Dynamic Amplification Factor for Moment Resisting Frames in Progressive Collapse

by

Renata Gomelskaya

B.E. Civil Engineering
The Cooper Union for the Advancement of Science and Art, 2013

Submitted to the Department of Civil and Environmental Engineering in Partial Fulfillment of the Requirements for the Degree of

Master of Engineering in Civil and Environmental Engineering
at the
Massachusetts Institute of Technology

June 2014

©2014 Renata Gomelskaya. All Rights Reserved.

The author hereby grants to MIT permission to reproduce and distribute publicly paper and electronic copies of this thesis document in whole or in part in any medium now known or hereafter created.

Signature of Author: ________________________________
Department of Civil and Environmental Engineering
May 9, 2014

Certified by: ____________________________________________________________________________
Pierre Ghisbain
Lecturer in Civil and Environmental Engineering
Thesis Supervisor

Certified by: ____________________________________________________________________________
Jerome J. Connor
Professor of Civil and Environmental Engineering
Thesis Co-Supervisor

Accepted by: ____________________________________________________________________________
Heidi M. Nepf
Chair, Departmental Committee for Graduate Students
Dynamic Amplification Factor for Moment Resisting Frames in Progressive Collapse

by

Renata Gomelskaya

Submitted to the Department of Civil & Environmental Engineering on May 9, 2014 in Partial Fulfillment of the Requirements for the Degree of Master of Engineering in Civil & Environmental Engineering

ABSTRACT

Progressive collapse has been a prevalent research topic since a gas explosion caused the collapse of the Ronan Point apartment building in 1968. Progressive collapse occurs when an instantaneous loss of a supporting element causes failure in adjacent members leading to a large scale collapse of the structure. The General Services Administration (GSA) and the Department of Defense (DoD) have provided design guidelines regarding progressive collapse mitigation. The guidelines provide requirements for setting up static and dynamic, linear and nonlinear analyses. In particular, a dynamic amplification factor (DAF) of 2 is recommended to account for the dynamic effects when performing a static analysis. Recent studies have determined that the GSA and the UFC guidelines provide an overly conservative dynamic amplification factor. In this thesis, various frames were analyzed using SAP2000 for all four analysis methods and the results were used to make recommendations regarding a more appropriate dynamic amplification factor. Additionally, the role of dynamic and nonlinear effects were observed and compared for the four different analysis methods.
ACKNOWLEDGEMENTS

First and foremost, I'd like to express my gratitude to my thesis advisor Dr. Pierre Ghisbain for his assistance throughout the thesis. He has provided guidance, advice and encouragement throughout this process and throughout the coursework this year. His willingness to always help his students is greatly appreciated. I would also like to thank Dr. Jerome J. Connor for his dedication and contributions to the High Performance Structures program.

Additionally, I would like to thank my parents for providing me with their support and encouragement to follow my dreams. I am forever thankful for their unconditional love and care. To my little sister, I appreciate her constant bragging about my accomplishments. Nobody has been more proud of me than her. I would also like to thank my grandmother for constantly reminding me to eat and sleep this year.

I would like to thank my beloved David for his faith that I can accomplish anything I set my mind to. He believed in me even when I didn’t believe in myself. He provided me with the strength to never give up during this challenging year, and has always been there for me, any time, day or night.

Lastly, I would like to thank my fellow MEng students. They provided me with friendships, laughs and great times. They make this program enjoyable and have left me with memories to last a lifetime.
# Table of Contents

1  **Background** ......................................................................................................................... 13

1.1 **Introduction** .................................................................................................................. 13

1.2 **Types of Building Collapse** .......................................................................................... 13

1.3 **Examples** ........................................................................................................................ 14

1.3.1 **Ronan Point** ............................................................................................................. 14

1.3.2 **Alfred P. Murrah Federal Building** ........................................................................... 15

1.3.3 **World Trade Center** ................................................................................................ 15

1.4 **Objective** ....................................................................................................................... 16

2  **Analysis Procedure** .......................................................................................................... 17

2.1 **Linear Static** .................................................................................................................. 18

2.2 **Nonlinear Static** .......................................................................................................... 18

2.3 **Linear Dynamic** ........................................................................................................... 18

2.4 **Nonlinear Dynamic** ...................................................................................................... 19

3  **Dynamic Amplification Factor** ........................................................................................ 20

4  **Implementation in SAP2000** ............................................................................................ 21

4.1 **Modeling Assumptions** ................................................................................................. 21

4.2 **Loading Conditions for Analysis** .................................................................................. 21

4.3 **Procedure for Linear Static Analysis** .......................................................................... 22

4.4 **Procedure for Nonlinear Static Analysis** .................................................................... 24

4.5 **Procedure for Linear Dynamic Analysis** .................................................................... 26

4.6 **Procedure for Nonlinear Dynamic Analysis** ............................................................... 29

5  **2D Frame Analysis Results** ............................................................................................... 31

5.1 **Introduction** .................................................................................................................. 31

5.2 **Specifications** ............................................................................................................... 31

5.3 **Modeling Removal of a Column** .................................................................................. 34

5.3.1 **Column Removal for a Soft Frame** ....................................................................... 34

5.3.2 **Column Removal for a Stiff Frame** ....................................................................... 36

5.4 **Results** .......................................................................................................................... 39

5.4.1 **2x2 Frame** .............................................................................................................. 39

5.4.2 **2x3 Frame** .............................................................................................................. 45
List of Figures

Figure 1: Partial Collapse at Ronan Point Apartment Building .................................................. 14
Figure 2: Propagation of Collapse of the World Trade Center .................................................. 15
Figure 3: Mass Source Definition (DEAD includes Self-Weight) ............................................... 23
Figure 4: Linear Static Analysis Definition .............................................................................. 23
Figure 5: Graphical Representation of FEMA 356 Plastic Hinges ............................................ 25
Figure 6: Nonlinear Static Analysis Definition .......................................................................... 26
Figure 7: Time History for Column Removal ............................................................................ 27
Figure 8: Linear Dynamic Analysis Procedure .......................................................................... 28
Figure 9: Nonlinear Dynamic Analysis Definition ..................................................................... 30
Figure 10: Dimensions of the 2D Frames ................................................................................... 32
Figure 11: Dimensions of the 2D Frames (cont.) ..................................................................... 33
Figure 12: DAF for Horizontal Deflection for Various Column Removal Times for Soft Frame35
Figure 13: DAF for Vertical Deflection for Various Column Removal Times for Soft Frame ... 35
Figure 14: DAF for Horizontal Deflection for Various Column Removal Times for Stiff Frame38
Figure 15: DAF for Vertical Deflection for Various Column Removal Time for Stiff Frame ... 38
Figure 16: Vertical Deflection at Free Node for a 2x2 Exterior Column Removal .................... 39
Figure 17: Moment in Member Above Removed Column for a 2x2 Exterior Column Removal 40
Figure 18: Plastic Hinge Comparison for 2x2 Exterior Removed Column ................................ 41
Figure 19: Vertical Deflection at Free Node for a 2x2 Interior Column Removal ....................... 42
Figure 20: Moment in Member Above Removed Column for a 2x2 Interior Column Removal 43
Figure 21: Plastic Hinge Comparison for 2x2 Interior Removed Column .................................. 44
Figure 22: Plastic Hinge Comparison for a 2x3 Exterior Column Removal ............................... 46
Figure 23: Plastic Hinge Comparison for a 2x3 Interior Column Removal ................................ 48
Figure 24: Plastic Hinge Comparison for a 2x4 Exterior Column Removal ............................... 50
Figure 25: Plastic Hinge Comparison for a 2x4 Interior Column Removal ................................ 52
Figure 26: Vertical Deflection at Free Node for a 3x2 Exterior Column Removal .................... 53
Figure 27: Moment in Member Above Removed Column for a 3x2 Exterior Column Removal 54
Figure 28: Plastic Hinge Comparison for a 3x2 Exterior Column Removal ............................... 55
Figure 29: Vertical Deflection at Free Node for a 3x2 Interior Column Removal ....................... 56
Figure 30: Moment in Member Above Removed Column for a 3x2 Interior Column Removal 57
Figure 31: Plastic Hinge Comparison for a 3x2 Interior Column Removal ............................... 58
Figure 32: Plastic Hinge Comparison for a 3x3 Exterior Column Removal ............................... 60
Figure 33: Plastic Hinge Comparison for a 3x3 Interior Column Removal ............................... 62
Figure 34: Plastic Hinge Comparison for a 3x4 Exterior Column Removal ............................... 63
Figure 35: Plastic Hinge Comparison for a 3x4 Interior Column Removal ............................... 65
Figure 36: Vertical Deflection at Free Node for a 4x2 Interior Column Removal ....................... 66
Figure 37: Vertical Deflection at Free Node for a 4x2 Exterior Column Removal ....................... 66
Figure 38: Moment in Member Above Removed Column for a 4x2 Exterior Column Removal 67
Figure 39: Plastic Hinge Comparison for 4x2 Exterior Removed Column

Figure 40: Vertical Deflection at Free Node for a 4x2 Interior Column Removal

Figure 41: Moment in Member Above Removed Column for a 4x2 Interior Column Removal

Figure 42: Plastic Hinge Comparison for 4x2 Interior Removed Column

Figure 43: Plastic Hinge Comparison for a 4x3 Exterior Column Removal

Figure 44: Plastic Hinge Comparison for a 4x3 Interior Column Removal

Figure 45: Plastic Hinge Comparison for a 4x4 Exterior Column Removal

Figure 46: Plastic Hinge Comparison for a 4x4 Interior Column Removal

Figure 47: Dynamic Amplification Factor for Various Number of Bays

Figure 48: Dynamic Amplification Factor for Various Number of Stories

Figure 49: Vertical Deflection at Free End for a 2x3 Exterior Column Removal

Figure 50: Moment in Member Above Removed Column for a 2x3 Exterior Column Removal

Figure 51: Vertical Deflection at Free Node for a 2x3 Interior Column Removal

Figure 52: Moment in Member Above Removed Column for a 2x3 Interior Column Removal

Figure 53: Vertical Deflection at Free End for a 2x4 Exterior Column Removal

Figure 54: Moment in Member Above Removed Column for a 2x4 Exterior Column Removal

Figure 55: Vertical Deflection at Free End for a 2x4 Interior Column Removal

Figure 56: Moment in Member Above Removed Column for a 2x4 Interior Column Removal

Figure 57: Vertical Deflection at Free End for a 3x3 Exterior Column Removal

Figure 58: Moment in Member Above Removed Column for a 3x3 Exterior Column Removal

Figure 59: Vertical Deflection at Free End for a 3x3 Interior Column Removal

Figure 60: Moment in Member Above Removed Column for a 3x3 Interior Column Removal

Figure 61: Vertical Deflection at Free End for a 3x4 Exterior Column Removal

Figure 62: Moment in Member Above Removed Column for a 3x4 Exterior Column Removal

Figure 63: Vertical Deflection at Free End for a 3x4 Interior Column Removal

Figure 64: Moment in Member Above Removed Column for a 3x4 Interior Column Removal

Figure 65: Vertical Deflection at Free End for a 4x3 Exterior Column Removal

Figure 66: Moment in Member Above Removed Column for a 4x3 Exterior Column Removal

Figure 67: Vertical Deflection at Free End for a 4x3 Interior Column Removal

Figure 68: Moment in Member Above Removed Column for a 4x3 Interior Column Removal

Figure 69: Vertical Deflection at Free End for a 4x4 Exterior Column Removal

Figure 70: Moment in Member Above Removed Column for a 4x4 Exterior Column Removal

Figure 71: Vertical Deflection at Free End for a 4x4 Interior Column Removal

Figure 72: Moment in Member Above Removed Column for a 4x4 Interior Column Removal
List of Tables

Table 1: DAF for Various Column Removal Durations for Soft Frame ........................................ 34
Table 2: DAF for Various Column Removal Durations for Stiff Frame ........................................ 36
Table 3: DAF for 2x2 Frame with Exterior Column Removal .................................................. 41
Table 4: DAF for 2x2 Frame with Interior Column Removal ................................................... 44
Table 5: DAF for 2x3 Frame with Exterior Column Removal .................................................. 45
Table 6: DAF for 2x3 Frame with Interior Column Removal ................................................... 47
Table 7: DAF for 2x4 Frame with Exterior Column Removal ................................................... 49
Table 8: DAF for 2x4 Frame with Interior Column Removal ................................................... 51
Table 9: DAF for 3x2 Frame with Exterior Column Removal ................................................... 55
Table 10: DAF for 3x2 Frame with Interior Column Removal ................................................ 58
Table 11: DAF for 3x3 Frame with Exterior Column Removal ................................................ 59
Table 12: DAF for 3x3 Frame with Interior Column Removal ................................................ 61
Table 13: DAF for 3x4 Frame with Exterior Column Removal ................................................ 63
Table 14: DAF for 3x4 Frame with Interior Column Removal ................................................ 64
Table 15: DAF for 4x2 Frame with Exterior Column Removal ................................................ 68
Table 16: DAF for 4x2 Frame with Interior Column Removal ................................................ 71
Table 17: DAF for 4x3 Frame with Exterior Column Removal ................................................ 72
Table 18: DAF for 4x3 Frame with Interior Column Removal ................................................ 74
Table 19: DAF for 4x4 Frame with Exterior Column Removal ................................................ 76
Table 20: DAF for 4x4 Frame with Interior Column Removal ................................................ 78
1 Background

1.1 Introduction

Progressive collapse refers to the phenomenon that occurs when a local failure in a building causes partial or full collapse of the rest of the building. ASCE 7 defines progressive collapse as “the spread of an initial local failure from element to element, eventually resulting in the collapse of an entire structure of a disproportionately large part of it” (ASCE 7-10, 2010). This phenomenon has been an issue many engineers have concerned themselves with since the collapse of the Ronan Point apartment building in 1968 where a gas explosion took out a load bearing wall causing partial collapse. Since then there have been many other instances of progressive collapse; however, it was cases of terrorism on the Alfred P. Murrah Federal Building in Oklahoma City in 1995 and the World Trade Center in New York City in 2001 that fostered intensive research on the topic.

1.2 Types of Building Collapse

The different types of progressive collapse have been described by Starossek (2009) as follows:

**Pancake Type Collapse**
This type of collapse occurs when there is an initial failure of a vertical load bearing element. This failure causes a separation of structural components which leads to an impact and failure on the floor below. This failure propagates vertically throughout the building causing a pancake type collapse.

**Zipper Type Collapse**
The zipper type collapse refers to the failure of tension elements. This is common in cable stayed or suspension bridges. The rupture of one cable causes the redistribution of forces and therefore overloads the adjacent cables. The combination of the redistribution of forces, the impulse due to sudden failure, and the concentration of static and dynamic forces in adjacent cables cause a failure prorogation in a transverse direction causing a zipper type collapse.

**Domino Type Collapse**
This type of collapse occurs when there is an initial overturning in one element where the fall of that element causes a lateral impact on the adjacent elements. The adjacent elements experience a horizontal force that is transmitted by a static and dynamic impact which causes that element to overturn as well. The failure propagates in an overturning direction.
Section-Type Collapse

When a member under bending moment or tension is cut, the internal force transmitted until the cut are redistributed into the remaining cross section. This corresponds to an increase in stress that results in a rupture further along the member.

Instability-Type Collapse

This type of failure occurs when there is a failure of a bracing or stiffening element that stabilizes load-bearing elements. As a result of this failure, the load-bearing element is no longer stable and there is a sudden failure due to small perturbations causing immediate collapse or failure propagation.

1.3 Examples

1.3.1 Ronan Point

In May 1968, in Newham, England, the Ronan Point apartment building collapsed due to disproportionate progressive collapse. A gas explosion on the 18th floor took out a load bearing precast concrete panel causing the partial collapse of a 22 story building. The loss of the column resulted in a chain reaction that caused partial collapse. The upper floors collapsing caused an impact on the floors below causing them to collapse as well as can be seen in Figure 1 below. The partial collapse of the corner bay caused disproportionate collapse of the whole building killing four people and injuring seventeen others. This collapse prompted interest in the engineering world and triggered research as well as significant changes in building codes (Pearson and Delatte 2005).

Figure 1: Partial Collapse at Ronan Point Apartment Building

Source: Pearson and Delatte (2005)
1.3.2 Alfred P. Murrah Federal Building

In April 1995, a bomb exploded near the façade of the Alfred P. Murrah Federal building in Oklahoma City, OK. The bomb damaged three perimeter columns causing the transfer girder to span three times the load it was designed for. As a result, the transfer girder failed causing the upper floors to collapse as well. More than half the floor area ended up collapsing. The damage was concentrated on the side of the building where the bomb exploded but part of the interior of the building also collapsed. The explosion killed 168 people and injured another 800 people. This attack increased the interest in progressive collapse research generating more studies to be conducted on preventing such failure (Ruparelia 2013).

1.3.3 World Trade Center

Since the September 11th terrorist attack on the World Trade Center in New York City, interest and research on progressive collapse has reached an all-time high. On September 11, 2001, the two twin towers were hit by Boeing 767 planes causing the progressive collapse of both towers. The large impact from the planes caused structural damage as well as fires on the floors that were struck. Although the towers were resilient to the impact of the planes, the fire heated the steel causing additional failures that the towers could not resist. As a result, the sections above the impact fell on the structure below as shown in Figure 2 below. This is an illustration of a pancake type collapse. The impact of the falling structure caused failure propagation all the way to the ground. The collapse of the World Trade Center caused the death of more than 3000 people, as well as extensive damage to numerous surrounding buildings (Seffen 2008).

![Figure 2: Propagation of Collapse of the World Trade Center](source: Hamburger et al. (2002))
1.4 Objective

One of the goals of this thesis is to analyze a set of frame varying in geometry and their ability to resist progressive collapse. The various geometry configurations can be found in Section 5.2. Each frame will be analyzed using four analysis methods: linear static (LS), nonlinear static (NLS), linear dynamic (LD), nonlinear dynamic (NLS). The four analysis methods are described in Chapter 2. The displacement and bending moment was observed for each of these frames for each analysis case. The maximum displacement and maximum moment were used to derive a dynamic amplification factor (DAF) for the static analysis cases. A static load was applied without a DAF and compared with the results of the NLD analysis to determine the amplification factor for these results.

Currently, a DAF of 2 is used for the static loading condition to take into account dynamic effects of the structure. According to recent studies, it has been determined that this DAF is overly conservative (Ruth et al. 2006). The frames will be analyzed using SAP2000. SAP2000 is an integrated software for structural analysis and design. This program was chosen because it was proven to be “the most integrated, productive, and practical general purpose structure program on the market today” (SAP2000 Overview). Chapter 4 describes the analysis procedure using SAP2000 for each of the analysis cases. The other goal of this thesis is to use SAP2000 to attempt to derive a more appropriate DAF to use for static analysis.
2 Analysis Procedure

The U.S. General Services Administration (GSA) issued general guidelines for evaluating a building’s progressive collapse potential. The document provides direct recommendations for the selection of the analysis procedure and evaluation of the results. According to GSA, progressive collapse analysis is accomplished by implementing an alternate path for the load. The alternate path method requires a structure “to be able to bridge over vertical load-bearing elements that are notionally removed one at a time at specific plan and elevation locations” (GSA 2000). This allows local failure to occur but provides the structure with alternate load paths by transferring loads to adjacent members.

GSA guidelines state the following loading combinations when analyzing progressive collapse:

Static Analysis: \[ P_u = 2*(DL + 0.25LL) \] (1)

Dynamic Analysis: \[ P_u = DL + 0.25LL \] (2)

where \( DL \) = dead load, which is generated by SAP2000 automatically using element properties and material density; and \( LL \) = live load, which is dependent on the occupancy of the building. A small safety factor is implemented for the live load because it is assumed that at a time of an extreme event the full live load will not be acting on the structure. The factor of 2 in front of the static equation is called the dynamic amplification factor. Since the static analysis does not take into account dynamic effects of the building, this factor is used to predict the maximum affect under static load.

There are four analysis procedures that could be carried out for progressive collapse each being more complex, regarding computational efficiency, than the previous: linear static, nonlinear static, linear dynamic, and nonlinear dynamic. The accuracy of the force and deflection predications increases with the complexity of the analysis. There are advantages and disadvantages to each analysis type that will be discussed below. Marjanishvili (2004) and Marjanishvili and Agnew (2006) provide more detail on the analysis methods.
2.1 Linear Static

Linear static (LS) analysis procedure is the simplest and quickest to run. The static load combination is applied to the structure and a dynamic amplification factor of 2 is used in order to predict the nonlinear and dynamic effects of the building. However, its accuracy is limited to relatively simple structure. GSA resists the application of the linear static approach to structures with 10 stories or less. In simple structures, the nonlinear and dynamic effects are intuitive and can be predicted. According to the GSA guidelines, the structure is evaluated based on the demand to capacity ratio (DCR). The DCR is the ratio of the maximum moment determined by the computer analysis to the maximum plastic moment inherent to the beam. If the DCR exceeds 2, the structure is susceptible to progressive collapse and should be redesigned.

2.2 Nonlinear Static

The nonlinear static (NS) procedure takes into account possible nonlinearities that could occur and allows for plastic hinges to form in the structure. The static load combination with the same dynamic amplification of 2 is applied to the structure. A stepwise increase of the load is applied to the structure until the full loads are attained or the structure collapses. Nonlinearities are monitored throughout every step of this analysis. The loads are increased until the maximum amplified load is reached or until the structure collapses. The advantage to this analysis case is its ability to take into account nonlinear effects. The disadvantage to this analysis case is its inability to predict dynamic effects. In addition, computational time can be great due to convergence issues. GSA guidelines recommend to only apply this procedure to structures where dynamic behavior can be easily predicted.

2.3 Linear Dynamic

Although dynamic analyses procedures are avoided due to their complexity, they are considered more accurate because they incorporate dynamic effects such as inertia and damping. For the linear dynamic (LD) analysis procedure, the dynamic load combination is applied to the structure and DCR’s are calculated to determine susceptibility of the structure to progressive collapse. The initial conditions are set as the deflected shape of the undamaged structure under the load combinations. However, for linear dynamic analysis the initial conditions can be
neglected because they are trivial. This analysis procedure is restricted to structures where nonlinearities can be predicted. The disadvantage of this analysis procedure is its inability to account for nonlinearities which are significant in larger structures where the nonlinear patterns are not easily predicted. The advantage of this analysis is its ability to accurately predict dynamic effects while maintaining a quick computational time.

2.4 Nonlinear Dynamic

Nonlinear dynamic (NLD) analysis procedure is the most complex and reliable method. This analysis case takes into account dynamic effects while considering nonlinear effect of the elements. The NLD analysis is the most thorough method because it models the dynamic removal of the column and analyzes the time history of the structural response and allows the structure to enter the inelastic range. The advantage of this method is that it takes into account both the dynamic effects as well as the material nonlinearities. This provides the user with more accurate information regarding larger deformations, energy dissipation through material yielding, cracking, and fracture. The disadvantage is the complexity of the method. The evaluation and verification of the results is a complicated process and may involve several computer analysis reruns until a stable solution is found. Evaluation and validation may be very time consuming and may require peer review requirements set by building codes (IBC 2003). This process is very time consuming and the computer analysis task requires a long analysis time.
3 Dynamic Amplification Factor

The static load combination includes a dynamic amplification factor that is meant to account for the dynamic effect. For linear and nonlinear static analyses, the current codes require the same amplification factor of 2 to be applied to all static progressive collapse load combinations. Using 2 for the DAF stems from the fact that the amplification factor really is 2 when a load is suddenly applied to a linear single degree-of-freedom system. There are some issues with this amplification factor for progressive collapse. The same factor is used for both the linear static and nonlinear static analyses. The LS model has to be amplified to take into account both dynamic and nonlinear effects. The NLS model takes into account the nonlinear behavior and therefore would only need to be amplified for dynamic effects. However, the same dynamic amplification factor is used for both procedures.

Since the LS method is advised to be used only for small scale structures where the nonlinear and dynamic effects are easily predicted, it is possible to use a dynamic amplification factor less than 2 (Ruth et al. 2006). An amplification factor of 2 is almost certainly conservative and can result in overdesigned structures. When LS analysis is used, the allowable load is essentially that at the first hinge formation. When using NLS analysis, plastic hinges are allowed to form and therefore the allowable load is the collapse load. Therefore, NLS allows larger loads and is more conservative than linear analysis (Marjanishvili 2004). However, the same dynamic amplification factor of 2 is used. Once again, this can result in overdesigned structures.

It is desirable from an economical standpoint to not over-design structures. As a result, it is prevalent to determine a dynamic amplification factor to be applied to static analyses for progressive collapse. The purpose of this thesis is to look at small scale structures to determine whether the dynamic amplification factor used in the LS and NLS analysis procedures is conservative and whether it can be reduced.
4 Implementation in SAP2000

SAP2000 was used to model 2D frames for several combinations of number of spans and stories. For each frame, two scenarios were looked at: a removal of an external column and a removal of an internal column. Linear static, nonlinear static, linear dynamic and nonlinear dynamic analyses procedures were implemented to predict the response of each frame to the removed column. The LS, NLS, and LD analysis cases were compared to the NLD analysis in order to determine a DAF.

4.1 Modeling Assumptions

When the frames were modeled, several assumptions were made in order to have consistent models and in order to simplify the analysis. The 2D frames considered are all moment connected. The connections to the foundation were modeled as fixed connections as well. Default hinge properties provided by SAP2000 which correspond to the hinge definition in FEMA 356 were used for nonlinear analyses. The damping ratio was assumed to be 1% for all dynamic analyses. An analysis time step of 0.001 seconds was used in order to ensure the dynamic effect was fully captured. A column removal time of 0.05 seconds was used as determined by Section 5.3. The Newmark time integration method was chosen with common values of 0.5 for gamma and 0.25 for beta. The Newmark integration method provides generally good results and convergence times (McKay, Gomez, Marchand 2009).

4.2 Loading Conditions for Analysis

The analysis was performed using the load combinations mentioned in Equations 1 and 2. The self-weight of the beams and columns were generated automatically based on element volume and material density by SAP2000. To estimate the dead load, a uniform concrete slab thickness of 4” was assumed with normal weight concrete density of 150 lb/ft$^3$. Since a 2D frame was analyzed, a 10’ tributary area was assumed and therefore, the concrete produced a dead load of 0.5 k/ft on each beam. The dead load of the concrete was included as a mass source in SAP2000. A gravity acceleration of 32.2 feet/s$^2$ was applied to the building. This provided the
ability to use the concrete as a mass source for dynamic effects while still applying the dead load of the concrete to the frame. The nature of the building is unspecified; therefore the maximum service live load of 100 psf was included. This produced a live load of 0.1 k/ft on each beam.

4.3 Procedure for Linear Static Analysis

The analysis was run using the linear static option in SAP2000. The displacements and moments were determined for the model. The analysis procedure involves the following steps:

1. Build a 2D or 3D model in SAP2000 computer program with the column removed.
2. Define the mass source to include the specified dead load pattern with a multiplier of 1 as shown in Figure 3 below.
3. Define a load case with a linear analysis type case and a static load case type.
4. Apply the static load combination without the dynamic amplification factor for the purpose of this thesis.
5. Apply the dead load in the form of acceleration in the vertical direction as can be seen in Figure 4 below. This is done to convert the mass to a force.
7. Evaluate the results. Typically, the results are evaluated based on DCR values. For the purpose of this thesis, displacement and moment values will be evaluated.
Figure 3: Mass Source Definition (DEAD includes Self-Weight)

Figure 4: Linear Static Analysis Definition
4.4 Procedure for Nonlinear Static Analysis

The analysis procedure involves the following steps:

1. Build a 2D or 3D model in SAP2000 with the column removed.
2. Define the mass source to include the specified dead load pattern with a multiplier of 1 as shown previously.
3. Define and assign nonlinear plastic hinges to ends and midspan of the beams and columns. Figure 5 below shows the graphical representation of the SAP2000 default plastic hinge definition which corresponds to the hinge definition in FEMA 356. Point A represents the origin, Point B represents yielding, Point C corresponds to plastic moment capacity, Point D indicates failure, and Point E is the residual strength.
4. Define a load case with a nonlinear analysis type case and a static load case type.
5. Apply the static load combination without the dynamic amplification factor for the purpose of this thesis. The load combination includes live load and gravity acceleration. The nonlinear static analysis definition can be seen in Figure 6.
6. Perform the nonlinear static analysis.
7. Verify and evaluate the results. Once again, the deflections and moments will be used to compare results for the purpose of this thesis.
Figure 5: Graphical Representation of FEMA 356 Plastic Hinges
4.5 Procedure for Linear Dynamic Analysis

This procedure includes the following analysis steps:

1. Build a full 2D or 3D model with column in place.
2. Perform a linear static analysis without dynamic amplification factors. Determine static, unamplified loads in the removed column.
3. Build a 2D or 3D model with removed column. Apply equivalent reaction forces to the joint where the column is removed. This models the presence of the column even though it has been removed.
4. Define the mass source as mentioned in the previous analysis cases.
5. Define a load case with a nonlinear analysis type case and a time history load case type and a direct integration solution type.

6. Uniform time histories are used to apply live load, gravity, and equivalent reaction forces to the structure.

7. Using the time history defined in Figure 7, the effect of the column is removed over time. The linear dynamic analysis definition is shown in Figure 8.

8. Evaluate the results. Typically, the results are evaluated based on DCR values of the peak value of the time history response. For the purpose of this thesis, displacement and moment plots will be evaluated.

Figure 7: Time History for Column Removal
Figure 8: Linear Dynamic Analysis Procedure
4.6 Procedure for Nonlinear Dynamic Analysis

The analysis requires the following steps:

1. Build a full 2D or 3D model with column in place.
2. Perform a linear static analysis without dynamic amplification factors. Determine static, unamplified loads in the removed column.
3. Build a 2D or 3D model with removed column. Apply equivalent reaction forces to the joint where the column is removed. This models the presence of the column even though it has been removed.
4. Define the mass source as mentioned in the previous analysis cases.
5. Define a nonlinear static analysis case applying the live load, gravity, and equivalent reaction forces. This is done to determine the deflected shape of the loaded, undamaged column. This will be used as the initial conditions for the nonlinear dynamic case.
6. Perform a nonlinear time history analysis using the initial conditions determined in the nonlinear static analysis. Only apply the time history function defined in Figure 7 above to remove the column. The nonlinear dynamic analysis definition can be seen in Figure 9 below.
Figure 9: Nonlinear Dynamic Analysis Definition
5 2D Frame Analysis Results

5.1 Introduction
Analyses of 2D frames have been performed for several combinations of bays and stories. The frames range from 2x2 to 4x4 spans. Each frame was analyzed using all of the analysis procedures discussed: linear static, nonlinear static, linear dynamic, nonlinear dynamic. All four analysis procedures were used to compare dynamic amplification factors to determine if a factor less than 2 can be used. For each frame span, two scenarios were considered. One model analyzed the removal of an exterior column and the other model analyzed the removal of an interior column. Vertical deflection at the free joint was observed as well as the moment in the beam directly above the collapse. These design criterions were used to compare the four analysis types to determine a dynamic amplification factor. Dynamic analyses time were varied depending on how long the frame took to stabilize but typically varied between 5 seconds to 20 seconds.

5.2 Specifications
The 9 frames shown in Figure 10 and Figure 11 were used for the analysis cases. The dimensions can be seen in the figures below. Each beam had a cross section of W12x26 and each column had a cross section of W8x18. The width of each span was 24’ and the height of each story was 12’. The structural steel used had a modulus of elasticity of 29,000 ksi and yield stress of 50 ksi. The plastic hinges defined in Figure 5 were assigned to every beam and columns at the ends and midspan. Using these specifications, each analysis was run and post processed and can be found in Section 0 below.
Figure 10: Dimensions of the 2D Frames
Figure 11: Dimensions of the 2D Frames (cont.)
5.3 Modeling Removal of a Column

An elastic static analysis was run with factored loads to determine the axial, shear, and moment in the column to be removed. The removal of the column is represented by deleting the column from the model and applying equivalent reaction forces determined from the static analysis. A time history function is defined in order to represent the presence of the column and its removal over time. Two separate tests were run, where the beam and column sizes are varied, to determine the appropriate duration over which the column is removed. A 3x3 frame was used with the analysis output time step.

5.3.1 Column Removal for a Soft Frame

The first model represented a soft frame and had W12x26 beams and W8x18 columns. The duration of the removal was varied from 0s to 4s in increments for both tests and the results for DAF were observed and can be seen in Table 1 below.

<table>
<thead>
<tr>
<th>Duration for Column Removal (s)</th>
<th>U1 (ft)</th>
<th>U3 (ft)</th>
<th>R2</th>
<th>V (k)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.273</td>
<td>1.834</td>
<td>1.963</td>
<td>0.968</td>
<td>1.451</td>
</tr>
<tr>
<td>0.0001</td>
<td>2.274</td>
<td>1.835</td>
<td>1.971</td>
<td>0.975</td>
<td>1.459</td>
</tr>
<tr>
<td>0.0005</td>
<td>2.274</td>
<td>1.835</td>
<td>1.969</td>
<td>0.974</td>
<td>1.458</td>
</tr>
<tr>
<td>0.001</td>
<td>2.274</td>
<td>1.835</td>
<td>1.968</td>
<td>0.974</td>
<td>1.458</td>
</tr>
<tr>
<td>0.005</td>
<td>2.275</td>
<td>1.838</td>
<td>1.969</td>
<td>0.971</td>
<td>1.452</td>
</tr>
<tr>
<td>0.01</td>
<td>2.273</td>
<td>1.834</td>
<td>1.963</td>
<td>0.968</td>
<td>1.451</td>
</tr>
<tr>
<td>0.05</td>
<td>2.237</td>
<td>1.778</td>
<td>1.905</td>
<td>0.927</td>
<td>1.392</td>
</tr>
<tr>
<td>0.1</td>
<td>2.138</td>
<td>1.607</td>
<td>1.725</td>
<td>0.839</td>
<td>1.253</td>
</tr>
<tr>
<td>0.5</td>
<td>1.344</td>
<td>1.193</td>
<td>1.229</td>
<td>0.629</td>
<td>0.934</td>
</tr>
<tr>
<td>1</td>
<td>1.230</td>
<td>1.133</td>
<td>1.155</td>
<td>0.596</td>
<td>0.885</td>
</tr>
<tr>
<td>2</td>
<td>1.140</td>
<td>1.097</td>
<td>1.113</td>
<td>0.583</td>
<td>0.864</td>
</tr>
<tr>
<td>3</td>
<td>1.146</td>
<td>1.082</td>
<td>1.097</td>
<td>0.572</td>
<td>0.847</td>
</tr>
<tr>
<td>4</td>
<td>1.111</td>
<td>1.082</td>
<td>1.096</td>
<td>0.573</td>
<td>0.849</td>
</tr>
</tbody>
</table>
When the column removal duration is less than analysis output time step, the DAF is greater than the case of the instantaneously removed column. In this case, looking at the DAF between $t=0s$ and $t=0.01s$, all the amplification factors are greater than those at $t=0s$ and $t=0.01s$ which have the same amplification factor. Since the output time step is greater than the time in which the column is removed, the analysis is not capturing the full dynamic effects of the removal. Therefore, it is necessary for the duration of the removal to be greater than the output time step of the analysis.

---

**Figure 12: DAF for Horizontal Deflection for Various Column Removal Times for Soft Frame**

**Figure 13: DAF for Vertical Deflection for Various Column Removal Times for Soft Frame**
As can be seen in Figure 12 and Figure 13, the dynamic amplification factor for horizontal and vertical displacements were plotted versus column removal duration. For a small duration for the removal, the dynamic amplification factor stays essentially the same. This means that a removal with a short duration is almost identical to removing the column instantaneously. As the removal duration increases, the DAF tends to 1. This is to be expected because when a column is very slowly removed, the structures deflection will not be amplified by any dynamic effect.

5.3.2 Column Removal for a Stiff Frame

The stiffness of the frame was changed to determine whether the effect of the column removal time is dependent on the structure’s properties. A 3x3 frame was used again with the same loading and the same analysis time step. The second model represented a stiff frame and had W18x86 beams and W12x50 columns. The duration of the removal was varied again from 0s to 4s in increments for both tests and the results for DAF were observed and can be seen in Table 2 below.

Table 2: DAF for Various Column Removal Durations for Stiff Frame

<table>
<thead>
<tr>
<th>Duration for Column Removal (s)</th>
<th>U1 (ft)</th>
<th>U3 (ft)</th>
<th>R2</th>
<th>V (k)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.292</td>
<td>1.731</td>
<td>1.881</td>
<td>0.951</td>
<td>1.461</td>
</tr>
<tr>
<td>0.0001</td>
<td>2.297</td>
<td>1.737</td>
<td>1.889</td>
<td>0.953</td>
<td>1.462</td>
</tr>
<tr>
<td>0.0005</td>
<td>2.298</td>
<td>1.737</td>
<td>1.889</td>
<td>0.953</td>
<td>1.462</td>
</tr>
<tr>
<td>0.001</td>
<td>2.298</td>
<td>1.737</td>
<td>1.886</td>
<td>0.953</td>
<td>1.462</td>
</tr>
<tr>
<td>0.005</td>
<td>2.292</td>
<td>1.731</td>
<td>1.881</td>
<td>0.951</td>
<td>1.461</td>
</tr>
<tr>
<td>0.01</td>
<td>2.289</td>
<td>1.729</td>
<td>1.881</td>
<td>0.950</td>
<td>1.458</td>
</tr>
<tr>
<td>0.05</td>
<td>2.158</td>
<td>1.585</td>
<td>1.711</td>
<td>0.847</td>
<td>1.292</td>
</tr>
<tr>
<td>0.1</td>
<td>1.810</td>
<td>1.255</td>
<td>1.328</td>
<td>0.613</td>
<td>0.931</td>
</tr>
<tr>
<td>0.5</td>
<td>1.313</td>
<td>1.186</td>
<td>1.225</td>
<td>0.612</td>
<td>0.928</td>
</tr>
<tr>
<td>1</td>
<td>1.295</td>
<td>1.102</td>
<td>1.127</td>
<td>0.586</td>
<td>0.880</td>
</tr>
<tr>
<td>2</td>
<td>1.182</td>
<td>1.086</td>
<td>1.106</td>
<td>0.571</td>
<td>0.861</td>
</tr>
<tr>
<td>3</td>
<td>1.119</td>
<td>1.046</td>
<td>1.057</td>
<td>0.553</td>
<td>0.831</td>
</tr>
<tr>
<td>4</td>
<td>1.095</td>
<td>1.032</td>
<td>1.041</td>
<td>0.547</td>
<td>0.820</td>
</tr>
</tbody>
</table>
For a duration for the column removal near the time for the analysis output time step, the DAF remains about the same meaning that the column is essentially instantaneously removed. As shown in Figure 14 and Figure 15 below, the DAF tends to 1 when the column removal time is slow. This shows that for both frames, a stiff or a soft frame, the effect of the column removal is the same. Therefore, it would be beneficially to know exactly how the column is removed. It is crucial not to overestimate the time for the removal of the column because an overestimation of the time can cause the structure to be under designed. If the duration of the removal of a column is overestimated, the DAF can be underestimated since it tends towards 1, which can lead to under designed structures. According to McKay, Gomez, and Marchand (2009), the column removal time should be modeled as 1/20 of the structures natural period. Using a sample 2x3 frame to test this recommendation, the natural period of the frame is 1.03 seconds. Using 1/20 of the structures natural period corresponds with a removal time of 0.05 seconds. Therefore, a column removal duration of 0.05s will be used for the remainder of the analyses.
Figure 14: DAF for Horizontal Deflection for Various Column Removal Times for Stiff Frame

Figure 15: DAF for Vertical Deflection for Various Column Removal Time for Stiff Frame
5.4 Results

Displacement response curves and moment response curves were plotted and looked at to determine dynamic amplification factors. Displacements and moments for the amplified and unamplified linear static analysis and linear dynamic analysis were compared to the nonlinear dynamic analysis to determine DAF. The amplified nonlinear static (ANLS) analysis was compared to the nonlinear dynamic by maximum displacement, maximum moment, and the number of hinges formed.

5.4.1 2x2 Frame

5.4.1.1 Exterior Column Removed for 2x2 Frame

Figure 16 shows the vertical deflection at the node where the column was removed. As previously mentioned, the linear static analysis was run without the dynamic amplification factor. The amplified linear static results were plotted on the graph as well. The linear static displacement of the free node is -0.82 feet and the amplified linear static deflection is -1.63 feet. The linear dynamic analysis shows that the initial removal of the column causes amplification but it ultimately stabilizes at the unamplified linear static deflection. The maximum displacement for the linear dynamic analysis is -1.29 feet and the stabilized deflection is -0.84 feet.

![Vertical Displacement vs Time for 2x2 Exterior Column Removal](image)

Figure 16: Vertical Deflection at Free Node for a 2x2 Exterior Column Removal
The displacement in the nonlinear dynamic analysis case exceeds the displacement shown for the linear dynamic analysis. The reason for this is the formation of plastic hinges in the frame. Once the plastic hinges form, the forces have to redistribute to adjacent members to transfer the load. The maximum displacement for the nonlinear dynamic analysis is -1.62 feet. In this case, the amplified linear static analysis case compares best to the nonlinear dynamic analysis. The maximum displacement for all the analysis cases can be seen in Table 3 below.

Figure 17 shows the moment response curve for the member above the removed column. The maximum unamplified linear static moment is -137 k-ft and the amplified linear static moment is -274 k-ft. As can be seen in Figure 17, the linear dynamic and nonlinear dynamic moments are significantly less than the amplified linear static moment.

![Moment vs Time for 2x2 Exterior Column Removal](image)

Figure 17: Moment in Member Above Removed Column for a 2x2 Exterior Column Removal
The maximum linear dynamic moment is -215 k-ft and the maximum nonlinear dynamic moment is -152 k-ft. These values are significantly lower than the amplified linear static moment of -274 k-ft. The dynamic amplification factors for the analysis cases can be found in Table 3 below.

**Table 3: DAF for 2x2 Frame with Exterior Column Removal**

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.816</td>
<td>-137.0</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-1.292</td>
<td>-215.1</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.993</td>
<td>-152.4</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.618</td>
<td>-176.0</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.98</td>
<td>1.28</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.58</td>
<td>1.57</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.63</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Another situation to look at is the plastic hinges that form in the nonlinear analysis. Since the nonlinear static model was run without an amplification factor to determine a DAF, another nonlinear static model was created with a DAF of 2 to compare the accuracy of the nonlinear static analysis to the nonlinear dynamic analysis. As can be seen in Figure 18, the nonlinear static analysis was able to form all except one plastic hinge in comparison to the nonlinear dynamic case. The amplified nonlinear static analysis shows a displacement of -1.34 feet which is a similar displacement to the linear dynamic case.

![Nonlinear Dynamic](image1)

![Nonlinear Static](image2)

*Figure 18: Plastic Hinge Comparison for 2x2 Exterior Removed Column*
5.4.1.2 **Interior Column Removed for 2x2 Frame**

Figure 19 shows the deflection at the midspan of the beam spanning the interior removed column. The unamplified linear static deflection is -0.37 feet and the amplified linear static deflection is -0.73 feet. The maximum linear dynamic displacement at midspan is -0.67 feet and stabilizes at the unamplified linear static deflection. The maximum nonlinear dynamic deflection is -0.73 feet. This is identical to the deflection determined by the linear static analysis using 2 for the DAF. The nonlinear dynamic case shows a permanent deformation of -0.55 feet which is greater than the linear dynamic case. This is due to the formation of plastic hinges in the nonlinear analysis.

![Image: Graph showing vertical displacement vs time for 2x2 Interior Removed Column.](image-url)

**Figure 19:** Vertical Deflection at Free Node for a 2x2 Interior Column Removal
Figure 20 shows the moment in the member being considered. The unamplified linear static moment is 114 k-ft and the amplified static moment is 228 k-ft. The maximum linear dynamic moment is 224 k-ft. The moment in the linear dynamic analysis is very similar to the linear static analysis with DAF included. The moment for the nonlinear dynamic analysis is smaller because when the plastic hinges form, the moment cannot exceed the plastic moment of that section. Therefore, for nonlinear dynamic analysis, the moment can never reach the linear dynamic moment because it is above the plastic moment.

Table 4 shows the maximum deflections and moment for the observed area. It also shows the dynamic amplification factor for the different analysis cases.
Table 4: DAF for 2x2 Frame with Interior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.366</td>
<td>113.9</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.671</td>
<td>224.2</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.369</td>
<td>114.7</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.732</td>
<td>193.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>DAF - NLD/LS</th>
<th>DAF - LD/LS</th>
<th>DAF - NLD/NLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.00</td>
<td>1.70</td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td>1.83</td>
<td>1.97</td>
<td>1.69</td>
</tr>
</tbody>
</table>

As mentioned before, the NS case was run without a dynamic amplification factor. It is necessary to look at this case with the amplification factor to determine the accuracy of using a DAF of 2 for this analysis case. The ANLS analysis shows a displacement of -1.38 feet which greatly exceeds all other cases for this frame. The deflection is large for the ANLS analysis because of all the plastic hinges that form. As seen in Figure 21, the members begin to yield and some begin to surpass immediate occupancy acceptance criteria. One hinge surpasses life safety acceptance criteria. The acceptance criterion is a function of the rotation about the plastic hinge. The detailed acceptance criteria can be found in FEMA 356. In this frame, the NLS analysis overestimates the response of the frame. The LD analysis would be reasonable to use in this case since the responses are relatively similar to the NLD case. The LS case is reasonable to use as well; however, it overestimates the deflection and moment which could lead to an overly conservative design.

Figure 21: Plastic Hinge Comparison for 2x2 Interior Removed Column
5.4.2 2x3 Frame

The comparison of the analysis methods was repeated for a 2x3 frame in order to determine the effect of geometry on amplification factors.

5.4.2.1 Exterior Column Removed for 2x3 Frame

By looking at the displacement response curves for the analysis cases, it is evident that the linear static analysis with the dynamic amplification factor overestimates the deflection and that the linear dynamic analysis underestimates the deflection. The maximum linear dynamic displacement is $-1.13$ feet and its permanent stabilized deflection is $-0.78$ feet. The maximum nonlinear displacement is $-1.36$ feet and its permanent deflection is $-1.15$ feet. The maximum deflections for the different analysis cases can be seen in Table 5. The graphs for vertical displacement and moment can be found in Appendix 8.1.

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.745</td>
<td>-130.6</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-1.134</td>
<td>-209.5</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.759</td>
<td>-130.1</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.357</td>
<td>-174.6</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>DAF - NLD/LS</td>
<td>1.82</td>
<td>1.34</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.52</td>
<td>1.60</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.79</td>
<td>1.34</td>
</tr>
</tbody>
</table>

The moment response curves show that the amplified linear static analysis significantly overestimates the moment. The moment for the linear dynamic analysis is similar to the moment for the nonlinear dynamic case. The maximum linear dynamic moment is $-210$ k-ft and its permanent stabilized deflection is $-125$ k-ft which is slightly less than the linear static moment. The maximum nonlinear moment is $-175$ k-ft and its stabilized moment is $-125$ k-ft. The linear dynamic analysis slightly overestimates the moment because it doesn’t take into account plastic hinges and therefore the moment in the beam can exceed the plastic moment.
Table 5 shows the maximum deflection and moment at the observed location. As can be seen, all values for dynamic amplification are below 2 for this case. The linear static analysis overestimates the deflection and moment by a significant amount.

The plastic hinges were compared for the nonlinear static and nonlinear dynamic cases to determine the accuracy of the nonlinear static analysis case. Since the deflection and moment values in Table 5 don’t include a DAF, another model was built with the DAF included to determine the location and values of the plastic hinges. The amplified nonlinear static analysis shows a displacement of -1.61 feet which exceeds the nonlinear dynamic deflection. Therefore, the nonlinear static analysis case produces a more conservative design. However, it was fairly accurate in determining the locations of the plastic hinges. The nonlinear static analysis displays most of the plastic hinges that the nonlinear dynamic case displays with the exception of two hinges.

Figure 22: Plastic Hinge Comparison for a 2x3 Exterior Column Removal
5.4.2.2 *Interior Column Removed for 2x3 Frame*

The maximum linear dynamic deflection is -0.68 feet and the frame stabilizes at -0.37 feet which is the unamplified linear static deflection. The maximum nonlinear deflection is -0.729 feet and its permanent deflection is -0.539 feet. The amplified linear static deflection is greater than the linear dynamic deflection and the nonlinear dynamic deflection at the same node. This demonstrates that the linear static analysis with a dynamic amplification of 2 overestimates the deflection at the damaged column. The displacement response curves can be found in Appendix 8.2.

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.371</td>
<td>112.7</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.679</td>
<td>222.5</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.373</td>
<td>113.0</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.729</td>
<td>193.0</td>
</tr>
</tbody>
</table>

Table 6: DAF for 2x3 Frame with Interior Column Removal

The amplified linear static analysis case overestimates the moment for the beam above the removed column. Due to the formation of plastic hinges, the maximum moment in the nonlinear dynamic analysis case is limited to the plastic moment. The maximum moment for the linear dynamic case is 223 k-ft and its final moment in the member is 120 k-ft. The maximum moment for the nonlinear dynamic case is 193 k-ft and the moment stabilizes at 125 k-ft for this case. The linear case overestimates the maximum moment which could lead to overdesign of the structure. The final stabilized moment for both the linear and nonlinear dynamic is on average 122 k-ft.
For this 2x3 span case where the interior column is removed, out of all the analysis tasks run, the linear dynamic analysis estimates the behavior of frame the best. The deflection is slightly underestimated but the moment is slightly overestimated in the linear dynamic analysis case. This would lead to a moderate design. Whereas, designing by linear static analysis with a dynamic amplification factor would lead to a conservative design. Table 6 show the dynamic amplification factors for each analysis case. As can be seen, the amplification factors are all less than 2.

Figure 23 shows the plastic hinges that form in the nonlinear dynamic analysis and the nonlinear static analysis. As can be seen, the nonlinear static analysis correctly depicts the location of the plastic hinges; however, this analysis case shows that the state of some of the plastic hinges exceed acceptance criteria by reaching the immediate occupancy and life safety limit states. Therefore, for this span frame, using a DAF of 2 for the nonlinear static analysis is very conservative.

Figure 23: Plastic Hinge Comparison for a 2x3 Interior Column Removal
5.4.3  2x4 Frame

5.4.3.1  Exterior Column Removed for 2x4 Frame

Using a dynamic amplification of 2 overestimates the deflection and moment for this span frame. The linear dynamic analysis underestimates the deflection and overestimates the moment compared to the nonlinear dynamic analysis. The maximum linear dynamic displacement is -1.14 feet and the frame steadies at a deflection of -0.68 feet which is slightly less than the linear static deflection. The graphs for vertical displacement and moment can be found in Appendix 8.3.

Table 7: DAF for 2x4 Frame with Exterior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.691</td>
<td>-127.3</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-1.141</td>
<td>-211.9</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.708</td>
<td>-126.6</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.257</td>
<td>-173.9</td>
</tr>
</tbody>
</table>

| DAF - NLD/LS         | 1.82    | 1.37     |
| DAF - LD/LS          | 1.65    | 1.66     |
| DAF - NLD/NLS        | 1.78    | 1.37     |

The maximum linear dynamic moment is higher than the nonlinear dynamic moment due to plastic hinge formation. The maximum linear dynamic moment is -212 k-ft and it stabilizes at -119 k-ft. The maximum nonlinear dynamic moment is -174 k-ft and the final moment in the member is -115 k-ft.
Table 7 compares deflection and moment values between the different analysis cases. The linear dynamic analysis response is the closest to the nonlinear dynamic case. As seen in Table 7, the dynamic amplification factor is less than 2 for all the analysis cases. In the case, the linear dynamic case would produce the least conservative design in comparison to the nonlinear dynamic case.

![Figure 24: Plastic Hinge Comparison for a 2x4 Exterior Column Removal](image)

The plastic hinge formation is compared in Figure 24 for the nonlinear dynamic and nonlinear static. The nonlinear dynamic analysis case shows a maximum displacement of -1.26 feet and a maximum moment of -174 k-ft. The nonlinear static analysis case with a dynamic amplification factor of 2 shows a deflection of -1.97 feet and a maximum moment of -179 k-ft. The deflection is overestimated with the ANLS analysis. The plastic hinges form in almost the same positions with a few exceptions. The nonlinear static analysis shows that some hinges exceed the immediate occupancy acceptance criteria meanwhile the nonlinear dynamic analysis case shows all the formed hinges have just yielded.
5.4.3.2 *Interior Column Removal for 2x4 Frame*

The maximum linear dynamic deflection is -0.65 feet and it comes to rest at the linear static deflection of -0.37 feet. The maximum nonlinear deflection is -0.71 feet which is slightly higher than the linear dynamic analysis deflection. The nonlinear dynamic analysis shows the structure coming to rest at a final deflection of -0.51 feet. The deflection response curves can be found in Appendix 8.4.

The nonlinear dynamic analysis moment response curve is very similar to the linear dynamic moment response curve. The linear dynamic curve for moment exceeds the nonlinear dynamic curve at all the peaks. This is due to the plastic hinges that form in the members in the nonlinear dynamic curve.

Table 8 shows the maximum displacement and maximum moment for the selected node and member considered. All the dynamic amplification factors are less than 2 for the frame. The linear dynamic analysis case shows results closest to the nonlinear dynamic case.

Table 8: DAF for 2x4 Frame with Interior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.367</td>
<td>111.2</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.652</td>
<td>212.9</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.369</td>
<td>111.3</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.706</td>
<td>190.3</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.92</td>
<td>1.71</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.78</td>
<td>1.92</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.91</td>
<td>1.71</td>
</tr>
</tbody>
</table>
Figure 25 shows the plastic hinges developed in the nonlinear dynamic and nonlinear static analysis case. The nonlinear static analysis with the dynamic amplification factor shows a maximum displacement of -0.58 feet. The nonlinear dynamic analysis case shows a maximum deflection of -0.71 feet. Based on deflection values, the nonlinear static underestimates the maximum deflection. The nonlinear static with the dynamic amplification factor shows a maximum moment of 171 k-ft which is slightly less than the maximum moment of 190 k-ft shown in the nonlinear dynamic case. The nonlinear static analysis displayed 14 out of 20 plastic hinges that form in the nonlinear dynamic case.
5.4.4 3x2 Frame

5.4.4.1 Exterior Column Removed for 3x2 Frame

Figure 26 shows the vertical displacement at the free end of the frame. The maximum linear dynamic displacement for this frame is -1.19 feet which is very close to the maximum nonlinear dynamic deflection of -1.21 feet. Even though the maximum values for the linear and nonlinear dynamic analysis cases are similar, the final permanent deflection varies for the two cases. For the linear dynamic analysis case, the stabilized deflection is -0.64 feet and for the nonlinear dynamic analysis case, the stabilized deflection is -0.92 feet. The permanent linear dynamic deflection is similar to the linear static case.

Figure 26: Vertical Deflection at Free Node for a 3x2 Exterior Column Removal
Figure 27 shows the moment response curves for linear static, linear dynamic, and nonlinear dynamic analysis cases. The linear dynamic moment exceeds the nonlinear dynamic maximum moment due to plastic hinges limiting the moment to the plastic moment. The maximum moment for the linear dynamic case is \(-215\ \text{k-ft}\) and its final moment at its stabilized position is \(-119\ \text{k-ft}\). The maximum moment for the nonlinear dynamic analysis case is \(-177\ \text{k-ft}\). The unamplified linear static analysis case has a constant moment of \(-133\ \text{k-ft}\) which is slightly higher than the final moment of the linear dynamic analysis. This is due to fact that the analysis was not run for long enough.

![Moment vs Time for 3x2 Exterior Removed Column](image)

Figure 27: Moment in Member Above Removed Column for a 3x2 Exterior Column Removal

Table 9 shows the maximum displacements and maximum moments for each analysis case. It also displays the dynamic amplification factors for the various cases. The dynamic amplification factor for this frame is significantly smaller than 2 and could aid in less conservative and therefore more economical results.
Table 9: DAF for 3x2 Frame with Exterior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.688</td>
<td>-133.2</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-1.187</td>
<td>-215.2</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.694</td>
<td>-132.6</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.212</td>
<td>-177.2</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.76</td>
<td>1.33</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.73</td>
<td>1.62</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.75</td>
<td>1.34</td>
</tr>
</tbody>
</table>

Figure 28 compares the plastic hinges that form in the nonlinear dynamic analysis and the ANLS case. The ANLS analysis case shows a maximum deflection of -2.67 feet which is significantly larger than the nonlinear dynamic analysis deflection of -1.21 feet. The ANLS shows a maximum moment of -195 k-ft. The moment is overestimated compared to the NLD analysis moment. The location of the plastic hinges are identical between the two cases, however, the nonlinear static case shows the hinges exceeding immediate occupancy and life safety acceptance criteria. The nonlinear static analysis also shows one hinge exceeding the plastic moment capacity. Nonlinear static analysis with a dynamic amplification factor overestimates the damage in the frame.
5.4.4.2 Interior Column Removed for 3x2 Frame

Figure 29 shows the vertical displacement for midspan of a beam spanning over an interior removed column. The maximum linear dynamic displacement is -0.65 feet and its final resting displacement is -0.35 feet which is identical to the linear static displacement. The maximum nonlinear dynamic deflection is -0.68 feet. The nonlinear dynamic displacement stabilizes at -0.49 feet.

![Vertical Displacement vs Time for 3x2 Interior Removed Column](image)

Figure 29: Vertical Deflection at Free Node for a 3x2 Interior Column Removal
Figure 30 shows the moment response curves for the different analysis cases. The maximum linear dynamic moment is 230 k-ft which is slightly higher than the maximum moment of 182 k-ft in the nonlinear dynamic case. As mentioned previously, it is expected for the linear dynamic analysis to develop a greater moment due to its inability to enter into the plastic range. Both moments for the linear and nonlinear dynamic analyses stabilize around 119 k-ft which is the linear static moment.

![Moment vs Time for 3x2 Interior Removed Column](image)

Figure 30: Moment in Member Above Removed Column for a 3x2 Interior Column Removal

Table 10 shows the maximum displacement and moments for this frame. The displacements for the linear dynamic and nonlinear dynamic analysis are similar and therefore either of these cases could be used to accurately predict the deflection of the frame. The dynamic amplification factor necessary to use for the linear static analysis to reproduce the results from the nonlinear dynamic analysis is less than 2.
Table 10: DAF for 3x2 Frame with Interior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.354</td>
<td>118.5</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.645</td>
<td>230.5</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.357</td>
<td>119.3</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.678</td>
<td>181.5</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.92</td>
<td>1.53</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.82</td>
<td>1.95</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.90</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Figure 31 shows the plastic hinges formed in the nonlinear static and nonlinear dynamic analysis cases. The amplified nonlinear static analysis shows a maximum deflection of -1.17 feet which is significantly higher than the nonlinear dynamic displacement of -0.678 feet. The amplified nonlinear static maximum moment is 221 k-ft which is larger than the nonlinear dynamic moment. This shows that using a dynamic amplification factor of 2 with the nonlinear static analysis overestimates the response of the frame. In addition, the nonlinear static analysis showed more hinges forming than the nonlinear dynamic. Six of those hinges exceed the immediate occupancy acceptance criteria and three of the hinges exceed the life safety acceptance criteria.

Figure 31: Plastic Hinge Comparison for a 3x2 Interior Column Removal
5.4.5 3x3 Frame

5.4.5.1 Exterior Removed Column for 3x3 Frame

The effects of the removed column on the frame were obtained by the 4 different analysis cases. The maximum deflection observed with the linear dynamic analysis case is -1.05 feet and the permanent deflection of the free end is -0.67 feet which is similar to the linear static deflection of -0.66 feet. The maximum displacement for the nonlinear dynamic case is -1.20 feet and the final resting position is a deflection of -0.94 feet. The displacement response curve can be found in Appendix 8.5.

The moment in the frame member above the removed column was looked at with the different analysis cases to determine maximum moments. The maximum moment for the linear dynamic analysis case was -174 k-ft and it stabilized at the linear static analysis moment which was -125 k-ft. The maximum moment for the nonlinear dynamic analysis case is -174 k-ft and the moment stabilizes at -124 k-ft. The linear dynamic analysis is accurate in this case because it gives the same result as the nonlinear dynamic method. However, the linear dynamic case overestimates the maximum moment due to its inability to enter into the plastic range. Table 11 shows the maximum deflection and moment for the frame.

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.657</td>
<td>-125.1</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-1.051</td>
<td>-206.7</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.666</td>
<td>-124.5</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.196</td>
<td>-174.3</td>
</tr>
</tbody>
</table>

Table 11: DAF for 3x3 Frame with Exterior Column Removal
The dynamic amplification factors are also listed in Table 11. A dynamic amplification factor of 2 provides a conservative response for this structure.

Figure 32 shows the comparison of the plastic hinges that form in the nonlinear dynamic and nonlinear static analyses cases. Nonlinear static analysis case shows a maximum deflection of -1.96 feet whereas the nonlinear dynamic analysis case shows a maximum deflection of -1.19 feet. This indicates that the nonlinear static analysis overestimates the displacement response of the frame when a column is removed. The maximum moment for the nonlinear static analysis case is -180 k-ft. This is slightly greater than the moment developed in the nonlinear dynamic case. This explains why the nonlinear static analysis shows hinges that exceed to immediate occupancy acceptance criteria. The nonlinear static model predicts the majority of the plastic hinge locations; however, the NLS case overestimates the state of the plastic hinges. It is possible that some of the additional hinges predicted by the nonlinear dynamic analysis are due to the other hinges forcing additional members to carry load which forms the additional hinges.

Figure 32: Plastic Hinge Comparison for a 3x3 Exterior Column Removal
5.4.5.2 Interior Column Removed for 3x3 Frame

Table 12 shows the maximum displacement and moments in the 3x3 frame with an interior column removed. The maximum displacement for the nonlinear dynamic analysis case is -0.51 feet and the maximum displacement for the linear dynamic analysis case is -0.43 feet. Both dynamic cases show the frame coming to rest at around -0.25 feet which is similar to the linear static deflection. The graphs for vertical displacement and moment can be found in Appendix 8.6.

The maximum moment for the nonlinear dynamic is 179 k-ft which is greater than the linear dynamic moment which is 161 k-ft. This is one of the few cases where the moment in the nonlinear dynamic case is larger than the moment in the linear dynamic case. The moments in the frame due to the loading are less than the plastic moment of the section; therefore, the nonlinear dynamic moment is not limited to the plastic moment and, as a result, exceeds the moments in the linear dynamic analysis. Although, if no plastic hinges form in the analysis, it is expected that the linear dynamic and nonlinear dynamic analysis results would match. However, there are hinges that form in other parts of the frame for the NLD analysis that cause a discrepancy of results in the LD and NLD analysis.

Table 12: DAF for 3x3 Frame with Interior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.257</td>
<td>91.0</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.427</td>
<td>161.4</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.258</td>
<td>90.2</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.506</td>
<td>179.9</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.97</td>
<td>1.98</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.66</td>
<td>1.77</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.96</td>
<td>1.99</td>
</tr>
</tbody>
</table>

Table 12 also shows the dynamic amplification factors for the different cases. As can be seen, a DAF of 2 is proven to be conservative.
The nonlinear static analysis with a dynamic amplification factor of 2 displays a maximum deflection of -0.48 feet which is an accurate representation of the maximum nonlinear dynamic displacement which is -0.51 feet. The moment represented by the nonlinear static case is 169 k-ft which is slightly less than the maximum moment of 179 k-ft represented by the nonlinear dynamic case. As a result, the nonlinear static analysis displays one fewer plastic hinge in the analysis which can be seen in Figure 33.

![Plastic Hinge Comparison](image)

Figure 33: Plastic Hinge Comparison for a 3x3 Interior Column Removal

5.4.6 3x4 Frame

5.4.6.1 Exterior Column Removed for 3x4 Frame

The displacements for the different analysis cases of the 3x4 span frame when the exterior column was removed are shown in Table 13. The maximum nonlinear dynamic displacement is -1.09 feet which agrees with the maximum linear dynamic displacement of -0.98 feet. The nonlinear dynamic analysis stabilizes at a permanent deflection of -0.75 feet. The linear dynamic analysis stabilizes at a deflection of -0.64 feet which corresponds to the linear static analysis deflection of -0.61 feet. The displacement response curve can be found in the Appendix. The nonlinear dynamic case shows a maximum moment of -173 k-ft and a permanent moment of -107 k-ft. The linear dynamic case shows a maximum moment of -196 k-ft which is greater than
the nonlinear dynamic moment. The graphs for vertical displacement and moment can be found in Appendix 8.7.

Table 13: DAF for 3x4 Frame with Exterior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.609</td>
<td>-116.1</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.979</td>
<td>-196.5</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.621</td>
<td>-115.4</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.094</td>
<td>-173.3</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.80</td>
<td>1.49</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.61</td>
<td>1.69</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.76</td>
<td>1.50</td>
</tr>
</tbody>
</table>

The dynamic amplification factors shown in Table 13 are all less than the value of 2 suggested by GSA. The frame in the amplified nonlinear static analysis case deflects -1.14 feet which is greater than the maximum nonlinear dynamic analysis deflection. The moment in the frame is -172 k-ft in the nonlinear static analysis is accurately represented. The plastic hinges formed in the nonlinear static case are shown in Figure 34. The locations of the plastic hinges vary between the two cases. Since the nonlinear dynamic analysis redistributes the forces every time a hinge forms, it is likely that the forces cause plastic hinges in different locations.

![Figure 34: Plastic Hinge Comparison for a 3x4 Exterior Column Removal](image-url)
5.4.6.2 Interior Column Removed for 3x4 Frame

The maximum deflections and moments were observed in the frame for the various analysis cases and the response curves can be found in the Appendix. The frame analyzed using the nonlinear dynamic analysis showed a maximum deflection of -0.62 feet and a permanent deflection of -0.41 feet. The linear dynamic analysis showed a maximum deflection of -0.60 feet with a final stabilized deflection of -0.33 feet. The final linear dynamic deflection corresponds to the unamplified linear static deflection. The graphs for vertical displacement and moment can be found in Appendix8.8.

The maximum moment in the nonlinear dynamic analysis is -173 k-ft and has a moment of -116 k-ft when the structure stabilized. The linear dynamic analysis overestimates the maximum moment because of its elastic behavior. The maximum moment for the linear dynamic case is 206 k-ft and a moment of 114 k-ft when the frame comes to rest.

Table 14 shows the dynamic amplification factors for the various analysis cases. Based on the table, it is evident that a dynamic amplification of 2 would overestimate the response of the frame.

Table 14: DAF for 3x4 Frame with Interior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.333</td>
<td>106.9</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.603</td>
<td>206.3</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.334</td>
<td>107.4</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.623</td>
<td>172.9</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.87</td>
<td>1.62</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.81</td>
<td>1.93</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.87</td>
<td>1.61</td>
</tr>
</tbody>
</table>
Figure 35: Plastic Hinge Comparison for a 3x4 Interior Column Removal

Figure 35 shows the plastic hinges formed in the nonlinear static and nonlinear dynamic analysis cases. The amplified nonlinear static analysis shows a maximum deflection of -1.23 feet which is significantly higher than the nonlinear dynamic displacement of -0.62 feet. The amplified nonlinear static maximum moment is 197 k-ft which is larger than the nonlinear dynamic moment. In addition, the nonlinear static analysis showed more hinges forming than the nonlinear dynamic. Thirteen of those hinges exceed the immediate occupancy acceptance criteria. This shows that using a dynamic amplification factor of 2 with the nonlinear static analysis overestimates the response of the frame.
5.4.7 4x2 Frame

5.4.7.1 Exterior Column Removed for 4x2 Frame

Figure 36 shows the vertical deflection at the node where the column was removed. The linear static displacement of the free node is -0.58 feet and the amplified linear static deflection is -1.16 feet. The linear dynamic analysis shows that the initial removal of the column causes amplification but it ultimately stabilizes at the unamplified linear static deflection. The maximum displacement for the linear dynamic analysis is -1.03 feet and it stabilizes at -0.58 feet.

The displacement in the nonlinear dynamic analysis exceeds the displacement shown for the linear dynamic analysis. The reason for this is the formation of plastic hinges in the frame. Once the plastic hinges form, the forces have to redistribute to adjacent members to transfer the load. The maximum displacement for the nonlinear dynamic analysis is -1.12 feet. In this case,
the amplified linear static analysis case compares best to the nonlinear dynamic analysis. The maximum displacement for all the analysis cases can be seen in Table 15 below.

Figure 38 shows the moment response curve for the member above the removed column. The maximum unamplified linear static moment is -124 k-ft and the amplified linear static moment is -249 k-ft. As can be seen in Figure 38, the linear dynamic and nonlinear dynamic moments are less than the amplified linear static moment.

![Moment vs Time for 4x2 Exterior Column Removed](image)

**Figure 38: Moment in Member Above Removed Column for a 4x2 Exterior Column Removal**

The maximum linear dynamic moment is -216 k-ft and the maximum nonlinear dynamic moment is -177 k-ft. These values are significantly lower than the amplified linear static moment of -249 k-ft. Therefore, a dynamic amplification factor less than 2 could be used to analyze this frame with the removed column. The dynamic amplification factors for the analyses cases can be found in Table 15 below.
Table 15: DAF for 4x2 Frame with Exterior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.582</td>
<td>-124.3</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-1.034</td>
<td>-215.9</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.597</td>
<td>-125.3</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.123</td>
<td>-177.4</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.93</td>
<td>1.43</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.78</td>
<td>1.74</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.88</td>
<td>1.42</td>
</tr>
</tbody>
</table>

Another factor to consider is the plastic hinges that form in the nonlinear analysis. Since the nonlinear static model was run without an amplification factor to determine a DAF, another nonlinear static model was created with a DAF of 2 to compare the ability to predict plastic hinge formation of the nonlinear static analysis to the nonlinear dynamic analysis. As can be seen in Figure 39, the nonlinear static analysis was able to form all except one plastic hinge in comparison to the nonlinear dynamic case. However, the nonlinear static model shows that the two of the hinges exceed life safety acceptance criteria, while the other two hinges reach plastic moment capacity. Therefore, it is evident that the nonlinear static analysis case overestimates the response of the structure to the removed column.

![Figure 39: Plastic Hinge Comparison for 4x2 Exterior Removed Column](image)
5.4.7.2 Interior Column Removed for 4x2 Frame

Figure 40 shows the displacement response curve at the midspan of the beam spanning the interior removed column. The unamplified linear static deflection is -0.35 feet while the amplified linear static deflection is -0.71 feet. The maximum linear dynamic displacement at midspan is -0.64 feet and stabilizes at the unamplified linear static deflection. The maximum nonlinear dynamic deflection is -0.68 feet. This is less than the deflection determined by the amplified linear static analysis. The nonlinear dynamic case shows a permanent deformation of -0.48 which is greater than the linear dynamic case. This is due to the formation of plastic hinges in the nonlinear analysis.

Figure 40: Vertical Deflection at Free Node for a 4x2 Interior Column Removal
Figure 41 shows the moment in the member spanning across the removed column. The unamplified linear static moment is 117 k-ft and the amplified static moment is 234 k-ft. The maximum linear dynamic moment is 228 k-ft. The moment in the linear dynamic analysis is very similar to the linear static analysis with DAF included. The moment for the nonlinear dynamic analysis is smaller because when the plastic hinges form, the moment cannot exceed the plastic moment of that section. Therefore, for nonlinear dynamic analysis, the moment can never reach the linear dynamic moment because of nonlinearities that occur in the member.

![Moment vs Time for 4x2 Interior Removed Column](image)

**Figure 41: Moment in Member Above Removed Column for a 4x2 Interior Column Removal**

Table 16 shows the maximum deflections and moment for the observed area. It also shows the dynamic amplification factor for the different analysis cases. As can be seen from the table, the dynamic amplification can be reduced for this case. A dynamic amplification factor of 2 overestimates the response of the frame.
Table 16: DAF for 4x2 Frame with Interior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.353</td>
<td>117.04</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.644</td>
<td>228.4</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.355</td>
<td>117.8</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.677</td>
<td>183.9</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.92</td>
<td>1.57</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.82</td>
<td>1.95</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.91</td>
<td>1.56</td>
</tr>
</tbody>
</table>

The amplified nonlinear static analysis shows a displacement of -0.89 feet which exceeds the displacement in the other cases for this frame. The deflection is large for the amplified nonlinear static because more plastic hinges form in this case. As seen in Figure 42, the nonlinear static case predicts two additional hinges than the nonlinear dynamic case. The maximum moment observed in the nonlinear static case is 199 k-ft which is larger than the nonlinear dynamic moment of 184 k-ft. This explains why more hinges form in the nonlinear static case because the forces are redistributed and cause hinges in other locations. Therefore, it is evident that a DAF of 2 is conservative.

Figure 42: Plastic Hinge Comparison for 4x2 Interior Removed Column
5.4.8  4x3 Frame

5.4.8.1 Exterior Column Removed for 4x3 Frame

By looking at the displacement response curves for the analysis cases, it is evident that the linear static analysis with the dynamic amplification factor overestimates the deflection and that the linear dynamic analysis underestimates the deflection. The maximum linear dynamic displacement is -1.01 feet and its permanent stabilized deflection is -0.60 feet which is similar to the linear static displacement. The maximum nonlinear displacement is -1.15 feet and its permanent deflection is -0.91 feet. The maximum deflections for the different analysis cases can be seen in Table 17. The graphs for vertical displacement and moment can be found in Appendix 8.9.

Table 17: DAF for 4x3 Frame with Exterior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.608</td>
<td>-122.3</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-1.007</td>
<td>-204.5</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.614</td>
<td>-121.7</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.149</td>
<td>-174.1</td>
</tr>
</tbody>
</table>

The moment response curves that the amplified linear static analysis significantly overestimates the moment. The moment for the linear dynamic analysis is similar to the moment for the nonlinear dynamic case. The maximum linear dynamic moment is -204 k-ft and its permanent stabilized deflection is -114 k-ft which is slightly less than the linear static moment. The maximum nonlinear moment is -174 k-ft and its stabilized moment is -126 k-ft. The linear dynamic analysis overestimates the moment because it does not take into account plastic hinges and therefore the moment in the beam can exceed the plastic moment.
Table 17 shows the maximum deflection and moment in the frame. As can be seen, all values for dynamic amplification are below 2 for this case. The linear static analysis overestimates the deflection and moment by a significant amount.

The plastic hinges were compared for the nonlinear static and nonlinear dynamic cases to determine the accuracy of the nonlinear static analysis case. The amplified nonlinear static analysis shows a displacement of -2.12 feet which greatly exceeds the nonlinear dynamic deflection. Therefore, this analysis case will produce a more conservative design. However, it was fairly accurate in determining the locations of the plastic hinges; although, not the state of the hinges. The nonlinear static analysis shows the hinges exceeds the immediate occupancy acceptance criteria and two hinges exceed the life safety acceptance criteria, while the nonlinear dynamic case shows that the beams have only yielded. Using 2 for a dynamic amplification factor for nonlinear static overestimates the response of the structure.

Figure 43: Plastic Hinge Comparison for a 4x3 Exterior Column Removal
5.4.8.2 Interior Column Removed for 4x3 Frame

The vertical deflection at the midspan of beam spanning over the removed column is observed and response curves can be found in the Appendix. The amplified linear static deflection is greater than the linear dynamic deflection and the nonlinear dynamic deflection at the same node. This demonstrates that the linear static analysis with a dynamic amplification of 2 overestimates the deflection at the damaged column. The maximum linear dynamic deflection is -0.63 feet and the frame stabilizes at -0.35 feet which is the unamplified linear static deflection. The maximum nonlinear deflection is -0.65 feet and its permanent deflection is -0.45 feet. The graphs for vertical displacement and moment can be found in Appendix 8.10.

Table 18: DAF for 4x3 Frame with Interior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.347</td>
<td>111.8</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.633</td>
<td>217.7</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.349</td>
<td>112.3</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.654</td>
<td>173.8</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.88</td>
<td>1.55</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.82</td>
<td>1.95</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.87</td>
<td>1.55</td>
</tr>
</tbody>
</table>

The amplified linear static analysis case overestimates the moment for the beam above the removed column. The moment response curves can be found in the Appendix. Due to the formation of plastic hinges, the maximum moment in the nonlinear dynamic analysis case is limited to the plastic moment. The maximum moment for the linear dynamic case is 218 k-ft and the permanent moment in the member is 120 k-ft. The maximum moment for the nonlinear dynamic case is 174 k-ft and the moment stabilizes at 106 k-ft for this case. The linear case overestimates the maximum moment which could lead to overdesign of the structure.

74
For this 4x3 span case where the interior column is removed, out of all the analysis tasks run, the linear dynamic analysis estimates the behavior of frame the best. The deflection is slightly underestimated but the moment is slightly overestimated in the linear dynamic analysis case. This would lead to a design appropriate for the loads and would result in a structure that is not over designed. Whereas, designing by the amplified linear static analysis would lead to a conservative design. Table 18 shows the dynamic amplification factors for each analysis case. As can be seen, the amplification factors are all less than 2.

Figure 44 shows the plastic hinges that form in the nonlinear dynamic analysis and the nonlinear static analysis. As can be seen, the nonlinear static analysis correctly depicts the majority of the locations for the plastic hinges; however, this analysis case shows that the state of some of the plastic hinges exceed acceptance criteria by reaching the immediate occupancy and life safety limit states. The nonlinear static analysis even shows that 2 of the hinges reach its plastic capacity. Therefore, for this span frame, nonlinear static analysis significantly overestimates the response of the system.
5.4.9 4x4 Frame

5.4.9.1 Exterior Column Removed for 4x4 Frame

The linear dynamic analysis underestimates the deflection and overestimates the moment compared to the nonlinear dynamic analysis. The maximum linear dynamic deflection is -0.95 feet and the frame comes to rest at a deflection of -0.61 feet which is slightly greater than the linear static deflection. By looking at the response curves in the Appendix, it is apparent that using a dynamic amplification of 2 overestimates the deflection and moment for this span frame. The graphs for vertical displacement and moment can be found in Appendix 8.11.

Table 19: DAF for 4x4 Frame with Exterior Column Removal

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.596</td>
<td>-110.7</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.952</td>
<td>-204.4</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.605</td>
<td>-111.8</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-1.074</td>
<td>-173.7</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.80</td>
<td>1.57</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.60</td>
<td>1.85</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.78</td>
<td>1.55</td>
</tr>
</tbody>
</table>

The maximum linear dynamic moment is higher than the nonlinear dynamic moment due to plastic hinge formation. The maximum linear dynamic moment is -204 k-ft and it stabilizes at -111 k-ft. The maximum nonlinear dynamic moment is -174 k-ft and the final moment in the member is -112 k-ft.

Table 19 compares deflection and moment values between the different analysis cases. The linear dynamic analysis compares the closest to the nonlinear dynamic case. As seen in Table 19, the dynamic amplification factor is less than 2 for all the analysis cases. In the case, the linear dynamic case would produce the least conservative design in comparison to the nonlinear dynamic case.
The plastic hinge formation is compared in Figure 24 for the nonlinear dynamic and nonlinear static. The nonlinear dynamic analysis case shows a maximum displacement of -1.26 feet and a maximum moment of -174 k-ft. The nonlinear static analysis case with a dynamic amplification factor of 2 shows a deflection of -1.97 feet and a maximum moment of -179 k-ft. The deflection is overestimated with the amplified nonlinear static analysis. The plastic hinges form in almost the same positions with a few exceptions. The nonlinear static analysis shows that some hinges exceed the immediate occupancy acceptance criteria meanwhile the nonlinear dynamic analysis case shows all the formed hinges have just yielded.

Figure 45: Plastic Hinge Comparison for a 4x4 Exterior Column Removal
5.4.9.2 Interior Column Removal for 4x4 Frame

The maximum linear dynamic deflection is -0.63 feet and it comes to rest at the linear static deflection of -0.35 feet. The maximum nonlinear deflection is -0.65 feet which is slightly higher than the linear dynamic analysis deflection. The nonlinear dynamic analysis shows the structure coming to rest at a final deflection of -0.44 feet. The deflection response curves can be found in Appendix 8.12.

The linear dynamic curve for moment exceeds the nonlinear dynamic curve at all the peaks. This is due to the plastic hinges that form in the members in the nonlinear dynamic analysis. Table 20 shows the maximum displacement and maximum moment for the node and member considered. All the dynamic amplification factors are less than 2 for the frame. The linear dynamic analysis case shows results closest to the nonlinear dynamic case.

<table>
<thead>
<tr>
<th></th>
<th>U3 (ft)</th>
<th>M (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Static (DL+0.25LL)</td>
<td>-0.348</td>
<td>111.5</td>
</tr>
<tr>
<td>Linear Dynamic</td>
<td>-0.633</td>
<td>216.4</td>
</tr>
<tr>
<td>Nonlinear Static (DL+0.25LL)</td>
<td>-0.349</td>
<td>111.8</td>
</tr>
<tr>
<td>Nonlinear Dynamic</td>
<td>-0.646</td>
<td>173.3</td>
</tr>
<tr>
<td>DAF - NLD/LS</td>
<td>1.86</td>
<td>1.55</td>
</tr>
<tr>
<td>DAF - LD/LS</td>
<td>1.82</td>
<td>1.94</td>
</tr>
<tr>
<td>DAF - NLD/NLS</td>
<td>1.85</td>
<td>1.55</td>
</tr>
</tbody>
</table>
Figure 46 shows the plastic hinges developed in the nonlinear dynamic and nonlinear static analysis case. The nonlinear static analysis with the dynamic amplification factor shows a maximum displacement of -0.79 feet. The nonlinear dynamic analysis case shows a maximum deflection of -0.65 feet. Based on deflection values, the nonlinear static overestimates the maximum deflection. The nonlinear static with the dynamic amplification factor shows a maximum moment of 136 k-ft which is less than the maximum moment of 173 k-ft shown in the nonlinear dynamic case. The nonlinear static analysis displays 20 plastic hinges which is greater than the 14 hinges shown in the nonlinear dynamic analysis. The nonlinear static analysis case overestimates the response and damage.
6 Conclusion

6.1 Dynamic Amplification Factor

The linear static procedure is the most commonly used analysis case in the workplace for progressive collapse. Currently, the linear static procedure uses a dynamic amplification of 2 which is conservative. If a more accurate dynamic amplification factor could be determined, this could result in fewer structures over-designed for progressive collapse prevention. Given the variety of building designs, it is interesting to look at how the dynamic amplification factor varies with the number of stories and the number of bays.

Figure 47 shows the dynamic amplification factor determined for the linear static case for the different number of bays. For the interior column removal, the dynamic amplification factor decreases as the number of bays increases. This is expected because when the frame has more columns, there are more paths to transfer the load. For the exterior removed column, it is unclear how the number of bays impacts the dynamic amplification factor.

![Dynamic Amplification Factor for Number of Bays for Linear Static Analysis](image)

Figure 47: Dynamic Amplification Factor for Various Number of Bays
Figure 48 shows the dynamic amplification factor varying with the number of stories in the frame. However, there is no clear correlation between the number of stories and the dynamic amplification factor necessary to make the linear static analysis as accurate as the nonlinear dynamic analysis case.

By looking at the different frames, it is evident that the dynamic amplification is consistently less than 2. This is true for all the frames analyzed except for the 2x2 span frame with an interior column removed where the DAF is exactly 2. It is reasonable for the dynamic amplification to be higher for a frame with two spans because when a column is damaged and removed in this frame, there are only two columns remaining to take all the load.

The dynamic amplification factors determined for the deflection is the governing case for these frames. The DAF is significantly larger for the deflections than the moments. The DAF for moment is less than 1.71 for all the frames discussed in this thesis except for the 3x3 frame with interior column removal where the DAF is 1.98. The DAF for bending moment is more likely to be relevant for collapse prevention when strength governs over stiffness. In the case where strength matters more than deflection, a DAF of 2 is too large for design applications.
6.2 Discussion

The four different analysis cases that can be used to estimate the progressive collapse of a structure each have their own advantages and disadvantages. The linear static analysis case is the quickest and simplest case to run and analyze. The linear dynamic analysis case is able to consider the real-time removal of a major structural component and take into account the dynamic effects of the removal. However, this analysis case does not account for material nonlinear behavior which can cause incorrectly calculated dynamic amplification for structures that exhibit large plastic deformations. Since nonlinear behavior is not considered in the case, the analysis time is quick. The nonlinear static analysis case considers the material nonlinear behavior but does not consider any damping effects. This analysis is overly conservative and could be time consuming due to the nonlinear analysis type which calculates a new stiffness matrix at each time step. The nonlinear dynamic analysis case is the most accurate analysis case to implement to assess progressive collapse of a structure. The nonlinear dynamic analysis considers the dynamic behavior as well as the material nonlinearity resulting in its most realistic results. However, it is also the most time consuming. Depending on the system properties of the computer the analysis is being run on, this analysis case can take upwards of 2 hours to fully run for the examples discussed in this thesis. It would take much longer for larger 2D models and especially 3D models.

According to some guidelines, the linear static analysis case and nonlinear static analysis case currently implement the same dynamic amplification factor. However, this is unnecessary because the dynamic amplification factor is included in the linear static analysis to compensate for its inability to predict dynamic or nonlinear effects on the structure. The dynamic amplification factor is included in the nonlinear static case to make up for its inability to predict any dynamic effects. Therefore, it is excessive to use the same dynamic amplification for both analysis cases. It is interesting to note that the linear dynamic analysis case requires no amplification factor even though the analysis is not capturing any of the nonlinear effects of progressive collapse.

It is difficult to assess a dynamic amplification factor for the nonlinear static analysis case. When a nonlinear static analysis is run without the dynamic amplification factor, the
analysis produces results that are essentially the same as the unamplified linear static analysis case. This is the case because the loads are not large enough to produce moments large enough to form plastic hinges in the frame. For most cases observed, the nonlinear static case with a dynamic amplification factor of 2 showed higher deflections and moments than the nonlinear dynamic case. In addition, the nonlinear static case occasionally showed that hinges surpassing immediate occupancy and life safety acceptance criteria meanwhile, the nonlinear dynamic analysis case showed those same hinges have only yielded. It is evident that the nonlinear static analysis case is very conservative.

Based on the different cases analyzed, it is clear that a dynamic amplification factor of 2 is a conservative value. It would be beneficial to determine a more accurate dynamic amplification factor to use with the linear static analysis case. A less conservative dynamic amplification factor could lead to more accurate designs preventing progressive design while keeping analysis time to a minimum. A consistent dynamic amplification factor was not determined from the scope of this analysis. Further research would have to be conducted to determine an actual value for a less conservative dynamic amplification factor.
6.3 Future Work

Further research would be required to determine a tangible value for a new amplification factor. The work described in this thesis was limited to a low rise moment frames. It would be necessary to look at how the dynamic amplification factor varies between moment frames and braced frames. Additionally, it would be essential to compute amplification factors for buildings of various heights. A comparison of amplification factors for low to medium rise buildings would aid in improving the conservativeness of the factor. It is unnecessary to analyze high rise buildings because the guidelines do not recommend using linear static analysis to assess progressive collapse of tall buildings. Additionally, it is possible that the guidelines are too conservative regarding this point. It would be beneficial to look at three dimensional buildings because in such a case, if one column is removed there is a plethora of other paths that the load could be transferred through. All these different tests would help determine a more accurate and less conservative dynamic amplification factor to be used with the linear static analysis case in order to assess the effects of progressive collapse in a building.
7 References


ASCE, "Minimum Design loads for Buildings and Other Structures", American Society of Civil Engineers (ASCE), Reston, VA, 2005


8 Appendix

8.1 2x3 Span Exterior Removed Column

Figure 49: Vertical Deflection at Free End for a 2x3 Exterior Column Removal

Figure 50: Moment in Member Above Removed Column for a 2x3 Exterior Column Removal
8.2 2x3 Span Interior Removed Column

**Figure 51:** Vertical Deflection at Free Node for a 2x3 Interior Column Removal

**Figure 52:** Moment in Member Above Removed Column for a 2x3 Interior Column Removal
8.3 2x4 Span Exterior Removed Column

Figure 53: Vertical Deflection at Free End for a 2x4 Exterior Column Removal

Figure 54: Moment in Member Above Removed Column for a 2x4 Exterior Column Removal
8.4 2x4 Span Interior Removed Column

Figure 55: Vertical Deflection at Free End for a 2x4 Interior Column Removal

Figure 56: Moment in Member Above Removed Column for a 2x4 Interior Column Removal
8.5 3x3 Span Exterior Column Removed

Figure 57: Vertical Deflection at Free End for a 3x3 Exterior Column Removal

Figure 58: Moment in Member Above Removed Column for a 3x3 Exterior Column Removal
8.6 3x3 Span Interior Removed Column

Figure 59: Vertical Deflection at Free End for a 3x3 Interior Column Removal

Figure 60: Moment in Member Above Removed Column for a 3x3 Interior Column Removal
8.7 3x4 Span Frame Exterior Removed Column

Vertical Displacement vs Time for 3x4 Exterior Removed Column

Figure 61: Vertical Deflection at Free End for a 3x4 Exterior Column Removal

Moment vs Time for 3x4 Interior Removed Column

Figure 62: Moment in Member Above Removed Column for a 3x4 Exterior Column Removal
8.8 3x4 Span Frame Interior Column Removal

**Figure 63: Vertical Deflection at Free End for a 3x4 Interior Column Removal**

**Figure 64: Moment in Member Above Removed Column for a 3x4 Interior Column Removal**
8.9 4x3 Span Frame Exterior Column Removed

Figure 65: Vertical Deflection at Free End for a 4x3 Exterior Column Removal

Figure 66: Moment in Member Above Removed Column for a 4x3 Exterior Column Removal
8.10 4x3 Span Frame Interior Column Removal

Vertical Displacement vs Time for 4x3 Interior Column Removed

Figure 67: Vertical Deflection at Free End for a 4x3 Interior Column Removal

Moment vs Time for 4x3 Interior Removed Column

Figure 68: Moment in Member Above Removed Column for a 4x3 Interior Column Removal
8.11 4x4 Span Frame Exterior Column Removed

**Vertical Displacement vs Time for 4x4 Exterior Column Removed**

![Graph showing vertical displacement vs time for a 4x4 exterior column removal.](image_url)

Figure 69: Vertical Deflection at Free End for a 4x4 Exterior Column Removal

**Moment vs Time for 4x4 Exterior Column Removed**

![Graph showing moment vs time for a 4x4 exterior column removal.](image_url)

Figure 70: Moment in Member Above Removed Column for a 4x4 Exterior Column Removal
8.12 4x4 Span Frame Interior Column Removed

Figure 71: Vertical Deflection at Free End for a 4x4 Interior Column Removal

Figure 72: Moment in Member Above Removed Column for a 4x4 Interior Column Removal