Simulation of Motion Responses of Spar Type Oil Platform

by

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B. S., Ocean Engineering, Massachusetts Institute of Technology (2001)

Submitted to the Department of Ocean Engineering in partial fulfillment of the requirements for the degrees of

Master of Science in Naval Architecture and Marine Engineering at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2001

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Abstract

Computer based simulations of the motion responses of spar type oil platforms were carried out. Theses simulations examined both the linear and nonlinear coupled responses in surge, heave, and pitch to plane progressive wave trains as well as random waves. A twelve line mooring system was also incorporated for more accurate modeling. These simulations showed the mooring system to limit only the platforms large-scale motions. Studies were made both at and far from the buoy's natural frequencies in both the linear and nonlinear cases. A pitch instability due to coupling between the pitch restoring term and the displaced heave position was also examined. The pitch instability was present only in the nonlinear simulation with all governing quantities evaluated at the local wave elevation. The magnitude of this instability was limited, but not eliminated, by the addition of the mooring system.

Thesis supervisor: Paul Sclavounos
Title: Professor of Naval Architecture

Acknowledgements

I would like to express my thanks to all the members of the Laboratory for Ship and Platform Flow in the Department of Ocean Engineering. In particular I would like to thank my advisor Prof. Paul Sclavounos for his guidance and answering my relentless questions, and Peter Sabin for his sense of humor and perspective when things looked bleak.

I would also like to my classmates and faculty at Webb Institute, without them I never would have made it to or made it through M.I.T.

I would like to thank my mother for all her support throughout my education. This work is dedicated in loving memory to her parents, Papa and Granny Boats, without whom I would have never have discovered my love for the sea.

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Chapter 1 Introduction

Spar buoy type oil platforms are being considered for many of the current generation of offshore oilrigs. Spar buoys are of particular interest for deployment in seas with extreme weather conditions and depth. Motion on an oil platform is not desirable from the standpoint of production where its effects can range from making the crew seasick to breaking risers and other vital pumping equipment. In shallow waters there are a number of solutions to this problem. One is the tension leg platform where the platform is held with high tension mooring lines to anchors mounted in the bottom; another is the jack-up platform where the platform is physically raised above the water's surface on rigid legs extending to the sea floor. These shallow water solutions are not feasible in water deep water or harsh sea states.

As shallow water oil reserves are being rapidly depleted the offshore industry is looking to move to water depths up to 10,000m and farther offshore where the weather conditions are less predictable and more punishing. The spar type platform is a competitive alternative to classical design for these difficult environments.

The classic shape of a spar buoy platform is that of a vertical cylinder. Traditionally they have a constant diameter of roughly 30m and a draft of approximately 200m. The motivation for the use of such an unusual shape with its large volume is to minimize the buoy's motion response in seas. The large volume to waterplane area ratio of the spar makes its response to changes in water elevation (waves) quite slow. In summary it is the shape of the platform, rather than the mooring system that is mainly responsible for the platform motions remaining small.

In the depth range for which spars are being considered the steel risers are extremely flexible, so horizontal (surge) motion is of little concern. The risers cannot, however, withstand huge stretching and compressive loads without rupturing or buckling, so vertical (heave) motions need to be minimized. Rotational (pitch) motions can also cause problems; small motions are associated with adverse working conditions, while extreme rotations could lead to capsizing and equipment breakage and loss.

The shape of the spar makes its natural frequencies in heave and pitch extremely low. With low natural frequencies the threat to spar platforms comes mainly from waves of long wavelengths. Waves begin to break when their slope is approximately $\frac{1}{7}$, so waves of high amplitude also have long wavelengths, a dangerous combination for spar platforms.

This research was aimed at predicting the motions of spar type oil platforms. This was done in a number of steps of computer-based simulation. The first simulations were based upon linear theory. These simulations predict the surge, heave, and pitch responses of a freely floating spar to one wave train of a specified amplitude and frequency. This was done to study the responses under specific, known conditions, such as around the natural frequencies. This simulation included the effects that one mode of motion may have on another, known as coupling.

Real oil platforms are not freely floating, they are moored in place, and so the next step was to add the effects of a mooring system to the linear simulation. Finally, a random wave generator was added to examine the responses to a wave more representative of a true ocean wave.

Next a nonlinear simulation was developed. This model evaluated governing quantities at the local free surface elevation, thus accounting for a number of effects that are neglected by linear theory. The same progression of adding a mooring system and random wave was also carried out.

Chapter 2 Mathematical Formulation

2.1 Coordinate System

Throughout this discussion a Cartesian coordinate system (x, y, z) will be used with the origin coinciding with the undisturbed free surface. It will be located at the centerline of the buoy with the positive z-axis pointing upwards. The elevation of the free surface will be defined by $\eta(x,y,t)$. Thus gravity (g) acts in the negative z-direction.

A body freely floating on the surface of a fluid is able to move in six modes of motion. Three modes, designated surge, sway and heave, coincide with translation motion parallel to the $x-y-z$ axes respectively. The other three modes, roll, yaw, and pitch, represent rotation about those axes. Since a spar is cylindrical, there are only three fundamentally different modes that must be studied, surge, heave, and pitch.

2.2 Linearized Free Surface Condition

Assuming an ideal and irrotational fluid, there will exist a velocity potential ϕ such that the fluid velocity is expressed as the gradient of this velocity potential

Equation 2.1

$$V = \nabla\phi$$

Due to conservation of mass, the divergence of this velocity potential must be zero,

Equation 2.2

$$\nabla^2\phi = 0$$

it must also satisfy both a kinematic and dynamic boundary condition on the free surface.

The kinematic boundary condition requires the velocity of the free surface to be equal that to that of the fluid particles of which it is comprised. The kinematic boundary condition is found by requiring that the substantial derivative of $(z-\eta)=0$ on the free surface. The result of which is

Equation 2.3

$$0 = \frac{D}{Dt}(z-\eta) = \frac{\partial\phi}{\partial z} - \frac{\partial\eta}{\partial t} - \frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x} - \frac{\partial\phi}{\partial y}\frac{\partial\eta}{\partial y}$$

The last two terms may be neglected because they are of second order and therefore much smaller than the first two, this results in the linearized kinematic boundary condition

Equation 2.4

$$\frac{\partial\eta}{\partial t} = \frac{\partial\phi}{\partial y}$$

The dynamic boundary condition requires that the pressure acting on the free surface from above be equal to the pressure acting from below. The dynamic boundary condition is found through the use of Bernoulli's equation,

Equation 2.5

$$-\frac{1}{\rho}(p-p_a)=\frac{\partial\phi}{\partial t}+\frac{1}{2}\nabla\phi\bullet\nabla\phi+gz=0$$

Substituting η for z and linearizing as in the kinematic boundary condition results in the linearized dynamic boundary condition

Equation 2.6

$$\eta=-\frac{1}{g}\frac{\partial\phi}{\partial t}$$

These two boundary conditions can be combined on the surface $z=0$ resulting in one boundary condition for the velocity potential

Equation 2.7

$$\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial y}=0$$

2.3 Plane Progressive Waves

Plane progressive waves are the simplest waves that satisfy the free-surface condition, and will be used extensively throughout this discussion. These waves are two-dimensional and have a single amplitude (A) and frequency (ω) and propagate with a sinusoidal motion in one direction.

Equation 2.8

$$\eta(x,t) = A\cos(kx - \omega t + \theta)$$

describes a plane progressive wave moving in the positive x-direction, a phase θ has been included but will be assumed to be zero. The value k is known as the wave number and is defined as

Equation 2.9

$$k = \frac{\omega}{V_p} = \frac{2\pi}{\lambda} = \frac{\omega^2}{g}$$

where V_p is the phase velocity (the velocity that a wave peak travels) and λ is the wavelength.

The velocity potential ϕ that satisfies both Equation 2.2 and Equation 2.7 and through the use of Equation 2.6 will result in

Equation 2.8 is given by

Equation 2.10

$$\phi = \frac{gA}{\omega} e^{kz} \sin(kx - \omega t)$$

In the ocean, waves are not two-dimensional, are not purely sinusoidal, and their amplitude and frequency cannot be described by single parameters. However, plane progressive waves are extremely useful for examining both simplified problems, and it will be discussed later how "real" ocean waves can be modeled as a superposition of many plane progressive waves each with differing amplitudes and frequencies.

2.4 Body Response in Plane Progressive Waves

As mentioned, a body freely floating on the surface of a fluid is free to move in all six modes of motion. This section will explore the response of a body in the three modes that have been identified as being of particular interest to the problem of a spar oil platform, namely surge, heave and pitch. First the equations of motion will be presented, followed by an examination of each of the terms contained within them including discussion of the coupling effects between surge and pitch. In these equations integrals will be carried out over the interval $\int_{-draft}^{surface} dz$ the actual z value represented by the term 'surface' will be discussed later.

The equation of motion given by

Equation 2.11

$$\sum_{j=1,3,5}[-\omega^2(M_{ij}+a_{ij})+i\omega b_{ij}+c_{ij}]\xi_i = AX_1$$

where the indices i, j are the modes of motion 1, 3 or 5 that corresponds with surge, heave, and pitch. When the indices $i \neq j$, potential coupling effects between modes of motion are represented. Some of the coupling terms represented in Equation 2.11 are equal to zero, indicating that even though coupling is possible, it does not actually occur. Non-zero coupling terms will be further addressed later. ξ_i terms are the body displacement in complex form.

In Equation 2.11 the M_{ii} terms are the components of the inertial force upon the body. M_{11} and M_{33} are simply the body mass, while M_{55} is the moment of inertia defined by

Equation 2.12

$$M_{55} = I_{22} = \iiint_V \rho_b y^2 dV$$

where ρ_b is the density of the body, and V is the body volume. There are no other non-zero mass terms associated with this problem.

2.4.1 Added Mass

The a_{ii} terms in Equation 2.11 are known as the added mass terms, since they are proportional to the body acceleration. Since this is the case, they are obviously dependent on the frequency of the motion. Through a method of images[1], added mass terms can be found analytically for a given geometry; however, due to the complexity of this method a combination of empirical values and strip theory were employed to estimate the added mass at low frequencies.

Empirical formulas exist determining the added mass of many common geometric shapes. Of importance to this discussion are those for a circle moving in a two dimensional fluid.

Equation 2.13

$$a_{11}^{2D} = \pi \rho r^2 \text{..........}(a)$$
$$a_{22}^{2D} = \pi \rho r^2 \text{..........}(b)$$

where a_{11} represents movement within the plane, and a_{22} is movement in and out of the plane. The added mass of a cylindrical spar buoy in heave is due to the flat bottom rising and falling; thus, Equation 2.13 (b) can be used directly to find the added mass of the buoy in heave.

Equation 2.13 (a) was used to find both the added mass in surge, pitch, and the coupling between the two. Strip theory allows a three-dimensional value to be found by integration of two-dimension values over a body, provided that the change in the two-dimensional values is small. The following illustrates how this was employed to find the surge added mass

Equation 2.14

$$a_{11}^{3D} = \int_{-draft}^{surface} a_{11}^{2D}(z)\,dz$$

In this discussion the buoys were assumed to be of constant diameter removing the dependence of a_{11}^{2D} on z, thus simplifying the problem further. The following two equations show how this method was used for determining the added mass in pitch, and the coupled added mass term between pitch and surge

Equation 2.15

$$a_{55}^{3D} = \int_{-draft}^{surface} z^2 a_{11}^{2D} dz$$

Equation 2.16

$$a_{51}^{3D} = a_{15}^{3D} = \int_{-draft}^{surface} z a_{11}^{2D} dz$$

There are no other non-zero added mass terms in this problem.

2.4.2 Damping Forces

The b_{ij} terms in Equation 2.11 are called the damping terms and are proportional to the velocity of the body. These forces are present do to the waves that are generated by the body that radiate outwards from the body dissipating energy. In the derivation of the damping terms ϕ is broken into eight components ϕ_i, with 1,2..6 representing the potential for each of the six modes of motion. ϕ_7 is called the diffraction potential and represents the body-generated waves. ϕ_0 is the incident wave potential. These waves must satisfy the free surface boundary condition Equation 2.7. From this fact can be derived the Haskind relations[1]

Equation 2.17

$$X_i = -\rho \iint_{S_B} \left(\phi_0 \frac{\partial \phi_i}{\partial n} - \phi_i \frac{\partial \phi_0}{\partial n} \right) dS$$

where S_B is the body surface and n is the normal vector of the body surface. The Haskind relations are used to derive expressions for the damping terms, which are related to the exciting force. It will be shown in 2.4.4 that the exciting forces are related to the frequency of the exciting wave; therefore, the damping terms are dependant on the wave frequency.

At this point in the simulation a simplification was made in determining damping coefficients. The general-purpose program SML™ was used to determine an appropriate constant value for damping coefficients.

2.4.3 Restoring Forces

The C_{ij} terms in Equation 2.11 are the restoring terms, which are responsible for trying to return the body back to it's original state. When examining the freely floating body these are all due to hydrostatics.

The restoring force in heave is the difference between the original displaced volume and the heaved displaced volume multiplied by the density of the fluid and the force of gravity

Equation 2.18

$$C_{33} = \rho g A_w$$

This term is multiplied by the heaved position to give the resultant force.

The pitch restoring term is comprised of two parts, the first is due to the redistribution of submerged volume that creates a moment. Another moment is created by pitch motion because the center of gravity (Z_G) and center buoyancy (Z_B) are no longer located within the same vertical plane. The center of gravity is located below the center of buoyancy (provided the body was initially stable), and the force acting at the center of gravity acts down while the force on the center of buoyancy acts upwards, since these two forces must be equal in magnitude but are now different distances from the origin they create another restoring moment proportional to the distance between the two. Figure 1 illustrates how these two terms are physically produced.

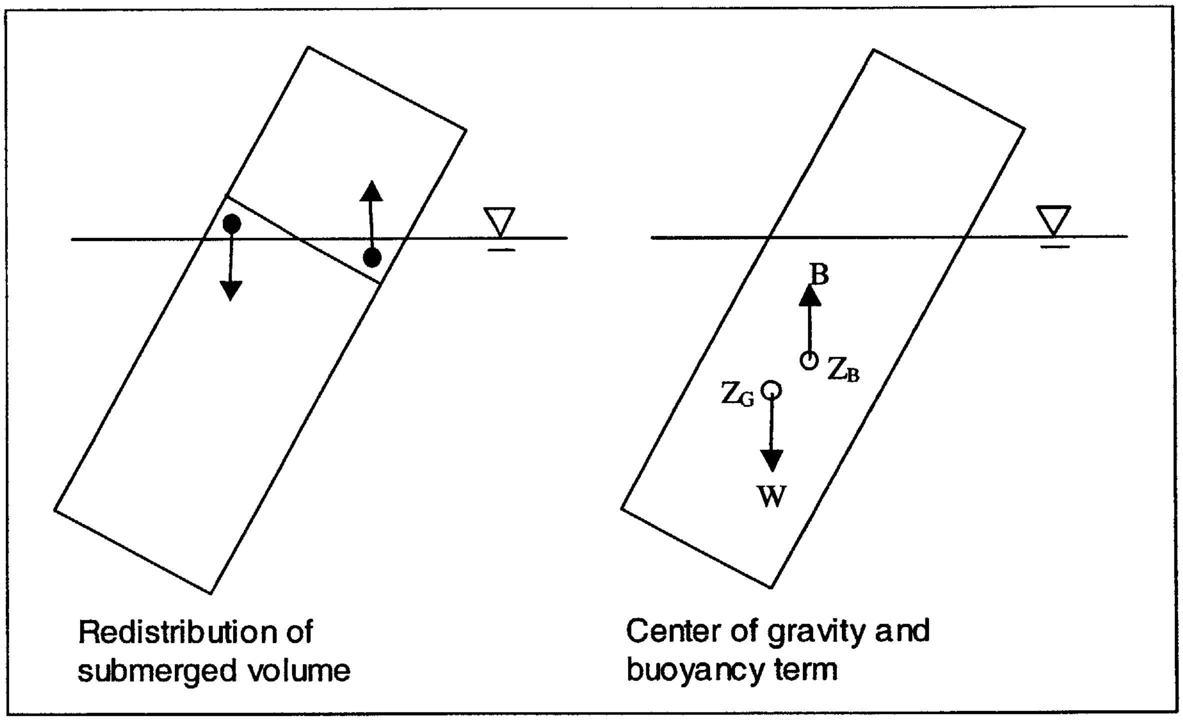

Figure 1 Illustration of the two pitch restoring force terms

These two terms can be seen in the following equation for C_{55}

Equation 2.19

$$C_{55} = \frac{\rho g \pi d^4}{64} + \rho g \forall (Y_B - Y_G)$$

where d is the buoy diameter, $\forall$ is the buoy volume, and Z_B and Z_G are the centers of buoyancy and gravity respectively. There are no other non-zero restoring forces.

2.4.4 Exciting Forces

Exciting forces for this problem were found using strip theory and G. I. Taylor's formula for the force of a two dimensional section

Equation 2.20

$$dX_i = (\forall + \frac{a_{ii}}{\rho})\frac{\partial}{\partial x_i}(\rho\frac{\partial \phi}{\partial t})$$

where x_i is the Cartesian axis [$(1,2,3)$ corresponds with (x,y,z)] parallel to the desired exciting force. Again strip theory was employed to integrate these slices over the entire buoy as follows for surge, heave, and pitch

Equation 2.21

$$X_1 = \int_{-draft}^{surface} (\forall + \frac{a_{11}}{\rho})\frac{\partial}{\partial x}(\rho\frac{\partial \phi}{\partial t})dz\Big|_{\substack{X=0\\Y=0}}$$

Equation 2.22

$$X_3 = (\forall + \frac{a_{33}}{\rho})\frac{\partial}{\partial z}(\rho\frac{\partial \phi}{\partial t})\Big|_{\substack{X=0\\Y=0\\Z=-draft}}$$

Equation 2.23

$$X_5 = \int_{-draft}^{surface} z(\forall + \frac{a_{11}}{\rho})\frac{\partial}{\partial x}(\rho\frac{\partial \phi}{\partial t})dz$$

2.4.5 Natural Frequency

If there exist no coupling and damping or external forces Equation 2.11 reduces to

Equation 2.24

$$-\omega^2(M_{ii}+a_{ii})\ddot{X}+c_{ii}=0$$

The solution of this equation produces what is known as the natural frequency for the i^{th} mode of motion

Equation 2.25

$$\omega=\pm\sqrt{\frac{c_{ii}}{M_{ii}+a_{ii}}}$$

Since negative frequencies are physically impossible we are only concerned with the positive value of Equation 2.25. The response of a body excited at a frequency that is extremely close to or exactly equal to its natural frequency will be extreme. Therefore, it is important that a physical structure not be excited at one of these frequencies.

Chapter 3 Linear Simulation

3.1 Linear Simulation of a Freely Floating Buoy

A time domain linear simulation of a freely floating spar buoy was created. This simulation was based upon the linear theory that has been presented. The spar buoy that will be used for all these simulations has a draft of 200m, with 30m freeboard, a diameter of 30m, and a center of gravity located 104m below the undisturbed free surface. With the exception of random waves the wave amplitude will always be 10m.

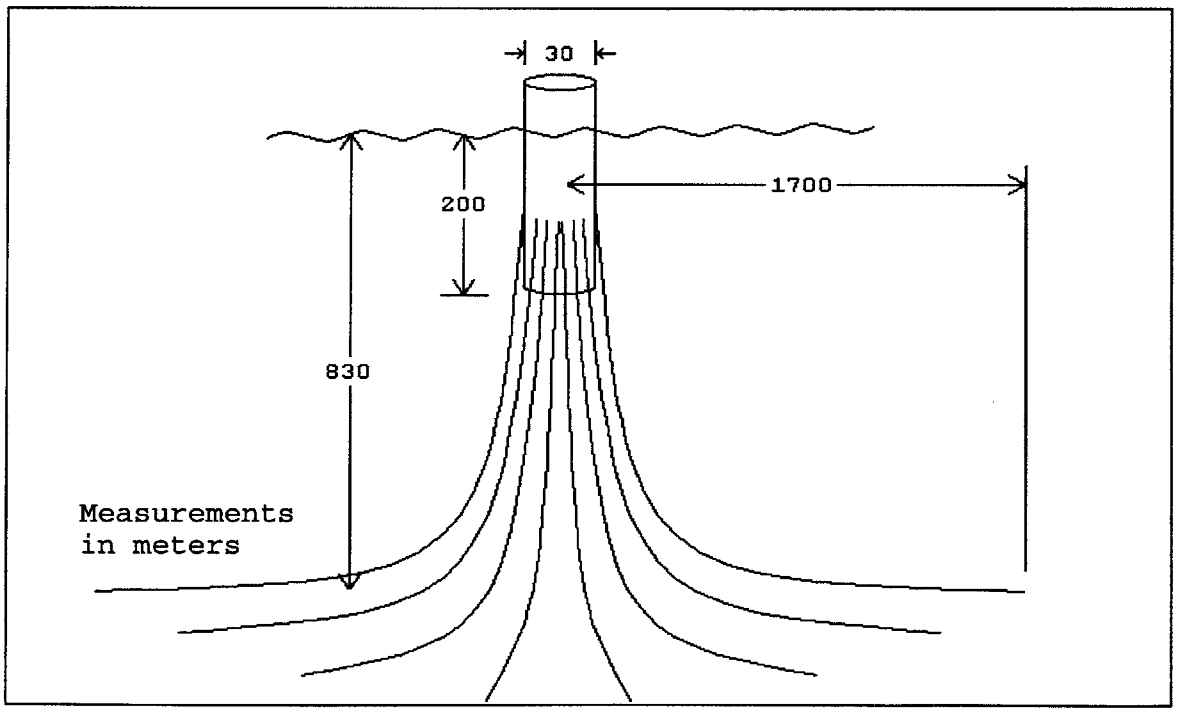

Figure 2 Spar buoy

Sample output from the simulation is presented in Figure 3

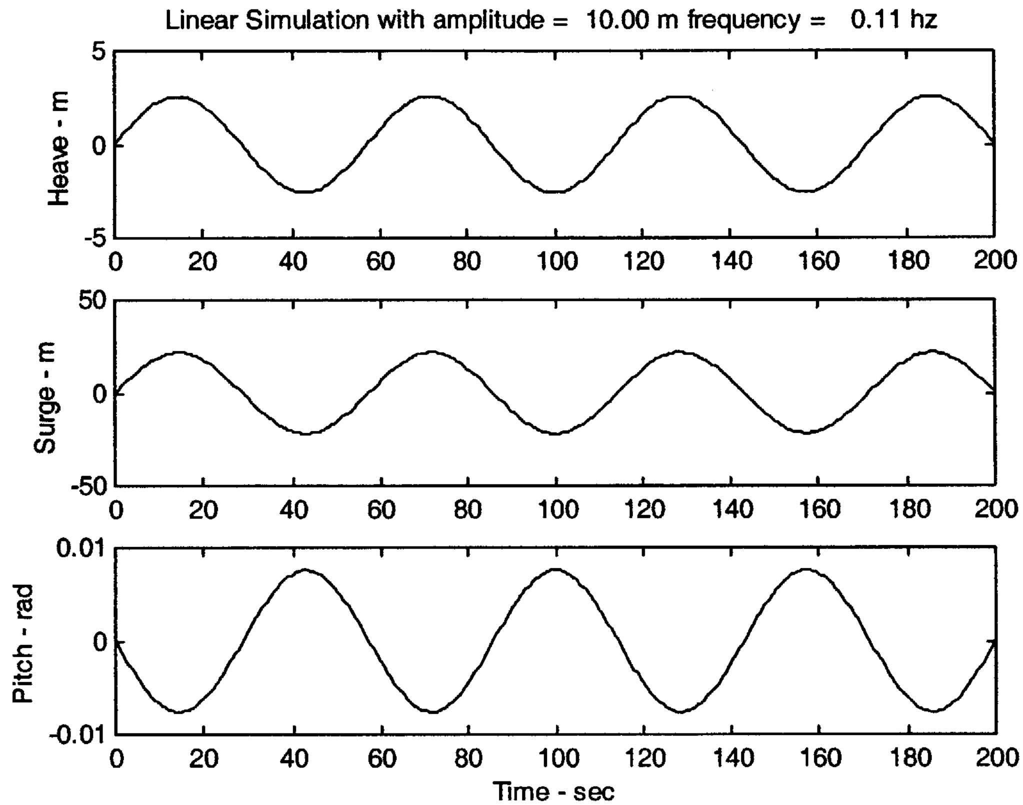

Figure 3 Linear simulation output

Figure 3 shows the sinusoidal motions of the buoy in heave, surge, and pitch. This simulation can also be used to examine the response near the buoy's natural frequencies. The natural frequency of the buoy in heave is $\omega = 0.2209Hz$, Figure 4 shows the responses at $\omega = 0.22Hz$. The magnitude of the heave motion is over 400m from a 10m wave, illustrating the effect of exciting the buoy near its natural frequency.

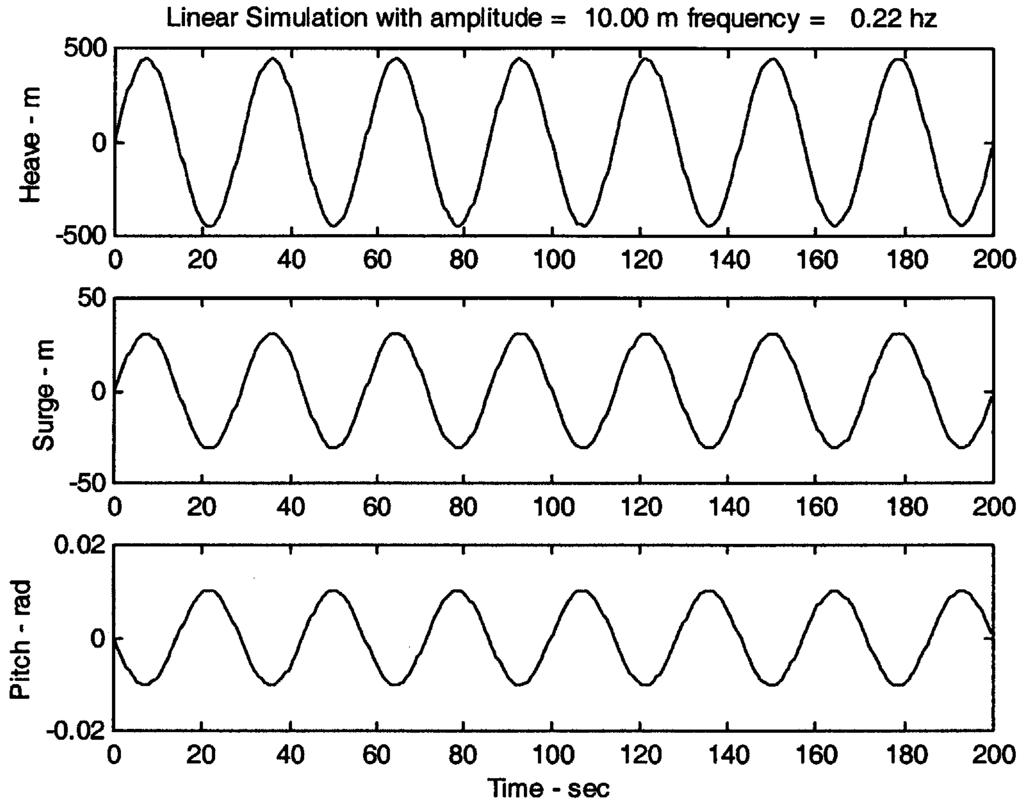

Figure 4 Responses near heave natural frequency

In Figure 4 the heave motion is so great that the buoy completely leaves the water. The effect of this event occurring is not included in this simulation. Although this behavior is indicative of the response to excitation near the natural frequency, it will be seen in 3.2 that the presence of a mooring system prevents this occurrence.

The natural frequency of the buoy in pitch is $\omega = 0.0491Hz$, Figure 5 shows the responses at $\omega = 0.05Hz$. The magnitude of the pitch response is extremely small. In this case the coupling between surge and pitch has overcome the effects natural frequency excitation as illustrated by Equation 3.1, which is the expanded form of Equation 2.11 for pitch including coupling effects

Equation 3.1

$$-\omega^2(M_{55}+a_{55})\xi_5-\omega^2(M_{11}+a_{11})+(i\omega b_{55})\xi_5+(i\omega b_{11})\xi_1+c_{55}\xi_5+c_{11}\xi_1=AX_5$$

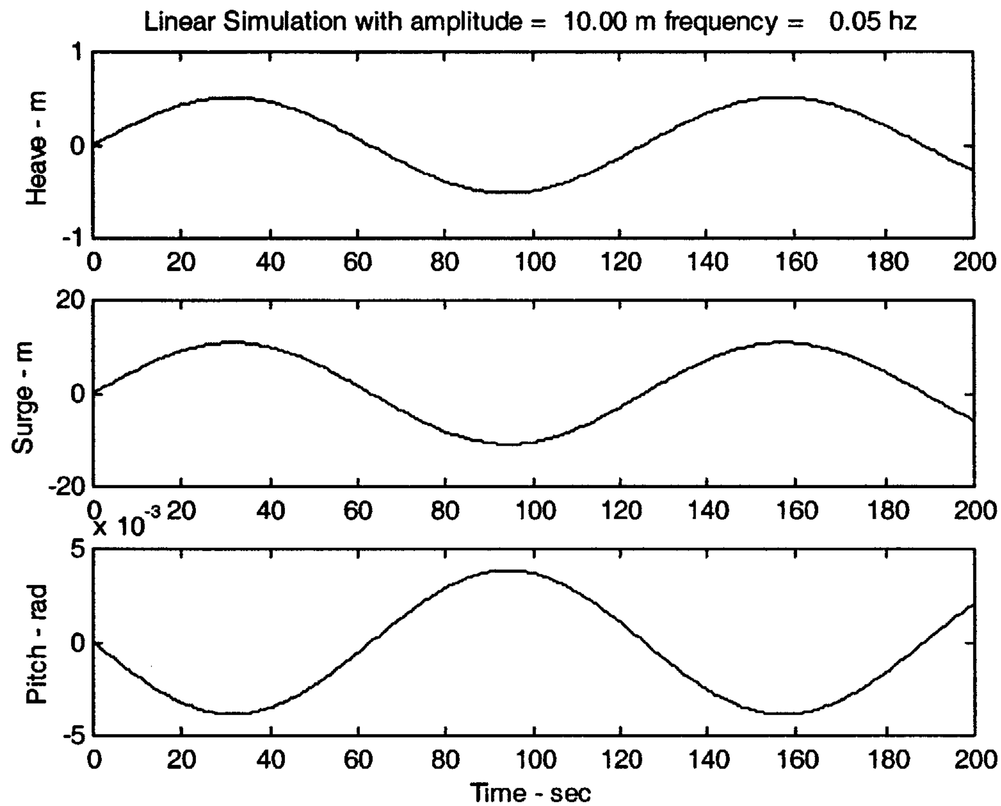

Figure 5 Responses near pitch natural frequency

3.2 Linear Simulation Coupled To Mooring System

Oil platforms are not freely floating bodies, they are held in place with mooring systems. Spar buoys are typically moored by ten to twenty mooring lines, spaced around the diameter of the buoy. These lines typically consist of three segments, the lower most that connects to the anchor is made of chain, the middle section is made of steel or synthetic rope, and the top section is also made of chain. As mentioned previously the purpose of the mooring system is to prevent large-scale motions, it would be unfeasible to hold the platform completely stationary. For this simulation the restoring effects of a 12-line mooring system were added. These effects were found through simulation using the Lines™ portion of SML™.

The buoy was moored in 830m of water by twelve equally spaced lines that were each 1850m in length. The lowest section was a 150m length of chain, followed by 1500m of polyester rope, followed by 150m of chain. The point at which the lines are moored to the buoy is defined as the fairlead; this point was varied from ½ the draft of the buoy to the bottom to examine the effects of fairlead location of platform motions.

Table 1 Mooring system restoring forces

Fairlead location below free surface (m)	C11 - kg/s^2	C33 - kg/s^2	C55 - kg.m/s^2	C51 - kg/s^2	C15 - kg/s^2
100	5.21E+05	1.96E+05	6.42E+09	-4.77E+07	-4.85E+07
110	3.59E+05	1.32E+05	5.75E+09	-4.44E+07	-3.75E+07
120	2.65E+05	9.60E+04	5.39E+09	-2.97E+07	-3.10E+07
130	2.27E+05	8.26E+04	5.32E+09	-2.77E+07	-2.78E+07
140	1.88E+05	6.76E+04	5.26E+09	-2.48E+07	-2.51E+07
150	1.54E+05	5.47E+04	5.13E+09	-2.19E+07	-2.23E+07
160	1.26E+05	4.43E+04	5.00E+09	-1.93E+07	-1.97E+07
170	1.05E+05	3.61E+04	4.88E+09	-1.71E+07	-1.75E+07
180	8.81E+04	2.97E+04	4.78E+09	-1.52E+07	-1.56E+07
190	7.50E+04	2.47E+04	4.71E+09	-1.37E+07	-1.40E+07
200	6.46E+04	2.08E+04	4.66E+09	-1.25E+07	-1.28E+07

twelve line mooring system in place. $w = 0.15Hz$ was selected because this frequency is spaced between the two natural frequencies, yet not close to either, so that the results would be in a realistic domain and natural frequency effects would be minimized. For all three modes of motion the maximum responses are diminished by the presence of the mooring system. Figure 6 also shows the effect that the fairlead location has upon these motions.

In heave as the fairlead location approaches the bottom of the buoy the motion approaches that of the unmoored buoy. Lines™ does take into account the elasticity, as well as the curvature, of the mooring lines, so as the lines are connected lower the heave restoring force they provide is lowered, as shown in Figure 6.

The results for surge and pitch do not show much dependence upon the fairlead location. In both cases this is due to the small relative magnitude of the motions. In surge the freely floating buoy moves only 26.84m, while the anchors for the mooring lines were positioned in a ring 1700m from the platform. As mentioned, a large amount of surge motion is acceptable, by positioning the anchors far away the mooring will prevent very large surge motion but allow motions on the order seen.

The effect of the mooring system in all three modes of motion is extremely small, as can be seen in Figure 7. In the heave plot the motions with the mooring line are just distinguishably lower than that of the unmoored buoy. In the surge and pitch plots the two motions are indistinguishable from one another.

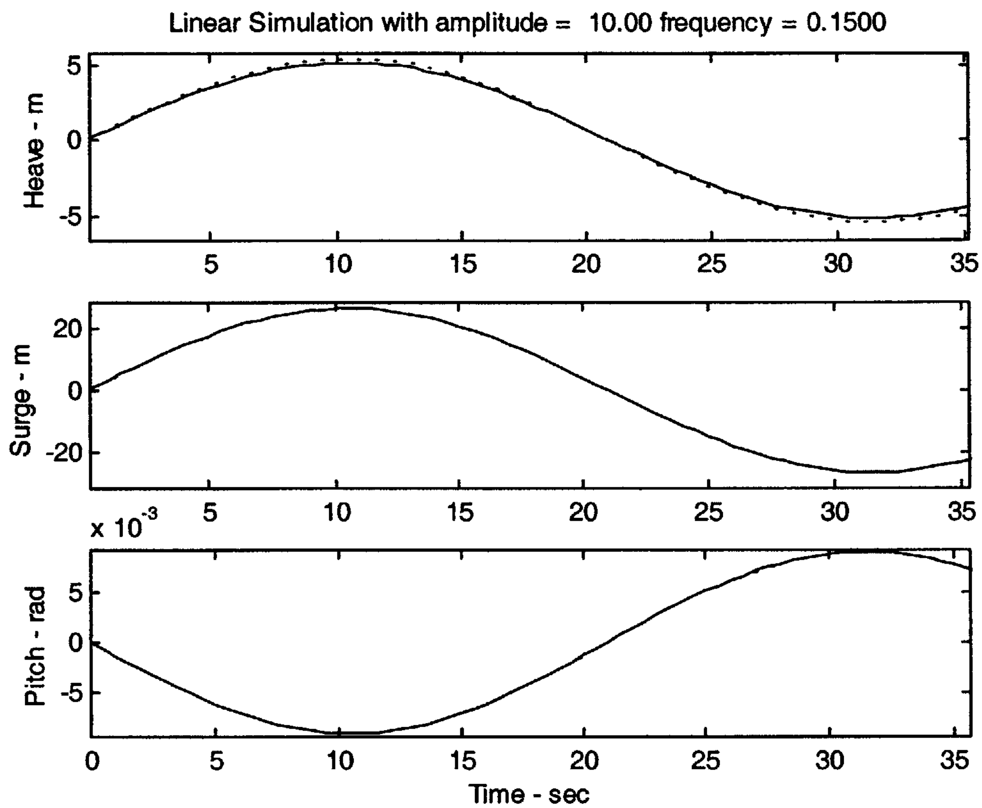

Figure 7 Responses with and without mooring system and a fairlead location of ½ the draft

3.3 Linear Simulation With A Random Excitation Wave and a Mooring System

True ocean waves cannot be described by a single amplitude and frequency. This is due mainly to the dispersion relationship, which was presented in Equation 2.9. The dispersion relationship states that waves of different frequencies travel at different velocities, as does the energy associated with them. Not only are ocean waves dispersive, they are random. These random waves can be modeled as a superposition of many plane progressive waves, each randomly selected amplitude, frequency and phase difference.

A random wave generator was created that randomly selects amplitude, frequency and phase, each over a range making physical sense. Frequencies were allowed to range from $0.05-0.45Hz$, amplitudes from $0-1m$, and phase from $0-2\pi$. Figure 8 shows the output from the random wave simulation, using the superposition of 200 waves. The first plot shows the wave elevation and the following three show the heave, surge, and pitch responses. This simulation used the same twelve line mooring system employed before with a fairlead location of ½ the draft. The heave motion exhibits responses larger than the wave elevation due to high amounts of wave energy that happen to be focused around the heave natural frequency. Note that this response is still far below that shown in Figure 4 where the buoy was excited at a single frequency that was very close to the natural frequency.

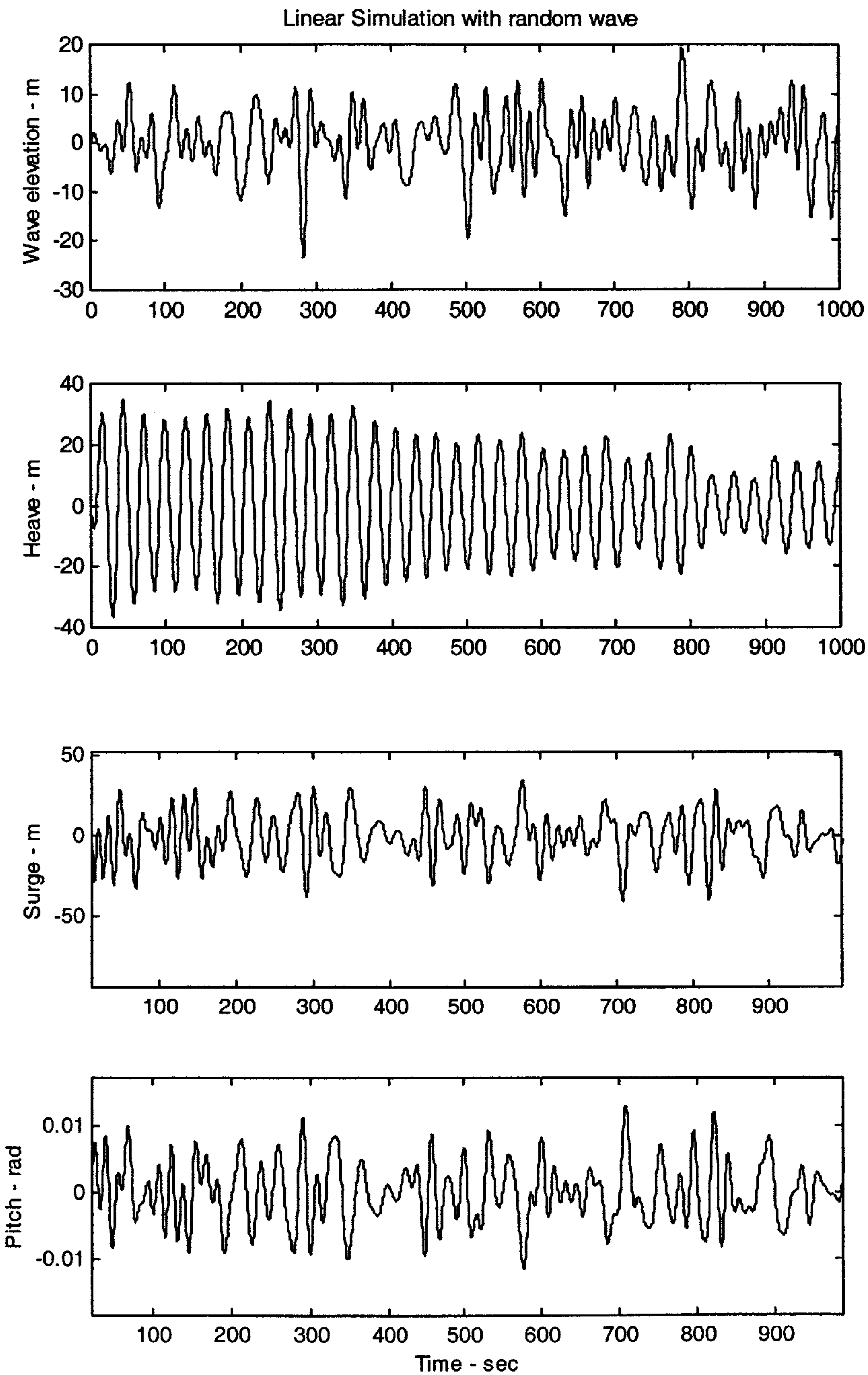

Figure 8 Linear responses to random wave with mooring system

Chapter 4 Nonlinear Simulation

4.1 Nonlinear Simulation of A Freely Floating Buoy

A nonlinear simulation was developed for more realistic modeling of the motions of a spar buoy. H. A. Haslum and O. M. Faltinsen presented a paper titled *Alternative Shapes of Spar Platforms for Use in Hostile Areas*[2] at the 1999 Offshore Technology Conference; within which an instability due to indirect coupling between heave and pitch was studied.

The instability was present because the pitch restoring term contains a term that is dependent on the distance between the centers of buoyancy and gravity. The center of gravity remains basically fixed to a point within the buoy; however, the center of buoyancy moves with the buoy's motion so it is always in the center of the submerged volume. In cases of large heave it is possible for the center of gravity to rise above the center of buoyancy making this part of the pitch restoring force negative. In extreme cases of heave it is possible for this term to become so negative as to make the pitch restoring term negative, making the buoy unstable in pitch.

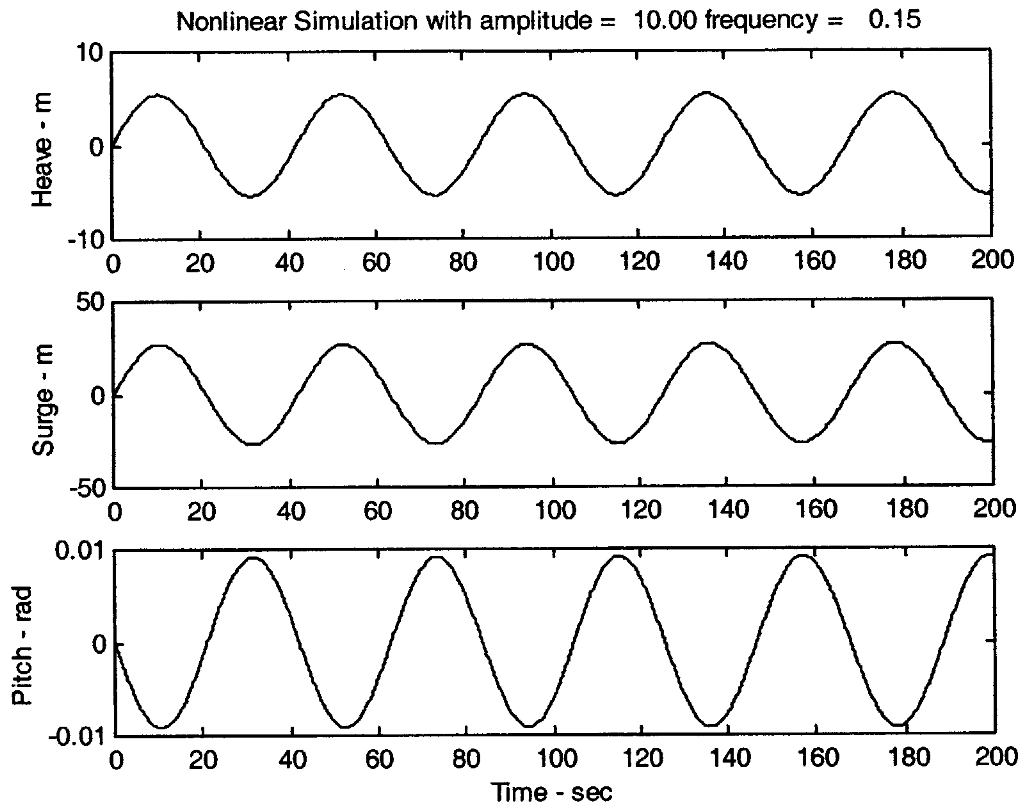

Figure 9 Nonlinear responses of freely floating buoy

Figure 9 shows the responses of the freely floating buoy with the pitch restoring term evaluated at the displaced heave position. The pitch instability is not evident in the simulated response even though the heave response is large enough for the pitch restoring term to become negative, as shown in Figure 10. This is due to the relative magnitude of the terms in the pitch equation of motion, each of which is at least one order of magnitude larger than the pitch restoring term.

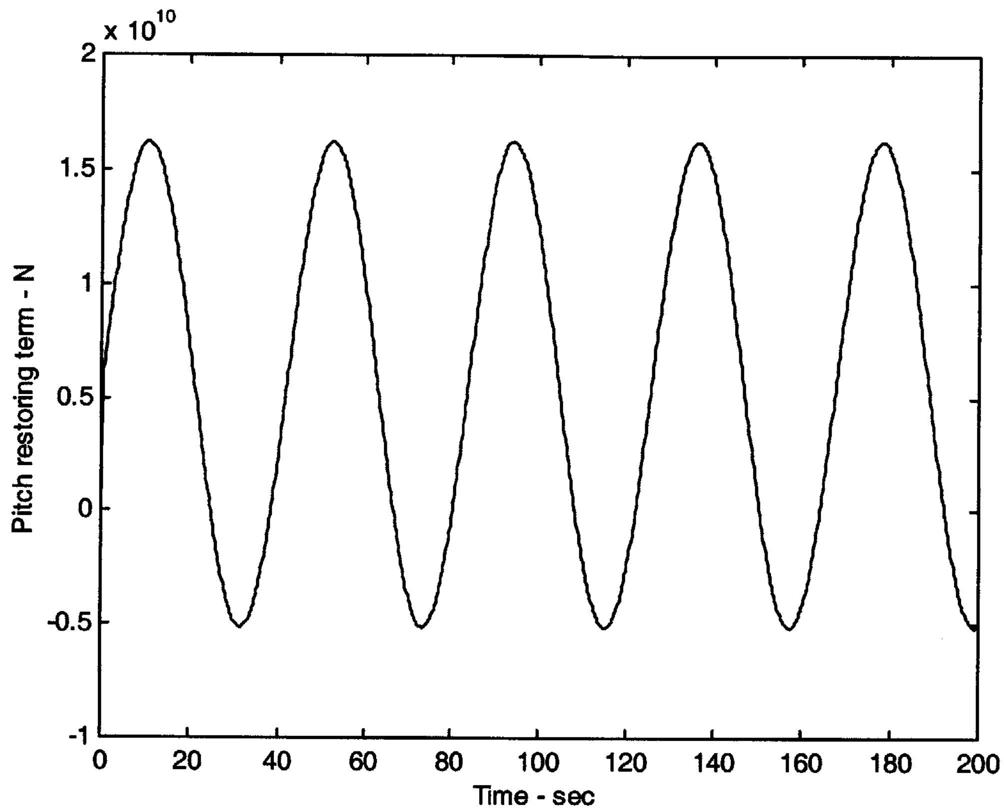

Figure 10 Pitch restoring term

Haslum and Faltinsen gave an equation for the excitation frequency, involving the natural frequencies in heave and pitch, which produces the pitch instability; this is given in Equation 4.1. Where $T_{N,pitch}$ and $T_{N,heave}$ refer to the natural periods in pitch and heave; these natural frequencies are $0.2209Hz$ and $0.0491Hz$ respectively, resulting in a $\omega_{critical} 0.2700Hz$.

Equation 4.1

$$T_{critical} = \frac{1}{\frac{1}{T_{N,pitch}} + \frac{1}{T_{N,heave}}}$$

Figure 11 shows the nonlinear responses at $\omega = 0.2700Hz$ as well as the value of the pitch restoring term. In this case the motions are small enough that the pitch restoring force never attains a negative value.

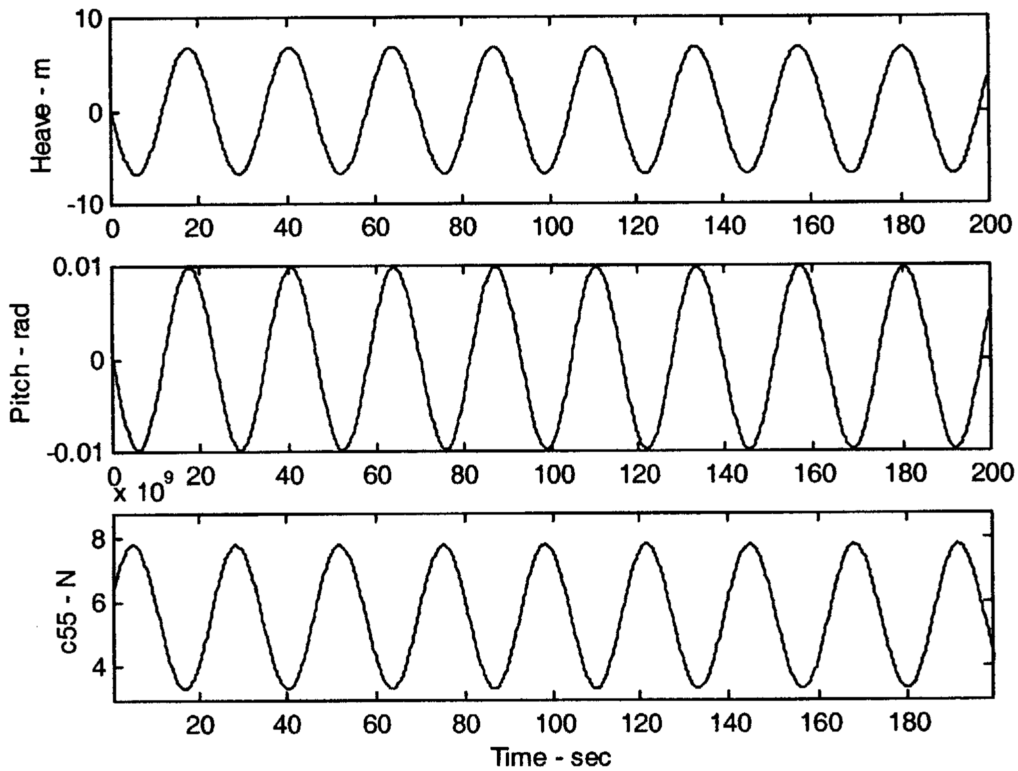

Figure 11 Nonlinear responses w = 0.2700Hz

Figure 12 shows the responses at $\omega = 0.2200Hz$. The heave response is very large because this value is close to the heave natural frequency. These large heave motions make the pitch restoring term (c55) follow the heave motion, reaching large negative values for nearly 50% of the time. These large negative restoring terms are still not enough to overcome the effects of the other terms in the pitch equation of motion.

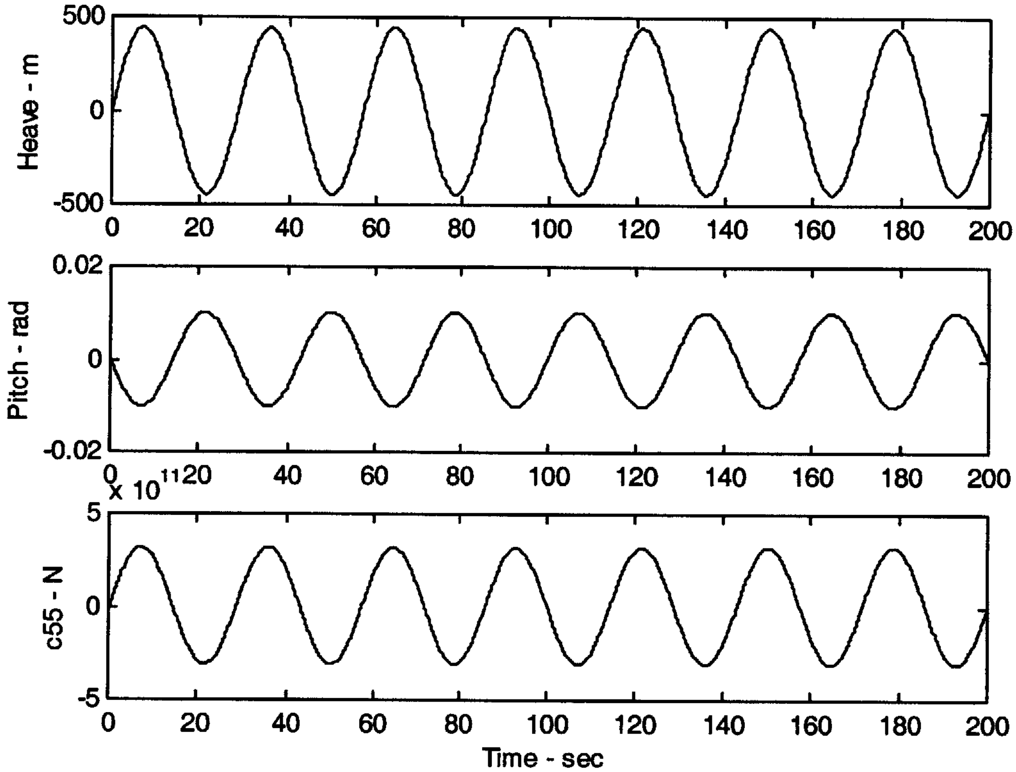

Figure 12 Responses with pitch restoring term evaluated at local wave elevation to excitation at w = 0.2200 Hz

As in the linear model the heave motion is so large that the buoy leaves the water; again this will be prevented by the mooring system.

Figure 12 shows that in the coupled simulation the evaluation of only the pitch restoring term at the displaced heave position does not produce the pitch instability that Haslum and Faltinsen studied.

The final nonlinear simulation evaluated all governing quantities at the local free surface. Again the simulation was run with a wave excitation at $\omega = 0.2700 Hz$ and $\omega = 0.2200 Hz$. These results are shown in Figure 13 and Figure 14. In Figure 13 the nonlinear behavior can be seen in both the surge and pitch motions; however, the magnitude of these motions is not increased over that of the linear simulation.

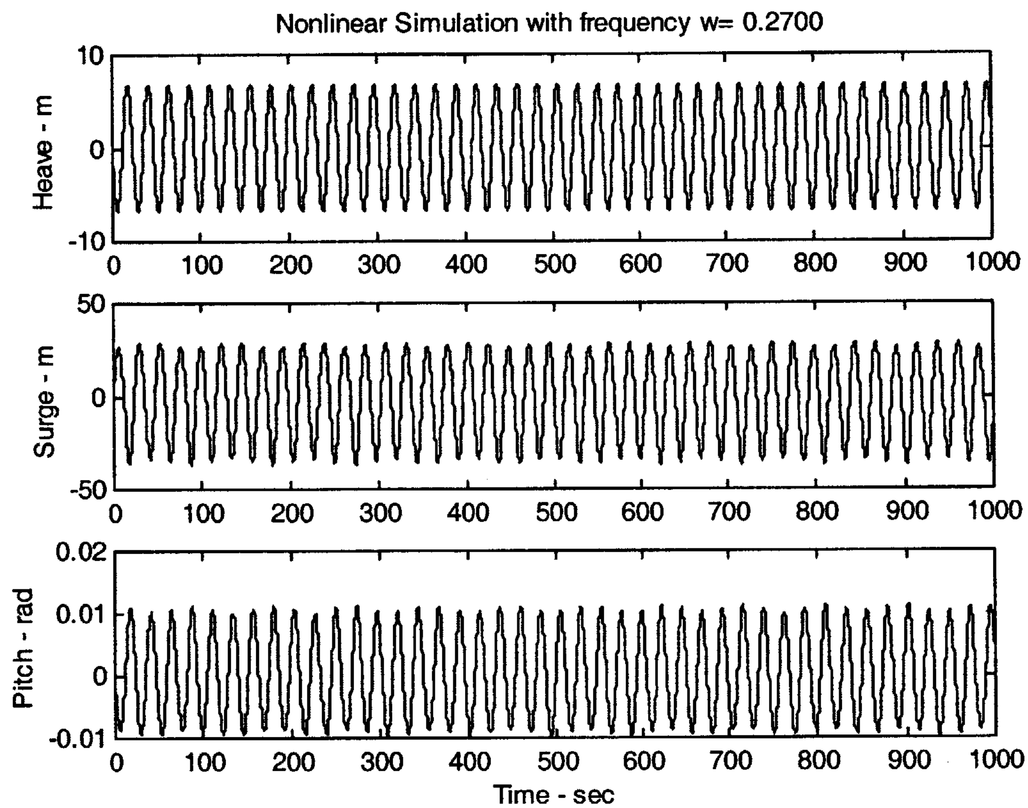

Figure 13 Nonlinear responses with all quantities evaluated at local wave elevation and excitation with w = 0.2700Hz

Figure 14 shows not only a more radical behavior but a pitch motion magnitude approximately double that of the linear simulation. The large heave motions are responsible for these large pitch motions as Haslum and Faltinsen indicated. Through evaluation of all the governing quantities at the local free surface a pitch instability was produced.

One item of interest is that the surge and pitch motions shown in Figure 13 are not symmetric nor are they centered around the origin, this is even more noticeable in Figure 14. The reason for this behavior is unknown and warrants further study of the nonlinear responses.

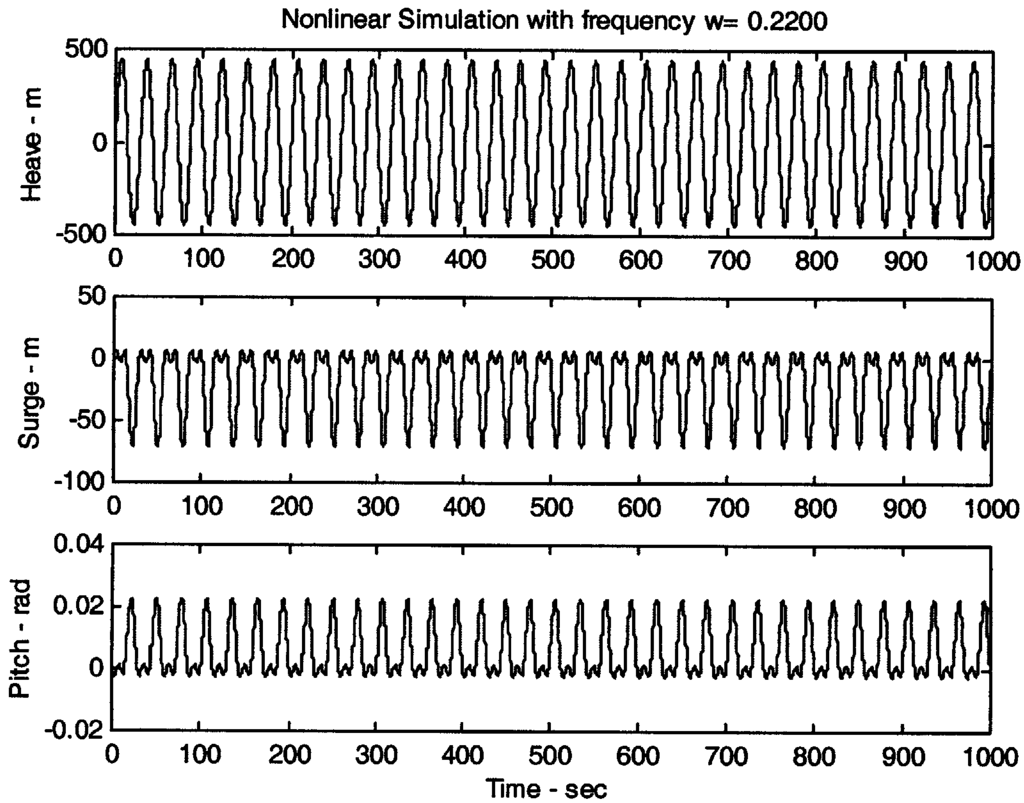

Figure 14 Responses with all quantities evaluated at local free surface elevation and excitation with w = 0.2200Hz

Nonlinear simulations were run with the twelve line mooring system also. These results are shown in Figure 15. The mooring system has greatly diminished the heave magnitude, dropping the maximum values from nearly 500m in Figure 14 to approximately 100m in Figure 15. The mooring system also limits the surge and pitch motions as well as smoothing the motion in both.

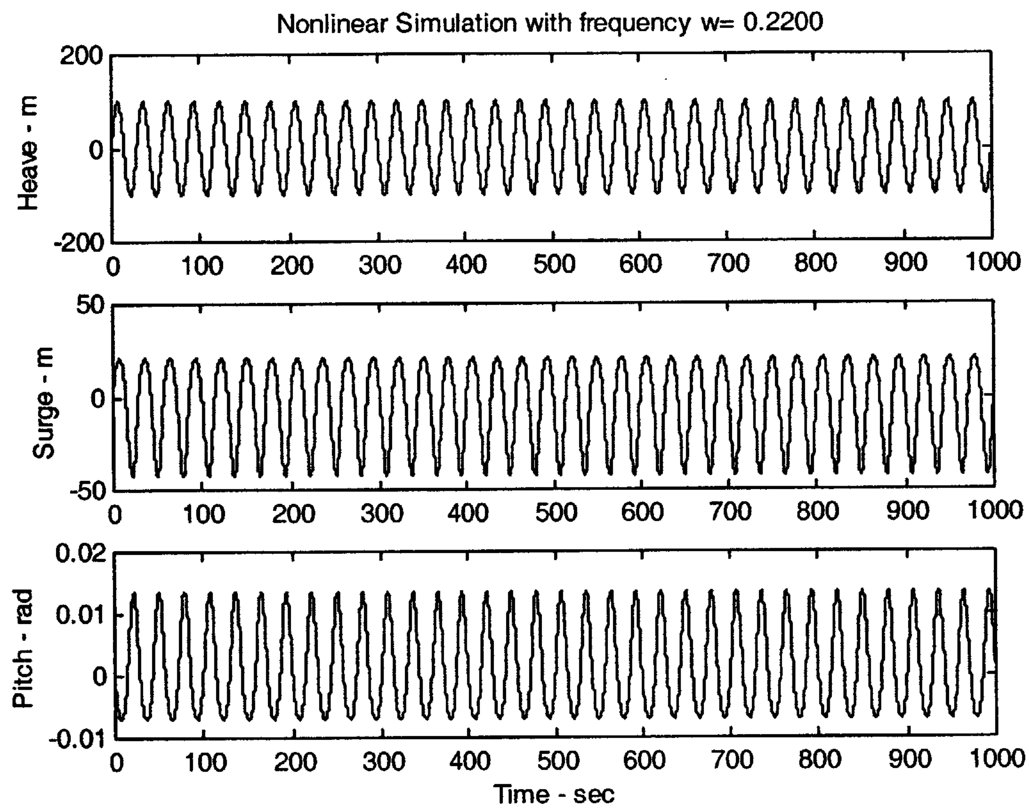

Figure 15 Nonlinear response to excitation of w=0.2200 Hz with a mooring system

Even with the mooring system in place to prevent the buoy fro heaving completely out of the water the pitch stability is still present. The magnitude of the pitch motion is approximately 0.0125 radians, compared with approximately 0.0095 radians in the linear model. Like the nonlinear unmoored simulation results, Figure 15 shows the surge and pitch motions to be centered around a non-zero position.

The random wave generator was added to the nonlinear simulation and was run with a wave composed with 200 plane progressive waves each with amplitudes ranging from $0-1m$, frequencies from $0.5-.45Hz$ and phases from $0-2\pi$. The responses with the mooring system in place are shown in Figure 16. In this trial the pitch instability is not evident. This is because there is not consistently large heave motion. Also, the surge and pitch motions do not appear to be biased to either side of the origin. The heave motion appears to diminish over the trial, this is just due to the random wave used, this behavior is not indicative of the behavior to all random waves.

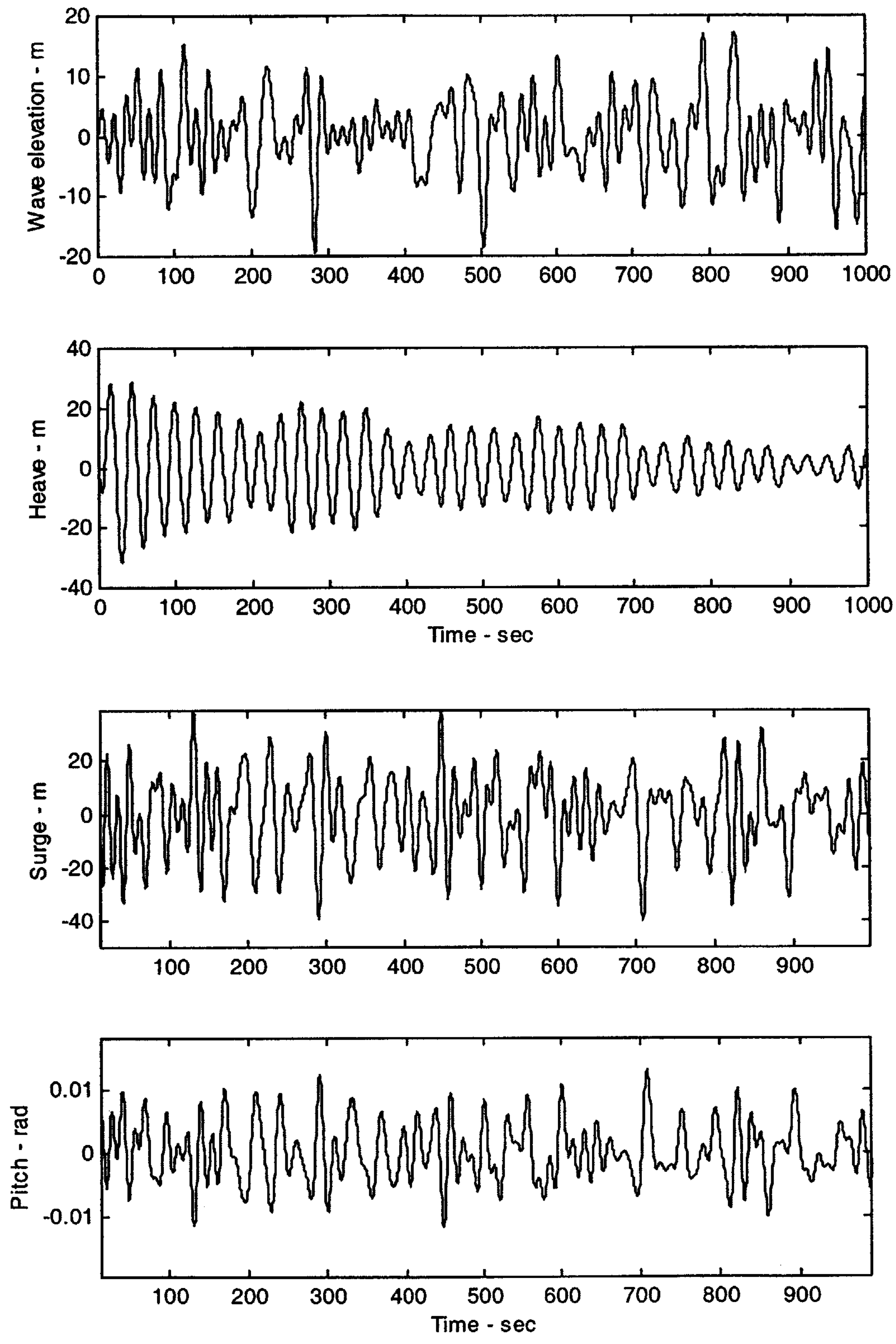

Figure 16 Nonlinear simulation with random wave excitation

Chapter 5 Conclusions

Computer based simulation of the motion of spar type oil platforms has been created to examine the three important modes of motion, surge, heave, and pitch. Linear simulation has been carried out with the addition of a realistic mooring system and the capabilities of examining the response to random waves.

Examples that examine the responses under a variety of circumstances have been presented for response comparisons, including response studies near the natural frequencies in heave and pitch. These examples showed that the heave motion of the buoy grows extremely large near the natural frequency.

Examples near the pitch natural frequency show that the coupling between surge and pitch is responsible for there being very little pitch motion at these frequencies.

The addition of the mooring system proved to have little effect on the motion of the buoy, except in extreme cases such as around the natural frequencies. This result was consistent with the motivation for using platforms of spar type design. This motivation being that the shape of the buoy is responsible for limiting the motions, not the mooring system.

A linear simulation was also created with a random wave, which is more representative of a true ocean wave. This showed the motions to be within ranges that were consistent with the linear single wave trials.

Additional simulations were carried out taking into account the effects of heave position on the pitch restoring force. These simulations showed the effect of this indirect coupling to be very small, not affecting the overall motions.

Simulations were carried out that evaluated all governing quantities at the local free surface elevation. In these trials the magnitude of the pitch motions were approximately doubled over those of the linear simulation. The addition of the mooring system to these simulations proved to limit these responses, resulting in over all motions of the magnitude seen in the linear trials.

Nonlinear simulations were also made with the mooring system and a random wave. The pitch instability was not present in these trials since the heave motion was random and not extreme. The overall magnitudes of these responses were consistent with the responses of the nonlinear, moored trials with sinusoidal excitation.

Future work on this topic should be done in the examination of platform responses to additional nonlinear effects. These include second order terms in governing equations.

Another important area for future work would be the addition of more complex spar buoy geometries. Many buoy designs have additional structures well below the surface to further lower the natural frequencies. These are important to model as are buoys with non-constant cross section.

Additional mooring system effects, such as drag a vibratory forces, should also be examined. A thorough study of mooring system configuration for optimum performance should also be included in this work.

Bibliography

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[2] Haslum, H. A. and Faltinsen, O. M., *Alternative Shapes of Spar Platforms for Use in Hostile Areas*, Offshore Technology Conference, 1999.