Measurement of event-plane correlations in \( \sqrt{s_{NN}} = 2.76 \) TeV lead-lead collisions with the ATLAS detector

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A measurement of event-plane correlations involving two or three event planes of different order is presented as a function of centrality for 7 \( \mu b^{-1} \) Pb + Pb collision data at \( \sqrt{s_{NN}} = 2.76 \) TeV, recorded by the ATLAS experiment at the Large Hadron Collider. Fourteen correlators are measured using a standard event-plane method and a scalar-product method, and the latter method is found to give a systematically larger correlation signal. Several different trends in the centrality dependence of these correlators are observed. These trends are not reproduced by predictions based on the Glauber model, which includes only the correlations from the collision geometry in the initial state. Calculations that include the final-state collective dynamics are able to describe qualitatively, and in some cases also quantitatively, the centrality dependence of the measured correlators. These observations suggest that both the fluctuations in the initial geometry and the nonlinear mixing between different harmonics in the final state are important for creating these correlations in momentum space.

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I. INTRODUCTION

Heavy-ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) create hot and dense matter that is thought to be composed of strongly interacting quarks and gluons. One striking observation that supports this picture is the large momentum anisotropy of particle emission in the transverse plane. This anisotropy is believed to be the result of anisotropic expansion of the created matter driven by the pressure gradients, with more particles emitted in the direction of the largest gradients [1]. The collective expansion of the matter can be modeled by relativistic viscous hydrodynamic theory [2]. The magnitude of the azimuthal anisotropy is sensitive to transport properties of the matter, such as the ratio of the shear viscosity to the entropy density and the equation of state [3].

The anisotropy of the particle distribution (\( dN/d\phi \)) in azimuthal angle \( \phi \) is customarily characterized by a Fourier series,

\[
\frac{dN}{d\phi} \propto 1 + 2 \sum_{n=1}^{\infty} v_n \cos n(\phi - \Phi_n),
\]

where \( v_n \) and \( \Phi_n \) represent the magnitude and phase (referred to as the event plane) of the \( n \)-th order azimuthal anisotropy (or flow) at the corresponding angular scale. These quantities can also be conveniently represented in a two-dimensional vector format or in the standard complex form [4,5]:

\[
\vec{v}_n = (v_n \cos n\Phi_n, v_n \sin n\Phi_n) \quad \text{or} \quad v_n e^{i n\Phi_n}.
\]

In noncentral collisions, the overlap region of the initial geometry has an almost elliptic shape. The anisotropy is therefore dominated by the second harmonic term, \( v_2 \). However, first-order \( (n = 1) \) and higher-order \( (n > 2) \) \( v_n \) coefficients have also been observed [6–8]. These coefficients have been related to additional shape components arising from the fluctuations of the positions of nucleons in the overlap region. The amplitude and the directions of these shape components can be estimated via a simple Glauber model [9] from the transverse positions \( r, \phi \) of the participating nucleons relative to their center of mass [10]:

\[
\epsilon_n = \frac{(r^n \cos n\phi)^2 + (r^n \sin n\phi)^2}{\langle r^n \rangle},
\]

\[
n\Phi_n^* = \arctan \left( \frac{\langle r^n \sin n\phi \rangle}{\langle r^n \cos n\phi \rangle} \right) + \pi,
\]

where \( \epsilon_n \) is the eccentricity and the angle \( \Phi_n^* \) is commonly referred to as the participant-plane (PP) angle. These shape components are transferred via hydrodynamic evolution into higher-order azimuthal anisotropy in momentum space. For small \( \epsilon_n \) values, one expects \( v_n \propto \epsilon_n \) and the \( \Phi_n \) to be correlated with the minor-axis direction given by \( \Phi_n^* \). However, model calculations show that the values of \( \epsilon_n \) are large, and the alignment between \( \Phi_n \) and \( \Phi_n^* \) is strongly violated for \( n > 3 \) owing to nonlinear effects in the hydrodynamic evolution [11].

Detailed measurements of \( v_n \) have been performed at RHIC and the LHC, and nonzero \( v_n \) values are observed for \( n \leq 6 \) [6–8,12–16], consistent with the existence of sizable fluctuations in the initial state. Further information on these fluctuations can be obtained by studying the correlations between \( \Phi_n \) of different order. If the fluctuations in \( \epsilon_n \) are small and totally random, the orientations of \( \Phi_n \) of different order are expected to be uncorrelated. Calculations based on the Glauber model reveal strong correlations between some PP angles such as \( \Phi_2^* \) and \( \Phi_1^* \) or \( \Phi_2^* \) and \( \Phi_6^* \) [10] and weak correlations between others such as \( \Phi_2^* \) or \( \Phi_4^* \) and \( \Phi_3^* \) [17]. Previous measurements at RHIC and the LHC support a weak correlation between \( \Phi_2 \) and \( \Phi_3 \) [18,19] and a strong correlation between \( \Phi_2 \) and \( \Phi_4 \) [20]. The former is
consistent with no strong correlation between $\Phi^3_T$ and $\Phi^3_T$ and the dominance of linear response for elliptic flow and triangular flow, i.e., $\Phi^3_T \approx \Phi^3_T$ and $\Phi^3_T \approx \Phi^3_T$. The latter is consistent with a significant nonlinear hydrodynamic response for quadrangular flow, which couples $v_3$ to $v_3^2$. The correlations among three event planes of different order have also been investigated in a model framework and several significant correlators have been identified [10,21–23]. However, no published experimental measurements on three-plane correlations exist to date. A measurement of the correlations between two and three event planes can shed light on the patterns of the fluctuations of the initial-state geometry and nonlinear effects in the final state.

II. ATLAS DETECTOR AND TRIGGER

The ATLAS detector [24] provides nearly full solid angle coverage of the collision point with tracking detectors, calorimeters, and muon chambers, which are well suited for measurements of azimuthal anisotropies over a large pseudorapidity range. This analysis primarily uses three subsystems to measure the event plane: the inner detector (ID), the barrel and end-cap electromagnetic calorimeters (ECals), and the forward calorimeter (FCal). The ID is contained within the 2-T field of a superconducting solenoid magnet, and measures the trajectories of charged particles in the pseudorapidity range $|\eta| < 2.5$ and over the full azimuth. A charged particle passing through the ID typically traverses three modules of the silicon pixel detector (“pixel”), four double-sided silicon strip modules of the semiconductor tracker (SCT), and a transition radiation tracker for $|\eta| < 2$. The electromagnetic energy measurement in the ECal is based on a liquid-argon sampling technology. The FCal uses tungsten and copper absorbers with liquid argon as the active medium and has a total thickness of about ten interaction lengths. The ECal covers the pseudorapidity range $|\eta| < 3.2$, and the FCal extends the calorimeter coverage to $|\eta| < 4.9$. The energies in the ECal and FCal are reconstructed and grouped into towers with segmentation in pseudorapidity and azimuthal angle of $\Delta \eta \times \Delta \phi = 0.1 \times 0.1$ to 0.2 $\times$ 0.2, which are then used to calculate the event plane. The procedure for obtaining the event-plane correlations is found to be insensitive to the segmentation and energy calibration of the calorimeters.

The minimum-bias Level-1 trigger [25] used for this analysis requires a signal in each of two zero-degree calorimeters (ZDCs) or a signal in either one of the two minimum-bias trigger scintillator (MBTS) counters. The ZDC is positioned at 140 m from the collision point, detecting neutrons and photons with $|\eta| > 8.3$, and the MBTS covers $2.1 < |\eta| < 3.9$. The ZDC Level-1 trigger thresholds on each side are set below the peak corresponding to a single neutron. A Level-2 timing requirement based on signals from each side of the MBTS is imposed to suppress beam backgrounds [25].

III. EVENT AND TRACK SELECTIONS

This paper is based on Pb $+$ Pb collision data collected in 2010 at the LHC with a nucleon-nucleon center-of-mass energy $\sqrt{s_{NN}} = 2.76$ TeV. The data correspond to an integrated luminosity of approximately $7 \, \mu$b$^{-1}$. To suppress noncollision backgrounds, an offline event selection requires a reconstructed primary vertex with at least three associated charged tracks reconstructed in the ID and a time difference $|\Delta \tau| < 3$ ns between the MBTS trigger counters on either side of the interaction point. A coincidence between the two ZDCs at forward and backward pseudorapidity is required to reject a variety of background processes, while maintaining high efficiency for non-Coulomb processes. Events satisfying these conditions are required to have a reconstructed primary vertex with $z_{\text{vtx}}$ within 150 mm of the nominal center of the ATLAS detector. The pileup probability is estimated to be at the $10^{-4}$ level and is therefore negligible. About $48 \times 10^6$ events pass the requirements for the analysis.

The Pb $+$ Pb event centrality is characterized using the total transverse energy ($\Sigma E_T$) deposited in the FCal over the pseudorapidity range $3.2 < |\eta| < 4.9$ and measured at the electromagnetic energy scale [26]. A larger $\Sigma E_T$ value corresponds to a more central collision. From an analysis of the $\Sigma E_T$ distribution after applying all trigger and event selection criteria, the sampled fraction of the total inelastic cross section has been estimated to be $(98 \pm 2\%)$ in a previous analysis [27]. The uncertainty in this estimate is evaluated by varying the trigger criteria, event selection, and background rejection requirements on the FCal $\Sigma E_T$ distribution [27]. The FCal $\Sigma E_T$ distribution is divided into a set of $5\%$-wide percentile bins, together with five $1\%$-wide bins for the most central $5\%$ of the events. A centrality interval refers to a percentile range, starting at $0\%$ for the most central collisions. Thus, the $0\%$–$1\%$ centrality interval corresponds to the most central $1\%$ of the events. A standard Glauber model Monte Carlo analysis [9] is used to estimate the average number of participating nucleons, $\langle N_{\text{part}} \rangle$, and its associated systematic uncertainties for each centrality interval [27]. These numbers are summarized in Table 1.

The event plane is also measured by the ID, using reconstructed tracks with $p_T > 0.5$ GeV and $|\eta| < 2.5$ [8]. To improve the robustness of track reconstruction in the high-multiplicity environment of heavy-ion collisions, more stringent requirements on track quality, compared to those defined for proton-proton collisions [28], are used. At least nine hits in the silicon detectors are required for each track, with no missing pixel hits and not more than one missing SCT hit, excluding the known nonoperational modules. In addition, at its point of closest approach the track is required to be within 1 mm of the primary vertex in both the transverse and the longitudinal directions [29]. The track reconstruction performance is studied by comparing data to Monte Carlo calculations based on the HIJING event generator [30] and a full GEANT4 simulation of the detector [31,32]. The track reconstruction efficiency ranges from 72% at $\eta = 0$ to 51%
TABLE I. The list of centrality intervals and associated \( \langle N_{\text{part}} \rangle \) values used in this paper. The systematic uncertainties are taken from Ref. [27].

<table>
<thead>
<tr>
<th>Centrality (%)</th>
<th>( \langle N_{\text{part}} \rangle )</th>
<th>( \langle N_{\text{part}} \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–5</td>
<td>400.6 ± 1.3</td>
<td>392.6 ± 1.8</td>
</tr>
<tr>
<td>5–10</td>
<td>386.2 ± 2.0</td>
<td>330.3 ± 3.0</td>
</tr>
<tr>
<td>10–15</td>
<td>281.9 ± 3.5</td>
<td>239.5 ± 3.8</td>
</tr>
<tr>
<td>15–20</td>
<td>10–15</td>
<td>15–20</td>
</tr>
<tr>
<td>20–25</td>
<td>5–10</td>
<td>281.9 ± 3.5</td>
</tr>
<tr>
<td>20–25</td>
<td>25–30</td>
<td>35–40</td>
</tr>
<tr>
<td>25–30</td>
<td>170.2 ± 4.0</td>
<td>141.7 ± 3.9</td>
</tr>
<tr>
<td>30–35</td>
<td>116.8 ± 3.8</td>
<td>95.0 ± 3.7</td>
</tr>
<tr>
<td>35–40</td>
<td>50–55</td>
<td>60–65</td>
</tr>
<tr>
<td>40–03</td>
<td>59.9 ± 3.3</td>
<td>46.1 ± 3.0</td>
</tr>
</tbody>
</table>

for \( |\eta| > 2 \) in peripheral collisions, while it ranges from 72% at \( \eta = 0 \) to about 42% for \( |\eta| > 2 \) in central collisions [33]. However, the event-plane correlation results are found to be insensitive to the reconstruction efficiency (see Sec. IV D).

IV. DATA ANALYSIS

A. Experimental observables

The \( n \)-th order harmonic has a \( n \)-fold symmetry in azimuth and is thus invariant under the transformation \( \Phi_n \to \Phi_n + 2 \pi / n \). Therefore, a general definition of the relative angle between two event planes, \( a_n \Phi_n + a_m \Phi_m \), has to be invariant under a phase shift \( \Phi_i \to \Phi_i + 2 \pi / l \). It should also be invariant under a global rotation by any angle. The first condition requires \( a_n (a_m) \) to be multiple of \( n (m) \), while the second condition requires the sum of the coefficients to vanish: \( a_n + a_m = 0 \). The relative angle \( \Phi_{n,m} = k(\Phi_n - \Phi_m) \), with \( k \) being the least common multiple (LCM) of \( n \) and \( m \), satisfies these constraints, as does any integer multiple of \( \Phi_{n,m} \).

The correlation between \( \Phi_n \) and \( \Phi_m \) is completely described by the differential distribution of the event yield \( dN_{\text{evts}} / [d(\Phi_n - \Phi_m)] \). This distribution must be an even function owing to the symmetry of the underlying physics and hence can be expanded into the following Fourier series:

\[
\frac{dN_{\text{evts}}}{d(\Phi_n - \Phi_m)} \propto 1 + 2 \sum_{j=1}^{\infty} V_j \cos j(\Phi_n - \Phi_m).
\]

The measurement of the two-plane correlation is thus equivalent to measuring a set of cosine functions \( \cos j(\Phi_n - \Phi_m) \) averaged over many events [22].

This discussion can be generalized for correlations involving three or more event planes. The multiplane correlators can be written as \( \langle \cos (c_1 \Phi_1 + 2c_2 \Phi_2 + \cdots + lc_l \Phi_l) \rangle \) with the constraint [21,23]:

\[
c_1 + 2c_2 + \cdots + lc_l = 0,
\]

where the coefficients \( c_n \) are integers. The two-plane correlators defined in Eq. (6) satisfy this constraint. For convenience, correlation involving two event planes \( \Phi_n \) and \( \Phi_m \) is referred to as “\( n-m \)” correlation, and one involving three event planes \( \Phi_n \), \( \Phi_m \), and \( \Phi_h \) as “\( n-m-h \)” correlation. The multiplane correlators can always be decomposed into a linear combination of several two-plane relative angles and they carry additional information not accessible through two-plane correlators [22].

Experimentally the \( \Phi_n \) angles are estimated from the observed event-plane angles, \( \Psi_n \), defined as the directions of the “flow vectors” \( \vec{q}_n \), which in turn are calculated from the azimuthal distribution of particles in the calorimeter or the ID:

\[
\vec{q}_n = (q_{x,n}, q_{y,n}) = \frac{1}{\Sigma u_i} \left[ \left( \Sigma [u_i \cos n \phi_i] - \left( \Sigma [u_i \cos n \phi_i] \right)_{\text{evts}} \right), \Sigma [u_i \sin n \phi_i] - \left( \Sigma [u_i \sin n \phi_i] \right)_{\text{evts}} \right].
\]

Here the weight \( u_i \) is either the \( E_T \) of the \( i \)th tower in the ECal and the FCal or the \( p_T \) of the \( i \)th reconstructed track in the ID. Subtraction of the event-averaged centroid, \( \left( \Sigma [u_i \cos n \phi_i] \right)_{\text{evts}}, \Sigma [u_i \sin n \phi_i] \right)_{\text{evts}} \) in Eq. (8) removes biases owing to detector effects [34]. A standard flattening technique [35] is then used to remove the small residual nonuniformities in the distribution of \( \Psi_n \). The \( \vec{q}_n \) defined this way, when averaged over events with the same \( \Phi_n \), is insensitive to the energy scale in the calorimeter or the momentum scale in the ID and to any random smearing effect. In the limit of infinite multiplicity it approaches the single-particle flow weighted by \( u \): \( \vec{q}_n \to (\vec{q})_{\text{evts}} = \Sigma u_i (\vec{v}_n) / \Sigma u_i \).

The correlators in terms of \( \Phi_n \) can be obtained from the correlations between the measured angles \( \Psi_n \) divided by a resolution term [22]:

\[
\langle \cos (c_1 \Psi_1 + 2c_2 \Psi_2 + \cdots + lc_l \Psi_l) \rangle = \frac{\langle \cos (c_1 \Psi_1 + 2c_2 \Psi_2 + \cdots + lc_l \Psi_l) \rangle}{\text{Res}[c_1 \Psi_1] \text{Res}[c_2 \Psi_2] \cdots \text{Res}[c_l \Psi_l]}
\]

The resolution factors \( \text{Res}[c_n \Psi_n] \) can be determined using the standard two-subevent or three-subevent methods [4], as discussed in Sec. IV B. To avoid autocorrelations, each \( \Psi_n \) needs to be measured using subevents covering different \( \eta \) ranges, preferably with a gap in between. Here a subevent refers to a collection of particles over a certain \( \eta \) range in the event. This method of obtaining the correlator is referred to as the event-plane, or EP, method.

\[\text{For example, a localized inefficiency over a } \phi \text{ region in the detector would lead to a nonzero average } \vec{q}_n. \text{ The subtraction corrects this bias.}\]
In Eq. (9), all events are given equal weights in both the numerator (raw correlator) and the denominator (resolution). It was recently proposed [36,37] that the potential bias in the EP method arising from the effects of event-by-event fluctuations of the flow and multiplicity can be removed by applying additional weight factors,

\[
\langle \cos(c_1 \Phi_1 + 2c_2 \Phi_2 + \cdots + l c_l \Phi_l) \rangle_w = \frac{\langle \cos(c_1 \Psi_1 + 2c_2 \Psi_2 + \cdots + l c_l \Psi_l) \rangle_w}{\text{Res} \{c_1 \Psi_1\} w, \text{Res} \{c_2 \Psi_2\} w, \cdots, \text{Res} \{l c_l \Psi_l\} w},
\]

\[
\langle \cos(c_1 \Psi_1 + 2c_2 \Psi_2 + \cdots + l c_l \Psi_l) \rangle_w = \left\langle \frac{\sqrt{\left| q_n^c \right|^2 \cos c_n (\Psi_n - \Phi_n)} \right\rangle_w^2,
\]

where the \( q_n \) represents the magnitude of the flow vector of the subevent used to calculate the \( \Psi_n \) [Eq. (8)], and the subscript “w” is used to indicate the \( q_n \) weighting. This weighting method is often referred to as the “scalar-product,” or SP, method [38]. Correspondingly, the weighted version of the PP correlators can be obtained by using the eccentricity \( e_n \) defined in Eq. (3) as the weight [21]:

\[
\langle \cos(c_1 \Phi_1^* + 2c_2 \Phi_2^* + \cdots + l c_l \Phi_l^*) \rangle_w = \frac{\langle \epsilon_1 \epsilon_2 \cdots \epsilon_l \cos(c_1 \Phi_1^* + 2c_2 \Phi_2^* + \cdots + l c_l \Phi_l^*) \rangle_w}{\sqrt{\left| \epsilon_1 \right|^2 \left| \epsilon_2 \right|^2 \cdots \left| \epsilon_l \right|^2}}
\]

In Eq. (10), events with larger flow have bigger weights in the calculation of the raw correlation and the resolution factors. Other than the weighting, the procedure for obtaining the raw signal and resolution factors is identical in the EP and SP methods. Hence, the discussion in the remainder of the paper should be regarded as applicable to both methods and the subscript “w” is dropped in all formulas, unless required for clarity.

It is worth emphasizing that the expression for the correlators in Eq. (10) is constructed to be insensitive to the details of the detector performance, such as the \( \eta \) coverage, segmentation, energy calibration, or the efficiency [4,34]. This is because the angle \( \Phi_n \) is a global property of the event that can be estimated from the \( \Psi_n \) from independent detectors, and the procedure for obtaining the correlators is “self-correcting.” A poor segmentation or energy calibration of the calorimeter, for example, increases the smearing of \( \Phi_n \) about \( \Phi_n \), and hence reduces the raw correlation [numerator of Eq. (10)]. This reduction in the raw correlation, however, is expected to be mostly compensated by smaller resolution terms \( \text{Res} \{c_n \Psi_n\} \) in the denominators.

A very large number of correlators could be studied. However, the measurability of these correlators is dictated by the values of \( \text{Res} \{j n \Psi_n\} \) (\( c_n \) replaced by \( j \) for simplicity). A detailed study in this analysis shows that the values of \( \text{Res} \{j n \Psi_n\} \) decrease very quickly for increasing \( n \), but they decrease more slowly with \( j \) for fixed \( n \) [23]. The resolution factors are sufficiently good for \( \text{Res} \{j n \Psi_n\} \) for \( n = 2 \) to 6 and \( j \) values up to \( j = 6 \) for \( n = 2 \). This defines the two- and three-plane correlators that can be measured.

**Table II.** The list of two-plane correlators and associated EP resolution factors that need to be measured.

| \( \langle \cos(4\Phi_2 - \Phi_1) \rangle \) | \( \text{Res} \{4\Psi_2\}, \text{Res} \{4\Psi_1\} \) |
| \( \langle \cos(8\Phi_3 - \Phi_1) \rangle \) | \( \text{Res} \{8\Psi_3\}, \text{Res} \{8\Psi_1\} \) |
| \( \langle \cos(12\Phi_1 - \Phi_4) \rangle \) | \( \text{Res} \{12\Psi_1\}, \text{Res} \{12\Psi_4\} \) |
| \( \langle \cos(16\Phi_2 - \Phi_1) \rangle \) | \( \text{Res} \{6\Psi_2\}, \text{Res} \{6\Psi_1\} \) |
| \( \langle \cos(20\Phi_3 - \Phi_1) \rangle \) | \( \text{Res} \{6\Psi_3\}, \text{Res} \{6\Psi_1\} \) |
| \( \langle \cos(24\Phi_4 - \Phi_1) \rangle \) | \( \text{Res} \{6\Psi_4\}, \text{Res} \{6\Psi_1\} \) |

Table II gives a summary of the set of two-plane correlators and resolution terms that need to be measured in this analysis for each centrality interval. The corresponding information for the three-plane correlators is shown in Table III. The first three correlators in Table II correspond to the first three Fourier coefficients (\( j = 1,2,3 \) in Eq. (6)) and are derived from the observed distribution \( dN_{\text{ev}}/d[4(\Psi_2 - \Phi_4)] \). All other correlators in Tables II and III only correspond to the first Fourier coefficient of the observed distribution. The two-plane and three-plane correlators are listed separately because different subdetectors are used (see Sec. IV B), and this requires separate evaluation of the resolution corrections.

**B. Analysis method**

For two-plane correlation (2PC) measurements, the event is divided into two subevents symmetric around \( \eta = 0 \) with a gap in between, so they nominally have the same resolution. Each subevent provides its own estimate of the EP via Eq. (8): \( \Psi_p^S \) and \( \Psi_m^S \) for positive \( \eta \) and \( \Psi_n^N \) and \( \Psi_m^N \) for negative \( \eta \). This leads to two statistically independent estimates of the correlator, which are averaged to obtain the final signal. Because of the symmetry of the subevents, the product of resolution factors in the denominator is identical for each measurement, and the event-averaged correlator can be written as

\[
\langle \cos k(\Phi_n - \Phi_m) \rangle = \frac{\langle \cos k(\Psi_p^N - \Psi_m^N) \rangle \langle \cos k(\Psi_m^N - \Psi_p^S) \rangle}{\text{Res} \{k \Psi_p^N\} \text{Res} \{k \Psi_m^N\} + \text{Res} \{k \Psi_m^N\} \text{Res} \{k \Psi_p^S\}}
\]

To measure a three-plane correlation (3PC), three nonoverlapping subevents, labeled as A, B, and C, are chosen to have approximately the same \( \eta \) coverage. In this analysis, subevents A and C are chosen to be symmetric about \( \eta = 0 \), and hence have identical resolution, while the resolution of subevent

**Table III.** The list of three-plane correlators and associated EP resolution factors that need to be measured.

| \( \langle \cos(8\Phi_4 - 5\Phi_1) \rangle \) | \( \text{Res} \{5\Psi_1\}, \text{Res} \{5\Psi_4\} \) |
| \( \langle \cos(-6\Phi_2 + 4\Phi_1) \rangle \) | \( \text{Res} \{2\Psi_1\}, \text{Res} \{2\Psi_4\} \) |
| \( \langle \cos(-4\Phi_2 + 4\Phi_1 + 6\Psi_1) \rangle \) | \( \text{Res} \{2\Psi_1\}, \text{Res} \{2\Psi_4\} \) |
| \( \langle \cos(-10\Phi_2 + 4\Phi_1 + 4\Psi_1) \rangle \) | \( \text{Res} \{2\Psi_1\}, \text{Res} \{2\Psi_4\} \) |
| \( \langle \cos(-10\Phi_2 + 6\Phi_1 + 4\Psi_1) \rangle \) | \( \text{Res} \{2\Psi_1\}, \text{Res} \{2\Psi_4\} \) |
B, in general, is different. There are $3! = 6$ independent ways of obtaining the same three-plane correlator. However, the symmetry between A and C reduces this to three pairs of measurements, which are labeled as Type1, Type2, and Type3. For example, the Type1 measurement of the correlation $2\Phi_2 + 3\Phi_3 - 5\Phi_5$ is obtained from $2\Psi_2^B + 3\Psi_3^A - 5\Psi_5^A$ and $2\Psi_2^B + 3\Psi_3^C - 5\Psi_5^C$, i.e., by requiring the $\Psi_2$ angle to be given by subevent B:

$$
\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle_{\text{Type1}} = \frac{\langle \cos (2\Phi_2^B + 3\Phi_3^A - 5\Phi_5^A) \rangle}{\text{Res}[2\Psi_2^B]} + \frac{\langle \cos (2\Phi_2^B + 3\Phi_3^C - 5\Phi_5^C) \rangle}{\text{Res}[2\Psi_2^B]} + \text{Res}[3\Psi_3^A \text{Res}[5\Psi_5^A] + \text{Res}[3\Psi_3^B \text{Res}[5\Psi_5^B]].
$$

Similarly, the Type2 (Type3) measurement is obtained by requiring the $\Psi_3$ ($\Psi_5$) to be measured by subevent B. Because the three angles in each detector, e.g., $\Psi_2^A$, $\Psi_3^A$, and $\Psi_5^A$, are obtained from orthogonal Fourier modes, the different types of estimates for a given correlator are expected to be statistically independent.

The resolution factors $\text{Res}\{jn\Psi_a\}$ are obtained from a two-subevent (2SE) method and a three-subevent (3SE) method [4]. The 2SE method follows almost identically the 2PC procedure described above: Two subevents symmetric about $\eta = 0$ are chosen and used to make two measurements of the EP at the same order $n$: $\Psi_n^P$ and $\Psi_n^N$. The correlator $\langle \cos jn(\Psi_n^P - \Psi_n^N) \rangle$ is then calculated, and the square root yields the desired resolution [8]:

$$
\text{Res}\{jn\Psi_a\} = \sqrt{\langle \cos jn(\Psi_n^P - \Psi_n^N) \rangle} \\
\equiv \text{Res}\{jn\Psi_n^P\} \equiv \text{Res}\{jn\Psi_n^N\}. \tag{14}
$$

In the 3SE method, the value of $\text{Res}\{jn\Psi_a\}$ for a given subevent A is determined from angle correlations with two subevents B and C covering different regions in $\eta$:

$$
\text{Res}\{jn\Psi_a^B\} = \sqrt{\langle \cos jn(\Psi_a^A - \Psi_a^B) \rangle \langle \cos jn(\Psi_a^A - \Psi_a^C) \rangle}. \tag{15}
$$

The 3SE method does not rely on equal resolutions for the subevents, and hence there are many ways of choosing subevents B and C.

In the case of the weighted correlators given by the SP method, the resolution terms defined by Eqs. (14) and (15) are instead calculated as [36]

$$
\text{Res}\{jn\Psi_a^w\} = \sqrt{\left(\langle q_n^a q_n^B \rangle^T \cos jn(\Psi_n^P - \Psi_n^N) \right).} \tag{16}
$$

and

$$
\text{Res}\{jn\Psi_a^w\} = \sqrt{\left(\langle q_n^a q_n^B \rangle^T \cos jn(\Psi_n^P - \Psi_n^C) \right).} \tag{17}
$$

### C. Analysis procedure

The large $\eta$ coverage of the ID, ECal, and FCal, with their fine segmentation, allows many choices of subevents for estimating the EPs and studying their correlations over about ten units in $\eta$. The edge towers of the FCal (approximately $4.8 < |\eta| < 4.9$) are excluded to minimize the nonuniformity of $E_T$ in azimuth, as in a previous analysis [8]. These detectors are divided into a set of small segments in $\eta$, and the subevents are constructed by combining these segments. A large number of subevents can be used for measuring both the raw correlation signal and the resolution corrections. A detailed set of cross checks and estimations of systematic uncertainties can therefore be performed.

The guiding principle for choosing the subevents is that they should have large $\eta$ acceptance, but still have a sufficiently large $\eta$ gap from each other. For 2PCs, the default subevents are ECal + FCal at negative ($-4.8 < \eta < -0.5$) and positive ($0.5 < \eta < 4.8$) $\eta$, with a gap of one unit in between. For 3PCs, the default subevents are ECal_p ($0.5 < \eta < 2.7$), FCal ($3.3 < |\eta| < 4.8$), and ECal_N ($-2.7 < \eta < -0.5$). As an important consistency cross check for the 2PC and 3PC analyses, subevents are also chosen only from the ID. These combinations are listed in Table IV. The resolution for each of these subevents is determined via the 2SE method and the 3SE method, and the latter typically involves measuring correlations with many other subevents not listed in Table IV, for example, using smaller sections of the ECal or ID.

Figure 1 shows the two-plane relative angle distributions for the 20%–30% centrality interval. The signal or “foreground” distributions are calculated by combining EP angles from the 20%–30% centrality interval. The signal or “foreground” distributions are obtained by dividing the foreground distributions are calculated by combining EP angles from the 20%–30% centrality interval. The signal or “foreground” distributions are obtained by dividing the foreground uncertainties can therefore be performed.

The background distributions provide an estimate of detector effects, while the foreground distributions contain both the detector effects and physics. The background distributions are almost flat, but do indicate some small variations at a level of about $10^{-5}$. To cancel these nonphysical structures, the correlation functions are obtained by dividing the foreground...
TABLE IV. Combinations of subevents used in 2PC and 3PC analysis. The calorimeter-based analysis is the default, while the ID-based result provides an important cross check.

<table>
<thead>
<tr>
<th>Subevents used for 2PCs and their ( \eta ) coverages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calorimeter-based ECalFCal ( \eta \in (0.5, 4.8) )</td>
</tr>
<tr>
<td>ID-based ( \eta \in (0.5, 2.5) )</td>
</tr>
<tr>
<td>ECalFCalN ( \eta \in (-4.8, -0.5) )</td>
</tr>
<tr>
<td>IDN ( \eta \in (-2.5, -0.5) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subevents used for 3PCs and their ( \eta ) coverages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calorimeter-based ECal ( \eta \in (0.5, 2.7) )</td>
</tr>
<tr>
<td>FCal (</td>
</tr>
<tr>
<td>ECalN ( \eta \in (-2.7, -0.5) )</td>
</tr>
<tr>
<td>ID ( \eta \in (-1.0, 1.0) )</td>
</tr>
<tr>
<td>IDN ( \eta \in (-2.5, -1.5) )</td>
</tr>
</tbody>
</table>

\((S)\) by the background distributions \((B)\):

\[
C[k(\Psi_n - \Psi_m)] = \frac{S[k(\Psi_n - \Psi_m)]}{B[k(\Psi_n - \Psi_m)]}.
\] (18)

The correlation functions show significant positive signals for \(4(\Psi_1 - \Psi_2), 6(\Psi_2 - \Psi_3), 6(\Psi_3 - \Psi_6),\) and \(6(\Psi_3 - \Psi_6)\). The observed correlation signals (not corrected by resolution) in terms of the cosine average are calculated directly from these correlation functions.

Figure 2 shows the centrality dependence of the observed correlation signals for various two-plane correlators. The systematic uncertainty, shown as shaded bands, is estimated as the values of the sine terms \(\langle \sin jk(\Psi_n - \Psi_m) \rangle\). Nonzero sine terms may arise from detector effects, which lead to nonphysical correlations between the two subevents. This uncertainty is calculated by averaging sine terms across the measured centrality range, giving uncertainties of \((0.2 - 1.5) \times 10^{-3}\) depending on the type of the correlator. This uncertainty is correlated with centrality and is significant only when the \(\langle \cos jk(\Psi_n - \Psi_m) \rangle\) term is itself small, as in the rightmost four panels of Fig. 2. This uncertainty is included in the final results (see Sec. IV D).

A large number of resolution factors \(\text{Res} [j/\Psi_n]\) needs to be determined using the 2SE and the 3SE methods, separately for each subevent listed in Table IV. For example, the resolution of ECalFCalN can be obtained from its correlation with ECalFCalP via Eq. (14) (2SE method) or from its correlation with any two nonoverlapping reference subevents at \(\eta > 0\) via Eq. (15) (3SE method) such as \(0.5 < \eta < 1.5\) and \(3.3 < \eta < 4.8\). Therefore, for a particular subevent in Table IV, there are

![Graphs showing relative angle distributions between two raw EPs from ECalFCalN and ECalFCalP defined in Table IV for the 20%-30% centrality interval for the foreground (open circles), background (open squares), and correlation function (solid circles) based on the EP method. The correlation functions give [via Eq. (12)] the two-plane correlators defined in Table II. The y-axis scales are not the same for all panels.](024905-6)
usually several determinations of Res\{jnΨs\}, one from the 2SE method and several from the 3SE method. The default value used is obtained from the 2SE method where available, or from the 3SE combination with the smallest uncertainty. The spread of these values is included in the systematic uncertainty, separately for each centrality interval. The relative differences between most of these estimates are found to be independent of the event centrality, except for the 50%–75% centrality range, where weak centrality dependencies are observed in some cases.

All the cosine terms in the 2SE and 3SE formulas are calculated from the distributions similar to those in Eq. (18), but at the same order \( n \),

\[
C[n(\Psi^A_n - \Psi^B_n)] = \frac{S[n(\Psi^A_n - \Psi^B_n)]}{B[n(\Psi^A_n - \Psi^B_n)]},
\]

where the background distribution is obtained by combining the Ψs of subevent A in one event with Ψs of subevent B from a different event with similar centrality and \( \varepsilon_{\text{vtx}} \). Furthermore, the nonzero sine values \( \langle \sin(n(\Psi^A_n - \Psi^B_n)) \rangle \) arising from the 2SE and 3SE analyses are also included in the uncertainty in the resolution factor. Once the individual resolution factors are determined for each subevent, the combined resolution factors are then calculated by multiplying the relevant individual Res\{jnΨs\} terms. They are shown in Fig. 3 as a function of centrality for the eight two-plane correlators listed in Table II. The systematic uncertainty is calculated via a simple error propagation from the individual resolution terms and is nearly independent of the event centrality. This uncertainty is included in the final results (see Sec. IV D).

The analysis procedure and the systematic uncertainties discussed above are also valid for the 3PC analysis. However, the 3PC is slightly more complicated because it has three independent measurements for each correlator, which also need to be combined. Figure 4 shows the relative angle distributions for various three-plane correlators from the Type1 measurement in the 20%–30% centrality interval. The observed correlation signals are calculated as cosine averages of the correlation functions in an obvious generalization of Eq. (18),

\[
C(c_n\Psi_n + c_m\Psi_m + c_h\Psi_h) = \frac{S(c_n\Psi_n + c_m\Psi_m + c_h\Psi_h)}{B(c_n\Psi_n + c_m\Psi_m + c_h\Psi_h)},
\]

where the background distribution is constructed by requiring that all three angles \( \Psi_n, \Psi_m, \) and \( \Psi_h \) are from different events. The correlation functions show significant positive signals for \( 2\Psi_2 + 3\Psi_3 - 5\Psi_5 \), \( 2\Psi_2 + 4\Psi_4 - 6\Psi_6 \), and \(-10\Psi_2 + 4\Psi_4 + 6\Psi_6 \), while the signal for \( 2\Psi_2 - 6\Psi_3 + 4\Psi_4 \) is negative, and the signals for the remaining correlators are consistent with zero.

Figure 5 shows the centrality dependence of the observed correlation signals (left panel), combined resolutions (middle panel), and corrected signals (right panel) for Type1, Type2, and Type3 combinations of \( \langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle \). The systematic uncertainty in the observed correlation signals is estimated from the values of \( \langle \sin(c_n\Psi_n + c_m\Psi_m + c_h\Psi_h) \rangle \) and is calculated by averaging these sine terms over the measured centrality range. This uncertainty is \( (0.2–1.5) \times 10^{-3} \) in absolute variation, depending on the type of three-plane correlator. The uncertainty in the combined resolution is obtained by propagation from those for the individual resolution factors. Both sources of uncertainties are strongly correlated with centrality, and they are included in the final
FIG. 3. (Color online) The combined resolution factors based on the EP method for two-plane correlators, \( \text{Res}(jk(\Psi_{an} - \Psi_{an})) = \text{Res}(jk\Psi_{an})\text{Res}(jk\Psi_{an}) \), as a function of \( \langle N_{\text{part}} \rangle \). The middle two panels in the top row have \( j = 2 \) and \( j = 3 \), while all other panels have \( j = 1 \). The error bars and shaded bands indicate the statistical and systematic uncertainties, respectively.

FIG. 4. (Color online) Relative angle distributions between three EPs from ECalN, FCal, and ECalP defined in Table IV for Type1 correlation in the 20%–30% centrality interval for foreground (open circles), background (open squares), and correlation function (solid circles) based on the EP method. The correlation functions give [via equations similar to Eq. (13)] the three-plane correlators defined in Table III. The y axis scales are not the same for all panels.
The error bars and shaded bands indicate the statistical and systematic uncertainties, respectively. Figure 5 shows that all three types of measurements (Type1, Type2, and Type3) have similar values for the observed signal and the combined resolution. This behavior is expected because the three subevents cover similar rapidity ranges. The three corrected results are statistically combined, and the spreads between them are included in the total systematic uncertainty.

The same analysis procedure is repeated for EP correlations obtained via the SP method. The performance of the SP method is found to be very similar to that of the EP method. The magnitudes of the sine terms relative to the cosine terms for both the signal distributions in $k(\Psi_n - \Psi_m)$ and $c_m n \Psi_m + c_m m \Psi_m + c_m h \Psi_h$, as well as the distributions in $n(\Psi_n^A - \Psi_n^B)$ for calculating the resolution factors are found to be nearly the same as those for the EP method. This behavior is quite natural as the effects of detector acceptance are expected to be independent of the strength of the flow signal. The resolution factors and their associated systematic uncertainties are calculated with the same detector combinations as those used for the EP method. The spreads of the results between various detector combinations are included in the systematic uncertainty for the resolution factors. These uncertainties are also found to be strongly correlated between the two methods. The uncorrelated systematic uncertainties between the two methods are evaluated by calculating a double ratio for each detector “X” listed in Table IV,

$$R_X = \frac{\text{Res}[jn\Psi_n]_m(X,\text{other})/\text{Res}[jn\Psi_n]_m(X,\text{ref})}{\text{Res}[jn\Psi_n](X,\text{other})/\text{Res}[jn\Psi_n](X,\text{ref})},$$

where the “ref” refers to the default detector combination used to calculate the resolution of X and “other” refers to other detector combinations used to evaluate the systematic uncertainties in the resolution of X via the 2SE and 3SE methods as discussed above [see the paragraph before Eq. (19)]. The spread of the $R_X$ values provides an estimate of the uncorrelated uncertainty between the two methods for resolution factor $\text{Res}[jn\Psi_n]$. This uncorrelated uncertainty is typically much smaller than the total systematic uncertainty in the resolution factor in either method.

D. Systematic uncertainties

The main systematic uncertainties in the result are introduced and discussed in Sec. IV C at various key steps of the analysis. This section gives a summary of these uncertainties and then discusses any additional systematic uncertainties and cross checks.

The systematic uncertainties associated with the analysis procedure are dominated by contributions from residual detector acceptance effects and uncertainties in the resolution factors. Most detector acceptance effects are expected to cancel in the raw correlation function by dividing the foreground and background distributions [Eqs. (18)–(20)]. The residual acceptance effects, estimated by the sine terms of the distributions, are found to be $0.2–1.5 \times 10^{-3}$ of the average amplitude of the correlation functions and are found to be independent of the event centrality. The uncertainties in the resolution factors are calculated from the differences between the 2SE estimate and various 3SE estimates, which are then propagated to give the total uncertainties for the combined resolution factor. These uncertainties are found to be quite similar in the EP and SP methods because they both rely on the same detector acceptance effects and uncertainties in the resolution factors. The total systematic uncertainty. The uncorrelated uncertainties are evaluated separately via Eq. (21) and are used for comparison between the two methods. The uncertainties in the resolution factors are found to depend only weakly on event centrality.

Additional systematic uncertainties include those associated with the trigger and event selections, as well as variations of resolution-corrected signals between different running periods. The former is evaluated by varying the full centrality range by $\pm 2\%$ according to the estimated efficiency of $(98 \pm 2)\%$ for selecting minimum-bias Pb + Pb events. The latter is evaluated by comparing the results obtained independently from three running periods, each with $1/3$ of the total event statistics. All these uncertainties are generally small and are quite similar between the EP and SP methods. Both types of uncertainties are found to be independent of the event centrality.

Tables V and VI summarize the sources of systematic uncertainties for 2PCs and 3PCs. The total systematic
TABLE V. Three sources of uncertainties for the two-plane correlators, $\langle \cos(\Sigma \Phi) \rangle$, where $\Sigma \Phi = jk(\Phi_a - \Phi_a)$. They are given as percentage uncertainties.

<table>
<thead>
<tr>
<th>$\Sigma \Phi$</th>
<th>4($\Phi_2 - \Phi_4$)</th>
<th>8($\Phi_2 - \Phi_4$)</th>
<th>12($\Phi_2 - \Phi_4$)</th>
<th>6($\Phi_2 - \Phi_4$)</th>
<th>6($\Phi_2 - \Phi_6$)</th>
<th>6($\Phi_1 - \Phi_6$)</th>
<th>12($\Phi_1 - \Phi_4$)</th>
<th>10($\Phi_2 - \Phi_5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution (%)</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>7</td>
<td>10</td>
<td>11</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>Trigger and event sel. (%)</td>
<td>1–2</td>
<td>1–4</td>
<td>3</td>
<td>3</td>
<td>1–2</td>
<td>1–2</td>
<td>&lt;1</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Run periods (%)</td>
<td>&lt;1</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

uncertainties are the quadrature sum of the three sources listed in these tables and the uncertainties associated with residual detector effects. The total uncertainties are found to be nearly independent of the event centrality over the 0%–50% centrality range, although a small increase is observed for some of the correlators in the 50%–75% centrality range. In most cases, the total systematic uncertainties are dominated by uncertainties associated with the resolution factors. The uncertainties in the resolution correction can become quite sizable when the angles $\Psi_5$ and $\Psi_6$ are involved. This is expected because the higher-order flow signals $\Psi_5$ and $\Psi_6$ are weak, leading to small values of $\text{Res} \{6\Psi_6\}$, $\text{Res} \{5\Psi_5\}$ and $\text{Res} \{10\Psi_5\}$ with large uncertainties.

One important issue in this analysis is the extent to which the measured correlations are biased by short-range correlations such as jet fragmentation, resonance decays, and Bose-Einstein correlations. These short-range correlations may contribute to the observed correlation signals and the resolution factors and hence affect the measured correlations. The potential influence of these short-range correlations is studied for the eight two-plane correlators with the EP method. The $\eta$ gap between the two symmetric subevents from ECalFCal, $\eta_{\text{min}}$, is varied in the range of 0 to 8. Seventeen symmetric pairs of subevents are chosen, each corresponding to a different $\eta$ separation. For each case, the observed correlation signals and the resolution factors are obtained using the correlations between these two subevents. Both the observed correlation signals and the combined resolutions decrease significantly (by up to a factor of four) as $\eta_{\text{min}}$ is increased. However, the final corrected correlation signals are relatively stable. For example, a gradual change of a few percent is observed for $\eta_{\text{min}} < 4$, where the statistical and systematic uncertainties are not very large. This observation strongly suggests that the measurement indeed reflects long-range correlations between the EPs. In most cases, the raw correlation signals decrease smoothly with $\eta_{\text{min}}$. In contrast, the estimated resolution factors have a sharp increase towards small $\eta_{\text{min}}$ in many cases, leading to a suppression of the corrected correlation signals at small $\eta_{\text{min}}$. This behavior suggests that short-range correlations can influence individual harmonics, and hence the resolution factors, but their influences are weak for correlations between EP angles of different order. In all cases, the influences of these short-range correlations are negligible for $\eta_{\text{min}} > 0.4$. The choices of the subevents in Table IV have a minimum $\eta$ gap of 0.6 and hence are sufficient to suppress these short-range correlations.

The EP correlators measured by the calorimeters are also compared with those obtained independently from the ID for both the EP method and the SP method (see Table IV for the definition of the subevents). Despite the larger fluctuations owing to the limited $\eta$ range of its subevents, the results from the ID are consistent with those from the calorimeters (see the Appendix). Because the ID is an entirely different type of detector and measures only charged particles, this consistency gives confidence that the measured results are robust. It is argued in Ref. [37] that the SP method as defined in Eq. (10) is insensitive to various smearing effects on the weighting factors, such as energy or momentum resolution or multiplicity fluctuations, and as long as these smearings are random and isotropic, they should cancel after averaging over events in the numerator and denominator of Eq. (10). This behavior was checked explicitly in the ID by calculating $\bar{q}_n$ given by Eq. (8) in several different ways: (1) instead of $u = p_T$ as in the default calculation, the charged particles are set to have equal weight $u = 1$; (2) the weight $u$ is randomly set to be zero for half of the charged particles; or (3) the $\bar{q}_n$ is redefined as $\bar{q}_n \Sigma u_i$ to include explicitly the event-by-event multiplicity fluctuations.

The results of all of these cross checks are consistent with the results of the default calculation.

TABLE VI. Three sources of uncertainties for the three-plane correlators, $\langle \cos(\Sigma \Phi) \rangle$, where $\Sigma \Phi = c_n \Phi_n + c_m \Phi_m + c_h \Phi_h$. They are given as percentage uncertainties.

<table>
<thead>
<tr>
<th>$\Sigma \Phi$</th>
<th>$2\Phi_2 + 3\Phi_3 - 5\Phi_5$</th>
<th>$2\Phi_2 + 4\Phi_4 - 6\Phi_6$</th>
<th>$2\Phi_2 - 6\Phi_3 + 4\Phi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution (%)</td>
<td>10</td>
<td>21</td>
<td>11</td>
</tr>
<tr>
<td>Trigger and event sel. (%)</td>
<td>1–2</td>
<td>1</td>
<td>3–4</td>
</tr>
<tr>
<td>Run periods (%)</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$\Sigma \Phi$</td>
<td>$-8\Phi_2 + 3\Phi_3 + 5\Phi_4$</td>
<td>$-10\Phi_2 + 4\Phi_4 + 6\Phi_6$</td>
<td>$-10\Phi_2 + 6\Phi_3 + 4\Phi_4$</td>
</tr>
<tr>
<td>Resolution (%)</td>
<td>13</td>
<td>24</td>
<td>16</td>
</tr>
<tr>
<td>Trigger and event sel. (%)</td>
<td>1–3</td>
<td>1–3</td>
<td>1–3</td>
</tr>
<tr>
<td>Run periods (%)</td>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

V. RESULTS AND DISCUSSIONS

Figures 6 and 7 show the centrality dependence of the two-plane and three-plane correlators, respectively. The results from both the EP method and the SP method are shown with their respective systematic uncertainties. These
systematic uncertainties are similar in the two methods and are strongly correlated across the centrality range. Strong positive values are observed in most cases and their magnitudes usually decrease with increasing \( \langle N_{\text{part}} \rangle \), such as \( \langle \cos(4\Phi_2 - \Phi_3) \rangle \), \( \langle \cos(8\Phi_2 - \Phi_4) \rangle \), \( \langle \cos(12\Phi_2 - \Phi_4) \rangle \), \( \langle \cos(6\Phi_2 - \Phi_6) \rangle \), \( \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle \), \( \langle \cos(2\Phi_2 + 4\Phi_4 - 6\Phi_6) \rangle \), and \( \langle \cos(-10\Phi_2 + 4\Phi_4 + 6\Phi_6) \rangle \). The value of \( \langle \cos(6\Phi_2 - \Phi_6) \rangle \) is small \((<0.02)\), yet exhibits a similar dependence on \( \langle N_{\text{part}} \rangle \). A small \( \langle \cos(6\Phi_2 - \Phi_3) \rangle \) value in this analysis is a consequence of dividing a small \( \langle \cos(6\Psi_2 - \Phi_3) \rangle \) signal \((\text{Fig. 2})\) by a relatively large combined resolution factor \((\text{Fig. 3})\). Two other correlators show very different trends: The value of \( \langle \cos(6\Phi_3 - \Phi_6) \rangle \) increases with \( \langle N_{\text{part}} \rangle \), and the value of \( \langle \cos(2\Phi_2 - 6\Phi_1 + 4\Phi_4) \rangle \) is negative and its magnitude decreases with \( \langle N_{\text{part}} \rangle \). The values of the remaining correlators are consistent with zero.

Figures 6 and 7 also suggest that the magnitude of the correlations from the SP method is always larger than that from the EP method. To better quantify their differences, Figs. 8 and 9 show the ratio \( \text{SP}/\text{EP} \) for some selected two-plane and three-plane correlators, respectively. As discussed in Sec. IV D, the nature of the systematic uncertainties is very similar in the EP and SP methods, and hence these uncertainties mostly cancel in the ratio. The results from the SP method are larger than those from the EP method, and their ratios reach a maximum at around the 100 < \( \langle N_{\text{part}} \rangle \) < 300 range or the 10%–40% centrality range. The maximum difference is about 10%–15% for most two-plane correlators, but reaches 20%–30% in midcentral collisions for \( \langle \cos(8\Phi_2 - \Phi_4) \rangle \) and \( \langle \cos(6\Phi_2 - \Phi_6) \rangle \). The differences are smaller for the three-plane correlators, except for \( \langle \cos(-10\Phi_2 + 4\Phi_4 + 6\Phi_6) \rangle \).

Figures 6 and 7 also compare the data with the correlators calculated using the PP angles defined in Eq. (4) from the Glauber model [9]. A total of 30 × 10^6 events were generated and grouped into centrality intervals according to the impact parameter. If each flow harmonic is driven solely by the corresponding geometric component and the \( \Phi_n \) aligns with the \( \Phi_n^* \), then the EP correlation and PP correlation are expected to have the same sign and show similar centrality dependence. The results in Figs. 6 and 7 show that for several correlators the centrality dependence of the Glauber model predictions show trends similar to the data, although in some cases the sign is opposite, e.g., \( \langle \cos(4\Phi_2 - \Phi_4) \rangle \) and \( \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle \). Indeed, large misalignments between \( \Phi_n \) and \( \Phi_n^* \) have been observed in event-by-event hydrodynamic model calculations for flow...
FIG. 7. (Color online) The centrality dependence of six three-plane correlators, \( \langle \cos(\Sigma \Phi) \rangle \), with \( \Sigma \Phi = c_{\text{n}} n \Phi_n + c_{\text{m}} m \Phi_m + c_{\text{h}} h \Phi_h \) obtained via the SP method (solid symbols) and the EP method (open symbols). The error bars and shaded bands indicate the statistical uncertainty and total systematic uncertainty, respectively. The expected correlations among PP angles from a Glauber model are indicated by the solid curves for weighted case [Eq. (11)] and dashed lines for the unweighted case.

harmonics with \( n > 3 \), and these have been ascribed to the nonlinear response of the medium to the fluctuations in the initial geometry [11,39]. The nonlinear effects are found to be small for lower-order harmonics [11,40], such that \( \Phi_n \approx \Phi_n^* \) and \( v_n \propto \epsilon_n \) for \( n = 2 \) and 3 or equivalently in the form introduced in Eq. (2):

\[
v_2 e^{i2\Phi_2} \propto \epsilon_2 e^{i2\Phi_2^*}, \quad v_3 e^{i3\Phi_3} \propto \epsilon_3 e^{i3\Phi_3^*}.
\]

(22)

Recently, motivated by the preliminary version [41] of the results presented in this paper, several theory groups calculated the centrality dependence of EP correlators based on hydrodynamic models [5,42–45]. The results of these calculations are in qualitative agreement with the experimental data. The dynamical origin of these correlators has been explained using the so-called single-shot hydrodynamics [42,44,45], where small fluctuations are imposed on a smooth average geometry profile, and the hydrodynamic response to these small fluctuations is then derived analytically using a cumulant expansion method. In this analytical approach, the \( v_4 \) signal comprises a term proportional to the \( \epsilon_4 \) (linear response term) and a leading nonlinear term that is proportional to \( \epsilon_2^2 \) [5,44],

\[
v_4 e^{i4\Phi_4} = \alpha_4 \epsilon_4 e^{i4\Phi_4^*} + \alpha_2 \epsilon_2^2 (\epsilon_2 e^{i2\Phi_2^*})^2 + \cdots
\]

\[
= \alpha_4 \epsilon_4 e^{i4\Phi_4^*} + \beta_2 \epsilon_2^2 e^{i2\Phi_2} + \cdots,
\]

(23)

where the second line of the equation is derived from Eq. (22), and the coefficients \( \alpha_4, \alpha_2, \) and \( \beta_2 \) are all weak functions of centrality. Because \( v_2 \) increases rapidly for smaller \( \langle N_{\text{part}} \rangle \) [8], the angle \( \Phi_2 \) becomes more closely aligned with \( \Phi_2 \). Hence, the centrality dependence of \( \langle \cos j4(\Phi_2 - \Phi_4) \rangle \) reflects mainly the increase of the \( v_2 \) as \( \langle N_{\text{part}} \rangle \) decreases.

Similarly, the correlations between \( \Phi_2 \) and \( \Phi_6 \) or between \( \Phi_3 \) and \( \Phi_6 \) have been explained by the following
FIG. 8. (Color online) The ratios of the SP-method correlators to the EP-method correlators, \( \langle \cos(\Sigma/\Phi) \rangle_w / \langle \cos(\Sigma/\Phi) \rangle \), for several two-plane correlators, i.e., with \( \Sigma/\Phi = jk(\Phi_n - \Phi_m) \). The error bars and shaded bands indicate the statistical uncertainties and total systematic uncertainties, respectively.

decomposition of the \( v_6 \) signal [5, 44]:

\[
v_6^{6/\Phi_6} = \alpha_6 \epsilon_6^{6/\Phi_6} + \alpha_{2,6} (\epsilon_2^{6/\Phi_2})^3 + \alpha_{3,6} (\epsilon_3^{6/\Phi_3})^2 + \cdots
\]

\[
= \alpha_6 \epsilon_6^{6/\Phi_6} + \beta_2 \epsilon_2^{6/\Phi_2} \epsilon_6^{6/\Phi_6} + \beta_3 \epsilon_3^{6/\Phi_3} \epsilon_6^{6/\Phi_6} + \cdots. \quad (24)
\]

Owing to the nonlinear contributions, \( \Phi_6 \) becomes correlated with \( \Phi_2 \) and \( \Phi_3 \), even though \( \Phi_2 \) and \( \Phi_3 \) are only very weakly correlated. The centrality dependencies of \( \langle \cos(6(\Phi_2 - \Phi_6)) \rangle \) and \( \langle \cos(6(\Phi_3 - \Phi_6)) \rangle \) are strongly influenced by the centrality dependence of \( v_2 \) and \( v_3 \): Because \( v_2 \) increases for smaller \( \langle N_{\text{part}} \rangle \) and \( v_3 \) is relatively independent of \( \langle N_{\text{part}} \rangle \) [8], the relative contribution of the second term increases and that of the third term decreases for smaller \( \langle N_{\text{part}} \rangle \); i.e., the collisions become more peripheral. This behavior explains the opposite centrality dependence of \( \langle \cos(6(\Phi_2 - \Phi_6)) \rangle \) and \( \langle \cos(6(\Phi_3 - \Phi_6)) \rangle \).

In the same manner, the correlation between \( \Phi_2 \), \( \Phi_3 \), and \( \Phi_5 \) has been explained by the following decomposition of the

FIG. 9. (Color online) The ratios of the SP-method correlators to the EP-method correlators, \( \langle \cos(\Sigma/\Phi) \rangle_w / \langle \cos(\Sigma/\Phi) \rangle \), for several three-plane correlators, i.e., with \( \Sigma/\Phi = c_{nn} \Phi_n + c_{mm} \Phi_m + c_{hh} \Phi_h \). The error bars and shaded bands indicate the statistical uncertainties and total systematic uncertainties, respectively.
A multiphase transport (AMPT) model [46] is frequently used to study the harmonic flow coefficients \(v_n\) and to study the relation of \(v_n\) to the initial geometry. The AMPT model combines the initial-state geometry fluctuations of the Glauber model and final-state interactions through a parton and hadron transport model. The AMPT model generates collective flow by elastic scatterings in the partonic and hadronic phase and was shown to reproduce the \(v_n\) values [47] and the particle multiplicity [48] reasonably well. As a full event generator, the AMPT model allows the generated events to be analyzed with the same procedures as in the data. Figures 10 and 11 compare some selected correlators (six two-plane correlators and four three-plane correlators) with a prediction [37] from the AMPT model. Good agreement is observed between the \(v_5\) signal [5,44]:

\[
v_5 \propto e^{i5\Phi_5} = \alpha_5 \delta \times e^{i5\Phi_5} + \alpha_{2,5} \epsilon_2 \times e^{i2\Phi_2} + \alpha_3 \epsilon_3 \times e^{i3\Phi_3} + \cdots
\]

\[
= \alpha_5 \delta \times e^{i5\Phi_5} + \beta_{2,5,3} v_5 v_3 \times e^{i(2\Phi_2 + 3\Phi_3)} + \cdots
\]

The coupling between \(v_5\) and \(v_2 v_3\) explains qualitatively the centrality dependence of the correlator \(\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle\).

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\]

\[
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the data and the calculation, and in particular the model predicts correctly the stronger signal observed with the SP method.

VI. CONCLUSIONS

Measurements of 14 correlators between two and three EPs, \( \langle \cos jk(\Phi_1 - \Phi_2) \rangle \) and \( \langle \cos (c_n \Psi_n + c_m m \Psi_m + c_h h \Psi_h) \rangle \), respectively, are presented using 7 \( \mu \)b\(^{-1}\) of Pb + Pb collision data at \( \sqrt{s_{NN}} = 2.76 \) TeV collected by the ATLAS experiment at the LHC. These correlations are estimated from correlations of observed EP angles measured in the calorimeters over a large pseudorapidity range \( |\eta| < 4.8 \) using both a standard EP method and a SP method. Significant positive correlation signals are observed for \( 4(\Phi_2 - \Phi_4), 6(\Phi_2 - \Phi_6), 2(\Phi_2 + 3\Phi_3 - 5\Phi_5), 2(\Phi_2 + 4\Phi_4 - 6\Phi_6), \) and \( -10\Phi_2 + 4\Phi_4 + 6\Phi_6 \). The correlation signals are negative for \( 2\Phi_2 - 6\Phi_4 + 4\Phi_6 \). The magnitudes of the correlations from the SP method are observed to be systematically larger than those obtained from the EP method. The centrality dependence of most correlators is found to be very different from that predicted by a Glauber model. However, calculations based on the same Glauber model, but including the final-state collective dynamics, are able to describe qualitatively, and in many cases also quantitatively, the centrality dependence of the measured correlators. These observations suggest that both the fluctuations in the initial geometry and the nonlinear mixing between different harmonics in the final state are important for creating these correlations in momentum space. A detailed theoretical description of these correlations can improve our present understanding of the space-time evolution of the hot and dense matter created in heavy-ion collisions.

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FIG. 12. (Color online) The comparison of the eight two-plane correlators between the calorimeters (default) and ID (cross check) as a function of \( \langle N_{\text{part}} \rangle \), both obtained from the EP method. The error bars and the shaded bands indicate the statistical and total systematic uncertainties, respectively.

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FIG. 13. (Color online) The comparison of the eight two-plane correlators between the calorimeters (default) and ID (cross check) as a function of \(\langle N_{\text{part}} \rangle\), both obtained from the SP method. The error bars and the shaded bands indicate the statistical and total systematic uncertainties, respectively.

FIG. 14. (Color online) The comparison of the six three-plane correlators between the calorimeters (default) and ID (cross check) as a function of \(\langle N_{\text{part}} \rangle\), both obtained from the EP method. The error bars and the shaded bands indicate the statistical and total systematic uncertainty, respectively.
FIG. 15. (Color online) The comparison of the six three-plane correlators between the calorimeters (default) and ID (cross check) as a function of $\langle N_{\text{part}} \rangle$, both obtained from the SP method. The error bars and the shaded bands indicate the statistical and total systematic uncertainties, respectively.

APPENDIX

Figures 12–15 compare results between the calorimeter and the ID for the two-plane and 3PCs. As discussed at the end of Sec. IV D, the results are consistent between the calorimeter and the ID within their respective systematic uncertainties.