Engineering Balance: The Conceptual Approach

by

Jonathan E. Abbott

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Abstract

This work presents a view of balance useful for mechanical engineers. Mechanical engineers often need to make quick intelligent decisions using conceptual and physical understanding. The typical mechanical engineering instruction usually provides a good basis for "back of the envelope" calculations, especially for mechanical systems; however, one exception to this case is in the field of dynamics and control. Dynamics and control is generally taught with much math, modelling most systems with differential equations. Although math is useful for designing control systems, when designing products for people who act as sophisticated controllers the engineer needs a more general understanding of balance. This work presents a conceptual intuitive way to break the act of balance into distinct mechanisms and thereby quickly evaluate how a system balances.

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Title: Professor of Mechanical Engineering

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Thank you to Pawel Zimoch for assistance and guidance throughout writing this thesis. Thank you also to Professor Anette (Peko) Hosoi for offering oversight. Lastly, thank you to Mr. James Fennemore who back in third grade first introduced me to unicycling and began my interest in balance.

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Chapter 1

Introduction to Balance

This work exists to enable engineers to make quick educated design decisions when encountering products and situations that involve balance. The examples presented in this work are generally products, such as a bicycle, that involve humans. Understanding how humans balance is important to be able to design any type of vehicle for humans. Although most of the examples will be vehicles intended for humans, the mechanisms of balance presented in this work are applicable to all systems and situations involving balance.

1.1 Relating Traditional Methods

The traditional engineer may first struggle with the definition of "balance" used here. Usually balance is interpreted as part of "dynamics and control." Traditional classes in dynamics and control generally depict a system using differential equations. Through system analysis and addition of artificial controllers the system becomes considered "stable." Stability refers to the idea that if a system slightly deviates from its desired state, the system will automatically correct itself and properly return to the desired state. This traditional definition of stability is not the "balance" that is discussed here.

The balance discussed here is about the ability, act, and practice of maintaining a desired course. The balance discussed here assumes that the controller (often a human) is already made or will be made using traditional methods. While a scientist might try to derive a mathematical relation that characterizes a human controller, a practising engineer often needs educated answers that focus on the critical ways in which a human controller works.

This work will not replace the math-heavy content of dynamics and control classes, but it will enable engineers on any level to have a deeper understanding of products, evaluate products faster and better, and generate novel design improvements simply based on a different perspective of balance.

1.2 Relating Previous Literature

Like previous literature, here the act of balancing is broken up into "mechanisms" of balance.[1][2] The first mechanism of balance is associated with the location of the center of pressure relative to the center of mass. The second mechanism of balance is associated with how one twists or reshapes their body. The third mechanism of balance in this work is associated with adding or subtracting mass from the system. Most cases of balance do not involve mass flow, so balance can thus be decomposed into the first two mechanisms of balance. The three mechanisms of balance are depicted visually in Figure 1-1.

The way in which the mechanisms of balance are derived and defined in this work is slightly different from previous literature. Rather than deriving the mechanisms of balance from an inverted pendulum model as in [1], a more intuitive force decomposition is used that provides easier understanding of the mechanisms of balance. The meaning of the mechanisms of balance is slightly different in that all external contact forces are grouped together and there is no separate mechanism specifically for external contact forces that are not from the "ground." Additionally, the effects of the body's rotational inertia are only considered in the second mechanism of balance.

This work also aims primarily for a conceptual understanding of balance. By keeping the mechanisms of balance simple and abstract, the mechanisms of balance described within are applicable to other domains.

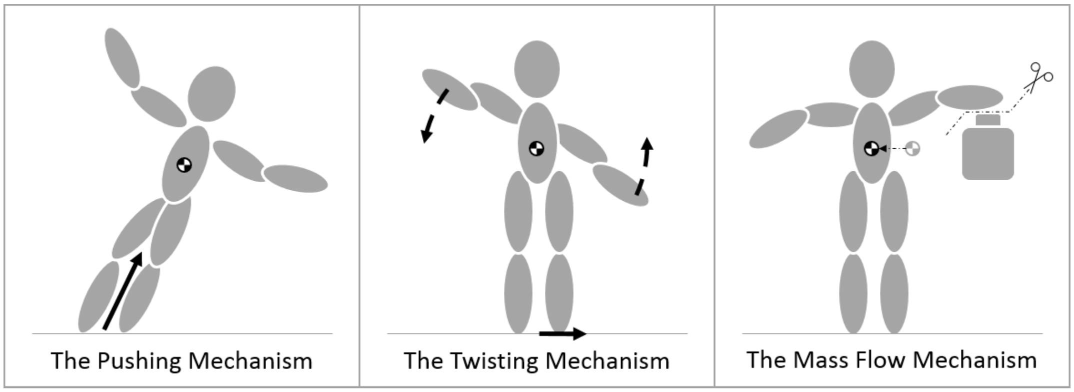

Figure 1-1: The three mechanisms of balance. The pushing mechanism has a force in line with the center of mass. The twisting mechanism has a perpendicular force here caused by arm twisting. The mass flow mechanism redefines the mass of the system here represented by releasing an object.

1.3 Contents

Chapter 2 derives the first two mechanisms of balance using a force decomposition. Chapter 3 explains each mechanism of balance in greater depth. Section 3.1 explains the pushing mechanism and especially its relation to countersteering. Section 3.2 explains the twisting mechanism. Chapter 4 offers case studies of using the mechanisms of balance to analyse systems and suggest possible improvements. Chapter 5 presents the concept of virtual grounds that change the placement of the center of pressure and thus the balance analysis. Chapter 6 states other factors that affect someone's ability to balance, and Chapter 7 concludes with a summary of this thesis.

Chapter 2

A Force Decomposition for Balance

In this chapter, a special force decomposition is presented that is ultimately the basis of the two main mechanisms of balance.

2.1 The Force Decomposition Derivation

One way of viewing the two main mechanisms of balance is to derive them from a simple force decomposition. The force decomposition in 2D is shown in Figure 2-1. The force decomposed is the net contact force which is the resultant of all external forces besides field forces like gravity. The net contact force is placed at the center of pressure at the level of the ground. Conceptually, the center of pressure is the location at which all forces from the ground can be concentrated to have the same effect.[1] The net contact force is a reaction force, meaning that the magnitude and direction of the force is intrinsically coupled to the way in which the system pushes and pulls on the ground.

The traditional decomposition for a reaction force is shown in (a), for which the normal force N is perpendicular to the bottom surface and the force of friction F_f is parallel to the bottom surface. (b) shows the new decomposition in which the net contact force is decomposed onto a rotated coordinate system for which one axis

[1]The details of how to precisely define the center of pressure is discussed more in Chapter 5, but the precise definition is usually not needed for conceptually understanding the mechanisms of balance.

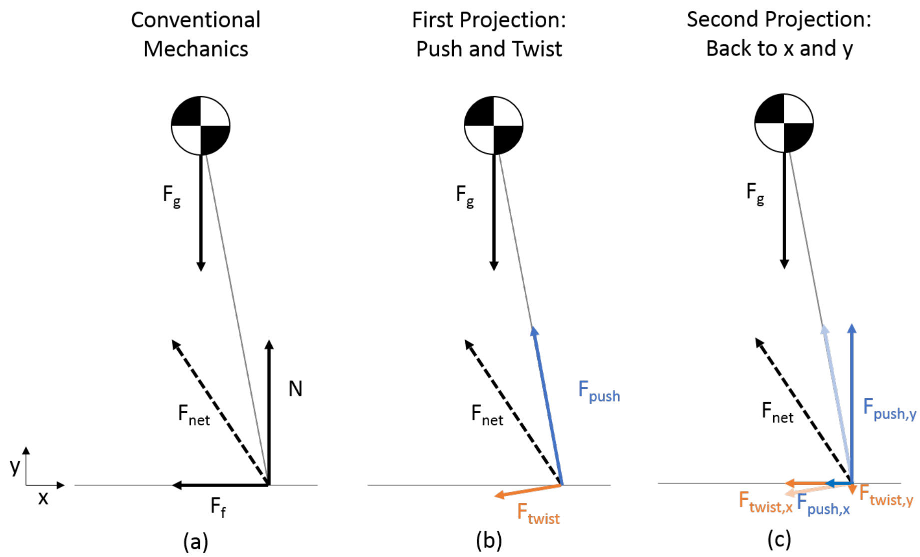

Figure 2-1: A meaningful 2D double projection of the net contact force. (a) shows the traditional standard decomposition for a reaction force with the normal and frictional forces. (b) shows a new decomposition with the "pushing" and "twisting" forces. (c) shows the pushing and twisting forces projected back onto the horizontal and vertical axes.

extends between the center of pressure and the center of mass. The names given here of these forces are the *pushing force* and the *twisting force*. (c) shows that the pushing and twisting forces can be projected back onto the original axes.

The pushing force is the force in the component of the net contact force that is in line with an axis extending from the center of pressure to the center of mass of the system. The pushing force is significant as it contributes to the dynamics of the system ignoring rotation about the system's center of mass.

The twisting force is the component of the contact force that lies in the plane perpendicular to the pushing force. The twisting force is significant as it accounts for much of the rotational dynamics of the system.[2] The twisting force also contributes to part of the translational dynamics of the system.

The pushing and twisting forces can be projected back onto the original axes. In Figure 2-1 only $F_{twist,x}$ is shown because Figure 2-1 is for the 2D case. In 3D F_{twist}

[2]In addition to the pushing force and twisting force, there may need to be a point torque to fully account for the rotational dynamics of the system. This extra point torque - which should be considered part of the twisting mechanism - is generally neglected in the analysis here, but it is discussed more in Chapter 5.

can be decomposed into both $F_{twist,x}$ and $F_{twist,z}$ where z is out of the page. The resultant or sum of $F_{twist,x}$ and $F_{twist,z}$ will be referred to as $F_{twist,hor}$ representing the magnitude of the projection of the twisting force on the horizontal plane. $F_{push,x}$ may be similarly referred to as $F_{push,hor}$.

The significance of this second projection is that it is possible to consider the net contact force in one direction and be able to quantify what effect each mechanism of balance (pushing or twisting) is having on the object. If the x-axis direction is defined such that the pushing force lies in the x-y plane, only the twisting force can contribute to acceleration in the z direction as shown in Equation 2.3.

$$F_{push,x} + F_{twist,x} = ma_x \tag{2.1}$$

$$F_{push,y} + F_{twist,y} = ma_y \tag{2.2}$$

$$F_{twist,z} = ma_z \tag{2.3}$$

The components of the pushing and twisting forces all have conceptual significance in understanding balance. For instance, figure 2-1 (c) shows $F_{push,y}$ directed upward and $F_{twist,y}$ directed downward. This means that the twisting in the system (due to the combination of rotational inertia and possible internal torques) actually attempts to direct the system downward, opposite the direction of the net contact force. One mechanism of balance can either add to or diminish the effect of the other mechanism.

2.2 Comparing Horizontal Force Magnitudes

One useful simplifying assumption is that the primarily focus is on movement in the horizontal plane. The grounds for this assumption is that once past the challenge of not falling, the focus usually becomes the challenge of propelling the system in a (typically horizontal) plane. For instance:

- a biker wants to steer left or right or speed up or slow down
- a human wants to walk forward or backward, possibly shuffling sideways

- an inverted pendulum wants to control its position left and right

In all of these cases, there exists a desire for system object to achieve a basic position in a specified plane - or if in 2D, in a specified linear direction. There is a desire to “control” the system object. Since in human balance there is generally only one plane of interest for movement, the horizontal component of the pushing and twisting forces within the plane is the primary focus.

For comparison across different systems, it could be helpful to normalize the horizontal component of the pushing and twisting forces against the force of gravity. The normalized forces could be particularly useful when plotting the pushing and twisting forces over time. With mass m and gravity g:

$$\hat{F}_{push,hor} = F_{push,hor}/mg \tag{2.4}$$

$$\hat{F}_{twist,hor} = F_{twist,hor}/mg \tag{2.5}$$

It would be convenient to have a nondimensional number that characterizes a system’s relative use of the pushing and twisting forces. The following two ratios are first steps towards describing a system over a period of time. These ratios use the root mean square (rms) of the forces since $F_{push,hor}$ and $F_{twist,hor}$ can be in opposite directions.

$$\text{The horizontal pushing ratio } \eta_{push_{hor}} = F_{push,hor,rms}/(F_{push,hor,rms} + F_{twist,hor,rms}) \tag{2.6}$$

$$\text{The horizontal twisting ratio } \eta_{twist_{hor}} = F_{twist,hor,rms}/(F_{push,hor,rms} + F_{twist,hor,rms}) \tag{2.7}$$

In practice, the above ratios should be checked alongside graphs of the horizontal pushing and twisting forces, as these ratios neglect the specific dynamics and directionality of the system over time. However, these ratios have the great conceptual benefit of adding up to 100%.

Chapter 3

Understanding Each Mechanism of Balance

The force decomposition from Chapter 2 highlights that the net contact force can be decomposed into two distinct forces: the pushing force and the twisting force. These forces can be associated with the two main mechanisms of balance: the pushing mechanism and the twisting mechanism.

3.1 The Pushing Mechanism

The pushing mechanism includes all the actions taken that directly affect the pushing force. To appreciate the pushing mechanism, it is necessary to understand countersteering.

Countersteering is classically defined as the act of steering in the opposite direction to initiate a turn.[3] Figure 3-1 shows countersteering as the path of the center of pressure along the ground before a turn. In this section, countersteering will be further explained, its significance for turning will be estimated, and its definition will be expanded.

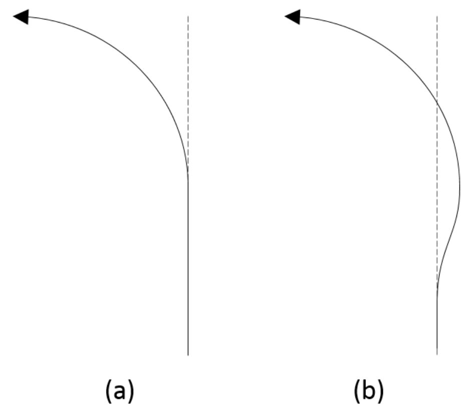

Figure 3-1: Turning left without and with countersteering. (a) shows a trajectory of the center of pressure in a case without countersteering. Usually in practice countersteering occurs as shown in (b). The countersteering - or steering initially in the opposite direction - helps to initiate the leaning needed for steady turning.

3.1.1 The Irony of Turning

Countersteering has traditionally been used in the context of initiating a turn. The word countersteering is most commonly found in the motorcyclist community, and it is less used outside the context of bicycle-like vehicles. Countersteering can be unintuitive because it claims that riders steer in the opposite direction to initiate a turn. Many riders claim to have difficulty perceiving countersteering.[3]

The mechanical engineer has the advantage of understanding the mechanics of countersteering. It is generally well accepted that to make a turn (on say a bicycle) the rider has to be leaning into the turn. The common language is that leaning is necessary to keep a rider from falling out of a turn. A more rigorous explanation for leaning is that leaning prevents a torque about the rider's center of mass due to the net contact force, often referred to as the "ground reaction force."[4] Eliminating the torque about a rider's center of mass means that the rider can continue turning without changing their angle slanted with respect to the ground. A free body diagram with no torque about the center of mass can be seen in Figure 3-2.

Clearly one has to lean during steady turning, and the next question becomes how does one begin leaning. The novice would say one just "leans" into a turn. The problem with this explanation is that it implies that the center of pressure does not

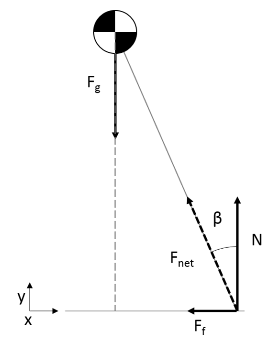

Figure 3-2: The basic force deposition during steady turning. Leaning is required to align the net contact force with the line that extends from the center of pressure to the center of mass, thereby eliminating any torque about the center of mass during a steady turn.

move laterally and the upper body is shifted over without any external force. From basic physics, the mechanical engineer knows that the body's center of mass cannot shift over without an external force. Countersteering can put a rider in a leaning orientation by moving the center of pressure laterally.

3.1.2 How Significant is Countersteering

Many riders do not notice that they countersteer, and so the first important question is then how significant is countersteering for initiating a turn. There are a number of factors that affect how much a rider countersteers and there are multiple ways to evaluate its significance. Here below are some general factors that affect how much a rider countersteers to initiate a turn:

1. Turn radius
2. Speed
3. Gravity
4. Time to initiate the turn
5. Use of the twisting mechanism
6. Coefficient of friction

7. Other outside forces (ex: wind)

The way each one of these factors changes the significance of countersteering can be quantitatively estimated. A simple order of magnitude estimate and a useful initial calculation can be derived from the basic angle needed for a steady turn based upon the rider's mass m, speed v, turn radius r, and gravity g. The following gives the frictional force F_f needed for a steady turn.

$$F_f = m\frac{v^2}{r} \tag{3.1}$$

The next step is to relate the angle β the rider leans to the force of friction. Here it is assumed that the normal force is equal to the force of gravity.

$$tan(\beta) = \frac{F_f}{mg} \tag{3.2}$$

Combining the above equations yields:

$$\beta = \arctan\frac{v^2}{gr} \approx \frac{v^2}{gr} \tag{3.3}$$

Equation 3.3 gives β for steady state turning. For example, travelling 5 m/s (11.2 mph) with a turn radius of 15 meters (49.2 ft) requires an angle of leaning of 9.65 degrees.

A smaller turn radius and a higher speed will generally require more countersteering because the rider will have to lean more for steady turning.

Gravity also plays a role in the steady state turning angle. Although changing gravity is not usually an option, sometimes an engineer can effectively change gravity in other ways. Here are four ways to achieve this effect (while staying on planet Earth):

1. Vertically accelerate the ground as if in an elevator. When the ground starts to accelerate upwards, the system feels there is a larger acceleration due to gravity.

2. Use a rotating reference frame. If the system is rotated quickly in a large circle,

the system will feel as if gravity is slightly larger and directed radially outward. This case is highly relevant when say a motorcycle is going around a banked turn in the road and has to countersteer to either turn sharper or end the turn.

3. The surface (e.g. a road) is at top of a hill or a valley. Riding over a surface that is concave up (like a valley) effectively results in a higher acceleration due to gravity.

4. Use a slanted surface. If the vertical is redefined as normal to a slanted surface the “vertical” component of gravity becomes less and the “horizontal” component of gravity can be simply treated as part of the contact force.

These four ways to effectively change gravity show the extent that one countersteers can often be dependent on the shape and orientation of the surface travelled on.

So far, the extent of countersteering needed has been described by the angle β with the vertical needed for steady state turning. It is reasonable to say that more leaning requires more countersteering, but it would be good to quantify how β is related to countersteering.

The relationship between countersteering and β as a function of time can be derived quantitatively. It is simplest to use an inverted pendulum model with no rotational inertia about the center of mass. Figure 3-3 shows a simple inverted pendulum in an inertial reference frame. Here x refers to the horizontal position of the center of mass, p refers to the horizontal position of the center of pressure, and l refers to the length from the center of mass to the center of pressure.

It is simplest to model the case for small deviations of β for which then $N \approx mg$ and thus

$$F_f \approx -mg\beta \tag{3.4}$$

Since β is approximately $\frac{p-x}{l}$, the one dimensional horizontal equation of motion is

$$\ddot{x} - \frac{g}{l}x = -\frac{g}{l}p \tag{3.5}$$

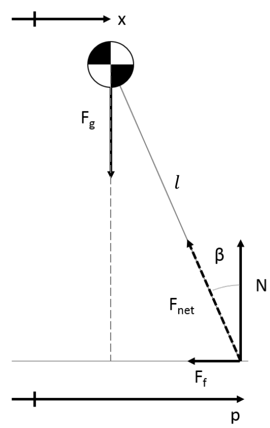

Figure 3-3: The inverted pendulum model. Little x refers to the horizontal location of the center of mass and p refers to the horizontal location of the center of pressure.

A Laplace transform of this equation of motion assuming rest initial conditions yields

$$(s^2 - \frac{g}{l})X = -\frac{g}{l}P \tag{3.6}$$

$$X = -\frac{g}{l}\frac{P}{(s^2 - \frac{g}{l})} \tag{3.7}$$

Partial fractions can be used to simplify the denominator on the right-hand side.

$$\frac{1}{(s+\sqrt{\frac{g}{l}})(s-\sqrt{\frac{g}{l}})} = \frac{A}{s+\sqrt{\frac{g}{l}}} + \frac{B}{s-\sqrt{\frac{g}{l}}} \tag{3.8}$$

By changing the value of s, it can be shown that

$$A = -\frac{1}{2}\sqrt{\frac{l}{g}} \tag{3.9}$$

$$B = \frac{1}{2}\sqrt{\frac{l}{g}} \tag{3.10}$$

Substituting in yields

$$X = -\frac{g}{l}\left(\frac{-P\frac{1}{2}\sqrt{\frac{l}{g}}}{s+\sqrt{\frac{g}{l}}} + \frac{P\frac{1}{2}\sqrt{\frac{l}{g}}}{s-\sqrt{\frac{g}{l}}}\right) \tag{3.11}$$

$$X = -\frac{1}{2}\sqrt{\frac{g}{l}}\left(\frac{-P}{s+\sqrt{\frac{g}{l}}} + \frac{P}{s-\sqrt{\frac{g}{l}}}\right) \tag{3.12}$$

Taking the inverse Laplace yields the time domain solution x(t):

$$x(t) = -\frac{1}{2}\sqrt{\frac{g}{l}}\left(-e^{-\sqrt{\frac{g}{l}}t} + e^{\sqrt{\frac{g}{l}}t}\right) * p(t) \tag{3.13}$$

where $*$ is the symbol for convolution, so

$$x(t) = -\frac{1}{2}\sqrt{\frac{g}{l}}\int_{0}^{t}\left(-e^{-\sqrt{\frac{g}{l}}t} + e^{\sqrt{\frac{g}{l}}t}\right) p(t-\tau)d\tau \tag{3.14}$$

Equation 3.14 gives the time domain solution to how a pendulum (with no rotational inertia) will move with small displacements in its point of contact. It can be noted that when the point of contact $p(t)$ moves in one direction, the system tends to fall the other direction (hence the initial negative). Additionally, the rate of movement increases with g and decreases with l. The convolution shows that the linearised impulse response to a displacement in the center of pressure is the combination of two exponentials that over time tend to be unstable.

The goal was to determine how much countersteering is needed to establish a given angle β. As stated before, $\beta \approx \frac{p-x}{l}$. Thus Equation 3.14 can be rewritten as

$$\beta(t) = \frac{p(t) - x(t)}{l} = \frac{p(t) + \frac{1}{2}\sqrt{\frac{g}{l}}\int_{0}^{t}\left(-e^{-\sqrt{\frac{g}{l}}t} + e^{\sqrt{\frac{g}{l}}t}\right) p(t-\tau)d\tau}{l} \tag{3.15}$$

Equation 3.15 highlights that β is a function of both the center of pressure at the moment $p(t)$ and in the past $p(t-\tau)$. If the time given to initiate the turn is short, the $p(t)$ term dominates. If the time given to initiate the turn is long, the $p(t-\tau)$ term dominates.

Fully understanding and appreciating the conceptual meaning of convolution is challenging without a full knowledge of linear time invariant systems. Rather than try to explain convolution, it is more expedient to just look at a visual representation of the effect of countersteering.

Figure 3-4 shows two graphs. The top graph represents an arbitrary path steering to the right. (The example here is $p(t) = -2 \times t^3 - 3 \times t^2 + 1$.) In one second, the center of pressure shifts gradually 1 cm to the right. The second graph shows the reverse impulse response and the contribution to the final leaning angle β. The contribution to the final leaning angle is determined by multiplying the steering path by the reverse impulse response.

To make the graph more interpretable, l is 1 m, and g is 9.8 m/s. Additionally, the horizontal axis is shifted by t to the left such that the moment of interest (right before a turn) is at t = 0. The graph should be interpreted such that $t = -1$ refers to 1 second before the turn. For the graph, β is in degrees because degrees are generally easier to conceptualize. Additionally, $p(t)$ is displayed using cm (rather than m) because the distance countersteered is on the order of cm.

The exponential shape of the reverse impulse response shows that small changes in the position of the center of pressure early on make a significant impact in the final β. This is one of the possible explanations why many riders do not perceive countersteering. By countersteering sufficiently in advance, the actual shift in the position of the center of pressure is very minimal.

It is also notable that the reverse impulse response is a function of time, not distance. This means that if travelling faster, more countersteering is needed to achieve the same β in a fixed distance. Countersteering becomes important when turning quickly in an emergency.[5]

Travelling 5 m/s (11.2 mph) with a turn radius of 15 meters (49.2 ft) requires an angle of leaning of 9.65 degrees, and using countersteering alone for just one second before the turn requires countersteering about 5.33 cm (2.10 in).

This basic analysis ignores the slight change in the component of velocity towards the inside of the turn while countersteering. Accounting for the velocity towards the inside of the turn ultimately requires less countersteering than estimated here. Additionally, use of the second main mechanism of balance, will also result in less countersteering needed than estimated here.

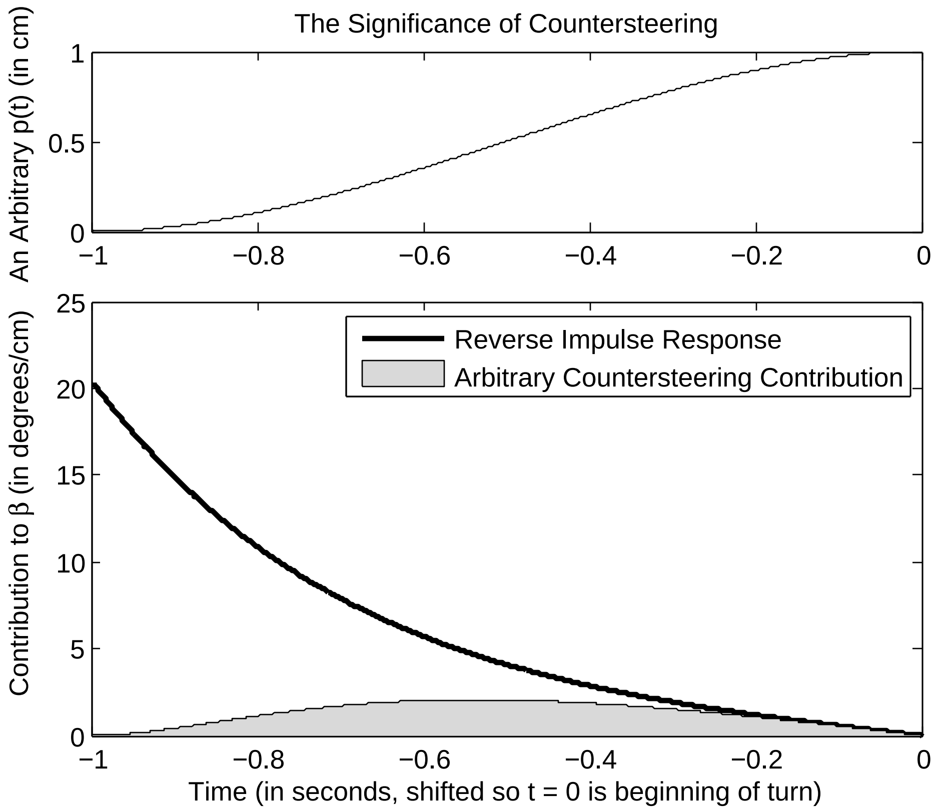

Figure 3-4: Countersteering's significance. The top plot shows an arbitrary countersteering action p(t) of steering 1 cm to the right over a one second interval. The bottom plot shows the reverse impulse response useful in the convolution and the area made by multiplying the arbitrary p(t) by the reverse impulse response. The shaded area integrating to 1.24 degrees means that gradually countersteering 1 cm over 1 second with a 1 m length to the center of mass contributes to about 1.24 degrees leaning from just the center of mass moving and about 1.81 degrees total including the final displaced center of pressure.

3.1.3 Expanding the Meaning of Countersteering

Although the term "countersteering" has traditionally been used in the context of turning for two-wheeled vehicles, the process of countersteering occurs in many circumstances. One can redefine countersteering broadly as *positioning the center of pressure relative to the center of mass.* Countersteering as a word is the concatenation of "counter" and "steering" and in general, with all else equal, any relative change in the position of the center of pressure tends to cause the system to accelerate more in the opposite direction of the change. Thus, it intuitively makes sense to broaden the definition of the word countersteering to be for all direct positioning of the center of pressure.

The definition of countersteering should be explicitly expanded in two ways:

1. Countersteering is also applicable to objects outside of motorcycles and bicycles.

2. Countersteering can be relevant for more than just turning left and right.

Traditionally countersteering has been associated with vehicles such as motorcycles or bicycles. This makes sense because the act of steering in the opposite direction to initiate a turn is used commonly with motorcycles and bicycles. However, here the definition of countersteering has been expanded to be the *positioning of the center of pressure relative to the center of mass*, and so countersteering becomes relevant to all objects that have contact with the ground. With this new definition, it is arguable that cars countersteer. The center of pressure during a turn quickly moves toward the outside of a turn.

Additionally, countersteering can be applied to even airplanes if a "virtual ground" on which the center of pressure rests is defined. Virtual grounds are analysed more in Chapter 5.

Countersteering is also relevant for more than just left and right turning. For instance a unicyclist can fall not just left and right but also forward and backward. When a forward-moving car brakes, the center of pressure quickly shifts towards the front wheels.

By expanding the meaning of countersteering to be *positioning the center of pressure relative to the center of mass* the word countersteering becomes an integral part of the pushing mechanism.

3.2 The Twisting Mechanism

The second main mechanism for balance is the twisting mechanism. The twisting mechanism includes all the actions taken that directly affect the twisting force.

Sometimes in the literature this mechanism of balance is referred to as "segment acceleration."[1] The motorcyclist community might refer to the corresponding actions as "body steering."[3] This mechanism of balance includes arm twisting, twisting at the hip, and leg extension, as all of these actions can directly affect the twisting force. This mechanism of balance also includes gyroscopic effects and the direct effects of rotational inertia about the center of mass.

3.2.1 How Arms Help Balance

It is well known that arms help humans to balance, but often not known why. Arm twisting ultimately leads to reaction forces that directly effect the net contact force. Figure 3-5 shows how arm twisting affects the acceleration of the center of mass by requiring a torque about the center of mass that is generated by the net contact force.

Arm twisting is effective when the arms are changing the angular momentum of the system. The angular momentum of the system changes when the arms are being accelerated either faster/slower or are changing the axis about which they rotate. The angular momentum of the system about its center of mass must change to have an external torque that could impact the net contact force.

Arm twisting is not the only way of reshaping the body to change the net contact force. It is possible to twist at the waist or simply kick a leg out as shown in Figure 3-6.

The interest here is accelerating segments of the body such that it changes the

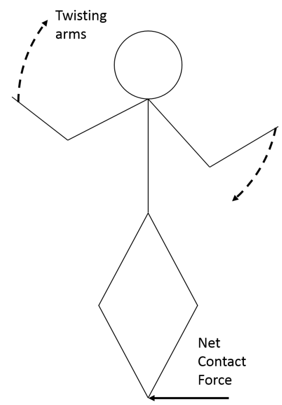

Figure 3-5: Arm twisting changes the net contact force. Here a person accelerates their arms in a clockwise rotation. To cause this rotational acceleration, there has to be a torque. The torque is caused by the net contact force. The arms effectively absorb the angular impulse from the net contact force.

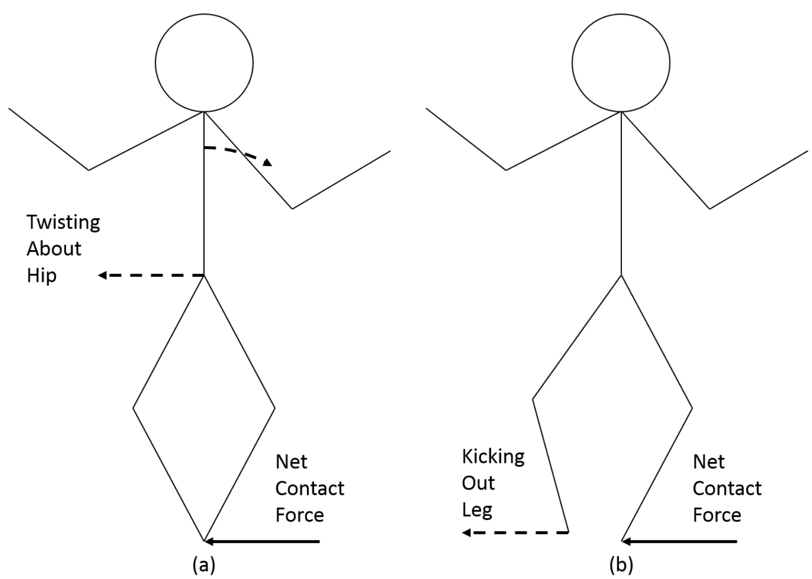

Figure 3-6: Other types of body reshaping to change the net contact force. (a) shows twisting at the waist. (b) shows kicking out a leg.

twisting force. Accelerating the arms upward when already upright would contribute to the pushing force that is associated with countersteering.

Additionally, shifting the points of contact with the ground often requires body twisting and bending, but only the bending and twisting that directly creates results in a torque about the center of mass is considered part of this mechanism of balance.

The equation central to this mechanism of balance is the following where $\vec{\tau}_{point}$ refers to the possible point torque needed at the center of pressure to fully account for the dynamics of the system:

$$\frac{d\vec{L}_{com}}{dt} = \vec{r}_{com,cop} \times \vec{F}_{twist} + \vec{\tau}_{point} \tag{3.16}$$

3.2.2 How Significant is the Twisting Mechanism

The twisting mechanism has many limitations. One limitation of the twisting mechanism is that arms can only sweep out so many degrees and move so quickly. This means that the impulse generated is limited in magnitude.

The classic example of this limitation is when someone is trying to stop themselves from falling into a pool. They stand on the pool's edge and once they realize they are falling forward, they quickly accelerate their arms to propel themselves back. However, since arms are humanly limited in their speed of rotation and acceleration, there is a limit beyond which they will fall into the pool.

Although the twisting force is naturally limited in instantaneous magnitude and total impulse, the pushing force can significantly amplify the effects of the twisting mechanism. If the twisting force can shift the center of mass such that the body has begun leaning in the desired direction, the pushing mechanism will take over and act upon the tilt.

Some heavy vehicles, such as motorcycles, limit the effectiveness of the twisting mechanism because the rotational inertia of the vehicle is significantly greater than the rotational inertia of human. Shown in Figure 3-7 is a simple two part model of arms and a body (which could include a vehicle such as a motorcycle). The arms are considered simply a mass lump centered around B. The body extends from a point on the ground A through the center of mass of the system and up to B. A simple linear analysis can relate how accelerations in $\ddot{\theta}_2$ directly affect accelerations in $\ddot{\theta}_1$.

One approach to relating $\ddot{\theta}_2$ and $\ddot{\theta}_1$ is to find the angular momentum about A.

$$L^A = I_1^A \dot{\theta}_1 + M_2 (L_1)^2 \dot{\theta}_1 + I_2^B \dot{\theta}_2 \tag{3.17}$$

Taking the derivative of L^A with respect to time yields

$$\frac{dL^A}{dt} = \left(I_1^A + M_2 (L_1)^2 \right) \ddot{\theta}_1 + I_2^B \ddot{\theta}_2 \tag{3.18}$$

If one assumes the point of contact with the ground at A is fixed, then the effect of deliberate changes in $\ddot{\theta}_2$ can be related to $\ddot{\theta}_1$ simply by ignoring all external torques about A such that $\frac{dL^A}{dt} = 0$. Thus,

$$\ddot{\theta}_1 = -\frac{I_2^B}{I_1^A + M_2 (L_1)^2} \ddot{\theta}_2 \tag{3.19}$$

Equation 3.19 shows that arm twisting is most effective when the arms have a large moment of inertia relative to the moment of inertia of the body with respect to the ground. Motorcycles have such a larger moment of inertia with respect to the ground that twisting one's body to help balance is of minimal use for balancing and thus countersteering has to be used to a much higher degree. For contrast, bicycles (that have a significantly less moment of inertia with respect to the ground) need less countersteering in practice simply because the bicycle weighs significantly less.

Equation 3.19 also shows why unicyclists are instructed to keep their arms extended outward. Extending the arms outward increases the moment of inertia of the arms. Similarly, tightrope walkers may hold a long pole while balancing that effectively increases the length of their arms.

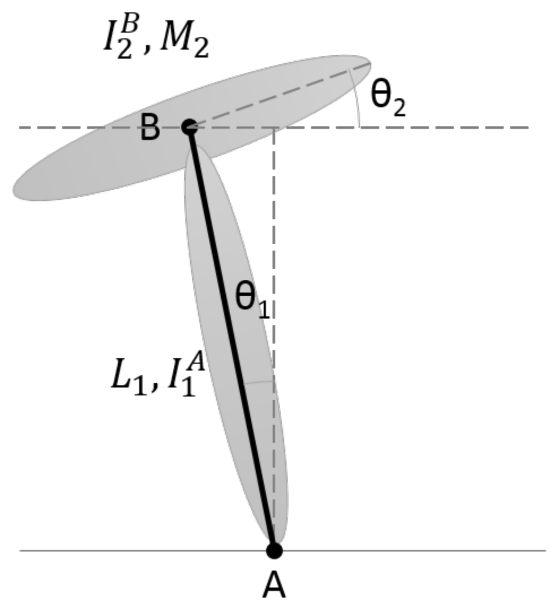

Figure 3-7: A two segment model of systems for arm twisting.

Chapter 4

Designing While Considering the Mechanisms of Balance

The basic conceptual analysis of balance consists of simply analysing how users will use the mechanisms of balance and the relevant factors in the design. Each design can differently weigh a variety of priorities, but by actively thinking about the mechanisms of balance, a designer can make intelligent quick design decisions that will benefit users. This chapter includes a couple of examples that demonstrate this quick analysis of balance with occasional commentary on possible improvements and general advice.

4.1 Basic Scooter

A scooter could be described as a bike without a seat, chain, and pedals. A scooter requires propulsion by pushing the ground backwards. A popular scooter at the time of this writing is the Razor A5 Lux which is shown in Figure 4-1. This section conceptually analyzes how humans balance on the Razor A5 Lux.

First, the scooter has only two points of contact corresponding to the front and back wheels. Since there are only two points of contact, the center of pressure (ignoring effects of wind resistance) is always going to be along a line extending between the front and back wheels.

Multiple points of contact means that there can be nearly instantaneous coun-

Figure 4-1: The Razor A5 Lux scooter (image from www.amazon.com)[6]

tersteering, at least forward and backward. Although someone might not usually consider falling forward and backward on a scooter, the scooter's handles help to quickly transfer forces forward and backward (say in the event of braking).

Of course the main challenge with balancing on a scooter is falling left and right. The scooter exhibits a number of interesting properties that enable one to maintain balance.

The wheels (although here larger than most scooter wheels to more easy transverse uneven terrain) are much smaller than bike wheels. This means that the front wheels have a very small rotational moment of inertia about the vertical axis. Additionally, the relative hardness of the wheels means that the two points of contact with the ground are highly concentrated, decreasing the torque needed to rotate the front wheel about the vertical. Bike tires and car tires have a larger area of contact and thus require more torque to overcome friction. The closeness of the handlebars provides less torque about the front wheel, but it also means that the rotational inertia of the hands about the steering axis is minimized. The net effect is that steering the front wheel left and right on this scooter is relatively easier and faster than on a bike.

Steering left and right easier has a number of effects; one of which is that it is easy to countersteer on a scooter. This is one reason why the scooter should be easier than a bike to ride at slow speeds.

The fact that the back wheel is fixed to not rotate about the vertical (although seemingly trivial) helps to provide horizontal forces. The rider's feet usually are towards the back of the foot plate and so the back wheel is intrinsic to the second mechanism of balance.

The second mechanism of balance is highly useful in the case of a scooter. The rider's hands are almost always touching the scooter handles, so there is little arm twisting. However, there can be much body twisting and even leg extension. The scooter is designed such that the center of mass of the scooter is exceptionally low and the rotational inertia of the scooter with respect to the ground is exceptionally low compared to a bike. The result is that the second mechanism of balance can be extremely effective.

Since the handlebars are close together twisting at the waist level is not significantly generated by the arms. For comparison, on a bicycle (especially a mountain bike with the handlebars spread out) the upper body can push one handlebar and pull the other to help achieve twisting at the waist level. For mountain bikes, this is especially important because the rider is often standing and would otherwise have little control over twisting the bike to utilize the second mechanism of balance.

The Razor A5 Lux noticeably has a short and narrow footplate. The footplate is short and narrow in the sense that both feet cannot fully fit on the footplate either aligned one after the other or beside each other. The rider instead generally rotates their feet on a diagonal. Having the rider stand on a diagonal to the direction of motion is quite non-trivial for balancing. Getting used to riding in this position takes time.

For an experienced rider, the significance of this small change in riding orientation can be shown by trying to ride twisted the other way with the opposite foot forward. The effect of having dominant and nondominant sides can play a huge role in one's ability to balance.

Furthermore, the scooter is designed such that the rider can gain forward propulsion by pushing backwards on the ground. Pushing backwards on the ground means that the rider has to shift their center of mass out from over the scooter or shift the scooter out from under their center of mass. Propulsion thus requires temporarily practically "losing one's balance" over the center of the scooter. However, this is not unlike walking or running which is well-known to be considered a controlled sequence of falls.

Given all this analysis, what might be improved? Well, first of all, this scooter is completely functional and most intended users would not claim it is difficult to balance. However, one thing that might be noticed is the odd location of the brake. The brake is placed over the back wheel, primarily out of mechanical convenience. However, to depress the brake requires pushing the back fender backwards, which awkwardly pushes the rider forwards and pressed against the handlebars. A more natural design might place the braking mechanism closer to the front such that the pushing mechanism naturally pushes the rider backward.

In defense of the current design, a close inspection of riders might reveal a tendency to naturally push the scooter forward (and thus push themselves backward) in an emergency. If this were the case, a brake in the back might be best for the initial response of riders. In any design, the braking mechanism ideally would primarily brake the back wheel to prevent the scooter from tipping forwards.

4.2 Modified Scooter

The following is a type of scooter whose unique design for balance has led to great popularity among young children. This scooter, made by Yvolution, is shown in Figure 4-2.

The unique design steers by shifting the handlebars left and right as opposed to rotating the handlebars about the steering axis. This design is useful for children because rotating handlebars to rotate a wheel is not natural. Balancing on a bike takes time to learn, and redesigning the way in which a rider steers to be more natural gives value to this design. It should be noted that there are three wheels, not just

Figure 4-2: The Yvolution Yglider scooter (image from www.yvolution.com)[7]

two. Since there are three points of contact, the user can perform nearly instant countersteering not only forward and backward, but also left and right. This makes it possible for the rider to stand stably on the scooter without even moving forward.

This scooter design illustrates how new designs might be generated by observing and analysing how users balance. The toolbox of recognizing the mechanisms of balance could help engineers arrive at designs such as this one.

4.3 Scootering Backward

Scootering backward illustrates how a simple design change can affect the ability of a user to balance. Scootering backward is definitely more difficult to learn than scootering forward. The reason why can be explained using the two main mechanisms of balance. The second main mechanism of balance, body twisting and reshaping, is nearly unaffected, but the rider's ability to countersteer is severely affected.

Countersteering is difficult while scootering backward because the ability to intuitively change the location of the center of pressure is impaired. The path of the center of pressure (COP) is shown in Figure 4-3. When riding forward, the front steering wheel tends to directly affect the placement of the center of pressure. When the steering wheel is turned to steer to the left, the front wheel moves to the left, the back wheel gradually follows, and the center of pressure correspondingly moves to the left.

However, for scootering backward, when the steering wheel is turned to steer left, the front wheel moves to the right, and the back wheel gradually turns to the left. Thus the center of pressure first moves to the right and then comes back to move towards the left. Having the center of pressure initially move in the opposite direction than desired is a major challenge for balancing.

The engineer is encouraged to generally implement systems that promote rather than inhibit a user's ability to employ the mechanisms of balance.

4.4 Skateboard

Skateboards also provide an interesting case study. Skateboards have four wheels, so the user has the ability to quickly shift the center of pressure within the quadrilateral made by the four wheels and thus nearly instantly countersteer. The board tilts about its length, so quickly countersteering left and right may still be challenging.

A skateboard usually requires a larger turning radius as compared to a scooter or a bicycle. As such, there a significantly limited ability to countersteer beyond the width of the skateboard.

The skateboard however, leaves the rider with a great ability to use body twisting and bending to help balance in the direction perpendicular to the length of the board. Although a skateboarder practically must keep two feet on the skateboard and this prohibits leg extension, the skateboarder is free to use arm twisting and bending at the hip in order to balance.

The skateboard illustrates a frequent compromise between the use of hands/arms in the pushing mechanism and the use of hands/arms in twisting mechanism. The skateboard frees the hands, improving the twisting mechanism in exchange for not taking advantage of potential use of the hands and arms for steering as in the case of a scooter or bike. The tendency for the twisting mechanism to potentially require more physical exertion is one consideration that engineers may want to consider.

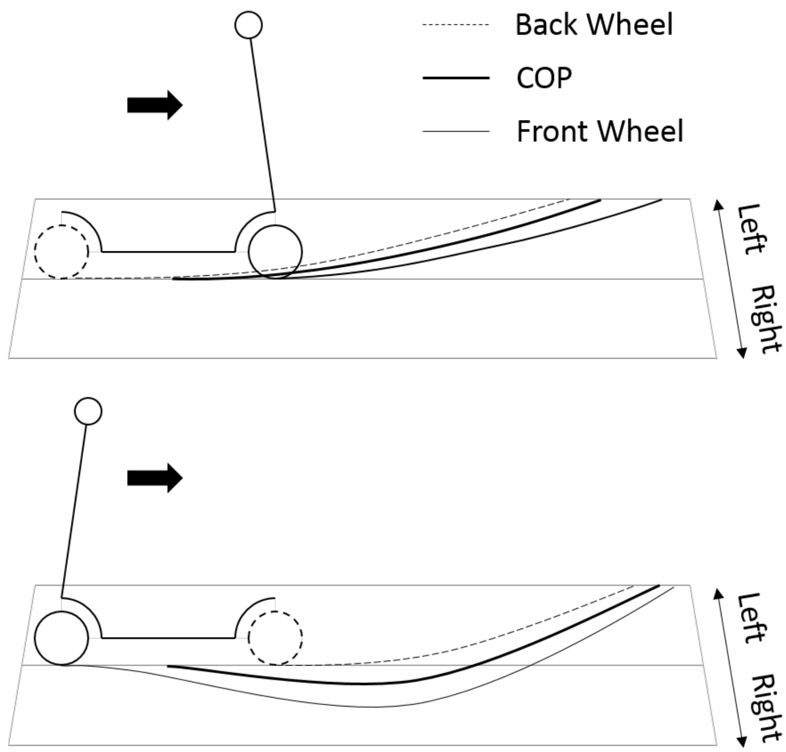

Figure 4-3: Steering left when scootering forward and backward. When riding forward (top), the center of pressure intuitively moves to the left. When riding backward (bottom), the center of pressure shifts in the opposite direction before shifting in the desired direction.

4.5 Luxury Vehicle Seats

The goal of this thesis is to enable engineers to make new designs using the analysis of the mechanisms of balance, and for illustration here is presented one last case study that presents a design derived from this balance analysis for luxury vehicle seats.

Vehicle seats are usually slightly plush and springy, but the seat has traditionally been purely passive. A quick balance analysis as shown in Figure 4-4 shows why there may be benefits for a slightly active control of vehicle seats that reacts to turns in the vehicle.

When the rider takes a turn, the center of pressure has to move towards the outside of the turn. For a passive seat, this requires the rider to tilt and effectively sink deeper into one side of their seat as shown in Figure 4-4 (b). An alternative solution that makes use of the need for countersteering is to actively pressurize or stiffen the side of the seat towards the outside of the turn. The added pressurization could be based upon the speed of the car and turn radius of the wheels. Of course, the degree of added pressurization would need to be optimized and might be adjustable by the rider. This effect could be applied to both the bottom of the seat as well as the back of the seat, especially if the seat were inclined backwards.

In addition to potentially increasing overall comfort, artificially increasing the rate of countersteering might improve the perceived response of the car without compromising the car's main suspension. This is a characteristic highly desirable in motor vehicles.

As this example shows, the mechanisms of balance offer a means to analyze balance and generate and improve designs.

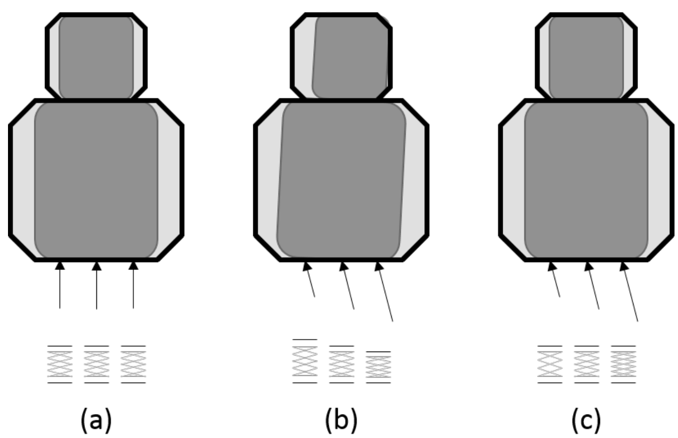

Figure 4-4: A vehicle seat design using active countersteering. These show the back view of a rider sitting on a vehicle seat, the seat's forces on the rider, and a visual representation of the springiness of the seat. (a) shows a normal vehicle seat on which sits a rider during while driving straight. (b) shows that when the vehicle makes a left turn the rider's center of pressure shifts right as the rider passively countersteers, leaning rightward and applying more pressure on the right side of the seat. (c) shows a new vehicle seat design for which the seat artificially augments the stiffness and pressure on the right side of the seat. This feature of active countersteering could prevent uncomfortable tilt and create the illusion of a more responsive vehicle.

Chapter 5

Doing More with the Center of Pressure

The mechanisms of balance presented in this thesis stem from the ability to locate the center of pressure. This chapter will present two different definitions for the center of pressure that can place the center of pressure in different places. Furthermore, it is possible to redefine the surface on which the center of pressure rests. This chapter calls the new surface a "virtual ground." Changing how the center of pressure is defined and on which surface the center pressure rests can change the interpretation of how a system balances.

5.1 Two Definitions

Conceptually, the center of pressure is the one point along the supporting surface where all outside contact forces can be concentrated to have the same dynamic effect; it's a physically equivalent situation with just one force and a point torque. For instance, Figure 5-1 shows a simple case of how the two normal forces on a bike can be combined as one force that acts at the center of pressure (in the figure drawn as a star). Additionally, Figure 5-2 shows how two forces and a point torque can be combined into one force that has the same net force and moment about the center of mass.

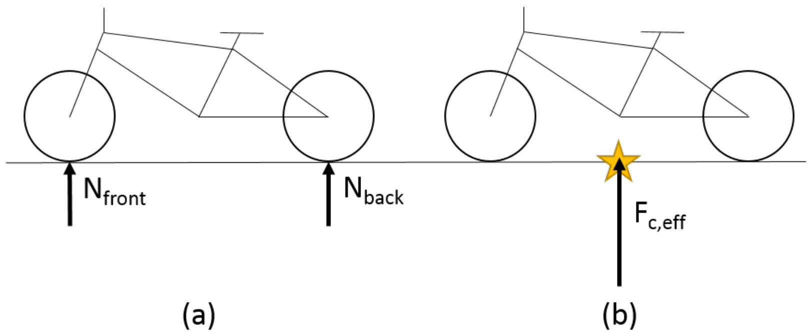

Figure 5-1: The center of pressure for a simple bike

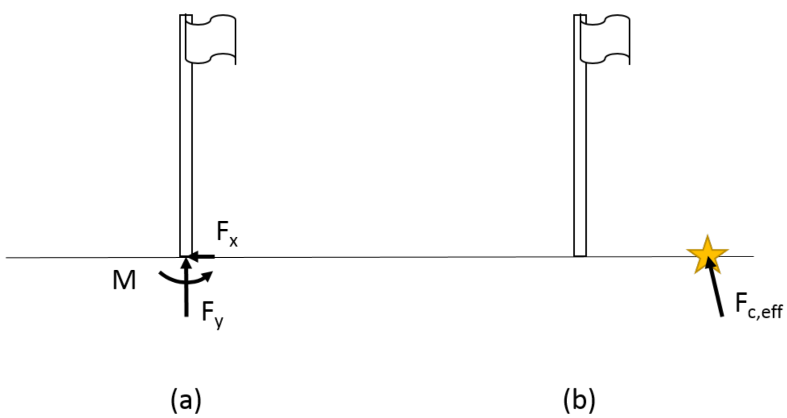

Figure 5-2: The center of pressure for a flag pole

The flag pole example in Figure 5-2 highlights that the center of pressure can be a point away from where the object physically touches the ground and not necessarily centered underneath the object.

For more complex systems, the exact positioning of the center of pressure is a little more challenging. Conceptually the center of pressure is such that:

1. the net contact force is the same:

$$\sum \vec{F}_c = \vec{F}_{c,eff} \tag{5.1}$$

2. and the net contact torque is the same:

$$\sum \vec{T}_{com} = \vec{T}_{com,eff} = \vec{r}_{com,eff} \times \vec{F}_{c,eff} + \vec{T}_{point} \tag{5.2}$$

Traditionally, the center of pressure has been defined with respect to the vertical such that it is the "projection on the ground plane of the centroid of the vertical force distribution."[8]

Choosing to define the center of pressure as the centroid of the *vertical* force distribution is intuitive in that the vertical is normal to the typically level ground. However, it is possible to define the center of pressure as the centroid of the force distribution that is *parallel with the net contact force*. The reason for this change becomes more apparent when the external forces do not lie in the same horizontal plane. Figure 5-3 shows the significance of these two meanings for the center of pressure and the change in the location of the center of pressure.

The center of pressure when defined with respect to the vertical and when all forces lie in a horizontal plane can be simplified with one net force and one torque about the vertical. Having a point torque solely about the vertical is advantageous for calculating the dynamics.

However, if the external forces do not lie in a horizontal plane, some of the elegance of the defining the center of pressure with respect to the vertical may be lost. Figure 5-4 shows three forces, all perpendicular, that cannot be represented by the one net

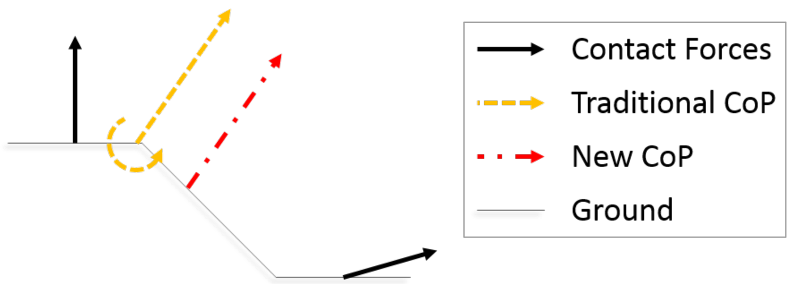

Figure 5-3: Redefining the center of pressure. The traditional definition of the center of pressure is based on just the vertical components of the forces. An alternative definition uses the component of the forces parallel to the net contact force, and this makes for a more "centered" placement of the center of pressure.

force through the traditional center of pressure with only a point torque about the vertical. Thus, there is some elegance for defining the center of pressure using the components of the forces parallel to the net contact force, which is guaranteed to only have a torque about the net contact force.

The choice of how to define the center of pressure is in one sense subjective. Since the position of the center of pressure can significantly change depending if it is defined with respect to the vertical or parallel to the net contact force, it should be explicitly mentioned in practice.

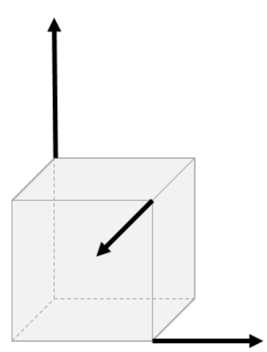

Figure 5-4: Three noncoincident forces. These three forces do not coincide and make it such that a center of pressure as defined with respect the vertical components of each force requires a torque that is not about the vertical.

5.2 The Slackline Model

The center of pressure is always located on a surface. Usually this surface is a flat, physical ground, but the surface could be redefined and a "virtual ground" could be made. To illustrate the potential usefulness of a virtual ground, the example of a slackline is presented. This section describes what a slackline is and how virtual grounds can change the analysis of balance.

A slackline is like a tightrope, but the rope is intentionally left with some slack. A visual representation of a slackline is given in Figure 5-5. The result of the rope's slack is that the feet rotate about a small arc while supporting the individual. This arc would initially appear like a curved ground that could represent a continuous range of points of contact. However, since the supporting force is perpendicular to the arc, the forces (in 2D) are coincident at one point.

It is useful to place a virtual ground at the level of this coincident point and then the system appears that it only has contact at this one point. The significance of the virtual ground and one point of contact becomes more apparent after a quick balance analysis of the system.

In the original system, without the virtual ground, it appears that the system has to exert both the pushing and the twisting force. This is shown in Figure 5-6 (a). However, by redefining the virtual ground as in Figure 5-6 (b), the horizontal component of the twisting force becomes much more significant than the horizontal component of the pushing force.

Although there is no "correct" level to place the ground, the raised virtual ground in the case of the slackline greatly simplifies the analysis. It is now arguable that a slackline is simply an offset tightrope, having only one true point of contact. The only difference between the slackline and the tightrope is that there are physical limitations that limit the ability of the human to quickly change the direction of the net contact force and utilize the second mechanism of balance.

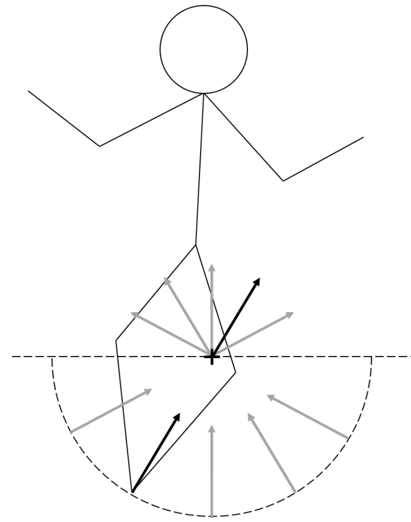

Figure 5-5: A visual of a slackline (a “tightrope” with slack). A slackline is a special case of body twisting and bending in which the point of contact moves along a semi-circle and the force of contact is pointed towards the central point. For simplicity, this can be modelled as if the point of contact is fixed at the central point.

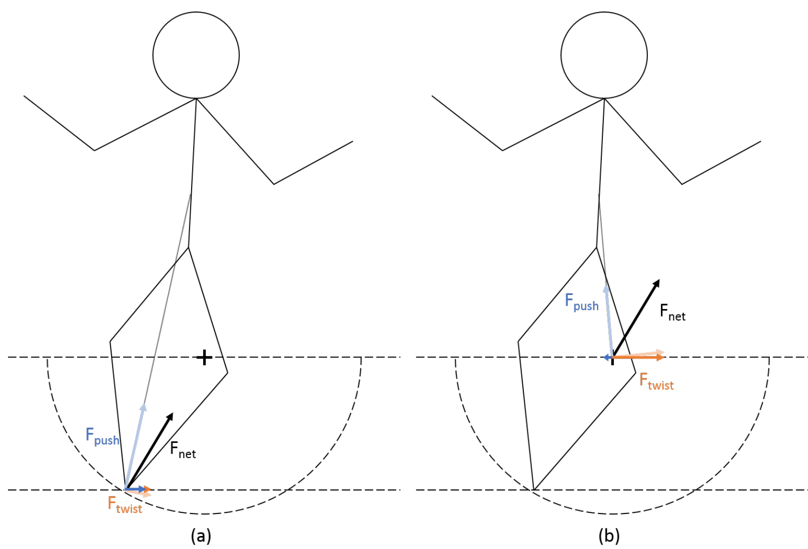

Figure 5-6: Redefining the placement of the ground can significantly affect which mechanisms of balance are considered active. (a) shows a balance analysis with the normal ground that suggests both mechanisms of balance are active. (b) shows a balance analysis with the virtual ground. In the second case, the twisting force becomes significantly more important than the pushing force.

5.3 Extending the Slackline Model

Many systems could potentially be modelled similar to the slackline model. These systems need not have an arced physical surface. Figure 5-7 is an example in which the physical ground is flat, but the forces are aligned such that they coincide at a fixed point. It is also useful in this case to define a virtual ground at the level of the coincident point.

The location of the convergent point helps to describe the system. A coincident point below the center of mass such as in Figure 5-7 or Figure 5-8 possess a degree of instability. A coincident point above the center of mass as in Figure 5-9 is inherently more stable.

Although most systems will not perfectly have a coincident point, some systems may come close, and then this balance analysis using the approximate coincident point may be relevant and useful.

For 3D systems, it may be that there is a set of axes such that the forces projected along each perpendicular vertical plane have different coincident points.

Finally, the virtual ground need not be a flat surface. It could be any shape on which the center of pressure can be placed. Virtual grounds thus greatly allow freedom in conceptual balance analysis.

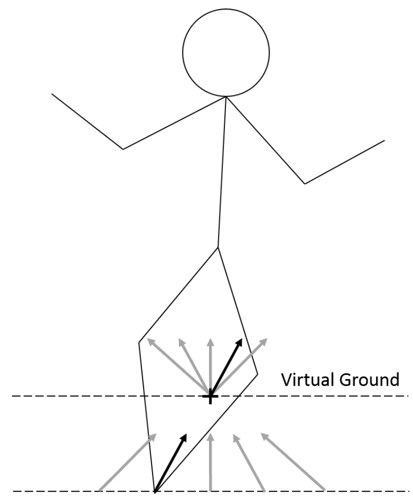

Figure 5-7: A virtual ground with a coincident point above the physical ground. The slackline analogy is useful for defining virtual grounds.

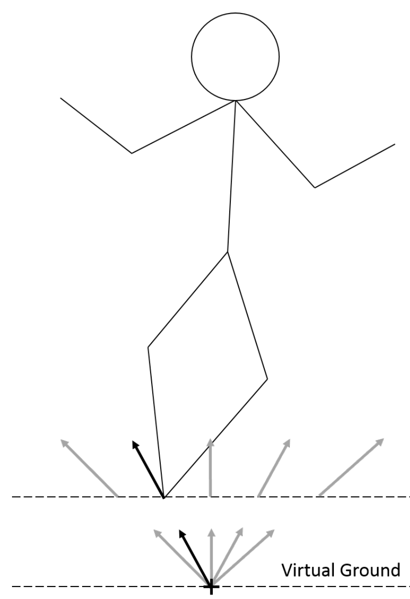

Figure 5-8: A virtual ground through a coincident point below the physical ground.

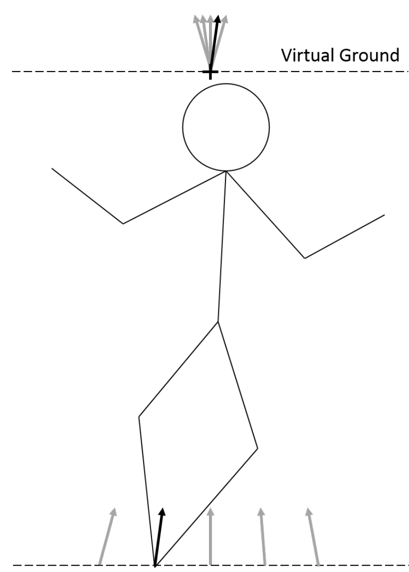

Figure 5-9: A virtual ground through a coincident point above the center of mass.

Chapter 6

Final Advice for Engineers

Understanding the concept of balance as presented here is useful for practical engineering design. Maximizing the potential to act upon each mechanism of balance can help design products better. This chapter presents a number of reasons why balance can be difficult for people, especially when first mastering a vehicle requiring balance. The following is a list of some of the many factors that may make balance challenging:

1. Sensory Input
2. Muscular Strength
3. Muscular Coordination
4. Muscle Memory, Response Time, and Cognition
5. Consistency to Past
6. Fear

These factors deserve special attention in practice. For instance, a walker or cane helps identify subtle bumps on the ground that might be difficult to see.

Additionally, humans tend to become mentally attached to the vehicles they use. For instance, if two cars collide a driver might say "he hit me" rather than "that other car hit mine." With time, the system in which humans define themselves expands to include the vehicles. There is usually an initial disconnect with the vehicle that plays a role in the cognition needed to balance.

Keeping mental models of balance consistent is very important. A bicyclist can often scooter and visa-versa because the physics is similar. Careful discretion should be used if deviating from standard balance models.

Fear could be a highly legitimate concern. Designing for when humans and systems fail to properly balance and fall may be central to the design.

Above all, designing for balance should be holistic. The mechanisms of balance are simply a framework for analysis to promote a more holistic view of balance.

Chapter 7

Conclusion

The pushing, twisting, and mass flow mechanisms give conceptual understanding to how systems balance. The pushing and twisting mechanisms can be derived from a simple force decomposition. The pushing mechanism, which is linked to countersteering, accounts for contact forces parallel to the axis that stem from the center of pressure to the center of mass. The twisting mechanism, which includes arm twisting, twisting at the hip, and leg extension, is often usually limited in direct impact, but the twisting mechanism may contribute to leaning that the pushing mechanism can act upon. By redefining the center of pressure and using virtual grounds, the engineer can sometimes simplify the analysis to become more meaningful. Ultimately, these mechanisms of balance can help engineers evaluate and improve design ideas.

Bibliography

[1] A. Hof, "The equations of motion for a standing human reveal three mechanisms for balance.," *Journal of Biomechanics*, vol. 40, no. 2, pp. 451 – 457, 2007.

[2] K. Halvorsen, "Comments on "the equations of motion for a standing human reveal three mechanisms for balance" (a. hof, vol. 40, pp. 451-457).," *Journal of Biomechanics*, vol. 43, no. 16, pp. 3244 – 3247, 2010.

[3] "The art of countersteering.," *Sport Rider*, vol. 19, no. 4, p. 92, 2011.

[4] M. H. Dickinson, C. T. Farley, R. J. Full, M. A. R. Koehl, R. Kram, and S. Lehman, "How animals move: An integrative view.," *Science*, vol. 288, no. 5463, p. 100, 2000.

[5] J. Forester, *Effective cycling / John Forester.* Cambridge, Mass. : MIT Press, c2012., 2012.

[6] Amazon.com, "Razor a5 lux scooter." `http://ecx.images-amazon.com/images/I/713jXDSRcUL._SL1500_.jpg`, Dec 2013.

[7] Yvolution.com, "Y glider original blue." `http://www.yvolution.com/img/products/thumb/carousel/Glider_Blue_Big.png`, Dec 2013.

[8] B. Benda, P. Riley, and D. Krebs, "Biomechanical relationship between center of gravity and center of pressure during standing," *Rehabilitation Engineering, IEEE Transactions on*, vol. 2, no. 1, pp. 3–10, 1994.