Essays on International Macroeconomics

by

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Abstract

This thesis examines several aspects of open economies. The first two chapters are about sovereign debt and its interactions with domestic financial markets. The third chapter, co-authored with my classmate Daniel Rees, studies volatility in terms of trade.

The first chapter studies how the introduction of new assets in sovereign debt markets can increase a country's level of investment and welfare. In the model presented in this chapter public debt has a liquidity purpose for the domestic private sector and is demanded as a saving vehicle by more patient international investors. The government commits to repay but is constrained by its fiscal capacity which is low when the private sector needs outside liquidity. I find that the government can increase domestic investment by tranching its fiscal capacity, increasing the number of assets supplied and introducing state-contingency or safe assets. In this chapter I also test the predictions of the model and find that domestic collateral constraints and international discount factor both play a significant role in determining the share of public debt held by non-residents and that there is a significant differential effect for countries that have introduced more financial innovation in sovereign debt markets.

The second chapter studies the implications of bailout policy tools for sovereign default and public default risk in a model where, similarly to chapter 1, public debt has a liquidity purpose in financial markets. In this chapter, I show that the government might default for strategic reasons if it can bailout its financial system. It does so when investment and output are low in the economy and when available credit in financial markets is below optimal. The model in this chapter delivers the empirical evidence that financial crises precede sovereign debt crises and it generates the qualitative evidence that, first, private credit drops before sovereign defaults, second, government defaults in periods of low output and, finally, bailout policies affect public debt sustainability.

The third chapter, co-authored with my classmate Daniel Rees, examines the consequences of changes in the volatility of commodity price shocks on commodity exporters. We first demonstrate the existence of time-varying volatility in the terms of trade of a selection of commodity-exporting small open economies. We then show empirically that increases in terms of trade volatility trigger a contraction in domestic consumption and investment and an improvement in the trade balance in these economies. Finally, we construct a theoretical model and demonstrate that it can replicate our empirical results.

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Chapter 1

Financial Innovation in Sovereign Borrowing and Public Provision of Liquidity

1.1 Introduction

The set of instruments that governments all over the world issue is large and has expanded over time. Governments issue debt with different maturities, bonds indexed to inflation or to some reference interest rate and some countries issue debt in different currencies. Financial innovation has transformed sovereign debt markets of advanced and emerging economies.

This process of financial innovation is still ongoing. To give a recent example, the United States approved in July 2013 the issuance of Floating Rate Notes (FRNs) indexed to the 13-week US Treasury bill auction rate and the first auction of this type of securities is expected to be held in January 2014.

The timing, circumstances and country characteristics of governments introducing financial innovations in sovereign debt markets differ widely. For instance, inflation-indexed bonds bonds are issued by emerging economies as well as advanced economies. Some of them
started issuing them in the nineties and 2000s and others as early as the forties. Moreover there is no systematic distinction in the timing across advanced and emerging economies (Borensztein et al. (2004)). A big proportion of emerging markets' borrowing is done in foreign currency but several advanced economies also issue part of their debt in a foreign currency. See Appendix 1. E.2 for some examples.

Another relevant characteristic of sovereign debt markets is that they are open to a large variety of investors. A common distinction made is between domestic and foreign holders of debt and within each of these whether it is the official sector, mostly Central Banks; the financial sector or the non-financial sector. These investors might differ in their degree of patience or in their rationale to hold public debt: as a vehicle to save, as a way to store liquidity or as a policy tool.

This paper combines the two previous observations and studies how the composition of public debt investor base, in particular local vis-a-vis foreign debt holders, can shape the government's financial innovations in sovereign debt markets.

I propose a model where the local private sector uses public debt to hoard liquidity for a future and uncertain liquidity shock in the spirit of Holmstrom and Tirole (1998), Woodford (1990), Gennaioli et al. (forthcoming) or Angeletos et al. (2012). In contrast with these models, I assume that sovereign debt markets are open to more patient risk-neutral international investors who demand the public assets as a savings vehicle. Finally, I assume that the government’s future fiscal capacity is uncertain which, absent financial innovation, renders public debt risky.

The main result of the paper is that the government can increase domestic investment if instead of issuing one bond which pays off its risky fiscal capacity in the next period, it tranches its fiscal capacity and issues two different assets. For instance a safe and a risky asset or two Arrow-Debreu securities. The intuition for this result is that these new asset combinations lower the cost of liquidity hoarding for the private sector. This increases domestic investment and welfare. The residual fiscal capacity is designed to attract risk-neutral international investors who do not have a liquidity motive for holding debt and are willing to hold riskier debt instruments.
This model is consistent with recent changes in ownership of public debt as a whole and differences in the investor composition for different debt instruments. First, it is consistent with the sudden reversal in the share of government debt held by non-residents for the Euro periphery as reported in Arslanalp and Tsuda (2012) and Merler and Pisani-Ferry (2012) and shown in figure 1.1. Through the lens of the model this can be explained by a drop in government’s fiscal capacity or by a tightening of domestic collateral constraints since both would bring about an increase in domestic demand for public debt.

Second, it is consistent with recent ownership shifts towards foreign investors of riskier debt instruments. Some relevant examples include the increase in local currency debt issued by emerging economies which is held by non-residents. Indeed, Du and Schreger (2013) report that the share of LC debt in total emerging market debt trading volume has increased from 35% in 2000 to 71% in the 2011 (see figure 1.2).

Regarding advanced economies, we have seen a similar behavior for inflation protected securities (TIPS) in the US. Between 2000 and 2008 this fraction fluctuated between 5% and 20% of total outstanding TIPS (Bondwave (2010)). As we see in figure 1.3 after the onset of the financial crisis in 2008 this proportion has increased steadily. It rose to almost 30% in 2009 and 2010 and has reached 36% in 2012.

\[1\] For the purpose of this paper, riskier sovereign debt instruments are those whose payment is cyclical, that is, they pay more in good states of the world.
Figure 1.2: Emerging markets offshore trading volume by instrument type, in Trillions of USD (Source: Du and Schreger (2013))

This shift towards riskier debt instruments in foreign holdings can be rationalized from the perspective of the model by increases in liquidity needs or drops in fiscal capacity.

In the empirical front this paper tests the relationships delivered by the benchmark model about proportion of debt held abroad and domestic liquidity conditions. Country-level data for advanced economies between 2004 and 2011 on investor base composition, credit conditions and risk premia confirm the predictions of the model.

The paper is organized as follows. The remainder of this section discusses the related literature. Section 1.2 introduces the model and presents the benchmark scenario with only one public bond. Section 1.3 presents the benevolent government’s general financial innovation problem. This section also proposes combination of assets that will implement the optimal allocation from the planner’s problem. Section 1.4 highlights the benefits of financial innovation in sovereign borrowing and its complementarities with financial integration. Section 1.5 presents some comparative statics results when the government issues more than one public asset. Section 1.6 tests empirically some of the comparative statics results from the benchmark model and the differential effect on financial innovators in sovereign debt markets and finally section 1.7 concludes.
This paper is related most directly to two strands of literature. First, the model I present builds upon the models about public provision of liquidity such as Holmstrom and Tirole (1998), Holmstrom and Tirole (2011) and Woodford (1990). It also relates to the models about optimal provision of liquidity using public debt such as Aiyagari and McGrattan (1998), Guerrieri and Lorenzoni (2009) or Angeletos et al. (2012). However all the aforementioned papers have studied the optimal quantity of debt and have assumed away default risk or any constraints on fiscal capacity and have also ruled out multiple debt instruments. This paper, by contrast, abstracts from the quantity of debt and focuses on the fiscal capacity dimension and shows how the issuance of different debt instruments can improve liquidity provision for a given quantity of debt.

Second, in the spirit of Allen and Gale (1994) I study how financial innovation can decrease the cost of liquidity hoarding and hence increase investment. To the best of my knowledge there has been little work about financial innovation in sovereign borrowing. Papers such as Sandleris et al. (2011) and Hatchondo and Martinez (2012) have studied GDP-linked bonds regarding sovereign debt sustainability, default incentives and risk-sharing benefits. However these models allow for sovereign default and consider a particular financial innovation, GDP-linked bonds, and their objective is not to solve a general financial innovation
problem for the government. This paper instead does not allow for default and imposes commitment on the government's side but studies a general financial innovation framework. Also, contrary to the papers mentioned above, the model presented in this paper features domestic debt and a liquidity purpose of public debt.

This paper also contributes to the debate about financial innovation in public debt where there have been several proposals to make governments' securities more state-contingency which would improve risk-sharing between debtors and creditors. The most relevant ones have proposed to make debt contingent on commodity prices or another external variable relevant to the country (Caballero (2002, 2003)) or to create securities indexed to GDP (Shiller (1993, 2003)). State-contigency in this model is advisable in this model due to the existence of two different types of investors who hold public debt for two different rationales: liquidity hoarding and saving.

This paper is also related to the literature on shortage of safe assets (Caballero and Krishnamurthy (2009), Caballero and Farhi (2013), Gourinchas and Jeanne (2012)) since the government increases domestic investment and welfare by issuing safe assets.

Finally this paper relates to recent papers on public debt ownership such as Broner et al. (2010), Erce (2012), Gennaioli et al. (forthcoming), Broner et al. (forthcoming) and Brutti and Saure (2013). Most have concentrated on investor base composition and default incentives, especially regarding creditor discrimination. This paper instead focuses on how investor base composition can shape the introduction of heterogenous debt instruments in sovereign debt markets.

1.2 One Defaultable Bond

1.2.1 Set-up

We consider a three period economy with time indexed $t = 0, 1, 2$ and a single good. There are three types of agents: an entrepreneur, a consumer and a foreign investor. All agents are risk-neutral and get utility from consumption in all three periods. The first two agents
have a discount factor of $\beta = 1$ whereas the foreign investor's discount factor is denoted by $\beta^* > 1$.

At date 0 the entrepreneur invests in a project of variable scale and chooses initial investment scale $I$. Her initial net worth equals $A$. The consumer has a big endowment that can lend to the entrepreneur. At date 1 the project is hit by a liquidity shock $s$ which makes the project require an injection of $s$ units of good per unit of initial investment to continue. The liquidity shock can take two values $\{s_H, s_L\}$, where $s_H > s_L$, with respective probabilities $\{\lambda, 1 - \lambda\}$. The entrepreneur after the project is hit by the liquidity shock can decide the continuation scale of the project $i(s) \in [0, I]$. At date 2 the project gives a private return to the entrepreneur of $R > 1$ for each unit of investment that was carried through to date 2. From this return only $\rho < R$ is pledgeable to consumers.

The entrepreneur's chooses initial investment $I > A$. Under $R > 1$ the project earns a higher return than the market rate. Therefore the entrepreneur wants to be a net borrower and invest more than her own wealth. At date 0 the entrepreneur borrows from consumers $I - A$ as well as the cost of insuring for the future liquidity shock. The foreign investor cannot lend to the domestic entrepreneur.

**Assumption 1.** *International investors cannot lend directly to domestic entrepreneurs.*

*Only domestic consumers can do so.*

This assumption is a reduced form way to capture that domestic consumers have a comparative advantage in lending to domestic entrepreneurs with respect to foreign investors. Borrowing from abroad would be too costly for entrepreneurs. This might capture domestic consumers having better information about the entrepreneur's project or the domestic conditions in which the entrepreneur's project is carried out, better monitoring capacity or a stronger protection by domestic bankruptcy laws.

At date 0 the government issues public bonds which give a return at date 1. The supply is fixed and normalized to one. The domestic entrepreneur will demand the bond as a way to insure against the future liquidity shock. Foreign investors also demand these bonds in an integrated sovereign debt market but they do not have a liquidity motive for holding debt. Instead, they buy bonds as a saving vehicle.
Assumption 2. The only available security for the entrepreneur to insure against the liquidity shock is the public bond. The supply of the public bond is fixed and it is issued in an integrated debt market, where entrepreneurs compete for the asset with more patient foreign investors.

The first part of this assumption acts only as a simplification. All the results of the model would still hold even if entrepreneurs had access to other assets as long as the value of the assets is not enough to fulfill the entrepreneur's liquidity needs. This simplification is especially applicable to financial crises when other asset prices collapse and sovereign debt becomes a highly valued asset due to its safety and liquidity (IMF (2012), Krishnamurthy and Vissing-Jorgensen (2012)) or when for regulatory reasons public debt is a relatively cheap asset as Acharya and Steffen (2014) analyze for Eurozone banks during the recent financial crisis.

The supply of bonds being fixed can be interpreted as the public borrowing needed to cover an exogenous and fixed level of government expenditures. The focus of this paper is the relative holdings of public debt between internationals and domestics and how the existence of both types of investors shapes the introduction of financial innovation in sovereign debt markets. Thus, we are going to abstract from the bond supply decision and take it as exogenous and fixed.

The last part of assumption 2, the fact that international investors are more patient than domestics, should not be taken literally. It is a reduced form to capture a higher foreign willingness to hold sovereign debt. When performing comparative statics, an increase in the international discount factor can be interpreted as an increase in world risk or as an increase in the available income internationally to invest in sovereign debt.

The government issues the bond at date 0 and receives $q$ units of good per bond which it transfers to consumers. It commits to repay and redeems the bond at date 1 by taxing consumers and repaying bond holders the face value of their bond. The government's taxation power or fiscal capacity at $t = 1$ is uncertain and perfectly correlated with the liquidity shock that hits the entrepreneur's project. In particular at date 1, the government can tax $\hat{\eta}$ when $s = s_L$ and $\eta$ when $s = s_H$, where $\eta < \hat{\eta} \leq 1$. 
Assumption 3. The fiscal capacity shock and the private sector liquidity shock are perfectly correlated.

Since the government commits to repay the bond issues in date 0, the payoff structure of the public bond is given by \((\eta, \bar{\eta})\) in states \((s_H, s_L)\) respectively. This is a risky bond that pays less in the state of the world where liquidity needs are high. In other sections we will introduce more than one asset with different payoff structures.

Assumption 3 is a reduced form way to capture the observed temporal connection between banking crises and sovereign debt crises reported in Reinhart and Rogoff (2009), Arellano and Kocherlakota (2012), Sosa-Padilla (2011), Balteanu and Erce (2012) and Borensztein and Panizza (2008).²

The model in this paper does not feature sovereign risk in the sense of willingness to pay. That is, a bond with a safe income stream in the future but subject to default risk due to government’s lack of commitment. Instead it concentrates on a bond that is issued as risky asset from date 0 and which the government commits to repay. However the payoff structure of the bond with commitment but risky fiscal capacity is observationally equivalent at date 1 to a model where the government issues one bond and imposes a haircut \(i - q\) in one of the states of the world.

Thus, according to this assumption in one of the states of the world the financial sector requires liquidity, which absence intervention could result in a banking crisis where profitable projects would need to be liquidated. It is in that same state of the world that the government is forced to repay a smaller amount than it would have if the financial sector would have not required liquidity, \(\eta < \bar{\eta}\).

This assumption makes the analysis of the model highly tractable and effective in capturing the empirical association between liquidity crises in the private sector and lower repayment of public debt. This comes at a cost, namely, I abstract away from strategic default and concentrate exclusively on ability to repay.

²Several of these papers explore whether banking crises preceded sovereign debt crises or vice versa. For the purpose of this model we assume that both happen simultaneously.
In Appendix 1. D I relax this assumption and study the case where the fiscal capacity and the private sector liquidity shock are positively but not perfectly correlated.

1.2.2 Demand from Investors

1.2.2.1 Demand from Domestic Investors

The entrepreneur maximizes her expected net return of the project. Since the entrepreneur wants to maximize the initial investment scale of the project it is optimal to assign all pledgeable returns $\rho$ to consumers, keeping the illiquid return for herself: $R - \rho$ of the amount that is carried through.

Denoting by $q$ the price of the liquid asset at $t = 0$, the entrepreneur’s problem is given by:

$$
\begin{align*}
\max_{\{I, i(s_H), i(s_L), z\}} & \quad (R - \rho)(1 - \lambda)i(s_L) + (R - \rho)i(s_H) \\
\text{s.t.} & \quad (\rho - s_L)(1 - \lambda)i(s_L) + (\rho - s_H)i(s_H) + (\lambda \eta + (1 - \lambda)\bar{\eta})z \geq I - A + zq \\
& \quad i(s_H)(s_H - \rho) \leq \eta z
\end{align*}
$$

where $i(s_L)$ and $i(s_H)$ are the continuation scales in both states and $z$ is the amount of bonds bought at $t = 0$.

The first constraint is the entrepreneur’s budget constraint by which the entrepreneur’s initial investment scale plus the purchase of the assets $qz$ need to be less or equal than the entrepreneur’s initial wealth plus the expected net return from the project and the expected return from the asset. It corresponds to the consumer’s participation constraint. In order for the consumer to be willing to lend to the entrepreneur at date 0 its expected return from the project must be at least what the entrepreneur borrowed at date 0, $I - A + zq$.

$^3$The contract between entrepreneur and consumer will always assign all liquid or pledgeable returns to the consumer in order to maximize the initial investment scale of the project. Thus, in state $s = s_L$ the consumer will get a repayment of $\eta z + (\rho - s_L)i(s_L)$ and in state $s = s_H$ the consumer obtains $\eta z + (\rho - s_H)i(s_H)$ which corresponds to the left hand side of the budget constraint.
The second constraint of the problem is the collateral constraint which imposes that the outside funds required for reinvestment in the high liquidity shock state are less or equal to the return from the liquid asset in that state of the world.

I assume that $s_L < \rho < s_H < R$ which implies that when the liquidity shock is low the project is self-financed and entrepreneur's inside liquidity is enough to withstand the liquidity shock. Instead when the liquidity shock is high the project needs prearranged financing. Thus in state $L$ full continuation is always optimal, $i(s_L) = I$ while in state $H$ full continuation might not be optimal. Denoting $\frac{i(s_H)}{\Pi} \equiv \chi$ and $\lambda \bar{\eta} + (1 - \lambda) \tilde{\eta} \equiv \Pi$ we can rewrite the entrepreneur’s problem as:

\[
\max_{I, \chi, z} \quad (R - \rho)(1 - \lambda + \lambda \chi)I \\
\text{s.t.} \quad (\rho - s_L)(1 - \lambda)I + \lambda \chi(\rho - s_H)I + \Pi z \geq I - A + zq \\
\chi I(s_H - \rho) \leq \Pi z
\]

When $q > \Pi$, both constraints bind. Therefore, the collateral constraint expresses the amount of bonds demanded by entrepreneurs at $t = 0$ in terms of the initial investment scale $I$ and the continuation scale $\chi$ as well as parameters:

\[
z = \frac{\chi I(s_H - \rho)}{\Pi}
\]

Intuitively the amount of bonds demanded is increasing in the continuation scale $i(s_H)$ and decreasing in the bond’s repayment fraction in the high liquidity need state of the world, $\Pi$.

Using this expression for $z$ in the budget constraint, the initial investment is given by:

\[
I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda \chi(\rho - s_H) + \frac{\chi(s_H - \rho) \Pi}{q - \Pi}}
\]

From (1.3) we see that $I'(\chi) < 0$, so the entrepreneur faces a scale-liquidity trade-off as in Holmstrom and Tirole (1998). If the entrepreneur wants to hold more liquidity to withstand
the future liquidity shock she has to choose a lower initial investment scale since both liquidity hoarding and initial investment scale are chosen at date 0.

The equivalent to maximizing the expected return to the investment is to minimize the unit cost of investment

\[
\min_{\chi} \quad c(\chi, q, \eta, \bar{\eta}, \lambda, \Theta) = \frac{1 + s_L(1 - \lambda) + s_H + \frac{\chi(s_H - \rho)}{\eta}[q - \Pi]}{1 - \lambda + \lambda \chi}
\]

where \( \Theta \equiv (s_H, s_L, \rho) \) is a vector of parameters regarding the project. The solution to this problem depends on the price of the bond \( q \):

\[
\chi(q, \eta, \bar{\eta}, \lambda, \Theta) = \begin{cases} 
1 & \text{if } q \in [\Pi, q^{\max}) \\
\in (0,1) & \text{if } q = q^{\max} \\
0 & \text{if } q > q^{\max}
\end{cases}
\]

where \( q^{\max} \) is given by

\[
\frac{\partial c(\chi, q^{\max}, \eta, \bar{\eta}, \lambda, \Theta)}{\partial \chi} = \lambda \eta + (1 - \lambda) \bar{\eta} + \frac{\lambda(1 + s_L(1 - \lambda) - s_H)\eta}{(s_H - \rho)(1 - \lambda)}.
\]

Thus, the demand for liquidity from local investors is given below and it is denoted by \( z^L \). It is decreasing in its price \( q \):

\[
z^L(q, \eta, \bar{\eta}, \lambda, \Theta) = \begin{cases} 
0 & \text{if } q > q^{\max} \\
\frac{\chi(s_H - \rho)}{\eta} & \text{if } q = q^{\max}, \text{ where } \chi \in (0,1) \\
\frac{I(s_H - \rho)}{\eta} & \text{if } q \in (\Pi, q^{\max})
\end{cases}
\]

where \( I \) is given by substituting the price \( q \) in (1.3).

### 1.2.2.2 Demand from International Investors

The demand from foreign investors, \( z^F \), is given by their valuation of the bond, which is determined by their discount factor and the bond’s expected payoff \( \lambda \eta + (1 - \lambda) \bar{\eta} = \Pi \). Their demand for bonds is perfectly elastic at \( q = \beta^* \Pi \), they will demand any positive amount of bonds as long as \( q = \beta^* \Pi \).
1.2.3 Market Clearing

Market clearing in the bond market at $t = 0$ implies that $z^L(q, \Pi, \bar{\eta}, \lambda, \Theta) + z^F(q, \Pi, \bar{\eta}, \lambda) = 1$. Necessarily $q \geq \beta^* \Pi$, otherwise the demand from international investors would be infinite. In this section we concentrate on the case where both types of investors hold part of the debt issued and the project is fully continued in both states of the world, $\chi = 1$. This is equivalent to making the following parametric assumptions about the fiscal capacity in the bad state of the world where the private sector is hit by the high liquidity shock:

$$\Pi > \frac{(s_H - \rho)(\lambda - (1 - \lambda)\bar{\eta})}{1 - (\rho - s_L)(1 - \lambda) + \beta^* \lambda (s_H - \rho)} \quad (1.4)$$

and about the foreign discount factor

$$\beta^* < 1 + \frac{\lambda (1 + s_L(1 - \lambda) - s_H) \Pi}{(s_H - \rho)(1 - \lambda) \Pi} \quad (1.5)$$

The first condition ensures that there is enough liquidity for both types of investors to hold the public debt and the second condition ensures that foreigners do not value public debt too much and crowd-out domestic demand for public debt and the liquidity hoarding motive. In Appendix 1. A we characterize the equilibria for the cases where (1.4) and (1.5) do not hold.

When (1.4) holds international investors hold part of the supplied public bonds. Thus, their valuation pins down the price of debt: $q = \beta^* \Pi$. The bond is sold at a premium, that is, $q - \Pi > 0$, because $\beta^* > 1$. If (1.5) also holds then $q = \beta^* \Pi < q^{max}$. Thus, at date 0 the demand from international investors is given by the section of the domestic demand for bonds where $q \in (\Pi, q^{max})$. In that case $\chi = 1$, the project is fully continued. Substituting this and the price for the public asset in (1.3) we obtain the initial scale of investment which is given by

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda (\rho - s_H) + (s_H - \rho)(\beta^* - 1)(\lambda + (1 - \lambda)\bar{\eta}) \Pi} \quad (1.6)$$
and is proportional to the entrepreneur's initial wealth $A$. It is multiplied by the equity multiplier

\[
\frac{1}{1-(s_H-p)(1-\lambda) - (\rho-s_H)(\beta^*-1)(\lambda+(1-\lambda)\frac{\eta}{\Pi})} > 1
\]

which defines the maximum leverage per unit of own capital.

We see that the maximum leverage per unit of own capital is increasing in the pledgeable return $\rho$. It is decreasing in the total expected cost of the project $1 + s_L(1 - \lambda) + s_H\lambda$ and the cost of liquidity hoarding which is the given by last summand in the denominator in (1.6).

Two points are worth highlighting regarding the cost of liquidity hoarding. First, for the liquidity hoarding to have an effect on the equity multiplier and decrease investment it is key that foreign investors are more patient than domestics. If $\beta^*$ were to equal 1 the level of investment would equal $\frac{A}{1-(\rho-s_L)(1-\lambda) - (\rho-s_H)}$ and would not be affected by the demand and cost of liquidity. The intuition for this is that if foreign investors were as patient as domestics, since they do not demand public debt as a way to hoard liquidity but in order to save, they would drive the premium at which public debt is sold, $q - \Pi$, to 0. The domestic entrepreneurs would be able to buy liquidity at no cost. This would increase the level of investment.

Second, the novel relationship that this model delivers is that the investment level is decreasing in the ratio of fiscal capacities in the low and high liquidity need states. With only one public bond the $\frac{\eta}{\Pi}$ ratio parametrizes the amount of wasted liquidity. Wasted liquidity is the amount of useless liquidity that the entrepreneur is forced to purchase when she does not need it ($\bar{\eta}$) for each unit of liquidity she buys for the state when she does need the return ($\eta$). Equilibrium investment is decreasing in the wasted liquidity, the lower this quantity is, the higher investment. The intuition for this is that wasted liquidity increases the cost of liquidity hoarding.

Finally the amount of bonds demanded by local entrepreneurs is $\frac{I(s_H-p)}{\Pi}$ where substituting $I$ for its expression from (1.6) and rearranging we obtain:

\[
\frac{z}{\Pi} = \frac{A(s_H - \rho)}{\eta[1 - (\rho - s_L)(1 - \lambda) - (\rho - s_H)] + (s_H - \rho)(\beta^* - 1)\Pi}
\]

which is decreasing in the asset returns $\eta$ and $\bar{\eta}$.
At price $q = \beta^*\Pi$ international investors will demand any residual bonds not demanded by locals. By market clearing the quantity of bonds demanded by internationals is the following:

$$z^F = 1 - z^L \quad (1.8)$$

### 1.2.4 Comparative Statics

A number of comparative statics are interesting to understand the workings of the model and will be relevant for the empirical analysis. An increase in $s_H$ and a decrease in $s_L$ such that the total cost per unit of investment, $1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)$, remains constant brings about an increase in the amount of debt held by locals and by market clearing, a decrease in the amount of debt in the hands of international investors. An increase in $s_H$ is akin to a tightening of domestic collateral constraints which increases the need for public liquidity, increasing the demand for bonds at home. Initial investment scale $I$ decreases because of the scale-liquidity trade-off: the higher the reinvestment shock in bad times, the higher the liquidity provision that the entrepreneur must make at date 0 and thus the lower the initial investment scale the entrepreneur can choose.

For the purpose of the comparative statics we can set $\bar{\eta} = 1$. This implies that in the good state when liquidity needs are low the government can fully redeem the public bond by taxing consumers. In the bad state when private liquidity needs are high the government bond pays less, the bond pays $\bar{\eta} \equiv \eta < 1$. An increase in the repayment fraction $\eta$ decreases the amount of bonds held by domestics, since local demand for public bonds is decreasing in the amount the bonds repay. By market clearing the amount of debt held by international investors increases. The total amount of liquidity held by domestic entrepreneurs, $\eta z_L$ also increases since

$$\eta z_L = \frac{A(s_H - \rho)}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)(\beta^* - 1)(\lambda + \frac{1 - \lambda}{\eta})}$$
when \( q = \beta^* \Pi \). This amount is increasing in \( \eta \). Investment increases because of the wasted liquidity force described before. An increase in \( \eta \) implies a return at \( s = s_H \) closer to 1 which lowers wasted liquidity purchased by the entrepreneur for state \( s = s_L \).

Finally, an increase in the patience of international investors parametrized by an increase in \( \beta^* \) decreases the local demand for bonds and lowers domestic investment. A higher international discount rate increases the price of the public bond for domestic entrepreneurs too because debt markets are integrated. Domestic investment is also lower when international investors become more patient. The increased demand from international investors crowds-out domestic demand for bonds and domestic investment. The comparative statics with \( \beta^* \) is a reduced form way to capture an increase in the foreign demand for domestic public bonds.

We summarize the comparative statics results in the following proposition:

**Proposition 4.** For a given level of expected cost of investment, \( 1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) \):

1. \( \frac{\partial I}{\partial p} > 0 \), \( \frac{\partial I}{\partial s} < 0 \) and \( \frac{\partial I}{\partial \eta} < 0 \); (ii) \( \frac{\partial I}{\partial p} < 0 \), \( \frac{\partial I}{\partial s} > 0 \) and \( \frac{\partial I}{\partial \eta} > 0 \); (iii) \( \frac{\partial I}{\partial \beta^*} < 0 \), \( \frac{\partial I}{\partial \beta^*} > 0 \) and \( \frac{\partial I}{\partial \beta^*} < 0 \).

### 1.3 Financial Innovation

The comparative statics for \( \beta^* \) implied that an increase in the foreign demand for public debt would crowd-out domestic investment and domestic demand for the public asset. I now discuss how the government can mitigate this effect, and more generally improve its provision of domestic liquidity, by introducing multiple debt instruments. In this section we suppose that the government issues two assets and chooses payoffs \((x_1^H, x_1^L)\) and \((x_2^H, x_2^L)\) respectively in states \( s = s_L \) and \( s = s_H \) to maximize total welfare.\(^4\)

\(^4\)In appendix 1. B we show that imposing capital controls and banning all competition from abroad will not increase welfare. As we discuss there, the reason for this is that there are no pecuniary externalities in the entrepreneur’s collateral constraint.
1.3.1 Entrepreneur's Problem

We now write the entrepreneur's problem with two assets available as liquidity vehicles, which is a generalization of Problem (1.1):

\[
\max_{\{x,z_1,z_2\}} \quad (R - \rho)(1 - \lambda + \lambda \chi)I \\
\text{s.t.} \quad \begin{align*}
(p - s_L)(1 - \lambda)I + \lambda \chi (\rho - s_H)I + \Pi_1 z_1 + \Pi_2 z_2 & \geq \\
I - A + z_1 q_1 + z_2 q_2 & \\
\chi I (s_H - \rho) & \leq x_1^H z_1 + x_2^H z_2
\end{align*}
\]

where \(\Pi_1\) and \(\Pi_2\) denote the expected payoffs of both assets: \(\Pi_1 = \lambda x_1^H + (1 - \lambda)x_1^L\) and \(\Pi_2 = \lambda x_2^H + (1 - \lambda)x_2^L\).

**Proposition 5.** When \(q_1 > \Pi_1\) and \(q_2 > \Pi_2\) both constraints bind.

**Proof.** To see this, note that \(q_1 \geq \Pi_1\) and \(q_2 \geq \Pi_2\). The price of the assets can never go below their expected values, otherwise consumers who are assumed to have a big endowment would want to postpone all their consumption to date 1. This would drive the price of the liquid assets to their date 1 values, which are the expected values. If \(q_1 > \Pi_1\) or \(q_2 > \Pi_2\), only entrepreneurs will demand the assets since they have a higher valuation for the assets. To see that if \(q_1 > \Pi_1\) and \(q_2 > \Pi_2\) the budget constraint must bind we rewrite it as:

\[
\rho(1 - \lambda + \lambda \chi)I - s_L(1 - \lambda)I - s_H \lambda \chi I \geq I - A + (q_1 - \Pi_1) z_1 + (q_2 - \Pi_2) z_2 \quad (1.10)
\]

Since \(\rho(1 - \lambda + \lambda \chi)I\) enters negatively the entrepreneur's objective function, she will make this term as small as possible choosing to just satisfy the constraint. The collateral constraint binds for \(q_1 > \Pi_1\) and \(q_2 > \Pi_2\) because the entrepreneur will choose \(z_1\) and \(z_2\) just enough to cover the liquidity shock in the high liquidity shock state. Demanding more than this amount would imply that the right-hand side of the budget constraint in (1.10) increases. Since the budget constraint binds this would increase \(\rho(1 - \lambda + \lambda \chi)\), which is
not in the entrepreneur’s interest given her objective function. Hence, from now on we will consider \( q > \Pi \) for both assets and both constraints will bind.

Denote by \( \ell \) the unit cost of liquidity. The entrepreneur will choose the asset that will minimize her unit cost of liquidity, that is: \( \ell = \min \left\{ \frac{q_1 - \Pi_1}{x_1^I}, \frac{q_2 - \Pi_2}{x_2^I} \right\} \). Suppose for concreteness that asset \( j \) minimizes \( \ell \) and that asset \( j \) provides enough liquidity in state \( s = s_H \) to cover all reinvestment needs. In that case \( z_{-j} = 0 \) since the entrepreneur does not want to purchase the liquidity using the asset that provides it at a higher cost.

We also know that \( q_j > \Pi_j \) because \( q_j \geq \beta^* \Pi_j \), with strict equality if international investors also hold part of asset \( j \) issued. Therefore as we proved above both constraints hold with equality. From collateral constraint we obtain the local demand for the relatively cheap asset:

\[
\frac{\chi I(s_H - \rho)}{x_j} = z_j
\]

Substituting this and \( z_{-j} = 0 \) in the budget constraint and solving for investment we obtain:

\[
I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda \chi (\rho - s_H) + \chi (s_H - \rho) \ell}
\]

The entrepreneur’s unit cost minimization problem is given below:

\[
\min_{\chi} \frac{1 + s_L(1 - \lambda) + s_H \lambda \chi + \chi (s_H - \rho) \ell}{1 - \lambda + \lambda \chi} = c(\ell, \chi)
\]

The solution for the continuation scale \( \chi \) is the following:

\[
\chi = \begin{cases} 
1 & \text{if } \ell \in (0, \ell_{\text{max}}) \\
\in (0, 1) & \text{if } \ell = \ell_{\text{max}} \\
0 & \text{if } \ell > \ell_{\text{max}}
\end{cases}
\]

where \( \ell_{\text{max}} \) is a threshold value. To see that this is the schedule for the continuation scale note that because the problem (1.9) is linear in \( I \) we only need to evaluate the utility
levels corresponding to $\chi = 0$ (continuing only when the shock is low) and $\chi = 1$ (always continuing). The unit cost for $\chi = 0$, $c(\chi = 0, \ell) = \frac{1 + s_L(1 - \lambda)}{1 - \lambda}$ and $c(\chi = 1, \ell) = 1 + s_L(1 - \lambda) + s_H \lambda + (s_H - \rho) \ell$. Comparing these we obtain that $c(\chi = 1, \ell) < c(\chi = 0, \ell)$ if and only if

$$\ell < \frac{\lambda}{(1 - \lambda)(s_H - \rho)} + \frac{(s_L - s_H) \lambda}{s_H - \rho} = \ell^{\text{max}}$$  \hspace{1cm} (1.13)

which equals $\frac{\partial c(\chi, \ell^{\text{max}})}{\partial \chi} = 0$. Therefore when $\ell < \ell^{\text{max}}$, investment as a function of $\ell$ is given by the following expression:

$$I(\ell) = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho) \ell}$$  \hspace{1cm} (1.14)

From expression (1.14) we see that $\frac{\partial I(\ell)}{\partial \ell} < 0$, that is investment level is decreasing in the unit cost of liquidity. We will impose throughout that $\ell < \ell^{\text{max}}$ and the project is continued at full scale in both states of the world.

### 1.3.2 Planner’s Objective and Constraints

Domestic welfare $W$ is given by the utility of consumption in the three periods for both types of agents in the economy, entrepreneurs and consumers, $U^E$ and $U^C$. Both agents have linear utility of consumption and do not discount future payoffs. We assume for this section that the government always wants to fully continue in both states of the world $\chi = 1$.\(^5\)

Entrepreneurs consume the expected rent from their investment at date 2. Consumers lend a part $A - I + q_1 z_1 + q_2 z_2$ of their endowment $E$ to finance the project initially and for the entrepreneur to prearrange for the future liquidity need. They obtain a return of $x^I_1 z_1 + x^I_2 z_2 + (\rho - s_L) I$ when the liquidity shock is low and obtain $x^H_1 z_1 + x^H_2 z_2 + (\rho - s_H) I$ when

\(^5\)As in section 1.2 for this to be optimal we impose an upper bound on the foreign discount factor $\beta^*$ which will ensure that the prices of the public assets are not too high. In particular, if we assume $\beta^* - 1 < \frac{1 + (s_L - s_H)(1 - \lambda)}{(s_H - \rho)(1 - \lambda)}$, $\chi = 1$ will be optimal for the two financial innovations problems that we study in this section.
the liquidity shock is high. Also, at date 0 they obtain the proceeds from the total asset issuance $q_1$ and $q_2$ and are taxed the face value of both assets at date 1.

Thus, the utility expressions are given by:

$$U^E = (R - \rho) I$$
$$U^C = E + A - I - q_1 z_1 - q_2 z_2 + (\rho - s_L)(1 - \lambda) I + (\rho - s_H) \lambda I$$
$$+ (q_1 - \Pi_1) + (q_2 - \Pi_2) + \Pi_1 z_1 + \Pi_2 z_2$$

and total welfare is given by

$$W = E + A + [R(1 - \lambda + \lambda \chi) - s_L (1 - \lambda) - s_H \lambda \chi - 1] I$$
$$+ (q_1 - \Pi_1)(1 - z_1) + (q_2 - \Pi_2)(1 - z_2)$$

By market clearing $1 - z_1$ equals the amount of asset held by international investors, $z_1^F$ and similarly for asset 2.

$$W = E + A + [R(1 - \lambda + \lambda \chi) - s_L (1 - \lambda) - s_H \lambda \chi - 1] I$$
$$+ (q_1 - \Pi_1) z_1^F + (q_2 - \Pi_2) z_2^F$$

The welfare expression in (1.16) is intuitive. The government wants to maximize the total net surplus from the investment and the liquidity premia, $q_1 - \Pi_1$ and $q_2 - \Pi_2$ obtained from international investors. The liquidity premium paid by entrepreneurs to consumers is a transfer across agents which cancels out in the welfare calculation and only the premia coming from abroad matter for welfare in the economy.

The government is constrained by its fiscal capacity, thus asset payoffs must satisfy:

$$x_1^H + x_2^H = \eta$$

$$x_1^H + x_2^H = 1$$
1.3.2.1 Monotonicity Requirement

If payoffs satisfy monotonicity it must be the case that for both assets:

\[ x^L \geq x^H \] (1.19)

We start our analysis without considering this restriction in section 1.3.3. Then, we add the more realistic assumption that public assets pay less in the state of the world that fiscal capacity is low.

In our discussion of assumption 3 in the set-up of the benchmark model in section 1.2 we argued that the state where fiscal capacity is low and the private liquidity shock is high is a state which would correspond to a state of twin crisis, banking and sovereign debt crisis.

Sovereign debt crises are resolved with a sovereign debt restructuring process. We know that after a debt restructuring process haircuts on debt instruments are positive. Allowing for violations of (1.19) would imply that haircuts on some debt instruments are negative. Although there is no systematic data about haircuts at the debt instrument level, this seems unrealistic.

We can imagine assets affected differently after a sovereign debt restructuring. For example, we can expect long-term debt more affected than short-term. Long-term debt due date can be adjourned and the payments will be rescheduled. It is more likely that short-term will be paid-off quicker and hence experience no haircut or a small one. In any case, it seems unlikely that it will have a negative haircut.

Also bonds issued under different laws might differ in the final recouped investment. Those under local law are normally hit stronger by a sovereign debt restructuring than those issued under the UK or US law where creditor litigation has increased dramatically the amount of recouped investment (Schumacher et al. (2013)). Again, however, bonds under UK or US law do not recoup more than they invested after a sovereign debt restructuring process which would be the implication of violating (1.19).
1.3.3 Planner’s Problem without Monotonicity Requirements

The planner maximizes expected welfare (1.16) choosing the asset payoffs and the minimum cost of liquidity hoarding for the entrepreneur, \( \ell \). In doing so the government is subject to the fiscal capacity constraints and internalizes the effect that its choice has on investment, demand for the public assets and prices. The government solves the following problem:

\[
\begin{align*}
\max_{\{x_1^H, x_1^L, x_2^H, x_2^L, \ell\}} \quad & W \equiv E + A + [R - s_L (1 - \lambda) - s_H \lambda - 1] I(\ell) \\
& + (q_1 - \Pi_1)(1 - z_1(\ell)) + (q_2 - \Pi_2)(1 - z_2(\ell)) \\
\text{s.t.} \quad & x_1^L + x_2^L \leq 1 \quad (1.21) \\
& x_1^H + x_2^H \leq \eta \quad (1.22) \\
& I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda (\rho - s_H) + (s_H - \rho) \ell} \quad (1.23) \\
& \ell = \min \left\{ \frac{q_1 - \Pi_1}{x_1^H}, \frac{q_2 - \Pi_2}{x_2^H} \right\} \quad (1.24) \\
& 0 \leq z_j(\ell) \leq 1 \quad (1.25) \\
& q_j \geq \beta^* \Pi_j \quad \text{with inequality only if } z_j(\ell) = 1 \quad (1.26)
\end{align*}
\]

where \( j = \{1, 2\} \) and \( \Pi_j \) are assets' expected payoffs defined above.

Constraints (1.21) and (1.22) are the fiscal capacity constraints. Equation (1.23) gives the expression for investment from the entrepreneur’s problem which depends on the unit cost of liquidity \( \ell \) which is defined in (1.24) and is also a constraint on the planner’s problem.

Constraints (1.25) and (1.26) impose market clearing considerations in the planner’s problem and short-selling constraints. Equation (1.25) imposes that the local demand for asset \( j \), \( z_j(\ell) \) which depends on the cost of liquidity, cannot be bigger than the total supply of asset \( j \) which is normalized to 1. Also, \( z_j(\ell) \geq 0 \) because agents cannot short-sell the public assets. Equation (1.26) is just saying that asset prices will be pinned down by international investors’ valuation if they hold the asset, that is if \( z_j(\ell) < 1 \) and will be strictly above
international investors' valuation only when all of the supplied asset \( j \) is held domestically \((z_j(\ell) = 1)\).

The approach to solve this problem is to solve a slightly modified version with fewer constraints and a modified objective and then check that the solution obtained satisfies initial constraints and that the objective takes the same value in the original objective. Thus, we start by modifying the objective \( W \).

From constraint (1.26) we see that the government is not free to choose any quantity for the liquidity premia coming from foreigners. The maximum the government can obtain from foreigners for each asset \( j \) is \((\beta^* - 1)\Pi_j\). A liquidity premium higher than that would imply that all the supplied assets are held domestically \((z_j(\ell) = 1 \) for both assets\) and the liquidity premia coming from foreigners goes to zero. Note also that \( \Pi_j \leq 1 \). This implies that we can bound the liquidity premia coming from international investors:

\[
0 \leq (q_1 - \Pi_1)(1 - z_1(\ell)) + (q_2 - \Pi_2)(1 - z_2(\ell)) \\
= (\beta^* - 1)(1 - z_1(\ell)) + (\beta^* - 1)(1 - z_2(\ell))
\]

where the first inequality in (1.27) holds with equality only when \( z_1 = z_2 = 1 \).

Therefore we can write a slightly modified welfare objective \( \tilde{W} \) which is an upper bound on \( W \) which is given by:

\[
\tilde{W} \equiv E + A + [R - s_L(1 - \lambda) - s_H\lambda - 1]I + \\
(\beta^* - 1)(1 - z_1(\ell)) + (\beta^* - 1)(1 - z_2(\ell))
\]

The constraint on asset prices contained in (1.25) also imposes a lower bound on the unit cost of liquidity \( \ell \), namely \( \ell \geq \frac{(\beta^* - 1)\Pi_j}{x_j^H} = \frac{(\beta^* - 1)\Pi_j}{x_j^H} = (\beta^* - 1) \left( \lambda + (1 - \lambda) \frac{x_j^H}{x_j^H} \right) \) where I have used the definition of \( \Pi_j \). The lower bound on \( \ell \) is given by:
\[ \ell \geq \lambda (\beta^* - 1) \]  

(1.29)

with equality only when \( x_j^I = 0 \) for asset \( j \).

Using the modified objective (1.28) and the lower bound on unit cost of liquidity (1.29) we can rewrite the previous planner problem (1.24) in terms of the implemented fiscal capacity allocation between domestics and foreigners in both states of the world.

Denote fiscal capacity allocations by \( F \). In \((F_D^L, F_D^H, F_F^H)\) superscripts denote state of the world and subscripts denote the holder. Then \( F_D^L \) denotes how much of the fiscal capacity in \( s = s_L \) is held domestically and \( F_F^H \) denotes how much of that capacity is held abroad and similarly for \( F_D^H \) and \( F_F^H \). Finally denote \((\ell^L, \ell^H)\) as the unit cost of liquidity in both states of the world.

\[
\max_{\{\ell^H, F_D^L, F_D^H, F_F^H\}} \quad E + A + [R - s_L (1 - \lambda) - s_H \lambda - 1] I + \lambda (\beta^* - 1) F_F^H + (1 - \lambda)(\beta^* - 1) F_D^L
\]

(1.30)

s.t.

\[
I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda (\rho - s_H) + (s_H - \rho) \ell^H}
\]

(1.31)

\[
F_D^H \geq I (s_H - \rho)
\]

(1.32)

\[
\ell^H \geq \lambda (\beta^* - 1)
\]

(1.33)

\[
F_D^H + F_F^H \leq \eta
\]

(1.34)

\[
F_D^L + F_F^L \leq 1
\]

(1.35)

The planner’s problem given by (1.36) above maximizes the returns from investment and the liquidity premia coming from abroad subject to the behavior coming from the entrepreneur’s problem. In particular (1.31) gives the expression for investment in terms of parameters and the liquidity premium in state \( s = s_H \). Constraint (1.32) rewrites the collateral constraint in terms of the new variables: it is just saying that the amount of fiscal capacity held by domestics in state \( s = s_H \) has to be greater or equal than the reinvestment
need in that state of the world. Constraint (1.33) gives the lower bound for the liquidity premium that we obtained above. Finally the government needs to satisfy the fiscal capacity constraints (1.34) and (1.35).

To solve this problem, first note that the objective function is increasing in investment \( I \). Since \( I'(\ell^H) < 0 \) the planner chooses the minimum possible \( \ell^H \), that is: \( \ell^H = \lambda (\beta^* - 1) \) from (1.33) with equality. This pins down investment \( I = \frac{A}{1-(\rho-s_H)(1-\lambda) + \lambda (\rho-s_H)(s_H-\rho)(\beta^* - 1)} \equiv I^{**} \). The planner’s aim is to make \( F^H_D \) as small as possible. From (1.34) we see that increasing \( F^H_D \) necessarily decreases \( F^L_D \) because fiscal capacity is limited and this lowers the objective function. Then (1.38) will hold with equality, \( F^H_D = I^{**}(s_H - \rho) \). Finally, it is optimal to make \( F^L_D = 0 \) in order to maximize the liquidity premium coming from abroad as we see from (1.41). Finally fiscal constraints (1.40) and (1.41) also hold with equality or else fiscal capacity would be wasted and the planner’s objective would be lower.

Therefore the optimal fiscal capacity allocation without monotonicity constraints is given in the proposition below.

**Proposition 6.** Without monotonicity constraints on asset payoffs, the optimal fiscal capacity allocation is given by:

\[
\begin{align*}
F^H_D &= I^{**}(s_H - \rho) \\
F^L_D &= 0 \\
F^H_F &= \eta - I^{**}(s_H - \rho) \\
F^L_F &= 1
\end{align*}
\]

where \( I^{**} = \frac{A}{1-(\rho-s_H)(1-\lambda) + \lambda (\rho-s_H)(s_H-\rho)(\beta^* - 1)} \) and it is the level of investment pinned down by \( I(\ell^H = \lambda (\beta^* - 1)) \).

The allocation provides just enough liquidity domestically to attain the desired level of investment. When the government is not constrained by monotonicity this implies that
the government does not supply any liquidity to domestics when the private liquidity is enough to achieve the desired level of investment. Hence the null allocation of liquidity in the low-liquidity need state of the world when the project is self-financed. The rest is allocated abroad to maximize the capital flows coming from abroad. Note that liquidity premia coming from abroad are a positive income flow for consumers who obtain the value of both assets from abroad at date 0 and then are taxed the payoff that the government needs to repay which is lower than what they obtained because $\beta^* > 1$.

1.3.4 Planner’s Problem with Monotonic Asset Payoffs

The problem when the planner is not subject to monotonicity requirements is simply (1.20) including a monotonicity constraint. To solve the problem in terms of fiscal capacity allocations we still solve a slightly modified version of the problem where we impose an upper bound welfare $\tilde{W}$ like in (1.28).

However the lower bound on unit cost of liquidity is now smaller than (1.29). To see this note that $\ell \geq \frac{(\beta^* - 1)\Pi_j}{x_j^H} = (\beta^* - 1) \left( \lambda + (1 - \lambda) \frac{x_j^L}{x_j^H} \right)$ which under monotonicity requirements $x_j^L \geq x_j^H$ cannot be smaller than $\ell \geq \beta^* - 1$. This condition holds with equality when $x_j^L = x_j^H$. We see that when the government is constrained by monotonicity the minimum payoff in state $s = s_L$ it can choose is the same as in $s = s_H$, which increases the minimum unit cost of liquidity it can attain.

The problem in terms of fiscal capacity allocations is the following:
The planner's problem given by (1.36) is very similar to the one with no monotonicity requirements. The only differences are a higher upper bound on the liquidity premium (1.39) and the monotonicity constraints (1.42). They require the amount of fiscal capacity held by both types of agents, domestics and foreigners, in the low liquidity shock state be greater or equal to what they hold in the high liquidity shock state.

The reasoning to obtain the solution is also very similar to above. The government wants to minimize \( \ell^H \) since \( I'(\ell^H) < 0 \). Thus the planner chooses the minimum possible that is: \( \ell^H = \beta^* - 1 \) from (1.39) with equality. This pins down investment \( I = \frac{A}{1-(\rho-s_L)(1-\lambda)-\lambda(\rho-s_H)+(s_H-\rho)\ell^H} \). The planner's aim is to make \( F^H_D \) as small as possible. From (1.40) we see that increasing \( F^H_D \) necessarily decreases \( F^H_F \) because fiscal capacity is limited and this lowers the objective function. Then (1.38) will hold with equality.

Also (1.42) for domestics must hold with equality \( F^L_D = F^H_D \). The argument is similar to above, making \( F^L_D \) higher than just necessary would lower \( F^L_F \) from (1.41) which again lowers the objective function. As before fiscal constraints (1.40) and (1.41) hold with equality. Under this allocation the monotonicity constraint for foreigners is slack.
Thus, the optimal fiscal capacity allocation is given in the following proposition.

**Proposition 7.** Under monotonicity constraints, the optimal fiscal capacity allocations are:

\[
F^H_D = F^L_D = I^*(s_H - \rho)
\]
\[
F^H_F = \eta - I^*(s_H - \rho)
\]
\[
F^L_F = 1 - I^*(s_H - \rho)
\]

where \( I^* = \frac{1}{1-(\rho-s_L)(1-\lambda)-\lambda(\rho-s_L)+(s_H-\rho)(\beta^*-1)} \) and it is the level of investment pinned down by \( I(\varepsilon^H = \beta^* - 1) \).

The optimal fiscal capacity allocation provides equal liquidity in both states of the world to domestics due to the monotonicity constraint. This allocation is intuitive, the government wants to provide just enough liquidity domestically to attain the desired level of investment. The residual fiscal capacity is allocated abroad in order to maximize the liquidity premia coming from foreigners.

### 1.3.5 Assets that Implement Optimal Fiscal Capacity Allocations

A payoff structure that implements the fiscal capacity allocation under no monotonicity corresponds to the Arrow-Debreu securities given by:

\[
\begin{align*}
\lambda^H_1 &= \eta, \quad \lambda^L_1 = 0 \\
\lambda^H_2 &= 0, \quad \lambda^L_2 = 1
\end{align*}
\]

in which fiscal capacity in each state of the world is supplied in the form of one asset. Papers such as Angeletos (2002) and Buera and Nicolini (2004) that non-contingent debt of different maturities can implement any Arrow-Debreu allocation.

Under monotonicity the payoff structure that implements the fiscal capacity allocation is a safe and a risky asset:
\[
\begin{align*}
x_1^H &= \eta, \ x_1^L = \eta \\
x_2^H &= 0, \ x_2^L = 1 - \eta
\end{align*}
\]

The empirical counterpart of these payoffs could be nominal debt and inflation-protected securities. The nominal debt would be the asset paying the same in both states of the world. The inflation-protected securities would be the asset paying-off in a cyclical manner, that is, paying a higher payoff in the good state of the world, in the model when the liquidity shock is small. Another combination of assets that we see in sovereign debt markets could be nominal debt and variable rate debt as the risky debt instrument.

1.3.5.1 Arrow-Debreu Securities

This combination implements the allocation obtained in subsection (1.3.3). In this case the entrepreneur will only demand asset 1, \( \Pi_1 = \eta \lambda \) which makes \( \ell = \lambda (\beta^* - 1) \). The investment level chosen by the entrepreneur is given by \( I^{**} = \frac{A}{1 - (\rho - r_1)(1 - \lambda) - \lambda (\rho - s_H) + (s_H - \rho) \lambda (\beta^* - 1)} \). The domestic demand for asset 1 equals \( z_1^D = \frac{I^{**}(s_H - \rho)}{\eta} \) and \( z_2^D = 0 \). The foreign demand for both assets equal \( z_1^F = 1 - z_1^D = 1 - \frac{I^{**}(s_H - \rho)}{\eta} \) and \( z_2^F = 1 \).

Plugging these and the payoffs in (1.43) and (1.44) for both types of investors we obtain the optimal fiscal capacity allocations.

1.3.5.2 Safe and Risky Asset

Indeed this combination implements the fiscal capacity allocations given in subsection (1.3.4). Given these two assets the entrepreneur will choose to hold asset 1 since it is the only asset that provides liquidity in state \( s = s_H \), thus minimizing the unit cost of liquidity. The expected payoff \( \Pi_1 = \eta \), thus \( \ell = \beta^* - 1 \). Investment level chosen by the entrepreneur equals \( I^* = \frac{A}{1 - (\rho - r_1)(1 - \lambda) - \lambda (\rho - s_H) + (s_H - \rho) (\beta^* - 1)} \).
The fiscal capacity allocated to domestics in both states of the world equals:

\[ F^H_D = z^D_1 x^H_1 + z^D_2 x^H_2 \]  \hspace{1cm} (1.43)

\[ F^L_D = z^D_1 x^L_1 + z^D_2 x^L_2 \]  \hspace{1cm} (1.44)

where \( z^D_1 \) and \( z^D_2 \) is the demand coming from domestic for each asset. Since the entrepreneur does not buy asset 2, \( z^D_2 = 0 \). From the collateral constraint (1.11) \( z^D_1 = \frac{I^*(\gamma H - \rho)}{\eta} \) and the fact that \( x^H_1 = x^L_1 = \eta \) we immediately see that the fiscal capacity allocation for domestics is the optimal one obtained in subsection 1.3.4.

The expressions for fiscal capacity allocated to foreigners \( F^H_F \) and \( F^L_F \) are symmetric to (1.43) and (1.44) where domestic demand is substituted by foreign demand. The foreign demand for asset 2 is all the asset supplied \( z^F_2 = 1 \) because domestics hold none of this asset and \( z^F_1 = 1 - z^D_1 = 1 - \frac{I^*(\gamma H - \rho)}{\eta} \). The fiscal capacity allocations that are attained with this combination of assets is exactly the optimal one from above.

1.4 The Benefits of Financial Innovation and Complementarities with Financial Integration

To start, it is worth noting that the planner problems given in subsections 1.3.3 and 1.3.4 were subject to the same fiscal capacity constraints as in the scenario with only one defaultable bond. Thus government revenues are identical in all scenarios. In the case with one bond the government revenues are given by \( \beta^*(1 - \lambda + \lambda \eta) \), which is identical to the level of government revenues in the other two scenarios when we add the revenues accruing from both assets. In the scenario with the safe and the risky asset, the revenues from the risky asset are given by \( \beta^*(1 - \lambda)(1 - \eta) \) and the revenues coming from the safe one are given by \( \beta^* \lambda \eta \) and the case with Arrow-Debreu securities the revenues coming from the security that pays in \( s = s_L \) equal \( \beta^*(1 - \lambda) \) and the revenues from the one that pays when \( s = s_H \) equal \( \beta^* \lambda \eta \).
Despite keeping fiscal capacity constant the government is increasing domestic investment when it issues two different securities. In particular we find the following:

**Proposition 8.** Let $I^{**}$ denote the investment level with Arrow-Debreu securities. Let $I^*$ denote the investment level with one safe and one risky bond. Finally let $I$ denote investment with one risky bond. We have $I^{**} \geq I^* \geq I$ with strict inequality when fiscal capacity $\eta > \frac{(s_H - \rho)(1 - (\beta^* - 1)\lambda)}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$ for the first inequality and $\eta > \frac{(s_H - \rho)(1 - (\beta^* - 1))}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$ for the second.

For the argument we concentrate on the case where the fiscal capacity conditions are met and thus both domestic and international investors hold part of the public asset held by domestics as a way to hoard investment. In this case financial innovation increases investment. Furthermore, investment is highest when the government issues Arrow-Debreu securities and lowest one it issues only one defaultable bond.

The intuition for this result is that by issuing two different assets the government tranches its fiscal capacity and reduces the wasted liquidity, that is, the amount of unneeded liquidity the entrepreneur purchases per unit of liquidity in the state of the world when liquidity is useful goes down. The lower the wasted liquidity, the higher the domestic investment. We see this by comparing the case with Arrow-Debreu securities and with one safe and one risky asset. In the former case, when the entrepreneur buys asset 1 she does not buy liquidity for the state when she does not need the public liquidity because that asset’s payoff is 0 in that state of the world. In the latter case, when the government is issuing a safe asset the entrepreneur has to buy some liquidity for the state she will not use it.

The cost of the wasted liquidity in this model comes from the existence of international investors. The model features a crowding-out effect coming from international investors’ demand for public bonds. This high demand from foreign investors is captured by the higher discount factor abroad $\beta^* > 1$ and drives up the price of the public bond. The government by tranching its fiscal capacity and decreasing wasted liquidity decreases the cost of liquidity hoarding for entrepreneurs without imposing capital controls. See Appendix 1. B for a discussion on capital controls in this model. Finally, financial innovation introduces assets especially designed to attract international investors who are risk-neutral and demand the assets as a savings vehicle.
It is worth noting that if the economy is in autarky then the following proposition holds:

**Proposition 9.** Under autarky, investment denoted as \( I^{Aut} \) does not depend on the ratio of fiscal capacities \( \bar{\eta}/\eta \). Thus, investment and welfare are equal under financial innovation than without.

In this model we see that the benefits of financial innovation arise when sovereign debt markets are integrated. In Appendix 1. C we find the equilibria when foreign investors cannot buy public debt in sovereign debt markets. In both equilibria, with high and low fiscal capacity, investment would not be affected by financial innovation.\(^6\) In the case where fiscal capacity is large enough, (1.4) holds, there is no cost of liquidity hoarding because the marginal holder of public debt is the consumer. Thus, the government cannot improve the allocation by changing the payoff structure.\(^7\)

### 1.5 Comparative Statics

In this section I perform comparative statics for the scenario with one safe and one risky asset. The comparative statics regard the relative holdings of safe to risky asset for different types of investors as well as the domestic investment level.

An increase in \( s_H \) and a decrease in \( s_L \) such that the total cost per unit of investment, \( 1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) \), remains constant brings about an increase in the international relative holdings of risky to safe asset. The intuition for this is that the tightening of collateral constraints increases the domestic demand for the safe asset. By market clearing, the amount of safe asset held by international investors decreases. The international holdings of risky asset are always equal to the amount supplied. Thus, the relative holdings of risky to safe increase because the denominator decreases.

\(^6\)More broadly we can think of sovereign debt markets open to different types of investors with different motives to hold debt and with a higher demand for the public bond. In the model presented in this paper these are foreign investors. This is consistent with the empirical evidence that shows a steady increase in the share of debt held by foreigners for most of the advanced economies in recent years (Arslanalp and Tsuda (2012)).

\(^7\)The same result holds when (1.4) does not hold. In that case, as we discuss in Appendix 1. A investment is increasing in \( \eta \), the fiscal capacity in the high liquidity shock state. However, the government cannot change its fiscal capacity by introducing financial innovation in sovereign debt markets. It can only change the relative payoffs across states.
An increase in fiscal capacity $\eta$ decreases the relative holdings of risky to safe asset. The reason for this is that an increase in $\eta$ lowers the domestic demand for the safe asset because now every bond pays more. Thus international investors need to hold more of the safe asset which decreases the ratio of risky to safe assets held by foreigners.

An increase in the patience of international investors parametrized in an increase in $\beta^*$ increases the foreign demand for the safe asset because being more patient they demand more of all assets. This decreases the ratio of risky to safe international holdings since as in the previous scenarios risky asset holdings are fixed and normalized to 1.

**Proposition 10.** Let $z^F_1$ denote the foreign holdings of the risky asset and $z^F_2$ the foreign holdings of the safe asset. Then for a given level of $1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)$, (i) $\frac{\partial(z^F_1/z^F_2)}{\partial s_H} > 0$, (ii) $\frac{\partial(z^F_1/z^F_2)}{\partial \eta} < 0$, and (iii) $\frac{\partial(z^F_1/z^F_2)}{\partial \beta^*} < 0$.

### 1.6 Empirical Analysis

#### 1.6.1 Objective and Data

In this section I first test the comparative statics in proposition 4 regarding public debt ownership using a dataset of 22 advanced economies between 2004 and 2011. The countries included in the sample are Australia, Canada, Japan, Korea, Switzerland, UK, US and several countries of the Euro area.\(^8\)

I test the predictions regarding public debt ownership and tightening of collateral constraints, fiscal capacity and foreign discount factor. For the first one I use local credit which decreases when collateral constraints are tighter in the domestic economy. For fiscal capacity I use tax revenues over GDP, where higher tax revenues over GDP imply a higher fiscal capacity of the government (Dincecco and Prado (2012)) or the economy's fiscal effort.

Finding a good measure of the extractive capacity of the state is hard. Tax revenues over GDP is a commonly used proxy which can be capturing changes of the party in power or

---

\(^8\)The included countries from the Euro area are Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Slovenia, Spain and Sweden.
changes in spending policies. Finally, as a measure of the inverse of the foreign discount factor I use the federal funds rate which I take as the world interest rate. Appendix 1. E.1 contains a list of variables and data sources.

Second, I study the differential effects of collateral constraints on sovereign debt ownership for countries that have introduced more financial innovations in their sovereign borrowing. I divide the sample of countries in two using the information contained in Appendix 1. E.2. As we see there all advanced economies issue debt with different maturities and many of them issue at least another type of bond, indexed to inflation, with a variable coupon or interest rate or in a foreign currency. I consider the countries that only issue bonds with different maturities as having introduced less financial innovation in sovereign debt markets and I label them as non-innovators in sovereign debt markets. These countries are Austria, Czech Republic, Greece, Ireland, Portugal, Spain and Switzerland.\(^9\)

### 1.6.2 Comparative Statics on Sovereign Debt Ownership

The first specification used to test the comparative statics regarding tightening of collateral constraints and fiscal capacity is the following:

\[
\%\text{non-resident}_t = \alpha_t + \lambda_t + \beta_1 \text{Local credit}_t + \beta_2 \text{Fiscal Capacity}_t + \gamma X_t + \epsilon_t \quad (1.45)
\]

where the dependent variable is the share of total outstanding public debt held by international investors and \(X_t\) contains total outstanding public debt over GDP and financial openness as controls.

According to the model, \(\beta_1 > 0\) and \(\beta_2 > 0\). The model predicts that a tightening of collateral constraints, lower local credit available, implies a repatriation of sovereign debt. Also, through the lens of the model higher fiscal capacity will increase the share of public debt held abroad.

\(^9\)Admittedly this is a crude measure of financial innovation and many improvements can be made to include heterogeneity in the degree of financial innovators. Constructing a better measure of financial innovation goes beyond the scope of this paper.
Specification (1.45) contains country and time fixed effects which control for time-invariant country specific factors and common shocks to all countries. It also includes two relevant control variables: financial openness and the level of debt over GDP. Table 1.1 in appendix 1. E.3 shows the results. The data confirms the predictions of the model. The effect of local credit is positive as predicted by the model and significant for the specification with controls. The point estimate of the coefficient for fiscal coefficient is also consistent with the model. However, the coefficient is not significantly different from zero in all specifications.

In order to test the prediction about the effect of the discount factor on the proportion of debt held abroad I run a specification without time fixed effects:

$$%_{\text{non-resident}} = \alpha_i + \beta \text{Federal Funds Rate}_i + \gamma X_{it} + u_{it}$$

(1.46)

According to proposition 4, $\beta < 0$ since a higher discount factor which in the model crowds-out domestic demand for bonds, corresponds with a lower market interest rate. Table 1.2 in appendix 1. E.3 shows that the data confirms this prediction.

1.6.3 Causal Effect of Local Credit on Sovereign Debt Ownership

The previous empirical analysis has only shown correlations that are consistent with the predictions of the model. Instrumental variable regressions will gauge at causality. I concentrate on local credit for this section for two reasons. First, the effect of tightening of collateral constraints on sovereign debt ownership is at the heart of the mechanism highlighted in this paper. Other mechanisms, such as the foreign discount factor can be obtained from many other models. Second, the fixed effects regressions gave significant point estimates consistent with the predictions of the model. This contrasts with tax revenues over GDP, which despite proxying a key ingredient of this model, did not give significant point estimates in the fixed effects regressions.

As an instrument for tightening of collateral constraints I use the financial crisis as a plausibly exogenous shock to collateral constraints in each of these countries. In order to capture
different intensities of the “financial crisis” treatment I rely on a Bartik-style instrumental variable (Bartik (1991)). In particular, I interact the financial crisis shock ($Post_{09}$) with the leverage level of the economy before the crisis hit. What is crucial for identification is that the leverage level of each economy before the shock is exogenous to the financial crisis shock. As an instrument for the federal funds rate I use the 2008 cut taken as a monetary policy decision by the Federal Reserve Board and plausibly exogenous to economic conditions everywhere except for the US ones.

The specification for local credit is given by:

\[
\text{%non-resident}_{it} = \alpha_i + \lambda_t + \beta \text{Local credit}_{it}^{08} \times Post_{09} + \gamma X_{it} + u_{it} \tag{1.47}
\]

where $Local \text{Credit}_{it}^{08} \times Post_{09}$ is a variable constructed interacting the level of local credit in 2008 for each country, that is, how exposed the country was to the treatment and $Post_{09}$, a dummy variable which takes a value of one for years after and including 2009. The vector of controls $X_{it}$ include financial openness, level of debt to GDP and local credit. As before (1.47) includes country and time fixed-effects. In this specification we expect $\beta < 0$: the shock to collateral constraints causes a repatriation of public debt since more public assets are needed by the domestic private sector to use as collateral. The table in appendix 1. E.4 confirms this prediction and the coefficient is significant when including local credit as one the controls.

### 1.6.4 Differential Effect of Collateral Constraints on Financial Innovators

In this subsection I explore whether public debt ownership of those countries that have introduced financial innovation in sovereign debt markets is differentially hit by the financial crisis or not. For this I divide the sample of 22 advanced economies in two, those that have introduced more financial innovation in sovereign debt markets and those who have introduced less. The latter group includes only Austria, Czech Republic, Greece, Ireland, Portugal, Spain and Switzerland who only issue bonds in different maturities and I label

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them as non-innovators. The inclusion of Czech Republic, Austria and Switzerland in the financial non-innovators group and the non-inclusion of other countries in the Euro periphery such as Italy minimizes the concerns about the financial non-innovators capturing other fundamentals. Also all regressions include country and time fixed-effects.

I run the following econometric specification:

\[
\text{\% non-resident}_{it} = \alpha_i + \lambda_t + \beta_1 Local credit_i^{08} * Post_i^{09} + \beta_2 Local credit_i^{08} * Post_i^{09} * Financial Innovator_i + \gamma X_{it} + \mu_{it}
\]

where \(Local Credit_i^{08} \ast Post_i^{09}\) is constructed as defined above and \(Financial Innovator_i\) is a dummy that takes a value of one if the country is a financial innovator and zero otherwise.

From the results in section 1.3 and 1.4 we expect the effects of the crisis to be less severe for financial innovators, that is a smaller public debt repatriation after the financial shock and a smaller hit to country investment. Since the dummy \(Financial Innovator_i\) takes a value of one when the country is a financial innovator and the coefficient of the financial crisis treatment in specification (1.47) is negative we expect \(\beta_2 > 0\). The results in Appendices 1. E.5 confirms this prediction and is significant when controlling for the level of debt and financial openness.

These results cannot be directly attributed to financial innovation in sovereign debt markets because the sample division can be capturing many other time-varying country-specific relevant variables not included in (1.48). However the differential degree of debt repatriation after the financial crisis shock that we see in the data is consistent with the benefits of financial innovation discussed in section in 1.4.

### 1.7 Conclusion

This paper has presented a model where public debt has a liquidity role, the government’s fiscal capacity is risky which renders public debt risky too and different types of investors
demand public debt in integrated sovereign debt markets. The paper has shown that finan-
cial innovation in sovereign debt markets can increase domestic investment and domestic 
welfare. The key assumption behind the investment increase is the liquidity role of public 
debt. The driver of this result is that financial innovation can make liquidity for the private 
sector cheaper by changing the payoff structure of the public assets and lowering liquidity 
while. For this to be possible there must be other types of investors willing to hold the 
residual risk not allocated to domestics.

The financial innovations proposed and the way of approaching the government's finan-
cial innovation problem have highlighted that the government can exploit the existence of 
different types of investors when designing assets. Especially the government can design 
assets taking advantage of the different degrees of patience in its investor base and the dif-
ferent rationales for holding public debt. The paper has shown that to provide liquidity 
optimally at home the government does not need to segment markets or tax foreigners. The 
model has shown that an appropriate asset design can result in market segmentation. The 
financial innovation problem has also highlighted that when there are investors willing to 
hold riskier tranches of the public fiscal capacity the government first allocates fiscal ca-
pacity to meet liquidity demands at home and after that allocates the residual riskier fiscal 
capacity to those investors willing to hold it. Finally it has also highlighted the comple-
mentarities between financial innovation and financial integration showing that the benefits 
of financial innovation are higher when sovereign debt markets are more integrated.

The model presented in this paper has delivered comparative statics consistent with size-
able shifts in composition of investor base for public debt as a whole that has been reported 
extensively in the recent years as well as shifts of relatively riskier debt instruments to 
foreigners. Moreover, recent data on debt ownership and local credit for a group of ad-
vanced economies is consistent with the predictions of the theoretical model and with the 
differential effects for financial innovators and non-innovators.

The framework and results presented in this paper point to a number of promising av-

enues for future research. Looking at investor base composition and debt ownership at a 
smaller level of granularity regarding debt instruments and different types of investors. For
instance, thinking about shifts of ownership between financial and non-financial types of investors or with different degrees of risk appetite. Introducing lack of commitment and allowing the government to default on its public debt can introduce relevant trade-offs in the planner’s financial innovation problem which have been ignored in this paper.
1. A Equilibrium characterization

First, we consider the case where $\eta > \frac{(s_H - \rho)(A - (1 - \lambda)\eta)}{1 - (p - s_L)(1 - \lambda) + \beta^*\lambda(s_H - \rho)}$ and $\beta^* > 1 + \frac{\lambda(1+s_L(1-\lambda)-s_H)\eta}{(s_H-p)(1-\lambda)\Pi}$. When $\beta^*$ is higher than this upper bound it is too expensive for the entrepreneur to hoard liquidity. Thus $z^L = 0$ and the project cannot continue when $s = s_H, \chi = 0$. The initial investment level is then given by

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda)}$$

All public debt is held by foreigners, $z^F$ and their valuation pins down the price of public debt $q = \beta^*\Pi$. As long as $\eta > \frac{(s_H - \rho)(A - (1 - \lambda)\eta)}{1 - (p - s_L)(1 - \lambda) + \beta^*\lambda(s_H - \rho)}$ international investors hold part of the public debt and $q = \beta^*\Pi$. In the knife-edge case where $\beta^* = 1 + \frac{\lambda(1+s_L(1-\lambda)-s_H)\eta}{(s_H-p)(1-\lambda)\Pi}$, the continuation scale $\chi \in (0, 1)$ and investment scale equals:

$$I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \chi\lambda(\rho - s_H) + \chi(s_H - \rho)(\beta^* - 1)(\lambda + (1 - \lambda)\frac{\eta}{\Pi})}$$

The domestic holdings of the public asset equal $z^L = \frac{\chi A(s_H - \rho)}{\eta [1 - (p - s_L)(1 - \lambda) - \chi\lambda(\rho - s_H) + \chi(s_H - \rho)(\beta^* - 1)(\lambda + (1 - \lambda)\frac{\eta}{\Pi})]}$ and international investors hold the rest $z^F = 1 - z^L$.

The last case to consider is the one where the return of the bond in $s = s_H$ is not big enough, that is when $\eta \leq \frac{(s_H - \rho)(A - (1 - \lambda)\eta)}{1 - (p - s_L)(1 - \lambda) + \beta^*\lambda(s_H - \rho)}$. Under this parametric condition if also the following condition on $A$ holds

$$\eta(1 - (p - s_L)(1 - \lambda) - \lambda(\rho - s_H)) < A < \frac{\eta(1 - (p - s_L)(1 - \lambda) - \lambda(\rho - s_H))}{s_H - \rho} + \frac{\eta\lambda(1 + s_L(1 - \lambda) - s_H)}{(1 - \lambda)(s_H - \rho)}$$

then the project is fully continued $\chi = 1, z^L = 1$ and $z^F = 0$.

The demand for bonds is given by $z^L = \frac{I(s_H - \rho)}{\eta}$. Combining this expression with (1.3) and equating it to total supply of bonds we can solve the price of the bond in this equilibrium:

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\[ q = \Pi + A - \frac{\Pi(1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H))}{s_H - \rho} \]  

The constraints on \( A \) given in (1.49) ensures that the price of the public bond (1.50) satisfies \( \Pi < q < q^{\text{max}} \) where \( q^{\text{max}} = \Pi + \frac{\lambda(1+s_L(1-\lambda)-s_H)\Pi}{(s_H-\rho)(1-\lambda)} \). Investment is given by:

\[ I = \frac{\Pi}{s_H - \rho} \]  

which as we see is increasing in \( \Pi \) but not in the ratio of fiscal capacities as in the section 1.2. It is increasing in \( \Pi \) because when \( \Pi \) is low the entrepreneur is constrained and cannot hoard as much liquidity as she would want to. Therefore, the slacker this constraint is the higher attainable investment will be.

**Proposition 11.** (a) An equilibrium in this model is given by a tuple \((q, I, z^L, z^F)\), consisting on price of debt, initial investment scale and domestic and foreign holdings of the asset; (b) when \( \Pi > \frac{(s_H-\rho)(\lambda-(1-\lambda)\Pi)}{1-(\rho-s_L)(1-\lambda)+\beta^*\lambda(s_H-\rho)} \) and \( \beta^* < 1 + \frac{\lambda(1+s_L(1-\lambda)-s_H)\Pi}{(s_H-\rho)(1-\lambda)\Pi} \), then the equilibrium is given by:

\[
q = \beta^*\Pi \\
I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + (s_H - \rho)(\beta^* - 1)(\lambda + (1 - \lambda)\frac{\Pi}{\Pi})} \\
z^L = \frac{I(s_H - \rho)}{\Pi} \quad z^F = 1 - z^L
\]

(c) When \( \Pi > \frac{(s_H-\rho)(\lambda-(1-\lambda)\Pi)}{1-(\rho-s_L)(1-\lambda)+\beta^*\lambda(s_H-\rho)} \) and \( \beta^* = 1 + \frac{\lambda(1+s_L(1-\lambda)-s_H)\Pi}{(s_H-\rho)(1-\lambda)\Pi} \), then the equilibria is given by for any \( \chi \in (0, 1) \)

\[
q = \beta^*\Pi \\
I = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H) + \chi(s_H - \rho)(\beta^* - 1)(\lambda + (1 - \lambda)\frac{\Pi}{\Pi})} \\
z^L = \frac{\chi I(s_H - \rho)}{\Pi} \quad z^F = 1 - z^L
\]
(d) When \( \eta > \frac{(s_H - \rho)(A - (1 - \lambda) \eta)}{1 - (\rho - s_L)(1 - \lambda) + \beta^* \lambda (s_H - \rho)} \) and \( \beta^* > 1 + \frac{\lambda(1 + s_L(1 - \lambda) - s_H)\eta}{(s_H - \rho)(1 - \lambda)\Pi} \), then the equilibrium is characterized by:

\[
\begin{align*}
q &= \beta^* \Pi \\
I &= \frac{A}{1 - (\rho - s_L)(1 - \lambda)} \\
\zeta_L &= 0 \quad \zeta_F = 1
\end{align*}
\]

(e) When \( \eta \leq \frac{(s_H - \rho)(A - (1 - \lambda) \eta)}{1 - (\rho - s_L)(1 - \lambda) + \beta^* \lambda (s_H - \rho)} \), then the equilibrium is as follows:

\[
\begin{align*}
q &= \Pi + A - \frac{\eta(1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H))}{s_H - \rho} \\
I &= \frac{\eta}{s_H - \rho} \\
\zeta_L &= 1 \quad \zeta_F = 0
\end{align*}
\]
1. B The Case of Capital Controls

As we have seen, the equilibrium investment level with international investors given in (1.6) is decreasing in the international investor's patience. A natural question is whether imposing capital controls which ban the arrival of international flows from abroad can increase welfare at home.

A first approximation to this question would be to calculate the consumption of both types of agents and compute domestic welfare as we discuss in section (1.3.2). Investment under open financial markets is given by (1.6). Under autarky since (1.4) holds the marginal buyer of the public bond is the domestic consumer which implies that the price of the bond has no liquidity premium, \( q = \Pi \) and investment under autarky is simply given by \( I = \frac{A}{1-(\rho-s_L)(1-\lambda)-\lambda(\rho-s_H)} \), since liquidity hoarding in autarky is costless. Thus, investment is higher under autarky. However, consumers lose from remaining in autarky since they now do not get the premia coming from abroad, \( q - \Pi \) which due to the higher patience abroad will always be greater than zero.

Now we allow for ex ante transfers between the entrepreneur and the consumer. The entrepreneur will transfer part of her initial wealth \( A \) to the consumer in order to make him indifferent between autarky and open financial markets. Then we will compute whether the final investment under autarky is higher than in open financial markets.

For convenience let's denote by \( \bar{R} \equiv R - s_L(1-\lambda) - s_H \lambda - 1 \), the net return from the domestic investment and by \( k \equiv 1-(\rho-s_L)(1-\lambda) - (\rho-s_H)\lambda \). As in the comparative statics part we simplify notation by assuming \( \bar{\eta} = 1 \). Under open financial markets investment and the transfer to consumers from abroad are given by:

\[
I^{Open} = \frac{A}{k + \frac{s_H - \rho}{\eta}(\beta^* - 1)(\lambda \eta + 1 - \lambda)} \tag{1.52}
\]

\[
T^{Open} = (\beta^* - 1)(\lambda \eta + 1 - \lambda) \left( 1 - \frac{I^{Open}(s_H - \rho)}{\eta} \right) \tag{1.53}
\]
For this to be an equilibrium (1.4) needs to hold, which we can rewrite in terms of the initial wealth \( A \) being below a threshold:

\[
A < \frac{\eta k}{s_H - \rho} + (\beta^* - 1)(\lambda \eta + 1 - \lambda)
\]

Therefore let assume that the entrepreneur transfers \( T^{\text{Open}} \) to consumers. After this transfer the consumer is indifferent between autarky and open financial markets. In autarky the entrepreneur’s investment choice after transfers is given by:

\[
I^{\text{Autarky},T} = \frac{\bar{A}}{k} = \frac{A - T^{\text{Open}}}{k}
\]

where the entrepreneur can leverage its initial wealth after transfers \( \bar{A} < A \) as much as \( 1/k \) since in the closed economy when (1.4) holds the consumer is the marginal holder of the asset who pins down the price. Thus, \( q = \Pi \) and there is no cost of hoarding liquidity for the entrepreneur.

**Proposition.** Under autarky the consumer is made worse-off, the entrepreneur is made better-off and total welfare is always smaller than under open financial markets. Open financial markets is also better than autarky when the entrepreneur makes a transfer to the consumer to make him indifferent between autarky and open financial markets.

**Proof.** For the first part, we need to calculate total welfare \( W = E + A + (R - s_L(1 - \lambda) - s_H \lambda - 1)I + (q - \Pi)z^E \) for both scenarios, open financial markets and autarky and show that \( W^{\text{Open}} > W^{\text{Autarky}} \). Denote \( \Pi = (\lambda \eta + 1 - \lambda) \).

Under open markets,

\[
I^{\text{Open}} = \frac{A}{k + \frac{s_H - \rho}{\eta}(\beta^* - 1)\Pi} \\
W^{\text{Open}} = E + A + \bar{R}I^{\text{Open}} + (\beta^* - 1)\Pi \left( 1 - \frac{I^{\text{Open}}(s_H - \rho)}{\eta} \right)
\]

where as we see \( W^{\text{Open}} \) welfare is comprised of the net return from investment and the financial flows from abroad. For this to be an equilibrium it must be that
\[ \eta > \frac{(s_H - \rho)(\lambda - (1 - \lambda))}{1 - (\rho - s_L)(1 - \lambda) + \beta^* \lambda (s_H - \rho)} \]  

(1.54)

Investment and welfare in the autarkic economy is given by:

\[ I_{\text{Autarky}} = \frac{A}{k} \]

\[ W_{\text{Autarky}} = E + A + \bar{R} I_{\text{Autarky}} \]

It follows directly from condition (1.54) that \( I_{\text{Autarky}} > I_{\text{Open}} \). For \( W_{\text{Open}} > W_{\text{Autarky}} \) it must be the case that

\[ \bar{R}(I_{\text{Autarky}} - I_{\text{Open}}) < (\beta^* - 1) \Pi \left(1 - \frac{I_{\text{Open}}(s_H - \rho)}{\eta}\right) \]  

(1.55)

Rearranging (1.55) we see that it always holds when \( I_{\text{Autarky}} > I_{\text{Open}} \), that is when (1.54) holds.

For the second part of the proposition we need to prove that \( I_{\text{Open}} - I_{\text{Autarky}, T} > 0 \).

Substituting investment for their expressions we obtain that

\[ \frac{A}{k + \frac{s_H - \rho}{\eta}(\beta^* - 1) \Pi} - \frac{\bar{A}}{k} > 0. \]

Rearranging this expression and substituting \( \bar{A} = A - T_{\text{Open}} \) we obtain \((\beta^* - 1) \Pi \left(\frac{\eta - I_{\text{Open}}}{\eta}\right) \eta k > -k(s_H - \rho)(\beta^* - 1) \Pi \). Cancelling out terms this expression becomes \( \eta + s_H - \rho > I_{\text{Open}} \). Using the expression for \( I_{\text{Open}} \) this is equivalent to

\[ \frac{A}{(\eta k + \frac{s_H - \rho}{\eta}(\beta^* - 1) \Pi)(\eta + s_H - \rho)} < 1 \]

which can be rewritten as

\[ A < \eta k + k(s_H - \rho) + (s_H - \rho)(\beta^* - 1) \Pi + \frac{(s_H - \rho)^2(\beta^* - 1) \Pi}{\eta} \]

This condition on \( A \) always holds when \( A < \frac{\eta k}{s_H - \rho} + (\beta^* - 1) \Pi \), which is the initial condition for an equilibrium given in section (1. B).

\[ \square \]

Capital controls which ban foreign investors from buying the public asset do not increase welfare in this model because there are no pecuniary externalities. A crucial ingredient
for capital controls to be welfare-improving is that a relative price affects constrained individuals, either directly by tightening collateral constraints or by decreasing wealth of constrained individuals (Caballero and Krishnamurthy (2001), Caballero and Krishnamurthy (2004), Aghion et al. (2001), Korinek (2011)).

From the entrepreneur’s problem (1.1) we see that this is not the case here. The price of the public debt affects the investment choice level \( I \) and in turn how much of the public bond available the entrepreneur demands but it does not affect how constrained the entrepreneur is directly nor by changing the entrepreneur’s net worth.
1. C Autarkic equilibria: Sovereign Debt Markets Closed to Foreign Investors

In this section we derive the solution of the model in autarky. We start by analyzing the case when (1.4) holds: $\eta > \frac{(s_H - \rho)(A - (1 - \lambda)\eta)}{1 - (\rho - s_L)(1 - \lambda) + \beta^*\lambda(s_H - \rho)}$. In that case we know $z^L$ coming from domestic entrepreneurs is lower than total supply of public bond. With integrated sovereign debt markets part of the bond would be held by foreign investors.

With sovereign debt markets only open to domestic agents, it will be consumers who will demand the public bond. Since they do not have a liquidity motive to hold debt and their discount factor $\beta = 1$ in equilibrium:

$$q^{Aut,I} = \Pi$$

When $q = \Pi$ the entrepreneur is indifferent between holding as much bonds as to cover liquidity needs and infinite: $z^L \in \left[\frac{I^L(s_H - \rho)}{\Pi}, \infty\right)$. We assume that the entrepreneur just holds enough to cover liquidity needs. Since $q < q^{max}$ defined in appendix 1. A, full continuation is optimal in both states $\chi = 1$ and liquidity needs are given by $\frac{I(s_H - \rho)}{\Pi}$. Plugging this and the price in the expression for investment (1.3) we obtain that the equilibrium level of investment is:

$$I^{Aut,I} = \frac{A}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$$

Therefore the level of investment is increasing in the level of entrepreneur's initial wealth and in the equity multiplier $\frac{1}{1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H)}$ which defines the maximum leverage per unit of own capital that the entrepreneur can obtain. As in section 1.2 it is decreasing in the expected cost of the project and increasing in the pledgeable return $\rho$.

However, under autarky when $\eta$ is large enough, investment does not depend on the cost of liquidity hoarding. This is due to the fact that when $\eta$ is large enough the marginal holder of the public bond is the domestic consumer who drives the liquidity premium to zero.
We now turn to the case when $\pi \leq \frac{(s_H - \rho)(A - (1 - \lambda)\tilde{\eta})}{1 - (\rho - s_L)(1 - \lambda) + \beta^2 \lambda(s_H - \rho)}$. In this case only the domestic entrepreneur will hold the bond. The analysis here will be identical to the case with integrated debt markets but $\pi$ low derived in Appendix 1. A. There we found that the price of the bond is given by:

$$q^{Aut,H} = \Pi + \lambda - \frac{\eta(1 - (\rho - s_L)(1 - \lambda) - \lambda(\rho - s_H))}{s_H - \rho}$$

and that the level of equilibrium investment is increasing in $\pi$:

$$\eta^{Aut,H} = \frac{\eta}{s_H - \rho}$$
1. D Financial Innovation under Positive Correlation

This appendix presents the optimal financial innovation under the following assumption:

**Assumption.** 3' *The fiscal capacity shock and the private liquidity shock are positively correlated.*

which substitutes Assumption 3 in section 1.2.

According to this new assumption there are four states of the world, \( s = \{HL, HH, LL, LH\} \) with probabilities denoted as \( \{AHL, AHH, ALL, ALH\} \). The first letter of each state denotes the government's fiscal capacity and the second denotes the liquidity shock. Thus, state \( HL \) corresponds to the state where the government's fiscal capacity is high and the private sector does not need outside liquidity because the liquidity shock is small \( s_L \), and so on. To span all the states of the world the government will issue four assets.

1. D.1 Financial Innovation without Monotonicity Constraints

As we saw in section 1.3.3 the optimal innovation is to issue Arrow-Debreu securities. These securities must satisfy the fiscal capacity constraints. Labeling the assets from 1 to 4, this implies that:

\[
\begin{align*}
\sum_{j=1}^{4} x_{HL}^j & \leq 1 \\
\sum_{j=1}^{4} x_{HH}^j & \leq 1 \\
\sum_{j=1}^{4} x_{LL}^j & \leq \eta \\
\sum_{j=1}^{4} x_{LH}^j & \leq \eta
\end{align*}
\]
The assets will have the following payoffs: \((1, 0, 0, 0), (0, 1, 0, 0), (0, 0, \eta, 0)\) and finally \((0, 0, 0, \eta)\). The investor base composition of assets 1 and 3 will be only international investors for the same rationale as the main body of the paper: entrepreneurs do not want to purchase an asset which gives a return when the liquidity shock is small, \(s_L\) and prices are \(q_1 = \beta^*\lambda_{HL}\) and \(q_3 = \beta^*\lambda_{LL}\). The other two assets will be held by domestics and internationals and prices will be pinned down by international investors' valuations: \(q_2 = \beta^*\lambda_{HH}\) and \(q_4 = \beta^*\lambda_{LH}\).

1. D.2 Financial Innovation under Monotonicity Constraints

Under imperfect correlation, monotonicity implies that each asset pays more when the government’s fiscal capacity is high. As in the main body of the paper the rationale for this is that the state with lower fiscal capacity corresponds observationally to a sovereign debt restructuring. This implies that:

\[
\begin{align*}
\lambda_{HL}^j & \geq \lambda_{LL}^j \\
\lambda_{HH}^j & \geq \lambda_{LH}^j \\
\lambda_{HL}^j & \geq \lambda_{LH}^j \\
\lambda_{HH}^j & \geq \lambda_{LL}^j
\end{align*}
\]

where \(j\) denotes the asset. The optimal asset combination is a safe asset and two Arrow-Debreu securities that satisfy the fiscal capacity constraints above: \((1 - \eta, 0, 0, 0), (0, 1 - \eta, 0, 0)\) and \((\eta, \eta, \eta, \eta)\). The investor base for the risky assets will only be international investors who will price the assets at \(q_1 = \beta^*\lambda_{HL}(1 - \eta)\) and \(q_2 = \beta^*\lambda_{HH}(1 - \eta)\). The safe asset will be held by both types of investors and its price will be \(q_3 = \beta^*\eta\).
1. E  Appendix to the Empirical Analysis

1. E.1  Data Sources and Definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonres</td>
<td>Public debt held by non-residents over</td>
<td>Arslanalp and Tsuda (2012)</td>
<td>2004-2011</td>
</tr>
<tr>
<td></td>
<td>total outstanding debt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local Credit</td>
<td>Private credit by deposit money banks other</td>
<td>Beck et al. (2009)</td>
<td>2004-2011</td>
</tr>
<tr>
<td></td>
<td>institutions over GDP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fed Funds</td>
<td>Effective federal funds rate</td>
<td>Federal Reserve Bank of St.</td>
<td>2004-2011</td>
</tr>
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<td></td>
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1. E.2 Financial Innovation in Sovereign Borrowing for Advanced Economies

<table>
<thead>
<tr>
<th>Country</th>
<th>Different Maturities</th>
<th>Indexed to Inflation</th>
<th>Variable rate/coupon</th>
<th>Different Currencies</th>
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<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td>Ireland</td>
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</tr>
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<td>Italy</td>
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<td>✓ 5</td>
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<td></td>
</tr>
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<td></td>
<td></td>
<td>✓ 7</td>
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<td></td>
<td></td>
<td>✓ 9</td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

Source: Debt Management Offices of each country

1,3,6,7,9 Foreign currency: US dollars. 2 Foreign currency: euro.
4 Indexed to European and French Inflation.
5 Indexed to European and Italian Inflation. 8 Other: kiwi bond (retail stock).
1. E.3 The Determinants of Proportion of Debt Held Abroad

Table 1.1 presents panel regressions for 22 advanced economies between 2004 and 2011. All regressions include country and year fixed effects and standard errors are clustered at the country level. The dependent variable is the percentage of debt held by foreign investors.

Table 1.1: Determinants of Proportion of Debt held Abroad

<table>
<thead>
<tr>
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<th>(4)</th>
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<td>nonres</td>
<td>nonres</td>
<td>nonres</td>
<td>nonres</td>
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<td>Local Credit</td>
<td>0.0770</td>
<td>0.0836*</td>
<td>0.0793*</td>
<td>0.0807*</td>
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<tr>
<td></td>
<td>(0.0469)</td>
<td>(0.0459)</td>
<td>(0.0419)</td>
<td>(0.0436)</td>
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<td>Fiscal Capacity</td>
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<td>0.646</td>
<td>0.638</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.486)</td>
<td>(0.490)</td>
<td>(0.476)</td>
<td></td>
</tr>
<tr>
<td>Financial Openness</td>
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<td>-4.528</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.031)</td>
<td>(6.162)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt</td>
<td>-0.00802</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0934)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time FE</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
</tr>
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<td>No. of Observations</td>
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<td>No. of Countries</td>
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<td>R-Squared</td>
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<td>6.627</td>
<td>6.396</td>
<td>6.961</td>
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</table>

Standard errors in parentheses clustered at the country level
* p < 0.1, ** p < 0.05, *** p < 0.01
Table 1.2 presents panel regressions for 22 advanced economies between 2004 and 2011. All regressions include country fixed effects and standard errors are clustered at the country level. The dependent variable is the percentage of debt held by foreign investors.

<table>
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<tr>
<th></th>
<th>Column (1)</th>
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<th>Column (3)</th>
<th>Column (4)</th>
<th>Column (5)</th>
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</thead>
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<td><strong>Federal Funds Rate</strong></td>
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<td>-0.879***</td>
<td>-0.750**</td>
<td>-0.465</td>
<td>-0.552*</td>
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<tr>
<td></td>
<td>(0.240)</td>
<td>(0.241)</td>
<td>(0.277)</td>
<td>(0.272)</td>
<td>(0.292)</td>
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<tr>
<td><strong>Financial Openness</strong></td>
<td>-2.886</td>
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<td>-2.439</td>
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<tr>
<td></td>
<td>(5.623)</td>
<td>(5.603)</td>
<td>(6.020)</td>
<td>(5.429)</td>
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<tr>
<td><strong>Debt</strong></td>
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<td>(0.0523)</td>
<td>(0.0654)</td>
<td>(0.0702)</td>
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<td><strong>Local Credit</strong></td>
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<td></td>
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<td>0.132***</td>
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<td>(0.0434)</td>
</tr>
<tr>
<td><strong>Fiscal Capacity</strong></td>
<td></td>
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<td>0.134***</td>
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<td>Yes</td>
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<td>161</td>
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<tr>
<td><strong>No. of Countries</strong></td>
<td>22</td>
<td>22</td>
<td>22</td>
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<td>22</td>
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<tr>
<td><strong>R-Squared</strong></td>
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</table>

Standard errors in parentheses clustered at the country level

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
1. E.4 Effect of Collateral Constraints on Proportion of Debt Held Abroad

The table presents panel regressions for 22 advanced economies between 2004 and 2011 to gauge the effect of tightening of collateral constraints on proportion of debt held abroad. All regressions include country and year fixed effects and standard errors are clustered at the country level.

The dependent variable is the percentage of debt held by foreign investors. $LC_{Post09t}$ is a variable which measures the intensity of the financial crisis shock for the different countries in the sample. It is constructed interacting $Local~Credit_i$ in 2008 and a dummy variable $Post09_t$ which takes value of 1 if time $\geq 2009$ and 0 otherwise.

Table 1.3: Effect of the Financial Crisis on Proportion of Debt held Abroad

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>nonres</td>
<td>nonres</td>
<td>nonres</td>
<td>nonres</td>
</tr>
<tr>
<td>Local Credit08*Post09</td>
<td>-0.0303</td>
<td>-0.0589</td>
<td>-0.0633</td>
<td>-0.0609*</td>
</tr>
<tr>
<td></td>
<td>(0.0292)</td>
<td>(0.0355)</td>
<td>(0.0369)</td>
<td>(0.0326)</td>
</tr>
<tr>
<td>Local Credit</td>
<td>0.113*</td>
<td>0.111*</td>
<td>0.107**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0550)</td>
<td>(0.0543)</td>
<td>(0.0496)</td>
<td></td>
</tr>
<tr>
<td>Debt</td>
<td>0.0259</td>
<td>0.0197</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0759)</td>
<td>(0.0777)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Openness</td>
<td>-3.332</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.333)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Time FE | Yes | Yes | Yes | Yes
Country FE | Yes | Yes | Yes | Yes
No. of Observations | 166 | 166 | 166 | 166
No. of Countries | 22 | 22 | 22 | 22
R-Squared | 0.326 | 0.395 | 0.397 | 0.403
F statistic | 5.732 | 6.816 | 6.709 | 5.845

Standard errors in parentheses clustered at the country level
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
1. E.5 Differential Effect of Collateral Constraints on Debt Held Abroad for Financial Innovators

The table presents panel regressions for 22 advanced economies between 2004 and 2011 to gauge the differential effect of tightening of collateral constraints on proportion of debt held abroad for financial innovators. All regressions include country and year fixed effects and standard errors are clustered at the country level.

The dependent variable is the percentage of debt held by foreign investors. $LCPost09 \ast Financial Innovator_{it}$ is a variable which measures the intensity of the financial crisis shock for the different countries as in table from section (1. E.4) for financial innovators (that is, all countries except Austria, Czech Republic, Greece, Ireland, Portugal, Spain and Switzerland).
<table>
<thead>
<tr>
<th>Table 1.4: Effect of Financial Crisis on Debt held Abroad for Financial Innovators</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>nonres</td>
</tr>
</tbody>
</table>
| Local Credit08*Post09 | -0.0348
(0.0317) | -0.0691* 0.0273
(0.0360) | -0.0677* 0.0362**
(0.0328) | -0.0769** 0.0368**
(0.0341) |
| Local Credit*Post09*Financial Innovator | 0.0273
(0.0179) | 0.0362** 0.0368**
(0.0160) | 0.0392** 0.0163
(0.0154) |
| Local Credit | 0.130**
(0.0533) | 0.125**
(0.0480) | 0.122**
(0.0479) |
| Financial Openness | -3.827
(4.626) | -3.489
(4.785) |
| Debt | 0.0486
(0.0572) |
| Time FE | Yes | Yes | Yes | Yes |
| Country FE | Yes | Yes | Yes | Yes |
| No. of Observations | 166 | 166 | 166 | 166 |
| No. of Countries | 22 | 22 | 22 | 22 |
| R-Squared | 0.354 | 0.442 | 0.450 | 0.456 |
| F statistic | 6.918 | 7.250 | 6.225 | 9.878 |

Standard errors in parentheses clustered at the country level

* p < 0.1, ** p < 0.05, *** p < 0.01
Chapter 2

Banking and Sovereign Debt Crises: The Bailout Mechanism

2.1 Introduction

Sovereign debt crises and banking crises often occur in a synchronized manner or one just before the other one. Reinhart and Rogoff (2011), using a comprehensive dataset that contains information on banking crises, currency crashes, default episodes, inflation crises and stock market crises for 70 countries between 1800 and 2010, conclude that banking crises often precede or accompany sovereign debt crises.

Arellano and Kocherlakota (2012) report that, for the shorter period between 1980 and 2003, out of the 22 sovereign default episodes in emerging markets 19 of those also involved an internal debt crisis and for the middle income countries 39 out of 63 sovereign default episodes were accompanied by an internal debt crisis. These authors also show that from those countries that experience both private and sovereign default episodes the private internal debt crisis typically precedes the sovereign default.

The evidence of a temporal connection between banking crises and sovereign default episodes does not imply any form of causation. However the tight link between domestic financial markets and public debt sustainability is nowadays indisputable.
For example, public bond holdings in banks’ balance sheets can act as a powerful transmission mechanism between both types of crises: a public default harms assets’ side of banks’ balance sheets which decreases credit availability, resulting in financial distress and potentially culminating in a more severe type of banking crises (for example, bank runs). At the same time, banking crises can also impose fiscal pressures on the government’s budget constraint due to a decrease in economic activity which very likely will cause a drop in tax revenues. Furthermore, government’s direct or indirect guarantees on private lending can undermine government’s debt sustainability when the financial system is in distress and those guarantees need to be executed.

The former transmission mechanism, banks’ balance sheets, can easily explain a sovereign debt crisis which precedes a banking crisis. The latter, decrease in tax revenues due to a drop in economic activity or due to guarantees; can account for banking crises preceding sovereign defaults. In reality both mechanisms are potentially relevant and the relative importance of one with respect to the other one will depend on the particular case we are focusing on.

The focus of this paper is another transmission mechanism: bailing out policies, in particular, banks’ recapitalizations. This measure is a very common intervention to mitigate periods of financial distress and banking crises. For the 2007-2009 financial crises, Laeven and Valencia (2010) list all the countries which experienced a systemic banking crisis and report that all of them used liquidity support as a policy intervention (table 1 in the paper). These authors also report the large quantitative importance of this policy: 70% of fiscal outlays directed as policy intervention to mitigate this banking crises corresponded to public sector recapitalization of financial intermediaries. They also highlight that this prevalence is common to the policy interventions used in earlier crisis episodes as well.

Public sector recapitalization of the banking system puts the government under fiscal pressure and affects the capacity and incentives of public debt repayment. As we see in figure 2.1 for the case of Ireland the announcement of a bailout had a clear discontinuous effect on sovereign CDS. For Spain, the highest peak in the CDS for the 10-year bonds, was within a period of two months after the bailout announcement in June 9, 2012.
The focus of this paper is to study how the availability of this tool affects strategic sovereign default as well as to study the timing in the default decision vis a vis the financial distress is affected by the existence of this policy tool.

The model in this paper is a stylized three-period model with two very differentiated areas of analysis. The first one is the sovereign debt problem: government needs to borrow from abroad to make an investment which gives a sure return the following period. The second one is the financial market where financial intermediaries (banks), who invest in a concave production technology, borrow from consumers who are endowed with a productive good. In this financial market borrowers are subject to a collateral constraint and thus banks in general will not be able to raise enough liquidity to invest the first-best level of capital.

The only connection between both areas of analysis is the bailing-out policy tool. At the point when the government has the option to use this tool its wealth is exogenous and it can choose whether to use this wealth to repay bonds to international investors or to recapitalize its banks. Defaulting on international investors has an exogenous cost.

The timing of the events is such that if the government bails out it does so after the first financial equilibrium and before banks go again to the financial market to raise funds. The
fact that there is a financial equilibrium before the default decision allows us to analyze how the previous financial conditions shape the government’s default decision when it can use the wealth freed-up from default to transfer it to its domestic financial system.

The model delivers two sets of results. First, it predicts that the government will optimally choose to default on its obligations with international investors and recapitalize its banks if by not doing so the deposits raised in the following period would not be enough to attain first-best level of investment in the economy. Therefore, the model delivers sovereign default even with no effect on tax revenues or fiscal pressure. Instead, the government finds it optimal to default when the benefit from recapitalizing its financial sector is larger than the cost of defaulting to its international investors.

Second, it delivers that after situations of financial distress, which I define as low credit availability in financial markets, government defaults and recapitalizes its banks. Therefore, the model can deliver the timing of events that has been reported in Arellano and Kocherlakota (2012) and Reinhart and Rogoff (2011). However, it abstracts from many interesting and empirically relevant issues.

First, it abstracts from the negative impact sovereign default has on banks’ balance sheets. This has been studied in other papers which are reviewed below (Gennaioli et al. (forthcoming); Sosa-Padilla (2011)). In practice the public default decision will depend on both: defaulting might allow the government to bailout its banks but at the same time it can have a negative impact on banks’ balance sheets. The relative importance of each of them will depend ultimately on the banking system’s exposure to sovereign debt. Second, the model abstracts from moral hazard. In reality, banks might change their leverage decision, the amount of capital they want to hold as reserves or the deposits they accept once they internalize the possibility of a future public bailout.

2.1.1 Literature review

This paper is related to several strands of the literature. First, the recent literature about the connection between sovereign debt crises and banking crises (Gennaioli et al. (forthcoming), Sosa-Padilla (2011) and Alessandro (2011)). These models think mostly about
default causing a drop in private investment and endogenous output drop through the effect of default on private debt holdings. Arellano and Kocherlakota (2012) studies the opposite timing of events: internal debt crises precede sovereign debt crises. However in this model the sovereign default is ultimately caused by a drop in tax revenues which forces the government to default on public debt as well. In contrast to this, my model delivers strategic default.

The present paper is also relates to the literature on strategic sovereign default (Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008)). However the methodology used in these is very different to the one used in this paper. Those papers are quantitative papers whereas this one is a three-period model which aims to isolate one particular channel for strategic sovereign default incentives: bailouts. In addition, those papers do not have bailouts as a available policy tool for the government.

Finally, this paper is related to the literature on public bailouts to the private sector (Acharya et al. (Forthcoming), Acharya and Yorulmazer (2008), Philippon and Schnabl (2013)). In contrast with those papers, in this paper in order to recapitalize the banking sector the government faces the trade-off of defaulting on its international investors. Also, the sovereign default in this paper is purely strategic, isolating this channel instead of combining a drop in tax revenues.

The paper is organized as follows. Section 2 presents the empirical evidence and stylized facts that the model tackles. Section 3 presents and solves the benchmark model. Section 4 concludes.

### 2.2 Empirical evidence

The stylized facts that this model captures are the following:

1. Financial distress occur *before* the default event: as we have mentioned in the introduction Reinhart and Rogoff (2011) report that banking crises (in the form of financial distress or bank runs) precede or accompany sovereign debt crises. Arellano and Kocherlakota (2012) report, using data from emerging economies between...
1980 and 2003, that sovereign defaults are often caused by fiscal pressures coming from domestic financial distress which causes a drop in tax revenues.

Looking at particular countries does not change the conclusion. In the appendix I present time series of private credit for Argentina, Uruguay, Venezuela and the Dominican Republic. As measures of private credit I use the “Private Credit by Deposit Money Banks and Other Financial Institutions by GDP” from the World Bank Database on Financial Development and Structure Beck et al. (2009), which captures credit allocation in the economy.

We observe that in all of these default episodes financial distress, in the form of lower credit in the economy, starts years before the final government’s default decision.

For example, the default episodes in Uruguay in 2003 and in Argentina in 2001 occur when the credit has dropped from its peak at least three years before. The case for Venezuela is starker: the default episodes in 1995 and 2004 occur when the economy is at its trough from the closest peak in private credit. The 1995 default occurs after a drop in credit which peaks in the early 90s. After the 1995 default private credit increases again in the end of the 90s, hitting again its trough when the government decides to default. The Dominican Republic shows this pattern even clearer: the 2005 default episode occurs after a big drop in credit which peaked in 2000. 1

2. Output drops occur before the default event: there is growing empirical evidence that output drops precede sovereign default episodes. Using quarterly data Levy-Yeyati and Pannizza (2011) conclude that, contrary to the conclusion that arises using yearly data, sovereign defaults tend to follow output contractions and defaults tend to occur in the trough of the recession.

1The Dominican banking system did not have a high exposure to sovereign bonds the year before sovereign default. Dominican Banks’ claims on government as percentage of deposits in 2004 was 12%. (Source: International Financial Statistics, International Monetary Fund).
Figure 2.2: Quarterly GDP before and after the event of default for Argentina (Source: Levy-Yeyati and Pannizza (2011))

Figure 1 shows the trend of quarterly output before and after the default episode in Argentina (dashed vertical line). Other default episodes such as Ecuador 1999, Indonesia, Russia and Pakistan all in 1998 show a similar behavior (see appendix in Levy-Yeyati and Pannizza (2011)).

3. Governments' bailing-out affects public debt sustainability: as we mentioned in the introduction banks' recapitalizations are a very widespread policy intervention and is by scale the most relevant measure after periods of financial distress accounting for around 70% of all fiscal outlays. Countries like Iceland or Ireland after the public interventions for the 2007-2009 financial crises saw their levels of public debt rise dramatically (Laeven and Valencia, 2010).

Other European countries such as Greece, Spain or France also used banks' recapitalization as a policy measure against the 2007-2009 financial crisis and have undergone severe debt crises afterwards. Greece forced haircuts on sovereign creditors in early 2012. Many rating agencies which focus on determining default events (for example, Standard & Poor's (S&P) or Moody's Investors Service) consider this event as sovereign default (Pescatori and Sy, 2007)
Countries like Spain or France saw the interest rates on their sovereign bonds rise in the summer of 2011. Despite not being episodes of sovereign default in strict terms we can interpret it as an increase in the sovereign default risk. Arellano and Kocherlakota (2012) look at the EMBI+ spread\textsuperscript{2} for each country as a measure of sovereign default risk. Other authors such as Pescatori and Sy (2007) propose to expand the definition of sovereign debt crises to the events when bond spreads are above a critical threshold.

In any case, the most recent empirical evidence points to the importance of banks' recapitalizations as policy measure against financial crises and its potential connection to public debt sustainability.

2.3 Benchmark model

2.3.1 Agents and available technologies

In this section I present a stylized model of a small open economy that lasts for three periods $t = 0, 1, 2$. There are three agents in this economy: consumers, entrepreneurs who own banks and a benevolent government.

The government needs to make an investment of size $G > 1$ at $t = 0$ which gives a return $RG$ at $t = 1$. To do so it issues public bonds to international investors at $t = 0$ which mature at $t = 1$. I assume that this economy is a small open economy and thus the government can borrow from abroad at a fixed interest rate $r^* = 1$ for all $t = 0, 1, 2$. At $t = 1$ the government has wealth $W_G = RG$ which it can use to repay international investors or to make a transfer to the banks. This model assumes that bailout is a costless policy tool available to the government in period 1.

Consumers are risk-neutral, do not discount future income streams and consume in periods 1 and 2. They have an initial endowment of $\omega_0$ units of a productive good which they obtain

\textsuperscript{2}The J.P. Morgan’s EMBI+ spread is defined as the difference between the yield of dollar denominated bonds relative to the yield of similar U.S. government bonds.
at the beginning of period 1 and a saving technology which can transfer any amount of the productive good from one period to another one to one.

Entrepreneurs are also risk-neutral. They own the banks, which are endowed with an initial level of net worth $n_0$ at $t = 1$ and have access to a concave production technology, $y_t = k_t^{a_t}$ for $t = 1, 2$ and also to the same saving technology that consumers have. Banks’ profits are consumed in both periods by the entrepreneurs owning the banks. In the rest of the paper I use ‘banks’ and ‘entrepreneurs’ interchangeably to refer to the production side of this economy.

In period 0, when government issues bonds, $n_0$ is unknown to the government and international investors. I assume for simplicity that in period 0 they only know that $n_0$ is distributed uniformly between 0 and 1: $n_0 \sim U[0, 1]$.

2.3.2 Sovereign risk and financial markets

The public bonds $b_0$ issued at $t = 0$ are in principle non-contingent financial claims which pay interest rate $r_b$ at $t = 1$. However, interest rate payments are subject to public default risk which makes public bonds effectively contingent. Defaulting has a cost $C$ for the economy which is a reduced-form way to capture the robust empirical evidence that countries suffer losses after sovereign default.

This losses can be due to a drop in trade, either because of direct trade sanctions or because after default there is a trade credit dry-up (Rose (2005); Martinez and Sandleris (2011)); government’s exclusion from international credit markets (Eaton and Gersovitz (1981); Alessandro et al. (2011)) or domestic costs of default such as an increase in financing costs for domestic firms (Mendoza and Yue (Forthcoming)), collapse of foreign credit to private sector (Arteta and Hale (2008)) or loss for residents in the defaulting country who had government bonds (Gennaioli et al. (forthcoming); Broner et al. (2010)).

Banks go to financial markets at time $t$ with net worth $n_{t-1}$ to raise liquidity from consumers, who go to financial markets with endowment $o_{t-1}$. Banks raise liquidity in the form of deposits $d_t$ for $t = 1, 2$ which pay a rate of return $r_t^d$ to consumers. Banks optimal
choice of deposits, \( d_t = k_t - n_{t-1} \), would be such that they could invest the first-best level of capital \( k_t^{FB} \) in the productive technology. This level of capital maximizes production minus the opportunity cost of capital which is equal to 1 (consumers and banks have access to saving technology).

Therefore,

\[
\max_k \{k^\alpha - k_t\}
\]

where the first order condition is given by \( \alpha k_t^{\alpha - 1} = 1 \) and hence \( k_t^{FB} = \alpha^{1\over 1-\alpha} \).

In general \( k_t^{FB} \) need not be invested in this model because the amount of liquidity banks can raise in financial markets is subject to a collateral constraint by virtue of which borrowing costs cannot exceed some fraction \( \theta \) of the banks' capital when writing the deposit contract. A way to interpret this borrowing constraint is to assume that private deposits are subject to imperfect court enforcement as in Gennaioli et al. (forthcoming): if a bank defaults only a share \( \theta \) of its capital can be seized by depositors.

### 2.3.3 Timing of the model

The model consists of three periods. The timing of the events in each of the three periods is given below.

#### Period 0

1. Government issues public bonds \( b_0 \).

#### Period 1

1. \( n_0 \) realizes, consumers get endowment \( \omega_0 \) and government gets \( W_G = R^G G \).
2. Financial markets open. Equilibrium in financial market \((d_1, r_1^d)\) is observed.
3. Banks invest \((n_0 + d_1)\) in the production technology and repay deposit returns \( r_1^d d_1 \).
4. Government uses its wealth \( W_G \) either to repay international investors \( R \) and/or to increase banks' wealth by making a transfer \( T \) (bailout). If government defaults, the default indicator variable takes a value of one \( \chi_1 = 1 \).

**Period 2**

1. Financial markets reopen. Equilibrium in financial market \( (d_2, r^2) \) is observed.

2. Banks invest \((n_1 + d_2)\) in the production technology and deposit returns, \( r^2 d_2 \), are paid out.

**2.4 Equilibrium**

First we will solve the equilibria in financial markets in \( t = 1 \) and \( t = 2 \). Knowing the equilibria in both periods we solve for the default decision at \( t = 1 \) where government maximizes consumption at \( t = 2 \) of all agents in the economy subject to its budget constraint and internalizing its effect on banks' net worth and on deposits at \( t = 2 \) through its effect on banks net worth.

Knowing in which cases the government decides to transfer the investment returns to banks we check whether \( E(r^b) \geq 1 \) and hence whether international investors are willing to lend to the government in the first place, that is, if public debt is sustainable. This problem will boil down to a fixed point problem.

**2.4.1 Equilibrium in financial markets at \( t = 1 \) and \( t = 2 \)**

In this section I solve for the equilibrium in financial markets.

Entrepreneurs maximize consumption in both periods, \( c^E_1 \) and \( c^E_2 \). Consumption comes from banks' operating profits in both periods: \( c^E_1 \leq k_1^\alpha - r^1 d_1 \) and \( c^E_2 \leq k_2^\alpha - r^2 d_2 \). Entrepreneurs will invest their net worth and the liquidity raised in financial markets in the production technology and will pay the costs of liquidity.
Hence entrepreneurs are ultimately maximizing profits subject to $t = 1$ and $t = 2$ budget constraints as well as the collateral constraints in both periods:

$$\max_{\{d_1, d_2\}} k_1^\alpha - r_1^d d_1 + k_2^\alpha - r_2^d d_2$$

s.t. $k_1 \leq n_0 + d_1$

$k_2 \leq \bar{n}_1 + d_2$

$r_1^d d_1 \leq \theta n_0$

$r_2^d d_2 \leq \theta n_1$

Banks' net worth at $t = 2$, $\bar{n}_1$, is defined as the net profits obtained from operating the period before plus the government's transfer, if any: $\bar{n}_1 = (n_0 + d_1)^\alpha - r_1^d d_1 + T$ and it is a given for the banks when they decide how much deposits to demand.

The first order conditions of this problem are identical to the ones from solving the problem independently for both periods. Therefore we use the following definition to solve for the equilibrium in financial markets.

**Definition 12.** *Equilibrium in financial markets at $t$ is interest rate on deposits and quantity of deposits $(r_t^d, d_t)$ such that banks maximize period by period profits from investment taking $r_t^d$ as given, consumers maximize consumption and markets clear.*

Banks maximize profits in each period:

$$\max_{d_t} k_t^\alpha - r_t^d d_t \quad (P1)$$

s.t. $k_t \leq n_{t-1} + d_t$

$r_t^d d_t \leq \theta n_{t-1}$

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Banks' demand for deposits is the following,

\[ d_t = \left( \frac{\alpha}{r_t^d} \right)^{\frac{1}{1-a}} n_{t-1} \quad \text{if} \quad n_{t-1} \geq \frac{[(r_t^d)^{2-\alpha}]^{\frac{1}{1-a}}}{1+\theta} \]

\[ d_t = \frac{\theta}{r_t^d} n_{t-1} \quad \text{otherwise} \]

The supply side of this market are consumers. Consumers' problem is to maximize utility of consumption subject to their intertemporal budget constraint. Consumers are risk-neutral, do not discount future utility of consumption and their savings technology gives a gross return equal to 1. Hence their problem at \( t = 1 \) is given by:

\[ \max_{\{d_1,d_2\}} \quad c_1 + c_2 \quad (P2) \]

subject to

\[ c_1 + c_2 = \omega_0 + (r_1^d - 1)d_1 + (r_2^d - 1)d_2 \]

where \( \omega_0 \) denote endowment received at the beginning of period 0.

Program (P2) delivers the following supply of funds schedule in financial markets for each period \( t \)

\[ d_1 = \omega_0; \quad d_2 = \omega_1 \quad \text{if} \quad r_1^d, r_2^d > 1 \]

\[ d_1 = [0, \omega_0]; \quad d_2 = [0, \omega_1] \quad \text{if} \quad r_1^d = r_2^d = 1 \]

\[ d_1 = d_2 = 0 \quad \text{if} \quad r_1^d, r_2^d < 1 \]

where \( \omega_1 \) denotes consumers' income at the beginning of \( t = 2 \) and it is defined as \( \omega_1 \equiv \omega_0 - d_1 + r_1^d d_1 \).

The equilibria in financial markets \((r_t^d, d_t)\) are the following:

1. Unconstrained equilibrium: \( r_t^{dU} = 1 \) and \( d_t^{dU} = \alpha^{\frac{1}{1-a}} - n_{t-1} \).

   This occurs whenever \( n_{t-1} \) is big enough: \( n_{t-1} \geq \frac{\alpha^{\frac{1}{1-a}}}{1+\theta} \equiv n_{t-1}^{FD} \). The intuition for this is clear: since deposits in equilibrium need to be collateralized by banks' wealth,
banks will be able to raise an unconstrained level of deposits if \( n_{t-1} \) is big enough to act as collateral.

\( n_{t-1}^{FD} \) denotes the cut-off level for “financial distress”.

2. Constrained equilibrium: \( r_{t}^{C} = 1 \) and \( d_{t}^{C} = \theta n_{t-1} \).

This happens when \( n_{t-1} < n_{t-1}^{FD} \).

For there to be equilibria where \( r_{t}^{C} = 1 \), consumers endowments need to be big enough so that demand and supply of deposits cross in the interval where the supply of funds is flat.

In the constrained equilibrium this requires that \( \theta n_{t-1} < \omega_{t-1} \), at \( t = 1 \) this condition is given in (A1) part (i). At \( t = 2 \) this condition is given by \( \omega_{0} > \theta n_{1} = \theta [k_{1}^{EB} - k_{1} + T] \). The maximum level of \( k_{1} = k_{1}^{EB} \) so the most restrictive version of this restriction is given in part (ii) of (A1).

**Assumption 13.** Assumptions on endowments: (i) \( \omega_{0} > \theta n_{0} \), (ii) \( \omega_{0} > \theta [\alpha^{1-\alpha} - \alpha^{1-\alpha} + T] \)

Since the unconstrained equilibrium values of deposits must be lower than the threshold in the collateral constraint, assumption 13 ensures that the unconstrained levels of deposits are smaller than total endowments.

### 2.4.2 Government’s bailout decision

When choosing whether to default or not, the benevolent government maximizes agents’ consumption in the economy. The economy’s resource constraint is given by the following expression which will hold with equality,

\[
c_{1}^{E} + c_{2}^{E} + c_{1} + c_{2} + k_{1} + k_{2} + R(1 - \chi_{1}) + C\chi_{1} = k_{1}^{G} + k_{2}^{G} + W_{G} + n_{0} + \omega_{0}
\]

where \( \chi_{1} \) is the indicator function for default: equals 1 if country defaults on international investors and 0 otherwise. \( R \) denotes the amount government decides to repay to international investors, \( R = r_{p}b_{0} \) which is the gross rate of return international investors get from
public bonds. $C$ denotes the output drop if there is sovereign default and $W_G, n_0$ and $\omega_0$ are the economy's endowments.

At the end of $t = 1$ the government can only affect the financial equilibrium in the following period, $t = 2$ and $\{c_1^F, c_1, k_1\}$ remain unaffected by the government's default policy. Denoting $\bar{W} = W_G + n_0 + \omega_0$ the total endowment in the economy we can rewrite the government's problem over $\chi_1$ ignoring the variables over which it has no control ($c_1^F, c_1, k_1, \bar{W}$):

$$\max_{\chi_1=\{0,1\}} \quad k_2^\alpha - k_2 - R(1 - \chi_1) - C\chi_1 \quad (P3)$$

$$s.t. \quad T = W_G - R(1 - \chi_1)$$

$$k_2 = k_2^{FB} = \frac{\alpha^{1-a}}{1+\theta} \quad if \quad n_1 + T \geq \frac{\alpha^{1-a}}{1+\theta}$$

$$k_2 = \theta(n_1 + T) \quad otherwise$$

The first constraint in (P2) is the government's budget constraint. This constraint binds because government wants to use up all its resources. The other two constraints denote the level of equilibrium capital in financial markets at $t = 2$, since when the government decides how much funds to transfer to banks it internalizes agents' behavior in financial markets at $t = 2$.

**Assumption 14.** Assumption on costs: $C > R$

Suppose first that banks' wealth and government's wealth net of repayment to international investors is big enough so that the equilibrium in financial markets at $t = 2$ is the unconstrained equilibrium, that is, suppose the following condition holds:

$$n_1 + W_G - R > \frac{\alpha^{1-a}}{1+\theta} \iff n_1 > \frac{\alpha^{1-a}}{1+\theta} + R - W_G \quad (2.1)$$

If 2.1 holds then surely the following inequality also holds:

$$n_1 > \frac{\alpha^{1-a}}{1+\theta} - W_G \quad (2.2)$$
Under conditions 2.1 and 2.2 the government's problem (P3) boils down to the following maximization problem,

$$\max\{(k_2^{FB})^\alpha - k_2^{FB} - R, (k_2^{FB})^\alpha - k_2^{FB} - C\} \quad (2.3)$$

where the first term in the brackets is the value of repaying investors and the second term is the value of default. Since we assume $C > R$, the government in 2.3 chooses to repay international investors.

Suppose now that we are in the opposite case: nor 2.1 nor 2.2 hold, that is, even if the government were to transfer all its wealth $W_G$ to banks they would not be able to raise enough deposits to invest the first-best level of capital. The most restrictive condition will be the one expressed below

$$n_1 < \frac{\alpha^\frac{1}{1-\alpha}}{1 + \theta} - W_G \quad (2.4)$$

In this case the government's problem is the following

$$\max\{(k_2^L)^\alpha - k_2^L - R, (k_2^H)^\alpha - k_2^H - C\} \quad (2.5)$$

where $k_2^L \equiv \theta(n_1 + W_G - R)$ and $k_2^H \equiv \theta(n_1 + W_G)$ and again the first term is the value of repaying and the second is the value of default. 2.4

Since $k_2^L, k_2^H < k^{FB}$ then $(k_2^H)^\alpha - k_2^H > (k_2^L)^\alpha - k_2^L$ because decreasing returns have not started to kick in when capital levels are below first-best level of capital.

Therefore the marginal benefit from defaulting on international investors equals the extra surplus from production using the banks' technology, that is,

$$(k_2^H)^\alpha - k_2^H - (k_2^L)^\alpha + k_2^L$$

The marginal cost from defaulting equals the additional cost the economy suffers by defaulting, that is,
Then if $(k^H)^a - k^H - (k^L)^a + k^L > C - R$ government chooses to default. Otherwise government prefers to repay its international investors. The intuition for this condition is clear: for government to default it has to be that the differential cost from defaulting is not too big in comparison to the benefits from mitigating the underinvestment in the productive sector of the economy.

We can rewrite this condition as

$$\text{Default} \Leftrightarrow [(k^H)^a - (k^L)^a] > C - (1 - \theta)R$$

(2.6)

Finally we study what is the optimal government’s decision when condition 2.2 holds but condition 2.1 is violated:

$$n_1 - R < \frac{\alpha \gamma}{1 + \theta} - W_G < n_1$$

(2.7)

When condition 2.4 holds then the government’s problem is similar to 2.3:

$$\max \{ (k^L)^a - k^L - R, (k^{FB})^a - k^{FB} - C \}$$

(2.8)

where $k^L$ and $k^{FB}$ are defined above. Again government decides to default when $(k^{FB})^a - k^{FB} - (k^L)^a + k^L > C - R$. We can rewrite this condition as follows

$$\text{Default} \Leftrightarrow [(k^{FB})^a - (k^L)^a] - [k^{FB} - k^H] > C - (1 - \theta)R$$

(2.9)

If condition 2.6 holds then necessarily condition 2.9 also holds. This is intuitive, if the marginal benefit from increasing investment from $k^L$ to $k^H$ is worth the additional cost from defaulting then surely increasing investment to first-best will also be worth the cost of defaulting.  

$^3$From the definition of $k^{FB}$ we know that it maximizes surplus, hence $(k^{FB})^a - k^{FB} > (k^H)^a - k^H \Rightarrow (k^{FB})^a - (k^L)^a - k^{FB} + k^H > (k^H)^a - (k^L)^a$. 

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I assume that 2.6 holds and hence as we have seen 2.9 follows. This means that the relevant cut-off for the default decision is defined in 2.1, \( n_1^D = \frac{a^{\frac{1}{1-a}}}{1+\theta} + R - W_G \) and hence whenever government can increase the production in the economy towards first-best it does so by bailing-out the banks and defaulting.

The following proposition summarizes the analysis up to now:

**Proposition 15.** If the government can use its wealth to make transfers to domestic financial institutions and (i) borrowing in financial markets is subject to a collateral constraint, (ii) banks have an optimal level of investment to make and (iii) costs of sovereign default are higher than repayment; then the government will optimally choose to default on public debt and recapitalize its banks only when future banks' wealth is below some threshold (given in 2.1).

The analysis of the government's default decision up to now was given in terms of \( n_1 \), net worth banks use as collateral and invest at \( t = 2 \). \( n_1 \) is defined in terms of output and costs the previous period, \( n_1 = (n_0 + d_1)^{\alpha} - d_1 \). Therefore we can also conclude how the default decision depends on initial net worth \( n_0 \) and on the equilibrium in the financial market at \( t = 1 \).

Banks' net worth \( n_1 \) can be at at two different levels. First, if banks' net worth was low enough to have a constrained number of deposits in equilibrium at \( t = 1 \), that is if \( n_0 < n_0^{FD} \), then banks' capital \( n_1 = (1 + \theta)^{\alpha}n_0^\alpha - \theta n_0 \) and it is increasing in \( n_0 \) for the range of \( n_0 \) we are considering.

For very low values \( n_0 \), \( n_1 \) will be for sure below the threshold level \( n_1^D \). If we assume:

\[
\alpha^{\frac{1}{1-a}} - \alpha^{\frac{a}{1-a}} < W_G - R \tag{2.10}
\]

then for sure \( n_1 \) is always below the recapitalizing threshold \( n_1^D \) when equilibrium deposits were constrained by banks' net worth in the period prior to default. Therefore, under the parametric restriction (10) government always bailsout the banking system when there was a period of financial distress prior to the bailing-out decision.
If instead banks were unconstrained at $t = 1$, that is if $n_0 \geq \tilde{n}_0^{FD}$, then banks were able to raise the optimal level of deposits and invest the optimal level of capital in the production technology. In this case,

$$ n_1 = \alpha^{1-a} - \alpha^{1-a} + n_0 $$

(2.11)

Then the condition of $n_1$ as defined in 2.11 being bigger or equal to $\tilde{n}_1^D$ can be rewritten in terms of $n_0$ as:

$$ n_0 \geq \frac{\alpha^{1-a} (2 + \theta)}{1 + \theta} + R - W_G - \alpha^{1-a} $$

(2.12)

If we assume 2.10 holds then necessarily condition 2.12 is more restrictive than $n_0 \geq \tilde{n}_0^{FD}$. This implies that even if there was no financial distress measured by low credit availability in the initial period the government can find it profitable to transfer resources to banks.

The following proposition summarizes the interaction between previous financial distress and sovereign default decisions when government has as policy tool bailing out the domestic financial system:

**Proposition 16.** If all assumptions in proposition 4 hold and additionally government’s wealth net of repayment is big enough, then government always defaults and bails out the domestic banking system when there was previously a period of financial distress with binding collateral constraints, low credit in equilibrium, low investment and low output.

The figure below is a graphical representation of the two propositions above under assumptions 2.6 and 2.10 which we reproduce here for convenience. In figure 2, $\tilde{n}_0^{FD} \equiv \frac{\alpha^{1-a}}{1+\theta}$ denotes the “financial distress” cut-off, for values of net worth lower than the cut-off the equilibrium deposits at $t = 1$ are the constrained ones. $\tilde{n}_0^D$ and $\tilde{n}_1^D$ are the default cut-offs, for net worths higher than them country does not default and are defined as

$$ \tilde{n}_0^D \equiv \frac{\alpha^{1-a} (2 + \theta)}{1 + \theta} + R - W_G - \alpha^{1-a} $$

and

$$ \tilde{n}_1^D \equiv \frac{\alpha^{1-a}}{1+\theta} + R - W_G. $$

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2.4.3 Public debt sustainability

For this to be an equilibrium where public debt is sustainable it must be that international investors at \( t = 0 \) are willing to lend to the government. Below we find the definition of when this is the case.

**Definition. Equilibrium in international markets:** Equilibrium in international bonds market \( \{b_0, r^b\} \) is such that \( b_0 \geq G \), so public investment is carried out and \( E(r^b) \geq r^* = 1 \) where \( E(r^b) \) is the expected return on the bond.

Recall that at \( t = 0 \), the realization of \( n_0 \) is unknown. I assume that \( n_0 \sim U[0, 1] \). Hence ex ante under 2.6 and 2.10 international investors have the following expected return from lending to this government:

\[
E(\text{repayment}) \cdot r^b = \left[ 1 - \frac{\alpha^{-\theta}(2 + \theta)}{1 + \theta} - R + W_G + \alpha^{-\theta} \right] r^b \tag{2.13}
\]

because international investors know that they will only be repaid if \( n_0 \) satisfies 2.12. Under the distributional assumption we made about \( n_0 \), the term in squared brackets in 2.13 is the probability of \( n_0 \) being bigger than the threshold in 2.12. If this condition is not satisfied

---

\( ^4 \)It must be the case that the threshold level \( \left[ \frac{\alpha^{-\theta}(2 + \theta)}{1 + \theta} + R - W_G - \alpha^{-\theta} \right] \in (0, 1) \). If it is greater than one government would always default and public debt would not be sustainable. If it is smaller than 0 then government never defaults and never bails out its banking system.
investors get zero because government defaults and transfers its wealth to its domestic financial system.

According to definition 4, 2.13 needs to be equal to 1, otherwise international investors won’t lend to the government. Recall that repayment \( R = r_b b_0 \). Hence the equilibrium value of interest rate on debt \( r^b \) in this economy will in general not be unique. Equilibria values of \( r^b \) will solve the following quadratic equation:

\[
Gr^b - \mu r^b + 1 = 0 \tag{2.14}
\]

where I have used the fact that \( b_0 \) will be the smallest possible value that satisfies \( b_0 \geq G \) and where for ease of notation I use \( \mu \equiv \left[ 1 + W^G + \alpha \frac{\tau_{1-a}}{1+\theta} \right] \).

The possible equilibrium interest rates on sovereign debt \( r^b \) that government pays to international investors when its domestic banking system is “healthy”, in the sense that banks are highly capitalized are the solutions to the quadratic equation 2.14 and are given below:

\[
r^b = \frac{\mu \pm \sqrt{\mu^2 - 4G}}{2G} \tag{2.15}
\]

which as long as \( \mu^2 > 4G \) gives two solutions for interest rates which necessarily satisfy \( r^b > 1 \) since international investors need to make expected returns equal to 1 \textit{ex ante} for them to lend to the government. If the government only repays in some states of the world and not in others the return the government pays when the realization of \( n_0 \) is high enough needs to compensate for the states when \( n_0 \) is low.

Another way of interpreting the question of debt sustainability which conveys more intuition is that of a fixed point problem. From 2.13 we can express the solution for \( r^b \) in terms of expected repayment,

\[
r^b = [E(\text{repayment})]^{-1} \tag{2.16}
\]
where expected repayment depends negatively on $r^b$, 
\[ \frac{\partial}{\partial r^b} \left[ 1 - \frac{\alpha^{1-a/(2+a)}}{1+a} - r^b b_0 + W_G + \alpha^{1-a} \right] < 0. \]

Hence if $r^b$ is high, expected repayment decreases and from 2.16 we know that this will drive up the equilibrium interest rate $r^b$ at which government borrows from international investors. The interest rates in 2.15 are the solutions to this fixed point problem.

In this model a high initial level of $n_0$, higher banks’ net worth, increases the probability of debt repayment and decreases $r^b$ and hence decreases public default risk as well.

### 2.5 Conclusion

This paper proposes a model to firstly analyze the optimal default decision when the government can use its funds to capitalize the domestic financial system and secondly to study the relationship between sovereign default and the situation in financial markets.

It concludes that periods of distress in financial markets will lead the government to strategically default on sovereign debt despite having a high cost of doing so if it can divert its funds to the productive technology in the economy.

Therefore the model proposed delivers the commonly observed timing of events that financial crises precede sovereign debt crises and the empirical evidence that sovereign defaults often follow periods of low credit availability, low investment and low output.

The chain of events that generates the timing we observe in the model starts with initial banks’ wealth. First, low levels of banks’ wealth brings about low credit availability in financial markets, due to the existence of collateral constraints. Second, low levels of initial wealth also bring about lower levels of net worth when the government takes the default/bailout decision. Lastly, low levels of banks’ net worth makes it impossible for the economy to attain first-best level of investment and output and this makes bailout the optimal usage of the public funds.

In this model more capitalized financial institutions increase probability of repayment which decreases public default risk as measured by the interest rate on bonds. In other
words, a capitalized financial system renders public debt sustainable and public default risk low.

In this model government's wealth, which is given by the return to a public investment, is exogenous. In reality most of government's wealth are tax revenues and these are chosen by the government. Endogeneizing the amount of public funds in the model by allowing government to levy taxes on consumers is a relevant avenue for future research which will capture relevant trade-offs faced by governments in reality when deciding whether to default on international investors or repay its debt.

Another empirically relevant extension of the model is to allow the domestic financial system to be exposed to government’s bonds. In a sense default would have an extra cost: the drop in banks’ net worth and hence default will require more funds to bail out banks. Introducing banks’ exposure to sovereign debt and exploring its “disciplining effect” regarding default when government can recapitalize its banking system is an interesting extension of the model presented in this paper.
2. A Private Credit over GDP for a selection of Countries

The following list of figures plot the variable "Private Credit by Deposit Money Banks and Other Financial Institutions by GDP" from the World Bank Database on Financial Development and Structure (Beck et al., 2009) for Argentina, Uruguay, Venezuela and Dominican Republic.
Chapter 3

Stochastic Terms of Trade Volatility in Small Open Economies

3.1 Introduction

The terms of trade of many commodity-producing small open economies are subject to large shocks that can be an important source of economic fluctuations. Alongside times of high volatility, however, these economies also experience periods in which their terms of trade are comparatively stable. The effect of shocks to the level of the terms of trade has been widely studied. But little is known about the impact of changes in the volatility of terms of trade shocks. We study the macroeconomic effects of these shocks and quantify their importance as a source of business cycle fluctuations.

Figure 3.1 shows the growth rate of the terms of trade for a selection of commodity-producing small open economies. At various times, each economy has experienced an increase or decrease in its terms of trade of more than 10 per cent in a quarter, while fluctuations of five per cent or more are common. The existence of these large shocks has motivated a substantial literature examining the impact of changes in the level of the terms of trade on these economies (Mendoza 1995, Kose and Riezman 2001, Broda 2004).

\[\text{This paper is co-authored with my classmate Daniel Rees.}\]
In addition to these large shocks, Figure 3.1 also suggests that the terms of trade volatility that small open economies experience varies over time. Each economy in the figure has undergone episodes of extremely high terms of trade volatility, including during the 1970s for Australia and New Zealand, 1980s for Brazil, Mexico and South Africa, and 2000s for Canada. But these economies have also experienced sustained periods in which their terms of trade were comparatively stable, such as the late 1990s for Australia, New Zealand and Mexico. This paper examines the economic relevance of these changes in terms of trade volatility.

To address this question, we first estimate the empirical process of the terms of trade for the six economies featured in Figure 3.1. We use the estimated time series of terms of trade volatility produced in this exercise to identify the effect of volatility shocks on output, consumption, investment, the current account and prices in a vector autoregression (VAR). We then set up and augment a small open economy real business cycle model to incorporate stochastic terms of trade volatility. We test whether this model can replicate the empirical responses produced by the VAR and use it to explore the theoretical causes and sectoral

Figure 3.1: Terms of Trade Growth: Selected Countries

Source: See Appendix 3. A.
impacts of these responses. Finally, we compute variance decompositions to quantify the importance of terms of trade volatility shocks as a source of macroeconomic fluctuations.

Our empirical results suggest that an increase in terms of trade volatility depresses domestic demand and leads to an improvement in the current account, leaving the response of aggregate output ambiguous. Our model successfully replicates these patterns. It also suggests increased terms of trade volatility causes a shift in the composition of output from non-tradeables to tradeables and a substitution in factor inputs from capital to labor.

The effects of terms of trade volatility shocks are generally small. But, interacted with shocks to the level of the terms of trade, variance decompositions suggest that they have an economically meaningful impact. For a typical small open economy we find that shocks to volatility account for around one quarter of the total impact of terms of trade shocks on the standard deviations of output, consumption and investment.

3.1.1 Literature review

Our paper is related to several strands of literature. Most directly, it complements other papers that have studied the real effects of uncertainty and time-varying volatility on macroeconomic aggregates. Examples here include Bloom (2009) who explores the short-run fluctuations of output, employment and productivity growth after a shock to macroeconomic uncertainty, Justiniano and Primiceri (2008) who shed light on the sources of changes in US macroeconomic volatility in the postwar period using a structural model and Christiano et al. (2009) who use a financial accelerator model to study the effects of an idiosyncratic risk shock to entrepreneurs' productivity. Closely related to our paper is Fernandez-Villaverde et al. (2011), who examine shocks to the volatility of sovereign debt interest rates and Fernandez-Villaverde et al. (2012), who study how changes in uncertainty about future fiscal policy affects aggregate economic activity. Our main contribution to this literature is empirical. We document time-varying volatility in a variable, the terms of trade, that has not previously been studied and explore the effects of changes in this volatility.

The paper also builds on the literature examining the macroeconomic effects of terms of trade shocks. Many papers in this literature have examined terms of trade shocks using
calibrated business cycle models. These typically conclude that terms of trade shocks are an important source of small open economy business cycles. For example, Mendoza (1995) concludes that terms of trade shocks account for around half of the fluctuations in GDP in developing countries and slightly less in developed economies. In a model calibrated to match features of a standard developing economy, Kose and Riezman (2001) find that terms of trade shocks account for 45 per cent of output volatility and 86 per cent of investment volatility. And, in a model calibrated for Canada, Macklem (1993) finds that a 10 per cent temporary deterioration in the terms of trade - a large but not unprecedented shock for the economies in Figure (3.1) - reduces real GDP by almost 10 per cent and investment by almost 20 per cent.

Other papers in this literature have adopted a more reduced form approach and examined the effects of terms of trade shocks in vector autoregression models. These papers typically find smaller effects of terms of trade shocks than those that rely on structural business cycle models. For example, using a panel VAR covering 75 developing countries, Broda (2004) concludes that a 10 per cent permanent deterioration in the terms of trade reduces the level of GDP by around one per cent, and that the terms of trade shocks explain between 10 - 30 per cent of GDP growth. Similarly, Collier and Goderis (2012) find that a 10 per cent rise in commodity prices increases the level of GDP by around one percentage point after two years for a typical developing country. Our contribution to this literature is to illustrate another channel through which the terms of trade can have macroeconomic effects. In particular, we show how changes in terms of trade volatility can have an impact even if the level of the terms of trade remains constant.

Alongside the literature examining the dynamic effect of shocks to the level of the terms of trade, another empirical literature documents a negative link between terms of trade volatility and long-run economic growth. Using a panel of 35 developed and developing economies over the period 1870 to 1939, Blattman et al. (2007) conclude that, for commodity producers, a one standard deviation increase in terms of trade volatility (in their sample, from 8 per cent to 13 per cent per year) causes a 0.4 percentage point reduction in annual per capita GDP growth. In related work, Williamson (2008) attributes much of
the gap in economic performance in the early 19th century between economies in Western Europe and those in Eastern Europe, the Middle East and East Asia to fact that the latter groups experienced more terms of trade volatility. Focussing on more contemporary patterns, Bleaney and Greenaway (2001) estimate a cross-country panel regression using data from 14 sub-Saharan African countries over the period 1980-95 and also conclude that terms of trade volatility, measured as the residuals from a GARCH model of the terms of trade, reduces GDP growth.

As well as long-run growth, papers in this literature have examined links between terms of trade volatility and the volatility of other macroeconomic variables. For example, using a panel of countries Easterly et al. (2000) show that times of high terms of trade volatility tends to be correlated with times of more volatile GDP growth, while Andrews and Rees (2009) also establish a link with consumption and inflation volatility. The theoretical grounding for these results was established in Mendoza (1997). Using a stochastic growth model, he demonstrates that terms of trade volatility can affect growth through its effects on households’ incentives to save, but that the direction of the effect depends on the degree of households’ risk aversion. He also shows that, regardless of its impact on growth, an increase in terms of trade volatility reduces welfare. Our paper complements this literature by illustrating the links between terms of trade volatility and macroeconomic outcomes in a fully-specified macroeconomic model and by tracing out the dynamic effects of changes in terms of trade volatility on output, external accounts and prices.

3.2 Estimating the Law of Motion for the Terms of Trade

In this section, we estimate the empirical process for the terms of trade for six small open economies: Australia, Brazil, Canada, Mexico, New Zealand and South Africa. We selected these countries based on two criteria. First, we focussed on commodity-producing small open economies whose terms of trade are both volatile and plausibly exogenous to domestic economic developments. Second, we required countries to have reasonably long time series data for the terms of trade and other macroeconomic variables.
Our claim that the terms of trade for these countries are exogenous may be controversial. To support our contention, Table 3.1 provides descriptive statistics about the size and export composition of each economy. The six economies each account for a small share of world GDP and merchandise trade. This suggests that economic developments within these countries are unlikely to have a substantial effect on world economic activity. Moreover, the exports of these countries are geared towards agriculture, fuels and mining - that is, commodities - with these goods accounting for more than 50 per cent of merchandise export values for each country, except Mexico. Commodity prices tend to be less differentiated, and more substitutable, than manufactured goods and commodity producers generally have less pricing power on world markets. Further evidence to support our contention comes from the numerous studies that have used statistical techniques to examine the exogeneity of the terms of trade for small open economies. For example, using Granger causality tests, Mendoza (1995) and Broda (2004) conclude that the terms of trade is exogenous for a large sample of small open economies, including Brazil, Mexico and Canada.

2 Even for Mexico, petroleum is the largest single export good at the three digit SITC 3 level, accounting for almost 12 per cent of total exports in 2010. Moreover, commodities accounted for the bulk of Mexico's exports in the early part of our sample, before the expansion of manufacturing exports that accompanied Mexico's trade liberalization in 1986 and entry into NAFTA in 1994 (Moreno-Brid et al. 2005).

3 While this is not strictly true for all commodity producers, such as large oil producers for example, it seems reasonable for the countries in our sample.

### Table 3.1: Summary Statistics: Merchandise Exports, 2010

<table>
<thead>
<tr>
<th>Share of world merchandise exports</th>
<th>Export Composition</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Food Items and Agricultural Raw Materials</td>
<td>Fuels, Ores and Metals</td>
<td>Manufactured Goods</td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>1.4</td>
<td>13.1</td>
<td>69.2</td>
<td>12.8</td>
</tr>
<tr>
<td>Brazil</td>
<td>1.3</td>
<td>34.8</td>
<td>29.5</td>
<td>35.8</td>
</tr>
<tr>
<td>Canada</td>
<td>2.5</td>
<td>13.5</td>
<td>35.6</td>
<td>47.8</td>
</tr>
<tr>
<td>Mexico</td>
<td>2.0</td>
<td>6.3</td>
<td>18.7</td>
<td>74.5</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.2</td>
<td>63.3</td>
<td>10.1</td>
<td>22.9</td>
</tr>
<tr>
<td>South Africa</td>
<td>0.6</td>
<td>10.5</td>
<td>47.4</td>
<td>39.2</td>
</tr>
</tbody>
</table>

Source: UNCTAD Handbook of Statistics 2011
3.2.1 Estimation

For each country, we specify that the terms of trade, $q_t$, follow an AR(1) process described by:

$$q_t = \rho_q q_{t-1} + \epsilon^{\sigma_{q,t}} u_{q,t}$$

(3.1)

where $u_{q,t}$ are normally distributed shocks with mean zero and unit variance. The log of the standard deviation of the terms of trade shocks, $\sigma_{q,t}$, varies over time, according to an AR(1) process:

$$\sigma_{q,t} = (1 - \rho_\sigma) \sigma_q + \rho_\sigma \sigma_{q,t-1} + \eta_q u_{\sigma,t}$$

(3.2)

where $u_{\sigma,t}$ are normally distributed shocks with mean zero and unit variance. To emphasize, innovations to $u_{q,t}$ alter the level of the terms of trade, while innovations to $u_{\sigma,t}$ alter the magnitude of shocks to the terms of trade, with no direct effect on its level. The parameter $\sigma_q$ is the log of the mean standard deviation of terms of trade shocks, while $\eta_q$ is the standard deviation of shocks to the volatility of the terms of trade. The parameter $\rho_\sigma$ controls the persistence of terms of trade volatility shocks. Throughout, we assume that $u_{q,t}$ and $u_{\sigma,t}$ are independent of each other.

Equations (3.1)-(3.2) represent a standard stochastic volatility model. Inference in these models is challenging because of the presence of two innovations, to the level of the terms of trade and to its volatility, that enter the model in a nonlinear manner. To overcome this issue, we follow Fernandez-Villaverde et al. (2011) and use a sequential Markov Chain Monte Carlo filter, also known as a particle filter, that allows us to evaluate the likelihood of the model using simulation methods. We estimate the model using a Bayesian approach that combines prior information with information that can be extracted from the data.

Denote the vector of parameters to be estimated as $\Psi = \{\rho_q, \rho_\sigma, \sigma_q, \eta_q\}$ and the log of the prior probability of observing a given vector of parameters $\mathcal{L}(\Psi)$. The function $\mathcal{L}(\Psi)$
summarizes what is known about the parameters prior to estimation. The log-likelihood of observing the dataset \( q^T = \{q_1, \ldots, q_T \} \) for a given parameter vector is denoted \( \mathcal{L} (q^T | \Psi) \).

The likelihood of the data given the parameters factorizes to:

\[
\exp (\mathcal{L} (q^T | \Psi)) = p (q^T | \Psi) = \prod_{t=1}^{T} p (q_t | q_{t-1}, \Psi)
\]

The final term in this expression expands as follows:

\[
\prod_{t=1}^{T} p (q_t | q_{t-1}, \Psi) = \prod_{t=1}^{T} \int p (q_t | q_{t-1}, \sigma_{q,t}; \Psi) p (\sigma_{q,t} | q_{t-1}^{-1}; \Psi) d\sigma_{q,t} \tag{3.3}
\]

Computing this expression is difficult because the sequence of conditional densities \( \{p (\sigma_{q,t} | q_{t-1}^{-1}; \Psi)\}_{t=1}^{T} \) has no analytical characterization. A standard procedure, which we follow, is to substitute the density \( p (\sigma_{q,t} | q_{t-1}^{-1}; \Psi) \) with an empirical draw from it. To obtain these draws, we follow Algorithm 1, which we borrow from Fernandez-Villaverde et al. (2011).

**Algorithm 17. Particle Filter**

**Step 0. Initialization:**

Sample \( N \) particles, \( \{\sigma_{q,t0}^i\}_{t=1}^{N} \) from the initial distribution \( p (\sigma_{q,t0}; \Psi) \).

**Step 1. Prediction:**

Sample \( N \) one-step ahead forecasted particles \( \{\sigma_{q,t|t-1}^i\}_{t=1}^{N} \) using \( \{\sigma_{q,t-1|t-1}^i\}_{t=1}^{N} \), the law of motion for the states (Equation 3.2) and the distribution of shocks \( u_{\sigma_{q,t}} \).

**Step 2. Filtering:**

Assign each draw \( (\sigma_{q,t|t-1}^i) \) the weight \( \omega_t^i \), where:

\[
\omega_t^i = \frac{p (q_t | q_{t-1}, \sigma_{q,t|t-1}^i; \Psi)}{\sum_{i=1}^{N} p (q_t | q_{t-1}, \sigma_{q,t|t-1}^i; \Psi)} \tag{3.4}
\]

**Step 3. Resampling:**
Generate a new set of particles by sampling \( N \) times with replacement from \( \left\{ \sigma_{q,t|t-1}^i \right\}_{i=1}^N \) using the probabilities \( \left\{ \omega_i \right\}_{i=1}^N \). Call the draw \( \left\{ \sigma_{q,t|t}^i \right\}_{i=1}^N \). In effect, this step builds the draws \( \left\{ \sigma_{q,t|t}^i \right\}_{i=1}^N \) recursively from \( \left\{ \sigma_{q,t|t-1}^i \right\}_{i=1}^N \) using the information on \( q_t \). If \( t < T \), set \( t = t + 1 \) and return to step 1. Otherwise stop.

Using the law of motion for the terms of trade in Equation 3.1, we can evaluate \( p \left( q_t|q_{t-1}, \sigma_{q,t|t-1}; \Psi \right) \) for any \( \sigma_{q,t|t-1}^i \). Moreover, from the Law of Large numbers we know that:

\[
\int p \left( q_t|q_{t-1}, \sigma_{q,t}; \Psi \right) p \left( \sigma_{q,t}|q_{t-1}; \Psi \right) d\sigma_{q,t} \approx \frac{1}{N} \sum_{i=1}^N p \left( q_t|q_{t-1}, \sigma_{q,t|t-1}^i; \Psi \right)
\]

Algorithm 1 provides a sequenced \( \left\{ \sigma_{q,t|t-1}^i \right\}_{i=1}^N \) for all \( t \). Consequently, the algorithm gives us the information needed to evaluate Equation (3.3).

To calculate the posterior distribution of the parameters, we repeat this procedure 25,000 times. At each iteration, we update our parameter draw using a random walk Metropolis-Hastings procedure, scaling the proposal density to induce an acceptance ratio of around 25 per cent. We discard the initial 5,000 draws and conduct our posterior inference on the remaining draws. For each evaluation of the likelihood we use 2,000 particles.

Other methods of modelling time-varying volatility processes, including Markov switching models and GARCH models, also exist. Although these methods have advantages in other contexts, we do not believe that they provide a satisfactory description of terms of trade volatility. For example, a GARCH model does not sharply distinguish between innovations to the terms of trade and its volatility. High levels of volatility are triggered only by large innovations to the terms of trade. In contrast, our methodology allows changes in the volatility of the terms of trade to occur independently of innovations to the level of the terms of trade. A Markov switching model would require us to restrict the number of potential realizations of terms of trade volatility in a way that seems inconsistent with the patterns in Figure 3.1.
3.2.1.1 Data

The terms of trade for each country are defined as the ratio of the export price deflator to the import price deflator and sourced from national statistical agencies.\(^4\) As we wish to estimate changes over time in the variance of the terms of trade, we require our data to be stationary. The stationarity of commodity prices (which drive the terms of trade for the countries in our sample) is a source of contention. Previous studies by Cashin et al. (2000), Powell (1991) and Lee et al. (2006), among others, have concluded that commodity prices are stationary. Others, including Kim et al. (2003), Newbold et al. (2005) and Maslyuk and Smyth (2008) have found that they are not.

In light of the disagreement in the literature, we adopt a compromise approach and detrend our data using a bandpass filter that excludes cycles of longer than 30 years. This preserves all but the lowest frequency movements in the terms of trade for each country while ensuring that the data is stationary.\(^5\)

3.2.1.2 Priors

Table 3.2 reports our priors for the parameters of the terms of trade process. For the persistence parameters, \(\rho_q\) and \(\rho_\sigma\), we impose a Beta prior with mean 0.9 and standard deviation of 0.1. The shape of this prior restricts the value of these parameters to lie between 0 and 1, consistent with economic theory. For the log of the mean standard deviation of terms of trade shocks, \(\sigma_q\), we impose a Normal prior. For each country, we set the mean of this prior equal to the OLS estimate of this parameter calculated assuming an AR(1) process for the terms of trade without stochastic volatility. For the standard deviation of terms of trade volatility shocks, \(\eta_q\), we use a Truncated Normal prior thus ensuring that this parameter is positive. We experimented with alternative priors and found that these had very little impact on our results.

\(^4\) Appendix 3, A includes a full list of data sources and descriptions.

\(^5\) We also estimated the models with HP filtered data (see Appendix 3, B for the results). The choice of detrending method has some effect on the estimated persistence of terms of trade shocks, but relatively little impact on the estimated magnitude of shocks to the terms of trade or its volatility.
Table 3.2: Prior Distribution of Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\rho_q$</th>
<th>$\sigma_q$</th>
<th>$\rho_\sigma$</th>
<th>$\eta_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior</td>
<td>$\beta(0.9,0.1)$</td>
<td>$\mathcal{N}(\hat{\sigma}_{OLS},0.4)$</td>
<td>$\beta(0.9,0.1)$</td>
<td>$\mathcal{N}^+(0.5,0.3)$</td>
</tr>
</tbody>
</table>

Notes: 1) $\beta$, $\mathcal{N}$ and $\mathcal{N}^+$ stand for Beta, Normal and truncated normal distributions.

3.2.1.3 Posterior estimates

Table 3.3 reports the posterior medians of the parameter estimates and associated confidence bands. The first row shows the posterior estimates of $\rho_q$, the persistence of the terms of trade processes. This parameter lies above 0.9 for all countries except for South Africa, indicating that shocks to the terms of trade for these countries tend to be highly persistent. The parameter estimates for $\sigma_q$ reveal substantial differences in the average size of shocks to the terms of trade between countries. Converting the parameters into standard deviations, the results suggest that the magnitude of the average terms of trade shock varies from around 1.2 per cent for Canada to 4.0 per cent for Brazil. The estimates for $\rho_\sigma$ indicate that shocks to the volatility of the terms of trade are highly persistent for Australia, Brazil, New Zealand and South Africa, but somewhat less so for Canada and Mexico. The final row of the table confirms that the magnitude of shocks to the volatility of the terms of trade differs between countries. Of the countries in our sample, Mexico has tended to experience the largest volatility shocks, while New Zealand and South Africa have experienced the smallest. To put these numbers in context, a one standard deviation shock to $u_{\sigma,t}$ increases the standard deviation of terms of trade shocks in Mexico from 3.2 per cent to 4.7 per cent and in South Africa from 3.7 per cent to 4.1 per cent.

To give a clearer insight into what our results imply for the time-varying terms of trade volatility, Figure (3.2) shows the model’s estimates of the evolution of the standard deviations of terms of trade shocks for each country. The average level of these series is higher for Brazil and Mexico than for the other countries in the sample, reflecting the fact that these countries have typically experienced larger terms of trade shocks. The changes in the level are also greatest for Mexico, as that country has experienced the largest shocks to $u_{\sigma,t}$.

---

6Recall, that the standard deviation of shocks to the terms of trade is equal to $\exp(\sigma_q)$.
the volatility of its terms of trade. In contrast, the standard deviation of shocks to Canada’s
terms of trade have typically been small and stable over time, at least compared to those
experienced by other commodity exporters. The experiences of Australia, New Zealand and
South Africa lie somewhere in between those of Canada and the Latin American countries.
They have typically experienced relatively large terms of trade shocks, with an average
standard deviation of around 3 per cent. They have also experienced periods of heightened
volatility, although not to the same extent as Brazil and Mexico.

The patterns of volatility suggested by Figure (3.2) broadly conform to our understanding
of macroeconomic developments over the sample. For example, the average magnitudes
of terms of trade shocks increased in most countries during the mid 1970s, mid 1980s and
late 2000s, while the 1990s was generally a period of low terms of trade volatility.

In sum, our results indicate that the volatility of shocks to the terms of trade for small open
economy commodity producers varies over time. Historically, the variation has been largest
for Latin American countries such as Brazil and Mexico, where the standard deviation of
terms of trade shocks has at times increased from an average level of around three per cent
to over 10 per cent. But countries like Australia, New Zealand and South Africa have also
experienced shocks that have increased the standard deviation of their terms of trade shocks from around three per cent to around six per cent.

### 3.3 The Impact of Volatility Shocks: Empirics

#### 3.3.1 Panel VAR

This section models the responses of real GDP ($y$), consumption ($c$), investment ($i$), the current account ($ca$) and the GDP deflator ($p$) to the terms of trade volatility shocks identified in the previous section. Because each economy in our sample has experienced only a relatively small number of sizeable volatility shocks, we pool the data for all six countries. The model can be expressed as a panel vector autoregression (VAR):

$$Y_{it} = v + A(L)Y_{it} + B(L)X_{it} + u_{it}$$  \hspace{1cm} (3.5)
where \( Y_t' = (y, c, i, ca, p) \) is a vector of stationary endogenous variables, \( v \) is a vector of constants, \( X_t' = (q_{it}, \sigma_{q_{it}}) \) is a vector containing the level of the terms of trade as well as its volatility, \( u_t' = (u_{it}, u_{it}^c, u_{it}^i, u_{it}^{ca}, u_{it}^p) \) is an error vector, \( A(L) \) and \( B(L) \) are matrix polynomials in the lag operator and \( \text{var} (u_{it}) = \Omega. \) Note that although the variables in \( Y_t \) respond to the terms of trade and its volatility, we do not include terms of trade variables as endogenous variables in the VAR. This is consistent with our assumption in Section 3.2 that the terms of trade is exogenous with respect to domestic economic developments for the small open economies in our sample.

The empirical model described in Equation (3.5) can be thought of as a simplified reduced form version of a DSGE model with stochastic volatility, like the one described in Section 3.4 below. Of course, the empirical model cannot fully capture the nonlinear relationships implied by a theoretical model. However, we argue that it nonetheless provides a meaningful indication of the relationships between exogenous terms of trade volatility shocks and macroeconomic variables that appear in the data and serves as a useful benchmark against which to compare the results of our theoretical model. In Appendix (3. C) we provide evidence to support this contention.

3.3.2 Results

To illustrate the consequences of a terms of trade volatility shock, we report the dynamic effects of an innovation to \( \sigma_{q} \) of 0.22, roughly equivalent to the average of \( \eta_q \) across the countries estimated in Section 3.2. After the initial shock, we allow \( \sigma_{q} \) to decay by 10 per cent per quarter, again broadly consistent with the estimates for Section 3.2.

Figure 3.3 shows the dynamic response of \( y_{it}, c_{it}, i_{it}, ca_{it} \) and \( p_{it} \) to an increase in the volatility of terms of trade shocks. Solid lines are the point estimates of the impulse response functions and dashed lines represent one standard deviation (16th and 84th percentile) of the empirical distribution of responses.

\(^7\)In the results below, we include four lags of the endogenous variables and the contemporaneous value and one lag of the terms of trade variables. Experiments with alternative lage structures produced broadly similar results.
The volatility shock reduces both consumption and investment on impact. Although the investment response is not initially significant, it becomes so in later quarters, eventually troughing two quarters after the shock. Consumption and investment both return to their original levels six - eight quarters after the shock, although there is some evidence of a small boom in domestic demand in later quarters. Aggregate output also decreases in the periods after the shock. The size of its response is smaller than the responses of consumption and investment, however, suggesting an offsetting response of net exports. This shows up in the current account-GDP ratio, which increases in the quarter in which the shock hits. It remains above its trend level for two subsequent quarters, before declining as domestic demand recovers. There is also a persistent decrease in the GDP deflator. As we have held the terms of trade constant in this exercise, this implies a fall in the price of non-tradeable goods relative to tradeables.

A possible criticism of our empirical approach is that pooling data conceals cross-country heterogeneity in the impact of volatility shocks. In particular, one might wonder whether economies in which households and firms are less able to hedge the risks associated with terms of trade volatility are more responsive to these types of shocks. As a first step to answering this question, Figures 3.4-3.5 show responses to volatility shocks when we separate
As Figure 3.4 shows, the effect of volatility shocks on output and its components is considerably greater when we estimate the model on a sample including only the emerging economies. The responses of these variables is roughly four times as large and the responses of investment and GDP are now significant from the quarter of impact. It also takes an additional quarter or two for these variables to return to trend after the shock. The current account-GDP ratio continues to increase following the shock, consistent with the decrease in domestic demand exceeding the decrease in GDP. The point estimate of the response of the GDP deflator is qualitatively similar to the pooled response, although the response is only marginally statistically significant.

Figure 3.5 reveals a somewhat different response to the shock among the developed economies. The responses of investment and consumption for these economies are not significantly different from zero, while the point estimates suggest that investment may actually increase immediately following the shock. The initial response of aggregate output is also positive, albeit only significant in the period in which the shock hits. In contrast, the response of the GDP deflator remains negative and significant, and is quantitatively
larger relative to the response of the other variables than in Figure 3.3. These economies also continue to experience a substantial improvement in their current account following the volatility shock.

In sum, the empirical results suggest that an increase in terms of trade volatility triggers a slump in domestic demand that is partly offset by an increase in net exports, leading to a relatively small impact on aggregate output. These shocks also cause a decrease in the domestic price level which, given that we have held the level of the terms of trade constant, suggests a relative decrease in the price of domestic non-tradeable goods. There is some evidence that the response of output and its components is larger in developing economies, while the price response is larger in developed economies. However, given the relatively small number of countries in our sample, we are reluctant to place too much weight on this conclusion. In the following section, we show that a standard international real business cycle model, augmented with stochastic terms of trade volatility, is broadly able to replicate these responses. We then use the model to shed light on the theoretical causes and sectoral implications of these responses.
3.4 The Impact of Volatility Shocks: Theory

In this section, we embed stochastic terms of trade volatility in an otherwise standard small open economy real business cycle model with incomplete markets. In the model, households choose consumption, saving and labor supply to maximize expected lifetime utility. Households consume three goods - non-tradables and home and foreign-produced tradeable goods - and can invest in three assets - a one-period risk-free bond traded in international capital markets and physical capital in the two domestic sectors. On the production side, firms produce output using capital, which is industry-specific, and labor, which is mobile across sectors, and aim to maximize profits. As well as terms of trade shocks, we also include productivity shocks in the model. These shocks help the model to match key features of the data, but play little role in the analysis.

3.4.1 Households

The economy features a representative household that maximizes its expected lifetime utility given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} B^t \left( \frac{c_i^{1-\sigma} - \frac{l_i^{1+\xi}}{1+\xi}}{1-\sigma} \right)$$

(3.6)

where $c$ is consumption of goods and $l$ is hours of work.

Consumption is a composite of tradable and non-tradable goods:

$$c_t \equiv \left[ \omega_T^{\frac{1}{\sigma}} (c_t^T)^{\frac{\sigma-1}{\sigma}} + (1 - \omega_T)^{\frac{1}{\vartheta}} (c_t^{NT})^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{1}{\vartheta-1}}$$

(3.7)

where the elasticity of substitution between tradables and non-tradables is $\vartheta$, the weight of tradables in the consumption basket is $\omega_T$ and $c^{NT}$ is the household’s consumption of non-tradables. $c^T$ is the household’s consumption of tradeable goods, which is itself a composite of home- and foreign-produced tradable goods:
where the elasticity of substitution between the two tradeable goods is $\eta$, the weight of home-produced goods is $\omega_H$, $c^H$ is the household’s consumption of home-produced tradable goods and $c^F$ is the household’s consumption of foreign-produced tradable goods.

To smooth consumption across time, households have access to three assets: a one-period risk-free bond, denominated in units of the foreign-produced tradeable good, and physical capital in the non-traded and home-tradeable sectors. Reflecting the fact that the domestic economy is small relative to the rest of the world, we assume that the interest rate faced by the economy on its debt issuance, $r$, is exogenous. Households face a portfolio adjustment cost from holding foreign debt at a different level than its steady-state level, $d$. This ensures that the economy’s foreign debt level is stationary and prevents precautionary savings diverging to infinity.\(^8\)

Household capital holdings, $k^{NT}$ and $k^H$, are sector specific. We assume that the price of all capital goods are denominated in units of the foreign-produced tradeable good.

We take the price of the foreign good as numeraire and set it equal to one. With this normalization, the household’s budget constraint is given by:

$$
c_t^T \equiv \left[ \omega_H^{\frac{1}{\eta}} \left( c_t^H \right)^{\frac{\eta-1}{\eta}} + (1 - \omega_H)^{\frac{1}{\eta}} \left( c_t^F \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} 
$$

(3.8)

where $\omega_H$ is the weight of home-produced goods, $c^H$ is the household’s consumption of home-produced tradable goods and $c^F$ is the household’s consumption of foreign-produced tradable goods. The terms of trade is exogenous in the model, while $p^{NT}$ is determined endogenously.

---

\(^8\) Portfolio adjustment costs are one of the several ad-hoc methods commonly used to close small open economy models. Others include a debtelastic interest rate premium or a time preference rate that varies with aggregate consumption. Schmitt-Grohe and Uribe (2003) show that all of these methods deliver almost identical dynamics at business-cycle frequencies. Another way of attaining a stationary asset distribution is to assume that the rate of time preference is smaller than the interest rate as in Aiyagari (1994).
$i^{NT}$ and $i^{H}$ are investment in the non-tradeable and home-tradeable sectors. And $w$, $r^{NT}$ and $r^{H}$ are the wage rate and return on capital in the non-tradeable and home-tradeable sectors. Note that as labor is mobile between the two sectors, firms in each sector pay the same wage. The final term on the right hand side of the equation represents the portfolio adjustment costs.

The capital stock of each sector evolves according to:

$$k_{t+1}^j = i_t^j + (1 - \delta)k_t^j - \frac{\phi}{2} \frac{(k_{t+1}^j - k_t^j)^2}{k_t^j}$$

(3.10)

for $j = \{NT, H\}$. The parameter $\delta$ represents the depreciation rate of capital, while the final term represents adjustment costs associated with changing the size of the capital stock. We include these to prevent the model from delivering unrealistically large movements in investment.

Household optimization implies that the demand for home- and foreign-produced tradeable goods is:

$$c_t^H = \omega_H \left( \frac{e^{\eta_t}}{p_t^T} \right)^{-\eta} c_t^T; c_t^F = (1 - \omega_H) \left( \frac{1}{p_t^T} \right)^{-\eta} c_t^T$$

(3.11)

where $p_t^T \equiv \left[ \omega_H (e^{\eta_t})^{1-\eta} + (1 - \omega_H) \right]^{\frac{1}{1-\eta}}$ is the traded goods price index. The demand for tradeable and non-tradeable goods is:

$$c_t^T = \omega_T \left( \frac{p_t^T}{p_t} \right)^{-\theta} c_t; c_t^{NT} = (1 - \omega_T) \left( \frac{p_t^{NT}}{p_t} \right)^{-\theta} c_t$$

(3.12)

where $p_t \equiv \left[ \omega_T \left( p_t^T \right)^{1-\theta} + (1 - \omega_T) \left( p_t^{NT} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$ is the consumer price index (CPI).

Using the optimal household decisions over different good types, we can re-write the household’s budget constraint as:
The household's optimal choice over consumption, labor supply and asset holdings implies the following intra- and intertemporal conditions:

\[
\begin{align*}
    c_t & = \frac{w_t}{p_t} \\
    c_t^{\sigma_{\text{int}}} & = \frac{w_t}{p_t} \\
    1 & = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left( 1 + r + \psi \left( d_{t+1} - d \right) \right) \right] \\
    \left[ 1 + \phi \left( \frac{k_{t+1}^H}{k_t^H} - 1 \right) \right] & = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left( 1 - \delta + r_{t+1}^H + \frac{\phi}{2} \left( \frac{k_{t+1}^H}{k_{t+1}^{NT}} \right)^2 \right) \right] \\
    \left[ 1 + \phi \left( \frac{k_{t+1}^{NT}}{k_t^{NT}} - 1 \right) \right] & = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left( 1 - \delta + r_{t+1}^{NT} + \frac{\phi}{2} \left( \frac{k_{t+1}^{NT}}{k_{t+1}^{NT}} \right)^2 \right) \right]
\end{align*}
\]

where \( \pi_{t+1} = p_{t+1}/p_t \) is the rate of CPI inflation between period \( t \) and period \( t + 1 \).

### 3.4.2 Firms

The home-tradeable and non-tradeable sectors both feature perfectly competitive firms that maximize profits given by:

\[
\begin{align*}
    \pi_t^H & = e^{q_t} y_t^H - w_t^H k_t^H \\
    \pi_t^{NT} & = p_t^{NT} y_t^{NT} - w_t^{NT} k_t^{NT}
\end{align*}
\]
Firms in each sector produce output using a Cobb-Douglas production function:

\[ y_t^H = e^{\alpha t} (k_t^H)^{\alpha} (l_t^H)^{1-\alpha} \]  
(3.20)

\[ y_t^{NT} = e^{\alpha t} (k_t^{NT})^{\alpha} (l_t^{NT})^{1-\alpha} \]  
(3.21)

where \( e^{\alpha t} \) is a productivity shifter that is common to both sectors.

Profit maximization by firms implies that factor prices are given by:

\[ w_t = (1 - \alpha) e^{\alpha t} \frac{y_t^H}{l_t^H} \]  
(3.22)

\[ w_t = (1 - \alpha) p_t^{NT} \frac{y_t^{NT}}{l_t^{NT}} \]  
(3.23)

\[ r_t^H = \alpha e^{\alpha t} \frac{y_t^H}{k_t^H} \]  
(3.24)

\[ r_t^{NT} = \alpha p_t^{NT} \frac{y_t^{NT}}{k_t^{NT}} \]  
(3.25)

### 3.4.3 Shock Processes

The model features three exogenous processes. First, productivity evolves according to an AR(1) process:

\[ a_t = \rho_a a_{t-1} + \varepsilon_t^a \]  
(3.26)

Second, the terms of trade and its variance evolve according to the processes described in the empirical section and repeated here for convenience:

\[ q_t = \rho q_{t-1} + e^{\alpha q_t} u_{q,t} \]  
(3.27)

\[ \sigma_{q,t} = (1 - \rho_\sigma) \sigma_q + \rho_\sigma \sigma_{q,t-1} + \eta_{q,t} \sigma_q \]  
(3.28)

The interpretation of the parameters is also given in the empirical section.
3.4.4 Equilibrium definition

The competitive equilibrium is given by an allocation \( \{c_t, l_t, i_t^H, i_t^{NT}, k_t^H, k_t^{NT}, i_t^H, i_t^{NT}, d_t \}_{t=0}^{\infty} \) and goods and factor prices \( \{w_t, r_t^H, r_t^{NT}, p_t^{NT}, p_t \}_{t=0}^{\infty} \) where (i) consumers’ satisfy their optimality conditions (3.14-3.17) and equations for evolution of capital in both sectors (3.10); (ii) firms’ zero-profit conditions given in Equations (3.22) to (3.25) hold; (iii) productivity and the terms of trade, \( a_t, q_t \) and \( \sigma_{q,t} \), follow the exogenously given processes in Equations (3.26)-(3.28) and (iv) factor and goods markets clear.

Regarding factor market clearing, labor is fully mobile across sectors. Hence, its market clearing condition is given by:

\[
l_t = l_t^H + l_t^{NT} \tag{3.29}\]

Capital is by assumption sector-specific. Market clearing is defined similarly: capital supplied by households has to equal capital demanded by firms.

Goods market clearing implies that all production in the tradeable and non-tradeable sectors is consumed:

\[
y_t^{NT} = c_t^{NT} \tag{3.30}\]

\[
y_t^H = c_t^H + c_t^{H*} \tag{3.31}\]

where \( c_t^{H*} \) is consumption of the home-produced tradable good by foreigners. The latter can be expressed in terms of home-variables only. To do so, we use the equation for the evolution of foreign debt \( d_{t+1} - d_t = rd_t - nx_t \) where \( nx_t \) denotes net exports. Net exports equals nominal exports minus nominal imports (where the latter include capital goods):

\[
\tau e^\eta c_t^{H*} = nx_t + c_t^F + i_t^{NT} + i_t^H \tag{3.32}\]
Substituting in the tradable goods market clearing condition and replacing \( n_t \) using the debt evolution equation we obtain the condition for home-produced goods market clearing in terms of home variables only:

\[
e^{\alpha_t} y_t^H = (1 + r) d_t - d_{t+1} + p_t^T c_t^T + i_t^{NT} + i_t^H + \frac{\psi}{2} (d_{t+1} - d)^2
\]  

(3.33)

3.4.5 Model solution and calibration

We solve the model using perturbation methods, taking a third-order approximation of the policy functions of the agents and the law of motion of the exogenous variables around the model’s steady state. As Fernandez-Villaverde et al. (2011) discuss, in models with stochastic volatility it is necessary to take a third-order approximation of the model to capture the effects of volatility shocks independent of the other innovations in the model.\(^9\)

We fix the value of a number of parameters in all calibrations using values generally found in the literature (Table 3.5). For the households, we set the discount rate, \( \beta \), equal to 0.99, the inverse of the elasticity of substitution, \( \sigma \), and the inverse of the Frisch elasticity, \( \zeta \), both equal to 2, which are all consistent with values commonly used in the literature. We base the values of \( \vartheta \) and \( \eta \) on available estimates for the elasticity of substitution between traded and non-traded goods. For the elasticity of substitution between tradeables and non-tradeables, \( \vartheta \), we use the estimate by Mendoza (1995), calculated for a sample of industrialized countries, and set that elasticity equal to 0.74.\(^10\) For the elasticity of substitution between home and foreign tradeables, \( \eta \), we use the estimate of Corsetti et al. (2008) and select a value of 0.85. We set the share of traded goods in the households’ consumption basket, \( \omega_T \), equal to 0.5, consistent with the estimates of Stockman and Tesar (1995). We also set the share of home goods in the tradable goods basket equal to 0.5. We

\(^9\) Specifically, a first order approximation eliminates all of the effects of volatility shocks as certainty equivalence holds. A second-order approximation captures the effects of volatility shocks only through their interaction with shocks to the level of the terms of trade. It is only in a third-order (or higher) approximation that stochastic volatility shocks enter as independent arguments in the policy functions.

\(^10\) For a sample of developed and developing countries, Stockman and Tesar (1995) estimate a lower elasticity of 0.44. We examine the sensitivity of our results to these elasticities in our robustness checks.
set the capital share of income, $\alpha$, equal to $1/3$ for both sectors. We follow Fernandez-Villaverde et al. (2011) in setting the persistence of productivity shocks, $\rho_a$, equal to 0.95. This choice has little effect on our results as we merely use this shock to calibrate the model. We set $\psi$, the portfolio adjustment cost of foreign debt, equal to $10^{-3}$ for all the countries. This small value ensures that the foreign debt level is stationary, without significantly affecting the dynamic properties of the model (Schmitt-Grohe and Uribe 2003; Fernandez-Villaverde et al. 2011).

Conditional on these choices, we pick the last three parameters to match moments of the ergodic distribution generated by simulating the model to moments of the data. The parameters are: (i) $\sigma_a$, the standard deviation of productivity shocks; (ii) $\phi$, the adjustment cost of capital; and (iii) $d$, the parameter that controls the average stock of foreign debt. The moments of the data that we match are: (i) output volatility; (ii) the volatility of investment relative to output; and (iii) the ratio of net exports to output. Because the moments are affected by a nonlinear combination of parameters, we choose the parameters to minimize

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
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<td>$\beta$</td>
<td>Discount factor</td>
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<td>Standard value.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Inverse of elasticity of substitution</td>
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<td>Standard value.</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Inverse of Frisch elasticity</td>
<td>2</td>
<td>Standard value.</td>
</tr>
<tr>
<td>$\omega_T$</td>
<td>Share of tradables in consumption basket</td>
<td>0.5</td>
<td>As in Stockman and Tesar (1995).</td>
</tr>
<tr>
<td>$\omega_H$</td>
<td>Share of home goods in tradable consumption basket</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution between tradable and non-tradable goods</td>
<td>0.74</td>
<td>As in Mendoza (1995).</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution between home and foreign tradable goods</td>
<td>0.85</td>
<td>As in Corsetti et al. (2008).</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share of income</td>
<td>$\frac{1}{3}$</td>
<td>Standard value.</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence of shock to productivity</td>
<td>0.95</td>
<td>As in Fernandez-Villaverde et al. (2011)</td>
</tr>
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</table>
the sum of the quadratic distance between the model moments and the moments from the data.\textsuperscript{11} Table 3.6 shows the calibrated parameter values.

### 3.5 Results

In this section, we analyze the quantitative implications of our model. First, we compare the moments of the model to those of the data. Second, we construct impulse response functions to illustrate how an innovation to the volatility of terms of trade shocks affects the other variables in the model. Third, we use variance decompositions to quantify the contribution of terms of trade volatility shocks to the variance of the key macroeconomic variables in the model.

#### 3.5.1 Moments

Table 3.7 compares the moments of the model to those of the data. The model matches the three calibrated moments - the variance of output, the relative variance of investment and the level of net exports relative to GDP - successfully for all countries. The model comes reasonably close to matching the correlation of investment and output and consumption and output. It is less successful at replicating some of the other moments of the data. In particular, the volatility of consumption relative to output is generally lower in the model than it is in the data. This is a common finding in small open economy real business cycle models and is generally resolved by assuming the absence of wealth effects on labor supply.

\textsuperscript{11}Specifically, for each economy, we simulate a sample of 200 observations and calculate moments based on these observations. We then repeat this procedure 200 times and calculate the mean of each moment across the 200 draws.
Table 3.7: Empirical Second Moments

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<table>
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<td>Model</td>
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<tr>
<td>$\sigma_y$</td>
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<tr>
<td>$nx/y$</td>
<td>-1.53</td>
<td>-1.54</td>
<td>0.80</td>
</tr>
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</table>

(as in Correia et al. 1995) or adding trend growth shocks (as in Aguiar and Gopinath 2007) to the model.\footnote{12} The model also produces too much correlation between net exports and income. The latter result might be due, in part, to our assumption that all investment goods are imported. We examine this issue in the robustness checks section below.

\footnote{12}{In our model, one can induce greater consumption volatility by increasing the magnitude of the portfolio adjustment cost, $\psi$. This makes it more costly for households to borrow and lend, which reduces consumption smoothing. However, we found that an extremely high value of $\psi$ - generally in the order of 0.1 - was required for the volatility of consumption in the model to match that found in the data. And, with $\psi$ at such a high level, the effect of portfolio adjustment costs on the model's dynamics cease to be negligible.}
3.5.2 Impulse response functions

We now turn to the dynamic response of the economies to a shock to terms of trade volatility. We focus and describe in detail the Brazilian case and consign the results for the other economies to Appendix 3. D. Figure (3.6) shows the response of the Brazilian economy to a one standard deviation shock to the volatility of the terms of trade, that is a shock to $\sigma_t$. Note that the shock has no effect on $q_t$, the level of the terms of trade. Despite this, the shock induces a decrease in consumption of almost 0.1 per cent on impact and a larger decrease in investment, of around 0.3 per cent. The current account-to-GDP ratio also increases by around 0.2 per cent following the shock, while the price level decreases. Because the terms of trade does not change following the shock, this implies a decrease in the GDP deflator. In sum, the model qualitatively matches the responses to a terms of trade volatility shock identified in the panel VAR. The magnitudes of the responses are slightly smaller than the VAR, although in the same ballpark as the ones found in Fernandez-Villaverde et al. (2011) and Fernandez-Villaverde et al. (2012).

The theoretical intuition for these responses comes from the household’s optimality conditions. Consider first the household’s Euler equation:
The shock to terms of trade does not affect the expected level of consumption directly. But it does make agents more uncertain about their future income flows, which increases the expected marginal utility of future consumption, \( E_t \{c_{t+1}^{-\sigma} \} \). As households prefer to smooth marginal utility across time, they reduce consumption today. This increases the marginal utility of consumption today and, by freeing up more resources for future consumption, reduces the expected future marginal utility of consumption. Moreover, the reduction in consumer demand lowers prices today relative to future prices, which increases \( E_t \{\pi_{t+1}\} \). Because the terms of trade is exogenous, the adjustment in prices must occur entirely through changes in the relative price of non-tradeable goods.

The decrease in current consumption and the reduction in the price level also affects labor supply through the household’s intratemporal optimality decision:

\[
c_t^{\sigma} l_t^\xi = \frac{w_t}{p_t}
\]

The decrease in current consumption increases the marginal utility of consumption. This brings about an increase in the labor supply because the utility cost of working is now lower. The increase in the labor supply causes the equilibrium wage rate to drop.

On the production side of the economy, the volatility shock brings about a change in the sectoral composition of output away from non-tradeables towards tradeables. The decrease in non-tradeable production follows directly from the decrease in consumption. Although consumption of tradeable goods also decreases, this effect is overwhelmed by an increase in exports, which helps agents to accumulate foreign assets.

The volatility shock also affects factor utilization. The increase in labor supplied by households lowers the capital-labor ratio, which reduces the real wage relative to the return on capital. This change in factor prices encourages firms to adopt more labor-intensive production methods. Combined with the changes in output, this reduces firms’ demand for
capital. The consequent reduction in investment also helps households to reduce domestic absorption, further contributing to their accumulation of foreign assets.

The qualitative behavior of variables in response to the terms of trade volatility shocks is similar for all the countries we study, although the magnitude of the responses are somewhat more modest.\textsuperscript{13}

It is also instructive to compare the response of the economy to a volatility shock to its response to a one standard deviation shock to the level of the terms of trade, shown in Figure (3.7). The shock brings about (i) a prolonged increase in consumption; (ii) an investment boom; (iii) an improvement in the current account; (iv) an increase in home-produced goods output and a temporary decrease in non-tradable goods production; and (v) an increase in hours worked and real wages. These results are consistent with the findings in Mendoza (1995).

Although both shocks lead to a boom in domestic tradeable output and a reduction in foreign debt, the terms of trade level shock is far more favorable to domestic agents. This

\textsuperscript{13}Appendix 3. D reports the IRFs for Mexico, Australia, New Zealand, South Africa and Canada to a shock to the volatility of terms of trade.
shock encourages firms to invest in order to increase production to take advantage of temporarily high relative goods prices. The resulting increase in the capital-labor ratio drives the increase in real wages, which triggers the expansion in labor supply.

3.5.3 Variance decompositions

In this section we study the contribution to aggregate fluctuations of each of the three shocks in our model. Because of our nonlinear approximation to the policy function, it is not possible to divide total variance among the shocks as in a linear model. Therefore, in this exercise, we set the realizations of one or two of the shocks to zero and measure the volatility of the economy when we simulate the economy with the remaining shocks.

We study four macro-aggregates: output, consumption, investment and net exports and explore four scenarios: (i) all shocks; (ii) terms of trade level shocks only; (iii) terms of trade volatility shocks only; and (iv) terms of trade level and volatility shocks jointly.14

Table 3.8 reports the variance decompositions for all six countries. For each of them, productivity shocks are the main contributor for output fluctuations, while shocks to the level and volatility of the terms of trade are key drivers of investment and net exports fluctuations.

By themselves, volatility shocks account for only a very small portion of the standard deviation of output and consumption for all of the countries in our sample. The impact of these shocks for investment and net exports is somewhat greater - with these shocks alone the standard deviation of Brazilian investment is estimated to be 0.36 and net exports 0.22 - although still modest given the high variance of these series.

However, interacted with shocks to the level of the terms of trade, volatility shocks are estimated to have a meaningful impact of macroeconomic outcomes. For example, with only shocks to the level of the terms of trade, the standard deviation of Brazilian investment is estimated to be 4.27 per cent. With shocks to the volatility as well as the level of the terms shocks.

14Note that in each decomposition agents in the model believe that the shocks are distributed according to the law of motion specified in the previous section. Consequently, they will respond to volatility shocks even when the realization of shocks to the level of the terms of trade is always zero.

123
<table>
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<th>TOT Level and TOT Volatility</th>
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<td>0.02</td>
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of trade, the standard deviation of investment is estimated to be 5.46 per cent - 28 per cent greater.

Indeed, for countries like Brazil, Mexico and Australia, the volatility of the key macroeconomic variables is between 20 and 30 per cent higher when there are both volatility and level shocks than it is when terms of trade level shocks operate alone. That is, for these countries between a fifth and a third of the effect of the terms of trade on macroeconomic volatility comes through changes in the volatility of terms of trade shocks. For countries like Canada, New Zealand and South Africa, the contribution of volatility to the overall effects of terms of trade shocks is smaller. However, even for these countries, our results suggest that between 10 and 20 per cent of the impact of the terms of trade on the key macroeconomic variables is due in part to volatility in the terms of trade.

Because of the nonlinear structure of our model, it is difficult to isolate the exact channels through which interactions between the level and volatility of the terms of trade affect the macroeconomy. However, much of the explanation may come from the fact that stochastic volatility increases the variance of the terms of trade and larger shocks to the terms of trade imply greater macroeconomic volatility. To see the impact of stochastic volatility on the variance of the terms of trade, first note that in the absence of stochastic volatility, that is if \( \sigma_{q,t} = \sigma_q \nu_t \), then the variance of the terms of trade, \( \text{var}(q_t) \) is:

\[
\text{var}(q_t) = \frac{\exp(2\sigma_q)}{1 - \rho_q^2}
\]  (3.34)

In contrast, when stochastic volatility is present, the variance of the terms of trade is:

\[
\text{var}(q_t) = \frac{\exp(2\sigma_q + 2\zeta^2)}{1 - \rho_q^2}
\]  (3.35)

where \( \zeta^2 = \frac{\eta^2}{1 - \rho_q^2} \). For Brazil, the presence of stochastic volatility increases the standard deviation of terms of trade shocks by a third, from 0.14 per cent to 0.19 per cent. Although other effects are likely to exist, this direct impact appears roughly large enough to explain
the change in macroeconomic volatility between the scenario with terms of trade level shocks only and the scenario with both terms of trade level and volatility shocks.

3.6 Robustness Checks

In this section we examine the robustness of our results to alternative parameter assumptions and modelling choices.

3.6.1 Alternative Parameter Values

As a first exercise, we test the sensitivity of our model’s dynamics to alternative parameter values. In particular we consider: (i) increasing the inverse Frisch elasticity, $\zeta$, from 2 to 100; (ii) increasing the inverse of the elasticity of substitution, $\sigma$, from 2 to 10; and (iii) increasing the parameter governing the sensitivity of the risk-free interest rate to the foreign debt level, $\psi$, from $10^{-3}$ to $10^{-2}$. We examine each of these alternative parameter choices separately, leaving the other parameters at the same level as in the baseline model presented above. Figure 3.8 shows impulse responses to a one standard deviation terms of trade volatility shock in Brazil under the alternative parameter values.

An increase in $\zeta$ makes labor supply less sensitive to changes in other macroeconomic variables. In fact, with $\zeta = 100$, aggregate hours worked are almost unchanged in response to a terms of trade volatility shock. Because of the muted labor supply response, home tradeables output increases by less following the shock than it does in the baseline case, while non-tradeables output decreases by more. The decrease in consumption and increase in the current account are also somewhat larger. The response of the current account reflects the fact that, when labor supply elasticity decreases, households are less able to smooth consumption by adjusting their working hours and are forced to rely more on changes in asset holdings. The decrease in consumption is required to square the smaller increase in tradeables output with a larger decrease in foreign debt.
Although macroeconomic models typically assume a low value for $\sigma$, microeconomic studies tend to report higher values (Hall (1988), Dynan (1993)). An increase in $\sigma$ reduces the willingness of agents to trade consumption intertemporally. As a consequence, the decrease in consumption following the terms of trade volatility shock is considerably smaller in this scenario than it is in the baseline case. The decrease in non-tradeables output is also smaller than in the baseline case. However, the responses of the other variables are broadly similar to the baseline case.

In our baseline results we assumed an extremely low value of $v$ in order to minimize the effect of this parameter on the dynamics of the model. Other papers that have estimated the value of this parameter for various countries have tended to find higher values (for example, Fernandez-Villaverde et al. (2011) for a selection of South American economies and Jaaskela and Nimark (2011) for Australia, while Justiniano and Preston (2010) calibrated this parameter to $10^{-2}$ in models of Australia, Canada and New Zealand). A higher value of $v$ penalizes the economy for accumulating foreign assets (or reducing its foreign debt) by reducing the interest rate that agents receive on those assets. Setting this parameter to $10^{-2}$ reduces the amount of time that it takes for the model to converge to its steady state.
However, the initial responses of the variables to the terms of trade volatility shock are broadly similar to the baseline model.

As a second parameter exercise, we also examined the effect of different elasticities of substitution between tradeable and non-tradeable goods, $\vartheta$, and between home- and foreign-produced goods, $\eta$. In our baseline scenario, we calibrated both $\vartheta$ and $\eta$ to values below one. This meant that these goods were complements in consumption. While this is a common assumption in the literature, other papers have also estimated values for these parameters above one, implying that these goods are substitutes in consumption.\footnote{For a review of empirical estimates of these parameters, see Bodenstein (2010).} To illustrate the sensitivity of our results to alternative values for these parameters, we simulated the model assuming that one, or both, of these parameters equaled three - a reasonable value in the literature. Figure 3.9 shows the results.

Changing the values of these parameters has little effect on aggregate consumption, investment or labor supply. It does, however, affect the composition of consumption between traded and non-traded goods and, consequently, the production of non-traded goods. Take the case where $\vartheta = 3$. In this scenario, traded and non-traded goods are substitutes, while home- and foreign-produced tradeable goods are complements. Relative to the baseline...
case, households now reduce their consumption of imports not only by reducing their overall consumption, but also by substituting towards non-tradeable goods.

Relative to the baseline case, a small change in the relative price of non-tradeable goods now leads to a large change in the demand for those goods. An increase in terms of trade volatility still reduces consumer demand for all goods, including non-traded goods - which reduces their price. However, consumers are now more willing to substitute from foreign-produced traded goods to non-traded goods. As a consequence, the contraction in the output of non-traded goods is smaller than in the baseline case, as is the decrease in the price of those goods. By shifting consumption from imports to non-traded goods, households are able to achieve a larger reduction in foreign debt despite a similar decrease in consumption. In contrast, increasing $\eta$ to three has a smaller impact on the results relative to the baseline case. This reflects the fact that while the increase in $\eta$ makes agents more willing to substitute between home- and foreign-produced tradeable goods, the shock to volatility does not change the relative price of these goods. Consequently, this parameter plays a relatively small role in influencing the response of the economy to a terms of trade volatility shock. In sum, altering the elasticities of substitution has relatively little qualitative effect on the results, although it has a quantitative effect on the responses in individual sectors of the economy.

3.6.2 Home-Produced Components of Investment

In our baseline model we assumed that the investment good was priced in units of the foreign tradeable good. This choice was motivated by the stylized fact that prices of investment goods differ less across countries than the prices of consumption goods (see Hsieh and Klenow (2007), Figures 4 and 5). In this exercise, we instead assume that the investment good is priced in the same units as the economy's consumption good. That is, we allow the prices of home-produced goods also to affect prices of investment goods. To do this, we assume that the investment good is a CES aggregate of tradeable and non-tradables goods and that the tradable component is itself an aggregate of home and foreign tradeable goods.
For simplicity, we set the weights of each good and elasticities of substitution between alternative goods to the same values as they are for the consumption aggregate.\footnote{To be precise: \( i_t = \left[ \omega_T \frac{1}{\pi} \left( \frac{i_t}{i_T} \right)^{\frac{1}{\pi-1}} + (1 - \omega_T) \frac{1}{\pi} \left( \frac{i_{tT}}{i_T} \right)^{\frac{1}{\pi-1}} \right]^{\frac{\pi}{\pi-1}} \) and \( i_t' = \left[ \omega_T \frac{1}{\pi} \left( \frac{i_t'}{i_T} \right)^{\frac{1}{\pi-1}} + (1 - \omega_T) \frac{1}{\pi} \left( \frac{i_{tT}'}{i_T} \right)^{\frac{1}{\pi-1}} \right]^{\frac{\pi}{\pi-1}} \).}

Because we are interested in how including home-produced investment goods affects the fit of the model, as well as the dynamics, we first recalibrate the new model to match the same moments of the data as we did with the baseline model. Table 3.9 shows the resulting model moments for Brazil. Including a home-produced component of investment goods reduces the correlation between consumption and output and increases the correlation between investment and output. This improves the fit of the model for Brazil in this dimension, although it would worsen the fit of the model for other countries. It also reduces the correlation between net exports and output, although by not nearly enough to match the correlation seen in the data.

<table>
<thead>
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<th>Table 3.9: Moments: Model with Home Produced Investment</th>
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<tbody>
<tr>
<td>( \sigma_y )</td>
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<tr>
<td>Brazil</td>
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</table>

Next, we examine the dynamic response of the economy to a terms of trade volatility shock, shown in Figure 3.10. Including a home-produced component of investment has very little effect on the response of the economy to a volatility shock. The response of consumption is slightly more muted, while the investment response is slightly larger. However, the labor supply response is somewhat larger, as is the output response. In sum, in terms of both model fit and dynamics there seems to be little to choose between the two model specifications.
3.7 Conclusion

This paper has contributed to the literature examining time-varying volatility in macroeconomics by studying the effects of changes in the volatility of the terms of trade, a plausibly exogenous price index for small open commodity-exporting economies. Our empirical section estimated the empirical process for the terms of trade for six commodity-producing small open economies. We demonstrate that the magnitude of terms of trade shocks varies considerably over time for all the economies in our sample. Using a panel vector autoregression we then demonstrated that a volatility shock reduces both consumption and investment. Aggregate output also decreases following the shock and the current account-GDP ratio increases when the shock hits, and remains above trend before decreasing as domestic demand recovers. There is also a persistent decrease in the price level. Dividing the sample into emerging and developed economies shows some differences between the two categories, with the volatility shock having a larger price effect in developed economies and a larger effect on quantities in emerging economies.

In our theoretical section, we set up a small open economy real business cycle model and demonstrate that it can replicate the responses to the volatility shock generated by the VAR.
We then use the model to further explore the mechanisms behind these responses and also to examine their sectoral impacts. In the model, a shock to terms of trade volatility reduces consumption, causes a boom in the tradable sector at the expense of the non-tradeable sector and triggers a shift in the factor intensity of production away from capital towards labor. The decrease in domestic absorption and the increase in tradeables production leads to an improvement in the trade balance that allows the economy to reduce its foreign borrowing.

Finally, the model allows us to quantify the direct contribution of terms of trade volatility shocks to the fluctuations of macro-aggregates. Although the direct contribution of terms of trade volatility shocks to the variance of key variables is rather small, we find that these shocks have a meaningful economic effect in interaction with shocks to the level of the terms of trade. Our estimates suggest that terms of trade volatility shocks account for between one-fifth and one-third of the total effect of the terms of trade on the volatility of output, consumption, investment and net exports in the countries in our sample.

Our results point to a number of promising avenues for further research. The disaggregated VAR results hint that, for emerging economies, the response to volatility shocks occur mainly through quantities while, for developed economies, the response occurs mainly through prices. More detailed empirical work using a larger sample of economies could shed light on the robustness of this result. And, if it does turn out to be robust, further theoretical work is needed to understand the economic drivers of this observation.
3. A Data Sources and Definitions

Terms of Trade Data

With the exception of Canada, all terms of trade data was sourced from national statistical agencies. We retrieved data for Canada from the OECD. For Australia, Brazil, New Zealand and South Africa, published terms of trade indexes were used. For Canada, we constructed a terms of trade index by dividing the exports of goods and services deflator by the imports of goods and services deflator. For Mexico, we constructed a terms of trade index by dividing the exports price index by the imports price index. The raw data for Australia, Canada, New Zealand and South Africa was quarterly. For Brazil and Mexico, we constructed a quarterly series using quarterly averages of monthly data. Samples and sources for the individual countries are:


National Accounts Data

For all countries, data for Gross Domestic Product and its components was sourced from the OECD economic outlook database (www.oecd.org). All national accounts data are HP-filtered using a smoothing parameter of 1600.
### 3. B Terms of Trade Processes: HP Filtered Data

**Table 3.10: Posterior Medians: HP Filtered**

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Brazil</th>
<th>Canada</th>
<th>Mexico</th>
<th>New Zealand</th>
<th>South Africa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_q$</td>
<td>0.84</td>
<td>0.77</td>
<td>0.83</td>
<td>0.78</td>
<td>0.83</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>[0.81, 0.89]</td>
<td>[0.65, 0.89]</td>
<td>[0.77, 0.91]</td>
<td>[0.71, 0.89]</td>
<td>[0.77, 0.92]</td>
<td>[0.60, 0.80]</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>-3.69</td>
<td>-3.36</td>
<td>-4.45</td>
<td>-3.46</td>
<td>-3.53</td>
<td>-3.38</td>
</tr>
<tr>
<td></td>
<td>[-4.19, -2.95]</td>
<td>[-3.87, -2.74]</td>
<td>[-4.82, -4.00]</td>
<td>[-3.88, -3.02]</td>
<td>[-3.96, -2.91]</td>
<td>[-3.95, -2.81]</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>0.94</td>
<td>0.93</td>
<td>0.89</td>
<td>0.87</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>[0.78, 0.99]</td>
<td>[0.78, 0.99]</td>
<td>[0.62, 0.99]</td>
<td>[0.66, 0.97]</td>
<td>[0.86, 1.00]</td>
<td>[0.88, 1.00]</td>
</tr>
<tr>
<td>$\eta_q$</td>
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<td>0.21</td>
<td>0.23</td>
<td>0.31</td>
<td>0.14</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>[0.12, 0.35]</td>
<td>[0.11, 0.38]</td>
<td>[0.11, 0.45]</td>
<td>[0.20, 0.46]</td>
<td>[0.08, 0.24]</td>
<td>[0.05, 0.21]</td>
</tr>
</tbody>
</table>
3. C What does the empirical VAR capture?

In this Appendix, we demonstrate the ability of our empirical vector autoregression exercise to capture the macroeconomic impacts of exogenous shocks to terms of trade volatility. To do this, we compare impulse responses from our empirical VAR estimated using simulated data to the impulse responses to exogenous terms of trade volatility shocks generated by our model. Specifically, we simulate our model for 200 periods setting all parameters at their baseline values for Brazil. We then estimate our empirical VAR using this data and calculate impulse responses to an innovation to the terms of trade volatility variable as in Section (3.3). We repeat this process 50,000 times to characterize the distribution of VAR responses.

Figure (3.11) shows the median, 5 and 95 per cent responses of the simulated VAR for each variable as well as the theoretical responses to a terms of trade volatility shock from the model. Despite its linear structure, the VAR comes extremely close to matching the theoretical model responses. This gives us confidence that our empirical model reflects a response to an exogenous terms of trade volatility shock.

Figure 3.11: Terms of Trade Volatility Shock: Model and VAR
3. D Theoretical Impulse Response Functions: Other Economies

Figure 3.12: Terms of Trade Volatility Shock: Australia
Figure 3.13: Terms of Trade Volatility Shock: Canada

[Graphs showing economic variables such as GDP, Consumption, Investment, CA/Y, CPI, Output, Capital, Labor, Wages, p^N, q, and p^N vs time]
Figure 3.14: Terms of Trade Volatility Shock: Mexico
Figure 3.15: Terms of Trade Volatility Shock: New Zealand
Figure 3.16: Terms of Trade Volatility Shock: South Africa
Bibliography


Du, W., Schreger, J., July 2013. Local currency sovereign risk. Harvard University mimeo.


