Valuation of Shipbuilding Option Contracts

By

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Submitted to the Department of Ocean Engineering
In Partial Fulfillment of the Requirements for the Degree of
Master of Science in Ocean Systems Management

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Abstract

This research develops the methodology for calculating the value of option contracts in the shipbuilding industry. Shipbuilding option contracts give the buyer the right to order a newbuilding at a pre-determined price. In practice these contracts are priced arbitrarily based on the shipyard's and buyer's beliefs.

The structure of these contracts is very similar to that of financial options whose market has exploded in recent years. Black-Scholes revolutionized the way traders price these options by developing a mathematical model governing the movement of the underlying asset: stock prices, interest rates, commodities, etc. in the case of financial options. The same risks that affect the value of an option also affect the value of the underlying asset and thus the option's price is contingent on the price of the underlying asset. Assuming a stochastic process for the underlying asset leads to a closed form solution for the price of the option contract. The difficulty presented in the case of shipbuilding option contracts is that newbuilding prices cannot be modeled in a well-defined stochastic process. Given the close correlation between newbuilding prices and freight rates the value of shipbuilding option contracts is calculated using freight rates as the underlying asset. Since freight rates exhibit mean reversion, that is movement around a fixed level, as do interest rates, a trinomial tree model is used to model their movement and calculate the price of shipbuilding option contracts. Using the option pricing methodology the parameters influencing the value of the option contracts are examined.
Acknowledgements

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Special thanks to my parents for always making every effort to provide me the best possible education and giving me all the tools necessary to face the challenges awaiting me. I would also like to thank my sister Marina and my cousin Stergios who reviewed my thesis and gave me helpful insights. This thesis is dedicated to all the people that have influenced me over the years and made this work possible.
# Table of Contents

Abstract ....................................................................................................................... 2  
Acknowledgements ..................................................................................................... 3  
Table of Contents ....................................................................................................... 4  
List of Figures ............................................................................................................. 5  
List of Tables ................................................................................................................ 6  

1. Introduction ............................................................................................................... 7  
   1.1 Thesis Objectives and Review of Previous Research ........................................... 10  
   1.2 Thesis Organization ........................................................................................... 14  

2. Financial Option Theory ........................................................................................ 16  
   2.1 Financial Options .............................................................................................. 16  
   2.2 Parameters Influencing the Value of an Option .................................................. 19  
   2.3 Option Valuation Using Binomial Trees ............................................................ 23  
   2.4 Stochastic Processes ......................................................................................... 28  
   2.5 Black-Scholes Formula ..................................................................................... 32  
   2.6 Term Structure .................................................................................................. 35  
   2.7 Interest Rate Models ......................................................................................... 37  
   2.8 Tree Building Procedure ................................................................................... 39  

3. Data .......................................................................................................................... 42  
   3.1 Data Source ....................................................................................................... 42  
   3.2 Data Testing ....................................................................................................... 45  
   3.3 Newbuilding Prices vs. Freight Rates ................................................................. 51  
   3.4 Time Charter Rates ........................................................................................... 54  

4. Option Pricing .......................................................................................................... 58  
   4.1 Option Pricing Using Newbuilding Prices ......................................................... 58  
   4.2 Option Pricing Using Freight Rates .................................................................... 61  
   4.3 Sensitivity Analysis ............................................................................................ 67  

5. Concluding Remarks ............................................................................................... 73  
   5.1 Recommendations for Future Research ............................................................. 77  

Bibliography ................................................................................................................ 79  
Appendix A.1 ............................................................................................................... 81  
Appendix A.2 ............................................................................................................... 82  
Appendix B .................................................................................................................. 83  
Appendix C .................................................................................................................. 84  
Appendix D .................................................................................................................. 85
List of Figures

Figure 2.1: Value of Call and Put Options Based on Boundary Conditions as a Function of the Underlying Asset’s Price ................................................................. 23
Figure 2.2: Asset and Call Option Movement in One Period Binomial Tree ................. 24
Figure 2.3: Asset and Call Option Movement in Two Period Binomial Tree ................ 26
Figure 2.4: Branching Methods in Trinomial Trees ................................................... 40
Figure 3.1: Newbuilding Prices and Freight Rates (Marsoft) ..................................... 43
Figure 3.2: Newbuilding Prices (Clarksons) ............................................................ 44
Figure 3.3: Freight Rates (Clarksons) ....................................................................... 44
Figure 3.4: Sample Paths of Ornstein –Uhlenbeck Processes ..................................... 48
Figure 3.5: Handymax Newbuilding Prices vs. Freight Rates ..................................... 53
Figure 3.6: Time Charter Rates ................................................................................ 55
Figure 3.7: Capesize Vessel’s Term Structure ............................................................. 57
List of Tables

Table 2.1: Effect of Different Parameters on Option Contract Prices ........................................ 21
Table 3.1: Newbuilding Prices and Freight Rates Regression Summary ........................................ 46
Table 3.2: Speed of Reversion and Fixed Level (Ornstein–Uhlenbeck Process) .......................... 48
Table 3.3: F-ratio for Newbuilding Prices and Freight Rates .................................................... 49
Table 3.4: Regression of Newbuilding Prices vs. Freight Rates ................................................... 52
Table 4.1: Option Prices Using Newbuilding Prices ........................................................................ 59
Table 4.2: Option Prices Using Freight Rates .................................................................................. 64
Table 4.3: Option Prices Using Freight Rates and Neutral Market Expectations ....................... 65
Table 4.4: Percentage Change in Option Price ................................................................................ 71
1. Introduction

Traditionally shipping has been a very risky business. The market’s cyclical nature in addition to the large number of players involved creates an environment that is highly volatile and uncertain. The future level of factors such as freight rates, ship prices, exchange rates, operating costs, liability claims, and world trade, which are an essential part of the shipping business, is very difficult to forecast. Traditionally shippers have transferred the risk of transporting their goods to the shipping market. A shipowner has to predict the shipping capacity the market will need and determine the optimal investment strategy. Forecasting the future level of all these factors is a very complex process since it depends very much on market expectations. The shipping business sounds more like a complex gambling game rather than a stable transport business.

Risk management, which involves the development of a strategy that is unaffected by the volatility of the factors influencing the shipping market, evens out the sharp movements of a shipping company’s revenues. Using different methods and instruments the shipowner can guarantee his company’s revenues no matter the movement of the market. Some of the ways a shipowner can reduce his exposure to the market’s movement involve traditional shipping practices and others involve modern financial securities. For example, time charters provide the shipowner protection against rapid movements in the freight rate market. In a time charter contract a shipowner agrees in advance to a rate of employment for his vessel for an extended period of time. No matter how the market moves, the income from operating the ship is fixed. The same kind of protection can be found in financial institutions. Future contracts that are traded in the Baltic International Freight Futures Exchange (BIFFEX) allow an investor to buy or sell
a freight rate index for an agreed price in the future. The futures' payoff is dependent on the movement of the freight rate index during the life of the contract. At maturity the seller of the contract pays the difference between the initial and final level of the freight rate index to the buyer. If the freight rate index has increased, the contract produces gains for the buyer. On the other hand if the freight rate index decreases during the life of the contract, the buyer incurs losses. Using these financial instruments a shipowner can, as in the case of time charters, protect his investment against movements in the freight rate market. For example, assume that a shipowner believes that the market will fall in the next year. If the market drops, then the gain generated from selling the future contracts compensates the loss from operating the ship in a depressed market. On the other hand, if freight rates increase, the loss due to the future contracts is compensated by the gain due to the operation of the vessel. No matter how the market moves the shipowner is guaranteed fixed revenues.

These contracts do not come for free. The protection time charters and future contracts offer against a downward movement in the freight rate market has the price of a limited gain in the case of an upward movement. The shipowner has managed to shed some of the risk involved in operating ships and, as in the case of insurances, has to pay a premium for it. Risk management is becoming indispensable in the highly volatile shipping market where bad decisions are disastrous. Another contract that offers some kind of protection in case of market downturns is shipbuilding option contracts.

An option gives the buyer the right but not the obligation to buy an asset at a given price before a specific date. In the case of shipbuilding option contracts, a shipyard gives the buyer the right to order a newbuilding at a pre-determined price. The decision to
exercise the contract and build the vessel lies with the contract’s holder. At the contract’s expiration date, if it is profitable for the holder of the contract to exercise his right, then he is able but not obliged to do so. On the other hand if at expiration it is not profitable for the buyer to build a ship at the pre-determined price then he will not exercise the option. Shipyards believe that by giving out option contracts they fill out the capacity of the yard. On the other hand shipowners believe that option contracts give them the ability to build vessels with some flexibility, depending on if it is profitable or not. Obviously there is some value in these contracts but it is not very clear, within the industry, who has the upper hand. In practice shipyards give the options for free within a shipbuilding contract. Having closed a deal with a buyer a shipyard might add an option to the contract allowing the buyer to build another ship at the same price within a specified period.

In the financial markets the buyer of an option contract has to pay for it. Obviously an option contract creates some value for the holder. As in the case of time charters shipbuilding option contracts offer protection to the holder in case of a downturn in the shipbuilding market. If the price of newbuildings falls then the holder of the option contract will not choose to exercise his right and build a ship at an inflated price, relative to the market. On the other hand, contrary to time charters in option contracts the holder gains from an increase in the shipbuilding market. If the price of newbuildings increases then the holder of the contract will exercise his right and build a ship at a price lower than the market price. Selling the ship at the market price immediately will generate instant profit for the holder of the option contract. Because the gain of these contracts is not limited, the holder may have to pay a price for the protection he gets. The value of option
contracts lies in the ability the holder has to wait and see the movement of the market before exercising his right. It often seems that shipyards are giving away free lunches.

Although the market for shipbuilding option contracts is not very developed in financial markets, trading financial options has exploded in recent years. The development of a methodology for the pricing of financial derivatives and their flexible use in hedging strategies has increased their demand considerably. In a revolutionary paper by Black and Scholes a closed form solution for the calculation of an option’s price was developed. Currently options are traded for stocks, interest rates, exchange rates, commodities and basically anything that is traded in a well-functioning market. Financial derivatives have become very complicated with floating exercise prices and complicated payoff functions, yet the methodology developed by Black, Scholes and others defines a framework within which all these complicated contracts can be priced.

1.1 Thesis Objectives and Review of Previous Research

Shipbuilding contracts have become popular in recent years. Yet their price is set arbitrarily and usually based on the shipyard and shipowner’s intuition. The existence of a market for financial options and a methodology for pricing them gives us an idea of how one could go about valuating such a contract. Although an option contract’s payoff, at maturity, depends on whether the option will be exercised or not, evidence from financial markets supports the fact that their value is fixed. A shipbuilding option contract’s value moves in relation to the value of the underlying asset, the price of a newbuilding. This correlation enables us to determine a formula that calculates the value of these contracts. Assuming a movement for the price of the underlying asset we can
determine the option contract’s payoff and thus calculate its price. Extensive work has been done on the valuation of financial derivatives and this methodology can be applied to shipbuilding option contracts.

At first glance the obvious way to go about valuating these option contracts is to assume that the underlying asset is the price of newbuildings. Since newbuildings are not a tradable asset this poses many difficulties. Firstly, the price of a newbuilding does not follow a well-defined stochastic process. Secondly, financial option pricing methodology assumes that the underlying asset can be bought or sold at any fraction, an unreasonable assumption for ships. Due to these limitations, pricing option contracts using newbuilding prices as the underlying asset will not give plausible results. The valuation of option contracts can also be performed using freight rates as the underlying asset. Freight rates are tradable assets quoted at the Baltic International Freight Futures Exchange (BIFFEX). Since the price of ships and consequently newbuildings is contingent on freight rates, their movement determines the value of option contracts. The movement of freight rates is very similar to that of interest rates exhibiting mean reversion. Freight rates fluctuate cyclically around a fixed level. The existence of time charters makes the modeling of the movement of freight rates even more realistic. Time charter rates give us some idea of the expected future level of freight rates. Incorporating them will produce a model that includes market expectations about the movement of the underlying asset. A trinomial tree model can be developed representing the possible paths freight rates can move to in the future. Based on the movement of the freight rates the option contract’s price can be determined.
This thesis develops a method for calculating the price of shipbuilding option contracts using both newbuilding prices and freight rates. For the reasons mentioned above, emphasis will be given to the valuation methodology using freight rates. The results will be compared and a sensitivity analysis will be performed. Although an exact value for these contracts will be calculated, absolute values must be viewed with caution in an illiquid market. The sensitivity of the option contract’s price to the movement of newbuilding prices, time, market expectations, and volatility will be examined using the option pricing methodology.

Black, Scholes and Merton (1973) developed the theory behind financial options valuation. They derived a closed form solution for the price of call and put options. Although their pricing model was revolutionary it is based on some simplifying assumptions that do not hold in the case of shipbuilding options. Simple geometric Brownian motion for and the tradability of the underlying asset are some of these assumptions. Hull and White (1990) expanded this methodology for the pricing of interest rates. Their model had several characteristics that are applicable in the case of freight rates. Firstly, the underlying asset is assumed to follow a mean reverting process. Secondly, their valuation assumes a term structure for the underlying asset. In the case of freight rates a term structure can be developed using time charter rates. In an important paper published by Cox, Ross, and Rubinstein (1979) the principals of risk neutral valuation and tree pricing were developed. According to this theory, which is the discrete equivalent of the Black-Scholes methodology, a tree of the possible future paths the underlying asset can take is developed. Knowing the contract’s payoff at maturity and using backward induction, the price of an option can be calculated.
Although the theory for financial options is quite extensive the case of real options is quite different. Real option theory refers to options imbedded in investment decisions. For example, choosing to invest in a ship can be seen as an option. The investor has the right but not the obligation to pay the price and get the ship or he can defer his decision. The underlying assets for real options are usually investments that are not traded in the market. Dixit and Pindyck (1993) applied the mathematical rigor used in pricing financial options to real projects. They developed a framework through which investments can be viewed as options as well as the methodology for pricing them. Using this pricing model they also examined optimal investment strategies. Trigeorgis (2000) researched the valuation of real options with examples from the shipping industry. His work gives us insight in how to model newbuilding prices and freight rates to price options. Bjeksund and Ekern (1992) examined more thoroughly the stochastic process followed by freight rates. According to their work freight rates exhibit mean reversion and can be modeled using the Ornstein-Uhlenbeck process.

Hoegh (1998) calculated the price of shipbuilding option contracts applying the theory for financial options. In his work newbuilding prices are assumed to be the underlying asset and a stochastic process representing their movement is determined. According to his findings none of the well-defined stochastic processes, namely geometric Brownian motion and Ornstein-Uhlenbeck process, describe accurately and completely the movement of newbuilding prices. Despite this discouraging finding the price of several option contracts is calculated using different option characteristics and parameters.
1.2 Thesis Organization

The thesis is organized as follows. Chapter 2 presents a general overview of the theory behind the valuation of financial options. Both the continuous (Black-Scholes) and the discrete (binomial) models are presented and the methodology for pricing options is analyzed. The basic theory behind stochastic processes is discussed and the geometric Brownian motion and the Ornstein-Uhlenbeck process are defined. These two processes are widely used in calculating the prices of stock and interest rate options. Freight rates, being the main focus of this thesis, can be modeled with the same tools as interest rates. The theory of interest rate modeling, using a term structure, and the methodology behind building a tree for the movement of interest rates are presented. The trinomial tree developed is used to price shipbuilding option contracts.

Chapter 3 presents the data used for the analysis and examines the process they follow. Newbuilding prices and freight rates are tested for geometric Brownian motion and Ornstein-Uhlenbeck process as defined in Chapter 2. The parameters governing these processes are calculated and assessed. The relationship between newbuilding prices and freight rates is also analyzed. A linear equation that determines the newbuilding price based on the current freight rate is calculated. Finally, charter rates are examined and a term structure for freight rates is developed. Both the linear equation and the term structure are essential in developing the trinomial tree that is used in the pricing of the option contracts in Chapter 4.

In Chapter 4 the pricing theory of financial options, presented in Chapter 2, is applied to shipbuilding option contracts. The price of option contracts using newbuilding prices as the underlying asset is calculated assuming both a geometric Brownian motion
and an Ornstein-Uhlenbeck process. The method of pricing option contracts using freight rates as the underlying asset is also presented. Freight rates are assumed to follow an Ornstein-Uhlenbeck process and a trinomial tree describing their movement is developed as defined in Chapter 3. A sensitivity analysis is performed and the parameters influencing the price of shipbuilding option contracts are analyzed. The results are presented and discussed qualitatively.

Finally, in Chapter 5 the implications of this thesis results to the shipbuilding industry are discussed. The limitations of the theory, some practical considerations, proposals for further research, and concluding remarks are presented.
2. Financial Option Theory

2.1 Financial Options

The market for financial options exploded with the foundation of the Chicago Board Options Exchange (CBOE) in 1973. Initially CBOE allowed investors to buy and sell stock options for individual shares. Since then options on indexes, commodities, bonds, foreign exchange etc. have been added. By establishing a standardized process the CBOE developed a market where different investors having different investment strategies could trade these option contracts. Without going into many details I will try to give an overview of what financial options are and how they are valued. The same methodology will be used to value shipbuilding options.

The two most basic option contracts are calls and puts. A call option, on a stock, gives the buyer the right but not the obligation to buy a stock at a specified exercise price within a specified exercise date. On the other hand, a put option gives the buyer the right to sell the stock. An important feature of options that distinguishes them from other contracts such as futures and forwards is that the decision to exercise or not to exercise the contract falls on the holder of the contract. So, if it is not profitable for the holder to exercise the option, he is not obligated to do so. This fact alone tells us something about the contract’s payoff, namely that it is limited to zero. In other words the holder of an option contract cannot lose more than what he has paid for the contract.

Consider, for example, a call option for a Microsoft stock with exercise price $45 and maturity one year from now. Having maturity one year from now means that if the option is not exercised within one year it expires worthless. Assuming that the price of
Microsoft’s stock is $40, at maturity, then it is not optimal for the holder of the option to exercise. If the option is exercised the holder will buy the stock at $45, which is not optimal since he could have bought it on the open market at $40. Let’s consider now an increase in Microsoft’s price. If the price rises to $50, the holder of the option can exercise his right to buy the stock at $45 and sell it on the market for $50 making an instant profit of $5. From the option contract’s payoff structure it is obvious that the payoff is closely related to the value of the underlying asset, in this case the price of Microsoft’s stock. If the price of the underlying asset is greater than the exercise price, then an increase in the price of the underlying asset leads to an increase in the call option’s payoff. The opposite is true for the payoff of a put option. In the case of a put option, the holder has the right to sell the underlying asset. Hence, a drop in the price of the underlying asset, below the exercise price, leads to an increase in the option’s payoff. In mathematical terms the payoff of a call option is:

\[ C = \max (S - K, 0) \]

and the payoff of a put option is:

\[ P = \max (K - S, 0) \]

where \( C \) and \( P \) are the payoffs of the call and put options, respectively, \( K \) is the exercise price and \( S \) is the price of the underlying asset.

At any given time if it is profitable for the option to be exercised then the option is said to be in the money (in the case of a call option when \( S > K \)). On the other hand, if it is not optimal to exercise, the option is out of the money. There exists a basic distinction among option contracts depending on when the holder has the right to exercise the option. In American options the holder has the right to exercise the option at any time
during its life. On the other hand for European options the holder has the right to exercise only at the exercise date. Since for shipbuilding options the holder has the right to exercise at any time up to the exercise date, I will concentrate on American options.

It is interesting to note that for non-dividend paying stocks it is never optimal to exercise an American option before the expiration date. Let $C$ be the value of the option contract. Then $C \geq S - K$. However, let’s assume that this is not the case; then $C < S - K$, which means that by buying the call and exercising it immediately one could make an instant profit $(S - K - C > 0)$. But since there are no arbitrage opportunities in a well functioning market $C \geq S - K$. So it is more profitable to keep the option alive than to exercise it. If we incorporate dividends into the model then the payoff formula becomes a bit more complicated. Dividends are paid to the holder of the underlying asset but not to the holder of the option; thus their value must be subtracted from the value of the option. The drop in the call value due to the distribution of dividends might also influence the timing of the exercise. For example consider the extreme case where a stock is about to pay its complete value in dividends. If an in the money call option holder does not exercise early, the value of the stock will go to zero, due to the dividend payout, and the option will be worthless at the exercise date. In this case the option contract holder should exercise early and capitalize the gains. In the case of shipbuilding options there are no obvious dividends since the building of a ship is a process that takes usually more than the life of the options. So exercising the option early will start the shipbuilding process but will not generate any income for the shipowner, holder of the option. The income the holder of the option contract gets is the same as the income the holder of the underlying asset, a ship under construction, gets.
2.2 Parameters Influencing the Value of an Option

It has been determined that the payoff of an option contract, at maturity, is contingent on the value of the underlying asset and the exercise price. Before the option’s expiration a number of other variables are also important. These variables will also be important in determining the value of a shipbuilding option. Some fundamental parameters influencing an option contract’s price are:

1. Current underlying asset price (S)
2. Exercise price (K)
3. Time to maturity or expiration (t)
4. Underlying asset volatility
5. Interest rates
6. Cash dividends

Looking at the call option’s payoff formula \( C = \max(S - K, 0) \) it is easy to see that the call option’s payoff, and hence its value, increases as the price of the underlying asset increases and the exercise price decreases. For the put option the value increases as the price of the underlying asset decreases and the exercise price decreases. In this respect the call and put options behave in opposite ways.

Time to maturity measures the amount of time remaining in the life of the option. Both call and put options increase in value as time to maturity increases. To see why this is true consider two options with different times to maturity. The option with the longer time to maturity offers more opportunities for exercising. The holder of the option with longer maturity can exercise it as long as the holder of the option with shorter maturity can, plus an extra amount of time. This extra flexibility is valuable and thus the option with longer maturity should be at least as valuable.

\[ ^1 \text{ Cox (1985) pp. 34} \]
Volatility is a measurement of how uncertain the movement of the underlying asset's price is. High volatility means that the asset's price fluctuates more and thus the final price level is more uncertain. An increase in volatility leads to an increase in the probability the asset will do very well or very badly. Since the option’s payoff is asymmetric this leads to greater value for the holder of an option contract. As mentioned earlier, the option’s final loss is limited to zero; thus, the holder of the contract does not lose more money if the asset’s price decreases a lot, assuming that the option is out of the money. On the other hand in the case of an increase in the asset’s price, since the holder of the contract captures this value, a greater increase leads to greater profit and thus greater value.

The effect of interest rates is different for put and call options. An increase in the interest rates leads to an increase in the asset’s expected payoff, according to the capital asset pricing model (CAPM)\(^2\). Moreover, a higher interest rate leads to a higher discount factor and the value of all future payoffs decreases. In the case of put options these two effects decrease the value of the contract. On the other hand for call options the first effect increases whereas the second decreases its value. Empirical evidence shows that the first effect dominates the second and that the call option’s value increases with an increase in interest rates.

Dividends decrease the value of the underlying asset. Paying dividends means that some of the underlying asset’s value is paid out as dividends. The effect of dividends on options is thus similar to a decrease in the price of the underlying asset. In the case of call

\(^2\) Bodie (1996) pp. 238
options dividends decrease the value of the contract whereas the opposite is true for put options. The effects of the mentioned parameters are summarized in Table 2.1:

<table>
<thead>
<tr>
<th>Determining Factor</th>
<th>Effect of Increase</th>
</tr>
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<tbody>
<tr>
<td>American Call</td>
<td>American Put</td>
</tr>
<tr>
<td>1. Current underlying asset price</td>
<td>↑</td>
</tr>
<tr>
<td>2. Exercise price</td>
<td>↓</td>
</tr>
<tr>
<td>3. Time to maturity</td>
<td>↑</td>
</tr>
<tr>
<td>4. Underlying asset volatility</td>
<td>↑</td>
</tr>
<tr>
<td>5. Interest rate</td>
<td>↑</td>
</tr>
<tr>
<td>6. Cash dividends</td>
<td>↓</td>
</tr>
</tbody>
</table>

Table 2.1: Effect of Different Parameters on Option Contract Prices

Having established the effect of these parameters on the value of call and put options the boundaries within which the exact value of an option must lie can be determined. The value of an American call option is limited by the price of the underlying asset. If this were not the case, an arbitrage opportunity would exist, namely an investor could buy the underlying asset and sell the call option and make an instant profit without any risk. Let c be the price of the call option. If $c > S$ then the mentioned strategy would generate a profit for the seller of the option contract. The seller will fulfill the option obligation with the acquired asset. Assuming a well functioning market the value of a call option has to be less than the underlying asset’s value ($c \leq S$). A lower boundary for the value of a call option is the value the holder gets by exercising the option immediately. If the value of the call option was less than that, an investor could buy the option exercise and enjoy an instant profit. So, $c \geq \max (S - K, 0)$. It is also apparent that the value of a call option asymptotically approaches zero as the price of the underlying asset decreases. If the call option is deep out of the money, then the possibility

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3 Cox (1985) pp. 37
of exercise is so slim that the contract does not have any value. On the other hand, as the price of the underlying asset increases the price of the call option asymptotically approaches the price of the underlying asset. If a call option is deep *in the money*, then exercise is certain and the contract payoff is the asset's price minus the exercise price. An increase in the price of the underlying asset leads to a direct increase in the price of the call option.

An upper boundary for the price of a put option is the exercise price. In the extreme case where the underlying asset's price falls to zero the holder of a put option will exercise the right to buy the asset for zero and sell it for the exercise price making a maximum profit equal to the exercise price ($p = K$). A lower boundary for the value of a put option is, as in the case of the call option, the value of immediate exercise. The non-existence of arbitrage opportunities leads to $p \geq \max(K - S, 0)$. For the same reasons as in call options the value of a put option will asymptotically approach zero, for large values of the underlying asset's price. The price of the put option will also asymptotically approach the exercise price minus the asset's price, for small values of the underlying asset's price. Figure 2.1 shows a graph of the price of a call and put option, respectively, as a function of the underlying asset's price. The shaded regions are restricted according to the mentioned boundary conditions.
So far some boundary conditions for the value of option contracts have been mentioned. In the next two sections two different methods for finding an exact price for these contracts will be examined.

2.3 Option Valuation Using Binomial Trees\(^4\)

The value of an option is closely correlated with the value of, and consequently the movement of, the underlying asset’s price. Since the option’s payoff is a function of the asset’s final price its value is influenced by the risks determining the asset’s price. Due to this correlation a portfolio including call options and the asset can be set up so that the portfolio’s return is riskless. A riskless asset should earn a return equal to the risk-free interest rate and since we know the cost of setting up the portfolio, the value of the call option can be easily calculated. Building a riskless portfolio is the essence behind the binomial valuation of options. To see how this works consider an asset whose price

\(^4\) This section is based on the theory presented in Hull (2000) pp. 201
today is $S_0$. In one period from now the price of this asset will either go up to $uS_0$ or
down to $dS_0$ ($u > 1$ and $d < 1$). Suppose there is a call option on this asset whose payoff is
$C_u$ if the underlying asset’s price goes up and $C_d$ if the asset’s price goes down. The time
to maturity for the call option is $T$. The binomial tree is illustrated in Figure 2.2.

Consider the following strategy of buying a fraction of the asset equal to $\Delta$ and
selling the call option. $\Delta$ is chosen so that the portfolio’s payoff is riskless. $\Delta$ is called the
delta or hedge ratio and is the change in the price of an option for a $\$1$ increase in the
underlying asset’s price\(^5\). To make the portfolio riskless the value at period one has to be
the same no matter if the price of the underlying asset has gone up or down. So,

$$\Delta uS_0 - C_u = \Delta dS_0 - C_d \iff \Delta = \frac{C_u - C_d}{uS_0 - dS_0}$$

Since the portfolio’s return is the risk-free rate then its present value is:

$$(\Delta uS_0 - C_u)e^{-rT}$$

and the portfolio’s cost is:

$$\Delta S_0 - C.$$ 

From these two equations we can deduce the price of the call option ($C$)

\(^5\) Bodie (1996) pp. 671
\[ C = e^{-rT} (pC_u + (1 - p)C_d) \]

where

\[ p = \frac{e^{rT} - d}{u - d}. \]

For a numerical example consider a call option for a stock with exercise price $50 and maturity in one year. Let's assume that in one year the price of the stock will either be $55 or $45 (\( u=1.1 \) and \( d=0.9 \)). Assuming a risk-free interest rate of 5% \( P_0 = 0.756 \) and \( C = e^{-0.05*1} (0.756 \times (55 - 50) + (1 - 0.756) \times 0) = \$3.6 \).

This method of valuating the price of call options is also called risk-neutral valuation. In a risk-neutral world investors are indifferent to risk and thus contrary to CAPM they do not require any compensation for taking up investments with uncertain payoff. This means that the expected return of all investments in this world is the risk-free rate. Calculating the expected return of the underlying asset based on the assumed probability \( p \) we can see why the binomial model assumes a risk-neutral world.

\[ E(S) = puS_0 + (1 - p)dS_0 \iff E(S) = p(u - d)S_0 + dS_0. \]

Substituting for the value of \( p \)

\[ E(S) = e^{rT} S_0. \]

Setting the probability of an increase in the price of the underlying asset equal to \( p \) is the same as assuming a risk-neutral world. According to this result, to price an option we can assume risk-neutrality. To calculate the price of an investment in such a world we can take the expected value of the payoff using the risk-neutral probability.
This model can be extended to many periods, which allows for a more realistic representation of the movement of the underlying asset's price. To generalize assume that the time to maturity is divided into \( n \) steps of equal time \( \Delta t \) (\( T = n\Delta t \)). Let's consider the case of a two period binomial tree illustrated in Figure 2.3

![Two Period Binomial Tree Diagram](image)

**Figure 2.3: Asset and Call Option Movement in Two Period Binomial Tree**

From the analysis in the one period case we can deduce the values for \( C_u \) and \( C_d \). We basically take the expected value using the risk-neutral probability

\[
C_u = e^{-r\Delta t} \left( pC_{uu} + (1-p)C_{du} \right)
\]

and

\[
C_d = e^{-r\Delta t} \left( pC_{du} + (1-p)C_{dd} \right).
\]

Working backwards and taking again the expected value, the price of a call option is calculated

\[
C = e^{-r\Delta t} \left( pC_u + (1-p)C_d \right) = e^{-r\Delta t} \left[ p^2 C_{uu} + 2p(1-p)C_{du} + (1-p)^2 C_{dd} \right].
\]

This result is consistent with the risk-neutral valuation. The price of the call option is the expected value of the payoff given the risk-neutral probability.
\[ p = \frac{e^{rT} - d}{u - d}. \]

This result can be generalized for any \( n \). Since the call option’s payoff is \( \max(S - K) \) the value of the call option is

\[
C = e^{rT} \left( \sum_{j=0}^{n} \frac{n!}{j!(n-j)!} p^j (1 - p)^{n-j} \max(u^j d^{n-j} S_0 - K) \right).
\]

The value of \( j \) shows how far up or down we are in the tree. If the price of the underlying asset, at maturity, is \( 5uS_0 \) then \( j = 5 \). There is a value for \( j \) for which the call option expires \textit{in the money} (\( u^j d^{n-j} S_0 > K \)). For all \( j < a \), \( \max(u^j d^{n-j} S_0 - K, 0) = 0 \) and for all \( j \geq a \), \( \max(u^j d^{n-j} S_0 - K, 0) = u^j d^{n-j} S_0 - K \). Therefore,

\[
C = S \left[ \sum_{j=a}^{n} \frac{n!}{j!(n-j)!} p^j (1 - p)^{n-j} (e^{-rT} u^j d^{n-j}) \right] - Ke^{-rT} \left[ \sum_{j=a}^{n} \frac{n!}{j!(n-j)!} p^j (1 - p)^{n-j} \right].
\]

The similarity of this formula to the Black-Scholes formula will be apparent in the next section.

The binomial model can be applied for the price calculation of put options and any derivative whose payoff is closely correlated with the movement of the underlying asset. According to this model having assumed a simple binomial movement for the underlying asset’s price the value of a derivative can be calculated. In practice \( u \) and \( d \) are chosen to match the volatility of the asset’s movement. A popular way of doing this is by setting \( u = e^{\sigma \sqrt{T}} \) and \( d = e^{-\sigma \sqrt{T}} \).

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6 Cox (1985) pp. 177


2.4 Stochastic Processes

To develop their formula Black and Scholes assumed a stochastic process for the movement of the underlying asset. Any variable whose value changes over time in an uncertain way is said to follow a stochastic process. Developing the right stochastic process for the given underlying asset is very important in pricing options since their value is contingent on the movement of the asset’s price. Many stock option pricing models assume a Markov process for the movement of the underlying asset’s price. A variable follows a Markov process if the future value of the variable is a function of the present state. The past is not important in predicting the future for this group of processes. A Wiener or Brownian process is a particular class of Markov processes. More specifically assuming that \( z \) is a variable following a Wiener process then the following properties must hold:

Property 1.
The change \( \Delta z \) during a small period of time \( \Delta t \) is

\[
\Delta z = \varepsilon \sqrt{\Delta t}
\]

where \( \varepsilon \) is a random drawing from a standardized normal distribution with mean 0 and standard deviation 1.

Property 2.
The values of \( \Delta z \) for any two different short intervals of time \( \Delta t \) are independent.

\( \Delta z \) itself is a normal random variable with mean 0 and standard deviation \( \sqrt{\Delta t} \). Since in most cases the expected return of the underlying asset is not 0 this process can be generalized to include more realistic scenarios. A generalized Wiener process for a variable \( x \) is defined as follows:

\[ \text{Property 1.} \]
\[ \text{Property 2.} \]

---

8 Hull (2000) pp. 218
\[ \Delta x = a \Delta t + b \Delta z \]

where \( \Delta z \) is a Wiener process as defined above. This process has two components. The term \( a \Delta t \) implies a drift. In other words the expected change of the process over time is \( a \). The term \( \Delta z \) on the other hand is a random movement added to the drift. There is some variability added to the movement of \( x \). In this case \( \Delta x \) follows a normal distribution with mean \( a \Delta t \) and standard deviation \( b \sqrt{\Delta t} \).

A further generalization within the class of Markov processes is the Ito process. In the case of the Ito process \( a \) and \( b \), as defined in the generalized Wiener process, are functions of \( x \) and \( t \). Assuming \( x \) is a variable following an Ito process then:

\[ \Delta x = a(x, t) \Delta t + b(x, t) \Delta z . \]

Two stochastic processes that fall under this category and are widely used for representing the movement of stocks and commodities are the geometric Brownian process and the Ornstein-Uhlenbeck process. The valuation of the shipbuilding option contracts will be done using these two processes. The geometric Brownian motion is defined as (in continuous time):

\[ dX = aX dt + bX dz . \]

The attractive feature of this process is that the percentage increments of \( X \) have constant mean and variance. This is especially important in stock modeling where the expected percentage return and volatility is independent of the stock’s price. No matter the level of a stock’s price the investors expect a constant return based on the risk of the stock. The same is true for the stock’s volatility. The equation for the geometric Brownian motion shows that \( \frac{dX}{X} \) (the percentage change of the stock) is normally distributed with mean
and standard deviation $b\sqrt{dt}$. Assuming that $dx$ follows a geometric Brownian motion we can deduce that $X$ has a lognormal distribution. A variable has a lognormal distribution if its log is normally distributed. To see why $X$ has a lognormal distribution we have to use Ito’s lemma derived by the mathematician K. Ito.\(^{10}\) Ito’s lemma is important because ordinary calculus is not valid for stochastic processes. To calculate the derivative of a variable that is a function of a stochastic process this lemma has to be used. Suppose that a variable $x$ follows an Ito process (in continuous time)

$$dx = a(x, t)dt + b(x, t)\epsilon\sqrt{dt}$$

a function $G$ of $x$ follows the process, according to Ito’s lemma

$$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}bd\epsilon$$

which is also an Ito process. Returning to the geometric Brownian motion, assuming $X$ follows a geometric Brownian motion, let $G = \ln X$; if $G$ is normally distributed then $X$ is lognormal. Using Ito’s lemma and:

$$\frac{\partial G}{\partial X} = \frac{1}{X}, \quad \frac{\partial^2 G}{\partial X^2} = -\frac{1}{X^2}, \quad \frac{\partial G}{\partial t} = 0$$

it follows that

$$dG = (a - \frac{b^2}{2})dt + bd\epsilon$$

\(^{10}\) K. Ito, “On Stochastic Differential Equation,” 151
G follows a normal distribution with mean \((a - \frac{b^2}{2})dt\) and standard deviation \(b\sqrt{dt}\). The mean of X is \(X_0e^{at}\) and the standard deviation is \(X_0e^{at}\sqrt{(e^{bt} - 1)}\), where \(X_0\) is the value of X at time 0.

The Ornstein–Uhlenbeck process is important in modeling the fluctuation in commodities prices because it exhibits mean-reversion. For assets such as oil and other commodities it can be argued that their price is contingent on a long-run cost of production. If there is a significant increase in the price of oil then supply will increase leading to a drop in the price. The opposite is true in the case of a price drop. Commodities move around a fixed level. The simplest mean reverting process is the Ornstein–Uhlenbeck process

\[
dX = k(a - X)dt + bdz
\]

\(k\) is the speed of reversion and \(a\) is the fixed level around which the variable moves. If \(k\) is large then the process exhibits large reversion, which means that if the level of the variable moves away from the fixed level then there is great pressure to bring the variable close to the fixed level. In other words, there is little fluctuation around the fixed level. Although there is no closed form solution for the distribution of X given an Ornstein–Uhlenbeck process we still know its mean and standard deviation. The mean of X is \(a + (X_0 - a)e^{-kt}\) and its standard deviation is \(\frac{b}{\sqrt{2k}}\sqrt{(1-e^{-2kt})}\), where \(X_0\) is the value of X at time 0.

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12 Dixit and Pindyck (1994) pp. 74
2.5 Black-Scholes Formula

In a paper published in 1973 Fisher Black and Myron Scholes revolutionized the way financial options were priced. In its essence the Black-Scholes formula uses the same argument as in the binomial valuation model. A riskless portfolio is built in continuous time leading to a partial differential equation that has to be satisfied by the price of the option. Solving this partial differential equation gives a closed form solution for the price of an option on an underlying asset. Assume that the underlying asset, a stock, follows a geometric Brownian motion $dS = aSdt + \sigma Sdz$. The value of a call option $C(S, t)$ is a function of the stock price and thus according to Ito’s lemma,

$$dC = \left( \frac{\partial C}{\partial S} aS + \frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial C}{\partial S} \sigma Sdz.$$

The stochastic terms of $dC$ and $dS$ are the same except for a scalar multiple $\frac{\partial C}{\partial S}$. This multiple is the hedge ratio. Consider now the following portfolio: sell the call option, buy $\frac{\partial C}{\partial S}$ shares, and borrow the difference to fund the investment. Assume a risk-free interest rate equal to $r$. The initial value of the portfolio $(P)$ is zero. The value of the portfolio in the next period is going to be:

$$dP = -dC + \frac{\partial C}{\partial S} dS - r(\frac{\partial C}{\partial S} s - C)dt = \left( -\frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 - rC + rS \frac{\partial C}{\partial S} \right) dt.$$

Since the portfolio does not depend on $dz$ it is riskless. The cost of setting up this portfolio is zero since the money needed to fund the investment was borrowed ($dP=0$). The partial differential that must be satisfied is:

$$\frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 - rC + rS \frac{\partial C}{\partial S} = 0.$$
The boundary conditions for a call option are: \( C(0,t) = 0 \), \( \lim_{S \to 0} C(S,t) = S \), and \( C(S,T) = \max(S-K,0) \), where \( T \) is the maturity date, \( t \) is the time to maturity and \( K \) is the exercise price. The solution to this partial differential equation is the Black-Scholes formula

\[
C(S,t) = SN(d_1) - Ke^{-rT} N(d_2)
\]

where

\[
d_1 = \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}
\]

\[
d_2 = d_1 - \sigma\sqrt{t}
\]

\( N(\cdot) \) is the normal cumulative distribution function and \( \sigma \) is the standard deviation of the annualized continuously compounded rate of return of the underlying asset.

The Black-Scholes formula confirms our intuition that the value of a call option is higher (1) the higher the value of the underlying asset, \( S \); (2) the longer the time to expiration, \( t \); (3) the lower the exercise price, \( K \); (4) the higher the risk-free interest rate, \( r \); and (5) the higher the standard deviation of the asset, \( \sigma \).\(^{13}\) For the Black-Scholes formula to be valid, certain assumptions must be made. The stock pays no dividends prior to expiration, is lognormally distributed, is traded in a frictionless market, without transaction costs and taxes, and has a constant standard deviation. Also, the interest rate is constant throughout the life of the option. Some of these assumptions can be relaxed and modified formulas can be developed.

A modification in the Black-Scholes formula has to be made in valuing real options. The difference between real and financial options is that in real options the underlying asset is not traded. An adjustment in the risk-neutral valuation methodology

\(^{13}\) Trigeorgis (1995) pp. 91
has to be made. In the risk-neutral world as defined earlier investors do not need any compensation for taking up risky investments. A portfolio can then be set up so its return is equal to the risk-free rate. Obviously for a non-tradable asset this portfolio cannot be built since non-integer positions in the underlying asset cannot be taken. For real options one can still use the risk-neutral pricing model adjusting for the expected growth of the underlying asset. The value of a call option is calculated using the Black-Scholes formula by changing the expected growth of the underlying from $\alpha$ to $\alpha - \lambda \sigma$, where $\lambda$ is the market price of risk\(^\text{14}\). The market price of risk can be calculated using the CAPM.

The Black-Scholes formula assumes that the underlying asset follows a geometric Brownian motion. An interesting question arises if we consider an Ornstein-Uhlenbeck process for the movement of the underlying asset. Trigeorgis developed a closed-form solution for this problem using Ito's lemma. Let

$$a^* = a - \frac{\sigma \lambda}{k} \quad \text{and} \quad m^* = e^{-kt} S + (1 - e^{-kt}) a^*$$

then the value of a call option on the underlying $S$ with exercise price $K$ and time to maturity $t$ is:

$$C(S, t) = Se^{-rt} \left[ (m^* - K) N(d) + \sigma_s n(d) \right]$$

where $N(\cdot)$ is the normal cumulative distribution function, $n(\cdot)$ is the normal probability density function, $d = \frac{m^* - K}{\sigma_s}$, and $\sigma_s = \sqrt{\frac{\sigma^2}{2k}(1 - e^{-2kt})}$\(^\text{15}\). The methodology for valuating an option on an underlying asset that follows either a geometric Brownian motion or an Ornstein-Uhlenbeck process has been presented.

\(^{14}\) Hull (2000) pp. 502

\(^{15}\) Hoegh (1998) pp. 78
3.6 Term Structure

For some assets, looking at the market and all the traded securities linked to that asset gives us information to build a better model for the asset's movement. If we know the price of the asset will be in the future, a stochastic process for its movement is not needed; knowing the future prices allows us to build a deterministic model. A deterministic model is very difficult to encounter in the real world since most assets have some randomness in their movement. In some cases, though, there is an indication of what the price level of the underlying asset might be in the future. An example of such an asset is interest rates. Zero-coupon bonds are bonds that pay the principal at the time of the contract’s maturity. So if an investor buys a zero-coupon bond, he is guaranteed a fixed return for the life of the bond. Say for example that an investor buys a zero-coupon bond paying $100 in 10 years at a price of $50. At maturity the investor will receive $100, and his return over the investment’s ten-year period will be 100%. Since there are tradable zero-coupon bonds for a variety of maturity dates, the value of the interest rate is known not only for one year but also for two, three, four, etc. From these values, the expected future interest rate can be derived. In the case of freight rates, time charter rates give us the same useful information.

The price of the zero-coupon bonds, which are assets traded in the market, defines the spot rate curve or term structure. What the spot rate curve shows is the annual rate we can achieve today for a long-term investment. If an investor lent his money for a period of five years, the spot rate curve would give him the annualized return he would achieve given the present market conditions. If $s_5$ is the five-year spot rate, then the total return for an investor lending money for a five-year period is $(1 + s_5)^5$. The forward rate curve,
deduced from the term structure, gives the rate an investor would get for lending his money for a period of time in the future. Let \( f_t \) be the forward rate at period \( t \), in other words the rate one would get for making a loan at period \( t \) with maturity at period \( t+1 \).

Deriving the forward rate curve from the term structure is a straightforward process. For the market to have no arbitrage opportunities the following equation must hold

\[
(1 + s_{s+1})^{t+1} = (1 + s_t)^t (1 + f_t)
\]

which means that \( f_t = \frac{(1 + s_{s+1})^{t+1}}{(1 + s_t)^t} - 1 \). To see how this works, consider lending money for a two-year period. There are two ways of doing this. First, one could enter a two-year contract using the two-year spot rate. The return of this investment is \( (1 + s_2)^2 \). Second, one could enter a one-year contract using the one-year spot rate, and one year from now enter a new contract with the proceeds, which would be \( (1 + s_1) \), using the forward rate \( (f_1) \). In an arbitrage-free world the two strategies should give the same return to the investor and thus

\[
(1 + s_2)^2 = \frac{(1 + s_1)^2}{(1 + s_1)} - 1.
\]

According to the expectation hypothesis, the forward rate is exactly equal to the market expectations for the future spot rate\(^{16} \). This means that from the existing market data and without assuming any stochastic process for the movement of interest rates the expected movement of interest rates is known. By developing a term structure and calculating the forward rates, one has a good idea of the movement of interest rates. It is interesting to note here that forward rates are not the actual future interest rates. In fact, the actual interest rates, in the future, could be very different from what the forward rates dictate. Forward rates are the expected value of interest rates given the current market conditions. The same methodology can be used to deduce the market expectations for

\(^{16}\) Luenberger (1998) pp. 81
freight rates. Time charter rates are the rates that a shipowner could achieve, employing his ship for a long period of time. A term structure for freight rates and a more realistic model for their movement can be built using time charter rates.

2.7 Interest Rate Models

With the extra knowledge provided by the term structure, a model for the movement of the interest rates can be built that incorporate this knowledge. Consider a binomial tree that represents the movement of interest rates. At each node we have the one-period rate for the next period. To complete the tree, we have to assign transition probabilities at each node. It is important to note here that in this model the risk neutral probabilities are not calculated but assumed. Without any loss of generality the transition probabilities are set to one-half. Having built the tree we can define the term structure. The value of a zero-coupon bond maturing in two years is equal to the expectation of the second period rates in the binomial tree, based on the probability function we have defined. Now, consider the whole process in reverse; assuming we have the term structure and the probability function, the binomial tree representing the movement of the interest rates is fixed. Building a model for interest rates that incorporates the term structure assumes that there are no arbitrage opportunities between the interest rates assumed by the tree and the current term structure.

The Hull and White model is one of the first no-arbitrage models that incorporates a term structure. The continuous time version of the model assumes that the following stochastic process governs the movement of interest rates

37
\[ dr = \alpha \frac{\theta(t)}{a} dt + \sigma dz \]

where \( \alpha \) and \( \sigma \) are constant. This process should remind us of the Ornstein-Uhlenbeck process with speed of reversion equal to \( \alpha \) and a fixed level equal to \( \frac{\theta(t)}{\alpha} \). In the Hull White model the fixed level is time-dependent since the initial term structure defines the path that the interest rates will take. If the term structure is upward sloping, then we expect interest rates to increase. The opposite is true if the term structure is downward sloping. The variable \( \theta(t) \) defines the direction that the interest rate moves at time \( t \) and can be calculated from the initial term structure using the formula

\[
\theta(t) = \bar{F}(0,t) + \alpha \bar{F}(0,t) + \frac{\sigma^2}{2\alpha} (1 - e^{-2\alpha t}).
\]

The last term of the equation can be ignored since it is relatively small. \( \bar{F}(0,t) \) is the forward rate with maturity \( t \). \( \bar{F}(0,t) \) is the partial derivative of the forward rate curve with respect to time. This means that the interest rate movement is a function of the forward rates' curve's slope.

This model can be applied to freight rates since there is a close relationship between the forward rate curve and time charter rates. If the freight rates follow the stochastic process defined in the Hull White model, then \( \bar{F}(0,t) \) is the time charter curve. Modeling freight rates this way incorporates the industry's expectations for their movement derived from time charter contracts. As mentioned earlier, freight rates exhibit mean reversion, a quality that is incorporated in the Hull and White model.

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17 Hull (2000) pp. 574
2.8 Tree Building Procedure\textsuperscript{18}

Hull and White have developed a way to construct a trinomial tree for the representation of their interest rate model. A trinomial tree is chosen instead of a binomial because the extra degree of freedom allows for the representation of the interest rates' mean reverting process. Assume that stochastic process followed by the interest rates is

\[ dr = (\theta(t) - ar)dt + \sigma dz. \]

The first step in building the tree is to construct the tree for interest rates without taking into account the current term structure. Let R be a variable that follows the stochastic process

\[ dR = -aR dt + \sigma dz. \]

To minimize the error in the modeling, \( \Delta R \), which is the spacing between interest rates, is set equal to \( \sigma \sqrt{3\Delta t} \). \( \Delta t \) is the time step on the tree. Define \( (i, j) \) as the node where \( t = i\Delta t \) and \( j = j\Delta R \). In this model \( j \) tells us how far up or down the tree we are, and \( i \) denotes the nodes' time period. To represent mean reversion in the tree, the branching must collapse into itself when the interest rates are very high or very low. Figure 2.4 shows the three branching methods that are going to be used.

\textsuperscript{18} This section is based on the theory developed by Hull and White (1998)
In most cases, the branching of Figure 2.4(a) will be used. For sufficiently large $j$ ($j_{\text{max}}$), it is necessary to switch to the branching of Figure 2.4(c) and for sufficiently negative $j$ ($j_{\text{min}}$), the branching of Figure 2.4(b) is appropriate. Hull and White showed that $j_{\text{max}}$ should be equal to the smallest integral greater than $\frac{0.184}{\alpha \Delta t}$ and that $j_{\text{min}}$ should be equal to $-j_{\text{max}}$. For the mean and standard deviation of the trinomial tree to equal those of the stochastic process, $p_u$, $p_m$, and $p_d$, being the probabilities of the up, middle and down movement, the following equations must be satisfied:

\[
\begin{align*}
 p_u \Delta R - p_d \Delta R &= -aj \Delta t \\
 p_u \Delta R^2 + p_d \Delta R^2 &= \sigma^2 \Delta t + \alpha^2 j^2 \Delta R^2 \Delta t^2 \\
 p_u + p_m + p_d &= 1.
\end{align*}
\]

Using $\Delta R^2 = 3\sigma^2 \Delta t$, the solution to these equations for the branching in Figure 2.4(a) is

\[
\begin{align*}
 p_u &= \frac{1}{6} + \frac{\alpha^2 j^2 \Delta t^2 - aj \Delta t}{2} \\
 p_m &= \frac{2}{3} - \alpha^2 j^2 \Delta t^2 \\
 p_d &= \frac{1}{6} + \frac{\alpha^2 j^2 \Delta t^2 + aj \Delta t}{2},
\end{align*}
\]

for the branching in Figure 2.4(b) the probabilities are
\[ p_u = \frac{1}{6} + \frac{a^2 j^2 \Delta t^2 + aj\Delta t}{2} \]
\[ p_m = -\frac{1}{3} - \frac{a^2 j^2 \Delta t^2 - 2aj\Delta t}{2} \]
\[ p_d = \frac{7}{6} + \frac{a^2 j^2 \Delta t^2 + 3aj\Delta t}{2} ; \]

and for the branching in Figure 2.4(c) the probabilities are

\[ p_u = \frac{7}{6} + \frac{a^2 j^2 \Delta t^2 - 3aj\Delta t}{2} \]
\[ p_m = -\frac{1}{3} - \frac{a^2 j^2 \Delta t^2 + 2aj\Delta t}{2} \]
\[ p_d = \frac{1}{6} + \frac{a^2 j^2 \Delta t^2 - aj\Delta t}{2} . \]

The second step of the tree building procedure is to incorporate the initial term structure and convert the tree for \( R \) into a tree for \( r \). A new tree is created for which the entry at node \((i, j)\) is equal to the entry at node \((i, j)\) in the \( R \)-tree plus

\[ a(t) = F(0, t) + \frac{\sigma^2 t^2}{2} . \]

\( F(0, t) \) can be derived from the forward rate curve as defined earlier.

This method establishes a mean reverting trinomial tree that can be used in valuating the price of interest rate derivatives using the backward expectation method defined for binomial trees. The price of shipbuilding option contracts will be calculated using a trinomial tree for the movement of freight rates.
3. Data

3.1 Data Source

In developing a model for the prices of shipbuilding option contracts, choosing the correct data set is very important. Not only does the underlying asset have to follow a well-defined process, but the calculated parameters from the data, such as mean, volatility, speed of reversion, etc. also have to represent the movement of the underlying asset accurately. An important consideration in calculating these parameters for financial options is the time chosen time horizon. The period considered should be long enough for the results to be statistically significant. On the other hand, going too deep in the past will lead to misleading calculations, since most of these parameters change over time. The standard deviation of newbuilding prices over the last 100 years might not accurately represent the existing market volatility. In practice most of these calculations are done for a time span equal to the contract’s life. If we need to calculate the value of a three-year call option, then the standard deviation over three-year periods is calculated.

For this analysis there are two sets of data used. The first data set was provided by Marsoft Inc., a consulting and market-forecasting firm based in Boston. Quarterly prices of newbuildings, freight rates and timecharter rates are quoted for Handymax (43,000 dwt) and Handysize bulk carriers (27,000 dwt). The prices are quoted for twenty years between 1980 and 2000 with the exception of Handysize freight rates that are quoted for ten years between 1990 and 2000. Figure 3.1 shows a plot of newbuilding prices and freight rates according to Marsoft’s data. The prices of the vessels are in million dollars.
and the freight rates are in dollars per ton corresponding to the grain trade route from the US Gulf to Rotterdam.

Figure 3.1: Newbuilding Prices and Freight Rates (Marsoft)

The second data set was provided by Clarksons & Co. Ltd., one of the largest ship brokerage firms based in London. The prices are quoted in monthly terms and cover a wide range of bulk carrier tonnage. Newbuilding prices are quoted for Handysize (23-30,000 dwt and 33-35,000 dwt), Handymax (40,000 dwt), Panamax (70,000 dwt), and Capesize (120,000 dwt and 150,000 dwt) bulk carriers. The period covered varies from fifteen to twenty-five years between 1976 and 2001. Freight rates are quoted monthly for Handymax, Panamax, and Capesize vessels for the period starting January 1990 up to January 2001. Figures 3.2 and 3.3 show the prices of newbuilding and freight rates according to Clarkson’s data. The prices of newbuildings are in million dollars and the
freight rates are in dollars per day according to the earnings a vessel of that tonnage would generate if it were operated in the spot market.

Figure 3.2: Newbuilding Prices (Clarksons)

Figure 3.3: Freight Rates (Clarksons)
3.2 Data Testing

The first step in modeling newbuilding prices and freight rates according to a stochastic process is to determine what general process they follow. As mentioned earlier, in practice, stocks and commodities are modeled according to the geometric Brownian motion or the Ornstein–Uhlenbeck (mean-reverting) process. A regression is run to determine which of these two processes, if either, is more appropriate. According to the theory, an important parameter in determining the validity of the results is the t-statistic. The t-statistic defines a region where the null hypothesis is rejected while comparing two independent samples. The region is specific to a confidence interval and the number of observations in the data set. If the two variables the regression is run on have different means, the rejection region is $|t| > t_{v-2}(a/2)$. 100$(1-\alpha)$ is the confidence interval, which in practice is usually 95%, and $m$ and $n$ are the number of observations in the two samples. $t_{v-2}$ is the t-distribution with $n+m-2$ degrees of freedom and can be looked up in a table\textsuperscript{19}. The critical value for the t-statistic is 1.96 for a 95% confidence interval and 80 degrees of freedom. If the t-statistic is greater than 1.96, the parameters calculated from the regression are significant. The regression for examining if the data follows a geometric Brownian motion is:

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = a + b \ln\left(\frac{X_{t-1}}{X_{t-2}}\right) + \varepsilon.$$  

According to the definition of the geometric Brownian motion, the difference between the logarithms of two consecutive price observations is normally distributed and does not depend on the prices’ history. For the null-hypothesis to hold, $b$ has to be zero, which

\textsuperscript{19} Rice (1995) pp. 392
would mean that there is no correlation between the level of the price now and the deep past. If the value of $b$ is close to zero and the $t$-statistic greater than 1.96, the data examined follows a geometric Brownian motion.

To examine if the data follows an Ornstein-Uhlenbeck process the following regression will be run:

$$X_t - X_{t-1} = a + bX_{t-1} + \varepsilon.$$

If the value of $b$ is different from zero and the $t$-statistic is greater than 1.96, then the data follows a mean reverting process. From the parameters $a$ and $b$, the process’s fixed level and speed of reversion are calculated. The following formulas are used:

\[ s = -\frac{a}{b}, \quad k = -\ln(1+b). \]

Table 3.1 summarizes the results of the regression analysis. The mean for the two data sets is also presented. Appendixes A.1 and A.2 have the complete results of the regression analysis.

<table>
<thead>
<tr>
<th></th>
<th>Brownian motion</th>
<th>Ornstein–Uhlenbeck process</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
</tr>
<tr>
<td>Newbuildings</td>
<td>0.0023</td>
<td>0.5085</td>
</tr>
<tr>
<td>(Marsoft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Newbuildings</td>
<td>0.0013</td>
<td>0.1765</td>
</tr>
<tr>
<td>(Clarksons)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freight rates</td>
<td>-0.0006</td>
<td>-0.0431</td>
</tr>
<tr>
<td>(Marsoft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freight rates</td>
<td>-0.0012</td>
<td>0.2174</td>
</tr>
<tr>
<td>(Clarksons)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Newbuilding Prices and Freight Rates Regression Summary

According to the regression analysis’s results, Brownian motion cannot be assumed for both newbuilding prices and freight rates. The results also show that the
Ornstein–Uhlenbeck process cannot be assumed for newbuilding prices, whereas the results are inconclusive for freight rates. For Brownian motion, the newbuilding prices' b is different from zero with a t-statistic showing a high level of significance. As expected, a newbuilding’s price today is contingent on the past level, and Brownian motion cannot be assumed. In the case of the freight rates, the results are different but support the same argument. For Marsoft’s data, b is very close to zero but the t-statistic is not large enough for the result to be significant. For Clarksons’ data, b is greater than zero with an average t-statistic of 2.5517, which means that the value for b is significant. In both cases the null hypothesis has to be rejected, and the movement of freight rates and newbuilding prices cannot be completely represented with a Brownian motion.

The results of the regression analysis for the Ornstein–Uhlenbeck process are also discouraging. In the case of newbuilding prices, the t-statistic is 1.0673 and -0.7087 for Marsoft’s and Clarksons’ data respectively. The t-statistic is below the critical value. For freight rates, the t-statistic is close to the critical value, which means that the calculated parameters are significant. Marsoft’s data results have an average b of −0.1267, and mean reversion can be assumed. On the other hand, mean reversion is not supported by Clarksons’ data where b is very close to zero.

To get a sense of the process assumed by these parameters we could calculate the fixed level and speed of reversion of the processes. These calculations for representative data sets reveal a rather small speed of reversion. If the speed of reversion is small, then the movement of the underlying asset is better modeled with a Brownian motion. Table 3.2 shows the fixed level and speed of reversion for the different data sets.
Handymax Newbuildings (Marsoft) & 21.7 & 0.0189 & 20.6 \\
Capesize Newbuildings (Clarksons) & 69.15 & 0.011 & 66.6 \\
Handymax Freight Rates (Marsoft) & 13 & 0.133 & 13.1 \\
Capesize Freight Rates (Clarksons) & 14,964.1 & 0.0664 & 15,277 \\

Table 3.2: Speed of Reversion and Fixed Level (Ornstein–Uhlenbeck Process)

Comparing the mean of the data sets to the fixed level calculated according to the Ornstein–Uhlenbeck process is a way of determining the sensibility of the results. In the case of freight rates these parameters are almost equal, a fact that supports the null-hypothesis. For newbuilding prices, although there is some difference in the parameters, it is not significant. The interesting fact is that the speed of reversion is rather small for all data sets. A speed of reversion equal to 0 corresponds to a Brownian motion without a drift. Figure 3.2 shows some sample paths generated with speed of reversions equal to 0.5, 0.1, 0.01 and 0.

Figure 3.4: Sample Paths of Ornstein–Uhlenbeck Processes
Further testing is necessary to determine which of the two stochastic processes is better for modeling freight rates and newbuilding prices. The limitation to the least square estimator (t-test) is that it is biased towards zero. The use of the t-statistic can lead one to incorrectly reject the null-hypothesis; in this case the null-hypothesis is that the underlying asset follows a geometric Brownian motion. An alternative test can be applied, namely the unit root test developed by Dickey and Fuller. According to this test the significance of the results can be determined by examining the F-ratio of the regression for the Ornstein-Uhlenbeck process. The F-ratio, under the null hypothesis, is not distributed as a standard F distribution but rather according to the distribution tabulated by Dickey and Fuller. The critical values for the rejection of the null-hypothesis depend on the number of observations. Table 3.3 shows four representative F-ratios

<table>
<thead>
<tr>
<th>F-ratio (Data)</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handymax Newbuildings (Marsoft)</td>
<td>0.8727</td>
</tr>
<tr>
<td>Capesize Newbuildings (Clarksons)</td>
<td>1.5622</td>
</tr>
<tr>
<td>Handymax Freight Rates (Marsoft)</td>
<td>5.4573</td>
</tr>
<tr>
<td>Capesize Freight Rates (Clarksons)</td>
<td>4.6666</td>
</tr>
</tbody>
</table>

Table 3.3: F-ratio for Newbuilding Prices and Freight Rates

What can be concluded from the F-test is that the null-hypothesis of simple geometric Brownian motion cannot be rejected in all cases. For newbuilding prices, the value of $b$ is different from zero but the F-ratio is not large enough to make the result significant. The case for the freight rates is not that clear. For Handymax and Capesize vessels, the F-ratio is close to the critical value, making the rejection of the null hypothesis more reasonable. It is interesting to note here that the F-ratio requires a large

\[20\] Pindyck and Rubinfeld (1998) pp. 507

49
number of observations to give results with some significance. If the speed of reversion is slow, then a small size data set will support geometric Brownian motion. Dixit and Pindyck performed an F-test on the prices of crude oil, which supported this fact. When the full 120-years data set was used, oil prices exhibited mean reversion. When the test was run using data for 30 years, the Brownian motion hypothesis could not be rejected.\(^\text{21}\) The results for Capesize vessels, where the number of observations is significantly greater, is a better indication of the process freight rates follow. Using more observations leads to the conclusion that freight rates are mean reverting.

A simple geometric Brownian motion assumes exponential growth of the underlying asset, an assumption that is not very sensible for the movement of newbuilding prices and freight rates. Perfect competition in the shipping industry allows players to enter and exit the market without many barriers. An equilibrium level must thus exist around which prices fluctuate. If freight rates are too high, then new players will enter the market, driving them down. The opposite is true if freight rates are too low. The Ornstein–Uhlenbeck process assumes a fluctuation around a fixed level but does not allow for a drift. This is an unrealistic assumption since prices increase due to inflation. Even though perfect competition assumes equilibrium, one expects that the price of a newbuilding will rise due to inflationary pressures.

The price of a newbuilding is a function of many parameters. The market players’ beliefs is one, but many others such as shipyard costs, interests rates, world economics, etc are also important. Determining a one variable stochastic process that accurately reflects the movement of newbuildings’ prices is quite unreasonable, and this is

\(^{21}\) Dixit and Pindyck (1994) pp. 77
supported by the test results, which are inconclusive. Although neither of the two processes is definitely rejected none of them are supported by the data. On the other hand, freight rates are traded and can be more accurately modeled with a simple stochastic process. The process followed by freight rates is somewhere in between the geometric Brownian motion and the Ornstein–Uhlenbeck process. According to option theory, that assumes a stochastic process for the underlying asset, calculating the price of shipbuilding option contracts by using freight rates and deducing the value of a vessel based on them is a more reasonable method.

3.3 Newbuilding Prices vs. Freight Rates

Newbuilding prices are very difficult to model with a well-defined stochastic process, making freight more attractive as the underlying asset for the option contract’s price calculation. A complication with this method arises since a shipbuilding option contract gives the holder the right to buy a vessel and not the freight rate. Examining these two variables, one sees that they are closely correlated, and thus the price of a newbuilding can be determined based on the level of the freight rate. The freight rate is the source of income for a shipowner, and the value of the ship, being the present value of the expected cash flow, is the discounted sum of the rates the vessel will achieve minus the operating costs. Since operating costs are usually fixed and do not fluctuate very much with the movement of the market, the value of a vessel is expected to move in accordance to the fluctuations in freight rate market. To find the correlation between newbuilding prices and freight rates, a simple regression is run. Again the main parameter of interest is the t-statistic, which determines if the parameters calculated from
the regressions are significant. Table 3.4 shows a summary of the regression analysis’ results.

<table>
<thead>
<tr>
<th></th>
<th>Marsoft</th>
<th></th>
<th>Clarksons</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>t-statistic</td>
<td>a</td>
</tr>
<tr>
<td>Handysize</td>
<td>14</td>
<td>0.27</td>
<td>2.3</td>
<td>-</td>
</tr>
<tr>
<td>Handymax</td>
<td>8.8</td>
<td>0.896</td>
<td>6.57</td>
<td>31.94</td>
</tr>
<tr>
<td>Panamax</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>41.59</td>
</tr>
<tr>
<td>Capesize</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>56.72</td>
</tr>
</tbody>
</table>

**Table 3.4: Regression of Newbuilding Prices vs. Freight Rates**

The t-statistic is high, in all cases above the critical value of 1.96, which means that the parameters calculated are significant. In other words there is linear relation between newbuilding prices and freight rates. For example, to calculate the price of a Handysize newbuilding, the formula \( N = 14 + 0.27F \) will be used where \( N \) is the price of the newbuilding and \( F \) is the freight rate. To calculate the price of a Capesize newbuilding, the formula \( N = 56.72 + 0.00095F \) will be used. It is important to note here that the inputs for the formulas calculated with Marsoft’s data are different from those calculated with Clarksons’ data. For Marsoft the freight rates are in dollars per ton whereas for Clarksons the freight rates are in dollars per day. Figure 3.5 is a scatter plot of the Handymax newbuilding prices versus the freight rate, showing the close correlation between the two variables.
An important statistic for determining the accuracy of the linear equations developed is $R^2$, which measures the goodness of the fit. $R^2$ is a measure of the correlation between the two variables. In the case of Handymax vessels, $R^2$ is equal to 0.35, which is large and confirms the correlation between newbuilding prices and freight rates. Note that $R^2$ is only a descriptive statistic\textsuperscript{22}. A large $R^2$ does not mean that freight rates predict the exact level of newbuilding prices since many factors influence the variables. Both freight rates and newbuilding prices, though, are affected by the same factors, which makes them move in the same way. A multivariable model of the two variables' relationship could be established; this would make the prediction of newbuilding prices based on the freight rates more accurate. It would make the calculation of the shipbuilding option contract’s price even more accurate.

The linear prediction of newbuilding prices based on freight rates gives conservative results. Appendix B compares real prices of newbuildings relatively to the

\textsuperscript{22} Pindyck and Rubinfeld (1998) p. 73
ones predicted by the linear model. When the real price is high, the prediction undershoots; when the real price is low, the prediction overshoots. In some sense the linear prediction averages out the sharp movements in real prices. The volatility of the predicted prices is less than that for real prices. Lower volatility will produce a more conservative value for the shipbuilding option contracts since the value of a call option decreases as volatility decreases.

3.4 Time Charter Rates

According to the model presented in Chapter 2 to build a trinomial tree representing the freight rates' movement, a term structure is essential. The term structure can be deduced from time charter rates. Time charter contracts allow the shipowner to hire his ship for a period extending up to a few years. In a time charter contract the shipowner gives the charterer operational control of the ship, while keeping the ownership and management of the vessel. The shipowner continues to pay the operating costs and gets a fixed freight rate for the life of the contract. Time charter rates can be viewed as freight rates with long maturity. Based on the time charter rates, the market’s expectations for the future level of spot rates can be determined.

Data provided by Clarksons & Co. Ltd will be used for the analysis. Charter rates for 6-month, 1-year, and 3-years periods are presented for Panamax and Capesize vessels, and charter rates for 6-month and 3-years periods are presented for Handymax vessels. Rates are quoted monthly covering the period from January 1989 to January 2001. Time charter contracts are not widely used in the dry bulk market, and data are not always available for all given time periods. In recent years the volume of time charter contracts
has decreased significantly not only for bulk carriers but also for oil tankers. In the early 1970s, about 80% of oil tankers were on time charters. In the 1990s, the percentage of oil tankers in time charters has decreased to 20%.\textsuperscript{23} Due to this lack of available data, Clarksons makes predictions for the periods when time charter contracts are not offered in the market. Figure 3.6 shows time charter rates for Panamax and Capesize vessels. Prices are quoted in dollars.

![Time Charter Rates Graph](image)

Figure 3.6: Time Charter Rates

The first step to finding a function that governs the relationship between time charter rates with different maturities is to establish the existence of such a function. By running regressions between time charter rates with different maturities, the correlation between them can be established. Time charter rates represent the market’s expectations for the future movement of the spot rate, and one expects that there is a close correlation between time charter rates with different maturities. This is supported by the data

\textsuperscript{23} Stopford (1997) pp. 85
analysis. A regression was run between every time charter period for all vessel capacities. Appendix C has the complete results of the regressions. The average $b$ calculated was 1.18, which is significantly different from zero. The average t-statistic being 32.25 is far greater than the critical value of 1.96. It is interesting to note that $R^2$ is very large. The minimum $R^2$ in the data sample is 0.72 while the largest value for $R^2$ is 0.95. A large $R^2$ means that there is good fit in the data. In other words the variables are highly correlated, and by knowing one of them we can accurately predict the others.

Time charter rates move in correlation with spot rates and can be thought of as an average value. If spot rates are very high, then time charter rates are low, representing an expectation of the market that freight rates will decrease. If spot rates are low, then time charter rates are relatively high. This relation, although interesting, is not important in determining the freight rate's term structure. To determine a term structure, the time charter rates do not have to move in a predictable pattern. What is needed is a strong relationship between time charter rates with different maturities. The term structure that will be defined takes the form of a time function. Hull and White's valuation model assumes that a function giving the value of time charter rates for any given maturity exists. Although time charter contracts do not exist for any given maturity, the function can be determined using the existing data. The close correlation between time charter rates of different maturities assures us that the predictions made will have some significance.

The percentage change of the value of time charter rates with different maturities is, in most cases, different. In other words, the derivative of the term structure function is different at the points of 6-month, 1-year, and 3-years periods. The term structure has
some convexity and cannot be a linear function. The best-fit logarithmic function will be used to determine the time function of the term structure. Figure 3.7 shows the time charter rates and the function of the term structure for Capesize vessels in January 2001.

![Graph of Capesize Vessel's Term Structure](image)

Figure 3.7: Capesize Vessel’s Term Structure

Time charter rates change over time and their values are contingent on the market’s expectations. To calculate the value of an option contract using a term structure, a specific time period with given time charter rates has to be defined. For this thesis’s calculations, the time charter rates for January 2001 will be considered and the value of call options for these dates will be calculated. The term structure function for Handymax vessels is \( TC(t) = -3840\ln(t) + 8972 \), where \( TC(t) \) is the time charter for Handymax vessels with maturity at time \( t \). The term structure function for Panamax vessels is \( TC(t) = -1151\ln(t) + 9389 \) and for Capesize vessels, \( TC(t) = -1155\ln(t) + 11344 \). Using this simple calculation for the term structure function means that the predicted values of time charter rates for different maturities might not represent actual market values. A more complicated function for the term structure could be calculated incorporating the current spot rate and historical trends. Since this function is used as an indication for the market’s expectations, this limitation is not important.
4. Option Pricing

4.1 Option Pricing Using Newbuilding Prices

Newbuilding prices are very difficult to model with a well-defined stochastic process. The data analysis in Chapter 3 showed that the tests for geometric Brownian motion and the Ornstein–Uhlenbeck process were inconclusive, and the parameters calculated not very significant. Nevertheless a price for shipbuilding option contracts based on the newbuilding prices is an indication of what the price level might be. The value of the option is calculated assuming both a Brownian motion and an Ornstein–Uhlenbeck process.

Option contracts in the shipbuilding industry have a life of no more than three years. They are given to buyers who already have shipbuilding commitments with the shipyard and their exercise price is usually equal to the price of the contract. For the calculation of the option’s price, the exercise price is assumed to be the same as the newbuilding price at the time of the contract (time 0). Time to maturity is two years. All relevant parameters such as the volatility (σ), fixed level (a), and speed of reversion (k) are calculated using the data provided by Clarksons.

In the case of options on assets not traded in the market, an adjustment has to be made to the Black-Scholes formula based on the risk profile of the investment. The expected rate of return for a newbuilding as well as the market price of risk have to be calculated. Although data for the expected rate of return of a newbuilding does not exist,
its value can be inferred from the expected rate of return of shipbuilding companies. According to the CAPM, the expected rate of return of a stock, over the risk-free rate, is proportional to the rate of return of the market. Knowing the β of a stock, the expected return based on its risk can be calculated. Appendix D shows the value of β for some major shipbuilding companies in the US, Japan, S. Korea, China, and Norway. The average β of the companies’ stocks is 0.47. This value is assumed to be the market price of risk.

Given these parameters, the price of the option contracts was calculated. Table 4.1 shows the prices of these contracts in million dollars.

<table>
<thead>
<tr>
<th>Price of option contract (in million dollars)</th>
<th>Brownian motion</th>
<th>Ornstein–Uhlenbeck process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handysize (23-30,000dwt)</td>
<td>1.81</td>
<td>1.46</td>
</tr>
<tr>
<td>Handysize (32-35,000dwt)</td>
<td>1.92</td>
<td>2.04</td>
</tr>
<tr>
<td>Handymax (40,000dwt)</td>
<td>2.37</td>
<td>-</td>
</tr>
<tr>
<td>Panamax (70,000dwt)</td>
<td>2.66</td>
<td>2.4</td>
</tr>
<tr>
<td>Capesize (120,000dwt)</td>
<td>3.87</td>
<td>3.29</td>
</tr>
<tr>
<td>Capesize (150,000dwt)</td>
<td>4.79</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.1: Option Prices Using Newbuilding Prices

The prices of option contracts on Handymax and Capesize (150,000dwt) vessels are not quoted for the Ornstein–Uhlenbeck process since there are not enough observations in the data to make the calculations significant. For these vessels’ sizes there are approximately 150 observations, whereas for Handysize, Panamax, and Capesize (120,000dwt) vessels there are at least 300 observations. The complete cycle of newbuilding prices cannot be observed in a limited data set and the option contract prices are misrepresented. From these calculations, it is obvious that the price of the option
contract is significant relative to the price of a newbuilding and ranges from 4.8% to 6.1% of the newbuilding’s price.

Except from the Handysize (32-35,000dwt) vessels case, the calculations based on the Ornstein-Uhlenbeck process are smaller than those for the Brownian motion. The stronger the reversion is, the less uncertain are the long-term revenues from operating a vessel. Consider the case where the speed of reversion is so large that there is almost no fluctuation in the movement of the underlying asset. In the short-run, there is always the possibility that there will be a jump and that leads to some volatility. On the other hand, in the long-run, the average price will be very close to the fixed level and therefore there is not much volatility. The price of an investment contingent on long-term prices should then increase with reversion since there is no uncertainty in the payoff. This would lead us to conclude that the price of call option contracts, whose payoff is contingent on the price of newbuildings in the future, would increase by incorporating reversion into the model. A decrease in uncertainty though directly influences the price of a call option, as mentioned in Chapter 2. Lower volatility decreases the value of a call option, which has an asymmetric payoff. If volatility decreases the potential gain decreases, whereas the potential loss remain the same; limited to zero. The effects of the decrease in volatility usually dominate and thus mean reversion leads to lower call option prices.

Increasing the life of the contract makes this effect even more apparent. With Brownian motion, as time to maturity increases, uncertainty increases proportionally. In the case of the Ornstein-Uhlenbeck process, since the movement of the underlying asset is pulled back to a fixed level, uncertainty does not increase as much. The price of a shipbuilding option contract for a Handysize (23-30,000dwt) vessel with maturity in three
years is $2.21 mill, for Brownian motion, and $1.64 mill, for Ornstein–Uhlenbeck process. The difference in the prices increases from 19% to 26%. Mean reversion becomes more important as the life of the contract increases. For shipbuilding option contracts whose maturity is short, the difference in prices, calculated with the two methods, although significant does not lead to large discrepancies.

A simple sensitivity analysis confirms the results outlined in Chapter 2 about the change in the price of the option contract given a change in the model’s parameters. For illustrative purposes, I will consider an option contract for a Capesize (120,000dwt) vessel and assume Brownian motion. Increasing the life of the contract from one year to three years increases the price of the option from $2.72 mill to $4.76 mill. Increasing the exercise price to 110% of the price of the newbuilding at the time of the contract’s initiation leads to a decrease in the value of the option contract from $3.87 mill to $1.59 mill. An increase in the volatility of the Capesize newbuilding prices from 10.3% to 11% leads to an increase in price from $3.87 mill to $3.93 mill. A more complete sensitivity analysis will be presented for the pricing of the option contracts using freight rates as the underlying asset.

4.2 Option Pricing Using Freight Rates

Using newbuilding prices as the underlying asset, as mentioned earlier, has many shortfalls. The prices do not follow a well-defined stochastic process and the option contract prices do not incorporate the market’s beliefs. The value of an option contract depends on the market’s expectations. If the market believes that the prices of newbuildings will increase, then the value of an option contract is higher. The possibility
that the option contract will end up in the money at expiration is higher and so is the expected payoff. On the other hand, if the market believes that newbuilding prices will decrease, then the value of the option contract is lower. In the case of financial securities, the market’s expectations are already incorporated in the market price of the underlying asset. The efficient market hypothesis assumes that all information available in the market for a traded asset is already represented in the asset’s value. Any movement in the price of a stock is due to new information. Assuming that the efficient market hypothesis is correct and that there is no way to know in advance what this information is going to be, stocks follow a random walk. A random walk is the discrete equivalent of geometric Brownian motion. The price of a stock the next period increases or decreases by the same amount and probability. Geometric Brownian motion is a reasonable assumption for stocks and the Black-Scholes formula is used. The case of newbuildings is quite different. Their market is not as liquid and random walk cannot be assumed. Different shipyards offer different prices based on labor and material costs, availability, personal contacts etc. The movement in newbuilding prices cannot be assumed as random. Time charter rates being long-term contracts, give us an idea of the market’s expectations for spot rates. Since there is a correlation between the value of a vessel and spot rates, the expectations for the prices of newbuildings can be derived. Using this valuable information, a reasonable value for shipbuilding option contracts is calculated.

The methodology presented in Chapter 2, used for interest rate option contracts, is used to derive the price of these contracts. The first step of this calculation is to derive a trinomial tree representing the movement of freight rates without considering charter rates. The results of Chapter 3 show that freight rates exhibit mean reversion. To
represent this characteristic, the trinomial tree's structure collapses into itself. In the first few nodes of the trinomial tree, freight rates can either go up, down or stay the same. The amount freight rates increase or decrease is derived from their historical volatility. Above a given freight rate level, the possible paths are changed to stay the same, go down one step or go down two steps. Below a given freight level, the paths are changed to stay the same, go up one step, or go up two steps. The maximum and minimum freight rate is defined by the speed of reversion of the process as calculated in Chapter 3. The tree structure is expanded for the life of the option contract.

The second step of the process is to incorporate in the trinomial tree time charter rates. A function for time charter rates calculated in Chapter 3 is used. Since logarithmic functions increase rapidly for values below the first observation point, in this case the 6-month time charter rate, a weighted average was used for the calculation of the 3-month time charter rate. The value of the time charter at each period is added to the initial tree and a mean reverting path structure is defined, given current time charter rates. In the third step, the values for freight rates in the trinomial tree are translated to ship prices. The function developed in Chapter 3 is used.

In the final step of the process, the value of the option contract is calculated. For each final node, the option contract’s payoff is calculated. For example, if the price of the ship in a given final node at the trinomial tree is $47 mill and the exercise price is $45 mill, then the option contract’s payoff is equal to $2 mill. The option contract’s price is the expected value of these payoffs based on the probability function defined in Chapter 2. For the calculation of the option’s prices, the exercise price is assumed to be the initial price of the newbuilding prices tree. Table 4.2 shows the prices of shipbuilding option
contracts using the time charter rates and freight rate of January 2001. Contracts maturing in two and three years are presented.

<table>
<thead>
<tr>
<th></th>
<th>2-year contract</th>
<th>3-year contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handymax (40,000dwt)</td>
<td>0.323</td>
<td>0.398</td>
</tr>
<tr>
<td>Panamax (65,000dwt)</td>
<td>0.159</td>
<td>0.220</td>
</tr>
<tr>
<td>Capesize (127,500dwt)</td>
<td>0.209</td>
<td>0.436</td>
</tr>
</tbody>
</table>

Table 4.2: Option Prices Using Freight Rates

Two things are striking at first glance. First, the prices of the option contracts using this methodology are considerably smaller than those calculated using newbuilding prices as the underlying asset. Second, the prices of the option contracts for Handymax vessels are larger than those for Panamax vessels and in some cases than Capesize vessels. Since the value of a Handymax newbuilding is less than that for a Capesize newbuilding, one expects that the price of the Handymax option should be less. Both of these "abnormalities" are due to the implied term structure derived from time charter rates. On the other hand, as expected, the value of option contracts maturing in two years is smaller than that for those maturing in three years.

Market expectations play a major role in the pricing of option contracts. The term structure used in these calculations for all vessel capacities was downward sloping in January 2001. The observed time charter rates correspond to a market’s belief that freight rates are high and should drop in the future. Due to these market expectations, the price of the option contracts is rather low. If we perform the same calculation for a Handymax vessel for November 1996, the price of the option contract is considerably higher. In November 1996, the spot rate was $7,798, the 6-month charter rate was $7,940, and the 3-year charter rate was $8,700. The term structure was upward sloping at that period,
exhibiting a positive market expectation. Freight rates were low and the market expected them to increase. A shipbuilding option contract at that period maturing in two years had a value of $1.33 mill. A contract maturing in three years had a price of $1.82 mill. To compare the prices calculated using freight rates as the underlying asset to those calculated using newbuilding prices as the underlying asset the expectations of the market have to be ignored. This is not the same as assuming a constant term structure. A constant term structure assumes that the market expects the freight rates to remain the same in the future. Brownian motion and the Ornstein–Uhlenbeck process assume a drift in the freight rates of $a$ and $k(a-X)$, respectively. To make the comparison, I will assume that time charter rates increase at a rate equal to the risk-free rate, as in the case of geometric Brownian motion. Table 4.3 shows the prices of the option contracts assuming a risk-free drift in the freight rates.

<table>
<thead>
<tr>
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<th>Price of option contract (in million dollars)</th>
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</thead>
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<tr>
<td></td>
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</tr>
<tr>
<td>Handymax (40,000dwt)</td>
<td>1.62</td>
</tr>
<tr>
<td>Panamax (65,000dwt)</td>
<td>1.67</td>
</tr>
<tr>
<td>Capesize (127,500dwt)</td>
<td>2.03</td>
</tr>
</tbody>
</table>

Table 4.3: Option Prices Using Freight Rates and Neutral Market Expectations

The prices of the option contracts are somewhat lower but comparable to those calculated with newbuilding prices as the underlying asset. To determine the newbuilding tree, the equation presented in Chapter 3 was used, which predicts the newbuilding price given the freight rate. As mentioned earlier, the predicted newbuilding prices have a smaller volatility than that of actual prices. A lower volatility leads to a lower option price and this is partly the reason for the discrepancy. The calculations in Table 4.3 were done using the January 2001 spot rates. If the mean freight rate is used, then the
difference in the option contract’s price, between the two methodologies, becomes even smaller. Despite their similar results, since freight rates are more accurately modeled with a stochastic process, this methodology’s calculations are more representative of the actual shipbuilding option contract’s price.

In Table 4.2, the price for a Handymax option was greater than that of a Panamax. One would expect that an option contract’s price for an asset that has a higher value would be greater. However, the data does not support this intuitive conclusion. The price of an option contract is not only a function of the value of the underlying asset but also of the expected payoff. If the option is deep out of the money, no matter how valuable the underlying asset is, the option’s price is going to be close to zero. On the other hand, if the option is deep in the money, then its price is going to be close to the price of the underlying asset. For the calculations in Table 4.2, the market’s expectations for the Panamax freight rate were downward sloping. The spot rate was $10,084, whereas the 6-month, 1-year, and 3-years time charter rates were $10,425, $9,000, and $8,275, respectively. The spot rate for a Handymax vessel on January 2001 was $8,925, whereas the 6-month and 3-year time charter rates were $9,238 and $8,550, respectively. Although the term structure in the case of Handymax vessels is also downward sloping, the slope is not as great. In the case of Panamax vessels, the difference between the 6-month and 3-year time charter rates is 2.12 standard deviations. For Handymax vessels, the difference is 1.26 standard deviations. On January 2001, the market expected a greater decrease in the Panamax freight rates, making the value of the option contract less than that of Handymax vessels. Looking at Table 4.3, where the same market
expectations are assumed for all vessel capacities, the prices of the option contracts increase as the value of the underlying asset increases.

As expected in all calculations, the value of the two-year option contract is less than that for the three-year contract. A three-year option contract offers more flexibility than a two-year contract. The holder of the three-year contract can exercise his right as long as the holder of the two-year contract can, plus an extra year. No matter what the market’s expectations are this flexibility adds value to the option contract. This result is in accordance with the option theory. A more extensive sensitivity analysis will be presented in the next section.

4.3 Sensitivity Analysis

Having calculated the option contract prices, it is interesting to see what parameters influence them and how. An absolute value for the price of an option contract must be viewed with caution since the shipbuilding market is illiquid. Selling a ship is not as easy as selling a stock and many factors such as taxes, bargaining games, world economy etc. make the transaction more complex. The option’s payoff is not guaranteed and examining how it is affected by different parameters will give us a better understanding of its range. According to the option pricing theory, presented in Chapter 2, the main parameters influencing the price of a financial option are: the price of the underlying asset, the exercise price, the time to maturity, the volatility of the underlying asset and interest rates. To these factors, market expectations as observed in the term structure will be added. For the sensitivity analysis of all other parameters, market
expectations are considered neutral. In other words, the term structure increases at the risk-free rate.

As the value of the underlying asset increases, then the price of the shipbuilding option contract also increases. A higher newbuilding price means that the option contract’s payoff is going to be higher. Consider a shipbuilding option contract maturing tomorrow and having an exercise price of $50 mill. If the current price of a newbuilding increases from $52 to $52.1, then the option contract’s payoff increases from $2 mill to $2.1 mill. Assuming a perfectly liquid market, the holder of the contract could build a ship for $50 mill and sell it for $52.1 mill. The option contract’s payoff is max (S-K, 0) where S is the price of the underlying asset. As S increases, it becomes more profitable to exercise the option contract. In the limit, if S is very high, it is always optimal to exercise the option contract since the probability that the option contract will be in the money at expiration is very high. In this case, there is a one-to-one correlation between an increase in the value of the underlying asset and the price of the option contract. A $1 increase in the value of the underlying asset leads to a $1 increase in the option contract’s payoff function that then leads to a $1 increase in the value of the option contract. On the other hand, if the price of S is very low, then the price of the call option is zero. These extreme cases are of no practical interest, since in most cases option contracts are neither deep in the money nor deep out of the money. For Handymax vessels, if the price of the spot rate is increased from $8,925 to $9,100, then the value of the two-year option contract increases from $1.62 mill to $1.65 mill.

The effect of the exercise price is exactly the opposite of that for the underlying asset’s price. A call option’s payoff increases as the exercise price decreases. If the
exercise price is so low that the call option will definitely end up in the money, then the correlation between the price of the underlying asset and the price of the call option is one-to-one. For Handymax vessels, if the exercise price decreases from $46.1 mill to $45 mill, then the price of the two-year shipbuilding option contract increases from $1.62 mill to $2.71 mill. Since the exercise price is usually set in the contract, its movement is restricted and of no particular interest. However, different contracts with different exercise prices have different values.

In section 4.2, the effect of time to maturity was analyzed. An option contract with longer maturity has greater flexibility and thus greater value. For Handymax vessels, the difference between the two-year and three-year contracts is $1.15 mill. As the value of the underlying asset increases, so does the difference. For Capesize vessels, the difference between the two contracts is $1.28 mill.

Volatility plays an important role in the value of an option contract. In the case of assets whose payoffs are symmetric, volatility does not increase the expected value. A stock’s price is equal to the present value of all expected future cash flows. If there is an increase in volatility, the future cash flows become more uncertain, but their expected value remains the same. The potential greater loss is compensated by a potential greater gain. The case of option contracts is not the same, since the payoff is asymmetric. In an option contract, the loss is limited to zero. If there is an increase in volatility, the potential gain increases but the potential loss remains the same. A low volatility means that the option contract’s value at expiration will be either low or zero. A high volatility means that the option contract’s value at expiration will be either high or zero. Obviously, the price of an option contract increases with volatility. For Handymax vessels, if the
volatility of freight rates increases from $544 to $600, the price of the two-year option contract increases from $1.62 mill to $1.65 mill. Volatility is calculated using historical data and must reflect the current market situation. The fact that in the 1970s the market was very volatile does not mean that the same holds for the 1990s. On the other hand, calculating volatility for a very short time period will not give statistically significant results.

Examining the Black-Shores formula, we see that as interest rates increase the price of a call option increases. The interest rate enters the formula inversely to the exercise price. In some sense, interest rates represent a positive market expectation. Risk-free assets are expected to grow at a higher rate when the interest rate is higher. For Handymax vessels, if the interest rate increases from 5% to 6%, the value of the two-year shipbuilding option contract increases from $1.62 mill to $1.92 mill.

The effects of market expectations were examined in section 4.2. The price of an option contract is directly influenced by what the market expects for the future movement of the underlying asset. A downward sloping term structure leads to lower prices for the option contracts. For the market situation in January 2001, where time charter rates were decreasing as maturity increased, the price of the option contract was far below than that of a neutral market. For Handymax vessels, if we assume that the market is constant, spot rates and time charter rates for all maturities are $8,925, the value of the option contract is $0.48 mill. If we assume that time charter rates increase by 1% each period, representing a positive market expectation, the price of the option contract increases to $0.66 mill.

24 Cox (1985) pp. 217
Having examined what is the general effect of the parameters in the option contract’s price, I will now examine what is the exact change, given the term structure in January 2001. Table 4.4 shows the percentage change in the price of the different option contracts, given a 1% change in each of the parameters. In the case of market expectations, the change in the option contract’s price is presented given that the term structure, namely time charter rates, is 1% higher than what it actually is.

<table>
<thead>
<tr>
<th></th>
<th>Spot rate</th>
<th>Exercise price</th>
<th>Volatility</th>
<th>Market Expectations</th>
</tr>
</thead>
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<tr>
<td></td>
<td>2-yr</td>
<td>3-yr</td>
<td>2-yr</td>
<td>3-yr</td>
</tr>
<tr>
<td>Handymax (40,000dwt)</td>
<td>2%</td>
<td>2%</td>
<td>-51%</td>
<td>-40%</td>
</tr>
<tr>
<td>Panamax (65,000dwt)</td>
<td>1%</td>
<td>1%</td>
<td>-49%</td>
<td>-47%</td>
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<tr>
<td>Capesize (127,500dwt)</td>
<td>0%</td>
<td>0%</td>
<td>-52%</td>
<td>-35%</td>
</tr>
</tbody>
</table>

Table 4.4: Percentage Change in Option Price

These calculations are contingent on the market situation. The fact that in January 2001 a decrease in the spot rate by 1% does not lead to an increase in the price of the Capesize option contract does not mean that this is going to be always the case. Yet, these calculations give us an idea of what the approximate effect of each variable is. The exercise price is the parameter that influences the value of the option contract the most. A 1% increase in the exercise price leads to a 47% decrease in the price of the option contract on average. Market expectations also play an important role. It is surprising that the spot rate and volatility do not considerably affect the price of the option contract. One has to understand, though, that all these parameters are interrelated. If the spot rate increases considerably, then one would expect that time charter rates and market expectations would follow. The same is true for volatility. Higher volatility means that the shipowner takes up more risk and would want a higher compensation for it. This
could potentially lead to an increase in time charter rates. Although looking at each parameter individually gives us good insights, one has to understand that all of them play an important role in the price of the option contract.

Although Shipyards do not charge the buyer a price for these option contracts, they sometimes increase the exercise price relative to the present newbuilding price. The buyer of the contract can build a ship at a price that is somewhat higher than the price of a newbuilding at the time of the contract’s initiation. An interesting parameter to calculate is the exercise price that makes the value of the option contract zero. In January 2001, if the exercise price was 3% higher than the price of a newbuilding, the value of the two-year option contract would have been less than $30,000 for all vessel capacities. Assuming an increasing term structure, for the option contract’s price to be close to zero, the value of the exercise price has to be increased by 7% relatively to the newbuilding price. For Handymax vessels, this is approximately $3 mill. If this value is discounted to the present, it is slightly higher than but comparable to the price of the option contract.
5. Concluding Remarks

Shipbuilding option contracts give the buyer the right but not the obligation to build a ship at a pre-specified price. The value of such a contract, although contingent on many factors, such as the current level of newbuilding prices, the exercise price, market expectations, etc., is fixed. According to the option pricing methodology, the price of the contract moves in accordance with the price of the underlying asset, which in this case is newbuilding prices. Using this relation, the exact price of the option is calculated. Having determined the movement of the underlying asset, the option contract’s expected payoff is defined and discounting it to the present gives the option contract’s price. In the shipping business, little is known about the methodology used to calculate the value of these contracts. This option contract mispricing leads to potential arbitrage opportunities. In this thesis, the methodology for pricing these contracts accurately was presented.

Several closed-form solutions for the price of financial options have been developed assuming that the underlying asset follows a well-defined stochastic process. For newbuilding prices, the data do not support such an assumption. An alternative method is presented for the pricing of option contracts. The freight rates a ship will achieve during its life determine its value and the value of the option contract. A forecasting method for the movement of freight rates is developed for the life of the contract, which is usually up to three years. The future movement of freight rates is determined using historical trends and time charter rates. Calculating the value of the option contract using freight and time charter rates yields a price that incorporates market expectations and current market conditions.
Freight rates, being cyclical in nature, can be modeled with an Ornstein–Uhlenbeck process, which exhibits mean reversion. Although the data supports this hypothesis, the parameters calculated show a very slow speed of reversion. According to these calculations, freight rates move around a fixed level but have long cycles; longer than the option contract’s life. This is practically the same as assuming geometric Brownian motion for freight rates leading to some unreasonable freight rate levels. Looking at the trinomial tree built in Chapter 4, freight rates for Handymax vessels are assumed to fluctuate between $16,620 and $4,380. Historically, freight rates have never been as high as $16,620 and it is unreasonable to assume that they will be that high in three years. More unreasonable is the assumption that freight rates will drop below operating costs to $4,380. Modeling freight rates in this way leads to a volatility that is higher than that observed in the market. The option contract’s price is inflated. This effect is somewhat dampened by the translation of freight rates to newbuilding prices. Newbuilding prices in the trinomial tree range between $39 and $58 mill. This is closer to actual market data. Calculating the price of option contracts, some assumptions about the movement of the underlying asset must be made. Since many of the factors influencing the movement of freight rates change over time, much caution must be taken in making these assumptions. To have a more complete picture of the option contract’s price a low, base, and high volatility case should be defined based on market expectations. The high volatility scenario will represent a situation where future freight rates are highly uncertain. The option’s price for each of these cases can be calculated, using the model, and based on the probability of each case occurring, a weighted average can be calculated.
The illiquidity of the market for ships poses another limitation to the model. Each year there are few ships built throughout the world and, unlike stocks, if an owner decides to sell the asset a buyer is not readily available. This model assumes that a newbuilding can be sold instantaneously at market price. In practice, the price of a newbuilding is determined by many factors other than the market’s beliefs, such as the reputation of the shipyard, the level of work done, and price games between the seller and the buyer. A correction to the option contract’s price must be made given the illiquidity of the market. The uncertainty involved decreases the contract’s value.

Shipbuilding option contracts are offered by shipyards to potential buyers to make their offer more attractive. Although the purpose of a shipyard is to build ships and work at full capacity, these contracts have significant value and should not be given for free. The value of these contracts varies significantly with market expectations and the exercise price defined in the contract. For an investment of $50 mill, the price of a two-year option contract ranges between $0.5 and $1.7 mill. A usual practice among shipyards is to set the exercise price somewhat higher than the newbuilding price at the time of the contract’s issuance. For example, if the price of a Panamax newbuilding is $46 mill, shipyards give the buyer the option to build a ship in the future for a price between $46 and $48 mill. The exact price is determined by the specifics of each contract. Using the option pricing methodology presented in this thesis, the exact exercise price for which the option contract’s price is zero can be calculated. Determining this price is important since, in this way, the shipyard does not give the buyer a free lunch. Since shipyards do not charge for these contracts, if the option contract’s price is different from zero, the buyer gets a valuable asset for free. In this case, the buyer by
selling the contract can make an instant profit. In addition, by exercising it the option contract will prove a profitable investment on average. Shipyards can take advantage of a correct method for pricing these option contracts in two ways. First, they can determine the exact exercise price for which the contracts will not produce losses for the shipyard. Second, option contracts although not the main focus of a shipbuilding company, are bargaining assets that, if used and priced accurately, can lead to more customers. On the other hand, arbitrary use of these contracts might lead to losses.

Shipbuilding option contract holders, usually being shipowners, should also be interested in a method that accurately prices them. Dealing with a shipyard, the shipowner should know the value that these offerings add to the contract. Moreover, although there is no over-the-counter market for this kind of option contracts, they are sometimes traded between shipowners. Holders of these contracts sell them for a price and exercise them giving the new buyers ownership of the vessel. Many complications due to the specifics of the contracts might arise but usually these complications are easily resolved. Since the option contract’s price depends on market expectations, it is important for the shipowner to have an up-to-date price for the contract. Selling these contracts when the market is optimistic will lead to considerable profits. Although the market is not well developed, there are profitable trading opportunities for the holders of these assets.

Three results are interesting in the calculations presented in Chapter 4. First, the option contract’s value is significant. For a shipyard, giving a $1 mil incentive might not be as important as having the yard working at full capacity, but for the holder of an option contract, if it is used properly, its value might generate a profit. Second, the value of the option contract is very sensitive to market expectations. Assuming that a shipowner...
gets hold of such a contract at a time when the prices of newbuildings were expected to fall, if the market expectations become more optimistic the contract’s value will increase considerably. This information can be used both in trading option contracts and in determining when it is optimal to exercise them. Finally, the appreciation of the exercise price relative to the current newbuilding price was calculated for the option contract to have zero value. If shipyards give buyers options to build ships at a price 3% to 7% above the current price of a newbuilding then the contract’s expected payoff is close to zero. Currently, the practice among shipyards is not to raise the exercise price as much. Option contracts seem like a profitable investment for shipowners. This is confirmed by the market’s attitude towards option contracts where shipowners demand them and shipyards are reluctant to give them.

5.1 **Recommendations for Future Research**

A more complete model for the movement of underlying assets, newbuilding prices and freight rates, can be developed representing more accurately the cyclical nature of these assets. Since freight rates, on average, move within a given range, floors and caps can be placed. Finding a closed form solution for the price of an option assuming a complex stochastic process is difficult since ordinary calculus rules cannot be applied. Using the tree approach, these restrictions can be more easily added to the model. Determining these levels, though, poses some difficulty since historical values do not always predict the future accurately. A more accurate prediction of newbuilding prices given the freight rate can also be developed. If the movement of the underlying
asset is modeled in a way representing reality more accurately, the calculated option prices will be more precise.

Using the prices calculated in this thesis, a trading strategy for the holders of these assets can be developed. Market conditions determine the profitability of these contracts. Comparing the option contracts’ price to the value they generate if exercised, an optimal strategy for shipowners can be developed. If the market is optimistic, these contracts have high value and selling them will produce considerable profits, which should be compared to the value generated by building the ship. A study can be done to determine the market for these contracts and how a standardized method for trading them can be developed. For financial option contracts, which are more liquid, exchanges exist with standardized contracts having different maturity dates.

The option pricing methodology can also be applied in analyzing strategic decisions facing a shipping company. The ability a shipping company has to invest in ships can be viewed as an option contract and its value can be calculated using the tools presented. By undertaking an investment, the value of this option is forgone. In deciding when it is optimal to invest, the traditional discounted cash flow (DCF) method does not incorporate this loss. Dixit and Pindyck have shown that for many kinds of investment decisions, incorporating this optionality leads to more conservative investment decisions. This result is in accordance with the arbitrarily high discount rates used in practice for analyzing the investment decisions with the traditional DCF method. The option pricing methodology can be used to determine when it is optimal for a shipping company to buy, sell, lay-up, or scrap its ships.
Bibliography


Appendix A.1

Testing newbuilding prices and freight rates for geometric Brownian motion.

<table>
<thead>
<tr>
<th>Brownian motion (newbuilding prices)</th>
<th>$a$</th>
<th>$b$</th>
<th>t-statistic</th>
</tr>
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<tbody>
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<td></td>
<td></td>
<td></td>
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</tr>
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<td><strong>1.37586</strong></td>
</tr>
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Appendix A.2

Testing newbuilding prices and freight rates for Ornstein–Uhlenbeck process and determining mean reversion parameters.

### Ornstein–Uhlenbeck process (newbuilding prices)

<table>
<thead>
<tr>
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<th>$b$</th>
<th>$t$-statistic</th>
<th>$f$-ratio</th>
<th>Fixed level</th>
<th>Speed of reversion</th>
<th># of observations</th>
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<tr>
<td>Marsoft</td>
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<td>Handysize (27,000dwt)</td>
<td>0.4687</td>
<td>-0.0270</td>
<td>-1.1998</td>
<td>1.4400</td>
<td>17.4</td>
<td>0.0274</td>
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<td>Handymax (43,000dwt)</td>
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<td>-0.0187</td>
<td>-0.9347</td>
<td>0.8700</td>
<td>21.7</td>
<td>0.0189</td>
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<td>Handysize (23-30,000dwt)</td>
<td>0.3283</td>
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<td>-1.4210</td>
<td>2.0195</td>
<td>34.6</td>
<td>0.0095</td>
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<td>Handysize (32-35,000dwt)</td>
<td>0.3579</td>
<td>-0.0093</td>
<td>-1.3923</td>
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<td>Handymax (40,000dwt)</td>
<td>1.4260</td>
<td>-0.0302</td>
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<td>Panamax (70,000dwt)</td>
<td>0.4946</td>
<td>-0.0101</td>
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<td>Capesize (120,000dwt)</td>
<td>0.7745</td>
<td>-0.0112</td>
<td>-1.2500</td>
<td>1.5622</td>
<td>69.2</td>
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<td>Capesize (150,000dwt)</td>
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<td><strong>Average</strong></td>
<td><strong>0.8254</strong></td>
<td><strong>-0.0178</strong></td>
<td><strong>-0.7983</strong></td>
<td><strong>3.1534</strong></td>
<td><strong>-</strong></td>
<td><strong>0.0179</strong></td>
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### Ornstein–Uhlenbeck process (freight rates)

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<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$t$-statistic</th>
<th>$f$-ratio</th>
<th>Fixed level</th>
<th>Speed of reversion</th>
<th># of observations</th>
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<td>Handysize (27,000dwt)</td>
<td>2.2665</td>
<td>-0.1289</td>
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<td>Handymax (43,000dwt)</td>
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<td>Handymax (40,000dwt)</td>
<td>438.6</td>
<td>-0.0503</td>
<td>-1.1930</td>
<td>3.7265</td>
<td>8719.1</td>
<td>0.0516</td>
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<td>Panamax (65,000dwt)</td>
<td>626.3</td>
<td>-0.0711</td>
<td>-2.3098</td>
<td>5.3351</td>
<td>8808.4</td>
<td>0.0738</td>
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<tr>
<td>Capesize (127,500dwt)</td>
<td>960.7</td>
<td>-0.0642</td>
<td>-2.1602</td>
<td>4.6666</td>
<td>14964.2</td>
<td>0.0664</td>
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<tr>
<td><strong>Average</strong></td>
<td><strong>-0.0878</strong></td>
<td><strong>-1.9183</strong></td>
<td><strong>4.3436</strong></td>
<td><strong>-</strong></td>
<td><strong>0.0925</strong></td>
<td><strong>-</strong></td>
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Appendix B

Linear prediction of newbuilding prices vs. actual vessel prices.

<table>
<thead>
<tr>
<th></th>
<th>Actual newbuilding prices $\sigma$</th>
<th>Predicted newbuilding prices $\sigma$</th>
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<tbody>
<tr>
<td><strong>Marsoft</strong></td>
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<tr>
<td>Handysize (27,000dwt)</td>
<td>1.82</td>
<td>0.62</td>
</tr>
<tr>
<td>Handymax (43,000dwt)</td>
<td>4.47</td>
<td>2.62</td>
</tr>
<tr>
<td><strong>Clarksons</strong></td>
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</tr>
<tr>
<td>Handymax (40,000dwt)</td>
<td>4.79</td>
<td>2.86</td>
</tr>
<tr>
<td>Panamax (65,000dwt)</td>
<td>6.61</td>
<td>3.47</td>
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<tr>
<td>Capesize (127,500dwt)</td>
<td>12.47</td>
<td>4.49</td>
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Appendix C

Regression analysis results for time charter rates.

<table>
<thead>
<tr>
<th>Time charter rates</th>
<th>$A$</th>
<th>$b$</th>
<th>$t$-statistic</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handymax (6-month vs 3-year)</td>
<td>-2717.03</td>
<td>1.2617</td>
<td>24.3284</td>
<td>0.81</td>
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<tr>
<td>Panamax (6-month vs 1-year)</td>
<td>-792.93</td>
<td>1.0849</td>
<td>50.1218</td>
<td>0.95</td>
</tr>
<tr>
<td>Panamax (6-month vs 3-year)</td>
<td>-1724.02</td>
<td>1.1611</td>
<td>20.0528</td>
<td>0.74</td>
</tr>
<tr>
<td>Panamax (1-year vs 3-year)</td>
<td>-1473.66</td>
<td>1.1297</td>
<td>30.7698</td>
<td>0.87</td>
</tr>
<tr>
<td>Capesize (6-month vs 1-year)</td>
<td>-1200.93</td>
<td>1.0709</td>
<td>52.8733</td>
<td>0.95</td>
</tr>
<tr>
<td>Capesize (6-month vs 3-year)</td>
<td>-4423.59</td>
<td>1.2866</td>
<td>18.9623</td>
<td>0.72</td>
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<tr>
<td>Capesize (1-year vs 3-year)</td>
<td>-3982.21</td>
<td>1.2786</td>
<td>28.6687</td>
<td>0.85</td>
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<tr>
<td><strong>Average</strong></td>
<td>-</td>
<td><strong>1.1819</strong></td>
<td><strong>32.2539</strong></td>
<td><strong>0.84</strong></td>
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</table>
Appendix D

Weekly β for shipbuilding companies for the period 1996-today.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Shipbuilding company</th>
<th>β</th>
<th>Expected return(^a)</th>
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<tbody>
<tr>
<td>NNS</td>
<td>Newportnews Shipbuilding</td>
<td>0.67</td>
<td>12.47 %</td>
</tr>
<tr>
<td>SMM</td>
<td>Sembcorp Marine Ltd.</td>
<td>0.60</td>
<td>11.72 %</td>
</tr>
<tr>
<td>TOD</td>
<td>Todd Shipyard Corp.</td>
<td>0.44</td>
<td>10.01 %</td>
</tr>
<tr>
<td>0945KS</td>
<td>Hyundai Mipo</td>
<td>0.36</td>
<td>9.15 %</td>
</tr>
<tr>
<td>3199KS</td>
<td>Daesun Shipbuilding</td>
<td>0.47</td>
<td>10.33 %</td>
</tr>
<tr>
<td>1014KS</td>
<td>Samsung Heavy Industries</td>
<td>0.64</td>
<td>12.15 %</td>
</tr>
<tr>
<td>7014JP</td>
<td>Namura Shipbuilding</td>
<td>0.43</td>
<td>9.90 %</td>
</tr>
<tr>
<td>7003JP</td>
<td>Mithui Shipbuilding</td>
<td>0.38</td>
<td>9.37 %</td>
</tr>
<tr>
<td>6302JP</td>
<td>Sumitomo Heavy Industries</td>
<td>0.64</td>
<td>12.15 %</td>
</tr>
<tr>
<td>7012JP</td>
<td>Kawasaki Heavy Industries</td>
<td>0.34</td>
<td>8.94 %</td>
</tr>
<tr>
<td>600685CH</td>
<td>Guangzhoo Shipbuilding</td>
<td>0.37</td>
<td>9.26 %</td>
</tr>
<tr>
<td>KVINO</td>
<td>Kvaener</td>
<td>0.57</td>
<td>11.40 %</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td><strong>0.49</strong></td>
<td><strong>10.57 %</strong></td>
</tr>
</tbody>
</table>

\(^a\) According to CAPM given a market return of 16% and a risk free rate of 5.3%

Source: Bloomberg L. P., April 2001