Observation of the Resonant Character of the $Z(4430)^-$ State

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Resonant structures in $B^0 \rightarrow \psi' \pi^- K^+$ decays are analyzed by performing a four-dimensional fit of the decay amplitude, using $p p$ collision data corresponding to 3 fb$^{-1}$ collected with the LHCb detector. The data cannot be described with $K^+\pi^-$ resonances alone, which is confirmed with a model-independent approach. A highly significant $Z(4430)^- \rightarrow \psi'\pi^-$ component is required, thus confirming the existence of this state. The observed evolution of the $Z(4430)^-$ amplitude with the $\psi'\pi^-$ mass establishes the resonant nature of this particle. The mass and width measurements are substantially improved. The spin parity is determined unambiguously to be $1^+$. DOI: 10.1103/PhysRevLett.112.222002 PACS numbers: 14.40.Rt, 13.25.Gv, 13.25.Hw, 14.40.Nd

The existence of charged charmonium-like states has been a topic of much debate since the Belle Collaboration found evidence for a narrow $Z(4430)^-$ peak, with width $\Gamma = 45^{+18+30}_{-13-13}$ MeV, in the $\psi'\pi^-$ mass distribution ($m_{\psi'\pi^-}$) in $B \rightarrow \psi'K\pi^-$ decays ($K = K_0$ or $K^+$) [1,2]. As the minimal quark content of such a state is $c\bar{c}d\bar{u}$, this observation could be interpreted as the first unambiguous evidence for the existence of mesons beyond the traditional $q\bar{q}$ model [3]. This has contributed to a broad theoretical interest in this state [4–20]. Exotic $\chi_{c1,2}\pi^-$ structures were also reported by the Belle collaboration in $B \rightarrow \chi_{c1,2}K\pi^-$ decays [21]. Using the $K^+ \rightarrow K\pi^-$ invariant mass ($m_{K\pi}$) and helicity angle ($\theta_{K^+}$) [22–24] distributions, the BABAR Collaboration was able to describe the observed $m_{\psi'\pi^-}$ and $m_{\chi_{c1,2}\pi^-}$ structures in terms of reflections of any $K^+$ states with spin $J \leq 3$ ($J \leq 1$ for $m_{K\pi} < 1.2$ GeV) without invoking exotic resonances [25,26]. However, the BABAR results did not contradict the Belle evidence for the $Z(4430)^-$ state. The Belle Collaboration subsequently updated their $Z(4430)^-$ results with a two-dimensional [27], and later a four-dimensional (4D), amplitude analysis [28] resulting in a $Z(4430)^-$ significance of 5.2$\sigma$, a mass of $M_{Z^-} = 4485 \pm 22^{+28}_{-11}$ MeV, a large width of $\Gamma_{Z^-} = 200^{+41+26}_{-46-35}$ MeV, an amplitude (defined further below) of $f_{Z^-} = (10.3^{+3.0+4.3}_{-3.5-2.3})\%$ and spin-parity $J^P = 1^{+}$ favored over the other assignments by more than 3.4$\sigma$. Other candidates for charged four-quark states have been reported in $e^+e^- \rightarrow \pi^+\pi^- \Upsilon(nS)$ [29,30], $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ [31,32], $e^+e^- \rightarrow \pi^+\pi^- h_c$ [33], and $e^+e^- \rightarrow (D^+D^*)^{0}\pi^+$ [34] processes.

In this Letter, we report a 4D model-dependent amplitude fit to a sample of $25,176 \pm 174$ $B^0 \rightarrow \psi'K^+\pi^-$, $\psi' \rightarrow \mu^+\mu^-$ candidates reconstructed with the LHCb detector in $p p$ collision data corresponding to 3 fb$^{-1}$ collected at $\sqrt{s} = 7$ and 8 TeV. The tenfold increase in signal yield over the previous measurement [28] improves sensitivity to exotic states and allows their resonant nature to be studied in a novel way. We complement the amplitude fit with a model-independent approach [25].

The LHCb detector is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, described in detail in Ref. [35]. The $B^0$ candidate selection follows that in Ref. [36] accounting for the different number of final-state pions. It is based on finding $(\psi' \rightarrow \mu^+\mu^-)K^+\pi^-$ candidates using particle identification information, transverse momentum thresholds, and requiring separation of the tracks and of the $B^0$ vertex from the primary $pp$ interaction points. To improve modeling of the detection efficiency, we exclude regions near the $K^+\pi^-$ vs $\psi'\pi^-$ Dalitz plot boundary, which reduces the sample size by 12%. The background fraction is determined from the $B^0$ candidate invariant mass distribution to be $(4.1 \pm 0.1)\%$. The background is dominated by combinations of $\psi'$ mesons from $B$ decays with random kaons and pions.

Amplitude models are fit to the data using the unbinned maximum likelihood method. We follow the formalism and notation of Ref. [28] with the 4D amplitude dependent on $\Phi = (m^2_{K^+\pi^-}, m^2_{\psi'\pi^-}, \cos\theta_{\psi'}, \phi)$, where $\theta_{\psi'}$ is the $\psi'$ helicity angle and $\phi$ is the angle between the $K^+$ and $\psi'$ decay planes in the $B^0$ rest frame. The signal probability density function (PDF) $S(\Phi)$ is normalized by summing over simulated events. Since the simulated events are passed through the detector simulation [37], this approach implements 4D efficiency corrections without use of a parametrization. We use $B^0$ mass sidebands to obtain a parametrization of the background PDF.

As in Ref. [28], our amplitude model includes all known $K^{*0} \rightarrow K^+\pi^-$ resonances with nominal mass within or slightly above the kinematic limit (1593 MeV) in $B^0 \rightarrow \psi'K^+\pi^-$ decays: $K_0^*(800)$, $K_0^*(1430)$ for $J = 0$; $K^*(892)$,
Collaboration [25,26], which does not constrain the model-independent approach developed by the LASS parametrization [39,40], in which the NR and $K^*_0(800)$ components are replaced with an elastic scattering term (two free parameters) interfering with the $K^*_0(1430)$ resonance.

To probe the quality of the likelihood fits, we calculate a binned $\chi^2$ variable using adaptive 4D binning, in which we split the data once in $[\cos \theta_{\psi}]$, twice in $\Phi$, and then repeatedly in $m_{K^+\pi^-}^2$ and $m_{\psi\pi}$, preserving any bin content above 20 events, for a total of $N_{\text{bin}} = 768$ bins. Simulations of many pseudoexperiments, each with the same number of signal and background events as in the data sample, show that the $p$ value of the $\chi^2$ test ($p_{\chi^2}$) has an approximately uniform distribution assuming that the number of degrees of freedom (NDF) equals $N_{\text{bin}} - N_{\text{par}} - 1$, where $N_{\text{par}}$ is the number of unconstrained parameters in the fit. Fits with all $K^*$ components and either of the two different $J = 0$ models do not give a satisfactory description of the data; the $p_{\chi^2}$ is below $2 \times 10^{-6}$, equivalent to 4.8$\sigma$ in the Gaussian distribution. If the $K^*_3(1780)$ component is excluded from the amplitude, the discrepancy increases to 6.3$\sigma$.

This is supported by an independent study using the model-independent approach developed by the BABAR Collaboration [25,26], which does not constrain the analysis to any combination of known $K^*$ resonances, but merely restricts their maximal spin. We determine the Legendre polynomial moments of $\cos \theta_{\psi}$, as a function of $m_{K^+\pi^-}$, from the sideband-subtracted and efficiency-corrected sample of $B^0 \rightarrow \psi' K^+\pi^-$ candidates. Together with the observed $m_{K^+\pi^-}$ distribution, the moments corresponding to $J \leq 2$ are reflected into the $m_{\psi\pi}$ distribution using simulations as described in Ref. [25]. As shown in Fig. 1, the $K^*$ reflections do not describe the data in the $Z(4430)^-$ region. Since a $Z(4430)^-$ resonance would contribute to the $\cos \theta_{\psi}$ moments, and also interfere with the $K^*$ resonances, it is not possible to determine the $Z(4430)^-$ parameters using this approach. The amplitude fit is used instead, as discussed below.

If a $Z(4430)^-$ component with $J^P = 1^+$ (hereafter $Z^+_1(1780)$) is added to the amplitude, the $p_{\chi^2}$ reaches 4% when all the $K^* \rightarrow K^+\pi^-$ resonances with a pole mass below the kinematic limit are included. The $p_{\chi^2}$ rises to 12% if the $K^*(1680)$ is added (see Fig. 2), but fails to improve when the $K^*_3(1780)$ is also included. Therefore, as in Ref. [28] we choose to estimate the $Z^+_1(1780)$ parameters using the model with the $K^*(1680)$ as the heaviest $K^*$ resonance. In Ref. [28] two independent complex $Z^+_1$ helicity couplings, $H^+_{1\lambda'}$ for $\lambda' = 0, +1$ (parity conservation requires $H^+_{10} = H^-_{10}$), were allowed to float in the fit. The small energy release in the $Z^+_1$ decay suggests neglecting $D$-wave decays. A likelihood-ratio test is used to discriminate between any pair of amplitude models based on the log-likelihood difference $\Delta(-2 \ln L)$ [41]. The $D$-wave contribution is found to be insignificant when allowed in the fit, 1.3$\sigma$ assuming Wilks’s theorem [42]. Thus, we assume a pure $S$-wave decay, implying $H^+_{11} = H^-_{11}$. The significance of the $Z^+_1$ is evaluated from the likelihood ratio of the fits without and with the $Z^+_1$ component. Since the condition of the likelihood regularity in $Z^+_1$ mass and width is not satisfied when the no-$Z^+_1$ hypothesis is imposed, use of Wilks’s theorem is not justified [43,44]. Therefore, pseudoexperiments are used to predict the distribution of $\Delta(-2 \ln L)$ under the no-$Z^+_1$ hypothesis, which is found to be well described by a $\chi^2$ PDF with NDF $= 7.5$. Conservatively, we assume NDF $= 8$, twice the number of free parameters in the $Z^+_1$ amplitude. This yields a $Z^+_1$ significance for the default $K^*$ model of 18.7$\sigma$. The lowest significance among all the systematic variations to the model discussed below is 13.9$\sigma$.

The default fit gives $M_{Z^+_1} = 4475 \pm 7$ MeV, $\Gamma_{Z^+_1} = 172 \pm 13$ MeV, $f_{Z^+_1} = (5.9 \pm 0.9)\%$, $f_{NR} = (0.3 \pm 0.8)\%$, $f_{K^*_0(800)} = (3.2 \pm 2.2)\%$, $f_{K^*(892)} = (59.1 \pm 0.9)\%$, $f_{K^*(1410)} = (1.7 \pm 0.8)\%$, $f_{K^*_0(1430)} = (3.6 \pm 1.1)\%$, $f_{K^*(1430)} = (7.0 \pm 0.4)\%$, and $f_{K^*(1680)} = (4.0 \pm 1.5)\%$, which are consistent with the Belle results [28] even without considering systematic uncertainties. Above, the amplitude fraction of any component $R$ is defined as $f_R = \int S_R(\Phi)d\Phi/\int S(\Phi)d\Phi$, where in $S_R(\Phi)$ all except the $R$ amplitude terms are set to zero. The sum of all amplitude fractions is not 100% because of interference effects. To assign systematic errors, we vary the $K^*$ models by removing the $K^*(1680)$ or adding the $K^*_3(1780)$ in the amplitude ($f_{K^*_3(1780)} = (0.5 \pm 0.2)\%$), use

![FIG. 1 (color online). Background-subtracted and efficiency-corrected $m_{\psi\pi}$ distribution (black data points), superimposed with the reflections of $\cos \theta_{\psi}$ moments up to order 4, allowing for $J(K^*) \leq 2$ (blue line) and their correlated statistical uncertainty (yellow band bounded by blue dashed lines). The distributions have been normalized to unity.](image-url)
the LASS function as an alternative $K^+$ S-wave representation, float all $K^+$ masses and widths while constraining them to the known values [38] within their measured uncertainties, allow a second $Z^-$ component, increase the orbital angular momentum assumed in the $B^0$ decay, allow a $D$-wave component in the $Z^-_1$ decay, change the effective hadron size in the Blatt-Weisskopf form factors from the default 1.6 [28] to 3.0 GeV$^{-1}$, let the background fraction float in the fit or neglect the background altogether, tighten the selection criteria probing the efficiency simulation, and use alternative efficiency and background implementations in the fit. We also evaluate the systematic uncertainty from the formulation of the resonant amplitude. In the default fit, we follow the approach of Eq. (2) in Ref. [28] that uses a running mass $M_R$ in the $(p_R/M_R)^2$ term, where $M_R$ is the invariant mass of two daughters of the $R$ resonance; $p_R$ is the daughter’s momentum in the rest frame of $R$ and $L_R$ is the orbital angular momentum of the decay. The more conventional formulation [38,45] is to use $p_R^{\pm 1}$ (equivalent to a fixed $M_R$ mass). This changes the $Z^-_1$ parameters via the $K^+$ terms in the amplitude model: $M_{Z^-_1}$ varies by $-22$ MeV, $\Gamma_{Z^-_1}$ by $+29$ MeV, and $f_{Z^-_1}$ by $+1.7\%$ (the $p_R^2$ drops to $7\%$). Adding all systematic errors in quadrature we obtain $M_{Z^-_1} = 4475 \pm 7_{-25}^{+15}$ MeV, $\Gamma_{Z^-_1} = 172 \pm 13_{-24}^{+37}$ MeV, and $f_{Z^-_1} = (5.9 \pm 0.9_{-1.3}^{+1.2})\%$. We also calculate a fraction of $Z^-_1$ that includes its interferences with the $K^+$ resonances as $f_{Z^-_1} = 1 - \int S_{no-Z^-_1}(\Phi)d\Phi/\int S(\Phi)d\Phi$, where the $Z^-_1$ term in $S_{no-Z^-_1}(\Phi)$ is set to zero. This fraction $(16.7 \pm 1.6_{-4.5}^{+4.5})\%$ is much larger than $f_{Z^-_1}$, implying large constructive interference.

To discriminate between various $J^P$ assignments we determine the $\Delta(-2\ln L)$ between the different spin hypotheses. Following the method of Ref. [28], we exclude the $0^-$ hypothesis in favor of the $1^+$ assignment at 25.7$s$ in the fits with the default $K^+$ model. Such a large rejection level is expected according to the $\Delta(-2\ln L)$ distribution of the pseudoexperiments generated under the $1^+$ hypothesis. For large data samples, assuming a $\chi^2$ (NDF = 1) distribution for $\Delta(-2\ln L)$ under the disfavored $J^P$ hypothesis gives a lower limit on the significance of its rejection [46]. This method gives more than 17.8$s$ rejection. Since the latter method is conservative and provides sufficient rejection, we employ it while studying systematic effects. Among all systematic variations described above, allowing the $K^+_2(1780)$ in the fit produces the weakest rejection. Relative to $1^+$, we rule out the $0^-$, $1^-$, $2^+$, and $2^-$ hypotheses by at least $9.7$s, $15.8$s, $16.1$s, and $14.6$s, respectively. This reinforces the $5.1$s ($4.7$s) rejection of the $2^+$ ($2^-$) hypotheses previously reported by the Belle Collaboration [28], and confirms the $3.4$s ($3.7$s) indications from Belle that $1^+$ is favored over $0^-$ ($1^-$). The
positive parity rules out the possibility that the \(Z(4430)^-\) state is a \(D^*(2007)\bar{D}_1(2420)\) threshold effect as proposed in Refs. [4,14].

In the amplitude fit, the \(Z_1^-\) is represented by a Breit-Wigner amplitude, where the magnitude and phase vary with \(m_{\psi'\pi^-}\) according to an approximately circular trajectory in the \((\text{Re } A^Z, \text{Im } A^Z)\) plane (Argand diagram [38]), where \(A^Z\) is the \(m_{\psi'\pi^-}\)-dependent part of the \(Z_1^-\) amplitude.

We perform an additional fit to the data, in which we represent the \(Z_1^-\) amplitude as the combination of independent complex amplitudes at six equidistant points in the \(m_{\psi'\pi^-}\) range covering the \(Z_1^-\) peak, 18.0–21.5 GeV\(^2\). Thus, the \(K^+\) and the \(Z_1^-\) components are no longer influenced in the fit by the assumption of a Breit-Wigner amplitude for the \(Z_1^-\). The resulting Argand diagram, shown in Fig. 3, is consistent with a rapid change of the \(Z_1^-\) phase when its magnitude reaches the maximum, a behavior characteristic of a resonance.

If a second \(Z^\pm\) resonance is allowed in the amplitude with \(J^P = 0^- (Z_0^-)\) the \(p_{\psi'\pi^-}\) of the fit improves to 26%. The \(Z_0^-\) significance from the \(\Delta(-2 \ln L)\) is 6\(\sigma\) including the systematic variations. It peaks at a lower mass 4239 \(\pm 18_{-10}^{+45}\) MeV, and has a larger width 220 \(\pm 47_{-108}^{+108}\) MeV, with a much smaller fraction, \(f_{Z_0^-} = (1.6 \pm 0.5_{-0.5}^{+1.0})\%\) \((f_{Z_0^-} = (2.4 \pm 1.1_{-0.7}^{+0.7})\%\) than the \(Z_1^-\). With the default \(K^+\) model, 0\(^\pm\) is preferred over 1\(^-\), 2\(^-\), and 2\(^+\) by 8\(\sigma\). The preference over 1\(^-\) is only 1\(\sigma\). However, the width in the 1\(^+\) fit becomes implausibly large, 660 \(\pm 150\) MeV. The \(Z_0^-\) has the same mass and width as one of the \(\chi_{c1}\pi^-\) states reported previously [21], but a 0\(^-\) state cannot decay strongly to \(\chi_{c1}\pi^-\). Figure 4 compares the \(m_{\psi'\pi^-}^2\) projections of the fits with both \(Z_0^-\) and \(Z_1^-\), or the \(Z_1^-\) component only. The model-independent analysis has a large statistical uncertainty in the \(Z_0^-\) region and shows no deviations of the data from the reflections of the \(K^+\) degrees of freedom (Fig. 1). Argand diagram studies for the \(Z_0^-\) are inconclusive. Therefore, its characterization as a resonance will need confirmation when larger samples become available.

In summary, an amplitude fit to a large sample of \(B^0 \rightarrow \psi'K^+\pi^-\) decays provides the first independent confirmation of the existence of the \(Z(4430)^-\) resonance and establishes its spin parity to be 1\(^-\), both with very high significance. The positive parity rules out the interpretation in terms of \(D^*(2007)\bar{D}_1(2420)\) [4,14] or \(D^*(2007)\bar{D}_2(2460)\) threshold effects, leaving the four-quark bound state as the only plausible explanation. The measured mass 4475 \(\pm 7_{-25}^{+35}\) MeV, width 172 \(\pm 13_{-34}^{+37}\) MeV, and amplitude fraction \((5.9 \pm 0.9_{-1.6}^{+1.5})\%\) are consistent with, but more precise than, the Belle results [28]. An analysis of the data using the model-independent approach developed by the BABAR collaboration [25] confirms the inconsistencies in the \(Z(4440)^-\) region between the data and \(K^+\pi^-\) states with \(J \leq 2\). The \(D^-\) wave contribution is found to be insignificant in \(Z(4440)^-\) decays, as expected for a true state at such mass. The Argand diagram obtained for the \(Z(4440)^-\) amplitude is consistent with the resonant behavior; among all observed candidates for charged four-quark states, this is the first to have its resonant character confirmed in this manner.

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FIG. 3 (color online). Fitted values of the \(Z_1^-\) amplitude in six \(m_{\psi'\pi^-}\) bins, shown in an Argand diagram (connected points with the error bars, \(m_{\psi'\pi^-}\) increases counterclockwise). The red curve is the prediction from the Breit-Wigner formula with a resonance mass (width) of 4475 (172) MeV and magnitude scaled to intersect the bin with the largest magnitude centered at (4477 MeV)\(^2\). Units are arbitrary. The phase convention assumes the helicity-zero \(K^+\) (892) amplitude to be real.

FIG. 4 (color online). Distribution of \(m_{\psi'\pi^-}^2\) in the data (black points) for \(1.0 < m_{\psi'\pi^-}^2 < 1.8\) GeV\(^2\) [\(K^+(892), K^+_2(1430)\) veto region] compared with the fit with two, 0\(^-\) and 1\(^-\) (solid-line red histogram) and only one 1\(^-\) (dashed-line green histogram) \(Z^\pm\) resonances. Individual \(Z^\pm\) terms (blue points) are shown for the fit with two \(Z^\pm\) resonances.
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[2] The inclusion of charge-conjugate states is implied in this Letter. We use units in which c = 1.
[42] See, e.g., Sec. 10.5.2 of Ref. [41] on asymptotic distribution of Δ(−2 ln L) for continuous families of hypotheses.
[44] With the mass and width floated in the fit a look-elsewhere effect must be taken into account.
[46] See Sec. 10.5.7 of Ref. [41] on testing separate hypotheses.
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