Risks And Returns Of Fixed Income Arbitrage Strategies In Varying Economic Environments: A Model Based on Empirical Considerations

By

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Abstract

I propose a discrete time model of financial markets in which an arbitrageur has investment opportunities but faces a number of financial constraints. Investment opportunities arise when the price discrepancy between a pair of similar assets becomes large enough. I propose an innovative way to model the effects of market liquidity and the arbitrage industry’s reversion force on a stochastic price discrepancy. I use empirical studies and common literature assumptions to build and calibrate the model. I then run a set of Monte-Carlo simulations to test the model’s response to the risks and returns of a number of arbitrage strategies in varying economic conditions. The model’s results are in line with a number of theories in the existing literature, and specifically confirm the role of the arbitrageur as a liquidity provider in disturbed market environments.

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# Table of Contents

Abstract ................................................................................................................................. iii
Acknowledgement .................................................................................................................. v
Table of Contents .................................................................................................................. vii
List of Figures ........................................................................................................................ viii
I) Introduction ....................................................................................................................... 10
   a. Definitions ..................................................................................................................... 10
   b. A Potentially Risky, Lucrative Business ................................................................. 16
   c. Interest of the Topic ................................................................................................. 23
II) Characteristics of Fixed Income Arbitrage Strategies .............................................. 25
   a. Role of Arbitrageurs in the Financial Markets ...................................................... 25
   b. Typology of Players ............................................................................................... 29
   c. Size and Trends of Hedge Fund Assets Under Management .................................. 31
   d. Operations: Funding and Leverage ....................................................................... 33
   e. Hedge Fund Styles and Strategies ......................................................................... 35
   f. Summary ..................................................................................................................... 45
III) Risks and Returns of Fixed Income Arbitrage ......................................................... 47
   a. Convergence Trading Risk Factors ....................................................................... 47
   b. Returns Characteristics .......................................................................................... 67
   c. Summary ................................................................................................................... 75
IV) Simulation of Arbitrage Strategies in Varying Economic Conditions ................. 77
   a. Objectives of the Simulation .................................................................................. 77
   b. Presentation of the Model ...................................................................................... 78
   c. Methodology and Results of the Simulation .......................................................... 96
   d. Additions to the Trading Algorithm ..................................................................... 136
V) Conclusion ....................................................................................................................... 160
Work Cited ............................................................................................................................ 164
Appendix ............................................................................................................................... 166
List of Figures

FIGURE 1: TIPS—TREASURY MISPRICING, EXPRESSED IN UNITS OF DOLLARS PER $100 NOTIONAL .......... 22
FIGURE 2: EURO - US DOLLAR CIP ARBITRAGE PROFITS DURING THE FINANCIAL CRISIS .......... 27
FIGURE 3: HEDGE FUND INDUSTRY AUM (BILLION DOLLARS) ................................................. 32
FIGURE 4: FIXED INCOME HEDGE FUND AUM (BILLION DOLLARS) ........................................... 32
FIGURE 5: ARBITRAGE ZERO-REVERTING FORCE AS A FUNCTION OF MISPRICING WIDTH ............. 64
FIGURE 6: ASSET PRICES TIME SERIES BASED ON MISPRICING GENERATION ......................... 81
FIGURE 7: ARBITRAGE ZERO-REVERTING FORCE AS A FUNCTION OF MISPRICING WIDTH ............. 86
FIGURE 8: TIME SERIES OF MISPRICING VALUE IN AMPLE SHOCK AND NO COUNTERSHOCK SCENARIO .. 86
FIGURE 9: TIME SERIES OF MISPRICING VALUE IN AMPLE SHOCK WITH COUNTERSHOCK SCENARIO .. 88
FIGURE 10: TIME SERIES OF NOISE MISPRICING UNDER NO-SHOCK ASSUMPTION ................. 89
FIGURE 11: CORRESPONDING TIME SERIES OF OBSERVABLE MISPRICING, SAME SCALE .......... 89
FIGURE 12: TIME SERIES OF ARBITRAGEUR WEALTH INDEX, EXPONENTIAL SCALE .................. 95
FIGURE 13: FOCUS ON 500 FIRST DAYS OF ARBITRAGEUR WEALTH INDEX .............................. 96
FIGURE 14: TIME SERIES OF DOLLAR MISPRICING OF SAME MATURITY Treasuries during the crisis .... 98
FIGURE 15: AVERAGE IRR AS A FUNCTION OF PARAMETERS B1 (COLUMNS) AND B2 (ROWS) .......... 103
FIGURE 16: IRR VOLATILITY AS A FUNCTION OF PARAMETERS B1 (COLUMNS) AND B2 (ROWS) .......... 103
FIGURE 16A: SCATTER GRAPH OF IRR VOLATILITY (Y AXIS) AS A FUNCTION AVERAGE IRR (X AXIS) .. 104
FIGURE 17: FUND SHARPE RATIO AS A FUNCTION OF PARAMETERS B1 (COLUMNS) AND B2 (ROWS) .......... 104
FIGURE 18: SUMMARY OF REGRESSION ANALYSIS ON VARYING PARAMETERS ......................... 105
FIGURE 19: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF NOISE TRADER EFFECT STANDARD DEVIATION (PARAMETER Σ) ......... 107
FIGURE 20: FUND SHARPE RATIO AS A FUNCTION OF Σ ......................................................... 107
FIGURE 21: FUND IRR DISTRIBUTION FOR Σ SET AT $8 AND CORRESPONDING NORMAL DISTRIBUTION .... 108
FIGURE 22: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF LIQUIDITY SHOCK SIZE (PARAMETER S) ........................................ 109
FIGURE 23: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER S ...................................... 110
FIGURE 24: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF LIQUIDITY SHOCK PROBABILITY (PARAMETER H) .................................. 112
FIGURE 25: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER H .................................... 112
FIGURE 26: IRR SKEWNESS AS A FUNCTION OF PARAMETER H .............................................. 113
FIGURE 27: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF LIQUIDITY SHOCK PROBABILITY (PARAMETER H), LARGER DATA RANGE .. 113
FIGURE 28: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF REVERSION FORCE POWER (PARAMETER F) ............................... 116
FIGURE 29: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER F .................................... 116
FIGURE 30: IRR SKEWNESS AS A FUNCTION OF PARAMETER F .............................................. 117
FIGURE 31: TIME-SERIES OF OBSERVABLE MISPRICING, F FACTOR SET AT 0.5% ....................... 117
FIGURE 32: TIME-SERIES OF OBSERVABLE MISPRICING, F FACTOR SET AT 10% ..................... 118
FIGURE 33: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF REVERSION FORCE POWER (PARAMETER F) ............................... 119
FIGURE 34: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER F .................................... 120
FIGURE 35: TIME-SERIES OF OBSERVABLE MISPRICING WITH LIQUIDITY SHOCK, F FACTOR SET AT 15% .... 120
FIGURE 36: CORRESPONDING TIME-SERIES OF ARBITRAGEUR WEALTH INDEX, LOGARITHMIC SCALE ... 121
FIGURE 37: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF AUTHORIZED LEVERAGE LEVEL (PARAMETER $A^1$) ........................................ 123
FIGURE 38: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER $A^1$ .................................................. 124
FIGURE 39: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF AUTHORIZED LEVERAGE SENSITIVITY (PARAMETER $A2$) .......................... 126
FIGURE 40: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER $A^2$ .................................................. 126
FIGURE 41: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF MARGIN CALL THRESHOLD (PARAMETER $G$) ............................................. 129
FIGURE 42: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER $G$ ..................................................... 130
FIGURE 43: FUND IRR AS A FUNCTION OF PARAMETER $G$, ACTUAL SCALE ............................................. 130
FIGURE 44: FUND IRR AND VOLATILITY AS A FUNCTION OF PARAMETER $G$, MODIFIED SCALE .......................... 131
FIGURE 45: FUND RETURNS SKEWNESS AS A FUNCTION OF PARAMETER $G$, MODIFIED SCALE .................. 131
FIGURE 46: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF INVESTOR WITHDRAWAL SENSITIVITY TO LOSSES (PARAMETER $R^2$) ......... 135
FIGURE 47: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER $R^2$ .................................................... 135
FIGURE 48: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 5,000), AS A FUNCTION OF CAPITAL INVESTED BY ARBITRAGEUR (PARAMETER $C^1$) .......................... 139
FIGURE 49: IRR (LEFT SCALE) AND RETURNS VOLATILITY (RIGHT SCALE) AS A FUNCTION OF $C^1$ ............. 139
FIGURE 50: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER $C^1$ ................................................... 140
FIGURE 50A: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 5,000), AS A FUNCTION OF CAPITAL INVESTED BY ARBITRAGEUR (PARAMETER $C^1$) .......................... 141
FIGURE 50B: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER $C^1$ .................................................. 142
FIGURE 51: AVERAGE IRR AS A FUNCTION OF PARAMETERS $C^1$ (COLUMNS) AND $C^2$ (ROWS) ............... 146
FIGURE 52: IRR VOLATILITY AS A FUNCTION OF PARAMETERS $C^1$ (COLUMNS) AND $C^2$ (ROWS) .......... 146
FIGURE 53: SCATTER GRAPH OF IRR VOLATILITY (Y AXIS) AS A FUNCTION AVERAGE IRR (X AXIS) ........... 146
FIGURE 54: FUND SHARPE RATIO AS A FUNCTION OF PARAMETERS $C^1$ (COLUMNS) AND $C^2$ (ROWS) ........ 147
FIGURE 55: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF CAPITAL INVESTED BY ARBITRAGEUR (PARAMETER $C^1$) .......................... 150
FIGURE 56: IRR (LEFT SCALE) AND RETURNS VOLATILITY (RIGHT SCALE) AS A FUNCTION OF $C^1$ ............. 150
FIGURE 57: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER $C^1$ ................................................... 151
FIGURE 58: AVERAGE IRR AS A FUNCTION OF PARAMETERS $C^1$ (COLUMNS) AND $C^2$ (ROWS) ............... 154
FIGURE 59: IRR VOLATILITY AS A FUNCTION OF PARAMETERS $C^1$ (COLUMNS) AND $C^2$ (ROWS) .......... 155
FIGURE 60: SCATTER GRAPH OF IRR VOLATILITY (Y AXIS) AS A FUNCTION AVERAGE IRR (X AXIS) ........... 155
FIGURE 61: FUND SHARPE RATIO AS A FUNCTION OF PARAMETERS $C^1$ (COLUMNS) AND $C^2$ (ROWS) ........ 156
FIGURE 62: IRR AND RETURNS VOLATILITY (LEFT SCALE), AND FUND FAILURES (RIGHT SCALE, OUT OF 1,000), AS A FUNCTION OF CAPITAL INVESTED BY ARBITRAGEUR (PARAMETER $C^1$) (LIQUIDITY WARNING STRATEGY) .......................................................... 158
FIGURE 63: IRR (LEFT SCALE) AND RETURNS VOLATILITY (RIGHT SCALE) AS A FUNCTION OF $C^1$ ............. 159
FIGURE 64: FUND SHARPE RATIO AS A FUNCTION OF PARAMETER $C^1$ ................................................... 159
I) Introduction

a. Definitions

i. Arbitrage

Arbitrage is the simultaneous purchase and sale of assets that pay the same cash flows in order to profit from a difference in their prices. In essence, arbitrage is riskless. Arbitrage opportunities exist because of market inefficiencies. Arbitrage ensures prices of securities with identical characteristics do not deviate substantially from each other for long periods of time.

In practice, arbitrage is widely used for trading strategies that are not riskless for two main reasons. First, the two assets used are often not identical, but very similar, hence introducing cash flow risk. Note that one arbitraged asset can be in some case the combination of several assets, the sum of the cash flow of which are very similar to that of the other arbitraged asset. I will use the term textbook arbitrage for all strategies that can (in theory) perfectly replicate cash flows, i.e. where the two assets arbitraged will deliver the same cash flows at the same time, and bear the same risks. I added “in theory” because some of these strategies are theoretically perfect, but limited in practice by some frictions. A good example of that is a dynamic hedging strategy that would require constant rebalancing. As it is impossible for an arbitrageur to trade on a permanent basis, the cash flow replication may be slightly imperfect. The second reason why arbitrage strategies are not riskless is that arbitrageurs tend to want to get out of their trades before all the cash flow have been paid (I call this convergence trading), thus introducing marked-to-market risk to their portfolio.
As I will show later, the interest I find in arbitrage is that – contrary to common beliefs – it is a risky endeavor and can generate very large returns, as well as spectacular crashes. In practice, arbitrage is therefore paradigmatic of the most fascinating foundation topic of finance: the relation between risk and return. Additionally, arbitrage requires three things that are very agreeable to conciliate in the studies of the financial markets: theoretical foundations, practical understanding of processes, as well as creativity and innovation. Indeed, arbitrage is at the heart of modern asset pricing theory; the arbitrage pricing theory was developed by Stephen A. Ross (today Franco Modigliani Professor of Financial Economics and a Professor of Finance at the MIT Sloan School of Management) in 1976 and widely used thereafter, in research and in practice. The practice of arbitrage is very different from the textbook view of a perfect world where capital markets are frictionless and investors are rational. In that respect, understanding the operations of an arbitrageur at work is challenging and exciting. Lastly, arbitrage strategies are changing and nimble; they require adaptation and overall understanding of macroeconomic factors and market specific limitations. For this reason, arbitrageurs are specialized, sophisticated investors, with specific, proprietary models and strategies that require innovation and constant rethinking.

ii. Fixed Income

A fixed income security is an investment that provides fixed periodic payments and the return of principal at maturity. Strictly speaking, fixed income securities are interest paying, fixed-rate bonds, but I will use a more general definition of fixed income, as is widely done in practice. In this document, fixed income securities include Treasury bonds, TIPS, Treasury FRNs, Municipal bonds, corporate bonds and FRNs, convertible bonds, Agency bonds, CDOs, CDS, and other option-bearing debt securities.
It is assumed that the reader knows the definitions of a bond, maturity, face value, interest rate, yield, default, leverage, commercial paper, federal funds rate, LIBOR, money market funds, time deposits, Eurodollars, futures, options, interest rate swaps, inflation swaps, caps, floors, and swaptions.¹

Treasury bonds are government bonds issued by the United States Treasury. They have maturities ranging from one month to thirty years. Treasury bills have a maturity of 1 month to one year, Treasury notes have a maturity of two to ten years, and Treasury bonds have a maturity of twenty and thirty years. I will, as is usually the case, use the term Treasury bond indifferently for any maturity. Treasury bonds typically earn a fixed rate of interest twice a year until maturity, when the principal is paid back, with the exception of Treasury bills, which pay no interest but are sold at a discount to their face value (they are zero-coupon securities). Treasury bonds are considered riskless securities.

TIPS (Treasury Inflation-Protected Securities) are Treasury bonds whose principal is protected against inflation. The principal value of the TIPS is adjusted using the Consumer Price Index, and interests are calculated and paid twice a year using the adjusted value of the principal amount. At maturity, TIPS pay out the largest of the original or adjusted principal.

Treasury FRNs are floating rate notes issued by the U.S. Treasury since January 2014. They have a two-year maturity and pay a floating rate interest quarterly. Interest rates are indexed on the discount rates in auctions of 13-week Treasury bills. Due to the recent apparition of these securities, they will not be used in my analyses and examples.

Municipal bonds are bonds issued by municipal governments.

¹ These definitions may be found in appendix 1.
Corporate bonds are bonds issued by corporations. They are typically divided into investment grade and high yield categories. Investment grade bonds are relatively low yield and low risk bonds, whereas high yield bonds have a lower credit rating (below Baa from Moody’s and BBB from Fitch and Standard & Poor’s), hence a higher yield and risk of default.

Corporate FRNs are floating rate notes issued by corporations. They may also be called floating rate bonds, and I will generally use the term corporate bonds for both fixed rate bonds and FRNs.

Convertible bonds are corporate bonds that can be converted into a predetermined amount of the company's equity at certain times during their life.

Agency bonds are bonds issued by a federal Agency, such as Fannie Mae, Freddie Mac, Ginnie Mae, FFCB, or Sallie Mae. The most common securities issued by these agencies are mortgage-backed securities (MBS) issued by Freddie, Freddie, or Ginnie. These MBS are pass-through securities (i.e., monthly mortgage payments are passed through to the MBS owner) that are secured by one or a collection of mortgages. The mortgages serve as collateral that can be seized by the owner of the MBS in case of default.

CDOs are structured financial products that pool together cash flow-generating assets and repackage this asset pool into discrete tranches that can be sold to investors. Strictly speaking, an example of a CDO would be a collateralized mortgage obligation (CMO), which is in effect a bond backed by pools of mortgages. Thus, CMOs are a type of MBS, and a type of CDO. The difference between MBS and CMOs is that CMOs contain varying classes of holders and maturities (tranches) that, as a whole, should form a less risky asset than taken separately. The process of pooling mortgages into a vehicle that issues bonds is securitization. Collateralized
Bond Obligations (CBOs) are another type of CDOs that are backed by a pool of non-mortgage, low-grade debt securities. Tranches are based on credit risk (i.e. the risk of default) level, rather than maturities. In practice, CDOs and CBOs are often used interchangeably, and MBS are not considered CDOs. Although in theory, these products are all asset-backed securities (ABS), the term ABS is mostly used to designate securitization issues backed by non-bond, non-mortgage types of debt, such as consumer credit (student loans, auto loans, and credit card loans) and Small Business Association (SBA)-guaranteed small business loans.

Subprime lending is the process of giving a loan to an individual who would under normal circumstances be considered a high-risk borrower because of its relatively low ability to maintain the repayment schedule. Typically, borrowers with such credit scores are low-income families, unemployed or divorced individuals. Subprime mortgages, for instance played an important part in the triggering of the 2008 financial crisis. Such loans did not meet the underwriting guidelines of Fannie Mae and Freddie Mac and were considered “non-conforming”. In the low interest rate and real estate bull market environment following the technology bubble at the beginning of the years 2000s, banks started taking more risk by extending variable rate mortgages to subprime borrowers and mixing them with standard prime mortgages in CMOs that received high credit ratings by virtue of diversification and overcollateralization (the process by which the issuer posts more collateral than the amount that the issuer borrows).

Credit default swaps (CDS) are credit derivative contracts, where the purchaser of the swap makes payments to the seller of the swap up until the maturity date of a contract. Payments are made to the seller of the swap. In return, the seller agrees to pay off a third party debt if this party defaults on the loan. CDS are usually considered insurance contracts against the risk of
default of an issuer. The CDS spread over a riskless security is a good indicator of the risk of default of an issuer.

Other debt securities that I will consider fixed income include all of the above, combined with any types of options. For instance, a convertible can be seen as the combination of a bond and a call option on the stock of the issuer. Similarly, a lot of fixed income hybrid products and variations exist. Options on or mixed with fixed income securities can include stock options, CDS, interest rate swaps, inflation swaps, bond futures, interest rate futures, floors, and caps.

I will focus on fixed income because most arbitrage strategies that do not use fixed income securities are far from being actual arbitrage opportunities as defined above. Although many fixed income arbitrage strategies are not purely arbitrage, fixed income securities present cash flow and risk characteristics that are relevant to arbitrage trading. Strategies that are usually called arbitrage but do not use fixed income securities include stocks and foreign exchange special arbitrage, depository receipts and dual listed arbitrage, private to public arbitrage, merger arbitrage, stock / future arbitrage, or ETF / constituents arbitrage.

iii. Crisis

The recent financial crisis has been very useful in examining risk and return opportunities for fixed income arbitrage in a constrained environment. The crisis has indeed provided significant empirical data that corresponds to the occurrence of rare events. As such, I will develop a number of examples from the crisis to illustrate in which way arbitrageurs are affected by stressed economic environments. By financial crisis, I mean the events caused by the US subprime mortgage crisis that profoundly disturbed the US and international banking sectors in 2008, and their consequences on the financial markets, but not the economic disturbances that
followed and affected most developed economies. I will thus consider for my analysis that the Eurozone crisis since late 2009 and the more recent recovery period of developed economies are not part of the 2008 financial crisis. Thus, the period I will look at more closely (although we may also pay attention to the effects of more recent events on our analysis) will stretch from August 2007 – when the first financial industry troubles appear in the form of the Bank of England having to support Northern Rock – to May 2009 – when the results of the US financial institutions stress tests ordered by the Federal Reserve of New York were released.

The reason for my focus on the core events of the banking crisis is that these events had an unprecedented impact on the liquidity and rationality on many fixed income securities that produced very large disturbances for arbitrageurs on these markets. These disturbances created both excellent opportunities for arbitrageurs that were able to seize them, and ample losses and crashes for arbitrageurs that were not ready and had to liquidate their portfolios at a very bad time. These abnormalities make that particular period very interesting to study in order to better understand the risks involved in fixed income arbitrage, and in particular the tail event risks these strategies bear.

Appendix 2 develops the events I previously mentioned in order to be able to tie them to financial market discrepancies.

b. A Potentially Risky, Lucrative Business

As I mentioned previously, one aspect of arbitrage that has attracted my interest is the deviation of such strategies from their original textbook definition. Indeed, although assets with very similar cash flow and risk characteristics should converge towards the same pricing,
evidence shows that convergence trading is inherently tied with many risk factors that make it potentially lucrative and dangerous. If arbitrageurs were in a riskless business, their returns would be small and continuous. In practice, because in normal times, returns are indeed small and relatively stable, arbitrageurs need to take on a lot of leverage to get an acceptable rate of return, and are able to take on such amounts of leverage because of the low risk profile of their trades. Such leverage makes returns much more sensitive to the timely convergence of asset pricing, and gives a lot of power to the leverage providers (the lenders). Indeed, in order to achieve high leverage, arbitrageurs must agree on some financial covenants that will usually allow the lenders to force the liquidation of the arbitrageur’s portfolio in case of large capital losses. Mitchell and Pulvino (2009) explain that, “because the benefits from an orderly liquidation accrue to hedge fund investors and not to hedge fund lenders, hedge fund lenders could force rapid liquidations.” Thus, Because there is a risk that the asset mispricing widens before it gets narrower, it is possible for the arbitrageur to be forced into liquidation and take large losses before he can make some profit on its trade.

In order to illustrate the riskiness and sophistication of arbitrage strategies, I will now develop an example of a large arbitrage hedge fund\(^3\) crash, and one of impressive arbitrage gains from a strategy implemented at the right time, on the right asset mispricing.

i. **Rise and Fall of LTCM**

Probably the most famous arbitrage fund meltdown was that of Long Term Capital Management (LTCM) in 1998. Nicholas Dunbar’s *Inventing Money: The Story of Long-Term Capital Management and the Legends Behind It* describes in details the founding and crash of the famous hedge fund.

\(^3\) For a definition of hedge fund, please refer to II) b) ii).
John W. Meriwether had had a splendid career as a bond trader, head of the fixed income arbitrage group, and vice-chairman at top Wall Street investment bank Salomon Brothers until he was caught in the Treasury bond scandal perpetrated by his colleague Paul Mozer in 1991. Meriwether’s success was partly due to his ability to hire the right finance academics in its arbitrage group, and give them the opportunity to develop pricing models and stimulate financial innovation at a time when arbitrage and derivative pricing theories were not widely spread and used in most investment banks. Academics Meriwether hired at Salomon include Robert C. Merton and Myron S. Scholes, Nobel Laureates in Economics for their work on option pricing at MIT, Eric Rosenfeld (PhD in Finance from MIT), and Greg Hawkins (PhD in Management from MIT).

In 1993, Meriwether and Rosenfeld decided to found LTCM, a hedge fund that would use the technology he and his PhD colleagues had developed at Salomon to place very leveraged fixed income relative value (i.e. convergence) trades in international capital markets. Salomon star traders Victor Haghani and Larry Hilibrand promptly joined the team, followed by Hawkins, Scholes, Merton, and Federal Reserve vice-chairman David W. Mullins, Jr. With the help of Merrill Lynch salesmen, the fund rose $1bn before it started its trading operations in early 1994.

Using classic Treasury bond arbitrage strategies such as on-the-run / off-the-run bond arbitrage (on-the-run Treasuries are being or have been auctioned recently, so they are very liquid and demand is high for them, making them a little more expensive than Treasuries with the same maturity but that were auctioned longer ago and were not traded as much. The arbitrageur can take advantage of that difference in pricing by buying off-the-run and short-
selling\footnote{Short selling is the process of selling a borrowed asset. In practice, the short-seller borrows the asset, sells it on the market, and buys it back at a later time (hopefully at a lower price) in order to give it back to the lender. In the case of arbitrage hedge funds, this is usually done via a reverse repurchase agreement, the process of which will be detailed below.} on-the-run treasuries), the fund returned 21%, 41%, and 43% net of fees (LTCM would charge 2% of management fees and 25% of the realized gains of the fund a performance fees) to its investors in its three first years of existence – truly outstanding results.

As time passed, LTCM started diversifying its range of strategies. It would not only arbitrage a wide range of fixed income products, but would also invest on merger arbitrage, dual listing arbitrage, or on non-arbitrage strategies. For instance, LTCM was short a lot of deeply out of the money put options on the S&P 500, meaning that the fund was selling insurance that the S&P would not face a major drop. The firm would place this kind of bets because of the famous mispricing of deeply out of the money options: investors were so afraid of tail events that they would be able to pay a hefty premium to buy insurance on these events. At the end of 1998, LTCM had almost $5bn of capital, but had taken on c.$125bn of debt, reaching a leverage ratio of 25 to 1! The company had an additional c.$1.25tr in off-balance sheet derivative positions, mostly in interest rate swaps to hedge its fixed income trades.

After the turbulences induced by the 1997 East Asian crisis, the results of LTCM begun to stumble, with the fund losing c.$500m of capital in mid-1998. Simultaneously, Salomon Brothers exited the arbitrage business. Salomon’s exit of large convergence positions had a similar effect to the placing of divergence trades (the bank was buying expensive assets and shorting cheap assets). This widened exiting mispricing gaps in the fixed income scope. In August-September 1998, the Russian government defaulted on its debt, creating a panic amongst fixed income investors, who started liquidating their positions in Japanese and European bonds.
in order to buy the safest asset on the planet: US Treasuries. The value of these bonds diverged further away from their fundamentals, thereby going against the convergence trades placed by LTCM. The firm lost c.2bn in capital in a few weeks, which lead it to forced liquidation of its trades at a very bad moment – thus increasing its losses. As LTCM’s investors learned about the debacle, they withdrew their capital in September, taking out $1.9bn of the remaining $2.3bn capital of the firm, boosting the leverage ratio of the fund to 250 to 1.

The partners did try to raise more capital. MacKenzie (2003), quotes Meriwether’s fax to its investors, just before the fund’s collapse:

“The opportunity set in these trades at this time is believed to be among the best that LTCM has ever seen. But, as we have seen, good convergence trades can diverge further. In August, many of them diverged at a speed and to an extent that had not been seen before. LTCM thus believes that it is prudent and opportunistic to increase the level of the Fund’s capital to take full advantage of this unusually attractive environment.”

Here, Meriwether shows the eternal conundrum of risk and return of arbitrage strategies: it is at the time of largest risk and losses that the opportunities and potential gains are the highest. As explained in Kondor (2009), Meriwether’s fax backfired as investors pulled back even more capital out of the fund, and competing hedge funds started to trade against LTCM, expecting their convergence trade to reverse. Because Wall Street firms that had been trading with and had provided to LTCM feared that the fund’s implosion would create a chain reaction and would provoke large losses among them, it was decided that LTCM needed to be bailed out. Goldman Sachs, AIG, and Berkshire Hathaway offered to buy out the partners at a very low value.
Meriwether did not take the deal in time, and the Fed was forced into organizing a $3.7bn bailout by most of the fund’s creditors.

ii. At the Right Time in the Right Trade

An interesting mispricing in the financial markets is that of TIPS to Treasury bonds, described by Fleckenstein, Longstaff, and Lustig (2011). In their paper, the authors show that the TIPS issued by the US Treasury are consistently underpriced, relative to the Treasury bonds of the same maturities. Indeed, TIPS can be seen as the combination of a Treasury bond, and an inflation swap. Ergo, there is an arbitrage opportunity if the value of Treasury bonds does not equal the value of the corresponding notional amount of inflation-swapped TIPS. The authors show that, in extreme circumstance, the mispricing can reach $20 per $100 notional, and that in normal circumstances, that gap is never significantly close to zero. Because the mispricing is not stable however, placing a convergence trade at a time when it is small and about the get much larger can lead to marked-to-market losses, and potentially to forced liquidation and the obligation to take those losses before convergence.

At least one hedge fund (Barnegat Fund Management) got into that trade at the right time, when the 2008 liquidity issues in the financial markets were at a maximum (just after September 15 Lehman bankruptcy), and the mispricing had reached unprecedented levels. Figure 1 shows the time series of the TIPS–Treasury mispricing, expressed in units of dollars per $100 notional, as calculated by Fleckenstein, Longstaff, and Lustig (2011).
The authors quote from Financial Times blogs by Kaminska (2010) and Jones and Kaminska (2010):

“As Barnegat explain: “We will buy the TIPS, short the nominal bond, and lock in the inflation rate with the inflation swap. The result is that the net initial payment is zero, but until 2014 this trade yields up to 2.5 percent per year of the notional.

For a small group of savvy traders the pricing discrepancies at their widest led to one of the most successful hedge fund trades in recent memory. One of the biggest beneficiaries was the low-profile New Jersey-based $450 million Barnegat fund founded in 1999. Barnegat acquired TIPS bonds shortly after the collapse of Lehman Brothers and then shorted—bet on a fall in rates—regular Treasury bonds of an equivalent maturity. As the pricing discrepancy narrowed, the fund realised huge gains. The fund returned 132.6 percent to investors in 2009.”
c. Interest of the Topic

i. Relation Between Risk, Return, and External Factors

As shown above, risks and returns of arbitrage strategies are tightly linked to the evolution of macroeconomic and financial factors. Studying the relation between security mispricings and external factors is very interesting because it leads to a better understanding of the risks inherent to convergence trading and therefore enables better risk management practices. Particularly, the recent financial crisis provides me with a good data set of what effects tail events may have on the pricing of fixed income assets, and on the financial system as a whole. I will thus use analyses and observations from existing literature to build and calibrate a model that is realistic and flexible. This model can be used to test existing theories and assumptions on risks and returns of fixed income arbitrage, and their relation with economic risk factors.

ii. Risk Management Tools for Arbitrageurs

The purpose of the modeling of one fixed income arbitrage strategy’s returns according to trading and investor behavior assumptions is to better understand the array of risks such strategies entail, and to potentially deduce adequate tools to manage these risks. It would also be interesting to see whether the relationships and rules between fund managers and their creditors and investors may evolve, and in which sense, in order to satisfy all parties in changing environments.

iii. Ambitions and Foundational Question

First, I would like to develop and organize the major aspects, theories, and findings on fixed income arbitrage strategies. Secondly, I would like to analyze risks and returns of one theoretical arbitrage strategy. In this analysis, I will develop a simple but innovative model that assumes specific trading rules and investor behavior towards capital allocation. I would also like
to answer the question of the value-added of the arbitrageur. Is the arbitrageur above all a liquidity provider who helps the markets transition through varying states and gets compensated for that, or is he closer to being a statistical trader, calibrating his models on uniform risk factors and taking losses when these are impacted by rare shock events?

The rest of this paper is organized as follows: section II explains core characteristics of the fixed income arbitrage world that are essential to the understanding of an arbitrageur’s operations; section III focuses on the specific risks a fixed income arbitrageur faces, and the return characteristics of arbitrage strategies; and section IV develops trading model based on a specific mispricing. The mispricing will be tied to external risk factors that can be calibrated to simulate any economic environment, and to a hypothetical arbitrage hedge fund’s investor behavior.

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5 I use the term statistical trader to describe a trader who uses statistical (empirical) models to figure out that, in most states of the world, prices probabilistically bound by some limitations, as opposed to traders who derive fundamental values from theoretical calculations and link deviations from theoretical values with external risk factors.
II) Characteristics of Fixed Income Arbitrage Strategies

a. Role of Arbitrageurs in the Financial Markets

i. Conceptual Framework: Enforcing the Law of One Price

As in Fung and Hsieh (2002), the law of one price (LOP) states that: “two assets with the same payoffs in every state of the world must have identical prices”. However conceptually trivial, this statement has plenty of consequences in regards to asset pricing and securities trading. Because it should bring the same return to invest in assets that will always have the same payoffs, the prices of these assets must remain the same at all time. If, for any reason, a pair of such assets breaks the LOP, then a textbook arbitrage opportunity arises. Indeed, going long on the more expensive asset and short on the cheapest one should bring the arbitrageur a riskless return, at no cost. Replicating this strategy as much as possible should make the arbitrageur richer and bring prices closer together. Because a lot of traders are always spotting small mispricings, the LOP should be close to perfectly respected at all times. In this state of the world where all mismatches are corrected in a timely manner by arbitrageurs, relative prices are always fair.

ii. From Riskless Textbook Arbitrage to Convergence Trading

In theory, arbitrageurs of fixed income securities are making a risk-free profit by buying and selling a pair of securities that generate the same cash flows but do not trade at the same price. Because the cash flows are the same, the trade is riskless if arbitrageurs keep both securities until maturity. In practice, arbitrageurs do not want to hold the mispriced pair of assets
to maturity, because holding a long/short portfolio is actually costly. Indeed, in order to be able to buy the cheap security, arbitrageurs have to borrow money. In practice, they will post the asset bought as collateral, in exchange for (a lesser amount of) debt financing. The amount by which the funds borrowed are smaller than the collateral value is called the haircut. Because of this haircut, holding an arbitrage portfolio requires the arbitrageur to freeze some of its capital. Although arbitrage trades are low risk, and therefore support high leverage (or low haircuts), liquidity providers will require some or the arbitrageur’s capital to be posted as collateral. This process induces an opportunity cost of capital for the arbitrageur: the longer the arbitrageur keeps its portfolio, the lower his return on capital will be.

For that reason, arbitrageurs tend to bet on the convergence of prices earlier than maturity, and place what are called convergence trades. As Fung and Hsieh (2002) put it: “Convergence trading bets on the relative price between two assets to narrow (or converge)”. Now that it is established that arbitrageurs aim at a timely liquidation of their portfolio at a gain, it can easily be show in what sense convergence trades are risky. Because arbitrageurs have to bet on the mispricing narrowing rapidly, they are subject to the risk that the price discrepancy does not narrow because of the market irrationality. As long as the gap does not narrow, the arbitrageur cannot exit his trade at a profit, and the expected return on his equity goes down. This risk is not the only one the convergence trader has to face, but the trader’s necessity to exit his trade in a timely manner is the main reason why his strategy may cause losses in some situations.

iii. How Well do Arbitrageurs do their Job?

For most fixed income assets, and in normal market situations, the LOP is firmly and consistently enforced. At some points in time however, it has been noticed that mispricings can increase to extreme, abnormal levels. One famous arbitrage opportunity arises when the covered
interest rate parity (CIP) is not respected. Mancini and Ranaldo (2011) devote a paper to the study of CIP arbitrage opportunities after Lehman went bankrupt. The authors show that at that time, arbitrage profits were large and persistent on this particular trade. The CIP states that the ratio of the risk-free returns of two countries must equal the ratio of the forward exchange rate of the two countries' currencies to their spot exchange rate. If that is not the case, then arbitrageurs may borrow in one country, buy spot and sell forward the second country's currency, invest in the second country for the period of time preceding the forward's maturity, and repay its initial debt making a riskless profit. Figure 2 shows the time-series of profits from CIP arbitrage as calculated by Mancini and Ranaldo (2011).

![CIP profits, secured and unsecured arbitrage, 1-week term, EURUSD](image)

**Figure 2: Euro - US Dollar CIP arbitrage profits during the financial crisis**

It is interesting to notice that, in normal times, spending capital on such well-known, universal parity is not profitable. In fact, transaction costs make the trade generate negative
returns. However, as seen previously for the TIPS to Treasury bond arbitrage strategy, getting on that trade right after Lehman’s bankruptcy is a winning bet. An interesting question is: what happened for CIP arbitrageurs to let the mispricing go so wide that it enabled such large profits to newcomers during the aftermath of Lehman’s bankruptcy?

iv. Can Arbitrageurs Hurt the Markets? Introduction to the Danger of Contagion

I have shown in what way arbitrageurs profit from price discrepancies in the financial markets, and how by doing so, they make the markets more efficient in the sense that they keep those pricing errors to a minimum. However, not only can arbitrageurs, in some instances, be incapable of maintaining the LOP, but they also may have a perverse effect on the pricing of the securities they are arbitraging. I call this effect contagion because it has a tendency to spread from one arbitrageur to the next (cross-player contagion), and eventually across a diverse set of markets (cross-market contagion). To illustrate what can go wrong when arbitrageurs fail at their task, one needs to understand the functioning of an arbitrageur’s funding. As seen previously, the arbitrageur receives leverage from posting the cheap security as collateral. However, the arbitrageur’s financier can only tolerate a reasonable amount of leverage, and uses the haircut to make sure the arbitrageur is creditworthy. When arbitrage fails, and the mispricing grows wider than at the time when the arbitrageur entered his trade, his portfolio shows marked-to-market losses, thereby reducing the actual value of his equity cushion. If that situation goes too far, the leverage provider has the ability to force the arbitrageur to liquidate his trade portfolio and take the losses before it is too late and the value of the collateral does not fully cover the leverage provider from potential losses.
This funding restriction on leveraged arbitrageurs is key to understanding the contagion phenomenon. If the arbitrage is relatively well known, there is a good chance it is a crowded trade, i.e. a lot of different investors have deployed capital on that particular opportunity. When financial disturbances bring a mispricing wider than expected, the most leveraged arbitrageur will receive a margin call from his leverage provider, forcing him to place more collateral. In order to do so, the arbitrageur will have to liquidate part of his portfolio (unfortunately, the more leveraged the trader is, the less capital he can free out of exiting a trade, the more assets he will have to sell to satisfy his margin call), and will thus place a series of divergence trades. As Mitchell, Pedersen, and Pulvino (2007) put it: “Rather than increasing investment levels when prices dip below fundamental values, arbitrageurs may, in the face of capital constraints, sell cheap securities causing prices to decline further.” In normal times, other arbitrageurs would be able to absorb the distressed arbitrageur’s reverse trades, but in this scenario, the other arbitrageurs have a reduced amount of capital due to marked-to-market losses and are most probably not looking to increase their leverage, or get into a trade is this volatile state of the market. The consequence is that the distressed arbitrageur’s liquidation of his portfolio will bring the mispricing even wider, causing larger losses for the other traders, who will start receiving margin calls too. It is easy to see how this process can get out of hand and create a chain reaction that could spread to initially unrelated markets, because of the diverse strategies implemented by arbitrageurs.

b. Typology of Players

i. Bank Proprietary Trading Arbitrage Desks

The first type of trader that will develop and implement arbitrage strategies is an investment bank’s proprietary trading desk. These desks employ research analysts who develop
models to value fixed income securities, and fixed income traders who use these models to come up with relative value trading strategies and implement them. These arbitrageurs use the bank’s own capital in order to place trades, and are aggressively compensated on their performance. A famous example of such a team is Meriwether’s arbitrage desk at Salomon Brothers (as told by Dunbar, 2001), during the 1980s. As I described earlier, Meriwether’s academics and traders developed innovative arbitrage strategies that lead to very successful years for the firm.

Since the financial crisis, and more specifically since the Dodd–Frank Wall Street Reform and Consumer Protection Act (Dodd-Frank) went into effect in July 2012, the vast majority of proprietary trading desks has been closed, and most proprietary traders have moved on to hedge funds or independent trading firms. In effect, § 619 of Dodd-Frank, also called the Volcker Rule, restricts the use of the banks’ capital for speculative investments that do not benefit their customers, resulting in a ban of proprietary trading activities.

ii. Specialized Hedge Funds

The second type of arbitrage investor is a hedge fund that usually specializes on fixed income arbitrage, or even on one type of arbitrage within the fixed income space. A hedge fund is an unregulated investment firm that invests in a wide range of traded securities. Hedge funds raise capital from external, sophisticated investors (Limited Partners, or LPs) – either high-net-worth individuals or institutional investors such as pension funds or university endowments. Because they are unregulated, hedge funds are free to follow sophisticated strategies, and to bet aggressively on specific risks. For instance, a hedge fund has the ability to short securities, to trade a vast array of derivatives, and to take on a lot of leverage. From that description, it is easy to see why hedge funds are good candidates for arbitrage strategies. A previously developed example of a hedge fund that started as a fixed income arbitrage investor is LTCM.
A last type of potential arbitrageur would be a proprietary trading firm. These firms are largely similar to hedge funds, but they do not raise capital from external investors. Instead, they trade using their own partners’ money.


The hedge fund industry’s assets under management (AUM) have been growing exponentially over the past decade and a half. According to BarclayHedge, a research and data provider on the alternative investments industry, hedge fund AUM have been multiplied by 18 from $118bn in 1997 to $2,157bn at the end of 2013. This corresponds to a 20% compound annual growth rate (CAGR). 2008 has seen a 32% decrease in AUM due to the financial crisis, making the years 2007 to 2013 a no-growth period. Figure 3 shows the evolution of hedge fund industry AUM according to BarclayHedge measures.

The fixed income hedge fund AUM has known similar trends, but with larger recent growth. According to BarclayHedge, AUM have been multiplied by 21 in the 1997-2013 period – from $16bn to $327bn; but after a 21% drop in 2008, AUM grew at a 18% CAGR until 2013, making 2013 AUM 1.8 times that of 2007. Figure 4 shows the evolution of fixed income hedge fund AUM according to BarclayHedge measures.6

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6 Note that this is not the fixed income arbitrage hedge fund AUM, as this information is not readily available at BarclayHedge.
Figure 3: Hedge fund industry AUM (billion dollars)

Figure 4: Fixed income hedge fund AUM (billion dollars)
In order to have an approximation of fixed income arbitrage hedge fund AUM, we can use Tremont/TASS (2005) Asset Flows Report, quoted by Duarte, Longstaff, and Yu (2006), that states that “the total amount of hedge fund capital devoted to fixed income arbitrage at the end of 2005 is in excess of $56.6 billion.” This represents 36% of total fixed income arbitrage AUM in 2005.

d. Operations: Funding and Leverage

As discussed earlier, there usually are two different players in the fixed income arbitrage business: investment banks proprietary trading desks (although they are far less active nowadays) and some specialized hedge funds. The fundamental difference between these two types of traders, as explained by Buraschi, Sener, and Mengütürk (2002), is the funding modalities of their trades. The collateral-based funding system I have already described (also called secured arbitrage) is mostly used by hedge funds, because they do not have access to the money market operations used by banks (unsecured arbitrage).

Because banks are large, creditworthy corporations, they are able to issue commercial paper at a very low cost. Money market funds buy these commercial papers because they provide a very safe (but not risk-free) source of revenue, at an interest rate higher than Treasuries. This source of funding is unsecured because the banks do not have to post collateral to issue commercial papers. This does not mean that proprietary trading activities are not subject to funding liquidity risks. Because commercial papers typically have a short maturity, the issuer faces rollover risk: when the bank needs to re-issue commercial paper to continue to fund its trading activities, it may have to do so at higher costs, or may simply be denied access to money markets if money market funds find risks are too high.
In order to fund their trading activities, hedge fund deal with prime brokers. Prime brokers are divisions of investment banks that provide a diverse set of services to hedge fund clients, such as portfolio and risk management, securities lending, or financing. In reality, prime brokers are an intermediary between hedge funds that need to borrow money, and other external investors (such as money market funds, banks, insurance companies, corporations) that need to invest money on a secured basis at a satisfying interest rate. The process by which the prime broker provides funding is rehypothecation. For a detailed description of the rehypothecation process, see Mitchell and Pulvino (2009). As shown before, hedge funds receive leverage from prime brokers by posting collateral including a haircut. The process by which the hedge fund grants first lien call on securities and cash held by the prime broker is called hypothecation. Hypothecation is in fact generally carried out by way of a repurchase agreement, or repo, where the hedge fund sells the asset to the prime broker\(^7\), and agrees on purchasing it back at a predetermined date and price. In order to finance this service, the prime broker will rehypothecate these securities in order to receive a loan from the final, external investor. This investor will receive the collateral, lend the money needed for the hedge fund operations, and be paid a small but higher than risk-free interest rate over the length of the trade. The prime broker will receive a margin fee from its hedge fund client, in the order of the federal funds rate + 20 to 30 basis points. The process by which the hedge fund short sells the other security of the pair he is arbitraging is a reverse repo. In such an agreement, the arbitrageur buys a security from his dealer, and agrees on selling it back at a predetermined date and price (which is in essence the same thing as borrowing the security). In the meantime, the arbitrageur sells the security to

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\(^7\) There is actual change of ownership of this asset, making a repo safer than classic hypothecation (or collateralization). Indeed, the repo lender can easily sell the asset to reimburse himself if the borrower defaults.
finance his purchase and buys it back at a later time to deliver it to the reverse repo counterparty. It is easy to see how a drop (resp. rise) in price of the security over the reverse repo (resp. repo) period becomes a capital gain to the arbitrageur. Of course, this type of funding also bears risks. For instance, repo haircuts have increased a lot during the financial crisis, forcing hedge funds to reduce their leverage\textsuperscript{8}, and liquidate some of their portfolios. Mitchell and Pulvino (2009) explain that after Lehman bankruptcy, repo haircuts on safe arbitrage strategies such as convertible arbitrage went from approximately 1% to 45% because panicking final investors had difficulty selling the illiquid securities (convertibles or high yield corporate bonds) they had received as collateral. A notable problem linked to the rehypothecation process is the fact that only half of the hedge fund’s arbitrage portfolio is transferred to the final investor as collateral (the other half being short-sold by the hedge fund). This means that rehypothecation does not transfer the portfolio hedges to the rehypothecation lender. The long only portfolio of fixed income securities that is transferred is much more volatile than the long-short portfolio theoretically held by the hedge fund. As a consequence, the rehypothecation lender is more likely to panic and fire-sell its collateral when the actual portfolio may be performing well.

e. Hedge Fund Styles and Strategies

i. Hedge Fund Styles

As in Fung and Hsieh (2002), hedge fund styles can be seen as a combination of location and strategy. The location is the asset the fund is trading (mostly stocks, bonds, commodities, or currencies). The strategy describes the trading patterns and rules that a hedge fund will follow.

\textsuperscript{8} As repos are typically overnight contracts, their terms can be renegotiated daily, and haircuts can increase substantially in a small amount of time.
Widely used strategies include buy-and-hold, long-only, long-short, trend-following, convergence, or passive spread trading.

Within fixed income hedge funds, most styles fall into 5 categories: long-only or long-short high yield funds, MBS funds that hedge out the interest rate and prepayment risks, convertible bonds arbitrage funds that buy convertibles and short the underlying stock, other fixed income arbitrage funds that exploit mispricings between fixed income securities and hedge out the interest rate risk, and diversified funds that use multiple strategies or develop niche strategies.

ii. Financial Innovation in the Fixed Income Arbitrage Business

Fixed income arbitrage is both an ancient practice (as it has always been noticed that the same securities trading in different markets could have a different price, thereby offering a risk-free profit to the trader able to spot the inefficiency and create a balanced long-short portfolio) and a relatively new form of investing. Indeed, most currently used arbitrage techniques were only invented in the 1980s, as the generalization of various fixed income derivatives allowed for theoretical mispricing to be arbitraged away in practice. Indeed, the Chicago Board of Trade (CBOT) launched Ginnie Mae futures in 1975, the Chicago Mercantile Exchange (CME) started trading Treasury bill futures in 1976, CBOT replied with Treasury bond futures in 1977, and CME did not trade Eurodollar futures until 1982. The first options on bonds appeared in 1983, followed by caps, floors and swaptions.

As Dunbar (2001) tells it, many trading strategies in the fixed-income space were invented within Meriwether’s proprietary trading desk at Salomon Brothers. Meriwether himself
started as a bond trader, and would arbitrage the on-the-run / off-the-run basis at the beginning of
his career. After Black and Scholes published their paper on the valuation of options in 1973, and
after Vasicek devised the first affine term structure model in 1977, it became easier for
arbitrageurs to understand the yield curve movements, and to find hidden-options in many fixed
income instruments. Arbitrageurs had the tools to price efficiently many securities and detect
accurately their price discrepancies.

As an example of the financial innovation that took place in the 1980s, Salomon’s
Hawkins found hidden options in MBS in the form of the right to refinance mortgages according
to interest rates. According to Hawkins’ models, the market overpriced these options. He then
bought MBS (i.e. wrote a call option on the underlying bond), hedged interest rate risk by selling
Treasury bonds futures, hedged the risk that the yield curve steepened by swapping bond
coupons into floating rate coupons (these two hedges make a duration hedge where the concave
MBS price / yield curve becomes “parallel” to the x axis). He bought treasury options to
replicate the hidden options in MBS (this makes a volatility hedge that suppresses the concavity
of the curve). He could thus capture a risk-free spread due to the overpricing of the hidden
option.

iii. Typology of Well-Known Fixed Income Arbitrage
Strategies

1. Swap Spread Arbitrage

In Duarte, Longstaff, and Yu (2006), Swap Spread arbitrage is described as follows: the
arbitrageur enters an interest rate swap (pays LIBOR L t, and receives fixed coupon CMS), shorts
Treasuries (pays fixed coupon CMT), and invests the proceeds in a margin account earning the
repo rate r t. In total, the arbitrageur receives the fixed annuity CMS – CMT and pays the floating
Because the LIBOR – Repo rate spread is considered very constant over time, entering this trade at a time when the fixed annuity is significantly larger than the floating spread should bring comfortably stable returns. However, there is a risk that the LIBOR spikes up when market liquidity is threatened (such as after Lehman’s bankruptcy). In such cases, Swap Spread arbitrageurs have suffered very large losses.

This strategy is far from being a textbook arbitrage as there is no guarantee that the observed pattern of stable LIBOR spread will remain during the length of the trade implementation.

2. CIP Arbitrage

CIP arbitrage strategy was previously described as a case in point of LOP transgression during the crisis. This strategy is a typical textbook arbitrage, but it is so widely known that it leads to negative excess returns in normal market periods.

3. Yield Curve Arbitrage

Yield curve arbitrage is the process of taking long and short positions on different points of the (usually Treasury) yield term structure. Typically, arbitrageurs would place steepener trades, flattener trades, or butterfly trades to bet on changes in the slope and curvature of the yield curve. Arbitrageurs would normally take positions that are hedged against change in the overall level of the yield curve, either by holding zero net duration portfolios, or zero net duration and convexity portfolios.

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9 Duration is the first order derivative of the price to the level of interest rates; as a consequence, small, parallel shifts in the interest rate term structures do no affect a zero-duration portfolio.
In order to detect points of the yield curve that are rich or cheap, arbitrageurs use several kind of models that fit a continuous term structure using actual market price data. A bootstrapping process or a linear regression on a number of bond prices to estimate coefficients that represent a set of discount factors can provide specific data points, and the fitting can be made by spline interpolation. Alternatively, a 3-factor affine model can be devised to fit data points by minimizing squared error. For more details on this process, refer to Nelson and Siegel (1987).

This strategy is not a textbook arbitrage either, as it simply relies on yield curve fitting rules that are not linked to the reality of bond pricing processes.

4. Mortgage Arbitrage

As seen in the financial innovation paragraph, mortgages can be seen as fixed income securities that come with a short position in a call option, in the sense that the issuer has the right to refinance his mortgage if interest rates go down. For that reason, with the help of proprietary models that link the refinancing rate with the level of interest rates, arbitrageurs can price the hidden options and hedge themselves for interest rate variations and volatility, thereby capturing the spread between the true value of the option and its market price.

If it is assumed that duration and volatility hedges are perfect (i.e. can be dynamically rebalanced with no frictions or transaction costs), then mortgage arbitrage is close to being a textbook arbitrage opportunity, with the exception that it still hinges on the accuracy of the arbitrageur’s prepayment model.

\textsuperscript{10} Convexity is the second order derivative of the price to the level of interest rates; a duration-convexity hedge allows reducing the price movements of a portfolio more efficiently (and for larger interest rate movements) than a simple duration hedge.
5. Volatility Arbitrage:

There is a widely recognized tendency for the implied volatility as backed out from option prices to be on average above realized volatility. For this reason, it is usually advantageous to be short volatility, i.e. to write (sell) options. A fixed income volatility arbitrageur simply sells options on fixed income securities, delta hedges\(^\text{11}\) his position in the underlying asset, and hopes for realized volatility to be less than implied volatility. In that sense, the arbitrageur is selling insurance on market volatility and consistently collecting insurance premia. It can be shown that the value of this premium (i.e. the return of the strategy) solves:

\[
g \approx \gamma^* (\sigma_r^2 - \sigma_i^2)
\]

where \(g\) is the return of the strategy, \(\gamma\) the gamma of the option, \(\sigma_r\) the realized volatility of the underlying asset over the option life, and \(\sigma_i\) the implied volatility of the underlying asset as backed out from the option price using the Black-Scholes formula.

As an illustration, Duarte, Longstaff, and Yu (2006) describe the use of this strategy on interest rate caps (that can be seen as a series of individual options on the LIBOR rate), delta hedged by Eurodollar futures.

This arbitrage strategy is not a textbook arbitrage, as it is merely based on an observed market inefficiency, and there is no guarantee that it will keep realizing.

6. Capital Structure Arbitrage

\(^{11}\) Delta hedging is mitigating the risk of an option by taking an offsetting position in the underlying asset. For example, the seller of an option will offset the risk that the asset price rises by taking a long position in the asset. The delta of the option is the first order derivative of the price of the option to the price of the underlying asset. Thus, the delta is used to compute an appropriate hedge ratio.
As defined by Duarte, Longstaff, and Yu (2006), capital structure arbitrage “refers to a class of fixed-income trading strategies that exploit mispricing between a company’s debt and its other securities (such as equity).”

a. Example 1: CDS Arbitrage

An example of such opportunities is the arbitrage between the theoretical and actual CDS spread of a corporation. Using the equity price and capital structure of an issuer, a model such as the CreditGrades model computes the issuer’s theoretical CDS spread. For more details on this model, refer to Duarte, Longstaff, and Yu (2006). The arbitrage strategy consists in shorting (resp. longing) the company’s CDS if the market spread is higher (resp. lower) than the theoretical spread, and hedge changes in the value of the CDS by shorting (resp. longing) the appropriate amount of the issuer’s equity until the theoretical and market spread converge.

This strategy is relatively far from being a textbook arbitrage (and in effect, it is closer to speculation) as it relies on the valuation of a derivative instrument as per one specific model. There is no guarantee that market prices will converge to theoretical values.

b. Example 2: Corporate Bond / CDS Spread Basis Arbitrage

A corporate bond interest rate can be seen as a risk-free rate and a credit risk premium. If it is assumed that a CDS contract carries no counterparty risk, then buying a corporate bond and a CDS on that bond should be the same as buying a risk-free bond (thus generating no excess return), because all credit risk is born by the CDS seller. For this reason, it is often considered that a CDS spread and corporate bond spread (over the risk-free rate) should be similar. In practice, the CDS spread tends to be smaller than the bond spread; the difference is called the
CDS / corporate bond basis. Fleckenstein, Longstaff, and Lustig (2011) put forward Duffie (2010)'s hypothesis of the slow-moving capital as an explanation for that basis: since buying a bond requires much more capital than buying the corresponding CDS, this mispricing may persist as long as there is not enough capital in the arbitrage business.

The strategy of buying the corporate bond and the corresponding CDS contract to take advantage of the basis is a passive spread strategy that bears counterparty risk in case of default, and liquidity (as explained in Buraschi, Sener, and Mengütürk (2010)) and idiosyncratic risk if not carried to maturity. As such, it is not a textbook arbitrage opportunity.

c. Example 3: Convertible Arbitrage

As seen previously, a convertible can be seen as a corporate bond embedded with a call option on the issuer's stock. This allows arbitrageurs to exploit price inefficiencies between the option's theoretical value and its value according to the price of the convertible bond. The pure convertible arbitrage as described in the literature consists in buying a convertible and shorting the underlying stock. In this strategy, the long-short position is hedged for large movements in the stock price. Indeed, in case of downside movement, the short position compensates for the loss in the price of convertible and the bond keeps paying its coupons, and in case of upward stock price trend, the conversion of the bond into stock compensates for the losing short position.

An alternative strategy is closer to the concept of arbitrage in the sense that it can hedge most of the risks carried by owning convertibles. Very similarly to MBS arbitrage, the arbitrageur can isolate the option premium from the other components of the convertible. LTCM's Haghani first implemented this strategy in Japan (see Dunbar (2001)). Haghani noticed the stock options embedded in Japanese convertibles were underpriced according to the Black-
Scholes-Merton model, so he bought the convertibles (financed by repo) and sold the correctly priced call options on the market. This way, he was only bearing the risk of the pure corporate bond. He hedged for the interest rate risk by swapping interest payments to floating. In theory, he could have been perfectly hedge by shorting the underlying corporate bond, but I assume there were not enough corresponding maturity bonds in the market. Alternatively, he could have gotten rid of the corporations’ credit risk by buying corresponding CDS.

Convertible arbitrage in the usual use of the term is not an arbitrage strategy, but more simply a long-short strategy that does not hedge for some factors such as interest rate risk, even though the right amount of short stock position does hedge for credit risk. It is rooted in the analysis of a mispricing (that of an option premium), but does not totally isolate that mispricing.

7. Municipal Bond Arbitrage

Municipal bond arbitrage profits from the inefficiencies that arise from the muni market. Since munis’ interest payments are exempted from tax, they tend to attract tax-paying long-only investors (such as individuals, pensions funds, fixed income mutual funds, insurance companies, or corporations). Because the muni market is very fragmented and crossover buyers (investors that do not have the mandate to generate tax-free returns, but may buy munis on an opportunistic basis, notably when they seem cheap in comparison with corporate bonds) erratically come and go into munis, many inefficiencies arise, allowing arbitrageurs to hold duration-neutral, long-short portfolios with positive excess return. These relative value strategies are not exactly arbitrage, and can be compared to market neutral long-short equity strategies that bet on relative stock performances.
Another strategy for tax-paying arbitrageurs is to hold a portfolio of long-maturity municipal bonds, hedged with short positions in similar maturity and credit quality corporate interest rate swaps. The higher slope of the muni term structure combined with the tax exemptions creates positive carry. Since the swaps are not perfectly correlated with the municipal bonds, there is basis risk in the sense that it is not guaranteed that the hedge will hold over the long term, especially in periods of high volatility. Additionally, munis are callable, and prepayment risk is generally not hedged in such strategies.

8. **Eurobond Arbitrage**

Eurobond arbitrage is an example of a strategy based on an actual pure violation of the LOP. As such, it is a textbook arbitrage (but the reservations I highlighted earlier regarding the implementation of convergence trading strategies on textbook arbitrage still hold). Eurobonds are bonds that are denominated in a currency other than that of the country it is issued from. As Buraschi, Sener, and Mengüçürk (2010) put it, “if expected recovery rates in different foreign currencies are the same, yield spreads of the same bond (across two currencies) must satisfy a simple no-arbitrage restriction. As a consequence, the price of two bonds with the same maturity and issuer, denominated in two different currencies, should be the same once the foreign exchange (FX) risk has been hedged in the FX swap market. Any exception to this rule creates a simple arbitrage opportunity.

The authors show that for a relatively long period of time after Lehman’s bankruptcy, the price difference between EUR-denominated and USD-denominated sovereign Eurobonds issued by emerging market (EM) governments was very large. A case in point is that Brazil's EUR yield spread was 25% higher than its USD spread at the end of 2008. This difference could not be explained by market liquidity, as Brazil is a very large issuer of frequently traded, same-
maturity bonds in both currencies. In addition, Brazil’s Eurobonds was far from the epicenter of the financial crisis, and its sovereign Eurobonds had previously consistently respected the LOP.

The strategy of spotting such price discrepancies and going long on the cheap bond, and short on the dear bond has to be done at the right time to prove lucrative for the arbitrageur.

9. Index / Average of Components Arbitrage

As contracts that are based on an index are typically traded separately, the market price of such a contract is not necessarily equal to the average value of the components of the index. For instance, the price of an ETF (Exchange Traded Fund) may not always be equal to the value of the ETF’s holdings. Because in theory, both instruments receive the same cash flows, the LOP states that they should have the same price, and exploiting any inconsistency is an arbitrage strategy. Fleckenstein, Longstaff, and Lustig (2011) describe the CDX arbitrage strategy as the convergence trade between the CDX index and the average CDS spreads for the 125 firms included in it.

Disregarding the liquidity and counterparty issues that arise from trading contracts on the based on the CDX Index, this strategy is a textbook arbitrage, with the risk that it may not bear fruit in a very long time.

f. Summary

Out of the numerous so-called arbitrage strategies that I have mentioned, only a handful is what I described as textbook arbitrage (i.e. a strategy base on a price discrepancy of two financial products that will with certainty receive the same cash flows until maturity). Namely, CIP, Eurobond, and Index / component strategies are textbook arbitrages, so is convertible arbitrage if hedges are perfectly computed and dynamically rebalanced. Mortgage arbitrage is
close, but is partially based on prepayment models, and thus imperfect. Muni arbitrage is a
relative value strategy (betting on the relative performance of different bonds). CDS arbitrage
and yield curve arbitrage heavily rely on model pricing, making them directional bets. Swap
spread arbitrage and corporate bond / CDS spread basis arbitrage are passive spread strategies
(spreads exist because the securities traded have different characteristics). Lastly, volatility
arbitrage is barely the collection of insurance premia.
III) Risks and Returns of Fixed Income Arbitrage

a. Convergence Trading Risk Factors

i. Fundamental Risk

According to Gromb and Vayanos (2009), fundamental risk is the risk that the two assets in the arbitrated pair do not pay the same cash flows. Buraschi, Sener, and Mengüçük (2010) call this the cash flow risk. This risk is very limited in the fixed income sphere, as by definition, fixed income cash flows are very reliable.

ii. Credit Risk

Credit risk is the risk that an issuer defaults on its debt. This risk affects an arbitrageur in two ways. The first is straightforward: if default risk is not exactly the same on the two sides of a convergence trade, then there exists the risk that one event of default happen on one security, and not on the other, making the arbitrage fail. Another way to see that is that, if hedges are not perfectly covering the credit risk differential between the two assets of a pair, there is a chance that the strategy will not work.

Secondly, credit risk appears in the form of counterparty default risk. There are both direct and indirect counterparty default risk. Mancini and Ranaldo (2011) mention a good example of direct counterparty default risk in the case of unsecured CIP arbitrage. As seen previously, prop desks’ unsecured arbitrage uses unsecured loans (contrarily to hedge fund secured arbitrage, where repos and reverse repos are used to finance long and short positions, thus placing collateral at the center of each trade). For that reason, bank prop desks tend to roll over short-term (usually overnight) money market positions to place their trades. Since in the
CIP strategy described above, the arbitrageur has to borrow in one money and lend in another, the arbitrageur runs the risk that its borrowing counterparty defaults on its paper. This would force the arbitrageur into closing his trade early and potentially suffer large losses. Because money market positions have so small maturities, this risk is usually considered very reasonable.

Because arbitrageurs deal with a lot of financial intermediaries that are interlaced by borrowing and lending, one counterparty default provokes the risk that a chain reaction occurs and prevents the arbitrageur from running its operations. This indirect risk is also known as contagion.

Fleckenstein, Longstaff, and Lustig (2011) use the 10-year swap spread and CDX index as a proxy for credit risk in the market.

### iii. Market Risk, Systematic Risk

Duarte, Longstaff, and Yu (2006) argue that any excess returns generated by swap spread arbitrage are compensation for the large amount of market risk this strategy bears. This is in my view true for other strategies because it is overall very difficult to hedge for all aspects of market risks. Market risk can be seen as a combination of a number of factors.

#### 1. Interest Rate Risk

In fixed income arbitrage, the first factor is interest rate risk. This risk can be divided into two: reinvestment risk, and price risk. If interest rates move up (resp. down) after a fixed income instrument has been bought, the market price of that instrument will go down (resp. up) and the future value of the reinvestment of cash flows the instrument will generate will go up (resp. down). These two movements do not offset each other, unless the portfolio of fixed income securities is properly and dynamically duration hedged. Because it is very difficult and costly to
perfectly hedge for interest rate risk, all fixed income strategies bear some. Additionally, changes in interest rates trigger other movements that are not as easy to predict as price and reinvestment risks. For instance, MBS are sensitive to a large drop in interest rates triggering prepayments, and proprietary prepayment models do not have perfect predictive power. To prove that arbitrageurs are concerned about interest rate risk, Buraschi, Sener, and Mengütürk (2010) cite the valuation risk due to stochastic discount factor to be one of the risks affecting the willingness of an arbitrageur to commit capital to exploit an arbitrage opportunity.

2. Market Event Risk (Tail Risk)

The market event risk is the risk that some large, unpredicted disturbance happens and spreads through the financial markets. Typically, an important central bank announcement, a large bankruptcy, bad economic news, a large geopolitical event can all trigger market events. Because they are rare, these events are taken into account in the (usually left) tails of the returns distribution. Such events tend to spread through asset classes and geographies, which explains their categorization as systematic. As Duarte, Longstaff, and Yu (2006) put it: “Financial-event risk may be an important source of the commonality in returns across different types of securities.”

Although arbitrage strategies are often considered to be market neutral, and thus uncorrelated with most investment strategies, it is clear that their market neutrality fails during market events such as the Russian debt crisis of 1998 or the Lehman bankruptcy of 2008. Interestingly, Fung and Hsieh (2002) show that the tail risk estimates of fixed income arbitrage strategies fluctuate a lot, and are correlated with the Baa/Treasury credit spread.
iv. Noise Trader Risk, Market Irrationality, Idiosyncratic Risk

An asset’s idiosyncratic risk is the part of its volatility that is not explained by overall market movements. Typically, these movements depend on the asset’s intrinsic characteristics, and the opinions traders have on that asset’s future characteristics. An asset’s price fluctuations are unpredictable because they are the result of a large number of trade decisions, made by a heterogeneous group of investors. In the case of arbitrage, arbitrageurs know the theoretical relative value of two assets, but other traders (that I call noise traders) do not, and most probably trade the assets independently from each other, for their own intrinsic value, and not as a pair. When the arbitrageur places a convergence trade, he runs the risk that the existing mispricing widens as a result of noise traders’ arbitrary perseverance in the wrong direction (which could in fact be the right direction for one asset, while the other remains wrongly priced). This market irrationality is one of the main sources of risk for convergence traders.

Even more than they do systematic risk, arbitrageurs are subject to idiosyncratic risk. Indeed, as Shleifer and Vishny (1995) explain, idiosyncratic volatility cannot be hedged, and arbitrageurs are not in practice really diversified. This lack of diversification exposes them strongly to the risk that one or more pairs do not converge in a timely manner. Gromb and Vayanos (2009) call this market unpredictability the supply risk, and define it as the risk that “the demand for liquidity may not be predictable.” This interesting perspective makes sense if one considers financial markets as needing liquidity, and the arbitrageur as a liquidity provider, who facilitates trades between otherwise segmented investors.

v. Liquidity Risks

1. Market Liquidity and Supply
Market liquidity is the ease with which market participants can buy and sell securities with minimum price impact. Factors that affect market liquidity include the notional size of the market, the frequency and size of transactions on this market (the depth of the market), and the number of players that actively trade on this market. It is usually considered that the most liquid market is that of international currencies. For instance, a trader placing an electronic order to buy a significant amount (in the order of tens of millions) of EUR for USD will see his order filled very fast (in a matter of seconds), and will probably not move the needle on the exchange rate. On the contrary, most corporate bonds are much less liquid because they are fragmented by issuer, maturity, and other intrinsic characteristics, and because market participants are often buy-and-hold investor types, that do not transact very often.

Market liquidity is key to arbitrageurs, because they aim at getting out of their trades fast, and with as little price impact as possible, when prices have converged. As seen previously, the arbitrageur may be considered as a liquidity provider, as he buys lower-priced assets that market participants are neglecting, and sells higher-priced assets that are in high demand. The trouble is, when they exit their trade, arbitrageurs play the role of liquidity takers, and run the risk of making prices diverge back before they can take in the profits of their strategy. Additionally, when liquidity dries up in a given market, volatility tends go spike up as buyers and sellers do not find matching orders and move the market significantly by transacting in larger chunks to get out of their positions, making markets less efficient. This may have the effect of widening mispricings and triggering marked-to-market losses for the arbitrageur. Kondor (2009) puts this phenomenon this way: “the average hedge fund should have positive exposure to liquidity shocks, that is, a positive liquidity beta.” According to Gromb and Vayanos (2009), supply risk
can also create a spillover effect into different markets and can be a vector for contagion: “supply shocks to one opportunity affect all opportunities’ risk premia.”

Mancini and Ranaldo (2011) use transaction costs (bid-ask spreads, or LIBOR-OIS\textsuperscript{12} (overnight indexed swap) spread as a proxy) as an indicator for market liquidity. Fleckenstein, Longstaff, and Lustig (2011) use repo fails (i.e. the inability for the borrower to find a specific security to deliver to the lender at the end of the repo period) and supply (in their case, the amount of TIPS in the market) as market liquidity indicators.

2. Funding Liquidity Constraints

Funding liquidity is a measure of the ability and cost for a market player to find the financing it needs to run his operations. In the case of unsecured arbitrage, funding comes from money market investors, and thus depends mainly on the credit perception of the bank in the markets, and on the money market investors’ willingness and ability to provide the funds. In the case of secured arbitrage, funding comes from the arbitrageur’s prime brokers, who in turn seek financing from rehypothecation lenders. The hedge fund’s funding liquidity will be based on its past performance and historical volatility, on the type of products it is trading, on its prime broker relationship, and on the marked-to-market value of its positions.

Funding liquidity is critical to an arbitrageur’s operations. Indeed, because of the low volatility and expected return of arbitrage portfolios, arbitrage strategies cannot deliver satisfying returns without being highly leveraged. However, funding liquidity raise very important risks as

\textsuperscript{12} OIS is the rate for overnight unsecured lending between banks (the fed fund rate in the US). The LIBOR indicates the risk premium of unsecured interbank lending over a relatively longer period than the OIS (typically 3 months), so the LIBOR-OIS spread is a measure of the credit quality of the banking sector. It is an important measure of risk and liquidity in the money market.
well. Indeed, rehypothecation lenders and money market investors in particular, are very risk
averse and provide low-cost leverage only insofar as they have the right to seize the collateral in
case the market moves too far against the trade, and force the arbitrageur into liquidating its trade
portfolio. For this reason, the sudden drying up of funding liquidity is very dangerous for the
arbitrage business. Moreover, as I have shown previously, the forced liquidation of arbitrage
portfolios can easily lead to a reinforced negative effect on convergence trades, putting an
increasing number of arbitrage funds at risk. As Buraschi, Sener, and Mengütürk (2010) put it:
“the model highlights how sensitive Eurobond arbitrage relationships are to spillover effects
from large funding capital markets.”

For Mancini and Ranaldo (2011), funding liquidity constraints are the number one
explanation for deviations from arbitrage. The authors use the Libor-OIS spread as a general
measure of funding liquidity. I will now provide details on some reasons why funding liquidity
could dry up.

a. Redemption Risk

As I have shown previously, being able to cover the repo dealer’s haircut is essential for
the secured arbitrage strategy to be carried out properly. In periods of financial turbulence, hedge
fund investors tend to withdraw their capital from hedge fund firms, for fear of capital losses.
These redemptions can be very detrimental to the arbitrageur as it reduces his ability to answer
prime brokers’ margin calls (or increase in haircuts) by posting more collateral. If the
arbitrageur’s capital is withdrawn too fast, the arbitrageur has to close some of its positions
before convergence. The fact that investors know their capital is more at risk when other
investors withdraw money can create a panic effect and lead to the fund’s liquidation. A good
example of that is the LTCM debacle, when investors withdrew more aggressively once
Meriwether announced he was short on cash. As Mitchell, Pedersen, and Puivino (2007) explain it: “shocks to capital matter if arbitrageurs with losses face the prospect of investor redemptions, particularly when margin constraints tighten during liquidity crises.” This aspect is essential, and I think it would be very interesting to dig deeper into the investor-hedge fund relationship and their mutual interest in AUM stability.

It is important to note that repo dealers do not consider that all collateral are made equal, especially in periods of capital scarcity. Consequently, arbitrageurs in less liquid securities will have to post more additional collateral in such times than, for example, Treasury bond arbitrageurs. As a result, the proxy for the capital drain risk that Mancini and Ranaldo (2011) use is the spread between Agency MBS and General Collateral (GC) repo rates.

b. Slow Moving Capital Theory as Funding Limitation

As I have shown, funding limitations are linked to capital limitations by the haircut (no fund can receive infinite leverage). The risk that an arbitrageur may not have enough capital to pledge may come from large redemptions, or more simply from the fact that there is not enough investor capital deployed in the arbitrage business. Indeed, arbitrage returns are not only a factor of the leverage the arbitrageur can afford, but also of the total amount of capital that is put into work on arbitrage strategies. The reason for that is that, if the money inflow into one type of convergence trade is not enough to cover the money outflow (i.e. the noise trader's relative value actions), then the mispricing gap will not narrow soon enough. Thus, there is a scale impact of arbitrage capital.
This point relates to the slow-moving capital (SMC) theory, as exposed by Mitchell, Pedersen, and Pulvino (2007). After a large selloff by convertible arbitrage hedge funds in the first quarter of 2005, the authors documented major and persistent price deviations from fundamental value, suggesting that capital can be slow to get into arbitrage opportunities following major capital dislocations. This phenomenon can be explained by several reasons. First, investors do not share arbitrage hedge funds’ knowledge of the markets, and therefore cannot spot arbitrage opportunities in case of market inefficiency, but do fear volatility and marked-to-market losses. This is the point made by Mitchell and Pulvino (2009): “because of this uncertainly on all fronts, capital inflows to low-risk highly profitable arbitrage strategies were very slow, causing prices of substantially similar securities to be substantially different for a long time.” In addition, investors tend to move large amounts of capital at once, because they share the same performance and risk metrics. As Shleifer and Vishny (1995) explain: “Arbitrageurs all attract or lose investors simultaneously, depending on the performance of their common arbitrage strategy.” Second, it is not a good tradeoff to keep capital dormant while waiting for a large, free meal type, arbitrage opportunity. Third, as I have shown previously, capital constraints become binding in times of market illiquidity, which prevents arbitrageurs from providing liquidity to an already struggling market.

The idea that the amount of capital available to arbitrageurs plays a role in the evolution of mispricings is not new, and effectively, as Keynes (1929) remarks: “This abnormal [spot vs. forward FX] discount can only disappear when the high profit of arbitrage between spot and forward has drawn fresh capital into the arbitrage business.” Further, there exists proof of such existing relation in the literature. For instance, Mancini and Ranaldo (2011) test the SMC theory by regressing the net asset value of hedge funds to the CIP mispricing and find a strong negative
correlation, significant at the 5% level. Similarly, Fleckenstein, Longstaff, and Lustig (2011) find a strong correlation between the TIPS mispricing and hedge fund available capital, and show that the mispricing does narrow as additional hedge fund capital flows into the market.

c. Lender’s Pressure to Cut Funding

I have described several reasons why leverage providers would want to cut funding to arbitrageurs, the most immediate of which is when value of the arbitrageur portfolio falls close to that of the collateral he has placed, and the arbitrageurs has trouble finding more collateral to post – thereby putting at risk the lender’s capital.

There also are external factors (i.e. reasons that are not linked with the arbitrageur’s performance or solvability) that could force the lender to cut funding. For example, Mitchell and Pulvino (2011) show that the 2008 convertible price dislocation was mainly caused by the rehypothecation lenders cutting funding to prime brokers, because they only had as collateral illiquid, long-only portfolios that were rapidly losing value. Another example would be the pressure for banks to deleverage. This typically happens in times of crisis, and 2008 was a case in point. For several years after the financial event, banks were pressured (not only by regulators, but also by their financial counterparties, and their own shareholders and risk departments) to bring overall leverage down, thereby forcing them into issuing less commercial paper and scaling down their balance sheets and rehypothecation business. Mancini and Ranaldo (2011) use the balance sheet size of financial intermediary as an indicator for the pressure to deleverage.

In many instances, funding liquidity is not completely suppressed, but the cost of funding spikes up, provoking additional frictions for arbitrageurs, and preventing them to efficiently deploy arbitrage capital. Fleckenstein, Longstaff, and Lustig (2011) show that haircuts on repo
arrangements went up significantly during the financial crisis, and that, interestingly, reverse-repo arrangements were also difficult to access for hedge funds in the period. Notably, the authors document that in 2008, regular Treasury reverse repo counterparties demanded repo rates as low as 0%.

d. Lender’s Hoarding Liquidity Away to Address its Own Funding Strains

Banks usually fund a lot of their assets by shorter-term, unsecured liabilities such as commercial papers. When funding markets dry up, banks face large liquidity issues and may have a hard time matching their liability requirements. For this reason, they usually tend to sacrifice some of their lending activity profits in order to face immediate liquidity threats. Mancini and Ranaldo (2011) also make the point that signaling dynamics are tied to this issue. Effectively, banks did not want to be seen as short on liquidity, in order to keep their lenders’ sentiment positive. Additionally, posting liquidity was important to maintain their credit rating and low cost of borrowing. The authors use the excess reserves at the Federal Reserve as a proxy for bank liquidity.

3. Impact of Policy Measures During the Crisis

I have described above most of the interventions the Federal Reserve made during and after the crisis, in order to prevent a liquidity and credit crisis to cause the financial markets to shut down. In the perspective of arbitrage, it is interesting to see how these measures have helped restore the LOP after the abnormalities I have discussed.

Buraschi, Sener, and Mengütürk (2010) run an analysis on the efficiency of Federal Reserve intervention during the period. Their results suggest that the swap line programs with
the ECB and SNB were significantly efficient in unlocking funding conditions, and consequently in reducing the EM sovereign Eurobond mispricings across all maturities. This result is line with Mancini and Ranaldo’s (2011) that arbitrage profits lowered at the same time as central banks were ramping up their USD swaps with the Federal Reserve. The same is true for the stress tests implemented in mid-2009.

vi. Risk Aversion as a Risk Factor

The risk that arbitrageurs do not come in and provide liquidity in some periods is an interesting game theory problem. Because arbitrageurs know that they do not have sufficient capital available to bring prices back to their fundamental relative values, they need to count on other arbitrageurs spotting the same opportunities and leaning in to reduce the gap. Thus, despite the fundamental riskiness of arbitrage positions, there exists a risk that other arbitrageurs do not play their role, or even trade against another’s arbitrage positions. Shleifer and Vishny (1995) remark that arbitrageur risk aversion could in some cases “make them likely to liquidate rather than double up when prices are far away from fundamentals.” Buraschi, Sener, and Mengütürk call this the arbitrageur cost of capital. Indeed, in some market conditions, the risks just do not match an arbitrageur’s large expected returns. To support this argument, Shleifer and Vishny (1995) show that markets that exhibit a high long-run, but a low short-run ratio of expected excess return to volatility are not attractive to arbitrageurs, especially since an arbitrageur’s reputation and available capital is affected by short-term performance. This result shows that, although the arbitrage business relies on market fluctuations to create mispricings, higher volatility is actually detrimental to arbitrage activities.

Gromb and Vayanos (2009) show how reinforcing effects of risk aversion and volatile returns can cause arbitrageurs to rapidly scale down their positions in more risky opportunities.
As a consequence, the more risky arbitrage trades are the first to suffer from arbitrage risk aversion. The authors also introduce the notion of opportunity cost of collateral. Because collateral posting is usually the binding resource for arbitrageur capital allocation, opportunity cost of collateral is a good way to look at the risk / return based capital allocation of an arbitrageur across different strategies and different market environment.

vii. Regulatory Constraints

1. Short Selling Ban

In most countries, there are no laws against short selling (although have been at several occasions in the history of stock markets, notably after the 1610 Dutch market crash, after the 1733 South Sea bubble, and in the US since the War of 1812 to the 1950s). However, short selling has been made difficult during the crisis, notably because of naked short selling\textsuperscript{13} bans in and outside of the US. In 2008, the SEC banned abusive naked short selling, and took notably emergency actions to limit naked short selling on Fannie and Freddie, from mid-July to August 12. Several other international exchanges (such as Euronext Amsterdam, the Tokyo Stock Exchange, or SWX Swiss Exchange) have put restrictions on naked short selling. Even though these bans usually concern stock markets, and not fixed income securities, other bans may risk interfering with the arbitrage business.

2. Regulatory Leverage Constraints

As I have shown before, leverage is essential to the operations of a fixed income arbitrage desk. Indeed, convergence trades are well hedged and show very little volatility, so they bring small, but fairly predictable and stable returns – making them good candidates for leveraged strategies.

\textsuperscript{13} Naked short selling is the action of short selling an asset without borrowing it first.
Because they are virtually unregulated, hedge funds’ leverage is not controlled by any regulation. However, the leverage of their financiers is tightly monitored and regulated. Indeed, capital requirements on financial institutions are in place in most countries, and have gotten much stronger since the financial crisis. As an illustration, Basel III reform, which is in the process of being implemented around the world, ramped up common equity requirements to 4.5% from 2%, and Tier I capital requirements to 6% from 4% in the previous Basel II regulation.

Other regulations more specifically limit the amount of leverage that can be provided to arbitrage hedge funds. Specifically, Rule 15c3-2 of the 1934 Securities Exchange Act regulates the rehypothecation process previously discussed. This process is at the center of the hedge fund funding from their prime brokers, and consists in the transfer of the collateral and loan made to the hedge fund to rehypothecation lenders such as money market funds. The rule states that the prime broker can rehypothecate up to 140% of a customer’s loan balance. Since the rest of the leverage will have to be done on the prime broker’s balance sheet, it will have an impact of its capital used, and consequently will be limited.

3. Taxation

Taxation is a very common friction in the financial markets, and it could explain many inconsistencies. For example, Fleckenstein, Longstaff, and Lustig (2011) examine a small tax discrepancy between the revenues of TIPS and Treasury Bonds to explain the existing mispricing. However, they have to note that most funds that invest in these instruments are tax-free vehicles.

viii. Other factors
1. **Rollover risk**

In the case of unsecured arbitrage, the arbitrageur usually receives funding from very short-term money market positions. The risk that the money markets shut down overnight because of difficult market conditions translates into a funding liquidity shortage for the arbitrageur, who may then be forced into liquidating his positions. This risk also exists for secured arbitrage, notably because it is typically difficult to negotiate long-term reverse repos. The arbitrageur thus has to roll over shorter tenor repo contracts during the length of his trade, and runs the risk of having to deliver the shorted security before price convergence.

2. **Basis risk**

Basis risk is the risk that a hedge using futures is imperfect. This risk can be due to imperfect correlation between the asset price to be hedged, and the asset underlying the futures, or by a mismatch between the contract maturity and the selling date. In fixed arbitrage, the two reasons are fairly common; as it is very frequent to hedge for interest rate risk, credit risk, or FX risk by using futures or swaps. For instance, I have shown how muni arbitrage's interest rate risk is usually hedged via corporate swaps that are similar but imperfectly correlated to munis. Furthermore, arbitrageurs usually sell their portfolios as soon as the mispricing has been sufficiently corrected. This makes the time of the sale unknown ex-ante, and hence makes the hedge imperfect.

3. **Contract risk**

Contract risk is the risk that the buyer abrogates the contract, but not because they are unable to pay. This risk is present in all derivatives contracts used to hedge trade positions, and

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14 Swaps can be seen as a series of futures.
therefore very important in arbitrage strategies. Specifically, the risk that forward or swap counterparties defaults is an example of contract risk.

4. Availability of Derivative Products

In some cases, arbitrage strategies require the use of derivative products that are not easy to find on the financial markets. This lack of availability (or of liquidity) of some products may explain the existence of some consistent mispricings in the financial markets, as they either prevent arbitrageurs from hedging properly their trades, or create large transaction costs if they are forced into trading over-the-counter (OTC) derivatives, i.e. non-standardized, non-exchange traded derivative contracts that are negotiated specifically with a broker.

5. OTC Microstructure

In some cases, the way the markets are made in different securities may affect their price discovery process or their market liquidity. Hence, relatively similar securities that are not traded by the same brokers or are traded in a different manner may show signs of price discrepancy. In the bond market, a good example of that is the specific over-the-counter microstructure of the Treasury bond market. Indeed, the Federal Reserve maintains a list of primary government securities dealers. In order to be on the list, primary dealers must respect a number of conditions. For instance, primary brokers must meet a USD 150m net capital requirement, must participate to all open market operations, and to all Treasury bond auctions. The fact that this list exists may create a discrepancy between the on the run Treasury, and other fixed income instruments market microstructure. Fleckenstein, Longstaff, and Lustig (2011) explore this possibility with regards to the TIPS – Treasury bond mispricings, but find that OTC microstructure is very similar in that case.
When Risks Trigger One Another: the Risk of Contagion

As I have presented above, contagion happens when the failure of a number of arbitrageurs boosts up mispricings, leading to new arbitrageurs failures, and eventually provoking price inconsistencies in a vast number of previously unrelated markets. As an illustration of the role of arbitrageurs and funding markets in cross-market contagion, it is interesting to look at the correlation between normally segmented markets during financial crises. For instance, Mitchell and Pulvino (2009) state that “prior to the 2008 financial crisis, the correlation between the CDS–bond basis and convertible debenture cheapness is 0.02 using weekly data from January 4, 2005 through September 12, 2008. However, the correlation spikes to 0.91 during the period September 19, 2008–March 31, 2009, highlighting the crucial role that debt financing plays in arbitrage strategies that have little fundamental risk.”

The price effect of arbitrage on a specific mispricing varies as a function of the width of that gap. As the gap grows, more arbitrageurs spot the mispricing, and the mispricing’s expected return on collateral goes up, hence more capital is drawn into convergence trades, so the zero-reverting power of arbitrage becomes increasingly powerful. If the gap continues to widen, less and less arbitrageurs have capital to invest on the opportunity (they are fully invested, and have maxed out their available leverage), so the arbitrage force reaches a peak. When the mispricing keeps on widening, the first funds that got into the mispricing (those that face the largest marked-to-market losses), or the more leveraged arbitrageurs receive margin calls, and start liquidating their portfolios; in the meantime, the more risk-averse investors take their losses and get out of the trade. As a result, the zero-reverting force decreases gradually (because the levels of risk aversion and capital buffers of arbitrageurs are diverse), but increasingly rapidly as the self-
fueling phenomenon of liquidations spreads. The arbitrage force can thus reach negative levels, forcing the mispricing into widening even more as all funds abandon their positions. Shleifer and Vishny (1995) describe this phenomenon: “in the extreme situation where noise trader shocks deepen starting from an already bad situation, arbitrageurs end up reducing their demand for the underpriced asset, and the price falls even further than it would without this adverse shift in arbitrageurs’ demand.” Figure 5 shows an illustrative attempt to capture the reverting force of arbitrage trades as a function of the mispricing width (scales are arbitrary).

Figure 5: Arbitrage zero-reverting force as a function of mispricing width

This model of the arbitrage force in function of the size of the mispricing has four parameters and four variables (the mispricing gap being one). I called the magnitude of the curve (i.e. the y axis scale) the force parameter because it influences directly the force of reversion
exercised by arbitrageurs. The force is influenced by two variables: the amount of capital available to the arbitrage industry (or at least available to arbitrageurs active on the market represented) and the amount of leverage taken on by arbitrageurs (i.e. its funding liquidity). As shown previously, the capital variable is considered to vary slowly and gradually. The funding liquidity may on the contrary change rapidly according to market conditions. I called the second parameter the peak factor. It is in fact the scale of the x axis (so it changes the point of the mispricing gap variable at which the peak of the force arrives). This parameter is also driven by capital and leverage, but in this case, more capital available means the peak happens further away (i.e. there will still be more force available at a larger gap), and leverage has the inverse effect (the force will decrease more rapidly if the gap widens further). The third and fourth parameters are amplification and acceleration, they behave directly as a function of market liquidity, and their effect is to amplify the force effect as the gap widens (the bottom of the peak is lower than the peak is high), and accelerate the movement as the gap widens (the force grows increasingly rapidly, and fall even faster).

In normal times, it is realistic to assume the force peak is largely sufficient to bring mispricings back to or close to zero rapidly after any kind of noise trader action (if it were not the case, then, over the long term, more arbitrage capital would be available to the industry, and the mispricing would be contained on the left side of the peak). This assumption raises the question of why the mispricing can go well beyond this peak, so far as to reverse the force of arbitrage. As Shleifer and Vishny (1995) put it: “arbitrage [should be] stabilizing, and more stabilizing in extreme circumstances, [instead, it] becomes ineffective in extreme circumstances, when prices diverge far from fundamental values.” In the framework described, the reason for
any price discrepancy to get so out of hand is the simultaneous occurrence of a combination of risk factors that I have described earlier, which leads to a player contagion in the market.

Unfortunately, risk factors are often correlated and can trigger one another easily. For instance, when a large financial shock such as Lehman’s bankruptcy occurs, the risk aversion of the lenders spikes up, affecting both interest rates and funding liquidity. At the same time, government intervention requires the recapitalization of financial institutions, which forces them into deleveraging and keeping liquidity to themselves. Simultaneously, market volatility goes up, market liquidity dries up, and noise trader risk is amplified. All these factors triggering each other may lead to a few hedge fund implosions, leading investors to suddenly withdraw arbitrage capital, hence starting a contagion process.

x. Conclusion: Why Can Arbitrage Fail? Limits of Arbitrage Theory

As I have shown, there are many examples where arbitrage fails to enforce the LOP for an extended period of time following a particularly large perturbation. Literature around the limits of arbitrage is diverse. I have already described a number of factors that can become limits of arbitrage is some market conditions, but I will now focus on a model that specifically looks to confirm limits of arbitrage hypotheses.

Shleifer and Vishny (1995) developed an agency model of arbitrage in order to support the limits of arbitrage theory. The model relies on the assumption that (1) highly specialized knowledge about arbitrage opportunities and financial resources are separated (the arbitrageurs rely on external, relatively unknowledgeable investors for capital). From this follows that (2) resources and funding are limited, (3) funds increase and decrease according to past performance
of funds (the authors call this “Performance Based Arbitrage”, PBA), and (4) markets are segmented and only a few professionals arbitrage a specific market. In this agency context, the authors show that arbitrageurs may fail to bring prices back to their fundamental values after unspecialized investor sentiment (or noise trading) has pushed them far away.

The consequences of the authors’ model assumptions are that in extreme circumstances, arbitrageurs are invested at their maximum leverage level, that no new capital can be brought in a specific arbitrage market from another arbitrage market due to the market segmentation, and that new capital is allocated according to past performances of a fund, due to the inability of investors to assess future potential opportunities. Liu and Longstaff (2004) also bring on this idea: “capital is provided to the arbitrageur by investors on the basis of past return performance (rather than future investment opportunities). This creates an agency conflict for the arbitrageur who attempts to maximize the amount of funds under management.” Under these assumptions, bringing prices back to their fundamental values becomes impossible in the events when noise traders have pushed prices very far apart. Indeed, funding constraints prevent arbitrageurs from increasing the aggressiveness of their convergence trades when they are already fully invested at intermediate levels of mispricing. The weak stabilizing effect of arbitrage described in the model does not take into account the contagion factor. When one adds the necessity of some hedge funds to liquidate their positions as the mispricing widens, arbitrageurs may indeed have a destabilizing effect on the market.

b. Returns Characteristics

i. Optimal Trading Strategies in Existing Equilibrium Models
Existing arbitrage trading models are numerous. I will describe four of them that focus on specific aspects of convergence trading.

I have already described Shleifer and Vishny’s (1995) agency model of arbitrage. It tests the ability for arbitrageurs to enforce the LOP when prices diverge abnormally. The model’s main contributions are Performance Based Arbitrage (PBA), and noise trader exogenous shocks.

Liu and Longstaff (2004) replicate a world with textbook arbitrages, but take into account the collateral requirements of implementing trading strategies, and the potential for forced liquidation before convergence. They show that it is often optimal for arbitrageurs to first underinvest in an opportunity in order to put aside capital (i.e. making the collateral constraint not binding) for cases of larger divergence.

Kondor (2009) uses a competitive dynamic equilibrium model of textbook arbitrage without exogenous shocks, but with random convergence timing. The author shows that competition changes the optimal capital allocation of arbitrageurs in a way that induces endogenous shocks in mispricing (the arbitrageurs’ actions create pricing shocks that would not have appeared otherwise). The consequence is a speculative dimension of arbitrage strategies, where arbitrageurs are trying to time price convergence. Kondor explains very clearly what I called the risk-return conundrum of arbitrage strategies: “the equilibrium possibility of a widening gap implies that arbitrage trading creates its own opportunity cost. Investing a unit today will lead to capital losses exactly at those states when investing would be the most profitable.”

Gromb and Vayanos’ (2009) model focuses on stocks in segmented markets, but has general enough assumptions to be relevant in the case of fixed income arbitrage. The authors
introduce a dynamic general equilibrium model where arbitrageur's liquidity provider abilities (and consequently price convergence effects) are driven by their capital available, and where price gap evolutions drive arbitrage expected returns, and therefore capital available. This dynamic interaction dictates optimal investment policies, market liquidity and price patterns. The authors show that opportunities with higher collateral requirement are more illiquid, have higher risk premia and offer higher excess return. The authors’ version of the conundrum is the following: “on the one hand, [arbitrageurs] must allocate their scarce capital across opportunities and over time. On the other hand, the performance of these investments affects their investment capacity.”

ii. The Impact of Financing Method on Arbitrage Returns

As previously explained, banks prop desks and arbitrage hedge funds uses different modalities to finance their trades. Ergo, it is legitimate to ask the question of the impact of financing methods on arbitrage returns. Mancini and Ranaldo (2011) address this problem. They find that during the crisis, profits from CIP arbitrage were similar for secured and unsecured arbitrage, because the main cause for the widening mispricing was funding liquidity constraints that affected both Hedge Funds and Banks proprietary desks. Looking into this in more details, and focusing on other strategies could be interesting.

iii. The Effect of Competition on Arbitrage Returns and the Speculative Dimension of Arbitrage

As mentioned above, Kondor (2009) models in the role of competition in arbitrage. He finds that the competition not only reduces the expected level of the price difference, but also makes the difference longer in time (the half-life of the gap gets longer). Indeed, the expected size of the future price difference diminishes at an increasingly slow rate as more arbitrageurs
enter the market. As the level of capital approaches its theoretical maximum level, the price difference approaches a martingale process and the half-life increases without bound. Intuitively, the assumption is that competition must have a negative effect on profits (or else, this would mean that there is an infinite amount of available profits in this market). Since arbitrageurs have the capacity to compensate decreasing levels of mispricings (and proportionally decreasing levels of risk, or noise volatility, and thus of collateral requirements) by increasing their leverage, assuming that the probability that the gap increases remains stable at equilibrium would mean that there is indeed no profit losses due to competition. As a consequence, the initial assumption forces the conclusion that in equilibrium, the probability of the gap widening (and the half-life of the gap) must increase as competition increases, hence tightening collateral constraints and reducing the expected return of the strategy. This process leads to consider arbitrage as a speculative bet (on the potential widening of the gap) where the risk-adjusted returns decrease as competition increases. In other words, competition must increase the risk in the market, and higher risk must cause higher required returns, which in turn require a higher price gap. By taking into account the attractiveness of future opportunities, the convergence trader times the gap variations in a speculative manner.

In the author’s words: “the main idea of the model is that if hedge funds with limited capital have to decide when to provide liquidity, in equilibrium the expected payoff of the arbitrage trade has to be the same at every point in time. This payoff might always be zero, in which case there is no arbitrage opportunity in the market, or it can be positive, in which case price divergence must be possible at every point in time, even though the aggregate capital constraint does not bind. This implies that arbitrageurs who follow their individually optimal strategies create losses endogenously even in the absence of any shocks. Their competition
reduces the predictability of relative price movements, transforming the arbitrage opportunity into a speculative bet.”

iv. **Skewness of Arbitrage Returns: Nickels in Front of a Steamroller?**

As noted by Duarte, Longstaff, and Yu (2006), the fact that arbitrage usually earns small positive returns, but sometimes suffers very large losses suggest that fixed income arbitrage strategies are not truly arbitrage, but are rather a form of tail-event insurance premium collection, or in other words, “picking up nickels in front of a steamroller”.

The literature widely agrees with the statement that fixed income arbitrage’s returns are merely the risk premia to carry important tail financial risk. For instance, Kondor’s (2009) model predicts a left-skewed distribution of returns. Alternatively, Liu and Longstaff (2004) find that “the highest returns occur along paths where there is a steady flow of small arbitrages that converge rapidly and where large widenings in the value of [the price gap] do not occur.”

However, Duarte, Longstaff, and Yu’s empirical study (2006) finds that the returns of arbitrage strategies (including swap spread arbitrage, yield curve arbitrage, mortgage arbitrage, volatility arbitrage, and capital structure arbitrage) have a positive skewness, suggesting the contrary of the conventional view. The only exception to that rule is volatility arbitrage, which prove to show negative return skewness, confirming that it is indeed an insurance strategy. The authors’ data set covers the 1989-2004 period. This period has witnessed important fixed income market perturbations (notably the 1997 Asian financial crisis and the 1998 Russian debt crisis followed by the meltdown of LTCM), but none of magnitude of the 2008 financial crisis. Thus, it would be interesting to run similar tests including data from the recent years.
v. The Key to Excess Return: Executing Complicated Strategies

All fixed income arbitrage strategies do not exhibit the same return characteristics. According to Duarte, Longstaff, and Yu (2006), out of the five strategies tested, the only strategies to generate positive excess returns after adjusting for market risks and accounting for transaction costs and fees are yield curve Arbitrage, mortgage arbitrage, and capital structure arbitrage. The authors note that these are the strategies that require the most "intellectual capital" to implement. Indeed, swap spread and volatility arbitrage are easily automated and do not hinge on any sophisticated models. On the other hand, yield curve Arbitrage, mortgage arbitrage, and capital structure arbitrage all require good models and intelligent implementation. If it is assumed that because these strategies are complicated, they can only be implemented by a smaller number of specialized arbitrageurs, then these results are in line with the theory that competition lowers risk-adjusted excess returns. Indeed, de facto lower levels of competition allow complicated strategies to bring true positive excess returns.

vi. Are Arbitrage Strategies Market Neutral?

Because they are long-short portfolios, arbitrage trades are often considered market neutral. In the case of fixed income strategies, although many other hedges can be implemented, market neutrality usually means that the duration of the short position offsets that of the long position. It is important to note that a duration (or even a duration and convexity) hedge is imperfect. Indeed, if not rebalanced dynamically, the hedge is not efficient. Moreover, when a large interest rate movement happens, the hedge is usually approximate. To finish, a duration hedge does not capture non-parallel shifts in the yield curve.
Despite their imperfect hedges, arbitrage strategies are in practice relatively uncorrelated to the markets. As an illustration, Fung and Hsieh (2002) show that the HFR fixed income arbitrage peer group average has low correlation with bond indices. However, they find that this group has non-directional exposure to interest rates and to spreads, indicating that arbitrage strategies do suffer from large swings in interest rate level and bond pricing.

Patton (2009) defines market neutrality as a combination of factors including correlation neutrality and tail neutrality. Tail neutrality is defined as the fact that the probability of extreme events should be unaffected by the market return. What Fung and Hsieh’s (2002) results, and many empirical evidence provided above suggest, is that – although usually correlation neutral – arbitrage strategies are not purely market neutral, in part because they do not exhibit tail neutrality characteristics.

vii. Diversification Characteristics and Portfolio Management

Although not perfectly market neutral, arbitrage strategies are relatively uncorrelated to other markets (in our case, essentially the bond market). It is hence reasonable to ask the question of whether holding exposure to arbitrage strategies is a good diversification tool in an investor’s portfolio. Their positive tail dependence is arbitrage strategies’ main shortcoming as a diversification tool. Indeed, as Patton suggests, risk-averse investors should prefer tail neutrality because tail dependence induces higher odds of a joint crash, and therefore increases the probability of a large negative return on a portfolio. Positive lower tail dependence will hence generally lead to a fatter left tail of the portfolio return distribution.
Arbitrage strategies provide exposure to a number of alternative risk factors that traditional fixed income investors cannot have access to. Fung and Hsieh (2002) introduce Asset Based Style (ABS) factors to assess the diversification characteristics of several fixed income hedge fund investment strategies. Asset Based Style factors are benchmark returns using observed asset prices. The returns are chosen in order to match a hedge fund style (i.e. a location and strategy, as defined earlier). For instance, an ABS factor for passive spread strategies would be the difference between two bond index returns (e.g. mortgage versus Treasury). For convergence trading strategies, the Asset Based Style factor is the payoff of a short position in a lookback straddle. Indeed, it can be shown that “the payout of the lookback straddle on an asset is the maximum any trend-following strategy can achieve.” A Lookback call (resp. put) option allows the buyer of the option to buy (resp. sell) the underlying security at its lowest (resp. highest) price over an agreed upon period (until the maturity date). Buying a straddle means buying a call and a put option with the same strike price. The further away from the strike price (in any direction) the final price of the security, the higher the payoff of this strategy. Intuitively, it can be seen that the payoff of buying such a lookback straddle is indeed a replication of the best possible outcome of a trend following strategy, i.e. that of a trader who chooses correctly to buy (resp. sell) the asset at the beginning of the period, and decides to sell (resp. buy) it back at exactly the best time to maximize his returns. Similarly, convergence trading strategies (that bet on a shortening of a mispricing, i.e. the reversing of the divergent spread trend) can be modeled as a short position in a lookback straddle on a spread (for instance, the Baa/Treasury yield spread).

Using ABS factors, the authors find that most fixed income hedge funds (including arbitrage funds) have considerable exposure to a large increase in credit spreads. They also
suggest that, because of the cross-market contagion effect, diversifying one’s exposure to apparently different fixed income hedge fund strategies is hardly efficient in mitigating the tail exposure of credit risk. Thus, arbitrage strategies offer interesting exposure to alternative risk factors (namely, exposure similar to that of a short lookback straddle), which most fixed income investors cannot get exposure, but seem to fail in providing tail risk diversification. This result confirms the intuition that arbitrage fails at the same time as large shocks affect the overall markets.

c. Summary

Arbitrage strategies have collateral constraints, and thus require capital. The economics of arbitrage are thus the same as all investment strategies, and lie in the trade-off between putting capital at risk and receiving on average excess return to compensate for that risk. Because there is no guarantee that an arbitrage trade will not lose money for a significant period of time before convergence happens, there is a game theory aspect to arbitrage strategies, where arbitrageurs try to time the right moment to get into a trade, knowing their competitors do the same. This speculative factor introduces the possibility that arbitrageurs create liquidity shortage without the effect of an external shock. In fact arbitrageurs following an optimal investment strategy almost invariably face losses early on. As Liu and Longstaff (2004) put it: “during the early stages of the arbitrage strategy, its returns may be observationally indistinguishable from those of a severely distressed conventional portfolio.” The authors add that according to their model, the probability that the strategy underperforms the risk-free asset at some point during the investment horizon is close to 100%.
Most importantly, a fund’s performance is dependent on its investor’s level of confidence. Because withdrawing funds at a time of distress can be so detrimental to the performance, it is essential for arbitrageurs to ensure their investors understand the risks associated with their strategies. According to Shleifer and Vishny (1995), this explains why established funds earn higher returns in the long run.
IV) Simulation of Arbitrage Strategies in Varying Economic Conditions

a. Objectives of the Simulation

i. Develop a Flexible, Realistic Model of Fixed Income Arbitrage Funds and their Ecosystem

Now that I have described the risk and return characteristics of fixed-income arbitrage strategies, and that I have shown in what way the literature has analyzed the financial crisis to pinpoint which risks were most critical for arbitrageurs in stressed environments, it would be interesting to try and replicate an arbitrageur’s returns in changing economic conditions, as well as the effect of other stakeholders’ behavior on the fund’s performance. The other stakeholders I will focus on are the other arbitrage funds or desks (the competition, whose effect on the theoretical mispricing will be essential), the arbitrageur’s leverage providers (whose margin call rules affect the fund’s ability to play its part), and the fund’s investors (who are able to withdraw their capital and force the fund into taking losses during stressed times).

My objective is to add enough factors to make the model realistic, but to keep it simple enough for the results to make sense in relation to the observations I have made in previous parts. For this reason, I will focus more on stakeholder relationships or behavior, and less on the vast array of risk factors I have previously discussed.

ii. Test Common Knowledge on the Risks and Returns Characteristics of Arbitrage Funds in Varying Environments

After developing this model, I will run a number of Monte Carlo simulation to test its robustness in regards to previous observations and to try to understand better the nature of the
relationship between arbitrage returns, risks, and stakeholder behavior. My model will allow me to test the limits of arbitrage theory and the resistance of a simple strategy to large swings in factors such as volatility and liquidity.

b. Presentation of the Model

i. Summary of the Model

Below is a summary of principal notations. A list of all notations can be found in Appendix 3.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_t$</td>
<td>Zero-reverting process</td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>Volatility process</td>
</tr>
<tr>
<td>$n_t$</td>
<td>Noise effects</td>
</tr>
<tr>
<td>$L_t$</td>
<td>Liquidity shocks</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Capital available to the arbitrage industry</td>
</tr>
<tr>
<td>$X_t$</td>
<td>Leverage available to the arbitrage industry</td>
</tr>
<tr>
<td>$C_t$</td>
<td>Value of countershock</td>
</tr>
<tr>
<td>$M_t$</td>
<td>Observable (final) mispricing affecter reverting effect</td>
</tr>
<tr>
<td>$b^1$</td>
<td>Arbitrageur buy-in parameter</td>
</tr>
<tr>
<td>$b^2$</td>
<td>Arbitrageur buy-out parameter</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Arbitrageur effective leverage</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Arbitrageur authorized leverage</td>
</tr>
<tr>
<td>$R_t$</td>
<td>Redemption size</td>
</tr>
</tbody>
</table>

My model contains 3 key steps to build the arbitrageur return, namely: modeling of the observable mispricing $M_t$ (key inputs: noise trader effect, market liquidity, reversion force), arbitrageur strategy, and financing / capital constraints (margin calls and redemptions).

1. Modeling of the Observable Mispricing $M_t$

The observable mispricing can be seen as being the sum of two processes: a zero-reverting process introduced by the arbitrage industry $\rho_t$, and a random volatility process $\beta_t$, introduced by noise traders and reinforced by random shocks in market liquidity.
\( (0) \forall t \in [1, T], M_t = \rho_t * M_{t-1} + \beta_t \)

Function \( \rho_t \) is the zero-reverting effect of the arbitrage industry's trades. It remains moderately smaller than 1 in most situations, but can become very small when the arbitrage industry invests large amounts of capital into correcting the mispricings, or can become larger than 1 when the arbitrage industry has to unwind positions following large losses and margin calls. Function \( \rho_t \) is a function of the mispricing size \( M_{t-1} \) and the arbitrage industry available capital \( K_t \) and leverage \( X_t \). \( K_t \) depends on the arbitrageur past performance and \( X_t \) depends on market volatility.

The volatility process \( \beta_t \) is a function of the noise trader effect \( n_t \), amplified by the market liquidity \( L_t \), and sometimes corrected by a countershock \( C_t \). The noise trader effect \( n_t \) is a random push of the mispricing in any direction. The market liquidity \( L_t \) determines the multiplier effect of the noise trader push. Liquidity shocks happen randomly and are persistent (with random duration). The countershock \( C_t \) has a probability to bring the mispricing back to a reasonable value when \( M_t \) is stuck at abnormally high values (i.e., when \( \rho > 1 \)).

2. Arbitrageur Strategy

The arbitrageur strategy is to buy and sell the same dollar value of both assets when the absolute value of the mispricing \( M_t \) is above a threshold \( b^1 \) and to liquidate his portfolio when \( \text{abs}(M_t) \) falls below the threshold \( b^2 \).

3. Financing and Capital Constraints

The financial constraints are margin calls and fund investor redemptions. Margin calls happen when the arbitrageur's authorized leverage \( A_t \) exceeds his effective leverage (calculated as the ratio of asset value to wealth index) by more than \( g \). Margin calls force the arbitrageur to
liquidate enough of his portfolio for his effective leverage to go back to his authorized leverage level. Redemptions of a size $R_t$ happen when the arbitrageur’s $z^3$-period losses exceed a certain level. Redemption boost up the arbitrageur’s effective leverage, potentially resulting in forced deleveraging.

ii. Elements of the Model

1. Simulation of Stochastic Noise Trader Mispricing

Because arbitrageurs typically bet on the mispricing width movements, it is simpler, for the purpose of this experiment, to model directly noise trader price effects on the mispricing, rather than on two correlated assets. Although it is possible to model correlated asset price paths, calculate the mispricing from these two time series, and figure out how much of the two assets to buy and sell, modeling only one time series simplifies the long/short ratio calculations. My assumption is that until the end of the simulation period $T$, noise traders generate random positive and negative pushes on the mispricing. The pushes, or noise effects ($n_t$) follow a normal distribution with mean of 0 and a standard deviation that represents the first parameter of the model. I call this parameter $\sigma$.

\[
(1) \forall t \in [1, T], n_t \sim \mathcal{N}(0, \sigma)
\]

2. Asset Price Relationship and Asset Volatility Calculation

Once the final mispricing $M_t$ is calculated\(^{15}\), I make the assumption that the average of the two underlying assets prices is always equal to a constant $i$. This assumption simplifies the

\(^{15}\) See section V)b)i)4)
calculations vastly because it allows me not to generate random price level of asset prices and focus on the relative value dimension. This assumption is largely realistic on small periods of simulation because the fixed income asset prices are very correlated to the level of interest rates and these tend to change progressively. It is however unrealistic over longer periods, but I assume that when price levels fluctuate, mispricings evolve in proportion (i.e., mispricings as a percentage of the assets' prices will not change on average). If that is true, there is no point in making average price levels dynamic.

Figure 6 plots an example of the prices of a pair of assets linked by the relationship I have just described. Here, i was set to the value of 1,500 and constitutes the symmetry axis of the time series.

![Figure 6: Asset prices time series based on mispricing generation](image)
From the time-series of asset prices, I can compute the asset price volatility $Z_t$. I use an exponentially weighted moving average (EWMA) to do so in order to be in line with industry practice when I want to assess the current volatility environment. I use a decay factor $\lambda$.

3. Simulation of Stochastic Liquidity Shocks and Effect on Noise Mispricing

As shown before, crises often create large and volatile mispricings for extended periods of time. This mispricing widening is the most dangerous risk in the arbitrage business. In order to model in this risk, I generate random liquidity shocks $L_t$ that boost the price effect of noise traders. In this model, I call liquidity the multiplier of noise trader effect. This definition is in accordance with the traditional description of liquidity as the price effect of trading. The Simulation of liquidity shocks depends on 4 parameters. The first one is the probability of a shock happening, $h$. The second one is the probability of the shock continuing on to the next period, $c$. The third on is the average size (i.e. amplitude) of the liquidity shock, $s$; and the last parameter is the liquidity shock size standard deviation, $d$. Equation (2) present the value of $L_t$.

\[
(2) \forall t \in [1, T] \cap [\text{Shock event}], L_t = l
\]

\[
\forall t \in [1, T] \cap [\text{Shock event}], L_t = \max (1, l_t), \text{ where } l_t \sim \mathcal{N} (s, d)
\]

Hence, the noise mispricing at time $t$ is the sum of all random pushes from time $1$ to $t$, multiplied by the liquidity factor. This series may take relatively large positive or negative values over the life of the simulation. The noise mispricing is not the variable that is observable to the arbitrageur of the model, because it will first go through the price effect of all other arbitrageurs active on that segment.
4. Arbitrage Industry Zero-Reversion Force Modeling

My assumption is that other specialized investors are active on this hypothetical market, and that their convergence trades have a zero-reverting price effect on the noise mispricing. The process by which this group of competitors progressively enters into convergence trades and progressively get out of them when the mispricing width decreases or when funding or capital constraints become binding was described in details in section III(a)ix). In order to model the reverting force of arbitrage, I used 4 parameters and 3 variables. It is a simplification of the model described previously, where acceleration and amplification are no longer driven by market liquidity but are considered constant (for purpose of efficient calibration). The four parameters are the power of the reversion effect $f$, the peak of the reversion $p$, the acceleration factor $a$, and the amplification factor $m$. The first variable is the size of the mispricing in the previous period $M_{t-1}$, the second one is the capital available to the arbitrage industry $K_t$, and the last one is the leverage available to the industry $X_t$.

I make the assumption that $K_t$ is derived from the past performance of arbitrage funds present on this market. This assumption is in line with observations and findings in the existing literature. As a proxy for past performance of the funds, I use the (lagged by period $A$) past performance of the fund I am modeling (its wealth index $W_t$). This assumption makes sense as most arbitrageurs present on one market will find and execute very similar trades and their respective performance will thus be correlated. Here, I am introducing a feedback loop in the model, as capital available to the industry has an effect on the mispricing that has an effect on the fund’s strategy and performance, which in turn influences the capital available to the industry. This loop is not problematic to the model because of its lagged effect and is realistic insofar as it
corresponds to the way performance and capital are linked in reality. In order to simulate the smoothening effect of all funds having different investor capital allocation time frames, I use a moving average of $K_t$ calculation based on $z$ periods. For $t$ values smaller than $\Delta$, $K_t$ is equal to 100%.

\[ (5) \forall \Delta > z, \forall t \in [\Delta, T], K_t = \max(80\%, \min(120\%, \text{average}(K_{r:t}, W_{r:t}/W_{s:t}))) \]

I make the assumption that $X_t$ is driven by the observable asset volatility $Z_t$. This assumption makes sense if I consider that lenders tend to update their margin call rules in relation with a value-at-risk calculation based on price volatility in the markets. I bounded both variables between 80% and 120% in order for the reversion force calibration to remain efficient. For similar reasons as previously, I also use a moving average of $z'$ periods. I only start calculation after a $\Delta'$ period for the EWMA calculation to make statistical sense. The leverage variable has two parameters, $l^1$ and $l^2$. Before $\Delta'$, $X_t$ is equal to 100%. $l^1$ must be calibrated so that is equals approximately the average value of the EWMA asset price volatility (for the leverage index to be 100% on the long-term average), and $l^2$ represents the sensitivity of leverage adaptation to values of EWMA that vary from their long-term average.

\[ (6) \forall \Delta' > z', \forall t \in [\Delta', T], \]

\[ X_t = \max(80\%, \min(120\%, \text{average}(K_{r:t}, l^1 + \text{sign}(l^1/Z_{r:t}) - l^1)^{l^2})) \]

Now that I have defined these two variables, equation (7) shows how they influence the arbitrage industry reversion force on the noise mispricing. As seen in section III)a)ix), such a model make sense according to the funding and capital binding limitations arbitrage funds suffer from.
(7) \( \forall t \in [1, T], V_t = f \times K \times X_t \times \sin(\text{abs}(M_t)^a) \times \text{abs}(M_t)^m \times (p \times K_t / X_t) \)

The sinusoidal aspect of the model makes it essential to calibrate properly all parameters (except from \( f \) that does not influence the shape of the reversion force) to fit the mispricing volatility produced by the noise traders. Indeed, a miscalibrated formula could produce aberrant reverting force patterns. It is also important to note that for very large mispricings, the shape of the force prevents the mispricing from going back to normal levels (indeed, the force is alternatively very negative, pushing the mispricing away from zero, and extremely positive (as the sinusoidal pattern come back up faster and higher for larger mispricing values). In a way, the mispricing value becomes trapped in the lower right valley of the force plot. To illustrate this effect, figures 7 and 8 show the force plot in function of the mispricing value and a scenario where a liquidity shock pushes the mispricing too far away from zero, leaving it in the valley for the rest of the period. Note the peak set at a mispricing value of c.125 and the valley in the 250-200 values. Also note the constant opposing forces effect on the mispricing when it has reached large values. These forces level are typically drawn from the two sides of the valley. This constant come-and-go of the mispricing is not in line with empirical evidence, but it has no effect on the arbitrageur strategy and returns because it is so far from normal trading values and the leverage and capital effect of the widening mispricing has already been taken into account when it has reached its peak.
Figure 7: Arbitrage zero-reverting force as a function of mispricing width

Figure 8: Time series of mispricing value in ample shock and no countershock scenario
This effect corresponds to the reality of the situation when arbitrage fails. Indeed, when cross-player contagion happens and capital flows out of arbitrage strategies, mispricings remain high for extended periods of time and remain very volatile until fresh capital is drawn to the opportunity hence created. I will factor in this new capital as a liquidity countershock $C_t$ (happening with probability $q$ when the mispricing exceeds the negative reversion threshold) that brings the mispricing back to a more reasonable value $\bar{M}$ ($\bar{M}$ is on the left side of the point when the sinusoidal force hits the value of zero). From this point on, the arbitrage industry goes back to work until the next big perturbation. In the meantime, if the arbitrageur has reasonable leverage levels and friendly investors, he may not have lost all capital and can take the large upside generated by the countershock. When the mispricing goes beyond the critical value where the force becomes negative, the countershock has a chance of being set off on every period. The higher the chance of the countershock happening, the smaller the average atypically high mispricing period. For illustration purposes, figure 9 plots the mispricing width evolution in a scenario where a large shock happens around day 1,800, and the countershock play its part on around day 2,200. The arbitrageur is forced to deleverage at time 1,800 and has to take most of its losses.

\begin{align*}
(3) \forall t \in [1, T] \cap [\text{Countershock event}], C_t &= 0 \\
\forall t \in [1, T] \cap [\text{Countershock event}], C_t &= \bar{M} - M_{t-1} \\
(4) \forall t \in [1, T], \beta_t &= L_t \ast n_t + C_t
\end{align*}
5. Actual Observable Mispricing Value

The mispricing value on which the modeled arbitrageur will base its trading strategy $M_t$ is the sum of the past noise effects (affected by liquidity shocks and countershocks) plus the sum of all past zero-reverting pushes from the arbitrage industry. The values taken by this variable are usually much closer to zero than a theoretical pure noise mispricing. Figure 8 and 9 were examples of this mispricing variable in a particular conditions. Figure 10 and 11 plot the noise mispricing and corresponding observable mispricing time-series under a no shock assumption.

\[ (8) \forall t \in [1,T], \rho_t = 1 - V_t \]

\[ (0) \forall t \in [1,T], M_t = \rho_t \cdot M_{t-1} + \beta_t \]

Figure 9: Time series of mispricing value in ample shock with countershock scenario
Figure 10: Time series of noise mispricing under no-shock assumption

Figure 11: Corresponding time series of observable mispricing, same scale
It is easy to see here that the assumption that at least some players of this market notice wide mispricings and try to correct them is essential to the model’s efficient working. If the arbitrageur based its strategy only on a random variable, he would in this case enter into 3 trades in the observation period (a ten-year period), and only close two of them, making its returns unsustainably low. This reverting effect is thus essential, and it is also in line with empirical observations on fixed income markets that it is very rare that a relatively large mispricing remains for an extended period of time.

6. Arbitrageur Authorized Leverage Calculation

The arbitrageur’s authorized leverage \( A_t \) is the variable I will use to compute how much of each asset the arbitrageur will buy and sell at the beginning of each of his trades. It is also the variable that will trigger margin calls in case the arbitrageur’s effective leverage goes too far above it. It is different from the arbitrage industry leverage is three ways. First, as it does not influence the arbitrageur force, there is no limit to its adequate level. In this sense, it can take values far more extreme than the industry’s, which I forced into staying between 80% and 120%. To be clear, these percentages are only an index and have no economic reality (they are not 1.8 or 2.2 times leverage values). Moreover, it is not only possible, but also interesting to allow the arbitrageur leverage to vary vastly to test the limits of the model and its relationship with empirical data. The second difference is that because I will assume that the fund only has one leverage provider (or a consortium of financiers following the same lending algorithm), the authorized leverage does not move progressively, but is brutally reset every given (\( z^{"} \)) periods. When authorized leverage is reset, the leverage provider can chose to ask for more collateral but will never provide more funding. Indeed, it does not seem realistic to assume that a hedge fund would try to releverage its trades before its closes them. Thirdly, this leverage figure has
economic meaning; it is indeed the inverse of the haircut the arbitrageur will have to post in its repo position. For instance, for a leverage of 5 times, if the arbitrageur’s wealth is $100, the arbitrageur will be able to buy $500 worth of asset 1, and finance it by repo ($500 worth of collateral will give him $400 in cash, enough to allow him to buy $500 worth of assets in the first place). It is interesting to see that the arbitrageur can in theory short as much of asset 2 as he likes, as there is usually no margin requirement in a reverse repo (since the reverse repo is similar to lending money). The trader will typically short approximately as much as he has bought, meaning that his total asset to capital leverage should be twice the figure described above. On the other hand, the arbitrageur will probably receive lower interest on its reverse repo than he has to pay on its repo, but this hasn’t been modeled in. Similarly to Industry leverage, there is an initial no-reset period $\Delta''$ at the beginning of the simulation period, to allow asset volatility EWMA calculation to become relevant. The variable has two parameters $a^1$ and $a^2$, which represent the average level and movement sensitivity of the leverage to asset volatility.

\[
(9) \forall t \in [\Delta'', T], \forall t = Mod(z''), A_t = a^1 / (z_t^a^2)
\]

7. Arbitrageur Strategy, Transaction Costs, Capital, P&L, Wealth Index, and Effective Leverage

The arbitrageur strategy is rather simple and depends on 2 parameters $b^1$ and $b^2$. These are effectively the buy-in and buy-out thresholds on the observable mispricing value. If for example $b^1$ is set at 20 and $b^2$ at 2, the arbitrageur would enter into a convergence trade when the observable mispricing goes below -$20 (resp. above $20) and would liquidate his position as soon as the mispricing goes back up (resp. back down) to -$2 or more (resp. $2 or less). The buy-out parameter could in theory be equal to zero or have negative values if the arbitrageur expects the mispricing value to swing around zero and the arbitrage industry to overshoot its reverting
effect, however I believe it is more realistic to model a risk-averse arbitrageur who gets out of his trade as soon as the mispricing is sufficiently close to zero, and before he expects the rest of the industry will unwind their convergence trades.

The arbitrageur will choose how much of both assets to buy and sell according to his authorized leverage (i.e. depending on past asset price volatility) and his available capital at the time of the trade. I made the assumption that the arbitrageur always has the same notional value of long and short positions (i.e. he will buy a little more of the cheaper asset than he shorts the more expensive one). This assumption was made in order to keep calculations simple but is rather realistic in the sense that (all else being equal) higher priced assets have a higher duration, making a duration-hedged portfolio slightly overweight on the cheaper asset. Every time the arbitrageur makes a transaction, he will pay transaction costs that represent a percent of the value of both assets adjusted by the liquidity variable value at the time of the transaction. The link between the liquidity variable and the transaction cost multiplier is linear. The capital available to the arbitrageur is the value of its portfolio when he last got out of a trade minus the transaction costs he has incurred ever since. The P&L of the arbitrageur is the marked to market gains and losses he is making on his portfolio in the present, and the arbitrageur’s wealth index \( W_t \) is the sum of its capital and P&L. Note that both the capital and wealth index variables are indices and do not correspond to dollar values. When redemptions occur, the arbitrageur has less capital in reality, but the model only shows the relative performance of the fund. In other words, comparing the wealth index at the beginning and at the end of the simulation period provides the returns of an investor who would have invested an amount of dollars in the fund and never withdrawn it. The actual size of the fund does not matter, as its trades do not have a price effect on the assets. The arbitrageur’s effective leverage is the market value of its portfolio divided by
two times its wealth index (to normalize for the double-leverage effect of the repo and reverse repo).

8. Margin Calls

Margin calls occur when the arbitrageur’s effective leverage surpasses the authorized leverage by more than a determined level g. g is expressed as a percent of the authorized leverage. Hence, higher asset price volatility and lower portfolio valuations (i.e. a widening mispricing) lead to a larger difference between the authorized leverage and the effective leverage and may cause the leverage provider to ask for more collateral. In the event of a margin call, the arbitrageur has to sell the appropriate amount of his portfolio to reset his effective leverage to the authorized leverage level. If the arbitrageur was indeed suffering from marked-to-market losses, he has to take the losses on the portion of the portfolio that he has to liquidate. Sometimes, the arbitrageur may actually take a profit by deleveraging if a peak in asset price volatility has decreased the authorized leverage (as opposed to effective leverage going up) and has caused the margin call.

9. Redemptions

Redemptions \( R_i \) are the last factor that will influence the arbitrageur trades and returns. Redemption values are calculated using three parameters \( z^3, r^1 \), and \( r^2 \), that respectively correspond to the marked-to-market losses calculation rolling period length, the level of first redemptions occurring, and the speed at which redemptions increase with losses. By default, first redemptions happen at a 5% loss, and increases in redemptions are calculated every 5% loss steps (loss steps are designated by \( S_i \)); however, a negative \( r^1 \) does not lead to negative withdrawals but instead increases the range of no-redemption (any negative withdrawal is replaced by a withdrawal of 0%). As an example, if the parameters are set to 5, -20%, and 2, then
no redemptions will happen until the 5-day rolling marked-to-market losses are above 20% (-20% and -20% + 1*2*5% = -10% will be calculated for losses of 5% and 10% and replaced by no withdrawals), but -20% + 3*2*5% = 10% of the capital will be withdrawn if losses are between 20% and 25%. If losses are between 25% and 30%, then a -20% + 4*2*5% = 20% capital withdrawal will happen. Redemption values will keep increasing by 10% every 5% loss step until they reach 100%. Investor redemptions immediately push up the fund’s effective leverage, potentially generating a margin call and forcing the fund to sell assets and take the significant losses it was suffering from according to marked-to-market pricing. Note that if the effective leverage to authorized leverage ratio was not very high at the moment of the withdrawal (i.e. if, despite some losses, a reduction in asset price volatility has lead to declining levels of authorized leverage), then a withdrawal may not trigger a margin call and lead to a releveraging of the fund, effectively allowing the fund to buy the dip and increased risk for the rest of the trade.

\[(10) \forall i \in [0,10], S_i = -i*10\%\]

\[\forall t \in [z^3+1,T], \exists i \in [0,9], (1 - W_{t,i}/W_{t-1}) \in [S_i,S_{i+1}], R_t = 0\]

\[\forall t \in [z^3+1,T] \exists i \in [0,9], (1 - W_{t,i}/W_{t-1}) \in [S_i,S_{i+1}], R_t = \text{Max}(0, \text{Min}(1, r^2((-5\%+S_i)+r^1)))\]

Figure 12 shows what the wealth index of the arbitrageur may look like according to all the assumptions of the model. Figure 13 shows the first 500 days of trading in more details. Notice on figure 13 the usual (staircase-like) wealth pattern of a no-trade flat period followed by a trade-in period where transaction costs and the mispricing widening create marked-to-market losses until the mispricing shrinks and the fund gets out of the trade by paying transaction costs. It is very interesting to look at the period around day 1850. The period shows a sudden loss for
the arbitrage fund. It is actually a liquidity shock that pushes the mispricing up at this point, allowing the arbitrageur to enter into a convergence trade. The day after he has entered, liquidity is still bad but noise traders push the mispricing towards a sudden larger increase, generating large marked-to-market losses for the fund (in the order of $200). The next day, some investors launch a stop-loss call and withdraw some (35% in this case) of their money from the fund, this boosts up the fund’s effective leverage from c.5.0x to c. 7.5x. In the meantime market volatility spikes up making authorized leverage much lower, and a margin call happens that forces the fund into taking close to 90% (combined effect of capital loss and volatility peak) of its current loss of c.$200. Additionally, very high transaction costs (because of high volatility) hit the fund on both days. The next day, the fund’s position in the trade is thus much lower but ends up making it some money until the fund can enter a new trade at a higher authorized leverage.

![Figure 12: Time series of arbitrageur wealth index, exponential scale](image-url)
c. Methodology and Results of the Simulation

i. Methodology and Calibration of the Simulation

In order to test the robustness of the model against findings described in the literature, I run a number of Monte Carlo simulations with a number of sets of varying parameters. I have parameters vary one by one, and I keep the others set on base case values that I will describe below. For every set of parameters, I have run the model 1,000 times and compute the average IRR$^{16}$ of the fund, the standard deviation (volatility) of that measure, the Sharpe ratio of the fund (computed using an arbitrary 4% return of the risk-free asset), the T-statistic of the IRR, the skewness of the IRR distribution, and the percent chance of fund failure. All IRR measures have very satisfying T-statistics (in the range of 5 to 50). My definition for a fund failure is a final IRR

$^{16}$ IRR is the internal rate of return of an investment, usually expressed as a CAGR.
of -50% or less. The fund failure statistics is only an indicator of left tail events, and it does not indicate that the fund was actually closed during the simulation. It is however likely that at such performance levels, investors have taken out all of the capital of the fund or that the fund managers have decided to close it. After these results are calculated, I regress the series of IRRs against the variable parameter to test a number of assumptions. I usually try 15 to 40 equally spaced values of varying parameters, although I have used a mobile scale on one occasion. This sometimes implies relatively few data points for the regression (and sometimes, non smooth data series) but these limitations have been imposed by the computing power of my pc.

I have based the calibration of the base case parameters on historical data, common sense, and literature observations. Sometimes, the calibration has been rather arbitrary due to lack of historical data and information on hedge fund strategies. When I could, I chose to tie the base case parameters to a mispricing observed in the market in order to illustrate how such a model could be calibrated. The data series I have used is that of the Treasury bond mispricing observed by Taliaferro and Blyth (2011). The respective bonds were T 4.25 08/15/15 and T 10.625 08/15/15. As Treasuries with the exact same maturity, they are a typical example of when the LOP should be enforced. However, the authors witnessed a very large mispricing between mid-2008 and mid-2009, for the reasons that we have detailed previously. Figure 14 shows the time series of dollar mispricing (per $100 notional) between the two bonds between late 2007 and early 2010. The yield data comes from the Bloomberg system. Notice the number of events surrounding the large and sustainable mispricing period of September 2008 to May 2009. These correspond to minor liquidity events that lasted for a few days and brought the bond yield apart

17 The model is very flexible and would allow any user to calibrate it with its own assumptions and judgment.
from each other. It typically took the yields two weeks to be brought back together by arbitrage capital.

Figure 14: Time series of dollar mispricing of same maturity Treasuries during the crisis

The base case parameter values are the following. I used 252 days a year and 10 years of data generation. The average asset value is set at $1,500. The noise trader daily effect standard deviation is set on 5, the liquidity shock probability on 20bps (resulting in an average of 5 liquidity events per 10 year period, typically 3 or 4 of which having a mild effect on the mispricing time series, and typically one or two of them generating a very significant deviation in the mispricing, which is in line with empirical observations in a relatively stressed period), its probability of continuing was 90% (resulting in an average liquidity event of 10 trading days, which is also in line with empirical observations), its average size was 12x with a standard deviation of 5.
deviation of 8x. These parameters have been calibrated using the maximum absolute value of mispricing to the daily volatility of mispricing ratio of the Treasury bond mispricing data series. After adjusting for the different coupon rates of the two bonds, this series provides a maximum dollar mispricing of $5.7 (or yield mispricing of 77bps), and a daily standard deviation over the period of $0.79, making the ratio 7.2x. The model has been run 1,000 times at a number of mispricing size parameters ranging from 6 to 18 and the average value of this ratio (over a period that corresponds to that of the Treasury time series) has been computed. An average liquidity multiplier of 12 provides an average ratio of 7.2x. Note that the treasury data time series consists of 7 years of Treasury yields since December 2006, and is thus heavily impacted by the financial crisis.

The countershock can be set off at a mispricing of $250 (this value corresponds to the average level of mispricing above which the reversion force becomes negative, according to base case parameters) with a probability $q$ of 2%. There is very little empirical data on which these assumptions are made, because the event of a mispricing of this order of magnitude is so rare historically. Concerning the reversion force, its power parameter is 2%, its amplification parameter 48%, its peak parameter 300, and acceleration parameter 1.2. Calibration was made by observations of the final mispricing data series produced by the model as compared to theoretical assumptions described previously. The industry leverage is calculated after 30 days (in order for the EWMA to gain significance), its first factor is 7, and second is 2, and the moving average period is 3 days. The sensitivity to volatility is set in order for the leverage of the industry to move progressively without being bounded by its limits too often. The average leverage was calibrated to correspond to the model’s average volatility. The industry capital calculation starts after 90 days (to give a minimum lag of capital effect of three months) and is averaged on 9 days
(to account for the high number of funds and investor types). The asset volatility EWMA λ is 0.94. The arbitrageur buy-in is at a $25 mispricing, and buy-out at $5 (a later analysis will test these choices), its transaction costs parameter is 10, meaning that the arbitrageur will pay 10bps multiplied by the liquidity variable on all of its transactions. This was set arbitrarily. The arbitrageur’s initial authorized leverage is 5x, the leverage calculation starts after 30 days, it is reset every 5 days and the two leverage-volatility factors are 13 and 0.5, respectively. These were calibrated to typically obtain an average leverage of 5x (corresponding to an asset to capital ratio of 1:10) and a 1x standard deviation of leverage (with consequently typical leverage values between 3x and 7x). Margin calls will happen if effective leverage is 30% or more higher than authorized leverage, marked-to-market losses are calculated on a 5-day rolling period and redemption factors are -20%, and 2.

ii. Presentation of Interesting Findings by Parameter

1. Optimal Trading Strategy

As a first parameterization test, I tried a number of buy-in and buy-out thresholds on the arbitrageur strategy. I used b^1 parameters ranging from 10 to 50 in 5 increments, and b^2 parameters from 0 to 8, and I ran the model 1,000 times per set of parameters. Figure 15 shows the heat map of all IRR outputs, figure 16 shows the standard deviation of all IRRs and figure 17 computes the resulting Sharpe ratios for all sets of parameters (using an arbitrary 4% return on the riskless asset).

Very interestingly, it is not easy to draw any surface trend from the IRR output. Indeed, any parameter pair from {0,10} to {8,30} seems to be yielding similar returns (overwhelmingly in the 25%-29% range). On the lower part of the map however, from {0,35} to {8,50}, it is clear that higher buy-in thresholds are very detrimental to returns. This can be explained by the fact
that, at such thresholds, it is relatively rare for the fund to enter (and get out of) a trade, making its trading profits, however larger ($50 per trade and per pair of assets arbitraging for \{0,50\} vs. only $2 per trade and per pair of assets arbitraging for \{8,10\}), much rarer. It is easy to notice in figure 11 that in the base case calibration, the mispricing size very rarely hits the $50 threshold.

It is very interesting to note that a "high frequency" strategy such as the \{8,10\} can be so profitable. In this strategy, it takes the arbitrageur 14.5 times (minus the compounding effect of trading more quickly) the number of trades of the \{30,1\} strategy to show the same 26% returns, even after much higher transaction costs (in the order of $3,000 vs. $400 for \{30,1\})! The buy-out threshold does not seem to be significantly impacting returns. This is surprising, as it is a very different strategy to wait for the mispricing to become very low (or even cross the x-axis) and to get out of trades as soon as the mispricing has started to go down. Indeed, the fact that waiting for the mispricing to go down to very low levels is not more effective than liquidating at relatively high levels of mispricings would suggest that, the arbitrageur would often have the occasion to place (at least) a second trade before the mispricing goes down completely, as the mispricing reversion is noisy and non-linear. This second trade would compensate the arbitrageur for not harvesting the end of the convergence profit. I interpret that fact as a consequence of the fast-decreasing reversion force at low levels of mispricings.

Looking at return volatility, there is a clear correlation between return and return volatility. A linear regression exhibits an R-Squared of 68%, with a statistically significant (T-Statistic of 13) beta coefficient of 0.5 and a statistically significant (T-Statistic of 6) intersection at 5%. The scatter graph (Figure 15a) seems to suggest a convex returns to volatility relationship, implying that very high and very low returns are attained with relatively high volatility compared to medium returns. Hence, volatility does not go down as fast as returns on the low side, which
explains the very low Sharpe ratios obtained on the high end of buy-out parameters. In a fashion similar to that of returns, volatility does not seem to be very affected by the buy-out parameter choice. Additionally, higher frequency strategies do not necessarily generate higher volatility of returns for reasonable buy-in parameters. The fact that volatility drops at the higher end of buy-in parameters (and together with returns) is in line with the assumption that getting into a convergence trade when prices are very far apart is a much safer strategy than doing so every time prices diverge a little from each other and risking to see the mispricing widen before it shrinks. The fact that the risk-adjusted excess returns are very low for high buy-in parameters may be partially explained by the fact that there is a higher probability to get into a trade during a liquidity shock period when the buy-in parameter is set very high. As a result, there is an increased chance that trades lose a large amount of money before they converge when the arbitrageur only trades on large market disruptions. The Sharpe ratio map is interesting because it proves that there is no clear winning strategy for the arbitrageur. It seems as though both low and high buy-out thresholds work well (and usually better than medium buy-out thresholds), and that every reasonable buy-in levels (from 20 to 35) produces good results. I ran the model 1,000 more times without obtaining satisfying smoothing results on the Sharpe surface, which would suggest that picking arbitrary trading thresholds within the previously defined reasonable range makes sense. In other, words, the strategy cannot be clearly optimized using the set of parameters tested. The base case parameters of {25,5} appear to be part of the optimal strategy sweet spot, with an average 26% IRR for 16% volatility, and a Sharpe ratio of 1.4.
Figure 15: Average IRR as a function of parameters $b^1$ (columns) and $b^2$ (rows)

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Figure 16: IRR volatility as a function of parameters $b^1$ (columns) and $b^2$ (rows)

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</tbody>
</table>
Figure 16a: Scatter graph of IRR volatility (y axis) as a function average IRR (x axis)

Figure 17: Fund Sharpe ratio as a function of parameters $b^1$ (columns) and $b^2$ (rows)

2. Summary of Regression Analysis
Figure 18 shows a summary of what parameters have been tested, what range they have been tested on, and the regression results against average arbitrageur IRR and arbitrageur IRR volatility. All alpha and beta figures are statistically significant at the 5% level (marked by two stars).

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable Range</th>
<th>Number of Observations</th>
<th>Independent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise Standard Deviation</td>
<td>[3.0; 11.2]</td>
<td>42</td>
<td>σ</td>
</tr>
<tr>
<td>Shock Size</td>
<td>[1; 21]</td>
<td>21</td>
<td>s</td>
</tr>
<tr>
<td>Shock Probability</td>
<td>[0.0%; 0.5%]</td>
<td>20</td>
<td>h</td>
</tr>
<tr>
<td>Reversion Force Power</td>
<td>[0.5%; 10%]</td>
<td>20</td>
<td>f</td>
</tr>
<tr>
<td>Reversion Force Power</td>
<td>[10%; 20%]</td>
<td>21</td>
<td>f</td>
</tr>
<tr>
<td>Authorized Leverage Level</td>
<td>[5; 35]</td>
<td>31</td>
<td>al</td>
</tr>
<tr>
<td>Authorized Lev. Sensitivity</td>
<td>[0.0; 2.0]</td>
<td>21</td>
<td>a2</td>
</tr>
<tr>
<td>Margin Call Threshold</td>
<td>[1%; 100%]</td>
<td>37</td>
<td>g</td>
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<tr>
<td>Redemption Size</td>
<td>[-100%; 100%]</td>
<td>21</td>
<td>r2</td>
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<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Regression Equation</th>
<th>R Squared</th>
<th>Alpha</th>
<th>Alpha T-Stat</th>
<th>Beta</th>
<th>Beta T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRR</td>
<td>IRR = 19% + 1.05% * σ</td>
<td>67%</td>
<td>19%</td>
<td>22.08 **</td>
<td>1.05%</td>
<td>9.04 **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 4% + 2.43% * σ</td>
<td>95%</td>
<td>4%</td>
<td>6.43 **</td>
<td>2.43%</td>
<td>29.11 **</td>
</tr>
<tr>
<td>IRR</td>
<td>IRR = 31% - 0.41% * s</td>
<td>92%</td>
<td>31%</td>
<td>90.54 **</td>
<td>-0.41%</td>
<td>(14.87) **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 2% + 0.96% * s</td>
<td>95%</td>
<td>2%</td>
<td>3.53 **</td>
<td>0.96%</td>
<td>19.37 **</td>
</tr>
<tr>
<td>IRR</td>
<td>IRR = 30% - 18.43 * h</td>
<td>98%</td>
<td>30%</td>
<td>163.3 **</td>
<td>(18.43)</td>
<td>(27.84) **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 9% + 29.09 * h</td>
<td>88%</td>
<td>9%</td>
<td>13.00 **</td>
<td>29.09</td>
<td>11.74 **</td>
</tr>
<tr>
<td>IRR</td>
<td>IRR = 21% + 2.06 * f</td>
<td>73%</td>
<td>21%</td>
<td>12.13 **</td>
<td>2.06</td>
<td>7.00 **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 14% + 1.26 * f</td>
<td>91%</td>
<td>14%</td>
<td>25.25 **</td>
<td>1.26</td>
<td>13.66 **</td>
</tr>
<tr>
<td>IRR</td>
<td>IRR = 114% - 6.36 * f</td>
<td>83%</td>
<td>114%</td>
<td>11.29 **</td>
<td>(6.36)</td>
<td>(9.67) **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 29% + 4.92 * f</td>
<td>97%</td>
<td>29%</td>
<td>4.92 **</td>
<td>23.13</td>
<td></td>
</tr>
<tr>
<td>IRR</td>
<td>IRR = 14% + 0.77% * a1</td>
<td>84%</td>
<td>14%</td>
<td>10.26 **</td>
<td>0.77%</td>
<td>12.46 **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 5% + 1.67% * a1</td>
<td>100%</td>
<td>5%</td>
<td>11.74 **</td>
<td>1.67%</td>
<td>78.06 **</td>
</tr>
<tr>
<td>IRR</td>
<td>IRR = 34% - 19.11% * a2</td>
<td>85%</td>
<td>34%</td>
<td>15.41 **</td>
<td>-19.11%</td>
<td>(10.26) **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 36% - 22.00% * a2</td>
<td>61%</td>
<td>36%</td>
<td>7.76 **</td>
<td>-22.00%</td>
<td>(5.48) **</td>
</tr>
<tr>
<td>IRR</td>
<td>IRR = 23% + 5.15% * g</td>
<td>57%</td>
<td>23%</td>
<td>69.68 **</td>
<td>5.15%</td>
<td>6.84 **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 15% + 3.35% * g</td>
<td>49%</td>
<td>15%</td>
<td>57.01 **</td>
<td>3.35%</td>
<td>5.77 **</td>
</tr>
<tr>
<td>IRR</td>
<td>IRR = 22% - 8.67% * r2</td>
<td>87%</td>
<td>22%</td>
<td>47.57 **</td>
<td>-8.67%</td>
<td>(11.35) **</td>
</tr>
<tr>
<td>Vol</td>
<td>Vol = 15% - 4.23% * r2</td>
<td>79%</td>
<td>15%</td>
<td>49.64 **</td>
<td>-4.23%</td>
<td>(8.37) **</td>
</tr>
</tbody>
</table>

Figure 18: Summary of regression analysis on varying parameters

3. Noise Traders

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable Range</th>
<th># Obs.</th>
<th>Ind. Var.</th>
<th>Dep. Var.</th>
<th>R Squared</th>
<th>Beta</th>
<th>Beta T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise Standard Deviation</td>
<td>[3.0; 11.2]</td>
<td>42</td>
<td>σ</td>
<td>IRR</td>
<td>67%</td>
<td>1.05%</td>
<td>9.04 **</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vol</td>
<td>95%</td>
<td>2.43%</td>
<td>29.11 **</td>
</tr>
</tbody>
</table>
To test the model’s response to varying price volatility environments, I made the noise trader pushes standard deviation vary from $3 to $11.2, all other parameters remaining equal to their base case values. The arbitrageur’s returns went up significantly as a function of this volatility, proving that arbitrageurs take advantage of relatively high volatility environments. Annual returns went up one percent per added dollar standard deviation in noise trader pushes. Average returns growth went down over the simulation range (giving the IRR plot a theoretical asymptotic maximum), as price volatility started generating some very large losses on occasions. This confirms the negative skewness aspect of arbitrage trading and explains the relatively low R-Squared of 67% for the IRR regression. The skewness of returns was around the level of -3 for all simulations. Figure 21 shows the distribution of returns when σ parameter is set at $8. A fat left tail is easily noticeable. As expected, returns volatility was almost completely explained by movements in the noise trader parameter, as shown by an R-Squared of 95%. IRR volatility went up 2.4% per added dollar volatility in noise trader price effect. Figure 19 shows clearly the average IRR reaching a ceiling at around 30%, corresponding to a σ factor of 5, when fund failures start compensating for otherwise growing IRRs. It also shows that IRR volatility keeps going up (almost linearly) when IRR stays stable, soon resulting in a declining Sharpe ratio (figure 20). Notice the very close correlation between IRR volatility and fund failures, as well as the interesting negatively correlated movements between volatility and returns (notably peaks and dibs at σ of 9.8 and 11).
Figure 19: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of noise trader effect standard deviation (parameter $\sigma$)

Figure 20: Fund Sharpe ratio as a function of $\sigma$
I tested the model’s response to varying levels of liquidity shocks, from a liquidity multiplier of 1 to 21, all other parameter being set at their base case values. IRR goes down slowly (less than 50 bps per increase in shock size) as shocks get bigger but this phenomenon is accelerating towards the higher end of shock sizes. This figure is associated with a satisfying 92% R-Squared. Volatility goes up significantly in the same manner, at an average rate of 1%
per added multiplier, for an R-Squared of 95%. These results seem to confirm that a lack of liquidity is detrimental to a fund’s returns even more so than to its risk-adjusted returns, as suggests the fund’s Sharp ratio plotted in figure 23. Increasing shock sizes make fund failures increase dramatically (faster than IRR volatility). Unsurprisingly, returns exhibit stronger negative skewness for large shocks than for small shocks (the measure progressively increasing from 0 to -2.5, hence confirming the left fat tail effect of large liquidity shortages.

Figure 22: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of liquidity shock size (parameter s)
I also looked at the effect of different values of shock probability, from 0% to 0.475% per day. As this probability increases, the fund has a higher chance of getting hit one or more times during the period of simulation. The regression suggests that the variation of shock probability explains 98% of the variation of IRR (this time, the factor has a better explanatory power for IRR than for volatility, standing only at 88%). There is a strongly negative and statistically significant beta coefficient of 18% per percent chance increase in shock. According to figure 24, the relation appears to be quasi-linear. I ran the regression on much higher values of shock chances (up to 4%) to see if the pattern would hold. Interestingly, for higher values of factor c, IRR picks up slowly and volatility hits a ceiling, as fund failures go down (see figure 27). Moreover, the IRR to factor c regression loses its statistical significance (with a T-Statistic of 1.3). The regression results would suggest that for highly probable liquidity shocks, the results of the fund begin to become more random as price jumps are generated randomly in both directions. Note however
that the randomness shown by the regression significance does not appear in volatility measures. The fact that IRR goes up and volatility does not at high parameter values suggests that the fund's strategy has been tuned conservatively, and that the fund is in reality able to sustain and take advantage of changing and volatile environments. My hypothesis is that the stop-loss effects of redemptions and margin calls play a large part in this apparent good resilience to varying liquidity environments. Skewness goes down (to more negative values) in this case, suggesting liquidity shocks still strengthen left tails in this scenario. In the first scenario (small incremental probability of shocks), shock liquidity and volatility are strongly and significantly positively correlated, as expected, and the Sharpe ratio of the fund goes down fast as the probability rises (figure 25). However, the skewness of the returns goes up significantly (from the very low level of -5) as the probability of a shock rises (figure 28). This result is surprising as fund failures (left tail events) rise significantly over the course of the simulation. According to the distribution plots of returns, it seems that the few very negative returns that happen when the probability of a shock is very low create very fat left tails in comparison with what would be expected by looking at returns volatility figures. For instance, for a 5bps parameter value, 0.9% of the returns were between -65% and -45%, for an average return of 29% and volatility of only 10%. According to a normal distribution, returns of this amplitude should happen only with a probability of $6 \times 10^{-11}$ %. This result shows that increasing probability of shocks makes volatility increase faster than fund failures.
Figure 24: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of liquidity shock probability (parameter $h$).

Figure 25: Fund Sharpe ratio as a function of parameter $h$. 
Figure 26: IRR skewness as a function of parameter $h$

Figure 27: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of liquidity shock probability (parameter $h$), larger data range
I tested the effect of changing the reversion force power from a value of 0.5% to 10%. This parameter is the multiplier of the effect on the reversion force on the mispricing. According to the regression analysis, the reversion force power acts as an ally of the arbitrageur. Indeed, IRR goes up by two points for each percent power of the reversion force. This data is very statistically significant and is associated with an R-Squared of 73%. This result would suggest that increasing competition in the market considered affects the fund’s performance positively. This has to be qualified by two aspects. First, as we have seen, my model hinges a lot on the zero-reverting action of competition. It is probable that a market with no active specialized arbitrageurs would behave in a similar way as my model does for a moderate to medium reversion force power, simply because normal investors would look for cheapest options in multiple markets and notice significant price discrepancies. Second, my simulation may have been done at the smaller end of the reasonable reversion force range, hence hiding the perverse effects of competition as seen in the literature (notably, in Kondor, 2009). To confirm this hypothesis, I ran the model on a second range of factor f, from 10% to 20%. I will analyze the results in the second part of this section. Focusing on the smaller end of force power, return volatility is clearly but slowly upward sloping as a function of reversion force, with a significant
1.26 beta associated with a 91% R-Squared. The relation is almost linear, as shown by figure 28. The fact that volatility of returns goes up as a function of reversion force power is somewhat surprising, because this force clearly reduces the volatility of the mispricing as it ties better to zero. It is understandable that, by bringing the mispricing back to zero more often, the force allows the fund to trade more often and increase its returns, but it is surprising that these returns become more volatile as the mispricing pattern should become increasingly predictable (i.e. small swings around the value of zero). As an illustration, compare figures 31 and 32 where the reversion force is 0.5% and 10%, respectively. My hypothesis is that, as the mispricing becomes closely tied to zero, the fund gets in and out of trades more quickly, meaning that, on average, it has a smaller chance of losing some money (and therefore of having to adapt its leverage) before a liquidity shock hits. As a result, the fund is on average more vulnerable to margin calls and redemptions than if it were trading on larger price differential swings. This would suggest that, by virtue of the stop-loss properties of margin calls and redemptions, a fund trading on larger, longer mispricings is effectively more risk-averse and can take larger liquidity shocks (not proportionately, but in total mispricing value) than a fund trading on fast-moving, smaller mispricings. This is confirmed by the increasingly negative skewness the fund’s results exhibit (figure 30), showing that as the average observable mispricing gets smaller and more rapidly brought back to zero, the fund’s returns correspond better to the phrase “picking up pennies in front of a steamroller”. The combination of these effects makes the fund’s Sharpe ratio go up fast and hit a ceiling as volatility shows a hiccup pattern and IRR growth slows down (figure 29).
Figure 28: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of reversion force power (parameter $f$)

Figure 29: Fund Sharpe ratio as a function of parameter $f$
Figure 30: IRR skewness as a function of parameter $f$

Figure 31: Time-series of observable mispricing, $f$ factor set at 0.5%
The regression analysis made with \( f \) values ranging from 10% to 20% tells a very different story. The IRR beta coefficient becomes very strongly negative (and remains statistically significant), such that returns end up strongly negative at the higher end of the spectrum. The regression has an \( R^2 \) of 83%. At this point, the volatility grows at a much faster rate as well (almost 4 times as fast as in the previous range). It is clear that at this level of reversion force, the perverse effects of competition catches up with both the returns and the risk-adjusted returns of the arbitrageur (see figures 33 and 34, where the Sharpe ratio goes down almost linearly). It seems that at these levels, the fund still trades a lot (the force is not strong enough to bring the mispricing to levels under the buy-in threshold), but it trades so fast that transaction costs go through the roof (from an order of magnitude of $100 to one of $500, over the period) and its leverage is rarely adapted to higher volatility environments, making liquidity
shock very dangerous. As an illustration, see the fund failure rates of almost 50% at the higher end of factor values. Figures 35 and 36 show the disastrous effect of a liquidity shock at the beginning of the simulation period for an otherwise very arbitrage-friendly mispricing data series. It is interesting to look at the range of values for the IRR data. At high levels of reversion force, the range of IRRs gets as large as [-100%; 322%] (for an f factor of 15%), showing how volatile returns become, probably due to the fund’s exposure to the random data generated by liquidity shocks. In this sense, fund returns start to look like a martingale process, confirming the findings of Kondor (2009).

![Graph](image)

*Figure 33: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of reversion force power (parameter f)*
Figure 34: Fund Sharpe ratio as a function of parameter $f$

Figure 35: Time-series of observable mispricing with liquidity shock, $f$ factor set at 15%
I tested the model’s behavior on the two arbitrageur authorized leverage factors $a^1$ and $a^2$. The first factor is the level of leverage. Dividing this constant by the volatility level to the power of the second factor generates the authorized leverage figure. As $a^1$ increases, the average authorized leverage increases linearly. The second factor is thus the sensibility of the authorized...
leverage to changes in asset volatility. As volatility is always superior or equal to 1, an increasing \( a^2 \) factor leads to less sensitive authorized leverage values. I tried values of \( a^1 \) ranging from 5 to 35, with \( a^2 \) staying constant at 0.5. This has to be compared with asset volatility in the order of magnitude of 5 to 8, leading to an average authorized leverage of 2 to 13.

There is a very clear upward direction of IRR to leverage factor, as exhibited by a positive beta coefficient of 0.8% and an R-Squared of 84%. Figure 37 shows this trend clearly but also suggests that IRR stops growing passed a factor value of 17 to 20. The IRR plot shows increasing hiccup effects as leverage goes up, hinting at an increasing level of IRR volatility. Unsurprisingly, volatility does go up when leverage goes up. As a matter of fact, the linear relation is perfect, with the regression showing an R-Squared of 100%, and a beta coefficient of 1.7% (statistically significant at all levels). As previously noted, fund failures increase in a similar manner, although apparently faster than volatility, suggesting that leverage increases the fund return fat tail effect. Once again, this is not confirmed by the skewness figure however (going up slowly but staying negative as leverage increases), probably because the distribution implied by very high levels of volatility is much more tolerant of left tails than lower volatility levels. Interestingly, IRR never grows fast enough to compensate for the linear growth of volatility, (except from the first data point where volatility is a little out of line), resulting in a steadily decreasing Sharpe ratio for the fund (figure 38). This result suggests that intrinsic leverage is mostly detrimental to the fund’s risk-adjusted excess returns, meaning that it is probably optimal for the fund’s investors to leverage their equity invested in the fund, and require the fund to have low leveraged strategies with very low volatility. This result has to be mitigated by the fact that the arbitrageur is able to finance its operations at a probably lower cost than its investors due to the collateralization process. It is probable that lower values of leverage
would have exhibited increasing Sharpe ratios, but such levels of leverage are unrealistic in the case of arbitrage strategies.

Figure 37: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of authorized leverage level (parameter $a^1$)
Figure 38: Fund Sharpe ratio as a function of parameter $a^l$

For parameter $a^2$, I used values from 0 to 2, leaving $a^l$ constant at 13, meaning the actual leverage of the fund would range between averages of 13 and 0.5. At high values, leverage would be low and volatile (i.e. very sensitive to volatility changes), and at low values, leverage would be high and constant. The double effect of changing this parameter makes results difficult to interpret, and its exponential dimension make the linear regressions less efficient (but still significant). There is a strong negative relationship between IRR and $a^2$. As expected, as leverage goes down, IRR goes down as well in an exponential manner. However, for very high and insensitive levels of leverage (at the left end of figure 39), IRR suddenly goes down fast and linearly. As similar levels of leverage had been reached in the $a^l$ simulation without causing a strong decrease in returns, this result shows that leverage that is not sensitive to volatility is detrimental to fund returns. In other words, this result suggests that the authorized leverage reset process engaged by the leverage provider is actually beneficial to a certain extent to the fund and
acts as an efficient stop-loss in disturbed environments. The fund’s Sharpe ratio (figure 39) suggests the same interpretation, as risk-adjusted excess returns (read from right to left) peak at a level of size and adaptation and subsequently plunge due to exponentially increasing volatility and (only later) decreasing results. On the other end of the spectrum (the right side of the graphs), volatility does not continue to go down exponentially, as IRR does. In fact, volatility hits a floor and even goes up in the 1.5-1.8 parameter region. This would suggest that too high a level of leverage sensitivity to asset volatility leads to unnecessary margin calls and subsequently more volatile returns. There is thus a perverse side of the stop-loss effect at high levels of leverage adaptation. To confirm this intuition, it is very interesting to have a look at fund failures. With reasonably low leverage level and moderate leverage sensitivity to asset price volatility (in the middle section of figure 39), there are zero to two fund failures per thousand 10-year returns. For even lower level of leverage but very sensitive authorized leverage to volatility (bottom right corner), this number picks up to three to four fund failures. As a result, some funds that would not have a very negative return with higher leverage and a less risk-averse leverage provider, go under with very conservative leverage and very risk-averse leverage providers that update authorized leverage too sensitively. These findings highlight the fact that authorized leverage rules should be properly calibrated in the mutual interest of the fund, its investors, and its lenders. Indeed, a good relationship between arbitrageur and lenders can bring higher returns with lower risk, and a bad one may cause lower returns with negative risk-mitigation. Also notice how, on the left end, fund failures increase much faster than volatility, confirming the positive stop-loss effect of reasonable leverage sensitivity on fund failures in the middle of the figure.
Figure 39: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of authorized leverage sensitivity (parameter $a_2$).

Figure 40: Fund Sharpe ratio as a function of parameter $a^2$. 
7. Margin Calls

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable Range</th>
<th># Obs.</th>
<th>Ind. Var.</th>
<th>R Var. Squared</th>
<th>Beta</th>
<th>Beta T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Margin Call Threshold</td>
<td>[1%; 100%]</td>
<td>37</td>
<td>g</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

IRR 57% 5.15% 6.84 **
Vol 49% 3.35% 5.77 **

To look deeper into the relationship between fund returns and leverage providers behavior, I ran the model on values of margin call thresholds ranging from 0.5% to 100%, keeping all other parameters constant (including reasonable authorized leverage parameters of 13 and 0.5). I did not use a uniform scale as I wanted to focus on low values for this parameter. I used 0.5% steps until 10% and 5% steps until 100%. At low levels of threshold, the leverage providers will ask for more collateral very often, every time asset price volatility goes up or the portfolio value goes down a little. At higher levels, margin calls will hardly happen and the arbitrageur will rarely have to take his losses in disturbed environments. He will however have to take an important loss at once when the threshold is met. I regressed the IRR and volatility results against the values of the factor from 1% to 100% as at the threshold of 0.5%, returns are slightly negative and constitute an outlier (the 0.5% values of IRR and volatility are plotted in figure 41). With a beta of 5%, there is a clear upward pattern of IRR to margin call threshold, although the 57% R-Squared for the regression suggests it is rather noisy. On the adjusted scale (where 0.5% steps are similar to 5% steps), the relation seems close to linear, suggesting that the actual pattern is more logarithmic looking in actual scale, as would suggest figure 43. Contrary to most of my results, volatility regression data is noisier than IRR data (with an R-Squared of only 49%). There is however a clear, statistically significant upward trend as well, although not as steep as in the case of IRR. On an adjusted scale, IRR and volatility seem tightly intertwined.
(figure 44), but IRR is actually slowing down, making the fund’s Sharpe ratio peak at a factor value of 2% and go down very slowly for the rest of the simulation range (figure 42).

The perverse effect of very tight collateralization rules is obvious at the lower end of factor g values. Indeed, a 0.5% leeway prohibits the fund from keeping any efficient leverage during the life of its trades and brings slightly negative returns (of -1%). This is consistent with returns going up to 16%, 22%, and 23% for a 1%, 1.5%, and 2% threshold. After this level, it seems that the fund is relatively indifferent to getting more margin call leeway. My hypothesis is that more reasonable thresholds have a positive stop-loss effect on risk and returns and that more leeway means higher returns but worse risk management in that sense. The two effects almost compensate each other, which explains an almost flat Sharpe ratio measure over the simulation range. Hence, it appears according to this measure that a relatively tight margin call policy (with a 2-5% threshold) is mutually beneficial to the fund’s lenders and investors. At the left end of returns distribution, it is difficult to conclude anything from fund failures across the simulation range. Although a tight policy somewhat mitigates the risk of fund failure (with data in the range of 8 to 20 per thousand), much longer leeway does not seem to be very detrimental in that respect (with number in the 16-26 range for no margin calls territory, less than the 26-35 figures in the middle of the range). Making the criterion for fund failures vary from -80% to -20% IRRs does not provide any more significant trend. For instance, for a fund failure threshold of -60% IRR, the variation of factor g does not explain at all variations in fund failures (R-Squared of 2%). Additionally, the beta factor loses all statistical significance (T-Statistic of -0.8) and is negative, which would suggest a smaller left tail as leverage provider becomes more lenient! The returns skewness does give more information as the skewness stay very negative and relatively stable (and noisy) over the simulation range (figure 45). A regression between returns skewness
and factor g values does provide a 14% R-Squared associated with a slightly positive 0.28 beta coefficient (as compared to skewness levels of -3 and g values between 0 and 1) with a satisfying T-Statistic of 2.4, suggesting that a tight leverage control does mitigate some of the left tail effect of fund returns.

Figure 41: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of margin call threshold (parameter g)
Figure 42: Fund Sharpe ratio as a function of parameter $g$.

Figure 43: Fund IRR as a function of parameter $g$, actual scale.
Figure 44: Fund IRR and volatility as a function of parameter g, modified scale.

Figure 45: Fund returns skewness as a function of parameter g, modified scale.

8. Redemptions
<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable Range</th>
<th># Obs.</th>
<th>Ind. Var.</th>
<th>Dep. Var.</th>
<th>R Squared</th>
<th>Beta</th>
<th>Beta T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redemption Size</td>
<td>[-100%; 100%]</td>
<td>21</td>
<td>$r^2$</td>
<td>IRR</td>
<td>87%</td>
<td>-8.67%</td>
<td>(11.35)**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vol</td>
<td>79%</td>
<td>-4.23%</td>
<td>(8.37)**</td>
</tr>
</tbody>
</table>

To test the importance of investor confidence for the arbitrageur’s returns, I used the base case parameters to model a changing environment (from mild to very volatile). I used a range of $r^2$ parameters from -100% to 100%. At a value of -100%, the first 10% withdrawal happens in case of a 60% 5-day loss, suggesting very patient investors. At 100%, the first withdrawal happens at a 5% loss and 100% of the fund’s capital is wiped out. Note that this does not mean that the fund’s wealth index goes down a lot. Indeed, a 100% withdrawal merely means that the fund will have to give all of its capital back to its shareholders, resulting in the total liquidation of its positions and the taking of all marked-to-market losses at the moment of the withdrawal. If however these losses are lower than 100% of the capital (which should be the case when investors call back their money every time the fund is losing 5% on a 5-day rolling period), the fund’s wealth index remains positive. Regardless of the actual size of the fund (which is not modeled here), the wealth index is the value of $100 invested in the fund at the beginning of the simulation period and never withdrawn. It is important to stress that making the second redemption parameter vary does not have a linear effect due to the fact that withdrawals cannot take negative values. When on the positive side of parameter values, any additional 10% in the parameter makes every redemption size go up by 10%. When on the negative side of the parameter, this effect is still true for already positive values of redemptions, but in addition, the loss at which the first withdrawal will happen goes down. The patience of the investors is thus double for negative values: they will give more leeway for the fund to make up for its losses (the
stop loss effect, starting at a 5% for $r^2 = 0$ loss can go up to a 60% loss), and they will also withdraw less money when this threshold is met.

Movements in the parameter explain 87% of the movements in the fund’s IRR, making the regression very relevant. There is a very statistically significant and negative beta of -9% associated to this regression, implying that returns go down fast as the parameter grows. This result confirms the perverse effects of Shleifer and Vishny’s (1995) performance-based allocation of capital to arbitrageurs (PBA). Indeed, if investors withdraw capital when they notice losses that are larger than should be, they force the arbitrageur into deleveraging at a bad moment and take unnecessary losses in an otherwise relatively safe trade. Figure 46 clearly show that the IRR decreases much faster after the critical 0% level of parameter $r^2$ is reached. This suggests that first withdrawal leeway is not as critical to the fund’s returns as the overall level of withdrawals. Clearly, small, progressive deleveraging in case of losses does not hurt returns as much as a sudden, large forced deleveraging. In this sense, it seems that the hurdle effect of large first redemptions hurts the fund’s IRR severely.

The R-Squared for the regression of return volatility against the parameter values is 79%, and exhibits a statistically significant, negative beta of -4%. It seems thus that the stop-loss effect of forced liquidation influenced positively return volatility. This makes intuitive sense as, in the event of a liquidity shortage, cutting losses early when the market is moving against the arbitrageur mitigates the risk of taking a very large downside if liquidity stays dry. On the other hand, it also prevents the fund from getting large upside when the market comes back to normal, thus reducing IRR volatility both ways. Very clearly, return volatility goes down much faster after the 0% point is reached. This confirms that redemption leeway is not as good at risk mitigation as is a tight redemption policy. Furthermore, volatility increases when negative values
of parameter $r^2$ increase. My hypothesis is that, for very negative values of the parameter, redemptions almost never happen, hence having no effect on IRR, whereas for less negative values of the parameter, redemptions start to happen, but at the exact worse time to deleverage (when the fund just hit a 30-40% theoretical loss), hence clearly boosting the downside effect of the stop-loss mechanism. This hypothesis is supported by the fund failures data. Indeed, notice the peak of fund failures when there is just enough leeway to take large losses occasionally (a -40% parameter means that redemptions start at a 30% level of losses). This confirms that investor withdrawals in time of distress accentuate the negative skewness of arbitrage funds return distribution. The fund’s Sharpe ratio remains high as long as investors don’t really intervene, but go to higher levels when investors use a progressive approach to redemptions (the peak is at 0%), it then goes down increasingly fast as IRR is crushed by incessant, large redemptions and volatility reaches a floor. Although these are theoretical, it is interesting to notice some very attractive states of the returns. For instance, for a parameter set at 60%, the fund attains a 17% IRR with as little as 11% volatility of returns. This shows that a good withdrawal policy (and having the trust of its investors) can be very beneficial for an arbitrageur. However, do withdrawals help a fund meets its risk and return targets? I believe not, as it seems easy for a fund to implement a progressive and tight stop-loss policy instead of waiting for investors to take the money away from them.
Figure 46: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of investor withdrawal sensitivity to losses (parameter $r^2$)

Figure 47: Fund Sharpe ratio as a function of parameter $r^2$
d. Additions to the Trading Algorithm

i. Saving Capital for Stressed Environments

Hitherto, the arbitrageur’s trading algorithm was very simple. The arbitrageur would “buy” into the mispricing when its absolute value was above a reasonable threshold and liquidate its portfolio when the absolute value of the mispricing would go back to another predetermined threshold. Additionally, the fund would put all of its capital to work anytime it would buy into the trading opportunity, and would keep its leverage at its initial authorized maximum until it would get out of the trade or receive a margin call.

Because of the collateral requirements of the strategy, the risk of having to sell the arbitrage portfolio at a loss is proportional to the leverage of the fund and can be significantly reduced if the fund decides to keep some capital aside in case of marked-to-market losses. In fact, Liu and Longstaff (2004) found that it is often optimal for arbitrageurs to first underinvest in an opportunity in order to put aside capital (i.e. making the collateral constraint not binding) for cases of larger divergence. The authors explain it this way: “If the investor suffers large losses in the early stages, he clearly has less wealth to exploit arbitrages at a later stage. By being too aggressive with small arbitrages, the investor risks finding himself in a state of the world where there is a large arbitrage, but his ability to exploit the arbitrage is severely reduced because of losses suffered as the arbitrage widened.”

In order to test this finding, I program the fund to only invest a portion of its available capital into an arbitrage opportunity. I run the model 5,000 times using base case parameters and a capital invested parameter $c$ varying from 30% to 100% in 5% increments. A small parameter signifies that the arbitrageur will only invest a small portion of his available capital in the opportunity. As a direct result, in case of transitory losses, the arbitrageur will be able to add
collateral without having to sell part of his portfolio at a loss. A 100% parameter results in a trading strategy exactly similar to that used in previous analyses. The methodology is the same to that used in IV)c).

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable Range</th>
<th># Obs. Var.</th>
<th>Ind. Dep. Var.</th>
<th>Beta</th>
<th>Beta T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Invested</td>
<td>[30%; 100%]</td>
<td>15</td>
<td>c¹</td>
<td>IRR</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vol</td>
<td>97%</td>
</tr>
</tbody>
</table>

Clearly, returns go down as less capital is invested in arbitrage opportunities. This is a direct consequence of the de facto lower total leverage\(^ {18}\) taken on by the fund. The relation is quasi-linear with a statistically significant 25% beta coefficient and a 100% R-Squared. However, according to Figure 48, returns do seem to pace down at the far end of c¹ parameters. This result suggests that when very little capital is set aside, occasional forced deleveraging lowers overall returns. The negative effect of left tail events on returns is not very clear on this graph as fund failures are only 70/5,000, i.e. 1.4% of total paths at their maximum. I run a second simulation with more aggressive shock probability and sizes to test this hypothesis. Results would also suggest that after a certain point, keeping more capital aside adds no (risk-adjusted) value as the events that would require such amounts of capital are too rare. Volatility is also very correlated to c¹, with a statistically significant beta coefficient of 16% and an R-Squared of 97%. As returns slow down, volatility catches speed on the high side of c¹ values, as figure 49 shows well. This illustrates that volatility is not only an effect of total leverage, but also benefits from the protection against large losses that higher capital provides. As a result of this catching up, the

\(^ {18}\) Total leverage is the leverage of the fund, i.e. the effective leverage of the fund’s current investment multiplied by the capital invested parameter.
fund’s Sharpe ratio goes down steadily for large $c^1$ values, after a peak in 50%-60% region, as shown in Figure 50.

The purpose of such strategy is to protect the fund against left tail events, as it allows it to not reduce risks when it suffers large losses. Contrarily to what I have noted for most simulations, fund failures go up much faster than volatility in this case. Indeed, note that fund failures go up very significantly with values of parameter $c^1$ above 75%. As a result, it is clear that setting a lot of capital aside effectively protects the fund against very low returns. Additionally, thanks to this strategy, the fund’s Sharpe ratio reaches new highs with a 2.26 record for a parameter value of 45%, and an excellent 2.22 value associated with a 17% IRR for 60% capital invested. In comparison, the base case strategy generates a 1.45 Sharpe ratio, and the record Sharpe ratio (when playing with the fund’s first leverage parameter) for the base strategy is 2.01 for a 13% IRR. These results confirm the findings of Liu and Longstaff (2004) that oftentimes, keeping capital aside improves an arbitrageur’s risk-adjusted returns.
Figure 48: IRR and returns volatility (left scale), and fund failures (right scale, out of 5,000), as a function of capital invested by arbitrageur (parameter $c^I$)

Figure 49: IRR (left scale) and returns volatility (right scale) as a function of $c^I$
To see better the effect of left tail events on fund returns and volatility, and understand why a fund manager could be incentivized to choose a lower c1 parameter, I run the same simulation, but I use a 1% probability of liquidity shock associated with a 15x average liquidity shock.

In this stressed scenario, it is clear that fund returns do not go up as fast as in the previous scenario when the equity cushion goes down, as can be seen with the 15% IRR beta, vs. 25% previously. In addition, it is more clear on Figure 50a that returns curve down at the end of the capital invested spectrum. On the other hand, volatility clearly goes up at a higher rate (21% beta, vs. 16% previously), and with apparent positive convexity. This time, left tail events (as approximated by fund failures) happen sufficiently often to have a direct impact on volatility and IRR, and the equity cushion protects as efficiently from pure volatility as it does against ample drawdowns. Interestingly, the maximum Sharpe ratio is now attained for a higher (65%) capital
invested parameter, suggesting that very high equity cushions are still not really efficient when liquidity is very disturbed.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable Range</th>
<th># Ind. Vari.</th>
<th>Dep. Var.</th>
<th>R Squared</th>
<th>Beta</th>
<th>Beta T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Capital Invested</em></td>
<td>[30%; 100%]</td>
<td>15</td>
<td>c₁</td>
<td>IRR 94%</td>
<td>14.57%</td>
<td>14.35 **</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vol 97%</td>
<td>20.55%</td>
<td>18.95 **</td>
</tr>
</tbody>
</table>

*Figure 50a: IRR and returns volatility (left scale), and fund failures (right scale, out of 5,000), as a function of capital invested by arbitrageur (parameter c₁)*
ii. 2-Phase Trading Strategy

A more sophisticated strategy would be to use the capital set aside after having suffered losses. This way, the fund increases risks when the mispricing is wide, thus piling on its investments when the expected return of the trade increases. This can be viewed as a 2-phase trading strategy where the arbitrageur partially buys into the mispricing at a given level (as in the previous strategy), and invests the rest of its available capital into the opportunity if the mispricing goes through a second threshold (this threshold is implied by the level of losses at which the trader wishes to increase his investment). The effect of this second investment is double. First, the fund’s average buy-in price is lower (in the sense that, on average, the fund has bought into the opportunity further away from a mispricing of zero), which will eventually bring higher returns (because the difference between the average mispricing size at which the trader entered the trade and the buy-out threshold is larger, and because more money is put into work in
total). Second the fund's effective leverage suddenly becomes very close to its authorized leverage, making it more at risk of forced liquidation if asset prices move further apart.

To simulate this strategy, I added a reinvestment parameter $c^2$. When a level of $\%$ P&L losses $c^2$ is reached, the arbitrageur put all of its capital left into the arbitrage opportunity for the remainder of the current trade, or until he has to sell some of its portfolio in order to satisfy margin calls or capital redemptions. As in part IV)c), I looked for a potential optimal strategy for a combination of $c^1$ and $c^2$ parameters between 20% and 100%, and between -0.75% and -6.75%, respectively.

Figure 51 shows the average fund IRR for every given set of parameters. The figure exhibits a very smooth surface where returns are growing as a function of capital invested. This is not surprising as capital invested has a direct effect on the fund’s total leverage. As noticed previously however, IRR does not grow linearly with capital invested and IRR growth slows down for high capital invested parameters. The relationship between loss reinvestment threshold and returns is very interesting. For low capital invested, high (absolute value) $c^2$ parameters yield very low returns, but for capital invested higher than 80%, the relationship is reversed, and the highest IRRs are found at the lower right corner of the heat map. For low capital invested, it is clear that allowing frequent reinvestments by using a low reinvestment threshold pushes up the average total leverage of the fund, hence the higher average returns. For high capital invested, on the one hand, frequent reinvestments do not change the average total leverage much as these reinvestments are small in size (the reinvestment size is most probably lower than $(1-c^1)$ because the trader’s effective leverage before reinvestment is higher than his authorized leverage due to marked-to-market losses); however, less frequent reinvestments have two positive impacts on returns: first, the fact that hey happen rarely means the equity cushion is almost permanent,
preventing the fund from cutting its positions at losses, and second, they are made when the mispricing is already very large, meaning they yield better returns on average.

Figure 52 shows the fund’s return volatility for the same values of parameters $c^1$ and $c^2$. The surface is less smooth because volatility is more sensitive to tail events that still happen at a rather noisy rate for 1,000 simulations. As observed in IV)d)i), volatility goes up when invested capital goes up, because of the leverage effect. Volatility does seem to increase faster for high invested capital values, confirming the assumption that lower invested capital protects better against very low returns. Interestingly, low reinvestment thresholds do not seem to add volatility to low invested capital (the opposite seems to be true). My theory is that reinvesting the (very large) rest of the capital often is still a much safer strategy than investing it all for every small mispricing, whereas reinvesting only for large mispricing increases the chance that reinvestment is made during a large market perturbation, and thus increases the chance of suffering large losses at maximum leverage. This last effect will be discussed in more details in the next section, as I will prevent the fund from reinvesting when liquidity conditions are unusual.

Figure 53 plots the volatility of IRRs as a function of their corresponding returns. A linear regression between the two data series exhibits an R-squared of 57%, a beta coefficient of 32%, associated with a very satisfying T-statistic of 10, and an alpha coefficient of 3%, associated with a T-statistic of 5, and implying that these strategies are still risky for very low returns. As seen in section IV)b)i), the relationship between risk and returns seems to have positive convexity. This observation suggests that very high and very low returns are achieved with relatively higher volatility than medium returns (as can be seen on the scatter graph: plots below the regression line are situated between 10% and 25% IRRs). The Sharpe ratio heat map confirms this (Figure 54). Indeed, medium-range capital invested parameters seem to yield the
best risk-adjusted returns. Additionally, lower reinvestment thresholds seem to work better on average, but for different reasons. They yield good risk-adjusted returns for low capital invested parameters because they increase a lot the average total leverage of the fund and boost returns significantly. For medium capital invested parameters, low reinvestment thresholds have a small positive effect on returns, that is combined with a small positive effect on volatility (meaning lower volatility), probably because reinvestments are less centered on liquidity shock situations. For high capital invested parameters, low reinvestments thresholds have a negative effect on returns because reinvestments are not significant enough to make a difference, especially if they are made at a small discount relative to the first investment. This effect is barely compensated by a very small positive effect on volatility (i.e. lower volatility), probably because high reinvestment parameters make the fund’s returns too sensitive to liquidity events. As a consequence, there seems to be two risk-adjusted returns sweet spots: one for very low capital invested parameters and reinvestment thresholds, and one for medium capital invested parameters and low to medium reinvestment thresholds.
Figure 51: Average IRR as a function of parameters $c^1$ (columns) and $c^2$ (rows)

Figure 52: IRR volatility as a function of parameters $c^1$ (columns) and $c^2$ (rows)

Figure 53: Scatter graph of IRR volatility ($y$ axis) as a function average IRR ($x$ axis)
Figure 54: Fund Sharpe ratio as a function of parameters $c^1$ (columns) and $c^2$ (rows)

I set the reinvestment parameter at a 225bps loss to examine in more details the effect of modifying the capital invested parameter.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable Range</th>
<th># Ind. Var.</th>
<th>Dep. Var.</th>
<th>R Squared</th>
<th>Beta</th>
<th>Beta T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Invested</td>
<td>[30%; 100%]</td>
<td>15</td>
<td>IRR</td>
<td>87%</td>
<td>15.90%</td>
<td>9.23 **</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Vol</td>
<td>77%</td>
<td>7.67%</td>
<td>6.62 **</td>
</tr>
</tbody>
</table>

Immediately, it is noticeable that both the IRR and volatility slopes are much smaller than when there is no reinvestment threshold. Indeed, the slope for IRR is now statistically significant at 16% (vs. 26% previously) and that of volatility is statistically significant at 8% (vs. 17% previously). These coefficients are associated with R-Squared of 87% and 93%, respectively (making the regression more noisy than before because of more random reinvestment events and higher average total leverage of the fund, and because of a less linear relations between returns and capital invested).
Figure 55 uses the same scales as figure 48 for comparison purposes. The new returns are concave as a function of capital invested. They are higher on the left hand side of the graph, because boosted by the larger average total leverage of the fund, and their slope decreases fast to almost zero, making the right hand side returns below that of figure 48, which shows that the reinvestment strategy is ineffective for large initially invested capital. The volatility of the returns is much flatter on figure 55, and is larger than that of figure 48 for all values of $c^1$ (except between 85% and 100% where these values are in the same order of magnitude). Fund failures follow volatility pretty well on this case and remain above or equal to that of the pure partially invested strategy. Clearly, this new strategy is more risky on the lower side of capital invested, and does not seem to be over performing the previous strategy for high capital invested parameters. Thus, it seems that doubling down on the investment when the market initially goes against the arbitrageur is not a winning strategy. My hypothesis is that the upside captured by a larger expected return due to a larger mispricing is not enough to compensate for the downside potential of increasing the probability that the arbitrageur increases risks when the markets are perturbed and liquidity is scarce. It is interesting to note that although I had noticed the volatility catching up with returns in the previous strategy, volatility and returns are in this case completely intertwined (Figure 56), making the Sharpe ratio of the fund more stable over the parameter range (Figure 57). Sharpe ratios peak at 2.05 for the reinvestment strategy, confirming it is not better than the previous one, on a risk-adjusted excess return basis. This result confirms Shleifer and Vishny' (1995) intuition that arbitrageurs’ cost of risk can sometimes cause them to actually liquidate their investments (and cut the losses), rather than doubling down in perturbed environments. This result is at the heart of the role of the arbitrageur in the markets: should the arbitrageur be a risk taker and liquidity provider when the markets are less efficient, or is the
arbitrageur simply taking profits on small statistical variations and withdraws when the predictability of the market is hurt?
Figure 55: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of capital invested by arbitrageur (parameter $c^i$)

Figure 56: IRR (left scale) and returns volatility (right scale) as a function of $c^i$
iii. Trading on Observed Market Liquidity

In order to try to remedy the problem that increasing the fund's investment after losses corresponds to taking maximum risk at times when the probability that the market liquidity is bad, I made the assumption that the arbitrageur could observe market liquidity in real time and changed its trading algorithm so that the trader avoids doubling down when the market liquidity variable is different than 1. By doing so, the arbitrageur should increase leverage when there is a good chance that the mispricing will decrease quickly, and should not increase leverage when there is no guarantee that the market will not move significantly further against him in the next trading days.

I run the same optimal strategy analysis as I have for the reinvestment strategy with no liquidity warning. I do not use the same set of $c^2$ parameters in order to focus on more realistic ones. Indeed, parameters above 5 never yielded interesting risk-adjusted returns in the previous
case, and such marked-to-market losses appear fairly scarcely in the simulation runs. The returns surfaces are very similar; however, the simple reinvestment strategy exhibits higher returns (with on average 0.2% higher IRR for all the parameter sets that match). This is especially true for a $c^2$ parameter of -150bps, where the average IRR is 50bps higher for the previous strategy (vs. 20bps for a -300bps parameter and -10bps for a -450bps parameter). This result is logical because the liquidity warning prevents much more opportunities to double down when the loss threshold is low than when it corresponds to rare states of the world. The fact that the new strategy yields marginally better results for a large -450bps parameter confirms the intuition that it is better to only reinvest on relatively rare but large mispricings when the market liquidity is good.

The new strategy seems to be on average more volatile than the previous one. Indeed, on the same parameter sets, the new strategy exhibits 1.2% additional volatility on average. The -150bps average excess volatility is 0.8%, the -300bps is 1.1%, and the -450bps is 1.8% on average. This result is surprising for two reasons. First, the new strategy is less leveraged than the previous one, as reinvestment occurs less often. Second, the new strategy was designed to prevent the trader from reinvesting during liquidity shocks, which should on average lower the amount of equity invested during such shocks, thus lowering returns volatility. My hypothesis is that volatility measures are too noisy at this point and that these differences are not necessarily significant.

The plot of returns against volatility (Figure 60) has the same appearance as that of the previous strategy. The R-Squared on the regression is 56%, the alpha 4% with T-statistic of 7, and the beta 30%, with T-statistic of 10 (vs. 3% and 32%, respectively, for the simple reinvestment strategy). Overall, the liquidity warning strategy seems to be most efficient for relatively low capital invested parameters (30% to 50%) and small reinvestment thresholds (-
50bps to -150bps), or for medium capital invested parameters (50% to 80%) and medium reinvestment thresholds (-150bps to -250bps). This is similar to the conclusions for the pure reinvestment strategy, but yields much lower Sharpe ratios on average (10 ratios are above 2.0 and the peak is at 2.3, vs. 20 and a 3.1 peak for the previous strategy). Thus is seems that the trader is better off not taking into account the liquidity state of the markets, rather than trading accordingly.\textsuperscript{19} This could be explained by three reasons. First, this strategy seems to mitigate a problem linked to large reinvestment thresholds, but large reinvestments thresholds being already non optimal because of their returns, the correction brought by the liquidity warning feature is not useful in generating better optimal strategy returns. Second, the protection of the liquidity warning goes two ways and may prevent positive trade outcomes almost as frequently as it does negative ones. Indeed, when a liquidity shock is on and the trader doubles down his investment, either of two things can happen: if in the next period, the noise traders change views on the relative value of the two assets, the mispricing goes back to low absolute values abruptly (or even opposite values) and the arbitrageur takes his proceeds by selling his portfolio. If the noise traders continue in the same direction, then the already large mispricing becomes very large, and the arbitrageur is forced into selling part of his portfolio because he has just invested his equity cushion. He takes an important loss and his new smaller arbitrage portfolio prevents him from making the money back when the mispricing goes back down. My hypothesis is the asymmetric negative effect of forced deleveraging may be compensated by the ample positive impact of the arbitrage industry zero-reverting effect when the mispricing widens. Indeed, when the arbitrageur reinvests and enters a trade at an already high mispricing, the effect of noise traders

\textsuperscript{19} This result is interesting because it would reinforce the intuition that arbitrageurs are indeed liquidity providers and are compensated for taking risks when the market liquidity is not optimal. In this sense, it is logical that not trading during liquidity events is a worse strategy than trading when the market needs liquidity, and providing it for a small amount of time.
(although multiplied by bad liquidity) may be compensated by a very aggressive zero-reverting force before the arbitrage industry starts failing. This hypothesis would suggest that arbitrageurs working together (i.e. placing a large number of similar trades at the same time) when market liquidity is bad mutually help each other and make large profits when the time to take back liquidity from the market comes. This means that my model implies that the optimal role of arbitrageurs is to be risk-takers and liquidity providers in transitional states of the markets, rather than statistical traders hoping for the market risk factors to remain fairly constant over time. Third, it is important to note that one reason to implement the liquidity warning feature is also to protect the arbitrageur against left tail events. Although this should lower overall returns volatility, it is interesting to look a fund failures for a given set of parameters.

![Figure 58: Average IRR as a function of parameters $c^1$ (columns) and $c^2$ (rows)](image-url)
### Figure 59: IRR volatility as a function of parameters $c^1$ (columns) and $c^2$ (rows)

<table>
<thead>
<tr>
<th>$c^1$ (%)</th>
<th>0%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
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<tr>
<td>-0.5%</td>
<td>8%</td>
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<td>9%</td>
<td>7%</td>
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<td>11%</td>
<td>10%</td>
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<tr>
<td>-1.0%</td>
<td>5%</td>
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<td>7%</td>
<td>11%</td>
<td>11%</td>
<td>14%</td>
<td>11%</td>
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<tr>
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<td>8%</td>
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<td>12%</td>
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</tr>
<tr>
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<td>11%</td>
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<tr>
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<td>6%</td>
<td>6%</td>
<td>10%</td>
<td>7%</td>
<td>8%</td>
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<tr>
<td>-3.0%</td>
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<td>10%</td>
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<tr>
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<td>7%</td>
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<tr>
<td>-4.5%</td>
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<td>7%</td>
<td>9%</td>
<td>10%</td>
<td>9%</td>
<td>13%</td>
<td>12%</td>
</tr>
</tbody>
</table>

### Figure 60: Scatter graph of IRR volatility (y axis) as a function average IRR (x axis)
Figure 61: Fund Sharpe ratio as a function of parameters \(c^1\) (columns) and \(c^2\) (rows)

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable Range</th>
<th># Ind. Var.</th>
<th>Dep. Var.</th>
<th>R Squared</th>
<th>Beta</th>
<th>T-Stat</th>
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</thead>
<tbody>
<tr>
<td>Capital Invested</td>
<td>[30%; 100%]</td>
<td>15</td>
<td>IRR</td>
<td>88%</td>
<td>16.75%</td>
<td>9.90 **</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Vol</td>
<td>81%</td>
<td>9.09%</td>
<td>7.38 **</td>
</tr>
</tbody>
</table>

I run the model 1,000 times for a \(c^2\) parameter of -225bps and a \(c^1\) parameter between 30% and 100%, in order to compare the new strategy with the previous one. The regression analysis results are above. As expected, the fund’s IRR and volatility are extremely similar from one strategy to another. To illustrate, I repasted the simple reinvestment strategy first (Figure 55) using the same scale. It thus appears that there is very little effect of the liquidity warning, proving that this feature is too rarely used on average. It also appears that the risk and return profile of the liquidity warning strategy is not worse than the simple strategy for this set of parameters. However, it is clear that the number of fund failures is smaller for the new strategy than for the previous strategy (total failures of 101 vs. 116 for the simple strategy). This result
confirms that the liquidity warning does help prevent left tail events, but that it probably prevents right tail events too, as overall returns are very similar in both cases.

Figure 55: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of capital invested by arbitrageur (parameter c1) (simple strategy)
Figure 62: IRR and returns volatility (left scale), and fund failures (right scale, out of 1,000), as a function of capital invested by arbitrageur (parameter $c^I$) (Liquidity warning strategy)
Figure 63: IRR (left scale) and returns volatility (right scale) as a function of $c^j$

Figure 64: Fund Sharpe ratio as a function of parameter $c^j$
V) Conclusion

The purpose of this thesis is to test existing theories and assumptions on risks and returns of fixed income arbitrage, and their relation with economic risk factors, and to derive a better definition of the value-added of the arbitrageur in varying economic environments. Below is a summary of a number of interesting results from my model that help answer these questions.

The model simulation shows no evidence that there exists an optimal simple arbitrage strategy based on a buy-in and a buy-out threshold. Indeed, high-frequency strategies and opportunistic strategies that consist in waiting for a larger mispricing to appear can both produce attractive risk-adjusted returns, but for different reasons. High-frequency strategies enable much faster, lower trading gains with high transaction costs, but are on average less likely to be implemented in low-liquidity market environments than higher trade-in threshold strategies. Additionally, the buy-out threshold does not seem to be significantly impacting returns. That waiting for the mispricing to go down to very low levels is not more effective than liquidating at relatively high levels of mispricings can be explained by the fact that the reversion force becomes very weak at low levels of mispricings.

I show that, across strategies, the relation between arbitrage risk and return exhibits positive convexity, suggesting that risk-adjusted excess returns are optimal for a range of moderately risky strategies. The model shows that arbitrageurs can take advantage of rather high volatility environments, thus suggesting that my assumptions replicate the returns of strategies fundamentally designed to provide liquidity to the markets, rather than simply take advantage of small price discrepancies that are unrelated to varying external risk factors.
All simulations highlight the negative skewness of arbitrage return distribution. Furthermore, highly volatile environments are proved to increase both average returns and fat tails characteristics of the returns. The model confirms the liquidity dependency of arbitrage returns. What is more, it suggests that liquidity has an even larger effect on left tail events than on pure returns volatility, thus confirming that the negative skewness of return distribution is partially caused by rare, large liquidity shortages. Interestingly, the probability of liquidity shocks has an inverse relation with return volatility and left tail fatness. Indeed, it seems that higher probability of shocks affect return volatility more than it does left tails of returns distribution.

An innovative contribution of this thesis is the modeling of the arbitrage industry reversion force. It leads to interesting considerations. First, it highlights the fact that, because of its small size, a single arbitrageur relies on other rational relative value investors to make money. As such, returns increase (up to a point) as a function of arbitrage industry reversion force. The effect of competition on volatility is however always negative, as trading on smaller, faster moving mispricings is an overall more risky strategy than trading on potentially very large, slowly reverting mispricings. Indeed, an arbitrageur trading on small, aggressively reverting mispricings is much more vulnerable to sudden changes in market liquidity. Finally, the model confirms that very competitive environments reinforce the negative skewness of return distribution, and bring arbitrage strategies close to being martingale processes.

The model’s output shows that arbitrage strategies’ intrinsic leverage increases return fat tail effects more than it does return volatility. Additionally, results suggest that intrinsic leverage is mostly detrimental to the fund’s risk-adjusted excess returns, meaning that it is probably
optimal for the fund’s investors to leverage their equity invested in the fund, and require the fund
to have low leveraged strategies with very low volatility.

The simulation suggests that the stop-loss effects of redemptions and margin calls play a
large part in the arbitrageur’s apparent good resilience to varying liquidity environments. Indeed,
reasonably indexing authorized leverage to market volatility is a good risk management practice
as it acts as an efficient stop-loss in disturbed environments. It is even more efficient at
mitigating the left fat tails of the returns than it is at generating better Sharpe ratios. Thus, a
relatively tight margin call policy (or, even better, an endogenous stop-loss policy) works in the
mutual interest of the fund, its investors, and its lenders. The model confirms the perverse effects
of performance-based arbitrage. Specifically, small, progressive deleveraging in case of losses
does not hurt returns as much as a sudden, large forced deleveraging. In this sense, it seems that
the hurdle effect of large first redemptions hurts the fund’s IRR severely. Moreover, investor
withdrawals in time of distress have been proved to accentuate the negative skewness of
arbitrage funds return distribution. Nevertheless, it appears that the stop-loss effect of
progressive forced liquidation positively influences arbitrage return volatility.

The additions to the model’s trading algorithm allow some interesting findings. The
simulation suggests that when very little capital is set aside, occasional forced deleveraging
lowers overall returns. It shows that, after a certain point, keeping more capital aside adds no
(risk-adjusted) value. It is clear that setting some capital aside effectively protects the fund
against very low returns (left tail events). Interestingly, it seems that doubling down on the
investment when the market initially goes against the arbitrageur is not a winning strategy. My
hypothesis is that the upside captured by a larger expected return due to a larger average traded
mispricing is not enough to compensate for the downside potential of increasing the probability
that the arbitrageur boosts his total leverage when the markets are perturbed and liquidity is scarce. This result indicates that an arbitrageur can only be a liquidity provider up to a point and that his cost of risk can become too high to play his role. Interestingly, results hint that the trader is better off not taking into account the liquidity state of the markets (although the liquidity warning does help prevent left tail events), rather than trading accordingly. My hypothesis is that the asymmetric negative effect of forced deleveraging may be compensated by the ample positive impact of the arbitrage industry zero-reverting effect when the mispricing widens. This means that my model implies that the optimal role of arbitrageurs is to be risk-takers and liquidity providers in disturbed states of the markets, rather than statistical traders hoping for the market risk factors to remain fairly constant over time.
Work Cited


Appendix

1) Additional definitions

A Bond is “A debt investment in which an investor loans money to an entity (corporate or governmental) that borrows the funds for a defined period of time at a fixed interest rate.” Bonds are market traded, whereas other forms of debt such as bank loans are far less liquid.

The maturity of a bond is the period of time for which it remains outstanding.

Also known as face value of a bond, the principal is the amount owned by the borrower to the lender. The principal of a bond is paid at maturity.

The interest rate is “the amount charged, expressed as a percentage of principal, by a lender to a borrower for the use of assets.”

The yield is the return an investor will realize on a bond.

Default is “the failure to promptly pay interest or principal when due. Default occurs when a debtor is unable to meet the legal obligation of debt repayment.”

Leverage is the level of indebtedness of a corporation, vehicle, or individual. It is usually defined as the ratio of debt to equity. For instance, an investment fund that raised $1bn of capital from investors and $2bn of debt from banks and other lenders is leveraged two to one.

A commercial paper is an unsecured, short-term note issued by large, creditworthy corporations to finance their short-term obligations.

The federal funds rate is the overnight, uncollateralized interest rate at which depository institutions trade balances at the Federal Reserve with each other.
The LIBOR (London Interbank Offered Rate) is an interest rate at which London large and creditworthy banks can borrow funds from other banks, on an unsecured basis (i.e. without posting collateral). The LIBOR serves as a benchmark for many variable interest rate calculations.

Money market funds are mutual funds that invest in short term, low risk fixed income securities such as Treasury bills and commercial paper. They usually have an initial Net Asset Value (NAV) of $1 and try to never lose money. When they do and their NAV is below $1, they are said to break the buck.

Time deposits are money deposits at a bank that cannot be withdrawn for a certain period of time.

Eurodollars are time deposits denominated in U.S. dollars at banks outside the United States.

A futures is a derivative contract that sets a predetermined future date and price for an asset to change hands.

An option is a derivative that offers its buyer the right to buy (call) or sell (put) an asset at an agreed-upon price during a predetermined period of time.

An interest rate swap is a contract whereby one stream of future interest payments is exchanged for another based on a specified principal amount. Usually, a floating and fixed stream will be exchanged.

An inflation swap is a derivative instrument that allows one party to exchange a fixed rate against a floating rate linked to the consumer price index (CPI).
An interest rate cap (resp. floor) is a derivative contract where the buyer receives payments at the end of each period in which the interest rate is higher (resp. lower) than the agreed strike price.

A swaption is an option on a swap.

2) Details of the 2008 financial crisis events

At the beginning of the crisis period, the markets became increasingly concerned with the potential failure of some of their counterparties, and consequently became hesitant in their day-to-day trading and interbank lending activities. The cause of this concern is the very large amount of US subprime MBS that many banks held on their books. As interest rates were going up from 2002 to 2008, subprime loans defaulted increasingly, leading to many foreclosures. US real estate market stumbled because of the higher interest rates and increasing amount of foreclosures. This led to concern regarding the true value of the issued MBS and their collateral. As a result, some banks’ large holdings in MBS became problematic and threatened their potential solvability, seizing up liquidity in interbank lending. As an illustration of the growing concerns around the liquidity of the interbank lending process, the 1-month LIBOR drifted up 19 basis points in August 2007.

In order to prevent the financial system from shutting down, the Federal Reserve felt the need to take a number of measures. As described in Taliaferro and Blyth (2011) and Buraschi, Sener, and Mengütürk (2010), in December 2007, the Federal Reserve was concerned about market liquidity and introduced a term auction facility (TAF) offering temporary funding to US financial institutions, and started accepting a very diverse set of securities as collateral. By being anonymous, the TAF was supposed to solve the discount window (DW) borrowing stigma. The
DW is used by the Fed to provide funding to illiquid but solvent banks, however, it has often been noticed that banks usually prefer alternative, more expensive sources of funding to the discount window because they do not want to be labeled as illiquid. Because the US dollar funding markets were disrupted overseas, the Federal Reserve also introduced swap lines with the European Central Bank (ECB) and the Swiss National Bank (SNB). In February 2008, Northern Rock collapsed. In March, the Federal Reserve facilitated the purchase of Bear Sterns by J.P. Morgan Chase. In September, Fannie and Freddie were placed into conservatorship and Lehman Brothers filed for bankruptcy and the swap lines initial caps of $20bn and $4bn, respectively, were raised.

The same month, the Reserve Primary Money Fund broke the buck and all banks struggled to replace their Lehman trading positions; they started trading in large chunks, pushing up market volatility. In October, the government passed the $700bn Troubled Asset Relief Program (TARP) to purchase assets from financial institutions. The Government wanted to directly address the credit risk of large financial institutions by increasing their available capital and strengthening their balance sheets. The Treasury purchased $125bn worth of preferred shares from the 9 largest US banks. The same month, the Federal Reserve launched the Commercial Paper Funding Facility (CPFF) in order to improve liquidity in short-term funding markets and thereby contribute to greater availability of credit for businesses and households. This was done through the purchase of commercial papers from money market funds. Lastly in October, AIG, which was on the verge of bankruptcy for having bought very large amounts of CDS on MBS, received a $38bn loan from the Federal Reserve, and the swap lines were entirely uncapped, and available to a wider range of central banks. In November, the Federal Reserve launched the Term Asset-Backed Securities Loan Facility (TALF). Over the two previous months, ABS issuances
for individuals and small businesses had declined and interest rates on such ABS had skyrocketed. In the Federal Reserve’s words, “the TALF is designed to increase credit availability and support economic activity by facilitating renewed issuance of consumer and small business ABS at more normal interest rate spreads”.

After addressing liquidity and credit risk concerns, the Federal Reserve focused on tail risk in the financial industry. In February 2009, the Federal Reserve implemented the Supervisory Capital Assessment Program (SCAP), a stress test that would assess the capacity of the largest US financial institutions (with assets of more than $100bn) to resist pessimistic macroeconomic scenarios. The purpose was to reduce uncertainty on the fundamental value of these financial institutions and regain the trust of market participants. The results on the 19 institutions tested were released on May 7, 2009. I will not develop other events (such as the Eurozone sovereign debt crisis) and policy measures (such as Quantitative Easing in the US, Europe, and Japan) related to the economic crisis that followed the financial debacle of 2008 because they do not fall under the pure definition of the financial crisis as I wish to look at it, and because these events and measures did not affect fixed income pricing in the same way as the 2008 events did.

3) Notations

<table>
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<tr>
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<td>Noise effect standard deviation</td>
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<td>L_t</td>
<td>Liquidity shocks</td>
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<td>Probability of a shock</td>
</tr>
<tr>
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<td>V_t</td>
<td>Zero-reverting force</td>
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<td>f'</td>
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<td>Peak of the reversion</td>
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<td>Acceleration factor</td>
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<td>X_t</td>
<td>Leverage available to the arbitrage industry</td>
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<td>i</td>
<td>Value of the arithmetic average of the two asset prices</td>
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<td>Wealth index of the modeled arbitrageur</td>
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<td>Δ</td>
<td>Lag period between fund performance and capital available to industry effect</td>
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<tr>
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<td>Industry capital smoothing moving average period</td>
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<td>Asset volatility EWMA decay factor</td>
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