The Effect of Staggered Buoyancy Modules on Flow-Induced Vibration of Marine Risers

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Submitted to the Department of Mechanical Engineering
in Partial Fulfillment of the Requirements for the Degree of
Master of Science in Mechanical Engineering

at the

Massachusetts Institute of Technology

June 2003

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ABSTRACT

This paper presents results from a 1997 joint industry project arranged by MIT and Marintek. The data was taken at the Marintek towing tank in Trondheim, Norway. The primary goal of the project was to evaluate how flow-induced vibration of a marine riser is affected by placement of staggered buoyancy along the riser. The riser was subjected to both sheared and uniform flow, and was tested with various configurations of buoyancy. Cross-flow measurements of acceleration were analyzed to determine dominant response frequency and vibration amplitude. Tension and flow velocity were also measured. It is shown that a riser with 50% staggered buoyancy or greater will have response dominated by vortex shedding on the large diameter buoyancy modules. It is also found that the addition of buoyancy may decrease fatigue damage rate, provided that measures are taken to minimize increases in the ratio of mass per unit length of the riser to mean tension. It is primarily the decrease in vortex shedding frequency due to the larger diameter that accounts for any reduction in fatigue damage rate. Unfortunately, the benefits gained from the addition of buoyancy may be undone by the typical increase in mass and decrease in riser tension that occur when buoyancy is present.

Two secondary goals of this paper are to evaluate the importance of estimating in-line acceleration when computing the fatigue damage rate, and to evaluate how much the higher harmonics of the vortex shedding frequency contribute to the fatigue damage rate. To estimate the importance of in-line measurements, these measurements are used to perform vector rotation, from which is found the direction and magnitude along which the riser experiences the most damage. This damage rate is then compared to the cross flow damage rate. To evaluate how much the higher harmonics of the vortex shedding frequency contribute to the damage rate, a comparison is made between the following two calculations of the damage rate. First, the damage is calculated using frequencies up to the primary cross flow and in-line vibration frequencies. Then, it is calculated from frequencies including up to the fourth harmonic of the vortex shedding frequency. In this secondary analysis, it is found that the cross flow damage parameter is a good estimate of the maximum damage parameter in the majority of cases; however, in a substantial number of tests the rotated maximum damage parameter is much larger than the cross flow damage parameter. It is also found that in the majority of tests the higher harmonics of the vortex shedding frequency make up more than half of the total contribution to the fatigue damage rate.

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Nomenclature

A = Response amplitude Factor [m]

b = A material parameter that characterizes the slope of a fatigue SN curve
    [dimensionless]

c = Wave propagation speed [m/s]

C = A material parameter that characterizes a fatigue life plot

D = Diameter [m]

DP = Damage Parameter [Hz (m/s²)ᵇ]

Dₒ = The outer riser diameter [m]

Dᵣ = Damage rate [1/(yrs to fail)]

E = The modulus of elasticity [Pa]

f = Frequency [Hz]

fᵣ = The natural frequency of the nth mode [Hz]

fᵥ = Vortex shedding frequency [Hz]

I = The area moment of inertia of the riser’s cross section [m⁴]

k = Wave number [1/m]

L = Length of the riser [m]

m = Mass per unit length [kg/m]

M = The bending moment [N x m]

n = An integer representing the mode number [-]

N = Number of cycles to failure [cycles]

P = A parameter used to characterize whether a beam’s vibration is tension or
    stiffness dominated [-]

PSD = The power spectrum, or power spectral density [(power like quantity)/Hz]

S = The stress range [Pa]

Sₘₙ = Shear parameter; it characterizes severity of shear in a current profile [-]

Sₛₘₛ = The root mean square stress [Pa]

Sᵣ = Strouhal number [-]

t = time [s]

T = The tension in the riser [N]

Tᵣₚ = The number of seconds in a year [s]

U = Current flow speed [m/s]

Uₘₐₓ = Maximum flow speed [m/s]

Uₘₑᵃₜ = Mean flow speed [m/s]

Uₘᵢₙ = Minimum flow speed [m/s]

x = A coordinate measuring distance along the length of the riser [m]

y = The coordinate transverse to the length of the riser, and measured from
    the neutral axis [m]

jᵣₛₘₛ = The root mean square acceleration of riser [m/s²]

φ = The counter clockwise angle of rotation [deg]

Γ = The gamma function [-]

vₒ = The up crossing frequency [Hz]

ω = frequency [rad/s]

σₘₐₓ = Maximum stress due to bending [Pa]

σᵣₛₘₛ = The root mean square of the maximum stress due to bending [Pa]
Chapter 1

Explanation of the Problem and Motivation for this Paper

In the offshore oil industry there are many types of marine risers. There are *catenary risers*, which are for exporting oil and gas from floating production platforms. There are *production risers*, which bring oil and gas from wells to the production platforms. There are also *drilling risers*, which are large vertical pipes through which the drill string is run. These are deployed from several kinds of fixed and floating exploration drilling rigs. All of these structures run from the bottom of the sea floor to the ocean surface, and all can experience vibration, due to vortex shedding, as ocean currents pass over these bluff structures.

In this research, drilling risers were the principal focus. Drilling risers are often heavy and large. The structural member is a 21 inch diameter steel pipe, which may be thousands of feet in length. The pipe including its contents typically has a specific gravity in excess of 2.0. The complete drilling riser is assembled from 50 to 75 foot long ‘joints’, which necessitate multiple connection points along the length of the riser. Because of their weight, drilling risers often require the application of external buoyancy modules. These modules are cylinders made of glass microspheres and epoxy, are typically 4 feet in diameter and have weight densities around 40 lbs/ft³. The modules are clamped as a pair of half-cylinders, one pair to each joint. There is a gap about 5.0 feet in length at each riser joint where no buoyancy is applied. Sometimes the buoyancy modules are not applied to the entire riser. Only sufficient modules are applied to meet
the need for additional buoyancy, which keeps the maximum top tension on the drill ship within acceptable limits.

The response and fatigue damage that accumulates on these buoyancy covered drilling risers provide the motivation for the analysis performed in this paper. Experimental data is examined in which the VIV response is evaluated under a variety of full and partial buoyancy coverage.
SECTION I:  
RISER RESPONSE AND SUBSEQUENT DAMAGE

Chapter 2

Introduction

The effect of buoyancy distribution on riser response and fatigue damage rate is not well established. It is known that an entirely bare riser will exhibit vortex-induced vibration (VIV) at a frequency governed by the Strouhal relationship, \( f_v = S U / D \). In particular the frequency is inversely proportional to the diameter. A bare riser will vibrate at a higher frequency than a riser completely covered by buoyancy of a much larger diameter. But what frequency dominates in the case of a riser with both bare and buoyant regions? In this case design guidelines would be helpful in choosing optimum buoyancy coverage patterns. The experimental results presented in this paper provide some guidance that will prove useful in making design choices.

In 1997 a model riser was constructed and tested by J. Kim Vandiver, and Halvor Lie at Marintek, in Trondheim Norway. The model was fitted with various configurations of staggered buoyancy, and tested using the Marintek rotating rig. The rotating rig enabled testing in both uniform and linearly sheared current profiles. Measurements of riser tension, velocity, and cross-flow acceleration at multiple points on the riser, were included.

The ultimate objective in “Section I” of this paper is to understand the effect of buoyancy distribution on fatigue damage rate. In the analysis of the data, a technique is
shown which allows the measured acceleration response to be interpreted in terms of stress and, ultimately, fatigue damage rate.
Chapter 3

Background

A Brief Introduction to Natural Frequencies and Mode Shapes

Understanding the vibratory motion of physical structures such as beams, tensioned cables, or marine risers can be aided by modeling such structures as a set of discrete mass elements interconnected by spring-like elements. When the mass elements of such a structure are displaced from their initial configuration by some external source, the spring-like elements provide a restoring force to those masses. If a structure is lightly damped and not subject to an ongoing input force, the masses will vibrate with successively smaller oscillation amplitudes until the structure returns to its original configuration [1].

Any such structure comprised of mass and spring elements will have configurations at which it will naturally tend to vibrate. These configurations are called modes or mode shapes, and the frequency at which a given mode will vibrate is known as its natural frequency. The vibratory motion of the structure can be decomposed as a linear combination, or superposition, of these mode shapes. The concept of natural frequencies and mode shapes is easily illustrated by considering the system consisting of two masses and three springs shown in Fig. 1. One mode of this system is when both masses move in the same direction. The second mode of this system is when the masses move in opposite directions. As stated previously, each of these modes will have its own natural frequency of vibration.
At Rest

Both Masses Vibrate Together in Mode 1 at a frequency (Hz) $f_1$:

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Masses Vibrate Opposite to one Another in Mode 2 at a frequency (Hz) $f_2$:

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{3k}{m}}$$

Figure 1: The concept of mode shapes and natural frequencies is illustrated by this system, which consists of two masses and three springs in between two immovable boundaries.
A very important point is that if a structure is driven by a periodic input at one of its natural frequencies, a phenomenon known as resonance occurs. At resonance, large oscillation amplitudes are obtained with very little input force.

A continuous system, which can be broken into an infinite number of infinitesimal masses and springs, will have an infinite number of mode shapes, each with a corresponding natural frequency. The mode shapes for such a structure can be described by a continuous function. For example, the mode shapes of a string under constant tension are sinusoids, and the natural frequency for each successive mode shape is simply an integer multiple of the first (or fundamental) natural frequency [1].

These concepts of natural frequencies, mode shapes, continuous systems, periodic input forces, and resonance, which are described above directly apply to the flow-induced vibration of marine risers.

**Vortex Shedding**

As flow passes over a bluff body such as a cylinder, the boundary layer separates on both sides of the body creating two shear layers that bound the wake. Within a shear layer the flow that is near the surface of the cylinder moves much more slowly than the flow that is near the free stream velocity, and thus vortices are formed and interact in the wake near the cylinder [2].
Figure 2: O.M. Griffin [3] picture showing periodic vortex shedding in the wake of an oscillating cylinder.

Figure 3: Blevins' [2] plot showing Strouhal number vs. Reynolds number relationship.
The way in which the vortices are shed depends on the Reynolds number, which is a function of the free stream velocity and the cylinder diameter.

\[ \text{Re} = \frac{U \cdot D}{\nu} \]  

In the above expression, \( U \) is the free stream flow velocity, \( D \) is the cylinder diameter, and \( \nu \) is the kinematic viscosity of the fluid. In general, periodic vortex shedding occurs for Reynolds number in the range \( \text{Re} > 10^2 \). This periodic shedding of vortices is shown in Fig.2. The frequency \( (f_v) \) in Hz at which vortices are shed from one side of the cylinder is given by the Strouhal number multiplied by the cylinder diameter and divided by the free stream velocity.

\[ St = \frac{f_v \cdot D}{U} \]

This number helps to characterize the vortex shedding for a particular flow speed and cylinder diameter. For stationary cylinders it has been found to be approximately 0.21 over the large range of Reynolds number for which periodic vortex shedding occurs [4]. This is shown in Fig.3. For moving cylinders, the movement may influence the vortex shedding process, resulting in greater coherence of the vortex shedding process along the length.

The periodic shedding of vortices has two major effects on the cylinder. It causes a lift force and corresponding cross flow vibration transverse to the free stream velocity,
and it causes a drag force with a corresponding in-line vibration parallel to the free stream velocity. The cross flow vibration occurs at or near the vortex shedding frequency. Each time the cross flow vibration is near a maximum or a minimum a vortex is shed [2]. The inline vibration, however, generally occurs at twice the cross flow vibration. This is because every time a vortex is shed (whether it be shed due to either a maximum or a minimum extreme point in the cross flow vibration) the in-line pressure drag increases and contributes to the periodic in-line drag force.

When the vortex shedding frequency is near one of the natural frequencies of the riser, a phenomenon called lock-in can occur. At lock-in, resonance in the structure occurs, the shedding of vortices correlate along the length of the riser, and the riser’s cross flow vibration amplitudes can range from 0.5 to 1.0 diameters in magnitude (or 1.0 to 2.0 diameters in peak to peak amplitude) [2]. Lock-in can occur over a range of frequencies around the in-air natural frequency, due to the effects of changes in the added mass. Added mass is a hydrodynamic effect that becomes more pronounced as the ratio of the fluid density to the density of the cylinder grows. Its effects are often accounted for with a term known as the added mass coefficient (Ca), and as the name would imply the natural frequency of a structure decreases with increases in the added mass coefficient. An important point concerning added mass is that at and near lock-in Ca decreases as the flow velocity increases. This allows the natural frequency of the cylinder to increase as the flow velocity increases, which allows the cylinder to remain locked-in over a larger range of flow velocities [5].
Chapter 4
Description of the Experiment

Riser and Buoyancy Configurations

The test riser was made of commercial PVC pipe. The PVC portion of the riser was 11.340 meters long with a wall thickness of 0.0023 m. The total riser length included the attached load cell and came to 11.479 m. The mass of the bare riser was 5.3 kg, and the mass per unit length was 0.47 kg/m. Additional material properties of the PVC riser are given in [6].

Plastic buoyancy modules were positioned along the riser with either 100% coverage, 50% coverage, 25% coverage, 15% coverage, or 0% coverage, as shown in Fig.4. The length of each buoyancy module was 0.5 m, and the outer diameter was 0.05 m. For 100% coverage, the lowest module began at 0.12 m from the lower end of the riser. Subsequent modules were spaced with gaps of 0.05 m. Other staggered buoyancy configurations were achieved by removing selected modules, as shown in Fig.4. Each buoyancy module had a mass of 0.290 kg in air, and a mass of 0.869 kg when filled with water. The modules were mounted so as to not increase the effective stiffness of the riser.
Figure 4: Buoyancy Configurations.
Test Arrangement.

The experimental data were taken from the Marintek rotating rig apparatus in Trondheim, Norway. The vertical axis rotating rig was in a tank 80 m long, 10 m wide, and 10 m deep. The rotating rig is described in [6], and a sketch is shown in Fig.5. The rig held the riser at any one of the three angles shown in Fig.5. This allowed for three different current flow profiles. For a given profile, the severity of the shear is expressed by the shear parameter, $S_h$, which is the difference in maximum and minimum flow speed divided by the average flow speed along the riser.

$$S_h = \frac{U_{\text{max}} - U_{\text{min}}}{U_{\text{mean}}}$$

Thus, in these experiments the rotating rig was used to create profiles with shear parameters of 0.0, 0.61, 1.2, corresponding to uniform, mildly sheared, and steeply sheared flow. A shear parameter of 0.0 corresponds to uniform flow. The mild and steep shear parameters correspond to ratios of minimum to maximum current speeds at the top and bottom of the riser of 0.5344 and 0.2542 respectively.

The riser tension was approximately constant for each test, but was affected from test to test by the total drag force on the system. The range of tension over all tests performed was 712 N to 1226 N. After a short initial acceleration phase the apparatus rotated at constant speed for one revolution. The data that was analyzed was taken after the startup transient and only from the first rotation of the system. This avoided problems that would result from encountering the wake from previous passage of the riser model. The flow speed for each test was governed by the rotation rate of the apparatus. The
rotation rate was varied systematically from 0.045 rad/s to 0.404 rad/s, resulting in maximum flow velocities of 0.2 to 1.9 m/s.

Cross flow acceleration was measured using nine accelerometers. The positions of the accelerometers are given in Fig.6. The signals were sampled at 200 Hz, and were low pass filtered at 80 Hz.
Figure 5: Sketch of the Marintek Rotating Rig, and close up of the bottom of the riser with attached buoyancy elements.
Figure 6: The location (in meters) of each cross flow and in-line accelerometer above the lower end of the riser.
Chapter 5

The Relationships Between Acceleration, Stress and Fatigue Damage Rate

The Relationship of Damage Rate to Stress

For estimation of fatigue damage of risers, common industry practice is to use S-N curves. The fatigue curve for the riser is characterized by the equation $N S^b = C$, with $N$ being the number of cycles to failure, $S$ being the stress range, and $b$ and $C$ being parameters that best fit the data for a given material. When the time history of the stress is a constant stress range sinusoidal process, the damage rate, $D_r$, is given by:

$$D_r = \frac{v_o^* T_{yr}}{C} (S)^b$$

(4)

If the stress history is a narrow-banded Gaussian random process, then the damage rate is, as adapted from Crandall [7]:

$$D_r = \frac{v_o^* T_{yr}}{C} \left( 2\sqrt{2} S_{rms} \right)^b \Gamma \left( \frac{b + 2}{2} \right)$$

(5)

In these expressions the damage rate is in units of (years to failure)$^{-1}$. $S_{rms}$ is the root mean square stress, $T_{yr}$ is the number of seconds in a year, and $\Gamma$ is the gamma function. The parameter $v_o^*$ is the mean upcrossing frequency of the stress time history, expressed
in Hz [7]. $v_0^*$ may be evaluated from the power spectral density function of the stress, PSD(\(f\)), as shown below and described in Crandall [7]:

$$v_0^* = \sqrt{\frac{\int f^2 PSD(f) \, df}{\int PSD(f) \, df}} \tag{6}$$

In either case of sinusoidal or narrow band random processes, the damage rate depends on the stress range raised to a power of ‘b’. Typically, ‘b’ ranges between 3 to 5 for steel, depending on the quality of the welds in the structures.

In these experiments, acceleration was measured, not stress. However, when the natural frequencies and mode shapes of a riser are primarily governed by the riser mass per unit length and tension, then a linear relationship exists between measured acceleration and bending stress. In other words, the bending stiffness of the riser has little influence on the mode shapes and natural frequencies of interest. In tension dominated systems, the damage rate formulation may be expressed in terms of measured acceleration in lieu of measured stress.

**The Relationship between Stress and Acceleration**

From elementary beam bending theory, the maximum stress in a riser due to bending is:

$$\sigma_{\text{max}} = \frac{M (D_o / 2)}{I} \tag{7}$$
In Eq. (7), \( M \) is the bending moment, \( D_o \) is the outer riser diameter (without buoyancy), and \( I \) is the area moment of inertia of the riser's load-bearing cross section. The bending moment is derived from the curvature.

\[
M = E I \frac{\partial^2 y}{\partial x^2}
\]  

(8)

In the above equation, \( E \) is the modulus of elasticity, and \( y \) is the coordinate transverse to the length of the riser and measured from the neutral axis. Thus the maximum stress on the outer surface of the riser is when \( y \) equals \( D_o/2 \):

\[
\sigma_{\text{max}} = \frac{E D_o \frac{\partial^2 y}{\partial x^2}}{2}
\]  

(9)

The key assumption in relating stress to acceleration is that at any particular frequency, the riser curvature is essentially sinusoidal in shape. In the case of an infinitely long, constant property beam under tension this is true, because vibration waves traveling along the riser can be described by the function \( \sin(kx - \omega t) \). In the case of shorter risers with slowly varying tension and constant mass per unit length, the mode shapes are locally sine functions with slowly varying wave lengths due to the variation in tension.

Consider either the situation in which the local vibration is dominated by one frequency and one mode shape, or the vibration is governed by waves traveling along the riser. The vibration amplitude time history at a point, \( x \), may be described for either the mode shape, shown first, or the traveling wave by:
\[ y = A \sin(kx) \cos(\omega t) \quad \text{or} \quad A \sin(kx \pm \omega t) \quad (10) \]

Here, \( A \) is an amplitude, \( k \) is the wave number, and \( \omega \) is the frequency of vibration. For a simple sinusoidal mode shape the wave number, \( k \), is given by \( k = n\pi/L \) where \( n \) is an integer representing the mode number. The sign in front of \( \omega t \) on the right side of Eq. (10) only denotes the direction of wave propagation. If the sign is negative then the waves are traveling in the positive \('x' \) direction and a positive sign means waves traveling in the negative \('x' \) direction. For the purpose of this example, we shall let the sign be positive. Taking two spatial derivatives of Eq. (10) yields the local curvature at \( x \).

\[
\frac{\partial^2 y}{\partial x^2} = -A k^2 \sin(kx) \cos(\omega t) \quad \text{or} \quad -A k^2 \sin(kx \pm \omega t) \quad (11)
\]

The transverse acceleration at the same location is obtained by taking two time derivatives of Eq. (10).

\[
\frac{\partial^2 y}{\partial t^2} = -A \omega^2 \sin(kx) \cos(\omega t) \quad \text{or} \quad -A \omega^2 \sin(kx \pm \omega t) \quad (12)
\]

By taking the ratio of the curvature, Eq. (11), to the acceleration, Eq. (12), an expression for curvature in terms of the acceleration is obtained.
\[ \frac{\partial^2 y}{\partial x^2} = k^2 \frac{\partial^2 y}{\partial \omega^2} \]  

(13)

In general the wave number 'k' is defined as \( k = \omega / c \), where 'c' is the wave propagation speed. Therefore, \( k^2 / \omega^2 = 1 / c^2 \). Substitution of the above result into Eq. (9) and (13) yields an expression for the local maximum stress in terms of the acceleration rather than curvature:

\[ \sigma_{\text{max}} = \frac{E D_o}{2} \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2} \]  

(14)

As shown from equations 10 through 12, the above result is valid for both a riser with a sinusoidal mode shape deformation, or for a riser with waves traveling along its length. The above expression does not require that the riser’s dynamic properties be dominated by tension, rather than bending rigidity. It requires only that the deflection shape of the riser be sinusoidal. It is especially useful to examine the tension dominated case, because in that scenario the stress is linearly proportional to acceleration, as shown below. It will be shown that in tension dominated cases the term \( 1 / c^2 \) is a constant, and the curvature is linearly proportional to acceleration. In cases with non-negligible bending rigidity, the ratio is frequency dependent.

The equation for the natural frequencies of a constant tension beam with pinned ends is given by Eq. (15). There are two terms, one related to the tension and the other to the riser bending rigidity. When one of these two terms is much larger than the other, then the riser is dominated by the properties of that term.
In this equation, \( n \) is the mode number, \( L \) is the riser length, and \( m \) is the mass per unit length, which includes added mass when submerged. By dividing the tension term in the above expression by the stiffness term, a parameter \( P \) is derived that may be used to characterize whether or not a system is tension dominated. When \( P \) is much greater than 1, the riser is tension dominated.

\[
P = \frac{T}{EI \left( \frac{n \pi}{L} \right)^2}
\]  

(16)

Eq. (15) can be substituted into Eq. (16) to give \( P \) in terms of frequency instead of mode number.

\[
P = \frac{2}{-1 + \sqrt{1 + \frac{4 EI m}{T^2}(2\pi f)^2}}
\]  

(17)

For the experimental results under discussion in this paper, the riser was tension dominated. At the dominant vortex shedding frequency, \( P \) was greater than 30 for 83% of the tests and it was never less than 13.
For tension dominated string-like behavior, the natural frequency in Hz of the nth mode is approximated by dropping the $EI$ term in the natural frequency expression of Eq. (15).

\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{m}} = \frac{n c}{2L} \]  \hspace{1cm} (18)

This expression is valid for a submerged beam, by letting $m$, the mass per unit length, include the added mass due to hydrodynamic sources.

As stated previously, for sinusoidal mode shapes the wave number $k$ is defined as $k = \frac{n \pi}{L}$, and the natural frequency in radians per second is simply $\omega = 2 \pi f_n$. Thus, the quantity $k^2/\omega^2$, or $1/c^2$, simplifies to $m/T$, and Eq. (14) can be used to determine the stress in terms of the local acceleration and other experimentally measured quantities. In the equation below we have chosen to show the stress and acceleration as RMS quantities.

\[ \sigma_{rms} = \frac{E D_0}{2} \frac{m}{T} \dot{y}_{rms} \]  \hspace{1cm} (19)

In this experiment, the cross flow acceleration was measured at nine locations. Taking the root mean square at each of these locations, and choosing the maximum from these values, the maximum RMS stress of the riser may be estimated from experimental data using Eq. (19). The result is then used in Eq. (5) to determine the damage rate for the riser.
This expression can be rewritten for the purpose of separating the fixed quantities from those that were varied in the experiments:

\[ D_r = \frac{\nu_o^* T_{yr}}{C} \left( \sqrt{2} \frac{E D_o}{2} \frac{m}{T} \dot{y}_{\text{rms}} \right)^b \Gamma \left( \frac{b+2}{2} \right) \]  

(20)

The expression in square brackets was constant throughout the experiments and did not influence the variation in the experimental outcome from one test to the next. The terms \( \nu_o^* \), \( m/T \), and \( \dot{y}_{\text{rms}} \) did vary from one experiment to the next. Their influence must be carefully evaluated and several required assumptions must be clearly understood.
Chapter 6

Important Assumptions for Evaluation of Experimental Results

The Effect of m/T on Mode Shape

In order to use these acceleration-based expressions for stress and damage rate in the analysis of the staggered buoyancy experiments, an additional assumption has been made. It has been assumed that with staggered buoyancy patterns, the mode shapes are still sinusoidal. This is at best an approximation, which is good at very low mode numbers and which will get worse with increasing mode number. Large abrupt changes in mass per unit length associated with buoyancy modules will lead to irregularities in the mode shapes. In this analysis it was assumed that the average mass per unit length could be used. This is a significant limitation to the quantitative interpretation of the results of the experiments described here. Nonetheless, some valuable general principles can be concluded from the data, which will help designers make decisions regarding the placement of buoyancy.

These experiments involved primarily modes 1 to 7. The data analysis did not attempt to evaluate mode participation factors. There were accelerometers at nine locations, which were used to estimate cross-flow response. For any given experimental run the accelerometer with the maximum observed response was assumed to be at a location which was closest to an anti-node of the mode shape for that case. The maximum response of all the accelerometers was used in the data shown here.
Hydrodynamic Influence on $v_o^+$ and RMS Acceleration

In these experiments, it was not possible to control $m/T$. Therefore, when the buoyancy configuration was altered, it was not possible to keep the natural frequencies and mode shapes from changing, even if the flow velocity was identical between the two tests. Nonetheless, it is still desirable to attempt to draw conclusions about the effect of the buoyancy configuration on damage rate. This is possible if one assumes that the VIV phenomena, rather than the mode shape, primarily governs the values of $v_o^+$ and $\ddot{y}_{rms}$. In other words, two cases with identical flow conditions but different values of $m/T$ would likely respond at approximately the same vortex shedding frequencies, but potentially at different mode numbers. Furthermore, both responses would have approximately the same maximum RMS response amplitudes. This is because the response amplitude is primarily controlled by the limit cycle relationship between lift coefficient and amplitude. This assumption is sure to be wrong in specific cases, but when all cases are viewed in aggregate some useful conclusions may be drawn from the trends in the data by invoking this assumption. These assumptions were necessary in evaluation of this data, because the parameter $m/T$ was not controlled, and the effect on mode shape of the spatial changes in $m/T$ were not accounted for.

The parameter $m/T$ was not constant between tests for several reasons. For instance, when buoyancy was added to the riser, the average mass per unit length of the riser changed significantly. Also, the addition of buoyancy caused the current to impart a greater drag force on the riser. This caused the tension to vary for every variation in flow velocity and buoyancy configuration. In hindsight, a more controlled experiment would have held $m/T$ constant by changing the tension by the same amount as the average mass
per unit length. In practice this is quite difficult, because the tension varies with mean drag force.

E, C, and b are material parameters that were held constant throughout the experiment. Since we are primarily interested in the damage rate that a steel riser model would have incurred had it undergone identical vibration to that of the PVC model riser, the parameters E, C and b, used in the following examples, were those for steel. Thus, for the example calculations shown in this paper E = 200 GPa, C = 7.52E+40, and b = 4.32.
Chapter 7

Presentation of Data and Results

Displacement Spectrum, Excitation Frequency, and RMS Values

The cross flow accelerometer data were used to find the RMS response and the excitation frequency. An example of a spectrum calculated from one of the accelerometer signals is given in Fig.7, and its corresponding displacement spectrum is shown in Fig.8. The “dominant excitation frequency” refers to the frequency of the dominant peak found in the displacement spectrum. The displacement spectrum was obtained from the acceleration spectrum by dividing the acceleration spectrum by frequency raised to the fourth power. Integration of the displacement spectrum over a range of frequencies results in an estimate of the mean square displacement. This is a simple way to obtain an estimate of the displacement without having to integrate the acceleration twice in the time domain. However, this method has one significant limitation. At very low frequencies, dividing by frequency raised to the fourth power causes the computed displacement spectrum to “blow up”, as seen in Fig.8. This is known as low frequency noise expansion. To avoid this problem, it is common to choose a low frequency cutoff. When integrating the spectrum to obtain the mean square displacement, one includes only those frequencies above the cutoff. This requires that the person doing the integration choose a cutoff before integrating, which involves a bit of guesswork. There is a simpler alternative.
Figure 7: An example acceleration spectrum (dashed) and the accumulated area under the spectrum (solid). The Data is from the accelerometer positioned 4.558 m from the bottom of the riser. There was a 1.11 m/s uniform flow, and 25% coverage of buoyancy.

Figure 8: An example displacement spectrum (dashed) and the accumulated area under the spectrum (solid). This spectrum is calculated from the acceleration spectrum shown in Fig.3.
Also shown in figures 7 and 8 is a line that represents the area beneath the spectrum, which is simply a numerical integration. This, is just an accumulation of the values of the spectrum. One may choose to accumulate the values of the spectrum from high frequency to low frequency, which is what has been done here. As shown in figures 7 and 8, the integration line begins at the maximum frequency and moves across the spectrum to the left. Thus the value of this integration line at any particular frequency is simply the area under the spectrum for all frequencies at and above that evaluation frequency.

The magnitude of the area under the spectrum is the mean square value of the spectral variable, such as acceleration or displacement. One is able to choose the cutoff frequency after plotting the result of the entire integral from high frequency to low. It is usually quite easy to tell which spectral features are real peaks and which ones are expanded noise. Once the mean square is evaluated, the square root can be taken to yield an RMS value. In this way, the RMS acceleration and RMS displacement were evaluated.

To eliminate contributions from high frequency noise, the reverse integral included only those frequencies below 42 Hz. The cut off at 42 Hz was three times the highest recorded excitation Strouhal frequency of any test, which ensured that frequency content caused by cross-flow vibration, in-line vibration at two times the crossflow, and the third harmonic of the cross-flow vibration would be included for all tests. To eliminate low frequency blow up, the RMS displacement included all frequencies above a chosen cut off, which varied for each test depending on the frequency at which noise expansion began to be a problem.
RMS Displacement Response

Figures 9, 10, and 11 show the maximum RMS displacement response amplitude plotted against flow speed for 0%, 25%, 50%, and 100% coverage of buoyancy, and for current flow with a shear parameter of 0.00, 0.61, and 1.20. The term maximum RMS displacement response indicates that the RMS displacement response was calculated for all nine cross flow accelerometers, and then the maximum out of the nine values for a particular case was plotted in the figures. These figures demonstrate that the riser’s response amplitude remains within the plus and minus one diameter typical limit cycle range for lock-in vibration. For all three current profiles, the riser with 100% buoyancy coverage had the largest RMS displacement; the riser with 50% coverage had the second largest. The bare riser and the riser with 25% coverage had approximately the same RMS displacements, and were less than the riser with greater coverage. Also note that the largest RMS response was experienced under uniform flow conditions.

Principal Excitation Frequencies

Excitation frequency was determined from the dominant peaks in the displacement power spectrum. By picking the highest peak in the displacement power spectrum, a plot of frequency versus flow speed was constructed for all cases subjected to uniform current flow. This plot is shown in Fig.12, where it is seen that the experimentally measured excitation frequency is linearly related to the flow speed. The two lines along which the data fall correspond to vortex shedding controlled by two different diameters. The Strouhal number is defined as the diameter multiplied by the frequency of vortex
shedding divided by flow speed. Thus, the Strouhal number for each line is determined by multiplying the slope of the line by the diameter (Eq.2).
Figure 9: Uniform flow RMS displacement. Each value corresponds to the accelerometer that yielded the largest RMS displacement for the particular flow speed, and buoyancy coverage.

Figure 10: RMS Displacement for flow with shear of 0.61. Each value corresponds to the accelerometer that yielded the largest RMS displacement for the particular flow speed, and buoyancy coverage.
Figure 11: RMS Displacement for flow with shear of 1.20. Each value corresponds to the accelerometer that yielded the largest RMS displacement for the particular flow speed, and buoyancy coverage.

Figure 12: The frequency of the dominant peak in each displacement spectrum is plotted against the corresponding flow speed for uniform flow.
The line with the smaller slope corresponds to vortex shedding on the large diameter at an average Strouhal number of 0.133. The line with the steeper slope corresponds to the small diameter excitation of the riser, at a best fit Strouhal number of 0.146. Note that when the riser had coverage of 100% and 50%, the dominant excitation frequency was always due to the large diameter buoyancy. Also note, the bare riser was always excited at the higher frequency, which is associated with the smaller riser diameter, as would be expected. At most flow speeds, the riser with 25% coverage was excited primarily at the higher frequency, corresponding to small diameter excitation; however, at some of the lower flow speeds the dominant peak appeared to be due to the low frequency large diameter excitation. Therefore, we conclude that 25% coverage marks an approximate boundary between dominance of large or small diameter regions.

Fig.13 summarizes the frequencies observed for the different buoyancy configurations. In Fig.13, if the spectrum for a given test contained both small and large diameter frequency peaks, then both frequencies were noted. In the uniform flow cases, the displacement spectra 100%, 50%, and 0% coverage were dominated by one significant peak in the spectrum, whereas the spectra for tests with 25% coverage tended to have two significant peaks. When the spectra contained two notable peaks, these peaks would correspond to the Strouhal frequencies, as shown in Fig.12, for the large and small diameter regions.
Figure 13: Dominant and secondary peaks in the displacement spectrum are plotted against flow speed for uniform flow.

Figure 14: The damage rate that a steel model riser would have experienced under uniform flow conditions.
In only two cases did the riser with 50% coverage contain frequency peaks corresponding to both the large and small diameter, and in both of these cases the small diameter peak height was significantly less than the large diameter peak height. In contrast, when the riser was set up with 25% coverage, there was significant response due to vortex shedding on both the large and small diameter portions of the riser in almost every case.

From figures 12 and 13, it is obvious that a riser with at least 50% alternating buoyancy coverage will exhibit dominant vibration at the excitation frequency corresponding to the large diameter of the buoyancy. Likewise, for coverage that is less than or equal to 25%, a riser will include vibration, which occurs at a frequency due to excitation of the small riser diameter.

**Damage Rate**

The damage rate is plotted against the maximum current speed in figures 14, 15, and 16. This damage rate uses the material properties for steel, and thus it is actually the damage that would have been accumulated on a model steel riser had it undergone identical vibratory motion and had identical dimensions as the model PVC riser. As discussed in the section “Important Assumptions for evaluation of experimental results”, and shown in Eq. (21), there are many parameters to account for within the equation for damage rate. The mass per unit length, the tension, the measured acceleration, and the characteristic up-crossing frequency of the acceleration spectrum all varied between the tests. Because of this, interpretation of the effects of staggered buoyancy on flow induced vibration can not be made solely by looking at the damage rates shown in figures 14, 15, and 16.
Figure 15: The damage rate that a steel model riser would have experienced, for current profiles having a shear of 0.61.

Figure 16: The damage rate that a steel model riser would have experienced, for current profiles having a shear of 1.20.
For example, for all three values of the shear, it would appear from these plots that the worst damage rate corresponded to the riser with 25% coverage. This seems counter-intuitive. The expected result might be that the completely bare riser would have a greater damage rate, due to a larger length of the riser exposed to possible lock-in. This discrepancy requires explanation.

The reason that the measurements do not support the expected result is because $m/T$, the mass per unit length divided by the tension, was not held constant during the experiment. This parameter varied significantly during the tests. The actual value of $m/T$ has been used to evaluate the damage rate from Eq. (21) and plot the results in figures 14, 15, and 16. The data can be plotted with the effects of this parameter removed if the assumption is made that for a given flow speed and corresponding excitation frequency, the maximum response amplitude is insensitive to $m/T$. This will be true as long as there is a mode available to lock-in, and as long as the response amplitude is primarily limited by its relationship to lift coefficient [8]. Figures 17, 18, and 19 show the parameter $\nu^+_o(\bar{y}_{rms})^b$ plotted against maximum flow speed for all tests. $\nu^+_o(\bar{y}_{rms})^b$ is that part of the equation for damage rate that varied from test to test, in addition to $m/T$. The remaining terms in Eq. (21) are simply constants.
Figure 17: The effects of changing m/T are removed from Fig. 14, and the remaining variables within the damage rate equation are plotted for uniform flow conditions.

Figure 18: The effects of changing m/T are removed from Fig. 15, and the remaining variables within the damage rate equation are plotted for current profiles having a shear of 0.61.
Figure 19: The effects of changing m/T are removed from Fig.11, and the remaining variables within the damage rate equation are plotted for current profiles having a shear of 1.20.
When only the quantity $v'(j_{max})^b$ is plotted versus maximum flow speed, the results make more intuitive sense. The worst case is generally the bare riser, followed by the 25% coverage case and then by the 50% and 100% cases, which are about the same. These plots tell us that if $m/T$ had been held constant during the tests, and that if the response amplitudes and frequencies had remained the same as observed, then the bare riser, followed closely by the 25% coverage case, would have had the worst case fatigue damage rate. The 50% and 100% coverage cases would have been about the same. In particular, note that the riser with 25% coverage or less had significantly higher damage than the riser with 50% coverage or more. With these results, it is possible to consider the effects of staggered buoyancy in the context of design scenarios.
Chapter 8

Considering Design Scenarios

Fixed Amount of Buoyancy

One common design scenario is when the total amount of buoyancy to be used is prescribed. For example, the plan may call for 50% of the total joints to be deployed to have buoyancy on them. In this case, the goal would be to keep the excitation frequency of the riser as low as possible, while also keeping the ratio of m/T as low as possible. As shown in Fig. 12, the excitation frequency is caused by, and linearly increases with the flow speed. Thus, the large diameter buoyancy should be placed in the regions of highest current flow, to reduce the likelihood of this current flow exciting the small diameter. If the buoyancy is staggered, it should not allow more than 50% of the riser to be bare in the high flow regions. Also, from the damage rates shown in figures 14, 15, and 16, and from Eq. (21), the tension in the riser should be increased as much as possible to decrease the value of m/T. Since m is prescribed in this instance, then only T can be adjusted to reduce m/T.

Variable Amounts of Buoyancy

A second design scenario is the case that the designer has some discretion in the amount of buoyancy to be used on a given riser. In this case, figures 12 and 13 would indicate that at least 50% coverage should be used in the high speed regions to ensure that the ensuing vibration is caused primarily from large diameter excitation. Hence, the
acceleration amplitude and frequency will be dominated by the large diameter regions, leading to low fatigue damage rates. This will be true as long as m/T is not allowed to increase by too much.

If no buoyancy is deployed in the low speed regions, then one must be careful to not allow the bare riser in the low speed region to reinforce the excitation on the large diameter in the high speed region. This will happen when the local vortex shedding frequency in both regions is the same. This may happen when the ratio of U/D is the same in both regions.
Chapter 9

Conclusions for SECTION I

Staggered buoyancy can reduce damage rates, but it cannot be blindly applied without considering the changes it will cause in the parameter m/T, the ratio of the riser’s mass per unit length to the tension. The addition of buoyancy tends to increase the response displacement amplitude; however, it also tends to decrease the excitation frequency if enough is applied. Specifically, it has been shown that with at least 50% buoyancy coverage the excitation frequency is dominated by the low frequency vortex shedding, which occurs on the large diameter of the buoyancy. The decrease in frequency results in lower modes being excited. A riser excited at a lower mode number has substantially lower curvature and hence lower fatigue damage rate. At the expense of larger displacement response, the use of buoyancy can actually reduce damage rate, providing the value of m/T has not been allowed to become too large.
SECTION II:
CONSIDERING IN-LINE RESPONSE AND VORTEX SHEDDING FREQUENCY HARMONICS

Chapter 10

Introduction

This section of the paper explores two additional topics that are particularly relevant to anyone using a response prediction program such as Shear7 to predict fatigue damage of a marine riser. Currently, many response prediction programs make damage predictions based only on cross flow response. It is also common for such programs to estimate the input force due to VIV on a riser with single component sinusoidal functions. It is therefore desirable to use the data obtained from the Marintek rotating rig, described in “Section I” of this paper, to try to answer the following two questions. First, how important is the in-line response when computing damage rates? Second, since VIV is in general periodic but not necessarily harmonic, how important are the higher harmonics when making damage rate predictions?

To answer the preceding questions, it is desirable to use both in-line and cross flow accelerations at a particular location along the riser. These two measurements represent two data components that are orthogonal to each other. This allows a composite vector to be formed from the two measured accelerations. With this composite vector, the reference frame in which the vector is measured can be rotated. The components of the composite vector in the rotated reference frame can be separated, and the orientation at which a desired parameter is maximized can be found. The reason for
doing this is to consider the orientation at which the damage parameter is maximized, and then use this maximized damage parameter to determine the relative importance of damage contributed by either in-line or cross flow acceleration. Additionally, before rotating the reference frame, the frequency content of the power spectrum obtained from in-line and cross flow acceleration signals can be restricted, enabling the rotated maximized damage parameter due to a particular range of frequencies to be found. This will allow the importance of higher harmonics caused by VIV to be evaluated.

One limitation of the Marintek data is that there were only two working in-line accelerometers. The locations of these accelerometers are shown in Fig.6. Additionally, the biaxial measurements required to perform the vector rotation described above can only be approximated since the in-line accelerometers and the cross flow accelerometers were not at the same location along the riser. In the calculations to follow, one biaxial pair was approximated using accelerometer il2 (in-line) and accelerometer cf6 (cross flow) that are located 5.040 meters and 5.520 meters, respectively, from the bottom of the riser. Another biaxial pair was approximated using accelerometers il3 and cf11 that are located 3.208 meters and 2.003 meters, respectively, from the bottom of the riser.
Chapter 11

Analysis Techniques: Vector Rotation and Choosing Frequency Bandwidths

The Meaning of Vector Rotation, and How it was Implemented

The purpose for using vector rotation in the analysis of the Marintek data was to obtain the maximum value for the damage parameter $\nu^o(x_{rms})$ at a particular location along the riser's length. This maximum value occurred along a direction, which did not necessarily correspond to either the cross flow or in-line direction (i.e., the directions along which accelerations were measured). This section gives details of how the vector rotation was performed, so that the orientation and magnitude of the maximum damage parameter could be found.

For any given biaxial pair of accelerometers, the following two-dimensional rotation matrix can be used to rotate the reference frame in which the original measurements of in-line and cross flow acceleration were made [9].

\[
\begin{bmatrix}
il' \\
cf'
\end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} il \\
cf
\end{bmatrix}
\]

(22)

In the above expression, \( il \) and \( cf \) represent the original measured in-line and cross flow acceleration signals respectively. At each instant in time at which an acceleration sample was measured, a vector is formed whose components are \( il(t) \) and \( cf(t) \). \( \phi \) represents the
counter clockwise angle of rotation of the original reference frame along which $i_l(t)$ and $c_f(t)$ were components. The new components in the rotated reference frame are then $i_l'(t)$ and $c_f'(t)$, which give the acceleration experienced along each of the component directions in the rotated reference frame. This process is depicted in Fig.20 for one instant in time.

The accelerations in this test were rotated in 5 degree increments over 360 degrees. At each angle of rotation, parameters such as the up crossing frequency and the damage parameter were evaluated along the component directions. Fig.21 shows an example, where the damage parameter $\nu_s^*(\ddot{y}_{rm})^b$ is calculated at each of the rotation increments. From this figure, both the orientation and magnitude of the maximum damage parameter can be simply read off the graph. For this particular example, the direction along which the damage parameter was maximized occurs at approximately 85 degrees counter clockwise from the in-line direction. The magnitude of the maximized damage parameter was approximately $2.04 \times 10^5 \text{Hz}(\text{m/s})^b$. 
Figure 20: An example of vector rotation of a reference frame. Note that the magnitude and direction of the composite acceleration vector have not changed; however, its components along the rotated frame have changed from the original components.

Figure 21: A composite acceleration vector is formed from in-line and cross flow acceleration measurements. The reference frame is rotated, and the new acceleration components are used to calculate the damage parameter along the rotated axes.
Choosing a Frequency Bandwidth

The damage parameter $\nu^*_a (\dot{\gamma}_{rm})^k$ is comprised of two parameters, each of which is calculated using a particular frequency bandwidth from the acceleration power spectrum. Hence, the damage parameter itself is calculated for a particular frequency bandwidth. This frequency bandwidth must be chosen, and it may vary depending on the investigation being carried out. This section explains which frequency bandwidths were used for the analysis in Section II of this paper, and it gives reasons why those bandwidths were chosen.

Any periodic function can be written as an infinite series of sine and cosine functions, known as a Fourier Series. The frequencies of the consecutive sine and cosine functions in such a series are integer multiples, or harmonics, of a fundamental frequency. As described in the “Background” of this paper, the forcing and subsequent vibration of a marine riser due to VIV is definitely periodic. As such, the vibration of a marine riser is comprised of frequency components that are approximately integer multiples of the fundamental frequency of vortex shedding, where the fundamental frequency of vortex shedding is governed by the Strouhal relationship of Eq. (2).

Figures 22-34 show the power spectrum for accelerometers i12 and cf6 for a maximum flow speed of approximately 1 m/s for each variation in current shear and coverage. In the figures, vertical lines are drawn indicating the first four integer multiples of the vortex shedding frequency. The vortex shedding frequency is found using the relationship of Eq. (2), and as shown in Fig.12 the mean Strouhal number was experimentally found for the large and small diameters to be 0.133 and 0.146 respectively. In the case of sheared flow, the shedding frequency was calculated using
the maximum current velocity for the flow profile, and thus represents a maximum vortex
shedding frequency. Figures 22-34 show that the cross flow accelerometer cf6 often
contains the first and third harmonic of the vortex shedding frequency, and the in-line
accelerometer il2 often contains the second and fourth harmonic of the vortex shedding
frequency. For 0% coverage, the vibration is clearly governed by vortex shedding from
the small diameter, whereas for 100% coverage the vibration is due to vortex shedding
from the large diameter. For 15% and 25% coverage, both the cross flow and in-line
vibration appear to be governed by vortex shedding from the small diameter, but for 50%
coverage, a very different and interesting result is shown. For 50% coverage, the in-line
acceleration appears to be governed by vortex shedding from the small diameter, whereas
the cross flow acceleration appears to be governed by vortex shedding from the large
diameter.
Figure 22: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of the small diameter Strouhal frequency.

Figure 23: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of the small diameter Strouhal frequency.
Figure 24: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of the small diameter Strouhal frequency.

Figure 25: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of either the small diameter Strouhal frequency (solid lines) or the large diameter Strouhal frequency (dashed lines).
Figure 26: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of either the small diameter Strouhal frequency (solid lines) or the large diameter Strouhal frequency (dashed lines).

Figure 27: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of either the small diameter Strouhal frequency (solid lines) or the large diameter Strouhal frequency (dashed lines).
Figure 28: An acceleration spectrum for iL2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of either the small diameter Strouhal frequency (solid lines) or the large diameter Strouhal frequency (dashed lines).

Figure 29: An acceleration spectrum for iL2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of either the small diameter Strouhal frequency (solid lines) or the large diameter Strouhal frequency (dashed lines).
Figure 30: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of either the small diameter Strouhal frequency (solid lines) or the large diameter Strouhal frequency (dashed lines).

Figure 31: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of either the small diameter Strouhal frequency (solid lines) or the large diameter Strouhal frequency (dashed lines).
Figure 32: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of the large diameter Strouhal frequency.

Figure 33: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of the large diameter Strouhal frequency.
Figure 34: An acceleration spectrum for il2 and cf6. Shear, coverage, and maximum flow speed are given. Vertical lines are multiples of the large diameter Strouhal frequency.
These results are used to determine which frequencies should be included when calculating the maximum damage parameter for this study’s investigations into the relative importance of estimating in-line vibration, and the relative importance of higher harmonics in the flow induced vibration of marine risers. In both of these investigations it is desirable to isolate and analyze only the VIV effects. For this reason, the damage parameter is, in general, calculated using frequency content up to 4.5 times the maximum vortex shedding frequency (based on figures 22-34). This will include the first and third harmonics present in cross flow vibration, and the second and fourth harmonics present in in-line vibration. When trying to isolate the first and second harmonic of the maximum vortex shedding frequency, the damage parameter will be calculated using frequency content up to 2.5 times the maximum vortex shedding frequency. This will keep the primary peak for the cross flow vibration, which is the vortex shedding frequency, and it will keep the primary peak for the in-line vibration, which is the second harmonic of the vortex shedding frequency. However, it will exclude the 3rd and 4th harmonics of the vortex shedding frequency.

A complication arises when trying to define the maximum vortex shedding frequency for the riser with 50% buoyancy coverage. For 50% coverage the in-line vibration appears to be dominated by vortex shedding from the small diameter and the cross flow vibration appears to be dominated by vortex shedding from the large diameter. Furthermore, the rotated maximum damage parameter, described in the preceding section, is defined using both in-line and cross flow acceleration measurements. Thus, it seems prudent to use the same frequency bandwidth for in-line and cross flow
accelerometers when calculating the rotated maximum damage parameter. This rotated maximum damage parameter is used to evaluate the importance of in-line vibration, by comparing it to the damage parameter calculated from the cross flow direction. For this reason, when trying to evaluate the importance of in-line vibration, the maximum vortex shedding frequency has been defined based on the small diameter for 0%, 15%, 25%, and 50% coverage, and has been defined based on the large diameter for 100% coverage.

This definition will not work, however, when trying to evaluate how much the higher harmonics of the vortex shedding frequency contribute to the maximum damage parameter. To make this evaluation, only tests where the cross flow damage parameter was greater than the in-line damage parameter will be used. And thus, for the purpose of evaluating the importance of higher harmonics in VIV, the maximum vortex shedding frequency will be defined based on the small diameter for 0%, 15%, and 25% coverage, and it will be defined based on the large diameter for 50%, and 100% coverage.
Chapter 12

Results

Evaluating the Importance of Estimating In-Line Acceleration

As stated previously, it is common practice in industry to evaluate fatigue damage of a marine riser based only on its cross flow response. This section seeks to examine the importance of measuring the in-line response so that a rotated maximum damage parameter can be evaluated. It will be shown that the maximum damage found by vector rotation exceeds the cross flow damage parameter for a significant portion of the tests analyzed.

For the biaxial pair of accelerometers il2 and cf6, the damage parameter \( DP = v_o'(\bar{\dot{\gamma}}_{rms})^b \) was greater in the cross flow direction in 107 out of the 201 tests (53% of tests). For the biaxial pair of accelerometers il3 and cf11 the damage parameter was greater in the cross flow direction in 162 out of the 201 tests (81% of tests). The fact that in the majority of cases the calculated damage parameter in the cross flow direction was greater than the damage parameter calculated from the in-line direction lends credence to using the cross flow measurement when determining the damage accrued on a marine riser. The question still remains, however, that if a damage parameter can be calculated, by means of vector rotation, which is larger than the damage parameter of the cross flow direction, then how important is it to have in-line measurements so that this larger damage parameter can be found? To help answer this question, the following ratio was constructed:
\[
\frac{DP_{\text{max}}(\phi) - DP(\text{cross flow})}{DP(\text{cross flow})}
\]

(23)

In the above ratio, \( DP_{\text{max}}(\phi) \) is the maximum damage parameter found by rotating the cross flow reference direction by an angle of \( \phi \), and \( DP(\text{cross flow}) \) is the damage parameter calculated from the cross flow accelerometer. When this ratio is 0.1, the maximum damage parameter exceeds the cross flow damage parameter by 10% of the cross flow damage parameter’s value. If the ratio is 1.0, the maximum damage parameter exceeds the damage parameter calculated in the cross flow direction by 100% of the cross flow damage parameter’s value (i.e., it is two times larger).

Figures 35 and 36 show histograms for each biaxial accelerometer pair. Each histogram considers all 201 tests. In Fig. 35, it is shown that the ratio in Eq. 23 was less than or equal to 0.1 in 48.8% of the tests, and Fig. 36 shows that the ratio was less than or equal to 0.1 in 70.1% of the tests. The differences in the distributions shown in the histograms of Fig. 35 and Fig. 36 raise the question of how might these histograms look had they been taken at another point along the riser. This once again demonstrates the need for future experiments to be performed in which biaxial data is taken along the entire length of a riser. Still, the results formed from the two available biaxial accelerometer pairs show valuable trends from which the importance of estimating in-line acceleration can be evaluated. Fig. 37 combines the results of figures 35 and 36 and shows a histogram which includes all 402 biaxial measurements. It shows that in the majority of measurements (≈59% of the measurements), the damage parameter of the cross flow direction was a good estimation for the damage of the riser, and was at least 90% of the maximum rotated damage parameter.
Figure 35: Histogram for biaxial pair i12 and cf6. The ratio \([\frac{DP_{\max}(\phi)-DP(cf)}{DP(cf)}]\) is a measure of how close the cross flow damage parameter is to the maximum damage parameter. If the ratio is equal to \(r\) then the cross flow damage parameter is \(1/(r+1)\) of the maximum damage parameter. To fall within a bin, the ratio must be less than or equal to the bin number, but greater than the preceding bin number.

Figure 36: Histogram for biaxial pair i13 and cf11. The same ratio of Figure 36 is plotted.
Figure 37: This histogram combines the results of figures 36 and 37. The same ratio from figure 36 is plotted. Once again, to fall within a bin, the ratio must be less than or equal to the bin number, but greater than the preceding bin number.
Fig. 38 shows the ratio of the damage parameter of the in-line direction divided by the damage parameter of the cross flow direction for the 59% of cases in which the cross flow damage parameter was at least 90% of the maximum rotated damage parameter. This figure shows that for approximately 80% of the time, the damage parameter of the in-line direction was less than or equal to 50% of the cross flow damage parameter.

Fig. 39 shows the ratio of the in-line to cross flow damage parameter for the 41% of cases in which the cross flow damage parameter was less than 90% of the maximum rotated damage parameter. This figure shows that for approximately 83% of these cases the damage parameter of the in-line direction was greater than 90% of the damage parameter of the cross flow direction.

These results, from figures 38 and 39, show that when looking at only one particular biaxial measurement of the in-line and cross flow damage parameter, it may not be clear whether the rotated maximum damage parameter is significantly higher than the damage parameter calculated in the cross flow direction, unless the vector rotation is performed. It is, however, highly likely that if the in-line damage parameter is substantially greater than the cross flow damage parameter that the calculated maximum rotated damage parameter will be substantially greater than the cross flow damage parameter. It is also important to note that in the majority of tests the cross flow damage parameter was a good estimate of the maximum damage parameter. However, the fact that in 27% of tests the maximum rotated damage parameter was greater than 150% of the cross flow damage parameter certainly points to the importance of estimating a riser’s inline response so that the maximum rotated damage parameter can be calculated.
Figure 38: Histogram showing distribution of values of the ratio il/cf for each biaxial accelerometer pairs, when the maximum damage parameter was less than 10% greater than the damage parameter of the cross flow direction. For a value to fall within a bin, it must be less than or equal to the bin number but greater than the preceding bin number.

Figure 39: Histogram showing the distribution of il/cf for each biaxial accelerometer pair for which the maximum damage parameter was 110% or greater than the damage parameter of the cross flow direction. For a value to fall within a bin, it must be less than or equal to the bin number but greater than the preceding bin number.
Evaluation of the Importance of Higher Harmonics

This section seeks to answer the question of how important is it to include frequency content from the acceleration spectrum that is substantially higher than the fundamental vortex shedding frequency, when calculating the damage accrued on a marine riser. This section will demonstrate that higher harmonics of the vortex shedding frequency often appear in the acceleration spectra, and that these harmonics contribute significantly to the damage rate calculated from those spectra.

Fig.22 is a good demonstration of the fact that integer multiples of the vortex shedding frequency appear in the acceleration spectra. To determine whether these higher harmonics offer substantial contributions to the damage accrued on the marine riser tested at Marintek, the following ratio has been developed:

\[
\frac{DP_{\text{max}} (4.5 \text{ filter}) - DP_{\text{max}} (2.5 \text{ filter})}{DP_{\text{max}} (2.5 \text{ filter})}
\]

(24)

In this ratio, \(DP_{\text{max}}\) denotes that the maximum rotated damage parameter was used. The (4.5 filter) and (2.5 filter) denote that frequency content up to either 4.5 times or 2.5 times the vortex shedding frequency was used when calculating the damage parameter. When this ratio is equal to 1.0, it denotes that one half of the total damage was caused by frequencies that were above the primary cross flow and in-line vibration frequencies (i.e., half of the damage was caused by the higher harmonics). In general, if the ratio is equal to \(r\), then \(r/(r+1)\) of the total damage was caused by the higher harmonics present in the acceleration spectrum. As stated in the section “Choosing a Frequency Bandwidth”, the histograms constructed using Eq.24 in this section are comprised only of the portion of
the 201 tests in which the cross flow damage parameter was greater than the in-line
damage parameter (107 cases for biaxial pair il2 and cf6 and 162 cases for the biaxial pair
il3 and cf11). This was done, in part, based on the reasoning explained in the section
“Choosing a Frequency Bandwidth”. It was also done because the majority of tests had a
cross flow damage parameter that was greater than the in-line damage parameter as
explained in the section “Evaluating the Importance of Estimating In-Line Acceleration”.

Fig.40 shows a histogram that has been constructed from the values of the ratio
given in Eq.24 for the biaxial accelerometer pair il2 and cf6. Fig.41 shows a similar
histogram for the biaxial pair il3 and cf11. In Fig.40 it is shown that for the biaxial pair
il2 and cf6 more than half of the total damage was caused by higher harmonics in the
acceleration spectrum in 72% of the tests. Similarly from Fig.41, for the biaxial pair il3
and cf11 more than half of the total damage was caused by higher harmonics in the
acceleration spectrum in 65% of the tests. These results overwhelmingly demonstrate
that for the model riser evaluated at Marintek, the higher harmonics had a substantial
contribution to the accrued damage.
Figure 40: Histogram showing the distribution of a ratio that evaluates the contribution of higher harmonics to the damage accrued in a riser for the biaxial pair i12 and cf6. When the ratio is equal to "r" the fraction of the total damage due to the higher harmonics is r/(r+1). For a value to fall within a bin it must be less than or equal to the bin number but greater than the preceding bin number.

Figure 41: Histogram of the same ratio evaluated in Fig.41, for the biaxial pair i13 and cf11.
Chapter 13

Recommendations and Summary for SECTION II

It is important to consider that the results of this section are based on, at most, 402 approximated biaxial accelerometer measurements, which were made at only two locations along the model riser. Despite this limited amount of data, the conclusions drawn from these results are quite revealing and important, and demonstrate the need for future work to be performed in order to verify these findings. These results bring into question the common industry practice of using only cross flow vibration to estimate the damage accrued on a riser due to VIV, and suggest that it may be important to measure or estimate in-line response. These results also indicate that higher harmonics of the vortex shedding frequency contribute significantly to the damage accrued in a marine riser experiencing VIV. Specifically, section II of this paper demonstrates the need to further investigate the flow-induced vibrations of a marine riser, by taking biaxial accelerometer measurements along the entire length of a marine riser experiencing VIV.

It has been shown that the direction along which damage builds up the fastest does not necessarily correspond to the in-line or the cross flow direction. A riser experiencing VIV will have vibration characteristics in both the in-line and cross flow directions that substantially contribute to the amount of damage accrued in the riser along that direction. In-line vibration tends to vibrate with higher frequency content than cross flow vibration, and cross flow vibration tends to vibrate with larger amplitudes than in-line vibration. While the damage parameter is usually larger in the cross flow direction, it can be substantial or even larger at times in the in-line direction.
Vector rotation can be used to find the direction along which the damage parameter is maximized, but in order to do so both in-line and cross flow acceleration measurements must be made. In the majority of tests, the damage parameter of the cross flow direction was a good estimate of the maximum damage occurring in the riser. However, there were a substantial number of tests in which the maximum damage was 150% of the cross flow damage.

Finally, the acceleration spectra in these tests often contained peaks that were higher harmonics of the vortex shedding frequency. Cross flow vibration contained the fundamental vortex shedding frequency and often the third harmonic of this frequency. In-line vibration often contained a primary peak at twice the vortex shedding frequency and one at four times the vortex shedding frequency. In the majority of tests more than half of the total damage was contributed from the frequencies that were higher harmonics of the fundamental vibration frequencies. This points to the need to include such frequencies when estimating the damage that a riser will accrue. It also demonstrates that when trying to isolate VIV effects while taking experimental measurements, filter choice should be made to include at least up to the third harmonic of the vortex shedding frequency for cross flow measurements, and up to the fourth harmonic of the vortex shedding frequency for in-line measurements.
Acknowledgements

This 1997 experiment was sponsored by ARCO Exploration and Production Technology, Amoco Production Company, BP Exploration Operating Company Ltd., Exxon Production Research Company, Mobil Business Resources Corporation, Norsk Hydro a.s, and STATOIL.

The recent analysis work was sponsored by the Office of Naval Research, Ocean Engineering division, Code 321; Award number N00014-95-1-0545, and by the MIT SHEAR7 JIP project, which includes as sponsors: BP, Chevron/Texaco, Exxon/Mobil, Intec Engineering, Norsk Hydro, Petrobras, Shell, Statoil, and Technip-Coflexip.
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