The Political Economy of Government Interventions in Financial Crises

by

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Abstract

Chapter 1 examines how political economy considerations affect the desirability of banking unions. It presents a model in which bank recapitalizations are carried out by rent-seeking policymakers. These policymakers face a trade-off between using public funds for needed recapitalizations and diverting them towards socially inefficient rents. In equilibrium, a banking union increases recapitalizations, but it can also increase rent-seeking and decrease consumer welfare. I consider two policy proposals for countering the reduction in welfare: better electoral accountability and limits on public debt. When used alone, neither policy can increase welfare for all countries in the banking union. When used together, the policies have complementary effects, and a Pareto improvement can be achieved in consumer welfare. Chapter 2 focuses on two key features of the bailout programs seen in the 2008-2009 financial crisis: first, the opposition of voters to these programs; and second, the implementation of a variety of interventions, ranging from targeted transfers that inject capital in particular institutions to untargeted transfers aimed at entire sectors. I argue that a shift towards untargeted transfers emerges in a political economy environment, when voters possess less information than the government about the shocks hitting the economy, and when firms can lobby the government for socially inefficient transfers. The model shows that the optimal incentives voters give to elected politicians lead to persistent effects of government interventions. Chapter 3 examines the optimal degree of centralization that can be achieved with respect to bailout policies when a central authority cannot supervise the entire banking system of the economy. Part of the banking system is supervised by a local authority that can observe local shocks and is biased towards local banks. The chapter presents a model of delegation in which a central authority can mandate the contribution of the local authority to bank bailouts as well as the size of the bailout fund. I derive conditions under which it is optimal for the local authority to be given full autonomy over bailout policies. The model shows that these conditions become more restrictive if the local authority can use public debt to increase the size of the bailout fund.

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Chapter 1

The Limits to Partial Banking Unions: A Political Economy Approach

1.1 Introduction

The recent banking and sovereign debt crises have renewed interest in creating common rules for government interventions in the banking sector. This has been particularly relevant for the Eurozone countries, given the large cross-border spillover effects of public bailouts. Naturally, the presence of such spillovers suggests that a banking union may deliver a Pareto improvement for all member countries. Domestic political economy constraints may, however, interfere with the functioning of such a supranational institution: once a banking union is in place, rent-seeking policymakers may divert resources towards socially inefficient rents. This raises the question of whether a banking union can improve consumer welfare and achieve a more efficient supranational coordination of government interventions.

The case of the Spanish savings and loan sector (the 'cajas')\(^1\) illustrates the role played by the politico-economic factor in the recent crisis. Spanish *cajas* were led by politically appointed policymakers.

\(^1\) Discussed in greater detail in Garicano (2012) and Cuñat and Garicano (2009).
executives, and the political pressures faced by these executives affected the types of loans that they extended. For example, regional governments used the cajas to fund projects that had little social benefit (e.g., airports with no flights, unused theme parks).\(^2\) In the absence of a banking union, the decisions to rescue undercapitalized cajas by merging them were made locally, and these mergers were based on political and regional motives rather than economic efficiency. These inefficient mergers led to the creation of larger troubled entities, increasing the cost of public bailouts and the pressure on public finances. Some of the public funds that were used to recapitalize the troubled cajas covered losses from large, unregulated payments taken by politically appointed board members just prior to the government intervention.\(^3\) The distortion introduced by political interference in the case of the Spanish cajas exemplifies the type of rent-seeking that this paper will examine.

This paper builds a model that captures the links between government recapitalizations and rent-seeking, and sheds light on the impact of a banking union in the presence of political economy distortions. I model a union of governments that are electorally accountable to voters, and that have policy objectives that differ from their voters'. Each government can carry out recapitalizations of distressed banks in its country, and these policies have cross-border spillover effects. The model considers a system of rules and transfers referred to as a 'partial banking union,' which centralizes intervention rules and facilitates cross-country transfers, but which leaves the decision of how to allocate bailout funds at the level of each country's government. Therefore, a partial banking union is an agreement that falls short of a full banking union because certain decisions – in this model, public recapitalizations – are not centralized. This framework captures some of the main features of the proposed European banking union, where member countries have reached agreement on a unique supranational supervisory authority (the Single Supervisory Mechanism), but recent proposals still leave the decision over bank recapitalizations with the national authorities in each country.\(^4\)

I embed a model of public liquidity provision in a principal-agent framework, in which in-

\(^3\) Ibid.  
\(^4\) A summary of progress on these proposals is provided by the German Ministry of Finance at http://www.bundesfinanzministerium.de/Content/EN/Standardartikel/Topics/Europe/Articles/2013-10-18-european-banking-union-takes-shape.html (accessed Nov 6, 2013).
centives must be provided to a non-benevolent policymaker who controls recapitalizations. The economy consists of two countries that form a union: a donor country that provides transfers and a home country that receives transfers. Households in both countries have endowments that they can either invest in banks or consume. To simplify, all banks are located in the home country and hold deposits from households in both countries. This creates a cross-country spillover effect of government interventions in the banking sector. Banks invest in risky projects funded by household deposits. Each period, banks’ projects are subject to a liquidity shock: a fraction of the projects become distressed and require reinvestment. When banks lack sufficient funds to reinvest, the home country policymaker has the option to intervene and recapitalize banks. But the policymaker faces a trade-off regarding the use of public funds: the public budget can also be used for political rents and non-financial public goods (e.g., infrastructure projects). The home and donor governments can form a partial banking union consisting of transfers and a proposed level of government intervention towards recapitalizations. Finally, the policymaker also faces an electoral constraint: citizens receive a random opportunity to replace the incumbent. All agents are assumed to have no commitment power, and policies and agreements are decided every period. The paper focuses on Markov Perfect Equilibria in this setup.

The first result of the model is that creating a partial banking union through a system of supranational rules and transfers can reduce consumer welfare. This happens because the banking union can give policymakers incentives to increase rent-seeking. In equilibrium, the contract between the two countries keeps the rent-seeking policymaker indifferent to participating in the banking union. This implies that, in a banking union, the country receiving transfers is required to increase government spending on recapitalizations and share some of the costs of higher recapitalizations. The policymaker can, however, divert public funds towards socially wasteful rents, and these rents cannot be observed separately from recapitalizations. Therefore, the required increase in recapitalizations also induces an increase in rent-seeking. More public funds are diverted towards rents at the expense of domestic public goods, and this leads to lower welfare in the receiving country.

The next set of results explores two possible resolutions to this political economy friction. I start by studying the role of better electoral accountability in increasing consumer welfare.
Electoral accountability is defined as voters' ability to threaten rent-seeking politicians with removal from office.\textsuperscript{5} Better electoral accountability allows voters to demand more public goods and services in order to keep an incumbent in power. In this model, it has two effects. First, it reduces the incentives for rent-seeking. Second, it forces the politician to choose policies closer to voters' preferences. This means spending more on both public goods and recapitalizations. Due to the cross-country spillovers, the benefits of bank recapitalizations do not fully accrue to domestic voters. Therefore, the level of recapitalizations preferred by voters is different from that preferred by the donor country. In the model, this second effect of improving electoral accountability makes recapitalizations costlier to the donor country, due to the higher pressure from voters to use more government funds for public goods rather than for recapitalizations of troubled banks. This means that higher transfers must be given to the receiving country in order to achieve a given level of recapitalizations. The consequence would be a reduction in the welfare of consumers in the donor country. Weak electoral or institutional control over politicians in the peripheral Euro countries has been indicated as one reason why cross-country transfers are difficult to achieve. These results, however, suggest that better electoral accountability for politicians in the receiving countries would not lead to higher welfare for the donor country, even if it increases the welfare of voters in the receiving country.

The next result highlights the role of fiscal rules. These are defined as supranational rules that constrain debt accumulation. Fiscal rules have the effect of reducing both overall spending and rents. While they reduce rents, they also constrain the ability of the policymaker to engage in desirable public spending - recapitalizations and public goods. The reason for this is that fiscal rules alone cannot restrict the spending on rents without also restricting spending in general. This results in both insufficient recapitalizations and insufficient public good provision in the country receiving transfers. Consequently, consumer welfare in the receiving country decreases. Although fiscal rules are beneficial for the donor country, they cannot increase welfare in the banking union because of the negative effect they have on the receiving country's welfare.

The model then shows that the negative welfare effects of the above two policies can be reversed if these policies are implemented \textit{together}, optimized for each other. Electoral account-

\textsuperscript{5}As in the models developed by Barro (1973) and Ferejohn (1986).
ability can be used to constrain the policymaker to reduce rent-seeking without decreasing spending on public goods and recapitalizations. Fiscal rules ensure that the higher spending on public goods is not done through increases in debt, but rather through larger decreases in rent-seeking. The outcome is that higher recapitalizations are achieved, while rent-seeking is controlled. Therefore, these two policies together can deliver a Pareto improvement over the case without a banking union.

The above results show how policies aimed at tackling one source of inefficiency can have negative welfare implications by augmenting other incentive problems. This seems particularly relevant for the Eurozone, which has not yet agreed upon a full banking union. The results suggest that these problems can be overcome through policies with complementary effects.

I also consider the way in which public debt affects how the donor country and the receiving country share the costs of recapitalizations. As public debt in the home country increases, the home country is more constrained in its ability to fund recapitalizations. Therefore, the donor country must take on a larger share of the cost of recapitalizations. This negatively affects both the equilibrium level of recapitalizations and the welfare of consumers in the donor country. The model shows that, as public debt increases, the benefits from forming a partial banking union shift more towards the home country and away from the donor country. This provides a framework for understanding why partial banking unions are harder to implement in high debt environments.

Related Literature. Several papers in this literature have analyzed the interplay between fiscal policy and monetary or financial integration. The main areas of focus within this literature include optimal fiscal policy coordination (Kehoe, 1987a; Chari and Kehoe, 1990; Beetsma and Lans Bovenberg, 1998), optimal fiscal and monetary policy (Dixit and Lambertini, 2001, 2003; Beetsma and Jensen, 2005; Gali and Monacelli, 2008), optimal fiscal rules in currency unions (Von Hagen and Eichengreen, 1996; Ferrero, 2009), and the role of fiscal transfers in providing efficient insurance within a currency union (Farhi and Werning, 2012b). These papers assume a benevolent government and focus on optimal policy, abstracting from any political economy distortions. Another strand of this literature emphasizes the role of political economy distortions in the context of fiscal or financial integration (Tabellini, 1990; Lohmann, 1993; Persson and
Tabellini, 1996a,b; Azzimonti et al., 2012). Whereas this strand focuses mainly on the effects of electoral institutions, my paper examines the effectiveness of supranational fiscal transfers and rules when the policymaker is motivated by rent-seeking incentives.

The motivation for this paper is akin to the discussion in Tabellini (1990) and Azzimonti et al. (2012), who show how fiscal or financial integration can lead to higher public debt due to political economy biases. In this paper, however, fiscal transfers lead to higher debt through the channel of higher incentives for current spending and rent-seeking, not due to lower costs of debt (as in Tabellini, 1990) or the aggregation of heterogeneous voter preferences (as in Azzimonti et al., 2012). Persson and Tabellini (1996a,b) study cross-country insurance and the effect of fiscal transfers on welfare under different political decision-making institutions, specifically direct voting versus bargaining. This paper complements their results by analyzing the effects of different levels of electoral accountability, not different institutions.

The modeling approach in this paper uses a principal-agent framework similar to those developed in Acemoglu (2005) and Acemoglu and Robinson (2006), which feature stochastic politician replacement costs, and in Yared (2010), which models electoral incentives as voters’ demand for a minimal utility level each period. The model also builds on the framework developed in Acemoglu et al. (2008) and Acemoglu et al. (2011) but differs from these models in two main ways. First, it focuses on Markov Perfect Equilibria as opposed to the best Subgame Perfect Equilibrium; and second, it considers an endowment economy without capital, but with public debt and supranational transfers and limits on spending and debt. Finally, this model links rent-seeking to recapitalizations using an approach similar to that of Milesi-Ferretti (2004). Supranational rules are imposed on spending measures which can differ from the true spending on recapitalizations, due to the presence of rents.

This paper also contributes to the larger political economy literature on public good provision with political economy distortions.6 Lizzeri and Persico (2001) and Besley and Coate (2003) focus on the effects of changes in voter electoral accountability on the government’s public good provision. This model complements their results by showing that, with rent-seeking

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6See Persson and Tabellini (2000) for a review.
politicians, better electoral accountability can result in lower public good provision and lower recapitalizations, even if voters are homogenous. The role of supranational controls over domestic policy is also discussed in Dewatripont and Seabright (2006). They present a mechanism by which higher electoral accountability can be detrimental to the goal of reducing wasteful spending, if no supranational controls are imposed. Their model is based on signaling by a politician whose type is unknown to voters, while in this model there is no private information and the politician has a direct preference for rent-seeking.

The effects and optimal form of fiscal rules has been the focus of a large literature, both theoretical (Milesi-Ferretti, 2004; Schmitt-Grohé and Uribe, 2007; Ferrero, 2009) and empirical (Von Hagen, 1991; Bayoumi and Eichengreen, 1995; Alesina and Bayoumi, 1996; Poterba and von Hagen, 1999; Von Hagen and Wolff, 2006). Several papers have studied these questions in environments with political economy distortions, including Battaglini and Coate (2008) and Azzimonti et al. (2010). In a model with heterogenous politicians, Besley and Smart (2007) show that fiscal limits can be desirable if they help voters select a good type of politician, and if this effect is larger than the induced increase in wasteful spending before the election. The role of fiscal rules in improving efficiency is also addressed in Bassetto (2006), which considers a rule that allows the government to issue debt only for the purpose of capital investments, and shows it can improve efficiency when policy is decided by majority rule. Considering the case of the European countries, Buiter et al. (1993) show through simulations that do not consider domestic political economy distortions, that fiscal rules are not desirable even in the presence of international spillovers. This paper contributes to the above results by showing that, with cross-country transfers, the desirability of supranational fiscal rules is linked to domestic electoral accountability. More specifically, restrictions to public debt are desirable in an environment with domestic rent-seeking policymakers, if current spending is constrained through domestic electoral accountability, so as to reduce the increase in wasteful transfers.

Finally, the relationship between fiscal discipline, decentralized decision-making, and public bailouts has also been addressed in the literature on fiscal federalism (Nicolini et al., 2002; Chari and Kehoe, 2007; Cooper et al., 2008; Sanguinetti and Tommasi, 2004a). Sanguinetti and Tommasi (2004a) and Chari and Kehoe (2007) show that fiscal rules may be optimal
when the central government or the central monetary authority lacks the power to commit to not bailing out regional governments. In this paper, the desirability of fiscal rules in a union emerges from their ability to reduce domestic rent-seeking, rather than their ability to achieve commitment. Cooper et al. (2008) show that, when the central government lacks commitment power, fiscal federalism is not desirable to autarky when there is a high correlation between shocks across regions. In this paper, regardless of the size of cross-country spillovers, the desirability of a banking union is determined by the existence of policy instruments that can limit the rent-seeking incentives of policymakers.

The rest of the paper is organized as follows. Section 1.2 presents the problem in a two-period model. Section 1.3 illustrates the main results of the model in the two-period setting. Section 1.4 gives the setup of the dynamic model. Section 1.5 presents the analysis of the model and the welfare effects of a partial banking union in the dynamic model. Section 1.6 analyzes the effects of higher electoral accountability and those of fiscal rules. Section 1.7 concludes, and the Appendix contains the proofs.

1.2 A Two-Period Model

This section presents a two-period model that illustrates the main results of the paper. It highlights the different driving forces of the model and the intuition behind the results. Later, the model is extended to a dynamic setting, which shows that the results continue to hold in a more general setting. Moreover, the dynamic model illustrates the effects of public debt accumulation, and leads to richer results regarding fiscal rules, beyond the baseline forces presented in the two-period case.

Consider a two-period economy, with \( t = 0, 1 \). The economy consists of two countries, a donor country and a home country, and a supranational authority which plays the role of a Principal that controls the interaction between the countries. Each of the two countries is made up of a continuum of mass 1 of identical households.
1.2.1 Households

At date 0, all households start with a perfectly diversified portfolio of risky projects, in the form of deposits in banks.\(^7\) Home households hold deposits \(\omega^H\), and donor households hold deposits \(\omega^F\). The assumption of different sizes for the deposits is made because this difference determines the size of the cross-country spillovers. The risky projects are owned by banks located in the home country, and the projects pay off at the end of period 0. At the beginning of period 0, an aggregate shock \(\theta \in (0, 1)\) is realized and observed by all agents. Following the shock, a fraction \(\theta\) of the project portfolio becomes distressed, and it pays off 0 unless additional funds \(x\) are reinvested, up to the original investment level \((x \leq \theta(\omega^H + \omega^F))\).\(^8\) I assume that banks have no access to a private borrowing market, so that reinvestment funds can only be provided by the government through public recapitalizations. Moreover, the reinvestment funds \(x\) can be supplied by the home country government only, while the donor government cannot directly recapitalize the banks in the home country. This feature is meant to capture the real world situation in which only the government of a country can use public funds to directly recapitalize institutions in that country. The liquidity shock therefore motivates the need for government intervention in this model. A key assumption is that reinvestment funds cannot be targeted, so both the home and donor households benefit from the reinvestment. This benefit is proportional to each country's share of deposits, where I denote by \(\sigma \equiv \frac{\omega^H}{\omega^H + \omega^F}\) the share of deposits held by the home country households. At the end of the period, the projects that continue after the shock yield a rate of return of \(R > 1\).

In the second period, all households hold safe deposits in banks, with values \(\omega^H\) for the home households and \(\omega^F\) for the donor households, and rate of return of 1. The assumption of a second period without aggregate shocks is made for simplicity. It creates a role for public debt in smoothing public good provision over time, as further shown below. The role of debt will be further motivated in the dynamic model.

Each period, households derive utility from private consumption equal to their deposit returns. They also derive utility from a domestic public good \(g^H\) provided by the government.

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\(^7\)An in-depth description of the banks is provided in the Appendix.

\(^8\)The liquidity shock is modeled as a simplified version of the one in the Holmström and Tirole (1998) model.
Their preferences are given by

\[ U^j(x, g^j, g^j_1) = u(R(1 - \theta)\omega^j + Rx^j) + w(g^j) + \beta \left(u(\omega^j) + w(g^j_1)\right), \]

where \( j = H, F, \) \( x^H = \sigma x, \) \( x^F = (1 - \sigma)x; \) \( \beta \in (0, 1) \) is the social discount rate and the inverse gross interest rate, and \( u(\cdot) \) and \( w(\cdot) \) are strictly concave, increasing, \( 0 < u'(0) < \infty, \) \( 0 < w'(0) < \infty, \) \( \lim_{g \to \infty} w'(g) = 0. \) Notice that both home and donor household utilities depend on the recapitalization \( x \) decided by the home government.

### 1.2.2 Donor Government

The donor country government’s preferences are assumed to be identical to the preferences of donor households. This implies that any political economy problems have been solved, to ensure that the government maximizes household utility:

\[ U^F(x, g^F, g^F_1) = u(R(1 - \theta)\omega^F + R(1 - \sigma)x) + w(g^F) + \beta u(\omega^F) + \beta w(g^F_1). \]

Each period, the donor government receives an endowment \( e^F. \) With this endowment, it can finance the domestic public good, and it can make transfers \( \tau \) to the supranational authority at date 0. The donor government does not have access to any storage technologies and cannot borrow or lend against the future. I make this assumption for simplicity, to limit the role of the donor government to only that of providing transfers. If transfers \( \tau \) are made, the donor government’s budget constraint at dates 0 and 1 is given by

\[ g^F + \tau \leq e^F, \]

\[ g^F_1 \leq e^F. \]

\(^9\)For ease of notation, I omit the subscripts for the period 0 variables, and keep only the subscripts for the period 1 policies.
1.2.3 Home Government

In the home country, government policy is decided by a rent-seeking politician, who maximizes a weighted sum of own utility from rents in period 0 and household utility (in both periods):

\[ V^H(r, x, g^H, g_1^H) = (1 - \gamma)v(r) + \gamma U^H(x, g^H, g_1^H), \]

(1.1)

where \( v(\cdot) \) is weakly concave and increasing, \( v' < \infty \), and \( \gamma \in (0, 1) \) represents the weight placed on household utility relative to rents.

The home government receives an endowment \( e \) each period and can take on one-period debt \( b_1 \) in period 0, at rate \( \beta \), with an exogenous lower limit \( b \) and upper limit \( b < e \). Assume period 0 starts with outstanding debt \( b = 0 \). The home government can also become part of a partial banking union in the current period. This involves receiving the transfer \( \tau \) from the supranational authority at date 0. In exchange, the government commits to an intervention level of \( \varepsilon \) towards bank recapitalizations. However, the spending on rents versus recapitalizations cannot be separately observed and verified by the supranational authority. Therefore, the intervention level \( \varepsilon \) can encompass both rents and recapitalizations; the required intervention level is satisfied as long as the following intervention constraint is satisfied:

\[ x + r \geq \varepsilon. \]

This constraint will never be slack, since the supranational authority will never prefer to set an intervention level below what the politician would choose in the absence of this required level. Such a choice will decrease recapitalizations, since the politician will always choose to balance the increase in \( x \) and \( r \). This result emerges because the politician’s utility is concave in both rents and recapitalizations, so any incentive to increase recapitalizations will also give the politician the incentive to increase rents. The only way for the supranational authority to increase recapitalizations is to increase the required intervention level beyond what the politician would prefer, and to accept an increase in both rents and recapitalizations.
The dynamic constraints for the home government in period 0 are:

\[ r + x + g^H \leq e + \beta b_1 + \tau, \quad (1.2a) \]
\[ r + x \geq \underline{x}, \quad (1.2b) \]
\[ b_1 \leq \bar{b}, \quad (1.2c) \]

and in period 1:

\[ g^H_1 \leq e - b_1. \quad (1.2d) \]

Since rents are discussed in relation to recapitalizations, they are assumed away in the second period. The dynamic model presented in the next section will consider the case of future rents as well, and discuss the implications of changes in debt on expected rent-seeking.

**Rent-seeking Process.** This reduced-form relationship between rents and recapitalizations can be motivated by the following rent-extraction process. The government can choose the degree of efficiency of its intervention in projects. The most socially efficient intervention provides reinvestment funds \( x \) for the distressed projects. The politician can also choose less efficient interventions. In this type of interventions, he provides reinvestment funds \( x \) but can also decide to expand the capacity of the project. Only the original project returns rate \( R \), while the expansion of the project has a rate of return of 1. Moreover, the proceeds from the expanded project go to the politician, in the form of political rents. A politician who values rents more will choose to engage in a more socially inefficient intervention scheme, in order to increase rents. The total intervention will be equal to \( x + r \), but only \( x \) will constitute true recapitalizations. A real-world motivation for this rent-extraction mechanism is provided by the example of the Spanish cajas mentioned in the introduction. Inefficient projects and payments were made as a consequence of political pressures, as described in the introduction. These correspond in the above model to expanding productive projects with extensions without added social value. The public funds used to recapitalize troubled banks were in part used to cover losses from these socially wasteful payments.
A transfer $\tau \geq 0$ and a level of intervention $x \geq 0$ are set by the supranational authority to maximize a weighted sum of home and donor household utilities, with weight $\eta$ on home households:

$$\max\{\eta U^H(x, g^H, g_1^H) + (1 - \eta)U^F(x, g^F, g_1^F)\} \quad (1.3)$$

A partial banking union requires the participation of both the home and donor governments. The donor government must agree to make the transfer $\tau$, and the home government must agree to implement the required intervention level in exchange for the transfer. Neither government can commit to participating in the union before the terms of the agreement are decided. Therefore, it is necessary that each government finds the banking union agreement to be preferable to autarky. The following participation constraints capture this requirement:

$$U^F(x^s, g^{Fs}, g_1^{Fs}) \geq U^F(x^0, g^{F0}, g_1^{F0}) \quad (1.4)$$

$$(1 - \gamma)u(r^s) + \gamma U^H(x^s, g^{Hs}, g_1^{Hs}) \geq (1 - \gamma)u(r^0) + \gamma U^H(x^0, g^{H0}, g_1^{H0}) \quad (1.5)$$

where $\{r^s, x^s, g^{Hs}, g_1^{Hs}\}$ are policy choices made under the agreement $(\tau, x)$, and $\{r^0, x^0, g^{H0}, g_1^{H0}\}$ are policy choices made in the absence of the agreement.

### 1.2.5 Timing

The timing of the model is as follows. In period 0, the supranational authority proposes a transfer $\tau$ and intervention level $x$. The donor government decides whether to accept the proposed agreement, and make transfer $\tau$, and the home government decides whether to accept the transfer in exchange for providing total intervention $x$. Finally, recapitalizations $x$, rents $r$, the domestic public good $g^H$, and debt $b_1$ are decided by the home government. In the second period, the governments provide the domestic public good, given the available budget after any debt repayments, and households consume the returns from second period deposits.
1.2.6 Politician's Problem

As a preliminary step, I assume that, absent transfers, the solution to the politician's problem is interior with respect to recapitalizations:

**Assumption 1** The following conditions are satisfied:

\[
\begin{align*}
\sigma R u'(R(1-\theta)\omega^H) &> \max\{w'(0), v'(0)\}, \\
\sigma R u'(R\omega^H) &< w'\left(e + \beta b - \theta (\omega^H + \omega^F)\right).
\end{align*}
\]

Assumption 1 states that positive recapitalizations are desirable for the politician. The second condition states that full reinvestment is never optimal, given the trade-off faced by the politician between recapitalizations and public good provision.

Consider the policy choices of the home government. The politician chooses \(\{r, x, g^H, b_1\}\) to maximize (1.1) subject to (1.2a)-(1.2d). To shorten notation in the rest of the analysis, define \(u^H(x, \theta) = u(R(1-\theta)\omega^H + \sigma Rx)\), and define \(u^F(x, \theta)\) analogously for the donor government. The following conditions emerge for an interior solution under no banking union, so ignoring constraint (1.2b) and setting \(T = 0\):

\[
\begin{align*}
(1-\gamma) v'(r^0) &= \gamma u^H(x^0, \theta), \\
r^0 + x^0 + g^H_0 &= e + \beta b_1^0.
\end{align*}
\]

Condition (1.6a) shows that the politician will choose rents \(r^0\) and recapitalizations \(x^0\) in order to equalize the marginal utilities from each of them. Since both rents and recapitalizations come at the same cost, conditions (1.6a) and (1.6b) show that any incentive to increase recapitalizations (for instance, through a higher government budget) will also give the politician the incentive to increase rents. Also from the first-order conditions to the politician's problem, it emerges that \(w'(g^H_0) = w'(g^H_1)\), where \(g^H_1 = e - b_1^0\). Therefore, the same level of public good \(g^H\) will be offered in both periods.
If the countries participate in a banking union, the following conditions come out of the politician’s maximization problem (given constraints (1.2a)-(1.2d)):

\[(1 - \gamma) v'(r^s) = \gamma u^{H'}(x^s, \theta), \quad (1.7a)\]
\[r^s + x^s \geq \bar{x}, \quad (1.7b)\]
\[r^s + x^s + g^{Hs} \leq e + \beta b^s + \tau. \quad (1.7c)\]

As before, recapitalizations and rents enter symmetrically in the constraints to the politician’s problem, as shown in (1.7b) and (1.7c). Therefore, any incentive to increase recapitalizations will also push the politician towards increasing rents, so that the marginal utilities from rents and recapitalizations are equal (as shown in constraint 1.7a). If \(\bar{x} > r^0 + x^0\), then the above constraints and the restriction \(\tau \geq 0\) imply that both rents and recapitalizations would be higher than the politician’s choices absent the agreement: \(r^s > r^0\) and \(x^s > x^0\). Finally, as before, the problem implies \(w'(g^{Hs}) = w'(g^{Hs})\) and \(g^{Hs} = e - b^s\), so the same level of public good \(g^H\) will be offered in both periods. From this last condition it follows that if the required increase in intervention is higher than the transfer, i.e., \(\tau - (\bar{x} - r^0 - x^0) < 0\), then the politician will also provide less public good \((g^{Hs} < g^{H0})\).

Given the policy choices made by each government, the supranational authority chooses a transfer \(\tau\) and an intervention rule \(\bar{x}\). If each government were bound to participate in the agreement, then the supranational authority would maximize (1.3) subject to the policy choices made by the politician, as described above. With its available instruments, the supranational authority can use the intervention rule to increase the level of recapitalizations \(x\) at the expense of lower domestic public good in the donor country or in the home country. It can use the transfer to provide the home government with more resources, the use of which is decided by the politician given the constraints described above.
1.2.7 Additional Assumptions

I assume additional restrictions to the model to ensure that positive transfers will be provided in equilibrium, so there is a reason for the banking union to exist. Second, I derive the necessary condition on the weight $\eta$ such that the participation constraint for the home government binds under a banking union.

First, I assume that the donor government has a sufficiently large endowment such that it always prefers to make transfers towards recapitalizations, even when $x$ is not binding.

**Assumption 2** The donor government endowment $e^F$ is sufficiently large so

$$u^F(x^0, \theta) \frac{\partial x^0}{\partial e} > w'(e^F),$$

where $x^0$ is the level of recapitalizations provided by the home government in autarky.

Assumption 2 says that the donor country benefits from positive transfers. It also implies that the supranational authority prefers to offer positive transfers. By making this stronger assumption, I ensure that transfers are desirable in the limit case in which $\eta = 0$, because they benefit the donor households, and not only the home households. Notice that the above condition is satisfied in the limit case $e^F \to \infty$, since this implies $w'(e^F) \to 0$. Given the continuity of $w'(e^F)$, this shows that indeed there exist values of endowment $e^F$ under which the condition of Assumption 2 is satisfied.

Next, I show there exist values of $\eta$ at which the participation constraint for the home government will bind in equilibrium. The following Lemma establishes this result

**Lemma 1** There exists $\eta^* \in (0, 1)$ such that $\forall \eta \leq \eta^*$, the participation constraint for the politician binds given the equilibrium policy $(\tau, x)$ set by the supranational authority to maximize (1.3) subject to constraints (1.4) and (1.5).
Proof. In the Appendix. ■

I restrict the analysis to the cases when $\eta \leq \eta^*$, as stated in the following assumption.

**Assumption 3** The parameter $\eta$ satisfies $\eta \leq \eta^*$, with $\eta^*$ as defined in Lemma 1.

Assumption 3 reduces the problem to a case in which the supranational authority places higher weight on the donor country than on the receiving country. In the limit case, when $\eta = 0$, the supranational authority can be thought of as the donor country setting the conditions for a loan to the home country. Then, the result of a binding participation constraint for the politician emerges immediately given the supranational authority’s problem.

### 1.3 Household Welfare under a Partial Banking Union

The following result captures the main inefficiency of the model.

**Proposition 2** Suppose Assumptions 1-3 hold. A partial banking union lowers household utility in the home country.

The intuition for this result is as follows. Under Assumptions 2 and 3, the supranational authority sets $\tau > 0$ and a binding intervention rule $x$. Then, the politician increases both rents and recapitalizations as a response to the binding limit $x$. The increase in rents implies that $v(r^s) > v(r^0)$. Under the binding participation constraint, resulting from the assumption about $\eta$, the politician gets the same utility under the partial banking union, with the higher rents, as he does without the partial banking union, with lower rents. The implication is that the supranational authority can get the politician to decrease the domestic public good compared to the outside option. To see this, notice that the binding participation constraint for the politician is given by

$$(1 - \gamma)v(r^s) + \gamma U^H(x^s, g^Hs, g^H_1) = (1 - \gamma)v(r^0) + \gamma U^H(x^0, g^H0, g^H_1).$$
Since $v(r') > v(r^0)$ and $u^H(x^s, \theta) > u^H(x^0, \theta)$, the above implies

$$U^H(x^s, g^{H_1}, g^{H_2}) < U^H(x^0, g^{H0}, g^{H0}).$$

This result shows that the supranational authority is willing to accept some increase in rent-seeking in order to achieve an increase in donor household utility. This increase in rent-seeking comes at the cost of lower home household utility, as home households have to suffer the cost of higher rents. The decrease in the utility of home households cannot be avoided because the supranational authority does not have the right instruments to deter the politician from engaging in more rent-seeking under the banking union. Proposition 2 opens the question of whether two different policy instruments - electoral accountability and fiscal rules- can revert the decrease in household welfare.

1.3.1 Role of Electoral Accountability

The first policy considered is that of domestic electoral accountability. With access to appropriate rewards and punishments, voters could develop a mechanism that limits the discretion of the politician and delivers the first-best. However, electoral accountability mechanisms in the real world are limited, and generally only involve removal from office. I consider a limited form of electoral accountability, in which voters can decide politician removal at the end of the first period, after policies are chosen, but before consumption happens. If removed, the incumbent gets a minimum attainable utility $V \to -\infty$ in the next period and is replaced with a politician chosen at random from a pool of identical politicians. A key limiting factor for voters is that, due to the timing of elections, the replacement decision is made after debt has been decided by the incumbent. Therefore, elections can offer the incumbent ex-post incentives, but cannot affect the policy choices ex-ante.\textsuperscript{11}

The electoral mechanism described above is represented by the following electoral constraint,

\textsuperscript{11}The model assumes that voters make the replacement decision collectively, and that they have solved any collective action problems ahead of the decision.
which reflects the problem faced by voters:

\[ u^H(x, \theta) + w(g^H) + \beta w(e - b_1) \geq \chi^V + \beta w(e - b_1), \]

where \( \chi^V \) an exogenous benefit from removal, described further below.

First, the above constraint highlights the fact that elections happen at the end of the first period, after debt has been decided, so replacing the incumbent with another politician does not change the policy outcomes in the second period. The reason why voters might still choose to replace the incumbent is because they receive a net benefit from replacement in the current period, denoted above by \( \chi^V \). The reelection strategy from the perspective of the voters is to replace the politician whenever their utility in period 0 is below \( \chi^V \). Any other strategy would not be credible, given that voters cannot punish the politician until the end of the first period, and they have no instruments for punishment in the second period. For the politician, the electoral incentives imply that he must provide voters with household utility equal to at least \( \chi^V \) in order to stay in power. Since being removed from power is strictly worse for the politician than continuing in the second period, he will satisfy the electoral constraint in equilibrium.

To better understand the electoral constraint, notice that it represents a present-bias in the behavior of voters. Voters demand more utility in the current period at the expense of higher debt, and therefore lower utility in the future. The assumption of an exogenous level of utility demanded by voters is not, however, inconsistent with rationality in this model, due to the timing of elections. The assumption of an non-pecuniary benefit of reelection has been made in the political economy literature (for example, in Yared (2010)). The timing of elections at the end of the period, which justifies why voters cannot punish the politician for taking on higher debt, has been used in Acemoglu (2005), Besley (2007) and is further discussed in Ch.4 of Persson and Tabellini (2000). The following replacement mechanism provides further motivation for \( \chi^V \). Assume voters derive a non-pecuniary benefit \( B \) from replacing the incumbent; however, replacement is costly, because the process of changing governments leads to delays in policy implementation. These delays are costlier if the government is doing more socially beneficial spending. I capture this by assuming that voters have to pay a utility cost equal to a fraction \( \phi \) of their current utility in order to replace the incumbent. Then, voters will replace
the politician only if \( B > \phi \left( u^H(x, \theta) + w(g^H) \right) \). This generates an electoral constraint for the politician: the incumbent is replaced unless voters receive a utility level of at least \( \chi^V \equiv B/\phi \).

With electoral accountability but without the banking union, the politician in the home country chooses \( \{r^0, x^0, g^{H0}, b^0_1\} \) to maximize (1.1) subject to (1.2a) and (1.2d) with \( \tau = 0 \), and the electoral constraint (1.8). I assume that the electoral demands made by voters, as captured by \( \chi^V \), are sufficiently small such that the incumbent could offer a feasible set of policies in the current period to satisfy the voter demands and get reelected; otherwise, the electoral constraints would be irrelevant for the policy choices made in the first period.

The binding electoral constraint modifies the optimal choices of the politician compared to the baseline case. In response to voters' electoral demands, the politician optimally increases the provision of both recapitalizations \( x^0 \) and public good \( g^{H0} \). Then, given the first-order conditions to the politician's problem and the budget constraint, the politician responds to the electoral demands by also increasing debt \( b^0_1 \) and decreasing rents \( r^0 \).

With a partial banking union, the politician faces the additional constraint (1.2b). Depending on the relative strength of electoral accountability, measured by the size of \( \chi^V \), several cases could emerge: very strong electoral accountability would make a banking union unnecessary, while very weak electoral accountability would make the electoral constraint not bind under the banking union.\(^{12}\) For the rest of this analysis, I consider the case in which the electoral demands are sufficiently high such that they continue to bind under a partial banking union, for any transfer \( \tau \) and corresponding equilibrium intervention rule \( \pi(\tau) \). Also, the analysis will be restricted to values of \( \chi^V \) that are still sufficiently low to make a banking union necessary.

**Assumption 4** The benefit from politician removal, \( \chi^V \), is sufficiently large for the electoral constraint to bind: \( u^H(x^*, \theta) + w(g^{H*}) < \chi^V \), where \( x^* \) and \( g^{H*} \) are the politician's choices from maximizing (1.1) given constraints (1.2a)-(1.2d), where the transfer \( \tau \) and corresponding equilibrium intervention rule \( \pi(\tau) \) are set in the absence of the electoral constraint.

\(^{12}\) For very low levels of \( \chi^V \), at which the electoral constraint still binds, the required increase in recapitalizations imposed under the banking union could increase voter utility in the current period sufficiently to make the electoral constraint redundant. This requires the special case in which the increase in utility due to recapitalizations is larger than the decrease in the period 0 utility from the public good, such that: \( [u^H(x^*, \theta) - u^H(x^0, \theta)] - [w(g^{H0}) - w(g^{H1})] \geq 0 \) and \( u^H(x^*, \theta) + w(g^{H1}) \geq \chi^V \).
The problem for the supranational authority is to maximize (1.3) subject to the participation constraints for both the home and donor governments, where the domestic policies are decided by the politician given his maximization problem. Assumptions 2 and 3 imply that in equilibrium $\tau > 0$ and $x > r^0 + x^0$. Then, a banking union leads to an increase in rents in the first period, due to the same forces as in the case without electoral accountability. Since the politician's participation constraint binds in equilibrium, the politician is indifferent to participating in the agreement, and so, the increase in rents under the banking union implies home household utility must decrease. The result and intuition from Proposition 2 are therefore upheld even when politicians are electorally accountable to voters. The intuition is that electoral accountability can guarantee more socially beneficial spending in the first period, but it cannot prevent the politician from borrowing more. Thus, the banking union still allows the politician to rent-seek more than under no banking union. The additional funds needed in order to also satisfy the voter demands for more public good are taken from the future, leading to higher debt and less public good in the second period.

The effects of electoral accountability are also reflected in the utility of the donor households.

**Proposition 3** Suppose Assumptions 1-4 hold. If the share $\sigma$ of deposits held by home households is sufficiently small, then a partial banking union in which politicians are electorally accountable to voters lowers donor household utility compared to a partial banking union without electoral accountability.

**Proof.** In the Appendix. 

The intuition for this result is as follows. Electoral accountability has two opposing effects on the utility of donor households. First, public debt increases in the home country, which has a negative effect on donor household utility. The electoral constraint forces the home politician to increase home household utility in the first period, in order to get reelected. Since voters decide removal at the end of the period, after debt has been decided, their electoral decision does not affect next period's utility. The timing of elections therefore allows the politician to increase household utility in the first period at the cost of higher debt. The higher debt makes it costlier for the supranational authority to set a binding intervention rule, because such a
rule requires even more debt to be taken on. Since it is harder to make the politician take on more debt, more of the cost of interventions shifts to the donor country. Second, the electoral constraint has a positive effect on donor households utility because of higher recapitalizations. Voters demand higher utility in the first period, and one way to satisfy their demands is through higher recapitalizations. The size of $\sigma$ then determines whether the negative effect of electoral accountability on donor country utility dominates the positive effect. If the share of deposits held by home households is sufficiently small, then any positive effect on recapitalizations coming from voter demands is small compared to the negative effect of higher debt. In other words, even if voters increase recapitalizations through electoral accountability, this increase is small compared to the higher cost of debt associated with their electoral demands, which makes any additional increases in recapitalizations harder to achieve. These costlier recapitalizations for the donor country lower donor household welfare compared to the case without electoral constraints.

The effect on donor household welfare can be seen more easily in the following example. Assume that $\eta = 0$ and that full recapitalizations are always preferred by the donor households. Then, an increase in electoral accountability in the home country implies that full recapitalizations require more transfers than before, since the politician now finds it more difficult to increase debt in order to finance part of these recapitalizations. Since, the politician has the outside option of not participating, he must be offered higher transfers in order to keep him in the union. The donor households then receive full recapitalizations, but must provide higher transfers to pay for them, which decreases their utility.\textsuperscript{13}

1.3.2 Role of Fiscal Rules

The second policy instrument that could be used to reduce rent-seeking is a limit on increases in the public debt. Consider the baseline setup, without electoral accountability. As discussed above, any required increase in intervention leads the politician to increase public debt, in order to smooth the costs over both periods. Fiscal rules can help limit the degree to

\textsuperscript{13}This example (as the rest of the model) assumes the donor government’s endowment is large enough to fully finance recapitalizations after the increase in electoral control.
which increases in rent-seeking in the first period can be financed at the expense of less domestic
public good in the second period. The type of fiscal rules considered are limits to how much
public debt can be increased. Fiscal rules are modeled by assuming that the supranational
authority can choose public debt $b^H_1$ under the partial banking union.

Under a partial banking union in which the supranational authority also chooses public
debt, the problem for the politician is to choose $\{x^s, g^{Hs}, r^s\}$ given the budget constraint and
the restriction on rent-seeking each period. This leads to the following first-order conditions
and budget constraints:

\begin{align}
(1 - \gamma) v'(r^s) &= \gamma u^H(x^s, \theta) = \gamma w'(g^{Hs}), \quad (1.9a) \\
\gamma r^s + x^s + g^{Hs} &\leq e + \beta b^H_1 + \tau, \quad (1.9b) \\
r^s + x^s &\geq \bar{\epsilon}, \quad (1.9c) \\
g^{Hs}_1 &\leq e - b^s_1. \quad (1.9d)
\end{align}

The problem for the supranational authority is to choose transfer $\tau$, intervention rule $\bar{\epsilon}$, and
debt $b^H_1$, to maximize (1.3) subject to conditions (1.9a)-(1.9d). Since the only use for debt in
the second period is the provision of public good $g^{Hs}_1$, the supranational authority will find it
optimal to choose the same level of debt as the politician. In doing so, it smooths the cost of
interventions over the two periods. Therefore, the analysis from Proposition 2 carries through
even in the presence of fiscal rules.

**Proposition 4** Suppose Assumptions 1-3 hold, and there are fiscal rules regarding public debt.
A partial banking union lowers household utility in the home country.

This result emerges because the second period is simply a consumption period, in which
debt only affects the home country consumption. The role of debt is to balance public good
provision between the first and second periods. Therefore, the objectives of the supranational
authority and those of the politician with respect to debt coincide. In the dynamic model
presented in the next section, this framework is enriched by having the supranational authority
and the politician value debt differently, due to the additional effects of debt on the future utility of the donor households.

1.3.3 Fiscal Rules and Electoral Accountability as Incentive Complements

Consider adding electoral accountability to the setup described above, in which the supranational authority controls public debt. In this case, the politician faces the additional constraint (1.8). The supranational authority chooses transfer $\tau$, intervention $x$, and debt $b_1$ to maximize (1.3) subject to the home government's policy choices and the participation constraint for each government. By controlling debt, the supranational authority can determine the utility of the politician in the second period. Voters, through the electoral demands $\chi^V$, determine the household utility in the first period. Therefore, the level of rent-seeking in the first period becomes a residual, determined by the constraints imposed by voters and the supranational authority. I assume, as above, that the necessary conditions are satisfied given the chosen parameters, such that the participation constraint for the politician binds in equilibrium. By entirely controlling the government budget, the supranational authority can force the politician to change his choice of rents in order to both satisfy the intervention rule and provide voters with enough household utility to guarantee reelection. As in the case without electoral constraints, the supranational authority and the politician value debt in the same way. This is due to the simple structure of the problem in the second period. Therefore, debt will not change compared to the case when the politician controls it. However, since the supranational authority implicitly controls rents, conditional on the politician's participation, it will set the intervention rule $x$ so that rents are minimal and the politician still participates in the agreement, i.e., the following condition holds:

$$
(1 - \gamma)v(r^s) + \gamma U^H(x^s, g^{Hs}, \theta^H_1) = (1 - \gamma)v(r^0) + \gamma U^H(x^0, g^{H0}, \theta^H_0) .
$$

(1.10)

Since public debt does not change under the banking union ($b^*_1 = b^0_1$) and, by Assumption 4, the electoral constraint binds both under the banking union and in the outside option, $w(g^{Hs}) + u^H(x^s, \theta) = w(g^{H0}) + u^H(x^0, \theta) = \chi^V$, this implies rents must be equal to their value
in the outside option: \( r^s = r^0 \). A sufficiently high intervention rule \( x \) can be set to achieve this result. This result is summarized in the following proposition.

**Proposition 5** Suppose Assumptions 1-4 hold, there is a fiscal rule over public debt, and politicians are electorally accountable to voters. A partial banking union does not lower household utility in the home country, and it increases household utility in the donor country, compared to the autarky allocation. Therefore, a partial banking union achieves a Pareto improvement.

Proposition 5 shows how fiscal rules and electoral accountability can act as incentive complements in controlling rent-seeking, and therefore achieving an increase in welfare without decreasing the utility of home consumers. Fiscal rules cannot by themselves increase household utility because they reduce overall government spending in the first period. This decreases rents and household utility. Electoral accountability cannot reduce rent-seeking under the banking union because voters cannot completely remove the rent-seeking incentives given to the politician by the banking union. However, taken together, fiscal rules and electoral accountability act as complements in reducing rents without allowing the politician to reduce socially beneficial spending.

As illustrated by the above results, the two-period setting does not allow for a richer role of fiscal rules in determining household welfare. This happens because in this setting debt has only a limited role, that of determining the domestic public good in the second period. The next section develops this framework further in a dynamic context, in which debt affects the future utilities of both donor and home households. This gives rise to a difference in incentives between the supranational authority and the politician, which allows for additional insights in the role of fiscal rules. Moreover, it provides a setting in which a stronger result can be obtained about the increase in home household welfare under a banking union.

**1.4 The Dynamic Model**

This section extends the results from the two-period model to an infinite horizon economy. Moreover, it highlights additional insights introduced by the dynamics of public debt. Debt
accumulation has different effects on the continuation utilities of home and donor households, which implies that the supranational authority's preferences over debt are different from those of the politician, an aspect that was not captured in the two-period model. This difference means that fiscal rules can lead to a different path of debt than in the absence of such rules. The dynamic model also shows how the costs and benefits of a banking union depend on the evolution of public debt in the home country. As public debt increases, the benefits from a banking union accrue more to the home country, while the donor country is forced to bear more of the costs.

1.4.1 Environment

Consider a similar setup to the one described in the two-period model. Time is discrete, with periods $t = 0, ..., \infty$, and discount rate $\beta \in (0, 1)$ for all agents. Each period, the households and the governments receive endowments $\omega^j$ and $e^j$, $j = H, F$. A more detailed description of the household problem is provided in the Appendix, where the household deposits each period are modeled as an endogenous choice between direct consumption and investment in risky projects.\(^{14}\) As before, banks invest the deposits in risky projects. These projects are subject to liquidity shocks: an aggregate i.i.d. shock $\theta_t$ is realized, $\theta_t \in \Theta = \{\theta^0, ..., \theta^N\}$ with probability $f^i$, and $\theta^0 > 0$, $\theta^N < 1$, and $\theta^i < \theta^k$ for $0 \leq i < k \leq N$.\(^{15}\) Following the shock, a fraction $\theta_t$ of the project portfolio becomes distressed, and it pays off 0 unless additional funds $x_t$ are reinvested by the home government, up to the original investment level.\(^{16}\) In this model, banks serve as a vehicle for pooling together the household endowments and investing them in projects. The banks' goal is to maximize expected household utility, where the preferences of households each period are given by: $u(R(1 - \theta_t)\omega^j + Rx_t^j) + w(g_t^j), j = H, F$, and $x_t^H = \sigma x_t, x_t^F = (1 - \sigma)x_t$.

The donor government can decide each period whether to accept or reject the supranational

\(^{14}\)For the purposes of simplicity, the necessary assumptions are made such that households decide to invest their entire endowment in deposits each period.

\(^{15}\)The assumption that $\theta^0 > 0$ is made for simplicity. If $\theta^0 = 0$ is assumed, then public interventions would happen only if $\theta$ is positive, and the analysis would be restricted to the positive realizations of $\theta$.

\(^{16}\)This type of technological liquidity constraint, which requires reinvestment before the project pays out, is similar to the one modelled in Holmström and Tirole (1998) and Farhi and Tirole (2012a), with the main modification that the current model considers the three stages of the project - investment, reinvestment and payoff - within the same electoral period.
agreement offered that period, and this decision is denoted by \( \varphi^F \in \{0, 1\} \), with \( \varphi^F = 1 \) for acceptance. The donor government derives utility

\[
J_0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ u^F(x_t, \theta_t) + w(e^F - \tau_t) \right].
\]

The home government also decides participation in the agreement each period, denoted by \( \varphi \in \{0, 1\} \), and derives utility: \(^{17}\)

\[
V_0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ (1 - \gamma)\nu(r_t) + \gamma \omega(g_t^H) + \gamma u^H(x_t, \theta_t) \right].
\]

The electoral process is extended in the following way. Each period, voters face a net stochastic benefit of politician replacement \( \chi \in \{0, \chi^V\} \), \( \Pr(\chi = \chi^V) = \pi \). This process allows for variation in the benefit to voters from incumbent removal. \(^{18}\) A real world correspondence for this assumption are periods in which elections are held versus periods in which there are no elections; the random shock also allows us to capture other ways through which voters may express demands beyond regular elections, for example protests, recall elections etc. At the end of each period, the representative voter can decide to replace the politician, a decision denoted by \( \rho_t \in \{0, 1\} \) with \( \rho_t = 1 \) for replacement. If replaced, the incumbent receives the minimum attainable continuation utility, \( V \to -\infty \).

Lastly, the supranational authority offers an agreement \((\tau_t, x_t)\) each period, and derives the following utility:

\[
\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \eta (u^H(x_t, \theta_t) + w(g_t^H)) + (1 - \eta) (u^F(x_t, \theta_t) + w(e^F - \tau_t)) \right],
\]

with the notation \( u^H(x_t, \theta_t) \) and \( u^F(x_t, \theta_t) \) used as in the two-period model.

\(^{17}\) An implicit feature of this specification is that the model allows for rents to be extracted even when \( x_t = 0 \). This would be equivalent to a limit case in the motivation provided for rent-seeking in the two-period model: the project would be inefficiently expanded even when true recapitalization funds are not provided. \(^{18}\) This assumption of a stochastic replacement with only two possible values for the replacement benefit can be seen as a simplified version of the models with stochastic replacement costs developed in Acemoglu (2005) and Acemoglu and Robinson (2006).
1.4.2 Timing

At each date $t$, the timing of events is as follows:

1. The households receive their respective endowments $\omega^H$ and $\omega^F$, and the governments receive endowments $e^H$ and $e^F$;

2. Banks make investments in projects;

3. Shocks $\theta_t$ and $\chi_t$ are realized and observed by all agents;

4. The supranational authority offers an agreement $(T_t, x_t)$ first to the donor country, then to the home country, and each government decides whether to accept or reject it;

5. The home government decides policies $\{x_t, g_t^H, r_t, b_{t+1}\}$;

6. Voters make politician replacement decision $\rho_t$; if $\rho_t = 1$, the incumbent is replaced with an identical politician.

1.4.3 Equilibrium Concept

I consider the pure strategy Markov Perfect Equilibria of this game, in which strategies only depend on the current state of the world and not on the entire history of the game. The current state of the world in period $t$ consists of the outstanding debt $b_t$, the liquidity shock in the current period $\theta_t$, and the benefit from removal, $\chi_t$. A Markov Perfect Equilibrium (MPE) is defined as a set of strategies $\{(\tau_t, x_t), \theta_t, x_t, g_t^H, r_t, b_{t+1}, \rho_t\}$ such that these strategies depend only on the current payoff-relevant state of the economy $\{b_t, x_t, \theta_t\}$ and on the prior actions within the same period as described in the timing of events. Therefore, an MPE is given by a set of strategies $\{\tau_t(b_t, x_t, \theta_t), x_t(b_t, x_t, \theta_t), g_t^H(b_t, x_t, \theta_t), r_t(b_t, x_t, \theta_t), b_{t+1}(b_t, x_t, \theta_t), \rho_t(b_t, x_t, \theta_t)\}$, where for notational simplicity I do not explicitly introduce the dependence of each strategy on the actions already taken in the same period.$^{19}$

$^{19}$Formally, the set of strategies is written as $\{\tau_t(b_t, x_t, \theta_t), x_t(b_t, x_t, \theta_t), g_t^H(\tau_t, x_t, x_t, b_t, x_t, \theta_t), \theta_t(\tau_t, x_t, x_t, b_t, x_t, \theta_t), r_t(\tau_t, x_t, x_t, b_t, x_t, \theta_t), b_{t+1}(\tau_t, x_t, x_t, b_t, x_t, \theta_t), \rho_t(\tau_t, x_t, x_t, x_t, g_t^H, r_t, b_{t+1}(\tau_t, x_t, x_t, \theta_t))\}$. 

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The focus on Markovian equilibria excludes any form of "consensual equilibria"\textsuperscript{20} in which the voters and the politician can use trigger strategies conditioned on the past realization of the investment shock $\theta$ or the electoral shock $\chi$. This restriction allows us to focus on the equilibria in which voters have limited means of punishing the incumbent, and electoral accountability is an imperfect tool for disciplining the incumbent.

The above framework with separable utilities functions, discount factor $\beta < 1$ and bounded instantaneous utilities (given to the maximum attainable resources $e + \beta \delta$) satisfies the conditions for the existence of a Markov Perfect Equilibrium to this game.\textsuperscript{21}

1.5 Analysis

1.5.1 Voters' Problem

The problem can be analyzed by studying each agent's problem, in the opposite order of each period's moves. Therefore, consider first the problem for the voters. As in the two-period model, voters can only punish the politician at the end of each period, after debt has been decided. The extent of the punishment depends on the benefit from removal. If $\chi = 0$, there is no benefit of removal, and the politician cannot be credibly constrained by voters. If $\chi = \chi^V$, voters demand at least utility $\chi^V$ in order to reelect the politician. Given the timing of elections, after debt has been decided, and given the restriction to Markov Perfect Equilibria, voters do not have the ability to offer ex-ante incentives to the politician. Electoral accountability is therefore limited to ex-post punishing at the end of each period.

1.5.2 Home Government's Problem

Next up, the politician decides domestic policy in the home country, given the agreement offered by the supranational authority. Each period, the state of the economy can be summarized by the outstanding debt $b$. The reelection benefit $\chi$ and the shock $\theta$ are observed before policy is decided. Let $V(b, \chi, \theta, \tau, x)$ denote the maximum expected utility for the politician at

\textsuperscript{20}As defined in Acemoglu (2005).
\textsuperscript{21}By Theorem 13.2 in Fudenberg and Tirole (1991)
the beginning of a period in which the state is given by \((b, \chi, \theta, r, x)\). The politician chooses a policy vector \(\alpha = \{r, x, g^H, b'\}\) with \(x \geq 0, g \geq 0, r \geq 0\), and a participation decision \(\varrho \in \{0, 1\}\) to solve the following problem:

\[
V(b, \chi, \theta, r, x) = \max_{\alpha, \varrho} \{(1 - \gamma)v(r) + \gamma w(g^H) + \gamma u^H(x, \theta) + \beta \mathbb{E}_{\chi', \theta'} [V(b', \chi', \theta', r'(b', \chi', \theta'), x'(b', \chi', \theta'))]\} \quad (1.11)
\]

s.t.

\[
\begin{align*}
    r + x + g^H & \leq e + \varrho r + \beta b' - b, \quad (1.12a) \\
    r + x & \geq \varrho x, \quad (1.12b) \\
    w(g^H) + u^H(x, \theta) & \geq \chi, \quad (1.12c) \\
    b' & \in [b, \bar{b}], \quad (1.12d) \\
    x & \leq \theta (\omega^H + \omega^F). \quad (1.12e)
\end{align*}
\]

Constraint (1.12a) is the resource constraint of the economy. Constraint (1.12b) is the required intervention \(x\) as part of the partial banking union. Inequality (1.12c) is the minimal voter utility that must be provided for the politician to stay in power. Finally, conditions (1.12d) and (1.12e) give the limits on debt and recapitalizations, respectively. The problem assumes that the agreement offered to the politician will be enforced if the politician accepts it. If the donor government does not accept the terms of an agreement offered by the supranational authority, then in that period the politician will not be offered an agreement consisting of positive transfers (the agreement will be \((0, 0)\) in such a case).

If the agreement is accepted in the current period, the politician’s problem can reduced to the case where \(\varrho = 1\) in all periods, given the equilibrium strategy of the supranational authority. Therefore, the utility of the politician from participating in the agreement is
\[ V(b, \chi, \theta, \tau, x) = \max_{\alpha} \{ (1 - \gamma)v(r) + \gamma w(g^H) + \gamma u^H(x, \theta) \]
\[ + \beta \mathbb{E}_{X', \theta'} [V'(b', \chi', \theta', \tau'(b', \chi', \theta'), x'(b', \chi', \theta'))] \} \] (1.13)

s.t.
\[ r + x + g^H \leq e - b + \tau + \beta b', \] (1.14a)
\[ r + x \geq x, \] (1.14b)
\[ w(g^H) + u^H(x, \theta) \geq \chi, \] (1.14c)
\[ b' \in [b, \bar{b}], \] (1.14d)
\[ x \leq \theta (\omega^H + \omega^F). \] (1.14e)

The constraints are the equivalents of constraints (1.12a)-(1.12e) with \( \varphi = 1 \). The supranational authority is expected to follow the equilibrium policy functions in all future periods, while this period’s agreement \( (\tau, x) \) can be a deviation from that.

If the politician does not participate in the agreement in the current period, let \( \alpha^{out} = \{ x^{out}, r^{out}, g^{H, out}, b^{out} \} \) denote the vector of policies chosen by the politician in the current period. The outside option for the home government is

\[ V^O(b, \chi, \theta) = \max_{\alpha^{out}} \{ (1 - \gamma)v(r^{out}) + \gamma w(g^{H, out}) + \gamma u^H(x^{out}, \theta) \]
\[ + \beta \mathbb{E}_{X', \theta'} [V'(b^{out}, \chi', \theta', \tau'(b^{out}, \chi', \theta'), x'(b^{out}, \chi', \theta'))] \} \] (1.15)

s.t.
\[ r^{out} + x^{out} + g^{H, out} \leq e + \beta b^{out} - b, \] (1.16a)
\[ w(g^{H, out}) + u^H(x^{out}, \theta) \geq \chi, \] (1.16b)
\[ b^{out} \in [b, \bar{b}], \] (1.16c)
\[ x^{out} \leq \theta (\omega^H + \omega^F). \] (1.16d)
The above problem accounts for the effects of the public debt on the choices made by the supranational authority in the future periods. Since the politician stays out of the banking union in the current period, the outside transfer is not received and there is no bound on current intervention.

The utility of the donor country in case of no agreement this period is given by

\[ J^O(b, \chi, \theta) = u^F(x^{out}, \theta) + w(e^F) + \beta \mathbb{E}_{\chi', \theta'} \left[ J(b'^{out}, \chi', \theta', \chi', \theta', x'(b'^{out}, \chi', \theta')) \right] \]  

(1.17)

### 1.5.3 Supranational Authority’s Problem

Lastly, the supranational authority seeks to maximize a weighted sum of household utilities.

The supranational authority chooses to offer a transfer-rule pair \((r, x)\), \(x \geq 0, r \geq 0\), given the policies that will be chosen by the politician according to program (1.13) and the outside options described by \(V^O(b, \chi, \theta)\) and \(J^O(b, \chi, \theta)\). Denote the politician’s choices by \(\{g^H(b, \chi, \theta, \tau, x), x(b, \chi, \theta, \tau, x), r(b, \chi, \theta, \tau, x), b'(b, \chi, \theta, \tau, x)\}\). Then the problem for the supranational authority is given by:

\[ S(b, \chi, \theta) = \max_{x, r} \left\{ \eta [u^H(x, \theta) + w(g^H)] + (1 - \eta) [u^F(x, \theta) + w(e^F - \tau)] \right\} + \beta \mathbb{E}_{\chi', \theta'} \left[ S(b', \chi', \theta') \right] \]  

(1.18)

s.t.

\[ V(b, \chi, \theta, \tau, x) \geq V^O(b, \chi, \theta), \]  

(1.19)

\[ J(b, \chi, \theta, \tau, x) \geq J^O(b, \chi, \theta). \]  

(1.20)

Constraint (1.19) represents the participation condition for the politician, where the outside option for the politician is described above. Constraint (1.20) is the participation constraint for the donor government.
1.5.4 Optimal Domestic Policy Choices

As in the two-period model, the benefit from reelection $\chi^V$ is assumed to be sufficiently small such that a solution with re-election every period exists.

**Assumption 5** The electoral demands $\chi^V$ are sufficiently low relative to the home government's endowment $e$ such that a feasible allocation with re-election every period exists under no banking union (i.e., $\rho_t = 0, \forall t$).

In order to characterize the politician's problem, the analysis restricts attention to the cases in which the value functions for the politician and the supranational authority are concave. The existence of functions $v(\cdot), u(\cdot),$ and $w(\cdot)$ that satisfy the conditions necessary for the value functions to be concave is established in the following Lemma. While this assumption restricts the set of possible utility functions, it helps provide a tractable framework under which the problem can be analyzed. In Section 1.6.3, a logarithmic form is assumed for functions $v(\cdot), u(\cdot),$ and $w(\cdot),$ and the problem is illustrated numerically under this particular functional form.

**Lemma 6** There exist concave functions $v(\cdot), u(\cdot),$ and $w(\cdot)$ such that the politician's value function $V(b, \chi, \theta, \tau(b, \chi, \theta), z(b, \chi, \theta))$ is concave and differentiable for $b \in (b, \bar{b})$ given the equilibrium policy functions $\tau(b, \chi, \theta)$ and $z(b, \chi, \theta),$ and the supranational authority's value function $S(b, \chi, \theta)$ is concave and differentiable in $b \in (b, \bar{b})$ given the equilibrium policies chosen by the politician.

**Proof.** In the Appendix. ■

The above Lemma allows for a characterization of the politician's problem. To begin with, I assume the conditions of Assumption 1 are satisfied $\forall \theta \in \Theta,$ so that positive recapitalizations can be part of the solution to the politician's problem. Denote by $\lambda(\chi, \theta), \kappa(\chi, \theta)$ and $\nu(\chi, \theta)$ the Lagrange multipliers on constraints (1.14a), (1.14b) and (1.14c), respectively. The first-order
conditions for an internal solution with respect to $r$, $x$, $g^H$, and $b'$ are:

\[
\begin{align*}
\lambda(\chi, \theta) - \pi(\chi, \theta) &= (1 - \gamma)v'(r), \quad (1.21\ a) \\
\lambda(\chi, \theta) - \pi(\chi, \theta) &= \gamma (1 + \nu(\chi, \theta)) \frac{\partial u^H(x, \theta)}{\partial x}, \quad (1.21\ b) \\
\lambda(\chi, \theta) &= \gamma (1 + \nu(\chi, \theta)) w'(g), \quad (1.21\ c) \\
\lambda(\chi, \theta) &= \mathbb{E} \left[ \frac{\partial V(b', x', \theta', \tau', \pi')}{\partial b'} \right]. \quad (1.21\ d)
\end{align*}
\]

The above conditions can be used to infer the effects of the electoral constraint and those of the intervention constraint. They show that the same effects described in the two-period model translate to the dynamic environment. When $\chi = 0$, the electoral constraint does not bind and $\nu(\chi, \theta) = 0$. If $\chi = \chi^V$, the electoral constraint binds and $\nu(\chi, \theta) > 0$. This has the effect of increasing public good $g^H$ and recapitalizations $x$ compared to the case without a binding electoral constraint ($\nu(\chi, \theta) = 0$). It also leads to lower equilibrium rents $r$ and higher public debt $b'$, through the effect of the multiplier $\lambda(\chi, \theta)$.

The effect of a binding constraint on interventions ($\pi(\chi, \theta) > 0$) is to increase recapitalizations $x$ and rents $r$ compared to the case without the binding intervention rule ($\pi(\chi, \theta) = 0$). This happens because the politician cannot be given incentives to increase recapitalizations without those incentives also acting towards increasing rents, due to the same forces described in the two-period model.

### 1.5.5 Additional Assumptions

As in the two-period model, necessary conditions must be assumed on the parameters of the model such that positive transfers are preferred by the supranational authority, and the participation constraint for the politician binds in equilibrium. The crucial difference in the dynamic model is that the supranational authority’s continuation value is affected by debt through the continuation utilities of both the home and the donor governments. These effects of debt extend the set of conditions necessary to obtain the dynamic equivalent to Assumption 3.
First, the following assumption is the equivalent to Assumption 2 in the dynamic environment. It requires that the donor government has a sufficiently large endowment such that positive transfers are optimal.

**Assumption 6 (2')** The donor government endowment $e^F$ is sufficiently large such that the following condition holds $\forall \theta \in \Theta, \chi \in \{0, \chi^V\}, b \in [b, \bar{b}]$ at $\tau = 0, \bar{x} = 0$:

$$\frac{\partial u^F(\bar{x}, \theta)}{\partial \bar{x}} \frac{\partial x^0(b, \chi, \theta)}{\partial \bar{c}} + \mathbb{E} \left[ \frac{\partial J(b^0, \chi', \theta')}{\partial b^0} \right] \frac{\partial b^0(b, \chi, \theta)}{\partial \bar{c}} > w'(e^F), \quad (1.22)$$

where $x^0(b, \chi, \theta)$ is the level of recapitalizations, and $b^0(b, \chi, \theta)$ is the debt chosen by the home government in autarky.

Since positive transfers increase the utility of home households, the above Assumption also implies that the supranational authority will prefer positive transfers. As discussed in the two-period model, existence of a sufficiently high value for $e^F$ such that the above condition holds is guaranteed by the $e^F \to \infty$ case.

Next, the I show there exist values of $\eta$ and $\bar{b}$ at which the participation constraint for the home government always binds in equilibrium.

**Lemma 7** There exist parameters $(\eta, \bar{b}) \in [0, 1] \times [0, \infty)$ such that the participation constraint for the politician binds $\forall \theta \in \Theta, \chi \in \{0, \chi^V\}, b \in [b, \bar{b}]$, and given the equilibrium policy $(\tau, \bar{x})$ set by the supranational authority to maximize (1.18) subject to constraints (1.19) and (1.20).

**Proof.** In the Appendix. □

The necessary condition required by the above Lemma comes from the maximization problem of the supranational authority. As shown in the Appendix, this condition implies a sufficiently small weight on the home country, along with a sufficiently small upper limit on debt. The existence of such parameters can be shown by examining the extreme case of $\eta = 0$ and $\bar{b} \to 0$. In this case, the supranational authority does not place any weight on the home country, and the cost of any debt is minimal. Therefore, it prefers to have the home country share as much of the cost of recapitalizations are possible given its participation constraint. Since the
condition is continuous in $\eta$ and $\bar{b}$, this implies that the condition holds for sufficiently low values of these parameters. I restrict the analysis to such parameters.

**Assumption 7 (3')** The parameters $\eta$ and $\bar{b}$ satisfy the condition of Lemma 7.

As shown in Lemma 6, the continuation values of the supranational authority and of the politician are monotonously decreasing in debt. This happens because higher debt decreases the available resources that can be used for both socially beneficial spending - recapitalizations and public goods - and for rents. Therefore, the cost of additional debt is increasing as the politician faces higher outstanding debt. The following lemma establishes a property that allows us to later on discuss the implications of weakening Assumption 7.

**Lemma 8** If the participation constraint for the politician binds in equilibrium for $b^* \in [b, \bar{b}]$, then it also binds for $\forall b < b^*$.

**Proof.** In the Appendix. ■

Lastly, I make the following assumption about the relative marginal utilities from rents, recapitalizations and the public good.

**Assumption 8** For any transfer $\tau > 0$ and associated intervention rule $x$, the following holds given the politician’s problem when $x = 0$ in the current period:

$$
\frac{\partial u^H(x, \theta)}{\partial x} \frac{\partial x(b, \chi, \theta, \tau, x)}{\partial x} + \frac{\partial w(g_H)}{\partial g_H} \frac{\partial g^H(b, \chi, \theta, \tau, x)}{\partial x} \leq 0.
$$

(1.23)

Assumption 8 says that a marginal increase in the intervention rule has a smaller positive effect on the instantaneous household utility from recapitalizations than the negative effect it has on the instantaneous household utility from the public good. This implies that the socially wasteful spending due to rent-seeking is sufficiently high such that an intervention rule would not increase instantaneous household utility in periods when there is no electoral accountability. It requires that the utility functions $u(\cdot)$, $w(\cdot)$, and $r(\cdot)$ are chosen such that an intervention rule has a sufficiently negative effect on the current provision of public goods. The assumption
ensures that the utility cost of interventions is split between the current period and the future, rather than it just being passed on to the future.

In the next sections, the welfare effects of a partial banking union are analyzed given the above Assumptions. The analysis shows how the results from the two-period model translate to the dynamic environment. After each result, a discussion is provided regarding the effects of relaxing Assumption 7 and allowing periods of high debt, in which the supranational authority might find it optimal to use transfers in order to decrease debt. Later, the dynamics of debt are discussed in more detail.

1.5.6 Partial Banking Unions and Household Welfare

This section considers the change in household welfare under a banking union compared to autarky. Given Assumption 7, a partial banking union achieves higher recapitalizations than under autarky. These outcomes are beneficial for the consumers of both countries. However, a banking union might also lead to increased rent-seeking, which could make home consumers worse off. The intuition for why welfare might decrease is similar to the one presented in the two-period model: the supranational authority does not value the home country public good as much as the home country consumers, so it is willing to accept a larger decrease in the public good in exchange for higher recapitalizations. Still, the dynamic model introduces another element in the decision problem of the supranational authority. Now the supranational authority places a different weight on decreases in the home public good in the current period versus decreases in the public goods in future periods. This happens because a decrease in the home public good today only affects the utility of home consumers, while a decrease in future home public goods affects all households, because it also implies a decrease in future recapitalizations, through the effects of higher debt. Therefore, the restrictions placed by Assumption 7 on the higher bound on debt make it possible for the results of Proposition 2 to be extended to the dynamic environment.

Proposition 9 Suppose Assumptions 6-8 hold. A partial banking union leads to lower expected household welfare in the home country than under no banking union.

Proof. In the Appendix. ■
The decrease in home household welfare happens because the politician is kept indifferent to participating in the banking union. The politician is given incentives to increase rent-seeking under the banking union, at the cost of public good provision. This happens because the incentives for increasing recapitalizations can only be given through transfers and the intervention rule, and both of these policies also act towards increasing rent-seeking.

The upper bound on debt is significant in determining whether the participation constraint for the politician binds in equilibrium. To better understand the role of debt, consider now the case when Assumption 7 is relaxed such that the upper bound on debt $\bar{d}$ is high enough for there to exist feasible debt values at which the participation constraint for the politician does not bind in equilibrium. At this level of debt, it becomes too costly for the supranational authority to further increase the costs passed on to the politician. Therefore, it optimally accepts an increase in the utility of the politician, such that his participation constraint becomes slack. Such a policy then increases the politician's utility from both rents and household utility. This implies that, in periods with high debt, a partial banking union would increase household utilities for both countries. The intuition is that, in high debt environments both countries are sufficiently hurt by the high level of debt that it becomes preferable for the supranational authority to accept lower recapitalizations in the current period in order not to further decrease future recapitalizations.

In an extreme case, for sufficiently high levels of debt, it becomes optimal for the supranational authority to fully cover the costs of higher recapitalizations through transfers from the donor households, so that part of the transfers are used towards decreasing debt in the home country. This aspect of the equilibrium dynamics could not be captured in the two-period model. In that setup, debt only affected the future utility of the home households. When the weight placed on the home country was $\eta = 0$ (or sufficiently small), the supranational authority found it optimal to keep the politician's participation constraint binding, because the high level of debt did not affect the donor household utility. The dynamic model shows how that result is overturned when the future utility of donor households is also a function of the home country's public debt.
1.6 Effects of Electoral Accountability and Fiscal Rules

1.6.1 Partial Banking Unions and Electoral Accountability

Proposition 9 gives the conditions under which, in an environment with rent-seeking and imperfect electoral accountability, a partial banking union cannot achieve a Pareto improvement. This section considers the effect of improvements in electoral accountability on the equilibrium partial banking union. The strength of electoral accountability in this model is given by the size of the benefit from reelection, $\chi^V$.

Consider the effect of a higher $\chi^V$ when the politician’s participation constraint binds in equilibrium. The electoral constraint makes the politician provide higher recapitalizations and more domestic public goods in order to get reelected. This demand for higher spending is financed by lowering rent-seeking and increasing public debt, given the politician’s maximization problem (1.13). An intervention rule imposed by the partial banking union aims at increasing recapitalizations, but in fact gives the politician incentives to increase both recapitalizations and rents. Therefore, the intervention rule has a positive effect on household utility through higher recapitalizations, and a negative effect that comes through the higher public debt taken on in order to finance higher rent-seeking. As public debt increases, the cost of taking on more debt also rises, making recapitalizations more expensive.

The following Lemma shows the effect of higher electoral accountability on the intervention rules set for each transfer level $\tau$. Under Assumption 7, the intervention rule for each transfer level is determined from the binding participation constraint for the politician.

**Lemma 10** For a given shock $\theta$ and transfer $\tau$, under Assumptions 6-8, higher electoral accountability (higher $\chi^V$) leads to lower supranational intervention rules, i.e., $z(\tau, \theta|\chi = \chi^V)$ decreases.

**Proof.** In the Appendix. ■

The next result considers the effects of higher electoral accountability on donor household welfare, and gives the dynamic equivalent of Proposition 3.
Proposition 11 Under Assumptions 4-8, if the share $\sigma$ of deposits held by home households is sufficiently small, then higher electoral accountability (higher $\chi^V$) in the home country lowers the expected donor household welfare under the partial banking union.

Proof. In the Appendix. ■

The intuition for the change in expected welfare is as follows. The effect of higher electoral demands in the home country is to increase public debt, since more funds are necessary to cover the voter demands in the current period. The higher debt increases the marginal cost of recapitalizations. This makes intervention rules costlier to implement, and therefore equilibrium interventions decrease. Moreover, because the home politician is more constrained, the donor households must bear more of the cost of recapitalizations. As in the two-period model, the condition on $\sigma$ ensures that the positive effect of electoral accountability - higher recapitalizations - is not sufficiently high to outweigh the negative effect of costlier debt. The higher cost of debt makes further increases in recapitalizations more expensive, and this lowers expected donor household welfare.

Proposition 11 continues to hold even if Assumption 7 is relaxed to allow for debt levels at which the politician’s participation constraint does not bind. In a high debt environment, in which the donor country bears the entire cost of higher recapitalizations, increases in electoral accountability still have the effect of increasing public debt by more. The higher public debt rises the cost of supranational interventions, and leads to lower equilibrium recapitalizations. Therefore, the welfare effects of electoral accountability on donor households are similar in both low debt and high debt environments.

The above results highlight that, while desirable from the perspective of home household utility, higher electoral accountability in the home country is not necessarily desirable from the perspective of the donor country. Higher electoral accountability makes transfers and recapitalizations harder to achieve under the partial banking union. This suggests that an improvement in domestic electoral institutions is not by itself sufficient in order to increase welfare in both countries.
1.6.2 Partial Banking Unions and Fiscal Rules

The results from the previous section show that higher electoral accountability leads to increases in public debt, in order to satisfy the need for higher government spending in the current period. This raises the question of whether fiscal rules that limit the increase in public debt could make home consumers better off. By reducing public debt, fiscal rules could limit rent-seeking. However, they can also inefficiently reduce spending on recapitalizations and the public good.

Consider the case of a stronger banking union, in which the fiscal rules are decided by the supranational authority. The type of fiscal rules considered are limits on the increase in public debt. This is modeled by assuming that the supranational authority controls debt. As before, the decision over the composition of domestic spending \((x, g^H, \text{and } r)\) belongs to the home government.

The supranational authority offers an agreement that both countries must accept in order for it to be implemented. Since the debt level is decided by the supranational authority, the problem for the politician under the banking union becomes just a static choice between recapitalizations, rents, and the domestic public good. Let the choices of the politician be denoted by \(\{x(b, b', \chi, \theta, \tau, x), r(b, b', \chi, \theta, \tau, x), g^H(b, b', \chi, \theta, \tau, x)\}\), given the outstanding debt \(b\), the new debt \(b'\) chosen by the supranational authority, shocks \(\chi\) and \(\theta\), and the agreement \((\tau, x)\). The problem for the supranational authority is given by:

\[
S(b, \chi, \theta) = \max_{\tilde{x}, \tilde{\tau}, \tilde{b}} \left\{ \eta \left[ u^H(x, \theta) + w(g^H) \right] + (1 - \eta) \left[ u^F(x, \theta) + w(e^F - \tau) \right] \right. \\
\left. + \beta \mathbb{E} \left[ S(b', \chi', \theta') \right] \right\} \tag{1.24}
\]

s.t.

\[
(1 - \gamma)u^F(x, \theta) + \gamma u^H(x, \theta) + \gamma w(g^H) + \beta \mathbb{E} \left[ V(b', \chi, \theta, \tau, \tilde{x}') \right] \geq V^O(b, \chi, \theta, \tau, x), \tag{1.25a}
\]

\[
u^F(x, \theta) + w(e^F - \tau) + \beta \mathbb{E} \left[ J(b', \chi, \theta, \tau, \tilde{x}') \right] \geq J^O(b, \chi, \theta, \tau, x), \tag{1.25b}
\]

\[b' \in [b, \bar{b}]. \tag{1.25c}\]
Constraint (1.25a) represents the participation constraint for the politician. The outside option for the politician is the same as in the previous section, since the politician is still free to choose debt in the outside option. Constraint (1.25b) is the participation constraint for the donor country. Finally, constraint (1.25c) gives the bounds on debt.

The following Lemma establishes that $S(b, \chi, \theta)$ is concave and differentiable, given the assumptions of the model.

**Lemma 12** The supranational authority's value function $S(b, \chi, \theta)$ is concave and differentiable in $b \in (b, \bar{b})$.

**Proof.** In the Appendix. $\blacksquare$

When the electoral constraint is not binding ($\chi = 0$), the politician retains some discretion in choosing current period policies. Even though the supranational authority controls debt increases, it cannot offer incentives for increasing recapitalizations without these same incentives also acting towards increasing rents.

When the electoral constraint binds ($\chi = \chi^V$), voters and the supranational authority essentially remove all politician discretion. Voters control the composition of intra-period spending, while the supranational authority controls how much the politician is allowed to borrow. The fiscal rule that can be set still depends on the outside option of the politician, due to the participation constraint. Yet incentives for higher recapitalizations can be offered without also enabling higher rent-seeking. This way, an allocation can be chosen that has weakly lower rents and public debt than in the outside option of the home government. Electoral accountability is therefore beneficial for home household welfare, both directly through the provision of utility $\chi^V$, and indirectly through lower rents and lower public debt. The next proposition sums up this result.

**Proposition 13** Suppose Assumptions 6 and 7 hold. In a partial banking union with optimized fiscal rules, periods of electoral accountability are characterized by weakly lower rents compared to the level of rents extracted in that same period under no banking union.
Proof. In the Appendix. ■

The intuition for the above result is based on the supranational authority's ability to fully control the policymaker's choices when there is electoral accountability. These periods allow the supranational authority to decrease public debt, and this way increase expected household utility. The result holds in both low debt and high debt environments, since in both cases the supranational authority does not have an incentive to increase debt above the level chosen by the politician in the outside option. Any increase above that level would decrease the utility of the supranational authority because of higher rents or inefficiently low future recapitalizations.

Corollary 14 Under Assumptions 6-8, expected home consumer welfare increases under a partial banking union with fiscal rules, if the decrease in debt in the periods of electoral accountability is sufficiently large.

Proof. In the Appendix. ■

The random nature of the electoral shock means that in some period, when $\chi = \chi^V$, fiscal rules can decrease rents and debt, while in other periods, when $\chi = 0$, fiscal rules cannot reduce rents and debt. Due to this variation, whether a Pareto improvement is achieved over having no banking union depends on how much debt decreases in the periods of electoral accountability versus how much it increases in the periods without electoral accountability.

As shown in the Appendix, the conditions under which a strict decrease in rents is obtained are endogenous. The regions in which household welfare increases require a sufficiently low marginal utility from rents and a low degree of substitution between $x$ and $g^H$ in the utility of voters. This means that a decrease in the public good $g^H$ must be offset by a large increase in recapitalizations $x$. This points to a case in which the domestic public good and socially wasteful rents are relatively more valuable to the politician than recapitalizations. An example is a case where the share of deposits held by home households is small, and the politician places high value on rent-seeking relative to voter utility.

The next set of results offer some comparative statics regarding the role of electoral accountability. First, Proposition 13 immediately leads to the following implication regarding the probability of a high benefit from incumbent removal.
Corollary 15 Under Assumptions 6 and 7, if periods of electoral accountability \( (\chi = \chi^V) \) lead to strictly lower rents and debt than in the outside option, then there exists a threshold probability of a high benefit from incumbent removal, \( \pi^*) < 1 \), such that for \( \pi > \pi^* \) the expected household utility under the banking union increases compared to under no banking union.

**Proof.** In the Appendix.

The size of the benefit from incumbent removal, \( \chi^V \), affects the balance between consumer welfare in the donor country and that in the home country. Starting from a high \( \chi^V \), further increases in \( \chi^V \) reduce equilibrium intervention rules, as described in the previous section. This decrease in recapitalizations reduces the welfare of donor households, and the results of the previous section carry through to the environment with fiscal rules.

Proposition 16 Under Assumptions 6 and 7, in a partial banking union with optimized fiscal rules, higher electoral accountability (higher \( \chi^V \)) lowers expected donor household welfare.

**Proof.** In the Appendix.

The welfare effects described in Proposition 16 also hold when Assumption 7 is relaxed to allow for higher debt levels. For high debt levels, the supranational authority can choose a strictly lower debt than the politician would in the outside option. This happens because high costs of debt can lead the supranational authority to prefer using the resources from the current period to increase future recapitalizations. Then, part of the supranational transfer is used towards politician rents, required to keep the politician participating in the agreement given the binding debt limit. Therefore, in high debt periods, in which the participation constraint for the politician does not bind, home consumer utility is increased.

The above results shed more light on the interaction between domestic electoral accountability and supranational agreements. First, electoral accountability is needed in order to achieve a Pareto improvement through the partial banking union. Second, the strength of the electoral accountability, measured by \( \chi^V \), determines the relative change in donor welfare versus home consumer welfare. While the supranational authority needs electoral accountability in order to lower rent-seeking, higher voter demands in the home country reduce its ability to achieve higher recapitalizations and to increase donor household welfare.
The dynamic model allows for a richer role for fiscal rules compared to the two-period case. Now, fiscal rules can be used to constrain the politician to lower debt. Under the conditions discussed above, they can help achieve an increase in the expected household welfare in both countries.

1.6.3 Debt Dynamics

The outstanding debt level and the equilibrium transfers and intervention rules affect current policy choices and the new level of debt. In turn, the new level of debt affects the future transfers and intervention rules that the supranational authority can set. This interaction gives an equilibrium path for public debt, transfers, and intervention rules. The following proposition shows that the sequence of debt distributions each period converges to an invariant distribution.

**Proposition 17** The equilibrium distribution of debt converges to a unique nondegenerate invariant distribution over \([\underline{b}, \overline{b}]\).

**Proof.** In the Appendix. ■

Public debt increases for higher liquidity shocks \(\theta\) or when the electoral constraint binds \((\chi = \chi^v)\). It decreases when the realization of \(\theta\) is lower or when the politician does not face binding electoral demands \((\chi = 0)\). This happens because the politician is using debt in order to smooth the costs of interventions and public good provision over time. When a high liquidity shock \(\theta\) or a high electoral shock \(\chi\) is realized today, the value of current spending increases, and the politician takes on more debt in order to finance higher current spending. When the liquidity shock \(\theta\) is low, or the electoral constraint is not binding \((\chi = 0)\), the need for government spending in the current period is lower, so the politician takes on less debt.

Under Assumption 7, the participation constraint for the politician binds at all feasible debt levels, so the politician is always sharing some of the costs of additional recapitalizations through decreases in the public good and through higher debt. As the debt level increases, the required share of the intervention cost borne by the politician decreases. This happens, as shown in Proposition 11, because it becomes harder for the supranational authority to give incentives to the politician, to ensure his participation in the banking union. Therefore, the
equilibrium intervention rule decreases, which means less overall spending on recapitalizations. Moreover, the relative cost of recapitalizations that must be covered by the politician is lower, as the more constrained politician cannot contribute as much to the banking union. Thus, higher debt hurts the donor country both because of lower equilibrium recapitalizations, and because of it has to cover a higher relative share of the cost of recapitalizations. This implies that the benefits from a banking union decrease for the donor country as the home country becomes more indebted.

Assume now the more general case when the upper bound on debt is high enough to allow for feasible debt values at which the participation constraint for the politician does not bind in equilibrium. In this case, denote by \( \hat{b} \) the highest level of debt at which Lemma 8 holds. To get a better sense of the effects of debt, consider the case in which the highest shock values, \( \theta^N \) and \( \chi^V \), are repeated over \( T \) periods. Then, after each period, outstanding debt \( b \) increases. While \( b < \hat{b} \), the participation constraint for the politician binds, and instantaneous home household utility decreases under the banking union. Therefore, at low debt levels, the cost of higher rent-seeking outweighs the benefits of a banking union for the home households. When \( b > \hat{b} \), the participation constraint is slack, home household utility increases under the agreement, while the increase in donor household utility is smaller. In this case, a banking union benefits the home households as well.

The above results regarding the role of debt show that, as the debt level increases, the benefits from a banking union shift more towards the home country and away from the donor country. Therefore, in periods of high liquidity shocks, high electoral demands and high debt, the donor country finances fewer recapitalizations, at a higher cost, while the receiving country shares less of the cost burden of recapitalizations. This model can provide a framework for understanding why partial banking unions might be harder to implement in high debt environments, if the bargaining power rests with the donor country.
Numerical illustration

The evolution of the public debt over time is illustrated using a numerical simulation. Consider the following specifications for the utility functions:

$$v(r) = r^d \log(r),$$

$$u^j(x) = x^d \log(R(1 - \theta^j)\omega^j + R\sigma x), \ j = H, F,$$

$$w(g) = g^d \log(g + g^c),$$

where $r^d, x^d, g^d, g^c \in \mathbb{R}_+$. Let the parameter values be given by $(r^d, x^d, g^d, g^c) = (0.02, 0.05, 0.46, 10)$, $R = 1.02$, $\sigma = 0.75$, $\beta = 0.95$, $\gamma = 0.7$ and $\eta = 0$. The endowments of the home and foreign countries are given by $(\omega^H, e^H, e^F) = (1, 1, 5)$. These parameter values ensure that the donor country has significantly higher resources compared to the home country. The weight placed by the supranational authority on the home country reflects the extreme case in which the supranational authority is simply a proxy for the donor country.

Consider the simple case in which the liquidity shock can only take two values, $\theta^H = 0.2$ and $\theta^L = 0.1$, with probabilities $f^H = 0.1$ and $f^L = 0.9$. The probability of the high electoral shock $\chi^V$ is $\pi = 0.1$. The shocks are i.i.d, as described in the setup of the model, and the probability of each shock is chosen to ensure that the high liquidity shocks and the high electoral shocks are sufficiently rare.

First, we compare the path of debt under no banking union to that under a partial banking union without fiscal rules. Figure 1-1 illustrates these results for a realized sequence of liquidity shocks and electoral shocks over 100 periods, starting from no initial debt. The vertical axis measures the public debt taken on each period relative to the home country endowment.

As shown in Figure 1-1, public debt increases over time in both cases. A high liquidity shock or a high electoral shock lead to temporary increases in debt, represented by the spikes seen on the path of debt. The combination of high liquidity and high electoral shocks leads to a much larger increase in debt, as shown in period 46. The fast decrease in debt following a
The high shock is seen because the shocks are not persistent, and the probability of a high liquidity shock together with a high electoral shock is small. The main difference between the path of debt under no banking union and the path of debt under a partial banking union is that debt accumulation is higher at all times under the partial banking union. The policymaker increases both recapitalizations and rents by borrowing more than under autarky. The model is characterized by a faster increase in public debt under the partial banking union, without any amplification or persistence effects.

The left-hand panel of Figure 1-2 illustrates the stationary distribution of debt under no banking union, and the right-hand panel of Figure 1-2 shows the stationary distribution of debt with a partial banking union without fiscal rules. The horizontal axis gives the level of public debt relative to the home country endowment, and the vertical axis shows the probability (×100) of the respective relative debt level. The distribution of debt illustrates that a partial banking union is associated with more debt-taking.

The numerical simulation is also used to illustrate the effects of adding fiscal rules to a
Figure 1-2: Stationary distribution of debt with and without a partial banking union

partial banking union. The path of debt under the partial banking union with fiscal rules is illustrated in Figure 1-3 along with the two cases shown previously – no banking union, and a partial banking union without fiscal rules. Figure 1-3 displays the path of debt under a sequence of realized \((\theta, \chi)\) shocks over 50 periods, starting from no initial debt. The role of fiscal rules in limiting excessive public debt becomes apparent. Public debt still increases in response to high liquidity or electoral shocks, but the overall debt accumulation is smaller. The supranational authority can use fiscal rules to limit rents without decreasing recapitalizations, and this translates into lower public debt.

Figure 1-4 illustrates the main effect of supplementing the partial banking union with optimally chosen fiscal rules: the rents diverted by the policymaker decrease in the periods of electoral accountability. Rent-seeking decreases when the electoral shock is realized, and this decrease is larger when fiscal rules are in place. When the electoral constraint is not binding, rents are the same with or without fiscal rules, which illustrates that fiscal rules are effective only when complemented by high electoral accountability.
Figure 1-3: The evolution of debt under no banking union, under a partial banking union with no fiscal rules, and under a partial banking union with fiscal rules.
1.7 Conclusion

This paper has studied the welfare effects of a partial banking union in the presence of rent-seeking and imperfect electoral accountability. It has shown that, under these conditions, implementing a partial banking union can have negative welfare effects. Within such a banking union, higher electoral accountability meant to reduce rent-seeking can also reduce needed recapitalizations and decrease donor country welfare. Strengthening the banking union with fiscal rules can limit rent-seeking, but such limits also reduce the ability of governments to engage in desirable spending. These results suggest that policies aimed at tackling one source of inefficiency might backfire by augmenting the other incentive problems. Yet the above results show how a policy that jointly sets intervention rules and fiscal rules, optimized for each other, could achieve a Pareto improvement in consumer welfare, in environments with sufficiently high electoral accountability. These implications seem relevant for the proposed banking union in
the Eurozone, in which not all decisions regarding interventions in the banking sector may be
centralized.

The framework presented in this paper opens several avenues for future research. First, the
model assumed a specific institutional structure, motivated by current policy proposals. A nat-
ural complement to this positive analysis is the study of what type of supranational agreements
are optimal in an environment with political economy distortions like the ones described in this
paper, and under what conditions supranational rules emerge as the optimal contract. The
current positive analysis can also be extended to incorporate a richer dependence of electoral
accountability on current policy variables. This would help create a two-way interaction be-
tween electoral accountability and supranational rules, which would enable the study of political
equilibria with endogenous domestic and supranational institutions. Another extension of the
model is to allow for a richer investment environment, including private storage technologies
that could be used for reinvestment along with public funds. Supranational intervention rules
could then be used to balance the political economy distortions from public recapitalizations
with the incentives for household savings and private recapitalizations. Exploring the interac-
tion between public and private transfer schemes within banking unions is a promising area for
future research.
1.A Appendix

1.A.1 Household Investment and Banks

This section endogenizes the household investment decisions, describes the banks in more detail and presents the necessary assumptions made such that households decide to invest their entire endowment in banks.

Households

Every household receives endowment $w^j$, $j = H, F$, at the beginning of each period. It can use this endowment for direct consumption, or it can decide to make a risky deposit $i^j_t$ in a bank. Households do not have access to any storage technology. The deposit has a risky return that depends on the aggregate shock $\theta_t$. The return is in terms of consumption goods, denoted as $c(i^j_t, \theta_t)$. Their instantaneous utility is given by

$$u(w^j - i^j_t + c(i^j_t, \theta_t)) + w(g^j_t).$$

Banks

Banks hold identical, risky investment projects. They do not have any equity and can fund projects exclusively using household deposits. Moreover, I make the simplifying assumption that the banks are owned by the same households that hold deposits in them, so that the objective of the bank is to maximize the expected household utility from private consumption. The investment technology exhibits constant returns to scale. The initial investment $(i^H_t + i^F_t) \geq 0$ determines the size of the project. The project return is subject to uncertainty. Following investment, an aggregated i.i.d. shock $\theta_t \in \Theta$ is realized. After the shock, a fraction $\theta_t$ of the investment is lost, while the remaining $(1 - \theta_t)$ fraction of the project is intact. The intact portion of the project has a rate of return $R > 1$ in the next period. The distressed portion does not produce any returns, unless additional funds are reinvested. Following the observation of $\theta_t$ and prior to the investment project completion, the bank can reinvest $x_t$ new funds into the project, such that the total size of the project is at most equal to the initial size: $x_t \leq \theta_t (i^H_t + i^F_t)$. Since the households and banks do not have access to any storage technology
and there is no loan market for the bank to access new reinvestment funds, all reinvestment
funds $x_t$ must be provided by the government. The government is the only agent who has access
to loan markets and also has an endowment that cannot be initially invested in private projects.
Therefore, the government is the only provider of liquidity in case of a shock to the project.
The project then returns $R((1-\theta_t)(i^H_t + i^F_t) + x_t)$ consumption units. This timing assumption
precludes the banks from having access to next period’s households’ endowment. In terms of
household consumption, the investment $i_t$ made by each households returns $R((1-\theta_t)i^H_t + x_t)$
consumption units, depending on the reinvestment $x_t$ made by the government.

The Household Problem

Each household is choosing whether to invest some part of its endowment. Given an
investment $i$, households receive expected utility: $E_{\theta} [u(\omega^j - i^j + R(1-\theta)i^j + Rx^j)]$, where
$j = H, F$. To simplify the problem, the following assumption is made so that households always
prefer to fully invest their endowment rather than directly consume:

Assumption 9 The rate of return $R$ is high enough such that the following condition holds:
$E_{\theta} [u(\omega^j - i^j + R(1-\theta)i^j)] > u(\omega^j) \forall i^j \leq \omega^j$.

1.A.2 Proof of Lemma 1

For any $\tau \leq e^F$, denote by $\overline{x}^E$ the value of $x$ at which the participation constraint for the
politician, constraint (1.5), holds with equality. Then, the participation constraint for the
politician binds in equilibrium if the following condition holds.

$$
\left[ \eta \frac{\partial u^H(x^s, \theta)}{\partial x^s} + (1-\eta) \frac{\partial u^F(x^s, \theta)}{\partial x^s} \right] \frac{\partial x^s}{\partial x} \left|_{x^s} \right. - \eta w' \left( e + \frac{\tau - \overline{x}^E}{1+\beta} \right) \\
\geq \left[ \eta \frac{\partial u^H(x^s, \theta)}{\partial x^s} + (1-\eta) \frac{\partial u^F(x^s, \theta)}{\partial x^s} \right] \frac{\partial x^s}{\partial \tau} + \eta w' \left( e + \frac{\tau - \overline{x}^F}{1+\beta} \right) - (1-\eta) w' (e^F - \tau)
$$

This condition says that the net marginal benefit to the supranational authority from in-
creasing $x$ is larger than the net benefit from increasing transfer $\tau$. Therefore, absent constraint
(1.5), the supranational authority could increase $x$ and decrease $\tau$, and this way it could achieve
higher utility.

The above condition can be re-written as:

\[
\left[ \eta \frac{\partial u^H(x^s, \theta)}{\partial x^s} + (1 - \eta) \frac{\partial u^F(x^s, \theta)}{\partial x^s} \right] \left( \frac{\partial x^s}{\partial x^s} |_{x^s} - \frac{\partial x^s}{\partial \tau} \right) \\
-2\eta w' \left( e + \frac{\tau - x^T}{1 + \beta} \right) + (1 - \eta) w' \left( e^F - \tau \right) \geq 0.
\] (1.26)

At \( \eta = 0 \), the above condition holds as \( \left( \frac{\partial x^s}{\partial x^s} |_{x^s} - \frac{\partial x^s}{\partial \tau} \right) > 0 \) given the setup of the politician’s problem. At \( \eta = 1 \), the above condition does not hold since from the first-order conditions to the politician’s problem it follows that \( \frac{\partial u^H(x^s, \theta)}{\partial x^s} \leq w' \left( e + \frac{\tau - x^T}{1 + \beta} \right) \) and \( \left( \frac{\partial x^s}{\partial x^s} |_{x^s} - \frac{\partial x^s}{\partial \tau} \right) \leq 1 \).

Finally, the left-hand side is continuous in \( \eta \). It it also monotonously decreasing in \( \eta \) since

\[
\frac{\partial u^H(x^s, \theta)}{\partial x^s} \left( \frac{\partial x^s}{\partial x^s} |_{x^s} - \frac{\partial x^s}{\partial \tau} \right) \leq \frac{\partial u^H(x^s, \theta)}{\partial x^s} \leq w' \left( e + \frac{\tau - x^T}{1 + \beta} \right),
\] \( \forall \tau \in [0, e^F] \). Therefore, \( \exists \eta^* \in (0, 1) \) such that condition (1.26) holds \( \forall \eta \leq \eta^* \).

1.A.3 Proof of Proposition 3

Assumption 3 implies that the participation constraint for the home government binds in equilibrium, positive transfers are made and \( x^s > x^0 + r^0 \). The binding participation constraint implies

\[
(1 - \gamma)v(r^s) + \gamma w(g^H) + \gamma u(R(1 - \theta)\omega^H + Rx^s) + \beta \gamma w(g^H_s) \geq \\
(1 - \gamma)v(r^0) + \gamma w(g^H_0) + \gamma u(R(1 - \theta)\omega^H + Rx^0) + \beta \gamma w(g^H_0).
\] (1.28)

Given the electoral constraint, the above constraint can be reduced to

\[
(1 - \gamma)v(r^s) + \beta \gamma w(g^H_s) \geq (1 - \gamma)v(r^0) + \beta \gamma w(g^H_0).
\] (1.29)
The policies \( r, x, g^H \) and \( g_1^H \) are given by the politician's first-order conditions:

\[
\lambda - \kappa = (1 - \gamma) v'(r),
\]
\[
\lambda - \kappa = \gamma (1 + \nu) u^H(x, \theta),
\]
\[
\lambda = \gamma (1 + \nu) w'(g),
\]
\[
\lambda = \gamma w'(g_1),
\]

where \( \lambda, \kappa, \) and \( \nu \) are the Lagrange multipliers on conditions (1.2a), (1.2b), and (1.8), respectively.

Denote by \( x(T) \) the intervention rule at which constraint (1.29) binds for a given \( \tau \). Also, denote by superscript \( e \) the policies and multipliers when the electoral constraint binds. From (1.29), by the Implicit Function Theorem,

\[
\frac{\partial x}{\partial \tau} \Big|_{x=\kappa^\nu} = \frac{\gamma w'(g_1^{se})}{\gamma w'(g_1^{Hse}) - (1 - \gamma) v'(r^{se}) (1 - \frac{\partial w'(g_1^{se})}{\partial x})},
\]

which using the first-order conditions, can be re-written as:

\[
\frac{\partial x}{\partial \tau} \Big|_{x=\kappa^\nu} = \frac{\gamma w'(g_1^{Hse})}{\gamma w'(g_1^{Hse}) - (1 - \gamma) v'(r^{se})},
\]

(1.31)

and therefore, using \( \frac{\partial x}{\partial \tau} = \frac{\partial x}{\partial x} \frac{\partial x}{\partial \tau} \):

\[
\frac{\partial x}{\partial \tau} \Big|_{x=\kappa^\nu} = \frac{\partial x^e}{\partial \xi} \frac{\gamma w'(g_1^{Hse})}{\gamma w'(g_1^{Hse}) - (1 - \gamma) v'(r^{se})},
\]

and from (1.28), and using the first-order conditions to the politician's problem:

\[
\frac{\partial x}{\partial \tau} \Big|_{x=0} = \frac{\gamma w'(g_1^s)}{\gamma w'(g_1^{Hse}) - (1 - \gamma) v'(r^s)},
\]

(1.32)

and

\[
\frac{\partial x}{\partial \tau} \Big|_{x=0} = \frac{\partial x}{\partial \xi} \frac{\gamma w'(g_1^{Hse})}{\gamma w'(g_1^{Hse}) - (1 - \gamma) v'(r^s)}.
\]
From the above first-order conditions

\[ \frac{u^{H}(x, \theta)}{w'(g^H)} = 1 - \frac{\kappa}{\lambda}. \]

Since \( \frac{u^{H}(x, \theta)}{w'(g^H)} < \frac{u^{H}(x, \theta)}{w'(g^H)} \), this implies \( \frac{\kappa}{\lambda} > \frac{\kappa}{\lambda} \). Therefore,

\[ \frac{\gamma w'(g^H) - (1 - \gamma) \nu'(\nu)}{\gamma w'(g^H)} > \frac{\gamma w'(g^H) - (1 - \gamma) \nu'(\nu)}{\gamma w'(g^H)} \], and from (1.31) and (1.32):

\[ \frac{\partial x}{\partial \tau} \bigg|_{\chi = 0} > \frac{\partial x}{\partial \tau} \bigg|_{\chi = \chi^v}. \]

For \( \frac{\partial x}{\partial \tau} \), \( \nu^* > 0 \) and \( \nu = 0 \), so \( \frac{\partial x}{\partial \tau} \bigg|_{\chi = 0} < \frac{\partial x}{\partial \tau} \bigg|_{\chi = \chi^v} \). The difference \( \frac{\partial x}{\partial \tau} \bigg|_{\chi = \chi^v} - \frac{\partial x}{\partial \tau} \bigg|_{\chi = 0} \) depends on \( \nu \), the shadow cost of the electoral demands and the marginal utility \( u^{H}(x, \theta) \). From the first-order conditions, \( (1 + \nu) = \frac{(1 - \gamma) \nu'(\nu)}{\gamma u^{H}(x, \theta)} \) and \( (1 + \nu) = \frac{w'(g^H)}{w'(g^H)} \). As \( u^{H}(x, \theta) \) is lower, \( \frac{\partial x}{\partial \tau} \bigg|_{\chi = \chi^v} \) is also lower. Under the assumed utility functions, \( u^{H}(x, \theta) \) is increasing in \( \sigma \), such that the benefit from recapitalizations is smaller at lower values of \( \sigma \). Then, the difference \( \frac{\partial x}{\partial \tau} \bigg|_{\chi = \chi^v} - \frac{\partial x}{\partial \tau} \bigg|_{\chi = 0} \) is increasing in \( \sigma \). In the limit case when \( \sigma \to 0 \), \( \frac{\partial x}{\partial \tau} \bigg|_{\chi = 0} \to \frac{\partial x}{\partial \tau} \bigg|_{\chi = \chi^v} \). Therefore, for sufficiently small \( \sigma \), \( \frac{\partial x}{\partial \tau} \bigg|_{\chi = \chi^v} \) is small enough relative to \( \frac{\partial x}{\partial \tau} \bigg|_{\chi = 0} \) such that \( \frac{\partial x}{\partial \tau} \bigg|_{\chi = 0} \geq \frac{\partial x}{\partial \tau} \bigg|_{\chi = \chi^v} \); the necessary condition for this is that \( \sigma \) is sufficiently small such that

\[ \frac{\partial x}{\partial \tau} \bigg|_{\chi = 0} < \frac{1 - \frac{u^{H}(x, \theta)}{w'(g^H)}}{1 - \frac{u^{H}(x, \theta)}{w'(g^H)}}. \]

Then, electoral accountability leads to a lower marginal benefit of transfers to the donor country; this implies (weakly) lower recapitalizations and transfers, and lower donor household utility.

1.A.4 Proof of Lemma 6

Below, I derive the conditions on the equilibrium policy functions \( \tau(b|\chi, \theta) \) and \( g(b|\chi, \theta) \) under which \( V \) is concave and differentiable. I then derive the conditions on \( b'(\tau, g|\chi, \theta) \) under which concavity and differentiability of \( V \) implies concavity and differentiability of \( S \). Finally, I show that there exist utility functions for which these properties of the policy functions are
consistent. This shows that an equilibrium can exist in which the value functions are concave and differentiable.

**Part 1**

Consider a feasible set of transfers, rules and debt, \( \{\tau, x, b, b'\} \), given \( \theta \) and \( \chi \). Such a feasible allocation satisfies:

\[
x \leq e - b + \tau + \beta b',
\]

\[
w(e - b + \tau + \beta b' - x) + u^H(\bar{x}, \theta) \geq \chi,
\]

\[
b' \in [\bar{b}, \bar{b}],
\]

where \( \bar{x} = \min\{\theta \left(\omega^H + \omega^F\right), x\} \).

Given the feasible allocation \( \{\tau, x, b, b'\} \), a feasible set of policies for the politician \( \{r, g^H, x\} \) satisfies

\[
r + x + g^H \leq e - b + \tau + \beta b',
\]

\[
r + x \geq x,
\]

\[
w(g^H) + u^H(x, \theta) \geq \chi,
\]

\[
x \leq \theta (\omega^H + \omega^F).
\]

Let \( \Lambda(\tau, x, b, b') \) denote the set of feasible allocations \( \{r, g^H, x\} \) given \( \{\tau, x, b, b', \chi, \theta\} \). Finally, the equilibrium policy functions \( \tau \) and \( x \) are functions of \( b \).

With \( \tau(b|\chi, \theta) \) and \( x(b|\chi, \theta) \), let \( \{r(b, b', \chi, \theta), g^H(b, b', \chi, \theta), x(b, b', \chi, \theta)\} \) \( \in \Lambda \) denote the solution to the following intra-period maximization problem faced by the politician given \( b, b', \chi, \) and \( \theta \).

\[
\{r, x, g\} = \arg \max (1 - \gamma)v(r) + \gamma u^H(x, \theta) + \gamma w(g^H)
\]

(1.37)
s.t.

\[ r + x + g^H \leq e - b + \tau(b|\chi, \theta) + \beta b' \tag{1.38a} \]

\[ r + x \geq x(b|\chi, \theta), \tag{1.38b} \]

\[ w(g^H) + u^H(x, \theta) \geq \chi, \tag{1.38c} \]

\[ x \leq \theta (\omega^H + \omega^F), \tag{1.38d} \]

\[ g^H \geq 0, x \geq 0, r \geq 0. \tag{1.38e} \]

Let \( r^0 \) and \( x^0 \) be the policies chosen by the politician without constraint (1.38b). The equilibrium policy \( \underline{x}(b|\chi, \theta) \) is set such that \( r^0 + x^0 \leq \underline{x}(b|\chi, \theta) \), because it is a weakly dominated strategy for the supranational authority to set the intervention bound to at least what the politician would choose without the bound (otherwise recapitalizations would decrease, which would strictly decrease the utility of the supranational authority). Therefore, constraint (1.38b) holds with equality. Then, the budget constraint is a linear function of \( \tau(b|\chi, \theta) \) and \( \underline{x}(b|\chi, \theta) \). Therefore, \( g^H = e - b + \tau - \underline{x} + \beta b' \), so \( g^H(b, b'|\chi, \theta) \) is a concave function of debt if \( \tau(b|\chi, \theta) - \underline{x}(b|\chi, \theta) \) is concave.

Denote the indirect utility of the politician by

\[ u^P(b, b'|\chi, \theta) = (1 - \gamma) v(r(b, b'|\chi, \theta)) + \gamma u^H(x(b, b'|\chi, \theta), \theta) + \gamma w(g^H(b, b'|\chi, \theta)). \]

If \( \chi = 0 \), then constraint (1.38c) does not bind, and \( u^P(b, b'|\chi, \theta) \) is concave if \( (1 - \gamma) v(r(b, b'|\chi, \theta)) + \gamma u^H(x^H(b, b'|\chi, \theta), \theta) \) is concave given \( \underline{x}(b|\chi, \theta) \).

Claim 18 If \( \chi = 0 \), there exist functions \( v(r(\underline{x}(b))) \) and \( u^H(x(\underline{x}(b)), \theta) \) that are weakly concave given \( \underline{x}(b|\chi, \theta) \) concave.

To show that the above claim holds, take, for example, \( v(\cdot) \) to be an affine transformation of \( u^H(\cdot) \); then, \( r \) and \( x \) are linear functions of \( \underline{x} \), and \( v(r(\underline{x}(b))) \) and \( u^H(x(\underline{x}(b)), \theta) \) are weakly concave given \( \underline{x}(b|\chi, \theta) \) concave.

Assuming \( v(r(\underline{x}(b))) \) and \( u^H(x(\underline{x}(b)), \theta) \) weakly concave, \( u^P(b, b'|\chi, \theta) \) is continuous and concave, by the Implicit Function Theorem. Moreover, if \( (\tau(b) - \underline{x}(b) - b) \) and \( \underline{x}(b) \) are decreasing in \( b \), then \( u^P(b, b'|\chi, \theta) \) is also decreasing in \( b \).
If \( \chi = \chi^V \), then constraint (1.38c) does binds and \( u^P(b,b'|\chi, \theta) = (1-\gamma)v(r(b,b'|\chi, \theta)) + \chi^V \).

From condition (1.38c), since \( g^H \) is a linear function of \( \tau - \bar{x} - b \), it follows that \( u^H(\cdot) \) is a convex function of \( \tau - \bar{x} - b \), \(^{22}\) and hence \( r \) is a concave function of \( \tau - \bar{x} - b \). Therefore, \( v(r) \) is a concave function of \( \tau - \bar{x} - b \), so it is concave for \( \tau(b|\chi, \theta) - \bar{x}(b|\chi, \theta) \) concave. The utility function \( u^P(b,b'|\chi, \theta) \) is concave in this case as well.

Finally, the set of feasible values for \( b' \) is given by the set of values at which \( e - b + \tau(b|\chi, \theta) - \bar{x}(b|\chi, \theta) + \beta b' \geq 0 \), and the constraints (1.38b) and (1.38c) are satisfied. Therefore, the set of feasible values of \( b' \), \( \Gamma(b|\chi, \theta) \), is compact, with upper bound \( \bar{b} \), and lower bound given by the minimum value of \( b' \) at which constraints (1.38a), (1.38b), and (1.38c) are satisfied given \( b, \chi \) and \( \theta \).

**Part 2**

a) Concavity of the value function:

Assuming concavity of \( \mathbb{E}[V(b', \chi', \theta', \tau'(b', \chi', \theta'), \bar{x}'(b', \chi', \theta'))] \) we can show concavity of \( V(b, \chi, \theta, \tau, \bar{x}) \) by induction.

Consider two feasible values \( b_1, b_2 \in [b, \bar{b}] \), and \( b_3 = \vartheta b_1 + (1-\vartheta)b_2, \vartheta \in (0,1) \). Then, the supranational policies are given by functions \( \tau_1 = \tau(b_1, \chi, \theta), \bar{x}_1 = \bar{x}(b_1, \chi, \theta), \tau_2 = \tau(b_2, \chi, \theta), \bar{x}_2 = \bar{x}(b_2, \chi, \theta), \tau_3 = \tau(b_3, \chi, \theta), \bar{x}_3 = \bar{x}(b_3, \chi, \theta). \) Let

\[
\begin{align*}
\{x_1, r_1, g_1, b_1'\} &= \arg \max V(b_1, \chi, \theta, \tau_1, \bar{x}_1), \\
\{x_2, r_2, g_2, b_2'\} &= \arg \max V(b_2, \chi, \theta, \tau_2, \bar{x}_2).
\end{align*}
\]

Let

\( b_3' = \vartheta b_1' + (1-\vartheta)b_2' \),

and

\[
\{x_3, r_3, g_3\} = \arg \max (1-\gamma)v(r) + \gamma u^H(x, \theta) + \gamma w(g^H),
\]

subject to constraints (1.38a)-(1.38c) given \( b_3, b_3', \tau_3, \bar{x}_3 \).

\(^{22}\)Since \( u^H(x, \theta) = \chi^V - w(g^H) \) and \( g^H = e + \beta b' - b + \tau - \bar{x}. \)
Value $b_3'$ is feasible given $\Gamma(b|\chi, \theta)$ compact, and $\{x_3, r_3, g_3\}$ are feasible given the above maximization problem. Since $u^P(b, b'|\chi, \theta)$ is concave under the assumptions from Part 1:

$$V(b_3, \chi, \theta, \tau, x) \geq u^P(b_3, b'_3|\chi, \theta) + \beta\mathbb{E}[V(b_3')]$$

$$\geq \theta [u^P(b_1, b_1'|\chi, \theta) + \beta\mathbb{E}[V(b_1')]] + (1 - \theta) [u^P(b_2, b_2'|\chi, \theta) + \beta\mathbb{E}[V(b_2')]] .$$

By induction, the value function $V(b_3, \chi, \theta, \tau, x)$ is therefore concave.

b) Differentiability of the politician's value function:

The policy function is continuous, given the compact set $\Gamma(b|\chi, \theta)$. The implicit utility function

$$u^P(b, b'|\chi, \theta) = (1 - \gamma)u(r(b, b'|\chi, \theta)) + \gamma u^H(x(b, b'|\chi, \theta), \theta) + \gamma w(g^H(b, b'|\chi, \theta))$$

is concave and differentiable in $b$. It then follows by Lemma 1 of Benveniste and Scheinkman (1979) that $V(b, \chi, \theta, \tau, x)$ is differentiable with respect to $b$ over $(b, b')$.

**Part 3**

Consider the effects of a marginal increase in $\tau$ or a decrease in $x$ on the politician's choices of $r$, $x$, and $g^H$, starting from an initial agreement with a binding $x$.

- If $\chi = \chi^V$, a marginal increase in $\tau$ or a marginal decrease in $x$ relaxes the budget constraint. Then,

  $$\frac{\partial (r + x)}{\partial \tau} = 0; \frac{\partial r}{\partial \tau} < 0; \frac{\partial g^H}{\partial \tau} = \frac{u'(x, \theta) \partial x}{w'(g^H) \partial \tau} > 0; \beta \frac{\partial b'}{\partial \tau} = \frac{u'(x, \theta) \partial x}{w'(g^H) \partial \tau} - 1 < 0,$$

  $$\frac{\partial (r + x)}{\partial x} = 1; \frac{\partial x}{\partial x} > 0; \frac{\partial r}{\partial x} > 0; \frac{\partial g^H}{\partial x} = \frac{u'(x, \theta) \partial x}{w'(g^H) \partial x} < 0; \beta \frac{\partial b'}{\partial x} = \frac{u'(x, \theta) \partial x}{w'(g^H) \partial x} + 1 > 0.$$

- If $\chi = 0$, then

  $$\frac{\partial g^H}{\partial \tau} - \beta \frac{\partial b'}{\partial \tau} = 1; \frac{\partial x}{\partial \tau} = 0; \frac{\partial r}{\partial \tau} = 0;$$

  $$-\beta \frac{\partial b'}{\partial x} + \frac{\partial g^H}{\partial x} = -1; \frac{\partial b'}{\partial x} > 0; \frac{\partial g^H}{\partial x} < 0; \frac{\partial x}{\partial x} + \frac{\partial r}{\partial x} = 1; \frac{\partial x}{\partial x} > 0; \frac{\partial r}{\partial x} > 0.$$
Part 4

Consider now the value function for the supranational authority. Denote the instantaneous utility function for the supranational authority as

\[
u^S(\tau, x, b|\chi, \theta, b'(\tau, x)) = \eta u^H(x(\tau, x, b|\chi, \theta), b'(\tau, x)), \theta) + \eta w(g^H(\tau, x, b|\chi, \theta, b'(\tau, x))) + (1 - \eta)u^F(x(\tau, x, b|\chi, \theta, b'(\tau, x)), \theta) + (1 - \eta)w(e - \tau).
\]

If \(X = 0\), then \(u^H(x(\tau, x, b)) = u^H(x(\chi))\) which is concave under the assumptions from Part 1. Since \(u^F(\cdot)\) is the same type of function as \(u^H(\cdot)\), concavity of \(u^H(\cdot)\) implies concavity of \(u^F(\cdot)\).

If \(X = \chi V\), then \(u^S(\tau, x, b|\chi, \theta, b'(\tau, x)) = \eta \chi V + (1 - \eta)u^F(x(\tau, x, b|\chi, \theta, b'(\tau, x)), \theta) + (1 - \eta)w(e - \tau).\) From the electoral constraint, \(u^H(x, \theta) = \chi V - w(g^H)\) Concavity of \(u^F(\cdot)\) requires \(b'(\tau, x)\) convex. Under the condition that \(b'(\tau, x)\) is convex, \(u^F(x(\tau, x, b'(\tau, x), b|\chi, \theta), \theta)\) is concave and \(u^S(\tau, x, b|\chi, \theta, b'(\tau, x))\) is concave.

Part 5

Consider feasible values \(\{b_1, \tau_1, x_1\}\) and \(\{b_2, \tau_2, x_2\}\). Let \(\{b_3, \tau_3, x_3\} = \theta \{b_1, \tau_1, x_1\} + (1 - \theta) \{b_2, \tau_2, x_2\}, \forall \theta \in (0, 1)\). Then, \(\{b_3, \tau_3, x_3\}\) is feasible and satisfies all constraints. Due to the concavity of \(u^S(\tau, x, b|\chi, \theta, b'(\tau, x))\), the concavity of \(S(b, \chi, \theta)\) follows by induction, analogous to the proof in Part 2: \(S(b_3, \chi, \theta) \geq \theta S(b_1, \chi, \theta) + (1 - \theta)S(b_2, \chi, \theta)\). Therefore, \(S(\cdot)\) is concave.

Part 6

Consider the sequence of feasible values \(b^j\) such that \(b^j \to b\); then there is also a corresponding sequence \(\{\tau^j, x^j\}\) which converges to \(\{\tau, x\}\) since the instantaneous utility \(u^S(\tau, x, b)\) is continuous in \(\{\tau, x\}\). Given the policy correspondence \(G(b^j, \tau^j, x^j)\), we want to show that if \(b^{j'} \in G(b^j, \tau^j, x^j)\), then \(\exists\) a convergent subsequence \(b^{j''} \to b'\) with \(b' \in G(b, \tau, x)\). Since \(\{\tau^j, x^j\}\) are defined over compact sets, \(\{\tau, x\}\) is feasible. Moreover, it implies a convergent subsequence \(\{b^{j''}\}\) must exist. Then, by the Dominated Convergence Theorem, \(b' = G(b, \tau, x)\). Therefore, the policy function is continuous.

Part 7
Consider the sequence \( \{\tau, x\} \) associated with the debt \( b \in (b, \bar{b}) \). Then, with \( S(b, \chi, \theta) \) concave and a continuous policy function, we can apply the argument of Lemma 1 of Benveniste and Scheinkman (1979) to show that \( S(\cdot) \) is differentiable in \( b \) over \((b, \bar{b})\).

**Part 8**

The supranational authority chooses to offer an agreement \( (\tau, x) \), \( x \geq 0 \), \( \tau \geq 0 \) each period to maximize program (1.18). Consider the case when the participation constraint for the home government binds in equilibrium (i.e., under the conditions given in Assumption 7). The participation constraint for the politician is given by

\[
(1 - \gamma) v(\tau) + \gamma w(g^H) + \beta \mathbb{E} [V(b', \chi', \theta', \tau'(b', \chi', \theta'), x'(b', \chi', \theta'))] = (1.39)
\]

\[
(1 - \gamma) v(\tau^0) + \gamma w(g^{H, \theta}) + \beta \mathbb{E} [V(b^0, \chi', \theta', \tau'(b^0, \chi', \theta'), x'(b^0, \chi', \theta'))].
\]

The above participation constraint gives the expression for \( x(\tau | \chi, \theta) \). So,

\[
\left[ \frac{\partial u^H(x, \theta)}{\partial x} + (1 - \eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \left( \frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial x} \frac{\partial x}{\partial \tau} \right) + \eta \frac{\partial w(g^H)}{\partial g^H} \left( \frac{\partial g^H}{\partial \tau} + \frac{\partial g^H}{\partial x} \frac{\partial x}{\partial \tau} \right) + (1 - \eta) \frac{\partial w(e^F - \tau)}{\partial \tau} + \beta \mathbb{E} \left[ \frac{\partial S'}{\partial \theta'} \right] \left( \frac{\partial b'}{\partial \tau} + \frac{\partial b'}{\partial x} \frac{\partial x}{\partial \tau} \right) = 0,
\]

- **if \( \chi = 0 \):**

\[
\left[ \frac{\partial u^H(x, \theta)}{\partial x} + (1 - \eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \frac{\partial x}{\partial \tau} + \eta \frac{\partial w(g^H)}{\partial g^H} \left( \frac{\partial g^H}{\partial \tau} + \frac{\partial g^H}{\partial x} \frac{\partial x}{\partial \tau} \right) + (1 - \eta) \frac{\partial w(e^F - \tau)}{\partial \tau} + \beta \mathbb{E} \left[ \frac{\partial S'}{\partial \theta'} \right] \left( \frac{\partial b'}{\partial \tau} + \frac{\partial b'}{\partial x} \frac{\partial x}{\partial \tau} \right) = 0,
\]

- **if \( \chi = \chi^V \):**

\[
(1 - \eta) \frac{\partial u^F(x, \theta)}{\partial x} \left( \frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial x} \frac{\partial x}{\partial \tau} \right) + \beta \mathbb{E} \left[ \frac{\partial S'}{\partial \theta'} \right] \left( \frac{\partial b'}{\partial \tau} + \frac{\partial b'}{\partial x} \frac{\partial x}{\partial \tau} \right) = (1 - \eta) w'(e^F - \tau). \quad (1.41)
\]

The above condition can be used to derive \( \tau(b | \chi, \theta) \). Outstanding debt \( b \) and transfer \( \tau \) enter in the expressions for \( x, x, g^H \) and \( b' \) as \( (\tau - b) \). The utility of the supranational authority from \( x \) and \( g^H \) can therefore be written as a concave function of \( (\tau - b) \), and the first-order condition leads to \( 0 < \frac{\partial \tau}{\partial b} < 1 \); moreover, \( \tau(b | \chi, \theta) \) concave can be obtained given the concavity of the
utility functions. Thus, conditions (1.40) and (1.41) are consistent with a concave function $\tau(b|x, \theta)$. For $\bar{x}(b|x, \theta)$, $\frac{\partial \bar{x}}{\partial \tau} = \frac{\partial \bar{x}}{\partial \theta} + \frac{\partial \bar{x}}{\partial b}$, since $\bar{x}$ is a function of $(\tau - b)$, $\frac{\partial \bar{x}}{\partial \theta}$ is concave for $\frac{\partial \bar{x}}{\partial b}$ concave (coming out of (1.39)). This is consistent with $\tau(b|x, \theta) - \bar{x}(b|x, \theta)$ concave for $\tau < \bar{x}$, and $b'(\tau, \bar{x}|x, \theta)$ convex. These properties can be shown to hold by example: the policy functions that emerge using the log specifications for utility functions in the numerical simulation of Section 1.6.3 satisfy these conditions.

Then, there exists an equilibrium in which the value functions for the politician and the supranational authority are concave and differentiable.

1.A.5 Proof of Lemma 7

For any $\tau < e^F$ denote by $\bar{x}^*$ the value of $\bar{x}$ at which the participation constraint for the politician, constraint (1.19), holds with equality. Then, the participation constraint for the politician binds in equilibrium if the following condition holds:

$$
\left[ \eta \frac{\partial u^H(x, \theta)}{\partial x} + (1 - \eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \left. \frac{\partial x(b, x, \theta, \tau, \bar{x})}{\partial \bar{x}} \right|_{\bar{x}^*} + \eta \frac{\partial w(g^H(0))}{\partial g^H(b, x, \theta, \tau, \bar{x})} \left. \frac{\partial x(b, x, \theta, \tau, \bar{x})}{\partial \tau} \right|_{\bar{x}^*} + \beta \mathbb{E} \left[ \frac{\partial S(b', x', \theta')}{\partial b'} \right] \left. \frac{\partial b'(b, x, \theta, \tau, \bar{x})}{\partial \tau} \right|_{\bar{x}^*} \geq 0.
$$

The condition says that an increase in the intervention rule and a decrease in transfers would achieve an increase in the utility of the supranational authority. The condition can be re-written as:

$$
\left[ \eta \frac{\partial u^H(x, \theta)}{\partial x} + (1 - \eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \left( \left. \frac{\partial x(b, x, \theta, \tau, \bar{x})}{\partial \bar{x}} \right|_{\bar{x}^*} - \left. \frac{\partial x(b, x, \theta, \tau, \bar{x})}{\partial \tau} \right|_{\bar{x}^*} \right) + \eta \frac{\partial w(g^H(0))}{\partial g^H(b, x, \theta, \tau, \bar{x})} \left( \left. \frac{\partial g^H(b, x, \theta, \tau, \bar{x})}{\partial x} \right|_{\bar{x}^*} - \left. \frac{\partial g^H(b, x, \theta, \tau, \bar{x})}{\partial \tau} \right|_{\bar{x}^*} \right) + \beta \mathbb{E} \left[ \frac{\partial S(b', x', \theta')}{\partial b'} \right] \left( \left. \frac{\partial b'(b, x, \theta, \tau, \bar{x})}{\partial \bar{x}} \right|_{\bar{x}^*} - \left. \frac{\partial b'(b, x, \theta, \tau, \bar{x})}{\partial \tau} \right|_{\bar{x}^*} \right) \geq 0.
$$
For $\eta = 0$, the above condition gives the upper limit $b^*(\eta)$ on $\bar{b}$ for which the supranational authority will prefer to increase the intervention rule, given the decrease in expected future utility. The condition holds for $\bar{b} = 0$, and given the continuity of $S(b, \chi, \theta)$, it holds in an interval around $\bar{b} = 0$.

For $\eta = 1$, the above condition does not hold given the first-order conditions of the politician’s problem, under which

$$\frac{\partial u^H(x, \theta)}{\partial x} \left( \frac{\partial x(b, \chi, \theta, \tau, \bar{x})}{\partial x} \right)_{\bar{x}}, \quad \frac{\partial x(b, \chi, \theta, \tau, \bar{x})}{\partial \tau} \right) - \frac{\partial w(g^H)}{\partial g^H} \left( \frac{\partial g^H(b, \chi, \theta, \tau, \bar{x})}{\partial x} \right)_{\bar{x}}, \quad \frac{\partial g^H(b, \chi, \theta, \tau, \bar{x})}{\partial \tau} \right) \leq 0.$$

The left-hand side of the condition is continuous in $\eta$ and $b$, and monotonous in $b$ and in $\eta$ for $\bar{b} < b^*(\eta)$. Therefore, there exist feasible values $(\eta, \bar{b})$ under which the above condition holds.

1.A.6 Proof of Lemma 8

Assume the condition of the lemma holds for $b^* \in [\bar{b}, \bar{b}]$ at $\bar{x}(\tau)$ at which the participation constraint (1.19) holds with equality given $\theta$, $\chi$, $b^*$, $\tau$. Then, given the politician’s problem, the functions $u^H(x, \theta)$ and $w(g^H)$ satisfy $\frac{\partial u^H(x, \theta)}{\partial \bar{x}} \leq 0$ and $\frac{\partial w(g^H)}{\partial \bar{x}} \leq 0$. The utility function $u^F$ has the same properties as $u^H$, it follows that $\frac{\partial S(b, \chi, \theta)}{\partial b} \leq 0$. Therefore, if the given condition holds for $b^*$, then it also holds for $b < b^*$.

1.A.7 Proof of Proposition 9

Under Assumption 7, the participation constraint for the politician binds in equilibrium for all debt levels $b \in [\bar{b}, \bar{b}]$, and the equilibrium rule $\bar{x}$ binds in every period.

---

23Since $g^H$ is a linear function of $b$ and $\bar{x}$, and $x$ increases in $\bar{x}$ and weakly increases in $b$ (depending on the realization of $\chi$).
If $\chi = 0$, constraint (1.19) takes the form:

$$
(1 - \gamma) v(r_{10}) + \gamma u^H(x_{10}, \theta) + \gamma w(g_{10}^H) + \beta \mathbb{E} \left[ V(b_{01}', \chi', \theta', \tau'(b_{01}', \chi', \theta'), x'(b_{01}', \chi', \theta')) \right] = \\
(1 - \gamma) v(r_{00}) + \gamma u^H(x_{00}, \theta) + \gamma w(g_{00}^H) \\
+ \beta \mathbb{E} \left[ V(b_{00}', \chi', \theta', \tau'(b_{00}', \chi', \theta'), x'(b_{00}', \chi', \theta')) \right],
$$

(1.42)

where the two subscripts denote that $\chi = 0$ and whether $\varphi = 1$ or $\varphi = 0$. A binding rule $r_{01} + x_{01} = \bar{x}_0$ implies $r_{01} > r_{00}$ and $x_{01} > x_{00}$, given the politician's first-order conditions.

When $\chi = \chi^V$, the binding participation constraint (1.19) takes the form:

$$
(1 - \gamma) v(r_{11}) + \gamma u^H(x_{11}, \theta) + \gamma w(g_{11}^H) + \beta \mathbb{E} \left[ V(b_{11}', \chi', \theta', \tau'(b_{11}', \chi', \theta'), x'(b_{11}', \chi', \theta')) \right] = \\
(1 - \gamma) v(r_{10}) + \gamma u^H(x_{10}, \theta) + \gamma w(g_{10}^H) \\
+ \beta \mathbb{E} \left[ V(b_{10}', \chi', \theta', \tau'(b_{10}', \chi', \theta'), x'(b_{10}', \chi', \theta')) \right],
$$

(1.43)

where the two subscripts denote that the electoral constraint binds and whether $\varphi = 1$ or $\varphi = 0$. A binding rule $r_{11} + x_{11} = \bar{x}_1$ implies $r_{11} > r_{10}$ and $x_{11} > x_{10}$ and $r_{11} > r_{10}$.

Starting at time $s$, the expected politician utility under a partial banking union (given no deviation):

$$
V_s = \max_{\theta, \chi} \left\{ \sum_{t=s}^{\infty} \beta^{t-s} \left[ (1 - \gamma) v(r_t) + \gamma w(g_t^H) + \gamma u^H(x_t, \theta) \right] \right\},
$$

(1.44)

and the politician utility under no partial banking union in any period is:

$$
V_s^0 = \max_{\theta, \chi} \left\{ \sum_{t=s}^{\infty} \beta^{t-s} \left[ (1 - \gamma) v(r_t^0) + \gamma w(g_t^{H, 0}) + \gamma u^H(x_t^0, \theta_t) \right] \right\}.
$$

(1.45)

In equilibrium, the agreement $$(\tau, x) \text{ is offered such that the participation constraint for the politician binds in every period. So } V_s = V_s^0$$:

$$
\mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} \left[ (1 - \gamma) v(r_t) + \gamma w(g_t^H) + \gamma u^H(x_t, \theta_t) \right] = \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} \left[ (1 - \gamma) v(r_t^0) + \gamma w(g_t^{H, 0}) + \gamma u^H(x_t^0, \theta_t) \right].
$$

(1.46)
For any outstanding debt level $b_t$, accepting an agreement with a binding rule $x$ implies a higher $b_{t+1}$ than under no agreement, given conditions (1.21a)-(1.21d). Then, the problem exhibits monotonicity in $b$: the policies $r$, $g^H$, and $x$ are non-increasing in $b$ (given (1.21a)-(1.21d)) and $b'$ is non-decreasing in $b$. So, a higher $b$ implies lower expected future utility for the politician, and lower expected future household utility:

\[
E \sum_{t=s+1}^{\infty} \beta^{t-s-1} \left[ w(g_t^H) + u^H(x_t, \theta_t) \right] \leq E \sum_{t=s+1}^{\infty} \beta^{t-s-1} \left[ w(g_t^{H,0}) + u^H(x_t^0, \theta_t) \right].
\]

(1.47)

If the electoral constraint binds in the current period, then the current-period household utility is fixed at $\chi^V$. Therefore, the above inequality extends to:

\[
E \sum_{t=s}^{\infty} \beta^{t-s} \left[ w(g_t^H) + u^H(x_t, \theta_t) \right] \leq E \sum_{t=s}^{\infty} \beta^{t-s} \left[ w(g_t^{H,0}) + u^H(x_t^0, \theta_t) \right].
\]

(1.48)

If the electoral constraint does not bind in the current period, then the change in the current-period utility of households is given by $[u^H(x_{01},\theta) + w(g_{01}^H) - u^H(x_{00},\theta) - w(g_{00}^H)]$. By Assumption 8, the decrease in instantaneous household utility due to $g^H$ is larger than the increase in instantaneous household utility due to $x$. Then inequality (1.47) can be extended to:

\[
E \sum_{t=s}^{\infty} \beta^{t-s} \left[ w(g_t^H) + u^H(x_t, \theta_t) \right] \leq E \sum_{t=s}^{\infty} \beta^{t-s} \left[ w(g_t^{H,0}) + u^H(x_t^0, \theta_t) \right].
\]

(1.49)

### 1.A.8 Proof of Lemma 10

Under Assumption 7, the participation constraint for the politician binds in equilibrium for all debt levels $b \in [\bar{b}, \overline{\bar{b}}]$, and the equilibrium rule $x$ binds in every period. Also, under Assumption 4, $\chi^V$ is large enough relative to $\bar{x}_1$ such that the electoral constraint still binds both with and without the agreement Assume a given transfer level $\tau$. For the given $\tau$, the participation constraint of the politician determines how high $x$ can be set. Let $\bar{x}(\tau|\chi = \chi^V)$ be the value of rule $\bar{x}(\tau)$ when $\chi = \chi^V$, and $\bar{x}(\tau|\chi = 0)$ the value of rule $\bar{x}(\tau) \text{ when } \chi = 0$.

When $\chi = 0$, constraint (1.19) takes the form (1.42) and when $\chi = \chi^V$ it is given by (1.43). The electoral constraint binding in the current period means

\[
u^H(x_{11},\theta) + w(g_{11}^H) = u^H(x_{10},\theta) + w(g_{10}^H) = \chi^V.
\]

(1.50)
Therefore, constraint (1.43) can be re-written as

\[(1 - \gamma) v(r_{11}) + \beta \mathbb{E} \left[ V(b'_{11}, \chi', \theta', \tau', \varepsilon') \right] = (1 - \gamma) v(r_{10}) + \beta \mathbb{E} \left[ V(b'_{10}, \chi', \theta', \tau', \varepsilon') \right]. \tag{1.51} \]

Consider an agreement \((\bar{x}_1, \tau)\) such that \(r_{11} + x_{11} = \bar{x}_1\). Then \(\{r_{11}, x_{11}, g_{11}^H, b'_{11}\}\) are chosen given the politician’s first-order conditions (1.21a)-(1.21d). Given these conditions, the binding limit \(\bar{x}_1\) implies both \(x_{11}\) and \(r_{11}\) increase, \(g_{11}\) decreases, and \(b'_{11}\) increases.

Given the binding electoral constraint (1.50), the value of the public good \(g_{11}^H\) is given implicitly by \(w(g_{11}^H) = \chi^V - u^H(x_{11}, \theta)\) and the value of \(g_{10}^H\) is given implicitly by \(w(g_{10}^H) = \chi^V - u^H(x_{10}, \theta)\).

In this case, let \(\delta\) denote the increase in \((x + \tau)\) caused by the agreement, \(\delta = \bar{x}_1 - x_{10} - r_{10}\), and \(\delta_x \equiv x_{11} - x_{10}, \delta_r = r_{11} - r_{10}\), so that \(\delta = \delta_x + \delta_r\). Let \(\delta_g = g_{10}^H - g_{11}^H\). Given (1.50) and the concavity of \(u^H(\cdot)\) and \(w(\cdot)\), it follows that \(\delta_g < \delta_x\). So, the increase in debt is given by \(\beta \delta_b = \delta_x + \delta_r - \delta_g - \tau\). Also, \(\delta_b > 0\) since constraint (1.43) binds.

Consider as before two values \(\chi_1^V\) and \(\chi_2^V\), \(\chi_1^V < \chi_2^V\) such that at both these values the participation constraint binds under the agreement:

\[
(1 - \gamma) v(r_{11}|\chi_j^V) + \beta \mathbb{E} \left[ V(b'_{11}, \chi', \theta', \tau'(b'_{11}, \chi', \theta'), \varepsilon'(b'_{11}, \chi', \theta')|\chi_j^V) \right] = (1 - \gamma) v(r_{10}|\chi_j^V) \\
+ \beta \mathbb{E} \left[ V(b'_{10}, \chi', \theta', \tau'(b'_{10}, \chi', \theta'), \varepsilon'(b'_{10}, \chi', \theta')|\chi_j^V) \right], \tag{1.52} \]

with \(j = 1, 2\).

Given the first-order conditions to the politician’s problem, (1.21a)-(1.21d), \(b'_{10}(\chi_2^V) > b'_{10}(\chi_1^V)\), where \(b'(\chi)\) denotes the debt taken on given electoral demands \(\chi^V\). Then, given the concavity of \(V(\cdot)\), increases in debt are more costly in terms of continuation utility when \(\chi = \chi_2^V\) than when \(\chi = \chi_1^V\), so \(\delta_r(\chi_2^V) < \delta_r(\chi_1^V)\) and \(\delta_b(\chi_2^V) < \delta_b(\chi_1^V)\), where the effect of a change in \(\chi^V\) on \((\delta_x - \delta_g)\) is second-order. Hence, \(\varepsilon(\tau|\chi = \chi_1^V) > \varepsilon(\tau|\chi = \chi_2^V)\). The values \(\chi_1^V\) and \(\chi_2^V\) were arbitrary conditional on satisfying Assumption 4. Therefore \(\varepsilon(\tau)\) decreases for higher values of \(\chi^V\).
1.A.9 Proof of Proposition 11

The supranational authority is choosing the transfer and intervention rule \((\tau, \mathbb{x})\) every period in order to solve program (1.18) subject to the participation of both governments. Under Assumption 7, the participation constraint for the politician binds in equilibrium for all debt levels \(b \in [b, \bar{b}]\), and the equilibrium rule \(\mathbb{x}\) binds in every period. Given the function \(\mathbb{x}(\tau, \chi)\) derived from the participation constraint of the politician (as shown in the proof to Proposition 10) the supranational authority chooses the value of \(\tau\) to maximize the following utility:

\[
S(b, \chi, \theta) = \max_{\tau} \eta \left[ u^H(x(b, \chi, \theta, \tau, \mathbb{x}(\tau)), \theta) + w(g^H(b, \chi, \theta, \tau, \mathbb{x}(\tau))) \right] + \beta \mathbb{E} \left[ S(b', \chi', \theta') \right].
\]

Given the first-order conditions described above, the equilibrium transfer \(\tau\) satisfies:

\[
\left[ \eta \frac{\partial u^H(x, \theta)}{\partial x} + (1-\eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \left( \frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial \mathbb{x}} \frac{\partial \mathbb{x}}{\partial \tau} \right) + \eta \frac{\partial w(g^H)}{\partial x} \left( \frac{\partial g^H}{\partial \tau} + \frac{\partial g^H}{\partial \mathbb{x}} \frac{\partial \mathbb{x}}{\partial \tau} \right)
+ (1-\eta) \frac{\partial w(e^F - \tau)}{\partial \tau} \left( \frac{\partial S'}{\partial \mathbb{x}} \left( \frac{\partial \mathbb{b}'}{\partial \tau} + \frac{\partial \mathbb{b}'}{\partial \mathbb{x}} \frac{\partial \mathbb{x}}{\partial \tau} \right) = 0, \right.
\]

\[(1.53)\]

- if \(\chi = 0\):

\[
\left[ \eta \frac{\partial u^H(x, \theta)}{\partial x} + (1-\eta) \frac{\partial u^F(x, \theta)}{\partial x} \right] \frac{\partial x}{\partial \tau} + \eta \frac{\partial w(g^H)}{\partial x} \left( \frac{\partial g^H}{\partial \tau} + \frac{\partial g^H}{\partial \mathbb{x}} \frac{\partial \mathbb{x}}{\partial \tau} \right)
+ (1-\eta) \frac{\partial w(e^F - \tau)}{\partial \tau} \beta \mathbb{E} \left[ \frac{\partial S'}{\partial \mathbb{x}} \left( \frac{\partial \mathbb{b}'}{\partial \tau} + \frac{\partial \mathbb{b}'}{\partial \mathbb{x}} \frac{\partial \mathbb{x}}{\partial \tau} \right) = 0, \right.
\]

\[(1.54)\]

- if \(\chi = \chi^V\):

\[
(1-\eta) \frac{\partial u^F(x, \theta)}{\partial x} \left( \frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial \mathbb{x}} \frac{\partial \mathbb{x}}{\partial \tau} \right) + \beta \mathbb{E} \left[ \frac{\partial S'}{\partial \mathbb{x}} \left( \frac{\partial \mathbb{b}'}{\partial \tau} + \frac{\partial \mathbb{b}'}{\partial \mathbb{x}} \frac{\partial \mathbb{x}}{\partial \tau} \right) = (1-\eta)w'(e^F - \tau). \right.
\]

\[(1.55)\]

For a shock \(\theta\) and \(\chi = 0\), let \((x_0, \tau_0)\) denote the equilibrium agreement at which constraint (1.53) holds, given \(\mathbb{x}^*(\tau_0)\) as derived in Proposition 10. Similarly, let \((x_1, \tau_1)\) denote the equilibrium agreement when \(\chi = \chi^V\).

Under Assumption 4, the electoral constraint is binding under the agreement as well as under no agreement, and the condition for \(\tau\) is given by (1.55).
Consider the case when \( x = \chi^V \). At a higher \( \chi^V \), \( x_1(\tau_1|\chi^V, \theta) \) decreases, as shown in Proposition 10. The higher \( \chi^V \) lowers the continuation utility for both the politician and the supranational authority. Then \( \frac{\partial E[V(\theta', x', \theta')]}{\partial x} < 0 \), and \( \frac{\partial h_f}{\partial \tau} < 0 \). The decrease in \( \frac{\partial E}{\partial \tau} \) implies \( \frac{\partial x}{\partial \tau} > 0 \), since \( x \) is an increasing function of \( \tau + \beta b' \) (the government budget). Also, \( \frac{\delta S(\theta' | \chi^V)}{\delta \theta} < 0 \) (by the Envelope Theorem, \( \frac{\delta S(\theta, \chi)}{\delta \theta} \) decreases as higher \( \theta \) increases the cost of future debt). Finally, assuming a small enough \( \sigma \) as explained in Proposition 3, \( \frac{\partial x}{\partial \tau} \mid _{\chi=\chi^V} \) decreases in \( \chi^V \). This implies that in the equilibrium agreement, \( \tau_1 \) and \( x_0 \) decrease in response to a higher \( \chi^V \).

Consider the case when \( x = 0 \) in the current period, in which case the condition for \( \tau \) is given by (1.54). An increase in \( \chi^V \) decreases the expected continuation utility for the politician, and for every \( \tau_0 \), it decreases \( x_0(\tau_0) \). Then \( \frac{\partial E[V(\theta', x', \theta')]}{\partial x} < 0 \), and \( \frac{\partial h_f}{\partial \tau} < 0 \). The decrease in \( \frac{\partial E}{\partial \tau} \) implies \( \frac{\partial x}{\partial \tau} > 0 \). By similar arguments as above, \( \frac{\delta S(\theta' | \chi^V)}{\delta \theta} < 0 \) decreases as well. A higher \( \chi^V \) then implies that in the equilibrium agreement, \( \tau_0 \) and \( x_0 \) decrease in response to a higher \( \chi^V \).

From the above, \( \frac{\partial x}{\partial \tau} < 0 \) and \( \frac{\partial x}{\partial \chi} < 0 \). By Assumption 7, the supranational authority always prefers higher recapitalizations \( x \) than the politician. So higher \( \chi^V \) reduces the utility of donor households by reducing \( x \). Therefore, expected donor household utility decreases:

\[
\sum_{t=s}^{\infty} [u^F(x_t, \theta|\chi^V) + w(e^F - \tau_t|\chi^V)] < \sum_{t=s}^{\infty} [u^F(x_t, \theta|\chi^V) + w(e^F - \tau_t|\chi^V)],
\]

\( \forall \chi^V > \chi^V \).

1.A.10 Optimal policy choices with fiscal rules

When the supranational authority is controlling debt, the politician faces a static problem under the agreement. The participation constraint for the politician is given by (1.42) when \( \chi = 0 \) and (1.43) when \( \chi = \chi^V \), with \( \theta' \) being the choice of the supranational authority.

The supranational authority chooses policies \((x, \tau, \theta')\), \( x \geq 0 \), \( \tau \geq 0 \), each period, to maximize program (1.24) given the politician’s choices for \( r, x, \) and \( g \) as functions of \((x, \tau, \theta')\) and shocks \( \theta, \chi \). Denote by \( \psi(\chi, \theta) \) the Lagrange multiplier to the politician’s participation constraint. The supranational authority’s problem leads to the following first-order conditions,
assuming an interior solution for \( x \) and \( b' \) (and suppressing coefficients for brevity):

\[
\left\{ \begin{array}{c}
(\eta + \gamma \psi(x, \theta)) u^H(x, \theta) \frac{\partial x}{\partial x} + (\eta + \gamma \psi(x, \theta)) w'(g^H) \frac{\partial g^H}{\partial x} \\
+ (1 - \eta) u^{F'}(x, \theta) \frac{\partial x}{\partial x} + \psi(x, \theta) (1 - \gamma) v'(r) \frac{\partial r}{\partial x} \end{array} \right\} = 0, \quad (1.56)
\]

\[
\left\{ \begin{array}{c}
(\eta + \gamma \psi(x, \theta)) u^H(x, \theta) \frac{\partial x}{\partial \tau} + (\eta + \gamma \psi(x, \theta)) u'(g^H) \frac{\partial g^H}{\partial \tau} + (1 - \eta) u^{F'}(x, \theta) \frac{\partial x}{\partial \tau} \\
- (1 - \eta) w'(e^F - \tau) + \psi(x, \theta) (1 - \gamma) v'(r) \frac{\partial r}{\partial \tau} \end{array} \right\} = 0, \quad (1.57)
\]

\[
\left\{ \begin{array}{c}
(\eta + \gamma \psi(x, \theta)) u^H(x, \theta) \frac{\partial x}{\partial y'} + (\eta + \gamma \psi(x, \theta)) w'(g^H) \frac{\partial g^H}{\partial y'} + (1 - \eta) u^{F'}(x, \theta) \frac{\partial x}{\partial y'} \\
+ \psi(\theta, x) (1 - \gamma) v'(r) \frac{\partial r}{\partial y'} + \psi(\theta, x) \beta \mathbb{E} \left[ \frac{\partial V'}{\partial y'} \right] + \beta \mathbb{E} \left[ \frac{\partial S'}{\partial y'} \right] \end{array} \right\} = 0, \quad (1.58)
\]

and the Envelope condition:

\[
\frac{\partial S}{\partial b} = (\eta + \gamma \psi(x, \theta)) u^H(x, \theta) \frac{\partial x}{\partial b} + (\eta + \gamma \psi(x, \theta)) w'(g^H) \frac{\partial g^H}{\partial b} \\
+ (1 - \eta) u^{D'}(x, \theta) \frac{\partial x}{\partial b} + \psi(x, \theta) (1 - \gamma) v'(r) \frac{\partial r}{\partial b}. \quad (1.59)
\]

From the above conditions (1.57) and (1.58), the supranational authority equalizes the marginal cost of interventions through transfers and through debt, \((1 - \eta) w'(e^F - \tau) = -\mathbb{E} \left[ \frac{\partial S'}{\partial y'} + \psi(\theta, x) \frac{\partial V'}{\partial y'} \right] \), since \( \tau \) and \( \beta b' \) have the same marginal effect on policies \( x, g^H \), and \( r \).

1.A.11 Proof of Lemma 12

The politician’s static choices of \( x(b, \chi, \tau, \bar{x}, b') \), \( g(b, \chi, \tau, \bar{x}, b') \), and \( r(b, \chi, \tau, \bar{x}, b') \) lead to concave and differentiable functions \( u^H(\cdot) \), \( u^F(\cdot) \) and \( w(\cdot) \). Then, by induction, the value function \( S(\cdot) \) is concave. Moreover, the policy functions are continuous, so by the standard arguments, \(^{24}\) \( S(\cdot) \) is differentiable over \((b, \bar{b})\).

\(^{24}\) Lemma 1 of Benveniste and Scheinkman (1979)
1.A.12 Proof of Proposition 13

As before, the participation constraints for the politician are given by (1.42) and (1.43). When $\chi = \chi^V$, and the electoral constraint binds under the agreement.

Claim 19 When $\chi = \chi^V$, the equilibrium allocation has the property that rents are weakly lower than without the agreement ($r_{11} \leq r_{10}$).

Proof. Assume an allocation $\{r_{11}, x_{11}, g_{11}, b'_{11}\}$ under the partial banking union. Consider first the case when $r_{11} > r_{10}$, where $r_{10}$ is the choice of rents without the agreement in the current period. The participation constraint for the politician must hold with equality. Therefore, at $\tau > 0$, $x_{11}$ is binding, and $b'_{11} > b'_{10}$. Assume a decrease by some small $\beta \varepsilon$ of $r_{11}$ and a decrease of $\varepsilon$ in $b'_{11}$, along with a decrease of $\beta \varepsilon$ in $x_{11}$. Without the agreement $(1 - \gamma)v'(r_{10}) = \mathbb{E} \left[ -\frac{\partial V(b', \theta', \sigma', \varepsilon', z')}{\partial b'_{10}} \right]$. Therefore, $(1 - \gamma)v'(r_{11}) < \mathbb{E} \left[ -\frac{\partial V(b'_{11}, \theta', \sigma', \varepsilon', z')}{\partial b'_{11}} \right]$, and the change would increase the utility of the politician. Moreover, it would not change the actual recapitalization level $x_{11}$ even though $x_{11}$ is decreased. The supranational authority’s utility decreases with debt $b'$, so a policy of decreasing $b'_{11}$ and $x_{11}$ by $\beta \varepsilon$ increases the utility of the supranational authority. Since $r_{11}$ and $b'_{11}$ were arbitrary, this argument holds for any allocation with $r_{11} > r_{10}$ and $b'_{11} > b'_{10}$ in which the participation constraint for the politician binds.

If $r_{11} = r_{10}$ and $b'_{11} < b'_{10}$, then the participation constraint is slack. Rents can be decreased by a small $\varepsilon$ and transfers decreased by the same amount. This does not change the other policy choices, the participation constraint of the politician still holds, while the utility of the supranational authority increases. Therefore, the original allocation was not optimal.  

When $\chi = 0$ and the rule $x$ is binding, a decrease in debt $b'$ while keeping $\tau$ fixed cannot be realized without a decrease in the provision of the domestic public good $g$ and in the rule $x$. This comes out of the binding participation constraint (1.43). The decrease in rule $x$ then implies a decrease in $\tau$ and $x$ given the politician’s first-order conditions to the static problem.

Under Assumption 7, the supranational authority prefers to increase debt $b'$ whenever positive transfers are made. Any equilibrium agreement satisfies the first-order conditions for the supranational authority. This requires that the marginal benefit from an increase in $x$ be equal to the marginal cost of transfers plus the marginal cost of higher debt to the supranational authority (given its continuation utility). Any decrease in debt to the level of the outside option
is therefore not optimal, since it would require an associated decrease in $x$, and the marginal cost of decreasing $x$ is higher than the marginal benefit decreasing debt, as shown in equations (1.56)-(1.59).

1.A.13 Proof of Corollary 14

Under the conditions of Proposition 13, the supranational authority would not set debt higher than $b'_{10}$. Below we derive the conditions under which the supranational authority would prefer to set debt below $b'_{10}$. Consider a transfer $\tau$, and an allocation $\{r_{11}, x_{11}, g^H_{11}, b'_{11}\}$ with $b'_{11} = b'_{10}$ (the value of the outside option), such that $r_{11}, x_{11},$ and $g^H_{11}$ are chosen to maximize the utility of the supranational authority given $\tau$ and $b'_{10}$ (the debt chosen by the politician in the outside option).

For $\chi = \chi^V$, the electoral constraint binds:

$$u^H(x_{11}, \theta) + w(g^H_{11}) = \chi^V,$$

which implies that any decrease in $g^H$ must be compensated by an increase in $x$ such that the electoral constraint is satisfied.

At the $\{r_{11}, x_{11}, g^H_{11}, b'_{11}\}$ allocation, assume the following marginal change: a decrease in $b'$ of $\Delta b'$ and a decrease in $r$ of $\Delta r_1 < \beta \Delta b'$, such that the change in the politician’s utility is 0:

$$(1 - \gamma) \left[ v'(r - \Delta r_1) - v'(r) \right] + \mathbb{E} \left[ V(b' - \Delta b', \phi', \theta', \tau', x') - V(b', \phi', \theta', \tau', x') \right] = 0.$$

Also, decrease $x$ by $\Delta x_1$ and increase $g^H$ by $\Delta g^H_1$, $\Delta g^H_1 < \Delta x_1$, such that the change in politician’s utility is 0:

$$w'(g^H_{11} + \Delta g^H_1) - w'(g^H_{11}) = u'(x_{11}, \theta) - u'(x_{11} - \Delta x_1, \theta).$$

Finally, we require $-\Delta r_1 + \Delta g_1 - \Delta x_1 = -\beta \Delta b'$. The condition for this to increase the utility of the supranational authority when $\eta \to 0$:

$$(1 - \eta) \frac{\partial u^F(x_{11}, \theta)}{\partial x} \Delta x_1 < \mathbb{E} \left[ -\frac{\partial S(b'_{10}, \phi', \theta', \tau', x')}{\partial b'_{10}} \right] \beta \Delta b'.$$
The above condition can be rewritten as

\[
\frac{\Delta x_1}{\beta \Delta b'_{1}} \leq \frac{\mathbb{E} \left[ -\frac{\partial S(b'_{10}, \phi', \theta', \gamma', x')}{\partial b'_{10}} \right]}{(1 - \eta) \mathbb{E} \left[ \frac{\partial u^F(x_{11}, \theta)}{\partial x} \right]}.
\]

(1.60)

Condition (1.60) requires a sufficiently small decrease in \(x_{11}\) in response to a decrease in \(b'_{1}\). In the limit \(\Delta r_{1} \rightarrow \beta \Delta b'_{1}\), so \(\Delta x_{1} \rightarrow 0\) and condition (1.60) holds, so a decrease in rents below the level of the outside option can be achieved. Since the condition is endogenous, we cannot infer sharper conditions on parameters.

1.A.14 Proof of Corollary 15

Compared to the allocation chosen by the politician in the case without a banking union, the politician’s utility from rents decreases when \(\chi = \chi'\), and it increases when \(\chi = 0\). Consider the equilibrium agreement \((x_{0}, \tau_{0}, b'_{0})\) for \(\chi = 0\) and \((x_{1}, \tau_{1}, b'_{1})\) for \(\chi = \chi'\). The politician’s expected continuation utility is given by

\[
\mathbb{E} \left[ V(b'_{1}, \chi', \theta', \tau', x') \right] \geq \mathbb{E} \left[ V(b'_{0}, \chi', \theta', \tau', x') \right],
\]

where \(b'_{1} \leq b'_{0}\). The politician’s participation constraint must bind in equilibrium, so rents are weakly lower under the banking union with fiscal rules and electoral accountability.

A Pareto improvement can be achieved if the expected home household welfare is at least as large as under no banking union:

\[
\mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} \left[ w(g_t^H) + u^H(x_t, \theta) \right] \geq \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} \left[ w(g_t^H, 0) + u^H(x_t^0, \theta) \right].
\]

Lower public debt under the banking union \((b'_{11} < b'_{10})\) increases expected household utility and expected future rents. An agreement when \(\chi = 0\) reduces household utility, due to the increase in both recapitalizations and rents, as shown in Proposition 9. Therefore, the overall effect on home household welfare depends on the expected increase in utility due to the lower outstanding debt relative to the decrease in utility due to rent-seeking when \(\chi = 0\).

A higher \(\pi\) implies an increase in the expected frequency of periods of electoral accountability, so increases the expected household utility. Moreover, the reduction in outstanding debt
under the partial banking union also implies higher expected future household utility under the partial banking union relative to the expected household utility under no banking union. In the limit case in which electoral accountability leaves rents equal to the rents in the outside option, \( \pi = 1 \) is necessary for a Pareto improvement. If rents decrease relative to the outside option when \( \chi = \chi^V \), then let \( \pi^* < 1 \) denote the value of \( \pi \) at which the expected household welfare under the agreement equals the expected household welfare under no banking union. It follows that for \( \pi \geq \pi^* \), a Pareto improvement can be achieved under the partial banking union.

1.A.15 Proof of Proposition 16

The proof is analogous to the proof to Proposition 11. If the electoral constraint binds under the agreement, then the rules \( \bar{x} \) that can be set in equilibrium decreases. This happens both when \( \chi = \chi^V \) and when \( \chi = 0 \), through the effects of lower expected future utility. It then implies a decrease in donor household welfare.

1.A.16 Proof of Proposition 17

The proof follows the same approach as in the proof of Proposition 3 in Battaglini and Coate (2008). Let \( \psi_t(b') \) denote the distribution function of the current level of debt at the beginning of period \( t \). The distribution function \( \psi_t(b') \) is exogenous and determined by the initial level of debt \( b_0 \). Let \( \hat{\Theta} = \{0, \chi^V \} \times \Theta \) and, since the shocks are independent, let \( \hat{P} \) denote the joint cumulative distribution over \( \hat{\Theta} \).

The correspondence implied by the politician’s equilibrium choices and the supranational authority’s equilibrium policy choices is given by \( T : [b, \bar{b}] \times [\bar{b}, \bar{b}] \rightarrow \hat{\Theta} : \)

\[
T(b, b') = \begin{cases} 
(0, \theta^0) & \text{if } b' < b'^{\min} \\
\min \left\{ (\chi, \theta) \in \hat{\Theta} : b' = \min \left\{ b' \mid (b, \chi, \theta) = \bar{x}(b, \chi, \theta) \right\} \right\} & \text{if } b' \in [b'^{\min}, b'^{\max}] \\
(\chi^V, \theta^N) & \text{if } b' > b'^{\max}
\end{cases}
\]

where \( b'^{\min} = b'(b, 0, \theta^0, \tau(b, 0, \theta^0), \bar{x}(b, 0, \theta^0)) \), \( b'^{\max} = b'(b, \chi^V, \theta^N, \tau(b, \chi^V, \theta^N), \bar{x}(b, \chi^V, \theta^N)) \).

The correspondence \( T(b, b', \tau, \bar{x}) \) gives the minimum combination of shocks under which the equilibrium new debt level would be \( b' \) given outstanding debt \( b \). Then, the transition function
is given by

$$H(b,b') = \tilde{P}(T(b,b')).$$

The function \(H(b,b',\tau,x)\) gives the probability that next period's debt will be less than or equal to \(b'\) given the current outstanding debt \(b\). Then, the distribution of debt at the beginning of any period \(t \geq 2\) is defined inductively by

$$\psi_t(b') = \int_b H(b,b')d\psi_{t-1}(b).$$

The sequence of distributions \(\psi_t(b')\) converges to distribution \(\psi(b')\) if \(\forall b \in [b,\bar{b}],\)

$$\lim_{t \to \infty} \psi_t(b') = \psi(b').$$

The limiting distribution is invariant if \(\psi^*(b') = \int_b H(b,b')d\psi^*(b).\)

To prove that the sequence of distributions converges to a unique invariant distribution, we must first prove that \(H(b,b')\) has the Feller Property and that it is monotonic in \(b\). By Theorem 12.12 in Stokey (1989), the following mixing condition must be satisfied: \(\exists \epsilon > 0\) and \(m \geq 1\), such that for any \(b^* \in [b,\bar{b}],\) \(H^m(\bar{b},b^*) \geq \epsilon\) and \(1 - H^m(b,b^*) \geq \epsilon\) where the function \(H^m(b,b')\) is defined inductively by \(H^1(b,b') = H(b,b')\) and \(H^m(b,b') = \int_z H(z,b')dH^{m-1}(b,z).\)

This condition requires that starting from the highest level of debt \(\bar{b}\), we will end up at debt \(b^*\) with probability greater than \(\epsilon\) after \(m\) periods, and if we start with the lowest level of debt, we will end up above \(b^*\) with probability greater than \(\epsilon\) in \(m\) periods.

The mixing condition can be shown to be satisfied given the monotonicity properties of the equilibrium policy functions, with respect to both \(b\) and the shocks \(\theta\) and \(\chi\).

For any \(b \in [b,\bar{b}]\) and \((\chi,\theta) \in \tilde{\Theta}\) define the sequence \((\phi_m(b,\chi,\theta))\) as follows: \(\phi_0(b,\chi,\theta) = b,\)

$$\phi_{m+1}(b,\chi,\theta) = b'(b,\chi,\theta),$$

assuming that the supranational authority is following the equilibrium policies \(\tau(b,\chi,\theta)\) and \(\pi(b,\chi,\theta)\). This means that \(\phi_m(b,\chi,\theta)\) is the level of new debt starting from outstanding debt \(b\), and assuming the same pair of shocks \((\chi,\theta)\) is repeated in periods 1 through \(m\). By the setup of the model there is a positive probability on each pair \((\chi,\theta)\), therefore

$$\hat{\pi}(\chi,\theta') - \hat{\pi}(\chi,\theta) > 0$$

for \(\theta' > \theta\). This implies that \(H^m(b,\phi_m(b,0,\theta^0)) - H^m(b,\phi_m(b,0,\theta^0)) > 0\) for \(\theta' > \theta^0\).

Using the above, it can be shown that \(H^m(\bar{b},b^*) > 0\). It suffices to show that for \(m\) sufficiently large, \(T(\phi_m(\bar{b},0,\theta^0),b^*) > (0,\theta^0)\). Then for any such \(m\), given the above property, \(T(\phi_m(\bar{b},0,\theta^0),b^*) > (0,\theta^0)\), for any \(\theta' > \theta^0\). From the politician’s first-order condi-
tions, the realization of shocks \((0, \theta^0)\) implies a decreasing debt \(b'\). Suppose that \(\phi_m(b, 0, \theta^0)\) converged to some \(b^{*\ast} > b\), then in the limit, by the continuity of the policy functions, 
\[
\lim_{m \to \infty} g'(\phi_m(b, 0, \theta^0), x, \theta) = g'(b^{*\ast}, x, \theta)
\]
for all pairs \((x, \theta)\). However, the policy \(g\) is are strictly decreasing in \(\theta\), which contradicts the above convergence assumption.

The analogous argument can be made starting from \(b\), given repeated \(\theta^N\) shocks, to show that \(1 - H^m(b, b^*) \geq \epsilon\).

Therefore, the necessary conditions are satisfied for a unique invariant distribution.
Chapter 2

Politically Feasible Public Bailouts

2.1 Introduction

The amount of public funds and the variety of programs used by governments to support the financial sector during the 2008-2009 crisis have been unprecedented.\(^1\) Governments intervened through programs ranging from specific, targeted transfers meant to inject capital in particular institutions, to broader, untargeted interventions aimed at supporting struggling industries and lowering the rates at which firms can borrow in the market.\(^2\) Two key characteristics stand out when examining these policy choices.

First, the bailouts of struggling financial institutions at the beginning of the crisis faced significant voter disapproval.\(^3\) Part of the negative public opinion involved the question of whether these bailout packages were a necessary response to the crisis or too large a rescue offered to lobbying banks.\(^4\) Voter backlash to these measures was reflected, for example, in the

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1. See Block (2010) for a description of the different programs and the estimated costs associated with them.
2. For example, permitting discount window loans to be collateralized by high grade private securities, opening the window for non-bank financial institutions, actions which essentially reduce the interest rate faced by borrowers.
3. These interventions included, in the US, taking Freddie Mac and Fannie Mae into government conservatorship or the capital injections into the largest private U.S. banks under the Troubled Asset Relief Program (TARP), passed in October 2008.
difficulty of passing another bailout program through the US Congress later on the crisis, in order to rescue troubled car makers.5

The second key feature that stands out is that, in spite of voter opposition, governments continued to intervene in the economy in order to avoid a systemic collapse, and they used a combination of targeted and untargeted interventions, along with a fragmentation of interventions among different "on-budget" and "off-budget" institutions, such that only part of the bailout programs counted as government spending.6 In the United States, the Treasury implemented capital assistance programs in conjunction with the Fed,7 it also developed several programs that were aimed at supporting struggling sectors rather than being targeted at particular firms,8 and it provided indirect assistance through special tax breaks.9 A similar pattern was also observed in Europe. For example, Germany began by directly rescuing its struggling banks using public funds.10 The government introduced a program to guarantee loans and help shore up equity in troubled banks in October 2008, followed by untargeted programs passed through the parliament in November 2008 and February 2009, both of which included a combination of untargeted support measures such as tax breaks, provisions for underwriting credit to struggling firms, and subsidies to industries.11

Given the above two features of the crisis response, the question that emerges is whether the observed government policies were a result of voter backlash followed by political stalemate,12 or something to be expected even in the absence of political stalemates. This paper argues

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5 A proposed bailout of the auto makers failed to pass a Senate vote in December, 2008. Afterwards, the President redirected funds meant for financial institutions from the TARP, in order to bail out the auto makers. For more details, see Stephen Labaton and David M. Herszenhorn, "White House Ready to Offer Aid to Auto Industry," The New York Times, December 12, 2008, page A1.

6 For more details on these programs and accounting methods, see Block (2010).

7 The most significant is the Term Asset-Backed Securities Loan Facility (TALF) set up in November 2008 to offer capital assistance to struggling firms; it was amended in December 2008 to extend loans with longer maturities and to accept a broader range of collateral.

8 For instance, the Car Allowance Rebate System to help car dealerships and car makers, and the Loan Modification Program for Homeowners, aimed at helping indebted households, both passed in 2009.


12 Mian et al. (2012) provide evidence about increased polarization and fractionalization following crises, which may explain the difficulties in reaching compromise.
that these two features of the crisis are an outcome that emerges in the absence of any political stalemate, given voters' available instruments for providing incentives to their elected officials. Such instruments are generally restricted to removal from office through elections, based on the information that voters possess about how appropriate the actions of the government are relative to the gravity of the crisis. Using the information available to them, voters evaluate whether their representatives are using the public funds in their constituents' interest, and they decide whether to reelect the incumbents. This paper builds a model that sheds light on why a shift over time from targeted to untargeted interventions can be in the best interest of voters.

The paper presents a principal-agent model in which government intervention is decided by an elected politician, who cares about maximizing a weighted sum of the economy-wide output. The output in the economy is produced by firms, and each period a fraction of firms is hit by a shock that causes a loss in their investment projects. After this shock hits, firms can reinvest funds in their projects; however, only the funds reinvested in projects that suffered losses give positive net returns, while investing additional funds in the healthy projects would be not cost effective. The funds for reinvestment can come from the firm's own reserves or from public funds, through the government. The firms hold identical projects ex-ante, but they differ based on whether or not they are connected to the government. Connected firms can lobby the politician for public transfers, and this leads to the politician placing more weight on the output of connected firms. It also results in a difference in objectives between the politician and voters, since voters do not benefit from the firms' lobbying activities. The government can choose to make public funds available to firms either through targeted transfers that are directed at a specific firm or through untargeted transfers that are given to all firms. The politician is biased towards using targeted transfers, which can be directed at connected firms. Finally, at the end of each period, elections are held, and voters can decide to remove the politician from office. They make this decision based on the information available to them about the size of the shocks hitting the economy and about the size of targeted versus untargeted transfers. Therefore, private information plays a key role in the model. Voters cannot directly observe the size of the shocks that hit the economy, or the fraction of connected firms that are not distressed but lobby the government, and they must rely on reports from the politician.
The main result of the paper is that, in equilibrium, voters provide dynamic incentives for the politician to limit the transfers made to firms who engage in lobbying activities, and for which the public transfers would be socially inefficient. Following a crisis period that requires a large government intervention, the politician is more restricted from using targeted transfers in the future. Voters implement the restriction by conditioning reelection on the politician reducing the funds allocated to targeted transfers. The voters’ restrictions are then relaxed after periods with low government transfers. The intuition for this result is that a politician who reports a high need for government intervention will initially be allowed to engage in public bailouts, even if voters know that some of these funds will be used for inefficient transfers to lobbying firms. This would be acceptable to voters given the severe decrease in household consumption in the absence of bailouts. Yet, if the same need for government transfers is reported next period as well, then voters are more likely to infer that the public funds will be used by the politician for inefficient transfers to lobbying firms rather than needed bailouts. Therefore, they will be less willing to keep the politician in power. Faced with a tougher condition for reelection, if the crisis indeed requires government intervention, then the politician will be forced to intervene in the economy through other types of transfers. This optimal incentives scheme shows why voters would prefer to make it more difficult for successive targeted bailout programs to be implemented, determining a switch towards untargeted transfers.

A second result of the paper is that the persistence effects of current policies on future reelection conditions can continue into the long run. Dynamic incentives continue to be provided over a long time horizon, leading to variation in the balance of targeted and untargeted transfers used by the government over time. In the long run, if the politician and voters are equally patient – they discount the future at the same rate – then the equilibrium policies chosen by the politician will converge to either using only targeted transfers or only untargeted transfers. This result mirrors the immiseration result obtained in Thomas and Worrall (1990) or Atkeson and Lucas (1992), and reflects the need for voters to commit to increasing rewards and punishments over time in order to offer incentives to the politician. Yet, if the politician is more impatient than the voters, then the need for increasing rewards and punishments is reduced, since the politician places a lower relative value placed on payoffs far in the future. Then the voters allow both types of transfers in the long run and there will not be convergence towards one type of
intervention.

The model also considers the role of public debt in affecting the politician's policy choices. The politician's ability to take on debt makes incentive provision more costly to voters, because it gives the politician more flexibility in determining the government intervention budget, while voters only have limited means of punishing inefficient transfers. The main result of this part is that debt limits the extent to which removal from office can be used as an effective instrument to influence politician behavior. High outstanding debt also creates a force for lower restrictions on targeted transfers. As the government has fewer overall resources in the current period, there are higher costs to using less efficient transfers in order to limit inefficient transfers. Yet, the overall effect of public debt on voter welfare is ambiguous. Public debt allows the politician to smooth the cost of government interventions over time, which is beneficial to voters. Yet, the cost of providing incentives to the politician is higher. The implication is that debt limits or budget balance requirements can be preferred by voters in high rent-seeking environments, in which inefficient transfers are very costly to voters. Otherwise, public debt is welfare increasing for both voters and the politician because it facilitates smoothing the cost of interventions over time.

Finally, an extension of the basic model considers the case in which the government can also provide a non-financial public good along with the transfers to firms. This non-financial public good is valued equally by both households and the politician. The presence of the non-financial public good makes the trade-off between the different types of transfers more complex. If voters decide to increase the restrictions on targeted transfers in order to provide incentives for the politician, then this might no longer have the effect of increasing untargeted transfers. In fact, voter restrictions on targeted transfers to financial firms result in higher provision of the non-financial public good, possibly without any increase in untargeted transfers to firms. Voters therefore have more control over the size of financial interventions. At the same time, however, it becomes harder to reduce inefficient transfers without reducing total support for the financial sector.

**Related literature.** The analysis of optimal government intervention in financial markets has in most cases assumed that decision-making is done by a benevolent government that
maximizes social welfare (Holmström and Tirole, 1998; Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; Farhi and Tirole, 2012b; Diamond and Rajan, 2012). Papers in this literature have focused on the responses of private agents to an expected government intervention in financial markets. Farhi and Tirole (2012b) highlight the moral hazard and strategic complementarities induced in the market by a time inconsistent government intervention policy. Philippon and Skreta (2012) also study the optimal government intervention in a market with adverse selection over private assets. They show that minimum cost public interventions can be achieved through targeted means, using debt contracts or debt guarantees. Similarly, Philippon and Schnabl (2013) study the optimal government intervention when the financial sector suffers from debt overhang. This paper also studies optimal government intervention, but it focuses on the incentive problems that emerge at the level of the government.

Several recent studies have provided substantial evidence that political economy considerations affect the decision making process of the government regarding interventions in financial markets. Lobbying and political rent-seeking have been shown to influence what type of intervention the government engages in (Mian et al., 2010), or which firms receive public bailouts (Faccio et al., 2006). Moreover, 'pork-barrel' expenses played an essential role in the passing of legislation regarding public purchases of private financial assets (Drazen and Ilzetzki, 2011). I contribute to this literature by studying the incentives problem that emerges when elected officials can derive political rents by making transfers to connected firms.

This paper builds a principal-agent model similar to Acemoglu et al. (2008). They study the dynamic incentives problem with a rent-seeking politician, in a standard neoclassical growth model. This paper studies a problem with similar constraints, but the analysis abstracts away from capital and taxation distortions. Instead, it focuses on the distortions due to the type of firms who receive transfers from the government as a result of lobbying. Moreover, it considers the existence of private information regarding the shocks affecting the economy.

The private information environment used in the paper is an extension of the models of Atkeson and Lucas (1992) and Thomas and Worrall (1990). If features two different shocks which are privately observed by the government each period. The two shocks capture the fraction of distressed firms and the fraction of connected firms which are distressed. While
the total effect of the shocks on aggregate output can be observed at the end of each period, the combination of shocks that led to this outcome cannot be perfectly inferred, resulting in a model with properties similar to Atkeson and Lucas (1992).

The paper is related methodologically to Farhi and Werning (2007), who study an optimal incentives provision problem in the presence of private information and different discount rates between principal and agent. Also, Sleet and Yeltekin (2008) consider the credibility of allocations in dynamic games with private information, and show that optimal politically credible allocations can be solved as virtual planning problems with social discount factors in excess of the private one. Another related paper is Ales et al. (2012), which adds private information to a mechanism similar to the one in Acemoglu et al. (2008). They focus on the issue of politician replacement on the equilibrium path. In this paper, the focus is on the variation in the incentives provided by voters to the elected politician, and not on the question of endogenous replacement.

Finally, there is a vast empirical literature studying the effects of lobbying, rent-seeking and electoral constraints on government policy. Mian et al. (2010) provide empirical evidence related to the role of lobbying and political influence in the policies adopted in the United States in the lead up to the recent financial crisis. They show that campaign contributions from the financial services industry were associated with a higher likelihood of congressional voting in favor of legislation that encouraged the subprime mortgage credit expansion. Duchin and Sosyura (2012) investigate the link between corporate political connections and government investment under the TARP. They find that politically connected firms are more likely to be funded, yet investments in politically connected firms underperform those in unconnected firms, which suggests that political connections lead to distortions in investment efficiency. Igan et al. (2011) study lobbying activities before the 2008 financial crisis, and they find that lobbying lenders faced higher probability of receiving a public bailout during the crisis, which again suggests that lobbying and political connections play an important role in government policy choices. Also, Cohen and Malloy (2010) show that personal connections amongst politicians have a significant impact on the voting behavior of U.S. politicians. The effects are significant when considering voting over bills that do not affect the representative’s constituents. This
provides further evidence on the inefficiencies that can emerge when the interests of politicians differ from those of their voters. For the European case, in a comparative study of four different government responses to the recent financial crisis, Grossman and Woll (2013) argue that the type of institutional business-government relationship existing in each country, more specifically whether banks negotiate with governments one-on-one or collectively, determines the composition of the bailout packages between direct government bailouts and other forms of indirect government support.

The rest of the paper is organized as follows. Section 2.2 outlines model. Section 2.3 provides the analysis of the model and the equilibrium. Section 2.4 considers the equilibrium policies when the government has access to public debt. Section 2.5 describes an extension that allows the government to also provide a non-financial public good along with the targeted and untargeted transfers. Section 2.6 concludes.

2.2 Model

2.2.1 Environment

The economy consists of a continuum of firms, a continuum of households and an elected politician selected from a pool of identical politicians.

Firms. Each firm holds a production technology \( f(k_t) \) that converts capital into consumption goods. The production function \( f(k) \) is concave, increasing and continuously differentiable in \( k \), with partial derivative denoted by \( f_k \), and \( \lim_{k\to0} f_k > 1 \), \( \lim_{k\to\infty} f_k = 0 \), and \( f(0) = 0 \). Capital fully depreciates at the end of each period. Firms are owned by households and are supplied with capital \( k_t \) each period. I assume that firms cannot enter into debt contracts with other parties (or with each other), so their only possible sources of funding are capital supplied by the owners and public funds from the government. While this is a strong assumption, it is made in order to isolate the inefficiencies emerging at the level of the government from the inefficiencies due to the capital market.

Each period \( t \), a fraction \( \theta_t \) of the projects become distressed. The fraction of distressed projects, \( \theta_t \), is an i.i.d. random variable whose distribution is discrete over \( \Theta = \{\theta^1, \theta^2, ..., \theta^N\} \).
with probability $\pi(\theta^n)$, $n = 1, \ldots, N$. If distressed, a project suffers a loss of $x$ of its initial capital, where $0 < x < \bar{k}$. If not distressed, the project pays off $f(k_t)$ consumption units.

The key ingredient of the model is that the firms also differ in terms of their ability to lobby the government. A fraction $\nu$ of firms are connected to the elected politician and therefore can lobby the politician for transfers. The politician derives political rents from any transfers that will be made to the connected firms who engage in lobbying, as described below. Each period, a fraction of the connected firms will be distressed, while the others will not suffer any distress. While the total share of distressed firms in period $t$ is $\theta_t$, the share of connected firms that become distressed can be different from $\theta_t \nu$. Denote the fraction of firms that are connected and not distressed in period $t$ by $\gamma_t$, and let $\gamma$ be distributed with $g(\gamma)$.

**Households.** Each household receives an endowment $\bar{k}$ of capital goods each period, and each household owns a firm. The endowment $\bar{k}$ is assumed to satisfy $f_k(\bar{k}) = 1$. Therefore, any investment above $\bar{k}$ would produce a return less than 1. Households cannot sell shares in their projects to other households. This assumption shuts down the channel for risk-sharing between households. The analysis would be identical if we allowed each household to build a perfectly diversified portfolio of shares in each project. Yet, by having the ownership of each firm differ, we can obtain the setup for the key assumption of this model – that some firms are connected to the government while others are not. This assumption has in the background the idea that some households are connected to the elected politician, while others not, and therefore the connected firms must be owned by the connected households.

Households do not have access to any storage technology that allows the transfer of resources between periods, so all output must be consumed in each period. At the end of each period, households receive consumption goods $c_t$ given the output of its firm. Each household cannot, however, observe the values of $\theta_t$, the fraction of firms that were distressed. The assumption that $\theta_t$ is not observable to households captures the idea that households cannot observe the total fraction of firms that are considered in need of funds. They can observe whether their firm is in need of funds, but not the needs of all other firms.\(^{14}\)

\(^{13}\)Although in expectation it is $E[\theta] \nu$.

\(^{14}\)In the current version of the model, distressed firms can only get additional funding from public sources, so
Households are risk-neutral, and each household $j$ receives instantaneous utility from consumption each period,

$$u_{t,j} = c_{t,j}.$$ 

They are infinitely-lived and discount the future at rate $\beta$. Therefore, their expected lifetime utility of each household $j$ is given by:

$$U_j = \mathbb{E} \sum_t \beta^t u_{t,j}.$$ 

**Elected politician.** The allocation of resources is delegated to an elected politician. The politician maximizes a weighted sum of household utilities (or firms' output), where households receive different weights depending on whether their firms are connected to the politician or not. Connected firms can lobby the government, which provides the politician with rents. The result of the lobbying process can be summed up in reduced form as the politician placing a higher relative weight on the output of connected firms. This reduced form outcome can be obtained from the standard Grossman and Helpman (1994) model. To simplify the analysis, we consider the case in which distressed firms lack the funds to engage in lobbying activities, so that only firms that are connected and not distressed can lobby the politician. The political rents that result from this process are reflected in higher weight being placed on the output of firms that lobby the politician. Let this relative weight be denoted by $R > 1$.

The role of lobbying in political decision-making, and the political rents it generates have been the subject of a large empirical literature, as described in the introduction. In the recent financial crisis, campaign contributions were shown to have played a significant role in the passing of legislation regarding public intervention in the financial sector. Also, targeted government interventions involve the purchase of assets from selected firms. Recent evidence by construction all distressed firms are in need of public liquidity. Yet, the model could easily be modified to include private borrowing, so that some distressed firms can borrow additional funds in the market, while others are constrained and cannot borrow in the market, requiring public liquidity. Then, households would observe which firms are distressed and which are not, but they would not observe who is in need of public liquidity versus who is able to borrow in the private market. Therefore, $\theta_i$ would capture the fraction of firms which cannot borrow in the private market and need public liquidity.

15 Mian et al. (2010) show that campaign contributions from the financial services industry were associated with a higher likelihood of congressional voting in favor of legislation supporting the subprime mortgage credit expansion.
from the government purchase programs during financial crisis shows that a significant portion
of the private assets were overvalued, and that politically connected firms benefited more from
targeted government purchases.\textsuperscript{16} Therefore, both campaign contributions as well as targeted
purchases of private assets can generate political rents, and this model captures this aspect
of the political process through the weighting of the output from different firms in the utility
function of the politician.

Each period, the elected politician receives government endowment $T$, which for now is taken
as exogenous (tax distortions are ignored). The politician observes the aggregate shock $\theta_t$, the
fraction of distressed firms, as well as $\gamma_t$, how many connected firms are not distressed and can
therefore lobby.

The politician can transfer funds to firms in period 1. Funds can be transferred directly
– targeted to specific firms – or indirectly through untargeted transfers to all firms. Denote
the targeted transfers to a firm $j$ by $g_{t,j}$ and the untargeted transfers to all firms by $g_t$. Since
firms only differ in whether they are distressed or not, and whether they are connected to
the government or not, the targeted transfers will only differ along these dimensions as well.
Therefore, let $g_t^d$ be the targeted transfer received by a distressed firm, and $g_t^c$ the targeted
transfer received by a connected firm that is not distressed. Finally, for completeness, we can
denote by $g_t^{nd}$ the targeted transfer received by a firm that is not connected and not distressed.
Yet, under the assumptions made below about the size of the government budget, the firms
that are not distressed and not connected will not receive any targeted transfers since these
transfers would be inefficient, and so $g_t^{nd} = 0$.

The model embeds the following political economy friction: the politician can derive political
rents by transferring funds to politically connected firms, through either targeted transfers or
through untargeted transfers; however, untargeted transfers cannot be restricted to only a
subset of firms, which makes them more expensive. The targeted transfers can bring political
rents at a lower cost, since they can be directed at the firms that are connected to the politician.
This setup is motivated by the fact that targeted interventions capture bailout programs under
\begin{footnote}{\textsuperscript{16}Duchin and Sosyura (2012) estimate empirically the effects of political connections on the funding made available to private firms though the TARP program.}\end{footnote}
which the government can choose the beneficiaries and overpay for the assets it buys. Untargeted transfers, on the other hand, are meant to capture the type of programs implemented by the Treasury in conjunction with the Fed, aimed at providing easier access to funds for all firms. The politician’s budget constraint each period is then given by:

\[ \theta_t g_t^d + \gamma_t g_t^c + (1 - \theta_t - \gamma_t) g_t^{nd} + g_t^i \leq \tau, \]  

or, incorporating \( g_t^{nd} = 0 \),

\[ \theta_t g_t^d + \gamma_t g_t^c + g_t^i \leq \tau. \]  

The politician’s instantaneous utility is then expressed as:

\[ v_t = \left[ \theta_t f(\bar{k} - x_t + g_t^d + g_t^c) + (1 - \theta_t - \gamma_t) f(\bar{k} + g_t^i) + R\gamma_t f(\bar{k} + g_t^d + g_t^c) \right]. \]  

The politician discounts the future at rate \( \tilde{\beta} \leq \beta \). The different discount rate captures the fact that, compared to the voters, the politician may care more about the present outcome than about the future. \(^{17}\) Therefore, the politician maximizes the following utility:

\[ V = \sum_t \tilde{\beta}^t v_t, \]  

or written in recursive form:

\[ V_t = v_t + \tilde{\beta} \mathbb{E} [V_{t+1}] \]  

**Private Information and the electoral process.** The politician is subject to the following form of electoral control. The households have the power to vote the incumbent politician out of office at the end of each period. We assume that the households have solved all collective action problems, and as voters, they maximize the sum of utilities of all households, with equal

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\(^{17}\) The justification for the difference in discount factors is also discussed in Farhi and Werning (2007), Acemoglu et al. (2008), and Sleet and Yeltekin (2008).
weight on each household’s consumption:

\[ u_t = \mathbb{E}_{\theta_t, \gamma_t} \left[ \theta_t f(k - x_t + g_t^d + \gamma_t^t) + (1 - \theta_t - \gamma_t) f(k + g_t^d) + \gamma_t f(k + g_t^d + g_t^c) \right]. \] (2.6)

The ability of the voters to punish the incumbent politician is limited by the information available to them each period. They cannot observe the values of shocks \( \theta_t \), the fraction of distressed firms, and \( \gamma_t \), the share of connected firms that are not distressed and therefore have the ability to lobby for funds. They can only observe the total tax revenue \( \tau \) and the total amount of funds disbursed towards targeted interventions \( (\theta_t g_t^d + \gamma_t g_t^c) \) versus untargeted transfers \( (g_t^c) \).

The politician can observe the shock \( \theta_t \) each period and send a message \( \hat{\theta}_t \) to voters about its value. After sending this message, the shock \( \gamma_t \) is observed by the politician, but it cannot be transmitted to voters, i.e., it is not observable or verifiable to voters. The assumption that \( \gamma_t \) is observed after the message \( \hat{\theta}_t \) is sent is meant to capture the fact that \( \gamma_t \) cannot be transmitted to voters, and also that the rents derived from connected firms are not known before the direct intervention is approved. Once the direct interventions are possible, firms will have an incentive to lobby for funds.

Voters set a replacement strategy at the beginning of the period and any replacement is done at the end of the period, after the report \( \hat{\theta} \) is sent, the total targeted intervention \( (\gamma_t g_t^c + \theta_t g_t^d) \) and the untargeted transfers \( (g_t^c) \) are observed.\(^\text{18}\) If replaced, the politician receives the exogenous utility \( V \). The value \( V \) is assumed to be low enough such that incentives can be offered to the politician, but also sufficiently high such that the voters’ preferred policy cannot be implemented. Denote by \( \rho_t \in \{0, 1\} \) voters’ replacement decision, where \( \rho_t = 0 \) stands for replacement.

\(^{18}\) An equivalent system would give voters the opportunity to replace the politician at the beginning of each period. The main characteristic of this electoral system is that it allows voters to condition their replacement decision on the message \( \hat{\theta}_t \) and the government policies in period \( t \).
2.2.2 Timing

Consider the following game. Each period, the economy starts with a politician in power and the following sequence of events happens:

1. Nature chooses shocks $\theta_t$ and $\gamma_t$;

2. Voters choose a strategy $T_t$ for politician replacement given the history of all reports $\hat{\theta}_j$, $j = 0, ..., t$;

3. The politician observes $\theta_t$ and sends message $\hat{o}_t$ about the current shock $\theta_t$;

4. The politician observes $\gamma_t$ and decides transfers $\{g_t^d, g_t^e, g_t^f\}$;

5. If $T_t = 1$, the incumbent is replaced with an identical politician;

6. Each household $h$ receives consumption goods $c_t^h$ according to the output of its firm.

2.3 Analysis

2.3.1 Equilibrium Concept

Let $h^0_t \equiv \{g_0^d, \hat{o}_0, ..., g_t^d, \hat{o}_t\}$ be the public history of the game up to and including period $t$. Let $h^1_t \equiv h^0_t \cup \{\theta_0, \gamma_0, g_0^d, ..., \theta_t, \gamma_t, g_t^d\}$ be the history of outcomes observed by the politician up to period $t$. Let $T_{|h^0_t}$ be the continuation strategy of the representative voter, and let $F_{|h^1_t}$ be the continuation strategy for the incumbent politician. We consider sustainable equilibria (Chari and Kehoe, 1993), in which both the voters’ and the incumbent politician's decision rules are required to be sequentially rational. The strategy for the voters $T$ solves the voter problem if for every $h^0_t$, $T_{|h^0_t}$ maximizes the expected sum of household utilities given $F$. The strategy $F$ solves the politician’s problem if for every $h^1_t$, the continuation strategy $F_{|h^1_t}$ maximizes the politician's expected utility given $T$. A sustainable equilibrium then consists of the set of strategies $\{T, F\}$ where $T$ solves the voters’ problem given $F$, and $F$ solves the politician’s problem given $T$. I focus on the best sustainable equilibrium from the perspective of voters, that maximizes the weighted sum of household utilities. By the Revelation Principle, we can restrict the analysis to equilibria with truthful reporting on the part of the politician.
Given the strategies defined above, I begin by characterizing the set of sustainable policies that can be supported in equilibrium. First, let $\Gamma(h_{t-1}, \hat{\theta}_t)$ denote the set of targeted interventions $- \gamma_t g_t^c + \theta_t g_t^d$ given report $\hat{\theta}_t$, after which voters would reelect the incumbent politician. Then, let $\phi_t = \{g_t^d, g_t^c, g_t^i\}$ be an allocation of the government budget between targeted transfers and untargeted transfers. The allocation $\phi_t$ is feasible if the following conditions are satisfied:

$$
\begin{align*}
g_t^c + \theta_t g_t^d + g_t^i & \leq \tau, \\
g_t^c & \geq 0; \quad g_t^d \geq 0; \quad g_t^i \geq 0.
\end{align*}
$$

The first condition limits the government expenses to the available government budget each period. Condition (2.8) bounds each type of transfer to be non-negative. Moreover, the politician will be reelected after proposing allocation $\phi_t$ if the total targeted interventions $- \gamma_t g_t^c + \theta_t g_t^d$ are in the set acceptable to voters, i.e.,

$$
\phi_t \in \Gamma(h_{t-1}, \hat{\theta}_t).
$$

Finally, the politician reports truthfully the need for funds if at allocation $\phi_t$ the following holds for each $t$:

$$
v(h_{t-1}, \theta_t, \gamma_t | \hat{\theta}_t) + \beta \mathbb{E} [V_{t+1}(h_{t-1}, \theta_t, \gamma_t | \hat{\theta}_t)] \geq v(h_{t-1}, \theta_t, \gamma_t | \hat{\theta}_t) + \\
\beta \mathbb{E} [V_{t+1}(h_{t-1}, \theta_t, \gamma_t | \hat{\theta}_t)], \quad \forall \hat{\theta}_t \in \Theta
$$

so that the politician is better off reporting the true need for liquidity rather than sending any other report $\hat{\theta}_t$.

The following proposition establishes that the above conditions must be satisfied for any allocation $\phi_t$ to be part of a sustainable equilibrium.

**Proposition 20** The sequence of allocations $\{\phi_t\}_{t=0, \ldots, \infty}$ is supported by sustainable equilibrium strategies if and only if conditions (2.7)-(2.10) are satisfied for each $\phi_t$, $t = 0, \ldots, \infty$.

**Proof.** In the Appendix. □
Given the results of Proposition 20, let \( \Phi \) denote the set of allocations \( \phi_t \) are supported by a sustainable equilibrium. For the rest of the analysis, we will consider only the allocations that belong to set \( \Phi \). To make further progress in analyzing the resulting equilibria, I proceed to analyze the constrained maximization problem in three steps: first, derive the properties of the reelection condition chosen by voters each period given a report \( \hat{\theta}_t \) and the observed policies \( \{g^t_t, (\tau - g^t_t)\} \); second, study the problem for the incumbent given the voters reelection condition; third, derive the inter-temporal problem for the voters given the incumbent politician’s response to their reelection strategy.

### 2.3.2 Voters’ Reelection Strategy

At the beginning of each period, voters set their reelection strategy. They can commit to this strategy, and at the end of the period, either keep or replace the incumbent politician. Each period, voters can condition their replacement decision on the report \( \hat{\theta}_t \) and the observed distribution of government funds, between untargeted transfers \( g^t_t \) and targeted interventions \((\tau - g^t_t)\).

**Lemma 21** For every report \( \hat{\theta}_t \), there exists \( g^*_t(h^0_{t-1}, \hat{\theta}_t) > 0 \) such that \( \forall g^*_t(h^0_{t-1}, \hat{\theta}_t) \geq \tau - g^*_t(h^0_{t-1}, \hat{\theta}_t), g^*_t(h^0_{t-1}, \hat{\theta}_t) \in \Gamma(h^0_{t-1}, \hat{\theta}_t) \) and \( \forall g^*_t(h^0_{t-1}, \hat{\theta}_t) < \tau - g^*_t(h^0_{t-1}, \hat{\theta}_t), g^*_t(h^0_{t-1}, \hat{\theta}_t) \notin \Gamma(h^0_{t-1}, \hat{\theta}_t) \).

**Proof.** In the Appendix. \( \blacksquare \)

Lemma 21 states that, for each report \( \hat{\theta}_t \), the voters will limit the total targeted interventions (targeted transfers and rents) to a maximum level of \( g^*_t(h^0_{t-1}, \hat{\theta}_t) \). A level of targeted interventions above this limit would be welfare-reducing for voters; it would lead to a decrease in untargeted transfers that would be costlier to voters than the gains from higher targeted transfers. This is due to the higher utility cost of decreasing untargeted interventions, specifically the increase in political rents.

### 2.3.3 Politician’s Problem

Given the results summarized in Lemma 21 regarding the voters’ reelection strategy, we can now analyze the problem for the incumbent politician. In case of removal from office, the
politician receives $V_t$, as described above. Therefore, for each report $\hat{\theta}_t$, the politician will offer policies $\phi_t \in \Gamma(h^0_t, \hat{\theta}_t)$, which by Lemma 21 implies that the following constraint must hold:

$$\gamma_t g_t^c + \theta_t g_t^d \leq g_t^* (\hat{\theta}_t).$$

Then, each period, the politician chooses $\{g_t^d, g_t^c, g_t^i, \hat{\theta}_t\}$ to maximize (2.5), taking into account the cutoff $g_t^* (h^0_{t-1}, \hat{\theta}_t)$ for targeted interventions above which voters would remove him from power. In recursive form, the politician's problem for each report $\hat{\theta}$ is given by:

$$V (\theta, \gamma, g^* (\hat{\theta})) \equiv \max_{\{g^d, g^c, g^i\}} \nu (\theta, \gamma, g^* (\hat{\theta})) + \beta E \left[ V' (\theta', \gamma', g''^* (\hat{\theta}')) \right]$$

subject to

$$\gamma g^c + \theta g^d \leq g^* (\hat{\theta}), \quad (2.12)$$

$$\gamma g^c + \theta g^d + g^i \leq \tau, \quad (2.13)$$

$$g^c \geq 0; \quad g^d \geq 0; \quad g^i \geq 0. \quad (2.14)$$

Constraint (2.12) is the re-election constraint imposed by voters given report $\hat{\theta}$. Constraint (2.13) is the resource constraint of the government, and constraint (2.14) is the non-negativity requirement for all government policies.

Denote by $\phi^P (\hat{\theta}) \equiv \{g^d (\theta, \gamma, g^* (\hat{\theta})), g^c (\theta, \gamma, g^* (\hat{\theta})), g^i (\theta, \gamma, g^* (\hat{\theta}))\}$ the set of policies that satisfy conditions (2.12)-(2.14). Given the concavity of the politician's instantaneous utility $\nu (\theta, \gamma, g^* (\hat{\theta}))$ and the linear constraints, it follows that the solution to the system (2.12)-(2.14) is unique, and $V (\theta, \gamma, g^* (\hat{\theta}))$ is a well-defined function. Since there are no inter-period linkages other than through the voters' strategies $\{g^* (\hat{\theta})\}$, the politician's problem given $g^* (\hat{\theta})$ is a static optimization over $\{g^d, g^c, g^i\}$. This leads to the following choice of
policies as a function of $g^*(\hat{\theta})$, assuming an interior solution:

$$g^i = \tau - g^*(\hat{\theta}),$$  
$$g^c = g^*(\hat{\theta}) - \theta g^d$$

and $g^d$ is given by the implicit equation

$$\theta f'(\bar{k} - x + g^d + \tau - g^*(\hat{\theta})) = R\gamma f'(\bar{k} + g^*(\hat{\theta}) - \theta g^d + \tau - g^*(\hat{\theta})).$$  \hspace{1cm} (2.17)

**Assumption 10** The government budget $\tau$ is sufficiently small and the loss $x$ is sufficiently large such that the following conditions hold $\forall \theta \in \Theta$, $\forall \gamma \leq \max\{1 - \theta, \nu\}$:

$$\theta f'(\bar{k} - x) > R\gamma f'(\bar{k} + \tau),$$  
$$\tau \leq \theta^1 x.$$  \hspace{1cm} (2.18)  
$$\tau \leq \theta^1 x.$$  \hspace{1cm} (2.19)

Assumption 10 states that in the absence of untargeted transfers ($g^i = 0$), the targeted transfers to distressed firm would be positive ($g^d > 0$). Moreover, the government budget available for public intervention is at most equal to the total loss of distressed firms. This assumption ensures that it will never be optimal to have untargeted transfers, since using up the entire budget for targeted transfers would be preferred.

### 2.3.4 Voters' Problem

Having derived the politician's problem, we can now study the optimal problem for the voters. By Lemma 21, the voters are choosing a sequence $\{g^*_t(h_{t-1}^0, \hat{\theta}_t)\}_{t=0}^{\infty}$ to solve their utility maximization problem, given the politician's strategy. Let

$$u^H(\theta, \gamma, g^*(\hat{\theta})) \equiv u(\phi^P(\hat{\theta}), \theta, \gamma)$$  \hspace{1cm} (2.20)

denote the instantaneous utility for voters at shock values $\theta, \gamma$, given the report $\hat{\theta}$ about the fraction of distressed firms, and the allocation $\phi^P(\hat{\theta})$ corresponding to the politician’s strategy.
Given the sequence sequence \( \{g_t(h_{t-1}, \hat\theta_t)\}_{t=0}^{\infty} \), let

\[
EV_{t+1}(\hat\theta_t) \equiv \mathbb{E} \left[ V_{t+1} \left( \theta_{t+1}, \gamma_{t+1}, g_{t+1}^* (\theta_{t+1}) \right) \right] \tag{2.21}
\]

be the continuation utility expected by the politician on the equilibrium path from period \( t + 1 \) on, after report \( \hat\theta_t \) in period \( t \) and with truthful reporting of funding needs in all periods after \( t \).

Define

\[
v^P(\theta_t, g_t^*(h_{t-1}^0, \theta_t)) \equiv \mathbb{E}_{\eta_t} \left[ v(\theta_t, \gamma_t, g_t^*(h_{t-1}^0, \theta_t)) \right], \tag{2.22}
\]

the expected value of politician’s instantaneous utility before the value of \( -Y_t \) is observed.

Then, the best sustainable equilibrium is a solution to the following program:

\[
\max_{g_t^*(h_{t-1}^0, \theta_t)} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t u^H(\theta_t, \gamma_t, g_t^*(h_{t-1}^0, \theta_t)) \right] \tag{2.23}
\]

subject to

\[
v^P(\theta_t, g_t^*(h_{t-1}^0, \theta_t)) + \bar{\beta} EV_{t+1}(\hat\theta_t) \geq v^P(\theta_t, g_t^*(h_{t-1}^0, \hat\theta_t)) + \bar{\beta} EV_{t+1}(\hat\theta_t), \quad \forall \theta_t \in \Theta, \tag{2.24}
\]

\[
g_t^*(h_{t-1}^0, \theta_t) \geq 0 \tag{2.25}
\]

\[
g_t^*(h_{t-1}^0, \theta_t) \leq \tau \tag{2.26}
\]

Constraint (2.24) is the incentive compatibility constraint for the incumbent politician. The politician must obtain weakly higher utility when reporting the true shock \( \theta_t \) today, than when deviating and reporting any other shock \( \hat\theta_t \). Consraints (2.25) and (2.26) are the lower bound and upper bound on the restriction that voters can set on targeted interventions.

To make progress in characterizing the equilibrium, we can restate the problem as a recursive program:

\[
U(EV) = \max_{\{g^*(\theta), EV'(\theta)\}} \mathbb{E}_{\theta, \gamma} \left[ u^H(\theta, \gamma, g^*(\theta)) + \beta U'(EV'(\theta)) \right] \tag{2.27}
\]
subject to

\[ EV = \mathbb{E}_{\theta, \gamma} \left[ v(\theta, \gamma, g^*(\theta)) + \hat{\beta} EV' (\theta) \right] , \]  
(2.28)

\[ v^p(\theta, g^*(\theta)) + \hat{\beta} EV (\theta) \geq v^p \left( \theta, g^* \left( \hat{\theta} \right) \right) + \hat{\beta} EV' (\hat{\theta}) , \quad \forall \hat{\theta} \in \Theta, \]  
(2.29)

\[ g^*(\theta) \geq 0, \]  
(2.30)

\[ g^*(\theta) \leq \tau, \]  
(2.31)

\[ EV'(\theta) \geq EV, \]  
(2.32)

\[ EV'(\theta) \leq EV. \]  
(2.33)

Constraint (2.28) represents the promise keeping constraint. It is the expected utility derived by the politician on the equilibrium path given the voters' ability to commit to their reelection strategy. Voters start the current period with a commitment to deliver to the politician expected utility \( EV \). This restricts the path of current and future reelection cutoffs. Constraint (2.29) is the incentive-compatibility constraint for the incumbent politician. This is the recursive-form equivalent of constraint (2.24), and it ensures that the politician will prefer to report the firms' true need for funding. Constraints (2.30) and (2.31) are the lower bound and upper bound, respectively, on \( g^*(\theta) \), the limit on targeted interventions. Constraints (2.32) and (2.33) are the lower bound and upper bound, respectively, on the continuation utility voters can promise the politician.

To derive the bounds \( EV \) and \( \overline{EV} \), note that the politician's utility increases as the cutoff \( g^*(\theta) \) increases. A higher cutoff \( g^*(\theta) \) gives the politician more freedom to choose policies without the threat of removal. Therefore, the utility that can be promised to the politician is bounded by the following lower and upper limits that correspond to removal from office and \( g^*(\theta) = \tau \), respectively. So

\[ EV = V, \]  
(2.34)

and

\[ \overline{EV} = \mathbb{E}_{\theta, \gamma} \left( \frac{v(\theta, \gamma, g^*(\theta))}{1 - \hat{\beta}} \right) \bigg|_{g^*(\theta)=\tau}. \]  
(2.35)
In the next paragraphs, I discuss the role of political rents and private information in problem (2.27) and characterize the equilibrium.

2.3.5 Benchmarks

The analysis of problem (2.27) relies on two key elements. First, the connection between a fraction of the firms and the politician allows for political rents, and leads to the politician having a different object function from that of the voters. Second, private information limits the ability of voters to punish the politician for any deviations from their preferred policy. In the absence of these two distortions, voters would be able to implement their preferred policy.

Proposition 22 \textit{The policy that maximizes household utility in the absence of connected firms and private information uses the available budget, $\tau$, solely for targeted transfers: $g_t^d = \tau$ and $g_t^i = g_t^c = 0 \forall t$.}

The voters’ preferred policy involves $g_t^d = \tau$ and $g_t^i = g_t^c = 0, \forall t$, since the government budget available for transfers is lower than the total loss of firms ($\tau < \theta^1 x$) and the marginal return from targeted transfers to distressed firms is higher than from untargeted transfers or from transfers to connected firms which are not distressed. Without connected firms that can lobby, the households and the politician have the same preferences, $u^H(\theta, \gamma, g^*(\theta)) = v(\theta, \gamma, g^*(\theta))$, so the politician would prefer the same policies as the voters: $g_t^d = \tau$ and $g_t^i = g_t^c = 0 \forall t$. Therefore, without connected firms there would be no agency problem, and the optimal policy that maximizes household utility would be implemented.

Without private information over $\theta$, but with connected firms, constraint (2.29) would not be part of the problem. Yet, voters would still be unable to observe how much is transferred to connected firms which are not distressed. In this situation, some level of untargeted transfers could still be optimal. Voters balance the costs of untargeted transfers - due to them funding undistressed firms - and the costs of political rents due to transfers to connected firms. Specifically, voters prefer positive untargeted transfers if the marginal benefit from providing untargeted transfers is higher than the marginal cost of reducing the budget for targeted interventions, and implicitly reducing $g^d$ and $g^c$. Deriving these marginal costs from the voters’
objective, given the politician’s transfer choices, we obtain the following condition for positive untargeted transfers to be preferred by voters:

\[
(1 - \theta - \gamma) > f'\left(\bar{k} - x + g^{du}\right) \left(\theta \frac{\partial g^{du}}{\partial g^*} \bigg|_{g^*=\gamma} - \theta + \frac{1}{R} \left(1 - \theta \frac{\partial g^{du}}{\partial g^*} \bigg|_{g^*=\gamma} - \gamma\right)\right),
\]  

(2.36)

where \(g^{du}\) is the value at which

\[
f'\left(\bar{k} - x + g^{du}\right) = R f' \left(\bar{k} + \frac{\tau - \theta g^{du}}{\gamma}\right).
\]  

(2.37)

I make the following assumption:

**Assumption 11** Parameters \(\{R, \bar{k}, \tau, x\}\) are chosen such that condition (2.36) holds given \(g^{du}\) defined implicitly in (2.37) above, \(\forall \theta \in \Theta, \gamma \in [0, 1 - \theta]\).

Assumption 11 states that a positive level of untargeted transfers will be preferable to voters given the cost associated with targeted transfers to connected firms. The following benchmark result is therefore reached in the case of no private information.

**Proposition 23** Without private information over \(\theta\), but with connected firms that can receive targeted transfers, voters’ optimal reelection strategy policy takes the form of a cutoff \(g^*_t(\theta) = \tau - g^*_t\), where \(g^*_t > 0\) under Assumption 11.

A key implication of Proposition 23 is that, in the absence of private information, voters’ optimal reelection strategy does not involve any history dependence. Shocks are independent, and hence there is no rationale for introducing history dependence. Yet, the results change once private information is taken into account, as it will be discussed in the next section.

### 2.3.6 Best Sustainable Equilibrium

We begin the analysis of problem (2.27) by showing that the function \(U(EV)\) is concave and differentiable. These results are captured in the following two lemmas.

**Lemma 24** The value function \(U(EV)\) is concave.
Proof. In the Appendix. ■

Lemma 25 The value function $U(EV)$ is differentiable for $EV \in (EV, EV)$. 

Proof. In the Appendix. ■

In order to simplify the analysis of the problem, I make the following additional assumptions about the utility functions:

Assumption 12 The function $v^p(\theta, g^*)$ defined in (2.22) satisfies: $v^p(\theta^{i+1}, g^*) - v^p(\theta^{i+1}, g^{*'}) \geq v^p(\theta^i, g^*) - v^p(\theta^i, g^{*'}) \forall g^*, g^{*'}$ with $g^{*'} < g^*, 1 \leq i < N$.

Under Assumption 12, if an incentive-compatible mechanism exists for problem (2.27) in which types are separated, then the only incentive compatibility constraints that bind in the set of constraints denoted by (2.29) are those for adjacent $\theta$–types, in the upward direction. The argument and proof for this result are given in the Appendix. To ensure that the types can be separated, I make the following assumption.

Assumption 13 Consider the case without private information over $\theta$. The following conditions hold $\forall 1 < n \leq N$, given the cutoff $g^*(\theta^n)$ that maximizes voters' utility:

\[
\frac{\partial u^H(\theta^n, \gamma, g^*)}{\partial g^*} \bigg|_{g^*(\theta^n)} - \frac{\partial u^H(\theta^{n-1}, \gamma, g^*)}{\partial g^*} \bigg|_{g^*(\theta^{n-1})} > 0, \forall \gamma,
\]

and

\[
\frac{\partial v^p(\theta^n, g^*)}{\partial g^*} \bigg|_{g^*(\theta^n)} - \frac{\partial v^p(\theta^{n-1}, g^*)}{\partial g^*} \bigg|_{g^*(\theta^{n-1})} > 0,
\]

where $v^p$ is defined in (2.22).

Assumption 13 ensures that the preferred policy for the voters, in the case without private information over $\theta$, is monotonically increasing in $\theta$. The second part of the statement also ensures that the politician's expected utility has the same property.

Lemmas 24 and 25 and the additional Assumptions 12 and 13 allow us to characterize the best subgame perfect equilibrium from the perspective of the voters. The first result in the analysis shows that the best sustainable equilibrium exhibits history dependence.
Proposition 26 In the best sustainable equilibrium described above, voters’ reelection decisions are history dependent: higher targeted transfers in the current period (higher $g^d_t$ and $g^r_t$) lead to more restrictions on targeted transfers in future periods, through lower future cutoffs $g^*_t, \forall t > t_0$.

Proof. In the Appendix. ■

Proposition 26 shows that government policies in the current period have persistent effects. They affect voters’ reelection strategies and therefore affect the set of future policies that a politician can engage in without risking removal from office. The intuition for this result is that voters want to induce the politician to truthfully report the firms’ current need for funding. To do this, voters punish reports of a high funding need, to discourage overreporting of funding needs, and conversely they reward reports of a low funding need. If a politician claims that many firms are distressed in the current period, and therefore need funding, voters will want to give the politician leeway to fund these firms, even if some of them are connected to the government. Otherwise, voters would risk large losses to their output. Yet, if next period the politician makes the same claim of high need for funding, then voters are less willing to accept the politician’s claims and give him leeway. If the shock is indeed high and the firms need government support in order to avoid high output losses, then the politician will be forced to provide this support through other means than just targeted transfers.

Voters’ reelection strategy involves a cutoff limit on targeted transfers, which restricts the politician’s ability to freely choose policies. The politician’s utility therefore increases as the cutoff on targeted interventions is set higher, expanding the set of policies permissible to a politician seeking reelection. Therefore, voters reward the politician following a low report $\hat{\theta}_t$ by committing to increase the cutoffs on targeted transfers in the current period and in future periods. Through their commitment to higher cutoffs in the periods following a low report, voters make current government policies have persistent effects. Moreover, they also create volatility in the future policies chosen by the politician, since each cutoff $g^*_t$ reflects both responses to reports of current shocks as well as commitments taken after past reports. This last remark is also summarized in the following corollary.

Corollary 27 Voters’ strategy in the best sustainable equilibrium introduces volatility in the policies undertaken by the government.
To see the main driver of these results, notice that, for an interior solution to $V'$, the first-order condition emerging from problem (2.27) is

$$\frac{\beta}{\beta} U_V(EV'(\theta)) + \mu + \sum_{\theta' \neq \theta} \phi_{\theta'\theta} \pi(\theta') - \sum_{\theta' \neq \theta} \phi_{\theta'\theta} \pi(\theta) = 0$$

(2.38)

where $\mu$ and $\pi(\theta)\phi_{\theta'\theta}$ are the Lagrange multipliers on constraints (2.28) and (2.29), respectively, and $U_V$ is the derivative of $U$ with respect to $EV'$. The envelope condition is given by

$$U_V(EV) = -\mu.$$  

(2.39)

Combining the above conditions and taking expectations over $\theta$ we obtain

$$\mathbb{E}[U_V(EV'(\theta))] = \frac{\hat{\beta}}{\beta} U_V(EV).$$

(2.40)

Equation (2.40) shows that $U_V(EV)$ is a martingale for $\hat{\beta} = \beta$ or a submartingale for $\hat{\beta} < \beta$. In order to satisfy the incentive compatibility constraint (2.29), the continuation value that must be promised to the politician, $EV'(\theta)$, must be higher following lower targeted spending in the current period (lower $g^*(\theta)$). Therefore, if $g^*(\theta)$ is increasing in $\theta$, then $EV'(\theta)$ must be decreasing with $\theta$. Equation (2.40) shows that, if $\hat{\beta} = \beta$, then, due to the concavity of $U$, the promised continuation value $EV'(\theta)$ will be higher than the current promised utility $EV$ following a low $\theta$ shock and it will be lower than $EV$ following a high shock.

**Corollary 28**  The future cutoffs that limit targeted transfers increase following a low shock $\theta$, and they decrease following a large shock $\theta$.

The above corollary summarizes the result that high shocks in the current period will reduce future targeted transfers, by making the voters commit to lower future cutoffs that limit targeted interventions. Through this commitment, voters punish the politician for reporting a high need for funding today, promising him a lower expected continuation value. This introduces persistence and volatility in the policy, even though the shocks themselves are not correlated.
2.3.7 Two Types Of \( \theta \)-Shocks And Two Periods

To illustrate the above result, consider a two period version of the above model. Assume that the model only has periods \( t = 0, 1 \) and there are only two possible values of the shock \( \theta \), \( \Theta = \{ \theta^L, \theta^H \} \). In each period, the sequence of events follows the timing described in 2.2.2 above. Each period, the politician chooses the transfers \( \{ g_t^0(\theta_t, \gamma_t, \gamma_t^*) \} \) according to the program (2.11) and conditions (2.12)-(2.14).

The maximization problem for voters is given by

\[
\max_{\{ g_0^*, g_1^* \}} \mathbb{E} \left[ u^H(\theta_0, \gamma_0, g_0^*) + \beta \mathbb{E} \left[ u^H(\theta_1, \gamma_1, g_1^*) \right] \right] \tag{2.41}
\]

subject to

\[
\begin{align*}
 v^p(\theta_0, g_0^*(\theta_0)) + \beta \mathbb{E} \left[ v^p(\theta_1, g_1^*(\theta_0, \theta_1)) \right] & \geq v^p \left( \theta_0, g_0^* \left( \tilde{\theta}_0 \right) \right) + \\
 v^p(\theta_1, g_1^*(\theta_0, \theta_1)) & \geq v^p \left( \theta_1, g_1^* \left( \tilde{\theta}_1 \right) \right), \quad \forall \tilde{\theta}_0, \tilde{\theta}_1 \in \Theta \tag{2.42a} \\
 g_0^*(\theta_0) & \geq 0, \quad g_1^*(\theta_0, \theta_1) \geq 0. \tag{2.42c}
\end{align*}
\]

The problem for the politician has the property that \( v^p(\theta, g^*) \) is an increasing function of \( g^* \) maximized at \( g^* = \tau \), \( \forall \theta \). The cutoff \( g^* \) is the best response for voters given the information they have access to, and the second period is the last period of the game. Therefore, an incentive compatible cutoff \( g_1^*(\theta_0, \theta_1) \) may vary only due to the reports from period 0, as separation of types cannot be achieved based on the report from the first period \( \theta_1 \); however, implementing a non-constant schedule for \( g_0^*(\theta_0) \) requires promising different continuation values to the politician such that the incentive compatibility constraint is satisfied. Therefore, the cutoff \( g_1^*(\theta_0, \theta_1) \) will be a non-constant function of \( \theta_0 \) whenever \( g_0^*(\theta_0) \) is non-constant.

Denote by \( \tilde{g}^* \) the constant limit that would be chosen by households each period. Given Assumption 11, \( \tilde{g}^* > 0 \). The following result captures the trade-off faced by voters when deciding their reelection strategy.
Proposition 29 Under Assumptions 11 and 13, there exists ratio $\frac{\theta_L}{\theta_H}$ such that:

- for $\frac{\theta_L}{\theta_H} < \frac{\theta_L^*}{\theta_H^*}$, a report $\theta_0$ of more distressed firms in period 0 will result in a lower cutoff $g_1^*(\theta_0, \theta_1)$ for targeted transfers in period 1;

- for $\frac{\theta_L}{\theta_H} \geq \frac{\theta_L^*}{\theta_H^*}$, the cutoff $g_1^*(\theta_0, \theta_1)$ on targeted transfers in period 1 will be independent of the report $\theta_0$ about the fraction of distressed firms in period 0;

Proof. In the Appendix. ■

The above proposition shows that when the size of shocks differs sufficiently between a crisis and a non-crisis, policies from period 0 will have persistent effects in period 1. Voters find it optimal to offer the politician incentives to truthfully report the firms' need for funding in period 0. The offer incentives by promising different cutoffs for targeted transfers in period 1, cutoffs above which they would remove the politician. Offering these incentives allows voters to bring the first period policies closer to their preferred values, compared to the policies that would be implemented with a non-contingent cutoff $g^*$. The large difference between shocks $\theta_L$ and $\theta_H$ makes the gains in utility from adapting the reelection cutoff to the report on $\theta$ exceed the costs of having to implement the promised reelection cutoffs in period 1. In order to offer incentives for a truthful report, voters must reward the report of a low shock with a higher promised cutoff on targeted transfers in the next period. Similarly, they must punish a report of a high shock with a lower promised cutoff next period. Therefore, the need to provide incentives in period 0 leads to a persistent effect of the first period policies, as well as volatility in the size of targeted transfers, even absent any change in shock values from one period to the next.

If the ratio $\frac{\theta_L}{\theta_H}$ is large, so that the difference in the share of distressed projects between the two states is small, then offering incentives for truthful reporting is not optimal for voters. The cost of providing such incentives, in terms of lost household utility in the second period, would be larger than the gain from moving policies closer to the voters' preferred level in the first period.

The above result is similar to the bunching versus separating result in Amador et al. (2006). In their framework, the threshold for separation is given by the degree of disagreement between
the selves. Here, too, the threshold is given by the degree of disagreement between two agents—voters and the politician—both in the current period, and in future periods. This disagreement comes from the higher weight placed by the politician on transfers to connected firms, relative to the value placed by voters on these transfers. As detailed in Amador et al. (2006), extending the above conclusion to more than two discrete $\theta$-types does not lead to a simple characterization of the optimal solution. Using a continuous distribution for $\theta$ can, under specific conditions, lead to a full characterization of the allocation.\footnote{See Amador et al. (2006) for further details.} The analysis in this model is restricted to the set of shocks over which separation is optimal, by imposing the conditions specified in Assumption 13. Therefore, the infinite horizon problem continues to assume a discrete distribution for the $\theta$-types.\footnote{An extension of the analysis to the case with a bunching threshold might require a continuous distribution of types.}

### 2.3.8 Long-run Implications

In this section, I study the long run implications of the model for the mix of targeted and untargeted transfers chosen by the politician. The main questions explored in this section are whether the expected continuation value obtained by the politician in the best sustainable equilibrium converges to a stationary distribution, and how do the cutoffs for targeted transfers vary in the long run.

If the discount factors are the same for voters and the politician ($\beta = \tilde{\beta}$), then the continuation utility given to the politician in the long run will be at either the high end or the low end of its possible values ($\overline{EV}$ and $\underline{EV}$ respectively), and it will converge almost surely. The argument for this is similar to the immiseration result obtained in Thomas and Worrall (1990) and Atkeson and Lucas (1992), and the details are given in the Appendix. The intuition for the result is that the rewards and punishments necessary to give incentives to the politician are optimally spread over the entire time horizon, which makes the expected continuation utility $EV$ behave like a random walk. Each period, a new shock accumulates, leading to more diverging rewards or punishments.
Proposition 30 With no difference in discount factors between voters and the politician \((\beta = \hat{\beta})\), the expected continuation value for the politician \((EV)\) converges almost surely to its boundary \((\bar{E}V\) or \(EV)\). Therefore, in the long run, the politician receives the maximum possible reward after a low reported need for funding and is punished maximally after a high reported need for funding.

**Proof.** In the Appendix. ■

Proposition 30 states that the long run distribution for the expected politician utility \((EV)\) has mass points only at \(EV\) and \(E\bar{V}\). The result emerges because voters optimally backload the incentives offered to the politician. Over time, voters increase the continuation value promised to the politician as a reward after a report of a low shock \(\theta\), and they decrease the continuation value promised after a report of a high shock \(\theta\). This is necessary in order to induce truthful reports about the state of the economy, and to optimally spread the cost of distortions over time. The intuition is that the longer the politician is in power, the more he must be rewarded for complying with the voters’ conditions. Since voters must make promises every period, these promises accumulate and increase the reward for the politician. Similarly, the punishments increase over time, the longer the politician is in power. In the long run, this amounts to maximal rewards and punishments. Since the maximum punishment and the maximum reward, \(EV\) and \(E\bar{V}\), respectively, correspond to either politician replacement or to having no cutoff for targeted transfers \((g^*_t = \tau)\), we obtain the following corollary.

**Corollary 31** With no difference in discount factors between voters and the politician \((\beta = \hat{\beta})\), the balance between targeted and untargeted transfers will be chosen by the politician without any binding restrictions from voters.

If the discount factors are different for voters and the politician, so that \(\hat{\beta} < \beta\), then the submartingale that characterizes the continuation value describes a mean-reversion process. The continuation value \(EV\) converges to an invariant distribution, as shown in Farhi and Werning (2007). The invariant distribution \(\psi^*\) has no mass at the bounds and

\[
\sum U_V(EV)\psi^*(EV) = 0.
\]

Therefore, in the long run, the optimal policy will feature both targeted and untargeted transfers, and the cutoff for targeted transfers will bind with positive
probability.

**Proposition 32** With different discount factors ($\beta > \tilde{\beta}$), the continuation value for the politician ($EV$) has an invariant distribution with no mass at the bounds $EV$ and $EV$. The optimal policy in the long run involves both targeted and untargeted transfers.

**Proof.** In the Appendix. ■

The intuition for Proposition 32 is that voters value their future consumption more than the politician. Because of this, it is not preferable for them to promise high rewards for the politician in the future. Since they value these rewards more than the politician, the cost to them is higher than the benefit to the politician. The promised rewards to the politician do not increase up to the maximum level. Similarly, the punishment for reporting a high fraction of distressed firms does not increase to the maximum. Voters prefer to offer more rewards to the politician in the short-term rather than backload political rents, as rents obtained further in the future are less valuable to the politician. In this case, even in the long-run, the politician is constrained by voters to choose a certain combination of targeted and untargeted transfers. 21

### 2.4 The Case With Public Debt

The model without debt highlights that the optimal cutoff on targeted transfers is history dependent even in an environment in which the periods are not linked through any additional state variables. However, with a fixed government budget each period, an increase in untargeted transfers necessarily leads to a decrease in targeted interventions. In this section, we allow for income to be shifted across periods through the use of public debt. The ability to take on debt allows the politician to shift resources from one period to the next. The question is then how the existence of debt affects voters' ability to restrict inefficient transfers and induce the politician to truthfully report the fraction of distressed firms. In particular, we can examine the change in voters' cutoffs on targeted interventions, above which they would replace the politician.

---

21The implications of the above result are based on a strong set of assumptions. Implementing the optimal contract requires permanent commitment from households and no renegotiation over time. This strong assumption simplifies the analysis, by restricting the voters to only using one possible instrument for incentives provision: the cutoff on targeted interventions. An extension of the model to allow for renegotiation would change the results.
Consider the case in which $\beta = \hat{\beta}$. The politician now has access to a debt market, and he can take on one-period non-contingent debt $b_{t+1}$ at rate $\frac{1}{\beta}$, where the value of $b_{t+1}$ is limited below by an exogenously given limit $b$, and it is limited above by the no-Ponzi condition. The debt is observable to households, and households can condition reelection on the size of $b_{t+1}$. As before, the problem can be split into a subproblem for the politician given the voters' reelection strategy and a subproblem for voters given the politician's observable policy choices and his report on shock $\theta$.

First, we establish the result that, with truthful reporting of shocks $\theta_t$, the equilibrium strategy for voters is to use a cutoff for debt taking, above which they replace the politician.

**Lemma 33** For every report $\hat{\theta}_t$ and $g^*_t(h^0_{t-1}, \hat{\theta}_t)$, there exists $b^*_t(h^0_{t-1}, \hat{\theta}_t) > 0$ such that

$$\forall b_{t+1}(h^0_{t-1}, \hat{\theta}_t) \leq b^*_t(h^0_{t-1}, \hat{\theta}_t), \rho_t(g_t, b_{t+1}|g^*_t \geq \tau - g^*_t) = 1, \text{ and } \rho_t = 0 \text{ otherwise.}$$

Lemma 33 states that the reelection strategy for the voters for every report $\hat{\theta}_t$ will involve setting a cutoff value $g^*_t$ for targeted transfers and a cutoff value $b^*_t$ for debt, above which the politician will be removed from office. The intuition for this result is similar to the intuition for the cutoff $g^*_t(h^0_{t-1}, \hat{\theta}_t)$. The voters' benefit from debt is lower than that of the politician, and therefore, they always prefer lower debt that the politician. Their utility varies monotonically with debt, and this gives the cutoff result in Lemma 33.

The politician's problem each period, given a schedule $\{g^*, b^*\}$:

$$V(\theta, \gamma, g^*(\hat{\theta}), b^*(\hat{\theta})) \equiv \max_{\{g^*, g^0, g^0, b^*\}} v(\theta, \gamma, g^*(\hat{\theta}), b^*(\hat{\theta}))$$

$$+ \beta \mathbb{E} \left[ V'(\theta', \gamma', g''(\hat{\theta}'), b''(\hat{\theta}')) \right]$$

(2.43)
subject to

\[ \gamma g^c + \theta g^d \leq g^* (\hat{\theta}), \] (2.44a)
\[ \gamma g^c + \theta g^d + g^i \leq \tau + \beta b' - b, \] (2.44b)
\[ b' \geq b, \] (2.44c)
\[ b' \leq b^* (\hat{\theta}), \] (2.44d)
\[ g^c \geq 0, \quad g^d \geq 0, \quad g^i \geq 0. \] (2.44e)

Constraint (2.44a) is the limit on targeted transfers imposed by voters as part of their reelection condition given report \( \hat{\theta} \). Constraint (2.44b) is the resource constraint faced by the politician when he can take on public debt. Constraint (2.44c) is the exogenous lower bound on debt. Constraint (2.44d) is the constraint on debt as part of voters’ reelection strategy. Finally, constraint (2.44e) is the non-negativity requirement for all government policies.

Given the concavity of the politician’s instantaneous utility \( v(\theta, \gamma, g^* (\hat{\theta}), b^* (\hat{\theta})) \) and the linear constraints, it follows that the solution to (2.12)-(2.14) is unique, and

\[ V (\theta, \gamma, g^* (\hat{\theta}), b^* (\hat{\theta})) \] is a well-defined function. Since the politician has a higher preference for debt in the current period than voters, due to the higher preference for transfers to connected firms, constraint (2.44d) will bind and \( b' = b^* (\hat{\theta}). \) We are therefore still in a situation in which there are no inter-period linkages other than through voters’ strategies \( \{g^* (\hat{\theta}), b^* (\hat{\theta})\}. \) The politician’s problem given \( g^* (\hat{\theta}) \) and \( b^* (\hat{\theta}) \) is a static optimization over \( \{g^d, g^c, g^i\}. \) This leads to the following choice of policies as a function of \( g^* (\hat{\theta}) \) and \( b^* (\hat{\theta}) \), assuming an interior solution:

\[ g^i = \tau + b^* (\hat{\theta}) - b - g^* (\hat{\theta}), \] (2.45)
\[ g^c = \frac{g^*(\hat{\theta}) - \theta g^d}{\gamma}, \] (2.46)

and \( g^d \) is given by the implicit equation

\[ \theta f' \left( \bar{k} - x + g^d + \tau + b^* (\hat{\theta}) - b - g^* (\hat{\theta}) \right) = \gamma R f' \left( \bar{k} + \frac{g^*(\hat{\theta}) - \theta g^d}{\gamma} \right). \]
Analogous to the previous section, let

\[ v^p(\theta_t, g^*_t(h^0_{t-1}, \theta_t), b^*_t(h^0_{t-1}, \theta_t)) = \mathbb{E} \left[ v^p(\theta_t, \gamma_t, g^*_t(h^0_{t-1}, \theta_t), b^*_t(h^0_{t-1}, \theta_t)) \right], \]

the expected utility of the politician in the current period before the realization of the \( \gamma_t \) shock.

The subproblem for the voters, taking into account the response from the politician, is given by:

\[
\max_{g^*_t(h^0_{t-1}, \theta_t), b^*_t(h^0_{t-1}, \theta_t)} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u^H(\theta_t, \gamma_t, g^*_t(h^0_{t-1}, \theta_t), b^*_t(h^0_{t-1}, \theta_t)) \tag{2.47}
\]

subject to

\[
v^p(\theta_t, g^*_t(h^0_{t-1}, \theta_t), b^*_t(h^0_{t-1}, \theta_t)) + \tilde{\beta} E V^h_{t+1}(\tilde{\theta}_t) \geq v^p(\theta_t, g^*_t(h^0_{t-1}, \tilde{\theta}_t), b^*_t(h^0_{t-1}, \tilde{\theta}_t)) + \tilde{\beta} E V^h_{t+1}(\tilde{\theta}_t), \forall \tilde{\theta}_t \in \Theta, \tag{2.48}
\]

\[ g^*_t(\theta_t) \geq 0, \tag{2.49} \]

\[ b^*_t(h^0_{t-1}, \theta_t) \geq b, \tag{2.50} \]

\[ \mathbb{E} \sum_{t=0}^{\infty} \beta^t b^*_t(h^0_{t-1}, \theta_t) \leq \frac{\tau}{1 - \beta}, \tag{2.51} \]

Constraint (2.48) is the incentive compatibility constraint for the politician, who must be induced to truthfully report the shock \( \theta_t \) each period. Constraint (2.49) is the lower bound for targeted interventions, \( g^*_t(\theta_t) \). Constraint (2.50) is the exogenous lower bound for debt-taking. Constraint (2.51) gives the intertemporal resource constraint, that limits maximum debt-taking to the lifetime resources available to the government. While there is no constraint for the upper limit for \( g^*_t(\theta_t) \), we can see from problem (2.43) that the politician will weakly prefer to choose \( g^*_t(\theta_t) \) equal to the all resources available in period \( t \) rather than setting \( g^*_t(\theta_t) \) above that value.

Using the strategy in Farhi and Werning (2007), let \( \eta \) denote the multiplier on the intertem-
poral resource constraint (2.51). We can then form the Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[ u^h(\theta_t, \gamma_t, \phi_t^{\star}(h_{t-1}^0, \theta_t), b_t^*(h_{t-1}^0, \theta_t)) - \eta b_t^*(h_{t-1}^0, \theta_t) \right].$$

Therefore, the problem reduces to studying the maximization of $\mathcal{L}$ subject to constraint (2.48). This problem is then equivalent to the maximization problem

$$K(V) = \sup_{g^*, b^*} \mathcal{L},$$

subject to constraints (2.48)-(2.51).

As in the case without debt, the above problem can be expressed in recursive form as

$$U(EV) = \max_{g^*(\theta), b^*(\theta), EV'(\theta)} \mathbb{E}_{\theta, \gamma} \left[ u^h(\theta, \gamma, g^*(\theta), b^*(\theta)) - \eta b^*(h_{t-1}^0, \theta) + \beta U(EV'(\theta)) \right],$$

subject to

$$EV = \mathbb{E}_{\theta, \gamma} \left[ v(\theta, \gamma, g^*(\theta), b^*(\theta)) + \beta EV'(\theta) \right],$$

$$v^p(\theta, g^*(\theta), b^*(\theta)) + \beta EV'(\theta) \geq v^p \left( \theta, g^*(\widehat{\theta}), b^*(\widehat{\theta}) \right) + \beta EV'(\widehat{\theta}), \forall \theta \in \Theta,$$  

$$g^*(\theta) \geq 0,$$  

$$b^*(\theta) \geq \bar{b},$$  

$$b^*(\theta) \leq \frac{\tau}{1 - \beta},$$  

$$EV'(\theta) \geq EV,$$  

$$EV'(\theta) \leq EV.$$

Constraint (2.55) represents the promise keeping constraint, constraint (2.56) is the incentive-compatibility constraint for the incumbent politician. Constraint (2.57) is the lower bound on direct interventions. Constraints (2.58) and (2.59) give the lower and upper bounds on debt. Similarly, constraints (2.60) and (2.61) give the lower and upper bounds, respectively, on the
expected continuation utility that voters can offer the politician. Since the utility of the politician is increasing in the cutoffs $g^*(\theta)$ and $b^*(\theta)$, we can, as in the case without debt, derive the lower and upper limits on the expected utility that can be promised to the politician. These limits correspond to removal from office and $g^*(0) = \tau$ and $b^*(0) = \frac{\tau}{1-\beta}$, respectively:

$$EV = V,$$  \hspace{1cm} (2.62)

and

$$EV = \mathbb{E}_\theta \left( \frac{v^P(\theta, g^*(\theta), b^*(\theta))}{1 - \beta} \right) \bigg|_{g^*(\theta) = \tau, b^*(\theta) = \frac{\tau}{1-\beta}}.$$  \hspace{1cm} (2.63)

Problem (2.53) is similar to the problem without debt in that it delivers the same martingale properties for the politician’s continuation value.

**Proposition 34** Consider the model with public debt. The best sustainable equilibrium has the property that voters’ reelection decisions are history dependent: higher targeted transfers in the current period (higher $g_t^\theta$ and $g_t^F$) lead to more restrictions on targeted transfers in future periods, through lower future cutoffs $g_j^*$ and $b_j^*$, $\forall j > t$.

**Proof.** In the Appendix.  ■

Proposition 34 shows that the presence of public debt delivers the same result as Proposition 26, that the cutoffs on targeted intervention decisions are history dependent. A report of a high shock $\theta$, so a high need for funding in the current period is punished by promising the politician a lower level of utility in the future. This is achieved by increasing the restrictions on the politician in future periods, through both a lower cutoff on targeted interventions and more limited access to debt.

While it does not change the qualitative features of the equilibrium, the existence of debt does affect voters’ choices of reelection cutoffs and therefore the equilibrium balance between targeted and untargeted transfers. Higher outstanding debt in a period will lead to a higher cutoff on targeted spending. Higher outstanding debt decreases the cost to voters of allowing more transfers to be given to connected firms (along with more targeted transfers to distressed firms) relative to the utility cost of using the scarcer available revenue for untargeted transfers.
Second, taking on more debt increases the politician’s relative preference for targeted transfers relative to untargeted transfers. This in turn affects voters’ ability to use reelection as an instrument for reducing inefficient transfers. The result of this is captured in the following Proposition.

**Proposition 35** *In the model with public debt, the politician must be promised a higher expected utility in equilibrium compared to the case without debt. The transfers targeted to connected firms are also higher than in the case without debt.*

**Proof.** In the Appendix. ■

The result in Proposition 35 emerges because both the politician, and voters derive utility from the output produced by firms. All transfers benefit both voters and the politician, but the politician places higher weight on transfers to connected firms. Therefore, with access to debt, the politician will increase all transfers, but he will increase the transfers to connected firms by more than voters would prefer. Each period, the utility of voters changes in the same way as the utility of the politician, but the utility of the politician increases by more relative to that of voters. The intuition is that the politician now has more access to funds that he can use for transfers to connected firms, while voters have limited means of restricting his behavior. The existence of public debt then makes the incentive compatible mechanism costlier to implement relative to the no debt case, because of the higher flexibility that the politician in accessing public funds. It does not change the result that the cutoff $g^*_t$ is history dependent, but it does change the costs to voters of providing incentives.

The above result also shows that the politician benefits from having access to public debt. The cost of providing incentives is higher for voters, but that does not necessarily imply that voter welfare goes down. On the one hand, public debt allows for a smoothing of intervention costs over time, which increases overall welfare. On the other hand, offering incentives to the politician imposes a higher utility cost on voters. The overall effect on voter welfare could go in either direction, depending on how much the politician’s preference for transfers to connected firms differs from that of voters. We therefore obtain the result that constraints on debt-taking are desirable in environments with a high divergence between voters and the politician, in which
the costs of offering incentives to the politician outweigh the benefits of smoothing the cost of interventions over time.

**Corollary 36** There exist environments (in terms of production functions \( f(\cdot) \) and weight \( R \) on connected firms) in which budget balance requirements increase voter welfare. These environments require a high weight \( R \) on transfers to connected firms. Otherwise, public debt is welfare increasing for both voters and the politician.

**Proof.** In the Appendix. ■

The intuition for Corollary 36 is that not having access to debt is only preferable to voters if debt reduces their ability to restrict transfers to connected firms which are not distressed, and transfers to connected but undistressed firms are highly inefficient for voters (given the production function \( f(\cdot) \)). These transfers benefit the politician more than they benefit voters, leading to welfare losses for voters and welfare gains to the politician. Restricting debt is desirable only if the loss from these inefficient transfers outweighs the benefits coming from having more public funds available in periods of severe crisis, when government intervention is most valuable.

### 2.5 Extension - Non-Financial Public Goods

So far, the model only considered the government's role as lender of last resort in a financial crisis. The Appendix presents an extension of the model in which a non-financial public good can also be provided by the government along with transfers to firms. I assume that this additional public good can only be provided by the government, and it offers the same utility to both households and the politician. This additional public good offers a dimension of policy along which households and the government have the same preferences, and it also allows us to consider a different use for public funds, other than transfers to firms. The analysis of the model presented in the Appendix shows that expanding the set of public goods provided by the government does not change the main implications of the model regarding the history dependence of the cutoffs on targeted transfers, nor the persistent effects of policies. Yet, we can show that adding such a non-financial public good can have a disciplining effect on the
politician. It can lead to cases where untargeted transfers are not offered in equilibrium. In this case, the government will prefer to respond to any cutoffs on targeted transfers by increasing the provision of the other public good rather than resorting to untargeted transfers.

2.6 Conclusion

This paper develops a political economy model of government intervention in financial markets when the government has private information about the shocks hitting the economy, and in which policy is decided by an elected politician. The politician is connected to some of the firms operating in this economy, and he derives political rents from making transfers to these firms. These lobbying firms are not distressed, and their use of public funds to expand output is socially inefficient. The government has access to two types of interventions: it can directly provide funds to firms through targeted transfers; or, it can make funds accessible to all firms through untargeted transfers. Making funds accessible to all firms means that some funds will go to undistressed firms, which would be an inefficient use of public funds.

The key friction of the model is that targeted transfers can be directed to connected firms that are not distressed, leading to an outcome that is socially inefficient, but beneficial for the politician, due to the political rents extracted from firms that lobby the government. In order to restrict the inefficient transfers to connected firms, voters can punish the incumbent politician with removal from power. Given these ingredients, the model shows that the policies chosen by the politician have persistent effects through the voters' reelection decisions in future periods. A politician who uses targeted transfers in the current period faces a stricter reelection condition from voters in future periods. If an incumbent wants to remain in office, he will then have to provide fewer targeted transfers in future periods, even as the same shocks hit the economy.

The model shows how reelection pressures affect the choice of government interventions in crises. It presents a mechanism that results in a combination of targeted and untargeted transfers that changes over time depending on the government's past actions. This pattern is maintained even if the government has access to public debt, or if the government is able to provide non-financial public goods along with transfers to firms. It shows how a targeted
intervention by the government in a financial crisis can have the side effect of making future targeted interventions less likely.

The model offers several avenues for future research. First, the paper does not consider the viability or solvability of distressed firms. The targeted transfers from the government could, however, be used to support inefficient projects. This would lead to an increased probability of default by these firms in the future, essentially creating more volatility in the economy. Another interesting extension of the model is to consider the difference in timing between targeted and untargeted interventions. Legislative procedures associated with targeted transfers are generally viewed as lengthier than policy decisions made outside of the legislative process. This difference in timing would introduce an even stronger incentive for voters to restrict targeted transfers and accept more untargeted transfers when faster intervention reduces firms' losses.

The model's implications provide a possible mechanism that explains the observed pattern of government intervention in the recent financial crisis. The implications of the model could be tested empirically by relating the type of government intervention to its timing relative to the duration of the crisis, and to the degree of electoral pressures faced by the government. These pressures can come in the form of scheduled elections or lobbying activities.
2.A Appendix

2.A.1 Proof of Proposition 20

Assume the allocation $\phi$ is supported by sustainable equilibrium strategies. Then, condition (2.10) must be satisfied for the politician to follow the equilibrium strategy of reporting the true shock $\theta$. The inequality in condition (2.10) ensures that reporting $\hat{\theta} \neq \theta$, with associated limit $g^*(\hat{\theta})$ and continuation value $V_{t+1}(\hat{\theta}_t, h_t)$, is a weakly dominated strategy. Conditions (2.7), (2.8) and (2.9) are necessary for feasibility.

Assume conditions (2.7)-(2.10) are satisfied. Then the allocation $\phi$ is feasible given conditions (2.7), (2.8) and (2.9). Finally, condition (2.10) implies that the politician does not have an incentive to deviate from the equilibrium strategy of reporting the true value of $\theta$.

2.A.2 Note on full information solution and Assumption 11

The condition in Assumption 11 is obtained from the following inequality, taking the marginal benefit and cost to voters of increasing $g^i$ from 0.

$$
\theta f' \left( k - x + g^{dv} \right) + (1 - \theta - \gamma) f' \left( k \right) + \gamma f' \left( k + \frac{\tau - \theta g^{dv}}{\gamma} \right) > \theta f' \left( k - x + g^{dv} \right) \frac{\partial g^{dv}}{\partial g^*} \bigg|_{g^* = \tau}
$$

$$
+ \frac{1}{R} \left( 1 - \frac{\theta g^{dv}}{g^*} \right) \frac{\partial g^{dv}}{g^*} \bigg|_{g^* = \tau},
$$

$$
\theta f' \left( k - x + g^{dv} \right) + (1 - \theta - \gamma) f' \left( k \right) + \gamma f' \left( k + \frac{\tau - \theta g^{dv}}{\gamma} \right) > \theta f' \left( k - x + g^{dv} \right) \frac{\partial g^{dv}}{\partial g^*} \bigg|_{g^* = \tau}
$$

$$
+ \gamma f' \left( k + \frac{\tau - \theta g^{dv}}{\gamma} \right) \frac{1 - \theta}{\gamma} \frac{\partial g^{dv}}{g^*} \bigg|_{g^* = \tau},
$$

$$
\theta f' \left( k - x + g^{dv} \right) + (1 - \theta - \gamma) f' \left( k \right) + \frac{\gamma}{R} f' \left( k - x + g^{dv} \right) > \theta f' \left( k - x + g^{dv} \right) \frac{\partial g^{dv}}{\partial g^*} \bigg|_{g^* = \tau}
$$

$$
+ \frac{f' \left( k - x + g^{dv} \right)}{R} \left( 1 - \theta \right) \frac{\partial g^{dv}}{g^*} \bigg|_{g^* = \tau},
$$

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where $g^{dv}$ is the value chosen by the politician when the limit on direct interventions is $\tau$:

$$f'(\bar{k} - x + g^{dv}) = R f'(\bar{k} + \frac{\tau - \theta g^{dv}}{\gamma}).$$

The first order condition for an interior $g^*(\theta)$, given conditions (2.15)-(2.17), must satisfy

$$\theta f'(\bar{k} - x + g^{dv} + \tau - g^*(\theta)) \left( \frac{\partial g^{dv}}{\partial g^*} - 1 \right) - (1 - \theta - \gamma) f'(\bar{k} + \tau - g^*(\theta))
+ \gamma f'(\bar{k} + \frac{g^*(\theta) - \theta g^{dv}}{\gamma} + \tau - g^*(\theta)) \left( \frac{1}{\gamma} - \frac{\theta}{\gamma} \frac{\partial g^{dv}}{\partial g^*} - 1 \right) = 0. \tag{2.64}$$

Given conditions (2.15)-(2.17), it also follows that:

$$\theta \left[ f''(\bar{k} - x + g^{dv} + \tau - g^*) \right] \left( \frac{\partial g^{dv}}{\partial g^*} - 1 \right) =
\gamma R \left[ f''(\bar{k} + \frac{g^*(\theta) - \theta g^{dv}}{\gamma} + \tau - g^*(\theta)) \right] \left( \frac{1}{\gamma} - \frac{\theta}{\gamma} \frac{\partial g^{dv}}{\partial g^*} - 1 \right),$$

so we can derive the following bounds for $\frac{\partial g^{dv}}{\partial g^*}$:

$$1 < \frac{\partial g^{dv}}{\partial g^*} < \frac{1 - \gamma}{\theta}.$$

Therefore, condition (2.64) can be satisfied for appropriate parameters $\{R, \bar{k}, x, \tau\}$ and functional forms.

### 2.A.3 Note of Incentive Compatibility

We simplify the problem by using the following Lemma.
Lemma 37 Under Assumption 12, if an incentive-compatible mechanism exists in which the 0-types are separated, then the only IC constraints that bind are those for adjacent 0-types, in the upward direction.

**Proof.** Define \( v^p(\theta^i, g^*(\theta^i)) \equiv E_\gamma [v(\theta^i, \gamma, g^*(\theta^i))] \).

Let

\[
C_{i,j} = v^p(\theta^i, g^*(\theta^i)) + \beta EV'(\theta^i) - v^p(\theta^i, g^*(\theta^j)) - \beta EV'(\theta^j).
\]

We first show that if \( C_{i,i+1} = 0 \), then \( C_{i,j} \geq 0 \) \( \forall j, i \).

\[
C_{i,i+1} = 0 \Leftrightarrow \beta EV'(\theta^i) - \beta EV'(\theta^{i+1}) = v^p(\theta^i, g^*(\theta^{i+1}))-v^p(\theta^i, g^*(\theta^{i+1})).
\]

By Assumption 12,

\[
v^p(\theta^{i+1}, g^*(\theta^{i+1}))-v^p(\theta^{i+1}, g^*(\theta^{i+1})) \geq \beta EV'(\theta^i) - \beta EV'(\theta^{i+1}),
\]

so

\[
v^p(g^*(\theta^{i+1}), \theta^{i+1}) - v^p(g^*(\theta^i), \theta^{i+1}) - \beta EV'(\theta^i) + \beta EV'(\theta^{i+1}) > 0.
\]

Thus,

\[
C_{i,i+1} = 0 \rightarrow C_{i+1,i} \geq 0.
\]

\( C_{i,i+1} = 0 \) means, \( \forall k > 0 \), with \( i + k \leq N \):

\[
\beta EV'(\theta^i) - \beta EV'(\theta^{i+1}) = v^p(\theta^i, g^*(\theta^{i+1}))-v^p(\theta^i, g^*(\theta));
\]

\[
\beta EV'(\theta^{i+1}) - \beta EV'(\theta^{i+2}) = v^p(\theta^{i+1}, g^*(\theta^{i+2}))-v^p(\theta^{i+1}, g^*(\theta^{i+2}));
\]

\[
(\ldots)
\]

\[
\beta EV'(\theta^{i+k-1}) - \beta EV'(\theta^{i+k}) = v^p(\theta^{i+k-1}, g^*(\theta^{i+k}))-v^p(\theta^{i+k-1}, g^*(\theta^{i+k})).
\]

Adding up the above equalities \( \Rightarrow \)

\[
\beta EV'(\theta^i) - \beta EV'(\theta^{i+k}) = \sum_{n=i}^{i+k-1} \{v^p(\theta^n, g^*(\theta^{n+1}))-v^p(\theta^n, g^*(\theta^n))\}.
\]
\[
\begin{align*}
\{ & v^P(\theta^i, g^*(\theta^i)) + \beta EV'(\theta^i) - v^P(\theta^i, g^*(\theta^{i+k}),) - \beta EV'(\theta^{i+k}) \}
\end{align*}
\]

\[
= \left\{ v^P(\theta^i, g^*(\theta^i)) - v^P(\theta^i, g^*(\theta^{i+k})) + \sum_{\theta=i}^{i+k-1} \{ v^P(\theta^n, g^*(\theta^{n+1})) - v^P(\theta^n, g^*(\theta^n)) \} \right\}
\]

By repeatedly applying the inequality in Assumption 12 in the above sum, it follows that

\[
v^P(\theta^i, g^*(\theta^i)) + \beta EV'(\theta^i) - v^P(\theta^i, g^*(\theta^{i+k})) - \beta EV'(\theta^{i+k}) \geq 0.
\]

Therefore \(C_{i,i+1} = 0\) implies \(C_{i,i+k} \geq 0\), \(1 < k \leq N - i\).

Then, we show that it must be the case that \(C_{i,i+1} = 0\). Notice that the utility of the voters is maximized when \(\overline{G}^B\) is implemented in period 2, which leads to an expected utility of \(EV^B\) for the government. As \(|EV'(\theta^i) - EV^B|\) increases, voters’ utility decreases, since it is a concave function that is maximized at the point corresponding to \(EV^B\). It follows then that \(EV'(\theta^N) \leq EV^B \leq EV'(\theta^1)\).

Assume that \(C_{i,i+1} > 0\). If \(EV'(\theta^{i+1}) \leq EV^B\), then increasing \(EV'(\theta^j)\) by a small \(\epsilon \forall j \leq i\) would still satisfy the incentive compatibility constraints, but would also increase the utility of voters. Similarly, if \(EV'(\theta^{i+1}) > EV^B\), increasing \(EV'(\theta^j)\) by a small \(\epsilon \forall j > i\) would still satisfy the incentive compatibility constraints and increase the utility of voters. Therefore, \(C_{i,i+1} > 0\) would not be optimal. ■

2.A.4 Proof of Lemma 24

The function

\[
u^H = \theta f(\kappa - x + g^d + g^i) + (1 - \theta - \gamma) f(\kappa + g^i) + \gamma f(\kappa + g^c + g^i)
\]

is strictly concave in transfers (since \(f(\cdot)\) is strictly concave). Given conditions (2.15)-(2.17), the equilibrium untargeted transfer \(g^i(\theta)\) is a linear function of \(g^*(\theta)\). Also, the first-order conditions
for the politician yield

\[ \theta f'(k - x + g^d + \tau - g^*(\theta)) = \gamma R f' \left( k + \frac{g^*(\theta) - \theta g^d}{\gamma} + \tau - g^*(\theta) \right). \]

Since the function \( f(\cdot) \) is strictly concave, it follows from the above equality that \( g^d(\theta, g^*(\theta)) \) is weakly concave. Then,

\[
\frac{\partial^2 u^H(\theta, \gamma, g^*)}{\partial g^*^2} = \left\{ \theta f'' \left( k - x + g^d(\theta, g^*) + g^i(\theta, g^*) \right) \left( \frac{\partial g^d(\theta, g^*)}{\partial g^*} - 1 \right)^2 + \theta f' \left( k - x + g^d(\theta, g^*) + g^i(\theta, g^*) \right) \frac{\partial^2 g^d(\theta, g^*)}{\partial g^*^2} + (1 - \theta - \gamma) f'' \left( k + g^i(\theta, g^*) \right) \left( \frac{1}{\gamma} - \frac{\theta}{\gamma} \frac{\partial g^d}{\partial g^*} - 1 \right)^2 + \gamma f'' \left( k + \frac{g^*(\theta) - \theta g^d}{\gamma} + \tau - g^*(\theta) \right) \left( -\frac{\theta}{\gamma} \frac{\partial^2 g^d}{\partial g^*^2} \right) \right\},
\]

which, given

\[ f' \left( k - x + g^d(\theta, g^*) + g^i(\theta, g^*) \right) = R f' \left( k + \frac{g^*(\theta) - \theta g^d}{\gamma} + \tau - g^*(\theta) \right), \]

is reduced to

\[
\frac{\partial^2 u^H(\theta, \gamma, g^*)}{\partial g^*^2} = \left\{ \theta f'' \left( k - x + g^d(\theta, g^*) + g^i(\theta, g^*) \right) \left( \frac{\partial g^d(\theta, g^*)}{\partial g^*} - 1 \right)^2 + \theta f' \left( k - x + g^d(\theta, g^*) + g^i(\theta, g^*) \right) \frac{\partial^2 g^d(\theta, g^*)}{\partial g^*^2} \left( 1 - \frac{1}{R} \right) + (1 - \theta - \gamma) f'' \left( k + g^i(\theta, g^*) \right) \left( \frac{1}{\gamma} - \frac{\theta}{\gamma} \frac{\partial g^d}{\partial g^*} - 1 \right)^2 \right\}.
\]

Since \( f'' < 0 \) and \( \frac{\partial^2 g^d(\theta, g^*)}{\partial g^*^2} < 0 \), it follows that \( \frac{\partial^2 u^H(\theta, \gamma, g^*)}{\partial g^*^2} < 0 \). So, the voters' utility for each shock value \( \theta - u^H(\theta, g^*(\theta)) \) is strictly concave. Then, \( u^H = E \left[ u^H(\theta, g^*) \right] \) is also strictly concave.
We use an inductive argument to show the concavity of $U$. Let $\alpha = \{g^*(\theta), EV'(\theta)\}_{\theta \in \Theta}$ denote the choice variables of the voters each period, for each reported $\theta$. Then

$$U_k(EV) = \max_{\alpha} \mathbb{E} \left[ u^H(\theta, g^*(\theta)) + \beta U_{k-1}(EV'(\theta)) \right].$$

Assume $U_{k-1}(EV')$ is strictly concave. Consider two continuation values $EV_a'$ and $EV_b'$ associated with corresponding solutions $\alpha_a = \{g_a^*(\theta), EV_a'(\theta)\}_{\theta \in \Theta}$ and $\alpha_b = \{g_b^*(\theta), EV_b'(\theta)\}_{\theta \in \Theta}$; $\alpha_a$ and $\alpha_b$ are feasible given the politician and the government constraints. Also, let $z \sim U[0,1]$ and $p \in (0,1)$. Let policy sequence $\alpha_c$ be defined as follows:

$$\alpha_c = \begin{cases} 
\alpha_a & \text{if } z \leq p \\
\alpha_b & \text{if } z > p 
\end{cases}.$$

Policy $\alpha_c$ is feasible since $\alpha_a$ and $\alpha_b$ are feasible. The expected continuation value for the government from policy $\alpha_c$ is

$$EV_c'(\theta) = pEV_a'(\theta) + (1 - p)EV_b'(\theta),$$

and voters get an expected continuation utility

$$pW(EV'_a(\theta)) + (1 - p)U(EV'_b(\theta)).$$

However,

$$U(EV'_c) = \max_{\alpha} \mathbb{E} \left[ u^H(\theta, \gamma, g^*(\theta)) + \beta W(EV'(\theta)) \right].$$

Let $\alpha^*$ be a solution to this maximization problem. It follows then that $U(EV'_c(\theta)) \geq pW(EV'_a(\theta)) + (1 - p)U(EV'_b(\theta))$. Therefore, $U(EV)$ is concave.

2.A.5 Proof of Lemma 25

Since $u^H(\cdot)$ and $v(\cdot)$ are concave and differentiable, we can use the argument from Lemma 1 of Benveniste and Scheinkman (1979) to show that $U(EV)$ is differentiable at $EV$ over
We need to show that there exists a function $Q(EV + \epsilon)$ for some small $\epsilon$, which is differentiable, weakly concave and satisfies $Q(EV + \epsilon) \leq U(EV + \epsilon)$, where $Q(EV + \epsilon) = U(EV + \epsilon)$ for $\epsilon = 0$. To do this, we construct the function $Q(EV + \epsilon)$ using the perturbation method from Ales et al. (2012).

Let $\alpha = \{g^*(\theta), EV'(\theta)\}$ be a solution to the maximization problem 2.27 given some $EV \in \{EV, EV\}$. Then, starting from an interior solution $\alpha$ we can construct a perturbed solution $\tilde{\alpha}(\epsilon)$ that satisfies the constraints of problem 2.27 and provides the government with expected utility $EV + \epsilon$. We construct $\tilde{\alpha}(\epsilon)$ such that the following condition is satisfied:

$$\tilde{g}^*(\theta, \epsilon) = g^*(\theta) + \xi^G(\theta, \epsilon).$$

Also, define functions $\xi^d(\theta, \epsilon), \xi^c(\theta, \epsilon)$ and $\xi^r(\theta, \epsilon)$ as:

$$\tilde{g}^d(\theta, \gamma, \tilde{g}^*, \epsilon) = g^d(\theta, \gamma, g^*(\theta)) + \xi^d(\theta, \epsilon);$$

$$\tilde{g}^c(\theta, \gamma, \tilde{g}^*, \epsilon) = g^c(\theta, \gamma, g^*(\theta)) + \xi^c(\theta, \epsilon);$$

$$\tilde{g}^r(\theta, \gamma, \tilde{g}^*, \epsilon) = g^r(\theta, \gamma, g^*(\theta)) + \xi^r(\theta, \epsilon).$$

The functions $\{\xi^d(\theta, \epsilon), \xi^d(\theta, \epsilon), \xi^c(\theta, \epsilon), \xi^c(\theta, \epsilon)\}_{\theta \in \Theta}$ are chosen such that the following conditions are satisfied:

$$\mathbb{E}_{\theta} \left[ v^\theta \left( \theta, \tilde{g}^* \right) \right] = \mathbb{E}_{\theta} \left[ v^\theta(\theta, g^*(\theta)|\alpha) \right] + \epsilon, \quad (2.65)$$

$$\theta f' \left( k - x + \tilde{g}^d \left( \theta, \gamma, \tilde{g}^*, \epsilon \right) + \tilde{g}^i \left( \theta, \gamma, \tilde{g}^*, \epsilon \right) \right) = \gamma R f' \left( \tilde{k} + \tilde{g}^c \left( \theta, \gamma, \tilde{g}^*, \epsilon \right) \right) + \tilde{g}^i \left( \theta, \gamma, \tilde{g}^*, \epsilon \right), \quad (2.66)$$

$$\theta \tilde{g}^d \left( \theta, \gamma, \tilde{g}^*, \epsilon \right) + \gamma \tilde{g}^c \left( \theta, \gamma, \tilde{g}^*, \epsilon \right) = \tilde{g}^*(\theta, \epsilon), \quad (2.67)$$

$$\tilde{g}^r \left( \theta, \gamma, \tilde{g}^*, \epsilon \right) = \tau - \tilde{g}^*(\theta, \epsilon), \quad (2.68)$$

$$v^\theta \left( \theta, \tilde{g}^* \right) + \beta V' \left( \theta \right) \geq v^\theta \left( \theta, \tilde{g}^* \right) + \beta V' \left( \theta' \right), \forall \theta, \theta' \in \Theta. \quad (2.69)$$

The above equations are sufficient to obtain solutions for $\{\xi^d(\theta, \epsilon), \xi^d(\theta, \epsilon), \xi^c(\theta, \epsilon), \xi^c(\theta, \epsilon)\}$ where $\xi^d(\theta, 0) = \xi^d(\theta, 0) = \xi^c(\theta, 0) = \xi^c(\theta, 0) = 0$. Since $V(\cdot)$ and $f(\cdot)$ are differentiable, then
it follows that the functions \( \{ \xi^d(\theta, \epsilon), \xi^c(\theta, \epsilon), \xi^i(\theta, \epsilon) \} \) coming out of the above equalities are also differentiable around \( \epsilon = 0 \).

Let \( Q(EV + \epsilon) \) denote the household utility obtained under policy \( \hat{\alpha}(\epsilon) \). Then at \( \epsilon = 0 \), \( \hat{\alpha}(0) = \alpha \) and \( Q(EV) = U(EV) \). By construction, the perturbed solution \( \hat{\alpha}(\epsilon) \) along with the solution to the politician sub-problem satisfy conditions (2.65)-(2.69), which implies \( \hat{\alpha}(\epsilon) \) satisfies all the constraints of problem (2.27) for \( \epsilon \to 0 \). Condition (2.69) implies that condition (2.29) is satisfied. Equality (2.65) implies that constraint (2.28) is satisfied, while equalities (2.66)-(2.68) directly imply that constraints (2.15) and (2.17) of the politician sub-problem are satisfied. Finally, feasibility condition that \( EV \in (EV, \bar{EV}) \) is satisfied by the assumption of a small perturbation around the interior solution \( \alpha \). It then follows that \( \hat{\alpha}(\epsilon) \) is a feasible solution to the voters' problem. This implies

\[
U(EV + \epsilon) \geq Q(EV + \epsilon). 
\]

Moreover, \( Q(EV) = U(EV) \) and (2.70) imply \( Q(EV + \epsilon) \) is locally concave around \( EV \). Then, by Lemma 1 of Benveniste and Scheinkman (1979), the value function \( U(EV) \) is differentiable for \( EV \in (EV, \bar{EV}) \).

2.A.6 Proof of Proposition 26

According to Lemmas 24 and 25, \( U(EV) \) is concave and differentiable, so the first-order conditions are necessary and sufficient for the maximization of problem of 2.27. Moreover, given Lemma 37, we can simplify the problem by only looking at the IC constraints where \( \hat{\theta} = \theta^{n+1} \) for \( \theta = \theta^n \).

Denote by \( \mu, \pi(\theta)\phi_{n,n+1}, \pi(\theta)\nu, \pi(\theta)\lambda, \pi(\theta)i \) the Lagrange multipliers on constraints (2.28), (2.29), (2.30), (2.31), (2.32) and (2.33) with messages \( \theta = \theta^n, \hat{\theta} = \theta^{n+1} \), and by \( U_{EV} \) the derivative of \( U \) with respect to \( EV \). Then, we have, the following first-order conditions:

\[
\frac{\beta}{\beta} U_{EV}(EV'(\theta^n)) + \mu + \phi_{n,n+1} - \phi_{n-1,n} \frac{\pi(\theta^{n-1})}{\pi(\theta^n)} - \xi + \tilde{\epsilon} = 0
\]
The envelope condition is given by

\[ U_{EV}(EV) \geq -\mu. \quad (2.73) \]

Taking expectations in (2.71), for \( EV'(\theta^n) < EV \), we obtain

\[ \mathbb{E} [U_{EV}(EV'(\theta^n))] + \mu \leq 0. \quad (2.74) \]

Combining with (2.73), we obtain

\[ \mathbb{E} [U_{EV}(EV'(\theta))] \leq U_{EV}(EV), \quad (2.75) \]

where the above equations hold with equality for an internal solution.

This leads to 2 possible cases:

1. We have an interior solution everywhere \( (EV' > EV, EV' < EV) \). In this case, taking expectations in the above expression, we can also infer that:

\[ \lambda = \mathbb{E} [U_{EV}(EV')] - U_{EV}(EV) = 0. \]

Equation (2.40) shows that \( U_{EV}(EV) \) is a martingale (for \( \hat{\beta} = \beta \)) or a submartingale (for \( \hat{\beta} < \beta \)). In order to satisfy the incentive compatibility constraint (2.29), \( EV'(\theta) \) must be decreasing with \( \theta \) (as \( g^*(\theta) \) increases with \( \theta \)).

If \( \hat{\beta} = \beta \), then from equation (2.40) and the concavity of \( U \), \( \exists \theta^*, \theta^* > \theta^1, \theta^* < \theta^N \) such that \( EV'(\theta) < EV \forall \theta > \theta^* \) and \( EV'(\theta) > EV \forall \theta < \theta^* \).

If \( \hat{\beta} < \beta \), then \( \mathbb{E}[U_{EV}(EV')] > U_{EV}(EV) \) for \( U_{EV}(EV) < 0 \), and \( \mathbb{E}[U_{EV}(EV')] < U_{EV}(EV) \) for \( U_{EV}(EV) > 0 \). That implies \( \exists \theta \) such that \( EV'(\theta) < EV \) if for \( U_{EV}(EV) < \)

\[ \{ \mathbb{E} \left[ \frac{\partial u^H(\theta^n, \gamma, g^*(\theta^n))}{\partial g^*} \right] + (\mu + \phi_{n,n+1}) \frac{\partial u^p(\theta^n, g^*(\theta^n))}{\partial g^*} \\
- \phi_{n-1,n} \frac{\pi(\theta^n)}{\pi(\theta^n)} \frac{\partial u^p(\theta^n-1, g^*(\theta^n))}{\partial g^*} \right) - \nu + \bar{\nu} = 0 \}. \quad (2.72) \]
0, but whether $EV' > EV$ for some $\theta$ is not immediately implied (the opposite is true for $U_{EV}(EV) > 0$).

2. The solution for $EV'$ binds for some $\theta^n$.

(a) First, consider the case $\hat{\beta} = \beta$. Assume $EV'$ binds for some $\theta^n$ at the lower bound. Since the IC solution for separability requires $EV'$ strictly decreasing in $\theta$, it must be the case that $\theta^n = \theta^N$ for $EV'(\theta^n) = EV$. The first-order condition for $EV'(\theta^N)$ is then

$$\frac{\beta}{\beta} U_{EV}(EV'(\theta^N)) + \mu - \phi_{N-1,N} + \xi = 0.$$ 

So

$$\frac{\beta}{\beta} U_{EV}(EV'(\theta^N)) = U_{EV}(EV) + \phi_{N-1,N} + \xi,$$

$$\frac{\beta}{\beta} U_{EV}(EV) = U_{EV}(EV) + \phi_{N-1,N} + \xi.$$ 

(2.76)

For condition (2.76) to hold, we need $(\phi_{N-1,N} + \xi) > 0$ if $\hat{\beta} = \beta$. Taking expectations of (2.71) over all $\theta$, we have

$$\frac{\beta}{\beta} E[U_{EV}(EV')] = U_{EV}(EV).$$ 

(2.77)

For $\hat{\beta} = \beta$ and $\xi > 0$, condition (2.77) implies $E[U_{EV}(EV')] < U_{EV}(EV)$ so $EV'(\theta^1) > EV$.

Assume we have a schedule that satisfies conditions (2.76), (2.77) and $EV'(\theta^N) = EV$. Then, following $EV = EV$, we have the following conditions for $EV'(\theta^N)$:

$$\frac{\beta}{\beta} U_{EV}(EV'(\theta^N)) = U_{EV}(EV) + \phi_{N-1,N} + \xi,$$

and

$$\frac{\beta}{\beta} E[U_{EV}(EV'(\theta^N))] = U_{EV}(EV).$$

The above conditions require $\phi_{N-1,N} \leq 0$ for $\hat{\beta} = \beta$, and $EV'(\theta^N) \geq EV$. For these conditions to hold, we must have $EV'(\theta^N) = EV$, $\phi_{N-1,N} \geq 0$ and $EV'(\theta^n) >
If $\bar{\beta} < \beta$, then the above conditions hold without the requirement $\phi_{N-1,N} \leq 0$, due to the reversal to the mean property implied by $\frac{\bar{\beta}}{\beta}$.

(b) Now, assume $EV'$ binds for some $\theta^n$ at the upper bound. Then, since the IC solution for separability requires $EV'$ strictly decreasing in $\theta$, it must be the case that $\theta^n = \theta^1$ for $EV'(\theta^n) = EV$. For $EV'(\theta^1)$ is then

$$\frac{\beta}{\bar{\beta}} U_{EV}(EV'(\theta^1)) + \mu + \phi_{1,2} - \bar{i} = 0$$

So

$$\frac{\beta}{\bar{\beta}} U_{EV}(EV'(\theta^1)) = U_{EV}(EV) - \phi_{1,2} + \bar{i}$$

$$\frac{\beta}{\bar{\beta}} U_{EV}(EV) = U_{EV}(EV) - \phi_{1,2} + \bar{i}$$

(2.78)

For condition (2.78) to hold, we need $(-\phi_{1,2} - \bar{i}) < 0$ if $\bar{\beta} = \beta$. Also, taking expectations of (2.71) over all $\theta$, we have

$$\frac{\beta}{\bar{\beta}} E[U_{EV}(EV')] = U_{EV}(EV)$$

(2.79)

For $\bar{\beta} = \beta$ and $EV$ is at the bound, condition (2.79) implies $E[U_{EV}(EV')] > U_{EV}(EV)$ so $EV'(\theta^1) < EV$. If $\bar{\beta} < \beta$, then the above conditions require a small enough $\bar{i}$ such that conditions (2.78) and (2.79) still hold given $\frac{\bar{\beta}}{\beta}$.

Assume we have a schedule that satisfies conditions (2.78), (2.79) and $EV'(\theta^1) = EV$. Then, following $EV = EV$, we have the following conditions for $EV'(\theta^1)$:

$$\frac{\beta}{\bar{\beta}} U_{EV}(EV'(\theta^1)) = U_{EV}(EV) - \phi_{1,2} + \bar{i}$$

and

$$\frac{\beta}{\bar{\beta}} E[U_{EV}(EV'(\theta^1))] = U_{EV}(EV) + \bar{\pi}(\theta^1).$$

The above conditions require $(-\phi_{1,2} + \bar{i}) \geq 0$ for $\bar{\beta} = \beta$, and $EV'(\theta^1) \leq EV$. For these conditions to hold, we must have $EV'(\theta^1) = EV$, $\phi_{1,2} > 0$ and $EV'(\theta^n) <$
If \( \hat{\beta} < \beta \), then the above conditions continue to hold given \( \hat{\beta} \). Therefore, the martingale property continues to hold at the boundary.

2.A.7 Proof of Proposition 29

Let the cutoff on targeted interventions be \( \bar{g} \) in both periods. First, notice that the problem of choosing a constant \( g^* \) is the same in each period, hence the same cutoff \( \bar{g} \) is optimal in each period. Then consider a small increase in \( \bar{g} \) by some \( \varepsilon > 0 \) when \( \theta_0 = \theta_H \) and a small decrease in \( \bar{g} \) by \( \zeta(\varepsilon) > 0 \) in period 1 after \( \theta_0 = \theta_H \), such that the incentive compatibility constraint holds for the \( \theta^L \) type.

Let

\[
\Delta v^p(\theta_0, g_0^*) = v^p(\theta_0, \bar{g} + \varepsilon) - v^p(\theta_0, \bar{g}) ;
\]

\[
\Delta v^p(\theta_1, g_1^*) = v^p(\theta_1, \bar{g} - \zeta) - v^p(\theta_1, \bar{g}) .
\]

Then, the binding IC constraint for the \( \theta^L \) type is:

\[
\Delta v^p(\theta^L, g_0^*) + \beta [\Delta v^p(\theta^H, g_1^*) \pi(\theta^H) + \Delta v^p(\theta^L, g_1^*) \pi(\theta^L)] = 0
\]

which can be expanded to

\[
\mathbb{E}_{\gamma} \{ \Delta u^H(\theta^L, \gamma_0, g_0^*) + (R - 1)\gamma_0 \Delta f(\bar{k} + g^c(g_0^*, \gamma_0) + g^b(g_0^*, \gamma_0)) \\
+ \beta [\Delta u^H(\theta^H, \gamma_1, g_1^*) + (R - 1)\gamma_1 \Delta f(\bar{k} + g^c(g_1^*, \gamma_1) + g^b(g_1^*, \gamma_1)) \pi(\theta^H) \\
+ \beta [\Delta u^H(\theta^L, \gamma_1, g_1^*) + (R - 1)\gamma_1 \Delta f(\bar{k} + g^c(g_1^*, \gamma_1) + g^b(g_1^*, \gamma_1)) \pi(\theta^L) \} = 0,
\]

with \( \Delta u^H \) and \( \Delta f \) defined analogous to \( \Delta v \).

The change in the household utility is given by:

\[
\Delta U = \mathbb{E}_{\gamma} \{ \Delta u^H(\theta^H, \gamma_0, g_0^*) \pi(\theta^H) \\
+ \beta [\Delta u^H(\theta^H, \gamma_1, g_1^*) \pi(\theta^H) + \Delta u^H(\theta^L, \gamma_1, g_1^*) \pi(\theta^H)] \pi(\theta^H) \}.
\]
Combining the two expressions above, we obtain

\[
\Delta U = \mathbb{E}_{\gamma}\{\Delta u^H (\theta^H, \gamma_0, g_0^{*H}) \pi(\theta^H) + \beta \pi(\theta^H)[-\Delta u^H (\theta^L, \gamma_0, g_0^{*H}) - \hat{\beta}(R - 1) \gamma_1 \Delta f (\bar{k} + g^c (g_1^{*H}, \gamma_1) + g^i (g_1^{*H}, \gamma_1)) \pi(\theta^L) - \hat{\beta}(R - 1) \gamma_1 \Delta f (\bar{k} + g^c (g_1^{*H}, \gamma_1) + g^i (g_1^{*H}, \gamma_1)) \pi(\theta^H) - (R - 1) \gamma_0 \Delta f (\bar{k} + g^c (g_0^{*H}, \gamma_0) + g^i (g_0^{*H}, \gamma_0))\}\}. 
\]

Notice that \( \Delta u^H (\theta^L, \gamma_0, g_0^{*H}) < 0 \) by the concavity of \( u^H (\theta, \gamma, g^*) \) and \( g_0^{*H} > \tilde{g}^B (\theta^L) \), where \( \tilde{g}^B (\theta^L) \) is the limit preferred by households at \( \theta^L \). By similar arguments, \( \Delta u^H (\theta^H, \gamma, g_0^{*H}) > 0 \), so

\[
\mathbb{E}_{\gamma}\{\Delta u^H (\theta^H, \gamma, g_0^{*H}) \pi(\theta^H) - \beta \pi(\theta^H)\Delta u^H (\theta^L, \gamma, g_0^{*H})\} > 0.
\]

Also, since \( \frac{\partial f(\cdot)}{\partial g^*} > 0 \), we have

\[
\Delta f (\bar{k} + g^c (g_1^{*H}, \gamma_1) + g^i (g_1^{*H}, \gamma_1)) < 0,
\]

and

\[
\Delta f (\bar{k} + g^c (g_0^{*H}, \gamma_0) + g^i (g_0^{*H}, \gamma_0)) > 0.
\]

Therefore,

\[
\mathbb{E}_{\gamma}\{-\hat{\beta}(R - 1) \gamma_1 \Delta f (\bar{k} + g^c (g_1^{*H}, \gamma_1, \theta^L) + g^i (g_1^{*H}, \gamma_1, \theta^L)) \pi(\theta^L) - \hat{\beta}(R - 1) \gamma_1 \Delta f (\bar{k} + g^c (g_1^{*H}, \gamma_1, \theta^H) + g^i (g_1^{*H}, \gamma_1, \theta^H)) \pi(\theta^H) - (R - 1) \gamma_0 \Delta f (\bar{k} + g^c (g_0^{*H}, \gamma_0, \theta^L) + g^i (g_0^{*H}, \gamma_0, \theta^L))\} > 0,
\]

which is a sufficient condition for \( \Delta U > 0 \).

Making the approximation

\[
\Delta f (\bar{k} + g^c (g_0^{*H}, \gamma_0, \theta^L) + g^i (g_0^{*H}, \gamma_0, \theta^L)) \sim -\Delta f (\bar{k} + g^c (g_1^{*H}, \gamma_1, \theta^L) + g^i (g_1^{*H}, \gamma_1, \theta^L))
\]

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reduces the above condition to

\[
\mathbb{E}_\gamma \left\{ -\Delta f \left( \bar{k} + g^c (g_{1H}^*, \gamma_I, \theta^H) + g^i (g_{1H}^*, \gamma_I, \theta^H) \right) 
+ \Delta f \left( \bar{k} + g^c (g_{1H}^*, \gamma_I, \theta^L) + g^i (g_{1H}^*, \gamma_I, \theta^H) \right) \right\} > 0. \tag{2.80}
\]

By Assumption 13, the expression \( \{ -[\Delta v^p (\theta^H, g_{1H}^*) - \Delta v^p (\theta^L, g_{1H}^*)] \} \) is positive and increasing in \( |\theta_H - \theta_L| \), which implies \( \mathbb{E}_\gamma \{ -\Delta f \left( \bar{k} + g^c (g_{1H}^*, \gamma_I, \theta^H) + g^i (g_{1H}^*, \gamma_I, \theta^H) \right) 
+ \Delta f \left( \bar{k} + g^c (g_{1H}^*, \gamma_I, \theta^L) + g^i (g_{1H}^*, \gamma_I, \theta^L) \right) \} > 0 \) and increasing in \( |\theta_H - \theta_L| \). So for a small enough \( \theta_L \) and large enough \( \frac{\pi(\theta^L)}{\pi(\theta^L)} \) such that the inequality from condition (2.80) holds, we have \( \Delta W > 0 \).

2.A.8 Proof of Proposition 30

The proof follows a similar argument as in Thomas and Worrall (1990). According to Lemmas 24 and 25, \( U(EV) \) is concave and differentiable, so the first-order conditions are necessary and sufficient for the maximization of problem of 2.27. Moreover, given Lemma 37, we can simplify the problem by only looking at the IC constraints for \( \hat{\theta} = \theta^{n+1} \) and \( \theta = \theta^n \).

Denote by \( \mu, \pi(\theta) \), \( \pi(\theta) \) and \( \pi(\theta) \phi_{n,n+1} \) the Lagrange multipliers on constraints (2.28), (2.32), (2.33), and (2.29), for \( \theta = \theta^n \) and \( \hat{\theta} = \theta^{n+1} \). Finally, denote by \( U_{EV} \) the derivative of \( U \) with respect to \( EV \). Then, we have:

\[
U_{EV}(EV'(\theta^n)) + \mu + \phi_{n,n+1} - \phi_{n-1,n} \pi(\theta^{n-1}) \pi(\theta^n) \leq 0 \tag{2.81}
\]

\[
\left\{ \mathbb{E}_\gamma \left[ \frac{\partial u^H(\theta^n, \gamma), g^*(\theta^n))}{\partial g^*} \right] + (\mu + \phi_{n,n+1}) \frac{\partial u^p(\theta^n, g^*(\theta^n))}{\partial g^*} 
- \phi_{n-1,n} \pi(\theta^{n-1}) \frac{\partial u^p(\theta^{n-1}, g^*(\theta^n))}{\partial g^*} + \bar{\ell} - \bar{i} \right\} \leq 0
\]

The envelope condition is given by

\[
U_{EV}(EV) \geq -\mu. \tag{2.82}
\]

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Taking expectations in (2.81),

$$\mathbb{E} \left[ U_{EV}(EV'(\theta^n)) \right] + \mu \leq 0.$$ 

Combining with (2.82), we obtain

$$\mathbb{E} \left[ U_{EV}(EV'(\theta)) \right] \leq U_{EV}(EV).$$

This implies that \( \{U_{EV}(EV_t)\}_{t=0}^{\infty} \) is a martingale and by the martingale convergence theorem, it converges almost surely to some random variable \( U_{EV} \). This implies \( \{EV_t\}_{t=0}^{\infty} \) must have a convergent subsequence \( \{EV_{tn}\}_{t=0}^{\infty} \) converging to \( EV_{\infty} \in [EV, \overline{EV}] \).

Consider the case \( EV_{\infty} \in (EV, \overline{EV}) \). Consider the case when state \( \theta^1 \) occurs infinitely often. Take a subsequence formed only of \( \theta^1 \). For the IC constraint (2.29) to hold with type separability, we must have \( EV(\theta^1) > \mathbb{E}[EV] \) for \( EV \in (EV, \overline{EV}) \), and

$$U_{EV}(EV'(1)) < U_{EV}(EV). \quad (2.83)$$

Denote the relationship between successive values of \( EV \) as \( EV_{t+1} = z(EV_t, \theta^1), t = 0, 1, \ldots \) where \( z \) is a continuous function. Then, as \( EV_t \) converges to \( EV_{\infty} \), \( z(EV_t, \theta^1) \) converges to \( z(EV_{\infty}, \theta^1) \) and \( EV_{t+1} \) converges to \( z(EV_{\infty}, \theta^1) \) as well. Therefore, we must have \( U_{EV}(EV_{\infty}) = U_{EV}(z(EV_{\infty}, \theta^1)) \), which contradicts (2.83). Therefore, \( EV_{\infty} \notin (EV, \overline{EV}) \).

Consider now the case where \( EV_{\infty} \in Bd [EV, \overline{EV}] \).

**Lemma 38** For any \( EV_t \in (EV, \overline{EV}) \) and \( \gamma \in (U_{EV}(EV), U_{EV}(\overline{EV})) \), if state \( \theta^N \) is repeated \( M \) times consecutively, then \( U_{EV}(EV_{t+M}) \geq \gamma \), for \( M \) large enough.

**Proof.** Consider a sequence in which state \( \theta^N \) is repeated \( M \) times consecutively. Then given the IC constraint (2.29), type separability implies \( EV_{t+M} < EV_{t+M-1} < \ldots < EV_t \). Then \( U_{EV,t+M} \geq U_{EV,t} \) since \( U \) is concave. Denote by \( \phi \) the value in \( (U_{EV}(EV), U_{EV}(EV)) \) such that \( U_{EV}(\phi) = \gamma \). Suppose there does not exist an \( M \) such that \( U_{EV,t+M} \geq \gamma \). Then
\lim_{M \to \infty} U_{EV,t+M} < \gamma \text{ and } \lim_{M \to \infty} U_{EV,t+M} > \phi. \text{ This implies } EV \text{ converges to a value in } (\phi, EV) \text{ which is a contradiction given the first part of the above proof (it implies } EV(\theta^N) = EV). \text{ So, } \forall \varepsilon > 0, \exists M \text{ subject to } U_{EV}(EV_{t+M}) \geq U_{EV}(EV) + \varepsilon. \blacksquare

Given Lemma 38, starting at any \( EV_t \in (EV, \overline{EV}) \), \( \exists \) a convergent subsequence \( \{EV_n\}_{n=0}^\infty \) that converges to \( EV_\infty = EV \). By a similar argument, \( \overline{EV} \) is also a limit point for some subsequence \( \{EV_n\}_{n=0}^\infty \). So \( EV_\infty \in Bd \, [EV, \overline{EV}] \), and \( \{EV_n\}_{n=0}^\infty \) converges to a distribution with mass points at the boundary of \( [EV, \overline{EV}] \).

### 2.A.9 Proof of Proposition 32

The proof follows the proof of Proposition 3 from Farhi and Werning (2007), in the case with bounded utility. The approach consists of defining a transition function \( Q \) for the derivative \( U_{EV}(EV) \) and constructing a sequence of distributions that converges weakly to an invariant distribution under \( Q \). Let the policy function for \( EV' \) be given by the continuous function \( g^v(EV, \theta) \) which maximizes the Bellman equation in problem (2.27). The derivative \( U_{EV}(EV) \) is continuous and strictly decreasing given the concavity of \( U \). Let \( U_{EV}(EV) = \overline{y} \) and \( U_{EV}(\overline{EV}) = y \), so we define the transition function \( Q(y, \theta) = U_{EV}(g^v((U_{EV})^{-1}(y), \theta)) \) for \( y \in [\underline{y}, \overline{y}] \). For any probability measure \( \mu \), let \( T_Q(\mu) \) be a probability measure defined by

\[
T_Q(\mu)(A) = \int 1_{\{Q(y, \theta) \in A\}} d\mu(y) d\pi(\theta),
\]

for any Borel set \( A \). Also, let

\[
T_{Q,n} = \frac{T_Q + T_Q^2 + \ldots + T_Q^n}{n}.
\]

Then \( T_Q, n(\delta_y) \) is the empirical average of \( \{U_{EV}(EV_t)\}_{t=1}^{n-1} \) over all histories of length \( n \) starting with \( U_{EV}(EV_0) = y \).

We want to show that for any \( y \) there exists a subsequence of distributions \( \{T_{Q, \phi(n)}(\delta_y)\} \) that converges weakly (i.e., in distribution) to an invariant distribution under \( Q \).

Let \( EV_L \) be the minimum of \( g^v \) over \( [EV, \overline{EV}] \). This minimum is attained since \( g^v(EV, \overline{\theta}) \) is continuous and \( \lim_{EV \to \overline{EV}} g^v(EV, \theta) = \overline{EV} \). Since \( g^v > V \), it must be that \( EV_L > EV \). The
transition function is continuous with $Q(y, \theta) \leq U_{EV}(EV_L) < \infty$.

We then need to show that the sequence $\{T_{Q,n}(\delta_y)\}$ is tight in that for any $\epsilon > 0$ there exists a compact set $A$ such that $T_{Q,n}(\delta_y)[A] \geq 1 - \epsilon$ for all $n$. The expected value of the distribution $\{T_{Q,n}(\delta_y)\}$ is $\mathbb{E}_{-1} [U_{EV}(EV_t(\theta^{t-1}))]$. From the first-order conditions, for an interior solution, we have that $\mathbb{E}_{-1} [U_{EV}(EV_t(\theta^{t-1}))] = \left(\frac{\partial}{\partial \theta}\right)^T U_{EV}(EV_0) \to 0$. This implies

$$\min \{U_{EV}(EV_0), 0\} \leq \mathbb{E}_{-1} [U_{EV}(EV_t(\theta^{t-1}))] \leq T_{Q}^{n}(\delta_y) \left( (y, A) \right) (A) + \left\{ 1 - T_{Q}^{n}(\delta_y) \left( (y, A) \right) (A) \right\} U_{EV}(EV_L),$$

for all $A$. This implies

$$T_{Q}^{n}(\delta_y) \left( (y, A) \right) \leq \frac{U_{EV}(EV_L) - \min \{y, 0\}}{A + U_{EV}(EV_L)}.$$

This implies that $\{T_{Q}^{n}(\delta_y)\}$ is tight and therefore $\{T_{Q,n}(\delta_y)\}$ is tight. Tightness implies that there exists a subsequence $\{T_{Q,\phi(n)}(\delta_y)\}$ that converges in distribution to some probability measure $\pi^*$. Then, given the continuity of $Q(y, \theta)$ and linearity of $T_{Q}$, it follows $T_{Q}(\pi^*) = \pi^*$. So $\pi^*$ is an invariant measure under $Q$.

2.A.10 Proof of Proposition 34

The proof is analogous to the proof of Proposition 26.

2.A.11 Proof of Proposition 35

Voters' utility each period is given by

$$u^H(\theta_t, \gamma_t, g^*_t, b^*_t) = \theta_t f(k - x + g^d_t + g^i_t) + (1 - \theta_t - \gamma_t) f(k + g^d_t) + \gamma_t f(k + g^e_t + g^i_t).$$

The utility of households is marginally increasing in all types of transfers, $\{g^d_t, g^e_t, g^i_t\}$. Consider the optimal policy without debt $\{g^0_t\}_{t=0}^{\infty}$ and the corresponding policy choices $\{g^0_t, g^0_t, g^0_t\}$. At this policy, the marginal benefit to voters from increasing $g^0_t$ equals the marginal cost of
decreasing \( g_t^0 \). Then, having the ability to take on debt implies that in any period \( t \) in which debt is taken on, all types of transfers must increase. Otherwise, an increase in \( g_t^0 \) only would make the marginal benefit from increasing \( g_t^0 \) higher than the cost of decreasing \( g_t^0 \). This implies that when debt is taken on, the optimal policy will relax the constraint on the politician's targeted interventions; the constraint will then be tighter when less debt is taken on.

Voters will take on debt if it increases their expected utility, and every period all types of transfers will be affected in the same way by debt. The politician's utility each period is

\[
v(\theta_t, \gamma_t, g_t^*, b_t^*) = u^H(\theta_t, \gamma_t, g_t^*, b_t^*) + (R - 1)\gamma_t f(h_t + g_t^c + g_t^i).
\]

Since \( u^H(\theta_t, \gamma_t, g_t^*, b_t^*) \) is a weighted sum of outputs, per the above discussion, any increase in \( u^H(\theta_t, \gamma_t, g_t^*, b_t^*) \) implies an increase in each component of this sum, it follows that \( v(\theta_t, \gamma_t, g_t^*, b_t^*) \) also increases when \( u^H(\theta_t, \gamma_t, g_t^*, b_t^*) \) increases. The increase in politician utility happens through both the utility from \( g_t^d \) and \( g_t^t \), as well as through the utility from transfers \( g_t^e \). This then implies that the politician will derive a higher expected utility from the contract with voters.

2.A.12 Proof of Corollary 36

Consider the case without debt. Voter utility is given by the maximization problem,

\[
U(EV) = \max_{\{g^*(\theta), EV'(\theta)\}} \mathbb{E}_{\theta, \gamma} \left[ u^H(\theta, \gamma, g^*(\theta)) + \beta U'(EV'(\theta)) \right].
\]

With debt, voters' utility is given by

\[
U(EV) = \max_{\{g^*(\theta), b^*(\theta), EV'(\theta)\}} \mathbb{E}_{\theta, \gamma} \left[ u^h(\theta, \gamma, g^*(\theta), b^*(\theta)) + \beta U'(EV'(\theta)) \right].
\]

The benefit for households of introducing public debt is the change in utility due to the increase in transfers \( g_t^d \), while the cost each period comes from higher \( g_t^c \) and \( g_t^i \), which are more
costly in terms of future utility than their benefit in the current period. Therefore, taking on
debt in the current period is preferred by voters if

$$
\theta f'(k - x + g^d + g^i) \left( \frac{\partial g^d}{\partial g^*} \frac{\partial g^*}{\partial b^*} + \beta - \frac{\partial g^*}{\partial b^*} \right) + (1 - \theta - \gamma) f'(k + g^i) \left( \beta - \frac{\partial g^*}{\partial b^*} \right) \\
+ f'(k + g^c + g^i) \left( (1 - \theta \frac{\partial g^d}{\partial g^*} \frac{\partial g^*}{\partial b^*} + \gamma \beta - \gamma \frac{\partial g^*}{\partial b^*} \right) - E \left[ \frac{\partial U'}{\partial b^*} \right] > 0.
$$

Using the politician's first-order conditions for the choice of $g^d$ versus $g^c$, we get

$$
f'(k + g^c + g^i) \left( \theta \frac{\partial g^d}{\partial g^*} \frac{\partial g^*}{\partial b^*} (R - 1) + R \theta \left( \beta - \frac{\partial g^*}{\partial b^*} \right) + \gamma \beta + (1 - \gamma) \frac{\partial g^*}{\partial b^*} \right) \\
+(1 - \theta - \gamma) f'(k + g^i) \left( \beta - \frac{\partial g^*}{\partial b^*} \right) - E \left[ \frac{\partial U'}{\partial b^*} \right] > 0,
$$
or

$$
f'(k + g^c + g^i) \left( \theta \frac{\partial g^d}{\partial g^*} \frac{\partial g^*}{\partial b^*} (R - 1) + R \theta \left( \beta - \frac{\partial g^*}{\partial b^*} \right) + \gamma \beta + (1 - \gamma) \frac{\partial g^*}{\partial b^*} \right) \\
+ [\lambda - (\theta R + \gamma) f'(k + g^c + g^i)] \left( \beta - \frac{\partial g^*}{\partial b^*} \right) - E \left[ \frac{\partial U'}{\partial b^*} \right] > 0,
$$

where $\lambda$ is the Lagrange multiplier on the politician’s budget constraint. Therefore,

$$
f'(k + g^c + g^i) \left( \theta \frac{\partial g^d}{\partial g^*} (R - 1) - 1 \right) \frac{\partial g^*}{\partial b^*} \\
+ \lambda \left( \beta - \frac{\partial g^*}{\partial b^*} \right) - E \left[ \frac{\partial U'}{\partial b^*} \right] \geq 0.
$$

Consider the effect of increasing $R$ in the above expression:

$$
\frac{\partial}{\partial R} = f''(k + g^c + g^i) \frac{\partial (g^c + g^i)}{\partial R} \left( \theta \frac{\partial g^d}{\partial g^*} (R - 1) - 1 \right) \frac{\partial g^*}{\partial b^*} \\
+ f'(k + g^c + g^i) \left( \theta \frac{\partial g^d}{\partial b^*} \frac{\partial (g^c + g^i)}{\partial R} (R - 1) - \frac{\partial g^*}{\partial b^*} \frac{\partial g^d}{\partial b^*} + \theta \frac{\partial g^d}{\partial b^*} \right) + o(2)
$$

Ignoring second order terms, the above expression is negative if $|f''| > f'$. In this case, a
lower value of $R$ in the current period corresponds to a higher benefit to voters from public
debt. Therefore, given a production function $f(\cdot)$ that satisfies this condition, and a sufficiently high value of $R$, the marginal benefit to voters of taking on debt is negative.

2.A.13 Extension - Adding a Non-Financial Public Good

Assume a public good $y$ can be supplied by the government each period, and it enters the utility of both the voters and the politician through the additive component $o(y)$, independent of $\theta$, where $o(\cdot)$ is increasing and concave.

The politician’s maximization problem (without debt) each period becomes:

$$
\bar{v}(\theta, \gamma, g^*(\theta)) = \max_{\{g^c, g^d, g^i, y\}} [v(\theta, \gamma, g^*(\theta)) + o(y)]
$$

subject to

$$
\gamma g^c + \theta g^d \leq g^*
$$

$$
\gamma g^c + \theta g^d + g^i + y \leq \tau
$$

The government policy ignoring constraint (2.85), satisfies:

$$
v_{g^d} \leq \theta \lambda
$$

$$
v_{g^c} \leq \gamma \lambda
$$

$$
o_y \leq \lambda
$$

$$
v_{g^i} \leq \lambda
$$

Given the underlying functions for $v(\cdot)$, this translates to

$$
g^d = 0,
$$

$$
f'(\bar{k} - x + g^d) = R f'(\bar{k} + g^c),
$$

$$
o'(y) = f'(\bar{k} - x + g^d),
$$

$$
\gamma g^c + \theta g^d + y = \tau.
$$
Denote by \( \delta = \{g^d(\theta), g^c(\theta), y(\theta)\} \) the solution to the above system. Then constraint (2.85) does not bind for \( g^* \geq \gamma g^c + \theta g^d \). The above system implies that \( [\gamma g^c(\theta) + \theta g^d(\theta)] - [\gamma g^c(\theta') + \theta' g^d(\theta')] > 0 \) for \( \theta > \theta' \) (as the equilibrium choice for \( y \) must decrease in \( g^d \)). We are thus in the case in which the preferred policy choice of the politician is different from that of the voters, but the preferred policies of both the politician and voters vary monotonically with \( \theta \). This makes it possible for a limit on targeted transfers in the current period to lead to a separation of types.

**Proposition 39** For each report \( \theta \), there exist a value of the limit on targeted intervention, call it \( g^*_C(\theta) \), such that:

- If the limit imposed by voters, \( g^* \), is low enough such that \( g^* < g^*_C(\theta) \leq \E \left[ \gamma g^c + \theta g^d \right] \), then positive untargeted transfers are provided along with targeted transfers (\( g^i > 0 \)).

- If the limit \( g^* \) is moderately high, so that \( g^*_C(\theta) < g^* < \E \left[ \gamma g^c + \theta g^d \right] \), no untargeted transfers are provided (\( g^i = 0 \)), so that all public intervention is accomplished with targeted transfers.

- If \( g^*_C(\theta) < \E \left[ \gamma g^c + \theta g^d \right] < g^* \), then the politician is unconstrained when shock \( \theta \) is realized, and only targeted transfers are be provided (\( g^i = 0 \)).

**Proof:**

The first-order conditions from the above problem when constraint (2.85) binds lead to the following system of equations for determining \( g^d, g^c, g^i \) and \( y \) as functions of \( g^* \):

\[
\begin{align*}
\frac{v_{g^d}}{\theta} &= \frac{v_{g^c}}{\gamma} \\
g^c &= \frac{g^* - \theta g^d}{\gamma} \\
o_y &= v_{g^i} \\
g^i + y &= \tau - g^*
\end{align*}
\]
which can be expanded to

\[
f'(\overline{k} - x + g^d + g^i) = Rf'\left(\overline{k} + \frac{g^* - \theta g^d + g^i}{\gamma}\right)
\]

\[
\theta f'(-x + g^d + g^i) + (1 - \theta - \gamma) f'(\overline{k} + g^i) + \gamma f'(\overline{k} + g^c + g^i) = \theta'(\tau - g^* - g^i)
\]

\[
y = \tau - g^* - g^i
\]

\[
g^c = \frac{g^* - \theta g^d}{\gamma}
\]

Denote by \(\delta_c = \{g^d, g^c, g^i, y\}\) the internal solution to the above system. A solution that features \(g^d > 0, g^c > 0,\) and \(y > 0\) exists because of the assumptions of the model while \(g^i > 0\) if \(g^*\) is small enough such that condition (2.36) holds. So we can have a region where \(g^i = 0,\) but we don’t have the unconstrained solution. More specifically, let the cutoff imposed by the voters be \(\overline{g}\). We will then have:

- **unconstrained solution:**

  - \(\overline{g} \geq \mathbb{E} [\gamma g^c + \theta g^d] ;\)
  - the cut-off will be \(\overline{g} = g^*(\theta)\).

- **constrained solution with \(g^i = 0\):**

  - the condition for \(g^i\) to be at a corner:

    \[
    \mathbb{E}_\gamma \left[ \theta f'(-x + g^d + g^i) + (1 - \theta - \gamma) f'(\overline{k} + g^i) + \gamma f'(\overline{k} + g^c + g^i) \right] < \lambda \forall g^i \geq 0,
    \]

    where \(\lambda\) is the Lagrange multiplier on the budget constraint. At the margin, \(g^i\) is positive when

    \[
    \mathbb{E}_\gamma \left[ \theta f'(\overline{k} - x + g^d) + (1 - \theta - \gamma) f'(\overline{k}) + \gamma f'(\overline{k} + g^c) \right] = \lambda.
    \]

  - this is equivalent to the following first-order condition being satisfied:

    \[
    \theta'(\tau - \overline{g}) = \mathbb{E}_\gamma \left[ \theta f'(\overline{k} - x + g^d) + (1 - \theta - \gamma) f'(\overline{k}) + \gamma f'\left(\overline{k} + \frac{\overline{g} - \theta g^d}{\gamma}\right) \right]. \tag{2.87}
    \]
the other first-order conditions are

\[ f'(k - x + g^d) = Rf'\left(\frac{\bar{g} - \theta g^d}{\gamma}\right), \tag{2.88} \]

\[ y = \tau - \bar{g}, \tag{2.89} \]

\[ g^c = \frac{\bar{g} - \theta g^d}{\gamma}. \tag{2.90} \]

therefore, condition (2.87) can be re-written as

\[ E_\tau \left[ f'(k - x + g^d) \left( \theta + \frac{\gamma}{R} \right) + (1 - \theta - \gamma) \right] = o' (\tau - \bar{g}), \tag{2.91} \]

where \( g^d \) is derived from the implicit equation (2.88).

from (2.88), as \( \bar{g} \) decreases, \( g^d \) decreases, which means \( f'(k - x + g^d) \) rises; on the other hand, as \( \bar{g} \) decreases, \( \tau - \bar{g} \) increases, which means \( o'(\tau - \bar{g}) \) decreases. Therefore, there exist a value of \( \bar{g} \), call it \( g^*_C \), such that condition (2.91) holds at \( g = g^*_C \). For \( \bar{g} < g^*_C \), condition (2.91) implies we must have \( g^i > 0 \) while for \( \bar{g} > g^*_C \), \( g^i \) is at the corner, \( g^i = 0 \).

- constrained solution with \( g^i > 0 \) : as shown above, for \( \bar{g} < g^*_C \), the solution will feature \( g^i > 0 \).
Chapter 3

Optimal Bailouts under Partially Centralized Bank Supervision

3.1 Introduction

The coordination of government interventions in the banking sector has been at the forefront of policy debates in the aftermath of the 2008 financial crisis. The need for more policy coordination has been motivated by both the increase in cross-border banking in the lead up to the financial crisis and the subsequent cross-border spillovers generated by public interventions. Yet, achieving a better coordination across different jurisdictions poses several challenges. It implies obtaining and piecing together information from different supervision and regulation authorities. Moreover, it requires designing policies that can be delegated to different authorities for implementation.

These challenges are particularly salient when considering public bailouts of banks, where local information collection is crucial for determining the real funding need of financial institutions and the viability of their investments. Therefore, a central policy question ahead of any coordination effort becomes to what extent policy decisions are better centralized, even at the cost of some information being lost, and, conversely, when is it preferable to give some autonomy to local authorities. This paper builds a model to shed light on this question. It aims to determine the conditions under which full autonomy of local authorities is optimal, and
what role do government finances, particularly public debt, have in determining whether local autonomy is optimal.

The importance and relevance of these questions is brought forward in the case of the European Banking Union. Following the financial crisis, the supervision and regulation of Eurozone banks was reformed in order to achieve two main objectives.\(^1\) First, common supervision and regulation by a central authority aimed to eliminate local authority biases towards local banks, as well as to allow for a better supervision of cross-border banks. Second, it was aimed at breaking the link between banks and sovereigns, the situation in which the cost of bailouts forced governments to take on debt, which increased their borrowing costs and reduced their ability to attract more funds. Yet, the supervision and regulation of banks was not fully entrusted to a central authority. Under the current system, the central authority, the Single Supervisory Mechanism, covers only the large, cross-border banks operating in the Eurozone, or about 130 banks, which hold about 85% of the total banking assets.\(^2\) The rest of the banks are supervised solely by local authorities. Therefore, under this system, information about the state of large banks is directly available to the central authority, while information on the smaller banks is only available to local authorities. Moreover, the current institutional reforms cover regulation and supervision, but the framework for intervention following a crisis has not yet been completed. This framework would have to include provisions regarding the autonomy of local authorities to engage in bailouts of banks. More importantly, it would need to establish when a local authority can access a common bailout fund rather than use its own funds for bailouts. A local authority could be given autonomy to make this decision based on the local information it has access to. Alternatively, it could be required to supply a certain level of funding to banks before any common bailout funds are used. This restriction would be preferable to the central authority – the European Central Bank in this context – if it were faced with a local authority that is biased towards its local banks. Such a local authority would allocate insufficient funds to the large, cross-border banks, passing the responsibility of funding them up to the common bailout fund. This particular issue has been pointed to as one of the main roadblocks in the

\(^1\)The European Central Bank's (ECB) perspective regarding the goals of these reforms is spelled out in the following document made available by the ECB: http://www.ecb.europa.eu/press/key/date/2013/html/sp130405.en.html, accessed March 31, 2014.

creation of a common bailout fund for the Eurozone.\textsuperscript{3} Therefore, examining the conditions under which autonomy for local authorities is optimal from the perspective of a central authority is a key first step in determining how much coordination can be achieved over bank bailouts.

This paper focuses on the issue of ex-post intervention, after a recession has occurred. The model considers the case of a central authority that can observe an aggregate shock – whether the economy is in a recession or a boom – but it cannot observe local shocks that affect local investments only. A local authority has the ability to observe all shocks and to intervene in the banking sector through bailouts. In case of a recession, the central authority can decide how much funding a local authority must provide to the large, cross-border banks, and how big the overall bailout budget can be. By restricting the overall budget, the central authority can essentially place an upper limit on how much a local authority can borrow in order to finance bailouts during a crisis.

The objective of the model presented in this paper is to examine the optimal ex-post policy from the perspective of the central authority that aims to maximize the total output in the economy. The local authority can observe the shock that hits local projects, and aims to maximize the total output that benefits households in its jurisdiction. Since the global banks are cross-border, some of their output goes to households in other jurisdictions. The local authority therefore values the output of global banks differently than the central authority, that values the entire value of the output funded by global banks. The model then features two main frictions: private information about the local shocks, and a bias of the local authority towards local banks. With these two frictions, the resulting problem each period has the structure of the model presented in Amador et al. (2006). The model of Amador et al. (2006) delivers the result that minimum intervention rules are optimal for a wide range of shock distributions. This would suggest that, if this model had the same structure, the optimal solution would call for setting a minimal contribution before the country can access any common funds. This would be equivalent to giving the local authority full autonomy, provided it used a minimal amount of funds for bailouts of cross-border banks. Yet, the present model is complicated by the fact

\textsuperscript{3}See, for example, the following interview with Sharon Bowles, Chair of the European Parliament’s Economic and Monetary Affairs committee:
that the local authority can take on debt each period, and the friction studied in the Amador et al. (2006) is repeated every period. The existence of debt changes the size of the total funds available for bailouts and modifies the conditions under which autonomy is optimal.

The first result of the paper is that the optimal policy features no autonomy for the local authority if the local shocks are high, so that the benefit from directing bailout funds to local banks is high. This happens because a high need for funds by local banks augments the conflict of interests between the local and the central authorities. The local authority has a strong incentive to direct more bailout funds to local banks. It then becomes optimal for the central authority to fully constraint the actions of the local authority, even if it means bearing the cost of not adjusting policy to local shocks.

The next result derives conditions under which it is optimal for the central authority to give full autonomy to the local authority. Full autonomy is optimal when there is a low conflict of interests between the two authorities. Moreover, the necessary conditions place additional restrictions on the nature of the conflict of interests, namely on how the objectives of the two authorities can differ. The implication of this result is that the goal of delinking sovereigns from banks also comes with the effect of making ex-post coordination over bailouts necessarily more constraining. As sovereigns have more freedom to borrow, the conflict of interests between them and the central authority increases, since the sovereigns can more easily assert their preferences over bailouts. Therefore, the freedom to borrow more easily comes at the cost of less autonomy to intervene in a crisis.

The paper also considers the effect of debt on the information rents extracted by the local authority. Having more outstanding public debt is shown to have no effect on the range of local shocks over which the local authority cannot extract any information rents. This happens because higher debt increases both the cost and benefit of giving autonomy to the local authority, such that the effects cancel out. Therefore, the range of local shock values over which information rents are derived does not change. The result changes, however, if the local authority can become borrowing constrained. In this case, binding borrowing constraints lead to a loss in local autonomy. This result has implications for the debate over the role of fiscal coordination or binding fiscal rules. It suggests that imposing such binding rules can lead to a
loss of local autonomy on other dimensions than borrowing – in this case the ability to tailor bailout policies to local shocks.

Finally, the paper shows that these implications hold both in a two-period version of the model and in a multi-period version. The results are illustrated using examples of two specific functional forms for the investment technology. The examples highlight the limits of full autonomy when public debt is available. In one case, full autonomy is never optimal given the functional form of the project technology, while in the other, it is always optimal. These two cases show how the degree of autonomy that can be given to the local authority depends on how large a conflict of interests there is between the two authorities. The larger the conflict of interests, the less autonomy is given to the local authority. The size of the conflict of interests depends on both the bias of the local authority and on the actual investment technology.

**Related literature.** The analysis of optimal policy in the context of indirect control has been the focus of a large theoretical literature (Hölmstrom, 1979; Athey et al., 2005; Amador et al., 2006; Alonso and Matouschek, 2008; Ambrus and Egorov, 2009; Amador and Bagwell, 2013; Frankel, 2014). The problem studied is that of a principal designing decision rules for an agent that is informed with regard to a private shock and is biased in his preferences. Papers in this literature derive conditions under which it is optimal to design decision rules that constraint the agent’s choices to an interval, or that set floors or ceilings on certain policies. In this paper, a similar problem is studied, in which a principal must design rules for an informed agent, but the focus is on the case in which the principal has direct control over a subset of the agent’s decisions, while it lacks control over others. Moreover, it studies a repeated interaction between the principal and the agent, and examines the role of debt in placing restrictions on the optimal contract that the principal can offer the agent. The model extends the setup described in Amador et al. (2006), and it uses the Lagrangian methods developed in this paper in order to establish the conditions under which full discretion for the agent is optimal. Amador et al. (2006) study the optimal delegation problem in a consumption-savings model, in which individuals expect taste shocks and suffer from temptation for higher present consumption. Their model is used in this paper to capture a local agent’s bias towards local projects and his private observation of local shocks. Unlike their model, the private information versus
temptation trade-off in this paper happens within each period, and the model is then repeated with this trade-off recurring each period.

The paper contributes to the literature on indirect control and partial decentralization. Miquel and Yared (2012) consider the issue of indirect control applied to the issue of governance in weakly institutionalized societies, in which the government delegates to local agents the authority to control insurgencies. They also develop a model of repeated moral hazard, but in which the principal has the ability to accomplish the same task as the agent through a different action. Our paper is motivated by a different type of interaction between structures of government, in which the principal does not have additional actions available. The focus is instead on the effects of public debt on the optimal policy. Another paper that builds on the Amador et al. (2006) framework is Halac and Yared (2012), which looks at the optimal policy in the case of persistent shocks. They also extend the Amador et al. (2006) model to a repeated game, and consider the case in which the agent has private information over persistent rather than i.i.d. shocks. Their focus is therefore different than that of this paper, since they do not consider the intra-period disagreement between principal and agent. This disagreement within each period, repeated over time, is the driving friction of the present model, and the inter-period links are achieved through public debt, without persistence in shocks. The issue of indirect control has also been studied in the case of policy design in unions. Chari and Kehoe (2008) study the optimality of rules for non-monetary policies in a union, when a monetary authority can intervene to set rules on what the non-monetary authorities can do. The frictions in their model are given by free riding and time inconsistency, while the focus in this paper is on private information about local shocks.

This paper is also related to the literature on partial decentralization and the division of supervision, regulation and intervention between the different levels of government. The interaction between different levels of government has been studied in the context of both subnational divisions of government (Nicolini et al., 2002; Tommasi et al., 2001), and unions of countries interacting with a supranational authority (Eichengreen and Von Hagen, 1995; Persson and Tabellini, 1993; Wildasin, 1990). Yet, many of the papers in this literature have

\(^{4}\)Ahmad (2006) provides a review of this literature.
focused on the adoption of policies in decentralized systems based voter preferences or on the aggregation of voter preferences in federal systems (Persson and Tabellini, 1996a,b; Dixit and Londregan, 1998; Besley and Coate, 2003; Hatfield and Padró i Miquel, 2012). Sanguinetti and Tommasi (2004b) consider a principal-agent model to study the trade-off between risk-sharing and incentives against local overspending. Their model features local authorities with private information about local shocks and preferences over local spending. Their focus, however, on the case in which a principal can make transfers to multiple agents, and the budget constraints of the agents are interrelated. They do not consider a multiple-period game, while this paper focuses on the role of debt and the repeated disagreement between principal and agent.

Finally, both the existence of biases for local versus federal regulators, and the link between banks and their respective governments have been examined in the empirical literature, and the findings of this literature motivate the setup and assumptions of this paper. Using data from the US state chartered commercial banks, Agarwal et al. (2012b) show that federal regulators are significantly less lenient than state regulators that alternately supervise the same institutions, which suggests the existence of a bias by local regulators towards local banks. Barth et al. (2004) use a database of bank regulation and supervision practices in 107 countries to examine the relationship between these policies and banking-sector development, efficiency, and fragility. They find that policies that force accurate information disclosure promote bank stability, which suggests that local private information could be an important concern when thinking about centralizing regulatory policies. Finally, recent evidence looks at the link between banks and their governments, and the effects of expected government support in case of a crisis. Correa et al. (2014) look at the joint effect of expected government support to banks and changes in sovereign credit ratings on bank stock returns using data for banks in 37 countries between 1995 and 2011. They find that sovereign credit rating downgrades have a large negative effect on bank stock returns for those banks that are expected to receive stronger support from their governments. A similar interdependence between banks and their governments is found by Demirgüç-Kunt and Huizinga (2013) when looking at the impact of government indebtedness and deficits on bank stock prices and credit default swap spreads. Therefore, evidence suggests that banks expect and value support by their governments.
The rest of the paper is organized as follows. Section 3.2 outlines the model. Section 3.3 provides the analysis of the model and the conditions under which local autonomy over bailouts is optimal. It also discusses the comparative statics with respect to public debt. Section 3.4 extends the model to an infinite horizon. Section 3.5 illustrates the results of the model with examples of specific functional forms. Section 3.6 concludes and discusses future research avenues.

3.2 Model

3.2.1 Environment

The economy consists of a continuum of households and two banks which differ by the type of investment projects they can finance - a local bank, which finances small, local projects, and a global bank, which finances large projects. Banks are supervised by either one or two types of public authorities - a local authority and a central authority. The central authority supervises a subset of the banks, as described below, while the local authority can supervise all banks and can provide public bailout funds to support distressed banks.

Banks and investment projects. The two types of banks, local versus global, differ in their ability to evaluate and finance projects. The technology for evaluating and financing each type of investment project is specific and exclusive to each type of bank. The local bank can only finance the local projects, denoted by \( L \), while the global bank can only finance the large projects, denoted by \( G \).

Investment projects are owned by households. Both types of projects are risky investment projects, in that their return depends on whether the economy is in a boom or a recession. The state of the economy is observed at the beginning of each period. A recession happens with probability \( \alpha \) and a boom with probability \((1 - \alpha)\). In a bloom, a large project produces

\[
 w^G(i^G + z) \geq 0, \quad \text{where} \quad z > 0, \quad i^G > 0 \quad \text{and} \quad w^G : \mathbb{R}^+ \rightarrow \mathbb{R} \text{ is a concave, increasing, continuously differentiable function with } \lim_{i \rightarrow 0} w^G(i) > 0, \lim_{i \rightarrow \infty} w^G(i) \leq 0 \quad \text{and} \quad w^G(i^G + z) < 1. \]

In a recession, the large projects become distressed and produce a return

\[
 w^G(i^G - z), \quad \text{where} \quad w^G(i^G - z) > 1. \]
The local projects are also affected by the aggregate state of the economy, but they can also be affected by local shocks. In a boom, a local project produces a return $u^L (i^L + z) \geq 0$, where $i^L > 0$ and the local technology $u^L : \mathbb{R}^+ \rightarrow \mathbb{R}$ is a concave, increasing, continuously differentiable function with $\lim_{i \rightarrow 0} u^L(i) > 0$, $\lim_{i \rightarrow \infty} u^L(i) \leq 0$, and $u^L(i^L + z) < 1$; in a recession, a fraction $\theta \in \Theta \subset (0, 1)$ of local projects become distressed and produce $u^L (i^L - z) > 0$, where $\theta u^L (i^L - z) > 1$, while a fraction $(1 - \theta)$ become fully compromised and produce $0$. The value of shock $\theta$ is observed by the local bank and by the local authority, as discussed below; however, which projects are compromised versus which projects are just distressed is not observable until the projects pay off. This specification allows for different types of projects to produce returns using potentially different technologies, and it allows for local projects to be affected by local shocks as well as aggregate shocks. Moreover, the model captures the fact that banks and governments can observe whether an asset needs liquidity support, but they cannot as readily evaluate the solvability of this asset.

Before the projects pay off, a reinvestment can be made in projects, which increases the size of the project, i.e., $i^l, J = L, G$. A reinvestment $y > 0$ in the global bank during a recession leads to total return $w^G (i^G - z + y)$ from its projects. A reinvestment $x > 0$ in the local bank in a recession returns $u^L (i^L - z + x)$ if the project is distressed and $0$ if the project is compromised. Since all local projects in the bank's portfolio are identical before they pay off, the reinvestment funds cannot be targeted specifically towards distressed projects.

Households. Each household holds an identical, perfectly diversified portfolio consisting of both local and large projects. Local projects are fully owned by households, while large projects are owned in proportion of $\delta \in (0, 1)$ by households and $(1 - \delta)$ by the central authority. The ownership by the central authority is a reduced-form representation of foreign ownership in global banks. Moreover, it is meant to capture the potentially different weight carried by global financial institutions in the objective function of local authorities versus global authorities. The latter may take into account the global effects of large banks, while local authorities are primarily concerned with the returns to local households and the effects on the local economy.

The households are risk-neutral, discount the future at rate $\beta$ and derive utility from consuming the project returns. Their expected per-period utility at the beginning of each period
is therefore given by

\[(1 - \alpha) \left[ u^L (i^L + z) + \delta w^G (i^G + z) \right] + \alpha \left[ \theta u^L (i^L - z + x) + \delta w^G (i^G - z + y) \right], \tag{3.1} \]

where \(x\) and \(y\) are the reinvestments made by banks in case of a recession. This expression incorporates the fact that, in case of a recession, \((1 - \theta)\) of local projects are compromised and return 0.

**Central and local authorities.** Banks are supervised by two types of public authorities. First, a central authority supervises global banks. The central authority can observe if the economy is in a recession, and it can observe any reinvestment \(y\) that is made by the global banks. It cannot, however, observe the local shock \(\theta\). It only knows the distribution of \(\theta, f(\theta)\). The central authority has the goal of maximizing the output of both local and global banks, taking into account the utility of households as well as its own utility from the global banks’ returns. Therefore, its per-period utility is given by

\[\mathbb{E} \{(1 - \alpha) \left[ u^L (i^L + z) + \delta w^G (i^G + z) \right] + \alpha \left[ \theta u^L (i^L - z + x) + \delta w^G (i^G - z + y) \right]\}.\]

After observing the state of the world in the current period, if the economy is in a recession, then its per-period utility is given by

\[\mathbb{E} \left[ \theta u^L (i^L - z + x) + \delta w^G (i^G - z + y) \right].\]

Second, a local authority can observe both the state of the world and the value of the local shock \(\theta\). Each period, the local authority receives a budget \(e\), which it can use to make transfers to banks or it can redistribute to households as consumption goods. The transfers are used for reinvestment by banks in their respective portfolios. In a recession, the local authority makes a transfer \(x\) towards the local bank and a transfer \(y\) towards the global bank. The transfers towards the global bank can be constrained by the central authority, as detailed below. The local authority can also increase the size of its available budget by borrowing against future income. It can take on short-term (one period) non-contingent debt \(b\) at rate \(\beta\). Therefore, the
budget constraint faced by the local authority is

\[ x + y \leq c + \beta b - b_{-1}, \]

where \( b_{-1} \) denotes any outstanding debt. The local authority seeks to maximize the utility of local households. Therefore, its per-period utility is given by (3.1).

The central authority can decide the value of \( y \), the transfer that must be made to global banks. This represents the contribution by local governments or national authorities to the bailout of global institutions that are under the supervision of the central authority. The model does not consider transfers from the central authority to the banks, but rather considers the problem for the central authority of deciding the funding \( y \) that it can constraint the local authority to make towards supporting global banks. It must decide the degree of autonomy that the local authority can enjoy in deciding bailouts of global banks. Since the central authority has complete information regarding the funding needs of the global banks, it can determine any additional contribution from the central authority’s funds after the local authority has made its contribution. Given the full information, this decision would become a straightforward problem, assuming the central authority has commitment power with respect to this contribution.

The mechanism described above would also be consistent with a local authority first engaging in a required bailout of the global banks and only afterwards accessing a common bailout fund to supplement any additional funding needs. Such a system for accessing common bailout funds has been proposed as one of pillars of the European Banking Union. Yet, concerns over the incentives problem of local authorities, which would contribute too little out of their own funds, have limited the scope of any such common bailout fund. Therefore, as argued in the introduction, focusing on the optimal degree of autonomy that can be given to a local authority is a key first step in understanding the limits of any common bailout policies among several constituencies.

The model considers the case in which the central authority can decide the size of the bailout fund that each local authority can operate with. This is equivalent to the central authority deciding public debt. A heavily indebted sovereign facing conditionality from outside
lenders would fit this scenario. At the other end of the spectrum, a sovereign could be able to borrow on the outside market, and therefore can easily evade any informal restrictions on how much to support local banks. In this case, the central authority must provide stronger incentives to mitigate the local authority's preference for support of local banks at the expense of global banks, and the regions where autonomy is optimal would be a subset of the case in which debt can be decided by the central authority.

3.2.2 Timing

The paper starts by studying the above problem in a two-period model, with \( t = 0, 1 \). It assumes that debt can be taken on by the local authority, but it is observed and can be limited by the central authority. The analysis considers the case in which the central authority must select an optimal mechanism to solve the ex-post intervention problem, once the economy is in a recession. The sequence of events is as follows:

1. Local shock \( \theta_0 \in \Theta \) is realized and observed by the local authority;
2. The central authority chooses a schedule \( \{y(\theta_0), b(\theta_0)\}_{\theta_0 \in \Theta} \);
3. The local authority reports a shock \( \hat{\theta}_0 \in \Theta \) to the central authority and it chooses policies \( \{x(\hat{\theta}_0), y(\hat{\theta}_0), b(\hat{\theta}_0)\} \);
4. Period 0 returns are realized;
5. Nature chooses the state of the economy in period 1;
6. If there is a boom, local households consume \( (e - b) \);
7. If there is a recession, local shock \( \theta_1 \in \Theta \) is realized and observed by the local authority;
8. The central authority chooses a schedule \( \{y(\theta_1)\}_{\theta_1 \in \Theta} \);
9. The local authority reports a shock \( \hat{\theta}_1 \in \Theta \) to the central authority and it chooses policies \( \{x(\hat{\theta}_1), y(\hat{\theta}_1), b(\hat{\theta}_1)\} \);
10. Period 1 returns are realized.
The model will then be extended to an infinite horizon in which the events from period 0 are repeated every period, provided that both agents lack commitment power.

### 3.3 Analysis Of The Two Period Model

#### 3.3.1 Two Types Of Local Shocks

Consider the case in which the local shock $\theta$ can only take two values, $\theta^H$ and $\theta^L$, with probabilities $\pi^H$ and $\pi^L$, respectively. To simplify notation, define

\[
\begin{align*}
    u(x) &amp;= u^L (i^L - z + x); \\
    w(y) &amp;= w^G (i^G - z + y).
\end{align*}
\]

In period 0, in a recession, the local authority’s utility is

\[
\theta_0 u(x_0) + \delta w(y_0) + \beta \{(1 - \alpha)(e - b) + \alpha V\}, \tag{3.2}
\]

where $V$ denotes the utility of the local authority in period 1, in case of a recession,

\[
V \equiv \pi^H [\theta^H u(x_1^H) + \delta w(y_1^H)] + \pi^L [\theta^L u(x_1^L) + \delta w(y_1^L)]. \tag{3.3}
\]

The utility of the local authority reflects the non-contingent nature of public debt, which must be paid off in period 1 regardless of whether the economy is in a boom or a recession.

The local authority is maximizing (3.2) subject to the budget constraints

\[
\begin{align*}
    x_0 + y_0 &amp;\leq e + \beta b, \tag{3.4} \\
    x_1^j + y_1^j &amp;\leq e - b, \tag{3.5}
\end{align*}
\]

$j = G, L$, and given the schedule offered by the central authority following the reports of shocks $\theta_0$ and $\theta_1$. 

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The central authority must choose a schedule \( \{y(\theta_0), b(\theta_0)\}_{\theta_0 \in \Theta} \) to be implemented given the report from the local authority about the value of the local shock \( \theta_0 \). By the Revelation Principle, we can focus on the mechanisms with truthful revelation. The central authority’s problem becomes

\[
\max_{\{x_0, y_0, y_1, b\}} \mathbb{E}[\theta_0 u(x_0(\theta_0)) + w(y_0(\theta_0)) + \beta (1 - \alpha) (e - b(\theta_0)) + \alpha U(b(\theta_0))],
\]

subject to

\[
\begin{align*}
\theta_0 u(x_0(\theta_0)) + \delta w(y_0(\theta_0)) &+ \beta [(1 - \alpha) (e - b(\theta_0)) + \alpha V(b(\theta_0))] \\
&\geq \theta_0 u(x_0(\theta'_0)) + \delta w(y_0(\theta'_0)) + \beta [(1 - \alpha) (e - b(\theta'_0)) + \alpha V(b(\theta_0))], \quad \forall \theta'_0 \in \Theta, (3.7)
\end{align*}
\]

\[
\theta_1 u(x_1(\theta_1)) + \delta w(y_1(\theta_1)) \geq \theta_1 u(x_1(\theta'_1)) + \delta w(y_1(\theta'_1)), \quad \forall \theta'_1 \in \Theta, (3.8)
\]

\[
x_0(\theta_0) + y_0(\theta_0) \leq e + \beta b(\theta_0), (3.9)
\]

\[
x_1(\theta_1) + y_1(\theta_1) \leq e - b(\theta_0), (3.10)
\]

where \( U(b(\theta_0)) \) denotes the utility of the central authority in period 1, in case of a recession,

\[
U(b(\theta_0)) \equiv \pi^H \left[ \theta^H u(x_1^H) + w(y_1^H) \right] + \pi^L \left[ \theta^L u(x_1^L) + w(y_1^L) \right].
\]

Constraint (3.7) is the incentive compatibility constraint for the local authority in period 0. The local authority must be given incentives to truthfully report the local shock \( \theta_0 \) that it observes in period 0. Constraint (3.8) is the incentive compatibility constraint for the local authority in period 1, which ensures that the local authority will truthfully report the local shock, if period 1 is a recession as well. Constraint (3.9) is the budget constraint that must be satisfied in period 0, given the local authority’s resources. Finally, constraint (3.10) is the budget constraint in period 1 in case of a recession.

The central authority’s problem illustrates the two sources of tension between the central and local authorities. Each period, the local authority derives less utility from the returns of the global bank than does the central authority. Moreover, the local authority has private
information on the value of the local shock $\theta$. The local authority then has the incentive to overreport the value of shock $\theta$ in order to increase the support that it is allowed to provide to the local bank relative to the global bank. The central authority’s problem is then a problem of balancing the need to provide reinvestment funds to the global bank with the need to provide the local authority with sufficient incentives for truthful reporting of local shocks. This reflects one concern regarding the creating of a common bailout fund that can be accessed by several local authorities, namely that it would create an incentives problem for local authorities, which are likely to value local banks more than global, cross-border institutions. This would give rise to pressures for directing bailout funds primarily towards local banks, leaving global banks more dependent on outside intervention through the common bailout fund.

In analyzing the problem for the central authority, notice first that in the absence of debt, problem (3.6) is a repeated version of the problem described in Amador et al. (2006). Without debt, the central authority is faced with a repeated problem of offering incentives for the local authority. In this case, as discussed in Amador et al. (2006), the optimal solution for the central authority each period is to offer the same schedule $y(\theta^H) = y(\theta^L) = y^*$ if $\delta \leq \delta^*$, and to offer a schedule with $y(\theta^H) < y(\theta^L)$ if $\delta > \delta^*$, with $\delta^* = \theta^L/\theta^H$. When considering the problem with public debt, the incentives problem in period 0 is generated by both a bias of the local authority towards more funding for the local bank in period 0, and a bias towards taking on more public debt than the level preferred by the central authority. The following proposition describes the optimal policy coming out of this problem.

**Proposition 40** There exists a value $\hat{\delta} \geq \delta$ such that the optimal policy for the central authority has the following form:

- if $\theta^L/\theta^H \geq \hat{\delta}$, the same reinvestment funds are mandated for the global bank regardless of the realization of the local shock, i.e., $y_0(\theta^H) = y_0(\theta^L)$ and $y_1(\theta^H) = y_1(\theta^L)$;
- if $\delta \leq \theta^L/\theta^H < \hat{\delta}$, fewer reinvestment funds are mandated for the global bank in period 0 following a high local shock, i.e., $y_0(\theta^H) < y_0(\theta^L)$; in period 1, the same reinvestment funds are mandated for global bank regardless of the realization of the local shock, i.e., $y_1(\theta^H) = y_1(\theta^L)$;
\* if $\theta_L / \theta_H < \delta$, fewer reinvestment funds are mandated for the global bank after a high local shock than after a low local shock, i.e., $y_0(\theta_H) < y_0(\theta_L)$ and $y_1(\theta_H) < y_1(\theta_L)$.

Proof. In the Appendix. □

Proposition 40 states that the optimal policy that maximizes the central authority’s expected utility is to impose the same allocation of funds when there is relatively low variation between the local shocks. In this case, the difference in preferences between the central and the local authorities is more significant than the difference in shocks, and the central authority finds it preferable to ensure that enough reinvestment funds are provided to the global bank even if policy is not perfectly adapted to the local shocks. Since the difference between the high and the low local shocks is relatively small, the cost of offering information rents is higher than the gain of offering more funding to the local bank when the shock $\theta$ is high. When there is relatively high variation between the local shocks, then the central authority benefits from having more reinvestment funds go to the local bank when the $\theta$ is high. In this case, offering information rents to the local authority is optimal.

The high ratio $\theta_L / \theta_H$ specified in Proposition 40 means local conditions to not vary much in a crisis. Once the central authority observes a crisis, it has an expectation that, at the local level, a fraction $E[\theta]$ of projects are viable, and it knows that the actual realization of the local shock is not far from that mean. Therefore, the relative benefit of finding out the actual value of the shock is low. The intuition is that if a banking crisis has similar effects for all types of banks, then authorities will find it cost-effective to implement the same standard bailout policy for all banks rather than pay the cost of tailoring policy to each type of bank.

The other result that is highlighted in Proposition 40 is that the cut-off value for the relative size of local shocks is lower than in the case without debt. When the local authority has access to public debt, the two dimensions of disagreement with the central authority – reinvestment in the local bank and public debt – increase the benefit to the central authority from choosing a schedule that depends on the value of local shocks. First, it gains in terms of current reinvestment in local projects when these projects are distressed, hence still productive. Second, it is able to spread the cost of offering information rents to the local authority by way
of the public debt. When public debt is no longer available, as is the case in period 1, the cost of offering information rents to the local authority is higher, which makes it less desirable to offer reinvestment schedules tied to local shock reports, if the variation in local shocks is not sufficiently large.

The driving force for the result in Proposition 40 is that public debt offers the central authority more flexibility in devising a schedule of reinvestments that is acceptable for the local authority. The central authority can compensate the local authority for providing more reinvestment funds to global banks when the local shock is high by allowing more public debt to help finance current period expenditures. At the same time, having to repay the debt in the next period gives the local authority the incentive to not overreport the shock value, because such a report comes at higher price in terms of future utility. Therefore, the existence of public debt expands the set of reinvestment schedules that can be offered to the local authority. The intuition is that a country that can borrow freely can more easily fund bailouts in periods of crisis. Requiring the country to spend more to support global banks does not impose as high a burden on the country as it does for a country that cannot borrow. Such a constrained country must sacrifice funds for local banks in order to comply with the requirement to contribute to the bailout of global banks. Therefore, compliance is more difficult to achieve for a country that cannot borrow, and the central authorities prefer to impose standard policies. These policies remove a country’s ability to supply inaccurate information about the state of its economy, in order to avoid compliance.

3.3.2 Continuous Distribution of Local Shocks

Problem (3.6) is now analyzed in the more general case when the distribution of local shocks $\theta$ has a continuous density $f(\theta)$ over the interval $\Theta \equiv [\underline{\theta}, \overline{\theta}] \subset [0,1]$. This allows us to move away from the simple two-type case, and show that the above result carries over to this general case. Moreover, the case with continuous case allows us to analyze the conditions under which full autonomy is optimal, conditions that could not be satisfied in the two-shock case.

Given a schedule $\{y(\theta_0), b(\theta_0)\}_{\theta_0 \in \Theta}$ in period 0, the local authority chooses a report $\tilde{\theta}_0$ and
a level of bailout funds for the local bank, $x_0(\hat{\theta}_0)$, to maximize its expected utility,

$$\max_{\theta_0,x_0(\theta_0)} \theta_0 u(x_0(\hat{\theta}_0)) + \delta w(y_0(\hat{\theta}_0)) + \beta \left\{ (1 - \alpha) \left( e - b(\hat{\theta}_0) \right) + \alpha V(b(\hat{\theta}_0)) \right\}, \quad (3.11)$$

subject to the budget constraint

$$x_0(\hat{\theta}_0) \leq e + \beta b(\hat{\theta}_0) - y_0(\hat{\theta}_0), \quad \forall \hat{\theta}_0 \in \Theta.$$  

Similarly, in period 1, in case of a recession, the local authority chooses a report $\hat{\theta}_1$ and local bailout $x_1(\hat{\theta}_1)$ to maximize

$$\max_{\theta_1,x_1(\theta_1)} \theta_1 u(x_1(\hat{\theta}_1)) + \delta w(y_1(\hat{\theta}_1)) \quad (3.12)$$

subject to the budget constraint

$$x_1(\hat{\theta}_1) \leq e - b(\hat{\theta}_1) - y_1(\hat{\theta}_1), \quad \forall \hat{\theta}_1 \in \Theta. \quad (3.13)$$

If the mechanism induces truth telling, then the local authority’s utility is $\theta_1 u(x_1(\theta_1)) + \delta w(y_1(\theta_1))$ and the incentive compatibility constraint in period 1 can be written as\(^5\)

$$\theta_1 u(x_1(\theta_1)) + \delta w(y_1(\theta_1)) = \int_\theta u(x_1(\theta)) d\theta + \theta u(x_1(\theta)) + \delta w(y_1(\theta)). \quad (3.14)$$

Assuming truthful reporting in period 1, the incentive compatibility constraint in period 0 for the local authority is then given by

$$\theta_0 u(x_0(\theta_0)) + \delta w(y_0(\theta_0)) + \beta \bar{V}(\theta_0) = \int_{\theta_0} u(x_0(\theta)) d\theta + \theta u(x_0(\theta)) + \delta w(y_0(\theta)) + \beta \bar{V}(\theta), \quad (3.15)$$

\(^5\)By integrating the Envelope Condition, as detailed in Milgrom and Segal (2002).
where

\[ \bar{V}(\theta_0) = (1 - \alpha) (e - b(\theta_0)) + \alpha V^* (b(\theta_0)) \]  

(3.16)

represents the expected utility for the local authority in period 1, given truth telling and the implementation of the optimal policy in that period. \( V^* (b(\theta_0)) \) is the value of the local authority’s utility in period 1, if the optimal policy is implemented in that period by the central authority. This implies \( V^* (b(\theta_0)) = \theta_1 u(x^*_1(\theta_1)) + \delta w(y^*_1(\theta_1)) \), and

\[ (x^*_1(\theta_1), y^*_1(\theta_1)) = \arg \max_{x_1(\theta_1), y_1(\theta_1)} \int \theta_1 u(x_1(\theta_1)) + w(y_1(\theta_1)) f(\theta_1) d\theta_1, \]

subject to constraint (3.13).

The problem of determining \((x^*_1(\theta_1), y^*_1(\theta_1))\) for a given level of outstanding debt \(b\) is the same problem as the one studied in Amador et al. (2006), with the endowment being \((e - b)\). The expected continuation utility for the local authority, \( \bar{V}(\theta) \) is a continuous and decreasing function of \(b(\theta)\), since it is a linear combination of \(V^* (b(\theta))\) and \(b(\theta)\). We follow the same procedure as in Amador et al. (2006) in order to incorporate the incentive compatibility constraint in the objective and to reduce the number of choice variables in the problem. Specifically, let \(X_t(u), Y_t(w),\) and \(B(V)\) be the inverse functions of \(u(x_t(\theta)), w(y_t(\theta)),\) and \(\bar{V}(\theta)\), respectively, for \(t = 0, 1\). Then, \(X(u), Y(w)\) are increasing are convex, and \(B(V)\) is decreasing and convex. Finally,

\[ \bar{U}(V) = \max (1 - \alpha) (e - B(V)) + \alpha U(B(V)) \]  

(3.17)

represents the expected utility for the central authority in period 1 given truth telling in period 1.

Using the variables defined above, the problem for the central authority in period 0 can be
written as:

\[
\max_{\{u(\cdot), w(y_0(\cdot), \tilde{\nu}(\cdot))\}} \left\{ \frac{\theta}{\delta} u(x_0(\theta)) + w(y_0(\theta)) + \frac{\beta}{\delta} \tilde{V}(\theta) + \right.
\]
\[
\int_{\tilde{\nu}} \left[ \frac{1}{\delta} \left[ 1 - F(\theta) - \theta (1 - \delta) f(\theta) \right] u(x_0(\theta)) + \beta \left[ \tilde{U}(\tilde{V}(\theta)) - \frac{V(\theta)}{\delta} \right] \right] d\theta \right\},
\]

subject to

\[
Y_0^{-1} \left( e + \beta B(\tilde{V}) - X_0(u) \right) + \frac{\theta_0}{\delta} u(x_0(\theta_0)) + \frac{\beta}{\delta} \tilde{V}(\theta_0)
\]
\[
- \int_{\theta}^{\theta_0} \frac{1}{\delta} u(x_0(\theta)) d\theta - \frac{\theta}{\delta} u(x_0(\theta)) - w(y_0(\theta)) - \frac{\beta}{\delta} \tilde{V}(\theta) \geq 0 \quad \forall \theta_0 \in \Theta, \quad (3.19)
\]

\[
u(x_0(\theta_0')) \geq u(x_0(\theta_0)), \quad \tilde{V}(\theta_0') \leq \tilde{V}(\theta_0) \quad \forall \theta_0' \geq \theta_0. \quad (3.20)
\]

Problem (3.18) incorporates the incentive compatibility constraint (3.15) and drops the choice over function \( w(y_0(\theta_0)) \) to its value at \( \tilde{\theta}, w(y_0(\tilde{\theta})) \). Constraint (3.19) incorporates the budget constraint and the incentive compatibility constraint (3.15), allowing us to once again drop the dependency on \( w(y_0(\theta_0)) \). Constraint (3.20) represents the requirement for monotonicity, necessary in order to obtain incentive compatibility.7

The first result that is obtained when analyzing problem (3.18) is that the cut-off property described in Proposition 40 carries over to the case with a continuous distribution of local shocks.

Proposition 41 There exists a value \( \theta^*_p \leq \tilde{\theta} \) such that the optimal policy for the central authority has the following form:

- if \( \theta \geq \theta^*_p \), the same reinvestment funds are mandated for the global bank, i.e., \( y(\theta) = y^* \) \( \forall \theta \geq \theta^*_p \);

6 See the Appendix for the detailed steps.
7 See Milgrom and Segal (2002) for a detailed derivation of the necessary conditions for incentive compatibility.
if $\theta < \theta_p^*$, fewer reinvestment funds are mandated for the global bank in period 0 following a report of a high local shock, i.e., $y(\theta') < y(\theta)$ for $\theta' > \theta$.

Proof. In the Appendix. ■

The above proposition shows that when the realization of the local shock is high, the local authority derives a high benefit from overreporting the shock and providing more support for distressed local projects. This happens because it values the returns of local projects relatively more than the returns of global projects. Then, the information rent that must be given to the local authority in order for it to report the real shock value is larger than the benefit to the central authority of responding accurately to the funding needs of the local bank. When the shock is below the threshold $\theta_p^*$, the local authority's preference for supporting the local bank as opposed to the global bank is relatively small. This makes it possible for the central authority to provide incentives for truthful reporting of local shocks. As shown in the Appendix, the threshold $\theta_p^*$ is the shock value at which the preferred policy for the local authority at $\theta_p^*$ is the same as the preferred policy for the central authority when the latter is constrained to offer a pooled policy schedule $\{y(\theta_p^*), b(\theta_p^*)\}$ for all $\theta > \theta_p^*$.

Proposition 41 establishes that the local authority is given no autonomy to choose bailout support for the local bank when local firms face a high need for reinvestment funds. The implication is that, in a severe crisis at the local level, it is not optimal to try to adapt policies to local information. Since local crises do not have spillover effects across regions, so they are self-contained, their impact weighs heavily on the local authority. It is then costly for the central authority to offer incentives to modify the response of the local authority. In practice, local governments suffer the high cost of local crises and are often under pressure to save local banks. Therefore, imposing policy measures that reduce the help the government gives to the local economy would come at a much higher cost. This cost discourages policies that require collaboration with local governments, and encourages setting standards that limit the role of local governments in the decision-making process. This result suggests that local crises might, in fact, simplify the ex-post agreement over bailout policies, since more general rules become optimal, as opposed to policies tailored specifically to local conditions.
When the local shocks are below the threshold described in Proposition 41, the local authority has a degree of autonomy, meaning that it can decrease the support it offers to the global bank as the local shock is larger. A natural question in this context is how much autonomy can optimally be given to the local authority, and under what conditions full autonomy is optimal. If full autonomy is optimal, then a central authority cannot improve on the allocation chosen by the local authority. The analysis of the problem in period 1 leads to the result obtained in Proposition 4 of Amador et al. (2006), that, under Assumption 14 below, full autonomy is optimal for local shocks below \( \theta_p \), where \( \theta_p \) is defined as the smallest value of \( \theta \) in \( \Theta \) for which the following condition holds:

\[
E \left[ \frac{\theta(\theta \geq \hat{\theta})}{\hat{\theta}} \right] \leq \frac{1}{\delta}.
\]

**Assumption 14** The expression \( F(\theta) + \theta (1 - \delta) f(\theta) \) is increasing for all \( \theta \in \Theta \).

In period 0, the analysis becomes more complex due to the presence of debt. The next result shows how debt severely limits the conditions under which full autonomy for the local authority is optimal.

**Proposition 42** A policy that offers the local authority full autonomy to choose its preferred policies in period 0 is optimal under Assumption 14 if

\[
\frac{\partial U(V)}{\partial V} \geq \frac{1}{\delta} \quad \text{and} \quad \frac{\partial U(V)}{\partial V} \text{ is a linear function of } V,
\]

where \( V \) and \( U \) are defined in (3.16) and (3.17), respectively.

**Proof.** In the Appendix. ■

Proposition 42 shows that full autonomy is optimal when the difference in expected continuation values between the central and the local authorities is relatively small. If the local authority places sufficient weight on the global bank, i.e., \( \delta \) is sufficiently large, then the central authority finds it optimal to offer the local authority full autonomy over policy. The local authority can then fine tune policy interventions to the local shocks. The intuition is that when the goals of the two authorities are not too far apart, it is valuable to take advantage of all

---

\(^8\) This assumption is more general than the Assumption made in Amador et al. (2006), in that it applies to all \( \theta \in \Theta \) rather than to just \( \theta \leq \theta_p \). This change was made such that the condition assumed here is used in the analysis of the optimal policy for period 0.
available information and adapt policy to local shocks. When $\delta$ is small, so the local authority does not have a large stake in the global bank, then the incentives of the local authority to direct funds primarily towards supporting local projects comes into conflict with the goals of the central authority. This leads to no autonomy being given to the local authority, and instead implementing the same policy based only on the information available to the central authority.

In the context of bailout policies, full autonomy being optimal implies that spending by local governments follows the patterns that serve the best interests of the central authority. If a common bailout fund were to be created at the level of a union of countries, in order for it to be accessed by a country in need of additional bailout funds for large banks, then no restrictions or conditionalities would be necessary for countries to access this fund. Their preferred policies would align with the policies preferred by the central authority. There would then be no concern that countries might access the common bailout fund too soon, before they have used up enough of their own funds to help global banks.

The implication of Proposition 42 is that there exist conditions under which it is optimal to require a minimum intervention from a government’s own funds before it requests access to any common funds. This result is the equivalent result to the minimum savings rule in Amador et al. (2006), and it is summarized in the following Corollary.

**Corollary 43** A policy that requires a minimum bailout intervention from a government’s own budget before any outside funds are accessed is optimal under the conditions outlined in Proposition 42.

Taken together with the previous results, Proposition 42 implies that public debt makes it more difficult for full autonomy to be part of the optimal solution. The central authority still finds it optimal to offer the local authority some information rents when the local shocks are relatively low, in order for interventions to be better adapted to local conditions. Yet, the information rents that it offers fall short of the information rents that can be extracted in the case without debt, as in period 1. Linking this implication back to the motivation of the paper, one of the goals of common supranational regulation and supervision of global banks was to prevent banking crises in one country from impairing the country’s ability to borrow.
The results presented above suggest that delinking sovereigns from banks might also result in tighter regulations on bailouts than was the case when governments had limited ability to borrow. This implies less local autonomy to respond to local shocks.

### 3.3.3 Role of public debt

The results of the model highlight the importance of public debt in determining the degree to which local authorities can be given autonomy to adapt bailouts to local shocks. The following comparative statics analyze the effects of having higher outstanding debt on the optimal policy implemented by the central authority.

**Proposition 44** Higher outstanding debt (or a lower net budget in the current period) lowers the size of the information rents that the central authority must pay to the local authority; however, it does not change the range of local shock values over which no autonomy for the local authority is optimal.

**Proof.** In the Appendix. ■

The intuition for Proposition 44 is that higher outstanding debt constrains both the central and the local authorities more, by decreasing the funds available to support banks. The lower available budget makes the allocation of funds according to the needs of the banks more valuable for the central authority. The preferred policies of the two authorities become closer, which reduces the size of the information rents, in absolute terms. The above result emphasizes the role of debt as a disciplining device, in that the lower cost of providing incentives allows the central authority to loosen the restrictions on bailout policies and give some autonomy for local authorities to tailor interventions to local shocks. The effect arises because of a change in the degree of disagreement between the two authorities, in absolute terms.

The second part of the proposition delivers the result that the cutoff $\theta^*_p$ above which the local authority has no autonomy does not change in response to a change in debt. This happens because the value of public debt changes both the cost and the benefit to the central authority of any policy changes. Therefore, the size of the information rents changes, as well as the benefit
from tailoring policy to the value of shock \( \theta \). These changes are equal in size under the optimal policy, which smooths the costs of any distortions across all policies. Yet, this result no longer holds if the local authority is borrowing constrained, for example if we assume that creditors limit debt-taking to an upper value \( \bar{b} \) below the maximum capacity to borrow given by the no-Ponzi condition. In this case, high outstanding debt and a binding borrowing constraint would lead to a lower cutoff \( \theta_p^* \).

**Corollary 45** Assume that the local authority is borrowing constrained, with a binding upper limit on new debt \( \bar{b} \). Higher outstanding debt (or a lower net budget in the current period) leads to an increase in the range of parameters over which no autonomy for the local authority is optimal.

**Proof.** In the Appendix. ■

The case with borrowing constraints places the optimal cutoff \( \theta_p^* \) between the value in the case with no debt at all and the case with full access to debt. It highlights the central role of borrowing constraints in determining the degree of autonomy that is optimal for the local authority to receive for ex-post interventions. These results show that the link between sovereigns and banks, that damages a sovereign’s ability to borrow, also affects the link between sovereigns and supranational authorities. It makes it more difficult for supranational authorities to rely on the information provided by local authorities, and increases the benefits of removing local autonomy.

### 3.4 Extension to Multiple Periods

The model presented above can be extended to more than two periods, in order to examine the variation over time in the importance of local information for centralized intervention policies. We consider the case in which neither the central authority nor the local authority can commit to future policies. Each authority can only commit to the policies decided in the current period. This is justified by the focus of the paper on the ex-post government intervention following a crisis. A model in which either authority could commit to future policies would be more appropriate for the study of ex-ante regulation, but the motivation of the current paper
is better served by focusing on the case without commitment. This assumption allows us to exclude equilibria based on trigger strategies. It also allows us to extend the results from the two period model to an infinite horizon setup.

Each period $t$, in case of a recession, the central authority offers the local authority a schedule $\{y_t(\theta), x_t(\theta)\}_{\theta \in \Theta}$, and the local authority reports the value of the privately observed local shock $\theta$. The problem for the central authority at any time $t$ can be written in recursive form as

$$U(b) = \max_{\{x(\theta), y(\theta), b'(\theta)\}} \mathbb{E}_{\theta, \gamma} \left[ \theta u(x(\theta)) + w(y(\theta)) + \beta \left[ (1 - \alpha) (e - b'(\theta)) + \alpha U'(b'(\theta)) \right] \right], \quad (3.21)$$

subject to

$$\theta u(x(\theta)) + \delta w(y(\theta)) + \beta \left\{ (1 - \alpha) (e - b'(\theta)) + \alpha V'(b'(\theta)) \right\} \geq \theta u(x(\theta')) + \delta w(y(\theta')) + \beta \left\{ (1 - \alpha) (e - b'(\theta')) + \alpha V'(b'(\theta')) \right\}, \quad \forall \theta' \in \Theta, \quad (3.22)$$

$$x(\theta) + y(\theta) \leq e + \beta b'(\theta) - b, \quad (3.23)$$

$$b(\theta) \leq \frac{e}{1 - \beta}, \quad (3.24)$$

where $V(b)$ is the value function for the local authority at the values $\{x^*(\theta), y^*(\theta), b'^*(\theta)\}$ chosen by the central authority:

$$V(b) \equiv \mathbb{E} \left[ \theta u(x^*(\theta)) + \delta w(y^*(\theta)) + \beta \left[ (1 - \alpha) (e - b'^*(\theta)) + \alpha V'(b'^*(\theta)) \right] \right].$$

Constraint (3.22) represents the incentive compatibility constraint for the local authority, which ensures that truth telling is the optimal response of this authority to the schedule offered by the central authority. Constraint (3.23) is the budget constraint in period $t$, and constraint (3.24) is the no-Ponzi condition on debt.

Under the assumptions of no commitment from either authority, monotonicity and strict concavity of the utility functions $u(x)$ and $w(y)$, the above problem is well defined. Moreover, under these conditions, the value functions for the two authorities are strictly concave and
Lemma 46 The value functions $U(b)$ and $V(b)$ are concave and differentiable over the interval $(b, e/(1 - \beta))$.

Given concavity and differentiability of the value functions, we can proceed to analyze the optimal policy that comes out of problem (3.21) using the standard approach. We can then show that the results from the two-period model extended to the multi-period model, as summarized in the following proposition.

Proposition 47 In each period $t$, there exists a value $\theta_t^* \leq \theta$ such that the optimal policy for the central authority has the following form:

- if $\theta_t \geq \theta_t^*$, the same reinvestment funds are mandated for the global bank, i.e., $y(\theta_t) = y^*$ for all $\theta_t \geq \theta_t^*$;
- if $\theta_t < \theta_t^*$, fewer reinvestment funds are mandated for the global bank in period 0 following a report of a high local shock, i.e., $y'(\theta_t') < y(\theta_t)$ for $\theta_t' > \theta_t$.

Proof. Follows directly from the proof to Proposition 2, with the budget each period taken as $(e - b)$. 

The intuition from Proposition 41 carries over to the results summarized in Proposition 47. The central authority finds it optimal to offer information rents to the local authority whenever the local shock is sufficiently small. As the local shock is larger, the local authority values support to local firms more, and it must receive higher information rents in order not to overreport the shock. It then becomes too expensive for the central authority to offer these information rents compared to the cost of not tailoring policy to local shocks. The last-period's borrowing is the only link across periods in this model, since all debt is short-term (one period). The outstanding debt then determines the available budget at the start of the period. Therefore, the trade-offs faced by the central authority within each period are the same at every date $t$. The outstanding debt only affects the relative size of these trade-offs, and implicitly the information rents given to the local authority.
The above result shows that the degree of discretion given to the local authority in choosing bailout policies varies over time. We examine next how the tailoring of policy to local shocks depends on the level of outstanding debt.

**Proposition 48** Assume the local authority faces no borrowing constraints. Then, the cutoff value \( \theta^*_t \) is constant over time, \( \theta^*_t = \theta^* \), \( \forall t \).

**Proof.** In the Appendix. □

Proposition 48 shows that the degree of autonomy given to the local authority on the extensive margin does not vary over time depending on the level of outstanding debt, which is the state variable each period. The extensive margin on autonomy refers to the range of shock values over which the local authority has no freedom to vary its bailout response to local shocks. This range of shocks is given by the condition \( \theta_t \geq \theta^*_t \) derived in Proposition 47. The intuition for the result is that the cutoff \( \theta^*_t \) is determined at the value at which the relative gain from tailoring the policy to local shocks equals the cost of offering the local authority information rents in order to extract the value of the local shock. A higher value of the outstanding debt affects both the aforementioned cost and benefit in the same matter, since the central authority is optimally smoothing the cost of any distortion across all available policy dimensions. Therefore, the cutoff \( \theta^*_t \) remains unchanged, and the model does not allow for any dynamic effects on the extensive margin of autonomy.

The implication of the above result is that the variation in debt holdings has no effect on a local authority’s ability to tailor policy to local shocks. What matters for the cutoff \( \theta^*_t \) is the existence – or availability – of public debt versus having a binding borrowing constraint. Similarly, we can show that if the local authority can take on debt, but it can become borrowing constrained at some debt ceiling \( \bar{b} \), then the cutoff \( \theta^*_t \) does change with the value of outstanding debt, whenever more debt leads the local authority to reach the debt limit. This result follows directly from Corollary 45. A local authority that has more difficulty borrowing also sees the cutoff \( \theta^*_t \) decrease. The range of shock values over which the local authority is given no autonomy increases. Local information is used less in the optimal policy if the central authority is faced with a local authority with less access to funding. The local authority has a higher
preference to use its scarce funds for reinvestments in local firms, which makes it costlier for the central authority to offer incentives for the truthful reporting of local shocks. We therefore end up with a case in which the local authority is more restricted in terms of the ability to tailor policy to local shocks. This result has implications for the debate over the role of fiscal coordination or binding fiscal rules in unions. It suggests that such binding rules can lead to a loss of local autonomy on other dimensions than public debt – in this case the ability to tailor bailout policies to local shocks.

Finally, notice that variations in the level of outstanding debt can, however, affect the degree of autonomy offered to the local authority on the intensive margin. More specifically, public debt affects the size of the incentive problem faced by the central authority, because it influences the relative preference of each authority for bailouts of local versus global banks, as well as the decision on how much to borrow in the current period. Whether more outstanding debt makes the incentives problem worse for the central authority depends on the relative change in payoffs from each type of bailout. If the increase in outstanding debt leads to a higher preference for local bailouts ($x(\theta)$) relative to bailouts of global projects ($y(\theta)$), then more public debt makes the incentives problem worse. The local authority values bailouts of global projects less, and has an even stronger incentive to overreport the value of the local shock $\theta$. Consider the case in which $\theta \in \{\theta^H, \theta^L\}$, $\delta < \theta^H$, $u(\cdot) = w(\cdot)$, and the high value of the local shock is realized. If the function $u(\cdot)$ is not isoelastic, then the preferences for bailouts of the two authorities change as the debt level changes. The incentives problem can then become worse as outstanding debt increases. This would then lead to less autonomy being optimal.

The analysis of the multi-period model has shown that whether a local authority has autonomy to tailor policy to local shocks depends on the limits or constraints on debt-taking. It does not depend on the variation of debt over time, as long as the local authority is not borrowing constrained. Therefore, the primary constraint of interest in both the two-period and the multi-period model is the ability of the local authority to borrow. Yet, repeated high realizations of the local shock, which increase debt, can lead to a worsening of the incentive problem and a decrease in autonomy for the local authority.
3.5 Illustration

Below, we discuss the paper’s results with reference to two specific functional forms. First, consider the case in which the project returns follow a logarithmic schedule for all projects. In case of a recession, the returns after any government reinvestment are given by:

\[
\begin{align*}
  u(x) &= \log(i^L - z + x), \\
  w(y) &= \log(i^G - z + y),
\end{align*}
\]

where \(i^L - z \geq 1\) and \(i^G - z \geq 1\). Also, assume that \(\Theta = \{\theta^L, \theta^H\}\) and \(\mathbb{E}[\theta] = \theta^M\).

In the case with logarithmic returns, in period 1, the local authority will be offered a schedule \(y(\theta^L) = y(\theta^H)\) if \(\delta < \frac{\theta^L}{\theta^H}\). It will be able to offer more funds to the local bank if \(\delta > \frac{\theta^L}{\theta^H}\), in which case \(y(\theta^L) > y(\theta^H)\). In period 0, we can derive the preferred policies for the local authorities and the preferred policies for the central authority, and we can infer that the local authority will be given no autonomy to differentiate interventions in response to local shocks if \(\delta < \tilde{\delta}\), where

\[
\tilde{\delta} = \frac{\theta^L + \theta^M}{\theta^H + \beta \theta^M + \beta - 1}.
\]

Notice that \(\tilde{\delta} > \frac{\theta^L}{\theta^H}\), whenever \(\theta^L < \theta^H\). The following result then follows.

**Remark 49** The existence of public debt allows the local authority to have more autonomy to differentiate its bailouts in response to local shocks.

The existence of debt in the two type case allows more autonomy to the local authority, for a wider variance in local shocks, as discussed in Proposition 40.

If we consider a continuous distribution of shocks \(f(\theta)\), then we can show that for a given value of \(\delta\), the conditions for full autonomy given in Proposition 42 may not be satisfied in this example. To provide the simplest illustration, assume that \(\alpha = 1\), so the economy is an recession for sure in period 1. Then, a necessary condition for full autonomy to be optimal is that \(\frac{\partial U(V)}{\partial V} \geq \frac{1}{\delta}\). Yet, we can derive \(\bar{V}\) and \(\bar{U}\) as the expected utilities in period 1 under the
optimal policy, and obtain
\[ \frac{\partial \bar{U}(V)}{\partial V} = \frac{\theta_M + 1}{\theta_M + \delta}. \]

The condition for full autonomy cannot, therefore, be satisfied for any distribution of shocks \( \theta \).

**Remark 50** A policy that gives the local authority full autonomy to choose transfers following local shocks in some subset of \( \Theta \) is not optimal in period 0. Yet, such a policy is optimal in period 1, when no public debt can be taken on.

To exemplify a situation in which full autonomy is optimal in period 0, consider another example, this time with quadratic production technologies for both types of projects. In case of a recession, the returns after any government reinvestment are given by:

\[
\begin{align*}
    u(x) &= (i^L - z + x) - (i^L - z + x)^2, \\
    w(y) &= (i^G - z + y) - (i^G - z + y)^2,
\end{align*}
\]

and \( i^L - z < \frac{1}{2}, i^G - z < \frac{1}{2} \).

In this case
\[ \hat{\delta} = \frac{\theta^L}{\theta^H}, \]
showing how public debt allows for differentiated reinvestments for the same range of local shock values as the case without debt. This happens because the preferred transfers for each authority are a fraction of the total budget, and therefore any effects introduced by debt are reflected similarly in the preferences of both authorities.

If we consider a continuous distribution of shocks \( f(\theta) \), we can derive
\[ \frac{\partial \bar{U}(V)}{\partial V} = \frac{1}{\delta}. \]

This shows that the conditions for full autonomy are satisfied in the case with debt whenever they are satisfied in the case without debt. The effect of debt cancels out, and full autonomy is
optimal for a subset of the shocks, \( \theta \leq \theta_p \), where \( \theta_p \) is the cut-off above which the local authority receives no autonomy. Taken together with the previous example, this result comes to show that the conditions for full autonomy are rather restrictive and dependent on the production technology. Public debt can in some cases help increase the range of local shocks under which the local authority is optimally given some autonomy. Yet, public debt can also lead to more restrictions to the ability of local authorities to have full autonomy, if it contributes to an increase in the conflict of interests between the two authorities.

### 3.6 Conclusion

This paper used a standard delegation model to examine the optimal degree of autonomy that should be given to a local authority that decides bailouts. The local authority possesses private information about local shocks, but it is also biased towards local banks. The model shows that the ability of the local authority to take on debt severely restricts the autonomy that it can be given for choosing bailouts. Yet, it is possible for full autonomy of the local authority to be optimal, as long as the conflict of interests between the central and the local authority is not too large. The paper derives implications about the role of debt in determining the degree of autonomy that can be given to a local authority, and shows that it is the constraint on borrowing and not the change in outstanding debt that can lead to losses of local autonomy.

The paper has focused on ex-post interventions by authorities, and it looked specifically at the role of the government as provider of funds to distressed banks when these banks cannot borrow in the market. An important complementary question is how to design policy ex-ante, more specifically how to design regulation ex-ante given that the supervision and regulation of banks is only partially done by a central authority. The rest of the system is still under the control of local authorities, who can impose their own regulatory frameworks. The interaction between the two authorities might lead to either insufficiently strict regulations, or to one authority being too strict in order to compensate for the conflict of interests with the other authority.

Another aspect of the problem that could be further examined is the interaction between local and global banks in the market. So far, the model assumes that project technology is
fixed and specific to each bank. Allowing investors to select which types of projects they want to hold would generate endogeneity in the market shares of local versus global banks. It would highlight the effect of cross-country coordination over bailouts on the evolution of local banks, and implicitly on the evolution of the conflict of interests between authorities. This extension could introduce a source of potentially interesting dynamics.
3.A Appendix

3.A.1 Proof of Proposition 40

Consider the problem for the principal (the central authority). In period 1, separation of the two types is possible whenever the principal's preferred transfer $y^P(0_H)$ after the realization of the high shock $\theta^H$ is less than or equal to the agent's preferred transfer $y(0_L)$ after the realization of the low shock $\theta^L$. Otherwise separating the two types would always be suboptimal for the principal, given her quasi-concave utility function (since $y^P(0_L) > y^P(0_H)$ and $y(0_L) > y(0_H)$). The expression for $y^P(0_H)$ is the solution to

$$\theta^H u(e - b - y^P) = w(y^P),$$

and the expression for $y(0_L)$ is the solution to

$$\theta^L u(e - b - y) = \delta w(y).$$

The separation of types will be optimal when $y^P(0_H) \leq y(0_L)$, which is equivalent to $\delta \geq \frac{\theta^L}{\theta^H}$.

In period 0, assume the same allocation is provided regardless of the local shock $\theta_0$. Denote this pooled allocation by $\{x^*, y^*, b^*\}$. Then, consider an increase in $b^*$ to $b^H$ along with a decrease in $y^*$ to $y^H$ after a report of $\theta^H$, such that the incentive compatibility constraint for the low type is binding, i.e.,

$$\theta^L u(x^*) + \delta w(y^*) + \beta \{(1 - \alpha)(e - b^*) + \alpha V(b^*)\},$$

$$= \theta_0 u(x^H) + \delta w(y^H) + \beta \{(1 - \alpha)(e - b^H) + \alpha V(b^H)\}. \quad (3.25)$$

Then, the incentive compatibility constraint for the high type is satisfied since $\theta^H > \theta^L$. The change in the principal's utility is given by:

$$\theta^H \left[ u(x^H) - u(x^*) \right] + \left[ w(y^H) - w(y^*) \right]$$

$$+ \beta \{(1 - \alpha) (b^* - b^H) + \alpha \left[ U(b^H) - U(b^*) \right] \}. \quad (3.26)$$
Re-writing \( [w(y^H) - w(y^*)] \) given the low type's incentive compatibility constraint, (3.26) becomes

\[
\left( \theta^H - \frac{\theta_L}{\delta} \right) [u(x^H) - u(x^*)] + \beta \left( 1 - \frac{1}{\delta} \right) [(1 - \alpha) (b^* - b^H)] \\
\quad + \beta \alpha \left( V(b^H) - U(b^*) \right) - \frac{\beta}{\delta} \alpha \left( U(b^H) - U(b^*) \right).
\]

Expanding \( U(b^H) - U(b^*) \) and \( V(b^H) - V(b^*) \) we obtain

\[
\left( \theta^H - \frac{\theta_L}{\delta} \right) [u(x^H) - u(x^*)] + \beta \left( 1 - \frac{1}{\delta} \right) [(1 - \alpha) (b^* - b^H)] \\
\quad + \beta \alpha \left( 1 - \frac{1}{\delta} \right) \{ \mathbb{E} [\theta u(x(b^H))] - \mathbb{E} [\theta u(x(b^*))] \}.
\] (3.27)

\( \mathbb{E} [\theta u(x(b^H))] - \mathbb{E} [\theta u(x(b^*))] < 0 \) since the transfers to banks are normal goods and \( b^H > b^* \).

Also, \( (1 - \frac{1}{\delta}) < 0 \) since \( \delta < 1 \). Hence,

\[
\beta \left( 1 - \frac{1}{\delta} \right) [(1 - \alpha) (b^* - b^H)] + \beta \alpha \left( 1 - \frac{1}{\delta} \right) \{ \mathbb{E} [\theta u(x(b^H))] - \mathbb{E} [\theta u(x(b^*))] \} \geq 0,
\]

and there exists a \( \tilde{\delta} \) such that (3.27) is positive, with \( \tilde{\delta} \leq \frac{\theta_L}{\theta^H} = \delta^* \).

3.A.2 Proof of Proposition 41

The period 1 problem is identical to the problem presented in Proposition 3 of Amador et al. (2006). It implies that there exists a cutoff \( \theta_p \) above which the same allocation is offered \( \forall \theta \geq \theta_p \).

In period 0, let \( \theta^*_p \in \Theta \) be one value of the local shock above which the same allocation is chosen. Then, for \( \theta \leq \theta^*_p \), the principal must find it optimal to offer an incentive compatible allocation. Such an allocation will be preferable for the principal whenever the pooled solution for \( \theta > \theta^*_p \) requires that \( y^* \leq y^A(\theta^*_p) \) or \( b^* \geq b^A(\theta^*_p) \), where where \( y^A(\theta^*_p) \) and \( b^A(\theta^*_p) \) are the preferred transfer and debt chosen by the agent under full autonomy. Otherwise, any incentive compatible policy would yield less utility to the principal than the pooled allocation.

Consider first the determination of \( y^A(\theta^*_p) \) and \( b^A(\theta^*_p) \). The first-order conditions for the
agent’s problem yield

\[ \theta_p^* u'(e + \beta b^A (\theta_p^*) - y^A (\theta_p^*)) = \delta w' (y^A (\theta_p^*)), \]

\[- (1 - \alpha) - \alpha V'(b^A (\theta_p^*)) = \delta w' (y^A (\theta_p^*)). \]

The conditions for determining \( y^* \) and \( b^* \) are

\[ \mathbb{E} \left[ \theta | \theta > \theta_p^* \right] u'(e + \beta b^* - y^*) = w' (y^*), \]

\[- (1 - \alpha) - \alpha U'(b^*) = w' (y^*). \]

Then, \( y^* \leq y^A(\theta_p^*) \) implies

\[ \frac{- (1 - \alpha) - \alpha V'(b^A (\theta_p^*))}{- (1 - \alpha) - \alpha U'(b^*)} \geq \delta, \]

\[ \frac{- (1 - \alpha)}{\alpha} (1 - \delta) - V'(b^A (\theta_p^*)) \geq - \delta U'(b^*), \]

which implies \( b^A (\theta_p^*) < b^* \). Since \( -V'(b) < -U'(b) \). Therefore, separation is optimal when \( y^* \leq y^A(\theta_p^*) \).

The condition that \( b^* \geq b^A (\theta_p^*) \) implies that \( \frac{- (1 - \alpha) - \alpha V'(b^A (\theta_p^*))}{- (1 - \alpha) - \alpha U'(b^*)} \geq \delta \), which is consistent with \( y^* > y^A(\theta_p^*) \). Denote by \( \theta_p \) the value of \( \theta \) at which \( y^* = y^A(\theta_p^*) \). Then, \( b^* \geq b^A (\theta_p^*) \) is consistent with \( y^* > y^A(\theta_p^*) \) for \( \delta < \frac{\theta_p}{\mathbb{E}[\theta \theta^* \theta_p^*]} \). The maximum value of \( \theta \) for which separation is optimal is given by the value at which \( b^* \geq b^A (\theta_p^*), y^* \geq y^A(\theta_p^*) \), and the principal is indifferent between the pooling and the separating allocation. Given the above conditions, this value of \( \theta \), denoted \( \theta_p^* \), must satisfy

\[ \delta < \frac{\theta_p^*}{\mathbb{E}[\theta \theta^* \theta_p^*]}. \]

Therefore, \( \theta_p^* > \theta_p \). The value \( \theta_p^* \in \Theta \) since \( \frac{\theta}{\mathbb{E}[\theta \theta^* \theta]} = 1 \) and \( \frac{\theta^*}{\mathbb{E}[\theta \theta^* \theta]} \) is increasing in \( \theta^* \).

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3.A.3 The maximization problem with continuous shocks

This section details the steps taken to reach the form of problem (3.18) presented in the text. The approach follows similar steps to Amador et al. (2006).

The problem for the central authority in period 0 is then given by

$$
\max_{\{u, w, \bar{V}\}} \int_{\theta} \left[ \theta_0 u(x_0(\theta_0)) + w(y_0(\theta_0)) + \beta \bar{V}(\theta_0(\bar{v})) \right] f(\theta_0) d\theta_0,
$$

(3.28)

subject to constraints (3.15),

$$
X_0(u) + Y_0(w) \leq e + \beta B(\bar{V})
$$

(3.29)

$$
u(x_0(\theta'_0)) \geq u(x_0(\theta_0)), \ \bar{V}(\theta'_0) \leq \bar{V}(\theta_0) \ \forall \theta'_0 \geq \theta_0
$$

(3.30)

Constraint (3.29) represents the budget constraint in period 0, and constraints (3.30) require that the utility from local projects is increasing in the local shock and the continuation utility for the local authority is decreasing with shock $\theta_0$ (given the choice of debt in period 0).

The assumption that $\bar{V}$ is the expected utility of the local authority in period 1, when the optimal policy is implemented, implies that in period 1, both the budget constraint,

$$
X_1(u) + Y_1(w) \leq e - B(\bar{V}),
$$

and the incentive compatibility constraint (3.14) are satisfied (along with the requirement that $u$ is increasing in $\theta$).
Problem (3.28) can be simplified and re-written in the following form:

\[
\max_{\{u(\cdot), w(y_0(\theta)), \overline{V}(\cdot)\}} \left\{ \frac{\theta}{\delta} u(x_0(\theta)) + w(y_0(\theta)) + \frac{\beta}{\delta} \overline{V}(\theta) + \int_\theta^\tilde{\theta} \left[ \frac{1}{\delta} \left[ 1 - F(\theta_0) - \theta_0 (1 - \delta) f(\theta_0) \right] u(x_0(\theta_0)) + \beta \left[ \overline{U}(\overline{V}(\theta_0)) - \frac{\overline{V}(\theta_0)}{\delta} \right] \right] d\theta_0 \right\},
\]

subject to

\[
Y_0^{-1} (e + \beta B(\overline{V}) - X_0(u)) + \frac{\theta_0}{\delta} u(x_0(\theta_0)) + \frac{\beta}{\delta} \overline{V}(\theta_0)
\]
\[
- \int_\theta^{\theta_0} \frac{1}{\delta} u(x_0(\theta)) d\theta - \frac{\theta}{\delta} u(x_0(\theta)) - w(y_0(\theta)) - \frac{\beta}{\delta} \overline{V}(\theta) \geq 0 \quad \forall \theta_0 \in \Theta,
\]

\[
u(x_0(\theta_0')) \geq u(x_0(\theta_0)), \quad \overline{V}(\theta_0') \leq \overline{V}(\theta_0) \quad \forall \theta_0' \geq \theta_0.
\]

3.A.4 Proof of Proposition 42

The proof follows the steps outlined in Amador et al. (2006). The problem faced by the principal is

\[
\max_{\{u(\cdot), w(y_0(\theta)), \overline{V}(\cdot)\}} \left\{ \frac{\theta}{\delta} u(x_0(\theta)) + w(y_0(\theta)) + \frac{\beta}{\delta} \overline{V}(\theta) + \int_\theta^\tilde{\theta} \left[ \frac{1}{\delta} \left[ 1 - F(\theta_0) - \theta_0 (1 - \delta) f(\theta_0) \right] u(x_0(\theta_0)) + \beta \left[ \overline{U}(\overline{V}(\theta_0)) - \frac{\overline{V}(\theta_0)}{\delta} \right] \right] f(\theta_0)d\theta_0 \right\},
\]

subject to

\[
Y_0^{-1} (e + \beta B(\overline{V}) - X_0(u)) + \frac{\theta_0}{\delta} u(x_0(\theta_0)) + \frac{\beta}{\delta} \overline{V}(\theta_0) - 
\]
\[
- \int_\theta^{\theta_0} \frac{1}{\delta} u(x_0(\theta)) d\theta - \frac{\theta}{\delta} u(x_0(\theta)) - w(y_0(\theta)) - \frac{\beta}{\delta} \overline{V}(\theta) \geq 0 \quad \forall \theta_0 \in \Theta,
\]

\[
u(x_0(\theta_0')) \geq u(x_0(\theta_0)), \quad \overline{V}(\theta_0') \leq \overline{V}(\theta_0) \quad \forall \theta_0' \geq \theta_0.
\]
Let $A(\theta_0)$ denote the Lagrange multiplier associated with the incentive compatibility constraint (3.31). The Lagrangian of the above program is

\[
\mathcal{L} = \frac{\theta}{\delta} u(x_0(\theta)) + w(y_0(\theta)) + \frac{\beta}{\delta} \bar{V}(\theta) +
\int_{\theta}^{\bar{\theta}} \left[ \frac{1}{\delta} \left[ 1 - F(\theta_0) - \theta_0 (1 - \delta) f(\theta_0) \right] u(x_0(\theta_0)) + \beta \left( \bar{U}(\theta_0) - \frac{\bar{V}(\theta_0)}{\delta} \right) \right] f(\theta_0) d\theta_0 +
\int_{\theta}^{\bar{\theta}} \left( Y^{-1}(\theta + \beta B(\bar{V}) - X_0(u)) + \frac{\theta_0}{\delta} u(x_0(\theta_0)) + \frac{\beta}{\delta} \bar{V}(\theta_0) \right) d\Lambda(\theta_0).
\]

The Lagrange multiplier is restricted to be non-decreasing and, without loss of generality, let $\Lambda(\bar{\theta}) = 1$ and assume that it is left continuous for all $\theta < \bar{\theta}$.

Integrating by parts we can re-write the Lagrangian as

\[
\mathcal{L} = \left[ \frac{\theta}{\delta} u(x_0(\theta)) + w(y_0(\theta)) + \frac{\beta}{\delta} \bar{V}(\theta) \right] \Lambda(\theta) +
\int_{\theta}^{\bar{\theta}} \left[ \frac{1}{\delta} \left[ \Lambda(\theta_0) - F(\theta_0) - \theta_0 (1 - \delta) f(\theta_0) \right] u(x_0(\theta_0)) + \beta \left( \bar{U}(\theta_0) - \frac{\bar{V}(\theta_0)}{\delta} \right) \right] d\theta_0 +
\int_{\theta}^{\bar{\theta}} \left( Y^{-1}(\theta + \beta B(\bar{V}) - X_0(u)) + \frac{\theta_0}{\delta} u(x_0(\theta_0)) + \frac{\beta}{\delta} \bar{V}(\theta_0) \right) d\Lambda(\theta_0).
\]

Denote by $\Phi$ the set of functions $(u, \bar{V})$ that satisfy condition (3.32). The rest of the proof is then based on the following extended lemma from Amador et al. (2006), based on the conditions outlined in Luenberger (1969).

**Lemma 51 (Amador et al. (2006))** If an allocation $(u^*, w^*(y_0(\theta)), \bar{V}^*)$ with $(u^*, \bar{V}^*) \in \Phi$ is optimal and $u^*$ and $\bar{V}^*$ are continuous then there exists a non-decreasing $\Lambda_0$ such that

\[
\mathcal{L}(u^*, w^*(y_0(\theta)), \bar{V}^*|\Lambda_0) \geq \mathcal{L}(u, w(y_0(\theta)), \bar{V}|\Lambda_0) \tag{3.33}
\]

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for all \((u, w(y_0(\theta)), V) \in \Phi \) with \(u\) and \(\bar{U}\) are continuous. Conversely, if (3.33) holds for some \(\Lambda_0\) for all \((u, w(y_0(\theta)), U) \in \Phi\) then \((u^*, w^*(y_0(\theta)), \bar{U}^*)\) is optimal.

The Lagrangian is concave, and it is maximized within a convex cone \(P\) if and only if the following first-order conditions hold:

\[
\partial \mathcal{L} \left( u^*, w^*(y_0(\theta)), \bar{V}^*; u^*, w^*(y_0(\theta)), \bar{V}^*|\Lambda_0 \right) = 0, \tag{3.34}
\]

\[
\partial \mathcal{L} \left( u^*, w^*(y_0(\theta)), \bar{V}^*; u, w, h_{\bar{V}}|\Lambda_0 \right) \leq 0, \tag{3.35}
\]

for all \((h_u, h_w, h_{\bar{V}}) \in P\), and if the Gateaux differentials are linear (Lemma 1 of (Luenberger, 1969), Chapter 8, page 227)

The Lagrangian is the sum of integrals over \(\theta\) of concave functions of \(u, w(y_0(\theta))\), and \(\bar{V}\), and therefore the Gateaux differential exists and can be computed, as shown in Amador et al. (2006). The Gateaux differential at the proposed allocation \((u^*, w^*(y_0(\theta)), \bar{V}^*)\) is given by

\[
\partial \mathcal{L} \left( u^*, w^*(y_0(\theta)), \bar{V}^*; u, w, h_{\bar{V}}|\Lambda_0 \right) = \left[ \frac{\theta}{\delta} h_u(\theta) + h_w + \frac{\beta}{\delta} h_{\bar{V}}(\theta) \right] \Lambda(\theta) +
\]

\[
\int_{\theta} \left[ \frac{1}{\delta} \left[ \Lambda(\theta_0) - F(\theta_0) - \theta_0 (1 - \delta) f(\theta_0) \right] h_u + \beta \left[ \bar{U}'(\bar{V}(\theta_0)) - \frac{1}{\delta} \right] h_{\bar{V}} \right] d\theta_0 +
\]

\[
\frac{\theta^*_{\bar{p}}}{\delta} \int_{\theta} \left( \frac{\theta_0}{\theta^*_{\bar{p}}} - 1 \right) h_u d\Lambda(\theta_0). \tag{3.36}
\]

For the above-mentioned conditions to hold such that the maximization of the Lagrangian is achieved, the Gateaux differential must be linear. This condition is not satisfied unless \(\bar{U}'(\bar{V}(\theta_0))\) is linear. In that case, the problem can be analyzed by defining

\[
\eta(\theta_0) = \left[ \bar{U}'(\bar{V}(\theta_0)) - \frac{1}{\delta} \right],
\]

and

\[
\gamma(\theta_0) \equiv \frac{1}{\delta} \int_{\theta_0} \left[ \Lambda^*(\theta) - g(\theta) \right] d\theta + \frac{\theta^*_{\bar{p}}}{\delta} \int_{\max\{\theta_0, \theta^*_{\bar{p}}\}} \left( \frac{\theta}{\theta^*_{\bar{p}}} - 1 \right) \Lambda(\theta).
\]
First, we must show that \( \left( u^*, w^*(y_0(\theta)), V^* \right) \) being optimal implies a non-decreasing \( \Lambda_0(\theta) \) exists such that conditions (3.34) and (3.35) hold. Condition (3.35) with \( h_u = h_V = 0 \) implies \( \Lambda_0(\theta) = 0 \) since \( h_w \) is unrestricted. Then, integrating (3.36) by parts, we obtain

\[
\partial \mathcal{L} \left( \left( u^*, w^*(y_0(\theta)), V^*; h_u, h_w, h_V \right| \Lambda_0 \right) = \gamma(\theta) h_u(\theta) + \eta(\theta) h_V(\theta) + \int_\theta^\theta \gamma(\theta_0) dh_u(\theta_0) + \int_\theta^\theta \eta(\theta_0) dh_V.
\]

Notice that \( h_V < 0 \) and \( h_u > 0 \). For condition (3.35) to hold, we then need \( \eta(\theta_0) \geq 0 \), or \( \bar{U}'(V(\theta_0)) \geq \frac{1}{\delta} \). Condition (3.34) then requires that

\[
\gamma(\theta) h_u(\theta) + \int_\theta^\theta \gamma(\theta_0) dh_u(\theta_0) = -\eta(\theta) h_V(\theta) - \int_\theta^\theta \eta(\theta_0) dh_V. \tag{3.37}
\]

This condition requires \( \Lambda_0(\theta) \) to be increasing in \( g(\theta) \equiv F(\theta) + \theta (1 - \delta) f(\theta) \) whenever \( \eta(\theta_0) \) is increasing in \( \theta \). Therefore, \( (F(\theta) + \theta (1 - \delta) f(\theta)) \) increasing and \( \bar{U}'(V(\theta_0)) \geq \frac{1}{\delta} \) along with \( \bar{U}'(V(\theta_0)) \) linear are necessary conditions for \( \left( u^*, w^*(y_0(\theta)), V^* \right) \) to be optimal.

The argument for sufficiency is identical to the argument made in the proof of Proposition 4 in Amador et al. (2006), with \( \Lambda_0(\theta) \) taken as an increasing function of \( g(\theta) \) which satisfies (3.37).

### 3.A.5 Proof of Proposition 44

Consider first the case with two types of shocks \( \theta \). If the two types can be separated, then the incentive compatibility constraint for the low type will bind, hence

\[
\theta^L u(x^L) + \delta w(y^L) + \beta \left[ (1 - \alpha) (e - b^L) + \alpha V(b^L) \right]
\]

\[
= \theta^L u(x^H) + \delta w(y^H) + \beta \left[ (1 - \alpha) (e - b^H) + \alpha V(b^H) \right]. \tag{3.38}
\]

The extent to which an incentive compatible allocation can be offered and the size of the information rents depend on the preferred policy of the agent when the shock is \( \theta^L \) relative to
the preferred policy of the principal when the shock is $\theta^H$. The preferred policy for the agent at $\theta^L$ is given by the following first-order conditions:

$$\theta^L u'(e + \beta b^A - b_0) = \delta w'(y^A),$$

$$\frac{1 - \alpha}{1 - \alpha} - \alpha V'(b^A) = \delta w'(y^A).$$

The preferred policy for the principal at $\theta^H$ is given by the following first-order conditions:

$$\theta^H u'(e + \beta b^P - b_0) = w'(y^P),$$

$$\frac{1 - \alpha}{1 - \alpha} - \alpha V'(b^P) = w'(y^P).$$

For $\delta > \frac{\theta^L}{\theta^H}$, the first-order conditions imply that an increase in $b_0$ leads to a larger change in $y^A$ than $y^P$. Then, the distance in utility terms between the preferred policy of the agent and that of the principal decreases. Hence, the agent receives an allocation closer to his preferred policy, and the difference from between the proposed allocation and the principal’s preferred allocation is decreased.

When the distribution of shocks is continuous, whether an incentive compatible allocation can be offered and the size of the information rents depend on the preferred policy of the agent relative to the pooled allocation. The pooled allocation has the properties

$$(\mathbb{E}_[\theta|\theta \geq \theta^*]) u'(e + \beta b^P - b_0) = w'(y^P),$$

$$(1 - \alpha) - \alpha V'(b^P) = w'(y^P),$$

where $\theta^*$ is the value of $\theta$ above which the types are pooled.

If $\delta \leq \mathbb{E}[\theta|\theta \geq \theta^*]$, i.e., $\theta^* = \theta_p$, such that the types are separated in both periods, then an increase in $b_0$ leads to a larger change in $y^A$ than $y^P$. The distance in utility terms between the preferred policy of the agent and that of the principal decreases, moving the equilibrium allocation closer to the agent’s preferred policy.

For the second part of the proposition, consider first the case with two shock values, $\theta \in \{L, H\}$. If $\delta > \frac{\theta^L}{\theta^H}$, then an increase in $b_0$ leads to a larger change in $y^A$ than $y^P$. The distance in utility terms between the preferred policy of the agent and that of the principal decreases, moving the equilibrium allocation closer to the agent’s preferred policy.
\( \{ \theta^L, \theta^H \} \). The condition for the cutoff \( \delta \) is

\[
\left( \frac{\theta^H - \theta^L}{\delta} \right) [u(x^H) - u(x^*)] + \beta \left( 1 - \frac{1}{\delta} \right) [(1 - \alpha) (b^* - b^H)] \\
+ \beta \alpha [U(b^H) - U(b^*)] - \frac{\beta}{\delta} \alpha [V(b^H) - V(b^*)] = 0,
\]

\[
\left( \frac{\theta^H - \theta^L}{\delta} \right) [u(x^H) - u(x^*)] + \beta \left( 1 - \frac{1}{\delta} \right) [(1 - \alpha) (b^* - b^H)] \\
+ \beta \left( \frac{1}{\delta} - 1 \right) \alpha \delta^M [u_1(b^*) - u_1(b^H)] = 0.
\]

Assume that \( \alpha = 1 \). Then

\[
\left( \frac{\theta^H - \theta^L}{\delta} \right) = -\frac{\beta (\frac{1}{\delta} - 1) \theta^M [u_1(b^*) - u_1(b^H)]}{[u(x^H) - u(x^*)]}.
\]

Consider an increase in outstanding debt \( b_0 \):

\[
\partial \left( -\frac{[u_1(b^*) - u_1(b^H)]}{[u(x^H) - u(x^*)]} \right) / \partial b_0 \approx \partial \left( -\frac{[u'(x^*) \frac{\partial x}{\partial b_1} \Delta b]}{[u'(x^*)] \Delta x} \right) / \partial b_0
\]

\[
sign \left\{ \partial \left( \frac{[u_1(b^*) - u_1(b^H)]}{[u(x^H) - u(x^*)]} \right) / \partial b_0 \right\} \approx \begin{align*}
&\text{sign} \left\{ \left( \frac{\partial x}{\partial b_1} \right) \frac{\partial b_1}{\partial b_0} \Delta b \right\} \Delta x - \frac{\partial x}{\partial b_0} \Delta x \frac{\partial x}{\partial b_1} \Delta b \\
&= \text{sign} \left\{ \left( \frac{\partial x}{\partial b_1} \right) \frac{\partial b_1}{\partial b_0} \right\} - \frac{\partial x}{\partial b_0} \\
&= \text{sign} \left\{ \left( \frac{\partial x}{\partial b_0} \right) \left( 1 + \beta \right) \left( \frac{1}{1 + \beta} \right) - \frac{\partial x}{\partial b_0} \right\} \\
&= 0.
\end{align*}
\]

Therefore, the cutoff for \( \left( \frac{\theta^H - \theta^L}{\delta} \right) \) at which separation of types is optimal does not change as outstanding debt \( b_0 \) increases.

In the case with a continuous distribution of types, the analysis is analogous to the two-shock case. The principal optimizing implies that the changes marginal utilities due to \( b_0 \) must be equalized across \( \theta u(x), \delta w(y) \), and \( \mathbb{E}[V(b')] \), such that the agent’s incentive compatibility constraint still binds with equality. Therefore, the overall effect on \( \theta^* \) must be zero, as the benefits and costs suffer the same marginal change.
3.A.6 Proof of Corollary 45

Let the upper limit of debt be \( \bar{b} < \frac{\gamma}{1+\beta} \). As before, the cutoff is given by the condition:

\[
\left( \frac{\theta^H - \theta^L}{\delta} \right) = -\frac{\beta \left( \frac{1}{\delta} - 1 \right) \theta^M}{\left| u(x^H) - u(x^*) \right|} \left[ u_1(b^*) - u_1(b^H) \right].
\]

Assume an increase in \( b_0 \) such that the limit \( \bar{b} \) becomes binding. In this case the increase in \( b_1 \) in response to the increase in \( b_0 \) is \( \frac{\partial b_1}{\partial b_0} < \frac{1}{1+\beta} \) since \( \frac{\partial b_1}{\partial b_0} \leq \bar{b} \). Then, following the steps in Proposition 44, it must be that

\[
\frac{\partial}{\partial b_0} \left( \frac{u_1(b^*) - u_1(b^H)}{\left| u(x^H) - u(x^*) \right|} \right) < 0.
\]

Then \( \left| \frac{\partial}{\partial b_0} \left( \frac{\theta^H - \theta^L}{\delta} \right) \right| < 0 \), which implies the cutoff \( \tilde{\delta} \) increases, since \( \theta^H - \frac{\theta^L}{\delta} < 0 \).

The analysis for the case with continuous shocks is analogous.

3.A.7 Proof of Proposition 48

Given the lack of commitment from both agents, the problem for the principal is the same every period, with the exception of the initial budget available each period. The only state variable in the principal's value function is the outstanding level of debt \( b_0 \). Yet, by Proposition 44, the level of debt does not affect the cutoff value \( \theta^* \). Hence, the region in which policy does not depend on the local shock is the region where \( \theta \geq \theta^* \).

The values of policies \( x_t, y_t, \) and \( b_{t+1} \) depend on the available budget, given the first-order conditions to the principal's problem. Hence, the size of the information rents changes over time as outstanding debt changes, but the cutoff \( \theta^* \) does not.
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