained for optimized diffusion plant operation exclusive of the reactor, because of the effect which R_W has on y_R and the effect of y_R on reactor performance. However, the AEC, in operating its diffusion plants for toll enrichment, is unable to adjust their R_W to satisfy the wishes of each customer supplying a feed stream, particularly since more than one feed stream might be present. We have assumed, in Section B, that the R_W for the plant is maintained by the AEC at the optimum value corresponding to natural uranium feed priced according to the current market level. The tails stream from the plant then has zero value, and we take $C_W = 0$. Since the price of natural uranium is a parameter in the study, R_W will vary correspondingly, e.g., $R_W = 0.002531/0.997469$ for the current cost of \$23.46/kg U for natural uranium as UF_6.

It follows from the above that the value of R_W used by the AEC will be independent of the U-236 content of feed material to the

plant. This is the same condition set by de la Garza, et al. (5), in their study.

For the present discussion, it is assumed that natural uranium has a single specified price and that R_W has the corresponding optimum value.

In addition to the effects cited in Section C which tend to lower the net feed value as y increases, we have here an increase in the Δ/F term as the U^{236} content in the makeup feed (hence, in the stream fed to the diffusion plant) increases. We are, therefore, further assured that $C_f(R,y) < C_f(R,0)$ for a given R, unless the price received for neptunium is very high.

By varying R and y, it is possible to obtain the variation of $C_f(R,y)$ for fixed C_N and fixed natural uranium price. The dependence of C_f on R and y will still be qualitatively similar to Figure 5. For the fuel cycle shown in Figure 6, the fuel value curves again drop to zero at some $R > R_W$; however, the points of zero fuel value occur at R considerably lower than for operation according to Figure 4, since re-enrichment of spent fuel is now performed in the diffusion plant and not by the net feed stream. The relative positions of the y = 0 and "AEC" lines are exactly as discussed in Section C, i.e., the two y = 0 lines are tangent at R* only if the $UF_6 \longrightarrow UO_2$ and $UO_3 \longrightarrow UO_2$ unit conversion costs are identical.

A second flowsheet which is applicable when upgrading of uranium is carried out by gaseous diffusion is shown, with appropriate nomenclature, in Figure 7. This flowsheet, which represents an extension of Figure 6, can be used to determine a second complete set of feed values for the range of uranium compositions considered in the analysis of Figure 6, and also provides a means for establishing fuel value for uranium having a very low ratio of U-235 to U-238. The PWR operator purchases UO_3 having a composition R',y', but now, instead of being used directly as feed to the fabricator, the material is first converted to UF_6 and fed into the diffusion plant for enrichment to R. This product stream, with composition R,y is then fed as UF_6 to the fuel

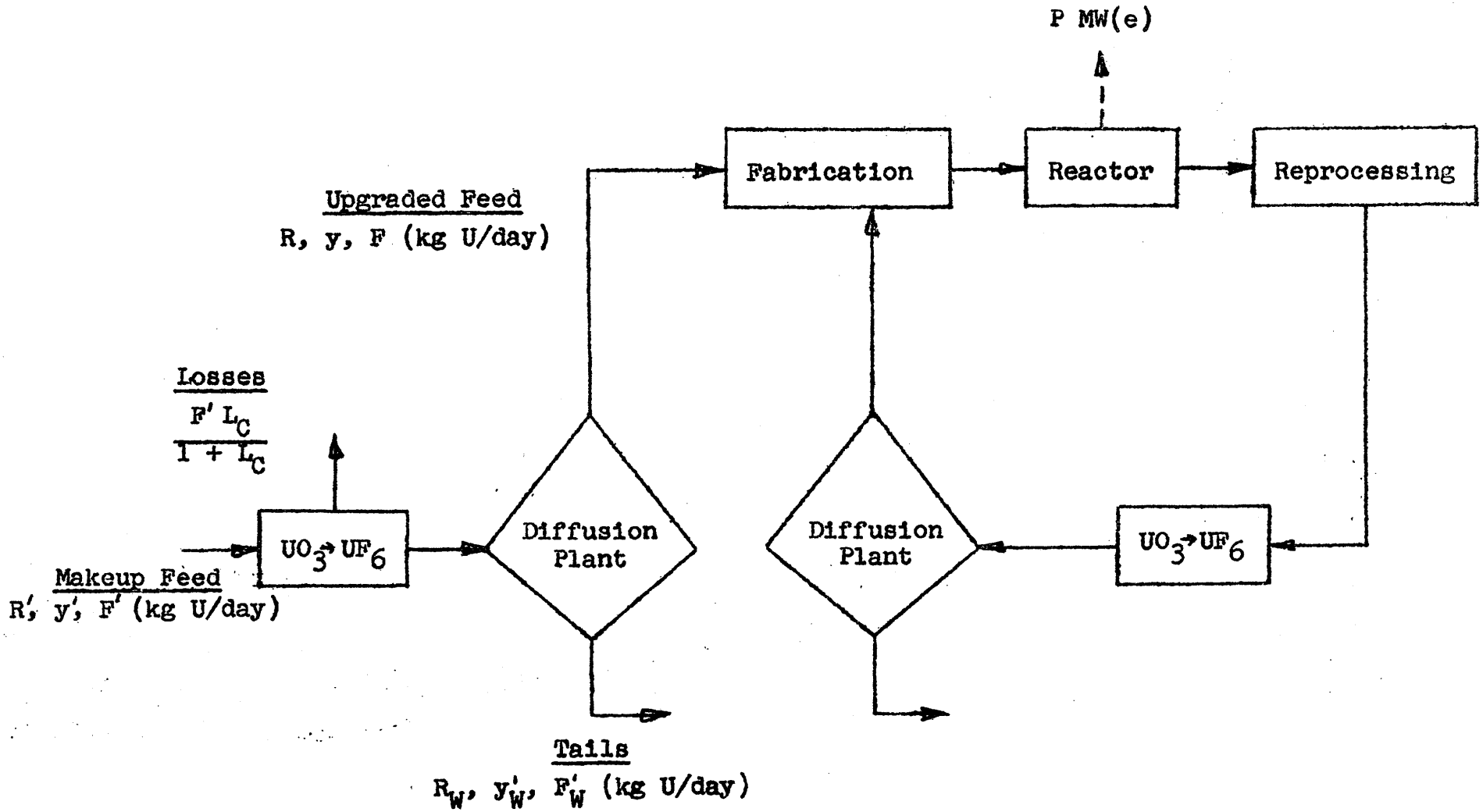


Figure 7. Recycle of Uranium Through Diffusion Plant - Makeup Feed Upgraded in Diffusion Plant

fabricator, after which the cycle is identical to that of Figure 6. Since the fuel value of the R,y material when it is used as upgraded feed is already known from the analysis of Figure 6, and since conversion costs and additional inventory and separative work costs can be easily calculated, it is possible to assign a fuel value to the R′,y′ feed stream. In this way, a fuel value can be attached to uranium over a range of R′ and y′, including material with low U-235 content which would not permit operation of the PWR if used as feed in the scheme of Figure 6 instead.

Figure 7 indicates the use of two diffusion plants, but only as a convenient means of illustrating the flowsheet as an extension of Figure 6. In reality, one plant could be used and the R′,y′ stream (in UF_6 form) could be fed to the cascade at the appropriate point. Thus, re-enrichment of both the R′,y′ and spent fuel streams could be performed in one diffusion plant. Such operation has the same separative work requirements as the use of two plants, when all are operated as "matched-R" cascades.

It should be noted that, for fixed Np-237 and natural uranium prices, the minimum fuel cycle cost, C_P^*, obtained in the analysis of Figure 6 represents the minimum cost when fuel is recycled through a diffusion plant, and that operation according to Figure 7 with toll enrichment of natural uranium feed would lead to a fuel cycle cost higher than C_P^*, since an extra inventory charge, $it_E FC_{AEC}(R)$, would be incurred in the latter case, with $C_{AEC}(R)$ being the price on the AEC scale. The fuel value of the makeup feed in Figure 7 must be such that the overall fuel cycle cost does not exceed C_P^*; hence, if natural uranium is used as makeup feed in Figure 7, its value would be somewhat lower than its price on the AEC scale.

For selected values of R′ and y′, y can be calculated for a series of specified R values, with the y′ values corresponding to those chosen for y in the study of Figure 6. With R_W maintained at the same value used for Figure 6, the other flow rates in Figure 7 can also be obtained. The equations used in this material balance calculation are presented in detail in Section F. It

follows that, for a given feed material in Figure 7 of composition R',y', it is possible to assign a series of fuel values, $C_d(R',y',R)$, corresponding to a series of selected R values. For each R investigated, y is calculable and the fuel value of the R,y stream, $C_f(R,y)$ can be determined by interpolation of the results for Figure 6. For a single set of R',y', R, and y values, $C_d(R',y',R)$ can be calculated as described below.

The separative work, Δ', expended in performing the enrichment is:

$$\Delta' = F\phi + F'_W \phi'_W - \frac{F'\phi'}{1+L_c} \quad , \qquad (9)$$

where separation potentials are defined as in Equation (6). The total value of the product stream, in \$/day, is given by:

$$FC_f(R,y) = \left[F'C_d(R',y',R)\right](1+it_c) + F'C_{CT} + \Delta' C_\Delta + it_E FC_f(R,y) \quad , \qquad (10)$$

where t_c is the time interval between the purchase of UO_3 and the receipt of UF_6 by the AEC, and where C_{CT} includes all unit costs incurred during t_c.

We can calculate the value of the makeup feed material as:

$$C_d(R',y',R) = \frac{1}{F'(1+it_c)}\left[(1-it_E)FC_f(R,y) - F'C_{CT} - \Delta' C_\Delta\right] \qquad (11)$$

For a given R' and y', there will be an R for which $C_d(R',y',R)$ is a maximum when the feed is used in the cycle of Figure 7. When the maximum $C_d(R',y',R)$, defined as $C_m(R',y')$, is determined over a range of R' and y' values, the variation is expected to be as shown by the solid line curves in Figure 8, for a single Np-237 price and a specified natural uranium price. The dashed-line curves represent the fuel value $C_f(R,y)$ when feed is sent directly to the fabricator, as in Figure 6, and are included to enable a qualitative comparison of the $C_m(R',y')$ and $C_f(R,y)$ lines.

One feature of the $C_m(R',y')$ curve for $y' = 0$ is that it is not tangent to the AEC price line at R^* and lies below it for all R'. Consequently, when $y' = y = 0$, the $C_m(R',0)$ curve will lie below the $C_f(R,0)$ curve over a range of abundance ratios; however, at sufficiently low R', $C_m(R',0)$ will become greater than $C_f(R,0)$ and will

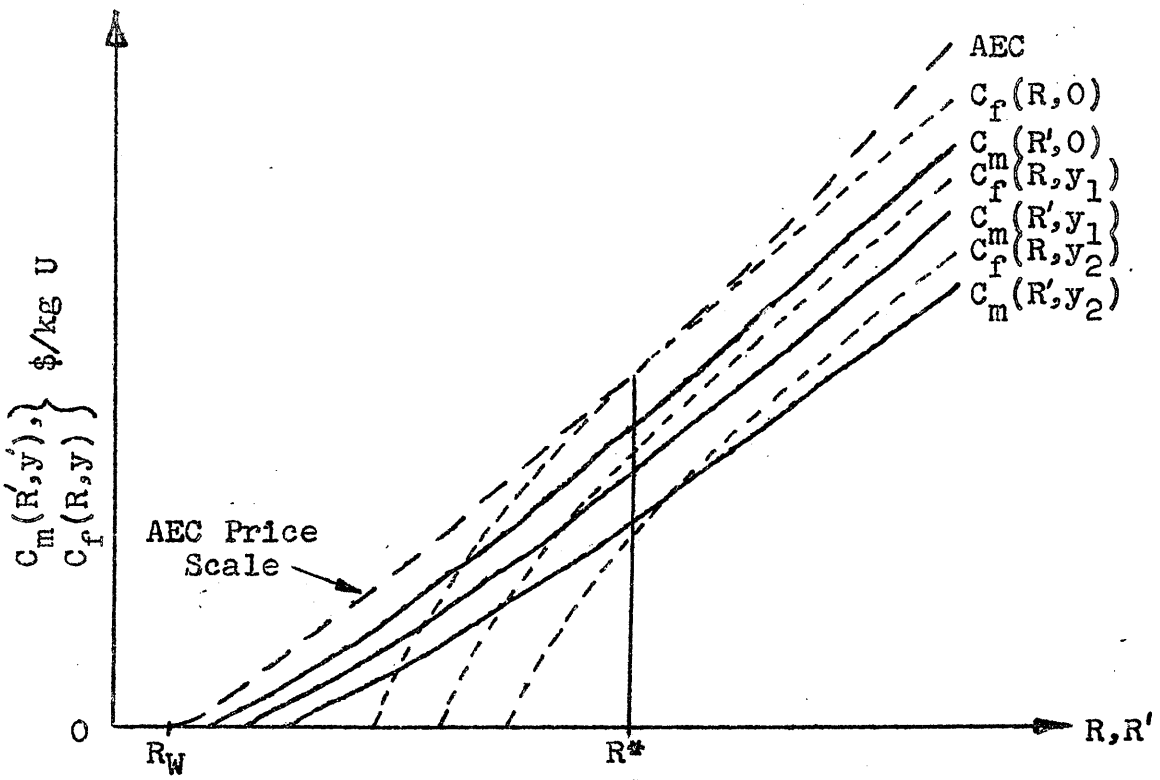


Figure 8

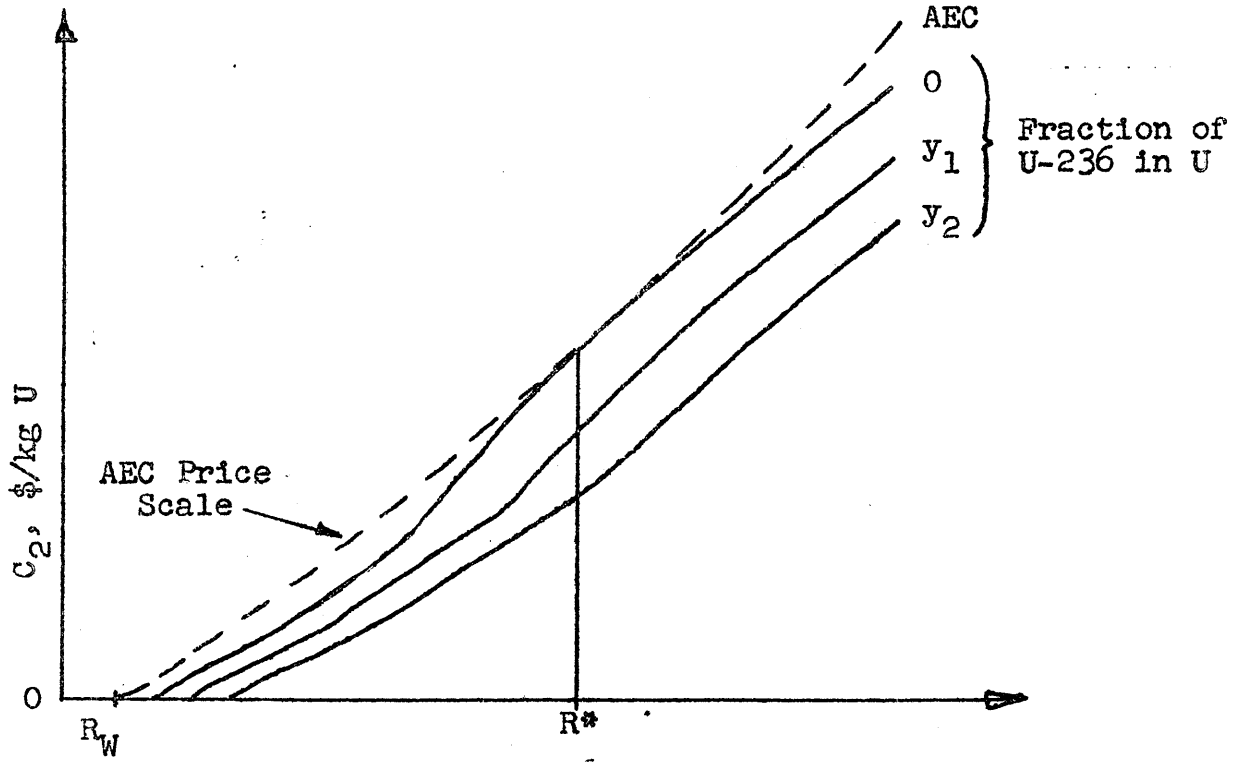


Figure 9

remain above zero for R' values below the R at which $C_f(R,0)$ drops to zero. Similar behavior is expected for fuel containing finite amounts of U-236. A more detailed discussion of the $y' = 0$ curves is included in Appendix I.

It follows that, for uranium of a given isotopic composition, if $C_f(R,y) > C_m(R',y')$, the maximum fuel value is obtained by operating according to Figure 6, while if $C_m(R',y') > C_f(R,y)$, the maximum fuel value is obtained by first upgrading feed before it is sent to the fuel fabricator, as shown in Figure 7. By choosing the larger of $C_m(R',y')$ and $C_f(R,y)$ for each uranium isotopic composition and plotting these maximum fuel values, designated as C_2, curves similar to those shown in Figure 9 are obtained. Figure 9 thus gives the highest obtainable value of uranium when it is used as makeup feed in a fuel cycle which utilizes a gaseous diffusion plant for re-enrichment of spent uranium.

At all points in Figure 9 for which C_2 is obtained by upgrading feed before it is sent to the fabricator, it is important to keep a record of the corresponding optimum R and the associated y so one knows exactly how to operate with the given feed material in order to attain its maximum fuel value.

The entire analysis leading to Figure 9 will be repeated for a series of Np prices and two or more natural uranium prices.

The case described in this section is of interest not only as an alternate procedure for recycling spent uranium but also as a means for estimating the fuel value of uranium feed containing U-236 and with low enrichment. Such fuel is typical of uranium recovered from most power reactors so the importance of estimating its value in a reactor type as common as the PWR is considerable.

E. Consideration of Neptunium Sale

The price at which a reactor operator can sell the Np-237 produced during irradiation can strongly influence the cost of power, the value of feed containing U-236, and the choice between the two methods of recycling uranium discussed in Sections C and D. Since the buildup of Np-237 will be generally greater when

uranium is recycled directly to the fabricator, the fuel cycle cost for that case will decrease faster with increasing Np-237 sale price than will the fuel cycle cost for recycle through the diffusion plant. Above some Np-237 price to be determined, it becomes more economical to recycle fuel directly to the fabricator, and for Np-237 prices below this level, it is more economical to recycle the uranium through the diffusion plant and permit the discharge of some U-236 with the tails stream.

As has been discussed previously, the price of Np-237 will affect the value of feed containing U-236 for both of the recycle schemes to be studied. As the Np-237 price increases, there will be an increase in the value of feed having a given U-236 content. This effect will be examined in detail by carrying out the calculations outlined in Sections C and D for a series of Np prices, C_N, ranging from zero to some arbitrary upper limit. Since an established price for Np-237 does not exist, and since this price is likely to vary considerably before stabilizing at some future date, it was felt that a range of prices should be examined. Rohrmann (6) has estimated a current market price of $500/g for Np-237, so the range considered for C_N will probably extend at least to $500,000/kg minus an estimated cost of separating Np-237 from processed fuel. Note that Equations (2) and (8) do not include any additional cost for separating Np-237 so that C_N represents the net credit to the reactor operator from selling Np-237.

By considering an arbitrary range of C_N values, it is implied that the price for Np-237 is set by considerations other than the cost of producing Np-237 by irradiation of U-236 in the PWR reference design. However, a comparison of fuel cycle costs for the two recycle schemes provides a way of determining the cost of producing Np-237 from U-236 in one possible irradiation scheme. The cost of producing Np-237 can be defined as that value of C_N for which the minimum fuel cycle costs for the two recycle schemes, Figure 4 and Figure 6, are equal. Figure 10 illustrates the principle in a hypothetical case for which the cost of producing Np-237 would be $200,000/kg.

With C_N set equal to the resulting cost for Np-237, the calculations of Sections C and D will be repeated to set fuel values for feed uranium for this specific Np-237 cost. With C_N set in this manner, the same fuel cycle cost will be attained regardless of whether the reactor operator recycles his uranium according to Figure 4 or Figure 6, as long as the value of feed uranium he purchases is properly adjusted for its isotopic composition. It then becomes a matter of indifference to him which fuel cycle he selects. If the value of C_N were greater or smaller than this "indifference" value, it would result in his choosing a recycle scheme which would increase or decrease his Np-237 production, respectively, and the resulting excess or deficiency of Np-237 would eventually tend to return C_N to the "indifference" value.

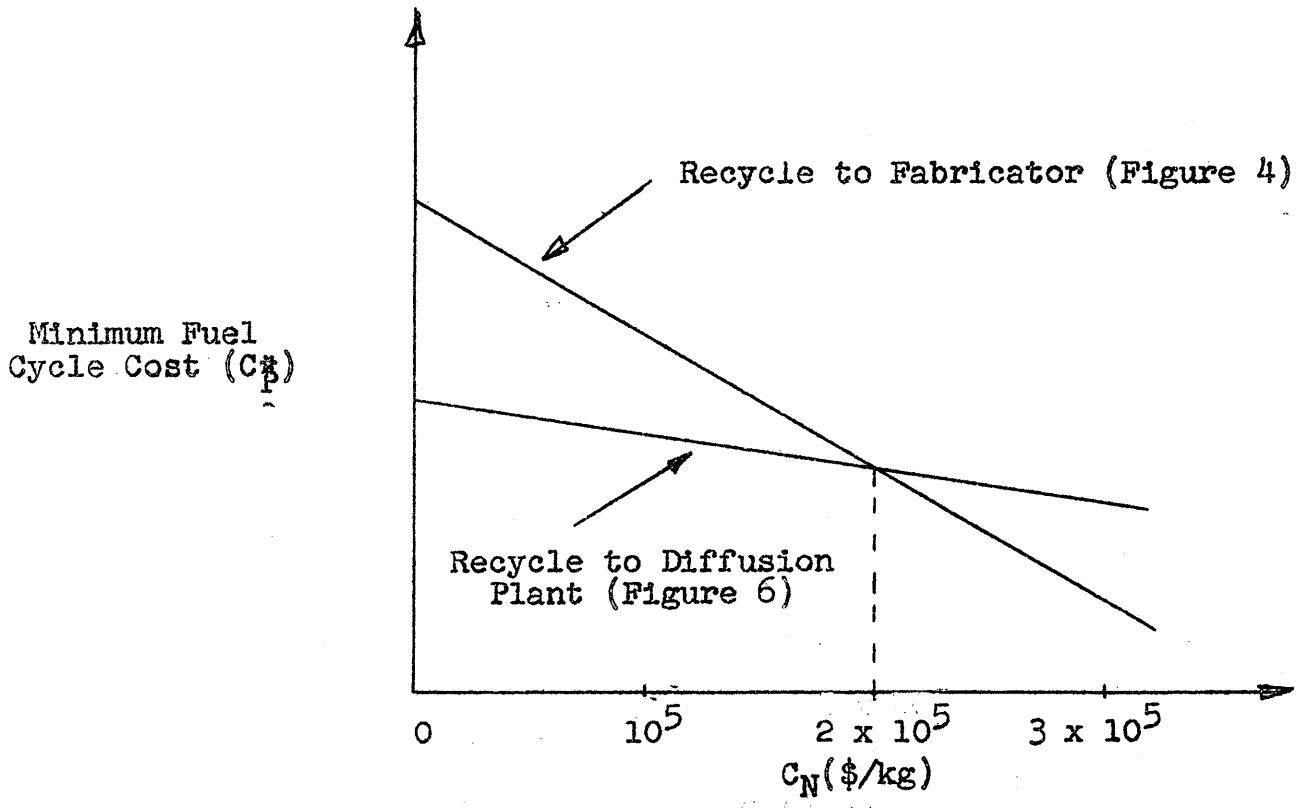


Figure 10

This "indifference" value of C_N will be determined for each of the natural uranium costs considered in the study.

Ransohoff (7) has suggested that the price obtainable for Np-237 could influence not only the choice of a recycling procedure, but also the choice between zircaloy and stainless steel cladding for power reactors. The economics of stainless steel cladding are favored more by a high Np-237 price than are the economics of zircaloy cladding, since Np-237 buildup is greater for higher enrichments; however, even at very high Np-237 prices, such an economic superiority for steel cladding is strictly artificial since it would be an easy task to poison a zircaloy-clad core so that its enrichment requirement would be similar to that of the stainless-steel-clad core, thereby bringing the Np-237 buildup to the same level for each cladding. In fact, target materials for the production of high-value isotopes could be used to supply this extra poisoning requirement, so that the zircaloy-clad reactor could actually yield lower fuel cycle costs than the stainless steel design, even at very high Np-237 prices.

Nevertheless, comparative results for stainless steel and zircaloy claddings might be of interest. The entire study described in this report will be performed for zircaloy-clad fuel and sufficient calculations could be repeated for stainless cladding to enable a comparison between claddings with respect to the effects of R,y, Np-237 price, and natural uranium cost on the value of net feed material for the reference reactor.

F. Method of Analysis

Determination of the steady-state fuel flow rates and compositions which correspond to a specified net feed material is a major part of the analysis for all fuel cycle flowsheets considered in the study. The steady-state cycle characteristics are necessary before the fuel value calculations outlined in Sections C and D can be accomplished. Steady-state operation of the reactor is reached only when fuel flow rates and compositions at every point in the fuel cycle become invariant with time. Such an operating condition insures that steady-state scatter refueling of the reactor is in effect, i.e. the fuel fed to the reactor and the

fuel discharged both have compositions which do not vary from one irradiation cycle to the next.

Since only the variation of steady-state characteristics with makeup feed composition is required for the economic analysis, there is no need to perform costly iterations on the feed enrichment required to give a certain initial reactivity or burnup for each of the transient cycles. For both Figures 4 and 6, the most obvious procedure would be to maintain a fixed net feed composition (R and y) and to follow successive batches of fuel through the reactor, during recycle, and in the re-enrichment (by gaseous diffusion and/or mixing with net makeup fuel) step, until the transient period terminates and all fuel batches possess identical histories through the fuel cycle. Such a procedure enables the determination of all steady-state characteristics as functions of R and y directly, as required for the fuel value calculation. However, as is discussed further in Section G, the fact that the reactor feed composition (R_R and y_R) would then change from one batch to the next during transient operation greatly complicates the burnup analysis when the CELLMOVE code (which has been selected as the major calculational tool) is employed.

An alternative method of achieving steady-state operation which utilizes the CELLMOVE code more efficiently has been chosen for the study. For both recycle schemes, this procedure begins with the assumption of reactor feed composition (R_R and y_R). Using CELLMOVE and maintaining this reactor feed composition, the scatter refueling scheme is brought to a steady state condition. The corresponding reactor feed rate and spent fuel flow rate can be evaluated, after which material balance considerations enable the determination of all other flow rates and uranium compositions throughout the fuel flowsheet; hence, the makeup feed composition (R and y) which corresponds to a fixed reactor feed composition (R_R and y_R) can be determined. The disadvantage of this simple procedure is obvious: there is no direct control over the R,y points for which one has the corresponding steady-state fuel cycle characteristics. All characteristics (including R and y) are

known directly only as functions of R_R and y_R. However, by choosing a series of R_R, y_R points spaced uniformly over an R_R/y_R grid and determining the steady-state characteristics of a given fuel cycle flowsheet at each point in the grid, sufficient data will be available to permit iterated double interpolation to find the R_R and y_R values which correspond to a desired R and y; then, for this R,y point, all other cycle characteristics can be determined by double interpolation using the corresponding R_R, y_R values.

Since cycle characteristics will not vary irregularly with R_R or y_R, the use of double interpolation is not expected to introduce significant error into the results. Although the determination of the R_R, y_R point which leads to a desired R and y may best be performed by graphical iteration, the subsequent double interpolations mentioned above can be easily performed by Lagrangian techniques on the computer.

Another reason for choosing the indirect method for determining steady-state characteristics is discussed in the final paragraph of this section.

As mentioned above, all steady-state fuel cycle characteristics can be determined once R_R and y_R have been fixed; however, the procedure for obtaining these characteristics differs considerably for the flowsheets of Figures 4, 6, and 7. The remainder of this section is devoted to a discussion of the equations available for each fuel cycle discussed in Sections C and D. The nomenclature used is identical to that utilized in those sections.

(1) Recycle to Fabricator

Reference should be made to Figure 4. Once the values for R_R and y_R have been arbitrarily chosen, the CELLMOVE code (described in Section G) can be used to determine the corresponding time-averaged values for F_R, R_S, y_S, $F_S/(1 - L_{RU})$, $K/(1 - L_{RP})$, and $N/(1 - L_{RP})$, when steady-state scatter refueling has been attained. The six remaining unknowns are F_S, N, K, R, y, and F. The first three are simply obtained from:

$$F_S = (1-L_{RU})\left(\frac{F_S}{1-L_{RU}}\right) \quad , \qquad (12)$$

$$N = (1-L_{RP})\left(\frac{N}{1-L_{RP}}\right) \quad , \qquad (13)$$

and

$$K = (1-L_{RP})\left(\frac{K}{1-L_{RP}}\right) \quad . \qquad (14)$$

The net feed characteristics (F, y, and R) required for steady-state operation can be determined by material balance relations for the fabrication plant.

$$F = (1+L_F)F_R - F_S \qquad (15)$$

$$yF = (1+L_F)y_R F_R - y_S F_S \qquad (16)$$

$$\left(\frac{R}{1+R}\right)(1-y)F = (1+L_F)\left(\frac{R_R}{1+R_R}\right)(1-y_R)F_R - \left(\frac{R_S}{1+R_S}\right)(1-y_S)F_S \qquad (17)$$

We see that, from the arbitrary choice of R_R and y_R, complete steady-state cycle characteristics can be determined.

(2) Recycle Through Diffusion Plant - Base Case

Reference should be made to Figure 6. As in Part (1), we first specify values for R_R and y_R and use CELLMOVE together with Equations (12), (13), and (14) to determine steady-state values for F_R, R_S, y_S, F_S, K, and N. As discussed in Section D, R_W is the optimum tails abundance ratio and is known once the costs of natural uranium and separative work have been specified. The eight remaining unknowns are y_W, F_W, R_P, y_P, F_P, R, y, and F. The steady-state values for these unknowns can be determined from Equations (3) and (4); the three mass balance relations for the diffusion plant given by (18), (19), and (20); and the three mass balance relations for the fabrication plant given by (21), (22), and (23).

$$R = R_P \qquad (3)$$

$$\frac{y_P F_P}{(R_P)^{1/3}} + \frac{y_W F_W}{(R_W)^{1/3}} = \frac{y_S F_S}{(1+L_C)(R_S)^{1/3}} \tag{4}$$

$$F_P + F_W = \frac{F_S}{1+L_C} \tag{18}$$

$$y_P F_P + y_W F_W = \frac{y_S F_S}{1+L_C} \tag{19}$$

$$\left(\frac{R_P}{1+R_P}\right)(1-y_P)F_P + \left(\frac{R_W}{1+R_W}\right)(1-y_W)F_W = \left(\frac{R_S}{1+R_S}\right)(1-y_S)\frac{F_S}{1+L_C} \tag{20}$$

$$F = (1+L_F)F_R - F_P \tag{21}$$

$$yF = (1+L_F)y_R F_R - y_P F_P \tag{22}$$

$$\left(\frac{R}{1+R}\right)(1-y)F = (1+L_F)\left(\frac{R_R}{1+R_R}\right)(1-y_R)F_R - \left(\frac{R_P}{1+R_P}\right)(1-y_P)F_P \tag{23}$$

Using Equation (3) in (23) to eliminate R and employing the result of subtracting (22) from (21), we can reduce (23) to the form

$$\frac{R_P}{1+R_P} = \frac{R_R}{1+R_R} \quad ,$$

which of course leads to

$$R_P = R_R \quad . \tag{24}$$

Hence, Equations (4), (18), (19), and (20) can be used to solve for the unknowns y_W, F_W, y_P, and F_P. The procedure is outlined below.

Equation (18) is used to eliminate F_P in (19), (4), and (20), resulting in the following equations:

$$y_P\left[\frac{F_S}{1+L_C} - F_W\right] + y_W F_W = \frac{y_S F_S}{1+L_C} \tag{25}$$

$$\frac{y_P}{(R_P)^{1/3}}\left[\frac{F_S}{1+L_C}-F_W\right]+\frac{y_W F_W}{(R_W)^{1/3}}=\frac{y_S F_S}{(1+L_C)(R_S)^{1/3}} \tag{26}$$

$$\left(\frac{R_P}{1+R_P}\right)(1-y_P)\left[\frac{F_S}{1+L_C}-F_W\right]+\left(\frac{R_W}{1+R_W}\right)(1-y_W)F_W=\left(\frac{R_S}{1+R_S}\right)(1-y_S)\frac{F_S}{1+L_C} \tag{27}$$

Multiplying (26) by $(R_P)^{1/3}$ and subtracting (25) from the resulting equation gives

$$y_W F_W\left[\left(\frac{R_P}{R_W}\right)^{1/3}-1\right]=\frac{y_S F_S}{1+L_C}\left[\left(\frac{R_P}{R_S}\right)^{1/3}-1\right] \tag{28}$$

Multiplying (27) by $(1 + R_P)/R_P$ and collecting terms gives

$$-y_P\left[\frac{F_S}{1+L_C}-F_W\right]+F_W\left[\frac{(1+R_P)R_W}{R_P(1+R_W)}(1-y_W)-1\right]=\frac{F_S}{1+L_C}\left[\frac{(1+R_P)R_S}{R_P(1+R_S)}(1-y_S)-1\right] \tag{29}$$

Adding (25) and (29) results in the equation

$$F_W(1-y_W)\left[\frac{(1+R_P)R_W}{R_P(1+R_W)}-1\right]=\frac{F_S(1-y_S)}{1+L_C}\left[\frac{(1+R_P)R_S}{R_P(1+R_S)}-1\right] \tag{30}$$

Dividing (28) by (30) results in the following relation

$$\frac{y_W}{1-y_W}=\frac{y_S}{1-y_S}\left[\frac{\frac{(1+R_P)R_W}{R_P(1+R_W)}-1}{\frac{(1+R_P)R_S}{R_P(1+R_S)}-1}\right]\left[\frac{\left(\frac{R_P}{R_S}\right)^{1/3}-1}{\left(\frac{R_P}{R_W}\right)^{1/3}-1}\right] \tag{31}$$

Equation (31) can be used to determine y_W, after which F_W can be calculated from (30). F_P can then be determined from (18) and y_P from (19).

Finally, Equations (21), (22), and (23) can be used to calculate F, y, and R, respectively. Again we see that from specification of R_R and y_R, all steady-state cycle characteristics can be evaluated.

(3) Recycle Through Diffusion Plant - Modified Case

Reference should be made to Figure 7. As discussed in Section D, this procedure is used to upgrade makeup feed material to uranium with composition R,y which has known fuel value from the analysis of the flowsheet in Figure 6. R_W is again the optimum tails abundance ratio governed by the cost of natural uranium. Three degrees of freedom exist in the solution of this portion of the problem. It is desirable to specify values for R' and y' so that direct control over the makeup feed composition is retained. The presence of R to the 1/3 power in Equation (35) below makes it particularly convenient to select R as the third arbitrary quantity. The five remaining unknowns are F, y, F', y'_W and F'_W. Four equations are available - three mass balance relations for the diffusion plant and the U-236 distribution equation.

$$F + F'_W = \frac{F'}{1+L_c} \tag{32}$$

$$yF + y'_W F'_W = \frac{y'F'}{1+L_c} \tag{33}$$

$$\left(\frac{R}{1+R}\right)(1-y)F + \left(\frac{R_W}{1+R_W}\right)(1-y'_W)F'_W = \left(\frac{R'}{1+R'}\right)(1-y')\frac{F'}{1+L_c} \tag{34}$$

$$\frac{yF}{(R)^{1/3}} + \frac{y'_W F'_W}{(R_W)^{1/3}} = \frac{y'F'}{(1+L_c)(R')^{1/3}} \tag{35}$$

The fifth relationship is that F is known as a function of both R and y (from the results of Part (2) above). The knowledge of F only as F(R,y) and the fact that R and y are not both specified seems to imply an iterative solution. However, iteration can be avoided if we divide Equations (32) - (35) by F and define two new variables A and B as:

$$A = \frac{F'_W}{F} \tag{36}$$

and

$$B = \frac{F'}{F} \tag{37}$$

We are left with four equations in the unknowns A, B, y, and y'_W as listed below.

$$1 + A = \frac{B}{1+L_c} \tag{38}$$

$$y + A y'_W = \frac{B y'}{1+L_c} \tag{39}$$

$$\left(\frac{R}{1+R}\right)(1-y) + \left(\frac{R_W}{1+R_W}\right)(1-y'_W)A = \left(\frac{R'}{1+R'}\right)(1-y')\frac{B}{1+L_c} \tag{40}$$

$$\frac{y}{(R)^{1/3}} + \frac{y'_W A}{(R_W)^{1/3}} = \frac{y' B}{(1+L_c)(R')^{1/3}} \tag{41}$$

The procedure for determining A, B, y, and y'_W is described next.

Equation (38) is used to eliminate $B/(1 + L_c)$ in (39), (40), and (41), resulting in the following equations:

$$y + A y'_W = y'(1+A) \tag{42}$$

$$\left(\frac{R}{1+R}\right)(1-y) + \left(\frac{R_W}{1+R_W}\right)(1-y'_W)A = \left(\frac{R'}{1+R'}\right)(1-y')(1+A) \tag{43}$$

$$\frac{y}{(R)^{1/3}} + \frac{y'_W A}{(R_W)^{1/3}} = \frac{y'}{(R')^{1/3}}(1+A) \tag{44}$$

Multiplying (44) by $(R_W)^{1/3}$ and subtracting the resulting equation from (42) gives

$$y\left[1 - \left(\frac{R_W}{R}\right)^{1/3}\right] = y'(1+A)\left[1 - \left(\frac{R_W}{R'}\right)^{1/3}\right] . \tag{45}$$

Multiplying (43) by $(1 + R_W)/R_W$ and using (42) to eliminate y'_W from the result gives the equation

$$(1-y)\left[\frac{R(1+R_W)}{(1+R)R_W} - 1\right] = (1+A)(1-y')\left[\frac{R'(1+R_W)}{(1+R')R_W} - 1\right] \tag{46}$$

Dividing (45) by (46) gives the following relation

$$\frac{y}{1-y} = \frac{y'}{1-y'} \left[\frac{\frac{R(1+R_W)}{(1+R)R_W} - 1}{\frac{R'(1+R_W)}{(1+R')R_W} - 1} \right] \left[\frac{1-\left(\frac{R_W}{R'}\right)^{1/3}}{1-\left(\frac{R_W}{R}\right)^{1/3}} \right] \qquad (47)$$

Equation (47) can be used to determine y, after which A can be calculated from (46). B can then be determined from (38) and y'_W from (39). Knowing R and y, F can be found by interpolation of the results from Part (2) above. Finally, F'_W and F' are calculated from Equations (36) and (37).

We see that the specification of R', y', and R enables the determination of the other steady-state characteristics in a simple way.

It should be noted that the range of R_R and y_R values chosen for the study can be the same for either method of recycling uranium since the reactor feed characteristics of interest are the same in Figures 4 and 6. Hence, the CELLMOVE calculations which provide F_R, R_S, y_S, F_S, N, and K as functions of R_R and y_R for steady-state scatter refueling need only be carried out once. The results can then be used to begin the analyses for both Part (1) and Part (2) outlined above. This is a distinct advantage of the "indirect" method of obtaining steady-state fuel cycle characteristics when applied to our study. Obviously, use of the "direct" method would necessitate a complete second set of CELLMOVE calculations - one for each recycle scheme - and would greatly increase the overall computer time required for the study. Of course, if the analysis is carried out for both Zry-4 and stainless steel claddings, the reactor characteristics will depend upon the cladding employed, and a set of CELLMOVE calculations will be required for each cladding.

G. Burnup Codes

All fuel depletion calculations and predictions of reactor characteristics at the steady-state scatter refueling condition

will be carried out with CELLMOVE, which is a modified version of FUELMOVE, a fuel management program written at M.I.T. (8). Two space dimensions are utilized in the diffusion theory calculation and energy dependence is described by a modified two-group model. A Wigner-Wilkins spectrum is calculated below the thermal cutoff energy. Two separate codes - CELL and MOVE - are actually involved. First, CELL is used to calculate the fuel composition as a function of thermal flux-time for each fuel material which is charged to the reactor. The MOVE code then performs the flux distribution calculations throughout the core lifetime, using the results from CELL to calculate the time-dependent characteristics at each mesh point in the reactor. The reactivity lifetime of the core is predicted, after which a variety of fuel management options are available for discharging and charging fuel to the core and repeating the core lifetime calculations until steady-state refueling is obtained.

If the fuel material charged to the reactor has the same composition for all transient cycles, it is necessary to perform the CELL calculation only once for each approach to steady-state refueling as performed by MOVE. Thus, one CELL run and one MOVE run are required to predict the characteristics of a reactor at steady-state refueling when the reactor feed composition is fixed. On the other hand, if the reactor feed composition changes from one transient cycle to the next (as is likely in a realistic fuel recycling procedure), a separate CELL run would be required for each transient cycle, thereby increasing the computer time and data handling requirement considerably. The relative simplicity which results from maintaining a fixed reactor feed composition strongly influenced the choice of the "indirect" method for obtaining steady-state fuel cycle characteristics for Figures 4 and 6, as discussed in Section F.

The modifications of the original FUEL code were largely governed by a desire to accurately predict the time-dependent characteristics of pressurized water reactors. The CELL code, which incorporates these modifications, will be described in

detail in the S.M. thesis of Mr. James Beaudreau. Some of the more important new features of CELL are listed briefly below.

1. complete rewriting of the FUEL program, resulting in a more concise and efficient program; respecification of input data requirements to minimize the amount of "off-line" calculations needed to describe the unit reactor cell; complete updating of microscopic data used in the calculations; convenient summary of isotopic densities as functions of fuel exposure.

2. use of the Wigner-Wilkins energy distribution of the thermal neutron flux instead of the Wilkins formulation.

3. employment of a more efficient Runge-Kutta-Gill solution for the isotope buildup equations; inclusion of Np-237 in the isotope chain.

4. calculation of cell disadvantage factors at each velocity point considered in the thermal flux energy distribution prediction and the recalculation of these disadvantage factors at each flux-time step; improvement in the cell homogenization procedure prior to calculating the Wigner-Wilkins distribution.

5. calculation of the U-238 resonance integral, including the effects of fuel temperature and Dancoff factor; calculation of ϵ and a transport-corrected form of the diffusion coefficient.

6. use of a time-dependent poison cross-section, which is read in, to simulate the presence of control material when performing the thermal spectrum calculation.

7. incorporation of a different scheme for calculating resonance integrals at each flux-time step than was used for FUEL; a large number of energy groups is used to describe the variation of nuclide cross-sections throughout the resonance region and the effective resonance integral for each nuclide (except U-238, which is treated separately) is obtained by integrating over the multigroup range; the effect of absorptions in other nuclides is accounted for when performing the resonance integral calculation for a given nuclide, i.e. the effect of resonance interference among all nuclides present is simulated in an approximate manner.

The availability of experimentally-determined characteristics for Core 1 of the Yankee reactor provided a means of evaluating the CELL code accuracy when applied to a PWR. Table 1 gives a comparison among certain results obtained from experiment, from FUELMOVE, and from CELLMOVE. Except for the initial reactivity of the clean core, the items compared do not involve the MOVE code, which is incapable of simulating the reactor control rod program used during the Yankee Core 1 lifetime. Since the actual rod program yielded significant power flattening over the core, the average discharge burnup was different from that which would result from a control scheme such as uniform (soluble) poisoning or a scheme in which rods are removed in a strictly out-in manner. It was thus decided to limit the comparison of CELLMOVE with Yankee results to isotopic buildup and initial reactivity, and to evaluate in another way the ability of the code to predict reactivity lifetime. The results in Table 1 are gratifying, especially those for the buildup of isotopes at high fuel burnup, and demonstrate a definite superiority of CELL over FUEL in this respect.

To insure that the burnup predictions of CELLMOVE are accurate, and to provide a further check on initial reactivity with and without equilibrium poison, a comparison was made with the results calculated by Westinghouse for the first core of the San Onofre reactor. It was felt that if good agreement could be obtained with the predictions of more sophisticated and detailed codes (which have been adjusted to improve their accuracy in calculating PWR's), then the use of CELLMOVE in the study of the San Onofre reactor would be justifiable. In addition, since the reactor utilizes soluble boron for control during irradiation, the MOVE code, which can simulate such a control scheme, could be properly evaluated.

Core 1 for the San Onofre reactor utilizes SS304 as cladding (Zry-4 will be used in replacement cores) and has an inner zone with fuel of 3.2% enrichment, a middle zone with 3.4% enriched fuel, and an outer zone having 3.8% enrichment. The three zones have very nearly equal volumes. Table 2 gives results from CELLMOVE

Table 1

Comparison of Calculated and Experimental Results for Yankee Core No. 1

	Experiment	FUELMOVE	CELLMOVE
k_{eff} (t = 0, clean)	1.1062	1.1466	1.1116
$\bar{\alpha}_{25}$ (t = 0)	0.24 - 0.27	0.216	0.246
@5100 MWD/MT:			
N_{25}/N_{28}^{o} (x 10^{2})	3.00	3.03	3.00
N_{26}/N_{28}^{o} (x 10^{3})	1.27	1.15	1.30
Pu/U mass (x 10^{3})	2.99	3.44	3.53
N_{49}/N_{28}^{o} (x 10^{3})	2.77	3.12	3.25
N_{41}/N_{28}^{o} (x 10^{4})	0.623	0.964	0.850
@23000 MWD/MT:			
N_{25}/N_{28}^{o} (x 10^{2})	1.70	1.81	1.71
N_{26}/N_{28}^{o} (x 10^{3})	3.66	3.21	3.61
Pu/U mass (x 10^{3})	10.9	10.4	11.3
N_{49}/N_{28}^{o} (x 10^{3})	7.63	6.86	7.80
N_{41}/N_{28}^{o} (x 10^{4})	12.3	13.3	13.3

Table 2

Comparison of CELLMOVE and Westinghouse
Calculated Results for San Onofre Reactor - Core No. 1

	ppm. B (t = 0, clean)	ppm. B (eq. poison)	Avg. Burnup (MWD/MT)
Westinghouse	2800	2200 (1)	13,500
CELLMOVE	2936	2194 (2)	12,711

	Fractional Energy Production	
	Westinghouse	CELLMOVE
U-235	0.73	0.709
U-238	0.08	0.079
Pu	0.19	0.212
Inner Zone	0.39	0.390
Middle Zone	0.36	0.372
Outer Zone	0.25	0.238

(1) Includes equilibrium Xe-135 and Sm-149

(2) Includes equilibrium Xe-135 and Sm-149, plus all other fission products with thermal $\sigma_a > 10{,}000$ barns.

and those calculated by Westinghouse. Reactivity results are given in terms of critical boron concentrations (ppm of B in H_2O, on a weight basis). Average fuel burnup for the first core is given, and for completeness, the fractional energy production per nuclide and per zone is also listed.

The agreement between CELLMOVE and Westinghouse calculations is quite close. It was decided that CELLMOVE predicts PWR characteristics with sufficient accuracy to justify its use in the present study.

The MOVE code is being modified to include the type of scatter refueling scheme described in Section B and to automatically predict the reactor characteristics at steady-state scatter refueling for a fixed reactor feed composition. Once steady-state refueling has been reached for a specified R_R and y_R, MOVE will carry out the calculations described in Section F, Parts (1) and (2), to give all flow rates and uranium compositions throughout the flowsheets of Figures 4 and 6 at steady-state operation. For Figure 6, the fuel cycle characteristics will be determined for each of a specified series of R_W values.

H. Summary of Procedure

A considerable number of parameters and procedures have been mentioned in the preceding sections and it would be helpful at this point to arrange them in their proper order, thereby arriving at a series of steps leading to completion of the study. As results become available they could influence the choice of subsequent cases to examine, so that the following list constitutes a "probable" course of action; however, each step is sufficiently general to make any changes in the list improbable. Reference to the proper sections should be made for definitions of nomenclature and for the additional details assumed in the following.

1. Complete the programming and checkout of the MOVE code modifications which permit the determination of steady-state scatter refueling characteristics and other characteristics of the steady-

state cycles of Figures 4 and 6, when reactor feed composition (R_R, y_R) is specified.

2. While carrying out Step 5 below, the major fuel cycle economics code will be written and checked out. The code will take a specified R and y and the corresponding R_R and y_R values (as found in Step 5 below) for either recycle scheme and will use the latter values to find all other cycle characteristics by double interpolation of built-in tables of F_i, y_i, and R_i vs R_R and y_R (these tables are formed in Step 5). Two versions of the code - one applicable to Figure 4 and one applicable to Figure 6 - will be written for convenience. Using the F_i, R_i, and y_i values arrived at, the code calculates M(R,y), which is the sum of all fuel cycle costs exclusive of the net feed cost. M(R,y) can be determined for a series of specified Np prices. For Figure 4, M(R,y) can also be determined for alternative natural uranium prices (the effects of which are felt only through changes in C_R and C_S); however, for Figure 6, M(R,y) can be determined only for the natural uranium price which corresponds to the value of R_W upon which the tables of F_i, R_i and y_i vs R_R and y_R are based. To automate the calculation of M(R,y), the code will calculate C_R, C_S, and C_D, given the optimum tails enrichment corresponding to the natural uranium prices to be considered, by incorporating the AEC price scale formula for mixtures of U^{235} and U^{238}. The code will have an option for reading in C_P^*, which would enable the calculation of feed value C_i, with i = 1 or f, from $FC_i(R,y) = 24\ LPC_P^* - M(R,y)$; obviously, C_P^* will be known for a given Np price and natural uranium price only after a sufficient number of cost calculations have been made on y = 0 feed which has been purchased from the AEC. To facilitate the determination of C_P^*, an option will be available when y = 0 to calculate $24LPC_P = M(R,0) + FC_{AEC}(R)$, with $C_{AEC}(R)$ calculated from the AEC price scale equation.

A second economics code will be written to perform the analysis of Figure 7 for a series of specified R′,y′ points and for a single Np price and a single natural uranium price. For each R′,y′ point, $C_d(R',y',R)$ can be calculated for a specified series of R values, using the equations developed in Section F, Part (3) and also Equation (11). In order to do this, tables of $C_f(R,y)$

and F as functions of R and y will be included and will be used in double interpolation once R and y are both known. These tables can be formed from results obtained in the analysis of Figure 6. Since the C_f vs R,y and F vs R,y tables read in will correspond to a single price for Np and for natural uranium, $C_d(R',y',R)$ values calculated during a single run can correspond only to these specified prices.

These codes will enable the rapid accumulation of fuel value data for varying Np price and varying natural uranium price, once the CELLMOVE cases have furnished the data for tables of F_i, R_i, and y_i as functions of R_R and y_R.

3. Select Zry-4 as the reference cladding material.

4. Select the natural uranium prices to be investigated. The current price of \$8.00/lb of U_3O_8 will be used and perhaps \$6.00/lb and \$4.00/lb would be reasonable alternatives if 2 additional prices are examined. The costs of natural uranium as UF_6 which correspond to these three U_3O_8 prices are \$23.46/kg U, \$18.17/kg U, and \$12.87/kg U, respectively. These UF_6 costs, C_{UF_6}, were determined from the following equation:

$$C_{UF_6} = (1+L_c)\left[(2.2046)\left(\frac{842}{714}\right)(1+it_c)C_{U_3O_8} + C'_{CT}\right], \qquad (48)$$

with $L_c = 0.01$, $i = 0.10/yr$, $t_c = 30/365$ yr, and $C'_{CT} = \$2.26/kg$ U, which includes the conversion of U_3O_8 to UF_6 and other charges incurred during t_c. C_{U3O8} is the price of U_3O_8 in \$/lb U_3O_8. For each price, the optimum R_W will be calculated, using the usual procedure for mixtures of U-235 and U-238.

5. CELLMOVE runs will be made to determine the steady-state reactor characteristics for a range of R_R and y_R values. These runs will also give the steady-state fuel cycle characteristics for each R_R, y_R point for the cycles of Figures 4 and 6. The cycle characteristics for Figure 6 will be found for each of the R_W values selected in Step 4. The ranges considered for R_R and y_R will be such that the ranges for the calculated R and y values will be adequate to enable fuel value determination over a considerable portion of the R,y plane for both recycle schemes.

6. Prepare graphs of R vs R_R (with y_R as a parameter) and of y vs R_R (with y_R as a parameter) for Figure 4 and for each R_W considered for Figure 6.

7. For y = 0, vary R over a reasonable range and, by graphical interation, find the values of R_R and y_R which correspond to each R considered. Repeat this for each pair of graphs prepared in Step 6, i.e. once for Figure 4 and for each R_W considered for Figure 6.

8. Specify a series of Np-237 prices to use in finding the "indifference" price discussed in Section E. For this purpose, C_N values ranging from \$0/kg to \$500,000/kg in steps of \$100,000/kg might be appropriate.

9. Using the economics code described in Step 2, calculate C_P as a function of R for both Figures 4 and 6, for each R_W specified in Step 4, and for each Np-237 price (C_N) chosen in Step 8, assuming the feed material to be purchased from the AEC. For both flowsheets, C_P^* can then be determined for each R_W, C_N combination.

10. Perform the analysis described in Section E to find the cost of producing Np-237 in the reactor for each R_W specified in Step 4. This cost will undoubtedly vary with R_W. At this point, select the C_N values to use in the remainder of the analysis. The "indifference" value will become one member of the set for each R_W and C_N = 0 will be a second member. One or two C_N values between zero and the "indifference" value and one or two C_N values greater than the "indifference" value will also be chosen, the selection of actual values being postponed until the "indifference" values are known. Except for the "indifference" value, the set of C_N values will be the same for all R_W values considered.

11. We now have C_P^* for Figures 4 and 6, as functions of C_N and R_W, and can proceed with the fuel value calculations. For Figure 4, choose a series of R,y points at which fuel values are desired and use the graphs of Step 6 to determine R_R and y_R for each point. Use the economics code to calculate $C_1(R,y)$ for each point, as functions of C_N and R_W, using the known C_P^* values (which,

again, differ for each C_N and R_W). Keep in mind that the set of C_N values will differ slightly for each R_W considered (as per Step 10). Graphs similar to Figure 5 can now be plotted for the flowsheet of Figure 4, for each C_N value and each R_W value.

12. Repeat Step 11 for Figure 6 to find $C_f(R,y)$ for various R and y values and as functions of C_N and R_W.

13. The second economics code described in Step 2 can now be used to calculate $C_d(R',y',R)$ for a series of R values at given R',y' points. The range of R' values considered will extend down to $R_W + \epsilon$, where ϵ is a small number. Runs will be carried out for the various C_N values and natural uranium prices to be investigated. From the results of these runs, it will be possible to specify $C_m(R',y')$ as a function of R',y',C_N, and natural uranium price.

14. From the results of Steps 12 and 13, the plots of $C_f(R,y)$ and $C_m(R',y')$, similar to Figure 8, can be made for each C_N and natural uranium price. Finally, plots resembling Figure 9 can be made by performing the fuel value "maximization" procedure described in Section D.

15. Using the fuel value graphs for Figure 4 and the "maximized" graphs for Figures 6 and 7, the following effects can be examined.

(a) For fixed C_N and R_W, the change of fuel value with increasing U-236 content, at a constant R or at a constant U-235 content, can be seen, and simple correlations of $C_i(R,0) - C_i(R,y)$ with R and y or $C_i(x,0) - C_i(x,y)$ with x and y will be attempted. (i = 1 or 2)

(b) For fixed R_W, the effect of increasing C_N on the fuel value results and on the $C_i(R,0) - C_i(R,y)$ and $C_i(x,0) - C_i(x,y)$ correlations can be determined.

(c) For fixed C_N, the effect of increasing R_W (i.e., decreasing natural uranium price) on the fuel value results and on the $C_i(R,0) - C_i(R,y)$ and $C_i(x,0) - C_i(x,y)$ correlations can be determined.

(d) Fuel value results can be compared not only with the AEC price scale (with U-236 considered as U-238) but also with the price scale developed by de la Garza, et al. (5), for mixtures of U-235, U-236, and U-238, which considers the effect of U-236 on diffusion plant separation costs.

Also, the C_P^* values determined in Step 9 can be used to compare the fuel cycle costs for the two recycle schemes at each R_W and C_N value and to see under what economic conditions it would be more favorable to use one scheme instead of the other.

16. Repeat Steps 4 through 15 for SS304 cladding. It probably is not necessary to perform a complete analysis for SS304, particularly since Zry-4 has been specified as the cladding for replacement cores, and the extent of the analysis will be decided upon later in the study. Due to the improbability of using SS cladding for steady-state operation, it may not be necessary to examine an alternative cladding. A comparison of Zry-4 and SS304 results at a single R_W and a single C_N might be suggested, but only a small saving in labor would result, since all CELLMOVE calculations of Step 5 would have to be carried out again for SS cladding, and Step 5 is by far the most time-consuming (computer, as well as calendar, time) portion of the analysis.

I. References

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APPENDIX I

By means of the analyses outlined in this appendix, it is possible to predict in a qualitative manner some of the more important features of the fuel value curves shown in Figures 5 and 8. In particular, the location of the y = 0 and y' = 0 fuel value curves with respect to the AEC price scale can be examined in detail. Although the curves for y > 0 are not as amenable to prediction as those for y = 0, the discussions following Equations (2) and (8) indicate that, for constant R, an increase in y will cause a decrease in fuel value except at high Np-237 prices.

A qualitative analysis of the feed fuel value for Figure 4 is identical to that for Figure 6 if we define M(R,y) as the total fuel cycle cost (including credits for Pu and Np) exclusive of charges for the makeup feed, when makeup feed has composition R,y. The value of the feed stream in Figure 4 is then:

$$FC_1(R,y) = 24LPC_p^* - M(R,y) \qquad (A\text{-}1)$$

where the feed rate, F, is known as a function of R and y from the analysis of the reactor and its recycle operation. C_p^* is the minimum unit fuel cycle cost with respect to R, for a specified Np-237 price, when y = 0 and when fuel cycle cost, $C_p(R)$, is evaluated from:

$$24LPC_p(R) = M(R,0) + FC_{AEC}(R) \qquad (A\text{-}2)$$

$C_{AEC}(R)$ is the unit cost of UF_6 containing no U-236 as given by the AEC price scale. The optimum R which corresponds to C_p^* is defined as R^*.

The value of feed with y = 0 is given by Equation (A-1) as:

$$FC_1(R,0) = 24LPC_p^* - M(R,0) \qquad (A\text{-}3)$$

Eliminating M(R,0) between (A-2) and (A-3) gives:

$$F\left[C_{AEC}(R) - C_1(R,0)\right] = 24LP\left[C_p(R) - C_p^*\right] \qquad (A\text{-}4)$$

Thus, for $R = R^*$,

$$C_{AEC}(R^*) = C_1(R^*, 0) \qquad (A\text{-}5)$$

and for $R \neq R^*$,

$$C_{AEC}(R) > C_1(R, 0) \qquad (A\text{-}6)$$

We see that the fuel value curve for $y = 0$ is tangent to the AEC price scale curve at R^* and lies below the AEC scale for all other R values. This is also obvious from the graphical representation of Equations (A-2) and (A-3), as shown in Figure A-1.

As R is decreased below R^*, for $y = 0$, the fuel value will become zero for one of two reasons. First, $C_1(R,0)$ becomes zero at some R value below which reactor operation can no longer be maintained. Second, if reactor operation can be maintained down to point R_o, shown in Figure A-1, then $C_1(R,0)$ becomes zero at R_o since the reactor operator can no longer afford to pay anything for the feed stream, i.e.,

$$M(R_o, 0) = 24LPC_P^* \qquad (A\text{-}7)$$

If $y = y_1 > 0$ and if the Np-237 price is low, then the $M(R,y_1)$ curve will lie above the $M(R,0)$ curve and will become equal to $24LPC_P^*$ at R_1, where $R_1 > R_o$. This is indicated in Figure A-1. If operation is possible at $R > R_1$, with $y = y_1$, then the fuel value $C_1(R,y_1)$ falls to zero at $R = R_1$.

These results enable one to predict qualitatively the fuel value curves of Figure 5, which applies to the flowsheets of both Figures 4 and 6. The points of zero fuel value will, of course, occur at higher values of R for operation according to Figure 4 than for Figure 6.

Analysis of the flowsheet in Figure 7 is complicated by the fact that for a specified feed composition R', y' a series of fuel values, $C_d(R', y', R)$, can be calculated for a series of R values. Figure A-2 represents this mode of operation. $M(R,y)$ and F are known as functions of R and y from the analysis performed for the flowsheet of Figure 6. For a specified R_W and Np-237 price, C_P^* is

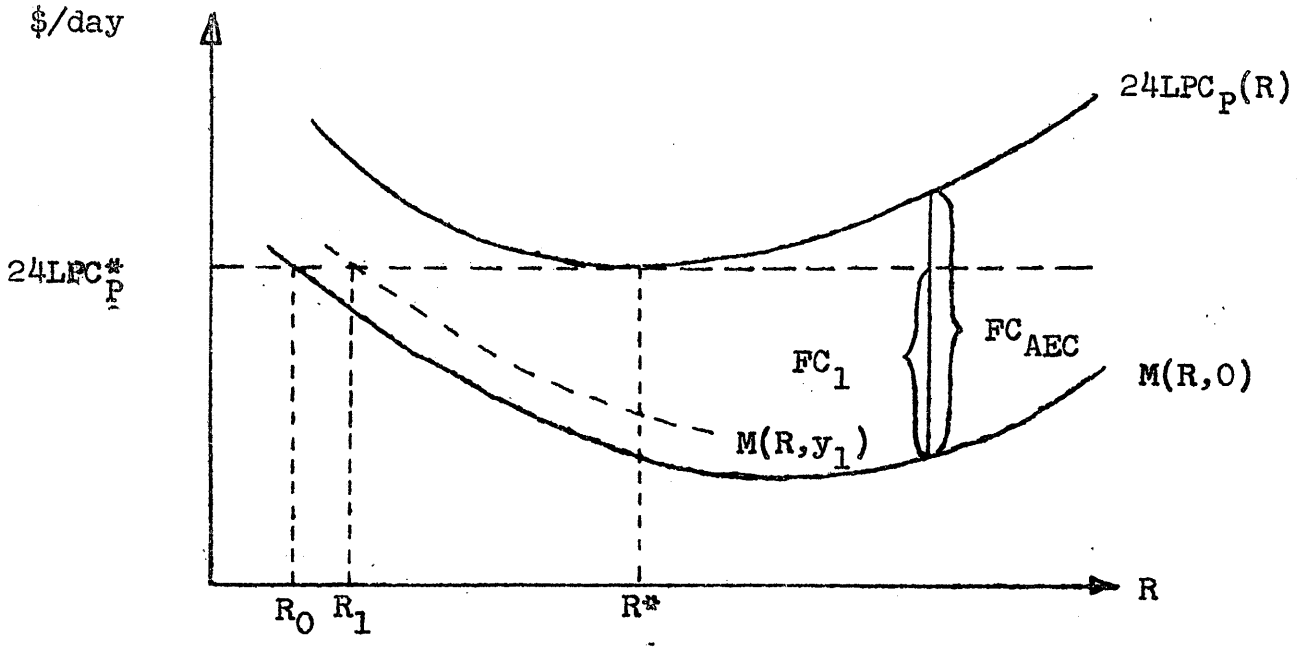

Figure A-1

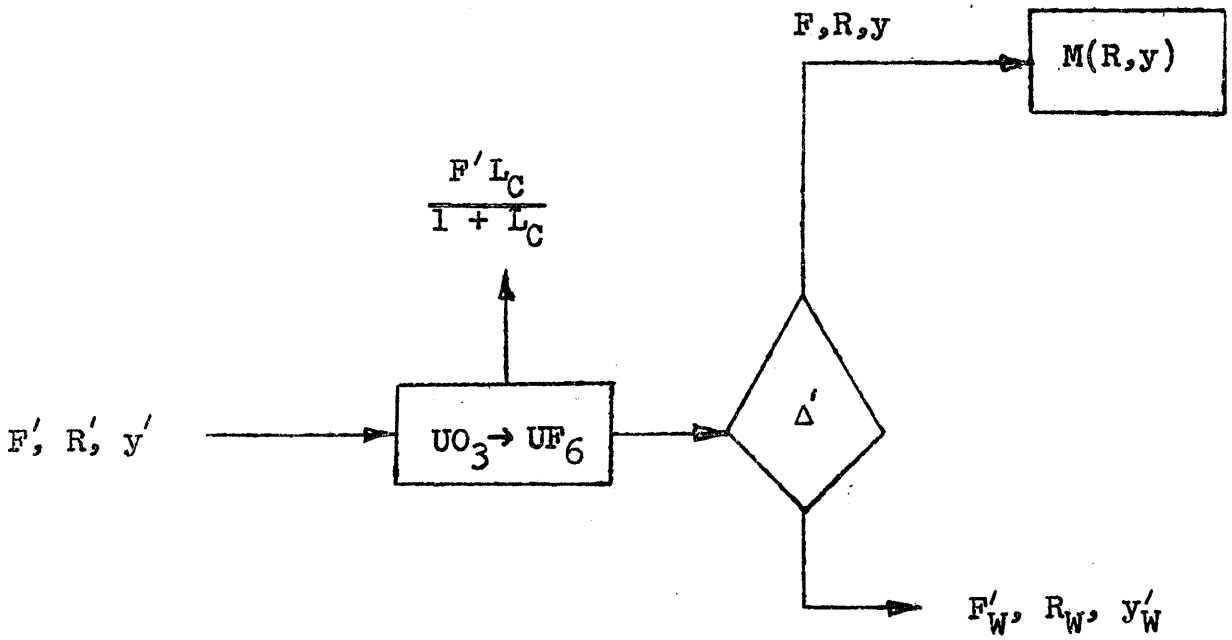

Figure A-2

the same for both Figure 6 and Figure A-2. All nomenclature is as defined in Section D.

The general expression for the value of the feed stream is:

$$F'C_d(R',y',R)=\frac{1}{1+it_c}\left\{\left[24LPC_p^*-M(R,y)\right](1-it_E)-F'C_{CT}-\Delta'C_\Delta\right\} \qquad (A\text{-}8)$$

For a specified R',y' we vary R until the maximum value of $C_d(R',y',R)$ is obtained. Define this maximum as $C_m(R',y')$. Note that only values of R for which $R \geq R'$ can be considered in determining $C_m(R',y')$.

Consider the value of material with $y' = 0$. In this case, $y = 0$ and the separative work requirement is such that

$$\Delta'C_\Delta = FC_{AEC}(R)-\frac{F'}{1+L_c}C_{AEC}(R') \qquad (A\text{-}9)$$

Inserting (A-9) into (A-8) gives, for $y' = 0$:

$$C_d(R',0,R)=\frac{1}{F'(1+it_c)}\left\{\left[24LPC_p^*-M(R,0)\right](1-it_E)-F'C_{CT}-FC_{AEC}(R)+\frac{F'}{1+L_c}C_{AEC}(R')\right\} \qquad (A\text{-}10)$$

Also,

$$\left[24LPC_p^*-M(R,0)\right]=FC_f(R,0) \qquad (A\text{-}11)$$

Using (A-11) in (A-10) and rearranging terms gives the equation:

$$C_{AEC}(R')-C_d(R',0,R)=\frac{F}{F'(1+it_c)}\left\{C_{AEC}(R)-C_f(R,0)\left[1-it_E\right]\right\}+\frac{C_{CT}}{1+it_c}+C_{AEC}(R')\left[1-\frac{1}{(1+it_c)(1+L_c)}\right] \qquad (A\text{-}12)$$

Since Equations (A-5) and (A-6) are true when $C_1(R,0)$ is replaced with $C_f(R,0)$, we see that the quantity in the curved braces of Equation (A-12) must be > 0 for all R. Hence, for a given R', we will have $C_{AEC}(R') > C_d(R',0,R)$ for any value of R we might select. This leads to the general inequality:

$$C_{AEC}(R') > C_m(R',0) \quad , \text{ for all } R'. \qquad (A\text{-}13)$$

Since the quantity in curved braces can never vanish, it is not possible to write a general expression for the R at which the

right side of Equation (A-12) is minimized, i.e. the R for which $C_d(R',0,R)$ is maximized at a given R'. This optimization is further complicated by the fact that the ratio F/F' will vary with R, at a given R'.

Consequently, for each R' value, we must vary R, keeping $R \geq R'$, until the quantity:

$$\frac{F}{F'(1+it_d)}\left\{C_{AEC}(R)-C_f(R,0)\left[1-it_E\right]\right\}$$

is a minimum. If this occurs at $R = R_m$, then

$$C_m(R',0)=C_d(R',0,R_m)\ ,\ R_m \geq R'. \qquad (A-14)$$

Although reactor operation can be maintained for all $R' > R_W$, we deduce from Equation (A-13) that $C_m(R',0)$ becomes zero when $C_{AEC}(R')$ is still greater than zero. Obviously, then, $C_m(R',0)$ becomes zero for $R' > R_W$.

APPENDIX II
NOMENCLATURE

C_A	unit cost of reprocessing, including conversion of UNH to UO_3, \$/kg fuel fed to reprocessing
$C_{AEC}(R)$	price of UF_6 with zero U-236 content and with abundance ratio R, based on the AEC scale, \$/kg U
C_C	unit cost of converting UO_3 to UF_6, \$/kg U fed to conversion
C_{CT}	cost incurred between purchase of UO_3 and end of conversion to UF_6, excluding inventory charges, \$/kg U fed to conversion
C'_{CT}	cost incurred between purchase of natural uranium as U_3O_8 and end of conversion to UF_6, excluding inventory charges, \$/kg U fed to conversion
$C_d(R',y',R)$	fuel value of UO_3 having composition R',y' when it is upgraded to a higher abundance ratio R in the first diffusion plant of Figure 7, \$/kg U
C_D	price of the product from toll enrichment of spent uranium, based on the AEC scale, with U-236 considered as U-238, \$/kg U
$C_f(R,y)$	fuel value of UO_3 (or UF_6) having composition R,y when used as makeup feed in the cycle of Figure 6, \$/kg U
C_F	unit cost of fabrication, including conversion of UO_3 or UF_6 to UO_2, \$/kg U leaving fabrication
C_K	unit price of fissile plutonium, \$/kg
$C_m(R',y')$	maximum fuel value of UO_3 having composition R',y' when it is used as makeup feed in the cycle of Figure 7, \$/kg U

C_N	unit price of Np-237, \$/kg
$C_P(R)$	fuel cycle cost when makeup feed having abundance ratio R and zero U-236 content is purchased as UF_6 on the AEC price scale, mills/kwhr
C_P^*	minimum fuel cycle cost realizeable when makeup feed having zero U-236 content is purchased as UF_6 on the AEC price scale, mills/kwhr
C_R	price of reactor feed, based on the AEC scale, with U-236 considered as U-238, \$/kg U
C_S	price of spent uranium, based on the AEC scale, with U-236 considered as U-238, \$/kg U
C_T	unit shipping cost for irradiated fuel, \$/kg fuel shipped
C_{UF_6}	cost of natural uranium as UF_6, \$/kg U
$C_{U_3O_8}$	price of natural uranium as U_3O_8, \$/lb U_3O_8
C_W	value of diffusion plant tails, \$/kg U
C_Δ	cost of separative work, \$/kg U
$C_1(R,y)$	fuel value of UO_3 having composition R,y when it is used as makeup feed in the cycle of Figure 4, \$/kg U
C_2	maximum fuel value of UO_3 of a given composition when it is used as makeup feed to a cycle involving the recycle of spent uranium through a diffusion plant, \$/kg U
F	time-averaged flow rate of makeup uranium fed to fabrication, kg U/day
F'	time-averaged flow rate of makeup feed uranium in the cycle of Figure 7, kg U/day
F_i	time-averaged flow rate of uranium in the stream designated by subscript i, kg U/day

F'_W	time-averaged flow rate of uranium in the tails stream from the diffusion plant used to upgrade makeup feed in the cycle of Figure 7, kg U/day
i	interest rate on working capital, yr^{-1}
I	total initial loading of uranium in the reactor, kg
K	time-averaged flow rate of fissile plutonium leaving reprocessing plant, kg/day
L	average load factor for power plant
L_C	fractional loss of uranium during chemical conversion of UO_3 or U_3O_8 to UF_6, based on product from conversion
L_F	fractional loss of uranium during fabrication, based on material leaving fabrication
L_{RP}	fractional loss of Pu and Np during reprocessing, based on material fed to the reprocessing plant
L_{RU}	fractional loss of uranium during reprocessing, based on uranium fed to the reprocessing plant
M(R,y)	sum of all fuel cycle costs, exclusive of makeup feed cost, when makeup feed having composition R,y is used in the cycles of Figures 4 and 6, $/day
N	time-averaged flow rate of Np-237 leaving reprocessing plant, kg/day
P	net electrical power output of plant, MW(e)
R	abundance ratio of U-235 to U-238 in makeup uranium fed to fabrication
R'	abundance ratio of U-235 to U-238 in makeup feed uranium in the cycle of Figure 7
R*	abundance ratio of U-235 to U-238 which gives minimum fuel cycle cost when makeup feed having zero U-236 content is purchased as UF_6 on the AEC price scale

R_i	abundance ratio of U-235 to U-238 in the uranium stream designated by subscript i
R_m	abundance ratio of U-235 to U-238 in upgraded uranium which gives maximum fuel value to UO_3 of a given composition, when the UO_3 is used as makeup feed in the cycle of Figure 7
R_o	abundance ratio of U-235 to U-238 at which $C_1(R,0)$ becomes zero, when reactor operation can still be maintained
t_C	time interval between purchase of UO_3 or U_3O_8 and completion of conversion to UF_6, years
t_E	time interval between the delivery of uranium to the AEC for toll enrichment and the receipt of product uranium, years
t_F	average pre-reactor fuel holdup time, years
t_{RP}	average post-reactor holdup time for Pu and Np, years
t_{RU}	average post-reactor holdup time for uranium, years
x_i	weight fraction of U-235 in the uranium stream designated by subscript i
y	weight fraction of U-236 in makeup uranium fed to fabrication
y'	weight fraction of U-236 in makeup feed uranium in the cycle of Figure 7
y_i	weight fraction of U-236 in the uranium stream designated by subscript i
y'_W	weight fraction of U-236 in the uranium tails stream from the diffusion plant used to upgrade makeup feed in the cycle of Figure 7

Δ	separative work requirement for the re-enrichment of spent uranium, kg U/day
Δ'	separative work requirement for the upgrading of makeup feed uranium in the cycle of Figure 7, kg U/day
ϕ	separation potential of makeup uranium fed to fabrication
ϕ'	separation potential of makeup feed uranium in the cycle of Figure 7
ϕ_i	separation potential of the uranium stream designated by subscript i
ϕ'_W	separation potential of the tails stream from the diffusion plant used to upgrade makeup feed in the cycle of Figure 7

Subscripts

P	denotes the product stream from the diffusion plant used to re-enrich spent uranium
R	denotes the reactor feed stream
S	denotes the spent uranium stream leaving the reprocessing plant
W	denotes the tails stream from the diffusion plant used to re-enrich spent uranium