Remote Sensing of Submerged Objects and Geomorphology in Continental Shelf Waters with Acoustic Waveguide Scattering

by

Purnima Ratilal

Submitted to the Department of Ocean Engineering
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Abstract

The long range imaging of submerged objects, seafloor and sub-seafloor geomorphology in continental shelf waters using an active sonar system is explored experimentally and theoretically. A unified model for 3-D object scattering and reverberation that takes into account the coupling between propagation and scattering in a stratified medium is developed from wave theory. The conditions necessary for scattering in a waveguide to become diffuse is derived directly from Green's Theorem. Simulations with the unified model indicate that the detection of submerged target echoes above diffuse seafloor reverberation is highly dependent upon waveguide properties, bandwidth, array aperture, measurement geometry, and the scattering properties of the target. Analysis with the unified model shows that it is theoretically plausible for coherent returns from the scattering of evanescent waves by extended but discrete sub-bottom geologic features to stand significantly above diffuse or incoherent returns arising from small-scale roughness of the waveguide boundaries. A long-range bistatic sonar system was deployed in a field experiment and used to image extensive networks of buried river channels and inclined sub-seafloor strata over tens of kilometers in near real time. Such a capability is of great advantage in geophysical applications. Since buried river channels are expected to be ubiquitous in continental shelf environments, sub-seafloor geomorphology will play a major role in producing "false alarms" or clutter in long-range sonar systems that search for submerged objects such as underwater vehicles. A generalized extinction theorem for object scattering in a stratified medium is derived that can be applied to detect and classify objects from the total field in the forward scatter direction in a waveguide. Analytic expressions are derived for the attenuation and dispersion in the forward propagated field due to scattering from random surface and volume inhomogeneities in a waveguide. The unified model is applied to show that the active sonar equation is not in general valid for scattering in a waveguide. It is shown that the sonar equation may be made approximately valid in a waveguide by lowering the active frequency of operation sufficiently for the given
measurement scenario to simplify analysis for target classification and localization.

Thesis Supervisor: Nicholas C. Makris
Title: Associate Professor
Executive Summary

The long range imaging of submerged objects, seafloor and sub-seafloor geomorphology in continental shelf waters using an active sonar system is explored experimentally and theoretically. In this problem, the sound field from the source travels hundreds of water-column depths in range to an object to be imaged. Due to frequent interaction with the waveguide boundaries along the propagation path, the field incident on an object is comprised of multipath arrivals or multiple modes. These arrivals are each coherently scattered from the object and then coherently superposed after additional multipath or multi-modal propagation to a distant receiver. The coupling between propagation and scattering then must be taken into account in the analysis of long range imaging in continental shelf environments.

A unified model for 3-D object scattering and reverberation in a stratified medium is developed from wave theory. The unified model provides a consistent approach for analyzing scattering from deterministic and stochastic scatterers. The conditions necessary for scattering in a waveguide to become diffuse is derived directly from Green's Theorem. An advantage of the unified approach is that it enables quantitative predictions to be made of the target-echo-to-reverberation ratio in an ocean waveguide. Simulations with the unified model indicate that the detection of submerged target echoes above diffuse seafloor reverberation is highly dependent upon water column and sediment stratification, bandwidth, array aperture, measurement geometry, and the scattering properties of the target and seafloor. Analysis with the unified model shows that it is theoretically plausible for coherent returns from the scattering of evanescent waves by extended but discrete sub-bottom geologic features to stand significantly above diffuse or incoherent returns arising from small-scale roughness of the waveguide boundaries. This implies that it is possible to image sub-bottom features from long range using an active sonar system.

The unified model was applied to optimise the design of a multi-national and multi-disciplinary acoustic field experiment sponsored by the Office of Naval Research and conducted in the New Jersey Continental shelf from April 27 to May 5, 2001. A
long-range bistatic sonar system was deployed and used to image extensive networks of buried river channels and inclined sub-seafloor strata over tens of kilometers in near real time. Such a capability is of great advantage in geophysical applications to rapidly map extended geomorphology over large spatial areas. Since buried river channels are expected to be ubiquitous in continental shelf environments, sub-seafloor geomorphology will play a major role in producing "false alarms" or clutter in long-range sonar systems that search for submerged objects such as underwater vehicles or marine mammals since returns from these geologic features can be confused with or camouflage those from the intended target.

The total power removed from an incident field as a result of scattering and absorption by an object is called extinction. Extinction is a consequence of shadow formation from destructive interference between the incident and scattered fields in the forward direction that survives as an active shadow remnant in perpetuity beyond the deep shadow. For planewave incidence in free space, the extinction caused by the object is proportional to the projected area of the object in the high frequency limit and so provides a robust method for estimating object size from incoherent power measurements. A generalized theorem for the extinction of the multi-modal field incident on an object in a stratified waveguide is derived. This theorem can be applied to detect and classify intruding objects that pass between an acoustic source and receiving array in shallow waters. Analytic expressions are also derived for the attenuation and dispersion in the forward propagated field due to scattering from random surface and volume inhomogeneities of arbitrary-size in a waveguide. This result can be applied to study the effect of scattering from internal waves and seafloor roughness in the forward propagated field in a waveguide.

The sonar equation is the most widely used tool for analyzing acoustic data in the ocean. The sonar equation is only valid when propagation and scattering effects are completely factorable from eachother. We show that the sonar equation is in general not valid for scattering in a shallow water waveguide and can lead to large errors and inconsistencies in estimating a target's scattering properties as well as it's limiting range of detection. By application of the unified model, a coherent
waveguide scattering theory, the conditions necessary for the sonar equation to become approximately valid in a shallow water waveguide are derived. These conditions are expressed in terms of the wavelength, target size and shape, and the waveguide properties. It is shown that the sonar equation may be made valid in a waveguide by lowering the active frequency of operation sufficiently for the given measurement scenario. This is often desirable because it greatly simplifies the analysis necessary for target classification and localization.
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Jai Sai Ram!
Dedicated to

my parents
Ratilal Chandulal and Pushpaben

and my Guru (spiritual guide)
Bhagawan Sri Sathya Sai Baba and Mata Amritanandamayi (Amma)
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Chapter 1

Introduction

1.1 Motivation

We explore the use of acoustic waves for rapidly imaging wide areas of the ocean environment. Acquiring information in the oceans is a challenging task making it a territory yet to be fully explored. For instance the bathymetry in much of the Earth’s oceans is only known to about 5 to 11 km resolution. In comparison, the surface elevation of heavenly bodies like our moon has been measured to at least 30 m resolution with electromagnetic waves. In the atmosphere, electromagnetic waves provide a very efficient means of imaging objects and acquiring information about vast areas of the environment. An example is the satellite imaging of the Earth surface with visible light. In water, electromagnetic waves are strongly attenuated and so do not travel far. Sound waves on the other hand travel thousands of kilometers in the oceans with very little attenuation at low frequencies. Low frequency sound waves are therefore our means for probing wide areas of the ocean. We explore the use of a low frequency active sonar, like an “eye” underwater, to rapidly image, detect and classify wide areas of the ocean environment. Our objects of interest include seafloor and sub-seafloor geomorphology, schools of fish, marine mammals, underwater vehicles like submarines, offshore structures and wreckages. The application for such a technology is immense. For instance, the present standard method for geophysical exploration involves combing the area of interest in a “lawn-mower” pattern where only data
within tens of meters of the survey vessel is profiled. It would take decades for present standard methods to acquire geologic data that covers all of the earth's ocean to finer resolution in the order of tens of meters. In naval operations, the ability to image and classify objects in the ocean from long ranges is vital for national defense. In marine mammal studies, such a technology can be applied to track and monitor populations of marine mammals over wide areas of the oceans.

In order to interpret and utilize images acquired underwater, and to further optimize imaging for a given application, it is essential to understand the physics of the imaging process. In this thesis, we conduct both theoretical modelling and field experiment to study, understand and test the process of image formation underwater using acoustic waves. The field scattered from all directions around the object, including the forward, will be studied and exploited for imaging and classification in the ocean.

Active systems are used for imaging, detecting and localizing objects that are non-radiating. An active system consists of a source that transmits an incident field that propagates to the object to be imaged, detected or localized. The object scatters the incident field that then propagates to a receiver. When imaging in the atmosphere or free space, the incident wave from the source to the object and the scattered wave from the object to the receiver follows a direct path. When an active system operates in a waveguide, the restriction imposed by the boundaries of the waveguide leads to effects that are not present in free space or in a halfspace. For instance, the long range propagation of signals from a point source in a waveguide is best described using orthogonal modes made up of standing waves in the vertical that travel in the horizontal. In an isovelocity layer, the modes can be decomposed into planewaves that propagate with discrete elevation angles. This is unlike in free space where the disturbance from a point source propagates with spherical symmetry. To understand the implications for scattering, it is convenient to decompose the incident field at an object in an iso-velocity layer, from the far field point source, into modal planewaves. Each incident planewave arrives with a specific elevation angle from the azimuth of the far field source and then scatters coherently from the target into outgoing planewaves.
in all elevation and azimuthal angles. At a far field receiver, the scattered field is the coherent sum of all scattered planewaves from all incident planewaves. The multimodal structure of waves in a waveguide causes propagation and scattering to be coupled. This fact must be taken into account in long range imaging analysis in a shallow water waveguide.

Unifield Model for Bistatic 3-D Reverberation and Object Scattering in a Stratified Medium

A common problem in the active imaging, detection and localization of an optical, radar or sonar target arises when scattered returns from the target become indistinguishable from returns from randomly rough boundaries, volume inhomogeneities, or other deterministic features or objects within the ocean environment. One goal of this research is to investigate the extent to which environmental reverberation limits the ability to image, detect and localize a target in an ocean waveguide, where methods developed for the radar half-space problem are inapplicable due to the added complications of multi-modal propagation and dispersion.

To this end, a unified model for 3-D reverberation and submerged target scattering in a stratified medium is developed from wave theory[48]. The model is fully bistatic and stems directly from Green's theorem based on the idea that the harmonic scattered field from an object can be expressed in terms of the waveguide Green function and the objects planewave scatter function[31, 46]. We provide analytic expressions for the 3-D field scattered bistatically by both stochastic and deterministic objects from a source with arbitrary time function, as well as the associated spatial and temporal covariances. These enable realistic modeling of the moments of the raw reverberant field received over extended spatial and temporal apertures as well as the output after subsequent processing with standard beamforming and broadband signal processing techniques.

While the unified model is consistent with certain narrowband results of previous "heuristic"[16] derivations [16, 6, 77, 37] for shallow water reverberation measured
with an omni-directional receiver that are based on the work of Bucker and Morris[6], it offers more insight and generality since it is developed from first principles with explicitly stated assumptions. For example, it clearly obeys reciprocity for source-receiver locations within a layered media, which is important in properly modeling the absolute level of returns from targets or surfaces within the seafloor, and it allows absolute comparison between reverberation and deterministic target returns. Such comparison led to inconsistencies in previous formulations as noted in Ref. [16].

We provide a method for simulating real sonar systems by putting the modelled reverberation through a beamformer and looking at the arrivals that are contained within the resolution footprint of the sonar system. This is achieved with the application of Parseval's Theorem when the integration time of the measurement system is sufficiently long to include the the dominant energy returned from the target or scattering patch. It allows analytic expressions for the statistical moments of the scattered field to be obtained directly. These moments can also be obtained numerically by sample averaging over realizations by Monte Carlo simulations, as for example is done for rough surface scattering in Ref. [68]. The relative merit of either approach depends on the relative difficulty in evaluating the analytically obtained moments or performing the Monte Carlo simulations for the given problem. The analytic approach has proven to be more advantageous and insightful for the illustrative examples of the present work.

The primary motivation for developing the unified model is to compare the absolute level of target echo returns with those from the seafloor and to investigate how these vary in both absolute and relative level as a function of water column and sediment stratification, receiving array aperture, and source, receiver and target locations in a shallow water waveguide. Another major focus of this research is to investigate the manner of scattering from both extended geomorphologic features and randomly rough patches of the seafloor and sub-seafloor. The latter typically makes up the incoherent or diffuse reverberant background that has an expected intensity that decays in time after the arrival of the direct signal waveform. The former typically leads to deterministic geological scattering, which is defined as any set of acoustic returns
from the seabed that stand significantly above the diffuse and temporally decaying reverberation background. Deterministic scattering from extended geologic features produces “false-alarm” or clutter in long-range active sonars in shallow water that search for submerged objects like an underwater vehicle or marine mammal. This is because the returns from geologic features can be confused with or camouflage returns intended from the submerged object. Deterministic scattering from geologic features can, however, be utilised for imaging in geophysical applications. What constitutes reverberation is therefore application specific.

A goal of this research is to determine plausible physical mechanisms for diffuse and deterministic scattering from the ocean environment by use of the unified model. To do so, both first order perturbation theory and empirical Lambert-Mackenzie[39] models are used to describe stochastic scattering from a randomly rough seafloor. Only the empirical Lambert-Mackenzie model is used in comparisons between diffuse seafloor reverberation and submerged-object returns since insufficient data on the requisite environmental parameters at low frequency are available to make a similar comparison with perturbation theory meaningful. Perturbation theory calculations are only used self-consistently to make inferences about the relative level of returns from different kinds of seafloor scatterers. These models, together with deterministic models for scattering from extended seafloor features, are used to determine scenarios where geologic scattering is significant. The present focus is on seafloor features with mean surfaces that are finite and inclined with the reflection properties of the layer to which they belong. These can be used to model the walls of buried river channels that are ubiquitous in continental shelf waters. While a large literature exists for scattering from 2-D features in a waveguide, the focus of the present work is on scattering from 3-D features in a waveguide. Apparently, the only previous work on deterministic scattering from 3-D seafloor features in a waveguide has been for acoustically compact ($ka \ll 1$) proturbances on perfectly reflecting bottoms[76], but compact targets are too weak to comprise geological clutter in a long-range active sonar system and are not relevant to the present analysis.

The unified model developed here is a generalization of Ingenito’s approach for
harmonic scattering from deterministic objects in a stratified medium by incorporating stochastic scatterers, diffuse reverberation, and time-dependent sources. A review of the general literature on 3-D scattering in an ocean waveguide is given in Ref. [42]. Here we summarize the points noted in that review. Prior to Ingenito's 1987 paper, work on scattering in the ocean waveguides were focussed on "adapting the T-matrix method to the waveguide problem. The most comprehensive work of this period is that of Hackman and Sammelmann[24], who developed a modal solution that includes the effect of multiple scattering between object and waveguide boundaries, but which requires use of the free-field Green function and is restricted to media with constant sound speed layers. As Hackman and Sammelman later note[25], Ingenito's use of the waveguide Green function greatly simplifies the problem when multiple scattering between the object and waveguide boundaries can be neglected. Moreover, Ingenito's single-scatter model[31] fully accounts for waveguide propagation effects, such as multiple reflections of the scattered field between waveguide boundaries, because it is based upon the waveguide Green function. As a result, multiple images of the object will appear at the receiver. However, there will be no re-scattering between these multiple images in the single-scatter model because it employs the free space scatter function for the object. The single-scatter approximation is typically valid when the object is not too close to the waveguide boundaries, relative to its scale and the wavelength[31, 46, 24, 25, 10, 60, 61, 56], and so is useful in many practical scenarios."[42]

The spectral or wavenumber integration formulation adopting Ingenito's approach for waveguide scattering was first developed by Makris, Ingenito and Kuperman in their 1994 paper[46] to "investigate the plausibility of detecting and localizing submerged objects by their perturbation of an ocean waveguide's ambient noise field."[42] An advantage of the spectral approach is that it enables the scattered field to be computed at closer ranges to the object than the modal formulation. The wavenumber integration approach was later applied by Schmidt and Lee in 1999 to investigate scattering from buried mines[68]. Perkins, Kuperman, Tinker, Heaney, and Murphy have employed Ingenito's modal approach to treat the problem of scattering from a
prolate spheroid in a deep ocean waveguide[60, 61]. A great practical advantage of the modal formulation of Ingenito and the wavenumber integration formulation of Makris et al. for scattering in a waveguide is that “it can be easily implemented by straightforward modification”[42] of existing normal-mode or wavenumber integration propagation softwares respectively.

A parabolic equation approach for 3-D object scattering in the ocean was described by Collins and Werby[10]. “The advantage of this approach is its ability to handle range-dependent waveguides. A primary disadvantage is that the 3-D PE must generally be used to properly handle diffraction about the object so that the entire 3-D field from the object must be marched to the range of the receiver even if the receiver is isolated at a single point in range, depth and azimuth. Additionally, Perkins et al.[60, 61] have used the adiabatic approximation to extend Ingenito’s modal approach for 3-D object scattering to weakly range-dependent waveguides” that is “valid when range from the source and receiver to the object is much greater than the waveguide depth.”[42]

**Effect of Environment, Measurement Geometry and Sonar Parameters on Imaging in a Shallow Water Waveguide**

We apply the unified model to investigate the possibility of remotely imaging extended sub-bottom features using low frequency acoustic waves. The model simulations show that coherent scattering from smooth but finite segments of inclined sub-seafloor strata, corresponding to buried river channels, can produce returns that stand significantly above diffuse or incoherent returns arising from small-scale roughness from the waveguide boundaries[48]. The energy returned from a coherent scatterer is proportional to the square of its projected area while that from an incoherent one is only proportional to its projected area, for scatterers large compared to the wavelength. Optically the former would correspond to “glints” from a polished surface while the latter to diffuse radiance from a matte surface such as paper. Along these lines, past experience from deep-water remote sensing suggests that geologic features are
likely to appear prominently in wide-area acoustic images when they project large area in the direction sound travels over the sonar resolution footprint\cite{43, 45, 8} We provide conditions for coherent returns from objects submerged in the water column or buried in the sediment to stand above diffuse returns from seafloor roughness\cite{48}. In the latter case, scattering of the evanescent component of trapped modes is found to be a primary mechanism for the imaging of sub-bottom features at long range. The evanescent component passes through the sediment by “tunneling”\cite{18} rather than propagating, where the level of the tunneled field typically falls off exponentially with depth below the water-sediment interface. Features buried closer to this interface are then expected to yield stronger returns.

The unified model results are used to calculate signal-to-diffuse-reverberation ratios (SRR) and subsequently the probabilities of detection and false alarm for submerged targets and geologic features in shallow waters. We show the effect of bandwidth on target detectability\cite{66}. We find that broadband measurements help reduce range and depth dependent variations in the scattered field from a submerged target caused by coherent interference between the waveguide modes. This in turn leads to more stable detection over range compared to narrowband measurements. Broadband signals also enhance detection of coherent scatterers over diffuse reverberation. Match filtering with coherent signals leads to better range resolution via a smaller sonar resolution footprint, less reverberation per footprint, and hence more reverberation rejection per footprint. It also leads to rejection of out-of-band additive ambient noise and supression of in-band noise. Higher SRR and signal-to-noise ratios (SNR), and longer detection ranges then can be achieved for coherent scatterers with broadband waveforms. The implication for active sonar systems that search for submerged objects like an underwater vehicle is that while the broadband waveform enhances the SRR and SNR of returns from the submerged object, it also enhances the coherent geologic clutter returns. Therefore broadband signals help mitigate diffuse reverberation but it leaves the sonar operator with enhanced geoclutter returns to deal with. The unified model is also applied to investigate the scattering from schools of fish in shallow waters.
Field Experiment to Remotely Image Sub-bottom and Seafloor Geomorphology in Shallow Water

A long-range bistatic sonar system was used to rapidly image geomorphology over wide areas of the New Jersey continental shelf south of the Hudson Canyon in a field experiment from April 27 to May 5, 2001[64, 49, 57]. The system consisted of a horizontally towed receiving array and two low-frequency vertical source arrays. Source signals were transmitted over hundreds of water-column depths in range to image sub-seafloor and seafloor geomorphology. Waveguide scattering [48, 42, 31, 46], and propagation[33] are therefore inherent to this new remote sensing technology.

The primary purpose of the field experiment, known as the Geological Clutter Acoustics Experiment, a part of the Office of Naval Research Geological Clutter Program[57], was to test the hypothesis that long-range, low-frequency active sonar systems can rapidly image sub-seafloor geomorphology over wide areas in continental shelf environments. This hypothesis has been proposed by a number of investigators to explain anomalous sonar returns in areas of level bathymetry [69, 50]. Theoretical work done in this thesis using the unified model had shown this hypothesis to be highly plausible as described above[48].

Other objectives of the experiment were to (1) understand the physical mechanisms that lead to prominent geologic returns in long range active sonar, (2) identify categories of seafloor and sub-seafloor geology that lead to prominent returns, (3) analyze the bistatic scattering properties of these natural features.

Prior to the experiment, a number of substantial 3-D geophysical surveys had previously characterized seafloor and sub-bottom features over wide areas in the New Jersey Stratafrom site, making it well-suited for the present acoustics experiment [14, 21, 13]. Data from these surveys were used to identify natural features of the seafloor and sub-seafloor that might stand out prominently in wide-area images produced by our remote acoustic sensing system. These include buried river channels, iceberg scours, surface erosional features and subsurface strata. Two acoustic targets[72], essentially cylindrical air-filled elastic tubes, with known scattering prop-
erties and locations were used to calibrate returns from the geologic features and confirm theories about waveguide scattering from extended objects and long-range imaging in continental shelf environments. Simulations using the unified model were used extensively to guide the design of the experiment.

During the Geoclutter Acoustics experiment two research vessels were used to acquire both monostatic and bistatic scattering data. Roughly 3000 waveforms were transmitted into the water column from vertical source arrays in the 390 to 440 Hz band and received by a horizontal towed array. A wide area image of the environment was generated for each transmission. On average between 10 to 100 discrete and localized scatterers were registered per transmission. From a geological clutter perspective, this gives a total of at least 30,000 scattering events that could be confused with that from a large submerged vehicle over the period of the experiment. In this context, geological clutter refers to scattering from geomorphological features of the seafloor and subseafloor that stand prominently above the diffuse and temporally decaying reverberation background.

Bathymetry throughout the New Jersey Strataform area is extremely benign with few discrete features. The vast majority of prominent and discrete scattering events returned from level areas where absolutely no bathymetric features are present in high resolution (30-m sampled) bathymetry data. In many cases, the scattering events registered well with buried river channel networks and other sub-bottom features characterized by previous geophysical surveys related to the Geoclutter Program [14, 21]. A large number of bistatic insonifications of areas infested with buried river channels revealed that the number and level of prominent scattering events is highly dependent on bistatic orientation. Registrations with known channels suggest that bistatic aspects that produce coherent “glints” from river channel walls with large projected area yield the strongest and most frequent target-like returns. Stronger returns were found to be correlated with less deeply buried channels. The level of returns was also found to be dependent on source and receiver depths, as well as sound speed structure. These measured results are consistent with those predicted by a unified model for reverberation and submerged-target scattering[48].
Wide area images acquired remotely during the Geoclutter Acoustics experiment have been used to identify the location and extent of previously unknown sub-bottom geomorphology whose existence was confirmed by subsequent geophysical surveys. An organized network of strong and highly aspect-dependent returns appeared in a region of flat bathymetry, where the sub-bottom had not yet been profiled[64, 49]. The returns were often stronger than and just as discrete in appearance as those from the calibrated targets deployed at similar range from the sonar system. Without prior knowledge, it would be impossible to distinguish the targets from these features. Subsequent geophysical surveys specifically designed to explore the sub-bottom in this area[1] revealed a large network of buried river channels at the location of these prominent acoustic features. In addition to discrete features, prominent lineated features were consistently imaged by the long-range sonar in regions where inclined sub-bottom strata approached the water-seabed interface. Much more prominent discrete features also appeared on these lineations in regions where buried river channels crossed the near surface expression of the sub-bottom layer. These findings show that a low-frequency sonar system can be used to remotely image previously unknown sub-bottom features over wide areas, tens of kilometers, in continental shelf waters in near real time.

In order to condense the information contained in data from large numbers of transmissions along a track-line of the receiver ship, a “hotspot” consistency chart was derived from the images of individual transmissions along each track. The hotspot consistency chart images the location of strong and persistent echo returns for transmissions along a track. In many instances, the hotspot charts provided significantly improved imaging of sub-bottom features such as river channel axes. They are also a useful tool for resolving the right-left ambiguity inherent in horizontal line array data when the bistatic range to the target is not much larger than the source-receiver separation. The hotspot charts also reduce charting errors due to waveguide dispersion since they combine information from a large number of transmissions.

Future work under the Geoclutter Program will involve correlating the long range scattered field from buried river channels with their morphology and to develop tech-
niques to estimate the shape, size and orientation of these channels from their scat-
tered fields. Estimating shape from shading is a topic that has been addressed in
free space imaging[27, 29] where the incident and propagated fields travel in straight
paths and in the deep oceans where one single incidence and scattered angle at the
target can be defined[43, 45, 8]. In a waveguide the shape and orientation information
is contained in the scatter function of the object that is convolved with propagation
in multi-path or multi-modal scattered fields, making this task of estimating shape
from shading challenging in a waveguide.

**Extinction Theorem for Scattering in a Waveguide applied to Object Classification**

Imaging is commonly achieved using the field scattered from an object in non-forward
directions. The total field in the forward scatter direction of the object forms a silhou-
ette that can be utilised to extract properties of the object useful for classification.

If an object is placed in the path of an incident wave, some of the intercepted power
is scattered in all direction and the remainder is absorbed. The total power removed
from the incident field as a result of scattering and absorption by the object is called
extinction[4]. Van de Hulst has shown, in what has become known alternatively as
the extinction theorem, optical theorem and forward scatter theorem, that the total
extinction of a planewave incident on an object in free space equals the imaginary part
of the forward scatter amplitude multiplied by the incident intensity and $4\pi/k^2$ where
$k$ is the wavenumber[74, 67, 75]. This remarkably simple relationship reflects the fact
that the extinction caused by the obstacle leads to shadow formation via destructive
interference between the incident and forward scattered fields. The permanance of
the extinction is maintained by the formation of a region of destructive interference
that survives as an active shadow remnant[67] in perpetuity beyond the deep shadow.

It should be stressed that “the actual extinction process is not a blocking of the
wave but a subtle interference phenomenon. The scattered wave removes some of the
energy of the original wave by interference.”[75]
Extinction measurements usually involve integrating the intensity of the total field over a screen sufficiently large to cover the full shadow remnant. The difference between the power intercepted on the screen with and without the object present is the extinction. This is applied, for instance, in optical astronomy where a telescope is used to measure the extinction of a distant star by a meteorite. For a telescope at range $z$ from a meteor, "the telescope will register the full extinction only if its diameter is much larger than $\sqrt{z\lambda}$. With increasing $z$, the linear size of the active area increases as $\sqrt{z}$. So the forward scattered wave has an ever weaker influence extending over ever wider circles in such a way that the total energy suppressed remains constant" [75] and is equal to the extinction.

The total power scattered by an object can be found by integrating the scattered intensity over a large control surface enclosing the object in the far-field. This integration is usually difficult to perform and makes an alternative approach attractive. For non-absorbing objects, the total power scattered by the object is the extinction [73, 53]. One great advantage of the extinction theorem is that it eliminates the need to integrate the scattered energy flux around the object.

The extinction theorem is typically applied to measure the extinction cross section of objects [75, 70]. This equals twice the object’s projected area in the high frequency limit, and so provides a useful method for estimating an object’s size from the incoherent power measurements. The free space extinction theorem is used in astronomy to estimate the size of meteorites from the extinction they cause to the light emanated by distant stars [75]. "This noteworthy paradox, that a large particle removes from the incident beam exactly twice the amount of light it can intercept, has attracted attention from various sides. Its paradoxical character is removed if we recall the exact assumptions that have been made in its derivation. They are (a) that all scattered light, including that at small angles, is counted as removed from the beam, and (b) that the observation is made at a very great distance, i.e., far beyond the zone where a shadow can be distinguished. A flower pot in a window prevents only the sunlight falling on it from entering the room, and not twice this amount, but a meteorite of the same size in interstellar space between a star and one of our big telescopes will
screen twice this light.”[75]

The extinction theorem has many diverse applications in acoustics, such as those given in Refs. [71] and [51]. It can also be used as a “burglar alarm” to detect and classify intruding objects that pass between a source and an acoustic receiver array.

In 1985 Guo [23] extended the extinction theorem to scattering by an object located next to an interface between two different acoustic halfspaces. He found an expression for the extinction of the incident planewaves in terms of the far-field scattered pressures in the specular reflection and transmission directions, determined by Snell’s law. In a waveguide, the effect of multi-modal propagation ensures that the field incident on the object will arrive from many incident elevation angles. Also, the effect of absorption loss by the medium in real waveguides will modify the extinction and subsequently the cross-section for scattering in the waveguide. The free space extinction theorem and the half-space extension of Guo are therefore not applicable in a waveguide.

Here we use wave theory to derive a generalized extinction theorem by developing a relation for the rate at which energy is extinguished from the incident wave of a far field point source by an object of arbitrary size and shape in a stratified medium. Like its free space analogue, the relation is again remarkably simple. The total extinction is shown to be a linear sum of the extinction of each waveguide mode. Each modal extinction involves a sum over all incident modes that are scattered into the given mode and is expressed in terms of the object’s planewave scatter function in the forward azimuth and equivalent modal planewave amplitudes. For the multiple incident planewaves in a waveguide, extinction is a function of not only the forward scatter amplitude for each of the incident planewaves but also depends on the scatter function amplitudes coupling each incident planewave to all other planewaves with distinct directions that make up the incident field. The final relation greatly facilitates extinction calculations by eliminating the need to integrate energy flux about the object.

Simulations in a shallow water show that an object’s cross section for the combined extinction of all the modes of the waveguide is highly dependent on measurement ge-
ometry, medium stratification, as well as its scattering properties. In addition, the combined cross section fluctuates rapidly with range due to coherent interference between the modes. The presence of absorption in the medium can also significantly modify a measurement of the total scattering cross section. The practical implication of these findings is that experimental measurements of the total scattering cross section of an obstacle in a waveguide may differ greatly from those obtained for the same obstacle in free space and may lead to errors in target classification if the waveguide effects are not properly taken into account.

For an object submerged in a waveguide, we also define modal cross sections of the object for the extinction of the individual modes of the waveguide. The modal cross section of an object for the extinction of mode 1 in a typical ocean waveguide was found to be nearly equal to the free space cross section of the object. A potential application of the extinction theorem in a waveguide is then the estimation of the size of an object submerged in the waveguide from a measurement of the extinction it causes to mode 1.

Diffuse Backscattering, Decorrelation of Waveguide Modes, and Forward Scattering from a Large Number of Random Scatterers

The conditions necessary for the scattering from a large number of stochastic scatterers to become diffuse in non-forward scattering is derived directly from Green's Theorem. In diffuse or incoherent scattering, decorrelation of the waveguide modes occurs which simplifies the expression for the intensity of the total scattered field. When these conditions are not met for a patch of seafloor, the scattering from the seafloor patch becomes coherent and leads to the formation of "reverberation rings".

The mean field scattered from the distribution of random scatterers in the forward azimuth in a waveguide is also derived. Analytic expressions are then obtained for the forward propagated field in a waveguide, the attenuation coefficient, and dispersion
due to scattering from arbitrary-sized surface and volume inhomogenities. This results
can be used to determine the attenuation due to volume and surface scattering of
guided waves propagating through stratified media such as the ocean or the earth's
crust. It can also be applied to study the effect of scattering from internal waves in
forward propagation.

Validity of Sonar Equation for Scattering in a Stratified Medium

The sonar equation is the most widely used analytical tool in applications of active
sonar.[73, 11, 33, 7, 35] It is typically employed to estimate a target's scattering
properties and limiting range of detection[73]. The sonar equation, in its active form
for an omni-directional source, rests on the assumption that received sound pressure
level in decibels can be written as a sum of four terms: source level, transmission loss
from the source to the target, target strength, and transmission loss from the target
to the receiver. This assumption has two related implications: (1) that propagation
and scattering effects are completely factorable from each other, and (2) that a linear
combination of the incoherent quantities, target strength, transmission loss and source
level completely specifies the sound pressure level at the receiver.

When this assumption is axiomatically adopted in an ocean waveguide, funda-
mental inconsistencies can occur when experimental data is examined. For example,
the target strength of an object, which should be invariant, can vary significantly with
range when experimentally estimated in a range-independent environment where the
direct return arrives together with multiple returns from the waveguide boundaries.
This has been noted in the classic text Physics of Sound in the Sea.[11] Regardless
of this inconsistency, target strength is still used in that text to describe the scatter-
ing properties of an object because a more fundamental approach had not presented
itself, just as it is still used today by many practitioners of ocean acoustics.

In this thesis, we show that the assumptions that the sonar equation rests on and
its implications are not generally valid for scattering in a shallow water waveguide.
Incorrect use of the sonar equation can lead to errors of tens of decibels in estimating
the scattered field level from a target. In the process we show that the invariant
scattering properties of an object in a waveguide cannot generally be described by
target strength, an incoherent quantity, but rather require a coherent representation
that arises from the fundamental waveguide scattering theory[31, 48] upon which our
analysis using the unified model is based.

First developed in World War II,[73, 11] the sonar equation, analogous to the radar
equation, is only valid when propagation and scattering dependencies are approxi-
mately separable. For example, given an omni-directional point source and receiver
in free-space, propagation and scattering are separable when the source and receiver
are in the far field of the target, where the incident as well as the scattered wave
may be approximated as planar. The sonar equation has a long history of legitimate
usage in deep water[73, 11] where these free-space conditions are effectively achieved
in many practical scenarios due to the significant time separation that often occurs
between direct and surface reflected arrivals and the adiabatic nature of refraction in
the ocean.

In shallow water waveguides, multiple reflections from the surface and bottom
typically overlap and coherently interfere with each other and the direct arrival. No
unique incident and scattered angle exists. To understand the implications for scat-
tering, it is convenient to decompose the incident field at a target, from a far field
point source, into planewaves. Each incident planewave arrives with a specific el-
evation angle from the azimuth of the far field source and then scatters coherently
from the target into outgoing planewaves in all elevation and azimuthal angles. The
target affects both the amplitude and phase of each scattered planewave. This phase
change cannot be described by an incoherent quantity such as target strength. At a far
field receiver, the scattered field is the coherent sum of all scattered planewaves from
all incident planewaves. Just as waveguide propagation models must account for the
coherent interference of multiple arrivals from a source to receiver to accurately de-
termine transmission loss, waveguide scattering models must account for the coherent
interference of all scattered waves from every wave incident on the target. Propaga-
tion and scattering are in this way coherently convolved for objects submerged in a
waveguide.
To establish when the sonar equation can be applied in a shallow water waveguide, we calculate the scattered field from a variety of target types in various shallow water waveguides using the physics-based waveguide scattering model, Eq. 2.1, that takes into account the coupling between propagation and scattering. We then compare the results to those predicted by the sonar equation. The waveguide scattering model expresses the scattered field in terms of the planewave scatter function of the object. The planewave scatter function depends on the absolute object orientation and direction of both the incident and scattered planewave and is the invariant quantity that describes the scattering properties of an object in a waveguide. This scatter function is a coherent quantity. The object's incoherent target strength is simply $20 \log$ of the scatter-function-magnitude-to-wavenumber ratio. Target strength then only contains the amplitude but not the essential phase information necessary to describe the scattering process in a waveguide.

We show analytically that if the scatter function of the object is approximately constant over the equivalent angles spanned by the waveguide modes for the given bistatic geometry, the scatter function of the object, which effectively couples the modes of the incident and scattered field, can be factored with little error. This leads to an approximation for the sound pressure level of the scattered field that is the sonar equation. Many rounded objects, such as spheres and certain spheroids, exhibit this behaviour in non-forward scatter. Flat homogenous objects, such as plates and discs, are the most highly directional convex targets. These have non-uniform scatter functions with strong main lobes in the forward and backscatter directions of diffraction-limited angular width $\lambda/L$, for $\lambda/L << 1$, where $L$ is the object’s length and $\lambda$ the wavelength. Other targets that are non-convex or inhomogenous, for example, may have a narrower main lobe due to interference from distinct parts of the target. In the limit, the scatter function of an object consisting solely of two point scatterers separated by $L$ has the narrowest main lobe of angular width $\lambda/2L$.

When the angular width of the object’s main lobe is much smaller than that spanned by the propagating modes $\pm \Delta \psi$ about the horizontal, which is often limited by the critical angle $\psi_c$ of the seabed beyond a few watercolumn depths in range, the modes
of the waveguide are scattered non-uniformly. This leads to strong coupling between propagation and scattering in both the forward and backscatter azimuths. The sonar-equation approximation is found to be in error, often by tens of decibels, when applied to such highly directional targets in shallow water waveguides.

These findings explain the physical basis for the discrepancy noted in Ref. 9 between the sonar equation and a waveguide scattering model. Some special cases were previously noted. For example, Ingenito pointed out that propagation and scattering become factorable in a waveguide that supports only a single mode.[31] Makris noted that this factorization is possible for compact objects, i.e. those with $ka = \pi L/\lambda \ll 1.[42]$ We note that the sonar equation is always valid for compact pressure-release objects since the scattered field is effectively omni-directional, but is more approximate for rigid compact objects since their scattered fields always maintain some directionality as $ka$ decreases.

As a general conclusion, we find that the sonar equation is valid when the target’s scatter function is roughly constant over the equivalent horizontal grazing angles $\pm \Delta \psi$ spanned by the dominant waveguide modes. This is approximately true (1) for all objects when $2\Delta \psi < \lambda/2L$ and (2) for spheres and certain other rounded objects in non-forward scatter azimuths even when (1) does not hold. For homogenous convex objects condition (1) is the less stringent $2\Delta \psi < \lambda/L$.

This conclusion is significant because, in an active scenario, the sonar operator has the ability to lower the frequency of transmission until the target’s scatter function becomes approximately constant over $\pm \Delta \psi$. The sonar equation then becomes a valid approximation when $f < c/(4L\Delta \psi)$. Operating in this frequency regime is desirable because when the sonar equation is valid, only a single parameter is necessary to characterize the scattering properties of the target for that measurement. This greatly simplifies target classification by making the classic approach of estimating a single-parameter target strength valid in a shallow water waveguide. It also simplifies other problems such as estimating target depth in a waveguide. When the sonar equation is not valid, the problem of classifying the target becomes much more complicated. Up to $2(2N)^2$ parameters would be necessary to characterize the scattering properties of
the target, for a waveguide that supports $N$ modes, because the amplitude and phase of the object’s scatter function would have to be determined for each incident and scattered pairing of each mode’s equivalent up and down-going planewave elevation angles.

Babinet’s principle maintains that the forward scattered fields of impenetrable objects that have identical projected areas with respect to a given incident planewave in free space are asymptotically equal for large $ka$.\cite{75, 4} This also holds true for some penetrable objects.\cite{75} For an object submerged in a waveguide, the incident and scattered fields are often characterized by a wide angular spectrum of planewaves. Despite this difference between waveguide and free space scattering, simulations in several typical shallow water waveguides with a variety of targets types show that Babinet’s principle can hold approximately in a waveguide in the forward-scatter azimuth if the equivalent propagation angles of the modes are sufficiently close to horizontal, as is often the case after long-range propagation in lossy media. By Babinet’s principle, objects that are large compared to the wavelength cast the same free space shadow as flat objects with the same projected area. Since flat objects of high $ka$ are the most directional, the sonar equation approximation breaks down rapidly in a shallow water waveguide as $ka$ increases beyond unity for scattering in the forward azimuth for all object shapes, including spheres. Extreme caution should then be used in applying the sonar equation in forward scatter.

Scattering from Stochastic Objects with Unknown or Fluctuating Orientation

Often in the oceans, we are interested in imaging or detecting submerged object with unknown orientation or whose orientation changes with time (fluctuates). Examples of fluctuating underwater objects include underwater vehicles like a submarine whose azimuth or aspect is unknown, marine mammals whose aspect may change rapidly, and bubble clouds from surf. Scattering from such objects can be studied by treating their bi-directional planewave scatter function as a random variable as is done in the
unified model. The expected scattered field from such an object can be calculated from its average bi-directional scatter function.

For a very large convex object, there is an argument that says that the scattering pattern caused by reflection from such an object with random orientation in 3-D is identical with the scattering pattern by reflection on a very large sphere of the same material and surface condition. For an object of unknown or complicated shape, its scatter function may be difficult to model. This theorem is useful because it allows us to estimate the statistics of the scattered field from a complicated object with random orientation from that of the equivalent sphere. Only one parameter, the length scale of the equivalent sphere is necessary to characterize the statistics of the scattering from complicated objects with random orientations.

We examine the average bi-directional planewave scatter function of a rigid square plate rotated in 3-D space. The rigid square plate is the most directional convex object that scatters strongly in both the forward and back scatter azimuths. Both the coherent and incoherent averages of the scatter function are computed. These are related to the mean and variance respectively of the scattered field. For 3-D rotation, we made use of Rodrigues formula to compute the scatter function for each orientation of the square plate in terms of an axis vector and a spin angle.

The numerical results show that for 3-D rotation the incoherent average scatter function of the square plate has properties that are similar to that of a sphere. For instance, the incoherent 3-D average scatter function of the square plate has a strong forward scatter and is omnidirectional in non-forward scatter directions. Its magnitude depends only on the cosine of the angle between the incident and scattered planewaves and not on the absolute angles of the plane waves, just like a sphere. The coherent 3-D average scatter function of the plate on the other hand does not have such properties.

Certain objects may not have completely random orientations. Their orientations may be limited to rotations in specific planes or angle. For instance, a submarine has unknown orientation only in the azimuth and limited roll, yaw and pitch in other directions. We extend the theorem to limited rotations of an object by comparing
its scattered field with that from the equivalent shape generated from the limited rotation of the object. We rotate the rigid square plate in 2-D about the axis normal to the plate, thus generating a circular disc. Both the coherent and incoherent 2-D average scatter functions of the square plate coincide with that of a circular disc for small dimensions of the plate relative to the wavelength. For large square plates, only the peak scatter function lobe of the coherent and incoherent 2-D average scatter function coincides with that of the circular disc. The side lobes however show some differences.

1.2 Thesis Organization

In Chapter 2, we derive the unified model for bistatic, 3D object scattering and reverberation in a stratified waveguide. Appendix A shows that the unified model is directly related to Green's Theorem. The conditions necessary for backscattering from stochastic scatterers to become diffuse in a waveguide is derived in Chapter 6. In Chapter 3, the unified model is applied to investigate the effect of the environment, measurement geometry, and sonar parameters on the imaging and detection of submerged objects and geomorphology in a shallow water waveguide. The mechanisms for scattering from extended geologic features are investigated in this Chapter. The results of the field experiment to remotely image sub-bottom and seafloor geomorphology are presented in Chapter 4.

In Chapter 5, we derive the generalized extinction theorem in a waveguide and apply it to classify submerged objects in shallow water. Analytic expressions for the forward propagated field in a waveguide, the attenuation coefficient and dispersion due to scattering from arbitrary scatterers is derived in Chapter 6. The validity of sonar equation and Babinet's principle for scattering in a waveguide is investigated in Chapter 7. Here, we also provide the conditions necessary for the sonar equation to become approximately valid in a waveguide. In Chapter 8, the statistics of scattering from objects with unknown or fluctuating orientation is examined.
Chapter 2

Unified Model for 3-D Reverberation and Submerged Object Scattering in a Stratified Ocean Waveguide

In this chapter, we formulate the unified model for 3-D reverberation and submerged object scattering in a stratified medium from wave theory.

2.1 Wave-theoretic model for 3-D scattering from an object of arbitrary shape in a stratified medium

We begin with the wave-theoretic normal mode model, based on Green’s theorem, for the field scattered by an object in a stratified medium, expressed as a linear function of the object’s planewave scatter function[31, 48]. A derivation of the formula is provided in Appendix A. The planewave scatter function [5] at frequency f is defined in Appendix A, where it’s relationship to Green’s theorem and traditional target strength measures of ocean acoustics commonly used with the sonar equation is explained.
In the formulation, the origin of the coordinate system is placed at the object centroid as shown in Fig. A-1 in Appendix A. The source coordinates are defined by \( r_0 = (x_0, y_0, z_0) \) and the receiver coordinates by \( r = (x, y, z) \) where the positive z-axis points downward and normal to the interface between horizontal strata. Spatial cylindrical \((\rho, \phi, z)\) and spherical systems \((r, \theta, \phi)\) are defined by \( x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta \) and \( \rho = x^2 + y^2 \). The horizontal grazing angle is \( \psi = \pi/2 - \theta \). The horizontal and vertical wavenumber components for the nth mode are respectively \( \xi_n = k \sin \alpha_n \) and \( \gamma_n = k \cos \alpha_n \), where \( \alpha_n \) is the elevation angle of the mode measured from the z-axis. Here, \( 0 \leq \alpha_n \leq \pi/2 \) so that the down and upgoing planewave components of each mode will then have elevation angles \( \alpha_n \) and \( \pi - \alpha_n \) respectively. The corresponding vertical wavenumber of the down and upgoing components of the nth mode are \( \gamma_n \) and \( -\gamma_n \) respectively, where \( \Re\{\gamma_n\} \geq 0 \). \( k^2 = \xi_n^2 + \gamma_n^2 \), and the wavenumber magnitude \( k \) equals the angular frequency \( \omega \) divided by the sound speed \( c \) in the object layer. The geometry of spatial and wavenumber coordinates is shown in Appendix A and is similar to that defined in Refs. [46, 31].

The spectral component of the scattered field from the object at the origin for a source at \( r_0 \) and a receiver at \( r \) from Eq. A.28 is

\[
\Phi_s(r|r_0) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_s^{(m,n)}(r|r_0),
\]

where

\[
\Phi_s^{(m,n)}(r|r_0) = \frac{(4\pi)^2}{k} \left[ A_m(r)A_n(r_0)S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi) + B_m(r)A_n(r_0)S(\alpha_m, \phi; \alpha_n, \phi_0 + \pi) + A_m(r)B_n(r_0)S(\pi - \alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi) + B_m(r)B_n(r_0)S(\alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi) \right],
\]

and

42
\[
A_m(r) = \frac{i}{d(0)}(8\pi \xi_m \rho)^{-1/2} u_m(z) N_m^{(1)} e^{i(\xi_m \rho + \gamma_m D - \pi/4)}, \quad \text{upgoing scattered}
\]

\[
B_m(r) = \frac{i}{d(0)}(8\pi \xi_m \rho)^{-1/2} u_m(z) N_m^{(2)} e^{i(\xi_m \rho - \gamma_m D - \pi/4)}, \quad \text{downgoing scattered}
\]

\[
A_n(r_0) = \frac{i}{d(z_0)}(8\pi \xi_n \rho_0)^{-1/2} u_n(z_0) N_n^{(1)} e^{i(\xi_n \rho_0 + \gamma_n D - \pi/4)}, \quad \text{downgoing incident}
\]

\[
B_n(r_0) = \frac{i}{d(z_0)}(8\pi \xi_n \rho_0)^{-1/2} u_n(z_0) N_n^{(2)} e^{i(\xi_n \rho_0 - \gamma_n D - \pi/4)}, \quad \text{upgoing incident}
\]

(2.3)

are the incident and scattered down and upgoing planewave amplitudes in the layer of the object, \(D\) is the depth of the object center from the sea surface, \(d(z)\) is the density at depth \(z\), \(u_n(z)\) are the mode functions, and \(S(\alpha, \beta; \alpha_i, \beta_i)\) is the object’s planewave scatter function. In Eq. 2.2, \((\phi_0 + \pi) = \beta_i\) and \(\phi = \beta\) where \(\phi_0\) is the source azimuth and \(\phi\) is the receiver azimuth. The definition of the planewave scatter function here follows that defined in Ref. [48] where the incident planewave on the object is described in terms of the direction it goes to so that for forward scatter in free space \(\alpha = \alpha_i, \beta = \beta_i\). The product of \(e^{-i2\pi ft}\) and the right hand side of Eq. 2.1 yields the time-harmonic scattered field. The mode functions are normalized according to

\[
\delta_{nm} = \int_{-D}^{\infty} u_m(z) u_n^*(z) \frac{d(z)}{d(z)} \, dz, \tag{2.4}
\]

and can be expressed in the layer of the object as

\[
u_m(z) = N_n^{(1)} e^{i\gamma_m (z+D)} - N_n^{(2)} e^{-i\gamma_m (z+D)}, \tag{2.5}
\]

where \(N_n^{(1)}\) and \(N_n^{(2)}\) are normalization constants. In a Pekeris waveguide,
\[ N^{(1)} = N^{(2)} = \frac{\sqrt{2}}{2i} \left[ \frac{1}{d} \left( H - \frac{\sin 2\gamma_n H}{2\gamma_n} \right) + \frac{1}{d_b} \frac{\sin^2 \gamma_n H}{\sqrt{c_n^2 - (\omega/c_b)^2}} \right]^{-1/2}, \quad (2.6) \]

where \( d_b \) and \( c_b \) are the density and sound speed of the bottom, and \( H \) is the thickness of the water column. If the mode functions are specified at any two depths, \( z_1 \) and \( z_2 \) within the target layer, the normalization constant can be readily obtained as

\[
\begin{bmatrix}
N^{(1)}_n \\
-N^{(2)}_n
\end{bmatrix} = \begin{bmatrix}
e^{i\gamma_n(z_1+D)} & e^{-i\gamma_n(z_1+D)} \\
e^{i\gamma_n(z_2+D)} & e^{-i\gamma_n(z_2+D)}
\end{bmatrix}^{-1} \begin{bmatrix}
u_n(z_1) \\
u_n(z_2)
\end{bmatrix}. \quad (2.7)
\]

Equations (2.1)–(2.6) differ from Ingenito’s formulation in a number of ways. The most substantial difference is that, by inclusion of Eqs. 2.5 and 2.7, they explicitly show how the scattered field for an arbitrarily shaped object can be computed in a stratified medium. Ingenito also defines the planewave scatter function differently than most standard texts by describing the incident plane wave in terms of the direction it comes from rather than the direction it goes to. The latter, standard approach, is adopted here.

The more standard mode function normalization of Ref. [33] is adopted here, so that Eq. (2.1) obeys reciprocity as defined in Appendix A2 of Ref. [33], so that 
\[ d(z_0)\Phi_s(r|r_0) = d(z)\Phi_s(r_0|r). \] Satisfaction of reciprocity becomes important for an approach if it is to yield accurate estimates of the scattered field when the source, receiver, and target are in layers that have significantly different densities. Obeying reciprocity is a natural consequence of the use of Green’s Theorem in the present formulation, but has been left unaddressed as an issue in previous heuristic reverberation formulations.[16, 6, 77, 37] The issue becomes of practical concern in modelling the level of returns from targets or surfaces buried in the seafloor from sonar systems operating in the water column above.

A more general expression than Eqs. (2.1)–(2.6) for the scattered field from an arbitrarily shaped object in a stratified medium is given in Refs. [46] and [42] in terms
of wavenumber integrals. A number of assumptions have to be satisfied for the above formulation to be valid as noted in Ref. [42]. In particular, the propagation medium is horizontally stratified and range independent, multiple scattering between the object and waveguide boundaries is negligible, the object lies within a layer of constant sound speed, and the range from the object to source or receiver must be large enough that the scattered field can be approximated as a linear function of the object's planewave scatter function and the field as sum of discrete modes. The present formulation and its spectral equivalent have been implemented for target scattering in a waveguide over the full $360^\circ$ span of bistatic angles in Refs. [31, 46, 47, 63]. It is noteworthy that this formulation includes the scattering of evanescent waves by analytic continuation of the scatter function, as has been previously discussed and implemented in Refs. [46, 47] as well as in Ref. [68] which uses a formulation similar to Ingenito's.

2.1.1 Interpretation of the Waveguide Scattering Model

In Eq. (2.1), the field radiated by the source is decomposed into modes incident on the object. Each incoming mode at the target location is composed of a pair of planewaves, one downgoing with amplitude $A_n(r_0)$ and one upgoing with amplitude $B_n(r_0)$ with incident elevation angles, $\alpha_n$ and $\pi - \alpha_n$ respectively. Each of the four terms in Eq. (2.1) represents the coherent scattering of one of the two incoming planewave components of the $n$th mode, into one of the two outgoing planewave components of the $m$th mode. The far field physics of the interaction is determined by the scatter function that depends on the elevation angles of both the incident and scattered planewaves. The scatter function is a coherent quantity that affects both the amplitude and phase of the scattered planewaves. The scattered fields from each incident planewave are coherently superposed to form the total scattered field at the receiver. The scattering process couples the modes so that propagation and scattering are coherently convolved for objects submerged in a shallow water waveguide. A conceptual diagram of scattering in a waveguide is given in Fig. 2-1 for a waveguide that excites only two modes.
Figure 2-1: Scattering in a waveguide with only two modes. Each mode is composed of a downgoing and an upgoing planewave. Each incoming planewave is scattered by the object into various outgoing planewaves. The scattered field from each incident planewave are coherently superposed to form the total scattered field at the receiver.
2.2 General saddle point approximation for the scattered field in time from a distant object

For a source with general time dependence

\[ q(t) = \int_{-\infty}^{\infty} Q(f) e^{-it2\pi f} df, \quad (2.8) \]

and spectrum

\[ Q(f) = \int_{-\infty}^{\infty} q(t) e^{it2\pi f} dt, \quad (2.9) \]

the scattered field as a function of time from an object with center at the origin becomes

\[ \Psi_s(r|r_0|t) = \int_{-\infty}^{\infty} Q(f) \Phi_s(r|r_0)e^{-it2\pi f} df. \quad (2.10) \]

Equation (2.1) can be rewritten as

\[ \Phi_s(r|r_0) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Lambda_{mn}(r|r_0, f) e^{i\psi_{mn}}, \quad (2.11) \]

so that

\[ \Psi_s(r|r_0|t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} Q(f) \Lambda_{mn}(r|r_0, f) e^{i\psi_{mn}(f)} df, \quad (2.12) \]

where
\[ \Lambda_{mn}(r|r_0, f) = \Phi_s^{(m,n)}(r|r_0)e^{-i\rho_0 \xi_n - i\rho \xi_m}, \]  

and

\[ \psi_{mn}(f) = \frac{\rho_0}{\rho} \xi_n + \xi_m - 2\pi f \frac{t}{\rho}. \]  

By application of the saddle point method discussed in Appendix C, for large \( \rho \), assuming \( \rho_0/\rho \) and \( t/\rho \) are fixed, Eq. 2.12 integrates to

\[
\Psi_s(r|r_0|t) \approx \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{l=1}^{L} \sqrt{\frac{2\pi}{\rho \psi''(f_{imn})}} Q(f_{imn}) \Lambda_{mn}(r|r_0, f_{imn}) e^{i\rho \psi_{mn}(f_{imn}) + i\pi/4}
\]

\[
= \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{l=1}^{L} Q(f_{imn}) \sqrt{\frac{2\pi}{\rho \psi''(f_{imn})}} \Phi_s^{(m,n)}(r|r_0) e^{-i2\pi f_{imn}t + i\pi/4},
\]  

where the relevant saddle points \( f_{imn} \) are determined by solving the equation

\[ \frac{d\psi_{mn}(f)}{df} \bigg|_{f=f_{imn}} = 0, \]  

and choosing the complex roots that lead to a finite solution of Eq. (2.12) as \( \rho \) and \( \rho_0 \) increase, where \( l \) is the index for multiple roots given \( n \) and \( m \). Such solutions do not exist before the wave packet has arrived or after it has passed the receiver. The frequencies \( f_{imn} \) for each \( n \) to \( m \) mode conversion must be evaluated at each time and source and receiver range. Equation (2.15) is also obtained if, analogously, \( \rho_0 \) is made large and \( t/\rho_0 \) held fixed in Eqs. (2.11)-(2.14). Typically, both \( \rho \) and \( \rho_0 \) will be sufficiently large for the saddle point method approximation to hold whenever the modal formulation of Sec. 2.1, which also requires large \( \rho \) and \( \rho_0 \), is valid.
2.2.1 Interpretation of the Saddle Point approach for Time-domain Scattered Field using Modal Group Velocities

At any time $t$, for fixed $\rho$ and $\rho_0$, the derivative of Eq. (2.14) with respect to frequency $f$ is

$$\frac{d\psi_{mn}(f)}{df} = \frac{\rho_0}{\rho} \frac{d\xi_n}{df} + \frac{d\xi_m}{df} - 2\pi \frac{t}{\rho}. \tag{2.17}$$

Eq. (2.17) can be written in terms of the modal group velocity defined by $c_n(f) = 2\pi df/d\xi_n$ as

$$\frac{d\psi_{mn}(f)}{df} = \frac{\rho_0}{\rho} \frac{2\pi}{c_n(f)} + \frac{2\pi}{c_m(f)} - 2\pi \frac{t}{\rho}. \tag{2.18}$$

The group velocity of each mode is a function of frequency. In a Pekeris waveguide with isovelocity water column of speed $c_w$ overlying a bottom halfspace of speed $c_b$, the dependence of the modal group velocity on frequency is sketched in Fig. 2-2 and the corresponding modal group slowness $1/c_n(f)$ in Fig. 2-3.

The solution to Eq. (2.18) satisfying Eq. (2.16) is

$$\frac{\rho_0}{c_n(f_{mn})} + \frac{\rho}{c_m(f_{mn})} = t$$

$$t_n + t_m = t, \tag{2.19}$$

where $t_n$ is the time taken for the incident mode $n$ to travel range $\rho_0$ from source to target and $t_m$ is the time taken for scattered mode $m$ to travel range $\rho$ from target to receiver. $f_{mn}$ is the frequency where this total travel time of the incident and scattered modes sum up to $t$. A sketch of Eq. (2.19) in Fig. 2-4 for fixed $\rho$ and $\rho_0$ shows that for $t < t_b$, the wave packet has not arrived at the receiver. For $t_b < t < t_e$, the incident and scattered mode pair contribute to the scattered field at frequencies.
Figure 2-2: Modal group velocity dispersion curves for a Pekeris waveguide comprising of a water column of speed $c_w$ overlying a bottom halfspace of speed $c_b$ that supports three propagating modes.

Figure 2-3: Group slowness dispersion curves for the modes of the Pekeris waveguide described in Fig. 2-2
The above analysis implies that for a broadband waveform, the scattered signal that arrives at time \( t \) for fixed \( \rho \) and \( \rho_0 \) can be composed by several discrete frequencies. These frequencies are the saddle points of Eq. (2.12) and they correspond to

\[ f_{\text{inm}}. \]
frequencies where the travel times of the incident and scattered mode pair sum to give exactly the time $t$. One advantage of the saddle point approach for broadband waveforms is that it allows the scattered field in time at any given time to be reconstructed from the harmonic scattered field at a small number of discrete frequency components within the signal bandwidth. In contrast, constructing the scattered waveform at a given time using Fourier synthesis following Eq. (2.10) requires the harmonic scattered field for all the frequencies within the full bandwidth of the waveform. The only requirement for the saddle point approach is that knowledge of the modal group velocity dispersion curves for the waveguide has to be known.

It should be noted that if the entire waveform of the scattered signal is to be constructed as a function of time, it would be much more efficient to use the Fourier synthesis approach following Eq. (2.10) for the reconstruction than the saddle point method. The is because the harmonic scattered field at each frequency component within the signal bandwidth would be required in both methods for the reconstruction of the entire scattered signal waveform. In the saddle point approach, additional knowledge of the saddle point frequencies making dominant contribution to the scattered field at each time from the modal group velocity dispersion curves is required. The Fourier synthesis approach, on the other hand, does not require any knowledge of the modal group velocity dispersion curves making it easier to implement when reconstructing the entire scattered waveform in time.

2.3 The field from general stochastic targets

By allowing the scatter function for the object to be a random variable, the single-scatter formulation of Secs. 2.1 and 2.2 applies to the more general problem of scattering from a stochastic target submerged in a waveguide. This approach is particularly valuable in modelling scattering from targets of unknown shape or orientation, randomly rough surface interfaces, or stochastic volume heterogeneities, all of which can contribute significantly to the reverberant field measured in shallow water.

The moments of the scattered field can be derived analytically to determine its
expected behaviour. The mean field, for example, becomes

\[
\langle \Phi_s(r|r_0) \rangle = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \langle \Phi_s^{(m,n)}(r|r_0) \rangle
\]

\[\text{(2.20)}\]

where

\[
\langle \Phi_s^{(m,n)}(r|r_0) \rangle = \frac{(4\pi)^2}{k} \left[ A_m(r) A_n(r_0) \langle S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi) \rangle \\
- B_m(r) A_n(r_0) \langle S(\alpha_m, \phi; \alpha_n, \phi_0 + \pi) \rangle \\
- A_m(r) B_n(r_0) \langle S(\pi - \alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi) \rangle \\
+ B_m(r) B_n(r_0) \langle S(\alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi) \rangle \right].
\]

\[\text{(2.21)}\]

where the expectations are taken over the scatter function. Similarly, the intensity and variance of the field can be expressed in terms of the second moment and variance of the object's scatter function. For instance, the mutual intensity of the field scattered for receivers at \( r \) and \( r' \) becomes

\[
\langle \Phi_s(r|r_0) \Phi_s^*(r'|r_0) \rangle = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{m'=1}^{\infty} \sum_{n'=1}^{\infty} \langle \Phi_s^{(m,n)}(r|r_0) \Phi_s^{*(m',n')}(r'|r_0) \rangle
\]

\[\text{(2.22)}\]

where
\[
\langle \Phi_s^{(m,n)}(r|r_0)\Phi_s^{(m',n')}(r'|r_0) \rangle \\
= \frac{(4\pi)^4}{k^2} \left[ A_m(r)A_n(r_0)A_{m'}^{*}(r')A_{n'}^{*}(r_0) \langle S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^*(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \\
- A_m(r)A_n(r_0)B_{m'}^{*}(r')A_{n'}^{*}(r_0) \langle S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^*(\alpha_{m'}, \phi'; \pi - \alpha_{n'}, \phi_0 + \pi) \rangle \\
- A_m(r)A_n(r_0)A_{m'}^{*}(r')B_{n'}^{*}(r_0) \langle S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^*(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \\
+ A_m(r)A_n(r_0)B_{m'}^{*}(r')B_{n'}^{*}(r_0) \langle S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^*(\alpha_{m'}, \phi'; \pi - \alpha_{n'}, \phi_0 + \pi) \rangle \\
- B_m(r)A_n(r_0)A_{m'}^{*}(r')A_{n'}^{*}(r_0) \langle S(\alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^*(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \\
+ B_m(r)A_n(r_0)B_{m'}^{*}(r')A_{n'}^{*}(r_0) \langle S(\alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^*(\alpha_{m'}, \phi'; \pi - \alpha_{n'}, \phi_0 + \pi) \rangle \\
- B_m(r)A_n(r_0)A_{m'}^{*}(r')B_{n'}^{*}(r_0) \langle S(\alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^*(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \\
+ B_m(r)A_n(r_0)B_{m'}^{*}(r')B_{n'}^{*}(r_0) \langle S(\alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^*(\alpha_{m'}, \phi'; \pi - \alpha_{n'}, \phi_0 + \pi) \rangle \\
- A_m(r)B_n(r_0)A_{m'}^{*}(r')A_{n'}^{*}(r_0) \langle S(\pi - \alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi)S^*(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \\
+ A_m(r)B_n(r_0)B_{m'}^{*}(r')A_{n'}^{*}(r_0) \langle S(\pi - \alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi)S^*(\alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \\
- A_m(r)B_n(r_0)A_{m'}^{*}(r')B_{n'}^{*}(r_0) \langle S(\pi - \alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi)S^*(\pi - \alpha_{m'}, \phi'; \pi - \alpha_{n'}, \phi_0 + \pi) \rangle \\
+ B_m(r)B_n(r_0)A_{m'}^{*}(r')A_{n'}^{*}(r_0) \langle S(\alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi)S^*(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \\
- B_m(r)B_n(r_0)B_{m'}^{*}(r')A_{n'}^{*}(r_0) \langle S(\alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi)S^*(\alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \\
- B_m(r)B_n(r_0)A_{m'}^{*}(r')B_{n'}^{*}(r_0) \langle S(\alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi)S^*(\pi - \alpha_{m'}, \phi'; \pi - \alpha_{n'}, \phi_0 + \pi) \rangle \\
+ B_m(r)B_n(r_0)B_{m'}^{*}(r')B_{n'}^{*}(r_0) \langle S(\alpha_m, \phi; \pi - \alpha_n, \phi_0 + \pi)S^*(\alpha_{m'}, \phi'; \pi - \alpha_{n'}, \phi_0 + \pi) \rangle \right] \\
(2.23)
\]

The spatial covariance, or cross spectral density, of the scattered field is

\[
\langle \Phi_s(r|r_0)\Phi_s^*(r'|r_0) \rangle - \langle \Phi_s(r|r_0) \rangle \langle \Phi_s^*(r'|r_0) \rangle \\
= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{m'=1}^{\infty} \sum_{n'=1}^{\infty} \left\{ \langle \Phi_s^{(m,n)}(r|r_0)\Phi_s^{*(m',n')}(r'|r_0) \rangle - \langle \Phi_s^{(m,n)}(r|r_0) \rangle \langle \Phi_s^{*(m',n')}(r'|r_0) \rangle \right\} . \\
(2.24)
\]

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Statistics of the scatter function of fluctuating objects with unknown or varying orientation is investigated in Chapter 8. Scatter function statistics of a random target that scatters diffusely or incoherently is discussed in Chapter 6.

2.3.1 The field scattered from a randomly rough or inhomogeneous seabed

When the scattering is due to a randomly rough seafloor patch, a great simplification occurs in the form of the mutual intensity. Of the sixteen parenthetical terms of Eq. 2.23, only one represents a down-going incident wave coupling to an up-going scattered wave. Accordingly, the field scattered into the waveguide from a randomly rough seafloor patch of area $\Delta A$ with characteristic dimension $L$, must have mutual intensity given by Eq. 2.22 with

$$\langle \Phi_s^{(m,n)}(r|r_0)\Phi_s^{*(m',n')}(r'|r_0) \rangle$$

$$= \left(\frac{4\pi}{k^2}\right)^4 A_m(r)A_n(r_0)A_{m'}^{*}(r')A_{n'}^{*}(r_0) \times \langle S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^{*}(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle,$$  \hspace{1cm} (2.25)

and cross spectral density given by Eq. 2.24 with

$$\langle \Phi_s^{(m,n)}(r|r_0)\Phi_s^{*(m',n')}(r'|r_0) \rangle - \langle \Phi_s^{(m,n)}(r|r_0) \rangle \langle \Phi_s^{*(m',n')}(r'|r_0) \rangle$$

$$= \left(\frac{4\pi}{k^2}\right)^4 A_m(r)A_n(r_0)A_{m'}^{*}(r')A_{n'}^{*}(r_0) \times \bigg\{ \langle S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi)S^{*}(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle$$

$$- \langle S(\pi - \alpha_m, \phi; \alpha_n, \phi_0 + \pi) \rangle \langle S^{*}(\pi - \alpha_{m'}, \phi'; \alpha_{n'}, \phi_0 + \pi) \rangle \bigg\}. $$  \hspace{1cm} (2.26)

The cross spectral density is more useful in describing the stochastic scattering properties of a randomly rough seafloor patch than the mutual intensity because deter-
ministic effects, such as specular reflection, coherent beaming and forward scattering are removed with the expected field. In diffuse surface scattering problems, where the surface scattering patch must be much larger than the wavelength, the expected value of the scattered field is typically negligible away from the specular direction due to random interference. The cross spectral density and mutual intensity then become effectively indistinguishable.

With the assumption of diffuse scattering described in Chapter 6, that is supported by a large amount of experimental evidence\[58, 73\], and that holds in a waveguide when the conditions given in Eqs. 6.14 and 6.12 are satisfied for \( L_x = L \), application of Eq. 6.17 yields

\[
\langle \Phi_s^{(m,n)}(r|\mathbf{r}_0)\Phi_s^{*(m',n')}(r'|\mathbf{r}_0) \rangle = \langle \Phi_s^{(m,n)}(r|\mathbf{r}_0) \rangle \langle \Phi_s^{*(m',n')}(r'|\mathbf{r}_0) \rangle
\]

\[
= \delta_{mm'}\delta_{nn'} C_{mn}(r, r'|\mathbf{r}_0) A_m(r) A_n(r') A_m^*(r) A_n^*(r_0) \frac{(4\pi)^4}{k^2}, \tag{2.27}
\]

which leads to a great simplification in the scattered field covariance,

\[
\langle \Phi_s(r|\mathbf{r}_0)\Phi_s^*(r'|\mathbf{r}_0) \rangle - \langle \Phi_s(r|\mathbf{r}_0) \rangle \langle \Phi_s^*(r'|\mathbf{r}_0) \rangle
\]

\[
= \frac{(4\pi)^4}{k^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_m(r) A_m^*(r') |A_n(r_0)|^2 C_{mn}(r, r'|\mathbf{r}_0). \tag{2.28}
\]

If the seafloor is taken as an aggregate of range-dependent scattering patches and if these are each small enough to have little effect on the mean forward field, Eq. 2.28 provides a good approximation to the cross spectral density after forward propagating through this mildly range-dependent waveguide and then scattering in a specified direction from the given patch.

It is noteworthy that a formulation in terms of wave number integrals is not convenient in describing the statistical equipartition of energy associated with diffuse scattering since the modes are the entities that describe the system's degrees of freedom rather than the wavenumber components.
We would like to point out some typographical errors in Ref. [48]. In Eqs. 14-17 and 37 of Ref. [48], the functions $B_m$ and $B^*_m$ should be replaced with $A_m$ and $A^*_m$, respectively, in Eqs. 41 and 43-45, the functions $B_t$ and $B^*_t$ should be replaced with $A_t$ and $A^*_t$ respectively, and in Eqs. 14 and 15, the angles $\alpha_m$ and $\alpha_m'$ should be replaced with $\pi - \alpha_m$ and $\pi - \alpha_m'$ respectively.

2.3.2 The field scattered from a school of fish

Consider a sheet of characteristic dimension $L$ containing a school of $N_{fish}$ fish. If the conditions in Eqs. 6.14 and 6.12 are satisfied for $L = L$, the total field scattered from the school is diffuse or incoherent. We assume that the size of each fish $a$ is much smaller than the wavelength $\lambda$, i.e. $a \ll \lambda$, then, the scatter function for each fish becomes omnidirectional and we let $S_{fish}(\alpha, \beta, \alpha_i, \beta_i) = S_0$.

The intensity of the total scattered field can be derived from Eqs. 6.16 and 2.23 expressed in absolute coordinates. The total scattered intensity can be written as a sum of 16 terms, the first of which is given by the first term in Eq. 6.10. Suming the 16 terms in the expression for the intensity, the resulting total intensity scattered from the school containing $N_{fish}$ fish is

$$I_{s, sum, fish}(\mathbf{R}_r - \mathbf{R}|\mathbf{R}_0 - \mathbf{R}) = (4\pi)^4 \left( \sum_{m=1}^{\infty} \frac{1}{d^2(Z)8\pi|\xi_m|\Omega} |u_m(Z_r - Z)|^2 |u_m(Z)|^2 e^{-2\Im{\xi_m}\Omega} \right) \times \left( \sum_{n=1}^{\infty} \frac{1}{d^2(Z_0)8\pi|\xi_n|\Omega_0} |u_n(Z_0 - Z)|^2 |u_n(Z)|^2 e^{-2\Im{\xi_n}\Omega_0} \right) \left( \frac{|S_0|^2}{k^2} \right) N_{fish}$$

Equation 2.29 can also be written in terms of the waveguide Green function, Eq. A.24, expressed in absolute coordinates as
\[
I_{s,\text{sum,fish}}(R_r - R|R_0 - R)) = (4\pi)^4 |G(R_r|R)|^2 |G(R|R_0)|^2 \left\langle \frac{|S_o|^2}{k^2} \right\rangle N_{\text{fish}} \tag{2.30}
\]

Since the scatter function of each fish is omnidirectional, the scattered field from each fish in the waveguide can be accurately expressed using the sonar equation A.12 as discussed in chapter 7. When the scattering from the fish school is diffuse, the mean intensity of the total scattered field, Eq. 2.30, is simply an incoherent sum of the mean scattered field intensity from the individual fish which can be easily obtained using the sonar equation.

\section*{2.4 An Absolute Reference Frame}

To compute reverberation from wide and heterogeneous areas of seafloor or a number of distributed scatterers, it is convenient to recast Eq. (2.1) in terms of an absolute, rather than target-centered spatial coordinate system. Let this system be defined by coordinates \( R = (X, Y, Z) \) whose axes are parallel to those of the \( x,y,z \) target-centered system, where the positive Z axis is again downward pointing, but whose origin lies at the ocean surface, for example, where \( z = -D \) in the target-oriented frame. In this more general frame, the source position is defined by \( R_0 = (X_0, Y_0, Z_0) \), the receiver positions by \( R_r = (X_r, Y_r, Z_r) \) and \( R_r = (X'_r, Y'_r, Z'_r) \), and the center of a given scattering patch by \( R = (X, Y, Z) \), where for example, \( X = R \sin \theta \cos \varphi \), \( Y = R \sin \theta \sin \varphi \), \( Z = R \cos \theta \), and \( R^2 = X^2 + Y^2 + Z^2 \). The origin of all these coordinate systems are colocated and the axes are parallel. Spatial coordinates are translated from the target-oriented to the absolute frame by substituting \( r = R_r - R \), \( r' = R'_r - R \), and \( r_0 = R_0 - R \) in Eqs. (2.1) to (2.3). This leads for example, to
\[
A_m(R_r - R) = \frac{i u_m(Z_r - Z) N_m^{(1)}}{d(Z) (8\pi \xi_m \sqrt{(X_r - X)^2 + (Y_r - Y)^2})^{1/2}} \times e^{i (\xi_m \sqrt{(X_r - X)^2 + (Y_r - Y)^2} + \gamma_m Z - \pi/4)},
\]

\[
B_m(R_r - R) = \frac{i u_m(Z_r - Z) N_m^{(2)}}{d(Z) (8\pi \xi_m \sqrt{(X_r - X)^2 + (Y_r - Y)^2})^{1/2}} \times e^{i (\xi_m \sqrt{(X_r - X)^2 + (Y_r - Y)^2} - \gamma_m Z - \pi/4)},
\]

(2.31)

by making the substitutions

\[
\begin{align*}
  x_0 &= (X_0 - X), \quad y_0 = (Y_0 - Y), \quad z_0 = Z_0 - Z, \\
  x &= (X_r - X), \quad y = (Y_r - Y), \quad z = Z_r - Z \\
  \rho_0 &= \sqrt{(X_0 - X)^2 + (Y_0 - Y)^2}, \\
  \rho &= \sqrt{(X_r - X)^2 + (Y_r - Y)^2}.
\end{align*}
\]

(2.32)

in Eqs. (2.1) to (2.3), where

\[
\begin{align*}
  \cos \phi_0 &= x_0 / \rho_0, \quad \sin \phi_0 = y_0 / \rho_0, \\
  \cos \phi &= x / \rho, \quad \sin \phi = y / \rho.
\end{align*}
\]

(2.33)

It must be stressed that the planewave amplitudes and vertical wavenumbers are evaluated in the layer of the scattering patch. The covariance of the scatter function for a given patch in the absolute, rather than object-oriented frame, then becomes

\[
C_{mn}(R_r - R, R_r' - R | R_0 - R) = C_{mn}(r, r'|r_0).
\]

(2.34)
2.5 Shallow Water Reverberation

Reverberation, as measured with an active sonar system, is taken to be any and all echoes returning from the environment rather than the intended target. The characteristics of reverberation then depend not only on the environment but also the geometry of the source and receiver as well as the signal waveform. In field measurements, reverberation is measured as a function of time. It can often be decomposed into two components. The most prevalent is a diffuse component. This has instantaneous intensity that typically undergoes random fluctuations that obey the central limit theorem about an expected value that decays uniformly with time. For reverberation to be diffuse, the scattering region that contributes to the intensity measured at a given instant must be large compared to the mean wavelength. This region, referred to as the system resolution footprint, will be considerably smaller for data beamformed with a high resolution array than for data received by an omnidirectional receiver. The second component, known as clutter, is here defined as any discrete temporal event, caused by an anomalous scatterer, that stands significantly above the diffuse reverberation background. Here 'significantly above' means much more than one standard deviation in sound pressure level. For certain systems that employ high resolution temporal processing and operate in weakly-dispersive waveguides, there may be no diffuse component to the reverberation. In this case coherent temporal oscillations may be found in reverberant intensity measurements that are due to modal interference as noted by Ellis [16]. Lepage has recently investigated similar coherent effects under a narrowband approximation for an omni-directional receiver [37].

2.5.1 Charting diffuse reverberation when system integration time spans dominant signal energy

A simpler analytic approach than the saddle point approximation can be employed to investigate system performance when the integration time of the measurement system $T$ is sufficiently long to include the dominant signal energy returned from the target.
or scattering patch. In this case, Parseval’s theorem can be applied to the Fourier integral of Eq. (2.10), converted to absolute coordinates, to obtain the time-averaged intensity expected at \( \mathbf{R}_r \) from a target or scattering patch at \( \mathbf{R} \) due to a source at \( \mathbf{R}_0 \).

\[
I(\mathbf{R}, \mathbf{R}_r, \mathbf{R}_0, t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} \langle |\tilde{\psi}_s(\mathbf{R}_r - \mathbf{R}|\mathbf{R}_0 - \mathbf{R}|t_0)\rangle^2 dt_0
\approx \frac{1}{T} \int_{-\infty}^{+\infty} \langle |\tilde{\psi}_s(\mathbf{R}_r - \mathbf{R}|\mathbf{R}_0 - \mathbf{R}|t_0)\rangle^2 dt_0
= \frac{1}{T} \int_{-\infty}^{+\infty} |Q(f)|^2 \langle |\tilde{\phi}_s(\mathbf{R}_r - \mathbf{R}|\mathbf{R}_0 - \mathbf{R})\rangle^2 df,
\tag{2.35}
\]

where \( t - T/2 \) is less than or equal to the arrival time of the scattered signal.

This type of incoherent integration is typically used in the reception of narrow band source waveforms, and is also often used in the analysis of broadband returns from explosive sources such as SUS[73] where the exact time function of the source is unknown. While it is equally valid for waveforms of arbitrary bandwidth, it does not take advantage of the full pulse compression possible for broadband waveforms. A center frequency approximation to Eq. 2.35 can often be made for narrow-band signals as discussed in Appendix B.

If the time spread of the signal due to dispersion in the waveguide \( \Delta \tau_s \) is small compared to the time duration \( T_s \) of the source signal, the expected horizontal range resolution \( \Delta \rho \) of the system will take roughly the same form as in free space \( \Delta \rho = \bar{c} T_s / 2 \), for narrowband signals, where \( \bar{c} \) is the mean horizontal propagation speed of the signal between source and receiver in the waveguide. In this case, the integration time \( T \) of the system can be set to its minimum value of \( T_s \). Both \( \Delta \tau_s \) and \( \bar{c} \) can be quantitatively defined in terms of the received field as in Ref. [3]. They depend on the acoustic properties of the waveguide, the signal time dependence and source-receiver geometry. For the narrowband examples of Chapter 3, simulations in Ref. [3] show that \( \bar{c} \approx 1500 \text{ m/s}, \ t\bar{c} \approx \rho + \rho_0 \), and \( \Delta \tau_s / T_s \) is small, where \( \rho \) and \( \rho_0 \) are defined in Eq. 2.32.
A typical bistatic sonar system will resolve a patch of seafloor \( A(\mathbf{R}, \mathbf{R}_r, \mathbf{R}_0) \), the dimensions of which depend on the receiving array aperture, frequency, and the bistatic geometry of the source, receiver and seafloor patch as discussed in Appendix C of Ref. [45] and Refs. [40] and [43]. For a monostatic measurement \( A = \rho \Delta \rho \Delta \varphi \), where \( \varphi = \lambda / L_A \) is the Rayleigh resolution of the horizontal receiving aperture of length \( L_A \).

For convenience, assume that the horizontal origin in an absolute reference frame is chosen to be at the center of the receiving array \( \mathbf{Z_r} = (0, 0, Z_r) \). Let the beamformed output of a receiving array located along the \( Y_r \)-axis, obtained by spatial Fourier transform of the time-harmonic scattered field across the array aperture, be denoted by

\[
\Phi_B(\varphi_s, Z_r, \mathbf{R}, \mathbf{R}_0) = \int_{-\infty}^{\infty} T(Y_r) Q(f) \Phi_s(\mathbf{R}_r - \mathbf{R}_0, \mathbf{R} - \mathbf{R}_0) e^{i k \sin \varphi_s Y_r} dY_r, \tag{2.36}
\]

where \( \varphi_s \) is the azimuth the array is steered towards, \( \varphi \) is the azimuth of the scattering patch, \( T(Y_r) \) is the array taper function. Suppose a uniform rectangular taper function is used with \( T(Y_r) = 1 / L_A \) for \(-L_A/2 \leq Y_r \leq L_A/2\) and zero elsewhere, and the seafloor scattering patch is in the far field of the array, such that \( |\mathbf{R}| > L_A^2 / \lambda \), and the scattering patch behaves as a point target to the array so that the angle it subtends at the array is less than \( \lambda / L_A \). Under these assumptions, the spectral density or field variance received from this patch can be well approximated by

\[
\langle |\Phi_B(\varphi_s, Z_r, \mathbf{R}, \mathbf{R}_0)|^2 \rangle - \langle |\Phi_B(\varphi_s, Z_r, \mathbf{R}, \mathbf{R}_0)|^2 \rangle^2 = \frac{4 \pi^4}{k^2} |Q(f)|^2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} |A_m(Z_r - \mathbf{R})|^2 |A_n(\mathbf{R}_0 - \mathbf{R})|^2 C_{mn}(Z_r - \mathbf{R}, Z_r - \mathbf{R} | \mathbf{R}_0 - \mathbf{R}) \times \left| \frac{\sin\{(L_A/2) \sin[\varphi(k - \Re\{\xi_m\})]\}}{(L_A/2) \sin[\varphi(k - \Re\{\xi_m\})]} \right|^2 \tag{2.37}
\]

upon substitution of Eq. 2.28 into Eq. 2.36 with \( \varphi_s = \varphi \) so the array is steered toward the patch. With the assumption that the resolution footprint \( A \) is much larger than
\( \Delta A \), the area of a given patch, and all patches are statistically independent, the total variance of the received field from seafloor within the system resolution footprint can be written as the sum of the variances of each patch following Eq. 6.16 where

\[ f(X', Y') = \frac{1}{n_x \Delta A} \]

as

\[ V_B(X, Y) = \int \int_{A(R, Z_r, R_0)} \left( |\Phi_B(\varphi_s, Z_r, R, R_0)|^2 - |\Phi_B(\varphi_s, Z_r, R, R_0)|^2 \right) \frac{1}{n_z \Delta A} dX'dY', \]

(2.38)

where \( \hat{n} = (n_x, n_y, n_z) \) is the surface normal at \( R \). Since the differential area \( dX'dY' \) must be normalized by the horizontal projected area of each potentially inclined patch at \( R' \) to allow horizontal integration, Eq. 2.38 does not allow vertical patches.

When Parseval's theorem is invoked again under the assumption that the integration time of the measurement system includes the dominant energy returned from the resolved patch of seafloor, after time-domain beamforming and finite time averaging by the receiver over period \( T \), the field variance from seafloor within the system resolution footprint of area \( A(R, R_r, R_0) \) centered at \( (X, Y) \) becomes

\[ \overline{V}_B(X, Y) = \frac{1}{T} \int_{-\infty}^{\infty} V_B(X, Y)df. \]

(2.39)

Equations 2.37-2.39 imply a reduction in reverberation level for off-broadside beams due solely to modal dispersion. Only at broadside does the phase speed of the incident waves match that expected under the non-dispersive assumptions of planewave beamforming. Only broadside beamforming is considered in the simulations of Chapter 3 to eliminate this effect from the analysis. The effects of modal dispersion on beamformed reverberation are investigated in Ref. [3].

Under the present assumptions, reverberation measured in time can be charted in space for any bistatic geometry using a look-up table comprised of the mean time delay from source to scattering patch \( \tau(r_t, r_0) \) and scattering patch to receiver \( \tau(r, r_t) \)
Similarly, reverberation modeled with the spatial formulation of Eqs. 2.38 and 2.39 can be made a function of time by reversing the procedure.

### 2.5.2 Lambert-Mackenzie scattering and reverberation

A number of simplifications are possible when the scattering surface has Lambertian behavior. The covariance of the scattering function for a Lambertian scattering patch of area $\Delta A$ with albedo $\epsilon$ takes the form of Eq. 6.17 with

$$C_{mp}(\mathbf{R}_r - \mathbf{R}, \mathbf{R}'_r - \mathbf{R}_0 - \mathbf{R}) = k^2 \frac{\epsilon}{\pi} |\hat{i}_{m, \text{scat}} \cdot \hat{n}| |\hat{i}_{p, \text{inc}} \cdot \hat{n}| \Delta A,$$

(2.40)

where $\hat{i}_{p, \text{inc}}$ and $\hat{i}_{m, \text{scat}}$ are the directions in which the downgoing component of the $p$th incident and upgoing component of the $m$th scattered modes propagate and $\hat{n}$ is the seafloor normal, pointing away from the water-column, in a scattering-patch-centered coordinate system where the positive-z axis points downward. The differential scattering cross section of the surface patch is then given by the product of $4\pi/k^2$ and the right hand side of Eq. 2.40.

Under Lambertian scattering, the cross spectral density of the scattered field given in Eq. 2.28 then becomes expressible in terms of single summations

$$\langle \Phi_s(\mathbf{r}|\mathbf{r}_0)\Phi_s^*(\mathbf{r}'|\mathbf{r}_0) \rangle - \langle \Phi_s(\mathbf{r}|\mathbf{r}_0)\rangle \langle \Phi_s^*(\mathbf{r}'|\mathbf{r}_0) \rangle = 4\pi^3 \epsilon \Delta A \left( \sum_{m=1}^{\infty} A_m(\mathbf{r}) A_m^*(\mathbf{r}') |\hat{i}_{m, \text{scat}} \cdot \hat{n}| \right) \left( \sum_{p=1}^{\infty} |A_p(\mathbf{r}_0)|^2 |\hat{i}_{p, \text{inc}} \cdot \hat{n}| \right).$$

(2.41)

The surface projection factors can be written in terms of the incident and scattered wave number components, the measurement geometry and the orientation of the surface patch via
\begin{align*}
|\hat{i}_{m,scat} \cdot \hat{n}| &= \left| \frac{1}{k} (\xi_m \cos \varphi, \xi_m \sin \varphi, -\gamma_m) \cdot (n_x, n_y, n_z) \right| \\
&= \left| \frac{\xi_m}{k} [n_x \cos \varphi + n_y \sin \varphi] - \frac{\gamma_m}{k} n_z \right|,
\end{align*}

(2.42)

\begin{align*}
|\hat{i}_{p,inc} \cdot \hat{n}| &= \left| \frac{1}{k} (\xi_p \cos (\varphi_0 + \pi), \xi_p \sin (\varphi_0 + \pi), +\gamma_p) \cdot (n_x, n_y, n_z) \right| \\
&= \left| \frac{\xi_p}{k} [n_x \cos (\varphi_0 + \pi) + n_y \sin (\varphi_0 + \pi)] + \frac{\gamma_p}{k} n_z \right|,
\end{align*}

(2.43)

where $+\gamma_p$ is the vertical wavenumber of the downgoing $p$th incident modal planewave and $-\gamma_m$ is the vertical wavenumber of the $m$th upgoing scattered modal planewave.

The scattered field covariance from a given seafloor patch in a target oriented frame is

\[\langle \Phi_s(r|\mathbf{r}_0)\Phi^*_s(r'|\mathbf{r}_0) \rangle - \langle \Phi_s(r|\mathbf{r}_0) \rangle \langle \Phi^*_s(r'|\mathbf{r}_0) \rangle \]

\[= 4^4 \pi^3 \epsilon \Delta A \left( \sum_{m=1}^{\infty} A_m(r) A^*_m(r') \left| \frac{\xi_m}{k} [n_x \cos \varphi + n_y \sin \varphi] - \frac{\gamma_m}{k} n_z \right| \right) \]

\[\times \left( \sum_{p=1}^{\infty} |A_p(r_0)|^2 \left| \frac{\xi_p}{k} [n_x \cos (\varphi_0 + \pi) + n_y \sin (\varphi_0 + \pi)] + \frac{\gamma_p}{k} n_z \right| \right),\]

(2.44)

where $n_x = n_y = 0$ for a bottom with zero mean inclination. In an absolute frame it becomes
\[
\langle \Phi_s(R_r - R|R_0 - R)\Phi_s^*(R'_r - R|R_0 - R) \rangle
- \langle \Phi_s(R_r - R|R_0 - R)\Phi_s^*(R'_r - R|R_0 - R) \rangle
= 4^4\pi^3\epsilon \Delta A \left( \sum_{m=1}^{\infty} A_m(R_r - R)A_m^*(R'_r - R) \left[ \frac{\xi_m}{k} n_x(X_r - X) + n_y(Y_r - Y) - \frac{\gamma_m}{k} n_z \right] \right)
\times \left( \sum_{p=1}^{\infty} |A_p(R_0 - R)|^2 \left[ \frac{\xi_p}{k} n_x(X - X_0) + n_y(Y - Y_0) + \frac{\gamma_p}{k} n_z \right] \right),
\]

where the components of the surface normal are now a function of the \( X, Y, Z \) position of the surface patch center. Under far field assumption, the field variance received from seafloor within the system resolution footprint centered at \( (X, Y) \) and averaged over time period \( T \) can be well approximated by

\[
\overline{V_B}(X, Y) = \frac{4^4\pi^3\epsilon}{T} \int_{-\infty}^{\infty} |Q(f)|^2 \int_{A(R_r, R'_r, R_0)} \left( \sum_{m=1}^{\infty} |A_m(Z_r - R')|^2 \left[ \frac{\xi_m}{k} n_x(X_r - X') + n_y(Y_r - Y') - \frac{\gamma_m}{k} n_z \right] \right)
\times \left[ \frac{\sin\{((L_A/2)\sin[\varphi(k - \Re\{\xi_m\})])^2\}}{(L_A/2)\sin[\varphi(k - \Re\{\xi_m\})]} \right)^2
dX'dY'df,
\]

when \( T \) is sufficiently large for the dominant energy of the scattered field to be received. This result for Lambertian seafloor offers significant advantages in implementation through the separation of the incident and scattered modal summations. For narrowband waveforms, terms within the modal summations of Eq. 2.46 often vary so slowly that they can be approximated as a constant function of frequency over the dominant portion of the spectral window \( Q(f) \). This greatly simplifies computations.
as shown in Appendix B.

We would like to point out some more typographical errors in Ref. [48]. In Eqs. 42-45, the vertical wavenumber \( \gamma \) should be replaced with \(-\gamma \) as it corresponds to an upgoing scattered modal planewave, and in Eqs. 42-45, the vertical wavenumber \(-\gamma_m \) should be replaced with \(+\gamma_m \) as it corresponds to a downgoing incident modal planewave.

### 2.5.3 Perturbation theory for diffuse rough surface scattering and reverberation

Perturbation theory can also be used to calculate the field scattered by a rough surface. The advantage of perturbation theory, when it is applicable, is that it is derived from first principles and so requires knowledge of only the geo-acoustic properties of the media, such as sound speed and density, as well as a second moment characterization of the statistical properties of the scattering surface.

Let the \( x \) and \( y \) components of the gradient of the surface \( z_s(x,y) \) be denoted by

\[
p = \frac{\partial z_s}{\partial x}, \tag{2.47}
\]

\[
q = \frac{\partial z_s}{\partial y}. \tag{2.48}
\]

The surface normal can be expressed as

\[
n = \frac{(-p, -q, 1)}{\sqrt{1 + p^2 + q^2}}, \tag{2.49}
\]

along with two orthonormal surface tangents.
\[ t_1 = \frac{(1, 0, p)}{\sqrt{1 + p^2}}, \]  

(2.50)

and

\[ t_2 = \frac{(-pq, 1 + p^2, q)}{\sqrt{(1 + p^2)(1 + p^2 + q^2)}}, \]  

(2.51)

where \( t_1 \) is obtained by taking an infinitesimal step along the surface on the \( x \) axis, and \( t_2 \) is the crossproduct of \( n \) and \( t_1 \).

The projections of the incident and scattered wavenumber vectors on the surface then become

\[ K_i = (k_i \cdot t_1) t_1 + (k_i \cdot t_2) t_2, \]  

(2.52)

\[ K = (k \cdot t_1) t_1 + (k \cdot t_2) t_2, \]  

(2.53)

where, for incident mode \( n \) and scattered mode \( m \),

\[ k_i = (\xi_n \cos(\phi_0 + \pi), \xi_n \sin(\phi_0 + \pi), \gamma_n), \]  

(2.54)

\[ k = (\xi_m \cos \phi, \xi_m \sin \phi, -\gamma_n), \]  

(2.55)

so that

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and, for example,

\[ |K_i|^2 = (k_i \cdot t_1)^2 + (k_i \cdot t_2)^2, \]  

\[ |K|^2 = (k \cdot t_1)^2 + (k \cdot t_2)^2. \]  

A planewave incident from medium 1 half-space that is reflected from strata below has total reflection coefficient[52]

\[ \Gamma(K_i) = \frac{\Gamma_{12}(K_i) + \Gamma'(K_i)e^{i2\gamma(2)h_1}}{1 + \Gamma_{12}(K_i)\Gamma'(K_i)e^{i2\gamma(2)h_1}}, \]  

where

\[ \Gamma_{12}(K_i) = \frac{\rho_2/\sqrt{k_2^2 - K_i^2} - \rho_1/\sqrt{k_1^2 - K_i^2}}{\rho_2/\sqrt{k_2^2 - K_i^2} + \rho_1/\sqrt{k_1^2 - K_i^2}} \]  

is the reflection coefficient from the medium 1 to medium 2 interface, \( \Gamma(K_i) \) is the total reflection coefficient from all strata below medium 2 for a planewave incident from medium 2, \( h_1 \) is the thickness of the layer containing medium 2 and \( \gamma(2) \) is the vertical wavenumber component of medium 2.

The differential scattering cross-section of a surface patch of area \( \Delta A \), from first order perturbation theory, can be expressed as [52]
\[
\sigma_{pert}(\alpha, \beta; \alpha_i, \beta_i) = 4\pi \Delta A \left(\frac{k_i^4}{4}\right) \left|\frac{\Gamma(K) + 1}{\Gamma(K_i) + 1}\right|^2 \\
\times \left|1 - \frac{\kappa^2}{d_{bt}} + \left(\frac{1}{d_{bt}} - 1\right) \left(\frac{K \cdot K_i}{k_i^2} - \frac{P_g^2(k, k_i, n)}{d_{bt}}\right)\right|^2 W(K - K_i),
\]

(2.61)

where

\[
P_g^2(k, k_i, n) = d_{bt}^2 \left|\frac{k_i \cdot n}{k_i}\right| \left|\frac{k \cdot n}{k}\right| \left(\frac{1 - \Gamma(K_i)}{1 + \Gamma(K_i)}\right) \left(\frac{1 - \Gamma(K)}{1 + \Gamma(K)}\right),
\]

(2.62)

and \(d_{bt} = d_b/d_t\), \(\kappa = \kappa_t/\kappa_b\), where \(d_t\) and \(d_b\) are the respective densities above and below the scattering interface and \(\kappa_t\) and \(\kappa_b\) are the respective wavenumber magnitudes above and below the scattering interface.

Following Moe and Jackson[52], the roughness of the given surface patch is assumed to follow the isotropic power law

\[
W(K) = w_2 |K|^{-\gamma}.
\]

(2.63)

With the assumption that the scattering patch is much greater than the wavelength and satisfies the conditions Eqs. 6.14 and 6.12 so that the incident and scattered modes are decorrelated by the scattering process, the covariance of the scatter function is given by Eq. 6.17 with

\[
C_{mn}(\mathbf{r}, \mathbf{r}'|\mathbf{r}_0) = \frac{k^2}{4\pi} \sigma_{pert}(\alpha_m, \phi; \alpha_n, \phi_0 + \pi).
\]

(2.64)

Upon substituting Eq. 2.64 into Eqs. 2.28 or 2.38 after beamforming, it is found that evaluation of the covariance of the field scattered from a rough surface patch that
obeys first order perturbation theory involves a double summation over the waveguide modes. Evaluating this is significantly more computationally intensive than the product of single modal summations found in the Lambert-Mackenzie formulation.

2.5.4 Coherent Scattering from Deterministic Extended Geologic Features and Highly-Resolved Stochastic Seafloor Patches

There are two general kinds of seafloor scatterers that do not decorrelate the incident or scattered modes. A seafloor scatterer of the first kind is a deterministic feature, with known or computable far field scatter function, that can have arbitrary size compared to the wavelength so long as it falls within the resolution footprint of the active sonar system. The feature must be distinct from the otherwise range-independent boundaries of the stratified medium in order to induce scattering.

A compelling canonical example of a seafloor scatterer of the first kind is a smooth flat inclined segment of the seafloor, such as a seafloor or sub-seafloor river channel, ice-berg scour, or submerged hillside, that can be modeled as a flat plate with scattering characteristics determined by its size, inclination and the local geo-acoustic properties of the interface. The 3-D scatter function for a rectangular surface patch with total reflection coefficient $\Gamma(K_i)$, for example, can be readily determined by applying Green’s Theorem, Eq. A.7, for a planewave, with wavenumber magnitude $k_i$, incident in the direction $(\alpha_i, \beta_i)$ and a far field receiver in the direction $(\alpha, \beta)$ with respect to the patch centroid. If the patch is assumed to be at inclination $\chi$ from horizontal, where the angle $\chi$ comprises a counter clockwise rotation about the y-axis, the scatter function takes the form
\[ S(\alpha, \beta, \alpha_i, \beta_i) = \frac{k_i^2}{4\pi} L_x L_y \left( [1 - \Gamma(K_i)] \left\{ \cos \alpha_i \cos \chi + \sin \alpha_i \sin \chi \cos \beta_i \right\} + [1 + \Gamma(K_i)] \left\{ \cos \alpha \cos \chi + \sin \alpha \sin \chi \cos \beta \right\} \right) \]
\[ \times \text{sinc} \left[ \frac{k_i L_x}{2} \left\{ \sin \alpha_i \cos \beta_i \cos \chi - \cos \alpha_i \sin \chi \right\} - \left( \sin \alpha \cos \beta \cos \chi - \cos \alpha \sin \chi \right) \right] \]
\[ \times \text{sinc} \left[ \frac{k_i L_y}{2} \left\{ \sin \alpha_i \sin \beta_i - \sin \alpha \sin \beta \right\} \right], \quad (2.65) \]

where \( \text{sinc}(x) \) is defined as \( \frac{\sin x}{x} \). The reflection coefficient can be determined from Eq (52) with the understanding that, in the present geometry, the squared magnitude of the transverse component of the incident wavenumber vector on the inclined surface patch is

\[ K_i^2 = k_i^2 - k_i^2 (\cos \alpha_i \cos \chi + \sin \alpha_i \sin \chi \cos \beta_i)^2 \quad (2.66) \]

In Eq. 2.65, it was assumed that the total field on the underside of the coherent scatterer is zero. This corresponds to perfect shadowing by the patch so that there is no power transmitted through the surface, i.e. there is perfect absorption by the scatterer.

Irregularities on the surface of the feature can make its scatter function deviate from that given in Eq. 2.65. For realistic seafloor and sub-seafloor riverbanks, however, it is reasonable to assume that, for a low frequency active system[43] at long-range in a shallow water waveguide where propagation is near horizontal, the product of the amplitude of such irregularities and the normal component of the wavenumber vector with respect to the surface will be small enough that the irregularities will have a negligible effect on the field from the riverbank.

A seafloor scatterer of the second kind is a randomly rough rather than deterministic feature but is appropriately modeled with completely coherent modes when
the ratio of wavelength to system range resolution, $\lambda/\Delta \rho$, is near or greater than the equivalent vertical propagation angle of the highest order trapped mode, which in many shallow water scenarios is roughly the bottom critical angle. This situation occurs for active sonar systems with high range-resolution and can lead to the formation of range-dependent rings in charted reverberant intensity caused by modal interference (Lepage[37] has recently described scenarios in which such rings can form even in narrowband reverberation at short ranges.) The level of returns can be estimated by appropriately modeling the seafloor scatter function. If the system resolution footprint extends over many wavelengths in any direction and the correlation length of surface roughness is much smaller than the system resolution footprint, then the scatter function for the seafloor over this area can be treated as a fluctuating target. If the resolution footprint is on the order of the wavelength or the correlation length of surface roughness, a quasi-deterministic description of the scattering process can be used.

2.6 Summary

One of the greatest challenges to active sonar operations in shallow water arises when echo returns from the intended target become indistinguishable from reverberation returned by the waveguide boundaries and volume. To determine conditions in which a typical low-frequency active sonar system may operate effectively in a shallow water waveguide, a unified model for submerged object scattering and reverberation is developed. The approach is to use a waveguide scattering model that follows directly from Green’s theorem but that takes advantage of simplifying single-scatter and farfield approximations that apply to a wide variety of problems where the source and receiver are distant from the target. To treat reverberation from randomly rough boundaries and stochastic volume inheterogeneities, the waveguide scattering model is generalized to include stochastic targets. Analytic expressions for the spatial covariance of the field scattered from a stochastic target are then obtained in terms of the waveguide Green function and the covariance of the target’s planewave scat-
ter function. This makes the formulation amenable to a wide variety of approaches for computing a stochastic target’s scatter function. For diffuse seafloor reverberation, two approaches are adopted, an empirical one of Lambert and Mackenzie and a fundamental one based on first order perturbation theory. It is most convenient to describe the diffuse component of distant seafloor reverberation with a modal formulation since the modes comprise the statistical entities of the field that the scattering surface may decorrelate. In Chapter 6, we derive the criteria necessary for scattering in a waveguide to become diffuse.

Since reverberation is measured in time but the waveguide scattering formulation is for harmonic field components, the time dependence of the field scattered by a distant object from a source of arbitrary time dependence is derived analytically using the saddle point method. The resulting expression is given in terms of modal group velocities, the frequency of which vary as a function of time and source, receiver and target position. A simpler analytic approach involving Parseval’s theorem can be applied when the integration time of the measurement system is sufficiently long to include the dominant energy returned from the target or scattering patch. This approach is used in the illustrative examples of Chapter 3.

A viewer-oriented reference frame is then adopted, translating from the traditional target-oriented frame of waveguide scatter theory, to incorporate the continuous distribution of scatterers encountered in waveguide boundary and volume reverberation. This enables analytic expressions to be developed for the reverberant field returned bistatically from seafloor within the resolution footprint of a typical active sonar system after narrowband beamforming with a horizontal array.
Chapter 3

Effect of Environment, Measurement Geometry and Sonar Parameters on the Imaging of Submerged Objects and Geomorphology in a Shallow Water Waveguide

In this chapter, we investigate several active detection scenarios in a shallow water waveguide for both deterministic and fluctuating objects, as well as geologic clutter features. These targets have to be detected against the background of diffuse reverberation from the environment which limits a target's detectability. The reverberant background depends on range and azimuth because it varies with the range and azimuth dependent resolution footprint of the sonar system. Scenarios involving an active source and a horizontal receiving array are investigated. We determine how an object's detectability varies with the properties of the environment, source-receiver location, source bandwidth, and object properties. Both the scattered field
level from the target and the diffuse reverberation level from the environment are computed using the unified model described in chapter 2. We also compute the probabilities of detection (POD) and false alarm (PFA) for some cases as a function of the target-signal-to-diffuse-reverberation ratio (SRR). The mechanisms for deterministic scattering from extended geologic features are investigated in this chapter. In addition, we also investigate the incoherent scattering from schools of fish.

3.1 Widely Representative Environment and Measurement Geometries

In all the illustrative examples of this section, a water column of 100-m depth is used to simulate a typical continental shelf environment. The sound speed structure of the water column varies from iso-velocity to downward refracting layers with constant density of 1 g/cm³ and attenuation of 6.0 × 10⁻⁵ dB/λ. The seabed is comprised of sand or silt half-spaces, with up to two sediment layers, comprised of sand or silt, over a sand or silt half-space. The density, sound speed and attenuation are taken to be 1.9 g/cm³, 1700 m/s, and 0.8 dB/λ for sand, 1.4 g/cm³, 1520 m/s, and 0.3 dB/λ for silt, and 1.2 g/cm³, 1510 m/s, and 0.3 dB/λ for light-silt. Scattering and reverberation calculations are made for a submerged target, roughness at the water-seabed interface, roughness at the interface between the upper seabed layer and lower half-space, as well as for anomalous features of the seafloor or sub-seafloor that return geological clutter. The latter are taken to be seafloor river banks at the water-seabed interface or sub-seafloor riverbanks at the interface between the upper sediment layer and lower half-space. The geometry of the waveguide is sketched in Fig. 3-1.

A horizontal line array with \( N = 32 \) equally spaced elements of length \( L_A = (N - 1)λ/2 \) at \( f = 300 \) Hz is used as a receiver and a cw pulse of \( T = 1/2 \) s duration centered at \( f = 300 \) Hz is used as a source waveform for all simulations of diffuse reverberation. Targets beyond \( L_A^2/λ \) are in the far field of the array, which begins at roughly 1.2 km. The beamformed field from an object that falls within the broadside
Figure 3-1: The geometry of the waveguide which has a water column comprised of upper layer sound speed $c_{w1}$ for $0 < Z < 25$, lower layer sound speed $c_{w2}$ for $35 < Z < 100$, and transition layer sound speed $c_{w1} - (c_{w1} - c_{w2})(Z - 25)/10$ for $25 < Z < 35$. The water column density is $d_w = 1000 \text{ kg/m}^3$ and the attenuation is $\alpha_w = 6.0 \times 10^{-5} \text{ dB/\lambda}$. The bottom can have up to two sediment layers. The upper and middle sediment layers have respective thicknesses, sound speeds, densities and attenuations of $h_1$, $c_{b1}$, $d_{b1}$, $\alpha_{b1}$, and $h_2$, $c_{b2}$, $d_{b2}$, $\alpha_{b2}$, overlying a sediment half-space of sound speed $c_{b3}$, density $d_{b3}$ and attenuation $\alpha_{b3}$. The monopole source is co-located with receiving array center, with array axis normal to the range-depth plane of the sketch. Source and receiver may be placed anywhere in the water column. The submerged target may be placed in the upper or lower layers of the water column where sound speed is constant as indicated in Fig. 3-3. Seafloor and buried riverbank features may also be included at the water-sediment and sediment-layer to sediment-half-space interfaces as indicated in Fig. 3-7. Squiggly lines indicate statistically rough interfaces.
beam of the array, in the absence of other sources or scatterers, equals the field received from that object by a single hydrophone at the array center when Eq. (2.36) is used with uniform taper \( T(Yr) = 1/L_A \). If the same object is placed at the same range but within an off-broadside beam, a reduction in the beamformed output may occur due to modal dispersion, as seen in Eq. 2.37. For simplicity, only objects and reverberation within the broadside beam are considered in the present paper. Only monostatic scenarios are considered, where the source is located at the center of the receiving array. This leads to a range-dependent resolution footprint \( A = \rho \Delta \rho \Delta \phi \), where \( d \rho = cT/2 = 375 \text{ m} \) and \( d \phi = \lambda/L_A \approx 3.7^\circ \) for the given array, frequency and cw pulse length. The range and azimuth dependent resolution footprint of a horizontal line array is illustrated in Fig. 3-2.

A center frequency approximation, at \( f = 300 \text{ Hz} \), is made for all scattering calculations. For reverberation calculations this approximation differs from the full spectral integration by less than 0.1 dB for the examples shown. As may be expected in coherent scattering from targets where modal interference is significant, some range-dependent nulls and valleys in the sound pressure level of the received field found in the single frequency calculation may be partially filled when the full bandwidth is used for the narrowband waveforms considered. Since this filling is window-dependent, as shown in Appendix B only center frequency calculations are presented in the main text. It is also shown in Appendix B that in some valleys of some single frequency calculations the target returns may fall below the expected reverberation level but will be above this level when the full bandwidth of a given narrowband window function is employed.

Only the empirical Lambert-Mackenzie model is used in comparisons between seafloor reverberation and submerged-object returns since insufficient data on the requisite environmental parameters at low frequency are available to make a similar comparison with perturbation theory meaningful. Perturbation theory calculations are only used self-consistently to make inferences about the relative level of returns from different kinds of seafloor scatterers.
Figure 3-2: The range and azimuth dependent resolution footprint of a horizontal line array. The range resolution is given by $\Delta \rho = c/2B$ where $B$ is the bandwidth of the signal. For a narrowband signal of duration $T$ with rectangular window in time, the bandwidth is $B = 1/T$. The cross-range resolution for the array steered to azimuth $\phi$ is $\rho \Delta \phi = \rho \frac{\lambda}{L \cos \phi}$. The cross-range resolution for a line array is smallest at broadside where $\phi = 0$ and it widens for off-broadside beams. The cross-range resolution is $\rho \Delta \phi = \frac{\lambda}{2L}$ at endfire where $\phi = \pi/2$. 
3.2 Submerged target echo versus diffuse reverberation as a function of source-receiver depth, target depth, water column, and bottom stratification

The geometry for active detection of a sphere submerged in an ocean waveguide is sketched in Fig. 3-3 for the illustrative examples of this section. The geometry is monostatic with co-located omni-directional point source and receiving array centers at 50-m depth. The sphere center is also at \( D = 50 \)-m depth at array broadside with variable horizontal range. The field back scattered from a pressure release sphere of radius \( a = 10 \) m at \( f = 300 \) Hz is shown as a function of range in Figs. 3-4(a)-(c) in decibels, i.e. \( 20 \log |\Phi_s| \), for various bottom types under a water column with constant sound speed \( c_w = 1500 \) m/s, where \( c_{w1} = c_{w2} = 1500 \) m/s. The scattered field is computed by Eq. (2.1), with scatter function of the sphere given in Ref. [5].

The variance of the field scattered from the seafloor within the range-dependent resolution footprint of the sonar system under the Lambert-Mackenzie assumption of Eq. (2.46) is also shown in Fig. 3-4 in decibels, i.e., \( 10 \log V_B \). Modal interference is absent due to the modal decoupling assumed in diffuse scattering from large seafloor patches. Ambiguous returns from both sides of the line array are included.

The scattered field from both the target sphere and seafloor is highly dependent on the geo-acoustic parameters of the bottom, as is evident in Fig. 3-4(a) where significant differences arise when the bottom type is changed from sand to silt. The differences arise primarily because the number of trapped modes is significantly larger for the sand half space due to the higher critical angle of \( 28.1^\circ \) for water to sand as compared with the \( 9.3^\circ \) of water to silt. This leads to a correspondingly higher mean level, of roughly 20 dB, for both target and seafloor backscatter and a shorter modal interference length-scale in the scattered field from the sphere.

In the Pekeris waveguide examples of Fig. 3-4(a), the target stands tens of decibels above the expected reverberation within the broadside beam regardless of whether the
Figure 3-3: Three scenarios for the active detection of a submerged pressure release sphere of radius $a = 10$ m. The water column is modeled as either having constant sound speed or as downward refracting. The bottom is composed of either a pure sediment half-space or a single sediment layer over a sediment half-space. (a) Monopole source and horizontal receiving array center are co-located at 50-m depth with target at 50-m depth also. (b) Source and receiving array center are co-located at 10-m depth with target at 50-m depth. (c) Source and receiving array center are co-located at 10-m depth with target at 15-m depth.
Figure 3-4: The scattered field from a submerged pressure-release sphere of radius $a = 10$ m, at $f = 300$ Hz and center at 50-m depth, and Lambert-Mackenzie reverberation from the seafloor within the broadside resolution footprint of the monostatic system as a function of range for a water column with constant sound speed of 1500 m/s, i.e. $c_{w1} = c_{w2} = 1500$ m/s. Monopole source and receiving array center are co-located at 50-m depth. Range increases along the x-axis and depth along the z-axis, with the array axis along the y-axis. Source level is 0 dB re 1 $\mu$Pa @ 1 m. Reverb modeled with $T = 1/2$ sec duration cw source signal at 300 Hz and receiving array resolution $\lambda/L = 3.7^\circ$. (a) Pekeris waveguide examples for bottom half-spaces composed of either sand or silt, i.e. $h_1 = h_2 = 0$. (b) Bottom has a silt layer of either $h_1 = 2$ m or $h_1 = 5$ m overlying a sand half-space, and $h_2=0$. (c) Bottom has a sand layer of either $h_1 = 2$ m or $h_1 = 5$ m overlying a silt half-space, and $h_2 = 0$. Error bars show the 5.6 dB standard deviation in reverb level.
bottom is composed of sand or silt. This signal excess is well above the reverberation level standard deviation of 5.6 dB assuming the seafloor scattering obeys circular complex Gaussian statistics, in accord with the central limit theorem[41]. If a single omni-directional hydrophone placed at the center of the receiving array replaces the full array, the reverberation levels are augmented by roughly $10 \log(2\pi/d\varphi) \approx 20$ dB in Fig. 3-4. The target sphere then no longer consistently stands above the expected reverberation even at short ranges, for example, within a few kilometers. A directional array is then necessary to spatially filter the target from omni-directional reverberation so that detection can be practically achieved in the given scenarios.

The effect of bottom properties on both submerged target scattering and reverberation is again evident when layered bottoms are considered. For the silt-over-sand scenarios, of Fig. 3-4(b), the characteristics of the field scattered from the target are a combination of those found for the silt and sand half-spaces. As the silt layer increases from roughly one-half to a full wavelength, the rate of modal interference decreases, as does the overall level of the scattered field from both the target and bottom. When the layer thickness reaches a full wavelength, the level of reverberation approaches that obtained for a pure silt bottom as range increases. The low critical angle between the water-silt interface enables greater bottom penetration than is possible with a water-sand interface. The high attenuation of the silt layer then leads to bottom loss that increases with the thickness of the layer.

For the sand-over-silt scenarios, of Fig. 3-4(c), the field scattered from the target greatly resembles that obtained for the pure sand bottom of Fig. 3-4(a). The match becomes better as the sand layer increases in thickness from one-half to a full wavelength, in which case the reverberation increases from a few decibels below to roughly the level found for a pure sand bottom. In the latter case, the silt half-space is effectively insulated from the watercolumn by evanescent decay of the trapped modes in the sand layer.

The absolute and relative levels of target and reverberation echo returns are highly dependent upon the water-column sound-speed structure as well as source, receiver and target depth. To illustrate this, consider the typical shallow water downward
refracting profile shown in Fig. 3-1, with \( c_{w1} = 1520 \text{ m/s}, \ c_{w2} = 1500 \text{ m/s} \) and a linear transition region in between, that is similar to what is found in continental shelf waters in late spring and summer months. Monostatic measurements of the field scattered from a 10-m radius pressure-release sphere are again made with the same array and cw tone used in the previous examples. The target is at array broadside and both target returns and reverb within the broadside beam are plotted as a function of range, where the reverb is computed again for a \( T = 1/2 \) s cw at 300 Hz center frequency. Three combinations of monostatic source-receiver and target depths are considered, as illustrated in Fig. 3-3.

First consider the case, in Fig. 3-5(a), where the source, receiving array and sphere center are at 50 m depth just as in the Pekeris waveguide examples. For the sand bottom, the levels are similar to those found in the corresponding example of Fig. 3-4(a), with the target standing out by tens of decibels. For the silt bottom, the target still stands tens of decibels above the reverberation but the absolute levels of the scattered fields decay more rapidly with range in the present scenario since the downward refracting profile causes more acoustic energy to penetrate into the bottom.

Loss of energy to the bottom is augmented when the source and receiver array are placed in the mixed layer, at 10 m depth, while the target remains with center at 50 m depth, as shown in Fig. 3-5(b). The absolute levels of both the field scattered from the target sphere and the seafloor are reduced by tens of decibels beyond a few kilometers range for the silt bottom. For the sand bottom, the reverberation level is not significantly changed by moving the source and receiver into the mixed layer. Returns from the target sphere, however, no longer stand prominently enough above the expected reverberation to insure detection, given a 5.6 dB standard deviation in reverberation level.

The situation for detection again changes when the target sphere is placed in the mixed layer, with sphere center at 15-m depth, along with the source and receiver at 10-m depth as shown in Fig. 3-5(c). This is especially so for the silt bottom, where the scattered field from the target sphere becomes so greatly reduced, when compared to the previous examples of this section, that its returns only stand above

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Figure 3-5: Same as Fig. 3-4 except water column is layered with $c_{w1}=1520$ m/s, $c_{w2}=1500$ m/s and monostatic source-receiver as well as target sphere are at variable depth. Only the cases of pure sand or pure silt bottom half-spaces are shown. (a) Source-receiver and sphere center are at 50-m depth. (b) Source-receiver are at 10-m depth while sphere center is at 50-m depth. (c) Source-receiver are at 10-m depth while sphere center is at 15-m depth.
the expected reverberation level within roughly 16 km range. It is interesting that for the sand bottom, the placement of the target and source-receiver in the mixed layer leads to more favorable conditions for detection, which should be possible beyond 50 km range, than if the only the source-receiver were placed in the mixed layer and the target was in the middle of the water column as in Fig. 3-5(b). This is because the higher order modes stimulated by the shallow source, receiver and target can be supported by the high-critical-angle sand bottom, but not by the silt bottom.

The exercise of changing the depths of the source-receiver and target is repeated in Fig. 3-6 for a constant sound speed water column. For the sand bottom, the level of the field scattered from the sphere is not affected significantly by moving the source-receiver and target depths. For the silt bottom, however, a significant decrease in the sphere’s echo-return level is found for shallow source-receiver and target placements. Apparently, these shallow placements stimulate higher order modes that are not supported by the silt bottom. These results should be compared to those found in Fig. 3-4(a) for source-receiver and target depths in the middle of the waveguide at 50 m.
3.3 Geological Clutter Versus Diffuse Seafloor Reverberation

Geomorphic features of the seafloor can return echoes that stand well above the diffuse reverberation background described in the previous section. Since these echoes appear as discrete events in time or range, they may be used to remotely image seafloor or sub-seafloor geomorphology in geophysical applications. They may, however, also be confused with returns from a submerged target in an active detection scenario.

Both coherent and incoherent scattering from the canonical seafloor and sub-seafloor features, shown in Fig. 3-7, are investigated. Both kinds of features are modeled as a flat 100-m by 100-m surfaces at an inclination of $10^\circ$. The dimensions and inclination are based on actual geophysical data characterizing seafloor and sub-seafloor riverbanks[34]. Seafloor and sub-seafloor river channels are commonly found in continental shelf waters after a sea level rise. The latter requires an additional influx of sedimentation. In all cases to be considered here, the waveguide is modeled as an iso-velocity water column overlying one or two sediment layers that cover a sediment half space.

For the coherent calculation, the riverbank is treated as a smooth but finite square surface with reflection coefficient appropriate to the given boundary conditions, including multiple reflection from various layers. The coherent scattered field from the riverbank in the layered waveguide follows when the scatter function for the smooth riverbank, given in Eq. (2.65), is inserted into Eq. (2.1). Coherent scattering from the riverbank is then completely determined by the boundary conditions at the riverbank and the riverbank geometry. For the incoherent calculation, the riverbank is modeled first as a diffusely scattering Lambertian surface with the empirically derived Mackenzie albedo and riverbank tilt angle incorporated as indicated in Eq. (2.41). The diffuse calculations are also made using perturbation theory by substituting Eq. (2.64) into Eq. (2.28). In both cases, the assumption is that the seafloor feature falls within the resolution footprint of the sonar system.

Illustrative examples are given in Figs. 3-8 to 3-10, 3-14, 3-20 and 3-21. The geom-
Figure 3-7: Scenarios for the active detection of seafloor and sub-seafloor riverbank features. The water column is modeled as having constant sound speed, i.e. $c_{w1} = c_{w2} = 1500$ m/s, and monopole source and horizontal receiving array center are co-located at 50-m depth in all cases. (a) Bottom is sediment half-space with seafloor feature, i.e. $h_1 = h_2 = 0$. (b) Bottom is composed of a single sediment layer with double-interface seafloor feature, i.e. $h_2 = 0$. (c) Bottom is composed of a single sediment layer with sub-seafloor feature, i.e. $h_2 = 0$. (d) Bottom is composed of two sediment layers with double-interface sub-seafloor feature.
etry is again monostatic with co-located omni-directional point source and receiving array centers at 50-m depth. The receiving array lies parallel to the $y$ axis. The square riverbank surface has two edges parallel to the $y$ axis, is centered at $y = 0$ and inclined 10° about the $x$ axis. All plots give the scattered field from the riverbank as a function of range from the monostatic sonar. For comparison, incoherent reverberation from the water-sediment interface within the resolution footprint of the sonar, based on the Lambert-Mackenzie model for an uninclined surface, is also plotted as a function of range in Figs. 3-8 to 3-10 and 3-14 and based on perturbation theory in Figs. 3-20 and 3-21. This is referred to as diffuse seafloor reverberation. The range and cross-range resolution of the sonar system resolution footprint are the same as those stated in Sec. 3.1. Similarly, incoherent reverberation from the sediment-layer to sediment-half-space interface, based upon the Lambert-Mackenzie model for an uninclined surface, is also plotted as a function of range in Figs. 3-8 to 3-10 and 3-14, and based on perturbation theory in Fig. 3-20 and 3-21. This is referred to as diffuse sub-seafloor reverberation. The far field of the coherent riverbank begins at roughly 2 km while the far field of the receiving array begins at roughly 1.2 km.

For the Pekeris waveguide scenario of Fig. 3-7(a), returns from the seafloor riverbank features stand well above diffuse seafloor reverberation from the silt bottom within ranges of roughly 20 km and from the sand bottom beyond ranges of 50 km when the riverbank is treated as a coherent scatterer, as shown in Fig. 7. The ordinate is in decibels, i.e. $20 \log |\Phi_s|$ for riverbank returns and $10 \log \bar{V}_B$ for diffuse reverberation.

For the single layered bottom scenarios of Fig. 3-7(b) and (c), returns from both the seafloor riverbank and sub-seafloor riverbank features can stand well above diffuse seafloor reverberation from the silt-over-sand bottom when the riverbank is treated as a coherent scatterer and the silt layer is 2-m or $2/5$ of a wavelength, as shown in Fig. 3-9(a) and (b). The sub-seafloor riverbank and seafloor riverbank return echoes at similar levels within roughly 5 km, where both features typically stand above the diffuse seafloor reverberation by roughly 10 dB, which exceeds the 5.6 dB standard deviation. The prominence of the sub-seafloor riverbank returns follows from
the greater impedance mismatch between the silt-sand interface than the water-silt interface incorporated in the riverbank scatter function. Beyond roughly 5 km, the sub-seafloor feature has returns that fall-off more rapidly than those of the seafloor feature. This follows from the stripping of higher order modes that propagate with high attenuation in the silt layer. Coherent returns from the riverbank arise because of its finite extent. Since the riverbank is modeled as a smooth flat surface, scattering is greatest in the specular direction and falls off in other directions in manner similar to the sidelobes of a phased array’s beampattern. In backscatter, for the given geometry, the riverbank returns increase in intensity with the square of its length, or cross-range extent. Longer riverbanks that fit within the sonar resolution footprint then yield significantly larger returns as a consequence of the coherent scattering assumption, and may stand well above diffuse seafloor reverberation beyond 10 km. Returns from such extended riverbanks, however, rapidly become more of a challenge to model since the near field moves out in range from the feature with the square of its length. As the range-extent of the riverbank increases, the coherent area increases but the side lobe level decreases for the present geometry, rendering the effect on the backscattered field less apparent than in cross-range augmentation.

When the thickness of the silt layer is increased to 5-m, or one wavelength, returns from the sub-seafloor riverbank features are somewhat reduced, as shown in Fig. 3-9(c), and again only stand above diffuse seafloor reverberation within roughly 5 km. This follows from a related increase in the stripping of the higher order modes that have propagating components in the sediment layer since the sediment layer has much higher attenuation than the water column. The seafloor riverbank feature stands above diffuse seafloor reverberation beyond 20 km but rarely in excess of the Gaussian field standard deviation of 5.6 dB.

When the sediment layer is composed of sand and the half-space below is made of silt, the situation changes drastically, as shown in Figs. 3-9(d) and (e). Returns from the sub-seafloor riverbank no longer stand above diffuse seafloor reverberation beyond 2 km because in the sand layer, which is much faster than the water column and silt half-space, the trapped modes become evanescent. Seafloor riverbank returns
stand well above diffuse seafloor reverberation, occasionally by 10 dB or more, even beyond 20 km for both the 2-m and 5-m thick sand layers, as expected given the large impedance contrast between water and sand. Older seafloor features, in fact, are more likely to be composed of consolidated material such as sand or limestone since such materials are better able to withstand erosion. Steeper seafloor features that are common in many continental shelves, such as glacier and iceberg scours, can yield even higher returns.

For the two-layered bottom of Fig. 3-7(d), the sub-seafloor riverbank returns stand roughly 10 dB above diffuse seafloor reverberation out to roughly 10 km for a 1-m light-silt layer over a 1-m silt layer over a sand half-space, as shown in Fig. 3-10(a). The double layer reflection coefficient from the light-silt to silt to sand interfaces leads to the increased prominence of the sub-seafloor riverbank returns, compared with those obtained with the single-layer reflection coefficient of Figs. 3-9(a) to (e). When the layering is altered to 1-m light-silt over sand over a silt half-space, returns from the sub-seafloor riverbank feature only stand above diffuse seafloor reverberation within roughly 5 km. This indicates that sediment stratification of the geomorphic feature can weigh in heavily in fixing its scattering amplitude.

The effect of bottom layering on the coherent scattering function of the inclined seafloor and sub-seafloor riverbank features of Fig. 3-7 is illustrated in Figs. 3-11 and 3-12 as a function of horizontal grazing angle, $\pi/2 - \alpha_i$ for the incident and $\alpha - \pi/2$ for the scattered wave, at fixed incident and scattered azimuths $\beta_i = 0$, $\beta = \pi$, as is appropriate to backscatter in a waveguide. The trapped modes for the Pekeris waveguide scenarios, Fig. 3-11(a) for a sand bottom and Fig. 3-12(a) for a silt bottom, have incident and scattered elevation angles that lie within the boxes shown, the dimensions of which correspond to the respective bottom critical angle. The boxes include all modes $n$ where $0.5 \text{ rad/km} > \text{Im}\{\xi_n\}$. This includes all and only trapped modes for the Pekeris waveguide scenario of Fig. 3-7(a). The latter criterion is used to segment modes that dominate the incident propagation by similar boxes for the more complicated layered bottom cases illustrated in Figs. 3-11 and 3-12. The value $\text{Im}\{\xi_n\}$ is plotted as a function of equivalent modal angle $\alpha$ in Fig.
Figure 3-8: The field at $f = 300$ Hz scattered from a coherently scattering rectangular patch of area $100 \times 100$ m$^2$ representing a seafloor riverbank for scenario shown in Fig. 3-7(a), constant sound speed water column over pure silt or sand half-spaces. Range increases along the $x$-axis and depth along the $z$-axis. The square riverbank surface has two edges parallel to the $y$-axis, and is inclined $10^\circ$ from the $x$-axis. Constant sound speed in the water column is assumed for all examples with $c_{w1} = c_{w2} = 1500$ m/s. Lambert-Mackenzie reverberation within the range-dependent resolution footprint of the monostatic system is also shown separately for the water-sediment interface (seafloor). Source level is 0 dB re 1 $\mu$Pa @ 1 m. Diffuse reverb modeled with $T = 1/2$ s duration cw source signal at 300 Hz and receiving array resolution $\lambda/L = 3.70^\circ$. 
Figure 3-9: Same as Fig. 3-8 except single layer bottom scenarios of Fig. 3-7(b)-(c) for coherent seafloor and sub-seafloor riverbank scattering are investigated. Lambert-Mackenzie diffuse seafloor reverberation within the sonar resolution footprint is also shown for the sediment-layer to sediment half-space interface (sub-seafloor). (a) Seafloor riverbank with the upper sediment layer composed of silt with $h_1=2$ m and the lower sediment half-space composed of sand. Coherent riverbank scattering is from the double interface of water to silt to sand. (b) Sub-seafloor riverbank with the upper sediment layer composed of silt with $h_1=2$ m and the lower sediment half-space composed of sand. Coherent riverbank scattering is from the single silt to sand interface. (c) Seafloor and sub-seafloor riverbanks as in (a) and (b) but with the upper sediment layer now at $h_1=5$ m thickness. (d) Same as (c) but with the upper sediment layer composed of sand with $h_1=2$ m and the lower sediment half-space composed of silt. (e) Same as (d) except $h_1=5$ m.
Figure 3-10: Same as Fig. 3-9 except two-layer bottom scenario of Fig. 3-7(d) is investigated for coherent scattering from sub-seafloor riverbank. (a) Sediment is comprised of light silt layer of \( h_1 = 1 \) m thickness over a silt layer of \( h_2 = 1 \) m thickness over a sand half-space. Coherent riverbank scattering is from the double interface of light silt to silt to sand. (b) Sediment is comprised of light silt layer of \( h_1 = 1 \) m thickness over a sand layer of \( h_2 = 1 \) m thickness over a silt half-space. Coherent riverbank scattering is from the double interface of light silt to sand to silt.

3-13 for the various waveguides considered. This makes it possible to see how the scattering functions of Figs. 3-11 and 3-12 are discretely sampled in the waveguide scattering theory defined by Eq. (2.1) and to estimate the attenuation of a given modal component as a function of range. Inspection of Figs. 3-11 and 3-12 reveals that seafloor and sub-seafloor riverbank features that backscatter most prominently in Figs. 3-8 to 3-10 have scatter functions with relatively large amplitudes at the equivalent angles of the propagating modes. While modes propagating at steeper angles suffer greater attenuation, as indicated in Fig. 3-13, these same modes are scattered much more efficiently by the slightly inclined riverbank features as indicated in Figs. 3-11 and 3-12, so that there is some balancing between the two effects that is unique to waveguide scattering. Higher order modes, with elevation angles less than 45° where 0° points downward, however, contribute negligibly to the field scattered from the riverbank features for the ranges and features investigated in the present paper.

When the riverbank is treated as an incoherent scatterer with the Lambert-
Figure 3-11: Magnitudes of the coherent scattering functions $|S(\alpha, \beta = \pi; \alpha_i, \beta_i = 0)|$, i.e. $20 \log |S|$ dB, for the 100-m by 100-m seafloor and sub-seafloor riverbank features at inclination $\chi = 10^\circ$ of Fig. 3-7 over bistatic horizontal grazing angle, $\pi/2 - \alpha_i$ for the incident and $\alpha - \pi/2$, for the scattered wave, as appropriate to backscatter in a waveguide. The boxes include all modes $n$ where 0.5 rad/km $> \text{Im}\{\xi_n\}$. This includes all and only trapped modes for the Pekeris waveguide scenario of Fig. 3-7(a). (a) Reflection coefficient for water to sand is used for scenario of Fig. 3-7(a). (b) Reflection coefficient of silt to sand is used for scenario of Fig. 3-7(c). (c) Double reflection coefficient of water to 2-m silt layer over sand is used for scenario of Fig. 3-7(b). (d) Double reflection coefficient of water to 5-m silt layer over sand is used for scenario of Fig. 3-7(b). (e) Double reflection coefficient of light silt to 1-m silt layer over sand is used for scenario of Fig. 3-7(d).
Figure 3-12: Same as Fig. 3-11 except (a) reflection coefficient for water to silt is used for scenario of Fig. 3-7(a). (b) Reflection coefficient of sand to silt is used for scenario of Fig. 3-7(c). (c) Double reflection coefficient of water to 2-m sand layer over silt is used for scenario of Fig. 3-7(b). (d) Double reflection coefficient of water to 5-m sand layer over silt is used for scenario of Fig. 3-7(b). (e) Double reflection coefficient of light silt to 1-m sand layer over silt is used for scenario of Fig. 3-7(d).
Figure 3-13: The horizontal wavenumber’s imaginary component $\text{Im}\{\xi_n\}$ is plotted as a function of horizontal grazing angle, $|\pi/2 - \alpha|$, for the various waveguides considered. Proper modes occur in Pekeris below the critical angle for $0.5 \text{ rad/km} > \text{Im}\{\xi_n\}$. (a) Pekeris with sand bottom and Pekeris with silt bottom. (b) Constant water column sound speed of 1500 m/sec over $h_1=2$-m and $h_1=5$-m silt layer over sand half-space. (c) Constant water column sound speed of 1500 m/sec over $h_1=2$-m and $h_1=5$-m sand layer over silt half-space. (d) Constant water column sound speed of 1500 m/s over 1-m light silt layer over 1-m silt layer over sand half-space and Constant water column sound speed of 1500 m/s over 1-m light silt layer over 1-m sand layer over silt half-space.
Mackenzie model of Eq. (2.41), returns from both the 100 x 100 m² seafloor and sub-seafloor riverbank features at 10° inclination never stand above the diffuse seafloor reverberation by more than a fraction of the expected 5.6 dB standard deviation, as shown in Fig. 3-14. Riverbank returns are again in decibels, i.e. 10 times the log of the covariance given in Eq. (2.41). This is still the case for the ranges shown in Fig. 3-14, except for the 2-m sand layer, even if the riverbank feature is extended laterally to fill the entire cross-range width of the system resolution footprint, as can be readily checked by noting that diffuse reverberation accrues in direct proportion to the area of the scattering patch. For the 2-m sand layer, the seafloor riverbank can have returns that exceed the diffuse reverberation background by more than 5.6 dB if it fills the entire resolution footprint in cross-range.

Diffuse sub-bottom reverberation, shown in Fig. 3-14, always returns at a lower level than diffuse seafloor reverberation if the same empirical Lambert-Mackenzie incoherent scattering law is used. This comparison highlights the differences in propagation to and from the seafloor and sub-bottom interfaces since the scattering function is held fixed. The comparison may be purely academic, however, because the Lambert-Mackenzie law serves as an empirical catch-all that describes the entire seabed scattering process and so already incorporates the effect of bottom layering and volume scattering in some average sense. There is, in other words, no reason to believe that scattering from the different interfaces can be modeled with exactly the same albedo and scattering law.

Perturbation theory offers a more fundamental approach to modeling rough surface scattering that can also be used to investigate potential mechanisms for geological clutter. While the impedance contrast at the scattering interface is fully accounted for in the perturbation theory formulation, additional parameters describing the roughness spectrum must be known. The perturbation theory formulation described in Sec. 2.5.3 is used with the spectral strength and power law parameters $w_2 = 0.04/(2\pi)$ and $\gamma = 4.0$, yielding frequency-independent scattering, following Essen[17]. These values are not based on physical measurements, since none are presently available in the present frequency range, but rather have been chosen so that the scattering strength
Figure 3-14: Same as Figs. 3-8 to 3-9 except Lambert-Mackenzie model is used to model scattering from inclined riverbank features. (a) Seafloor riverbank over sand and silt half-spaces. (b) Seafloor and sub-seafloor riverbank scattering with the upper sediment layer composed of silt with $h_1=2$ m and the lower sediment half-space composed of sand. (c) Same as (b) except $h_1=5$ m. (d) Seafloor and sub-seafloor riverbank scattering with the upper sediment layer composed of sand with $h_1=2$ m and the lower sediment half-space composed of silt. (e) Same as (d) except $h_1=5$ m.
Figure 3-15: Scattering strength in free-space backscatter as a function of surface grazing angle for a diffusely scattering surface obeying Lambert-Mackenzie and first order perturbation theory scattering laws. The first order perturbation theory curves are for cases where the planewave is incident from an upper to a lower medium, where the upper and lower media can be either are water, sand or silt.

that perturbation theory yields is near that of the empirical Lambert-Mackenzie model for the various single and multiple reflection interfaces considered here, as shown in Fig. 3-15. In all curves where scattering arises from a wave incident from a slower medium, a discontinuity in slope is found at the critical angle. Beyond this a significant reduction in scattering occurs until roughly 45° where shallow angle assumptions of 1st order perturbation theory are no longer valid for the given surface roughness parameters, and the curves increase dramatically in an unphysical manner. Differences in the perturbation theory curves away from the critical angles arise principally from the impedance contrasts between the media considered. An exception occurs for fast sand over slow silt where no critical angle exists and transmission into the silt is significant even at very shallow angles, where low level scattering results.
The effect of scattering and reflection from multiple layers can be significant as shown in Figs. 3-16 to 3-17 for an inclined riverbank surface and Figs. 3-18 to 3-19 for general uninclined seafloor, where the perturbation theory scattering strength is presented for the bistatic scattering scenario relevant to backscatter in a waveguide, as was done in Figs. 3-11 to 3-12 for the coherent scatter function.

Empirical values for the spectral strength and power law parameters of first order perturbation theory have been obtained for various seafloor types by Jackson[32] over the short spatial scales relevant to the analysis of high frequency scattering in the ten kilo-Hertz range and beyond, where this author has shown perturbation theory to match experimental data well. When these same values are used at low frequency, specifically $f = 300$ Hz, the resulting scattering law falls more than an order of magnitude below the empirical Lambert-Mackenzie curve shown in Fig. 3-15. Unrealistically high roughness values for the spectral strength, as obtained for rough, rocky surfaces in the high frequency analysis of Ref. [32], are necessary for first order perturbation theory to match the empirical seafloor scattering strength curve of Mackenzie in low frequency regime of interest here. Since the Mackenzie curve summarizes the entire seafloor scattering process, and is not limited to interface scattering or the more restrictive type of interface scattering described by 1st order perturbation theory, it is reasonable to conclude that either the assumptions of first order perturbation theory are inadequate to properly model seafloor scattering at low frequency, a significantly different set of spectral strength and power law parameters must characterize seafloor interface scattering at low frequency, or a more sophisticated modeling of the seabed layering and sound speed gradients is necessary. It is also possible that scattering from volume heterogeneities may yield significant reverberation. This is most likely to be the case where a propagating, rather than evanescent, component of the modal spectrum exists in the layer where the volume heterogeneities are present.

When the riverbank feature is treated as an incoherent scatterer using first order perturbation theory, as in Figs. 3-20 to 3-21, only seafloor riverbank returns from a single sand layer can stand above diffuse seafloor reverberation by more than 5.6 dB, and this only occurs when the riverbank feature is extended laterally to fill the entire
Figure 3-16: Scattering Strength $SS(\alpha, \beta = \pi; \alpha_i, \beta_i = 0)$, based on first order perturbation theory for the seafloor and sub-seafloor riverbank features at inclination $\chi = 10^\circ$ of Fig. 3-7 over bistatic horizontal grazing angle, $\pi/2 - \alpha_i$ for incident and $\alpha - \pi/2$ for scattered wave, as appropriate to backscatter in a waveguide. The boxes include all modes $n$ where $0.5 \text{ rad/km} > \text{Im}\{\xi_n\}$. This includes all and only trapped modes for the Pekeris waveguide scenario of Fig. 3-7(a). (a) Reflection coefficient for water to sand is used for scenario of Fig. 3-7(a). (b) Reflection coefficient of silt to sand is used for scenario of Fig. 3-7(c). (c) Double reflection coefficient of water to 2-m silt layer over sand is used for scenario of Fig. 3-7(b). (d) Double reflection coefficient of water to 5-m silt layer over sand is used for scenario of Fig. 3-7(b). (e) Double reflection coefficient of light silt to 1-m silt layer over sand is used for scenario of Fig. 3-7(d).
Figure 3-17: Same as Fig. 3-16 except (a) reflection coefficient for water to silt is used for scenario of Fig. 3-7(a). (b) Reflection coefficient of sand to silt is used for scenario of Fig. 3-7(c). (c) Double reflection coefficient of water to 2-m sand layer over silt is used for scenario of Fig. 3-7(b). (d) Double reflection coefficient of water to 5-m sand layer over silt is used for scenario of Fig. 3-7(b). (e) Double reflection coefficient of light silt to 1-m sand layer over silt is used for scenario of Fig. 3-7(d).
Figure 3-18: Scattering Strength $SS(\alpha, \beta = \pi; \alpha_i, \beta_i = 0)$ based on first order perturbation theory for level seafloor, $\chi = 0^\circ$, over bistatic horizontal grazing angle, $\pi/2 - \alpha_i$ for the incident and $\alpha - \pi/2$ for the scattered wave, as appropriate to backscatter in a waveguide. The boxes include all modes $n$ where $0.5 > \text{Im}\{\xi_n\}$. This includes all and only trapped modes for the Pekeris waveguide scenario of Fig. 3-7(a). (a) Reflection coefficient for water over sand is used. (b) Double reflection coefficient of water over 2-m silt over sand is used. (c) Double reflection coefficient of water over 5-m silt layer over sand is used. (d) Triple reflection coefficient of water over 1-m light silt layer over 1-m silt layer over sand is used.
Figure 3-19: Same as Fig. 3-18 except (a) reflection coefficient for water over silt is used. (b) Double reflection coefficient of water over 2-m sand over silt is used. (c) Double reflection coefficient of water over 5-m sand layer over silt is used. (d) Triple reflection coefficient of water over 1-m light silt layer over 1-m sand layer over silt is used.
cross-range extent of the system resolution footprint. Returns from the sub-seafloor riverbank only stand above diffuse seafloor reverberation by more than 5.6 dB for the two-layered bottom in the light-silt over sand over silt scenario, and this only occurs if the feature is extended to fill the resolution footprint.

Comparison of Figs. 3-8 to 3-10 and Fig. 3-14 shows that coherent returns greatly out-weigh incoherent returns from the riverbank feature. This finding is advantageous since only deterministic physical and geometrical parameters of the seafloor are necessary in the coherent model, whereas either empirical data or a stochastic representation of the seafloor is necessary in the incoherent model. The environmental description necessary for the coherent model is then easier to obtain and rests on far fewer supporting assumptions than the incoherent one.

3.4 Probability of Detection and False Alarm

In this section, we compute the probabilities of detection (POD) and false alarm (PFA) as a function of target-signal-to-diffuse-reverberation ratio (SRR) for a spherical target in two different waveguides. We present the statistical model used to compute these probabilities.

We model diffuse reverberant field from the environment as a circular complex Gaussian random (CCGR) variable whose amplitude follows the Rayleigh distribution and whose phase follows a uniform distribution. The instantaneous intensity of the CCGR field follows the exponential distribution,

\[ P_I(I) = \begin{cases} \frac{1}{\sigma_N^2} e^{-I/\sigma_N^2} & \text{for } I > 0 \\ 0 & \text{otherwise} \end{cases} \]  

with mean equal to the reverberation variance \( \langle I \rangle = \sigma_N^2 \). For a deterministic signal, such as that returned from an object or an extended geologic feature, submerged in CCGR reverberation field, the amplitude of such a signal follows the Rician distribution.[12, 38] The intensity follows the following distribution.
Figure 3-20: Same as Figs. 3-8 to 3-9 except first order perturbation theory is used to model scattering from the inclined riverbank features. (a) Seafloor riverbank over sand and silt half-spaces. (b) Seafloor and sub-seafloor riverbank scattering with the upper sediment layer composed of silt with $h_1=2$ m and the lower sediment half-space composed of sand. (c) Same as (b) except $h_1=5$ m. (d) Seafloor and sub-seafloor riverbank scattering with the upper sediment layer composed of sand with $h_1=2$ m and the lower sediment half-space composed of silt. (e) Same as (d) except $h_1=5$ m.
Figure 3-21: Same as Fig. 3-10 except first order perturbation theory is used to model scattering from the inclined sub-seafloor riverbank for the two-layer bottom scenario of Fig. 3-7(d). (a) Sediment is comprised of light silt layer of $h_1=1$-m thickness over a silt layer of $h_1=1$-m thickness over a sand half-space. Diffuse riverbank scattering is from the double interface of light silt to silt to sand. (b) Sediment is comprised of light silt layer of $h_1=1$-m thickness over a sand layer of $h_2=1$-m thickness over a silt half-space. Diffuse riverbank scattering is from the double interface of light silt to sand to silt.
\[
P_t(I) = \begin{cases} 
\frac{1}{\sigma_N^2} e^{-(I+|s|)/\sigma_N^2} & \text{for } I > 0 \\
0 & \text{otherwise}
\end{cases}
\] (3.2)

with mean \( \langle I \rangle = |s|^2 + \sigma_N^2 \).

The detection threshold is selected to be 10 times the instantaneous reverberation intensity at each range, \( D_T = 10\sigma_N^2 \). Since the instantaneous reverberation intensity has a standard deviation of 5.6 dB, this detection threshold is roughly two standard deviations above the instantaneous reverberation intensity. POD and PFA are calculated by integrating Eqs. (3.1) and (3.2) respectively from \( D_T \) to infinity. The unified model described in Chapter 2 is used to calculate the target signal and diffuse reverberation intensities.

Calculations of SRR, POD and PFA are illustrated for the 10 m radius pressure-release sphere target in Figs. 3-22 to 3-24 for the scenarios illustrated in Fig. 3-3 with a downward refracting sound speed structure in the water column. The mean intensity of the back-scattered field from the sphere target and diffuse seafloor reverberation are plotted as function of increasing separation between the sonar and target in Figs. 3-5(a)-(c). The SRR at each range in Figs. 3-22 to 3-24 is obtained as the ratio of the magnitude square of Eq. 2.1 which gives the mean backscattered intensity from the sphere over Eq. 2.46 which gives the mean intensity of the diffuse seafloor reverberation. The SRR in decibel units can also be obtained by subtracting the diffuse reverberation level from the target echo level in decibels in Figs. 3-5(a)-(c) respectively. We observe from Fig. 3-22 that the sphere target can be detected out to more than 50 km range in both the sand and silt bottom waveguides when the sonar and sphere target center are at 50 m depth where the returns from the target stands more than 20dB above the diffuse seafloor reverberation at most ranges. When the sonar is placed in the mixed layer at 10 m depth while the target remains with center at 50 m depth, the target’s detectability is reduced to about 15 km in the sand bottom waveguide as seen in Fig. 3-23. In the silt waveguide, except for some reduction in POD initially at small ranges, the target stays highly detectable up to
ranges beyond 50 km. The detectability of the sphere target reverses in the two waveguides when the target sphere is placed in the mixed layer with center at 15 m depth along with the source and receiver at 10 m depth as shown in Fig. 3-24. In the silt bottom waveguide, the POD of the target falls drastically to zero within 15 km range while the sphere remains highly detectable out to more than 50 km range in the sand bottom waveguide. As discussed in Sec. 3.2, the high order modes are stimulated by the shallow source, receiver and target, and these are well supported by the high-critical-angle sand bottom, but not the silt bottom.

3.5 Narrowband versus Broadband, Effect of Bandwidth on Detectability of Geoclutter Targets

Here we investigate the effect that bandwidth of a signal has on the detection of returns from deterministic coherent scatterers, such as extended geologic features as well as submerged man-made objects. The returns from these targets have to be detected in the presence of incoherent background reverberation that limits detectability. A broadband waveform allows match filtering[2, 12, 38] to be performed on the return signal from a coherent scatterer which improves SRR over narrowband measurements. The enhancement in SRR arises due to a better (smaller) range resolution and smaller sonar resolution footprint from match filtering. This leads to a smaller amount of reverberation per footprint and hence higher reverberation rejection per footprint. Match filtering also enhances signal-to-noise ratio (SNR) through rejection of out-of-band additive noise and suppression of in-band-noise.

The gain $G$ in the SRR from match-filtering for a broadband waveform is proportional to the ratio of the resolution footprint before $\Delta\rho$ and after $\Delta\rho_{mf}$ the match filtering operation.

$$G = 10 \log \frac{\Delta\rho}{\Delta\rho_{mf}}$$  \hspace{0.5cm} (3.3)
Figure 3-22: Target-signal-to-diffuse-reverberation ratio (SRR) for a 10-m radius pressure-release sphere in the downward refracting waveguides with (a) sand, and (c) silt halfspaces. The geometry of the measurement is shown in Fig. 3-1(a) with the source-receiver at 50 m depth and the sphere target at 50 m depth in the water column. The target signal and diffuse reverberation intensity is obtained from Fig. 3-5(a) for computing the SRR. POD and PFA for detection of the sphere in the downward refracting waveguide with (b) sand, and (d) silt halfspaces.
Figure 3-23: Target-signal-to-diffuse-reverberation ratio (SRR) for a 10-m radius pressure-release sphere in the downward refracting waveguides with (a) sand, and (c) silt halfspaces. The geometry of the measurement is shown in Fig. 3-1(b) with the source-receiver at 10 m depth and the sphere target at 50 m depth in the water column. The target signal and diffuse reverberation intensity is obtained from Fig. 3-5(b) for computing the SRR. POD and PFA for detection of the sphere in the downward refracting waveguide with (b) sand, and (d) silt halfspaces.
Figure 3-24: Target-signal-to-diffuse-reverberation ratio (SRR) for a 10-m radius pressure-release sphere in the downward refracting waveguides with (a) sand, and (c) silt halfspaces. The geometry of the measurement is shown in Fig. 3-1(c) with the source-receiver at 10 m depth and the sphere target at 15 m depth in the water column. The target signal and diffuse reverberation intensity is obtained from Fig. 3-5(c) for computing the SRR. POD and PFA for detection of the sphere in the downward refracting waveguide with (b) sand, and (d) silt halfspaces.
As discussed in Sec. 2.5.1, for a broadband source of duration $T_s$, if the time spread $\Delta \tau_s$ of the signal due to dispersion in the waveguide is small compared to $T_s$, the horizontal range resolution is approximately $\Delta \rho = \bar{c} T_s / 2$ where $\bar{c}$ is the mean horizontal propagation speed of the signal between the source and receiver in the waveguide. The range resolution after match filtering $\Delta \rho_{mf}$ is dependent upon the signal replica used in the match filtering operation. If an exact replica of the return signal from the object that has propagated through the dispersive waveguide is used, the range resolution decreases to $\Delta \rho_{mf} = \bar{c} / 2B$ where $B$ is the signal bandwidth. This type of matching provides the full match filter gain obtainable and is

$$G = 10 \log \frac{\bar{c} T_s / 2}{\bar{c} / 2B} = 10 \log T_s B.$$ (3.4)

Here, $T_s B = \mu$ is the time-bandwith product of the signal.[12, 38, 41] Often however, it is complex to model the exact scattered signal from an object that has propagated through a dispersive waveguide. This is because it requires an accurate knowledge of the acoustic properties of the object, waveguide as well as the measurement geometry. In real measurement scenarios, the waveform transmitted from the source is used instead for match-filtering. This leads to some degradation in the match-filter gain which depends on how much the scattered signal differs from that transmitted by the source. In Ref. [36], it is shown that the degradation in the match filter gain is of the order of 3 to 4 dB in a more dispersive sand waveguide and is almost negligible in a less dispersive silt waveguide.

We illustrate the effect of bandwidth on the detection of the coherently scattered field from a sub-seafloor riverbank surface of dimension $100 \times 100$ m$^2$ inclined at 10° in the direction facing the sonar as shown in Fig. 3-7(d). We consider two different waveforms, a 1/2 s Linear Frequency Modulated (LFM) signal from 390 to 440 Hz, and a 1/2 s CW tone at 415 Hz which is at the center frequency of the LFM signal. These waveforms were used in the 2001 Geoclutter Acoustic experiment which is discussed in Chapter 4. The waveguide is modeled as an isolvelocity water column
with sound speed $c_w = 1500 \text{ m/s}$ overlying a bottom composed of two layers of sediment over a halfspace. We consider two different sea bottom types, similar to the environment in Fig. 3-10. The first has a 1 m light-silt layer over 1 m silt layer over a sand halfspace, while the second has a 1 m light-silt layer over 1 m sand layer over a silt halfspace. Coherent riverbank scattering is from the double interface of sediment layer to the bottom halfspace. The monostatic source-receiver is at 50-m water depth. The receiver is a horizontal line array of 32 elements at half wavelength spacing at 415 Hz.

Fig. 3-25 (a) and (b) show the scattered returns from the riverbank calculated using Eq. 2.1 and 2.65 as well as diffuse reverberation from the seafloor calculated using Eq. 2.46 as a function of increasing range separation between monostatic source-receiver and riverbank feature. The results are plotted for a tukey windowed LFM signal from 390-440Hz and a CW tone at 415 Hz. Both the transmitted waveforms are normalised to unit amplitude. For the LFM waveform, we plot the output of the match filter at its peak value for each range which is given by

$$\frac{1}{T} \int_{t-T/2}^{t+T/2} \Psi_c(R_r - R|R_0 - R|t)s^*(t)dt$$  \hspace{1cm} (3.5)$$

where $s(t)$ is the replica signal employed in the match filtering. The range resolution of the 1/2-s CW signal is $cr/2 = 375$ m. For the simulation in this section, the exact scattered signal from the object is used in the matching $s(t) = \Psi_c(R_r - R|R_0 - R|t)$ so that the range resolution after match-filtering is $c/(2B) = 15$ m. This corresponds to having an exact acoustic model of the waveguide and object, and accurate knowledge of the the measurement geometry. A smaller range resolution implies that a smaller amount of diffuse reverberation is returned to the array at each range cell which improves SRR after match-filtering for the LFM signal. The perfect match filter gain in this case is 14.0 dB.

The SRR for the CW and LFM signals are plotted in Fig. 3-26 (a) and (c) respectively for the waveguide in Fig. 3-25(a) and in Fig. 3-27 (a) and (c) respectively for
the waveguide in Fig. 3-25(b). The SRR is much higher for the LFM signal which also has less fluctuation as a function of range in both waveguides.

Figure 3-26 (b) and (d) shows the POD and PFA plotted for the CW and LFM waveforms respectively for the waveguide in 3-25(a) and in Fig. 3-27 (b) and (d) respectively for the waveguide in Fig. 3-25(b). We see that the LFM signal gives more stable detection and over longer ranges than the CW signal. There is more fluctuation in the CW signal detection due to the coherent interference between the modes of a waveguide. For the broadband signal, this interference averages out over range due to the contribution of a large number of modes from the frequencies over the bandwidth that makes the detection more stable over range. The ability to perform match-filtering with LFM data enhances the returns from a coherent scatterer over Gaussian random reverberation and improves detectability over longer ranges than the CW waveform.

3.6 Scattering from Fish School

In this section, we investigate whether the incoherent scattering from schools of fish is significant enough to stand above diffuse seafloor reverberation in shallow waters at the frequencies used in the Geological Clutter Acoustic Experiment 2001 discussed in Chapter 4. The data we use here for the type and distribution of fish is based on measurements made by Nero et. al. [55] at the New Jersey Stratafrom area roughly a month after the Geoclutter Acoustics Experiment. They measured the spatial distribution of fish schools using a 38 kHz echo sounder while a lower frequency echosounder from 2-10kHz was used to study the scattering characteristics of selected schools. Here, we use bubble scattering theory to estimate the target strength of an individual fish. Equation 2.29 is used to estimate the intensity of the total field scattered incoherently by schools of fish within the sonar resolution footprint in a shallow water waveguide.

Of the variety of demersal and pelagic fish found in the Strataform area, the type of fish school that could lead to strong scattered returns at the frequency range from
Figure 3-25: The field scattered from a coherently scattering rectangular patch of area $100 \times 100$ m$^2$ representing a subseafloor riverbank for scenario shown in Fig. 3-7(d) for two different waveforms, a 1/2 s duration tukey shaded LFM signal from 390 to 440 Hz and a 1/2 s duration CW tone at 415Hz. The environment consists of a water column with constant sound speed $c_w = 1500$ m/s overlying a bottom composed of two sediment layer over a halfspace. (a) Sediment is comprised of light silt layer of $h_1=1$ m thickness over a silt layer of $h_2=1$ m thickness over a sand half-space. Coherent riverbank scattering is from the double interface of light silt to silt to sand. (b) Sediment is comprised of light silt layer of $h_1=1$ m thickness over a sand layer of $h_2=1$ m thickness over a silt half-space. Coherent riverbank scattering is from the double interface of light silt to sand to silt. Range increases along the $x$-axis and depth along the $z$-axis. The square riverbank surface has two edges parallel to the $y$-axis, and is inclined $10^\circ$ about the $x$-axis. Lambert-Mackenzie reverberation within the range-dependent resolution footprint of the monostatic system is also shown separately for the water-sediment interface (seafloor). Source level is 0 dB re 1 $\mu$Pa @ 1 m. Diffuse reverb modeled with the two different source waveforms, a $T = 1/2$ s duration tukey shaded LFM from 390 to 440 Hz and a $T = 1/2$ s CW tone at 415 Hz and receiving array resolution $\lambda/L = 3.70^\circ$. The LFM signal with $B = 50$ Hz bandwidth has a smaller (better) range resolution.
Figure 3-26: Signal-to-diffuse reverberation ratio (SRR) and probabilities for detection (POD) and false alarm (PFA) for the detection of the subseaflor riverbank in Fig. 3-25(a) in the environment were the bottom is composed of $h_1=1$ m thick light silt layer over a $h_2=1$ m thick silt layer over a sand half-space. The detection scenario is investigated in (a) and (b) for the $T = 1/2$ s CW signal and in (c) and (d) for the $T = 1/2$ s LFM signal from 390-440 Hz. The broadband waveform leads to higher SRR and larger POD over longer ranges than the CW tone.
Figure 3-27: Similar to fig. 3-26 except that the bottom is composed of $h_1=1$ m thick light silt layer over a $h_2=1$ m thick sand layer over a silt half-space.
390-440 Hz is scups of average length about 17-cm. They occur in tight schools at depths of about 5m off the bottom during mid day and in loose schools that are higher up in the water column at night. The scattering from this fish is mainly due to the presence of a swimbladder which is essentially an air-filled bubble within the fish.

The resonant frequency of the swimbladder of this fish was estimated to be about 900-Hz at depths of between 75 to 95 m. Figure 3-28 shows the distribution of fish in the area. The mean fish density was measured to be about 0.1 fish/m². The mean fish school radius was estimated to be about 20-m giving an average school population of 126 fish/school. The average number of fish schools found within a 1-km² area was about 50.

An air-filled bubble with resonance frequency \( f_r \) has a radius \( a \) given by:

\[
a = \frac{3.26}{f_r} \sqrt{1 + 0.0984D},
\]

where \( D \) is the depth of the bubble below the sea surface in meters. For scups with swimbladder resonance frequency of 900-Hz at a mean depth of 85 m, the radius of the equivalent bubble is approximately 1.1 cm. At 415 Hz, the radius of the swimbladder is small compared to the wavelength of the acoustic wave which is 3.6 m. Each fish is therefore a compact scatterer with an omnidirectional scatter function that can be estimated from that of a submerged bubble. The target strength of the bubble at 415-Hz is calculated to be -50.5 dB using:

\[
TS = 10\log \frac{a^2}{(f^2/f_r^2 - 1)^2 + \delta^2},
\]

where \( \delta = 0.15 \) is the damping constant for bubble oscillation. For an area \( A \) in meters within the sonar resolution footprint, the average number of fish contained within the area is
Figure 3-28: Distribution of average fish density within a school (individuals/m²) against the school radius from measurements made at the New Jersey Strataform area[55, 54].
\[ N_{fish} = \frac{A}{1 \text{ km}^2} \times (126 \text{ fish/school}) \times (50 \text{ schools/km}^2) \]
\[ = \frac{A}{10^6} \times 6300 \text{ fish.} \quad (3.8) \]

Substituting Eq. 3.8 into Eq. 2.29 we obtain the intensity of the total scattered field from the \( N_{fish} \) fish within the sonar resolution footprint.

Figure 3-29 (a) and (b) show the total scattered field intensity at 415-Hz summed incoherently from the fish at 85-m depth within the sonar resolution footprint as a function of increasing range of the fish schools from a monostatic sonar. The environment, source-receiver depth and array configuration are similar to those in Fig. 3-25(a) and (b) respectively of Sec. 3.5. The diffuse reverberation from the seafloor is also plotted at each range. Both the scattered intensity from the fish and the diffuse reverberation intensity from the bottom have a 5.6-dB standard deviation. The scattering from the fish school based on the average distribution statistics is insignificant at 415 Hz compared to the reverberation from the seafloor.

In Fig. 3-28, there are a very few large fish schools\[55\] that have radii of approximately 1-km\(^2\) and density of 1 fish/m\(^2\). These schools provide the maximum number of fish that can occur within the sonar resolution footprint at long ranges. Figure 3-29 (a) and (b) show that the incoherently scattered intensity from such a large school can stand above the diffuse reverberation from the seafloor. This implies that scattering from large and high density fish schools are also possible sources of clutter in active sonar systems that search for underwater vehicles. But, as Fig. 3-28 shows, such large and high density fish schools are rare compared to the mean fish school distribution.

### 3.7 Summary

The unified model is used to investigate typical low-frequency active detection scenarios in shallow water. Sample calculations for finite-duration cw source signals
Figure 3-29: Figures (a) and (b) show the field scattered incoherently from schools of fish at 80-m water depth contained within the sonar resolution footprint in the environments similar to Fig. 3-25 (a) and (b) respectively. The array configuration and source-receiver geometry are similar to that in Fig. 3-25. The results are plotted for the mean density of fish and the maximum density of fish within the sonar resolution footprint. These densities are based on measurements of fish school distribution at the New Jersey Strataform area reported in Ref. [55] and shown in Fig. 3-28.

indicate that the maximum range at which echo returns from a submerged target stand unambiguously above diffuse seafloor reverberation is highly dependent upon the watercolumn and sediment stratification, as well as the receiving array aperture, source, receiver and target location, and the scattering properties of the target and seafloor.

The model is also applied to determine conditions in which discrete morphological features of the seafloor and sub-seafloor return echoes that stand prominently above diffuse seafloor reverberation. Simulations for finite-duration cw source signals indicate that typical seafloor and sub-seafloor riverbank features, ubiquitously found throughout continental shelf waters, can return echoes that stand significantly above the diffuse component of seafloor reverberation in the operational ranges of typical low-frequency active sonar systems. This finding is significant since returns from these discrete features can be confused with returns from an intended submerged target. The relative prominence of this kind of geological clutter is highly dependent on the
waveguide properties, measurement geometry and scattering characteristics of the geological feature and surrounding seafloor. The finding that sub-seafloor features can cause significant clutter is particularly troubling for active sonar operations because it greatly increases the environmental characterization necessary to make accurate predictions of the expected clutter. The coherent component of the field scattered from the riverbank features examined, arising from the features’ finite size, is found to far outweigh the diffuse component arising from random roughness of the features.

The methods and findings in this Chapter were used to help design the Geoclutter field experiment discussed in Chapter 4 to experimentally investigate the physical mechanisms for scattering from geologic features in shallow-water. We also show that coherent scattering from submerged objects and geologic features can be enhanced over seafloor reverberation when using a broadband source. Therefore broadband waveforms used in navy system only improve detection over diffuse reverberation but leads to enhanced clutter from geologic features.

Incoherent scattering from schools of fish was also investigated using the unified model. It was found that incoherent scattering from mean distribution of fish schools is insignificant compared to seafloor reverberation at the frequencies of interest in long-range imaging. However, a rare occurrence of a very large and high density fish school can give rise to returns that stand above the seafloor reverberation. Such fish schools are therefore a source of clutter for sonar systems that search for underwater vehicles.
Chapter 4

Remote Acoustic Imaging of Sub-bottom and Seafloor Geomorphology in Shallow Water: The Geoclutter Experiment 2001

In this chapter, we present the results of the Geoclutter Acoustic Experiment of April-May 2001 where sub-bottom and seafloor geomorphology over wide areas in the continental shelf waters off the coast of New Jersey were rapidly imaged using a long-range active sonar system. In Sec. 4.1, we provide a description of the New Jersey Stratafrom area geology and a detailed description of the Geoclutter Acoustic Experiment. In Sec. 4.2, we explain how long-range acoustic data is processed to generate wide area images. Images of geomorphologic features are presented in Sec. 4.4. Oceanographic data collected during the experiment, such as sound speed structure, are presented in Ref. [64].
4.1 Design and Implementation of the Geoclutter Acoustics Experiment

A number of geophysical surveys\cite{14, 21, 13} at the New Jersey Strataform area shown in Fig. 4-1 prior to the acoustic experiment characterized the seafloor and sub-bottom features over wide areas. Figure 4-2 shows the water depth at the strataform site where bathymetric data is available at 30-m horizontal resolution.\cite{21} Seabed and sub-bottom features identified from the geophysical surveys are overlain on the bathymetry. The candidate features for prominent and coherent scattering include incised or buried river channels, relict iceberg scours and surface erosional features on the seafloor, and surface or near surface outcroppings of seismically reflective subsurface strata within the seabed called “R-reflectors”. Figure 4-3 shows a seismic profile of several river channels buried at different depths from the seafloor intersecting with sub-bottom strata. Apart from these coherent geologic scatterers, incoherent aggregates of compact scatterers such as gravel deposits on the seafloor, gas pockets in the seabed and very large, high-density schools of fish were also identified as possible sources of prominent scattering.

The experiment was conducted using the Research Vessels (RV) Endeavour and the Nato Reasearch Vessel (NRV) Alliance, Fig. 4-4. RV Endeavour was used mainly as a source ship for bistatic measurements. It deployed the Multistatic Active Program’s MACE source system, consisting of a 7 element array made up of XF-4 transducers, beamed to transmit at broadside during the whole experiment. RV Endeavour was fastened to moorings at three specific sites where a fixed transmission location was maintained for the bistatic measurements. NRV Alliance was the only ship that deployed a horizontal receiving array. It also deployed a two element MOD40 transducer source system that was towed for effectively monostatic measurements. A tow speed of 2-m/s was maintained throughout the measurements by NRV Alliance. The horizontal receiving array was a 256 element line array with three nested apertures each consisting 128 sensor elements evenly spaced at 0.5, 1 or 2 m. Only data from 128 elements at 1m spacing is analyzed in the present paper. For a sound speed
Figure 4-1: Figure taken from Ref. [21] showing the New Jersey continental shelf in the vicinity of the Strataform area highlighted in gray. The depth contours are in meters.
Figure 4-2: Bathymetry of the Strataform area sampled at 30-m interval. Candidate features identified from previous geophysical survey\cite{14, 21} that would give prominent and coherent scattered returns includes incised or buried river channels (green), relict ice-berg scours and erosion pits on the seafloor (blue), and surface or near surface expression of seismically reflective subsurface strata within the seabed (white) called “R-reflector”. Coordinates of south-west corner: 38° 53.00’N, 73° 15.00’W.
Figure 4-3: Figure taken from Ref. [14] showing a seismic profile and line drawing interpretation of the bottom stratigraphy surveyed at the New Jersey Strataform area within site 1. The figure shows numerous buried river channels intersected by highly reflective sub-bottom strata.

of 1500 m/s, this subaperture corresponds to an array cut for 750 Hz. Each of the two calibrated targets, deployed at a selected site, were moored to the bottom in about 80-m water at 18-m from the seafloor. They stood vertical in the water column spanning roughly 32 to 62-m depth from their own buoyancy.

Figure 4-5 shows the bathymetric gradient at the Strataform site. Inspection of Fig. 4-5 shows that the experiment site has mostly benign slopes of < 1/2° with few discrete features on the seafloor. Even the seafloor features that are noticeable, such as the ice-berg scour and erosion pits, have extremely small slopes typically less than 3°. The tracks traversed by NRV Alliance, mooring locations of RV Endeavour, and locations of the two calibrated targets and sub-bottom features are also plotted in Fig. 4-5. Acoustic transmissions were centered about three distinct sites of the Stratafrom area called sites 1, 2 and 3. At site 1, the tracks of NRV Alliance and the location of RV Endeavour source were designed to image the buried river channels and the R-reflectors. In particular, locations where the buried river channels transect the R-reflectors were singled out as probable spots for obtaining prominent returns[30]. At site 2, the tracks were spaced to measure scattered returns from erosion pits, R-reflectors, as well as the calibrated targets. While the tracks at site 3 were designed to
NRV Alliance

GPS

Variable 23 to 65 m

2-element Vertical Source Array (Monostatic)

Variable 0 to 140 m

Horizontal Receiving Array

MF aperture 127 m

Full aperture 254 m

Variable 21.5 to 68.5 m

Variable 35.4 to 83.8 m

7-element Vertical Source Array (Bistatic)

Seafloor

Sub-bottom Strata

Buried River Channel

Figure 4-4: A sketch of NRV Alliance towing a two element vertical source array at 1.875 m spacing and a 256 element horizontal receiving array with spacing between elements of 0.5, 1 and 2m. Only data from 128 elements of the receiving array at 1-m spacing is analyzed in this paper. The mean depth of the NRV Alliance two-element source was varied between 23 to 65 m. The mean depth of the receiver array was varied from 21.5 to 68.5 m. RV Endeavour was moored at each site where measurements were collected and it deployed the MACE source system consisting of a 7 element array spaced at 1.625-m. The mean depth of the MACE source was varied between 35.4 and 83.8-m and it was beamed to transmit at broadside throughout the experiment.
image the ice-berg scours and more subsurface reflectors, the tracks were also placed close to the Hudson canyon to obtain scattered returns from the walls of the canyon.

The large number of tracks at each site were necessary to not only study the range and azimuth dependence of the scattering but also to separate returns from the various candidate geoclutter targets (at a variety of ranges and azimuths), as well as to minimize the ambiguity inherent in line array measurements. The prominence of returns from geologic features above diffuse background reverberation was found to be highly dependent on the source and receiver orientation and location relative to the sound speed profile as will be shown in Sec. 4.4.6. These results are consistent with those predicted from theory[48] and illustrated in Chapter 3.

The water depth at the three sites ranged from roughly 70 to 130-m. During the experiment the mean depth of the RV Endeavour 7-element source array varied between 35.4 m and 83.8 m while the mean depth of the NRV Alliance two-element source array varied between 23 m and 65-m. The mean depth of the NRV Alliance receiving array varied between 21.5 m to 68.5 m.

The sources transmitted both linear frequency modulated (LFM) and sinusoids or “continuous wave” (CW) signals of varying duration in the frequency range from 390 to 440 Hz.[57] The LFM signals were shaded with a Tukey window while the CW signals were shaded with a hanning window for the signals transmitted by RV Endeavour. For the monostatic measurements, a rectangular window was used in all transmissions for both the LFM and CW signals. The standard length of the NRV Alliance track line is roughly 10-km and the waveforms were transmitted at every 50 or 100-s interval. With a speed of 2-m/s for the receiver ship, data from a total of roughly 50 or 100 transmissions were measured along each track.
Figure 4-5: Directional derivative of bathymetry at the Strataform site with respect to a source in the north. The seafloor is mostly level locally with slopes of $< 1/2^\circ$. There are very few discrete features such as ice-berg scours and erosion pits on the seafloor with slopes of at most $3^\circ$. Acoustic transmissions are centered about three distinct sites in the Strataform area. Overlain are the tracks traversed by NRV Alliance (white lines), mooring locations of RV Endeavour (red stars), location of the two calibrated targets (white stars) and sub-bottom features (blue and pink lines). Coordinates of south-west corner: $38^\circ 53.00'N$, $73^\circ 15.00'W$. 
4.2 Generating Wide-Area Images of the Ocean Environment in Near-Real Time

During the Geoclutter Acoustics Experiment, a wide area image of received sound pressure level as a function of horizontal position over tens of kilometers was generated for every transmission. For a given transmission, two-way travel time was used to determine the range of returns and beamforming to determine the azimuth. The process has been previously described in Refs. [43, 44, 62]. It follows the same principles used in high frequency side-scan sonar, medical ultrasound and radar image processing except that the present imaging process has to deal with the complexities of multipath propagation, waveguide scattering and dispersion.

Echo returns from mono- and bistatic LFM transmissions of varying duration measured with NRV Alliance's horizontal line array with 128 elements at 1m spacing are exclusively analyzed in this paper. The raw time series data for each hydrophone was filtered, demodulated and decimated. This decimated array data was then converted to beam-time data by time-domain beamforming. Since the horizontal line array forms the narrowest beam at broadside, it provides the best cross-range resolution at broadside where the signals arrive almost perpendicular to the array axis. The broadest beam from the line array occurs close to the endfire direction and so a broader cross-range resolution is obtained for reception of signals that arrive almost parallel to the array axis.

A Hanning spatial window function was applied in the beamforming to reduce sidelobe levels where the first sidelobe level is down 30 dB from the main lobe. Following Eqs. 1 and 2 of Ref. [44], the 3-dB beamwidth of the array \( \beta \) is approximated using

\[
\beta(\varphi) \approx 1.3 \frac{\lambda}{L \cos \varphi}
\]

(4.1)

for steering angles from broadside \( \varphi = 0 \) to a transition angle \( \varphi_t \) near endfire, \( \varphi = \pi/2 \).
As \( \varphi \) approaches \( \varphi_t \), ambiguous beamwidths each reach a value approximately equal to that at endfire, and begin to merge until they completely overlap at endfire where the beamwidth is

\[
\beta(\varphi = \pi/2) \approx 2.8\sqrt{\lambda/L}.
\]

For the array aperture of 127 m, the optimal 3 dB resolution is about 2.1\(^\circ\) at broadside and 27\(^\circ\) at endfire. This corresponds to a cross-range resolution \( \Delta \rho = \rho \beta \) of 364 m at broadside and 4.7 km at endfire at a range of \( \rho = 10 \) km which is an average range for detecting the clutter events during the experiment.

The beam-time data are linearly converted to beam-range data by multiplying the total two-way travel time with half the mean sound speed which was taken to be 1475 m/s. To improve on the range resolution and signal-to-additive-noise ratio, the LFM data from 390 to 440 Hz were match filtered with a replica of the source signal to give an effective range resolution of \( c/2B \approx 15 \) m where \( B \) is the bandwidth of 50 Hz. This data was then averaged to 30-m resolution and then mapped to a Cartesian grid with a similar 30 \( \times \) 30 m\(^2\) grid increment as the high resolution bathymetry data available. The mapping procedure accounts for beam overlap by an incoherent averaging of adjacent beams.[40]

The standard deviation of a pixel value in an acoustic image is now estimated.[43] We assume that the transmitted waveform's interaction with the seafloor scattering area completely randomizes the return such that the real and imaginary temporal components of the instantaneous scattered field are identically distributed and uncorrelated zero-mean Gaussian random variables. The instantaneous intensity of the return is then exponentially distributed and the time-averaged intensity is gamma distributed with degrees of freedom \( \mu \) corresponding to the time-bandwidth product \( TB \) of the scattered field.[22, 41] The time bandwidth product is an approximate measure of the number of independent and instantaneous intensity fluctuations averaged over the measurement time \( T \). The bandwidth of the LFM transmissions analyzed
in this paper is $B = 50$ Hz while the averaging time used is $T = 0.04$ s leading to degrees of freedom of $\mu = 2.0$. (We ignore averaging of overlapping beams because the measurements are not independent.) If the reverberation level in dB re 1 $\mu$ Pa for a given pixel in the acoustic image is $R = 10 \log$ (mean square pressure), the standard deviation of the reverberation level is[44]

$$\sigma = 4.34 \sqrt{\zeta(2, \mu)}$$

(4.3)

where

$$\zeta(\nu, \mu) = \sum_{k=0}^{\infty} \frac{1}{(\mu + k)^\nu}, \quad \text{for} \quad \nu > 1, \quad \mu \neq 0, -1, -2, -3, \ldots$$

(4.4)

is the Riemann’s zeta function where $\zeta(2, 1) = \pi^2/6$. For the LFM transmissions, the standard deviation at a given pixel is roughly 3 dB. This standard deviation comprises a small fraction of the range of values spanned in the acoustic images to be presented. The prominent returns that register with well-defined geomorphology typically stand above the background reverberation by tens of decibels (many standard deviations) and are therefore considered to be deterministic. It is noteworthy that while temporal averaging reduces the variance, it also reduces the range resolution.

4.3 Interpretating Wide Area Images Generated with Low Frequency and Long Range Towed Array Sonar

Figure 4-6 shows a wide area image of the ocean environment at site 2 obtained from a single bistatic transmission with the receiver ship NRV Alliance along track 17. The image shows both the diffuse background reverberation level in decibels from
the ocean environment as well as strong scattered returns from geologic features and
submerged objects. For each transmission, the signal first measured by the receiving
array correspond to those that travel directly from the source to the receiving array.
This direct arrival is strong and gives rise to the blast-out region which appears as a
red ellipse surrounding the source and the receiver in Fig. 4-6. The diffuse background
reverberation scattered from random rough patches of the ocean environment arrives
after the direct signal has passed. This diffuse reverberation has a mean intensity
that decays with range due to spreading and absorption loss in the environment. The
rate of decay depends on the the properties of the environment, such as the sound
speed structure and attenuation in the water column and bottom, as well as the
measurement geometry. The decay in the diffuse reverberation provides vital inform-
ation about the environment needed to model propagation and diffuse scattering.
In Fig. 4-6, we do not average out the trend in the data since it throws out this vital
information.

A horizontal line array has left-right ambiguity that is expressed differently in
monostatic and bistatic charts. Prominent returns are ambiguously charted in an
almost symmetrical fashion about the receiving array axis in monostatic geometries.
For bistatic geometry, ambiguity occurs about an ellipse with a major axis that passes
through the source and receiver. A diagram illustrating the two-way travel time ellipse
for bistatic measurements is provided Ref. [45] where it is shown that distortion in the
image may occur as the ambiguous returns are charted to either a smaller or more
extensive spatial extent. The methods used in this paper to resolve the left-right
ambiguity in the measurement is discussed further in Sec. 4.4.7.

In order to measure the frequency of occurence of a strong scattering event from
a target or feature along a given track, we generated a hotspot consistency chart for
each track. First a moving local peak detector was applied to a single reverberation
image to detect pixels where the reverberation level stood more than 10dB above
the average for a 1.8 × 1.8-km² sub-image and these pixels were assigned a value of
1. Pixels in the image that do not satisfy this criteria were assigned a value of 0.
Pixels in the blast-out region are assigned a value of 0 as we are only interested in
detecting targets and reverberation from the environment. This binary matrix from each reverberation image was then summed for all the images along a track to form the hotspot consistency chart for each track. An example of a hotspot consistency chart is shown in Fig. 4-8.

The standard deviation for the time averaged diffuse reverberation is 3-dB[41]. The local peak detector algorithm essentially picks out pixels with returns that are about three standard deviations above the local mean time averaged diffuse reverberation within each sub-image.

Scattered returns from a fixed target as well as the diffuse background reverberation decay with range in a real ocean waveguide due to spreading and absorption loss. Fix threshold detectors require the general trend of decay of the reverberation to be removed or corrected for before the detector can be applied. For real measurements, it is usually difficult to detrend the data accurately. One advantage of this local peak detector is that it looks for pixels with echo values that are locally higher by 10-dB from their immediate neighbours and it therefore takes the range decay into account, thus circumventing the problem faced with using a fixed threshold detector. The local peak detector is also independent of the strength of the source making it versatile and easy to implement in measurements.

4.4 Remotely Imaging Sub-seafloor Geomorphology, Seafloor Geomorphology and Calibrated Targets in the Strataform Area

In this section we present an overview of the major experimental findings of the Geoclutter Acoustics Experiment. Roughly 3000 waveforms were transmitted into the water column and wide area image of the reverberation from the ocean environment was generated in near real time for each transmission. On average between 10 to more than 100 discrete and localized scatterers were registered for each image, giving a total of at least 30,000 scattering events that could be confused with that from
Figure 4-6: A bistatic wide area image along track 17 at site 2. Travel time to range conversions are done by multiplying the two-way travel time with half the mean sound speed of 1475-m/s. All returns are mirrored about the array axis (71°) due to right-left ambiguity. Two prominent and discrete scattering events > 20 dB above the background co-register well with the location of the calibrated targets approximately 8.5-km to the south of the source and receiving array. Numerous other prominent scattering events from features are also observed that can be confused with returns from the calibrated targets. Comparison with Fig. 4-7 breaks the receiver line array's right-left ambiguity and places the true location of these features to the south within the dotted white trapezoid. (Track 17, file:hla2001121130425, transmission: 1s duration LFM from 390 to 440-Hz. Mean source depth: 55-m, mean receiver depth: 42-m, array axis: 71°. Coordinates of south-west corner: 38° 55.80'N, 73° 13.42'W.)
a large submerged vehicle over the period of the experiment. The vast majority of prominent and discrete scattering events appear in areas where the bathymetry is locally level indicating that they could be of sub-bottom origin. A large number of these target-like returns co-registered with buried river channel networks previously characterized under the Geoclutter Program. In addition, new channel networks were discovered during subsequent geophysical surveys in areas where prominent scattering events were recorded in the wide area reverberation images.

A hotspot consistency chart was generated for each track combining the information contained in images acquired along a track. The hotspot chart shows the location of strong and persistent echo returns from transmissions along a track. A large number of the scattering events at site 2 were as prominent and consistent as those from the calibrated targets. Site 1 however had many more scattering events that fade in and out along a track due to the larger bistatic separation and orientation of source and receiver array relative to the prominent geologic features. Registrations with known channels suggest that bistatic aspects that produce coherent "glints" from river channel walls with large projected area yield the strongest and most frequent target-like returns. Stronger returns were found to be correlated with less deeply buried channels. The level of returns was also found to be dependent on source and receiver depths, as well as sound speed structure.

Moderately strong and lineated returns from inclined sub-bottom strata or R-reflectors were consistently observed in regions where the reflector approached the water-seabed interface. They were imaged at sites 1 and 3 from long range, up to 30 km. Some of these lineated returns co-registered well with existing R-reflector traces from geophysical surveys. In some instances, the wide-area images provided more extensive mapping of the R-reflect surfaces expression than that available from geophysical surveys. Much stronger discrete features also appeared on those lineations, in regions where buried river channels crossed the near surface expression of the sub-bottom layer.

Seafloor features, such as the ice-berg scours and the walls of the hudson canyon were also imaged as expected at site 3. The ice-berg scours were only imaged at very
small ranges and their echo returns were much weaker than those from sub-bottom features. Returns from the canyon walls are very strong, but extended and are not target-like in appearance.

These findings show that a low frequency active acoustic system can be used to remotely image a network of previously unknown sub-bottom and seafloor features over a wide area at roughly 5 to 30-km standoff in near real time. Current geophysical survey methods, on the other hand, using a chirp or seismic source system only provide information about the seabottom directly beneath the survey ship. To provide similar coverage of the sea bottom over a wide area as that provided by the low frequency active sonar system, current geophysical survey methods require much longer time duration for data collection as the entire area has to be combed like a “lawn-mower”.

The hotspot consistency charts were used extensively in the experimental data analysis. Apart from their use in clutter classification to distinguish consistent echos from fluctuating ones, they were also used to resolve the right-left ambiguity inherent in horizontal line array data when the bistatic range to the target was not much larger than the source-receiver separation. In many instances, the hotspot charts provided significantly improved imaging of sub-bottom features such as river channel axes compared to the individual reverberation images. The hotspot charts also reduce charting errors due to dispersion in the waveguide since they combine information from a large number of transmissions.

### 4.4.1 Discovery of a Prehistoric Buried River Channel Network with a Long-range Sonar System

In this section, we present wide area images showing an organized network of prominent and highly aspect-dependent scattering events in a region of flat bathymetry at site 2, where the sub-bottom had not yet been profiled. These scattering events were often more prominent than and just as discrete in appearance as those from the calibrated targets deployed at similar range from the sonar system. Without prior knowledge, it would be impossible to distinguish the calibrated targets from
these features. Subsequent geophysical surveys, specifically designed to explore the sub-bottom in the vicinity of these prominent scattering events,[1] revealed a large network of buried river channels at the location of these events.

In Fig. 4-6, which shows a wide-area image from track 17 at site 2, we observe two prominent scattering events approximately 8.5km away to the south of the source and receiving array that register well with the calibrated targets. These events stand out by more than 20 dB above the reverberation in adjacent beams. Since this difference is much larger than the received level standard deviation of 3-dB, the scattering from the calibrated targets is deterministic. The receiving line array has right-left ambiguity about the array axis. We therefore see the ambiguous returns from the calibrated targets charted to the west in Fig. 4-6.

We observe numerous other prominent and discrete scattering events in the image shown in Fig. 4-6. Since many of these features are just as prominent as the calibrated target, standing out by more than 20 dB above the reverberation in adjacent beams, they can be confused with the calibrated targets if the precise location of the targets were unknown. Without prior knowledge it would be impossible to distinguish the calibrated targets from the features in the image.

To identify the region the features originate from, we need to break the ambiguity due to the receiving line array. We do so by comparing Fig. 4-6 with Fig. 4-7 which shows an acoustic image from a bistatic transmission along track 14 which has a slightly different orientation from track 17. In both Figs. 4-6 and 4-7, we see consistently strong and prominent events within the trapezoid to the south of the source and receiving array showing that this is the true location of the feature. The ambiguous events are charted to the west in both figures and are not in the same location. The differing array orientation in tracks 14 and 17 causes the ambiguous returns from the features to be charted to different regions.

To examine the consistency of the prominent scattering events within the trapezoid as well as those from the calibrated targets, a hotspot consistency chart, as described in Sec. 4.3, was derived for track 17 and the result is shown in Fig. 4-8. A total of 49 bistatic acoustic images were combined to form this composite. The results are
Figure 4-7: Similar to Fig. 4-6 except that data is from track 14. Right-left ambiguity of the prominent scattering events from the features of interest can be resolved by comparing this figure with Fig. 4-6. (Track 14, file: hla2001121113455, transmission: 2s duration LFM from 390 to 440-Hz. Mean source depth: 55-m, mean receiver depth: 22-m, array axis: 225.5°. Coordinates of south-west corner: 38° 55.80’N, 73° 13.42’W.)
plotted as $10 \log N$ where $N$ is the number of images that register a strong scattering event $> 10$ dB above the local average within $1.8 \times 1.8$-km$^2$ area of a given pixel location. Figure 4-8 displays the consistency of events that are 10dB above the local average of the event. From Fig. 4-8, we observe that the events corresponding to the calibrated targets are consistently prominent over variation in receiver location along the track appearing as a local maxima in all 49 images along the track. The scattering events within the trapezoid in Fig. 4-8 are also prominent and consistent along the track appearing in most of the charts with levels $>10$ dB above the local average. It should be pointed out that the scattering events of interest within the trapezoid are not all alike. In the top half of the trapezoid, the prominent events are discrete and target-like in appearance. These are the events that can be confused with those from the calibrated targets. The hotspot event in the lower half of the trapezoid is large, extending over 3-km, and is less likely to be confused with a discrete target like an underwater vehicle, but it can camouflage an underwater vehicle from detection if the vehicle were to be located within it.

In Fig. 4-9, we overlay the prominent events from Fig. 4-8 on the color gradient of bathymetry. Only events with levels that are 10dB above the local average and are consistent in at least 10 transmissions along the track are overlain. From Fig. 4-9, we observe that the prominent and consistent scattering events within the trapezoid do not originate from the seafloor because the seafloor in this region is level. Some of these events probably originate from sub-bottom features submerged beneath the seafloor and are hence not detectable using a conventional depth sounder.

Since no prior survey of the sub-bottom existed in this region, RV Endeavour explored the bottom during the Geoclutter Acoustic Experiment with a hull-mounted chirp sub-bottom profiler in the vicinity of the prominent scattering events in the trapezoidal region in Fig. 4-9. A more detailed geophysical survey was carried out in this region using both deep-tow and hull-mounted sub-bottom profiling systems in August 2001 under the Geoclutter Program.[1] Both surveys found a dense network of buried river channels that rise up close to the seafloor at the location of the discrete scattering events in the top half of the trapezoid as shown in Fig. 4-10. Fig. 4-11
Figure 4-8: Hotspot consistency chart from 49 bistatic LFM transmissions along Track 17. The results are plotted as $10 \log N$ where $N$ is the number of images that register a strong scattering event $> 10$ dB above the local average within $1.8 \times 1.8$-km$^2$ area of a given pixel location. Scattering events within the trapezoid as well as those from the calibrated targets are consistently prominent in most of the reverberation charts along track 17. Events within the top half of the trapezoid are discrete and target-like in appearance and they may be confused with those from the calibrated targets. The hotspot event in the lower half of the trapezoid is large, extending over 3-km, and is less likely to be confused with a discrete target like an underwater vehicle, but it can camouflage an underwater vehicle from detection if the vehicle were to be located within it. (Mean source depth: 55-m, mean receiver depth: 42-m, array axis: 71°. Coordinates of south-west corner: 38° 55.79'N, 73° 13.44'W.)
Figure 4-9: Prominent events from LFM transmissions on Track 17 shown in Fig. 4-8 are overlain in white on the directional derivative of the bathymetry calculated with respect to the source location and the receiver location in the middle of track 17. Only prominent (>10-dB above local average) and repeatable events that occur in at least 10 charts out of 49 are overlain. Scattering events of interest within the trapezoid do not originate from the seafloor because the seafloor in this region is level. High resolution sub-bottom profiling revealed the existence of a dense network of buried river channels that coincide with the location of the discrete scattering events in the top half of the trapezoid as shown in Figs. 4-10 and 4-11. Coordinates of south-west corner: 38° 55.80′N, 73° 13.42′W.)
shows a deep-tow chirp profile of several buried river channels found in this area from the geophysical survey in August 2001. The extent and distribution of the channels match the distribution observed for the clutter events in the top half of the trapezoid.

Another major geophysical survey was conducted under the Geoclutter Program within the trapezoidal region providing much finer resolution of the sub-bottom in May 2002. This is the first survey to provide detail coverage of the sub-bottom in the southern half of the trapezoid where the acoustic scattering events imaged were more extended in appearance. The data from this survey is presently being analyzed. Other possible sources of origin for the extended scattering event in the lower half of the trapezoid are also being investigated. One possibility is the presence of sub-bottom gas entrapments (pockets) that have strong acoustic impedance contrast with the neighbouring sedimentation that could lead to strong scattered returns. Fish schools are known to exist close to the seafloor in this region.[55] Simulations using the unified model in Sec. 3.6 showed that scattering from mean distribution of fish schools within the Strataform area is insignificant compared to the seafloor reverberation. However, a rare occurrence of a very large and high density fish school can give rise to returns that stand above the seafloor reverberation. Gravel deposits may also exist on the seafloor. Scattering from schools of fish and gravel deposits is incoherent and omnidirectional. More studies are being done presently to study the coherency of the measured clutter events in the lower region of the trapezoid and to model the various sources of clutter mentioned above to shed some light on the measured data.

**Repeatability of clutter events**

Apart from tracks 14 and 17, scattering events in the region within the trapezoid shown in Figs. 4-6-4-9 were also registered in other tracks. An example is provided in Fig. 4-12 which shows the hotspot consistency image composed from the bistatic LFM transmissions along track 23x. Fewer bistatic signals were transmitted along track 23x where the data was collected on the same day as track 17 and 14. Fig. 4-12 shows scattering events within the same trapezoidal region as that on tracks 17 and
Figure 4-10: Location of buried river channels at site 2 mapped out by the August 2001 geophysical survey[1] using chirp sub-bottom profiling systems is overlain on the hotspot consistency chart for Track 17. The channels have varying sizes, from small (in white stars) to medium and large (pink stars). Some of these channels at site 2 come right up close to the water-sediment interface and are less deeply buried than those at site 1. Coordinates of south-west corner: 38° 55.58'N, 73° 8.77'W.
Figure 4-11: Deep-tow sub-bottom chirp profile from the geophysical survey of August 2001[1] at site 2 showing three buried river channels in the vicinity of the prominent and discrete scattering events in the top half of the trapezoid of Fig. 4-9. These river channels are located between 39° 1.3684′N, 73° 2.9619′W and 39° 2.2878′N, 73° 2.9624′W.
4.4.2 Clutter Co-registration with Previously Identified Buried River Channels Networks

Site 1 had been the focus of previous geophysical surveys where an extensive network of buried river channels had been mapped and characterized prior to the Geoclutter Acoustic Experiment. In this section, we present wide-area single transmission images as well as hotspot consistency images from several tracks where the prominent scattering events recorded at site 1 co-registered with previously identified river channel networks.

Buried river channel networks often have complicated morphology. The wide area images obtained from consecutive single transmissions along a track at site 1 reveal prominent scattering events that appear to be “glints” originating from distinct sections of the channel wall that would fade in and out depending on the bistatic orientation of the source and receiver. Two of these images are shown in Figs. 4-13(a)-(b). An analogy can be drawn with the “glints” observed on a spoon. Due to the curved surface of the spoon, the glints on a spoon migrate when the bistatic orientation of the observer changes.

The hotspot consistency charts, obtained along a track, Figs. 4-14(a)-(c), show not only the repeatability of these prominent scattering events, but also provide a better image of the channel extent since it combines “glints” accumulated throughout a track. Comparing Figs. 4-14(a) and (b), we observe that bistatic aspects that lead to large projected areas from the river channels produce the most prominent “glints” from those sections of the channel walls. This can be observed by comparing the hotspot consistency image from track 86 in Fig. 4-14(b) which registered prominent and consistent scattering events from channel branches to the south of the source and receiving array with the hotspot image from track 85 in Fig. 4-14(a). The orientation of track 86 allows a larger area of these southern river channel axis to be projected in the direction of the receiving array than in track 85.
Figure 4-12: Similar to Fig. 4-8 but for 15 bistatic transmissions along track 23x. Data for this track was collected on the same day as track 17. Scattering events are located within the same trapezoidal region as that on tracks 17 and 14 shown in Figs. 4-6-4-9. (Mean source depth: 55-m, mean receiver depth: 43-m, array axis: 209°. Coordinates of south-west corner: 38° 55.80’N, 73° 13.44’W.)
Figure 4-13: Similar to Fig. 4-6 except that the image is from (a) track 1c (b) track 85 at site 1. "Glints" from walls of the buried river channels can be observed in both figures. These originate from channel sections that are less deeply buried under the water-sediment interface. Compare with Fig. 4-14(d). "Glints" in the images from track 85 fade in and out along a track depending on the bistatic orientation. ((a) Track 1c, file:hla20011118160015, transmission: 1s duration LFM from 390 to 440-Hz. Mean source depth: 46.1-m, mean receiver depth: 36.5-m, array axis: 285°. Coordinates of south-west corner: 39° 13.06’N, 72° 54.97’W. (b) Track 85, file:hla2001124192958, transmission: 1s duration LFM from 390 to 440-Hz. Mean source depth: 40.9-m, mean receiver depth: 39-m, array axis: 293°. Coordinates of south-west corner: 39° 13.98’N, 72° 56.09’W.)
Apart from bistatic orientation, long-range imaging of buried river channels also depend on the depth the channels are buried below the water-sediment interface. Comparing Figs. 4-14 (a) and (b) with Fig. 4-14(d), [20] we observe a high correlation between the location of prominent scattering events in tracks 85 and 86 with sections of the channel wall that are closer to the water-sediment interface. This implies that we are better able to image channel sections that are less deeply buried. The propagating modes of a waveguide are usually evanescent in the sediment layer with modal amplitudes that decay exponentially with depth below the sediment. The less deeply buried river channels would therefore have higher amplitude incident waves which would lead to stronger scattered field from these channels than from the more deeply buried ones. In Fig. 4-13(a), we see particularly strong returns from two sections of the buried river channel that are close to the water-sediment interface.

4.4.3 Long Range Imaging of Sub-bottom R-reflectors

The surface expression of the sub-bottom reflectors at site 3 were imaged bistatically from long ranges >30-km on tracks 61 and 62, as seen in Figs. 4-15(a)-(b) respectively. We observe scattered returns from a long section of the sub-surface horizon that extends much further out in range from that indicated in previous geophysical surveys maps. Comparing Figs. 4-15(a) and (b) allows the ambiguity about the line array to be resolved. The true location where the sub-surface horizon outcrops close to the sea surface is to the south-west in both charts.

The R-reflectors at site 1 were also imaged as shown in Figs. 4-16(a)-(b) where lineated echo returns more than 10 dB above the background extending 3-km outward to the left and right of the receiving array can be observed. (The surface expression of the R-reflector was found from geophysical survey to exist within the region indicated by the dotted white lines.) In addition more prominent and discrete events with levels 20 dB above the background appeared on these lineations, in regions where the buried river channels crossed the near surface expression of an R-reflector, as seen in the north-east region of the charts. Since the array has left-right ambiguity, the echos from these crossings have ambiguity charted to the south-west of the receiving array.
Figure 4-14: Similar to Fig. 4-8 but for (a) 87 bistatic transmissions along track 85 with array heading 293°, (b) 79 bistatic transmissions along track 86 with array heading 97°. (c) 12 monostatic transmissions along track 12 with array heading 228.6°. Prominent scattering events coincide with axis of buried river channels that have been previously characterized.[14, 21] The occurrence and level of returns depend on bistatic orientation with the strongest and most frequent target-like returns coinciding with features that project the largest area in the direction of source and receiving array. Less deeply buried river channel sections are also much better imaged than deeply buried ones. RV Endeavour imaged monostatically in (c) as shown by the red star. (d) Depth of buried river channels below water-sediment interface. [20] Shallower, less deeply buried segments are better imaged than more deeply buried ones. Compare (d) with (a) and (b). Coordinates of south-west corner: (a) 39° 12.95′N, 72° 58.11′W, (b) 39° 13.32′N, 72° 56.88′W, (c) 39° 14.20′N, 72° 59.64′W (d) 39° 12.69′N, 72° 59.82′W.
Figure 4-15: (a) and (b) are similar to Fig. 4-6 except that data is from tracks 61 and 62 respectively at site 3. Scattered returns from surface expression of seismically reflective horizon called R-reflector is imaged out to ranges of 30-km and extends further from what is indicated from previous geophysical surveys. Comparing Figs. (a) and (b) breaks the receiver line array's right-left ambiguity and places the true location of the R-reflector's surface expression to the south-west. ((a): Track 61, file:hla2001122121135, transmission: 2s duration LFM from 390 to 440-Hz. Mean source depth: 83.8-m, mean receiver depth: 60-m, array axis: 357.5°.) (b) Track 62, file:hla2001122160815, transmission: 2s duration LFM from 390 to 440-Hz. Mean source depth: 83.8-m, mean receiver depth: 34-m, array axis: 133°. Coordinates of south-west corner in (a) and (b): 38° 59.17'N, 72° 59.13'W.)
Figure 4-16: Monostatic wide area images along track 3 at site 1. Moderately strong lineated returns from the near surface expression of the R-reflector extending 3km outward from the array axis can be observed. More prominent and discrete echos appear in regions where buried river channels cross the R-reflector in the north-east of the array axis. The two white dotted lines show the approximate region where the R-reflector approaches the water sediment interface. (Track 3, (a) file:hla2001119183615, transmission: 0.5s duration, (b) file:hla2001119183935, transmission: 2s duration, LFM from 390 to 440-Hz. Mean source depth: 30-m, mean receiver depth: 36-m, array axis: 133°. Coordinates of south-west corner in (a) and (b): 39° 11.78’N, 72° 59.07’W.)
4.4.4 Imaging of Seafloor Features

Two prominent seafloor features were also imaged at site 3. Figures 4-17(a)-(b) show the hotspot consistency images formed from data acquired along tracks 61 and 62 respectively. For these hotspot images, levels that are 10 dB above the local average over an area $2.7 \times 2.7 \text{ km}^2$ are selected as peaks. The ice-berg scour on the seafloor about 2km away from the source was imaged along both tracks repeatedly. Furthermore, strong scattering from the walls of the hudson canyon more than 20 km away from the source and receiving array was also registered. It is noteworthy that the R-reflectors discussed in Sec. 4.4.3 were detected with levels that are less than 10 dB above the reverberation in adjacent beams in Figs. 4-17(a)-(b). As a result, these returns are not very prominent in the hotspot images which picks out peaks that are 10 dB above the local average.

4.4.5 Images with High Density of Scattering Events at Site 1

Many of the images obtained from the bistatic measurements at site 1, such the one shown in Fig. 4-18(a) from track 81, contain a high density of prominent and discrete scattering events, but few of these register with the location of existing mapped sub-bottom or surface geologic features. Even though each individual image is dense, few of these scattering events are repeatable as seen from the hotspot consistency chart, Fig. 4-18(b) One reason for this is that many of the sub-bottom features like the buried river channel morphology in many regions of site 1 have not been fully mapped out.

4.4.6 Effect of Source and Receiver Location and Orientation on Imaging

Scattering events that co-register with the calibrated target locations were stable and consistent for all tracks at site 2 when the targets were actively deployed. Scattering
Figure 4-17: (a) Hotspot consistency charts from 15 bistatic LFM transmissions along track 61 and at site 3. The results are plotted as $10 \log N$ where $N$ is the number of images that register a strong scattering event $> 10$ dB above the local average within $2.7 \times 2.7$-km$^2$ area of a given pixel location. (b) Similar to (a) but for 21 bistatic transmissions along track 62 at site 3. Comparing Fig. (a) and (b) breaks the receiver line array’s right-left ambiguity. It registers the prominent scattering from the walls of the Hudson canyon located to the west in both figures, as well as scattering events from the ice-berg scour close to the location of the bistatic source. (a) Track 61, mean source depth: 83.8-m, mean receiver depth: 60-m, array axis: 357.5°. (b) Track 62, mean source depth: 83.8-m, mean receiver depth: 34-m, array axis: 357.5°. Coordinates of south-west corner in (a) and (b): 38° 59.50’N, 72° 59.18’W.)
Figure 4-18: (a) Similar to Fig. 4-6 except that the image is from track 81 at site 1. (b) Similar to Fig. 4-8 but for 89 bistatic transmissions along track 81. Prominent and discrete scattering events are densely distributed throughout the image in (a) from individual transmissions, but few of these events are repeatable in (b). (Track 81, (a) file: hla2001124121318, transmission: 1s duration LFM from 390 to 440-Hz. (a) and (b) Mean source depth: 41-m, mean receiver depth: 35-m, array axis: $351^\circ$. Coordinates of south-west corner: (a) 39° 11.97'N, 72° 58.45'W, (b) 39° 11.97'N, 72° 58.47'W.)
events originating from features located within the trapezoid in Fig. 4-8, however, showed much variability depending on source and receiver location and orientation.

Figure 4-19 shows the hotspot consistency chart for the 46 bistatic transmissions along track 14. Prominent and discrete events are registered from the calibrated targets along the full length of track 14. Scattering events from features within the trapezoid, however, were prominent with levels of more than 10-dB above the background in at most seven images along track 14. These images were acquired when NRV Alliance was located towards the southern end of the track. Less prominent event was registered in the region within the trapezoid when NRV Alliance was located towards the northern half of track 14. Tracks 17 and 14 are located at roughly the same range from the calibrated targets and the features located within the trapezoid, but they differ in their orientation by approximately 10°. The receiver array was at a shallower depth in the water column of about 22-m on track 14, while it was deeper at 42-m depth on track 17. The bistatic source depth were similar for data acquired along these two tracks.

Figure 4-20(a) shows an image from track 18 which has a similar mean receiver array depth of 42-m, but a different location and orientation from track 17. This image was acquired a day before that on track 17. The scattering events originating from the calibrated targets are prominent in Fig. 4-20(a) with levels of more than 20 dB above the background in adjacent beams. In Fig. 4-20(a) however, no distinguishable events are visible within the trapezoidal region that had been observed along track 17 and part of track 14. A similar result is seen from Fig. 4-20(b) which shows the hotspot consistency chart for track 18.

As discussed in Sec. 4.1 the 2 calibrated targets are cylindrical tubes that span close to half the water column from 32 to 62-m depth and have an omnidirectional scatter function in the azimuth. They were registered as events with equally strong signal-to-background-noise ratio in all the tracks at site 2, independent of the source and receiver location and orientation. The scattering events from features showed strong dependence on bistatic location and orientation of source and receiver. The effect of the sound speed profile is presently being investigated with simulation using
Figure 4-19: Similar to Fig. 4-8 but for 46 bistatic transmissions along track 14. Prominent scattering events with levels 10-dB above the background co-register with the location of the calibrated targets along the full length of track 14. Scattering events originating from features within the trapezoid are not registered as consistently along the track 14. It should be noted that track 14 had a shallower receiver depth than track 17 in Fig. 4-8. This hotspot image shows that detection of scattering events from features is dependent on receiver depth and orientation. (Mean source depth: 55-m, mean receiver depth: 22-m, array axis: 225.5°. Coordinates of south-west corner: 38° 54.11'N, 73° 15.11'W.)
Figure 4-20: (a) Similar to Fig. 4-6 and except that data is from track 18. (b) Similar to Fig. 4-8 but for 47 bistatic transmissions along track 18. Prominent events > 20 dB above the background are registered from the two calibrated targets approximately 8.5-km to the south of the source and receiving array. No distinguishable scattering events are visible within the trapezoidal region that had been observed along tracks 17 in Fig. 4-8 and part of track 14 in Fig. 4-19. These image illustrate the fact that scattering events from features is dependent on bistatic location and orientation of source and receiver. (Track 18, (a) file:hla2001120154635, transmission: 1s duration LFM from 390 to 440-Hz. (a) and (b) Mean source depth: 55-m, mean receiver depth: 42-m, array axis: 340°. Coordinates of south-west corner: (a) 38° 55.80'N, 73° 13.42'W, (b) 38° 55.79'N, 73° 13.44'W.)
the unified model.

4.4.7 Ambiguity Resolution

Two methods are employed in the present analysis to resolve the left-right ambiguity in the returns from the horizontal line array. The first method involves comparing the events from images obtained on tracks where the array orientation differs.[43] For instance, the location of the prominent scattering events from the calibrated targets and features in the images from track 17 were resolved by comparing them with the images on track 14, which had a different orientation for the array at approximately the same range from track 17 (see Figs. 4-6 and 4-7). Another example is the ambiguity resolution of the lineated returns from the surface expression of a sub-bottom strata measured along tracks 61 and 62 (see Figs. 4-15(a)-(b)). These tracks varied in their orientation and the ambiguous returns from the R-reflector were identified because they were charted to different locations.

The second method resolves the ambiguity for measurements made along a single track with the use of the hotspot consistency chart for the track. This method is applicable in bistatic scenarios when the range to the feature is not much larger than the source-receiver separation. It exploits the elliptical shape of the travel time locus to break the ambiguity in the line array measurement. Figure 4-21(a), from site 1, shows a set of prominent and discrete scattering events charted to the north-west and another set of more elongated events charted to south-east. Only one of these is the true scattered return from the features while the other is ambiguous. From Fig. 4-21(b), which shows the hotspot consistency chart for the track, the events from the south-east are consistently charted to the same area leading to a strong reinforcement of these events in the composite chart. The events to the north-west are distributed over a wide area in the composite chart with little repeatability showing that these events are the ambiguous ones. The true scattered returns therefore originate from the south-east. Geophysical survey from August 2001 indicates that these events, normal to the seafloor inclination, are likely to originate from acoustic impedance differences due to sediment variation on the seafloor.
Figure 4-21: (a) Similar to Fig. 4-6 except that image is from track 73 at site 1. (b) Similar to Fig. 4-8 but for 12 bistatic transmissions along track 73 at site 1. In (a) set of of prominent and discrete scattering events charted to the north-west and another set of more elongated events charted to south-east. Only one of these is the true scattered return from the features while the other is ambiguous. Comparing (a) and (b) shows that the events to the south-west are the true returns. (Track 73, (a) file:hla2001123162835, transmission: 2s duration LFM from 390 to 440-Hz. (a) and (b) Mean source depth: 40.9-m, mean receiver depth: 38-m, array axis: 219.5°. Coordinates of south-west corner: (a) 39° 8.88′N, 73° 3.48′W, (b) 39° 8.87′N, 73° 3.50′W.)
4.5 Summary

An active sonar system was used to remotely and rapidly image geomorphology over wide areas in continental shelf waters by long-range echo sounding. The bistatic system, deployed in the strataform area south of Long Island in April-May of 2001, imaged extensive networks of buried river channels and inclined sub-seafloor strata over tens of kilometers in near real time. Bathymetric relief in the strataform area is extremely benign. The vast majority of features imaged correspond to sub-bottom geomorphology that sound waves apparently reach after tunneling as well as propagating through the overlying sediment. Returns from buried river channels were often found to be as discrete and strong as those from calibrated targets placed in the water column. Since buried river channels are expected to be ubiquitous in continental shelf environments, sub-seafloor geomorphology will play a major role in producing “false alarms” or clutter in long-range sonar systems that search for submerged objects such as underwater vehicles or marine mammals since the returns from the geologic features may be confused with those from the submerged object. Waveguide scattering and propagation are inherent to this new remote sensing technology because source signals are transmitted over hundreds of water-column depths in range to image sub-seafloor and seafloor geomorphology.
Chapter 5

Extinction Theorem for Object Scattering in a Stratified Medium

In this chapter, we derive the generalized extinction theorem that provides a relation for the rate at which energy is extinguished from the incident wave of a far field point source by an object of arbitrary size and shape in a stratified medium. We begin our waveguide extinction derivation in Sec. 5.1 with the unified model for the time-harmonic scattered field from an object in a waveguide, Eq. (2.1), which is expressed in a normal mode formulation. Since the full extinction is maintained in the region of active interference and this region extends into the farfield where separation of variables can be invoked, energy fluxes necessary for the derivation can be calculated in the far field in terms of waveguide modes and the object’s planewave scatter function.[48, 42]

The extinction cross section of an object is defined as the ratio of its extinction to the rate at which energy is incident on unit cross sectional area of the object [4]. The extinction cross section reduces to the scattering cross section for non-absorbing objects, and is useful in actively classifying targets since, as the ratio of the total extinction to the incident intensity, it depends only upon scattering properties of the target in free space. This definition, however, is ambiguous in a waveguide because both the incident and scattered fields are comprised by superpositions of plane waves. Here scattering and propagation effects are not generally separable since
they are convolved together in the extinction. Additionally, the incident intensity is not spatially constant. In spite of these difficulties, we find it convenient to interpret the extinction cross section for an object in a waveguide in Sec. 5.3 as the ratio of the extinction to the incident energy flux per unit area in the radial direction at the object's centroid. This definition is sensible when the object is in a constant sound speed layer and in the far field of the source.

In Sec. 5.5 we show using calculations for a shallow water waveguide that an object's cross section for the combined extinction of all the modes of the waveguide is highly dependent on measurement geometry, medium stratification, and its scattering properties. The combined cross section fluctuates rapidly with range due to coherent interference between the modes making it difficult to estimate an object's size from the combined cross-section. We introduce a modal cross-section of the object for the extinction of the individual modes of the waveguide. We show that the size of an object in a waveguide can be estimated from its modal cross-section for mode 1.

5.1 The Generalized Extinction Theorem

In this section, we derive the extinction in the incident field of a far field point source due to an obstacle of arbitrary size and shape in a stratified medium. The general approaches for calculating extinction are discussed in Appendix D.1. Here, we adopt the intuitive approach of Van de Hulst [74, 75, 4] which involves integrating the energy flux, or intensity, over a screen placed a distance away from the object sufficiently large to register Fraunhofer diffraction, Eq. (D.11). In the absence of the object, the total energy flux across the screen is maximal. In the presence of the object, the total energy flux across the screen is diminished by the shadow remnant. For a sufficiently large screen, the difference between these fluxes is the total extinction.

We focus on the Van de Hulst screen method for calculating extinction because it is of more practical use since it represents the type of measurement that can be made with a standard 2-D planar or billboard array. This is discussed further in Sec. 5.4. The other approach for calculating extinction using a control surface that encloses
Figure 5-1: The geometry of the problem showing an object in a stratified medium composed of a water column of thickness $H$ overlying a bottom. The origin of the coordinate system is at the center of the object and the source is located at $(-x_0, 0, z_0)$. The screen has width $L$ and is semi-infinite in the $z$-direction penetrating into the bottom with an edge at the top of the water column.
the object in a stratified waveguide is discussed in Appendix D.1. A control volume measurement would be very difficult to implement since it would require an array that completely encloses the object.

The relevant geometry is shown in Fig. 5-1. As a reminder, the origin of the coordinate system is placed at the object centroid with $z$ axis vertically downward, and $x$ axis parallel to the boundaries. The coordinates of the source are defined by $\mathbf{r}_0 = (-x_0, 0, z_0)$. The screen is positioned in forward azimuth on the $y-z$ plane at a horizontal range $x$ from the object centre. The width of the screen is $L$ along the $y$ direction and is semi-infinite in the $z$ direction with an edge at the surface of the waveguide. Let $\mathbf{r} = (x, y, z)$ be the coordinates of a point on the screen. As discussed in Appendix D.1 to measure the full extinction in the waveguide, we require $L > \sqrt{\lambda x}$, where $x$ is the horizontal range of the screen from the object.

Assuming that the object is far from the source and the screen so that the range from the screen to the source is large, the incident field at location $\mathbf{r}$ on the screen for a source at $\mathbf{r}_0$, can be expressed as a sum of normal modes,

$$
\Phi_i(\mathbf{r}|\mathbf{r}_0) = (4\pi) \frac{i}{d(z_0)\sqrt{8\pi}} e^{-i\pi/4} \sum_{l=1}^{\infty} u_l(z)u_l(z_0) \frac{e^{i\xi_l|\rho - \rho_0|}}{\sqrt{\xi_l}|\rho - \rho_0|}
$$

where $u_l(z)$ and $\xi_l$ are the $l$th modal amplitude and horizontal wavenumber respectively, and $d(z)$ is the density at depth $z$. Equation 5.1 is the incident field from a point source with unit amplitude, $A = 1$.

To calculate the extinction using the general formula of Eq. (D.11), we first evaluate the integrand for the point $\mathbf{r}$ on the screen. The first term in the integrand of Eq. (D.11) using Eqs. (D.2), (5.1), (2.1) and (2.3) is
In the above expression, the terms representing absorption by the waveguide have been factored out explicitly to avoid confusion when conjugating the fields and also to keep track of absorption losses due to the waveguide. The exact expressions for $|\rho - \rho_0| = \sqrt{(x + x_0)^2 + y^2}$ and $|\rho| = \sqrt{x^2 + y^2}$ were kept in the terms that determine the phase of the integrand while the approximations $|\rho - \rho_0| \approx (x + x_0)$ and $|\rho| \approx x$ were used in the spreading and absorption loss factors, since $x, x_0 >> y$ can be satisfied for a screen that measures the full extinction.

Next we integrate Eq. 5.2 over the area of the screen. With the screen lying parallel to the $y - z$ plane, an area element of the screen is $dS = \mathbf{i}_x dy dz$. We use the orthogonality relation in Eq. (2.4) between the modes $u_i^*(z)$ and $u_m(z)$ to integrate Eq. (5.2) over the semi-infinite depth of the screen in the waveguide. This reduces the triple sum over the modes to a double sum.
\[
\iint_{S_c} V^*_i \Phi_s \cdot dS \\
= \int_{-L/2}^{L/2} \int_{-D}^{\infty} V^*_i \Phi_s \cdot i_x dz dy \\
= \frac{8\pi^2}{\omega k} \frac{1}{d(z_0)d(0)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\xi_m}{\xi_n} \sqrt{x(x_0 + x)} u^*_m(z_0) \\
\times \left\{ \int_{-L/2}^{L/2} \left[ N^{(1)}_m e^{i \mathcal{R}(\gamma_m) D} A_n(r_0) S(\pi - \alpha_m, \phi; \alpha_n, 0) \\
- N^{(2)}_m e^{-i \mathcal{R}(\gamma_m) D} A_n(r_0) S(\alpha_m, \phi; \alpha_n, 0) \\
- N^{(1)}_m e^{-i \mathcal{R}(\gamma_m) D} B_n(r_0) S(\pi - \alpha_m, \phi; \alpha_n, 0) \\
+ N^{(2)}_m e^{-i \mathcal{R}(\gamma_m) D} B_n(r_0) S(\alpha_m, \phi; \pi - \alpha_n, 0) \right] \\
\times e^{i \mathcal{R}(\xi_m) (\sqrt{x^2 + y^2} - \sqrt{(x_0 + x)^2 + y^2})} dy \right\} e^{-\mathcal{R}(\xi_m) (x_0 + 2x) e^{-\mathcal{R}(\gamma_m) D}} \quad (5.3)
\]

In the above expression, the scatter function is dependent on \( y \) via the azimuth angle \( \phi = \tan^{-1}(y/x) \). As discussed in Appendix D.1, the angular width of the active area on the screen in azimuth is of the order of \( \sqrt{\lambda/x} \). We can therefore approximate the scatter function with its value at \( \phi = 0 \) and factor it from the integral above since \( x \) is large. We also expand the exponent involving the variable \( y \) according to

\[
\sqrt{(x_0 + x)^2 + y^2} \approx x_0 + x + \frac{y^2}{2(x_0 + x)} \quad (5.4)
\]

\[
\sqrt{x^2 + y^2} \approx x + \frac{y^2}{2x} \quad (5.5)
\]

Applying the result of the following asymptotic integration over the width of the screen
\[ \int_{-L/2}^{L/2} S(\pi - \alpha_m, \phi; \alpha_n, 0) e^{iR(\xi_m)} \left( \sqrt{\alpha^2 + y^2 - \sqrt{\alpha_0 + x}^2 + y^2} \right) dy \]

\[ = e^{-iR(\xi_m) x_0} S(\pi - \alpha_m, 0; \alpha_n, 0) e^{i\pi/4} \sqrt{\frac{2\pi x_0(x_0 + \alpha)}{\Re(\xi_m)}} , \quad (5.6) \]

to Eq. (5.3), the integration of the first term in Eq. (D.11) over the area of the screen in the waveguide becomes

\[
\iint_{S_c} V_s^* \Phi_s \cdot dS
\]

\[ = \frac{4\pi^2}{\omega k} \frac{i}{d^2(z_0)d(0)} \frac{1}{x_0} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\xi_m^*}{|\xi_m| \sqrt{\Re(\xi_m)}} u^*_n(z_0) u_n(z_0) e^{iR(\xi_n - \xi_m) x_0} \]

\[ \times \left[ N^{(1)}_m N^{(1)}_n e^{iR(\gamma_m + \gamma_n) D} S(\pi - \alpha_m, 0; \alpha_n, 0) \
- N^{(2)}_m N^{(1)}_n e^{iR(-\gamma_m + \gamma_n) D} S(\alpha_m, 0; \alpha_n, 0) \right. \\
- N^{(1)}_m N^{(2)}_n e^{iR(-\gamma_m - \gamma_n) D} S(\pi - \alpha_m, 0; \pi - \alpha_n, 0) \\
+ N^{(2)}_m N^{(2)}_n e^{iR(-\gamma_m - \gamma_n) D} S(\alpha_m, 0; \pi - \alpha_n, 0) \right] \\
\times e^{-\Im(\xi_m + \xi_n) x_0} e^{-\Im(2\xi_m) x} e^{-\Im(\gamma_m + \gamma_n) D}. \quad (5.7) \]

Similarly, we can evaluate the second term in Eq. (D.11) which gives
\[
\begin{align*}
\int \int_{S_c} V_s \Phi_i \cdot dS &= \frac{4\pi^2 i}{\omega k d^2(z_0)d(0)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\xi_m^*}{|\xi_m| \sqrt{\Re \{\xi_m\}}} u_m^*(z_0) u_n^*(z_0) e^{-iR\{\xi_m - \xi_n\}z_0} \\
& \times \left[ N_m^{-*} N_n^{-*} e^{-iR\{\gamma_m + \gamma_n\}D} S^*(\pi - \alpha_m, 0; \alpha_n, 0) \\
& - N_m^{+*} N_n^{+*} e^{-iR\{\gamma_m + \gamma_n\}D} S^*(\alpha_m, 0; \alpha_n, 0) \\
& - N_m^{-*} N_n^{(2)} e^{-iR\{\gamma_m - \gamma_n\}D} S^*(\pi - \alpha_m, 0; \pi - \alpha_n, 0) \\
& + N_m^{+*} N_n^{(2)} e^{-iR\{\gamma_m - \gamma_n\}D} S^*(\alpha_m, 0; \pi - \alpha_n, 0) \right] \\
& \times e^{-\Theta(\xi_m + \xi_n)x_0} e^{-\Theta(2\xi_m)x_0} e^{-\Theta(\gamma_m + \gamma_n)D}.
\end{align*}
\]

(5.8)

When we sum Eqs. (5.7) and (5.8), taking only the negative of the real part of the sum following Eq. (D.11), we obtain the range dependent extinction \( \mathcal{E}(x|\mathbf{r}_0) \) of the incident field in a waveguide due to an object at the origin measured by a screen at distance \( x \) from the object with source at \( \mathbf{r}_0 \),

\[
\begin{align*}
\mathcal{E}(x|\mathbf{r}_0) &= \frac{8\pi^2 i}{\omega k d^2(z_0)d(0)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sqrt{\Re \{\xi_m\}}}{|\xi_m|} \Im \left\{ \frac{1}{\sqrt{\xi_n}} u_m^*(z_0) u_n^*(z_0) e^{iR\{\xi_m - \xi_n\}z_0} \right\} \\
& \times \left[ N_m^{(1)} N_n^{(1)} e^{iR\{\gamma_m + \gamma_n\}D} S(\pi - \alpha_m, 0; \alpha_n, 0) \\
& - N_m^{(2)} N_n^{(1)} e^{iR\{\gamma_m + \gamma_n\}D} S(\alpha_m, 0; \alpha_n, 0) \\
& - N_m^{(1)} N_n^{(2)} e^{iR\{\gamma_m - \gamma_n\}D} S(\pi - \alpha_m, 0; \pi - \alpha_n, 0) \\
& + N_m^{(2)} N_n^{(2)} e^{iR\{\gamma_m - \gamma_n\}D} S(\alpha_m, 0; \pi - \alpha_n, 0) \right]\} \\
& \times e^{-\Theta(\xi_m + \xi_n)x_0} e^{-\Theta(2\xi_m)x_0} e^{-\Theta(\gamma_m + \gamma_n)D}.
\end{align*}
\]

(5.9)

From Eq. (5.9), we see that the total extinction is a linear sum of the extinction of each waveguide mode. The extinction of mode \( m \) involves a sum over all incident
modes \( n \) that are scattered into that extinguished mode and is expressed in terms of the object's plane wave scatter function in the forward azimuth and equivalent planewave amplitudes of the modes. The extinction decreases with source-object range \( x_0 \) in a waveguide due to geometrical spreading, and with source-object and object-receiver ranges, \( x_0 \) and \( x \), due to absorption loss in the medium.

### 5.1.1 Effect of Multiple Incident Planewaves

To understand the implications of Eq. (5.9), we consider several cases and examine the resulting expression for the extinction in each case.

#### A.1 single mode excited by source

First we consider a source that excites only a single mode \( p \). The incident field on the object and at the screen is determined by this single mode \( p \). The triple sum in Eq. (5.2) reduces to a single sum over \( m \) in this case since both \( l \) and \( n \) can only take on the integer value \( p \). The orthogonality relation between the modes \( u_1^*(z) \) and \( u_m(z) \) eliminates the sum over \( m \) leaving just a single term where \( m = p \) in Eq. (5.3). Consequently, the expression for the extinction will have only one term corresponding to \( m = n = p \), the mode excited by the source,

\[
E(x|x_0) = \frac{8\pi^2}{\omega k} \frac{1}{d^2(z_0)d(0)} \frac{1}{x_0} \frac{1}{|\xi_p|} |u_p(z_0)|^2 \mathfrak{R}\left\{ \frac{1}{\sqrt{\xi_p}} \right\} 
\times \left( N_p^{(1)} e^{i\Re(2\gamma_p)} D S(\pi - \alpha_p, 0; \alpha_p, 0) - N_p^{(2)} N_p^{(1)} S(\alpha_p, 0; \alpha_p, 0) \right) 
\times \left( -N_p^{(1)} N_p^{(2)} S(\pi - \alpha_p, 0; \pi - \alpha_p, 0) + (N_p^{(2)})^2 e^{i\Re(-2\gamma_p)} D S(\alpha_p, 0; \pi - \alpha_p, 0) \right) 
\times e^{-\Im(2\xi_p)(x_0+x)} e^{-\Im(2\gamma_p)D}.
\]

Even though the scattered field from the object is composed of multiple modes \( m \), only one of these can interfere destructively with the single incident mode \( p \) on the screen and it is precisely the scattered mode that has the same elevation angle as the
incident mode.

Mode \( p \) is made up of an upgoing and a downgoing planewave. Two of the four terms in Eq. 5.10 arise from the forward scatter of the up and down going planewaves of mode \( p \), while the remaining two terms arise from the scatter of the incident downgoing planewave of mode \( p \) to an upgoing planewave of the same mode and vice versa. This shows that when we have multiple planewaves incident on the object, the extinction will depend on not only the scatter function in the forward direction but also depend on the scatter function amplitudes coupling the incident planewave to the other planewave directions that make up the incident field.

A.2 many modes excited by source

For a general harmonic source that excites many modes, the incident field on the screen is a sum of the contribution from various excited modes. Each of these incident modes on the screen will only interfere destructively in the forward azimuth with the corresponding scattered mode from the object with the same elevation angle. The scattering process causes the various incoming incident modes at the object to be coupled to each outgoing scattered mode through the scatter function and this leads to a double sum in the expression for the extinction in Eq. (5.9).

A.3 large object-receiver range, \( x \)

Next we consider the scenario where the screen is placed at a sufficiently large distance from the object that only the first mode survives for both the incident field on the screen from the source and the scattered field from the object, i.e. \( l = m = 1 \) in Eq. (5.2). The field incident on the object is still comprised by a sum over the modes \( n \) excited by the source since the range of the source from the object is not too large. The expression for the extinction in Eq. (5.9) then reduces to a single sum over the incident modes \( n \) on the object that are scattered into the outgoing mode \( m = 1 \) that survives at the screen.

A.4 large source to object range, \( x_0 \)
If the source is placed at large distances away from the object, the field incident on the object and on the screen will be determined by the single mode \( l = n = 1 \) that survives while the rest of the modes are stripped due to absorption in the waveguide. The extinction in this case has a single term in Eq. (5.9) corresponding to \( m = n = 1 \), the mode that survives at the screen. The expression for the extinction is given by Eq. (5.10) with \( p = 1 \).

These examples illustrate the fact that it is really the interference between the incident field and the scattered field on the screen that determines the extinction. Only scattered field directions that have a fixed phase relationship with the incident field will contribute to the extinction. In the literature, the extinction is often stated to be directly proportional to the forward scatter amplitude of a planewave in free space. For multiple incident planewaves, however, the extinction is not simply a function of the forward scatter function for each incident direction but also depends on the scatter function amplitudes coupling each incident plane wave to the other planewave directions that make up the incident field. Guo’s [23] result for the extinction of planewaves due to an object placed near an interface between two media can be similarly interpreted.

### 5.1.2 Effect of Absorption by the Medium

The extinction of the incident field due to an object in the far field of a point source in free space with absorption in the medium is derived in Appendix C. Comparing the expression for extinction in a waveguide, Eq. (5.9), with that in free space, Eq. (D.27) in Appendix D.3, we see that absorption in the medium lowers the extinction that we would otherwise measure in a lossless medium. In free space, the term due to absorption by the medium is separable from the properties of the object in the formula for the extinction. These terms, however, are in general, convolved in a waveguide with multi-modal propagation. The convolution arises because the absorption loss suffered by each mode varies from mode to mode. Furthermore, the modes have varying elevation angles and they are thus scattered differently by the
object depending on the elevation angle of the mode. In the waveguide, the absorption loss term can be separated from the term due to the object only if a single mode is incident on the object as seen from Eq. (5.10), which is the extinction caused by a single mode. One way this arises naturally in a waveguide is when the source to object separation is large enough that only mode 1 survives in the incident field on the object.

5.2 Total Scattered Power in the Waveguide

The total power scattered by an object in a waveguide can be obtained by integrating the scattered field intensity \( V_s^* \Phi_s \) around a closed control surface enclosing the object, as described in Eq. (D.8). We let the control surface be a semi-infinite cylinder of radius \( R \) with a cap at the sea surface where \( z = -D \). The axis of the cylinder is parallel to the \( z \) axis and passes through the object centroid.

The sea surface is a pressure-release surface where the total field vanishes. Since the incident field in the absence of the object is zero at the sea surface, the scattered field has to vanish as well. The scattered energy flux through the cap of the cylinder at \( z = -D \) is zero. We need only integrate the scattered intensity over the curved surface of the cylinder to obtain the total scattered power.

From Eqs. (2.22) and (2.23), we see that the scattered intensity at the surface of the cylinder can therefore be expressed as a sum of 16 terms, the first of which is

\[
(V_s^* \Phi_s)_1 = \frac{32\pi^3}{\omega k^2} \frac{i}{d(0)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} u_p(z) \left[ \frac{\partial}{\partial z} u_m^*(z) \right] \frac{e^{iR(\xi_p - \xi_m)R}}{\sqrt{\xi_m \xi_p R^2}} \times N_{m-n} N_{p-1} e^{iR(\gamma_p - \gamma_m)D} A_n^*(r_0) A_q(r_0) S^*(\pi - \alpha_m, \phi; \alpha_n, 0) S(\pi - \alpha_p, \phi; \alpha_q, 0) \times e^{-\Theta(\xi_m + \xi_p)R} e^{-\Theta(\gamma_m + \gamma_p)D}.
\]

(5.11)

An area element on the curved surface of the cylinder is given by \( dS = i_p R d\phi dz \). Making use of Eq. (2.4), the orthogonality relation between the modes, we integrate Eq. (5.11) over the semi-infinite depth of the cylinder and the resulting expression is
\[
\int \int (V_x^* \Phi_x)_1 \cdot dS = \int_{0}^{2\pi} \int_{-\infty}^{\infty} (V_x^* \Phi_x)_1 \cdot i_{\rho} dz d\phi \\
= \frac{32\pi^3}{\omega k^2} \frac{1}{d^2(0)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{q=1}^{\infty} \frac{\xi_{m}^*}{|\xi_{m}|} |N_{m}^{(1)}|^2 A_{n}^{*}(r_{0}) A_{q}(r_{0}) \\
\times \int_{0}^{2\pi} S^*(\pi - \alpha_{m}, \phi; \alpha_{n}, 0) S(\pi - \alpha_{m}, \phi; \alpha_{q}, 0) d\phi \\
\times e^{-2\Im(\xi_{m})} R e^{-2\Im(\gamma_{m})} D. 
\] (5.12)

The above integral cannot be further evaluated without specifying the scatter function of the object. In general the total scattered power in the waveguide is a complex expression with a triple sum of 16 integrals. The real part of Eq. (5.12) gives the triple sum of just the first integral.

If there is no absorption by the object, the extinction caused by the object is due entirely to scattering. If the object is in a perfectly reflecting waveguide or a waveguide with small absorption loss, the total scattered power is the extinction. In that case, the complicated expression with triple sum of 16 integrals discussed above reduces to the simple expression of a double sum and no integral of Eq. (5.9). In a lossy waveguide, if we measure the extinction around a small control surface enclosing the object, the absorption loss inside the control volume is small and the above holds as well. Therefore, the extinction formula eliminates the need to integrate the scattered energy flux about the object in a waveguide when determining the scattered power.

### 5.3 Combined and Modal Extinction Cross Sections

The ratio \( \sigma_T \) between the rate of dissipation of energy and the rate at which energy is incident on unit cross sectional area of an obstacle is called the extinction cross section of the obstacle [4]. In the waveguide, the intensity of the incident field on the
object at the origin from a source at \( r_0 \) is

\[
I_i(0|r_0) = \Re \left\{ \mathbf{V}_i^*(0|r_0) \Phi_i(0|r_0) \right\} \\
= \Re \left\{ \frac{i2\pi}{d^2(z_0)d(0)\omega} \sum_{p} \sum_{q} \int_{-\infty}^{\infty} u_p^*(z_0) \left[ \frac{\partial}{\partial z} u_p^*(z) \right]_{z=0} \partial \mathbf{u}_q(z_0)u_q(0) \\
\times e^{i\Re\{\xi_{p} - \xi_{q}\}x_0} e^{-3\Im\{\xi_{p} + \xi_{q}\}x_0} \right\}. 
\]  

(5.13)

In our derivation, the screen is positioned normal to the \( x \) axis and it measures the extinction of the energy flux propagating in the \( x \) direction. We therefore normalise this extinction by the component of the incident intensity in the \( x \) direction to obtain the extinction cross section \( \sigma_T \) of the object in the waveguide,

\[
\sigma_T(x|r_0) = \frac{\mathcal{E}(x|r_0)}{I_i(0|r_0) \cdot \hat{1}_x}
\]

\[
= \left( \frac{4\pi}{\kappa} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sqrt{\Re\{\xi_m\}} \right) \left\{ \frac{1}{\sqrt{\xi_n}} u_m^*(z_0)u_n(z_0)e^{i\Re\{\xi_n - \xi_m\}z_0} \\
\times \left[ N_m^{(1)} N_n^{(1)} e^{i\Re\{\gamma_m + \gamma_n\}D} S(\pi - \alpha_m, 0; \alpha_n, 0) \\
- N_m^{(2)} N_n^{(1)} e^{i\Re\{-\gamma_m + \gamma_n\}D} S(\alpha_m, 0; \alpha_n, 0) \\
- N_m^{(1)} N_n^{(2)} e^{i\Re\{-\gamma_m - \gamma_n\}D} S(\pi - \alpha_m, 0; \pi - \alpha_n, 0) \\
+ N_m^{(2)} N_n^{(2)} e^{i\Re\{-\gamma_m - \gamma_n\}D} S(\alpha_m, 0; \pi - \alpha_n, 0) \right] \\
\times e^{-\Im\{\xi_{m} + \xi_{n}\}z_0} e^{-\Im\{\gamma_{m} + \gamma_{n}\}D} \right) \\
\div \left( \sum_{p} \sum_{q} \Re \left\{ u_p^*(z_0)u_p^*(0)u_q(z_0)u_q(0) \sqrt{\xi_p/\xi_q} e^{i\Re\{\xi_p - \xi_q\}z_0} e^{-\Im\{\xi_p + \xi_q\}z_0} \right\} \right). 
\]

(5.14)

Equation 5.14 is due to the combined extinction of all the modes of the waveguide by the object and we define it to be the combined extinction cross section. This combined cross section of an object depends on the properties of the object which
are convolved with the properties of the waveguide, as well as the source and object locations.

For a source that excites only a single mode $p$, the incident intensity on the object in the $x$ direction is

$$I_i(0|x_0)_p = \frac{i2\pi}{d^2(z_0)\omega x_0}|u_p(z_0)|^2|u_p(0)|^2 \frac{\Re \{ \xi_p^* \}}{|\xi_p|} e^{-\Im(2\xi_p)x_0}. \quad (5.15)$$

Dividing the extinction of mode $p$ by the object in Eq. (5.10) with the intensity of the incident field composed solely of mode $p$ in Eq. (5.15), we obtain the cross section of the object for the extinction of mode $p$,

$$\sigma_p(x) = \frac{4\pi}{k} \frac{1}{\sqrt{\Re \{ \xi_p \}|u_p(0)|^2}} \times \Im \left\{ \frac{1}{\sqrt{\xi_p}} \left[ N_p^{(1)} e^{i\Re(2\gamma_p)D} S(\pi - \alpha_p, 0; \pi - \alpha_p, 0) - N_p^{(2)} N_p^{(1)} S(\alpha_p, 0; \alpha_p, 0) \right. \right.$$

$$- N_p^{(1)} N_p^{(2)} S(\pi - \alpha_p, 0; \alpha_p, 0, 0) + \left( N_p^{(2)} \right)^2 e^{i\Re(-2\gamma_m)D} S(\alpha_p, 0; \pi - \alpha_p, 0) \right\} \left. \right\} \times e^{-\Im(2\xi_p)x} e^{-\Im(2\gamma_p)D}. \quad (5.16)$$

We define Eq. (5.16) as the modal cross section of the object for the extinction of the individual modes of the waveguide. Analogous to planewaves in free space, the modes in a waveguide are the entity that propagate in the waveguide and determine the energy of the acoustic field in the waveguide. It therefore becomes meaningful to quantify the extinction caused by an object of the individual modes of the waveguide and subsequently the cross section of the object as perceived by the individual modes of the waveguide.
5.4 Estimation of Object Size from Extinction Theorem in an Ocean Waveguide

The extinction formula can be used to estimate the size of an object by measuring the extinction it causes in an incident beam. For instance, in astronomy, the size of a meteorite is estimated from the extinction it causes in the light reaching a telescope when the meteorite is in interstellar space between a star and the telescope, so long as the telescope is large enough to measure the entire shadow remnant.[75]

For an object that is large compared to the wavelength, its extinction cross section in free space, according to Babinet’s principle, is equal to twice its geometrical projected area.[75] If we let $T_p$ be the projected area of the object in the direction of an incident planewave in free space, we obtain

$$\frac{4\pi}{k^2} \Im\{S_f\} = 2T_p. \quad (5.17)$$

The size of the object is therefore directly related to the free space forward scatter function of the object for objects that are large compared to the wavelength. The forward scatter function can be determined from a measurement of the extinction caused by the object.

Extinction measurements usually involve integrating the intensity of the incident and total fields over a sufficiently large screen that registers the full extinction caused by the object. We measure the incident power on the screen in the absence of the object and the total power in the presence of the object. The difference between these two energy fluxes on the screen is the extinction.

An intensity measurement at a single point in space in the forward scatter direction is typically inadequate. This can be seen from Eq. (D.16) for free space, and Eq. (5.2) in the waveguide, where the interference intensity $\mathbf{V}_i^\ast \Phi_s$ at a point depends very sensitively on the source and receiver positions which cause rapid fluctuation in the phase term. To determine the forward scatter function from a single receiver in the
forward direction then requires extremely accurate knowledge of the source, object and receiver locations. In practical measurements, it may also be difficult to precisely locate the point sensor in the forward direction. This is especially true for large objects as they have very narrow forward scatter function peaks. Equation (D.27) for the extinction in free space on the other hand has no phase dependence involving the source or screen position. Extinction measurement over a screen is therefore a more robust method for estimating the forward scatter amplitude and hence the size of an object. For measurements in a shallow water waveguide, the screen over which the intensity is integrated can be either a sufficiently large planar array, or a billboard array whose spacing between the sensor elements satisfies the Nyquist criterion for sampling the field in space.

In a waveguide, the extinction caused by an object, Eq. (5.9), depends not only upon the properties of the object through the scatter function, but also the properties of the waveguide and the measurement geometry. They are, in general, convolved in the expression for the extinction and are separable only when the incident field is composed of a single mode as evident in Eq. (5.10). This suggests a possible scenario for extinction measurements in a waveguide to extract the scatter function’s forward amplitude and subsequently to estimate the size of an object.

For large source to object separation $x_0$, the mode that survives in the incident field is mode 1. Mode 1 of any waveguide propagates almost horizontally and we can approximate its elevation angle as $\alpha_1 \approx \pi/2$. In this case, the four scatter function amplitudes in Eq. (5.10) can be approximated as $S(\pi/2, 0, \pi/2, 0)$ and factored out of the equation for the extinction. Using the fact that for mode 1, $\Re\{\xi_1\} \gg \Im\{\xi_1\}$ we rewrite the extinction for mode 1 as

$$
\mathcal{E}(x|x_0) = \frac{8\pi^2}{\omega k} \frac{1}{d^2(z_0)} \frac{1}{d(0)} \frac{1}{x_0 \Re\{\xi_1\}} \left| u_1(z_0) \right|^2 \\
\times \Im \left\{ S(\pi/2, 0; \pi/2, 0) \left[ (N_1^{(1)})^2 e^{i\Re(2\gamma_1)D} - 2N_1^{(2)} N_1^{(1)} + (N_1^{(2)})^2 e^{i\Re(-2\gamma_1)D} \right] \right\} \\
\times e^{-\Im(2\xi_1)(x_0 + x)} e^{-\Im(2\gamma_1)D} \\
= \mathcal{E}(x|x_0)$$

(5.18)
In a Pekeris waveguide,[33, 19] with

\[ N_1^{(2)} = N_1^{(1)} \approx \frac{1}{i} \sqrt{\frac{d(0)}{2H}}, \]  

(5.19)

using Eq. (2.5), we see that

\[ |u_1(0)|^2 = \left[ (N_1^{(1)})^2 e^{i\mathcal{R}(2\gamma)D} - 2N_1^{(2)}N_1^{(1)} + (N_1^{(2)})^2 e^{i\mathcal{R}(-2\gamma)D} \right] e^{-\mathcal{R}(2\gamma)D}. \]

(5.20)

The extinction formula for mode 1 therefore leads to

\[ \mathcal{E}(x|x_0) = \frac{8\pi^2}{\omega k} \frac{1}{d^2(z_0) d(0)} \frac{1}{x_0} \text{Re}\{\xi_1\} |u_1(z_0)|^2 |u_1(0)|^2 \text{Im}\{S(\pi/2, 0; \pi/2, 0)\} e^{-\mathcal{R}(2\gamma)(z_0 + x)}. \]

(5.21)

Equation (5.16) for the modal cross section of the object for mode 1 in the Pekeris waveguide, simplifies to

\[ \sigma_1(x) = \frac{4\pi}{k} \text{Re}\{\xi_1\} \text{Im}\{S(\pi/2, 0; \pi/2, 0)\} e^{-2\mathcal{R}(\xi_1)x}. \]

(5.22)

Since mode 1 propagates close to the horizontal, \( \text{Re}\{\xi_1\} \approx k \). The cross section of an object for the extinction of mode 1 in a Pekeris waveguide, Eq. (5.22), is almost identical to the cross section of the object for the extinction of planewaves in free space, Eq. (D.29).

In Eqs. (5.21) and (5.22) the properties of the target are separated from the waveguide and geometric parameters. If we can measure the extinction of mode 1 caused by the object in the waveguide, we can estimate the free space forward scatter amplitude of the object and subsequently, the size of the object. A knowledge of the waveguide properties, and location of source, object and screen is necessary to correct
for the spreading and absorption loss in the waveguide, as well as the amplitude of mode 1 at the source and object depths. Experimentally, we can estimate the source to object range $x_0$ from the arrival of the back scattered field from the object using a sensor that is co-located with the source.

As discussed in Eq. (5.17), the object size is related to the forward scatter function amplitude. The extinction of the higher order modes of the waveguide, apart from mode 1, depend on the scatter function amplitude in other directions in addition to the forward. It is therefore much more difficult to extract information about the size of the object from modes higher than mode 1 unless the object is compact as will be discussed in Sec. (5.5.5). For objects that are buried in sediments that are faster than water, mode 1 excited by a source in the water column does not penetrate into the bottom due to total internal reflection. The above method will therefore not be useful in estimating the size of objects buried in fast bottoms.

5.5 Illustrative Examples

In all the illustrative examples, a water column of 100 m depth is used to simulate a typical continental shelf environment. The sound speed structure of the water column is iso-velocity with constant sound speed of 1500 m/s, density of 1 g/cm$^3$ and attenuation of $6.0 \times 10^{-5}$ dB/λ. The seabed is either perfectly reflecting or comprised of sand or silt halfspaces. The density, sound speed and attenuation are taken to be 1.9 g/cm$^3$, 1700 m/sec, and 0.8 dB/λ for sand, 1.4 g/cm$^3$, 1520 m/sec, and 0.3 dB/λ for silt. Calculations are made of the combined and modal extinction, incident intensity on the object, and the combined and modal cross sections in various waveguides for different objects as a function of source, object and screen locations. The object size and frequency is also varied. Except for Sec. (5.5.5), the frequency used in all other examples is 300 Hz.
5.5.1 Combined Extinction Cross section in different Waveguides

First, we examine how the combined extinction of all the modes, caused by a pressure-release sphere of radius 10 m, in a Pekeris waveguide with either sand or silt halfspace varies as a function of source to object range at a source frequency of 300 Hz. The source and sphere centers are located at \( D = 50 \) m in the middle of the water column. The combined extinction measured by the screen Eq. (5.9), the incident intensity on per unit area of the sphere Eq. (5.13), and the combined cross section of the sphere Eq. (5.14) in the waveguides are plotted as a function of source to object separation \( x_0 \) in Fig. 5-2 (a)-(c) respectively. At each \( x_0 \), the separation of the screen from the object is the same as that of the source from the object, i.e. \( x = x_0 \). The combined extinction is calculated using Eq. (5.9) with the scatter function for the sphere given by Eqs. (8,9) of Ref. [42] with \( f(n) \) replaced by \((-1)^n f(n)\) to convert from Ingenito's definition to the standard one described in Ref. [48].

The combined extinction and incident intensity fluctuate with range due to the coherent interference between the modes. The resulting combined cross section of the sphere also fluctuates with range. The incident intensity and extinction are larger in the waveguide with sand bottom. The fluctuations in the fields are also greater in the sand bottom waveguide as compared to the silt bottom waveguide. The difference arise primarily because the number of trapped modes is larger for the sand halfspace due to the higher critical angle of \( 28.1^\circ \) for the water to sand interface as compared with the \( 9.3^\circ \) of water to silt leading to larger fields and fluctuations in the waveguide with sand bottom.

We find it useful to approximate the combined extinction measured by the screen and the incident intensity on the sphere as a single incoherent sum over the modes which provides an average trend to the curves as a function of range. Taking the ratio of the incoherent combined extinction and incident intensity, we obtain the incoherent combined cross section. The combined extinction, incident intensity and combined cross section of the sphere calculated incoherently, using Eqs (5.9), (5.13) and (5.14)
Figure 5-2: The combined extinction Eq. (5.9) of all the modes, caused by a pressure release sphere of radius 10 m centered at 50 m depth, in a Pekeris waveguide composed of 100 m water with either sand or silt halfspace is plotted as a function of $x_0$, its range from a point source of frequency 300 Hz also placed at the same depth in the waveguide. The separation of the screen from the object is the same as that of the source from the object at each source to object range, $x = x_0$. (b) The incident intensity on the sphere Eq. (5.13). (c) The combined cross section of the sphere Eq. (5.14). Both the coherent and incoherent approximation of the quantities are plotted in each subfigure. Source level is 0 dB re 1 µPa @ 1 m.
Figure 5-3: Incoherent combined cross section of a 10 m radius pressure release sphere at 300 Hz source frequency in a Pekeris waveguide with sand bottom halfspace, Pekeris waveguide with silt bottom halfspace, perfectly reflecting waveguide, and free space as a function of source to object range $x_0$. For this plot, $x = x_0$. In the waveguides, the source and sphere centre are located at 50 m water depth. The incoherent combined cross section is calculated using Eq. (5.14) by replacing the double sum over the modes with a single sum.
respectively by replacing the double sum with a single sum over the modes are plotted in Fig. 5-2 (a)-(c). From the incoherent plots, we see that the extinction and the incident intensity decay with range due to geometrical spreading and absorption loss in a real waveguide.

In a perfectly reflecting waveguide, there is no absorption in the waveguide. Subsequently, an incoherent approximation of $\sigma_T$ is independent of range as can be seen from Eq. (5.14). The decay in the extinction due to spreading loss is compensated by spreading loss in the flux incident on the object which keeps the cross section a constant. In this case, the extinction measured by the screen is due entirely to the object. Figure 5-3 shows $\sigma_T$, calculated incoherently, plotted for a pressure-release sphere of radius 10 m in a perfectly reflecting waveguide as a function of $x_0$. In this figure $x = x_0$. The incoherent combined cross section of the object in free space with no absorption and in the Pekeris waveguide examples considered so far are also plotted for comparison.

For a screen placed a fixed range from the object, it is the coherent extinction and cross section that we measure experimentally. From Fig. 5-2 (c), we see that the coherent combined cross section of the object varies rapidly with range in the waveguide. Subsequently, it would be difficult to extract information about the size of the object from a measurement of the combined extinction caused by the object of all the modes of the waveguide.

### 5.5.2 Modal Cross section in different Waveguides

In this section, we will investigate how the modal extinction cross section of the 10 m pressure release sphere varies for the individual modes in various waveguides at 300 Hz. Figure 5-4 (a) and (b) show the amplitudes of the modes at the source depth of 50 m in the Pekeris waveguide with sand and silt bottom respectively. Only the propagating modes are plotted because these are the modes that compose the incident field on the object in the far-field. These are the mode amplitudes at the object depth because the target is also at 50 m depth. The amplitude of the modes in the perfectly reflecting waveguide are plotted in Fig. 5-4 c. Only the even number
Figure 5-4: Modal amplitude at the source and target depth of 50 m in (a) Pekeris waveguide with sand halfspace, (b) Pekeris waveguide with silt halfspace, and (c) perfectly reflecting waveguide for a frequency of 300Hz.
Figure 5-5: Extinction of the individual modes Eq. (5.10) of the waveguide by the 10 m radius pressure release sphere at 50 m water depth with the source separated from the sphere by (a) 1 km range in a Pekeris sand halfspace waveguide, (b) 25 km range in a Pekeris sand halfspace waveguide, (c) 1 km range in a Pekeris silt halfspace waveguide, (d) 25 km range in a Pekeris silt halfspace waveguide, and (e) 1 km range in a perfectly reflecting waveguide, (f) 25 km range in a perfectly reflecting waveguide. The source depth is also 50 m and the source frequency is 300 Hz. The screen measuring the extinction is separated the same distance from the object as the source in each case. Source level is 0 dB re 1 µPa @ 1 m.
Figure 5-6: Modal cross section Eq. (5.16) at 300 Hz of the 10 m radius pressure release sphere at 50 m water depth for the extinction of the individual modes in a (a) Pekeris sand halfspace waveguide, (b) Pekeris silt halfspace waveguide, and (c) perfectly reflecting waveguide. We set $x = 0$ in Eq. (5.16) to remove the effect of absorption by the waveguide. The modal cross section of the sphere for mode 1 in each waveguide is almost equal to its free space cross section.
modes are excited by the source at 50 m depth and they have the same amplitude.

The extinction of each individual mode in the Pekeris waveguide with sand bottom, calculated using Eq. (5.10), is plotted in Fig. 5-5 (a) and (b) at the source to object range of 1 km and 25 km respectively. The screen is placed the same distance away from the object as the source in each case. The modal extinction in the Pekeris waveguide with silt bottom at 1 km and 25 km are plotted in Fig. 5-5 (c) and (d) respectively. Comparing Fig. 5-5 with Fig. 5-4, we see a dependence of the extinction of each mode on its amplitude at the object depth, with the more energetic modes being extinguished the most. The extinction of the modes vary with range due to spreading and absorption loss suffered by the modes. Absorption loss suffered by each mode as a result of absorption in a real waveguide is more severe for the high order modes due to their steeper elevation angles. The higher order modes get gradually stripped with range and at sufficiently long ranges, the extinction caused by the object is very much limited to the extinction of the first few propagating modes. For the perfectly reflecting waveguide in Fig. 5-5 (e) and (f) at 1 km and 25 km respectively, as there is no absorption loss, the extinction for each of the modes decays only with source to object range \( x_0 \). There is no mode stripping effect in a perfectly reflecting waveguide and the relative magnitude of the extinction across the modes remains the same, independent of range.

Figure 5-6 (a)-(c) shows the modal cross section of the sphere, calculated using Eq. (5.16), for the extinction of the individual modes in the Pekeris sand, silt and perfectly reflecting waveguides respectively. We set \( x = 0 \) in Eq. (5.16) to obtain the modal cross section of the object corrected for absorption in the waveguide. In each of the waveguides illustrated in Fig. 5-6 we see that the modal cross section of the sphere for the extinction of mode 1 is very close to its cross section for the extinction of planewaves in free space. For the higher order modes, the modal cross section of the object can be much larger or smaller than its free space value depending on the waveguide. We can calculate the forward scatter function amplitude of the object from a measurement of the extinction of mode 1 as discussed in Sec. 5.4 which allows us to estimate the size of the object.
Figure 5-7: Incoherent combined cross section of a 10 m radius pressure release sphere at 300 Hz source frequency in a Pekeris waveguide with sand bottom halfspace, Pekeris waveguide with silt bottom halfspace, perfectly reflecting waveguide, and free space as a function of source to object range \( x_0 \). For this plot, \( x = x_0 \). In the waveguides, the source and sphere centre are located at 52.5 m water depth. The incoherent combined cross section is calculated using Eq. (5.14) by replacing the double sum over the modes with a single sum.

### 5.5.3 Dependence of Modal Cross Section on Object Depth

The modal cross section of an object depends on the depth of the object in the waveguide. We investigate how the modal cross section of the 10 m pressure release sphere varies when we lower its depth by half a wavelength distance to 52.5 m in the Pekeris silt or sand halfspace waveguides, and in the perfectly reflecting waveguide. We also lower the source depth to 52.5 m so that all the modes in the perfectly reflecting waveguide are excited by the source. The source frequency is 300 Hz.

Figure 5-7 shows the incoherent combined cross section of the sphere in the three waveguides. In the perfectly reflecting waveguide, the incoherent combined cross
Figure 5-8: Modal amplitude at the object depth of 52.5 m in (a) Pekeris waveguide with sand halfspace, (b) Pekeris waveguide with silt halfspace, and (c) perfectly reflecting waveguide for a frequency of 300Hz.
Figure 5-9: Modal cross section Eq. (5.16) at 300 Hz of the 10 m radius pressure release sphere at 52.5 m water depth for the extinction of the individual modes in a (a) Pekeris sand halfspace waveguide, (b) Pekeris silt halfspace waveguide, and (c) perfectly reflecting waveguide. We set $x = 0$ in Eq. (5.16) to remove the effect of absorption by the waveguide. The modal cross section of the sphere for mode 1 in each waveguide is almost equal to its free space cross section.
section of the sphere is now larger than its free space value. Figure 5-8 (a)-(c) shows the modal amplitudes in the three waveguides and Fig. 5-9 (a)-(c) shows the modal cross sections, Eq. (5.16). In the perfectly reflecting waveguide Fig. 5-9 (c), all the modes that exist in the waveguide are scattered by the object to form the scattered field when it is at the shallower depth of 52.5 m, unlike in the previous example of Fig. 5-6 where it was at 50 m depth and only the excited odd number modes were scattered by the object. Comparing Fig. 5-9 with Fig. 5-6, we see that the modal cross section of most of the modes vary with the object depth. For mode 1 however, in all the three waveguides, the modal cross section of the object remains close to its free space value.

5.5.4 Modal Cross Section of various Object Types

The cross section of the 10 m pressure release sphere is compared to that of a hard disk of radius 10 m in the waveguide. In free space, with the plane of the disk aligned normal to the direction of propagation of the incident waves, it is well known that its plane wave extinction cross section is equal to twice its projected area, which is 628.3 m² in this example. The cross section of a sphere in free space depends on the size of the sphere relative to the wavelength of the incident waves, i.e. the term \( ka = 2\pi a/\lambda \) where \( a \) is the radius of the sphere. The dependence of the extinction cross section of a pressure release or hard sphere on \( ka \) in free space is plotted in Ref. [5]. For a large pressure release sphere, high \( ka \), the extinction cross section is roughly twice the projected area which is the same for both the sphere and the disk. For a compact pressure release sphere, small \( ka \), the cross section of the sphere begins to exceed twice its projected area. For the present example, at 300 Hz source frequency, \( ka = 12.6 \) and the extinction cross section of the sphere in free space is 736.7 m².

The incoherent combined cross section of the 10 m hard disk in the three different waveguides, calculated using Eq. (5.14), is plotted in Fig. 5-10. The modal cross section Eq. (5.16) of the disk for each mode of the Pekeris sand, silt, and the perfectly reflecting waveguide is plotted in Fig. 5-11 (a)-(c) respectively. In free space, the cross section of the sphere at 300 Hz is only a little larger than that of the disk of the
Figure 5-10: Incoherent combined cross section of a hard disk of radius 10 m at 300 Hz source frequency in a Pekeris waveguide with sand bottom halfspace, Pekeris waveguide with silt bottom halfspace, perfectly reflecting waveguide, and free space as a function of source to object range $x_0$. For this plot, $x = x_0$. In the waveguides, the source and disk center are located at 50 m water depth with the disk aligned in the $y - z$ plane. The incoherent combined cross section is calculated using Eq. (5.14) by replacing the double sum over the modes with a single sum.
Figure 5-11: Modal cross section Eq. (5.16) at 300 Hz of the 10 m radius hard disk at 50 m water depth for the extinction of the individual modes in a (a) Pekeris sand halfspace waveguide, (b) Pekeris silt halfspace waveguide, and (c) perfectly reflecting waveguide. We set $x = 0$ in Eq. (5.16) to remove the effect of absorption by the waveguide. The modal cross section of the disk for mode 1 in each waveguide is almost equal to its free space cross section.
same radius. Comparing Figs. 5-3 and 5-10, we see that in the perfectly reflecting waveguide, the incoherent combined cross section of the sphere is much larger than that of the disk. The elevation angle of each mode of the waveguide increases with the mode number. Since the disk is aligned with its plane parallel to the y-z plane, the projected area of the disk perceived by each mode decreases as the elevation angle of the mode increases. For the sphere, however, each mode sees the same projected area, regardless of the elevation angle of the mode. Therefore the combined extinction of the modes by the sphere is much larger than by the disk. In the real waveguide, absorption by the waveguide alters the amplitude of each mode with the higher order modes suffering greater absorption losses than the lower order modes. The higher order modes are less important in determining the combined extinction in the real waveguide. Consequently, in a real waveguide, the incoherent combined cross section of the sphere is only slightly larger than that of the disk. From Fig. 5-11, we see once again that the modal cross section of the object for the extinction of mode 1 is almost equal to its free space cross section. In the present example, the cross section of the disk is equal to twice its projected area. This example further illustrates that we can obtain a measure of the size of an object from the extinction of mode 1 in a waveguide.

5.5.5 Dependence of Modal Cross Section on Object Size and Frequency

Here we investigate how the modal cross section Eq. (5.16) of a pressure release sphere at 50 m water depth compares with its free space cross section when we vary the size of the sphere and the frequency of the incoming waves. Figure 5-12(a)-(d) show the result in a Pekeris sand waveguide, plotted as a function of $ka$. The corresponding result in the Pekeris silt and perfectly reflecting waveguides are plotted in Figs. 5-13 and 5-14 respectively.

For a large sphere with the high $ka$ of 62.8, we see from Figs. 5-12-5-14 (d) that the modal cross section of the sphere for the high order modes fluctuates and departs
drastically from the free space cross section for most of the modes. The modal cross section for mode 1, however, remains nearly equal to the free space cross section of the large sphere in each waveguide. For the compact sphere with the small $ka$ of 0.1 on the other hand, Figs. 5-12-5-14 (a), the modal cross section of most of the modes are fairly close to the free space cross section of the object.

Figure 5-15(a)-(d) shows the scatter function amplitude plotted as a function of elevation angle of the modes at various $ka$. Compact objects scatter like point targets and they have an omnidirectional scatter function $S_0$. In Eq. (5.16), we see that the modal cross section depends on not only the forward scatter amplitude, but also the scatter function amplitude in non-forward directions. For a compact object, since the scatter function amplitude is a constant, independent of azimuth or elevation angles, we can factor it out in Eq. (5.16). Furthermore, in a perfectly reflecting waveguide, since[33, 19]

\[ N_p^{(2)} = N_p^{(1)} = \frac{1}{i} \sqrt{\frac{d(0)}{2H}}, \quad (5.23) \]

$N_p$ can be factored out of the equation as well. Subsequently, for a compact object in the perfectly reflecting waveguide, Eq. (5.16) for the modal cross section reduces to

\[ \sigma_p(x) = \frac{4\pi}{k} \Re \{ \xi_p \} \Im \{ S_0 \} e^{-2\Im(\xi_p)x} \quad (5.24) \]

which resembles the expression for the free space cross section of the object in Eq. (D.29). The modal cross section of the compact object in the waveguide will, however, be slightly larger than the free space cross section because of the dependence on the horizontal wavenumber of the mode $\xi_p$ in the denominator of Eq. (5.24) instead of $k$ as in Eq. (D.29) for free space. The real part of the horizontal wavenumber decreases as the mode number increases. We see a gradual increase in the modal cross section in Figure 5-14 (a) with increase in mode number for the compact sphere in the perfectly reflecting waveguide.
Figure 5-12: Modal cross section Eq. (5.16) of a pressure release sphere at 50 m water depth for the extinction of the individual modes in a Pekeris sand halfspace waveguide with (a) sphere radius 0.1 m, 300 Hz source frequency, $ka = 0.1$, (b) sphere radius 1 m, 300 Hz source frequency, $ka = 1.3$, (c) sphere radius 10 m, 300 Hz source frequency, $ka = 12.6$, and (d) sphere radius 10 m, 1500 Hz source frequency, $ka = 62.8$. We set $x = 0$ in Eq. (5.16) to remove the effect of absorption by the waveguide. Only the propagating modes are illustrated in each plot. The modal cross section of the sphere for mode 1 in each case is almost equal to its free space cross section.
Figure 5-13: Same as Fig. 5-12, but in a Pekeris silt waveguide.
Figure 5-14: Same as Fig. 5-12, but in a perfectly reflecting waveguide.
Figure 5-15: The magnitude of the free space planewave scatter function $S(\alpha, \phi = 0^\circ, \alpha_i = 90^\circ, \phi_i = 0^\circ)$ is plotted as a function of $\alpha$, the elevation angle for a pressure release sphere at (a) $ka = 0.1$, (b) $ka = 1.3$, (c) $ka = 12.6$, and (d) $ka = 62.8$. The forward scatter peak is at $\alpha = 90^\circ$. 


In a real waveguide, $N_p$ is usually complex. For the lower order modes, $N_p$ has a large imaginary component and we can still factor it out as we did for the perfectly reflecting waveguide. We also observe a trend of increase in modal cross section with mode number for the compact sphere in the real waveguide examples of Figs. 5-12-5-13 (a). This implies that for a compact object in a waveguide, in addition to mode 1, we can also use the higher order modes to extract its omidirectional scatter function amplitude from modal extinction measurements. Once the scatter function amplitude of a compact object is known, its size can be estimated.

5.6 Summary and Discussion

A generalized extinction theorem for the rate at which energy is extinguished from the incident wave of a far field point source by an object of arbitrary size and shape in a stratified medium has been developed from wave theory. In a waveguide, both the incident and scattered fields are comprised by a superposition of planewaves or equivalently by modes. The total extinction is shown to be a linear sum of the extinction of each waveguide mode. Each modal extinction involves a sum over all incident modes that are scattered into the given mode and is expressed in terms of the objects's planewave scatter function in the forward azimuth and equivalent modal plane wave amplitudes.

For multiple incident planewaves, extinction is a function of not only the forward scatter amplitude for each of the incident directions but also depends on the scatter function amplitudes coupling each incident planewave to one of the other planewave directions that make up the incident field. Our derivation greatly facilitates scattering calculations by eliminating the need to integrate energy flux about the object. The only assumptions are that multiple scattering between the object and waveguide boundaries is negligible, and the object lies within a constant sound speed layer. Our extinction calculations are made in the far field in analogy with those made in free space because the analysis is simpler and the modal cross sections are conserved regardless of range.
Two extinction cross sections were defined for an object submerged in a waveguide. The first is the combined cross section which is defined as the ratio of the combined extinction of all the modes of the waveguide to the total incident intensity in the radial direction at the object’s centroid. Calculations for a shallow water waveguide show that both the combined extinction and the combined cross section of an object are highly dependent on measurement geometry, medium stratification, as well as the scattering properties of the object. They also fluctuate with range due to the coherent interference between the modes. Both are significantly modified by the presence of absorption in the medium. The presence of absorption typically means that the extinction and corresponding cross section of the obstacle in a real waveguide will be smaller than it’s value in free space. The practical implications of these findings is that experimental measurements of the total scattering cross section of an obstacle in a waveguide may differ greatly from those obtained for the same obstacle in free space and may lead to errors in target classification if the waveguide effects are not properly taken into account.

We also define the modal cross section of an object for the extinction of the individual modes of a waveguide. Simulation in several shallow water waveguides show that for an object submerged in a typical ocean waveguide, after correcting for absorption loss in the medium, its modal cross section for the extinction of mode 1, is almost identical to its free space cross section. This finding can be used to robustly estimate the size of objects submerged underwater from extinction measurements involving mode 1 which is often the dominant mode after long range propagation in a shallow water waveguide.
Chapter 6

Diffuse Backscattering, Decorrelation of Waveguide Modes, “Reverberation Rings”, Forward Scattering, Attenuation and Dispersion from Random Surface and Volume Inhomogeneities

Here we derive the conditions necessary for the total field backscattered from a large number of independent and identically distributed random scatterers in a waveguide to become diffuse. In diffuse or incoherent scattering, the mean of the total scattered field tends to zero\[58\]. Furthermore, for diffuse scattering in a waveguide, decorrelation of the waveguide modes occurs that leads to a simplification in the the expression for the intensity of the total scattered field\[48\]. We also show that when these conditions are not met for scattering from a seafloor patch, it leads to the formation of coherent “reverberation rings”.

We also derive the field scattered from the distribution of random scatterers in
the forward azimuth in a waveguide and show that the mean forward scattered field is non-zero. This result is then used to derive analytic expressions for the forward propagated field in a waveguide, the attenuation coefficient and dispersion due to scattering from arbitrary-sized scatterers. This result can be applied to study the effect of scattering from internal waves and seafloor roughness in the forward propagated field in a waveguide.

6.1 Conditions for Diffuse Backscattering

Consider a target sheet of area $\Delta A$ in the $x$-$y$ plane containing a large number of random scatterers distributed throughout the sheet. An example of such a sheet would be a patch of seafloor of area $\Delta A$ contained with the sonar resolution footprint of area $A$. The seafloor patch $\Delta A$ is composed of a large number of incremental seafloor patches of elemental area $\delta A = \ell \times \ell$, each with wavelength scale random roughness. Another example would be a sheet containing a school of fish. The target sheet can be of arbitrary shape, but for convenience, we assume that it is rectangular with dimension $L_x \times L_y = \Delta A$.

Here we investigate the field backscattered from the target sheet. It is convenient to use an absolute reference frame as defined in Sec. 2.4, where we let the center of the sheet be at $(X_t, Y_t, Z_t)$. We can describe the distribution of statistically independent scatterers over the sheet using an areal density function $f(X, Y)$ that describes the number of scatterers contained per unit area at location $(X, Y)$. The form of the density function depends on the scatterers. For a school of fish, $f(X, Y) = \sum_{k=1}^{N_{fish}} \delta(X - X_k)\delta(Y - Y_k)$ where $(X_k, Y_k, Z)$ is the location of the $k$th fish and there are a total of $N_{fish}$ fish in the target sheet. For a large seafloor patch, $f(X, Y) = \frac{1}{n_x \delta A}$ where $\hat{n} = (n_x, n_y, n_z)$ is the surface normal to the patch at $(X, Y, Z)$.

For convenience, assume that the horizontal origin is chosen to be at the location of an omnidirectional receiver, $R_r = (0, 0, Z_r)$, and that the source is colocated in the horizontal plane with the receiver, $R_0 = (0, 0, Z_0)$. For convenience, we also assume that $Y_t = 0$ and $P_t = X_t$ is the horizontal range of the target sheet center from the
Let the mean field scattered from each elemental random scatterer be \( \langle \Phi_s(R_r - R | R_0 - R) \rangle \) which is given by Eqs. 2.20 and 2.21, but expressed in the absolute reference frame as discussed in Sec. 2.4. For simplicity, let us consider the first term \( \langle \Phi_s^{(1)}(R_r - R | R_0 - R) \rangle \) in the double modal sum of \( \langle \Phi_s(R_r - R | R_0 - R) \rangle \), i.e.,

\[
\langle \Phi_s^{(1)}(R_r - R | R_0 - R) \rangle = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(4\pi)^2}{k} A_m(R_r - R) A_n(R_0 - R) \langle S_{m,n}^{(1)}(\varphi) \rangle,
\]

where \( \langle S_{m,n}^{(1)}(\varphi) \rangle = \langle S(\pi - \alpha_m, \varphi + \pi, \alpha_n, \varphi) \rangle \) and \( \varphi \) is the azimuth of the elemental scatterer in the absolute reference frame. In the coordinate system of the scatterer, \( \varphi \) is the azimuth of the incident planewave while \( \varphi + \pi \) is the azimuth of the scattered planewave in backscattering.

We assume that the elemental scatterers within the target sheet have identically distributed scatter functions. Let the total mean field scattered from the target sheet be \( \langle \Phi_{s,\text{sum}}(R_r - R_t | R_0 - R_t) \rangle \) which has a first term given by

\[
\langle \Phi_{s,\text{sum}}^{(1)}(R_r - R_t | R_0 - R_t) \rangle
= \iint_{\Delta A} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(4\pi)^2}{k} A_m(R_r - R) A_n(R_0 - R) \langle S_{m,n}^{(1)}(\varphi) \rangle f(X, Y) dX dY.
\]

Substituting Eqs. 2.31 and 6.1 into Eq. 6.2, we obtain

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where \( P = \sqrt{X^2 + Y^2} \) is the horizontal range of the scatterer at \((X, Y)\) from the source-receiver, and \( f(P, \varphi)PdPd\varphi = f(X, Y)dXdY \). We assume that for large enough \( P_t \), the functions of \( P \) in Eq. 6.3 are slowly varying from \( P_t - L_x/2 \) to \( P_t + L_x/2 \) and can be approximated by their mean value over this range interval, except for the exponential function in \( P \) which carries the phase information. For an elemental scatterer of size \( \ell \) where \( \ell < L_y \), the minimum angular width of its scatter function lobe by diffraction is \( \lambda/\ell \). For \( \lambda/\ell > L_y/P_t \) or equivalently \( P_t > \ell L_y/\lambda \), the receiver is in the far-field of the elemental scatterer so that the angle subtended by the elemental scatterer at the receiver is less than \( \lambda/\ell \). The scatter function then can be factored out of the integral involving \( \varphi \) in Eq. 6.3. The range criteria \( P_t > \ell L_y/\lambda \) is automatically satisfied at ranges in the far field of the target sheet given by \( P_t > L_y^2/\lambda \). The mean total scattered field from the target sheet becomes

\[
\langle \Phi^{(1)}_{s,\sum_m}(R_r - R_t| R_0 - R_t) \rangle \\
= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} 2\pi \frac{i}{k} \frac{1}{d(Z_0)d(Z)\sqrt{\xi_m\xi_n}P_t} u_m(Z_r - Z)u_n(Z_0 - Z)N_m^{(1)}N_m^{(1)}e^{(\gamma_m + \gamma_n)Z} \\
x \langle S_{m,n}^{(1)}(\varphi) \rangle f(P, \varphi)e^{-3(\xi_m + \xi_n)P_t} \int_{\varphi - L_y/2P_t}^{\varphi + L_y/2P_t} P_t d\varphi \int_{P_t - L_x/2}^{P_t + L_x/2} e^{i\Re(\xi_m + \xi_n)P} dP \\
= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} 2\pi \frac{i}{k} \frac{1}{d(Z_0)d(Z)\sqrt{\xi_m\xi_n}P_t} u_m(Z_r - Z)u_n(Z_0 - Z)N_m^{(1)}N_m^{(1)}e^{(\gamma_m + \gamma_n)Z} \\
x \langle S_{m,n}^{(1)}(\varphi) \rangle f(P, \varphi)e^{-3(\xi_m + \xi_n)P_t} e^{i\Re(\xi_m + \xi_n)P_t} \Delta Asinc \left[ \Re\left(\xi_m + \xi_n\right) \frac{L_x}{2} \right],
\]
the target sheet, \( \bar{f}(P, \varphi) = \frac{N_{\text{fish}}}{\Delta A} \) is the mean fish density. For a patch of seafloor within the sonar resolution footprint, \( \bar{f}(P, \varphi) = \frac{1}{\delta A} \). \( \varphi_t \) is the azimuth of the target sheet center in Eq. 6.5.

Next, we obtain an expression for the intensity \( I_{s, \text{sum}}(R_t - R_0 - R_t) \) of the total backscattered field from the target sheet using Eqs. 2.22 and 2.23 expressed in the absolute reference frame. The expression for the intensity is a double sum of 16 terms, the first of which is
\[ I^{(1)}_{s, \text{sum}}(R_r - R_t | R_0 - R_t) = \int \int_{\Delta A} \langle \Phi_s^{(1)}(R_r - R|R_0 - R) \Phi_s^{(1)}(R_r - R|R_0 - R) \rangle f(X, Y) dX dY \]  
\[ = \int \int_{\Delta A} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{m'=1}^{\infty} \sum_{n'=1}^{\infty} \frac{(4\pi)^4}{k^2} A_m(R_r - R) A_n(R_0 - R) A_{m'}^*(R_r - R) A_{n'}^*(R_0 - R) \times \langle S_{m,n}^{(1)}(\varphi) S_{m',n'}^{(1)}(\varphi) \rangle f(X, Y) dX dY \]  
\[ = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{m'=1}^{\infty} \sum_{n'=1}^{\infty} \int_{\psi_{t-L_y/2}^{t+L_y/2}} \int_{\psi_{t-L_x/2}^{t+L_x/2}} \frac{4\pi^2}{k^2} d^2(Z_0) d^2(Z) \frac{1}{\sqrt{\xi_m \xi_{m'} \xi_n \xi_{n'}} P^2} \times u_m(Z_r - Z) u_{m'}^*(Z_r - Z) u_n(Z_0 - Z) u_{n'}^*(Z_0 - Z) N_n^{(1)} N_n^{(1)} N_{m'}^{(1)} N_{m'}^{(1)} \times e^{i\Re[(\gamma_m - \gamma_{m'}) + (\gamma_n - \gamma_{n'})] Z} e^{-\Im[\gamma_m + \gamma_{m'} + \gamma_n + \gamma_{n'}] P} \times \langle S_{m,n}^{(1)}(\varphi) S_{m',n'}^{(1)}(\varphi) \rangle f(P, \varphi) P dP d\varphi \]  
\[ + \sum_{m} \sum_{n} \sum_{m'} \sum_{n'} \sum_{m''} \sum_{n''} \int_{\psi_{t-L_y/2}^{t+L_y/2}} \int_{\psi_{t-L_x/2}^{t+L_x/2}} \frac{4\pi^2}{k^2} d^2(Z_0) d^2(Z) \frac{1}{\sqrt{\xi_m \xi_{m'} \xi_n \xi_{n'}} P^2} \times u_m(Z_r - Z) u_{m'}^*(Z_r - Z) u_{n'}^*(Z_0 - Z) u_{n''}^*(Z_0 - Z) N_n^{(1)} N_{n'}^{(1)} N_{m'}^{(1)} N_{m''}^{(1)} \times e^{i\Re[(\gamma_m - \gamma_{m'}) + (\gamma_n - \gamma_{n'})] Z} e^{-\Im[\gamma_m + \gamma_{m'} + \gamma_n + \gamma_{n'}] P} \times \langle S_{m,n}^{(1)}(\varphi) S_{m',n'}^{(1)}(\varphi) \rangle f(P, \varphi) P dP d\varphi. \]  
\[ (6.8) \]  

The first term in Eq. 6.9 is obtained for \( m = m', n = n' \) where \( e^{i\Re[(\xi_m - \xi_{m'}) + (\xi_n - \xi_{n'})] P} = 1 \). As before, we assume that for large enough \( P_t \), the functions of \( P \) in Eq. 6.9 are slowly varying from \( P_t - L_x/2 \) to \( P_t + L_x/2 \) and can be approximated by their mean...
value over this range interval, except for the exponential function in $P$ which carries the phase information. Moreover, we assume $P_t > \ell L_y / \lambda$. Equation 6.9 becomes

\[
I_{s,\text{sum}}^{(1)}(R_r - R_t | R_0 - R_t)) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{2\pi^2}{k^2} \frac{1}{d^2(Z_0)d^2(Z)} \sqrt{\frac{1}{\xi_m^* \xi_n^* P_t^2}} |u_m(Z_r - Z)|^2 |u_n(Z_0 - Z)|^2 \times |N_n^{(1)}|^2 |N_m^{(1)}|^2 e^{-2\Im(\gamma_m + \gamma_n)Z} e^{-2\Im(\xi_m + \xi_n)P_t} \langle |S_{m,n}^{(1)}(\varphi_t)|^2 \rangle \Delta A f(P, \varphi)
\]

\[
+ \sum_{m} \sum_{m'} \sum_{n} \sum_{n'} 4\pi^2 \frac{1}{k^2} \frac{1}{d^2(Z_0)d^2(Z)} \sqrt{\frac{1}{\xi_{m'} \xi_n \xi_{n'} P_t^2}} \times \langle S_{m,n}^{(1)}(\varphi_t) S_{m',n'}^{(1)}(\varphi_t) \rangle f(P, \varphi) e^{-2\Im(\xi_{m'} + \xi_{n'} + \xi_{n} + \xi_{m})P_t} \]

\[
\times e^{i \Re[(\gamma_{m'} + \gamma_{n} - \gamma_{m} + \gamma_{n'})Z]} e^{-2\Im(\xi_{m'} + \xi_{n'})P_t} \Delta \text{sinc} \left[ \Re \left\{ (\xi_{m'} - \xi_{m}) + (\xi_{n} - \xi_{n'}) \right\} \frac{L_x}{2} \right]. \tag{6.10}
\]

For the second term in Eq. 6.10 to be negligibly small compared to the first term, we require \(\text{sinc} \left[ \Re \left\{ (\xi_{m} - \xi_{m'}) + (\xi_{n} - \xi_{n'}) \right\} \frac{L_x}{2} \right] \ll 1\). This occurs for the sinc function when

\[
\Re \left\{ (\xi_{m} - \xi_{m'}) + (\xi_{n} - \xi_{n'}) \right\} \frac{L_x}{2} \gg \pi. \tag{6.11}
\]

We substitute $\xi_m = k \sin \alpha_m$ and $k = 2\pi / \lambda$ into Eq. 6.11 to obtain

\[
\left[ (\sin \alpha_m - \sin \alpha_{m'}) + (\sin \alpha_n - \sin \alpha_{n'}) \right] \gg \frac{\lambda}{L_x}. \tag{6.12}
\]

We note that when the criteria given in Eq. 6.12 is satisfied, the quadrupole sums over the modes in Eq. 6.7 reduces to a double sum over the modes given by the first term in Eq. 6.10 in the expression for the total mean intensity of the scattered field.
from the target sheet. Therefore, Eq. 6.12 provides the criteria needed for scattering in a waveguide to decorrelate the waveguide modes resulting in a simplification in the expression for the mean scattered intensity.

For the square of the mean scattered field given by the square of Eq. 6.5 to be negligibly small compared to the mean intensity given by the first term of Eq. 6.10, we require

\[ \left| \text{sinc} \left[ \mathcal{R}(\xi_m + \xi_n) \frac{L_x}{2} \right] \right|^2 \ll 1 \]

This occurs for the sinc function when

\[ \mathcal{R}(\xi_m + \xi_n) \frac{L_x}{2} \gg \pi. \]  

(6.13)

Substituting \( \xi_m = k \sin \alpha_m \) and \( k = 2\pi/\lambda \) into Eq. 6.13 leads to

\[ L_x (\sin \alpha_m + \sin \alpha_n) \gg \lambda. \]  

(6.14)

According to the criteria in Eq. 6.14, when the target sheet is many times larger than the projected wavelength of the modal plane wave on the target sheet, the mean backscattered field from the target sheet is negligibly small. The criteria implies that a statistically large number of scatterers are required in order for the mean backscattered field to average to zero.

The above derivation can be readily extended to scattering in other non-forward azimuths apart from backscatter to obtain the criteria expressed in Eqs. 6.14 and 6.12 for the scattering to become diffuse in these azimuths as well. It should be stressed that diffuse scattering only occurs in non-forward scatter. This can be understood from our everyday experiences with scattering from rough surfaces, such as the scattering of light from the surface of a road. For instance, the dry road appears to reflect like a mirror or a pool of water at very shallow grazing angles in forward scatter. “When people see rough surfaces they can tell that they are rough by the way that they scatter from all angles except forward at shallow grazing angles. But even rough surfaces look "smooth" in shallow grazing angles in forward scatter. We
call this a mirage because we don’t understand it ... or rather it is an ambiguity since we can’t distinguish smooth and rough surfaces in this case.” (Nicholas Makris)

6.1.1 Summary

In summary, for a random target of characteristic dimension \( L \) in a waveguide, if it satisfies the conditions in Eqs. 6.14 and 6.12, the mean field in non-forward scatter azimuths,

\[
\langle \Phi_{s,sum,non-forward}(R_r - R|R_0 - R) \rangle \approx 0, \tag{6.15}
\]

and the waveguide modes decorrelate in the scattering process which results in the quadrupole sum over the modes for the intensity reducing to a double sum over the modes. Comparing Eqs. 6.10 and 6.7, the first out of 16 terms in the intensity of the scattered field in non-forward azimuths can be expressed as a double sum over the modes given by

\[
I_{s,sum}^{(1)}(R_r - R_t|R_0 - R_t)) = \int \int_{\Delta A} \langle \Phi_s^{(1)}(R_r - R|R_0 - R)\Phi_s^{(1)}(R_r - R|R_0 - R) \rangle f(X,Y)dXdY
\]

\[
\approx \int \int_{\Delta A} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(4\pi)^4}{k^2} |A_m(R_r - R)|^2 |A_n(R_0 - R)|^2 \langle S_m^{(1)}(\varphi)S_m^{(1)}(\varphi) \rangle f(X,Y)dXdY. \tag{6.16}
\]

Since the mean field goes to zero, \( I_{s,sum}(R_r - R_t|R_0 - R_t) \) as given in Eq. 6.16 is also equal to the variance of the field.

If we let \( S_{sum}(\alpha_m, \beta, \alpha_n, \beta_i) \) be the scatter function of the sheet of characteristic dimension \( L \), then comparing Eq. 6.15 with Eq. 2.20 implies that \( \langle S_{sum}(\alpha_m, \beta, \alpha_n, \beta_i) \rangle \approx 0 \) for \( \beta \neq \beta_i \). Also, if we compare Eq. 6.16 with Eqs. 2.22-2.24, we find that the covariance of the scatter function of the target in non-forward azimuths is
\[
\langle S_{\text{sum}}(\alpha_m, \beta, \alpha_n, \beta_i) S_{\text{sum}}^*(\alpha'_m, \beta', \alpha'_n, \beta_i) \rangle - \langle S_{\text{sum}}(\alpha_m, \beta, \alpha_n, \beta_i) \rangle \langle S_{\text{sum}}^*(\alpha'_m, \beta', \alpha'_n, \beta_i) \rangle 
= \delta_{nn'} \delta_{mm'} C_{s,\text{sum,}mn}(r, r'|r_0),
\]

where

\[
C_{s,\text{sum,}mn}(r, r'|r_0) 
= \langle S_{\text{sum}}(\alpha_m, \beta, \alpha_n, \beta_i) S_{\text{sum}}^*(\alpha'_m, \beta', \alpha'_n, \beta_i) \rangle - \langle S_{\text{sum}}(\alpha_m, \beta, \alpha_n, \beta_i) \rangle \langle S_{\text{sum}}^*(\alpha'_m, \beta', \alpha'_n, \beta_i) \rangle,
\]

and \( \beta \neq \beta_i \).

### 6.2 Coherent “Reverberation Rings”

When the conditions given in Eqs. 6.14 and 6.12 are not satisfied, the scattering from a target sheet becomes coherent. The quadrupole sum over the modes in Eq. 6.10 for the intensity of the scattered field cannot be neglected in this case. This quadrupole sum has range dependent phase terms due to the coherent interference between the waveguide modes that give rise to nulls and peaks in the scattered field from the target sheet. For scattering from a patch of seafloor, this quadrupole term leads to coherent “reverberation rings” that is often observed in high-resolution sonar systems.

### 6.3 Forward Scattering, Attenuation and Dispersion

To investigate the forward scattered field from the target sheet, it is convenient to place the origin of the coordinate system at the center of the target sheet. In this target-centered reference frame, as defined in Sec. 2.1, we let the coordinates of the
source be \((-x_0, 0, z_0)\), the coordinates of the receiver be \((x, 0, z)\), and the coordinates of a point on the target sheet be \((x', y', 0)\). Similar to Eqs. 6.1 and 6.2, we can express the total mean field scattered by the random scatterers in the forward azimuth of the target sheet as a sum of four terms, the first of which is given by

\[
\left\langle \Phi_{s, \text{sum}}^{(1)}(r|r_0) \right\rangle = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \int_{-L_y/2}^{L_y/2} \int_{-L_x/2}^{L_x/2} \frac{i}{k} \frac{1}{d(z_0) d(0)} u_m(z) u_n(z_0) N_n^{(1)} N_m^{(1)} e^{i(\gamma_{m}+\gamma_{n})D} \\
\times e^{i(\xi_m + \xi_n \rho_0)} S_m^{(1)}(y') f(x', y') dx' dy',
\]

where \(\rho_0 = \sqrt{(x_0 + x')^2 + y'^2}\) is the horizontal range from the source to a scatterer within the target sheet, \(\rho = \sqrt{(x - x')^2 + y'^2}\) is the horizontal range of the scatterer to the receiver, and \(\left\langle S_m^{(1)}(y') \right\rangle = S(\pi - \alpha_m, -\frac{y'}{\rho}, \alpha_n, \frac{y'}{\rho_0})\). We expand \(\rho_0 \approx (x_0 + x') + \frac{y'^2}{2x_0}\) and \(\rho \approx (x - x') + \frac{y'^2}{2x}\) in the exponent, and approximate \(\rho_0 \approx x_0\) and \(\rho \approx x\) in the denominator. We also approximate the functions of \(x'\) and \(y'\) by their mean value over the integration region, except for the exponential function which carries the phase information. The mean total scattered field in the forward azimuth of the target sheet becomes

\[
\left\langle \Phi_{s, \text{sum}}^{(1)}(r|r_0) \right\rangle = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \int_{-L_y/2}^{L_y/2} \int_{-L_x/2}^{L_x/2} \frac{2\pi}{k d(z_0) d(0)} u_m(z) u_n(z_0) N_n^{(1)} N_m^{(1)} e^{i(\gamma_{m}+\gamma_{n})D} f(x', y') \\
\times e^{i(\xi_m + \xi_n \rho_0)} \left\langle S_m^{(1)}(y') \right\rangle e^{\frac{k^2}{2x_0}(\frac{y'^2}{x_0} + \frac{y'^2}{x})} dy'.
\]

(6.20)

When \(\sqrt{x_0} < L_y < \frac{\lambda_0}{k}\), the integration region is sufficiently wide to include the full active region of interference (see Sec. D.1) and it also allows the scatter function to be approximated as a constant in forward azimuth over the distribution of scatterers in the target sheet. We can then use the following asymptotic integration,
\[
\int_{-L_y/2}^{L_y/2} \left(S_{m,n}^{(1)}(y')\right) e^{i\frac{\mu^2}{2} \left(\frac{\mathbb{R}(\xi_n + \xi_m)}{x_0} + \frac{\mathbb{R}(\xi_m)}{z'}\right)} dy' = \left(S_{m,n}^{(1)}(0)\right) \sqrt{\frac{2\pi x_0}{\xi_n x + \xi_m x_0}} e^{i\pi/4}, \quad (6.21)
\]

in Eq. 6.20, to obtain,

\[
\langle \Phi_{s,\text{turn}}(r|R_0) \rangle \\
= \sum_{m=1}^{\infty} \frac{2\pi}{k} \frac{i}{d(z_0)d(0)} \frac{1}{\xi_m \sqrt{x_0}} u_m(z) u_m(z_0) (N_m^{(1)} e^{i2\gamma_m D} \langle S_{m,m}^{(1)}(0) \rangle f(x', y')
\times e^{i\xi_m(x+x_0)} L_x \sqrt{\frac{2\pi x_0}{\xi_m (x_0 + x)}} e^{i\pi/4} \\
+ \sum_{m} \sum_{n \neq m} \frac{2\pi}{k} \frac{i}{d(z_0)d(0)} \frac{1}{\xi_m \xi_n x_0} u_m(z) u_n(z_0) N_m^{(1)} N_n^{(1)} e^{i(\gamma_m + \gamma_n) D} \langle S_{m,n}^{(1)}(0) \rangle f(x', y')
\times e^{i\xi_m x + \xi_n x_0} L_x \text{sinc} \left[ \Re(\xi_n - \xi_m) \frac{L_x}{2} \right] \sqrt{\frac{2\pi x_0}{\xi_n x + \xi_m x_0}} e^{i\pi/4}, \quad (6.22)
\]

where \( \langle S_{m,n}^{(1)}(0) \rangle = \langle S(\pi - \alpha_m, 0, \alpha_n, 0) \rangle \) is the scatter function in the forward azimuth. We observe that the second term in Eq. 6.22 is negligibly small compared to the first term when \( |\text{sinc} \left[ \Re(\xi_n - \xi_m) \frac{L_x}{2} \right]| \ll 1 \) or

\[
L_x (\sin \alpha_n - \sin \alpha_m) \gg \lambda. \quad (6.23)
\]

Under this condition, the mean forward scattered field from the target sheet can be obtained from the first term in Eq. 6.22. Combining all four terms in the expression for the mean forward scattered field, we obtain
\[
\langle \Phi_{s,\text{sum}}(r|\mathbf{r}_0) \rangle = \sum_{m=1}^{\infty} \frac{2\pi}{k} \frac{i}{d(z_0)d(0)} \frac{1}{\xi_m} u_m(z_0) u_m(z) f(x', y') e^{i\xi_m(2x_0 + x)} L_x \left( \frac{2\pi}{\xi_m} \right) e^{i\pi/4} \times \left[ (N^{(1)}_m)^2 e^{i2\gamma_m D} \langle S(\pi - \alpha_m, 0, \alpha_m, 0) \rangle - N^{(2)}_m N^{(1)}_m \langle S(\alpha_m, 0, \alpha_m, 0) \rangle \right. \\
- N^{(1)}_m N^{(2)}_m \langle S(\pi - \alpha_m, 0, \pi - \alpha_m, 0) \rangle + (N^{(2)}_m)^2 e^{-i2\gamma_m D} \langle S(\alpha_m, 0, \pi - \alpha_m, 0) \rangle \right].
\] (6.24)

The total field \( \langle \Phi_{T,L_z}(r|\mathbf{r}_0) \rangle \) in the forward azimuth of the target sheet is the sum of the incident field \( \Phi_i(r|\mathbf{r}_0) \) from the source and the forward scattered field \( \langle \Phi_{s,\text{sum}}(r|\mathbf{r}_0) \rangle \) from the target sheet. Using Eq. 5.1 for the incident field and Eq. 6.24 for the scattered field in the forward azimuth from the target sheet, the total field can be expressed as

\[
\langle \Phi_{T,L_z}(r|\mathbf{r}_0) \rangle = \Phi_i(r|\mathbf{r}_0) + \langle \Phi_{s,\text{sum}}(r|\mathbf{r}_0) \rangle
\] (6.25)

\[
= (4\pi) \frac{i}{d(z_0)\sqrt{8\pi}} e^{-i\pi/4} \sum_{m=1}^{\infty} u_m(z_0) u_m(z) \frac{e^{i\xi_m(x_0 + x)}}{\sqrt{\xi_m(x_0 + x)}} \left( 1 + \frac{2\pi}{k} \frac{i}{d(0)} \frac{1}{\xi_m} L_x f(x', y') \right) \times \left[ (N^{(1)}_m)^2 e^{i2\gamma_m D} \langle S(\pi - \alpha_m, 0, \alpha_m, 0) \rangle - N^{(2)}_m N^{(1)}_m \langle S(\alpha_m, 0, \alpha_m, 0) \rangle \right. \\
- N^{(1)}_m N^{(2)}_m \langle S(\pi - \alpha_m, 0, \pi - \alpha_m, 0) \rangle + (N^{(2)}_m)^2 e^{-i2\gamma_m D} \langle S(\alpha_m, 0, \pi - \alpha_m, 0) \rangle \right].
\] (6.26)

\[
= (4\pi) \frac{i}{d(z_0)\sqrt{8\pi}} e^{-i\pi/4} \sum_{m=1}^{\infty} u_m(z_0) u_m(z) \frac{e^{i\xi_m(x_0 + x)}}{\sqrt{\xi_m(x_0 + x)}} \left( 1 + i\nu_m L_x \right),
\] (6.27)

where
The quantity \( \nu_m \) provides a measure of the attenuation and dispersion in the \( m \)th mode of the forward propagating field as a result of scattering from the horizontal target sheet of width \( L_x \).

The effect of attenuation and dispersion on the forward propagating field from a continuous sheet of scatterer extending from the source at \( r_0 \) to a receiver at \( r \) can be found by integrating the effect of each incremental sheet of width \( L_x \). If we express Eq. 6.27 for the forward propagated field after scattering from the target sheet of size \( L_x \) as

\[
\langle \Phi_{T,L_x}(r|r_0) \rangle = \sum_{m=1}^{\infty} \Phi_{T,L_x}^{(m)}(r|r_0) = \sum_{m=1}^{\infty} \Phi_i^{(m)}(r|r_0) (1 + i \nu_m L_x),
\]

(6.29)

where \( \Phi_i^{(m)}(r|r_0) \) is the incident field from mode \( m \) in the absence of the scatterer given by

\[
\Phi_i^{(m)}(r|r_0) = (4 \pi) \frac{i}{d(z_0)\sqrt{8\pi}} e^{-i\pi/4} u_m(z_0) u_m(z) \frac{e^{\xi_m(x+x_0)}}{\sqrt{\xi_m(x_0 + x)}},
\]

(6.30)

we can write Eq. 6.29 for each mode \( m \) as

\[
\frac{\langle \Phi_{T,L_x}^{(m)}(r|r_0) \rangle - \Phi_i^{(m)}(r|r_0)}{\Phi_i^{(m)}(r|r_0)} = i \nu_m L_x.
\]

(6.31)

Eq. 6.31 can be simply expressed in the form
\[
\frac{\Delta \Phi}{\Phi} = i\nu_m L_x. \quad (6.32)
\]

Integrating Eq. 6.31 over \( L_x \) for a continuous sheet of scatterer extending from the source at \( r_0 \) to a receiver at \( r \), we obtain

\[
\int_{\text{field with target sheet}} \frac{\Delta \Phi}{\Phi} = \int_0^{x_0+x} i\nu_m L_x \quad (6.33)
\]

\[
\langle \Phi_{2}^{(m)}(r|r_0) \rangle = \Phi_{1}^{(m)}(r|r_0) e^{i\nu_m (x+x_0)}. \quad (6.34)
\]

Summing up the contribution from each mode and using Eq. 6.30 in Eq. 6.34 for \( \langle \Phi_{i}^{(m)}(r|r_0) \rangle \), we obtain

\[
\langle \Phi_{T}(r|r_0) \rangle = (4\pi) \frac{i}{d(z_0)\sqrt{8\pi}} e^{-i\pi/4} \sum_{m=1}^{\infty} u_m(z_0) u_m(z) \frac{e^{i(\xi_m + \nu_m)(x+x_0)}}{\sqrt{\xi_m(x_0 + x)}}, \quad (6.35)
\]

so that \( \Im\{\nu_m\} \) is the attenuation coefficient for mode \( m \) arising from scattering off the sheet of random scatterers and \( \Re\{\nu_m\} \) is related to the dispersion due to scattering.

If instead of a target sheet in the x-y plane, we have a volume of scatterers extending from the water surface \( z' = -D \) to the seafloor \( z' = H - D \). The distribution of scatterers within this volume can be described using a volumetric density function \( f(x', y', z') \). The effect of attenuation and dispersion from propagating through this volume of scatterers can be obtained from
\[ \nu_m = \int_{-D}^{H-D} \frac{2\pi}{k} \frac{1}{d(z')} \frac{1}{\xi_m} f_{z'}(x', y') \times \left[ \left( N_m^{(1)} \right)^2 e^{i2\gamma_m(D+z')} \langle S(\pi - \alpha_m, 0, \alpha_m, 0) \rangle - N_m^{(2)} N_m^{(1)} \langle S(\alpha_m, 0, \alpha_m, 0) \rangle 
\right. \\
\left. - N_m^{(1)} N_m^{(2)} \langle S(\pi - \alpha_m, 0, \pi - \alpha_m, 0) \rangle + (N_m^{(2)})^2 e^{-i2\gamma_m(D+z')} \langle S(\alpha_m, 0, \pi - \alpha_m, 0) \rangle \right] dz'. \] 

(6.36)

where \( f_{z'}(x', y') \) is the average value of the density function over the target coordinates in \( x' \) and \( y' \) for a given \( z' \), i.e. \( f_{z'}(x', y') = \frac{1}{\Delta A} \int f(x', y', z') dA, dA = dx' dy' \).

For a distribution of compact scatterers, the scatter function \( S \), is a constant and can be factored out of the integral in Eq. 6.36. The resulting equation can be simplified using Eq. 2.5 to obtain

\[ \nu_m = \frac{2\pi S_o}{k} \frac{1}{\xi_m} f(x', y', z') \int_{-D}^{H-D} \frac{u_m^2(z')}{d(z')} dz'. \] 

(6.37)

In Eq. 6.37, \( f(x', y', z') \) is the mean volumetric density function, i.e. \( f(x', y', z') = \frac{1}{\Delta V} \iiint f(x', y', z') dV, dV = dx' dy' dz' \). Making use of modal orthogonality described in Eq. 2.4, Eq. 6.37 can be integrated over the depth of the scatterers to obtain

\[ \nu_m \approx \frac{2\pi S_o}{k} \frac{1}{\xi_m} f(x', y', z'). \] 

(6.38)

From Eq. 6.38, we observe that the attenuation coefficient for scattering from a large number of compact scatterers is a linear sum of the attenuation coefficient from a single scatterer. This result for compact scatterers in a waveguide is similar to that for scattering in free-space[75].
Chapter 7

Validity of the Sonar Equation and Babinet’s Principle for scattering in a Stratified Medium

In this chapter we investigate the validity of the sonar equation and Babinet’s principle for scattering in shallow water waveguides. In Sec. 7.1, we show how the sonar equation is commonly applied to analysis in a waveguide. By comparing the sonar equation with the waveguide scattering model of Chap. 2, we show when propagation and scattering become decoupled in a waveguide and when incoherent target strength is sufficient to describe the scattering properties of an object in a waveguide. Examples illustrating the conditions under which the sonar equation becomes approximately valid in a waveguide are presented in Sec. 7.2. Babinet’s principle and issues involved with applying it in a waveguide are discussed in Sec. 7.3.

7.1 The Sonar Equation

7.1.1 In Free Space

The sonar equation model for the field scattered from an object in free space in derived in Appendix A and is given by Eq. (A.10). In Eq. (A.12), the sonar equation
is expressed in terms of the free space Green function and the objects planewave scatter function. From Eq. (A.12), we see that the Green functions that describe the propagation of waves from source to object $G(0|r_0)$ and from object to receiver $G(r|0)$ are decoupled from the scattering function $S(\alpha, \beta, \alpha_i, \beta_i)$ of the object that governs the scattering process. Only the directions of the source and the receiver relative to the object matter for far field scattering in free space where propagation and scattering effects become factorable from each other. The approximation given in Eq. (A.12) is always valid in the far field, where $r, r_0 > L^2/\lambda$, and may be valid at much closer ranges for certain targets, such as spheres[42]. The incident wave effectively arrives at the target as a planewave propagating from the direction of the source and the scattered wave at the point receiver behaves as a planewave propagating from the target centroid. The scatter function of the target determines how a planewave from the source is scattered in the direction of the receiver.

It is important to notice that Eq. (A.12) is in the frequency domain for a time harmonic source. If the source was broadband with spectrum $Q(f)$, the received field would be the inverse Fourier transform of the product of $Q(f)$ and the right hand side of Eq. (A.12). For a broadband source signal, it then is impossible to separate scattering from propagation even in free space. For a narrowband source, $A \approx Q(f)df$.

The sonar equation in its more familiar form where the level of the scattered field in decibels is expressed as a linear combination of incoherent source level, transmission loss, and target strength is provided in Eq. (A.13). The incoherent target strength is obtained from the magnitude of the free-space scatter function following Eq. (A.16). It contains only the amplitude but not the phase information of the coherent scatter function and depends only on the direction of the source and receiver, relative to the target. A conceptual diagram of free space scattering is given in Fig. 7-1. This diagram should be contrasted with that in Fig. 2-1 which illustrates scattering in a waveguide.
7.1.2 Application of the Sonar Equation in a Waveguide

It is common practice when using the sonar equation in a waveguide to replace the transmission loss in free space with that in the waveguide [11]. This can be done analytically by replacing the free-space Green function with the waveguide Green function in Eq. (A.12). Using a modal formulation, the Green function in the waveguide between a point at the origin 0 and a field point at \( r \) can be expressed as a sum of normal modes,

\[
G(r|0) = \frac{i}{d(0)}(8\pi)^{-1/2}e^{-i\pi/4} \sum_{m} u_m(z)u_m(0)\frac{e^{i\xi_m \rho}}{\sqrt{\xi_m \rho}}. \tag{7.1}
\]

In Eq. 7.1 \( M_{\text{max}} \) is the mode number at which the series can be truncated and still accurately represent the field. Using Eqs. (2.3) and (2.5), we can express the Green function in the waveguide, Eq. (7.1), as
\[ G(r|0) = \sum_{m}^{M_{\text{max}}} [A_{m}(r) - B_{m}(r)]. \]  

(7.2)

By reciprocity,

\[ G(0|r_0) = G(r_0|0) = \sum_{n}^{M_{\text{max}}} [A_{n}(r_0) - B_{n}(r_0)]. \]  

(7.3)

Substituting the waveguide Green functions, Eqs. 7.2 and 7.3, into Eq. A.12, we obtain the sonar equation approximation for the scattered field from an object in a waveguide.

\[
\Phi_s(r|r_0) = A(4\pi)^2 \left( \sum_{n}^{M_{\text{max}}} [A_{n}(r_0) - B_{n}(r_0)] \right) \left( \sum_{m}^{M_{\text{max}}} [A_{m}(r) - B_{m}(r)] \right) \frac{S(\alpha, \beta, \alpha_t, \beta_t)}{k} 
\]  

(7.4)

The wave-theoretic model for object scattering in a waveguide Eq. (2.1) differs significantly from the sonar equation model in Eq. (7.4). In the waveguide scattering model, the scattered field depends on the direction of each incoming and outgoing modal planewave. Each incoming planewave is \textit{coherently} scattered to each outgoing planewave by the object depending on the scatter function, which can vary with the azimuth and elevation angles of the incoming and outgoing planewaves. In the sonar equation model, since propagation and scattering are assumed to decouple, the scattered field depends only on the direction of the source and receiver relative to the object and not the direction of the individual modal planewaves.

The sonar equation (7.4) is a special case of the general coherent scattering formulation for a waveguide of Eq. (2.1), and so is only valid under restrictive conditions. If the scatter function remains constant over the horizontal grazing angle span of
the waveguide modes $\pm \Delta \psi$ for the given measurement scenario, the scatter function factors from the modal sums of the waveguide scattering model, Eq. (2.1), that then reduces to the sonar equation, Eq. (7.4). Propagation and scattering are then separable, and the sonar equation becomes valid in a waveguide where $\alpha$ and $\alpha_i$ are approximately $\pi/2$ in Eq. (7.4). Target strength, along with the other incoherent terms of the sonar equation, $SL$ and $TL$, then become sufficient to determine the scattered field level in decibels. We approximate the horizontal grazing angle span of the waveguide modes by

$$\pm \Delta \psi = \pm \left(\frac{\pi}{2} - \alpha_{M_{\max}}\right), \quad (7.5)$$

where

$$\alpha_{M_{\max}} = \tan^{-1} \frac{\xi_{N_{\max}}}{\gamma N_{\max}}. \quad (7.6)$$

Here $M_{\max}$ and $\Delta \psi$ are range-dependent even in realistic range-independent waveguides and tend to decrease with range due to attenuation from absorption and scattering in the ocean, following the process known as "mode stripping." This is significant because the sonar equation approximation improves as $\Delta \psi$ decreases for fixed $\lambda/L$.

### 7.2 Analytic Conditions Necessary for the Sonar Equation to Become Valid for Scattering in a Waveguide

We now use examples to illustrate the fact that the sonar equation is valid when the scatter function is roughly constant over the equivalent horizontal grazing angles spanned by the dominant waveguide modes. We show that the sonar equation is generally a good approximation (1) for all objects when $2\Delta \psi < \lambda/2L$, for homogenous
convex objects when $2\Delta \psi < \lambda / L$ and (2) for spheres and certain other rounded objects in non-forward scatter azimuths even when (1) does not hold. We proceed by analyzing active sonar examples for a variety of target types and shallow water waveguides with both the sonar equation and the waveguide scattering model.

In all the illustrative examples, a water column of 100-m depth is used to simulate a typical continental shelf environment. The sound speed in the water column is isovelocity at 1500 m/s with constant density of 1 g/cm$^3$ and attenuation of $6.0 \times 10^{-5}$ dB/\(\lambda\). The seabed is either perfectly reflecting or comprised of sand or silt halfspaces. The density, sound speed and attenuation are taken to be 1.9 g/cm$^3$, 1700 m/sec, and 0.8 dB/\(\lambda\) for sand, 1.4 g/cm$^3$, 1520 m/sec, and 0.3 dB/\(\lambda\) for silt. The receiver is either co-located with the source, in which case we calculate the backscattered field where $\beta_i = 0$, $\beta = \pi$, or in the forward azimuth of the object where $\beta_i = \beta = 0$ and we calculate the forward scattered field. The scattered fields from a circular disc, square plate, sphere, and composite target are computed as a function of increasing range between source-receiver and object. In all the examples, range increases along the $x$-axis and depth along the $z$-axis.

### 7.2.1 Effect of Bottom Type on the Validity of the Sonar Equation

We use examples to illustrate how the validity of the sonar equation depends on bottom type through the grazing angle span of the waveguide modes $\Delta \psi$. In a Pekeris waveguide, $\Delta \psi$ is bounded by the critical grazing angle of the bottom $\psi_c$ beyond a few waveguide depths in range where the leaky modes no longer contribute significantly. A bottom with a large sound speed contrast relative to the water column has a correspondingly large $\psi_c$, $M_{max}$ and $\Delta \psi$. So for fixed source frequency and object size, the sonar equation approximation is expected to improve as this sound speed contrast decreases. We show this by examining Pekeris waveguides with silt, sand and perfectly reflecting bottoms that respectively exhibit an increase in sound speed contrast.
The backscattered field from a homogenous convex object, an upright 10-m radius rigid circular disc at 300-Hz, is plotted in Fig. 7-2 (a)-(c) for the three bottom types. For this example the product $ka = \pi L/\lambda$ is 12.6, where $a$ is the radius of the disc, and $ka$ is the ratio of the object circumference to the wavelength. By applying Green's Theorem, the scatter function for the rigid circular disc is found to be

\[ S(\alpha, \beta, \alpha_i, \beta_i) = \frac{i}{2} \left( \frac{kL}{2} \right)^2 \sin \alpha \cos \beta \text{circ} \left[ \frac{kL}{2} \sqrt{\left( \sin \alpha_i \sin \beta_i - \sin \alpha \sin \beta \right)^2 + \left( \cos \alpha_i - \cos \alpha \right)^2} \right]. \] (7.7)

where $L = 2a$ is the diameter of the disc and $\text{circ}(x) = 2J_1(x)/x$. An alternative but equivalent derivation and form for the scatter function can be found in Ref. [5].

As expected, the sonar equation matches the waveguide scattering model well in the Pekeris silt waveguide of Fig. 7-2(a) beyond roughly 5-km range. In the Pekeris sand waveguide, Fig. 7-2(b), the difference between the sonar equation and the waveguide scattering model is as much as 10-dB within a few kilometers range. The match however improves as range increases. In the perfectly reflecting waveguide, Fig. 7-2(c), the sonar equation overestimates the scattered field level by as much as 20 dB. This error is more than half of the object's maximum target strength of 36-dB re 1 m, which occurs monostatically at broadside. The scattered field shows greater fluctuation with range for the sonar equation approximation than for the waveguide scattering model. This is true in all three waveguides.

In practice, measurements are often averaged over space, time or frequency to reduce the fluctuations that arise from waveguide interference. Less fluctuation with range is observed in both of the depth-averaged scattered fields of Fig. 7-3 than in Fig. 7-2(b) for the sand bottom. The discrepancy between the sonar equation and waveguide scattering model remains, however, with differences as large as 10-dB still occurring within a few kilometers range.

The reason that the sonar equation is found to be a good approximation for the
silt beyond roughly 5 km but not the sand or perfectly reflecting waveguides is that
the condition $2\Delta \psi < \lambda/L$ only holds for the silt waveguide. This can be seen by
examining Fig. 7-2(d) where the ratio $2\Delta \psi/(\lambda/L)$ is plotted. Here $\Delta \psi$ is computed
from Eqs. (7.5) and (7.6) at a given receiver depth by determining the minimum
value for $M_{\text{max}}$ at which the modal sum of Eq(1) differs less than 1-dB from that of
an infinite sum. The $\Delta \psi$ used in the figures is the average value over receiver depth
throughout the watercolumn. For ranges beyond roughly 5 km in the silt waveguide,
the ratio is less than unity and the condition for the sonar equation to be valid holds.
Figure 7-2(d) also shows that the condition is generally not satisfied in the sand
waveguide for the ranges shown. The condition is never satisfied in the perfectly
reflecting waveguide because the ratio $2\Delta \psi/(\lambda/L)$ is always 12.6. As expected,
the performance of the sonar equation improves with range in realistic ocean waveguides
because $M_{\text{max}}$ and $\Delta \psi$ decrease due to modal stripping, as is evident in Figs. 7-2(a),
(b) and (d). Note that $\Delta \psi$ can be determined from Fig. 7-2(d) since $\lambda/L = 0.25$
radians or 14.3° as can be seen from Figs. 7-4 and 7-5. Here the scatter function
magnitude is plotted for the upright disk where the main lobe has a minimum width
of $\lambda/L$ for a planewave incident at broadside where $(\pi/2 - \alpha_i) = 0$.

To visualize why the sonar equation is not valid when the condition $2\Delta \psi < (\lambda/L)$
does not hold, it is instructive to plot the width of the bottom critical angle across the
main lobe for each bottom type as is done in Fig. 7-5. When the condition does not
hold, the object scatters the dominant incident modes with widely varying amplitudes
and the scatter function cannot be approximated as a constant over $\pm \Delta \psi$. In this
case, both the magnitude and phase variations of the scatter function are important
in describing the scattering process. The sonar equation overestimates the level of
the scattered field because it depends only on two directions, those of the source and
receiver relative to the object. The two relevant directions for the upright disc are
$(\pi/2 - \alpha_i) = 0$ and $\beta_i = 0$ for the incident and $(\pi/2 - \alpha) = 0$ and $\beta = \pi$ for the
scattered field. These correspond to global maxima in both the scatter function and
target strength, as can be seen in Fig. 7-2, which is inappropriately assigned to all
incoming and outgoing directions by the sonar equation.
Figure 7-2: The backscattered field from an upright 10-m radius rigid circular disc at 300 Hz in Pekeris waveguides with (a) silt, (b) sand, and (c) perfectly reflecting bottoms respectively, calculated using the waveguide scattering model, Eq. (2.1), and compared to the sonar equation, Eq. (7.4). The water depth is 100 m with source-receiver and object at 50-m depth in the middle of the water column. Range increases along the x-axis and depth along the z-axis. The circular disc is aligned with its plane normal to the x-axis. The results are plotted in decibels, i.e., $20 \log |\Phi_s|$, as a function of increasing range between object and monostatic source-receiver. Source strength is $0 \text{ dB re } 1 \mu Pa \text{ @ } 1 \text{ m}$. $ka$ is 12.6 for this example. (d) The ratio $2\Delta \psi/(\lambda/L)$ for the examples given in (a) and (b). For the perfectly reflecting waveguide $2\Delta \psi/(\lambda/L)$ is 12.6. The sonar equation provides a good approximation to the scattered field in the waveguide when $2\Delta \psi/(\lambda/L) < 1$. 

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Figure 7-3: Similar to Fig. 7-2(b) except the scattered field in the back azimuth is averaged over depth throughout the water column from 0-m to 100-m.
Figure 7-4: The magnitude of the planewave scatter function $20 \log |S(\alpha, \beta, \alpha_i = 90^\circ, \beta_i = 0^\circ)|$ for an upright rigid circular disc of $ka = 12.6$ is plotted as a function of horizontal grazing $(90^\circ - \alpha)$ and azimuth $\beta$ angles of scattered planewaves for an incident planewave travelling in the direction $(90^\circ - \alpha_i) = 0^\circ, \beta_i = 0^\circ$. The scatter function is anti-symmetric about the plane of the disk and as can be seen from Eq. (7.7).
Figure 7-5: The magnitude of the planewave scatter function $|S(\alpha, \beta, \alpha_i, \beta_i = 0^\circ)|$ for an upright rigid circular disc of $ka = 12.6$ is plotted as a function of horizontal grazing angle $(90^\circ - \alpha)$ of scattered planewaves in the back scatter azimuth $\beta = 180^\circ$ for several incident planewaves with horizontal grazing angles $(90^\circ - \alpha_i) = 0^\circ, 30^\circ, 60^\circ, \text{ and } 90^\circ$. The solid curve in this figure for broadside incidence ($(90^\circ - \alpha_i) = 0^\circ$) is a slice through Fig. 7-4 at the backscatter azimuth $\beta = 180^\circ$ of the scattered planewaves. The width $\lambda/L$ of the scatter function main lobe for broadside incidence is $14.3^\circ$ or 0.25 radians. Also shown is the critical grazing angle $\psi_c$ of the seabed in the Pekeris silt waveguide of $9.3^\circ$, Pekeris sand waveguide of $28^\circ$ and the perfectly reflecting waveguide of $90^\circ$. 
7.2.2 Effect of Object Size and Frequency on the Validity of the Sonar Equation

In this section, we investigate sonar equation performance as a function of object size and frequency for a rigid circular disc in various waveguides. At the high $ka$ of 62.8 shown in Fig. 7-6(a), (c), and (e), the object is large compared to the wavelength and the sonar equation significantly over-estimates the scattered field level. This is to be expected from Fig. 7-7(a) where the condition $2\Delta\psi < \lambda/L$ is not satisfied in any of the waveguides. At the lower $ka$ of 1.3, Fig. 7-6(b), (d), and (f), the sonar equation provides a good approximation in all except the perfectly reflecting waveguide which is consistent with the results of Fig. 7-7(b) where the condition is satisfied for the sand and silt waveguides for all ranges shown. This shows that the sonar equation can be made valid for a given object and measurement geometry by lowering the frequency of operation. The scatter functions for the high and low $ka$ cases are plotted in Figs. 7-8(a) and (b) respectively. For the low $ka$ case in the perfectly reflecting waveguide, the width of the scatter function lobe is only slightly smaller than the full grazing angle span of the waveguide modes so the sonar equation provides only an order-of-magnitude approximation in backscatter.

In general, for pressure-release objects that are compact, small compared to the wavelength ($ka < 1$), the sonar equation approximation is always good since the scattered field is effectively omni-directional. For rigid objects that are compact, the sonar equation is still a good approximation even though their scattered field always maintains some directionality as $ka$ decreases.

7.2.3 Effect of Orientation and Shape of Homogenous Convex Targets on the Validity of the Sonar Equation

In this section, we investigate the effect of orientation and shape on the validity of the sonar equation for homogenous convex objects. The effect of orientation is investigated by rotating the upright disk ($ka = 12.56$) of Sec. 4A clockwise by 18° and 34° respectively about the $y$-axis. This places the zero of the scatter function at
Figure 7-6: Similar to Figs. 7-2(a)-(c) but for an upright rigid circular disk of (a) high and (b) low $ka$ in the Pekeris silt waveguide, (c) high and (d) low $ka$ in the Pekeris sand waveguide, and (a) high and (b) low $ka$ in the perfectly reflecting waveguide, The high $ka$ case corresponds to a disc of 10-m radius at 1500-Hz with $ka = 62.8$ while the low $ka$ case corresponds to a disc of 1-m radius at 300-Hz with $ka = 1.3$. 

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Figure 7-7: Figures (a) and (b) are similar to Fig. 7-2d but for \(ka\) of 62.8 and 1.3 respectively for the cases shown in Figs. 7-6(a)-(d) in the Pekeris silt and sand waveguides. The ratio \(2\Delta \psi/(\lambda/L) = 62.8\) for Fig. 7-6(e) with \(ka\) of 62.8 and \(2\Delta \psi/(\lambda/L) = 1.3\) for Fig. 7-6(f) with \(ka\) of 1.3 in the perfectly reflecting waveguide.

Figure 7-8: Similar to Fig. 7-5 but for an upright rigid circular disk of (a) \(ka = 62.8\), and (b) \(ka = 1.3\). The width \(\lambda/L\) of the scatter function main lobe for broadside incidence ((90° - \(\alpha_t\)) = 0°) is 2.9° or 0.05 radians for \(ka = 62.8\) and 143° or 2.5 radians for \(ka = 1.3\).
the horizontal as can be seen by inspection of Fig. 7-2 so that the scatter function experiences a phase change about the horizontal. The difference between the scattered field and sonar equation for the two rotated cases is almost identical to the unrotated case shown in Figs. 7-2(a)-(c). The variation across the cases is less than roughly 1-dB for ranges beyond 1 km, showing that the condition $2\Delta \psi < \lambda / L$, as shown in Fig. 7-2(d), holds regardless of object orientation. This makes sense because the condition states that the sonar equation will be a good approximation if the scatter function undergoes no more than one oscillation within $\pm \Delta \psi$. This minimizes the destructive interference possible in multimodal propagation and scattering.

There are two limiting object shapes for homogeneous convex objects, flat and rounded ones. Flat objects, such as the disk examined in the previous section, are highly directional scatterers when $ka$ is large. In free space, they scatter the strongest in specular and forward scatter directions. The sonar equation is only valid for flat objects when the condition $2\Delta \psi < \lambda / L$ holds, as demonstrated in the previous section for an upright disk and in this section for the rotated disk. We note that the scattered field level in the forward azimuth is identical to that given in the back azimuth in Figs. 7-2, 7-3 and 7-6 because the scatter function for the upright disk is anti-symmetric about the plane of the disk, as can be seen in Eq. (7.7) and Fig. 7-4. The phase of the scatter function is constant over the main lobe, as can be seen in Eq. 7.7. This constancy over the main lobe is characteristic of flat objects where the narrowest lobe is always the main lobe for broadside incidence.

The rigid square plate is another example of a flat object that behaves similar to the disk. Simulations for an upright rigid square plate of sides 20 m by 20 m at 300 Hz is shown in Figs. 7-9-7-11 for the three bottom types considered in previous sections. The scatter function for the rigid square plate can be obtained by applying Green's Theorem, and is found to be
Figure 7-9: Figures (a)-(c) are similar to Figs. 7-2(a)-(c) but for an upright rigid square plate of sides 20 m by 20 m at 300 Hz with $ka = 12.6$.

\begin{align}
S(\alpha, \beta, \alpha_i, \beta_i) &= \frac{i}{2\pi} k^2 L^2 \sin \alpha \cos \beta \text{sinc} \left[ \frac{kL}{2} (\sin \alpha_i \sin \beta_i - \sin \alpha \sin \beta) \right] \\
&\quad \times \text{sinc} \left[ \frac{kL}{2} (\cos \alpha_i - \cos \alpha) \right].
\end{align}

(7.8)

(7.9)

When the condition $2\Delta\psi < \lambda/L$ does not hold, the sonar equation can still be extremely accurate in non-forward scatter azimuths for spheres and certain other rounded or smoothly varying convex objects. For these objects, the scatter function
Figure 7-10: Similar to Fig. 7-4 but for an upright rigid square plate of $ka = 12.6$. The scatter function is anti-symmetric about the plane of the plate and as can be seen from Eq. (7.9).
Figure 7-11: Similar to Fig. 7-5 but for an upright rigid square plate of $ka = 12.6$. The solid curve in this figure for broadside incidence ($90^\circ - \alpha_i = 0^\circ$) is a slice through Fig. 7-10 at the backscatter azimuth $\beta = 180^\circ$ of the scattered planewaves. The width $\lambda/L$ of the scatter function main lobe for broadside incidence is $14.3^\circ$ or 0.25 radians.
is approximately uniform in non-forward directions but has a main lobe in the forward direction of width $\lambda/L$. This uniformity makes it possible to approximate the scatter function in non-forward azimuths as a factorable constant over $\pm \Delta \psi$ making the sonar equation valid. The sonar equation, however, will still only be valid in the forward scatter azimuth for spheres and rounded objects when $2\Delta \psi < \lambda/L$ holds because they behave like flat objects in forward scatter by Babinet's principle. These issues will be illustrated for a smoothly varying convex object, the sphere in the same three waveguides examined in previous sections. Babinet's principle will be discussed further in Sec. 7.3.

A expected, the sonar equation predicts the backscattered field much more accurately than the forward scattered field for the sphere as shown in Fig. 7-12 for the various waveguides. This can be explained by examining the scatter function of a sphere, given in Ref. [5] and plotted in Figs. 7-13 and 7-14. Figure 7-14(a) shows that the magnitude of the scatter function in the backscatter azimuth for the sphere is approximately constant over $\pm \Delta \psi$ in the Pekeris silt and sand waveguides, where $\Delta \psi$ can be determined from Fig. 7-2(d). Figure 7-14(b) shows that the phase of the scatter function varies by less than $\pi/4$ over $\pm \Delta \psi$ and so can be considered effectively constant. This means that the scatter function can be factored from the modal sum in Eq. (2.1) so that propagation is decoupled from scattering. The sonar equation then becomes valid in the backscatter direction for the sphere in the Pekeris sand and silt waveguides.

In the perfectly reflecting waveguide, $\Delta \psi$ is $\pi/2$ rads or $90^\circ$. In this case, some higher order modes at very steep incident grazing angles near $90^\circ$, for example, scatter much stronger in the backscatter azimuth than the other modes of lower order that scatter more uniformly, as shown in Fig. 7-14(a). The phase of the scatter function also varies by more than $180^\circ$ over $\Delta \psi$. As a result, scattering is not completely de-coupled from propagation and so the sonar equation provides only a crude approximation in the backscatter azimuth as shown in Fig. 7-12(e).

In the forward azimuth, the sphere scatters approximately as a flat object with the same projected area, by Babinet's principle, and so behaves like the disk of Fig.
7-2. Example of another rounded convex object, the prolate spheroid is given in Ref. [63].

The sonar equation can sometimes under-estimate the scattered field depending on the object type, size, waveguide and frequency. This is examined in Ref. [63].

7.2.4 Non-Convex Objects and Fluctuating Objects

The scatter function lobes for homogeneous convex objects have minimum widths limited to $\lambda/L$ by diffraction. For more general, potentially inhomogeneous and non-convex objects, the scatter function lobes are limited by both diffraction and interference from different parts of the object. For example, a situation may arise where it is convenient to consider an aggregate of disjoint scatterers as a "single object" with a single scatter function. In the limiting case of two point scatterers separated by length $L$ normal to the incident wave, the main lobe of this "single object's" scatter function has the narrowest width possible $\lambda/(2L)$. The condition for the sonar equation to be come valid in a waveguide then is the most stringent $2A0 < \lambda/2L$.

The scatter function for a "single object" comprised of two point scatterers separated in depth by length $L$ is $2\cos(\frac{\pi L}{\lambda}[\cos \alpha - \cos \alpha])$ if the scatter function of each point is unity. The field scattered from this "single object" is given in Figs. 7-15 (a)-(b) for point separations of $L = 2$ m and $L = 10$ m respectively at 1500 Hz using both the sonar equation and waveguide scattering model. For $L = 2$ m the sonar equation is valid and matches the waveguide model because the condition $2A0 < \lambda/2L$ is satisfied as can be seen in Fig. 7-15 (c). This condition is not satisfied for the $L = 10$ m case where the sonar equation is not valid as can be seen in Figs. 7-15 (b) and (c).

For an object whose orientation is unknown or constantly varying, like a fluctuating target, a conservative criterion for the sonar equation's validity in a waveguide is $2A0 < \lambda/2L_{max}$ where $L_{max}$ is the largest spatial extent of the object, which may be composed of an aggregate of disjoint scatterers.
Figure 7-12: Similar to Figs. 7-2 (a)-(c) but for a 10-m radius pressure-release sphere at 300-Hz in the (a) back and (b) forward azimuths in Pekeris silt waveguide, the (c) back and (d) forward azimuths in Pekeris sand waveguide, and the (e) back and (f) forward azimuths in the perfectly reflecting waveguide. $ka$ is 12.6 for this example.
Figure 7-13: Similar to Fig. 7-4 but for a pressure-release sphere of $ka = 12.6$. The scatter function for the sphere is given by Eqs. (8,9) of Ref. [42] with $f(n)$ replaced by $(-1)^n f(n)$ to convert from Ingenito's definition to the standard one described in Ref. [48]. It can also be obtained from Ref. [5].
Figure 7-14: Figures (a) and (c) are similar to Fig. 7-5 but for a pressure-release sphere of $ka = 12.6$ plotted in the back ($\beta = 180^\circ$) and forward ($\beta = 0^\circ$) azimuths respectively of the scattered planewaves. The solid curves in Fig. 7-14 (a) and (c) for $(90^\circ - \alpha_i) = 0^\circ$ are slices through Fig. 7-13 in the back and forward scatter azimuths. The width $\lambda/L$ of the scatter function main lobe is $14.3^\circ$ or 0.25 radians in Fig. 7-14(c). The phase of the scatter function for the solid curve (horizontal incidence, $(\alpha_i - 90^\circ) = 0^\circ$) in (a) and (b) is shown in (b) and (d) respectively which are in the back and forward scatter azimuths.
Figure 7-15: Similar to Fig. 7-2(a) but for a "single object" comprised of two point scatterers with separation of (a) $L = 2$ m and (b) $L = 10$ m respectively at 1500Hz. (c) The ratio $2\Delta\psi/(\lambda/2L)$ for the examples given in (a) and (b). The sonar equation provides a good approximation to the scattered field in the waveguide when $2\Delta\psi/(\lambda/2L) < 1$. 

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7.2.5 Forward Scatter Function and Projected Area for Homogenous Convex Objects

According to the extinction or forward scatter theorem in free space,[75, 4, 65] the scatter function of an object in the forward direction at a given frequency is proportional to the object’s projected area normal to the direction of propagation of the incident wave for high $ka$. The projected area of the 10-m radius rigid circular disc is largest for planewaves incident on the disc at horizontal grazing $(\pi/2 - \alpha_i) = 0)$ as shown in Figs. 7-5 and 7-8(a). At other incident grazing angles, the projected area of the disc is smaller and the peak value of the scatter function corresponding to forward scatter decreases. In addition the width of the forward scatter lobe broadens since it is inversely related to the projected area. For a compact pressure-release object ($ka \ll 1$) the scatter function becomes omni-directional.

For the 10-m radius pressure-release sphere in Fig. 7-14(c), the forward scatter function’s peak is constant because the projected area of the sphere is independent of the angle of incidence.

7.3 Conditions for Babinet’s Principle to Become Valid in a Waveguide and Forward Scattered Field

Babinet’s principle maintains that the forward scattered fields from impenetrable objects in free space with identical projected areas are equal for large $ka$.[75, 4] This also holds true for some penetrable objects.[75] The forward scattered field from a large object in free space interferes destructively with the incident field to form a shadow directly behind the object. In the far field, the intensity of the forward scattered field is the same pattern as that diffracted through a hole of the same projected area as the object in a rigid wall. The forward scattered field then only depends on the projected area of the object. It is therefore possible to replace the
3D object by the largest 2D cross section of the object normal to the incident wave. This is Babinet's principle. For example, in free space, a sphere and a circular disc of the same radius, with the disc aligned normal to the incident wave vector, have the same projected area and hence identical forward scattered fields when \( ka \gg 1 \).

We find that Babinet's principle is approximately valid in forward \( \text{azimuth} \) in a waveguide if the projected area of the object does not vary significantly for incident planewaves over \( \pm \Delta \psi \). We stress that this holds in the forward \( \text{azimuth} \) because there is generally no unique forward direction in a waveguide. This result is illustrated in Figs. 7-16-7-18 for a sphere and a disc and in Ref. [63] for a spheroid and a disc. The projected areas for an upright disc, sphere and prolate spheroid are shown as a function of horizontal grazing angle in Fig. 7-16. The values of \( \Delta \psi \) over range can be determined from Fig. 7-2(d) for the sand and silt waveguides, and are typically less than 30°, where the variation in projected area is small for all three objects. For the perfectly reflecting waveguide, the variation in projected area is very large for all three objects since \( \Delta \psi \) is 90°. Babinet's principle is then expected to be valid for these objects in the silt and sand waveguides at high \( ka \), but not in the perfectly reflecting waveguide. This is found to be the case in Figs. 7-17-7-18, where in Fig. 7-17 \( ka \) is not large enough for Babinet's principle to provide better than a crude approximation even in free-space.

When Babinet's principle holds in a waveguide, the sonar equation will only be valid in the forward scatter azimuth if the condition \( 2\Delta \psi < \lambda/L \) is satisfied regardless of object shape. This means that rounded objects such as spheres have no advantage over flat objects in attaining the sonar equation in the forward azimuth in a waveguide.

### 7.4 Summary

As a general conclusion, we find that the sonar equation is valid when the target's scatter function is roughly constant over the equivalent horizontal grazing angles \( \pm \Delta \psi \) spanned by the dominant waveguide modes. This is approximately true (1) for all objects when \( 2\Delta \psi < \lambda/2L \) and (2) for spheres and certain other rounded objects in
Figure 7-16: Projected areas of an upright 10-m radius rigid circular disc, a 10-m radius pressure-release sphere and a pressure-release prolate spheroid of aspect ratio 2 with a major axis of 40-m and a minor axis 0f 20-m are plotted as a function of horizontal grazing angle $|\alpha - 90^\circ|$ of the incident planewaves. The spheroid is aligned such that the incoming planewaves are incident at the bow aspect where $ka = 12.6$. 
Figure 7-17: The forward scattered field at 300 Hz calculated using the waveguide scattering model, Eq. (2.1), from a 10-m radius pressure release sphere is compared to that from a rigid circular disc of radius 10-m in (a) Pekeris silt, (b) Pekeris sand, and (c) perfectly reflecting waveguides. $ka$ is 12.6 for these examples. The geometry of the set-up is similar to Figs. 7-2 and 7-12, except that the receiver is in the forward azimuth.
Figure 7-18: Similar to Fig. 7-16 but at 1500 Hz. $ka$ is 62.8 for these examples.
non-forward scatter azimuths even when (1) does not hold. For homogeneous convex objects condition (1) is the less stringent $2\Delta\psi < \lambda/L$.

The sonar operator then has the ability to lower the frequency of transmission until the target's scatter function becomes approximately constant over $\pm\Delta\psi$. The sonar equation then becomes valid when $f < c/(4L\Delta\psi)$. Operating in this frequency regime is desirable because when the sonar equation is valid, only a single-parameter, target strength, is necessary to characterize the scattering properties of the target. This greatly simplifies target classification in a shallow water waveguide by making the traditional approach[73, 11] valid.

We find that Babinet's principle is approximately valid in forward azimuth in a waveguide if the projected area of the object does not vary significantly for incident planewaves over $\pm\Delta\psi$. When Babinet's principle holds in a waveguide, the sonar equation will only be valid in the forward scatter azimuth if the condition $2\Delta\psi < \lambda/L$ is satisfied regardless of object shape. This means that rounded objects such as spheres have no advantage over flat objects in attaining the sonar equation in the forward azimuth in a waveguide.
Chapter 8

Average Scatter Function of Convex Objects with Unknown or Fluctuating Orientation

In this Chapter, we examine the scattering from objects with unknown or fluctuating orientation. We use a rigid square plate of length $L$ as the object whose orientation is unknown or fluctuating. When the plate is rotated in the plane perpendicular to its surface normal in 2D, a circular disc is generated. When the plate is rotated in 3D over all orientation and spin angles, a sphere is generated.

In Sec. 8.1, the bi-directional scatter function of the rigid square plate is expressed in terms of the incident and scattered planewave components. We discuss the 2D rotation of the plate in Sec. 8.2 and compute the coherent and incoherent average scatter function of the plate numerically. The results are compared with that of a circular disc. In Sec. 8.3, the plate is rotated in 3D making use of Rodrigues formula and rotation matrices.[28] The coherent and incoherent average scatter function from the 3D rotation are compared with that of a sphere.
Figure 8-1: Square plate of sides $L$ lying in the $x-y$ plane with surface normal to the $z$ axis and sides parallel to the $x$ and $y$ axis. Also drawn are the elevation $\alpha$ and azimuth $\beta$ angles of a scattered planewave with wavevector $\mathbf{k}$.

8.1 Bi-Directional Scatter Function of a Rigid Square Plate

The bi-directional scatter function of an object for a given plane wave incidence can be calculated using Green’s theorem[53, 19, 15] For a rigid square plate, we use the fact that the normal derivative of the total field has to vanish on the surface of the plate in Green’s theorem to find the planewave scatter function which is valid in the far-field. Here we present the results for a rigid square plate.

We let the origin of the coordinate system coincide with the center of the square plate as shown in Fig. 8-1. For a rigid square plate of length $L$ in the x-y plane with its surface normal to the z-axis and sides parallel to the x and y axis, the resulting scatter function calculated using Green’s theorem and expressed in terms of the incident and scattered wavenumber components is
\[ S(\alpha, \beta, \alpha_i, \beta_i) = -\frac{j}{2\pi} (k_z L) (k L) \text{sinc} \left( \frac{L}{2} (k_z - k_z) \right) \text{sinc} \left( \frac{L}{2} (k_{yi} - k_y) \right), \quad (8.1) \]

where the wave-vector \( \mathbf{k}_i = (k_{xi}, k_{yi}, k_{zi}) \) is in the direction of the incident planewave and the wave-vector \( \mathbf{k} = (k_x, k_y, k_z) \) is in the direction of the scattered planewave. We can express the wavenumber components in terms of the elevation \( \alpha \) and azimuth \( \beta \) angles for the incident and scattered planewaves respectively as

\[
\begin{align*}
  k_{xi} &= k \sin \alpha_i \cos \beta_i, & k_{yi} &= k \sin \alpha_i \sin \beta_i, & k_{zi} &= k \cos \alpha_i, \\
  k_x &= k \sin \alpha \cos \beta, & k_y &= k \sin \alpha \sin \beta, & k_z &= k \cos \alpha.
\end{align*}
\]

(8.2)

The scatter function formula in Eq. 7.9 is for an upright rigid square plate whose surface is normal to the x-axis and is expressed in terms of the elevation \( \alpha \) and azimuth \( \beta \) angles. From Figs. 7-10 and 7-11 for the upright rigid square plate, we observe that the rigid square plate scatters the strongest in the forward and specular directions as discussed in Sec. 7.2.3

### 8.2 2D rotation of the Rigid Square Plate

Suppose we rotate the square plate by \( \tau \) radians about the z-axis as shown in Fig. 8-2. We then define a new coordinate system \( x', y', z' \) such that \( z' = z \), and \( x' \) and \( y' \) are now parallel to the edges of the square plate. \( x' \) and \( y' \) are rotated from \( x \) and \( y \) respectively by \( \tau \) radians. We express the incident and scattered planewaves relative to the new coordinate system as

\[
\begin{align*}
  k'_{xi} &= k \sin \alpha_i \cos (\beta_i - \tau), & k'_{yi} &= k \sin \alpha_i \sin (\beta_i - \tau), & k'_{zi} &= k_{zi} = k \cos \alpha_i, \\
  k'_{x} &= k \sin \alpha \cos (\beta - \tau), & k'_{y} &= k \sin \alpha \sin (\beta - \tau), & k'_{z} &= k_{z} = k \cos \alpha.
\end{align*}
\]

(8.3)
Figure 8-2: Geometry for 2D rotation. Square plate of sides $L$ lying in the $x-y$ plane with surface normal to the $z$ axis. The plate is rotated by $\tau$ radians about the $x$ axis in the $x-y$ plane.

The scatter function of the rotated square plate becomes

$$S(\alpha, \beta, \alpha_i, \beta_i, \tau) = -\frac{j}{2\pi}(k_x' L)(k L) \text{sinc} \left( \frac{L}{2}(k_x' - k_x) \right) \text{sinc} \left( \frac{L}{2}(k_y' - k_y) \right), \quad (8.4)$$

The incoherent 2D average scatter function is calculated using

$$\overline{S}_{\text{incoh,2D}}(\alpha, \beta, \alpha_i, \beta_i) = \left[ \frac{1}{2\pi} \int_0^{2\pi} |S(\alpha, \beta, \alpha_i, \beta_i, \tau)|^2 d\tau \right]^{1/2}, \quad (8.5)$$

while the coherent 2D average scatter function is calculated using

$$\overline{S}_{\text{coh,2D}}(\alpha, \beta, \alpha_i, \beta_i) = \frac{1}{2\pi} \int_0^{2\pi} S(\alpha, \beta, \alpha_i, \beta_i, \tau) d\tau. \quad (8.6)$$

We compare the average scatter function of the 2D rotated square plate with that of a rigid circular disc. The scatter function of a rigid circular disc of diameter $L$ lying
in the x-y plane with the normal in the z direction can be calculated using Green's theorem to be[15]

\[ S(\alpha, \beta, \alpha_i, \beta_i) = -Q \frac{j}{2} (k_z L/2)(kL/2) \text{circ} \left[ P \frac{L}{2} \sqrt{(k_{xi} - k_x)^2 + (k_{yi} - k_y)^2} \right] \] (8.7)

where \( \text{circ}[x] = \frac{2J_1(x)}{x} \) and \( Q = P = 1 \). We may need to scale and skew the scatter function of the circular disc of diameter \( L \) to match with the average scatter function of the square plate. We introduce the parameters \( Q \) and \( P \) which we adjust to obtain as good a fit as possible between the incoherent and coherent average scatter function of the square plate with that of the circular disc.

The 2D rotation of the square plate was implemented using Matlab. The integration over the rotation angle \( \tau \) was calculated using the quad function in Matlab with a tolerance of 0.05 specified for the integration accuracy.

Figure 8-3(a)-(c) plots the incoherent 2D average scatter function of the square plate for \( kL/2 = 1, kL/2 = 10 \) and \( kL/2 = 20 \) respectively. \( kL/2 = \pi L/\lambda \) is a measure of the size of the plate relative to the wavelength of the incident planewave. Figure 3(a)-(c) shows the coherent 2D average scatter function of the square plate for \( kL/2 = 1, kL/2 = 10 \) and \( kL/2 = 20 \) respectively. The results are plotted in decibel scale where the scatter function magnitude is defined as \( 20 \log_{10} |S_{\text{incoh},2D}(\alpha, \beta, \alpha_i, \beta_i)| \) for the incoherent case and is similarly defined for the coherent average, as well as for the circular disc.

The scatter function of the circular disc was scaled such that the peak and first minima of its main lobe coincided with that of the incoherent and coherent 2D average scatter function of the square plate. The scatter function of the circular disc in Figs. 8-3 and 8-4 are plotted for \( Q = 4/\pi \) and \( P = 1.6^{1/4} \) in Eq. (8.7). The value of \( Q \) is obtained from matching the peak of the scatter function main lobes by equating the two scatter function formulas for \( \alpha_i = \alpha = \beta_i = \beta = 0 \). \( P \) is found by curve-fitting to match the location of the first minima in the main lobe. Figure 8-5 shows a comparison of the scatter function for other angles of incidence of the incoming
Figure 8-3: Incoherent 2D average scatter function of the rigid square plate is compared to that of the rigid circular disc for (a) $kL/2 = 1$, (b) $kL/2 = 10$ and (c) $kL/2 = 20$ where $\alpha_i = \beta_i = \beta = 0$. The square plate is rotated in 2D about the axis normal to the surface of the plate, generating the shape of a circular disc in the process. The incoherent average scatter function for the square plate is obtained using Eq. 8.5. The circular disc scatter function is plotted in the figure using Eq. 8.7 with $Q = 4/\pi$ and $P = 1.6^{1/4}$. 
Figure 8-4: Similar to Fig. 8-3 but for the coherent average of the scatter function of the square plate in 2D calculated using Eq. 8.6.
Figure 8-5: Similar to Figs. 8-3 and 8-4 except that the (a) incoherent and (b) coherent 2D average scatter function of the rigid square plate is compared to that of the rigid circular disc for $kL/2 = 10$ where the angle of incidence is $\alpha_i = \pi/4$ and $\beta_i = \beta = 0$ for this case.

For a square plate that is small compared to the wavelength, i.e. with small $kL/2$, the incoherent and coherent 2D average scatter function of the square plate coincide well with that of the circular disc. When the plate size gets larger compared to the wavelength, large $kL/2$, the 2D average scatter function of the square plate coincides with that of the disc only for the main lobe. The side lobes do not coincide and the mismatch increases $kL/2$ gets larger.

### 8.3 3D rotation of the Rigid Square Plate

In order to rotate the rigid square plate in 3D space, we first define the orientation of the square plate using an axis vector $\mathbf{w}$ and a spin angle $\tau$ as shown in Fig. 8-6. The axis $\mathbf{w} = (w_x, w_y, w_z) = (\sin \chi \cos \eta, \sin \chi \sin \eta, \cos \chi)$ is a unit vector which is specified by the elevation $\chi$ and azimuth $\eta$ angles. We define a new coordinate system $(x', y', z')$ that is centered on the square plate with the $x'$ and $y'$ axes parallel to the edges of the square plate and the $z'$ axis normal to the plate. The directions of the incident and scattered planewaves relative to the new coordinate system is found
Figure 8-6: Geometry for 3D rotation. Square plate of sides $L$ with surface normal to unit vector $\mathbf{\hat{w}}$. The plate is rotated by $\tau$ radians about the axis vector $\mathbf{\hat{w}}$.

using Rodrigues formula\[28, 27\]

\[k'_i = Rk_i = k_i \cos \tau + (1 - \cos \tau)(\mathbf{k}_i \cdot \mathbf{\hat{w}})\mathbf{\hat{w}} + \mathbf{\hat{w}} \times k_i \sin \tau\]

\[k' = R\mathbf{k} = \mathbf{k} \cos \tau + (1 - \cos \tau)(\mathbf{k} \cdot \mathbf{\hat{w}})\mathbf{\hat{w}} + \mathbf{\hat{w}} \times \mathbf{k} \sin \tau\] (8.8)

where $R$ is the rotation matrix given by

\[
R = \begin{bmatrix}
\cos \tau + (1 - \cos \tau)w_z^2 & (1 - \cos \tau)w_yw_x - w_z \sin \tau & (1 - \cos \tau)w_zw_x + w_y \sin \tau \\
(1 - \cos \tau)w_xw_y + w_z \sin \tau & \cos \tau + (1 - \cos \tau)w_y^2 & (1 - \cos \tau)w_zw_y - w_x \sin \tau \\
(1 - \cos \tau)w_xw_z - w_y \sin \tau & (1 - \cos \tau)w_yw_z + w_x \sin \tau & \cos \tau + (1 - \cos \tau)w_z^2
\end{bmatrix} \tag{8.9}
\]

The scatter function of the plate with orientation defined by $\mathbf{\hat{w}}$ and $\tau$ is
\[ S(\alpha, \beta, \alpha_i, \beta_i | \mathbf{w}, \tau) = S(\alpha, \beta, \alpha_i, \beta_i | \chi, \eta, \tau) \]
\[ = \frac{j}{2\pi} (k'_z L) (k L) \text{sinc} \left[ \frac{L}{2} (k'_{zi} - k'_z) \right] \text{sinc} \left[ \frac{L}{2} (k'_{yi} - k'_y) \right] \] (8.10)

The Jacobian for integration in 3D over all orientations is \( \frac{1}{2} \sin^2(\frac{\tau}{2}) \sin(\chi) \). A derivation for the Jacobian is provided in Appendix E. When we integrate the scatter function over all orientation angles, the normalization factor in computing the averages is

\[ \int_0^{2\pi} \int_0^{2\pi} \int_0^{\pi} \frac{1}{2} \sin^2(\frac{\tau}{2}) \sin(\chi) d\chi d\eta d\tau = 2\pi^2. \] (8.11)

The incoherent 3D average scatter function is obtained from

\[ \overline{S}_{\text{incoh, 3D}}(\alpha, \beta, \alpha_i, \beta_i) = \left[ \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^{\pi} |S(\alpha, \beta, \alpha_i, \beta_i | \chi, \eta, \tau)|^2 \sin^2(\frac{\tau}{2}) \sin(\chi) d\chi d\eta d\tau \right]^{1/2}, \] (8.12)

while the coherent 3D average scatter function is obtained from

\[ \overline{S}_{\text{coh, 3D}}(\alpha, \beta, \alpha_i, \beta_i) = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^{\pi} S(\alpha, \beta, \alpha_i, \beta_i | \chi, \eta, \tau) \sin^2(\frac{\tau}{2}) \sin(\chi) d\chi d\eta d\tau. \] (8.13)

We will compare the incoherent and coherent averaged scatter function of the square plate rotated in 3D with that of a pressure-release sphere of radius \( r \). The scatter function of the sphere is given by \([5, 46]\)
Figure 8-7: (a) Incoherent 3D average scatter function of the rigid square plate of \( kL/2 = 1 \) is plotted for \( \alpha_i = \beta_i = \beta = 0 \). The numerical integration was implemented using Eq. 8.11 for progressive increments of the angles \( d\chi, d\eta \) and \( d\tau \) of 2°, 4° and 8° to show the stability of the numerical integration. (b) The scatter function of a pressure-release sphere of \( kr = 1 \) calculated using Eq. 8.14.

\[
S(\alpha, \beta, \alpha_i, \beta_i) = \sum_{n=0}^{\infty} j(-1)^n(2n+1) \frac{j_n(kr)}{h_n^{(1)}(kr)} P_n[\cos \alpha \cos \alpha_i + \sin \alpha \sin \alpha_i \cos(\beta - \beta_i)],
\]

(8.14)

where \( P_n(x) \) is the associated Legendre functions.

The 3D rotation of the square plate was implemented in Fortran instead of Matlab as the triple integral over the orientation angles, \( \chi, \eta \) and \( \tau \) could be computed with much faster speed in Fortran.

Figure 8-7(a) shows the incoherent 3D average scatter function of the square plate for \( kL/2 = 1 \). The integration was done for different increments of the angles \( d\chi, d\eta \) and \( d\tau \) to show the stability of the numerical integration. Figure 8-7(b) plots the scatter function of a sphere of \( kr = 1 \). Except for some scale factors, Figs. 8-7(a) and (b) show similar trends of a strong forward scatter where \( \alpha = 0^\circ \) and weak backscatter where \( \alpha = 180^\circ \). Figure 8-8(a) and (b) plot the corresponding results for a square plate with \( kL/2 = 10 \) and a sphere of \( kr = 10 \). We again observe similar
Figure 8-8: Similar to Fig. 8-7 but for a rigid square plate of $kL/2 = 10$ and a pressure-release sphere of $kr = 10$.

trends in Figs. 8-8(a) and (b) of a strong forward scatter and a weak back scatter. Comparing Figs. 8-7 and 8-8, we see that the scale factor needed for the incoherent 3D average scatter function of the plate with different $kL/2$ to coincide with the scatter function of the sphere depends on the dimension of the plate and sphere relative to the wavelength.

Figure 8-9 plots the incoherent 3D average scatter function of the square plate for varying angles of the incident plane wave. The incoherent 3D average scatter function depends only on the cosine of the angle between the incident and scattered planewaves and the not the absolute value of the angles. This is a property that is exhibited by the scatter function of spherical scatterers. The incoherent 3D average scatter function of the square plate has similar characteristics as that of a spherical scatterer.

The coherent 3D average scatter function of the square plate of $kL/2 = 10$ for two different angles of incidence of the incoming plane wave is plotted in Fig. 8-10. The coherently 3D averaged scatter function of the square plate has strong forward and backscatter peaks, unlike a sphere. It also depends on the absolute angle of the incident and scattered planewaves and does not behave like a spherical scatterer. Comparing Figs. 8-9 with 8-10, we note that the coherently averaged scatter function
values are much smaller than the incoherently averaged ones. The theorem mentioned in Hulst[75] is therefore only applicable to incoherent 3D averages of the scatter function of a convex object.

8.4 Summary

The incoherent and coherent average scatter functions from the 2D and 3D rotations of a rigid square plate of various dimension compared to the wavelength $kL/2$ was calculated. For 3D rotation, we made use of Rodrigues formula to compute the scatter function for each orientation of the square plate in terms of an axis vector and a spin angle.

The numerical results show that for 3D rotation, except for some scale factors, the incoherent average scatter function of the square plate resembles that of a sphere. The incoherent 3D average scatter function of the square plate has a strong forward scatter, a weak backscatter and the scatter function magnitude depends only on the cosine of the angle between the incident and scattered planewaves and not on the absolute value of the angles of the plane waves. The coherent 3D average scatter function of the plate does not behave like that of a spherical scatterer.

For 2D rotations of the square plate, both the coherent and incoherent average of the square plate scatter function coincides with that of a circular disc for small dimensions of the plate relative to the wavelength. For large square plates, only the peak scatter function lobe of the coherent and incoherent 2D average scatter function coincides with that of the circular disc. The side lobes however show some differences.

This analysis is relevant to estimating the scattered field from objects whose orientation is unknown or fluctuating, e.g. a submarine. For such objects, we can treat their scatter function as a random variable and the expected scattered field can be calculated from the 2D and 3D averages of its scatter function.[48] In real applications, it may not be neccessary to take an average over all 3D or 2D rotations. The object may have some limited roll or yaw and the average can be restricted to a smaller rotation angle.
Figure 8-9: Incoherent 3D average scatter function of the rigid square plate of $kL/2 = 10$ is plotted for different elevation $\alpha_i$ and azimuth $\beta_i$ angles of the incident planewave as well as the azimuth angle $\beta$ of the scattered planewave. The incoherent 3D average scatter function depends only on the cosine of the angle between the incident and scattered planewaves and the not the absolute value of the angles.
Figure 8-10: Coherent 3D average scatter function of the rigid square plate of $kL/2 = 10$ is plotted for different elevation $\alpha_i$ and azimuth $\beta_i$ angles of the incident planewave. The coherent 3D average scatter function depends on the absolute angle of the incident and scattered planewaves and does not behave like a spherical scatterer.
For large and complicated objects with both convexities and concavities, the scatter function of such an object would have very complicated lobe pattern. The mean and variance of the scatter function for such objects may be obtained by treating them as Gaussian random particles which is a topic for further investigation.
Chapter 9

Conclusion

In this thesis, we investigated the remote imaging of the ocean environment with low frequency acoustic waves through theoretical model development, numerical simulation and field experiment. This work takes into account the coupling between propagation and scattering that occurs in ocean waveguides with multi-modal field structure, such as in the continental shelf environment.

Through our theoretical work[48], we unified deterministic and stochastic scattering in waveguides. The conditions necessary for scattering in a waveguide to become diffuse were explicitly derived. Our model also accounts for coherent “reverberation rings” frequently observed in high-resolution sonar systems. This is the first time in ocean waveguide acoustics that it is possible to analyze both scattering from targets and reverberation from the environment using a consistent theory developed directly from first principles, namely Green's Theorem.

We showed with both theory and experiment that acoustic waves that either “tunnel” or propagate through the sediment can scatter from extended sub-bottom geologic features[48, 64]. This scattering is both coherent and deterministic, and can stand significantly above incoherent or diffuse seafloor reverberation. This proves that sub-bottom and seafloor geologic features are a major source of clutter in long range active sonar systems that search for submerged objects such as underwater vehicles or marine mammals. Broadband waveforms are frequently used to improve signal-to-noise and signal-to-diffuse-reverberation ratios via match-filtering in the detection
of coherent scatterers such as underwater vehicles. Since scattering from extended geologic features is also coherent, broadband waveforms lead to enhanced geologic clutter for sonar systems that search for underwater vehicles such as submarines[66].

Through our field work, we developed a technique to rapidly and remotely image sub-bottom and seafloor features over tens of kilometers in shallow water waveguides.[64] Our method provides substantial time-saving in acquiring information about underwater geomorphology. Current geophysical survey techniques on the other hand have to comb through an area to be surveyed in a “lawn-mower” fashion thus requiring much longer time duration for data collection.

We derived a theorem for calculating the power removed from the multi-modal incident field by objects in a waveguide. The size of an object in a waveguide can be estimated from the total field in the forward scatter direction of the object through the generalized extinction theorem. This theorem can be applied to design harbour surveillance systems for intruder detection and classification.[65] We also derived analytic expressions for the attenuation and dispersion in the forward propagated field due to scattering from surface and volume inhomogeneities in a waveguide. This result can be applied to study the effect of scattering from internal waves and seafloor roughness in the forward propagated field in shallow waters.

We showed that the most widely used tool for underwater acoustic analysis, the sonar equation, is in general not valid for waveguide scattering. The conditions necessary for the sonar equation to become approximately valid in a shallow water waveguide were derived. It was shown that the sonar equation may be made valid in a waveguide by lowering the active frequency of operation sufficiently for the given measurement scenario. This is often desirable because it greatly simplifies the analysis necessary for target classification and localization.[63]

In conclusion, this work shows that a long range sonar system can be deployed like an “eye” underwater to image, detect, and classify objects and features in continental shelf waters.
Appendix A

Reciprocity, Planewave Tracking Rule and Deterministic Object Scattering in Free Space and in a Waveguide

In this appendix, we discuss how to track a planewave in both free space and in a waveguide. We show that as a consequence of reciprocity of Greens function, the direction of a planewave at a point in space cannot be unambiguously specified using just the Green function exponential expression. Additional knowledge of whether the spatial point corresponds to a source or receiver point is necessary to uniquely identify the direction of a planewave. We develop a planewave tracking rule and apply it in the derivation of the model for deterministic object scattering in both free space and the waveguide. We show how to track the coefficients of the up and down going planewaves incident on and scattered from a target in both free space and in a waveguide. This analysis is neccessary to correctly identify the planewave coefficients to be used in formulating a model for the scattering from a patch of seafloor or sea surface in a waveguide which will be discussed in Chapter 2.
A.1 Consequence of Reciprocity

We use the geometry for planewaves and field points in space similar to that described in Ref. [46] with the positive z axis pointing downward as shown in Fig. A-1. A planewave \( \mathbf{k} \) in free space is uniquely defined by its magnitude \( k \) and direction \((\alpha, \beta)\) where \( \alpha \) is the elevation angle and \( \beta \) is the azimuth angle, i.e. \( \mathbf{k} = (k, \alpha, \beta) \). A point in space \( \mathbf{r} \) is at a distance \( r \) away from the origin in the direction \((\theta, \phi)\) where \( \theta \) is the elevation angle and \( \phi \) the azimuth angle, i.e. \( \mathbf{r} = (r, \theta, \phi) \). The dot product between the wave vector \( \mathbf{k} \) and the spatial vector \( \mathbf{r} \) is

\[
\mathbf{k} \cdot \mathbf{r} = kr \eta(\alpha, \beta, \theta, \phi) \tag{A.1}
\]

where \( \eta(\alpha, \beta, \theta, \phi) \) is the cosine of the angle between the wave vector and the spatial vector,

\[
\eta(\alpha, \beta, \theta, \phi) = \cos \alpha \cos \theta + \sin \alpha \sin \theta \cos(\beta - \phi). \tag{A.2}
\]
Consider two spatial points \( r_1 = (r_1, \theta_1, \phi_1) \) and \( r_2 = (r_2, \theta_2, \phi_2) \) and the two scenarios I and II illustrated in Fig. A-2. First consider scenario I where a planewave \( k \) travels in the direction from \( r_1 \) to \( r_2 \). \( r_1 \) is the source point where the planewave originates and \( r_2 \) is the receiver point. Making use of the free space Green function, the field at \( r_2 \) can be expressed as

\[
G(r_2|r_1) = \frac{1}{4\pi|r_2-r_1|} e^{i\mathbf{k} \cdot (r_2-r_1)} = \frac{1}{4\pi|\mathbf{r}_2-\mathbf{r}_1|} e^{i\mathbf{k} \cdot (r_2-r_1)} \\
= \frac{1}{4\pi|\mathbf{r}_2-\mathbf{r}_1|} e^{i(k r_2(\alpha,\beta,\phi_2)-kr_1(\alpha,\beta,\phi_1))}. \tag{A.3}
\]

Here, the planewave vector \( \mathbf{k} = (k, \alpha, \beta) \) points in the direction from \( r_1 \) to \( r_2 \) and is therefore parallel to the vector \( (r_2 - r_1) \). The product of \( e^{-i2\pi f t} \) and the right hand side of Eq. A.3 yields the time-harmonic field, so that \( \Re\{G(r_2|r_1)e^{-i2\pi f t}\} \) is a wave propagating from \( r_1 \) to \( r_2 \).

Now consider scenario II where a planewave \( \mathbf{k}' = -\mathbf{k} \) travels in the direction from \( r_2 \) to \( r_1 \). Now \( r_2 \) is the source point where the planewave originates and \( r_1 \) is the receiver point. The free space Green function describing the field at \( r_1 \) is
Here, the planewave vector \( k' = (k, \pi - \alpha, \beta + \pi) \) points in the direction from \( r_2 \) to \( r_1 \) and is therefore parallel to the vector \( \mathbf{r}_1 - \mathbf{r}_2 \).

We observe that \( G(\mathbf{r}_2|\mathbf{r}_1) = G(\mathbf{r}_1|\mathbf{r}_2) \), i.e. the Green function is invariant to an exchange in source and receiver locations. The Green function is symmetric in \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) which satisfies the principle of reciprocity. It is noteworthy that the planewaves in scenarios I and II are travelling in opposite directions and are therefore two distinct planewaves. The same exponential is used to describe scenarios I and II in Fig. A-2 as a result of reciprocity. This implies that the Green function exponentials cannot specify the direction of a planewave uniquely. To identify the direction of a planewave from the Green function exponentials, we need to specify the source and receiver locations.

\[ G(\mathbf{r}_1|\mathbf{r}_2) = \frac{1}{4\pi|\mathbf{r}_2 - \mathbf{r}_1|} e^{ik'(|\mathbf{r}_1 - \mathbf{r}_2|)} = \frac{1}{4\pi|\mathbf{r}_2 - \mathbf{r}_1|} e^{ik'((\mathbf{r}_1 - \mathbf{r}_2))} = \frac{1}{4\pi|\mathbf{r}_2 - \mathbf{r}_1|} e^{ik'((\mathbf{r}_1 - \mathbf{r}_2))} = \frac{1}{4\pi|\mathbf{r}_2 - \mathbf{r}_1|} e^{i(\mathbf{k}_r \eta(\alpha, \beta, \theta_2, \phi_2) - \mathbf{k}_r \eta(\alpha, \beta, \theta_1, \phi_1))}. \] (A.4)

A.2 Planewave Tracking Rule

In free space, consider the field from a point source at location \( \mathbf{r}_0 = (r_0, \theta_0, \phi_0) \) incident on a target centered at the origin as shown in Fig. A-3. Let the coordinates of a point on the target surface be \( \mathbf{r}_t = (r_t, \theta_t, \phi_t) \). For \( r_0 \gg r_t \) the field incident on the target is approximately planar and we let the incident planewave vector be \( \mathbf{k}_i = (k, \alpha_i, \beta_i) \). The incident planewave direction \( \alpha_i \) and \( \beta_i \) is related to the point source direction \( \theta_0 \) and \( \phi_0 \) by \( \alpha_i = \pi - \theta_0 \) and \( \beta_i = \phi_0 + \pi \). Moreover, the dot product between the incident planewave vector and the source coordinate is \( \mathbf{k}_i \cdot \mathbf{r}_0 = kr_0 \eta(\alpha_i, \beta_i, \theta_0, \phi_0) = -kr_0 \). The field incident on the target at \( \mathbf{r}_t \) can be expressed using the free space Green function as
Figure A-3: A planewave $k_i$ is incident on the target from a far-field source $r_0$. The target scatters the field $k$ to a far-field receiver $r$. This figure is used to derive the tracking rule for a fixed planewave when $k_i = k$.

\[
G(r_t|r_0) = \frac{1}{4\pi} \frac{e^{ik|r_t-r_0|}}{|r_t-r_0|} \approx \frac{1}{4\pi r_0} e^{ikr_0} e^{ikr_t} (\alpha, \beta, \theta, \phi).
\]  

(A.5)

The target scatters the signal in all directions. At a receiver $r = (r, \theta, \phi)$ sufficiently far from the object, the scattered field at the receiver for $r \gg r_t$ is approximately planar. Let the scattered planewave vector be $k = (k, \alpha, \beta)$. The scattered planewave direction $\alpha$ and $\beta$ is related to the receiver direction $\theta$ and $\phi$ by $\alpha = \theta$ and $\beta = \phi$. The dot product between the scattered planewave vector and the receiver coordinate is $k \cdot r = k r \eta(\alpha, \beta, \theta, \phi) = k r$. The field scattered from a target point $r_t$ to the distant receiver can be expressed using the free space Green function,
To track a fixed planewave incident on and scattered from the target point, we let $k_i = k$ where $\alpha_i = \alpha$ and $\beta_i = \beta$ in Eqs. A.5 and A.6. In Eq. A.5 for the incident field, the target point $r_t$ behaves like a receiver and the exponential expression contains a term that is a dot product between the wavevector and the target point coordinates. In Eq. A.6 for the scattered field, the target point $r_t$ behaves like a source and the exponential expression contains a term that is the negative of the dot product between the wavevector and the target point coordinates.

This leads to a general rule for tracking a planewave direction at a point in space. For a receiver point, we can identify the planewave direction from the positive dot product of the unique wave vector and the spatial vector of the point in the exponential term containing the spatial point coordinates. For a source point, we identify the planewave direction from the negative dot product of the unique wave vector with the spatial vector of the point in the exponential term containing the spatial point coordinates. This rule explains how the direction of the planewaves in scenarios I and II in Sec. A.1 can be obtained from the Green function exponentials.

\[ G(r|r_t) = \frac{1}{4\pi |r - r_t|} e^{ik|r - r_t|} = \frac{1}{4\pi |r - r_t|} e^{ik(r - r_t)} \]

\[ \approx \frac{1}{4\pi r} e^{ikr} e^{-ikr_t} = \frac{1}{4\pi r} e^{ikr} e^{-ikr_t\eta(\alpha, \beta, \theta, \phi)} \]  

(A.6)

A.3 Sonar Equation Model for Deterministic Object Scattering in Free Space

Here we derive the active sonar equation model which describes object scattering in free space from first principles using Green's Theorem and some steps from from Appendix A of Ref. [48]. Keeping in mind that the object centroid is at the origin of the coordinate system, the harmonic field scattered by an object and measured at receiver location $r$ can be expressed using the Helmholtz-Kirchoff integral equation[53]
as

$$
\Phi_s(r) = -\iint_{A_t} \left\{ \left[ \Phi_i(r_t) + \Phi_s(r_t) \right] \frac{\partial G(r \mid r_t)}{\partial n_t} - G(r \mid r_t) \frac{\partial}{\partial n_t} \left[ \Phi_i(r_t) + \Phi_s(r_t) \right] \right\} dA_t,
$$

(A.7)

where $G(r \mid r_t)$ is the medium’s Green function and $\Phi_i(r_t)$ is the incident field on the surface of the object at $r_t$, each satisfying the Helmholtz equation driven by a source at angular frequency $\omega = 2\pi f$. The area integral encloses the scatterer and the surface normal points into the enclosed volume.

The field incident on the object surface at $r_t$ can be obtained from Eq. A.5 as

$$
\Phi_i(r_t) = A \frac{1}{r_0} e^{ikr_0} e^{ikr_0(\alpha_i, \beta_i, \theta_i, \phi_i)}
$$

(A.8)

where $A$ is the source amplitude. For a narrowband source, $A \approx Q(f) df$ where $Q(f)$ is the source spectrum. Substituting Eq. A.8 for the incident field and Eq. A.6 for the Green function into Eq. A.7, we obtain

$$
\Phi_s(r) = -A e^{ikr_0} \frac{e^{ikr}}{r_0 4\pi r} \iint_{A_t} \left\{ \left[ e^{ikr_0(\alpha_i, \beta_i, \theta_i, \phi_i)} + \frac{r_0}{A} e^{-ikr_0} \Phi_s(r_t) \right] \frac{\partial}{\partial n_t} e^{-ikr_0(\alpha, \beta, \theta, \phi)} - e^{-ikr_0(\alpha, \beta, \theta, \phi)} \frac{\partial}{\partial n_t} \left[ e^{ikr_0(\alpha_i, \beta_i, \theta_i, \phi_i)} + \frac{r_0}{A} e^{-ikr_0} \Phi_s(r_t) \right] \right\} dA_t.
$$

(A.9)

By definition of the planewave scatter function $S(\alpha, \beta, \alpha_i, \beta_i)$, however, Eq. (A.9) can also be written as

$$
\Phi_s(r) = A \frac{e^{ikr_0} e^{ikr}}{r_0} \frac{S(\alpha, \beta; \alpha_i, \beta_i)}{k}
$$

(A.10)
in an object-centered coordinate system, which leads to the equality

$$S(\alpha, \beta; \alpha_i, \beta_i) = -\frac{k}{4\pi} \int_A \int \left\{ e^{i kmr(\alpha, \beta, \theta, \phi_t)} + \frac{r_0}{A} e^{-ikro} \Phi_s(r_t) \right\} \frac{\partial}{\partial n_t} e^{-ikrn(\alpha, \beta, \theta, \phi_t)} dA_t,$$

that relates Eq. (A.9) directly to Green's theorem when $r, r_0 \gg r_t$. Using the free space Green function, Eqs. (A.5) and (A.6), we can write Eq. (A.10) as

$$\Phi_s(r) = A(4\pi)^2 G(0|r_0)G(r|0) \frac{S(\alpha, \beta; \alpha_i, \beta_i)}{k}.$$  \hspace{1cm} (A.12)

Equation (A.10) can be recast as a sonar equation by taking $10 \log$ of the squared magnitude of both sides,

$$10 \log \left( \frac{|\Phi_s(r)|^2}{P_{ref}^2} \right) = SL - TL(0|r_0) + TS - TL(r|0),$$  \hspace{1cm} (A.13)

where $P_{ref} = 1 \mu Pa$, $r_{ref} = 1 \text{ m}$, and

$$SL = 20 \log \left| \frac{A}{P_{ref} r_{ref}} \right| \text{ dB re } 1 \mu Pa \text{ at } 1 \text{ m} \hspace{1cm} (A.14)$$

$$TL(0|r_0) = 20 \log \frac{r_0}{r_{ref}} \text{ dB re } 1 \text{ m} \hspace{1cm} (A.15)$$

$$TS = 20 \log \left| \frac{S(\alpha, \beta; \alpha_i, \beta_i)}{kr_{ref}} \right| \text{ dB re } 1 \text{ m} \hspace{1cm} (A.16)$$

$$TL(0|r) = 20 \log \frac{r}{r_{ref}} \text{ dB re } 1 \text{ m.}$$ \hspace{1cm} (A.17)

Following the sonar equation, the radiated sound has a source level of $SL$, which is
the sound pressure level measured at 1 m from the source. This is reduced by the transmission loss $TL(r_0|0)$ from source to target centroid. The level is augmented by the target strength $TS$, and further diminished by transmission loss $TL(0|r)$ from target centroid to receiver. This is the sonar equation model expressed in decibels that describes the scattering from a deterministic object in free space conditions.

To track how the object scatters a fixed planewave defined by a unique wave vector $k$ incoming and outgoing at the object, we let $\alpha_i = \alpha$ and $\beta_i = \beta$ in Eq. A.11. We observe consistency with the planewave tracking rule discussed in Sec. A.2. When the target surface coordinate $r_t$ is a receiver point, we have the positive dot product of the wave vector $k$ with $r_t$ in the exponential term in Eq. A.11. A negative dot product of the wave vector with $r_t$ appears in the exponential term in Eq. A.11 when $r_t$ acts like a source point.

**A.4 Wave-theoretic Model for Deterministic Object Scattering in a Waveguide**

Here we derive the wave-theoretic normal mode model for the 3-D scattering by a deterministic object in a stratified waveguide from Green’s Theorem following some steps from Refs. [31] and [46].

As discussed above and in Chapter 2, we let the origin of all coordinate systems be placed at the object centroid with positive $z$ axis pointing downward and normal to the interface between horizontal strata. We will make use of spatial cartesian $r = (x, y, z)$, spherical $(r, \theta, \phi)$, and cylindrical systems $(\rho, \phi, z)$ defined by

$$
x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta, \quad \text{and} \quad \rho = \sqrt{x^2 + y^2} = r \sin \theta.
$$

(A.18)

The wavenumber cartesian $k = (\xi_x, \xi_y, \gamma)$, spherical $(k, \alpha, \beta)$, and cylindrical systems $(\xi, \beta, \gamma)$ are defined by
\[ \xi_x = k \sin \alpha \cos \beta, \quad \xi_y = k \sin \alpha \sin \beta, \quad \gamma = k \cos \alpha, \quad \text{and} \quad \xi = \sqrt{\xi_x^2 + \xi_y^2} = k \sin \alpha. \]  

(A.19)

In cylindrical systems, we can also write \( \mathbf{r} = (\rho, z) \) for a spatial vector and \( \mathbf{k} = (\xi, \gamma) \) for a wave vector. The dot product between wave vector \( \mathbf{k} \) and spatial vector \( \mathbf{r} \) is expressed in cylindrical systems as

\[ \mathbf{k} \cdot \mathbf{r} = \xi \cdot \rho + \gamma z = kr \eta(\alpha, \beta, \theta, \phi), \]  

(A.20)

where the last equality follows from Eq. A.1. In this discussion, the elevation angle of the nth mode of the waveguide is defined by \( \alpha_n \) where \( 0 \leq \alpha_n \leq \pi/2 \). The down and upgoing planewave component of each mode will then have elevation angles \( \alpha_n \) and \( \pi - \alpha_n \) respectively. The corresponding vertical wavenumber of the down and upgoing components of the nth mode are \( \gamma_n \) and \( -\gamma_n \) respectively, where \( \Re \{\gamma_n\} \geq 0 \).

In the waveguide, the field incident at a point on the target surface \( \mathbf{r}_t = (\rho_t, z_t) \) from a far-field point source at \( \mathbf{r}_0 = (\rho_0, z_0) \) can be expressed as a sum of normal modes.

\[ \Phi_i(\mathbf{r}_t|\mathbf{r}_0) = (4\pi A) \frac{i}{d(z_0)} \frac{1}{\sqrt{8\pi}} e^{-i\pi/4} \sum_{n=1}^{\infty} u_n(z_t) u_n(z_0) \frac{e^{i\xi_n |\rho_t - \rho_0|}}{\sqrt{\xi_n |\rho_t - \rho_0|}} \]  

(A.21)

The mode functions \( u_n(z) \) are normalized according to Eq. 2.4 and can be expressed in the layer of the object as Eq. 2.5. It should be noted that the direction of the planewave component of the mode, whether it is up or down going cannot be known from Eq. 2.5 alone as discussed in Sec. A.2. The planewave tracking rule will be used later in the discussion to determine the up and down going planewave components of the incident and scattered modes.

From Fig. A-4, we observe that \( \xi_n \), the horizontal projection of the incident modal
plane wave, is approximately parallel to \((\rho_t - \rho_0)\) and \(\xi_n \cdot \rho_0 = -\xi_n \rho_0\). We can express 
\[e^{i\xi_n \cdot (\rho_t - \rho_0)} = e^{i\xi_n \cdot (\rho_0 - \rho_0)} \approx e^{i(\xi_n \cdot \rho_t + \xi_n \rho_0)}.\]
Making use of Eq. 2.5 at the target depth \(z_t\), we can rewrite Eq. A.21 for \(\rho_0 \gg \rho_t\) as

\[
\Phi_i(r_t|r_0) \\
\approx (4\pi A) \frac{i}{d(z_0) \sqrt{8\pi}} e^{-i\pi/4} \sum_{n=1}^{\infty} u_n(z_0) \left[ N_n^{(1)} e^{i\gamma n(z_t + D)} - N_n^{(2)} e^{-i\gamma n(z_t + D)} \right] \frac{e^{i(\xi_n \cdot \rho_t + \xi_n \rho_0)}}{\sqrt{\xi_n \rho_0}} \\
= (4\pi A) \sum_{n=1}^{\infty} \left[ A_n(r_0) e^{i(\xi_n \cdot \rho_t + \gamma n z_t)} - B_n(r_0) e^{i(\xi_n \cdot \rho_t - \gamma n z_t)} \right] \\
= (4\pi A) \sum_{n=1}^{\infty} \left[ A_n(r_0) e^{ikr_0 \eta(\alpha_n, \beta_t, \theta_t, \phi_t)} - B_n(r_0) e^{ikr_0 \eta(\pi - \alpha_n, \beta_t, \theta_t, \phi_t)} \right], \quad (A.22)
\]

where the last equality follows from Eq. A.20, \(\beta_t = \phi_0 + \pi\), and
\[ A_n(r_0) = \frac{i}{d(z_0)} \frac{1}{\sqrt{8\pi\xi_n\rho_0}} u_n(z_0) N_n^{(1)} e^{i(\xi_n\rho_0 + \gamma_n D - \pi/4)}, \] downgoing incident

\[ B_n(r_0) = \frac{i}{d(z_0)} \frac{1}{\sqrt{8\pi\xi_n\rho_0}} u_n(z_0) N_n^{(2)} e^{i(\xi_n\rho_0 - \gamma_n D - \pi/4)}, \] upgoing incident.

(A.23)

Since \( \alpha_n < \pi/2 \) and the target surface point \( r_t \) acts like a receiver for the incident field in Eq. A.22, we make use of the planewave tracking rule in Sec. A.2 to conclude that \( A_n(r_0) \) represents the amplitude of the downgoing modal planewave incident on the target point \( r_t \) while \( B_n(r_0) \) represents the amplitude of the upgoing modal planewave incident on the target point \( r_t \).

The waveguide Green function for a source at \( r_t \) and a receiver at \( r \) expressed in terms of a modal sum is

\[ G(r|r_t) = \frac{i}{d(z_t)} \frac{1}{\sqrt{8\pi}} e^{-i\pi/4} \sum_{m=1}^{\infty} u_m(z) u_m(z_t) \frac{e^{i\xi_m|\rho - \rho_t|}}{\sqrt{\xi_m|\rho - \rho_t|}} \] (A.24)

From Fig. A-4, we observe that the horizontal projection of the scattered modal planewave \( \xi_m \) is approximately parallel to \( (\rho - \rho_t) \) and \( \xi_m \cdot \rho = \xi_m \rho \). We can express \( e^{i\xi_m|\rho - \rho_t|} = e^{i\xi_m \rho - \xi_m \rho_t} \approx e^{i(\xi_m \rho - \xi_m \rho_t)} \). Making use of Eq. 2.5 at the target depth \( z_t \), we can rewrite Eq. A.24 for \( \rho \gg \rho_t \) as

\[ G(r|r_t) \approx \frac{i}{d(0)} \frac{1}{\sqrt{8\pi}} \sum_{m=1}^{\infty} u_m(z) \left[ N_m^{(1)} e^{i\gamma_m(z_t + D)} - N_m^{(2)} e^{-i\gamma_m(z_t + D)} \right] \frac{e^{i(\xi_m \rho - \xi_m \rho_t)}}{\sqrt{\xi_m \rho}} \]

\[ = \sum_{m=1}^{\infty} \left[ A_m(r)e^{-i(\xi_m \rho_t - \gamma_m z_t)} - B_m(r)e^{-i(\xi_m \rho_t + \gamma_m z_t)} \right] \]

\[ = \sum_{m=1}^{\infty} \left[ A_m(r)e^{-ikr_t \eta(\pi - \alpha_m, \beta, \theta_t, \phi_t)} - B_m(r)e^{-ikr_t \eta(\alpha_m, \beta, \theta_t, \phi_t)} \right], \] (A.25)

where the last equality follows from Eq. A.20, \( \beta = \phi \), and
\[ A_m(r) = -\frac{i}{d(0)\sqrt{8\pi\xi_m\rho}}u_m(z)N_m^{(1)}e^{i(\xi_m\rho+\gamma_mD-\pi/4)}, \text{ upgoing scattered} \]
\[ B_m(r) = -\frac{i}{d(0)\sqrt{8\pi\xi_m\rho}}u_m(z)N_m^{(2)}e^{i(\xi_m\rho-\gamma_mD-\pi/4)}, \text{ downgoing scattered}. \]

(A.26)

Since \(\alpha_m < \pi/2\) and the target surface point \(r_t\) acts like a source for the scattered field in Eq. A.25, we make use of the planewave tracking rule in Sec. A.2 to conclude that \(A_m(r)\) represents the amplitude of the upgoing modal planewave scattered from the target point \(r_t\) while \(B_m(r)\) represents the amplitude of the downgoing modal planewave scattered from the target point \(r_t\).

Substituting Eq. A.22 for the incident field and Eq. A.25 for the scattered field into the Helmholtz-Kirchoff integral theorem, Eq. A.7, we obtain,

\[ \Phi_s(r) = -\oint_{A_t} \left\{ \left( 4\pi A \sum_{n=1}^{\infty} \left[ A_n(r_0)e^{ikr(p)(\alpha,m,\beta,\theta_t,\phi_t)} - B_n(r_0)e^{ikr(p)(\pi-\alpha,m,\beta,\theta_t,\phi_t)} \right] + \Phi_s(r_t) \right) \right\}
\]
\[ \times \frac{\partial}{\partial n_t} \left( \sum_{m=1}^{\infty} \left[ A_m(r)e^{-ikr(p)(\pi-\alpha_m,\beta,\theta_t,\phi_t)} - B_m(r)e^{-ikr(p)(\alpha_m,\beta,\theta_t,\phi_t)} \right] \right)
\]
\[ - \left( \sum_{m=1}^{\infty} \left[ A_m(r)e^{-ikr(p)(\pi-\alpha_m,\beta,\theta_t,\phi_t)} - B_m(r)e^{-ikr(p)(\alpha_m,\beta,\theta_t,\phi_t)} \right] \right)
\]
\[ \times \frac{\partial}{\partial n_t} \left( 4\pi A \sum_{n=1}^{\infty} \left[ A_n(r_0)e^{ikr(p)(\alpha_n,\beta,\theta_t,\phi_t)} - B_n(r_0)e^{ikr(p)(\pi-\alpha_n,\beta,\theta_t,\phi_t)} \right] + \Phi_s(r_t) \right) \right\} dA_t. \]

(A.27)

Comparing Eq. A.27 with the expression for the free space scatter function of the object Eq. A.11, we can rewrite Eq. A.27 as
\[
\Phi_s(\mathbf{r}) = A \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(4\pi)^2}{k} [A_m(\mathbf{r})A_n(\mathbf{r}_0)S(\pi - \alpha_m, \beta; \alpha_n, \beta_t) - B_m(\mathbf{r})A_n(\mathbf{r}_0)S(\alpha_m, \beta; \alpha_n, \beta_t) \\
- A_m(\mathbf{r})B_n(\mathbf{r}_0)S(\pi - \alpha_m, \beta; \pi - \alpha_n, \beta_t) + B_m(\mathbf{r})B_n(\mathbf{r}_0)S(\alpha_m, \beta; \pi - \alpha_n, \beta_t)],
\]  

(A.28)

where, for instance,

\[
S(\pi - \alpha_m, \beta; \alpha_n, \beta_t) = -\frac{k}{4\pi} \oint \oint \left\{ \left[ e^{ik\mathbf{r}_i \cdot \mathbf{n}_i(\alpha_n, \beta_i, \beta_t, \phi_t)} + \frac{1}{4\pi AA_n(\mathbf{r}_0)} \Phi_s(\mathbf{r}_t) \right] \frac{\partial}{\partial r_i} e^{-ik\mathbf{r}_i \cdot \mathbf{n}_i(\pi - \alpha_m, \beta, \beta_t, \phi_t)} \\
- e^{-ik\mathbf{r}_i \cdot \mathbf{n}_i(\pi - \alpha_m, \beta, \beta_t, \phi_t)} \frac{\partial}{\partial r_i} \left[ e^{ik\mathbf{r}_i \cdot \mathbf{n}_i(\alpha_n, \beta, \beta_t, \phi_t)} + \frac{1}{4\pi AA_n(\mathbf{r}_0)} \Phi_s(\mathbf{r}_t) \right] \right\} dA_t.
\]  

(A.29)

In Eq. A.28 the first term represents the scattering of a down going incident planewave at the target to an upgoing scattered planewave which is relevant for scattering from a patch of the seafloor when the field is measured in the water column. The fourth term represents a upgoing incident planewave scattered by the target to a downgoing scattered planewave which is relevant for modelling scattering from the sea surface when the field is measured in the water column.
Appendix B

Single Frequency Approximation Versus Full Bandwidth in Narrowband Scattering Calculations

A single frequency approximation is used for all the narrowband scattering calculations of Secs. 3.2 and 3.3. The time averaged, expected mutual intensity of Eq. 2.35,

\[ I(R, R_r, R_0, t) = \frac{1}{T} \int_{-\infty}^{+\infty} |Q(f)|^2 \langle |\Phi_s(R_r - R|R_0 - R)|^2 \rangle df, \quad (B.1) \]

is approximated as

\[ I(R, R_r, R_0, t) \approx \langle |\Phi_s(R_r - R|R_0 - R)|^2 \rangle \frac{1}{T} \int_{-\infty}^{+\infty} |Q(f)|^2 df, \quad (B.2) \]

where \( \langle |\Phi_s(R_r - R|R_0 - R)|^2 \rangle \) is calculated at the center frequency \( f_c \) of the narrowband waveform \( q(t) \), where \( f_c = 300 \text{ Hz} \) for the examples of Sec. 3.2 and 3.3.
In practice, the narrowband waveform or window function \( q(t) \) has a spectrum \( Q(f) \) with either a narrow main lobe such as the rectangular window, or a broader main lobe with lower side lobes such as the Bartlett, Hanning, and Hamming windows.[59] The window functions are normalised according to

\[
\frac{1}{T} \int_{-\infty}^{+\infty} |Q(f)|^2 df = 1
\]  

(B.3)

so that the rectangular window, with a main lobe half-power bandwidth of \( 0.886/T \) and a first side lobe 13.4 dB down peak-to-peak from the main lobe, becomes

\[
q(t) = \begin{cases} 1 & \text{for } -T/2 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases}
\]  

(B.4)

and the Hamming window, for example with main lobe half-power bandwidth of \( 1.30/T \) and a first side lobe 42.7 dB down peak-to-peak from the main lobe, becomes

\[
q(t) = \begin{cases} \sqrt{1/0.3974} \left( 0.54 + 0.46 \cos\left[2\pi t / T\right]\right) & \text{for } -T/2 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases}
\]  

(B.5)

where \( T = 1/2 \) for the examples of Secs. 3.2 and 3.3.

For the reverberation calculations of Secs. 3.2 and 3.3, calculations show that this approximation differs from the full spectral integration by less than 0.1 dB.

As may be expected in the coherent scattering from targets where modal interference is significant, some range-dependent nulls and valleys in the sound pressure level of the received field found in the single frequency calculation may be partially filled when the full bandwidth is used for the narrowband waveforms considered. This is exhibited in Figs. B-1 and B-2 where the filling is shown to be window-dependent and more negligible for bottoms that support fewer trapped modes. It is noteworthy that for narrowband transmissions at the given center frequency and duration,
the sphere target may have returns that fall below the expected reverberation level, but the quantity and location of these expected deep "fades" of the target is highly dependent on the window function used.
Figure B-1: A comparison of scattering using the single frequency approximation versus the full bandwidth of the given window function. Reverberation calculated using the single frequency approximation is indistinguishable from that calculated with the full bandwidth. (a) Same as Fig. 3-4(a) for sphere in waveguide with reverb except only sand bottom case is shown. Single frequency approximation is compared to rectangular window. (b) Same as (a) except Hamming window is used instead of rectangular window. (c) Same as (a) except only silt bottom case is shown. (d) same as (c) except Hamming window is used instead of rectangular window.
Figure B-2: A comparison of scattering using the single frequency approximation versus the full bandwidth of the given window function. Reverberation calculated using the single frequency approximation is indistinguishable from that calculated with the full bandwidth. (a) Same as Fig. 3-8(a) for seafloor riverbank with reverb except only sand bottom case is shown. Single frequency approximation is compared to rectangular window. (b) Same as (a) except Hamming window is used instead of rectangular window. (c) Same as (a) except only silt bottom case is shown. (d) same as (c) except Hamming window is used instead of rectangular window.
Appendix C

Asymptotic Integration using Method of Stationary Phase

Given an integral of the form

$$\int_{-\infty}^{+\infty} e^{i\lambda f(x)} g(x) dx,$$

(C.1)

where \( \lambda \) is large \((\lambda \gg 1)\), if \( f(x) \) changes a little, the phase of the exponent changes by a huge amount. The integrand oscillates a lot leading to cancellation, following Riemman Lesbesgue Lemma, except for contribution from points where the variation of the phase is least rapid. These points, \( x_{s_n} \), are the stationary phase points where \( f'(x_{s_n}) = 0 \) for \( n = 1, 2, 3, \ldots \). For each point of stationary phase, we expand \( f(x) \) about that point

$$f(x) \approx f(x_{s_n}) + \frac{(x - x_{s_n})^2}{2} f''(x_{s_n}) + \ldots$$

(C.2)

and substitute into Eq. (C.1) to obtain
\[
\int_{-\infty}^{+\infty} e^{i\lambda f(x)} g(x) dx
\approx \sum_{n} \int_{x_{sn} - \epsilon}^{x_{sn} + \epsilon} e^{i\lambda f(x)} \left( \frac{(x-x_{sn})^2}{2} + \lambda f''(x_{sn}) \right) g(x_{sn}) dx
\]
\[
= \sum_{n} e^{i\lambda f(x_{sn})} g(x_{sn}) \int_{x_{sn} - \epsilon}^{x_{sn} + \epsilon} e^{i\lambda f''(x_{sn})} \frac{(x-x_{sn})^2}{2} dx. \quad (C.3)
\]

Let \( u^2 = \lambda f''(x_{sn}) \frac{(x-x_{sn})^2}{2} \), and \( du = \sqrt{\frac{\lambda f''(x_{sn})}{2}} dx \). Substituting \( u \) and \( du \) into Eq. (C.3), making use of the fact that \( \lambda \gg 1 \), and \( \int_{-\infty}^{+\infty} e^{iu^2} du = \sqrt{\pi} e^{i\pi/4} \), leads to the following final result for the asymptotic integration:

\[
\int_{-\infty}^{+\infty} e^{i\lambda f(x)} g(x) dx
\approx \sum_{n} e^{i\lambda f(x_{sn})} g(x_{sn}) \sqrt{\frac{2}{\lambda f''(x_{sn})}} \int_{-\infty}^{+\infty} e^{iu^2} du
\]
\[
= \sum_{n} e^{i\lambda f(x_{sn})} g(x_{sn}) \sqrt{\frac{2\pi}{\lambda f''(x_{sn})}} e^{i\pi/4} \quad (C.4)
\]

Note: The Riemann Lesbesgue Lemma.

\[
\lim_{\lambda \to \infty} \int_{0}^{2\pi} \sin(\lambda t) g(t) dt = 0 \quad (C.5)
\]
\[
\lim_{\lambda \to \infty} \int_{0}^{2\pi} \cos(\lambda t) g(t) dt = 0 \quad (C.6)
\]
\[
\lim_{\lambda \to \infty} \int_{0}^{2\pi} e^{i\lambda t} g(t) dt = 0 \quad (C.7)
\]
Appendix D

Further Notes on Extinction Theorem

D.1 General Approach for Calculating Extinction

There are two approaches to calculate extinction in an incident field due to absorption and scattering by an object. In the first approach, we define a closed control surface $C$ that encloses the object, but excludes the source. We let the origin of the coordinate system be at the object centroid. Let $r_0$ be the position of the source and $r$ is the position of a point on the control surface.

In the absence of the object, only the incident field $\Phi_i$ exits. The intensity of the incident field at location $r$ on the control surface from a source at $r_0$ is

\[
I_i(r|r_0) = \Re \{ V_i^*(r|r_0) \Phi_i(r|r_0) \} \tag{D.1}
\]

where $V_i(r|r_0)$ is the velocity vector of the incident field which, from Newton's law, can be expressed as

\[
V_i(r|r_0) = \frac{1}{i\omega d(r)} \nabla \Phi_i(r|r_0) \tag{D.2}
\]
for a harmonic field at frequency $\omega$ with $d(r)$ being the density at location $r$. Integrating the incident intensity over the entire control surface $C$ gives the net incident intensity flux $\mathcal{F}_i$ through the control surface,

$$\mathcal{F}_i = \Re \left\{ \iint_C V_i^* \Phi_i \cdot dS \right\}. \quad (D.3)$$

In a lossless media, the incident energy flux entering the control surface has to equal that leaving the surface. Therefore in a lossless media,

$$\mathcal{F}_i = 0. \quad (D.4)$$

In the presence of the object, the total field at location $r$ on the control surface for a source at $r_0$ is the sum of the incident pressure field from the source and the scattered field from the object,

$$\Phi_T(r|r_0) = \Phi_i(r|r_0) + \Phi_s(r|r_0). \quad (D.5)$$

The intensity of this total field at $r$ on the control surface is

$$I_T(r|r_0) = \Re \{V_T^*(r|r_0)\Phi_T(r|r_0)\}. \quad (D.6)$$

The total intensity integrated over the entire control surface $C$ is the total energy flux $\mathcal{F}_T$ through $C$, or the total intercepted power,
Using Eqs. (D.3), (D.4) and (D.8) in Eq. (D.7), the extinction $E$ due to the object in a lossless media is

$$E = W_a + W_s = -R \left\{ \oint C (V_i^* \Phi_s + V_s^* \Phi_s) \cdot dS \right\}. \quad (D.9)$$

From Eq. (D.9) we see that extinction is a result of the interference between the incident and scattered fields over the control surface. For a planewave in free space, the active region of the control surface over which the incident and scattered fields have a fixed phase relationship to interfere destructively lies within an angular width $\sqrt{\lambda/r}$ of the forward direction, where $\lambda$ is the wavelength of the incident wave, and $r$ is the distance of the control surface from the object centroid in the forward direction.
This region comprises the shadow remnant. Outside of this region, the integrand in Eq. (D.9) fluctuates too rapidly to contribute to the extinction.

Consequently, instead of integrating the interference flux over the entire control surface, we can replace the enclosed control surface by a screen $S_c$ in the forward direction. From Eq. (D.7), if we integrate the interference flux over the area of the screen, instead of the enclosed control volume, we obtain

$$-\Re \left\{ \iint_{S_c} (V_i^* \Phi_s + V_s^* \Phi_i) \cdot dS \right\} = \Re \left\{ \iint_{S_c} (V_i^* \Phi_i - V_T^* \Phi_T + V_s^* \Phi_s) \cdot dS \right\}. \quad (D.10)$$

The first term on the right hand side of Eq. (D.10) is the incident flux $\mathcal{F}_i$ through the screen, which is the flux through the screen in the absence of the object. The second term is $\mathcal{F}_T$, the total flux through the screen in the presence of the object. The last term is the scattered flux through the screen. If we place the screen sufficiently far from the object so that $V_i^*$ and $\Phi_s$ become small relative to $V_i^*$ and $\Phi_i$ due to spreading loss, the scattered flux becomes negligible. For instance, for planewaves in free space, the spherical spreading of the scattered field causes the scattered field intensity to decrease with range with a $1/r^2$ dependence, while the incident intensity remains constant.

At any given range $r$ of the screen from the object, to measure the full extinction caused by the object, the screen has to be much wider than $\sqrt{\lambda r}$. For a sufficiently large screen, the extinction $\mathcal{E}$ is, from Eq. (D.10),

$$\mathcal{E} = \mathcal{F}_i - \mathcal{F}_T = -\Re \left\{ \iint_{S_c} (V_i^* \Phi_s + V_s^* \Phi_i) \cdot dS \right\}, \quad (D.11)$$

the difference between the incident flux measured by the screen in the absence of the object and the total flux in the presence of the object. This is the approach due to
Van de Hulst for calculating the extinction by placing a sufficiently large screen in the forward direction to register the full extinction.

D.2 Extinction, Absorption and Scattering Cross Sections

The extinction, absorption and scattering cross sections can be viewed as fictitious areas that intercept a portion of the incident power equal to the extinguished, absorbed or scattered power respectively. The extinction cross section $\sigma_T$, from definition, is the ratio between the rate of dissipation of energy $E$ and the rate at which energy is incident on unit cross-sectional area of the object $I_i$,

$$\sigma_T = \frac{E}{I_i}.$$  \hfill (D.12)

From Eq. (D.9), we can also express the above as

$$\sigma_T = \frac{W_a + W_s}{I_i} = \sigma_a + \sigma_s,$$  \hfill (D.13)

where $\sigma_a$ and $\sigma_s$ are the absorption and scattering cross sections respectively. For a non-absorbing object, $\sigma_a = 0$ and the extinction cross section is then equal to the scattering cross section, $\sigma_T = \sigma_s$.

D.3 Extinction Formula for Scattering in an Infinite Lossy Unbounded Media

We derive the formula for the extinction of an incident planewave in the far field of a point source by an object in an infinite unbounded medium with absorption loss. We
will derive the expression using both the control surface method and the Hulst screen method discussed in Appendix D.1 and compare the resulting expressions. Let \( \nu \) be the coefficient for absorption in the medium. We write the magnitude of the complex wave vector as \( k = \kappa + i\nu \), where \( \kappa = \omega/c \).

The object centroid coincides with the center of the coordinate system and we place the source at \( r_0 = (0, 0, -z_0) \). First we derive the formula using the control surface method, Eq. (D.9). We let the control surface be a spherical surface of radius \( R \) centered at the object centroid. At any point \( r \) on the control surface, the incident field from a unit amplitude source \( (A) \) can be obtained from the free space Green's function,

\[
\Phi_i(r| r_0) = \frac{e^{ik|r-r_0|}}{|r-r_0|}. \tag{D.14}
\]

Since the object is in the far field of the point source, the incident field at the object can be approximated as composing of planewaves with amplitude \( e^{ikz_0}/z_0 \). Following Eq. A.10, the scattered field from the object at ranges far from the object can be expressed as

\[
\Phi_s(r| r_0) = \frac{e^{ikz_0}e^{ikr}}{z_0} S(\theta, \phi; 0, 0). \tag{D.15}
\]

The first term in the integrand of Eq. (D.9) for this case is

\[
V_i^s \Phi_s = \frac{(\kappa - i\nu)}{\omega d} \frac{e^{-i\kappa \sqrt{x^2+y^2+(z+z_0)^2}} (r_1 + z_0 i_z) e^{ikz_0}}{z_0} \frac{e^{ik\sqrt{x^2+y^2+z^2}} S(\theta, \phi; 0, 0)}{(\kappa + i\nu) \sqrt{x^2+y^2+z^2}} e^{-\nu \sqrt{x^2+y^2+(z+z_0)^2}} e^{-\nu z_0} e^{i\nu \sqrt{x^2+y^2+z^2}}. \tag{D.16}
\]

In the above expression, we explicitly factor out the term representing absorption in the medium to avoid confusion when conjugating the fields and to keep track of
absorption losses in the medium. On the control surface, \( r = \sqrt{x^2 + y^2 + z^2} = R \).

We will assume that \( z_0 \gg R \) since the object is in the far field of the point source.

We use the approximation \( \sqrt{x^2 + y^2 + (z + z_0)^2} \approx z + z_0 \) in the term that determines the phase of the integrand, and the approximation \( \sqrt{x^2 + y^2 + (z + z_0)^2} \approx z_0 \) in the spreading loss factor. The resulting expression becomes,

\[
V_i^* \Phi_s = \frac{(\kappa - i\nu)}{\omega d(\kappa + i\nu)} \frac{e^{-i(\kappa + \nu)z}}{z_0^3} (z_0 \hat{i}_z + R \hat{i}_r) \frac{e^{i\kappa R}}{R} S(\theta, \phi; 0, 0) e^{-2\nu z_0} e^{-\nu R}.
\] (D.17)

Next, we integrate Eq. (D.17) over the area of the enclosed spherical surface. An area element on the surface is \( dS = i R^2 \sin \theta d\theta d\phi \). Since \( z_0 \gg R \) we need only consider the first term in Eq. (D.17) and note that \( i_z \cdot i_r = \cos \theta \). With \( z = R \cos \theta \) on the spherical surface, we obtain

\[
\oint \oint_C V_i^* \Phi_s \cdot dS = \frac{(\kappa - i\nu)}{\omega d(\kappa + i\nu)z_0^3} \frac{e^{i\kappa R}}{R} \int_0^\pi \int_0^{2\pi} e^{-(i(\kappa + \nu)R \cos(\theta))} S(\theta, \phi; 0, 0) \cos \theta \sin \theta d\theta d\phi \times e^{-2\nu z_0} e^{-\nu R}.
\] (D.18)

Making use of asymptotic integration,

\[
\oint \oint_C V_i^* \Phi_s \cdot dS = i2\pi \frac{1}{\omega d z_0^2 \kappa + i\nu} e^{-2\nu z_0} \left( S(0, 0; 0, 0) e^{-2\nu R} + S(0, \pi; 0, 0) e^{i2\kappa R} \right).
\] (D.19)

Similarly, we integrate the second term of Eq. (D.9) over the control surface and obtain

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\[
\oint \oint \mathbf{V}_s \Phi_i \cdot dS = \frac{-i2\pi}{\omega dz_0^2} \kappa + iv e^{-2\nu z_0} (S^*(0,0;0,0)e^{-2\nu R} - S^*(0,\pi;0,0)e^{-i2\nu R}). \tag{D.20}
\]

Summing Eqs. (D.19) and (D.20), taking only the negative of the real part of the sum, we obtain the extinction caused by an object in an infinite unbounded lossy medium using the control volume method,

\[
\mathcal{E}_C(r|r_0) = \frac{4\pi e^{-2\nu(z_0+R)}}{\omega d} \frac{\kappa}{\kappa^2 + \nu^2} \mathfrak{R}\{S(0,0;0,0)\} - \frac{\nu}{\kappa^2 + \nu^2} \mathfrak{R}\{S(0,\pi;0,0)e^{i2\nu R}\} e^{2\nu R}. \tag{D.21}
\]

Next, we derive the extinction using the Van de Hulst screen method, Eq. (D.11). We start with the expression in Eq. (D.16). We place a square screen of length \(L\) a sufficiently large distance from the object in the forward direction, parallel to the \(x - y\) plane a distance \(z\) away from the object. As discussed in Appendix D.1, we require \(L > \sqrt{\lambda z}\). Since \(z\) is large, we assume that for points on the active region of the screen, \(z \gg \rho\) where \(\rho = \sqrt{x^2 + y^2}\). We use the approximations \(\sqrt{x^2 + y^2 + (z + z_0)^2} \approx z + z_0 + \frac{\rho^2}{2(z+z_0)}\), and \(\sqrt{x^2 + y^2 + z^2} \approx z + \frac{\rho^2}{2z}\) in the terms that determine the phase of the integrand, and the approximations \(\sqrt{x^2 + y^2 + (z + z_0)^2} \approx z + z_0\) and \(\sqrt{x^2 + y^2 + z^2} \approx z\) in the absorption and spreading loss factors. Equation (D.16) simplifies to

\[
\mathbf{V}_s^* \Phi_s = \frac{(\kappa - iv)}{(\kappa + iv) \omega dx_0 z(z_0 + z)} e^{i\kappa x_0 \frac{z_0}{z_0 + z}} \rho^2 S(\theta, \phi; 0, 0)e^{-2\nu(z_0 + z)}. \tag{D.22}
\]

We integrate Eq. (D.22) using Eq. (D.11) over the area of the screen. With the screen lying normal to the \(z\) axis, an area element of the screen is \(dS = i_z dx dy\).
\[
\int \int_{S_c} V_t^* \Phi_s \cdot dS = \frac{(\kappa - i\nu)}{(\kappa + i\nu)} \frac{1}{\omega dz_0 z(z_0 + z)} e^{-2\nu(z_0 + z)} \\
\times \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} e^{i\kappa \frac{z}{2z_0(z_0 + z)}} \rho^2 S(\theta, \phi; 0, 0) dxdy
\]  

(D.23)

As discussed in Appendix D.1, the angular width of the active area on the screen is of the order of \(\sqrt{\lambda/z}\) which is small for large \(z\). We therefore approximate the scatter function with its value at \(\theta = \phi = 0\) and factor it out of the integral above. Integrating the resulting expression using asymptotic integration, we obtain

\[
\int \int_{S_c} V_t^* \Phi_s \cdot dS = \frac{i2\pi}{\omega dz_0 \kappa} S(0, 0; 0, 0) \frac{(\kappa - i\nu)}{(\kappa + i\nu)} e^{-2\nu(z_0 + z)}. 
\]  

(D.24)

Similarly, we integrate the second term in Eq. (D.11) to obtain

\[
\int \int_{S_c} V_s^* \Phi_t \cdot dS = \frac{-i2\pi}{\omega dz_0 \kappa} S^*(0, 0; 0, 0) e^{-2\nu(z_0 + z)}. 
\]  

(D.25)

Adding the two expressions and taking only the negative of the real part of the sum, we obtain the extinction caused by an object in an infinite unbounded lossy medium using the screen method,

\[
\mathcal{E}_{Sc}(r|r_0) = \frac{4\pi e^{-2\nu(z+z_0)}}{\omega d z_0^2} \kappa \left( \mathcal{R}\{S(0, 0; 0, 0)\} - \nu \mathcal{R}\{S(0, 0; 0, 0)\} \right). 
\]  

(D.26)

The expression for the extinction using the control volume method Eq. (D.21) and that obtained using Van de Hulst screen method Eq. (D.26) are equal if \(\nu = 0\). The second term in both equations arise due to absorption by the medium. The expressions for the absorption loss term differ because we integrate the energy flux over different surfaces. If \(\nu\) is small compared to \(\kappa\), \(\nu/\kappa << 1\), we can ignore the
second term in both equations, and letting $z = R$, the resulting expressions for the extinction are identical and become

$$E(r|r_0) = \frac{4\pi}{\omega d\kappa} \Im \left\{ S(0, 0; 0, 0) \right\} e^{-2\nu(z+z_0)} \frac{e^{-2\nu(z+z_0)}}{z_0^2}. \quad (D.27)$$

This derivation shows that the screen method gives the true extinction only if the absorption loss in the medium is small. From Eq. (D.27), we see that absorption in the medium lowers the extinction that we would otherwise measure in a lossless medium. The $1/z_0^2$ factor is due to the spherical spreading of the incident field from the source to the object.

The incident intensity on the object in the $z$ direction for $\nu/\kappa << 1$ is

$$V_i^*\Phi_i = \frac{\kappa}{\omega d} e^{-2\nu z_0}. \quad (D.28)$$

Dividing the extinction in Eq. (D.27) with the incident intensity on the object Eq. (D.28), we obtain the extinction cross section of the object in the infinite unbounded lossy media.

$$\sigma_T(z) = \frac{4\pi}{\kappa^2} \Im \left\{ S(0, 0; 0, 0) \right\} e^{-2\nu z} \quad (D.29)$$

Equation (D.29) shows that a measurement of the cross section of an object in a lossy medium will be smaller than in a lossless medium. To obtain the true cross section of the object, independent of the medium, we have to correct for absorption in the lossy medium.
D.4 Extinction Formula for Scattering in a stratiﬁed waveguide calculated using a control surface that encloses the object

Let the control surface be a semi-inﬁnite cylinder of radius $R$ with a cap at the sea surface where $z = -D$, similar to that deﬁned in Sec. 5.2. The axis of the cylinder is parallel to the $z$ axis and passes through the object centroid. The source is located at $r_0 = (-x_0, 0, z_0)$, and we assume that $R \ll x_0$.

As discussed in Sec. 5.2, the sea has a pressure-release surface where both the incident and scattered ﬁelds vanish. The contribution of the interference ﬂux through the cap at the sea surface $z = -D$ is zero. We need only integrate the interference ﬂux in Eq. (D.9) over the curved surface of the cylinder to obtain the extinction caused by the object.

Using Eqs. (5.1), (2.1), (2.2) and (D.2), the ﬁrst term in the integrand of Eq. (D.9) on the curved surface of the cylinder, $R = (x, y, z)$ is

$$V^* \Phi_s = \frac{8\pi^2}{\omega k} \frac{i}{d(z)d(z_0)d(0)} \sum_{i=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} u_i^*(z_0) u_m(z) \left[ \frac{\partial}{\partial z} u_i^*(z) i_z - i \xi^*_i u_i^*(z) i_z \right]_{m,n} e^{-iR(x_0+x)} e^{iR(x_m)R}$$

$$\times \frac{\sqrt{\xi^*_i x_0}}{\sqrt{\xi_m R}}$$

$$\times \left[ N_m e^{-iR(\gamma_m)D} A_n(r_0) S(\pi - \alpha_m, \phi; \alpha_n, 0) 
- N^+_m e^{-iR(\gamma_m)D} A_n(r_0) S(\alpha_m, \phi; \alpha_n, 0) 
- N_m e^{iR(\gamma_m)D} B_n(r_0) S(\alpha_m, \phi; \pi - \alpha_n, 0) 
+ N^+_m e^{iR(\gamma_m)D} B_n(r_0) S(\alpha_m, \phi; \pi - \alpha_n, 0) \right] \times e^{-\Im(\xi_i) x_0} e^{-\Im(\gamma_m)D}.$$  (D.30)

In the above expression, the terms representing absorption by the waveguide have been factored out explicitly to avoid confusion when conjugating the ﬁelds and also to keep track of absorption losses due to the waveguide. Since $R \ll x_0$, the expansion

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\(|p - p_0| = x_0 + x\) was used in the terms that determine the phase of the integrand while the approximation \(|p - p_0| \approx x_0\) was used in the spreading and absorption loss factors. We ignore the absorption loss term \(e^{-\Omega(x)R}\) since it is small compared to \(e^{-\Omega(x)z_0}\).

Next we integrate Eq. (D.30) over the curved surface of the cylinder. An area element on the surface of the cylinder is \(dS = i_p R d\phi dz\). We use the orthogonality relation in Eq. (2.4) between the modes \(u^*(z)\) and \(u_m(z)\) to integrate Eq. (D.30) over the semi-infinite depth of the cylinder in the waveguide. This reduces the triple sum over the modes to a double sum.

\[
\oint_C \oint \nabla^* \Phi_s \cdot dS = \oint_0^{2\pi} \oint_{-D}^{\infty} V^*_i \Phi_s \cdot i_p R dz d\phi \\
= \frac{8\pi^2}{\nu k} \frac{1}{d(z_0)d(0)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\xi_m^*}{|\xi_m| \sqrt{x_0 R}} u^*_m(z_0) e^{i\Omega_\mu(X_0)} R e^{-i\Omega(x_0)z_0} \\
\times \left\{ \int_0^{2\pi} \left[ N_m e^{i\Omega_\mu(X_0)} A_n(r_0) S(\pi - \alpha_m, \phi; \alpha_n, 0) \\
- N_m e^{-i\Omega_\mu(X_0)} A_n(r_0) S(\alpha_m, \phi; \alpha_n, 0) \\
- N_m e^{i\Omega_\mu(X_0)} B_n(r_0) S(\pi - \alpha_m, \phi; \pi - \alpha_n, 0) \\
+ N_m e^{-i\Omega_\mu(X_0)} B_n(r_0) S(\alpha_m, \phi; \pi - \alpha_n, 0) \right] \\
\times e^{-i\Omega(x_0)R \cos \phi \cos \phi R d\phi} \right\} e^{-\Omega(x)z_0} e^{-\Omega(x)D} \quad (D.31)
\]

The integral involving \(\phi\) can be evaluated using the method of stationary phase. There are two stationary phase points corresponding to the forward azimuth \(\phi = 0\), and the back azimuth \(\phi = \pi\). Applying the result of the following stationary phase integration over the azimuth angle \(\phi\)
\begin{equation}
\int_0^{2\pi} S(\pi - \alpha_m, \phi; \alpha_n, 0) e^{-iR\{\xi_m\} R \cos \phi} \cos \phi d\phi
= \sqrt{\frac{2\pi}{\mathcal{R}\{\xi_m\} R}} e^{i\pi/4} \left[ e^{-iR\{\xi_m\} R S(\pi - \alpha_m, 0; \alpha_n, 0)} + i e^{iR\{\xi_m\} R S(\pi - \alpha_m, \pi; \alpha_n, 0)} \right],
\end{equation}

(D.32)

to Eq. (D.31), the integration of the first term in Eq. (D.9) over the curved surface of the cylinder in the waveguide becomes

\[ \oint V_i^* \Phi_s \cdot dS \]

\begin{equation}
= \frac{4\pi^2}{\omega k} \frac{1}{d^2(z_0)} \frac{1}{d(0)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\xi^*_m}{\mathcal{R}\{\xi_m\} \xi_n} u^*_m(z_0) u_n(z_0) e^{iR\{\xi_n - \xi_m\} x_0}
\times \left( i \left[ N^-_m N^- e^{iR\{\gamma_m + \gamma_n\} D} S(\pi - \alpha_m, 0; \alpha_n, 0) \\
- N^+_m N^- e^{iR\{\gamma_m + \gamma_n\} D} S(\alpha_m, 0; \alpha_n, 0) \\
- N^-_m N^+ e^{iR\{\gamma_m - \gamma_n\} D} S(\pi - \alpha_m, 0; \pi - \alpha_n, 0) \\
+ N^+_m N^+ e^{iR\{\gamma_m - \gamma_n\} D} S(\alpha_m, 0; \pi - \alpha_n, 0) \right] \\
- e^{iR\{2\xi_m\} R} \left[ N^-_m N^- e^{iR\{\gamma_m + \gamma_n\} D} S(\pi - \alpha_m, \pi; \alpha_n, 0) \\
- N^+_m N^- e^{iR\{\gamma_m + \gamma_n\} D} S(\alpha_m, \pi; \alpha_n, 0) \\
- N^-_m N^+ e^{iR\{\gamma_m - \gamma_n\} D} S(\pi - \alpha_m, \pi; \pi - \alpha_n, 0) \\
+ N^+_m N^+ e^{iR\{\gamma_m - \gamma_n\} D} S(\alpha_m, \pi; \pi - \alpha_n, 0) \right] \right)
\times e^{-\Theta\{\xi_m + \xi_n\} x_0} e^{-\Theta\{\gamma_m + \gamma_n\} D}.
\end{equation}

(D.33)

Similarly, we can evaluate the second term in Eq. (D.9) which gives

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\[ \oint_C \mathbf{V}_s' \Phi_i \cdot dS = \frac{4\pi^2}{\omega k} \frac{1}{\alpha_0^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\xi_n^* u_m(z_0) u_n^*(z_0) e^{-i\Re(\xi_n - \xi_m) x_0}}{\xi_m - \xi_n} \]

\[ \times \left( i \left[ N_m^* N_n^{-*} e^{-i\Re(\gamma_m + \gamma_n) D} S^*(\pi - \alpha_m, 0; \alpha_n, 0) 
- N_m^* N_n^{-*} e^{-i\Re(\gamma_m - \gamma_n) D} S^*(\alpha_m, 0; \alpha_n, 0) 
- N_n^* N_m^* e^{-i\Re(\gamma_m + \gamma_n) D} S^*(\pi - \alpha_m, 0; \pi - \alpha_n, 0) 
+ N_m^* N_n^* e^{-i\Re(\gamma_m - \gamma_n) D} S^*(\alpha_m, 0; \pi - \alpha_n, 0) \right] 
- e^{-i\Re(2\xi_m) R} \left[ N_m^* N_n^{-*} e^{-i\Re(\gamma_m + \gamma_n) D} S^*(\pi - \alpha_m, \pi; \alpha_n, 0) 
- N_m^* N_n^{-*} e^{-i\Re(\gamma_m - \gamma_n) D} S^*(\alpha_m, \pi; \alpha_n, 0) 
- N_m^* N_n^* e^{-i\Re(\gamma_m - \gamma_n) D} S^*(\pi - \alpha_m, \pi; \pi - \alpha_n, 0) 
+ N_m^* N_n^* e^{-i\Re(\gamma_m - \gamma_n) D} S^*(\alpha_m, \pi; \pi - \alpha_n, 0) \right] \right) \]

\[ \times e^{-\Re(\xi_m + \xi_n) x_0} e^{-\Re(\gamma_m + \gamma_n) D} \] (D.34)

We then sum Eqs. (D.33) and (D.34), taking only the negative of the real part of the sum following Eq. (D.9). This leads to the range dependent extinction \( E(R|r_0) \) of the incident field in a waveguide due to an object at the origin measured by a cylinder of radius \( R \) centered on the object with source at \( r_0 \).
After comparing the expression for the extinction calculated using the control surface method Eq. (D.35), with that obtained using the Van de Hulst screen method Eq. (5.9), we see that they are identical only in the perfectly reflecting waveguide where $\Im\{\xi_m\} = 0$. If the absorption loss in the waveguide is small, we can neglect the second term in Eq. (D.35) and the resulting expression will be similar to Eq. (5.9). The differences in Eqs. (5.9) and (D.35) arise because of absorption loss in the medium and also because we integrate the energy fluxes over different surfaces for the two methods.

The Van de Hulst screen method is of more practical use because it represents the type of measurement that can be made with a standard 2-D planar or billboard array. A control volume measurement, on the other hand, would be very difficult to implement since it would require an array that completely encloses the object.
Appendix E

Jacobian for Integration in 3D over all Orientations

In this section, we derive the Jacobian for the integration over all orientations of an object in 3D space. The orientation of a general object in 3D space has 3 degrees of freedom that can be specified using a unit axis vector and a spin angle or with unit quaternions. A quaternion is an entity with four degrees of freedom and can be thought of as a vector with four components, $\mathbf{q} = (q_0, q_x, q_y, q_z)$. It can also be viewed as a sum of a scalar and a vector part.[27]

$$\mathbf{q} = q + \mathbf{q} \tag{E.1}$$

Quaternion math is discussed further in Refs. [27, 28]. For instance the dot product of two quaternions $\mathbf{p}$ and $\mathbf{q}$ is

$$\mathbf{p} \cdot \mathbf{q} = pq + \mathbf{p} \cdot \mathbf{q} \tag{E.2}$$

The unit quaternion $\mathbf{q}$ corresponding to rotation $\tau$ about the unit vector $\hat{\mathbf{w}}$ is given by
\[ \tilde{q} = \cos \frac{\tau}{2} + \hat{w} \sin \frac{\tau}{2}, \]  

(E.3)

where \( ||\tilde{q}|| = \sqrt{\tilde{q} \cdot \tilde{q}} = 1. \)

When the quaternion is viewed as a vector with four components, we let \( i_0, i_x, i_y, \) and \( i_z \) be four orthogonal unit quaternions in the four component directions. Then we can express a general quaternion as

\[ \tilde{q} = q_0 \hat{i}_0 + q_x \hat{i}_x + q_y \hat{i}_y + q_z \hat{i}_z. \]  

(E.4)

Similar to cartesian and spherical coordinate systems in 3D space, we can define a quaternion in 4D in terms of a new coordinate system defined by \( r, \chi, \eta, \) and \( \zeta, \)

\[ \tilde{q} = (r \cos \zeta, r \sin \zeta \sin \chi \cos \eta, r \sin \zeta \sin \chi \sin \eta, r \sin \zeta \cos \chi), \]  

(E.5)

where

\[
\begin{align*}
q_0 &= r \cos \zeta, \\
q_x &= r \sin \zeta \sin \chi \cos \eta, \\
q_y &= r \sin \zeta \sin \chi \sin \eta, \\
q_z &= r \sin \zeta \cos \chi,
\end{align*}
\]  

(E.6)
\[
\|\mathbf{q}\| = \sqrt{q_0^2 + q_x^2 + q_y^2 + q_z^2},
\]
\[
\tan \eta = \frac{q_y}{q_x},
\]
\[
\tan \chi = \frac{\sqrt{q_x^2 + q_y^2}}{q_z},
\]
\[
\tan \zeta = \frac{\sqrt{q_x^2 + q_y^2 + q_z^2}}{q_0}.
\]

(E.7)

Let \( \hat{i}_r, \hat{i}_x, \hat{i}_\eta, \) and \( \hat{i}_\zeta \) be the four quaternions in the \( r, \chi, \eta, \) and \( \zeta \) directions. Then the tangent vectors in the \( r, \chi, \eta, \) and \( \zeta \) directions are given by [26]

\[
\begin{align*}
\hat{h}_r \hat{i}_r &= \frac{\partial \hat{q}}{\partial r} = \cos \chi \hat{i}_0 + \sin \chi \cos \eta \hat{i}_x + \sin \chi \sin \eta \hat{i}_y + \sin \zeta \cos \chi \hat{i}_z, \\
\hat{h}_x \hat{i}_x &= \frac{\partial \hat{q}}{\partial \chi} = r \sin \chi \cos \cos \eta \hat{i}_x + r \sin \chi \sin \eta \hat{i}_y - r \sin \zeta \sin \chi \hat{i}_z, \\
\hat{h}_\eta \hat{i}_\eta &= \frac{\partial \hat{q}}{\partial \eta} = -r \sin \chi \sin \chi \hat{i}_x + r \sin \chi \sin \chi \cos \eta \hat{i}_y, \\
\hat{h}_\zeta \hat{i}_\zeta &= \frac{\partial \hat{q}}{\partial \zeta} = -r \sin \chi \hat{i}_0 + r \cos \zeta \sin \chi \cos \eta \hat{i}_x + r \cos \zeta \sin \chi \sin \eta \hat{i}_y + r \cos \zeta \cos \chi \hat{i}_z,
\end{align*}
\]

(E.8)

where

\[
\begin{align*}
\hat{h}_r &= \|\frac{\partial \hat{q}}{\partial r}\| = 1, \\
\hat{h}_x &= \|\frac{\partial \hat{q}}{\partial \chi}\| = r \sin \chi, \\
\hat{h}_\eta &= \|\frac{\partial \hat{q}}{\partial \eta}\| = r \sin \chi \sin \chi, \\
\hat{h}_\zeta &= \|\frac{\partial \hat{q}}{\partial \zeta}\| = r.
\end{align*}
\]

(E.9)

It can be noted from Eq. E.8 that \( \hat{i}_r, \hat{i}_x, \hat{i}_\eta, \) and \( \hat{i}_\zeta \) are orthogonal.
Orientations in 3D space using unit quaternions are specified by the variables \( \chi \), \( \eta \), and \( \zeta \) where \( r = 1 \). The element of volume in rotation space is obtained from the volume of the parallelepiped determined by the vectors[26]

\[
\begin{align*}
    d\tau &= (i_\chi \times i_\eta \cdot i_\zeta)h_\chi h_\eta h_\zeta d\chi d\eta d\zeta = r^3 \sin^2 \zeta \sin \chi d\chi d\eta d\zeta. \\
    (E.10)
\end{align*}
\]

Comparing Eqs. E.3 with Eq. E.5, we can equate \( \zeta = \tau/2 \). For the unit quaternion representing orientation, \( r = 1 \), and hence the Jacobian for integration is the volume of the parallelepiped representing an element in orientation space given by

\[
\begin{align*}
    d\tau &= \frac{1}{2} \sin^2 \left(\frac{\tau}{2}\right) \sin \chi d\chi d\eta d\tau. \\
    (E.11)
\end{align*}
\]
List of Figures

2-1 Scattering in a waveguide with only two modes. Each mode is composed of a downgoing and an upgoing planewave. Each incoming planewave is scattered by the object into various outgoing planewaves. The scattered field from each incident planewave are coherently superposed to form the total scattered field at the receiver. 46

2-2 Modal group velocity dispersion curves for a Pekeris waveguide comprising of a water column of speed $c_w$ overlying a bottom halfspace of speed $c_b$ that supports three propagating modes. 50

2-3 Group slowness dispersion curves for the modes of the Pekeris waveguide described in Fig. 2-2. 50

2-4 Sum of travel times of incident mode $n$ over range $\rho_0$ and scattered mode $m$ over range $\rho$ plotted as a function of frequency. At time $t_b < t < t_e$, contributions by incident $n$ and scattered $m$ mode pair to the scattered field occurs at discrete frequencies that vary in number from one (e.g. at $t_1$) to at most four (e.g. at $t_2$). For $t < t_b$, the wave packet has not arrived at the receiver. For $t > t_e$, the incident $n$ and scattered $m$ mode pair make no further contribution to the scattered field at the receiver. 51
The geometry of the waveguide which has a water column comprised of upper layer sound speed \( c_{w1} \) for \( 0 < Z < 25 \), lower layer sound speed \( c_{w2} \) for \( 35 < Z < 100 \), and transition layer sound speed \( c_{w1 - (c_{w1} - c_{w2})(Z - 25)/10} \) for \( 25 \leq Z \leq 35 \). The water column density is \( d_w = 1000 \text{ kg/m}^3 \) and the attenuation is \( \alpha_w = 6.0 \times 10^{-5} \text{ dB/} \lambda \). The bottom can have up to two sediment layers. The upper and middle sediment layers have respective thicknesses, sound speeds, densities and attenuations of \( h_1, c_{b1}, d_{b1}, \alpha_{b1} \), and \( h_2, c_{b2}, d_{b2}, \alpha_{b2} \), overlying a sediment half-space of sound speed \( c_{b3} \), density \( d_{b3} \) and attenuation \( \alpha_{b3} \). The monopole source is co-located with receiving array center, with array axis normal to the range-depth plane of the sketch. Source and receiver may be placed anywhere in the water column. The submerged target may be placed in the upper or lower layers of the water column where sound speed is constant as indicated in Fig. 3-3. Seafloor and buried riverbank features may also be included at the water-sediment and sediment-layer to sediment-half-space interfaces as indicated in Fig. 3-7. Squiggly lines indicate statistically rough interfaces.

The range and azimuth dependent resolution footprint of a horizontal line array. The range resolution is given by \( \Delta \rho = c/2B \) where \( B \) is the bandwidth of the signal. For a narrowband signal of duration \( T \) with rectangular window in time, the bandwidth is \( B = 1/T \). The cross-range resolution for the array steered to azimuth \( \varphi \) is \( \rho \Delta \varphi = \rho \frac{\lambda}{L \cos \varphi} \). The cross-range resolution for a line array is smallest at broadside where \( \varphi = 0 \) and it widens for off-broadside beams. The cross-range resolution is \( \rho \Delta \varphi = \sqrt{\frac{\lambda}{L}} \) at endfire where \( \varphi = \pi/2 \).
Three scenarios for the active detection of a submerged pressure release sphere of radius $a = 10$ m. The water column is modeled as either having constant sound speed or as downward refracting. The bottom is composed of either a pure sediment half-space or a single sediment layer over a sediment half-space. (a) Monopole source and horizontal receiving array center are co-located at 50-m depth with target at 50-m depth also. (b) Source and receiving array center are co-located at 10-m depth with target at 50-m depth. (c) Source and receiving array center are co-located at 10-m depth with target at 15-m depth.

The scattered field from a submerged pressure-release sphere of radius $a = 10$ m, at $f = 300$ Hz and center at 50-m depth, and Lambert-Mackenzie reverberation from the seafloor within the broadside resolution footprint of the monostatic system as a function of range for a water column with constant sound speed of 1500 m/s, i.e. $c_{w1} = c_{w2} = 1500$ m/s. Monopole source and receiving array center are co-located at 50-m depth. Range increases along the $x$-axis and depth along the $z$-axis, with the array axis along the $y$-axis. Source level is 0 dB re 1 $\mu$Pa @ 1 m. Reverb modeled with $T = 1/2$ sec duration cw source signal at 300 Hz and receiving array resolution $\lambda/L = 3.7^\circ$. (a) Pekeris waveguide examples for bottom half-spaces composed of either sand or silt, i.e. $h_1 = h_2 = 0$. (b) Bottom has a silt layer of either $h_1 = 2$ m or $h_1 = 5$ m overlying a sand half-space, and $h_2 = 0$. (c) Bottom has a sand layer of either $h_1 = 2$ m or $h_1 = 5$ m overlying a silt half-space, and $h_2 = 0$. Error bars show the 5.6 dB standard deviation in reverb level.
3-5 Same as Fig. 3-4 except water column is layered with $c_{w1}=1520$ m/s $c_{w2}=1500$ m/s and monostatic source-receiver as well as target sphere are at variable depth. Only the cases of pure sand or pure silt bottom half-spaces are shown. (a) Source-receiver and sphere center are at 50-m depth. (b) Source-receiver are at 10-m depth while sphere center is at 50-m depth. (c) Source-receiver are at 10-m depth while sphere center is at 15-m depth.

3-6 Pekeris waveguide with varying source-receiver and target depth. (a) Same as Fig. 3-4(a) except source-receiver is at 10-m depth and target is at 50-m depth. (b) Same as Fig. 3-4(a) except source-receiver is at 10-m depth and target is at 15-m depth.

3-7 Scenarios for the active detection of seafloor and sub-seafloor riverbank features. The water column is modeled as having constant sound speed, i.e. $c_{w1} = c_{w2} = 1500$ m/s, and monopole source and horizontal receiving array center are co-located at 50-m depth in all cases. (a) Bottom is sediment half-space with seafloor feature, i.e. $h_{1} = h_{2} = 0$. (b) Bottom is composed of a single sediment layer with double-interface seafloor feature, i.e. $h_{2} = 0$. (c) Bottom is composed of a single sediment layer with sub-seafloor feature, i.e. $h_{2} = 0$. (d) Bottom is composed of two sediment layers with double-interface sub-seafloor feature.
3-8 The field at $f = 300$ Hz scattered from a coherently scattering rectangular patch of area $100 \times 100$ m$^2$ representing a seafloor riverbank for scenario shown in Fig. 3-7(a), constant sound speed water column over pure silt or sand half-spaces. Range increases along the $x$-axis and depth along the $z$-axis. The square riverbank surface has two edges parallel to the $y$-axis, and is inclined 10$^\circ$ from the $x$-axis. Constant sound speed in the water column is assumed for all examples with $c_{w1} = c_{w2} = 1500$ m/s. Lambert-Mackenzie reverberation within the range-dependent resolution footprint of the monostatic system is also shown separately for the water-sediment interface (seafloor). Source level is 0 dB re 1 $\mu$Pa @ 1 m. Diffuse reverb modeled with $T = 1/2$ s duration cw source signal at 300 Hz and receiving array resolution $\lambda/L = 3.70^\circ$.

3-9 Same as Fig. 3-8 except single layer bottom scenarios of Fig. 3-7(b)-(c) for coherent seafloor and sub-seafloor riverbank scattering are investigated. Lambert-Mackenzie diffuse seafloor reverberation within the sonar resolution footprint is also shown for the sediment-layer to sediment half-space interface (sub-seafloor). (a) Seafloor riverbank with the upper sediment layer composed of silt with $h_1 = 2$ m and the lower sediment half-space composed of sand. Coherent riverbank scattering is from the double interface of water to silt to sand. (b) Sub-seafloor riverbank with the upper sediment layer composed of silt with $h_1 = 2$ m and the lower sediment half-space composed of sand. Coherent riverbank scattering is from the single silt to sand interface. (c) Seafloor and sub-seafloor riverbanks as in (a) and (b) but with the upper sediment layer now at $h_1 = 5$ m thickness. (d) Same as (c) but with the upper sediment layer composed of sand with $h_1 = 2$ m and the lower sediment half-space composed of silt. (e) Same as (d) except $h_1 = 5$ m.
3-10 Same as Fig. 3-9 except two-layer bottom scenario of Fig. 3-7(d) is investigated for coherent scattering from sub-seafloor riverbank. (a) Sediment is comprised of light silt layer of $h_1$=1 m thickness over a silt layer of $h_2$=1 m thickness over a sand half-space. Coherent riverbank scattering is from the double interface of light silt to silt to sand. (b) Sediment is comprised of light silt layer of $h_1$=1 m thickness over a sand layer of $h_2$=1 m thickness over a silt half-space. Coherent riverbank scattering is from the double interface of light silt to sand to silt.

3-11 Magnitudes of the coherent scattering functions $|S(\alpha, \beta = \pi; \alpha_i, \beta_i = 0)|$, i.e. $20 \log |S|$ dB, for the 100-m by 100-m seafloor and sub-seafloor riverbank features at inclination $\chi = 10^\circ$ of Fig. 3-7 over bistatic horizontal grazing angle, $\pi/2 - \alpha_i$ for the incident and $\alpha - \pi/2$, for the scattered wave, as appropriate to backscatter in a waveguide. The boxes include all modes $n$ where $0.5 \text{ rad/km} > \text{Im}\{\xi_n\}$. This includes all and only trapped modes for the Pekeris waveguide scenario of Fig. 3-7(a). (a) Reflection coefficient for water to sand is used for scenario of Fig. 3-7(a). (b) Reflection coefficient of silt to sand is used for scenario of Fig. 3-7(c). (c) Double reflection coefficient of water to 2-m silt layer over sand is used for scenario of Fig. 3-7(b). (d) Double reflection coefficient of water to 5-m silt layer over sand is used for scenario of Fig. 3-7(b). (e) Double reflection coefficient of light silt to 1-m silt layer over sand is used for scenario of Fig. 3-7(d).

3-12 Same as Fig. 3-11 except (a) reflection coefficient for water to silt is used for scenario of Fig. 3-7(a). (b) Reflection coefficient of sand to silt is used for scenario of Fig. 3-7(c). (c) Double reflection coefficient of water to 2-m sand layer over silt is used for scenario of Fig. 3-7(b). (d) Double reflection coefficient of water to 5-m sand layer over silt is used for scenario of Fig. 3-7(b). (e) Double reflection coefficient of light silt to 1-m sand layer over silt is used for scenario of Fig. 3-7(d).
3-13 The horizontal wavenumber's imaginary component \( \text{Im}\{\xi_n\} \) is plotted as a function of horizontal grazing angle, \( |\pi/2 - \alpha_i| \), for the various waveguides considered. Proper modes occur in Pekeris below the critical angle for \( 0.5 \text{ rad/km} > \text{Im}\{\xi_n\} \). (a) Pekeris with sand bottom and Pekeris with silt bottom. (b) Constant water column sound speed of 1500 m/sec over \( h_1=2\)-m and \( h_1=5\)-m silt layer over sand half-space. (c) Constant water column sound speed of 1500 m/sec over \( h_1=2\)-m and \( h_1=5\)-m sand layer over silt half-space. (d) Constant water column sound speed of 1500 m/s over 1-m light silt layer over 1-m silt layer over sand half-space and Constant water column sound speed of 1500 m/s over 1-m light silt layer over 1-m sand layer over silt half-space.

3-14 Same as Figs. 3-8 to 3-9 except Lambert-Mackenzie model is used to model scattering from inclined riverbank features. (a) Seafloor riverbank over sand and silt half-spaces. (b) Seafloor and sub-seafloor riverbank scattering with the upper sediment layer composed of silt with \( h_1=2\) m and the lower sediment half-space composed of sand. (c) Same as (b) except \( h_1=5\) m. (d) Seafloor and sub-seafloor riverbank scattering with the upper sediment layer composed of sand with \( h_1=2\) m and the lower sediment half-space composed of silt. (e) Same as (d) except \( h_1=5\) m.

3-15 Scattering strength in free-space backscatter as a function of surface grazing angle for a diffusely scattering surface obeying Lambert-Mackenzie and first order perturbation theory scattering laws. The first order perturbation theory curves are for cases where the planewave is incident from an upper to a lower medium, where the upper and lower media can be either are water, sand or silt.
Scattering Strength $SS(\alpha, \beta = \pi; \alpha_i, \beta_i = 0)$, based on first order perturbation theory for the seafloor and sub-seafloor riverbank features at inclination $\chi = 10^\circ$ of Fig. 3-7 over bistatic horizontal grazing angle, $\pi/2 - \alpha_i$ for incident and $\alpha - \pi/2$ for scattered wave, as appropriate to backscatter in a waveguide. The boxes include all modes $n$ where $0.5 \text{ rad/km} > \text{Im}\{\xi_n\}$. This includes all and only trapped modes for the Pekeris waveguide scenario of Fig. 3-7(a). (a) Reflection coefficient for water to sand is used for scenario of Fig. 3-7(a). (b) Reflection coefficient of silt to sand is used for scenario of Fig. 3-7(c). (c) Double reflection coefficient of water to 2-m silt layer over sand is used for scenario of Fig. 3-7(b). (d) Double reflection coefficient of water to 5-m silt layer over sand is used for scenario of Fig. 3-7(b). (e) Double reflection coefficient of light silt to 1-m silt layer over sand is used for scenario of Fig. 3-7(d).

Same as Fig. 3-16 except (a) reflection coefficient for water to silt is used for scenario of Fig. 3-7(a). (b) Reflection coefficient of sand to silt is used for scenario of Fig. 3-7(c). (c) Double reflection coefficient of water to 2-m sand layer over silt is used for scenario of Fig. 3-7(b). (d) Double reflection coefficient of water to 5-m sand layer over silt is used for scenario of Fig. 3-7(b). (e) Double reflection coefficient of light silt to 1-m sand layer over silt is used for scenario of Fig. 3-7(d).
3-18 Scattering Strength $SS(\alpha, \beta = \pi; \alpha_i, \beta_i = 0)$ based on first order perturbation theory for level seafloor, $\chi = 0^\circ$, over bistatic horizontal grazing angle, $\pi/2 - \alpha_i$ for the incident and $\alpha - \pi/2$ for the scattered wave, as appropriate to backscatter in a waveguide. The boxes include all modes $n$ where $0.5 > \text{Im}\{\zeta_n\}$. This includes all and only trapped modes for the Pekeris waveguide scenario of Fig. 3-7(a). (a) Reflection coefficient for water over sand is used. (b) Double reflection coefficient of water over 2-m silt over sand is used. (c) Double reflection coefficient of water over 5-m silt layer over sand is used. (d) Triple reflection coefficient of water over 1-m light silt layer over 1-m silt layer over sand is used.

3-19 Same as Fig. 3-18 except (a) reflection coefficient for water over silt is used. (b) Double reflection coefficient of water over 2-m sand over silt is used. (c) Double reflection coefficient of water over 5-m sand layer over silt is used. (d) Triple reflection coefficient of water over 1-m light silt layer over 1-m sand layer over silt is used.

3-20 Same as Figs. 3-8 to 3-9 except first order perturbation theory is used to model scattering from the inclined riverbank features. (a) Seafloor riverbank over sand and silt half-spaces. (b) Seafloor and sub-seafloor riverbank scattering with the upper sediment layer composed of silt with $h_1=2$ m and the lower sediment half-space composed of sand. (c) Same as (b) except $h_1=5$ m. (d) Seafloor and sub-seafloor riverbank scattering with the upper sediment layer composed of sand with $h_1=2$ m and the lower sediment half-space composed of silt. (e) Same as (d) except $h_1=5$ m.
3-21 Same as Fig. 3-10 except first order perturbation theory is used to model scattering from the inclined sub-seafloor riverbank for the two-layer bottom scenario of Fig. 3-7(d). (a) Sediment is comprised of light silt layer of \( h_1 = 1 \)-m thickness over a silt layer of \( h_1 = 1 \)-m thickness over a sand half-space. Diffuse riverbank scattering is from the double interface of light silt to silt to sand. (b) Sediment is comprised of light silt layer of \( h_1 = 1 \)-m thickness over a sand layer of \( h_2 = 1 \)-m thickness over a silt half-space. Diffuse riverbank scattering is from the double interface of light silt to sand to silt.

3-22 Target-signal-to-diffuse-reverberation ratio (SRR) for a 10-m radius pressure-release sphere in the downward refracting waveguides with (a) sand, and (c) silt halfspaces. The geometry of the measurement is shown in Fig. 3-1(a) with the source-receiver at 50 m depth and the sphere target at 50 m depth in the water column. The target signal and diffuse reverberation intensity is obtained from Fig. 3-5(a) for computing the SRR. POD and PFA for detection of the sphere in the downward refracting waveguide with (b) sand, and (d) silt halfspaces.

3-23 Target-signal-to-diffuse-reverberation ratio (SRR) for a 10-m radius pressure-release sphere in the downward refracting waveguides with (a) sand, and (c) silt halfspaces. The geometry of the measurement is shown in Fig. 3-1(b) with the source-receiver at 10 m depth and the sphere target at 50 m depth in the water column. The target signal and diffuse reverberation intensity is obtained from Fig. 3-5(b) for computing the SRR. POD and PFA for detection of the sphere in the downward refracting waveguide with (b) sand, and (d) silt halfspaces.
3-24 Target-signal-to-diffuse-reverberation ratio (SRR) for a 10-m radius pressure-release sphere in the downward refracting waveguides with (a) sand, and (c) silt halfspaces. The geometry of the measurement is shown in Fig. 3-1(c) with the source-receiver at 10 m depth and the sphere target at 15 m depth in the water column. The target signal and diffuse reverberation intensity is obtained from Fig. 3-5(c) for computing the SRR. POD and PFA for detection of the sphere in the downward refracting waveguide with (b) sand, and (d) silt halfspaces.

3-25 The field scattered from a coherently scattering rectangular patch of area \(100 \times 100\) m\(^2\) representing a subseafloor riverbank for scenario shown in Fig. 3-7(d) for two different waveforms, a 1/2 s duration tukey shaded LFM signal from 390 to 440 Hz and a 1/2 s duration CW tone at 415 Hz. The environment consists of a water column with constant sound speed \(c_w = 1500\) m/s overlying a bottom composed of two sediment layer over a halfspace. (a) Sediment is comprised of light silt layer of \(h_1 = 1\) m thickness over a silt layer of \(h_2 = 1\) m thickness over a sand half-space. Coherent riverbank scattering is from the double interface of light silt to silt to sand. (b) Sediment is comprised of light silt layer of \(h_1 = 1\) m thickness over a sand layer of \(h_2 = 1\) m thickness over a silt half-space. Coherent riverbank scattering is from the double interface of light silt to sand to silt. Range increases along the \(x\)-axis and depth along the \(z\)-axis. The square riverbank surface has two edges parallel to the \(y\)-axis, and is inclined 10° about the \(x\)-axis. Lambert-Mackenzie reverberation within the range-dependent resolution footprint of the monostatic system is also shown separately for the water-sediment interface (seafloor). Source level is 0 dB re 1 \(\mu\)Pa @ 1 m. Diffuse reverberation modeled with the two different source waveforms, a \(T = 1/2\) s duration tukey shaded LFM from 390 to 440 Hz and a \(T = 1/2\) s CW tone at 415 Hz and receiving array resolution \(\lambda/L = 3.70^\circ\). The LFM signal with \(B = 50\) Hz bandwidth has a smaller (better) range resolution.
3-26 Signal-to-diffuse reverberation ratio (SRR) and probabilities for detection (POD) and false alarm (PFA) for the detection of the subseaflloor riverbank in Fig. 3-25(a) in the environment were the bottom is composed of $h_1=1$ m thick light silt layer over a $h_2=1$ m thick silt layer over a sand half-space. The detection scenario is investigated in (a) and (b) for the $T=1/2$ s CW signal and in (c) and (d) for the $T=1/2$ s LFM signal from 390-440 Hz. The broadband waveform leads to higher SRR and larger POD over longer ranges than the CW tone.

3-27 Similar to fig. 3-26 except that the bottom is composed of $h_1=1$ m thick light silt layer over a $h_2=1$ m thick sand layer over a silt half-space.

3-28 Distribution of average fish density within a school (individuals/m$^2$) against the school radius from measurements made at the New Jersey Strataform area[55, 54].

3-29 Figures (a) and (b) show the field scattered incoherently from schools of fish at 80-m water depth contained within the sonar resolution footprint in the environments similar to Fig. 3-25 (a) and (b) respectively. The array configuration and source-receiver geometry are similar to that in Fig. 3-25. The results are plotted for the mean density of fish and the maximum density of fish within the sonar resolution footprint. These densities are based on measurements of fish school distribution at the New Jersey Strataform area reported in Ref. [55] and shown in Fig. 3-28.

4-1 Figure taken from Ref. [21] showing the New Jersey continental shelf in the vicinity of the Strataform area highlighted in gray. The depth contours are in meters.
4-2 Bathymetry of the Strataform area sampled at 30-m interval. Candidate features identified from previous geophysical survey\cite{14, 21} that would give prominent and coherent scattered returns includes incised or buried river channels (green), relict ice-berg scours and erosion pits on the seafloor (blue), and surface or near surface expression of seismically reflective subsurface strata within the seabed (white) called "R-reflector". Coordinates of south-west corner: 38° 53.00'N, 73° 15.00'W.

4-3 Figure taken from Ref. \cite{14} showing a seismic profile and line drawing interpretation of the bottom stratigraphy surveyed at the New Jersey Strataform area within site 1. The figure shows numerous buried river channels intersected by highly reflective sub-bottom strata.

4-4 A sketch of NRV Alliance towing a two element vertical source array at 1.875 m spacing and a 256 element horizontal receiving array with spacing between elements of 0.5, 1 and 2m. Only data from 128 elements of the receiving array at 1-m spacing is analyzed in this paper. The mean depth of the NRV Alliance two-element source was varied between 23 to 65 m. The mean depth of the receiver array was varied from 21.5 to 68.5 m. RV Endeavour was moored at each site where measurements were collected and it deployed the MACE source system consisting of a 7 element array spaced at 1.625-m. The mean depth of the MACE source was varied between 35.4 and 83.8-m and it was beamed to transmit at broadside throughout the experiment.
Directional derivative of bathymetry at the Strataform site with respect to a source in the north. The seafloor is mostly level locally with slopes of $< 1/20^\circ$. There are very few discrete features such as ice-berg scours and erosion pits on the seafloor with slopes of at most $3^\circ$. Acoustic transmissions are centered about three distinct sites in the Strataform area. Overlain are the tracks traversed by NRV Alliance (white lines), mooring locations of RV Endeavour (red stars), location of the two calibrated targets (white stars) and sub-bottom features (blue and pink lines). Coordinates of south-west corner: 38° 53.00'N, 73° 15.00'W.

A bistatic wide area image along track 17 at site 2. Travel time to range conversions are done by multiplying the two-way travel time with half the mean sound speed of 1475-m/s. All returns are mirrored about the array axis ($71^\circ$) due to right-left ambiguity. Two prominent and discrete scattering events $> 20$ dB above the background co-register well with the location of the calibrated targets approximately 8.5-km to the south of the source and receiving array. Numerous other prominent scattering events from features are also observed that can be confused with returns from the calibrated targets. Comparison with Fig. 4-7 breaks the receiver line array's right-left ambiguity and places the true location of these features to the south within the dotted white trapezoid. (Track 17, file:hla20011211130425, transmission: 1s duration LFM from 390 to 440-Hz. Mean source depth: 55-m, mean receiver depth: 42-m, array axis: $71^\circ$. Coordinates of south-west corner: 38° 55.80'N, 73° 13.42'W.)

Similar to Fig. 4-6 except that data is from track 14. Right-left ambiguity of the prominent scattering events from the features of interest can be resolved by comparing this figure with Fig. 4-6. (Track 14, file: hla2001121113455, transmission: 2s duration LFM from 390 to 440-Hz. Mean source depth: 55-m, mean receiver depth: 22-m, array axis: 225.5°. Coordinates of south-west corner: 38° 55.80'N, 73° 13.42'W.)
4-8 Hotspot consistency chart from 49 bistatic LFM transmissions along Track 17. The results are plotted as $10 \log N$ where $N$ is the number of images that register a strong scattering event $> 10$ dB above the local average within $1.8 \times 1.8$-km$^2$ area of a given pixel location. Scattering events within the trapezoid as well as those from the calibrated targets are consistently prominent in most of the reverberation charts along track 17. Events within the top half of the trapezoid are discrete and target-like in appearance and they may be confused with those from the calibrated targets. The hotspot event in the lower half of the trapezoid is large, extending over 3-km, and is less likely to be confused with a discrete target like an underwater vehicle, but it can camouflage an underwater vehicle from detection if the vehicle were to be located within it. (Mean source depth: 55-m, mean receiver depth: 42-m, array axis: 71°. Coordinates of south-west corner: 38° 55.79’N, 73° 13.44’W.)

4-9 Prominent events from LFM transmissions on Track 17 shown in Fig. 4-8 are overlain in white on the directional derivative of the bathymetry calculated with respect to the source location and the receiver location in the middle of track 17. Only prominent ($> 10$-dB above local average) and repeatable events that occur in at least 10 charts out of 49 are overlain. Scattering events of interest within the trapezoid do not originate from the seafloor because the seafloor in this region is level. High resolution sub-bottom profiling revealed the existence of a dense network of buried river channels that coincide with the location of the discrete scattering events in the top half of the trapezoid as shown in Figs. 4-10 and 4-11. Coordinates of south-west corner: 38° 55.80’N, 73° 13.42’W.)
4-10 Location of buried river channels at site 2 mapped out by the August 2001 geophysical survey[1] using chirp sub-bottom profiling systems is overlain on the hotspot consistency chart for Track 17. The channels have varying sizes, from small (in white stars) to medium and large (pink stars). Some of these channels at site 2 come right up close to the water-sediment interface and are less deeply buried than those at site 1. Coordinates of south-west corner: 38° 55.58'N, 73° 8.77'W.

4-11 Deep-tow sub-bottom chirp profile from the geophysical survey of August 2001[1] at site 2 showing three buried river channels in the vicinity of the prominent and discrete scattering events in the top half of the trapezoid of Fig. 4-9. These river channels are located between 39° 1.3684'N, 73° 2.9619'W and 39° 2.2878'N, 73° 2.9624'W.

4-12 Similar to Fig. 4-8 but for 15 bistatic transmissions along track 23x. Data for this track was collected on the same day as track 17. Scattering events are located within the same trapezoidal region as that on tracks 17 and 14 shown in Figs. 4-6-4-9. (Mean source depth: 55-m, mean receiver depth: 43-m, array axis: 209°. Coordinates of south-west corner: 38° 55.80'N, 73° 13.44'W.)
4-13 Similar to Fig. 4-6 except that the image is from (a) track 1c (b) track 85 at site 1. “Glints” from walls of the buried river channels can be observed in both figures. These originate from channel sections that are less deeply buried under the water-sediment interface. Compare with Fig. 4-14(d). “Glints” in the images from track 85 fade in and out along a track depending on the bistatic orientation. ((a) Track 1c, file:hla2001118160015, transmission: 1s duration LFM from 390 to 440-Hz. Mean source depth: 46.1-m, mean receiver depth: 36.5-m, array axis: 285°. Coordinates of south-west corner: 39° 13.06′ N, 72° 54.97′ W. (b) Track 85, file:hla2001124192958, transmission: 1s duration LFM from 390 to 440-Hz. Mean source depth: 40.9-m, mean receiver depth: 39-m, array axis: 293°. Coordinates of south-west corner: 39° 13.98′ N, 72° 56.09′ W.)

4-14 Similar to Fig. 4-8 but for (a) 87 bistatic transmissions along track 85 with array heading 293°, (b) 79 bistatic transmissions along track 86 with array heading 97°. (c) 12 monostatic transmissions along track 12 with array heading 228.6°. Prominent scattering events coincides with axis of buried river channels that have been previously characterized.[14, 21] The occurrence and level of returns depend on bistatic orientation with the strongest and most frequent target-like returns coinciding with features that project the largest area in the direction of source and receiving array. Less deeply buried river channel sections are also much better imaged than deeply buried ones. RV Endeavour imaged monostatically in (c) as shown by the red star. (d) Depth of buried river channels below water-sediment interface. [20] Shallower, less deeply buried segments are better imaged than more deeply buried ones. Compare (d) with (a) and (b). Coordinates of south-west corner: (a) 39° 12.95′ N, 72° 58.11′ W, (b) 39° 13.32′ N, 72° 56.88′ W, (c) 39° 14.20′ N, 72° 59.64′ W (d) 39° 12.69′ N, 72° 59.82′ W.
4-15 (a) and (b) are similar to Fig. 4-6 except that data is from tracks 61 and 62 respectively at site 3. Scattered returns from surface expression of seismically reflective horizon called R-reflector is imaged out to ranges of 30-km and extends further from what is indicated from previous geophysical surveys. Comparing Figs. (a) and (b) breaks the receiver line array’s right-left ambiguity and places the true location of the R-reflector’s surface expression to the south-west. ((a): Track 61, file:hla2001122121135, transmission: 2s duration LFM from 390 to 440-Hz. Mean source depth: 83.8-m, mean receiver depth: 60-m, array axis: 357.5°.) (b) Track 62, file:hla2001122160815, transmission: 2s duration LFM from 390 to 440-Hz. Mean source depth: 83.8-m, mean receiver depth: 34-m, array axis: 133°. Coordinates of south-west corner in (a) and (b): 38° 59.17′N, 72° 59.07′W.)

4-16 Monostatic wide area images along track 3 at site 1. Moderately strong lineated returns from the near surface expression of the R-reflector extending 3km outward from the array axis can be observed. More prominent and discrete echos appear in regions where buried river channels cross the R-reflector in the north-east of the array axis. The two white dotted lines show the approximate region where the R-reflector approaches the water sediment interface. (Track 3, (a) file:hla2001119183615, transmission: 0.5s duration, (b) file:hla2001119183935, transmission: 2s duration, LFM from 390 to 440-Hz. Mean source depth: 30-m, mean receiver depth: 36-m, array axis: 133°. Coordinates of south-west corner in (a) and (b): 39° 11.78′N, 72° 59.07′W.)
4-17 (a) Hotspot consistency charts from 15 bistatic LFM transmissions along track 61 and at site 3. The results are plotted as $10 \log N$ where $N$ is the number of images that register a strong scattering event $> 10$ dB above the local average within $2.7 \times 2.7$-km$^2$ area of a given pixel location. (b) Similar to (a) but for 21 bistatic transmissions along track 62 at site 3. Comparing Fig. (a) and (b) breaks the receiver line array’s right-left ambiguity. It registers the prominent scattering from the walls of the Hudson canyon located to the west in both figures, as well as scattering events from the ice-berg scour close to the location of the bistatic source. ((a) Track 61, mean source depth: 83.8-m, mean receiver depth: 60-m, array axis: 357.5°. (b) Track 62, mean source depth: 83.8-m, mean receiver depth: 34-m, array axis: 357.5°. Coordinates of south-west corner in (a) and (b): 38° 59.50'N, 72° 59.18'W.)

4-18 (a) Similar to Fig. 4-6 except that the image is from track 81 at site 1. (b) Similar to Fig. 4-8 but for 89 bistatic transmissions along track 81. Prominent and discrete scattering events are densely distributed throughout the image in (a) from individual transmissions, but few of these events are repeatable in (b). (Track 81, (a) file: hla2001124121318, transmission: 1s duration LFM from 390 to 440-Hz. (a) and (b) Mean source depth: 41-m, mean receiver depth: 35-m, array axis: 351°. Coordinates of south-west corner: (a) 39° 11.97'N, 72° 58.45'W, (b) 39° 11.97'N, 72° 58.47'W.)
4-19 Similar to Fig. 4-8 but for 46 bistatic transmissions along track 14. Prominent scattering events with levels 10-dB above the background co-register with the location of the calibrated targets along the full length of track 14. Scattering events originating from features within the trapezoid are not registered as consistently along the track 14. It should be noted that track 14 had a shallower receiver depth than track 17 in Fig. 4-8. This hotspot image shows that detection of scattering events from features is dependent on receiver depth and orientation.

(Mean source depth: 55-m, mean receiver depth: 22-m, array axis: 225.5°. Coordinates of south-west corner: 38° 54.11'N, 73° 15.11'W.)

4-20 (a) Similar to Fig. 4-6 and except that data is from track 18. (b) Similar to Fig. 4-8 but for 47 bistatic transmissions along track 18. Prominent events > 20 dB above the background are registered from the two calibrated targets approximately 8.5-km to the south of the source and receiving array. No distinguishable scattering events are visible within the trapezoidal region that had been observed along tracks 17 in Fig. 4-8 and part of track 14 in Fig. 4-19. These image illustrate the fact that scattering events from features is dependent on bistatic location and orientation of source and receiver. (Track 18, (a) file:hla2001120154635, transmission: 1s duration LFM from 390 to 440-Hz. (a) and (b) Mean source depth: 55-m, mean receiver depth: 42-m, array axis: 340°. Coordinates of south-west corner: (a) 38° 55.80'N, 73° 13.42'W, (b) 38° 55.79'N, 73° 13.44'W.)
4-21 (a) Similar to Fig. 4-6 except that image is from track 73 at site 1. (b) Similar to Fig. 4-8 but for 12 bistatic transmissions along track 73 at site 1. In (a) set of of prominent and discrete scattering events charted to the north-west and another set of more elongated events charted to south-east. Only one of these is the true scattered return from the features while the other is ambiguous. Comparing (a) and (b) shows that the events to the south-west are the true returns. (Track 73, (a) file:hla2001123162835, transmission: 2s duration LFM from 390 to 440-Hz. (a) and (b) Mean source depth: 40.9-m, mean receiver depth: 38-m, array axis: 219.5°. Coordinates of south-west corner: (a) 39° 8.88'N, 73° 3.48'W, (b) 39° 8.87'N, 73° 3.50'W.)

5-1 The geometry of the problem showing an object in a stratified medium composed of a water column of thickness $H$ overlying a bottom. The origin of the coordinate system is at the center of the object and the source is located at $(-x_0, 0, z_0)$. The screen has width $L$ and is semi-infinite in the $z$-direction penetrating into the bottom with an edge at the top of the water column.

5-2 The combined extinction Eq. (5.9) of all the modes, caused by a pressure release sphere of radius 10 m centered at 50 m depth, in a Pekeris waveguide composed of 100 m water with either sand or silt halfspace is plotted as a function of $x_0$, its range from a point source of frequency 300 Hz also placed at the same depth in the waveguide. The separation of the screen from the object is the same as that of the source from the object at each source to object range, $x = x_0$. (b) The incident intensity on the sphere Eq. (5.13). (c) The combined cross section of the sphere Eq. (5.14). Both the coherent and incoherent approximation of the quantities are plotted in each subfigure. Source level is 0 dB re 1 $\mu$Pa @ 1 m.
5-3 Incoherent combined cross section of a 10 m radius pressure release sphere at 300 Hz source frequency in a Pekeris waveguide with sand bottom halfspace, Pekeris waveguide with silt bottom halfspace, perfectly reflecting waveguide, and free space as a function of source to object range $x_0$. For this plot, $x = x_0$. In the waveguides, the source and sphere centre are located at 50 m water depth. The incoherent combined cross section is calculated using Eq. (5.14) by replacing the double sum over the modes with a single sum.

5-4 Modal amplitude at the source and target depth of 50 m in (a) Pekeris waveguide with sand halfspace, (b) Pekeris waveguide with silt halfspace, and (c) perfectly reflecting waveguide for a frequency of 300 Hz.

5-5 Extinction of the individual modes Eq. (5.10) of the waveguide by the 10 m radius pressure release sphere at 50 m water depth with the source separated from the sphere by (a) 1 km range in a Pekeris sand halfspace waveguide, (b) 25 km range in a Pekeris sand halfspace waveguide, (c) 1 km range in a Pekeris silt halfspace waveguide, (d) 25 km range in a Pekeris silt halfspace waveguide, and (e) 1 km range in a perfectly reflecting waveguide, (f) 25 km range in a perfectly reflecting waveguide. The source depth is also 50 m and the source frequency is 300 Hz. The screen measuring the extinction is separated the same distance from the object as the source in each case. Source level is 0 dB re 1 $\mu$Pa @ 1 m.

5-6 Modal cross section Eq. (5.16) at 300 Hz of the 10 m radius pressure release sphere at 50 m water depth for the extinction of the individual modes in a (a) Pekeris sand halfspace waveguide, (b) Pekeris silt halfspace waveguide, and (c) perfectly reflecting waveguide. We set $x = 0$ in Eq. (5.16) to remove the effect of absorption by the waveguide. The modal cross section of the sphere for mode 1 in each waveguide is almost equal to its free space cross section.
5-7 Incoherent combined cross section of a 10 m radius pressure release sphere at 300 Hz source frequency in a Pekeris waveguide with sand bottom halfspace, Pekeris waveguide with silt bottom halfspace, perfectly reflecting waveguide, and free space as a function of source to object range $x_0$. For this plot, $x = x_0$. In the waveguides, the source and sphere centre are located at 52.5 m water depth. The incoherent combined cross section is calculated using Eq. (5.14) by replacing the double sum over the modes with a single sum. .......................... 192

5-8 Modal amplitude at the object depth of 52.5 m in (a) Pekeris waveguide with sand halfspace, (b) Pekeris waveguide with silt halfspace, and (c) perfectly reflecting waveguide for a frequency of 300Hz. .............. 193

5-9 Modal cross section Eq. (5.16) at 300 Hz of the 10 m radius pressure release sphere at 52.5 m water depth for the extinction of the individual modes in a (a) Pekeris sand halfspace waveguide, (b) Pekeris silt half-space waveguide, and (c) perfectly reflecting waveguide. We set $x = 0$ in Eq. (5.16) to remove the effect of absorption by the waveguide. The modal cross section of the sphere for mode 1 in each waveguide is almost equal to its free space cross section. ......................... 194

5-10 Incoherent combined cross section of a hard disk of radius 10 m at 300 Hz source frequency in a Pekeris waveguide with sand bottom halfspace, Pekeris waveguide with silt bottom halfspace, perfectly reflecting waveguide, and free space as a function of source to object range $x_0$. For this plot, $x = x_0$. In the waveguides, the source and disk center are located at 50 m water depth with the disk aligned in the $y - z$ plane. The incoherent combined cross section is calculated using Eq. (5.14) by replacing the double sum over the modes with a single sum. ... 196
5-11 Modal cross section Eq. (5.16) at 300 Hz of the 10 m radius hard disk at 50 m water depth for the extinction of the individual modes in a (a) Pekeris sand halfspace waveguide, (b) Pekeris silt halfspace waveguide, and (c) perfectly reflecting waveguide. We set \( x = 0 \) in Eq. (5.16) to remove the effect of absorption by the waveguide. The modal cross section of the disk for mode 1 in each waveguide is almost equal to its free space cross section.

5-12 Modal cross section Eq. (5.16) of a pressure release sphere at 50 m water depth for the extinction of the individual modes in a Pekeris sand halfspace waveguide with (a) sphere radius 0.1 m, 300 Hz source frequency, \( ka = 0.1 \), (b) sphere radius 1 m, 300 Hz source frequency, \( ka = 1.3 \), (c) sphere radius 10 m, 300 Hz source frequency, \( ka = 12.6 \), and (d) sphere radius 10 m, 1500 Hz source frequency, \( ka = 62.8 \). We set \( x = 0 \) in Eq. (5.16) to remove the effect of absorption by the waveguide. Only the propagating modes are illustrated in each plot. The modal cross section of the sphere for mode 1 in each case is almost equal to its free space cross section.

5-13 Same as Fig. 5-12, but in a Pekeris silt waveguide.

5-14 Same as Fig. 5-12, but in a perfectly reflecting waveguide.

5-15 The magnitude of the free space planewave scatter function \( S(\alpha, \phi = 0^\circ, \alpha_i = 90^\circ, \phi_i = 0^\circ) \) is plotted as a function of \( \alpha \), the elevation angle for a pressure release sphere at (a) \( ka = 0.1 \), (b) \( ka = 1.3 \), (c) \( ka = 12.6 \), and (d) \( ka = 62.8 \). The forward scatter peak is at \( \alpha = 90^\circ \).

7-1 Scattering in free space. Scattering by the object depends only on the direction of the source and receiver relative to the target in free space.
The backscattered field from an upright 10-m radius rigid circular disc at 300 Hz in Pekeris waveguides with (a) silt, (b) sand, and (c) perfectly reflecting bottoms respectively, calculated using the waveguide scattering model, Eq. (2.1), and compared to the sonar equation, Eq. (7.4). The water depth is 100 m with source-receiver and object at 50-m depth in the middle of the water column. Range increases along the x-axis and depth along the z-axis. The circular disc is aligned with its plane normal to the x-axis. The results are plotted in decibels, i.e., $20 \log |\Phi_s|$, as a function of increasing range between object and monostatic source-receiver. Source strength is 0 dB re 1 $\mu$Pa @ 1m. $ka$ is 12.6 for this example. (d) The ratio $2A\phi/(\lambda/L)$ for the examples given in (a) and (b). For the perfectly reflecting waveguide $2A\phi/(\lambda/L)$ is 12.6. The sonar equation provides a good approximation to the scattered field in the waveguide when $2A\phi/(\lambda/L) < 1$.

Similar to Fig. 7-2(b) except the scattered field in the back azimuth is averaged over depth throughout the water column from 0-m to 100-m.

The magnitude of the planewave scatter function $20 \log |S(\alpha, \beta, \alpha_i = 90^\circ, \beta_i = 0^\circ)|$ for an upright rigid circular disc of $ka = 12.6$ is plotted as a function of horizontal grazing $(90^\circ - \alpha)$ and azimuth $\beta$ angles of scattered planewaves for an incident planewave travelling in the direction $(90^\circ - \alpha_i) = 0^\circ, \beta_i = 0^\circ$. The scatter function is anti-symmetric about the plane of the disk and as can be seen from Eq. (7.7).
The magnitude of the planewave scatter function $|S(\alpha, \beta, \alpha_i, \beta_i = 0^\circ)|$ for an upright rigid circular disc of $ka = 12.6$ is plotted as a function of horizontal grazing angle $(90^\circ - \alpha)$ of scattered planewaves in the back scatter azimuth $\beta = 180^\circ$ for several incident planewaves with horizontal grazing angles $(90^\circ - \alpha_i) = 0^\circ, 30^\circ, 60^\circ,$ and $90^\circ$. The solid curve in this figure for broadside incidence ($(90^\circ - \alpha_i) = 0^\circ$) is a slice through Fig. 7-4 at the backscatter azimuth $\beta = 180^\circ$ of the scattered planewaves. The width $\lambda/L$ of the scatter function main lobe for broadside incidence is $14.3^\circ$ or 0.25 radians. Also shown is the critical grazing angle $\psi_c$ of the seabed in the Pekeris silt waveguide of $9.3^\circ$, Pekeris sand waveguide of $28^\circ$ and the perfectly reflecting waveguide of $90^\circ$.

Similar to Figs. 7-2(a)-(c) but for an upright rigid circular disk of (a) high and (b) low $ka$ in the Pekeris silt waveguide, (c) high and (d) low $ka$ in the Pekeris sand waveguide, and (a) high and (b) low $ka$ in the perfectly reflecting waveguide, The high $ka$ case corresponds to a disc of 10-m radius at 1500-Hz with $ka = 62.8$ while the low $ka$ case corresponds to a disc of 1-m radius at 300-Hz with $ka = 1.3$.

Figures (a) and (b) are similar to Fig. 7-2d but for $ka$ of 62.8 and 1.3 respectively for the cases shown in Figs. 7-6(a)-(d) in the Pekeris silt and sand waveguides. The ratio $2\Delta\psi/(\lambda/L) = 62.8$ for Fig. 7-6(e) with $ka$ of 62.8 and $2\Delta\psi/(\lambda/L) = 1.3$ for Fig. 7-6(f) with $ka$ of 1.3 in the perfectly reflecting waveguide.

Similar to Fig. 7-5 but for an upright rigid circular disk of (a) $ka = 62.8$, and (b) $ka = 1.3$. The width $\lambda/L$ of the scatter function main lobe for broadside incidence ($(90^\circ - \alpha_i) = 0^\circ$) is $2.9^\circ$ or 0.05 radians for $ka = 62.8$ and $143^\circ$ or 2.5 radians for $ka = 1.3$.

Figures (a)-(c) are similar to Figs. 7-2(a)-(c) but for an upright rigid square plate of sides 20 m by 20 m at 300Hz with $ka = 12.6$. 

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7-10 Similar to Fig. 7-4 but for an upright rigid square plate of $ka = 12.6$.

The scatter function is anti-symmetric about the plane of the plate and as can be seen from Eq. (7.9).

7-11 Similar to Fig. 7-5 but for an upright rigid square plate of $ka = 12.6$.

The solid curve in this figure for broadside incidence ($(90^\circ - \alpha_i) = 0^\circ$) is a slice through Fig. 7-10 at the backscatter azimuth $\beta = 180^\circ$ of the scattered planewaves. The width $\lambda/L$ of the scatter function main lobe for broadside incidence is $14.3^\circ$ or 0.25 radians.

7-12 Similar to Figs. 7-2 (a)-(c) but for a 10-m radius pressure-release sphere at 300-Hz in the (a) back and (b) forward azimuths in Pekeris silt waveguide, the (c) back and (d) forward azimuths in Pekeris sand waveguide, and the (e) back and (f) forward azimuths in the perfectly reflecting waveguide. $ka$ is 12.6 for this example.

7-13 Similar to Fig. 7-4 but for a pressure-release sphere of $ka = 12.6$. The scatter function for the sphere is given by Eqs. (8,9) of Ref. [42] with $f(n)$ replaced by $(-1)^n f(n)$ to convert from Ingenito’s definition to the standard one described in Ref. [48]. It can also be obtained from Ref. [5].

7-14 Figures (a) and (c) are similar to Fig. 7-5 but for a pressure-release sphere of $ka = 12.6$ plotted in the back ($\beta = 180^\circ$) and forward ($\beta = 0^\circ$) azimuths respectively of the scattered planewaves. The solid curves in Fig. 7-14 (a) and (c) for $(90^\circ - \alpha_i) = 0^\circ$ are slices through Fig. 7-13 in the back and forward scatter azimuths. The width $\lambda/L$ of the scatter function main lobe is $14.3^\circ$ or 0.25 radians in Fig. 7-14(c). The phase of the scatter function for the solid curve (horizontal incidence, $(\alpha_i - 90^\circ) = 0^\circ$) in (a) and (b) is shown in (b) and (d) respectively which are in the back and forward scatter azimuths.
7-15 Similar to Fig. 7-2(a) but for a “single object” comprised of two point scatterers with separation of (a) \( L = 2 \text{ m} \) and (b) \( L = 10 \text{ m} \) respectively at 1500Hz. (c) The ratio \( 2\Delta\psi/(\lambda/2L) \) for the examples given in (a) and (b). The sonar equation provides a good approximation to the scattered field in the waveguide when \( 2\Delta\psi/(\lambda/2L) < 1 \).

7-16 Projected areas of an upright 10-m radius rigid circular disc, a 10-m radius pressure-release sphere and a pressure-release prolate spheroid of aspect ratio 2 with a major axis of 40-m and a minor axis of 20-m are plotted as a function of horizontal grazing angle \( |\alpha_i - 90^\circ| \) of the incident planewaves. The spheroid is aligned such that the incoming planewaves are incident at the bow aspect where \( ka = 12.6 \).

7-17 The forward scattered field at 300 Hz calculated using the waveguide scattering model, Eq. (2.1), from a 10-m radius pressure release sphere is compared to that from a rigid circular disc of radius 10-m in (a) Pekeris silt, (b) Pekeris sand, and (c) perfectly reflecting waveguides. \( ka \) is 12.6 for these examples. The geometry of the set-up is similar to Figs. 7-2 and 7-12, except that the receiver is in the forward azimuth.

7-18 Similar to Fig. 7-16 but at 1500 Hz. \( ka \) is 62.8 for these examples.

8-1 Square plate of sides \( L \) lying in the \( x - y \) plane with surface normal to the \( z \) axis and sides parallel to the \( x \) and \( y \) axis. Also drawn are the elevation \( \alpha \) and azimuth \( \beta \) angles of a scattered planewave with wavevector \( k \).

8-2 Geometry for 2D rotation. Square plate of sides \( L \) lying in the \( x - y \) plane with surface normal to the \( z \) axis. The plate is rotated by \( \tau \) radians about the \( x \) axis in the \( x - y \) plane.
Incoherent 2D average scatter function of the rigid square plate is compared to that of the rigid circular disc for (a) \( kL/2 = 1 \), (b) \( kL/2 = 10 \) and (c) \( kL/2 = 20 \) where \( \alpha_i = \beta_i = \beta = 0 \). The square plate is rotated in 2D about the axis normal to the surface of the plate, generating the shape of a circular disc in the process. The incoherent average scatter function for the square plate is obtained using Eq. 8.5. The circular disc scatter function is plotted in the figure using Eq. 8.7 with \( Q = 4/\pi \) and \( P = 1.6^{1/4} \).

Similar to Fig. 8-3 but for the coherent average of the scatter function of the square plate in 2D calculated using Eq. 8.6.

Similar to Figs. 8-3 and 8-4 except that the (a) incoherent and (b) coherent 2D average scatter function of the rigid square plate is compared to that of the rigid circular disc for \( kL/2 = 10 \) where the angle of incidence is \( \alpha_i = \pi/4 \) and \( \beta_i = \beta = 0 \) for the this case.

Geometry for 3D rotation. Square plate of sides \( L \) with surface normal to unit vector \( \mathbf{w} \). The plate is rotated by \( \tau \) radians about the axis vector \( \mathbf{w} \).

(a) Incoherent 3D average scatter function of the rigid square plate of \( kL/2 = 1 \) is plotted for \( \alpha_i = \beta_i = \beta = 0 \). The numerical integration was implemented using Eq. 8.11 for progressive increments of the angles \( d\chi \), \( d\eta \) and \( d\tau \) of \( 2^\circ \), \( 4^\circ \) and \( 8^\circ \) to show the stability of the numerical integration. (b) The scatter function of a pressure-release sphere of \( kr = 1 \) calculated using Eq. 8.14.

Similar to Fig. 8-7 but for a rigid square plate of \( kL/2 = 10 \) and a pressure-release sphere of \( kr = 10 \).
Incoherent 3D average scatter function of the rigid square plate of \( kL/2 = 10 \) is plotted for different elevation \( \alpha_i \) and azimuth \( \beta_i \) angles of the incident planewave as well as the azimuth angle \( \beta \) of the scattered planewave. The incoherent 3D average scatter function depends only on the cosine of the angle between the incident and scattered planewaves and the not the absolute value of the angles.

Coherent 3D average scatter function of the rigid square plate of \( kL/2 = 10 \) is plotted for different elevation \( \alpha_i \) and azimuth \( \beta_i \) angles of the incident planewave. The coherent 3D average scatter function depends on the absolute angle of the incident and scattered planewaves and does not behave like a spherical scatterer.

Figure taken from Ref. [46] showing the geometry of spatial and wave-number coordinates. All coordinates have origin at the object center.

Two scenarios each with a unique planewave vector.

A planewave \( k_i \) is incident on the target from a far-field source \( r_0 \). The target scatters the field \( k \) to a far-field receiver \( r \). This figure is used to derive the tracking rule for a fixed planewave when \( k_i = k \).

Object scattering in a waveguide. View from the horizontal \( x-y \) plane.

A comparison of scattering using the single frequency approximation versus the full bandwidth of the given window function. Reverberation calculated using the single frequency approximation is indistinguishable from that calculated with the full bandwidth. (a) Same as Fig. 3-4(a) for sphere in waveguide with reverb except only sand bottom case is shown. Single frequency approximation is compared to rectangular window. (b) Same as (a) except Hamming window is used instead of rectangular window. (c) Same as (a) except only silt bottom case is shown. (d) same as (c) except Hamming window is used instead of rectangular window.
B-2 A comparison of scattering using the single frequency approximation versus the full bandwidth of the given window function. Reverberation calculated using the single frequency approximation is indistinguishable from that calculated with the full bandwidth. (a) Same as Fig. 3-8(a) for seafloor riverbank with reverb except only sand bottom case is shown. Single frequency approximation is compared to rectangular window. (b) Same as (a) except Hamming window is used instead of rectangular window. (c) Same as (a) except only silt bottom case is shown. (d) same as (c) except Hamming window is used instead of rectangular window.
Bibliography


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