Stochastic Prediction of Mooring Line Fatigue and Failure

by

Peter B. Sabin

B.S. Economics
University of Florida, 1991

Submitted to the Department of Ocean Engineering in Partial Fulfillment of the Requirements for the Degree of

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Signature of Author: __________________________________________
Department of Ocean Engineering
July 13, 2001

Certified by: ________________________________________________
Paul D. Sclavounos
Professor of Naval Architecture
Thesis Supervisor

Accepted by: ________________________________________________
Henrick Schmidt
Professor of Ocean Engineering
Chairman, Committee for Graduate Students
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Abstract

Offshore structures maintain their position above the ocean floor through the use of mooring lines. As exploration for new resources pushes drilling into deeper depths and heavier seas the service lifetime of these lines is of primary concern. This thesis will examine, for a given sea state, how these intact lines will fatigue over time and the expected time until failure.

The response of a truss-spar structure in 830m of water was modeled, with existing software, in both the frequency and time domain. This unique analysis allowed for both linear and nonlinear effects to be considered in the calculation of the fairlead tension. The time record of the fairlead tension was used as the basis for the calculation of fatigue.

Stochastic techniques were used to compile and manipulate the data in such a way that simple methods for defining fatigue could be used for calculating the accumulation of damage and the prediction of failure. Probability mass and density functions were then created for lines in various positions around the structure and used in the comparison of both fatigue and response to the input signal.

Thesis Supervisor: Paul D. Sclavounos
Title: Professor of Naval Architecture
Acknowledgements

I would also like to thank the family and friends that I owe
I ain’t namin’ names cuz they know
where they stand.
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Chapter 1
Introduction

As the demand for petroleum continues to grow so does the demand to retrieve it from deeper depths and harsher environments. A component of designing a structure that will be successful in such an environment is an understanding of the fatigue of the mooring lines. This research considered the fatigue, failure and ultimately the lifetime of the mooring lines of a truss-spar-type oil-drilling platform. There are essentially three states in which mooring lines exist: intact, transient and damaged. This paper examines the intact mooring line as it experiences fatigue. Specifically the fatigue life of generic steel spiral-strand cables will be examined from a stochastic perspective. The tools of probability theory allow evaluation of the data in such a way that it can be compiled and examined as a statistic such that a prediction of long-term effects of the structure existing in a given environment can then be made.

Data was collected from software developed to determine the response of floating structures in a given sea state. This software calculates the response of the structure both in the time and frequency domain, therefore both linear and non-linear sea loads acting on the structure were considered. The hydrodynamic forces responsible for the large amplitude responses are non-linear in nature and are dominated by second order effects. Two types of forces must be known for the calculation of the large amplitude slow drift responses of the structure, the slow drift excitation and wave drift damping. These as well as viscous excitation, damping and the mooring line restoration effects are all handled by this program. The input signal and tension history of an approximately three-hour period was the main focus of analysis. Using these data sets it was
possible to create a statistical description of the sea state as well as the tension history of individual mooring lines. From the tension history, probability mass functions (pmf) and probability density functions (pdf) of the peak values were created and described. These being useful for the analysis of fatigue as well as gaining insight to the relationship between input and response. Simple principles of fatigue were applied to the tension history data to determine the fatigue incurred during the sampled time period. Linear extrapolation was then used to calculate fatigue over time and ultimately the time until failure.
Chapter 2
Pertinent Probability Concepts

A few basic ideas need to be introduced before beginning the analysis of the data. These concepts will allow a statistical description, compilation and analysis of the data. It will be shown that the input signal, i.e. the sea state incident upon the platform, is comprised of independent random variables and that its density function will be Gaussian as expected by the Central Limit Theorem. This reassures us that we have a large enough sample and the results from the tension analysis are useful. It will be shown, as a byproduct of determining convergence, that the process can be considered stationary; proving that the density function is independent of time. Since a single record of the response (or tension history) to this sea state is not as meaningful as a statistical description we do ‘trails’ to accumulate data. Each trial can be considered a random process. By doing multiple trials a whole family of time histories was created that could equally as well have been the outcome of the same experiment. These trials were then averaged together to get a set of data that could be used. In SML this is made possible by a feature which allows the seed number to be specified for the random number generator; selecting a different seed number for each trial generates statistically independent time simulations. The sea state is shown to be an ergodic process. Proving ergodicity is useful because it then allows the statistical description of the wave field. The probability density function of the maxima of the input signal will also be revealed to be a Rayleigh distribution.
Chapter 3
Acquisition of Data and Configuration of Platform

3.1 SML

The Swim-Motion-Lines (SML) suite of programs developed by M.I.T. Naval Architecture Professor Paul Sclavounos generated the data analyzed for this research. SML analyzed the hydrodynamic properties and responses of floating structures in a given sea-state. Swim treated linear and second-order frequency-domain hydrodynamics. Motion carried out time domain simulations and Lines determined the nonlinear mooring line effects upon the structure.\(^2\)

3.2 Spar and Mooring Line Configuration

3.2.1 Spar

The truss-spar considered has a draft of 200 m and radius of 20 m. There are two circular plates of area 1000 m\(^2\) respectfully located 150 m and 200 m below the surface. There is a moonpool with area 78.5 m\(^2\). The fairleads, where the mooring lines attach to the spar, are attached at 100 m below the surface. Figure 1 illustrates this spar.

Figure 1
3.2.2 12 Mooring Line Configuration

The 12 mooring line arrangement can be seen in Figure 2.

It is a symmetrical configuration subjected to a unidirectional sea state that is incident upon the structure such that line 7 is oriented directly to the weather and line 1 directly to the lee. This configuration was placed in a water depth of 830 m. Each of the 12 lines, being identically constructed, had three segments with the properties described in Table 1.
### Table 1

<table>
<thead>
<tr>
<th>Component</th>
<th>Length (m)</th>
<th>Diameter (m)</th>
<th>EA (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Chain</td>
<td>150</td>
<td>0.15</td>
<td>$1 \times 10^9$</td>
</tr>
<tr>
<td>Spiral Strand Wire</td>
<td>1500</td>
<td>0.15</td>
<td>$1.5 \times 10^9$</td>
</tr>
<tr>
<td>Top Chain</td>
<td>200</td>
<td>0.15</td>
<td>$5.0 \times 10^8$</td>
</tr>
</tbody>
</table>

### 3.2.3 4 Mooring Line Configuration

Ultimately the goal was to create a mooring system (referring to the number and configuration of mooring lines) and test it over a period of time and enough trials that would allow for convergence to a representative probability density function; this being the basis for the calculation of fatigue. Since the CPU time for a large mooring configuration can be substantial, it was necessary to simplify the mooring system to allow for quicker processing hence a faster determination of convergence. Therefore a 4-line system was developed to expedite the process. The 12 mooring line system was used as the basis for the modified 4 line system. The 4 mooring line arrangement can be seen in Figure 3. Again, it is a symmetrical configuration subjected to a unidirectional sea state that is incident upon the structure such that line 2 is oriented more towards the weather.
To create the 4ML configuration, the 12 mooring lines were considered as a ‘system’ with a given stiffness and given weight. The stiffness is defined by:

\[ S = EA \]  
\text{Eqn. 1}

where \( E \) is the modulus of elasticity for the material and \( A \) is the sectional area of the line. The total stiffness is defined as:

\[ TS = \text{number of mooring lines} \times EA \]  
\text{Eqn. 2}

By determining the total stiffness for the 12 line system and then using that value for a 4 line system an equivalent cross sectional area was backed out to use for each line of the 4 line system. A few of the important changes are given in Table 2 for comparison to the original values. As before each of the four lines will be identically constructed.
Table 2

<table>
<thead>
<tr>
<th>Component</th>
<th>Length (m)</th>
<th>Diameter (m)</th>
<th>EA (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Chain</td>
<td>150</td>
<td>0.26</td>
<td>3.0x10^9</td>
</tr>
<tr>
<td>Spiral Strand Wire</td>
<td>1500</td>
<td>0.26</td>
<td>4.5x10^9</td>
</tr>
<tr>
<td>Top Chain</td>
<td>200</td>
<td>0.26</td>
<td>1.5x10^9</td>
</tr>
</tbody>
</table>

The result of this process being that the 12 line system and the 4 line system had the same stiffness and weight. Note that because each mooring line has three segments (chain – wire – chain) with different properties that this process had to be completed for each segment. The 4 mooring line configuration is tested in 830 m water depth.

3.3 Sea State

The sea state environment that the spar was tested in can be described by a directional JONSWAP (Joint North Sea Wave Project) spectrum. This spectrum is typically used for water where there will be limited fetch for the storm to develop. Good examples of such a situation where there is currently drilling activity are the Gulf of Mexico and the North Sea. Our storm was created with a significant wave height of 10 m, a peak period of 13 seconds, and gamma of 3.3. These are considered very rough to high seas. The structure was also subjected to a wind as
well as a current with the same direction as the storm. The current speed varied with depth from

\[ 2 \frac{m}{s} \text{ at the surface to } 1 \frac{m}{s} \text{ 800 m below the surface.} \]

The wind velocity was a steady 20 \( \frac{m}{s} \).
Chapter 4
Analysis of Output

4.1 Convergence

For a moment I would like to use information from a later section to justify the time interval used for these trials. Figure 4 shows the relative frequency of the peak distribution for a given mooring line for two distinct time intervals.

![Figure 4](image)

**Figure 4**

The details of how these plots were created will be discussed in Section 5.3. For now it is only necessary to notice that the shape of the distribution has been fairly well established at 4095 seconds and by comparing this to the distribution at 8190 seconds it is clear that we have established convergence. Note that the extreme similarity of these curves taken over different time periods allow us to assume stationarity. Now that we are comfortable that we have a representative distribution we can run trials for 8190 seconds and later be confident with linearly extrapolating the damage accrued for longer periods as well as ultimately determining the time
until failure. Also, 8190 seconds is the closest time interval allowed by SML to the 3-hour model (for short storms) recommended by API\textsuperscript{3}. Having converged on a distribution for the 4 mooring line system we infer that this time period will also be convergent for the 12-line system. Consider all trails for this research to be run for 8190 seconds, also note that the time increment was 2 seconds.

### 4.2 Input Signal

The information received from running the simulations then needed to be processed. The first thing checked was the input wave signal. A history of the wave height as a function of time is provided by SML. A plot from an individual trial from the 4 mooring line configuration of this first order free surface elevation is seen below in Figure 5.

![Figure 5](image-url)
To get a probability density function for the wave height of this signal, it was first necessary to create a histogram; this was done by developing code that would create a user-specified number of bins (a bin representing a range of values to which the count of wave heights is incremented as the time history is read). Eqn. 3 determines the uniform bin size

\[
\text{Eqn. 3} \quad \frac{\text{Max Tension} - \text{Min Tension}}{\text{User Specified Number of Bins}}
\]

The bins, being of equal size, range from the minimum to maximum value of the wave height. The code then read through the time history stopping at each time step to read the wave height and determine which bin it belonged in. The total count in each bin was then divided by the total number of wave heights of the record. This determined the probability of the wave being in a given bin. Considering each trial as a statistic, it was then necessary to converge on a distribution. In this case the histogram for each trial was very similar and it was apparent that it wouldn't take very long to converge on a representative plot. After five trials it was clear that the average of the pmf’s had been established. However since ten trails were available they were averaged and the result, with a comparative Gaussian curve superimposed for reference, is shown in Figure 6 on the next page. Because the pdf is Gaussian, this confirms the theory that the input signal is random. We can consider the collection of bins to be discrete random variables and represent, at this point, a probability mass function. However as the number of bins gradually increased the representation begins to resemble a probability density function. Fitting a line to the pmf gives us a very good approximation of a pdf for this set of data. Figure 7 shows the peaks of the input signal to have a Rayleigh distribution, which will be used in discussion later.
Cumulative Relative Frequency of Signal Input Peaks From 10 Trials

Figure 6
4.3 Tension History

The next thing to look at is the tension history at the fairlead for mooring lines of interest. The tension in Newtons is provided as a function of time by the SML software. For the 4 mooring line configuration the plot of the entire tension history for line 1, mean tension of $1.295 \times 10^7$ N is shown below in Figure 8. Note that for all cases the first twenty seconds of the time history were thrown out because of extreme values in the record due to startup.
An enlarged view reveals more detail of the response as can be seen in Figure 9.
Similarly we have the tension history for line 2 at the fairlead provided below in Figure 10. Note that the average tension in line 2 ($1.970 \times 10^7$ N) is higher than in line 1, this checks intuitively by recalling that configuration of the mooring system relative to the directional sea state.

Mooring line 2 is on the weather side of the spar.

![Figure 10](image)
Chapter 5
Fatigue Analysis

5.1 Fatigue Concepts

5.1.1 General

There are several ways that failure of a mooring line can occur including tension reaching a
certain high level or by remaining above a certain level for a period of time. However we will
look at failure due to an accumulation of damage. This accumulation of damage is known as
fatigue. When a test specimen is subject to fluctuating tension many small changes take place in
the material. In this discussion we will not delve into the particular physics that take place; rather
we will rely on data that has been collected for the material of interest together with stochastic
analysis to determine fatigue and predict failure. Referring to the enlarged view of the tension
history in Figure 11 it is seen that the value of tension moves around the mean.

![Tension History Graph](image-url)
We will call the maximum of each excursion above the mean a peak. Each peak reached causes a small amount of damage that is proportional to the amplitude of the peak, the amplitude being measured relative to the mean tension. As these excursions accumulate the material experiences fatigue. When the accumulation of these damages reaches a fixed number we expect failure. It has been determined that if the mean stress is not too large, approximately one third of the fluctuation amplitude, that the mean stress has a relatively weak effect on the fatigue life.\(^4\) An average tension of 0.3\(^\ast\)\((\text{BreakingStrength})\) is considered to be representative for conventional mooring systems\(^5\) (see Table 4). Because the tensions vary for each line in the system a specific design curve will be considered for each line, this will be derived for each line using Eqn. 5 and Table 3.

### 5.1.2 Fatigue Curve

A fatigue curve for a given material, also known as a T-N curve, represents how a material behaves under a range of tension amplitudes. T represents the tension amplitude and N is the number of cycles until fracture under uniform amplitude. This ‘curve’ represents the mean of the statistical distribution of fatigue lives for given stress amplitudes. However this work will assume that there is a single well-defined curve for the material. As the application of stress moves from a sinusoidal to a random process the prediction of fatigue and ultimately failure becomes substantially more difficult. As a solution we turn to the linear damage accumulation hypothesis proposed by Palmgren and Miner\(^6\).

\(^a\) API contends that the following is one method that can account for the mean load effect: For each sea-state, determine the mean load and corresponding design curve that is then used to calculate the fatigue damage for that sea-state. This requires using different design curves for different sea-states.
5.2  Palmgren and Miner Hypothesis

5.2.1  General Theory

According to this hypothesis when \( n \) cycles of tension amplitude \( T \) have occurred, then a certain fraction of the material's fatigue life has been depleted. This fraction is calculated as \( \frac{n}{N} \). The hypothesis claims that the fractional damage that occurs at a given stress level can be added to the damage incurred at other amplitudes to obtain the total damage experienced. So if the specimen experiences \( n_i \) cycles of amplitude \( T_i \) for \( i = 1, 2, 3, \ldots \) the total cumulative damage fraction can be defined as

\[
D = \sum \frac{n_i}{N_i}
\]

Eqn. 4

According to Palmgren-Miner the specimen will undergo fatigue failure when \( D = 1 \). Since the hypothesis does not specify the order in which the stresses must be applied it may then be used for any process including a random one. Crandall notes that this process may have shortcomings but that it provides a model with results similar to the real situation.

5.2.2  Calculation of Variables

T-N curves should be based on fatigue test data and a regression analysis for the specific material to be used. When a T-N curve is not available for the exact material being considered for use in a mooring line, Eqn. 5 is considered an acceptable alternative.

\[
NR^M = K
\]

Eqn. 5

where \( N \) is the number of cycles to failure; \( R \) is the ratio of tension range, double the tension amplitude (taken at the peak), to nominal breaking strength; \( M \) is the slope of the T-N curve; and
K is the intercept of the T-N curve. To calculate K, it is necessary to know \( Lm \), the ratio of mean load to breaking strength (see Table 4, Section 5.3.2). Table 3 can be used as a guide to calculate M and K for some generic sample materials.

### Table 3

<table>
<thead>
<tr>
<th>Component</th>
<th>M</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common chain link</td>
<td>3.36</td>
<td>370</td>
</tr>
<tr>
<td>B&amp;K Connecting link</td>
<td>3.36</td>
<td>90</td>
</tr>
<tr>
<td>Six/multi strand rope</td>
<td>4.09</td>
<td>( 10^{(3.20-2.79Lm)} )</td>
</tr>
<tr>
<td>Spiral Strand Rope</td>
<td>5.05</td>
<td>( 10^{(3.25-3.43Lm)} )</td>
</tr>
</tbody>
</table>

### 5.3 Line properties with respect to fatigue life

#### 5.3.1 General

Referring to the basic Palmgren-Miner equation we need to know two things: \( n \) and N. Code was run that extracted \( n \) from our tension history, however we still need to know which bin it will belong to and the N value assigned to that bin. Every peak encountered between the mean and maximum was assigned a bin. These bins are equally spaced between the mean and maximum tension values; Figure 12 illustrates this. Note that the calculation of the bin size is the same as Eqn. 3 but uses mean tension rather than minimum tension. Any number of bins can be specified, however in this research 10 bins were used because it described the output well without dramatically increasing the number of calculations.
Before solving for N we need to determine several variables. First we need to determine R, the ratio of tension range (double the amplitude) to nominal breaking strength. To determine this tension amplitude each peak is assigned to a bin. To simplify the calculation, all peaks that fall into a given bin are assigned the average tension value of that bin. The breaking strength for the material may be obtained from reference material (see next section). The ratio R can then be calculated. From Table 3 we get values for M and K. Now we know N for each bin by solving Eqn. 5 and can determine a design curve. Figure 13 is the curve for one trial of line 7 from the 12 mooring line configuration.
5.3.2 4 line data issues

To design the 4 mooring line system each line was calculated to have a diameter slightly larger than would commonly be found in production. To determine a representative breaking strength, it was necessary to extrapolate information from the table of Appendix A. Figure 14 is a plot of line diameter versus breaking strength. A line was fit to these points and then the breaking strength could be extrapolated for a diameter of .26 m, approximately 43,605,000 N.
Now that the breaking strength is known the needed ratio can be calculated. Table 4 has been provided to show the ratio of mean tension to breaking strength. Please use this table for reference to the discussion of section 5.1.1.

Table 4

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Average Tension</th>
<th>Breaking Force</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 ML – Line 1</td>
<td>$1.295 \times 10^7$ N</td>
<td>$4.3605 \times 10^7$ N</td>
<td>0.29</td>
</tr>
<tr>
<td>4 ML – Line 2</td>
<td>$1.970 \times 10^7$ N</td>
<td>$4.3605 \times 10^7$ N</td>
<td>0.45</td>
</tr>
<tr>
<td>12 ML – Line 1</td>
<td>$4.010 \times 10^6$ N</td>
<td>$2.0410 \times 10^7$ N</td>
<td>0.19</td>
</tr>
<tr>
<td>12 ML – Line 4</td>
<td>$5.262 \times 10^6$ N</td>
<td>$2.0410 \times 10^7$ N</td>
<td>0.25</td>
</tr>
<tr>
<td>12 ML – Line 7</td>
<td>$7.420 \times 10^6$ N</td>
<td>$2.0410 \times 10^7$ N</td>
<td>0.36</td>
</tr>
</tbody>
</table>
5.4 Calculation of Fatigue

Now all the information has been collected to determine fatigue. Because the 12 mooring line system is the most realistic we will look at it more closely. Since we have only considered a unidirectional sea state we only need to examine lines from one side of the structure, results for other lines can be inferred from symmetry, particular attention will be paid to the windward and leeward lines (lines 7 and 1). Figure 15 shows the average damage that was incurred by line 7 for the 12 mooring line configuration over 10 trials, Figure 15a shows the results of the individual trials for reference.

![Figure 15](image-url)
Essentially what is seen in these figures is the result of the Palmgren-Miner calculation. Each bin shows the $\frac{n}{N}$ results and the final column shows the sum of the damage accrued in each bin.
Comparing Figure 15 to Figure 20 shows that there is very high sensitivity to the peak tensions that fall in the highest range bin. Note that while 50% of the peaks fall in bin 1 that they contribute the least to the total damage accrued. In Figure 22 it can be seen that the frequency of peaks drops smoothly in bins 1 through 9 but there is a slight increase in bin 10. Note how this increase appears in Figure 15. There is a much higher sensitivity to peak count in the upper range bins due to the lower N values. See Figure 16 to see how N behaves as a function of the bin being considered.

![Figure 16](image)

Similarly for line 1 of the 12 line configuration:

![Figure 17](image)
Initial calculations consider only the 8190-second record that was generated by SML. Naturally failure was not expected to occur within this short time period. After the fraction representing the total damage for the trial has been calculated it can be used to determine the total lifetime of the structure. Our criteria for failure is, referring to Eqn. 3, \( D = 1 \). Therefore calculating the time until failure can be done as follows

\[
\frac{1}{\text{Total Fractional Damage Incurred During Trial}}(\text{Length of Trial (sec)}) \quad \text{Eqn. 5}
\]

Table 5 shows calculated time until failure for the lines we have considered.
### Table 5

<table>
<thead>
<tr>
<th>Mooring Line (4 Line Config)</th>
<th>Expected Lifetime – years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1 - Leeward</td>
<td>55,646</td>
</tr>
<tr>
<td>Line 2 – Weather</td>
<td>78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mooring Line (12 Line Config)</th>
<th>Expected Lifetime – years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1 - Leeward</td>
<td>597,841</td>
</tr>
<tr>
<td>Line 4</td>
<td>868,363</td>
</tr>
<tr>
<td>Line 7 – Weather</td>
<td>132</td>
</tr>
</tbody>
</table>

Note that the weather line is the limiting factor of the design for this unidirectional storm scenario. Because the unidirectional sea state the lines off the windward side are unrealistically loaded. However the expected lifetime of the weather line makes sense considering that it should be designed to be approximately three times the design service life.

### 5.5 Creation of PDF’s

As explained in detail above, the number of peaks that fall in each bin are collected and define the pmf plots for each trial, then the trials are averaged for one representative plot. A line is fit, in the least squares sense, to the average pmf plot and is described by the equation given in each figure. Although not perfect, and of differing orders, these equations look to be fairly reasonably fit on the interval considered. After the line has been described it is then possible to check the distribution of peaks for the given number of bins. It is also then possible to refine the analysis of
the tension ranges by increasing the number of bins looked at (effectively narrowing each range).
So it is possible to look at, for example, the probability being in any of twenty different bins. In
effect it is possible to determine the probability of being in any tension range or determine the
probability a discrete tension.

5.5.1 4 Mooring Line Configuration

Figure 18 shows the distribution of peaks above the mean, by bin, for line 1. Note that this is the
leeward line and its resemblance to the input signal peak distribution.

For line 2, the weather line, we see that the weight of where the most peaks fall has shifted
dramatically towards the mean.
5.5.2 12 Mooring Line Configuration

In this configuration we only will look at three of the 12 lines: line 1 is the lee, line 4 is 90 degrees from the weather, and line 7 is directly towards the weather (Figures 18 –20). It is more clearly seen from this configuration how the weight of the function shifts into the first bin as we move from the leeward to windward line. Note that the transform from the input signal to the tension response of the various lines is non-linear. Recall Figure 7, the perfect Rayleigh distribution of the input signal and notice how the leeward line most resembles it as a response and the windward the least. Because this is the more realistic configuration of the two, the pmf’s for the individual trials have been included below the plot of the averaged results.
PMF for 10 Trials and Equation for PDF (12 Line System - Line 1)

PDF Eqn.: \(0.0000662x^5 - 0.0022656x^3 + 0.0291959x^2 - 0.1687058x + 0.3710013 - 0.0006064\)

Figure 20

PDF
PMF

Figure 20a
PMF for 10 Trials and Equation for PDF (12 Line System - Line 4)

PDF Eqn.: 0.000046x^6 - 0.0001966x^7 + 0.003424x^8 - 0.031257x^9 + 0.1535001x^{10} - 0.3679349x^{11} + 0.2262619x^{12} + 0.3791966x^{13} - 0.0000282

Figure 21

Figure 21a
PMF for 10 Trials and Equation for PDF (12 Line System - Line 7)

PDF Eqn.: \(-0.00000008x^6 + 0.0000361x^7 - 0.0006518x^6 + 0.0000000000000000193\)

Figure 22

Figure 22a
Chapter 6
Conclusions and Future work

The purpose of this work was to investigate the fatigue and failure of the mooring lines of offshore structures. Having the benefit of SML, a program that considered non-linear effects, to provide realistic and reliable tension histories was fundamental to the results of this paper. Another benefit of SML was the ability to run many trials for the same scenario that allowed for the stochastic treatment of the data. Based on the convergence discussion of section 4.1 the number as well as length for the trails run provided a good data set. Marginal error may have been introduced by assumptions such as average bin values for the calculation of the double amplitude value. The results of the fatigue calculation showed that there is high sensitivity to peaks that fall into the bins of the highest tension range. Comparing the incremental damage seen in Figure 15 with the probability distribution of Figure 20 shows that there is a great need to design for and manage the number of times tensions reach the high range bins. This would be of primary concern for the designer and could be more carefully considered using the pdf curves developed. The comparison of the pdf’s of the lines to the pdf of the input signal showed that position of the line relative to the weather was very important. The weather line having a completely nonlinear relationship to the input signal and becoming more linear as the lee line position is approached. Although no general theory exists for the statistical description of nonlinear responses, we will assume that the response is ergodic and stationary. When considering the pdf of the windward vs. leeward lines it makes sense that the windward line, under greater average tension, would have a large percentage of the peaks located right around
the mean. Whereas the leeward would have more slack and the tendency to ‘jerk’ to higher
tensions. Because the techniques used were relatively simple and the processing time relatively
quick, they combine to make a good tool for the analysis of a very important question facing the
engineer on such a project.

However, there are a few things that could improve the results of this paper. The first being that
it looks at storm conditions for 8190 seconds and then extends the calculation of damage to
determine the lifetime of use. This is essentially like calculating the lifetime of the structure in a
multi-year storm. Even in the worst environments storms do not last for years on end. Ideally
there would be a probability distribution for the types of storms encountered so that the pdf’s
revealed here could be applied to accurately describe the lifetime environment of the structure.
Another aspect of the environment that limits this model’s usefulness is the fact that the storm is
coming precisely from one direction, there should be a correction for three-dimensionality.

The next step in this line of research would be looking into the non-linear transfer function to
describe the relationship between the input signal and the response. Also investigating the
damaged condition due ultimately to fatigue failure and its transition to equilibrium. A correction
for deformation of the material would also refine this research.
### Appendix A

#### SPIRAL STRAND CONSTRUCTION

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Minimum Breaking Load (tf)</th>
<th>Weight / Metre in air (kgs)</th>
<th>Weight / Metre Submerged (kgs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Unsheathed</td>
<td>Sheathed</td>
</tr>
<tr>
<td>3.00&quot; (76mm)</td>
<td>585</td>
<td>28</td>
<td>31</td>
</tr>
<tr>
<td>3.25&quot; (84mm)</td>
<td>690</td>
<td>35.2</td>
<td>38.7</td>
</tr>
<tr>
<td>3.50&quot; (90mm)</td>
<td>800</td>
<td>39.5</td>
<td>42.5</td>
</tr>
<tr>
<td>3.75&quot; (96mm)</td>
<td>900</td>
<td>45</td>
<td>49.5</td>
</tr>
<tr>
<td>4.00&quot; (102mm)</td>
<td>1000</td>
<td>51</td>
<td>54</td>
</tr>
<tr>
<td>4.25&quot; (108mm)</td>
<td>1150</td>
<td>57</td>
<td>62</td>
</tr>
<tr>
<td>4.50&quot; (114mm)</td>
<td>1250</td>
<td>65</td>
<td>70</td>
</tr>
<tr>
<td>4.75&quot; (121mm)</td>
<td>1400</td>
<td>71</td>
<td>76</td>
</tr>
<tr>
<td>5.00&quot; (127mm)</td>
<td>1550</td>
<td>80</td>
<td>85</td>
</tr>
<tr>
<td>5.25&quot; (133mm)</td>
<td>1700</td>
<td>88</td>
<td>94</td>
</tr>
<tr>
<td>5.50&quot; (140mm)</td>
<td>1900</td>
<td>96</td>
<td>101</td>
</tr>
<tr>
<td>5.75&quot; (146mm)</td>
<td>2000</td>
<td>106</td>
<td>111</td>
</tr>
<tr>
<td>6.00&quot; (151mm)</td>
<td>2200</td>
<td>114</td>
<td>120</td>
</tr>
</tbody>
</table>

1 Lecture Notes 13.42, Design Principles for Ocean Vehicles, Spring 2001, © A.H. Techet


3 API Recommended Practice 2SK; Second Edition, December 1996


5 API


7 API