Hydrodynamic Design
of High Speed Catamaran Vessels

by

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SUBMITTED TO THE DEPARTMENT OF OCEAN ENGINEERING IN PARTIAL
FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN NAVAL ARCHITECTURE AND MARINE ENGINEERING
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

FEBRUARY 2003

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Submitted to the department of Ocean Engineering on January 17, 2003
in Partial Fulfillment of the Requirements for the degree of
Master in Science in Naval Architecture and Marine Engineering

Abstract

This thesis examines the hydrodynamic design of a slender, semi-displacement, generic catamaran with a variable separation ratio (the ratio of the transverse distance between the demi-hulls to the length of the vessel) and the influence of this separation ratio upon the vessel behavior at sea. The hydrodynamic aspects studied were the hull calm water resistance, the seakeeping characteristics of the vessel with and without passive lifting appendages, and the structural loads developed as a result of an incident wave spectrum. Some measurements were compared with a similar monohull (a monohull with exactly the same displacement and length). The catamaran used for the research had a generic hull form and displaced 7500 ton. The hydrodynamic characteristics of this vessel were analyzed by a general purpose, potential flow, time domain, Rankine Panel Method, FORTRAN code, called SWAN-2 (Ship Wave Analysis). SWAN-2 is a state-of-the-art Computational Fluid Dynamics (CFD) code, developed in MIT in recent years, and was utilized practically as a numerical towing tank.

Thesis Supervisor: Paul D. Sclavounos
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Acknowledgments

The following thesis was written during the year 2002 (January – December) in the ‘Laboratory for Ship and Platform Flow’, at the Ocean Engineering Department, MIT. I would like to express my appreciation to all who helped me while preparing this paper and supported me throughout my graduate studies.

I thank my thesis advisor, Professor Paul Sclavounos, for his help and guidance. His insights and advice throughout this last year inspired me towards the completion of this paper.

I thank Dr. Sungeun Kim who helped me with SWAN and other hydrodynamic and technical related problems. His help is greatly appreciated.

I thank my lab partners, Yile Li, Talha Ulusoy and Kwang Lee for their good advice and bright ideas.

Finally I thank my family who ‘suffered’ and supported me in the last eighteen month of my graduate studies at MIT, my wife Elana, and my daughters Yarden and Noa.

Shiran Purvin
Cambridge, Massachusetts, December 23, 2002.
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Chapter 1: Introduction

Semi-displacement sea vehicles and particularly fast, slender catamarans became recently a popular way of transportation at sea. The broad deck enables the catamaran to transport larger number of travelers and commerce in comfort and faster than ever. Naval applications of these hull shapes are also considered, especially in light of their capabilities to carry more man, support and armor into harms way.

Catamarans were used throughout history as a favorite way of transportation over seas. Primitive catamarans were found in the South Pacific and off the coast of India. In some regions this vessel was made of three logs lashed together (a trimaran design). The middle log was longer than the other two and was pointed to form a prow. In other areas the catamaran consisted of two canoe hulls attached parallel to each other.

In recent years the catamaran design was built out of composites or aluminum in demand to lighter vessels with superior speed at sea. The catamaran has larger deck area compared to the traditional monohull and has a better transverse stability features. Its main disadvantage is the need to withstand a large torsion moment, caused due to loads created over the demi hulls; and acting as a moment on the vessels low cross section area around the centerline. This area suffers consequently from strains, cracks and breaks.

Computational naval-architecture methods were developed in the last two decades along with the development of personal computers. This development allowed more ship hydrodynamic and structure work to be done on computers and less by experiments and model testing, although the stage of eliminating these completely was not reached yet. Advance marine vehicles as the catamaran, possesses different characteristics than the conventional monohull and by that present new technical challenges. These challenges include operating in high speeds, geometrical complexities (as transom sterns) and
multiple hulls. Experimental data for these unique vessels is limited and typically proprietary. Numerical and computational techniques that can evaluate the performance of this unique vessel in calm water and in waves are thus invaluable to the designer.

Before computational methods appeared, hydrodynamic problems were solved analytically using approximations like the two-dimensional strip theory algorithm. Strip theory utilizes two-dimensional boundary value problem solutions, and integrates them over the ship length, to find the three-dimensional solution. This method is limited to slender bodies advancing at relative slow speeds. A more accurate solution to the ship response problem, require solution of the fully three-dimensional problem. This can be done solely using powerful computer platforms utilizing fluid dynamics – panel methods algorithms. Such methods discretize the boundaries of the fluid into elements with associated singularity strength; impose appropriate boundary conditions; and most of them use linear, potential flow theory. In recent years a code based on the Rankine Panel Method was developed at the Massachusetts Institute of Technology by the name SWAN (Ship Wave Analysis). At first the frequency domain was used to ease calculations, but later the time domain was developed and included in recent upgrades of SWAN by the name of SWAN-2. The Rankine Panel Method is a subgroup of the aforementioned panel methods, which employ the Rankine Source as the elementary singularity.

Overview

The research presented in the following paper, invokes computational methods in predicting ship responses while advancing with forward speed in calm water and in waves. The responses predicted were the seakeeping heave and pitch (and roll) motions and the body loads, shear force, bending moment and torsion moment of the vessel about its center-line. A generic catamaran was created and was examined as a case study in this thesis. Three different separation ratios between the demi-hulls of the catamaran were evaluated in order to establish a pattern of response and to analyze the influence of the interference between the hulls on their behavior at sea. The sea spectrum chosen to test the ship was the ISSC (International Ship and Offshore Structures Congress) spectral
Chapter 1. Introduction.

formulation for fully developed sea. A significant wave height of 4 meters and a modal period of 12 seconds were used with this spectrum. The vessel analyzed was a generic, slender hull, round bilge, deep transom catamaran. A slender, round bilge, deep transom monohull with similar length and displacement values was examined as well for comparison purposes. All vessels were evaluated in the semi-displacement regime ($Froude Number = 0.5 - 1.0$).

The catamaran examined in this paper was evaluated as if it was tested in a towing tank. The benefits of the three dimensional, time-domain, computational fluid dynamic codes are obvious. The vessel evaluated here was developed on a spreadsheet using polynomial equations, and was never built, yet a set of ‘numerical’ model tests were conducted upon it. Not only the vessel behavior at sea was evaluated, but also an analysis of alternatives of improving its seakeeping characteristics by attaching to it lifting appendages, was conducted.

The monohull that was analyzed for comparison purposes had the same overall length and displacement as the catamarans. The length and displacement of a vessel determine a great deal of its measure of effectiveness. Characteristics of its effectiveness could be the weight of the payload, the vessel ability to maneuver in harbors, or its cost. It was interesting to compare, for instance, between two ways to deliver the same payload across the Atlantic Ocean. It turned out that the catamarans are much more efficient than the monohull in wave making resistance. This result is off course expected because of the slenderness of the demi-hulls in contrast of the ‘bulkiness’ of the monohull. SWAN, as a potential flow code, could not simulate viscous effects; hence the total resistance of the vessels was estimated manually.

Chapter two of the thesis lays the theoretical background for the physical problem at hand. The chapter presents the boundary conditions for the problem and the mathematical model that simulates the physical world. The influence of the lifting appendages on the hull is also presented. Some sections deal with the method used to solve the mathematical model – *Rankine Panel method*. 
Chapter three presents the time domain, three-dimensional-code, SWAN-2. It elaborates the of implementing the mathematical model over the physical problem by the code (for example by linearization of the boundary conditions).

Chapter four elaborates in details the results for the steady problem. The Kelvin wake pattern and the ideal fluid resistance of the vessels were evaluated in calm water. The results are compared with a model test. The dynamic trim and sinkage of the vessels were evaluated and analyzed as well. The ideal fluid resistance of the catamaran is compared with twice the resistance of one demi-hull.

Chapter five presents the results of the seakeeping simulation predictions for all catamarans and monohull. The standard deviation of the response was found and analyzed. The effect of several alternatives of lifting appendages over the seakeeping response of the vessels was examined.

Chapter six examines wave load effects on the catamaran hull. All three catamarans were evaluated at bow waves and oblique waves. The significant one third highest shear force, bending moment and torsion moment along the center line of the vessels were calculated.

Chapter seven is a short summary and discussion of all above. Several suggestions for future work are elaborated.

Chapter eight is a nomenclature table.

Chapter nine is a list of references that were used while writing this paper.

Three annexes are attached to this paper: (1) Annex A elaborates the characteristics of the vessels evaluated; (2) Annex B contains the seakeeping RAO’s for heave, pitch and roll modes obtained by SWAN-2; and (3) Annex C includes the wave loads non-dimensional RAO’s calculated by SWAN-2.
Chapter 2: Theoretical Background

The following paper investigates the hydrodynamic behavior of a catamaran, namely, its calm water resistance, seakeeping performances and wave load characteristics. The research is using a time domain Rankine Panel Method CFD, SWAN-2. The hull used for demonstration purposes is a generic catamaran, slender with a deep transom stern, typical for a semi-displacement vessels. Several catamaran types with different separation ratio were studied. Also tested a monohull with similar displacement and length. The following chapter lays the theoretical background for the physical problem to be solved further on.

2.1 Linearized Boundary Conditions

The following paragraph deals with the boundary conditions defined to solve the flow regime around a vessel with a forward translation speed $U$. A potential flow model is introduced and utilized to establish the required boundary conditions. The flow thus is assumed ‘linear’, i.e. ideal and irrotational. Viscosity and non-linear effects can be neglected. Satisfying conservation of mass, the velocity potential flow can be written as:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

This is known as the Laplace Equation. It provides the governing partial differential equation to be solved for the function $\phi$, where $\phi(\vec{X},t)$ is the total fluid velocity potential. The fluid velocity vector can be defined as:

$$\vec{V}(\vec{X},t) = \nabla \phi$$
\( \overline{X} = xi + yj + zk \) is the translation vector with respect to the vessel’s local coordinate system. By integrating conservation of momentum equations, an explicit formulation for the pressure, known as the Bernoulli Equation, is introduced. The pressure around a ship with forward speed \( U \) is defined as:

\[
(2.3) \quad p - p_a = -\rho \left( \frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi - \frac{1}{2} U^2 + gz \right)
\]

\( p_a \) is the atmospheric pressure and \( g \) is the gravity acceleration.

Two different types of boundary conditions must be discussed: kinematic condition corresponding to the velocity of the fluid on the boundaries and dynamic condition corresponding to the forces act on the boundaries. Unlike viscous fluid there are fewer conditions to impose since no shear stresses exist in the inviscid potential flow.

2.1.1 Kinematic Boundary Condition

The physically kinematic boundary condition for a fluid flow at the boundary of a rigid body moving at speed \( U \) is that the normal component \( V \cdot \overline{n} \) of the fluid velocity must be equal to the normal velocity \( U \cdot \overline{n} \) of the boundary surface itself, meaning no fluid particle can flow through the boundary surface (no flux). In terms of velocity potential this condition becomes:

\[
(2.4) \quad \frac{\partial \phi}{\partial n} = U \cdot \overline{n} \text{ for } \overline{X} \in S_b
\]

\( \frac{\partial \phi}{\partial n} \) is the derivative of the potential in the direction of the normal to the fluid and \( \overline{X} \) is a point on the body surface, \( S_b \). The sea floor is subjected to the same no-flux condition for all time.
where $H$ is the sea depth.

The fluid domain is bounded by the free surface which can be defined by its elevation $z = \eta(x,y,t)$. The kinematic boundary condition on the free surface states that a fluid particle on the free surface remains there for all times. Mathematically this condition requires that the substantial derivative of the quantity $z - \eta$ will vanish on the free surface:

$$\frac{D}{Dt}(z - \eta) = 0 \Rightarrow$$

$$\frac{\partial}{\partial t} \left( \frac{\partial}{\partial z} - (\nabla \cdot \nabla \phi) \cdot \nabla \right) \eta = \frac{\partial \phi}{\partial z} \Rightarrow$$

$$\frac{\partial \phi}{\partial z} \frac{\partial \eta}{\partial \phi} - \frac{\partial \phi}{\partial \phi} \frac{\partial \eta}{\partial x} \frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial \phi} \frac{\partial \eta}{\partial y} \frac{\partial \phi}{\partial y} = 0$$

The last two terms are of second order and thus can be neglected in the linearized kinematic boundary condition which therefore can be reduced to:

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z}$$

This approximate boundary condition states that the vertical velocities of the free surface and the fluid particles are equal; hence a fluid particle on the surface stays there for all times.

### 2.1.2 Dynamic Boundary Condition

The dynamic boundary condition on the free surface is obtained from integrating equations of momentum conservation, known as the *Bernoulli's Equation*, assuming that the pressure
of the fluid must be equal to the atmospheric pressure. The dynamic condition for a ship advancing in a speed $U$ is expressed as:

\[
-\frac{1}{\rho} (p - p_a) = \frac{\partial \phi}{\partial t} - \frac{1}{2} U^2 + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gz = 0
\]

For the free surface, $z = \eta$ so:

\[
\eta = -\frac{1}{g} \left( \frac{\partial \phi}{\partial t} - \frac{1}{2} U^2 + \frac{1}{2} \nabla \phi \cdot \nabla \phi \right)
\]

As before, the second order terms can be neglected.

The linearized equation for the free surface dynamic boundary condition is then:

\[
\eta = -\frac{1}{g} \frac{\partial \phi}{\partial t}
\]

2.1.3 Combined Condition

Both conditions for the free surface can be combined for a single formulation and can be written as:

\[
\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \text{ on } z = 0
\]

The last condition is a general case of condition (2.5) and requires that the velocity potential gradient decays to zero when $z$ approaches infinity for all finite $t$,

\[
|\nabla \phi| \to 0, |z| \to \infty
\]
2.2 Plane Progressive Wave

The free surface elevation defined by a plane progressive wave formulation is simple yet significantly practical representation. The propagating wave has amplitude, it is sinusoidal in time with a radian frequency and it moves in phase velocity. The wave elevation can be described in the following form:

\[ \eta(x, y, t) = A \cos(kx \cos \beta + ky \sin \beta \pm wt + \varepsilon) \]

or in the complex form:

\[ \eta(x, y, t) = A \cdot \Re \left\{ e^{i(kx \cos \beta + ky \sin \beta \pm wt + \varepsilon)} \right\} \]

By using linear theory, a long-crested irregular sea can be written as sum of wave elevations:

\[ \eta(x, y, t) = \sum_{j=1}^{N} A_j \cos(k_j x \cos \beta + k_j y \sin \beta \pm wt + \varepsilon_j) \]

where \( A \) is the wave amplitude, \( w \) is the wave frequency, \( \beta \) is the wave direction relative to the x-axis, \( \varepsilon \) is a random phase angle and \( k \) is the \textit{wavenumber} defined as the number of waves per unit distance or:

\[ k = \frac{2 \pi}{\lambda} \]

where \( \lambda \) is the wave length. The wave speed is defined as the velocity of the wave crests (i.e. the number of crests which travel per unit time from a specific point). It is signed by \( V_p \) and is equal to the relation between the wave frequency and the \textit{wavenumber}. 
The velocity potential can be described as:

\[ \phi(x, y, z, t) = A \Re \left\{ \frac{ig}{w} e^{i (kx \cos \beta + ky \sin \beta \pm \omega t + \phi)} \frac{\cosh[k(z + H)]}{\cosh[kH]} \right\} \]

In order to satisfy (2.11) the wave frequency should be:

\[ w^2 = g k \tanh[kH] \]

This relation is known as the dispersion relation since it relates the wavenumber with the wave frequency. For deep water \( \tanh[kh] \rightarrow 1 \) so,

\[ w^2 = g k \]

The following figure, borrowed from Lloyd [15], describes clearly several characteristics of the propagating wave.

Figure 2.1: The three dimensional propagating wave
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It should be noticed that the frequency \( w \), is an absolute 'global' frequency, different from the encounter frequency which is the frequency that a moving vessel with a forward speed \( U \) 'feels'. The encounter frequency is given by:

\[
(2.20) \quad w_e = w - \frac{w^2}{g} U \cos \beta
\]

Figure 2.6 demonstrates clearly the advancing vessel (with local coordinates) and the unidirectional propagating wave.

2.3 Ship Resistance

The resistance of a ship advancing with a forward speed \( U \), is the force required to tow the ship at that speed in calm water, assuming no interference from the towing ship. In case the ship has no appendages, this magnitude is called 'The bare hull total resistance'. The power required to overcome this resistance is called the effective power and is usually notated as EHP (Effective Horse Power). The total resistance is a sum of several different components, which are caused due to a variety of factors. These factors also interact with each other; hence changing one factor affects several resistance components. In order to simplify this problem, the total calm water resistance is usually considered a sum of four major components:

1) Frictional (viscous) resistance caused due to the movement of the ship through a viscous fluid. Contribute mainly a tangential component to the resistance.

2) Wave making resistance. Caused mainly due to the energy 'wasted' by the ship, generating surface waves. This quantity is closely related to the amplitude of the waves the ship leaves behind in its wake, as will be outlined below.

3) Induced resistance that is caused due to eddies shed from the hull or appendages and due to special and unique hull forms. Streamlining the hull and appendages can reduce this component.
4) *Air resistance* is experienced by the above water part of the hull and the superstructure of the ship. This part of the total resistance will not be discussed here. Details can be found in *PNA* [1].

The wave making resistance and the induced resistance are commonly taken together and are known as the *residuary resistance*.

### 2.3.1 Frictional Resistance

Frictional resistance is the largest single component of the total resistance of a ship. It varies from 80%-85% of the total resistance for slow speed ships to 50% of the total resistance for high-speed ships. Frictional resistance is basically the result of the tangential fluid force exerted from the ship's movement in water. Frictional resistance depends on the roughness of the surface, on the overall wetted area of the ship and on the ship's speed. Any roughness of the wetted surface will increase the frictional resistance appreciably over that of a smooth surface. Any increase of wetted surface (mainly due to over loading the ship) will cause an increase in the frictional resistance component. The frictional resistance force is usually presented by:

\[
D = \frac{1}{2} \cdot C_f \cdot \rho \cdot S_B \cdot U^2
\]

When \( D \) is the frictional resistance force, \( C_f \) is the frictional resistance coefficient, \( \rho \) is the mass density of water, \( S_B \) is the wetted area of the ship and \( U \) is the ship's speed.

Over the years, lots of model and full-scale experiments were performed in order to assess the magnitude of this component and the frictional resistance coefficient. The International Towing Tank Conference (ITTC) in Madrid in 1957 adopted the following formula for the frictional resistance coefficient for a flat plate:

\[
C_f = \frac{0.075}{(\log_{10} R - 2)^2}
\]
where $R_n$ is a non-dimensional value and is known as the Reynolds Number:

$$\begin{align*}
R_n &= \frac{U \cdot L}{\nu} \\
\end{align*}$$

$U$ is the ship’s speed; $L$ is the ship’s length and $\nu$ is the kinematic viscosity of the water. This line is known as the “ITTC 1957 model-ship correlation line”.

### 2.3.2 Three Dimensional Ship Waves

The most general wave elevation distribution in three dimensions, transformed to a reference system, moving with a vessel in the positive $x$-axis direction with velocity $U$, is given by:

$$\eta(x, y, t) = \Re \left\{ \int_{0}^{2\pi} \int_{0}^{\infty} A(w, \theta) \cdot e^{-ik(x\cos\theta + y\sin\theta) + iw(w - kU\cos\theta)} \, dw \, d\theta \right\}$$

This formulation represents oblique wave (with angle $\theta$) distribution in the ship’s wake. For a steady motion the second exponential term vanishes, hence:

$$kU \cos \theta - w = 0$$

the wave phase velocity becomes:

$$V_p = \frac{w}{k} = U \cos \theta$$

The expression in (2.24) for a steady wake pattern behind a moving ship becomes:
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\[
\eta(x, y) = \Re \left\{ \frac{\pi}{2} \int_{-\pi/2}^{\pi/2} \bar{A}(\theta) e^{-ik(y \cos \theta + x \sin \theta)} d\theta \right\}
\]

This expression is known as the free-wave distribution.

For a very large distance downstream from the location of the ship, this expression can be simplified and the classic ship-wave pattern can be obtained as derived by Kelvin in 1887. The following figure describes the Kelvin wave pattern in the wake of a moving vessel.

![Figure 2.2: Kelvin wave pattern in a ship wake](image)

The waves generated in the wake of a ship are confined to a symmetrical sector about the negative x-axis, which includes semiangles of \( \frac{y}{x} = \pm 19^\circ 28' \). At this maximum, the wave is directed with a corresponding angle of \( \theta = \pm 35^\circ 16' \) above or below the x-axis. For all other \( \frac{y}{x} \) angles in the wake between \(-19^\circ 28'\) to \(+19^\circ 28'\) there are two corresponding values of the wave angle, \( \theta \). One represents a transverse wave pattern, the other represents a diverged wave. More Details and the complete formulation of the pattern can be found at Newman [6].
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Figure 2.3 demonstrate the Kelvin wave pattern in a monohull wake. The vessel translates in a uniform speed $U$ in calm water. The figure was produced by SWAN-2 for a 100 m, 7500 ton monohull at Froude Number 0.74.

![Kelvin wave pattern in a monohull wake](image)

**Figure 2.3: Kelvin wave pattern for a monohull at Fn=0.74**

Figure 2.4 describes the Kelvin wave pattern at the wake of a 7500 ton, 100m catamaran with a separation ratio of 0.4 at Froude Number 0.74. This figure is a realization of the fluid and body domains produced by SWAN-2.

On each demi-hull wake a Kelvin wave pattern develops. Understanding these wave patterns and measuring their energy is the key to accurate predictions of the vessels wave making resistance.
2.3.3 Calm water Wave Making Resistance

Wave making resistance of a ship is the net fore and aft force exerted from the fluid pressure, acting normally on all parts of the hull. For a surface ship, movements of the ship through water generate waves, which change the pressure regime along the hull. The resultant net fore and aft force along the hull is the wave making force. The magnitude of this resistance component energy must be equal to the energy required to maintain surface waves system.

The waves contain energy both potential and kinetic. Considering the wave energy per wavelength, per unit breadth, per period,

\[
(2.28) \quad \text{potential energy: } \quad P.E. = \int_0^T \int_0^L \frac{1}{2} \rho g \frac{\eta^2}{T} dx dt
\]
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kinetic energy: \[ K.E. = \iiint_{0}^{T} \frac{1}{2} \rho |\nabla \varphi|^2 \, dz \, dx \]

The x-axis coincides with the direction of the waves so the y-dependence is dropped. After integration the following results are obtained:

\[ \text{(2.30)} \quad P.E. = \frac{1}{4} \rho g A^2 \quad \text{and} \quad K.E. = \frac{1}{4} \rho g A^2 \]

and the total energy density is given by:

\[ \text{(2.31)} \quad E = \frac{1}{2} \rho g A^2 \]

The traveling energy is best expressed as wave power per unit width averaged over a wave period and can be found to be:

\[ \text{(2.32)} \quad P = \frac{1}{2} \rho g A^2 V_p \left( 1 + \frac{2kH}{\sinh(2kH)} \right) \]

where \( V_p \) is the wave phase velocity as defined earlier and the term \( \frac{1}{2} V_p \left( 1 + \frac{2kH}{\sinh(2kH)} \right) \) is known as the group velocity, \( V_g \). Group Velocity is defined as the velocity by which the energy of the wave travels. In a fixed reference frame, the energy of each plane wave component moves in direction \( \theta \) with a group velocity \( V_g \). The velocity of energy transfer across the control surface, which moves through the fluid with velocity \( U \), is given by \( V_g \cos \theta - U \). Multiplying by the energy density and integrating along the width of the control surface, the total energy flux can be expressed as:
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\[(2.33) \quad \frac{dE}{dt} = \frac{1}{2} \rho g \int_{-\infty}^{\infty} A^2 \left(V_s \cos \theta - U \right) \, dz \]

Using the stationary-phase approximation the total wave resistance of a moving vessel can be derived, as shown in Newman chapter 6 [3]:

\[(2.34) \quad D = \frac{1}{2} \pi \rho U^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A(\theta)^2 \cdot \cos^3 \theta \cdot d\theta \]

A thin-ship theory was developed by J. H. Michell in 1898. The essential assumption is that the hull is thin, i.e. its beam is small relative to other length characteristics of the ship. Wave resistance of a thin ship can be expressed in terms of a distribution of sources along the center line of the hull, where the local source strength is proportional to the longitudinal slope of the hull. Using this theory, the wave resistance of a thin-ship can be expressed in the form of Michell's integral:

\[(2.35) \quad D = \frac{4 \rho g^2}{\pi U^2} \int_{0}^{\frac{\gamma}{2} \sec^3 \theta} \left| \int \frac{\partial \xi}{\partial x} e^{\left(\frac{g}{U^2}\right) \sec^3 \theta (z - \xi \cos \theta)} \right| dx \, dz \, d\theta \]

where \( y = \pm \xi(x, z) \) defines the local half-beam of the hull surface.

2.3.4 Induced Resistance

The turbulent frictional belt around a ship consists of eddies caused due to changes of form, appendages or other projections. This interference to the flow around the hull is the component of resistance known as eddy or induced resistance. This component is usually combined with the wave making resistance and called ‘residuary resistance’. Semi-displacement vessels experience eddy resistance mainly due to transom sterns and separation of flow there. At high speeds, the fluid leaves the transom dry and so the pressure there is practically atmospheric. The bow of the ship is submerged so an
unfavorable pressure gradient is generated and a force is acting in the opposite direction to the ship movement. This force can be evaluated as the integrated hydrostatic pressure over the transverse projected dry area of the transom.

### 2.4 Ship Response in Sea Waves

The simplest way to discuss ship response at sea waves is to assume regular, small amplitude, sinusoidal type waves. Actual ocean waves are highly irregular, but even so, they can be related to regular waves by a linear superposition of sinusoidal components as shown in equation (2.15). In that case the irregular sea can be described by a sea spectrum, $S(w_j)$:

\[
\frac{1}{2} A_j^2 = S(w_j) \Delta w
\]

where $A$ is wave amplitude, and $\Delta w$ is a small, constant increment, between successive frequencies. Another important assumption is that the ship behaves as a Linear, Time Invariant System. This definition provides the designer the opportunity to treat the ship response as linear and by that to find the total response of a ship by superposing all single responses. By virtue of linearity, the response due to each component of the wave spectrum can be analyzed separately and the total response would simply be a linear superposition of all individual responses. The response is also time-invariant, hence depend on past responses. This definition is correct for all ship responses; motions (heave, surge, etc...) and body loads (shear force, bending moment, etc...). More details can be found in PNA [2], Faltinsen [6], Hughes [8], and Sclavounos [9].

#### 2.4.1 Sea Spectrum

The wave spectrum is estimated from wave measurements. It assumes that the sea can be described as a stationary random process and is referred as short term statistics. For an open sea conditions the 15th ITTC (International Towing Tank Conference) recommended
the use of the ISSC (International Ship and Offshore Structures Congress) spectral formulation for fully developed sea:

\[
S(w) = \frac{0.11}{2\pi} \left( \frac{wT_1}{2\pi} \right)^{-5} \cdot e^{-0.44\left( \frac{wT_1}{2\pi} \right)^{-1}} \cdot H_{\xi} \cdot T_1
\]

where \( H_{\xi} \) is the significant wave height, defined as the mean of the one third highest waves and \( T_1 \) is the mean wave period defined as:

\[
T_1 = 2\pi \frac{m_0}{m_1}
\]

where the 'moments' are defined as:

\[
m_k = \int_0^\infty w^k S(w)dw
\]

Figure 2.5 illustrates a wave spectrum which is actually a statistical representation of its wave components.

Figure 2.5: Wave spectrum illustration
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The sea spectrum defined above is used in the following paper and utilized to analyze the ship motion responses.

### 2.4.2 Ship Response

For the problem discussed in this section, a plane progressive wave of amplitude $A$ and direction $\beta$ is incident upon the ship as illustrated in figure 2.6. The ship in response is free to move in six degrees of freedom, three translational motions: *surge, sway, heave* and three rotational motions: *roll, pitch, and yaw*.

![Ship motions diagram](image)

*Figure 2.6: Ship motions, six degrees of freedom and the incident wave*
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The corresponding velocities \( U_j(t) \) are sinusoidal in time oscillating with the frequency of encounter as defined on equation (2.20):

\[
(2.40) \quad U_j(t) = \text{Re}\left(i w_0 \zeta_j e^{i\omega t}\right), \quad j = 1,2,\ldots,6
\]

\( \zeta_j \) is the \( j^{th} \) response motion. \( \zeta_j \) is defined as:

\[
(2.41) \quad \zeta_j = |\zeta_j| e^{i\chi_j} e^{i\omega t}, \quad j = 1,2,\ldots,6
\]

\( |\zeta_j| \) is the amplitude and \( \chi \) is the phase angle of the \( j^{th} \) mode of the response. Here \( \zeta_j \) represent the total response in each direction. The ratio of the response amplitude, \( |\zeta_j| \), to the incident wave amplitude, \( A \), is known as the Response Amplitude Operator (RAO) and is one of the ship characteristic. The response spectrum of the ship to the incident wave is then a function of the ocean wave spectrum and the ship’s RAO.

\[
(2.42) \quad S_{R_j}(w) = |H_j(w)|^2 \cdot S_w(w), \quad j = 1,2,\ldots,6
\]

where \( H(w) \) is the Transfer Function. It should be noticed that the sea spectrum is given in absolute frequency terms, while the ship’s transfer function is sometimes given in encounter frequency terms. The variance of the response can be found by:

\[
(2.43) \quad \sigma_{R_j}^2 = \int_0^\infty S(w)|H_j(w)|^2 dw, \quad j = 1,2,\ldots,6
\]

and the standard deviation, \( \sigma_j \), is the square root of the variance.

2.4.3 Velocity Potential

The total three dimension velocity potential can be presented by:
(2.44)

\[ \phi(x, y, z, t) = [-Ux + \phi_S(x, y, z)] + \left( \sum_{j=1}^{6} \xi_j \phi_{Rj}(x, y, z) + \phi_D(x, y, z) + \phi_I(x, y, z) \right) e^{i\omega t} \]

where the terms in the first parenthesis are steady, \( U \) is the ship's speed and \( \phi_S \) is the perturbation potential due to steady translation.

The boundary condition concerning the steady potential on the ship hull are similar to those obtained by (2.4):

(2.45) \[ \frac{\partial \phi_S}{\partial n} = \bar{U} \cdot \bar{n} \quad \text{on} \quad S_B \]

where \( \bar{n} = (n_1, n_2, n_3) \) is the unit vector, normal to the ship hull, pointing out of the fluid domain. The terms on the second parenthesis in (2.44) are the unsteady part of the potential. \( \phi_{Rj} \) represent the radiation potential of a rigid-body motion due to unit amplitude motion in calm water (in the absence of incident waves) in the \( j^{th} \) direction. \( \phi_D + \phi_I \) represent the potential due to the incident waves and their interaction with the body. Assuming linearity this motion is independent of the body motion and can be defined as if the body is fixed. \( \phi_I \) is the incident wave potential and \( \phi_D \) is the diffraction potential represents the disturbance of the incident waves by the fixed body. Boundary conditions for these potentials are:

(2.46) \[ \frac{\partial (\phi_D + \phi_I)}{\partial n} = 0 \Rightarrow \frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n}, \quad \text{on} \quad S_B \]

Each potential has to satisfy also the Laplace equation, stated in eq. (2.1). On the sea bottom, \( \phi \to 0 \) for \( z \to -\infty \) as defined in eq. (2.5) and (2.12). On the free surface the boundary condition follows equations (2.11) and (2.44) to be:
\[ (2.47) \quad -\frac{w^2}{g} \phi_j + \frac{\partial \phi_j}{\partial z} = 0 \quad \text{on } z = 0 \quad \text{for } j = 1,2,...,6 \text{ and } \phi_i, \phi_D \]

### 2.4.4 Wave exciting Forces

The general form of the basic Linearized equations in six degrees of freedom using the local ship coordinates is given by:

\[ (2.48) \quad F_j(t) = \sum_{k=1}^{6} \Delta_{jk} \ddot{\xi}_k(t), \quad j = 1,2,...,6 \]

where \( \Delta_{jk} \) is the general inertia matrix of the ship containing components of mass and mass moments of inertia. \( F_j(t) \) represent the total exciting forces or moments acting on the hull in the \( j^{th} \) direction. \( \ddot{\xi}_k \) are the hull accelerations in the \( k^{th} \) mode.

\( F_j(t) \) can be expressed in terms of the \( j^{th} \) direction hydrostatic and hydrodynamic fluid forces:

\[ (2.49) \quad F_j(t) = \sum_{k=1}^{6} \Delta_{jk} \ddot{\xi}_k(t) = F_j^{HS} + F_j^{HD} \]

\( F_j^{HS} \) is the net hydrostatic force and includes gravitational and hydrostatic forces. It can be expressed by:

\[ (2.50) \quad F_j^{HS} = -\sum_{j=1}^{6} C_{jk} \ddot{\xi}_k e^{iw_j} \]

where \( C_{jk} \) is the hydrostatic restoring force coefficient. Exact values of \( C_{jk} \) can be found in PNA [2]. \( \ddot{\xi}_k e^{iw_j} \) are the six motion responses of the ship as defined in equation (2.41).
The hydrodynamic force due to unsteady translations is given by:

\[ F_{j}^{HD} = F_{j}^{EX} + F_{j}^{R} \]  

where \( F_{j}^{EX} \) is the excitation force in the \( j^{th} \) direction and is equal to:

\[ F_{j}^{EX} = (F_{j}^{I} + F_{j}^{D}) e^{iw_{t}} \]

\( F_{j}' \) is the complex amplitude of the exciting force due to incident waves, known also as the Froude-Krylov exciting force. It results from the pressure integration over the body surface, which would exist in the wave system if the ship were not present. It equals to:

\[ F_{j}' = -\rho \int_{S_{a}} n_{j} \left( iw_{e} - U \frac{\partial}{\partial x} \right) \phi_{i} dS \]

\( F_{j}^{D} \) is the complex amplitude of the exciting force due to diffracted waves, called also the diffracted force. It is caused by the diffraction of the incident wave due to the presence of the vessel. It equals to:

\[ F_{j}^{D} = -\rho \int_{S_{a}} n_{j} \left( iw_{e} - U \frac{\partial}{\partial x} \right) \phi_{n} dS \]

\( F_{j}^{R} \) is the hydrodynamic force in the \( j^{th} \) direction due to forced motion. It results from the radiation of waves away from a vessel that is forced to oscillate. It is given by:

\[ F_{j}^{R} = \sum_{k=1}^{6} \left[ -\rho \int_{S_{a}} n_{j} \left( iw_{e} - U \frac{\partial}{\partial x} \right) \phi_{k} dS \right] \bar{e}_{k} e^{iw_{t}} \]
\( \xi \) are the six motion responses of the ship as defined in equation (2.41). Solving for the integral in equation (2.55) can simplify the radiation force expression to:

\[
F^R_j = \sum_{k=1}^{6} \left( w_e^2 A_{jk} - iw_e B_{jk} \right) \xi_k e^{iw_j}
\]

where \( A_{jk} \) is the added mass in the \( j^{th} \) mode due to unit motion in the \( k^{th} \) direction and \( B_{jk} \) is the damping coefficient in the \( j^{th} \) mode due to unit motion in the \( k^{th} \) direction.

### 2.4.5 Ship Equation of Motion

Taking into account the various components of forces as introduced above, the total fluid force on the ship becomes:

\[
F_j(t) = F_j^{HS} + F_j^{EX} + F_j^R
\]

Plugging in equation (2.57) the components of the forces as were defined above:

\[
\sum_{k=1}^{6} \left[ -w_e^2 (\Delta_{jk} + A_{jk}) + iw_e B_{jk} + C_{jk} \xi_k \right] = F_j^i + F_j^D, \quad j = 1, 2, \ldots, 6
\]

Equation (2.58) is the linearized governing equation of the three dimensional motions for an unrestrained vessel in sinusoidal waves. Using complex number analysis, this equation can be used to find, for example, the heave motion RAO of a vessel:

\[
\left| \frac{\xi_3}{A} \right| = \frac{|F_j^i + F_j^D|}{\sqrt{\left(C_{33} - w_e^2 (\Delta_{33} + A_{33}) \right)^2 + w_e^2 B_{33}^2}}
\]

The three dimensional added mass and damping coefficients can be found using experimental results or analytical calculations. Using strip theory (for slender bodies) this
characteristics can be derived from the two-dimensional coefficient describing cross-section elements of the hull. For example, in order to find \( A_{33} \) (the added mass value of the hull in the ‘3’ direction due to force in the ‘3’ direction) of a transom stern vessel the following formulation can be used.

\[
A_{33} = \int_{s_r}^{s_b} a_{33}(x)dx - \frac{U}{W_e} b_{33}^s
\]

or \( B_{33} \) of a transom stern vessel (the damping coefficient of the hull in the ‘3’ direction due to force in the ‘3’ direction) can be found using the following equation:

\[
B_{33} = \int_{s_r}^{s_b} b_{33}(x)dx + Ua_{33}^s
\]

where \( a_{33} \) and \( b_{33} \) are the two-dimensional added mass and damping coefficient respectively, and \( a_{33}^s \) and \( b_{33}^s \) are the two-dimensional added mass and damping coefficient of the ship’s stern (in case of transom stern). The curious reader will be able to find more details describing the three-dimensional components in PNA [2], Faltinsen [6], and Lloyd [15].

**2.4.6 Hull loads**

All floating objects are subjected to loads that result from the body weight, its buoyancy and the environment; wind, current and waves. As discussed above, waves excite ship motion in three translational and three rotational directions. Waves also excite three ‘translational’ and three ‘rotational’ body loads.

A way of classifying loads is according to how they vary in time: static, slowly varying or rapidly varying. Three ways of calculating loads effect correspond to these loads: static, quasi-static and dynamic. An example to static loads is all the *stillwater* loads, internal and
external pressures (buoyancy and weight) and thermal loads. Examples to slowly varying loads are the wave induced dynamic pressure distribution along the hull as a result of wave encounter and ship motion, green seas, sloshing of liquids in ship tanks and inertia loads especially on masts or other elongated structures. Some examples for rapidly varying loads are slamming and mechanical vibrations.

Almost any irregular dynamic or quasi-static loading can be represented as a combination of regularly varying loads especially if the force-displacement ratios are linear or almost linear. The problem of calculating the load effects can then be solved ‘in the frequency’ domain with frequency as the principal independent variable instead of time. The frequency based distribution of loads or load effects is presented in a shape of spectrum, so wave spectrum causes a load response spectrum.

2.5 Lifting Foils

As part of the research, several alternatives of flat lifting foils were adjacent to the vessel, trying to reduce heave and pitch responses. The following section lays the theoretical fundamentals to evaluate this problem.

2.5.1 Prandtl’s lifting line theory

The three-dimensional lifting and induced drag forces for a foil with span $s$, chord $c(y)$ and advancing speed $U$ are given respectively as:

\[ (2.62) \quad L = \frac{1}{2} \rho \cdot C_L \cdot S \cdot U^2 \quad D = \frac{1}{2} \rho \cdot C_D \cdot S \cdot U^2 \]

where $S$ is the planform area of the foil, $C_L$ is the three-dimension lifting coefficient and $C_D$ is the three-dimension drag coefficient. The next figure, borrowed from Newman [3], explains clearly the variables and the coordinate system concerned with the foil.
The lift and drag coefficient are difficult to evaluate. *Newman* chapter 5 [3] derives a general expression for these quantities under the assumptions that the foil circulation $\Gamma(y)$ and its derivative $\Gamma'(y)$ are continuous along the span and that $\Gamma(y)$ vanishes smoothly at the foil tips $\pm s$ (*Kutta-condition*).

\begin{align*}
C_L &= \frac{2}{US} \int_{-\frac{s}{2}}^{\frac{s}{2}} \Gamma(y) dy \\
C_D &= \frac{1}{2\pi U^2 S} \int_{-\frac{s}{2}}^{\frac{s}{2}} \Gamma(y) dy \int_{-\frac{s}{2}}^{\frac{s}{2}} \frac{d\Gamma(\kappa)}{d\kappa} \frac{d\kappa}{y-\kappa} d\kappa
\end{align*}

where $k$ is a dummy variable utilized to determine the integrated circulation at the transverse position, $y$. In order to solve for the coefficient introduced above, it is essential to solve *Prandtl's lifting line equation* to determine the circulation distribution along the span of a lifting surface.
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\begin{equation}
\Gamma(y) = \Gamma_{2D}(y) + \frac{1}{4} c(y) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\kappa \frac{d\Gamma(\kappa)}{d\kappa} \frac{d\kappa}{\kappa - y}
\end{equation}

where $\Gamma_{2D}(y)$ is the two-dimension circulation of the foil given as the point vortices along the chord.

\begin{equation}
\Gamma_{2D}(y) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \gamma(\zeta) d\zeta
\end{equation}

where $\zeta$ is a dummy variable.

Using the circulation distribution and the lift and drag coefficient defined above, simple expressions can be evaluated for an elliptic circulation distribution along the span of a symmetric foil.

\begin{equation}
C_L = 2\pi \frac{AR}{AR + 2} \alpha, \quad C_D = 4\pi \frac{AR}{(AR + 2)^2} \alpha^2 = \frac{C_L^2}{\pi AR}
\end{equation}

where $AR$ is the three-dimension aspect ratio of the foil and is given by the ratio of the span to the mean chord. $\alpha$ is the angle of attack relative to the x-axis.

Prandtl’s lifting line theory is valid mainly for large aspect ratios but has proven to give good results also for relative small aspect ratios. It is valid for ‘thin’ foils for small angle of attack. For angles greater then 12-13% stall phenomena occurs and lift is lost.

\textbf{2.5.2 The effect of Lifting Foils on the Equation of Motion}

A vessel with lifting foils has different damping and restoring forces then those obtained without these foils, hence different motions and forces response. The lifting force as a result of a foil attached to a vessel can be defined as:
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\[ L(t) = \text{Re}\{F_3 e^{i\omega t}\} \]

While the vessel and the attached foil advance on sea with ambient waves, oscillatory displacements occur and the foils effective angle of attack alters with time. The time dependent change in the angle of attack is given by:

\[ \alpha(t) = -\zeta_3 + \frac{\phi_f - \dot{\zeta}_3 + x_{foil} \dot{x}_3}{U + \dot{\zeta}_1 - \frac{\partial \phi_f}{\partial x}} \]

where \( \zeta_1, \zeta_3, \zeta_5 \) are the surge, heave and pitch total responses respectively and their doted values represent the motion velocity in this direction, \( \dot{\zeta}_j = i \omega \zeta_j e^{i\omega t} \). \( x_{foil} \) is the longitudinal distance between the location of the foil and the origin of the coordinate system located at midship. \( \phi_f \) is the incident wave potential. Combining equations (2.41), (2.68) and (2.69), assuming surge speed, \( \dot{\zeta}_1 \), and wave speed in the x-direction, \( \frac{\partial \phi_f}{\partial x} \), are negligible relative to the ships speed, gives:

\[ F_3 L = F \left[ \left( \frac{x_{foil}}{U} i \omega_c - 1 \right) \zeta_5 - \frac{i \omega_c}{U} \zeta_3 + \frac{1}{U} \frac{\partial \phi_f}{\partial z} \right] \]

\( e^{i\omega t} \) canceled on both sides, where \( F \) is given by Prandtl’s Lifting theory (2.62), (2.67):

\[ F = \frac{\pi \rho S U^2 AR}{AR + 2} \]

these expressions lay on the assumption of slowly varying flow around the lifting foil. This assumption is correct for small reduced frequencies, \( \Omega \), where:
for hydrofoil vessels this assumption is almost always correct since the vessels speed is much higher than $w_c$.

The vertical lifting force obtained in (2.70), can be combined in the heave equation of motion (2.58):

\begin{equation}
(2.73) \quad -w_c^2 (A_{33} + M) \xi_3 - w_c^2 (A_{35} + M) \xi_5 + i w_c B_{33} \xi_3 + i w_c B_{35} \xi_5 + C_{33} \xi_3 + C_{35} \xi_5 =
\end{equation}

\[ F_3^I + F_3^D + F \left[ \left( \frac{x_{foil}}{U} i w_c - 1 \right) \xi_5 - \frac{i w_c}{U} \xi_3 + \frac{1}{U} \frac{\partial \phi_I}{\partial z} \right] \]

similarly, the pith moment contribution from the foil can be expressed as:

\begin{equation}
(2.74) \quad F_{5l} = -x_{foil} F_{5l}.
\end{equation}

this result can be combined in the pitch equation of motion:

\begin{equation}
(2.75) \quad -w_c^2 (A_{55} + I_{55}) \xi_5 - w_c^2 (A_{53} + I_{53}) \xi_3 + i w_c B_{55} \xi_5 + i w_c B_{53} \xi_3 + C_{55} \xi_5 + C_{53} \xi_3 =
\end{equation}

\[ F_5^I + F_5^D - x_{foil} F \left[ \left( \frac{x_{foil}}{U} i w_c - 1 \right) \xi_5 - \frac{i w_c}{U} \xi_3 + \frac{1}{U} \frac{\partial \phi_I}{\partial z} \right] \]

In equations (2.73) and (2.75) heave and pitch responses are coupled and surge responses are neglected. Collecting terms from (2.73) and (2.75) relative to the ships heave and pitch motions give:
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(2.76)

\[-w_e^2 (A_{33} + M) x_3 - w_e^2 (A_{53} + M) x_5 + \left( i w_e B_{33} + \frac{i w_e x_{foil}}{U} F \right) x_3 + \left( i w_e B_{53} - \frac{i w_e x_{foil}}{U} F \right) x_5 + C_{33} x_3 \]

\[+ (C_{53} + F) x_5 = F e^I + F^D + F \left[ \frac{1}{U} \frac{\partial \phi_I}{\partial z} \right] \]

\[-w_e^2 (A_{53} + I_{53}) x_3 - w_e^2 (A_{53} + I_{53}) x_5 + \left( i w_e B_{53} + \frac{i w_e x_{foil}}{U} F \right) x_3 + \left( i w_e B_{53} - \frac{i w_e x_{foil}}{U} F \right) x_5 \]

\[+ (C_{55} - x_{foil} F) x_5 + C_{55} x_3 = F e^I + F^D - x_{foil} F \left[ \frac{1}{U} \frac{\partial \phi_I}{\partial z} \right] \]

The damping and restoring coefficients of the ship with the foil can be presented by the following relations:

(2.77)

\[B_{33}^{foil} = B_{33} + \frac{F}{U} \]

\[B_{53}^{foil} = B_{53} - \frac{x_{foil} F}{U} \]

\[B_{55}^{foil} = B_{55} + \frac{x_{foil} F}{U} \]

\[C_{33}^{foil} = C_{33} \]

\[C_{35}^{foil} = C_{35} + F \]

\[C_{53}^{foil} = C_{53} \]

\[C_{55}^{foil} = C_{55} - x_{foil} F \]

The exciting forces with the foil can be presented by:
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(2.78)

\[
\left( F_3^I + F_3^D \right)_{\text{foil}} = \left( F_3^I + F_3^D \right) + \frac{F}{U} \frac{\partial \phi}{\partial z} = \left( F_3^I + F_3^D \right) + \frac{F}{U} i \omega e^{-kT_{\text{foil}}} \cos \beta
\]

\[
\left( F_5^I + F_5^D \right)_{\text{foil}} = \left( F_5^I + F_5^D \right) - x_{\text{foil}} \frac{F}{U} \frac{\partial \phi}{\partial z} = \left( F_5^I + F_5^D \right) - x_{\text{foil}} \frac{F}{U} i \omega e^{-kT_{\text{foil}}} \cos \beta
\]

where \( T_{\text{foil}} \) is the foils’ draft, \( \beta \) is the wave heading, \( F \) is given by (2.71) and \( U \) is the vessels speed.

2.6 Panel Method

Panel methods are developed to solve complex, open form, three-dimensional fluid dynamic problems where greater accuracy is required. Panel methods are based on potential flow theory where oscillating amplitudes of the fluid and the body are small relative to the dimensions of the body cross-section.

Panel method relies on the assumption that any irrotational, incompressible flow can be represented by a proper distribution of sources, sinks or doublets over its bounding surfaces. A source is defined as a point from which a fluid is imagined to flow out uniformly in all directions. A sink is a ‘negative’ source, where fluid is ‘sucked’ in uniformly. A doublet is a combination of source and sink. \( G \) represent the potential of a source at an arbitrary point inside a control volume such that:

\[
(2.79) \quad G = -\mu \frac{1}{4\pi} \cdot \frac{1}{r}
\]

\( r \) is the distance from the source to the arbitrary point where the potential is to be evaluated and \( \mu \) is the ‘strength’ of the source, defined as the total flux outwards (or inwards) across a small closed surface surrounding the arbitrary point.
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The governing mathematical identity utilized to solve fluid hydrodynamic problems is called *Green's second Identity*. This identity is derived from the *divergence line*.

\[ \int_{\text{control volume}} \nabla \cdot \bar{U} \cdot dV = \int_{\text{control surface}} \bar{n} \cdot \bar{U} \cdot dS \]  

(2.80)

where \( \bar{U} \) is a vector. \( \bar{U} \) can be replaced by the vector \( \phi \nabla G - G \nabla \phi \), where \( \phi \) is the velocity potential inside the control volume such that \( \bar{V} = \nabla \phi \) (\( \bar{V} \) is the body velocity) and \( G \) is the source potential. The *Green's second Identity* becomes:

\[ \int_{\text{control volume}} \nabla \cdot (\phi \nabla G - G \nabla \phi) dV = \int_{\text{control surface}} \bar{n} \cdot (\phi \nabla G - G \nabla \phi) dS \]  

(2.81)

This identity basically relates the governing equation of the physical problem to the velocity potential on the bounding surfaces of the boundary value problem. On the left hand side of the identity the term \( \int_{\text{control volume}} G \nabla^2 \phi dV \) turns into zero due to *Laplace Equation* (2.1). This fact indicates that an infinite control volume problem in space is reduced to a finite closed form problem over the bounding surfaces of the body.

### 2.6.1 Rankine Panel Method

*Rankine Source* potential with unit strength takes the form of:
\[
G(\vec{x}, \vec{\xi}) = \frac{1}{4\pi} \frac{1}{r} = \frac{1}{4\pi} \frac{1}{|\vec{x} - \vec{\xi}|} = \\
\frac{1}{4\pi} \frac{1}{\left[ (x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 - \xi_3)^2 \right]^{\frac{1}{2}}}
\]

\( \vec{x} \) is the vector aiming to an arbitrary point and \( \vec{\xi} \) is the vector aiming to the source point.

Figure 2.7 demonstrates the coordinate system and the distances from the origin satisfying equation (2.82). The simple Rankine Source is used to model free surface flows since it is difficult to evaluate the wave Green Function. The penalty for using this source is the necessity to discretize also the free surface in addition to the body surface.

![Figure 2.8: Rankine Source and an arbitrary point coordinate system](image)

Using the Rankine Source as the Green Function in equation (2.81), the Green identity becomes:

\[
(2.83) \quad -\frac{1}{2} \Phi(\vec{\xi}, t) + \int_{S_b+S_f} \Phi(\vec{x}, t) \frac{\partial G(\vec{x}, \vec{\xi})}{\partial n} dS - \int_{S_b+S_f} G(\vec{x}, \vec{\xi}) \frac{\partial \Phi(\vec{x}, t)}{\partial n} dS = 0
\]
\( \Phi(\bar{x}, t) \) is the unknown velocity potential of the problem stated in section 2.1, \( S_B \) is the surface plane of the body, \( S_F \) is the free surface and \( \bar{\xi} \) is the position of the Rankine Source.

The contribution from a closing surface at infinity vanishes due to the decay of \( \Phi(\bar{x}) \) and \( G(\bar{x}, \bar{\xi}) \) as \( \bar{x} \to \infty \) for fixed values of \( \bar{\xi} \) and \( t \). Over \( S_B \), \( \Phi_n \) is known and can be found using the body boundary conditions. Over \( S_F \) the linearized boundary conditions establish \( \Phi_z = \Phi_n \). Substitution of these boundary conditions reduces (2.83) into two integro-differential equations for \( \Phi \) over \( S_B \) and \( (\Phi, \bar{\xi}) \) over \( S_F \) which is solved by SWAN and described on the next chapter.
Chapter 3: SWAN-2 Implementation

SWAN-2 (Ship Wave Analysis) is computational fluid dynamic software, developed in the Ocean Engineering department at the Massachusetts Institute of Technology in recent years. A potential flow, Rankine Panel Method for a body with or without uniform speed is used to solve the three-dimensional time-domain ship-wave interaction problem. SWAN-2 finds the wave making resistance of a translating body, solves its seakeeping problem in ambient waves and calculates the different induced loads acts on it. The following chapter elaborates the theory and assumptions behind the computer code. More details regarding the code can be found in Sclavounos [4] and in the theory and user manuals of SWAN [11], [12].

3.1 Background and Linearization of the Boundary Value Problem

The linearized boundary conditions defined in section 2.1 are solved by SWAN to find the total velocity potential of the free flow around the hull. The program uses the coordinate system defined in figure 2.5. As elaborated on section 2.4.3, the total potential of the flow is constructed of three components:

1) A steady potential due to steady translation of the ship in calm water,
2) An unsteady potential due to unit amplitude motion in calm water (in the absence of incident waves). Arises from steady dynamic sinkage and trim of the ship and wave radiation.
3) The potential due to the incident waves and their interaction with the body.

The steady flow past a ship and its positive image above the free surface is selected to be the basis-flow and is referred to as the double-body flow. The advantage of this choice over the uniform stream, $U\bar{i}$, is that the effects of ship thickness are better modeled by the basis-flow potential, hence better results are obtained.
Linearization of the boundary conditions (as demonstrated in section 2.1) is justified if the next two conditions hold: (1) the ambient wave slope is small and (2) the hull is sufficiently thin, slender or flat. For the zero speed case, linearization exists by dropping all quadratic terms. For the forward speed case linearization assumptions follows that the fluid disturbance and wave amplitudes (caused by the ships translation) are much smaller compared to the ships speed $U$. This also means that the free surface elevation due to the disturbance is much smaller relative to the free surface elevation due to the double-basis flow.

### 3.1.1 The Free Surface

Using the *Bernoulli Equation* stated in (2.3) the basis flow surface elevation can be expressed as:

$$
\eta_0 = \frac{U}{g} \frac{\partial \phi_s}{\partial x} - \frac{1}{2g} \nabla \phi_s \cdot \nabla \phi_s
$$

(3.1)

$\phi_s$ is the steady basis flow potential or the linear first order component of the potential.

Substituting equation (3.1) in the non-linear form of the free-surface boundary conditions, (2.6) and (2.9) leads to the following formulation of the unsteady radiated potential and elevation.

$$
\left[ \frac{\partial}{\partial t} \left( \nabla \phi_s \right) \cdot \nabla \right] \eta = \frac{\partial^2 \phi_s}{\partial z^2} \eta + \frac{\partial \phi_R}{\partial z} \quad \text{on } z = 0
$$

(3.2)

$$
\left[ \frac{\partial}{\partial t} \left( \nabla \phi_s \right) \cdot \nabla \right] \phi_R = -g \eta + \left[ \nabla \cdot \nabla \phi_s - \frac{1}{2} \nabla \phi_s \cdot \nabla \phi_s \right] \quad \text{on } z = 0
$$

(3.3)

$\phi_R$ is the radiated unsteady potential, or the second order non-linear potential. It is assumed that the basis wave elevation, $\eta_s$, is sufficiently small to be transferred on the $z = 0$ plane with small error.
3.1.2 Body Boundary

Assuming small radiated amplitudes, the boundary conditions for the steady and unsteady potentials on the mean vessel position are given in section (2.4.3). The total unsteady potential can be expressed as:

\[
\phi(x, y, z, t) = \sum_{j=1}^{6} \phi_{Dj} + \phi_i
\]

where \( \phi_i \) and \( \phi_D \) are the incident and radiated potentials respectively.

Using (2.46) the radiated potential can take the form:

\[
\frac{\partial \phi_{Dj}}{\partial n} = \sum_{j=1}^{6} \left( \frac{\partial \zeta_j}{\partial t} n_j + \zeta_j m_j \right) \quad \text{on } S_B
\]

where \( \zeta_j(t), \ j = 1, 2, \ldots, 6 \), is the ship oscillatory motions in six degrees of freedom, \( \vec{n} = (n_1, n_2, n_3) \) is the unit vector, normal to the ship hull, pointing out of the fluid domain (into the hull) and

\[
(n_4, n_5, n_6) = \vec{x} \times \vec{n}
\]

\[
(m_1, m_2, m_3) = (\vec{n} \cdot \nabla) \left( \vec{U} - \nabla \phi_s \right)
\]

\[
(m_4, m_5, m_6) = (\vec{n} \cdot \nabla) \left( \vec{x} \times \left( \vec{U} - \nabla \phi_s \right) \right)
\]

The formulation presented here along with the conditions presented in sections 2.1 and 2.4.3 complete the boundary value problem calculated by SWAN.
3.2 Ship Response in Calm Water and in Ambient Waves

3.2.1 Ship Translating in Calm Water

The basic case of zero speed is treated by assigning $U = 0$ and $\phi_s = 0$. In the case of a ship starting to translate with a forward uniform speed $U$, the steady and unsteady potentials obey the conditions (2.11), (2.45). The transient wave flow induced by the ship forward speed is subject to the free surface conditions stated in section 3.1.1. The vessel motions, $\zeta_j(t) = 0$ except for $\zeta_3(t)$ which corresponds with the ship steady dynamic sinkage and $\zeta_5(t)$ which correspond with the ship steady dynamic trim. The steady motions are produced after the transient effect die out.

3.2.2 Ship Seakeeping with Forward Speed

This is the most general case where ambient waves exist ($\phi_D, \phi_t \neq 0$ as opposed to the case in section 3.2.1). It involves solving the complete set of boundary conditions, namely (2.11), (2.45) for the steady potential and (2.46), (3.2)-(3.5) for the unsteady, time dependent potential. Solving the potential flow enables finding values of interest as the ship motions in six degrees of freedom. More details and a test case are examined in Kring [16].

3.2.3 Thin Wake Sheets – Transom conditions

The majority of the modern ship have transom sterns. For such stern shapes, flow separation at the sharp lower edge of the transom is triggered by viscous effects and requires implicit enforcement of the correct behavior of the potential flow.

SWAN-2 allows modeling of thin sheets in the wake of vertical and horizontal objects as struts or transom sterns. Lifting surface theory methods are utilized. The velocity potential is forced to satisfy the Kutta-Condition at the vessels transom: the fluid is enforced to detach the stern in a smooth manner. As an outcome, the fluid velocity is finite and the rest
Chapter 3. SWAN-2 Implementation

of the transom is dry and causes induced drag as introduced in section 2.3.4. The total potential at the wake should satisfy the Laplace equation.

Figure 3.1 illustrates the free surface wake shed by a transom stern vessel at Froude Number 0.74. On the horizontal wake surface, the free surface conditions (3.2) and (3.3) are enforced. The wave elevation right after the transom is enforced to be equal to the vessels draft (flow separates at Trailing Edge1).

The Trailing Edge0 is defined to continue with the \( z = 0 \) plane. The wave slope at the transom must be equal to the slope of the hull at the transom and the velocity potential at the transom and its normal derivative are equal on the vessel and in the free surface wake.

![Figure 3.1: A thin wake sheet](image)
The dynamic *Kutta-Condition* imposes a dynamic pressure on the free surface that balances the hydrostatic pressure due to a given transom draft. Along with the kinematic *Kutta-Condition* this ensures continuity in pressure along the detaching streamlines of the transom.

### 3.2.4 Motion Control Lifting Appendages

SWAN-2 allows modeling of motion control lifting appendages attached to the ship in order to improve its attitude at sea. In most cases the appendages dimensions are small relative to the hull, hence the flow around the device does not change significantly the hull potential. SWAN-2 ignores the effects of the hull disturbance potential due to the presence of these appendages. The lift force achieved by these devices is appreciated using the ship forward speed and angular displacement and the incident wave velocity vector. The overall force is calculated by summation of the steady force (as stated at equations (2.62) and (2.67)) and the time dependent lifting force (as stated at equations (2.70), (2.71) and (2.74)).

\[
L_{\text{Total}}^3 = \rho \pi U^2 S \frac{AR}{AR + 2} \left( \alpha_{\text{steady}} + \alpha(t) \right)
\]

where \( L_{\text{Total}}^3 \) is the total lifting force acting vertically, \( \alpha_{\text{steady}} \) and \( \alpha(t) \) are the steady and time dependent angles of attack. The total lifting force is assumed to act vertically at the center point of the lifting surface. Its contribution to the total force and moment acting on the hull is included in the derivation of the equation of motions introduced at (2.75)-(2.78).

### 3.3 Numerical Solution

*Green's Second Identity* (equation 2.81) is being solved by SWAN-2 using the boundary conditions introduced in sections 2.1 and 3.1 for the velocity potential and its normal derivatives over the fluid domain boundaries. A *Rankine point source* (equation 2.82) is invoked as the unit strength *Greens function* and leads to the integral equation...
introduced in equation (2.83). This equation is utilized to solve all linearized boundary value problems over all surfaces: body surface, free surface (including the surface between the hulls of a twin hull vessel) and wake sheet.

### 3.3.1 Spatial Discretization

The vessel boundary and the free surface within well defined boundaries are divided into a large number of quadrilateral panels. Over the surfaces the velocity potential, \( \phi \) and the wave elevation, \( \eta \) are approximated by a bi-quadratic spline variation:

\[
\phi(x, t) \approx \sum_j \phi_j(t) B_j(x)
\]

\[
\eta(x, t) \approx \sum_j \eta_j(t) B_j(x)
\]

where the basis functions, \( B_j(x) \), is centered in the \( j^{th} \) panel and provides continuity of the potential and elevation and their tangential gradient. The coefficients \( \phi_j \) and \( \eta_j \) are not equal to the potential and wave elevation in the center of each panel but are related to them linearly through the spline coefficients of the relevant panel.

### 3.3.2 Meshing

SWAN-2 utilizes panel shapes elements through a well defined fluid domain and hull boundaries. The surfaces are discretized using rectangular topology within cartesian or polar coordinates. The shape of the body is crucial in determining which coordinate system is to be used (for example: offshore cylindrical platforms are modeled using polar coordinates, slender ship hulls are models using cartesian coordinates). Transom stern vessels are modeled in SWAN-2 using three different types of panel sheets.

1. A free surface panel sheet. Includes all the free surfaces around the vessel. The boundary conditions introduced in equation (3.2) and (3.3) together with the Greem's Identity (2.83) are applied for the velocity potential and the wave elevation.
At the boundaries of this sheet the second derivative of the potential and the wave elevation are enforced to zero. The free surface can be modeled using rectangular or oval panels. The free surface panel discretization includes several components:

a. A thin wake sheet behind the vessel where the transom stern Kutta-Conditions are also imposed.

b. An artificial beach on the fluid domain boundaries to absorb (damp) the wave energy. Damping forces are measured and are the wave making resistance of the vessel.

c. A thin wake sheet behind a vertical solid boundary as strut.

2. A submerged body panel sheet. The submerged part of the hull or components of it are modeled by this type of sheet. Body boundary conditions are applied (2.45), (2.46) and (3.5). It also include panel sheets used to discretize thin vertical shapes as struts.

3. A transom stern panel sheet. Imposes the Kutta-conditions on the trailing edges to resume continuity with the leading edges of the wake behind it.

SWAN-2 utilizes each sheet automatically based on input data entered by the program user.

A common feature of all types of sheets is that all of them have a rectangular topology. More details can be found in *SWAN Theory Manual* [11].

The next figure demonstrates a discretization of the fluid domain and of the left side of a twin hull vessel in a cartesian coordinate system. Only half the vessel in modeled (in case of a monohull – half the hull) for symmetry reasons. ‘Cutting’ half the problem saves calculation time.
3.3.3 Temporal Discretization

In the time domain, a time-marching scheme is selected to approximate the time derivative of the state variables: the potential, $\phi$ and the wave elevation, $\eta$ or equivalently their spline coefficients as defined by equations (3.8) and (3.9). The Euler time scheme as defined here is used by SWAN-2.

$$
(3.10) \quad \left( \frac{\partial \phi}{\partial t} \right)^n \approx \frac{\phi^{n+1} - \phi^n}{\Delta t}
$$
where \( n \) marks the time step. Equation (3.10) is substituted in the free surface condition, equations (3.2) and (3.3), and in the Green’s Identity, equation (2.83). The remaining terms in (3.2) and (3.3) which does not contain time derivatives of the state variables can be enforced either by the \( n^{\text{th}} \) or the \( (n+1)^{\text{st}} \) time step. SWAN-2 enforces the kinematic boundary condition to be satisfied by the explicit \textit{Euler} discretization for the past solutions at time \( t = t_n \), and enforces the dynamic boundary condition to be satisfied by the implicit \textit{Euler} discretization at present time where \( t = t_{n+1} \). The resulting is a mixture of both explicit and implicit discretization methods and is referred to as the explicit \textit{Euler} discretization.

### 3.3.4 Numerical dispersion, damping and stability

A discretization error throughout the free surface mesh could distort the propagating wave pattern to the point where the solution becomes inaccurate. This error is quantified by the \textbf{numerical dispersion} and \textbf{damping}. These quantities measure the discrepancies in the phase and amplitude between the discrete and the continuous wave patterns respectively. The condition required for a numerical error to vanish in the limit as the panel size goes to zero is known as the \textbf{numerical stability} criterion.

The free surface discretization is characterized by the \textbf{panel aspect ratio}, \( \alpha \) and the \textbf{grid Froude Number}, \( F_h \) where

\begin{align}
\alpha &= \frac{h_x}{h_y} \\
F_h &= \frac{U}{\sqrt{gh_x}}
\end{align}

The panel dimensions in the stream-wise and transverse direction are presented by \( h_x \) and \( h_y \) respectively.
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The stability analysis of marching schemes in time domain utilizes a criterion which restricts the choice of the non-dimensional time step:

\[
\beta = \sqrt{\frac{h_x}{g \Delta t}}
\]

(3.13)

The stability properties of a broad range of marching time schemes were studied by Nakos [17] and Vada & Nakos [18]. The result, borrowed from Sclavounos [4], illustrates the critical values of \( \beta \), is presented in the following figure.

![Figure 3.3: Stability criterion for implicit Euler time marching schemes.](image)

The diagram plots the values of \( \beta \) as functions of \( \alpha \) and \( F_h \). It elaborates the upper boundary of the time step, \( \Delta t \). It is noted that for higher speeds a smaller time step is required, thus longer execution time.
3.4 Radiation Condition

The proper enforcement of the radiation condition is essential for the success of Rankine Panel Methods in simulating ship flows. SWAN-2 enforces radiation condition by the design of an absorbing dissipative beach located at the outer end of the free surface mesh. Over the dissipative beach the following conditions are enforced:

\begin{align}
\left( \frac{\partial}{\partial t} - U \frac{\partial}{\partial x} \right) \eta &= \frac{\partial \phi}{\partial z} - 2\nu \eta + \frac{\nu^2}{g} \phi \\
\left( \frac{\partial}{\partial t} - U \frac{\partial}{\partial x} \right) \phi &= -g \eta
\end{align}

\( \phi \) is the wave disturbance potential, and \( \nu \) is the strength of the damping parameter. \( \nu \) is chosen such that its value increases towards the outwards of the absorbing beach. Over the inner edge of the beach its value is zero for continuity reasons. At zero speed, these conditions lead to the dispersion relation:

\[ w = i\nu \pm \sqrt{gk} \]

More details about the construction and performances of the dissipative beach can be found in Nakos, Kring & Sclavounos [19].

3.5 Flow Solver

Enforcing the free and body surfaces boundary conditions into the Green's Identity, leads to an integral equation in space and a system of ordinary differential equations in time. The unknowns are the basis flow spline coefficients, \((\phi_i)(t)\) for \(i = 0,1,...,N\), the
wave disturbance floe spline coefficients, $\phi_i(t)$ for $i = 0, 1, \ldots, N$ and the free surface elevation spline coefficients, $\eta_j(t)$ for $j = 0, 1, \ldots, N_f$. $N$ is the overall number of panels over the domain and $N_f$ is the number of free surface panels.

The velocity potential and wave elevation are then solved for each time step of the implicit Euler scheme described in section 3.3.3 at each panel according to equations (3.8) and (3.9).

### 3.6 Hydrodynamic Pressure

The hydrodynamic pressure follows from the application of Bernoulli's Equation (2.3) for the total potential (2.44). SWAN-2 decomposes the total potential into two components:

$$
(3.17) \quad \phi(\vec{x}, t) = \psi(\vec{x}) + \varphi(\vec{x}, t)
$$

$\psi(\vec{x})$ is defined as the impulsive potential and has the following boundary conditions:

$$
(3.18) \quad \psi(\vec{x}) = 0 \quad \text{on the free surface}
$$

$$
(3.19) \quad \frac{\partial \psi}{\partial n} = \overrightarrow{U}_b \cdot \vec{n} \quad \text{on the body surface}
$$

Since $\psi$ vanishes at the free surface, all wave effects are included in the second component, the residual wave potential, $\varphi(\vec{x}, t)$. This component satisfies the following condition:

$$
(3.20) \quad \frac{\partial \varphi}{\partial n} = 0 \quad \text{on the boy surface}
$$
This condition is obtained by substituting (3.17) into equations (3.2) and (3.3). This decomposition is essential to ensure the stability of the time marching scheme, invoked to treat the vessel equations of motion, discussed later.

For a fixed point in the fluid, the hydrodynamic pressure can be expressed in terms of the Bernoulli Equation for a coordinate system translating with the ship in speed \( U \) as follows:

\[
(3.21) \quad p(\bar{x}, t) = -\rho \left\{ \frac{\partial}{\partial t} \left( \mathbf{U} - \nabla \phi_0 \right) \cdot \nabla \phi + \frac{1}{2} \nabla \phi \cdot \nabla \phi + g_z \right\}
\]

where \( \phi_0 \) is the basis-flow potential and \( \phi \) is the total potential.

The total potential can be decomposed, hence the pressure can be obtained in terms of the potential components. It is noted that the impulsive potential (or the basis-flow potential) are functions of space and independent of time.

Calculating the pressure over the hull surface is more difficult, since the hull position changes in time due to a time dependent sinkage and trim, and oscillations caused by waves. \( \bar{\delta}(t) \) is denoted as the displacement vector of a fixed point on the body surface where:

\[
(3.22) \quad \bar{\delta}(t) = \bar{\zeta}(t) + \bar{\alpha}(t) \times \bar{x}
\]

\( \bar{\zeta}(t) \) is the vessels linear translation motion, \( \bar{\zeta}(t) = (\zeta_1, \zeta_2, \zeta_3) \), \( \bar{\alpha}(t) \) is the vessels rotational motions defined as \( \bar{\alpha}(t) = (\alpha_4, \alpha_5, \alpha_6) \) and \( \bar{x} \) is the position of the point, or as defined by SWAN-2 - the center of a particular panel.

For small values of \( \bar{\delta}(t) \) a Taylor series expansion is used to evaluate the time dependent pressure of the center of each panel on the hulls surface as:
The spatial gradient of the hydrodynamic pressure, defined in (3.21), is calculated by first and second spatial gradients derivations of the velocity potential. The potential is evaluated by the B-Spline approximation, equation (3.8).

Expression (3.21) is the fundamental form of Bernoulli's Equation used by SWAN-2 to evaluate the steady state as \( t \to \infty \) and the time dependent forces acting on the vessel.

### 3.7 Hydrodynamic Forces

The hydrodynamic forces exerted by the surrounding fluid on the ship hull are necessary for the derivation of the vessel's equation of motion and for evaluation of the structural loads exerted on the body.

The vector \( \bar{n} \) represents a unit normal vector pointing into the hull. By virtue of Taylor Series, the instantaneous position of the hull surface is given by:

\[
(3.24) \quad \bar{n}(t) = \bar{\alpha} \times \bar{n} + O(\delta^2)
\]

The total hydrodynamic force acting on the hull is expressed by:

\[
(3.25) \quad \bar{F}(t) = \iint_{S_B(t)} p(\bar{x}, t) \bar{n}(t) dS
\]

where \( S_B(t) \) is the instantaneous wetted surface of the hull, \( p(\bar{x}, t) \) is given by (3.23) and \( \bar{n}(t) \) by (3.24). The corresponding moment about the reference coordinate system is given by:
(3.26) \[ \bar{M}(t) = \int\int_{S_B(t)} \rho(\bar{x}, t)(\bar{x} + \bar{\delta}) \times \bar{n}(t) \, dS \]

where \( \bar{x} = x\hat{i} + y\hat{j} + z\hat{k} \) and \( \bar{\delta} \) is defined in (3.22).

Linearization of equations (3.25) and (3.26) about the mean position of the ship hull \( S_B \) is required for two reasons:

1. All quantities evaluated by SWAN-2 are known over panels located on \( S_B \).
2. Linearization will reveal the steady state, linear and quadratic components of the force and moment as individual effects.

The time dependent wetted surface can be expressed similarly as the hydrodynamic pressure about its mean position.

(3.27) \[ S_B(t) = \bar{S}_B + dS(t) + O(\delta^2) \]

\( dS(t) \) is the differential wetted surface arises near the water line and can be expressed by:

(3.28) \[ dS(t) = dl[\eta(t) - \bar{\delta} \cdot \bar{k}]_{\text{water-line}} \]

where \( dl \) is a differential length of a segment along the vessels mean water-line. \( \eta(t) \) is the wave elevation along the water line, accounts for all wave elevation effects such as the incident wave disturbance. \( \bar{\delta} \) is defined by (3.22), and accounts for all the vessels displacements as sinkage and trim, and the wave oscillations. \( \bar{k} \) is the unit vector pointing in the z-direction.

The resulting time dependent, linearized expressions for the induced force and moment on the body are:
Chapter 3. SWAN-2 Implementation

(3.29) \[ \vec{F}(t) = \int_{S_p} p(\bar{x},t)\vec{\eta}(t)\,dS + \int_{\text{water-line}} p(\bar{x},t)\vec{\eta}(t)\cdot\vec{k} \,dl \]

(3.30) \[ \vec{M}(t) = \int_{S_p} p(\bar{x},t)(\bar{x} + \vec{\delta})\times\vec{\eta}(t)\,dS + \int_{\text{water-line}} p(\bar{x},t)(\bar{x} + \vec{\delta})\times\vec{\eta}(t)\cdot\vec{k} \,dl \]

SWAN-2 Invoke Taylor's Series expansions for \( p(\bar{x},t) \) and \( \vec{\eta}(t) \) from equations (3.23) and (3.24), in order to evaluate the force and moment as expressed above. It employs them to evaluate the vessels small amplitude displacements, structural loads, wave resistance and wave added resistance.

3.8 Equations of Motion

The vessel undergoes time-dependent motions, due to the hydrodynamic forces and moments presented in the last section. Applying Newton’s law results in the following set of equations for dynamic equilibrium.

(3.31) \[ \sum_j \left[ \vec{M}_{ij} \ddot{\xi}_j(t) + \vec{C}_{ij} \dot{\xi}_j(t) \right] = \vec{F}_i(\ddot{\xi}_j, \dot{\xi}_j, \xi_j, t) \quad i = 1...6, \quad j = 1...6 \]

\( M_{ij} \) is the vessels inertia matrix and \( C_{ij} \) are the vessels hydrostatic coefficients.

As time increases surface waves are generated. Theses waves propagate outward from the body, but still continue to affect the fluid pressure around the hull and the body force for all subsequent times. Memory Effects are introduced. Under the assumption of small vessel motions and wave slopes, linearization of the equation of motion is justified. The forces acts on the hull can be decomposed into a local component due to the instantaneous motion, and to a time dependent component due to memory effects. For an arbitrary time-dependent motion, \( \xi_j(t) \), the time-dependent component of the pressure force acts on the hull can be expressed in terms of a convolution integral.
\[ F_j(t) = X_j(t) - \sum_{j=1}^{6} \int_{0}^{t} K(t-\tau) \cdot \dot{\zeta}_j(\tau) \cdot d\tau \quad j = 1...6 \]

\( X_j(t) \) is the hydrodynamic exciting force and the kernel \( K(t-\tau) \) can be interpreted as the force, at time \( t \), due to a delta-function body velocity at an earlier time \( \tau \). In short it can be referred to as the velocity impulse response function.

The following equation of motion is accepted.

\[ \sum_j \left[ (M_y + a_y) \ddot{\zeta}_j(t) + b_y \dot{\zeta}_j(t) + (C_y + c_y) \zeta_j(t) \right] = F_j(\zeta_j, \zeta_j, t) \]

\( i = 1...6, \quad j = 1...6 \)

\( a_y, b_y \) and \( c_y \) represent the local force coefficients. Solving this equation requires determining the values of these coefficients and the time-dependent force. This is achieved by solving the boundary value problem stated at section 3.1 for the instantaneous force.

The left hand side includes all inertia, 'hydrostatic' and "effective" impedance forces induced by the impulsive potential (as defined in section 3.6). The right hand side includes all wave effects, namely calm water radiation, ambient wave induced radiation and diffraction effects, wave induced excitation and the wave induced lifting forces and moments on motion control devices considered in sections 2.5 and 3.2.4.

SWAN-2 solves equation (3.33) for each time step, yielding the vessels kinematics and forces. The solution of the boundary value problem and evaluation of the hydrodynamic forces and responses happen simultaneously for each time step. The result is a time record for the vessels response, \( \zeta_j(t) \), and corresponding forces and structural loads.
3.9 Applications

3.9.1 Wave making Resistance, Sinkage and Trim

SWAN-2 calculates the vessels ideal fluid resistance, sinkage and trim motions while translating at calm water, by invoking the boundary conditions presented in section 3.1 and solving the equations of motion introduced by equation (3.33). The solution of these equations, provides the time record of the sinkage and trim time-dependent displacements of the vessel.

The wave resistance is evaluated by direct pressure integration, as introduced in equation (3.29), for the positive x-direction. As time grows, the force converges and gain a steady value, which is extracted by SWAN-2 as the vessels wave resistance. Sinkage and trim affects the vessels wave resistance appreciably. This is particularly true for high speed semi-displacement vessels at Froude Numbers 0.5 and higher.

3.9.2 Linear Seakeeping in the time domain

The ambient wave potential, given by equation (2.17), is included in the seakeeping boundary value problem described in section 3.1, and in the solution of the equations of motions, equation (3.33). The resulting quantities of interest include the vessels response and structural loads in the time domain, excited by a wave spectrum, defined by the software user. The solution of the seakeeping problem can not be separated from the sinkage and trim of the vessel due to its speed. However these quantities are small compared to the response due to the incident waves, hence can be neglected.

3.9.3 Evaluation of Frequency Domain RAO’s

SWAN-2, as a time domain seakeeping program, solves for the vessels response for all degrees of freedom, excited by an incident polychromatic wave record, with velocity potential defined as:
All wave components are assumed to have the same heading, $\beta$, but their amplitudes, $A_m$, wave number, $k_m$, wave frequency, $w_m$, and random phase angle, $\varepsilon_m$, may be different. SWAN-2 simulations for the response provides a polychromatic signal for large time interval. This signal is analyzed by a Fourier Transform to extract the amplitude and phase of each response.

### 3.9.4 Wave Induced Loads

The local hydrostatic and hydrodynamic pressure distribution over the hull is given by expression (3.23) evaluated over the mean ship position. This pressure distribution is required as an input for structural analysis calculations, to evaluate structural loads. These loads are often needed for preliminary design of a ship. These loads include mainly the bending moment, shear force and torsion moment. The time-dependent load records are obtained corresponding to a steady forward translation in calm water or in waves.
Chapter 4: The Steady Problem

4.1 Introduction

The time domain, *Rankine Panel Method* code, SWAN-2 (Ship Wave Analysis) was used to evaluate the vessels calm water resistance. This was done by specifying no ambient waves, hence calm water sea – steady state solution. The extent of the interaction between the two demi-hulls of the catamarans was examined for three separation ratios between these demi-hulls, at various forward speeds. All the catamarans examined had symmetric demi-hulls, and a deep and wide transom shape, where the draft of the transom was equal to the draft of the keel, and its beam was equal to the maximum beam of the demi-hull.

Transom stern vessels often experience hydrostatic induced resistance at high speeds, as a result of flow separation at the transom. The pressure over the transom in these cases is atmospheric, and since the rest of the vessel is half-submerged, there exists an un-favorable net force against the direction of ship advance. If the transom stern is deep and wide, the drag penalty has a significant magnitude. In cases of transom stern vessels, SWAN-2 always assumes dry transom. This assumption is correct for the high speed vessel evaluated in this paper.

SWAN-2 calculates the total resistance of the vessel by direct integration of the pressure along the hull, as stated in section 3.9.1. This value is thus includes the wave making resistance and the induced resistance component over the dry transom. All resistance results presented in this chapter are the summation of these two quantities and are referred to as the **ideal fluid resistance**.
SWAN-2, as a potential flow code, does not predict viscous resistance values. The ideal fluid resistance calculated by SWAN-2, is thus only a part of the total resistance of the vessel. Friction drag can be approximated by solving the boundary value problem including viscous effects, by performing model tests, or as will be shown below, by estimating the friction drag on the basis of the flat plate 1957 ITTC line and an estimated form factor.

4.2 SWAN-2 Execution

The vessel's ideal fluid resistance is sensitive to the hull geometry. It is important therefore, to establish a spatial mesh, which is as dense as possible, to assure close similarity to the real physical problem. SWAN-2 allows the user to establish a number of panels to be used in the analysis, in a prescribed domain, hence to establish the size of the domain. Several sensitivity checks were made over the catamaran hull. It was found that a combination of 28 panels along the hulls length, and 7 panels along the hulls width, gave the optimum solution. This combination gave good and reasonable results on one hand, and saved computer calculation time (comparing to other more dense combinations) on the other. This panel combination over the hull surface, and the free surface domain, resulted in about 3700 elements. This combination was left constant for all ideal fluid resistance simulations.

The size of the fluid domain is important. It is required to choose a domain that is large enough, to avoid wave reflections from the artificial beach. A large domain ensure square panels in the vicinity of the free surface, which is important to the correct representation of the steady wave pattern, downstream the vessel. The domain chosen for all catamaran simulations included $0.5L_{pp}$ upstream from the bow to the free surface boundary, $1.5L_{pp}$ from center-line of the port demi-hull to the transverse free surface boundary, and $2L_{pp}$ downstream from the vessels stern to the free surface boundary. $L_{pp}$ represents the length between perpendiculars of the vessel, or the design water line length.
Chapter 4. Calm Water Resistance

SWAN-2 simulations use only the port side of the symmetric hull, in order to save calculation time. In the case of the catamaran, only the port demi-hull was modeled, including the free surface between the hull and the center-line of the entire ship. This parameter was changed for the three different separation ratios.

The selection of the time step is crucial for stability of the steady state simulation. For all simulations the default value of time step, calculated by SWAN-2, was used. As stated in section 3.3.4, stability of convergence depends on the size of the panel and the chosen time step, where the critical value of $\beta$ is given in equation (3.13). SWAN-2 chose as a default, a time step which coincides with $2\beta$, such that stability is always ensured with a safety margin.

4.3 Wave Patterns Predictions

The steady wave patterns for all separation ratios, at all speeds were predicted. Some of these patterns are presented in the following figures. The flow around each demi-hull is asymmetric due to interference from the second demi-hull. The interference increases as the separation ratio decreases due to increasing proximity between the hulls. The wave patterns are all within the Kelvin Wave Pattern sector as described in section 2.3.2. From the figures it can be noticed that as the speed of the vessel increases, more diverging waves are seen. For lower speeds, more transverse waves exist.

The following figures present several results of the simulations performed at the semi-displacement regimen. The figures describe Kelvin Wave Patterns for the catamarans separation ratios of $s/L=0.3$, $s/L=0.4$ and $s/L=0.5$, at three different forward speeds, which comply with Froude Numbers 0.57, 0.74 and 0.90.

Green portions of the figure present free surface with zero elevation or a very low elevation (positive or negative). Yellow to red spots describe positive elevation, where the darker color stands for higher waves. Light blue and blue present negative elevation, i.e. wave troughs. As the color becomes darker the trough is deeper.
A deep trough exists at the wake of all vessels close to the transom, similar to what can be observed at the wake of ships, traveling at high speeds. This trough is followed by a high crest at the wake panel sheet and around it.

Figure 4.1: wave pattern for $s/L=0.3$, at Froude Numbers 0.57, 0.74 and 0.90
Figure 4.2: wave pattern for s/L=0.4, at Froude Numbers 0.57, 0.74 and 0.90
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Figure 4.3: wave pattern for $s/L=0.5$, at Froude Numbers 0.57, 0.74 and 0.90

Figure 4.3: wave pattern for $s/L=0.5$, at Froude Numbers 0.57, 0.74 and 0.90
4.4 Dynamic Sinkage and Trim

The dynamic sinkage \( (\varsigma_3) \) and trim \( (\varsigma_5) \) of the vessel influence the vessel’s calm water ideal fluid resistance especially at high speeds. It is essential that the values of the sinkage and trim would converge to steady values, when the ideal fluid resistance is measured. Convergence is achieved by running calm water simulation several times, each time with the sinkage and trim results of the previous run, until the difference between input and output values is negligible. Usually 7-8 runs were required for convergence. The sinkage and trim angles are calculated at the origin of the coordinate system, hence at midship. Negative sinkage means that the ship draft increases, positive trim means a trim angle by the stern. Another characteristic of the hull, calculated during calm water simulation, is the ship’s wetted area, which is important for calculating viscous effects.

The following graphs present the converged sinkage and trim for all separation ratio catamarans, and for a similar monohull (with the same displacement / length ratio).

![Dynamic Sinkage Graph](image)

**Figure 4.4: Dynamic sinkage for all catamarans and a monohull as a function of Fn**
Chapter 4. Calm Water Resistance

Figure 4.5: Dynamic trim for all catamarans and a monohull as a function of \( Fn \)

The monohull has lower dynamic sinkage and trim values than all catamarans. This is mainly due to a lower wetted surface (the monohull wetted surface is smaller in about 25\% the average wetted surface of the catamarans), hence lower force exists due to pressure distribution and by that, a lower motion displacement appears.

The dynamic trim for all catamaran types is increasing for higher speeds, i.e. the steady pitch angle increases for higher Froude Numbers. A negative trim (bow up, stern down) locally increases the draft of the vessel at the stern, hence increase the induced resistance (due to dry transom). For large transom stems as exist for the catamarans investigated, an increase of the draft has a significant impact on the overall ideal fluid resistance of the vessel. At higher Froude Numbers, the trim angle gets relatively high values, which are not practical for design purposes. This pattern is caused due to the catamarans round bilge shape. The catamaran has a displacement hull shape, and it translates at the semi-displacement regime. For this type of hull shapes, at high speeds, the vessels bow is 'sucked' into the water. A very large pressure differential exists in the bow area and causes this phenomenon.
Chapter 4. Calm Water Resistance

The dynamic sinkage of the catamarans does not have a steady pattern at the semi-displacement region. The mean of all sinkage values, is close to zero sinkage. The catamarans have higher sinkage value than the monohull, meaning that their drafts do not increase as much as the monohull drafts, i.e. at Froude Numbers of 0.5 to 1.0, the monohull gain more wetted surface than the catamarans. As will be shown below, the wetted surface of the monohull, regardless of the large negative sinkage, is still smaller than these of the catamarans.

4.5 Calm Water Resistance

4.5.1 Preface

The ‘numerical’ catamaran evaluated in this paper, was examined at speed range of 35-60 knots with a 5 knots step. All results at calm water and in waves are presented as a function of Froude Number, a non-dimensional quantity which is defined by the vessels speed and length.

\[
Fn = \frac{U}{\sqrt{gL}}
\]

where \( g \) is gravity acceleration. All Froude Numbers evaluated in this research were in the semi-displacement regimen, \( Fn = 0.5 - 1.0 \).

Couser [20] breaks down the resistance of a vessel into the following components: (1) a tangential stress, due to viscousity and form effects and (2) a normal stress, due to pressure distribution (wave making resistance, induced drag over the transom and around the hull and pressure due to viscousity). The induced drag of catamaran ships includes also the affect of the interface between the demi-hulls. This effect is not treated numerically yet, but was measured by model tests. A particular problem for catamarans at high Froude-Numbers is that, due to their slenderness, the total resistance is dominated by the viscous
resistance component. This fact is highlighted in Couser [20]. For Froude-Numbers at the range of 0.5-1.0, the ratio between the wave making resistance component and the viscous resistance component (containing a normal component due to pressure) is about 1:4. This is unfortunate since the friction is calculated empirically based on the friction of a flat plate, and is modified to obtain the viscous resistance component.

SWAN-2 measures the normal resistance, obtained by integrating the pressure distribution along the hull. Since a bare hull was evaluated (with no other appendages), no other normal components exists. The wave making resistance and the induced drag due to dry transom are included in this magnitude.

4.5.2 Ideal Fluid Resistance

The component of resistance, calculated by SWAN-2, was referred here (section 4.1) as the ideal fluid resistance.

Predictions of the ideal fluid resistance for the three catamarans and the monohull, as a function of their Froude Number, are detailed in figure 4.6.

The ideal fluid resistance is calculated by integrating the pressure distribution along the hull, thus including an induced resistance component at the dry transom. The induced resistance can be calculated by integrating the hydrostatic pressure over the transverse projected area of the wetted transom. The draft of the catamarans and the area of the ‘wetted’ transom changes as a function of the catamarans speed, thus related to alterations of sinkage and trim. At zero speed, the draft of the transom at calm water is 5.75 m, the ‘wetted’ area at zero speed is about 31 m², and so the induced drag is estimated by 1800 KN. This component can be deducted from the ideal fluid resistance in order to get the wave making resistance results.

Figure 4.6 also include an ideal fluid resistance prediction for ‘twice demi-hull’. This prediction was conducted by measuring the drag over one demi-hull (modeled in SWAN-2.
as a monohull), and doubling the results. This measurement is very much like measuring the resistance of a catamaran with a separation ratio of: $s/L \to \infty$.

![Figure 4.6: Ideal fluid resistance coefficient results](image)

The separation between the demi-hulls does not affect appreciably on the catamarans’ **ideal fluid resistance**. For the most parts of the semi-displacement range, the larger separation ratio has higher resistance values then the smaller separation ratios, but the magnitude of this difference is small comparing to the absolute value of the resistance. The drag predicted for ‘twice demi-hull’ was measured in four speeds, and its values were larger then the resistance for the widest catamaran. The phenomena of increase in resistance as the distance between the demi-hulls grows, is caused due to a favorable interaction between the demi-hulls. The *Kelvin* wave pattern, exerted from one demi-hull, ‘wets’ the transom stern of the adjacent demi-hull and vice versa. The induced, adverse drag force is decreasing, since less area of the stern is dry, hence the overall ideal fluid resistance is smaller.
Chapter 4. Calm Water Resistance

The monohull examined in this paper has the same length and displacement values as the catamarans. It had larger beam, in order to keep the same displacement. The length and the displacement of a vessel have a large effect on its measure of effectiveness (for instance, the amount of payload the vessel is capable to transform, etc.) and cost. It was interesting to compare different hull forms with similar requirements. The monohull had higher values of resistance then the catamarans. The main reason is that the catamarans demi-hulls are much slender then the monohull. Their hulls are more streamlined and surface piercing, the monohull has larger transom area, higher B/T ratio and higher B/L ratio. It can be concluded that all the catamarans, evaluated in this paper, are more effective then the monohull, in a sense that they require less power to gain the same speeds (or less fuel to gain the same range). It should be noted that although the ideal fluid resistance component is lower for the catamarans, the friction component of resistance is probably higher, since the overall wetted surface of the catamarans is larger in 25% from the overall average wetted surface of the monohull.

The catamaran hull forms developed in this paper, has a superior ideal fluid resistance characteristic over a monohull with a similar mission. It is also cost-effective relative to its demi-hulls components due to favorable interaction between the hulls.

4.5.3 Wave Making Resistance

For each catamaran, at each speed, the results of the wave making resistance were isolated by subtracting the induced drag at the transom.

Calculating the induced drag, due to dry transom, shows that the induced drag component is significantly large relative to the wave making resistance values, mainly since the vessels has a ‘deep transom’. The values of the induced drag were calculated separately for each speed, and their magnitudes varied due to the trim and sinkage effects over the transom draft and ‘wetted’ area. This component of the induced resistance was easy to approximate. Other components of the induced resistance (or normal drag as referred above) due to the shape of the hull, are harder to appreciate, but it is assumed that their influence is negligible, due to the hull smooth lines. The area of the ‘dry’ transom was
calculated as if there were no interactions between the demi-hulls, and the wake does not ‘wet’ the adjacent demi-hull transom. Calculating this variable is somewhat difficult, and since the ideal fluid resistance magnitude of all catamarans was similar it was decided to neglect this favorable force.

Figure 4.7 emphasizes the catamarans wave making resistance coefficient after deducting the influence of the induced drag at the transom.

![Wave making resistance coefficient](image)

**Figure 4.7: Wave making resistance coefficient**

In figure 4.8, the resistance coefficients were analyzed as a function of the vessels dynamic trim. As the dynamic trim increases, the vessels resistance coefficient decreases, hence as the bow is submerged deeper in the water, the values of these coefficients are decreasing.
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The trend of the wave making resistance curves is similar to the trend of the ideal fluid resistance curves. Generally when the speed increases, the resistance decreases. Although the vessel has a displacement hull, it behaves at high speeds as a planning hull. Its draft values decreases, (an indication for planning) so its induced drag at the transom is lower and the vessel ‘invests’ less energy in producing waves. As noticed above, the separation between the demi-hulls has a minimal effect on the wave making resistance coefficient values.

The wave making resistance results were compared to several model tests conducted for catamarans with similar separation ratios. Figure 4.9 presents the results of the comparison.

Model test results were obtained from Steen [21] and from Bruzzone [22]. Steen [21] presented resistance results for several catamaran models, tested by MARINTEK. The curve shown in the graph above is a mean value of the wave resistance coefficients, measured by MARINTEK.
Chapter 4. Calm Water Resistance

Figure 4.9: Wave making resistance coefficient – comparison chart

Bruzzone [22] presented several separation ratio results of a 40m catamaran. The curve above presents a separation ratio of s/L=0.3. The resistance coefficients predicted by SWAN-2 are slight higher than those obtained from the tests at the range of Froude Numbers 0.55 to 0.85. At high speeds, Fn = 0.85-1.0, SWAN-2 predicts lower values of wave making resistance. The differences from model tests are due to differences in hull forms and demi-hull dimensions. Despite these differences, the trend of the lines is similar and their values are in the same order of magnitude.

4.5.4 Viscous Drag

The ITTC 1957 line presented in equation (2.22), is valid for viscous drag over a flat plate. A form factor was needed to be established in order to estimate the ratio between the flat plate viscous drag and the viscous component of the resistance of the vessel in hand.

\[ 1 + k = \frac{C_f^{\text{real}}}{C_f^{\text{flat plate}}} \]
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$k$ is the form factor and the $C_f$'s are the viscous drag coefficients. The difference between the real frictional resistance and the flat plate resistance is mainly due to the curvature of the hull.

Steen [21], suggested an empirical method to find the form factor, $k$, based on several model tests of high speed, displacement, catamaran hulls. The form factor was found to be a function of the length-displacement ratio of the ship between values of 6 to 12. The length-displacement ratio of the catamaran examined here is less then 6, but with no other better way to estimate friction drag, this was the method used.

According to Steen [21], the form factor can be approximated as follows:

\[
1 + k = 3.4275 \cdot \left( \frac{L}{\sqrt[3]{V}} \right)^{-0.443}
\]

Length displacement value for the catamaran is: \( \frac{L}{\sqrt[3]{V}} = \frac{100}{7500^{\frac{1}{3}}} = 5.108 \), the form factor is then: \( 1 + k = 3.4275 \cdot 5.108^{-0.443} = 1.6641 \). This result was verified by the suggested form factors table for a high-speed, round-bilge catamaran forms, presented in Couser [20]. The form factors in that table were defined as functions of the vessels length over cubic root of the underwater volume coefficient. The results of this representation were extrapolated to find the form factor corresponds to the catamarans \( \frac{L}{\sqrt[3]{V}} \) coefficient.

Figure 4.10 presents the correlation between the vessels \( \frac{L}{\sqrt[3]{V}} \) coefficient and its form factor, and the extrapolation using a second order polynomial trend line. This form factor was used to 'normalize' the flat plate viscous drag approximation and to calculate the viscous drag coefficient as presented in figure 4.11.
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Figure 4.10: Form factor correlation line

Figure 4.11: Friction resistance coefficient
The friction drag coefficient, in figure 4.11, was estimated as constant for all separation ratios. Its values were determined as functions of the ships *Reynolds Number* and an estimated form factor, without any influence of the separation between the demi-hulls.

### 4.5.5 Total Resistance and Verification

A summation of the resistance components evaluated so far, leads to a total drag coefficient of the ship. It should be noted that the total drag coefficient evaluated here does not take into account the wind or air components of the drag. Figure 4.12 presents the total drag coefficient.

As stated above, there is no significant difference in the drag results, between the three separation ratios vessels evaluated here, although the smaller separation ratio shows slightly better performances.
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Very few model tests and experiments regarding the behavior and resistance of catamarans at sea were available. The numerical results obtained here were verified by comparing them to the few results found, most of them in the 'FastShip Proceedings papers' from recent years.

Figure 4.13 presents the total drag coefficient results for the s/L = 0.3 catamaran and a comparison with a routine model test conducted for a similar s/L = 0.3 catamaran for the semi-displacement range of speeds. The results were obtained from Bruzzone [22]. There is a good agreement between the results obtained by SWAN-2 and by the model test.

![Comparison chart of s/L = 0.3 total resistance results](image)

Figure 4.13: Comparison chart of s/L = 0.3 catamaran total resistance results
Chapter 5: Seakeeping Evaluation

5.1 Introduction

In recent years, greater attention has been given to vessels’ seakeeping performances due to several factors: (1) the increase in use of high-speed semi-displacement vessels; (2) increasing demand for comfort (especially for passenger ferries); (3) the development and use of a delicate, sophisticated systems onboard (especially on naval ships); (4) growing pressure from the public and regulatory bodies for safer vessels; (5) fast advancement in computational, predictive, simulation and analysis tools for seakeeping.

As the speed of vessels increases, their seakeeping characteristics become more important. This is especially true for the growing passenger ferries market, and also for several naval applications. When the speed grows, the momentum of the advancing vessel grows, hence every small diversion of the vessels course has the potential of structural damage to the ship and a risk to the operating personal and to the passengers. Predicting the vessels motions and particularly suppressing them has therefore a great importance during the vessels design process.

The seakeeping problem can be separated into three parts: (1) estimation of the environmental conditions encountered by the vessel (i.e. sea state); (2) prediction of the vessels response characteristics; (3) comparison of the vessels response with specified motion criteria. For passenger vessels this criteria would consider mostly the comfort of the passengers, while for naval vessels this criteria would probably consider equally the
wellbeing of the instruments and weapon systems onboard. This kind of comparison was not in the scope of this paper.

The seakeeping heave and pitch motion responses of the generic, ‘numerical’ catamaran, introduced in Annex A, are presented in the following chapter. These characteristics were obtained by the time domain, computational fluid dynamic code, SWAN-2. Seakeeping computations were carried out in order to evaluate the effect of different separation ratios between the demi-hulls of the catamarans heave and pitch motions, in a unidirectional, irregular sea spectrum. The frequency dependent added mass and damping coefficients of the vessel were derived from the forced heave and pitch oscillations at some prescribed frequencies, starting from rest at \( t = 0 \). The resulting forced records converged to a harmonic signal, which upon Fourier Transform, lead to heave and pitch frequency dependent, hydrodynamic coefficients. The ship motion response standard deviation was measured invoking equation (2.43) and the ISSC spectral formulation, introduced in section 2.4.1. Lifting appendages in various sizes were attached to a constant location of the ship, and their influence on the ships’ motions was examined as well.

5.2 Heave and Pitch Responses at unidirectional, irregular sea

The seakeeping characteristics examined in this section were the heave and pitch motions of a variable separation ratio catamaran. Three different separation ratios were tested: \( s/L = 0.3 \), \( s/L = 0.4 \) and \( s/L = 0.5 \). A similar monohull with the same displacement and length characteristics was investigated as well. The monohull is wider then the demi-hulls, but is narrow compared to the overall breadth of the catamarans. The characteristics of the monohull and the catamarans are elaborated in Annex A. The vessels were evaluated at six different speeds, at four different wave headings: bow waves (waves that advance towards the bow of the ship with an angle of 180°, as defined by SWAN-2, with respect to the negative x-axis), oblique waves (waves that approach the ship from the first quarter, towards the port or starboard sides of the ship, with an angle of 120° and 150° with respect to the negative x-axis) and beam waves (waves that approach from the side with an angle of 90° with respect to the direction of the ship advance). The ships speed were at the semi-
displacement range \((Froude Numbers = 0.5-1.0)\). The following figure is a remainder of the heave, pitch and roll motions of a vessel. It also defines the ships coordinate system and the wave headings angle as defined by SWAN-2.

![Coordinate system, heave and pitch motion and wave headings]

\[
\begin{align*}
\beta = 180' &\rightarrow \text{Bow waves} \\
\beta = 150' &\rightarrow \text{Oblique waves} \\
\beta = 120' &\rightarrow \text{Oblique waves} \\
\beta = 90' &\rightarrow \text{Beam waves}
\end{align*}
\]

Figure 5.1: Coordinate system, heave and pitch motion and wave headings

Most of the figures in this chapter present the motions RAO. As discussed in chapter 2, the RAO (Response Amplitude Operator) is the ratio between the response modulus and the incident wave amplitude. For the vessels examined in this paper, thirty different waves (different in their frequencies) were generated and invoked thirty different ship responses. The response amplitudes were isolated and the RAO of the ship heave and pitch motions was derived. SWAN-2 measures ship response amplitude at the encounter frequency.
Response motions are measured at the origin of the coordinate systems, i.e. at midship, at the centerline, on the design water line of the ship.

The incident wave amplitude was chosen to be 2 meters. The incident wave frequencies were chosen at the range of 5 to 25 [rad/sec]. The frequency step was usually a constant at the size of 0.04 [rad/sec]. At the peak response of each motion, more measurements were taken to verify results and to achieve ‘fine tuning’ of the response. At this range, usually smaller frequency step was used. The abscissa of the graphs presents the non-dimensional magnitude of \( \lambda/L \), wavelength over ships length. The y-axis presents the heave or pitch RAO in units of \([m/m]\), non-dimensional units in case of heave and \([\text{deg/m}]\) in case of pitch.

The following figure present a seakeeping simulation sample of bow waves, 180°, and an incident wave with time period of 7 [sec] and amplitude of 2 [m] applied upon a catamaran advancing forward with a speed of 45 knots.

![Seakeeping simulation of a vessel advancing in 45 knots (Fn=0.74)](image)
Figure 5.3 present the heave motion RAO of a catamaran with separation ratio of 0.3, i.e. the ratio of the distance between the centerlines of the demi-hulls to the length of the vessel is 0.3. The catamaran response is to bow waves. Six different speeds were examined. The peak response decays (for the lowest speeds), as the forward translating speed decrease. At the high end of the ship speed range ($F_n = 0.82-0.99$), the peak response increases as the speed increases. The natural frequency of the response for $s/L=0.3$ increases as the ships Froude Number increases. The responses at $F_n = 0.90-0.99$ have similar peak values and peak frequencies.

![Figure 5.3: Heave RAO for bow waves for s/L=0.3](image)

Figures 5.4 present the heave response for bow waves for a catamaran with separation ratio of 0.4. For this catamaran the peak heave response decays smoothly as the speed of the vessel increases. $A/L$ values of the peak response (at each speed) grow as the Froude Number of the vessel grows. For this catamaran, as for the $s/L=0.3$, $F_n = 0.90-0.99$ have similar peak response, this peaks have similar $A/L$ values.
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**Figure 5.4:** Heave RAO for bow waves for s/L=0.4

**Figure 5.5:** Heave RAO for bow waves for s/L=0.5
Figures 5.5 present the heave responses for bow waves, at the same six ship speeds, for the same hulls, with a separation ratio of 0.5. The peak heave response of this vessel increases for $Fn = 0.57-0.74$ and then decreases for higher value of Froude Number. Heave responses of $Fn = 0.82-0.90$ has a similar peak values and similar natural frequencies.

Several conclusions for the bow wave heave motions can be made after a brief analysis of the graphs.

1. Slow catamaran response in bow waves is higher then a fast catamaran responses for the same wave heading. This rule is exact for the s/L=0.4 catamaran. Exceptions are seen and elaborated for the s/L=0.3 and s/L=0.5 catamarans.
2. Peak responses for the semi-displacement catamaran are at $\lambda/L$ range of 1.5 to 2.5.
3. Higher catamaran speeds produce a peak response at higher frequencies.
4. At low $\lambda/L$ (up to 1.5) slower catamarans gain higher heave response. At high $\lambda/L$ (above 2.5) faster catamarans gain higher heave response.
5. Peak response occurs usually at the same $\lambda/L$ value for all separation ratios.
6. As expected, for large wavelength the ship response amplitude equals the incident wave amplitude, i.e. the RAO curve goes to 1. The ship gains the exact wave motion. For infinitely large frequency (low values of $\lambda/L$), the excitation is so quick so that is no time for respond, hence the ratio between the response amplitude and the wave amplitude (the exciting amplitude) is vanishes.
7. At small wavelength numbers the response vanishes.

Figure 5.6 present the pitch motion response of an s/L=0.3 catamaran at the semi-displacement speed range for bow waves. The heave and pitch responses for all separation ratios were measured at the same frequencies.

For the s/L=0.3 catamaran, slower catamaran provokes higher pitch responses. An exception is at $Fn = 0.99$. Peak response is decaying for the range of $Fn = 0.57-0.82$ where the highest response is for the slowest speed. At $Fn = 0.90-0.99$ the peak grows as the speed increases.
Figure 5.6: Pitch RAO for bow waves for s/L=0.3

Figure 5.7: Pitch RAO for bow waves for s/L=0.4

Figure 5.7 presents the pitch response of an s/L=0.4 catamaran for bow waves.
For the s/L = 0.4 catamaran, as before, slow speed invokes high response. For a response of \( F_n = 0.74 \) and higher, two peak responses are clearly spotted. Peak response is decaying for \( F_n = 0.57-0.66 \). At \( F_n = 0.74-0.82 \) peak response is similar (although in different frequencies). For \( F_n = 0.90-0.99 \) the peak response increases as the speed grows.

Figure 5.8 presents the pitch response of an s/L = 0.5 catamaran for bow waves. For this separation ratio, the pitch peak response decays for \( F_n = 0.57-0.82 \) as the speed grows and increases for \( F_n = 0.90-0.99 \). For the former Froude Numbers, the peak response is similar.

Several conclusions can be made from the pitch RAO curves.

1. The pitch response is not a smooth curve as the heave response curve. Two peak responses are often seen. The second natural frequency is due to the hydrodynamic influence of one hull over the other.

2. Pitch peak responses of a catamaran at the semi-displacement speed range for bow waves is at the \( \lambda / L \) range of 1.5 to 2.5.

3. Peak responses of higher speeds are usually generated at higher frequencies of encounter.
4. Peak pitch response at \( F_n = 0.57 - 0.66 \) is generated at higher \( \lambda / L \) values for higher separation ratios.

5. At lower \( \lambda / L \) ratios (up to 1.5), low speed generate higher response then high speed catamarans. For high \( \lambda / L \) ratios (above 2.5) high speed catamarans generate the higher response.

6. The pitch response vanishes for short and long waves. For the long wave extremes, the slope of the incident wave tends to zero hence no pitch RAO.

Figure 5.9 present seakeeping heave and pitch motions of the three catamarans (s/L=0.3, s/L=0.4, s/L=0.5) and a monohull at four wave headings at Frond Number of 0.99. All other monohull / catamaran comparison charts are attached in Annex B. The monohull dimensions are elaborated in Annex A.

Several conclusions can be made from figure 5.9:

1. Heave motion:
   a. Monohull heave response has approximately the same natural frequency as the catamarans for wave headings of 120°, 150° and 180°.
   b. At wave heading of 90° the shapes of the response are different. The catamaran are oscillating with peaks at several natural frequencies were the s/L=0.5 has the highest peak response as expected. The monohull has a smooth, non-oscillating response shape which peaks at higher frequency and has larger response amplitude value.
   c. Catamaran natural frequencies are getting smaller as the wave heading advance towards beam waves.
   d. All responses start from zero at very short waves and go to one at long waves.
   e. The monohull amplitude of response is lower then this of the catamarans for wave headings of 150° and 180°.
Figure 5.9: Heave and Pitch RAO's for Fn=0.99 including monohull performances
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2. Pitch motion:
   a. The response of the s/L=0.3 catamaran has the largest amplitudes of the catamarans for the bow and oblique waves. At beam waves it has the lower response amplitude.
   b. All the catamarans at 120° and 90° wave headings have several natural frequencies.
   c. Pitch motion of the monohull has similar response at wave headings of 120° and 180° as the catamarans. It has similar natural frequencies although at 180° it has lower response amplitude and in the 150° heading it has higher amplitudes than the s/L=0.4 and s/L=0.5 catamarans.
   d. At 120° heading, the monohull has the highest response amplitude but only one peak. The catamarans have several natural frequencies.
   e. At 90° heading the response amplitudes of the catamarans are larger significantly than those of the monohull. The catamarans response has several natural frequencies.

Figures 5.10, 5.11 and 5.12 presents the heave and pitch RAO’s for 150°, 120° and 90° wave headings for all three different separation ratios.
Figure 5.10: Heave and Pitch RAO's for 150° wave headings at six different Froude Numbers for catamaran separation ratio of s/L=0.3, s/L=0.4, and s/L=0.5
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Figure 5.11: Heave and Pitch RAO's for 120° wave headings at six different Froude Numbers for catamaran separation ratio of \( s/L = 0.3 \), \( s/L = 0.4 \), and \( s/L = 0.5 \)
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Figure 5.12: Heave and Pitch RAO’s for 90° wave headings at six different Froude Numbers for catamaran separation ratio of s/L=0.3, s/L=0.4, and s/L=0.5

Froude Numbers for catamaran separation ratio of s/L=0.3, s/L=0.4, and s/L=0.5
Some observations can be made from these charts.

**Heave:**

1. One natural frequency exists for all heave motions for the separation ratios at wave headings of 150° and 180°.
2. Heave response at these waves is higher for the low ship speeds. At *Froude Number* 0.57 and 0.66 response amplitudes are maximal.
3. Hydrodynamic interaction between the hulls is observed at oblique waves (120° wave headings) and beam waves (90°), two separate modal frequencies are noticed at the oblique waves at higher frequencies. Their response amplitude decays as the distance between the demi-hulls diminishes.
4. At beam waves, several heave response peaks are shown. At separation ratio of s/L=0.3, the forward speed of the ship, has small impact on its heave RAO, almost all frequency ranges (except $A/L$ between 0.4 and 0.6) has the same response amplitude. At separation ratio of s/L=0.4 a distinction exists between the heave motions at different forward speeds. At low *Froude Numbers* (0.57, 0.66) three peak responses are observed. At higher *Froude Numbers* another peak exists. For the s/L=0.5 separation ratio four response peaks are noted for the fastest speeds (Fn = 0.90, 0.99). At lower speeds only three peaks are observed.

**Pitch:**

5. Hydrodynamic interaction between the hulls is noticed at the pitch response for bow waves. At Fn = 0.74 there are two peak responses for the s/L=0.3 catamaran. As separation ratio increases, two peaks are seen for higher *Froude Numbers*.
6. At oblique waves this interaction is well observed. At 150° headings, at least three *Froude Numbers*, has double peak response. At 120° headings all speeds has at least two peaks. The small peak exists at high frequency range similar to the situation for heave motion at oblique wave.
7. For beam waves the pitch response pattern for each separation ratio repeats itself for all *Froude Numbers*. Same number of peaks exists for all speeds. At s/L=0.3 the main peak appear at about $A/L = 0.5$ and is maximal for Fn = 0.74, 0.82 and 0.90. At s/L=0.4, two distinct peaks are observed. The highest response is generated by the fastest speed, the lowest response by the lowest speed. Same thing happens at
Chapter 5. Seakeeping Evaluation.

s/L=0.5. At this separation ratio the highest response appear in two different frequencies. The high speed vessels (Fn = 0.82, 0.90, 0.99) peak at different wavelength from the low speed vessels (Fn = 0.57, 0.66, 0.74).

5.3 Motion Response - Standard Deviation

The heave and pitch responses variance and standard deviation were developed. The wave spectra used to analyze the responses was the ISSC (International Ship and Offshore Structures Congress) spectral formulation for fully developed sea. The formulation stating this sea spectrum is elaborated in section 2.4.1. The formulation as stated in section 2.4.2 was used to calculate the variance and standard deviation of the responses:

\[
\sigma_j = \sqrt{\int_0^\infty S(w)H_j(w)^2 \, dw}, \quad j = 1, 2, \ldots, 6
\]

where \( \sigma_j \) is the standard deviation.

Figures 5.13 and 5.14 are examples of the calculated variance (\( \sigma_j^2 \)).

The heave motion has one peak amplitude response for all catamarans at the same natural frequency, where s/L = 0.3 catamaran has the highest peak, and s/L = 0.5 catamaran has the lowest peak. Theoretically they should have had the same response for all frequencies, yet the growing distance between the demi-hulls damps the response and each hull behaves as a single monohull.

The pitch response for beam waves has two obvious natural frequencies for the catamarans. The peak response decays as the distance between the demi-hulls grows. As for the heave motion, it is shown that the response variance decays as the distance between
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the demi-hulls grows. The monohull has much superior response variance then the catamarans examined here.

**Figure 5.13:** The variance of the heave response spectrum at $F_n = 0.57$, bow waves ($180^\circ$)

**Figure 5.14:** The variance of the pitch response spectrum at $F_n = 0.74$, beam waves ($90^\circ$)
The following figures present a three-dimensional surface view of the standard deviation values, for the heave and pitch responses, at four wave headings and six Froude Numbers (heave in [m] units, pitch in [deg] units).

Figure 5.15: 3D view of the heave and pitch standard deviation of a s/L=0.3 catamaran
Figure 5.16: 3D view of the heave and pitch standard deviation of a s/L=0.4 catamaran
Figure 5.17: 3D view of the heave and pitch standard deviation of a s/L=0.5 catamaran
5.3.1 Analysis of results

The standard deviation of the response is the most practical and useful parameter when analyzing a vessel's response to a sea state. Twice the motion’s standard deviation is a very good approximation of the vessel’s significant one third highest responses. The graphs shown above, present a comprehensive and thrill view of all the standard deviation values for the semi-displacement regimen, for a round bilge catamaran, at all wave headings between bow and beam waves. The standard deviation three-dimensional surfaces were analyzed. Six polynomial equations of the forth power, were developed, one for each surface using the least square method. The independent variables were the ships Froude Numbers and the waves heading \( \left( \pi / 2 \rightarrow \pi \right) \). The polynomials enable calculating a good approximation of the heave or pitch standard deviation, at a prescribed wave heading, for the semi-displacement range, without the use of any fluid dynamic code. The following equations present a polynomial representation of each standard deviation plane. \( x \) represent the Froude-Number \((0.57 - 0.99)\), \( y \) represent the wave heading \( \left( \pi / 2 \rightarrow \pi \right) \).

Equation 5.2 presents the heave response standard deviation for a catamaran with a separation ratio of \( s/L = 0.3 \).

\[
(5.2) \quad st.\text{dv} = 3.1218 - 7.2017 \cdot x - 5.1223 \cdot y + 281608 \cdot x \cdot y - 186892 \cdot x^2 + 211994 \cdot x^3 - 9.1842 \cdot x^4 \cdot y \\
-10.2314 \cdot x \cdot y^2 + 0.8961 \cdot y^3 - 8.3601 \cdot x^4 + 3.5165 \cdot x^3 \cdot y + 0.4021 \cdot x^2 \cdot y^2 + 1.3856 \cdot x \cdot y^3 - 0.1861 \cdot y^4
\]

Equation 5.3 presents the pitch response standard deviation for a catamaran with a separation ratio of \( s/L = 0.3 \).

\[
(5.3) \quad st.\text{dv} = 12.2972 - 59.3273 \cdot x - 9.0814 \cdot y + 52.4701 \cdot x \cdot y + 50.2794 \cdot x^2 - 51.8678 \cdot x^3 + 6.6776 \cdot x^4 \cdot y \\
-26.9204 \cdot x \cdot y^2 + 2.7084 \cdot y^3 + 14.1437 \cdot x^4 + 3.3513 \cdot x^3 \cdot y - 2.4701 \cdot x^2 \cdot y^2 + 4.2914 \cdot x \cdot y^3 - 0.6145 \cdot y^4
\]
Equation 5.4 presents the heave response standard deviation for a catamaran with a separation ratio of $s/L=0.4$.

\begin{equation}
\text{std.} \text{d}v = 6.3698 - 5.8148 \cdot x - 10.324 \cdot y + 43.3073 \cdot x \cdot y - 43.12 \cdot x^2 + 50.66 \cdot x^3 - 18.6816 \cdot x^2 \cdot y - 13.4193 \cdot x \cdot y^2 + 1.4684 \cdot y^3 - 19.4171 \cdot x^4 + 6.2381 \cdot x \cdot y^2 + 1.6854 \cdot x \cdot y^3 - 0.29 - y^4
\end{equation}

Equation 5.5 presents the pitch response standard deviation for a catamaran with a separation ratio of $s/L=0.4$.

\begin{equation}
\text{std.} \text{d}v = -9.0454 + 47.1918 \cdot x - 5.5041 \cdot y + 31.6508 \cdot x \cdot y - 123.997 \cdot x^2 + 106.7237 \cdot x^3 - 3.6441 \cdot x \cdot y - 15.4472 \cdot x \cdot y^2 + 1.8328 \cdot y^3 - 37.0484 \cdot x^4 + 4.7096 \cdot x^3 \cdot y - 0.8175 \cdot x^2 \cdot y^2 + 2.4313 \cdot x \cdot y^3 - 0.4165 \cdot y^4
\end{equation}

Equation 5.6 presents the heave response standard deviation for a catamaran with a separation ratio of $s/L=0.5$.

\begin{equation}
\text{std.} \text{d}v = 12.8538 - 30.0638 \cdot x - 12.3850 \cdot y + 45.9155 \cdot x \cdot y - 0.1218 \cdot x^2 + 14.1992 \cdot x^3 - 19.0007 \cdot x \cdot y - 14.2509 \cdot x \cdot y^2 + 1.7879 \cdot y^3 - 8.0792 \cdot x^4 + 6.5843 \cdot x^3 \cdot y + 0.8646 \cdot x^2 \cdot y^2 + 1.8141 \cdot x \cdot y^3 - 0.3537 \cdot y^4
\end{equation}

Equation 5.7 presents the pitch response standard deviation for a catamaran with a separation ratio of $s/L=0.5$.

\begin{equation}
\text{std.} \text{d}v = -1.0906 + 10.6090 \cdot x - 5.3467 \cdot y + 27.6056 \cdot x \cdot y - 52.1748 \cdot x^2 + 51.2927 \cdot x^3 - 7.0362 \cdot x^2 \cdot y - 11.8827 \cdot x \cdot y^2 + 1.4853 \cdot y^3 - 17.4245 \cdot x^4 + 0.6019 \cdot x^3 \cdot y + 1.8545 \cdot x^2 \cdot y^2 + 1.3138 \cdot x \cdot y^3 - 0.2908 \cdot y^4
\end{equation}
5.4 The influence of lifting appendages on Heave and Pitch RAO’s

Reduction of heave and pitch motion RAO was one of the objectives of this paper, especially at the vessel’s natural frequency. After determining the shape of the response, passive lifting appendages were ‘attached’ to the hull for extra ‘damping’. The appendages were attached to each catamaran at similar longitudinal positions at their bow and stern, at a constant vertical position under the keel, with a zero angle of attack. Two foils were located at the bow of the vessel (each foil under the keel of a demi-hull) and two foils at the stern (same configuration). The foils had a rectangular shape with a constant span and chord, they were not cambered. The foils were evaluated at two different wave headings: bow waves (180°) and oblique waves (120°) at three forward speed conditions (Froude Numbers: 0.66, 0.82 and 0.99). The total area of the foils was 80 m², which was about 4.7% of the water-plane area of the ship and about 2.5% of its wetted surface. Eight options of foils were evaluated; aspect ratio and area of the foils were changed although the total area of the foils, as already stated, was kept 80 m². For symmetry reasons, two adjacent foils had the same area and aspect ratio. The next table summarizes eight foil combinations.

<table>
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<th>Option #</th>
<th>Fwd foils Aspect ratio</th>
<th>Fwd foils Area [m²]</th>
<th>Aft foils aspect ratio</th>
<th>Aft foils area [m²]</th>
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<td>9.61</td>
<td>1</td>
<td>30.25</td>
</tr>
</tbody>
</table>

Table 5.1: Lifting appendages alternatives
The following figure presents the location of the foils along the hull.

![Location of lifting appendages along the hull](image)

**Figure 5.18: Location of lifting appendages along the hull**

The curves in the following figures, present reduction in the heave and pitch motion responses of all catamarans, to an incident, unidirectional sea state.

Figures 5.19 and 5.20 presents the affect of the lifting appendages over the three different separation ratio catamarans, for bow and oblique waves, at $Fn = 0.66$.

Figures 5.21 and 5.22 presents the effect of the lifting appendages on the ship at $Fn = 0.82$.

Figures 5.23 and 5.24 presents the effect of the lifting appendages on the ship at $Fn = 0.99$. 
Chapter 5. Seakeeping Evaluation.

**Figure 5.19**: Lifting appendages at $F_n=0.66$ at bow waves
Chapter 5. Seakeeping Evaluation.

Figure 5.20: Lifting appendages at Fn=0.66 at oblique waves
Figure 5.21: Lifting appendages at Fn=0.82 at bow waves
Chapter 5. Seakeeping Evaluation.

Figure 5.22: Lifting appendages at $F_n=0.82$ at oblique waves
Figure 5.23: Lifting appendages at Fn=0.99 at bow waves
Chapter 5. Seakeeping Evaluation.

Figure 5.24: Lifting appendages at Fn=0.99 at oblique waves
From visual inspection of the heave and pitch RAO's, several conclusions can be made.

1. All foils reduce the heave and pitch responses of all catamarans.

2. Heave response:
   a. The lowest reduction of the heave peak response was achieved by the foil combinations 4 and 8. These combinations consists a large area foil at the stern. Combination number 4 is the only foil combination where the forward foil has shorter span then chord. The span of all the foil of combination 8 is equal to its chord.
   b. The highest reduction of heave RAO was obtained for foil combinations 5 and 7. These combinations reduce significantly the response amplitude for all catamarans at all wave headings / forward speeds. These foils have larger foil area at the bow. Combination 5 has a span / chord ratio of 1.67 for the rear foil. The aspect ratio of all the foil of combination 7 is one.

3. Pitch response:
   a. At Fn = 0.66, the best foils combinations were 1 and 5 for bow waves and 1, 3, 5 and 7 for oblique waves. At Fn = 0.82, the best combinations were 1, 3 and 5 for bow waves and 1, 3, 5 and 7 for oblique waves. At Fn = 0.99, the best combinations were 1, 3, 5 and 8 for all wave headings except for s/L = 0.5 where combination 8 achieved the lowest reduction of RAO.
   b. The pitch response of the catamarans without lifting appendages contained more then one natural frequency, especially at oblique waves. Some foils often reduced the number of natural frequencies, examples can be found at Fn = 0.99, oblique waves, for s/L = 0.4 and s/L = 0.5.
   c. Occasionally, for the same catamaran at the same forward speed and wave heading, some foils combinations peak at one natural frequency and some at another frequency. For example, at s/L = 0.3, Fn = 0.66, bow waves: foils 6 and 7 peak at the same frequency as the first natural frequency of the catamaran without the foils. Foils 4 and 8 peak at the same frequency as the second natural frequency of the catamaran without foils.
Chapter 5. Seakeeping Evaluation.

The standard deviation of the ships’ response with all foils combinations was calculated. It was found that the lowest standard deviation for most of the responses (heave and pitch) was achieved by foils combination 5. The highest standard deviation was calculated for combination 4. Upon these findings, it is suggested that hydrofoil catamarans would be designed with a larger foil at the bow and a smaller, narrow foil near the stern.

A theoretical check of the maximum foil area, where the decrease of the response is negligible, was made. The s/L = 0.3 catamaran at F_n = 0.99, bow waves was used. All four foils had equal area and their aspect ratio was one. Total area of foils was raised starting at 160 m^2. Convergence achieved at area of 1200 m^2. This is not a practical foil area from a design point of view.

The following figure presents the increase of foils area and the decrease of heave and pitch motion until convergence.

![Graphs showing the increase of foils area and the decrease of heave and pitch motion until convergence.](image)

Figure 5.25: Large area lifting appendages at F_n=0.99, bow waves, s/L=0.3
Chapter 6: Structural Loading Analysis

6.1 Preface

Wave excited loads can be analyzed in three different ways.

1. An instantaneous local hydrodynamic pressure loads as a result of the ship motions or the ship interaction with waves.
2. A continues load, obtained by integration of the instantaneous pressure loads over time. These loads yield the hulls shear force, bending moment and torsion moment.
3. An impulsive, local pressure loads that cause vibratory response (loads like slamming, whipping, springing etc.).

6.1.1 The exciting forces

The following chapter discusses the continuous shear force, longitudinal bending moment and torsion moment, applied upon the vessel as a result of ambient waves. The overall force can be evaluated by equation (2.57), stated again here.

\[ F_j(t) = F_j^{HS} + F_j^{EX} + F_j^R \]

\( F_j^{HS} \) is the net hydrostatic force and includes gravitational and hydrostatic components. \( F_j^{EX} \) is the excitation force in the \( j^{th} \) direction and contains the incident wave force, known also as the Froude - Krylov force, and the diffracted wave force. \( F_j^R \) is the hydrodynamic force in the \( j^{th} \) direction due to forced motions. It results from the radiation of waves away from a vessel that is forced to oscillate.
The component of interest in this chapter is $F_{EX}^i$, the wave exciting forces. In order to exert these forces, it is necessary to solve the equations of motion (2.58), (3.33), to get the motions response amplitude operators. Then, the incremental vertical force, in excess of or less than the still water buoyant force, at any instance of time, along the hull, can be calculated. Evaluation of shear forces and longitudinal moments, and especially transverse moments, in irregular seas, has important consequences on the structural design of catamaran hulls. Theoretically, longitudinal bending moments tend towards zero for both very long and very short waves and often are 'double humped', where one peak corresponds with the motions resonance and the other corresponds with the wavelength that cause the largest re-distribution of buoyancy. In practice, longitudinal bending moment exists also in the steady state, where no interaction with an ambient wave occurs from two reasons: (1) the interaction between the opposite and coupled weight and buoyant forces, and (2) the sinkage and trim the ship withstand when translating forward. The Same theory holds for the shear force along the hull. In long waves, similar to the steady state situation, the longitudinal bending moment or the shear force are thus tends to a finite value which is not zero. In case of a port - starboard symmetric ship, where the ship does not develop any steady heal or list throughout its motion at sea, its torsion moment vanishes at very short and very long waves.

6.1.2 Force RAO's

Time dependent forces, in all six degrees of freedom, were evaluated by SWAN-2. A Fourier transform was conducted for three of the forces: $F_{3}^{EX}$, the exciting force in the $z$-direction, to find the shear force; $F_{4}^{EX}$, the exciting moment around the $x$-axis, to find the torsion moment; and $F_{5}^{EX}$, the exciting moment around the $y$-axis, to find the longitudinal bending moment. The results were non-dimensionalized by:

$$
(6.2) \quad RAO(w_x) = \frac{F_{3}^{EX}}{A \cdot \rho \cdot g \cdot A_{wp}}; \quad \text{non-dimensional shear force RAO.}
$$
Chapter 6. Structural Loading Analysis

\( RAO(w_e) = \frac{F^{EX}_4}{A \cdot \rho \cdot g \cdot A_{WP} \cdot L} \),

non-dimensional bending moment RAO.

\( RAO(w_e) = \frac{F^{EX}_5}{A \cdot \rho \cdot g \cdot A_{WP} \cdot L} \),

non-dimensional torsion moment RAO.

Where, \( A \) is the incident wave amplitude, \( A_{WP} \) is the ships water plane area and \( L \) is the ships length. All forces and moment were evaluated at five stations along the center line of each catamaran.

### 6.1.3 Ship weight distribution

The weight distribution of the ship was modeled for simulation purposes to comply with the ships longitudinal center of gravity and it’s over all displacement. The following graph present the weight distribution applied along the ship.

![weight distribution](image)

**Figure 6.1: Simulation of weight distribution along the hull**
Chapter 6. Structural Loading Analysis

The weight distribution was constructed such that the ship would not gain a least angle. Due to the deep transom of the vessel, it was assumed that most of the weight is concentrated at the aft portion, hence the rising trend of the graph towards the transom.

6.2 Significant Forces

Significant forces and moments are defined as the average of the one third highest values of forces and moments applied upon the vessel. These values were obtained by calculating the non-dimensionalized forces standard deviation and multiplying it by two. Standard deviation of the force was calculated as described in section 2.4.2 and as applied for the motion responses in section 5.3. The significant forces and moments were evaluated at two wave headings: 180° and 120°, for three ship speeds, or Froude Number values of 0.66, 0.82 and 0.99. All three separation ratio catamarans were evaluated at five stations along the hull: station 5 - fore perpendicular; station 4 - fore quarter; station 3 - midship; station 2 - aft quarter; and station 1 - aft perpendicular. The ambient 'wave' was constructed from 27 waves with different periods between 1 second to 23 seconds divided into three period intervals: (1) from periods of 1 second to 7 seconds the difference between the waves was of 1 second; (2) Between 7 seconds and 15 seconds, the difference was of 0.5 second; and (3) from period of 15 seconds to 23 seconds with period ‘step’ of two seconds.

The following charts present the results for wave heading of 180°.
Torsion moments do not exist for a symmetric port – starboard ship at wave headings of 180° as stated above. For higher speeds, larger shear forces and bending moments develop along the hull center line. For each speed the magnitudes of the significant force and moment, at each station, are similar for all three separation ratios. The shear force and moment...
bending moment distribution, generally tend to higher values towards the transom of the vessels as the weight distribution curve.

![Graphs of shear force and bending moment](image)

**Figure 6.4**: Significant shear forces and bending moments at 180° wave headings for \( F_n = 0.66, 0.82, 0.99 \), for \( s/L = 0.5 \)

The following graphs present the results obtained for wave heading of 120°.

![Graphs of shear force, torsion moment, and bending moment](image)

**Figure 6.5**: Significant shear force, bending and torsion moments at 120° wave headings for \( F_n = 0.66, 0.82, 0.99 \), for \( s/L = 0.3 \)
At wave heading of 120°, some differences are observed between the magnitudes of the forces, although the trend of the curves is similar. The trend of the shear forces and bending moments is similar to this of the 180° wave heading and follows the weight distribution curve, larger forces and moments develop at the stern, while smaller forces develop at the bow.

The catamarans with separation ratios of s/L = 0.4 and s/L = 0.5 obtain similar values of forces at Fn = 0.82 and Fn = 0.99, and the slower speed (Fn = 0.66) gets significantly lower values of forces. In general, as for the 180° wave heading, larger forces are
developed in higher speeds. It is also noted that the torsion moment values are larger at larger separation ratio.

For the s/L = 0.3 catamaran, the torsion values are approximately in one order smaller from the bending moment values for the same catamaran at the same station, while for the s/L = 0.5 catamaran the torsion and bending moment values are in the same order of magnitude. As stated earlier, this is one of the biggest disadvantages of the catamaran hull type, as the separation between the demi-hulls increases, the torsion moment that acts upon the center line, is getting higher.

Annex C presents the RAO’s curves for each speed, wave heading and separation ratio. These RAO’s were used to construct the significant forces graphs.
Chapter 7: Discussion and Conclusions

A hydrodynamic 'numerical' model test was conducted upon a 7500 ton, round bilge, deep transom stern catamaran. The test was developed throughout this paper and compared with a 'numerical' model test conducted upon a similar length – displacement monohull. Several catamaran shapes were examined, all had the same demi-hulls, but different separation ratios. The following chapter summarizes the work done so far and makes some suggestions for future work.

7.1 Conclusions

The vessels steady state was evaluated. The ideal wave resistance, the Kelvin wave pattern, and the sinkage and trim were measured when advancing at a constant forward speed, at calm water. It was shown that for higher advancing speeds, the waves created at the wake tend to divert, while for slower speeds, more transverse waves are seen at the wake.

A monohull with the same length (100 meters) and displacement (7500 ton) as the catamarans was evaluated as well. It was shown that the monohull had lower dynamic sinkage and trim values then all catamarans. This is mainly due to a lower wetted surface. The dynamic trim for all catamaran types is increasing for higher speeds, i.e. the steady pitch angle increases for higher Froude Numbers. A negative trim (bow up, stern down) locally increases the draft of the vessel at the stern, hence increase the induced resistance (due to dry transom). For large transom sterns as exist for the catamarans investigated, an increase of the draft has a significant impact on the overall ideal fluid resistance of the vessel.
Chapter 7. Conclusions.

The following figure was derived from figure 4.6 and presents a comparison of the ideal fluid resistance component of the three catamarans, and the resistance results obtained from two times the ideal fluid resistance of one demi-hull.

![Ideal fluid resistance coefficient graph](image)

**Figure 7.1: Ideal fluid resistance**

For the most parts of the semi-displacement range, the larger separation ratio has higher resistance values than the smaller separation ratios, and the \( s/L \rightarrow \infty \) (twice the demi-hull results) has the highest resistance results. The phenomena of increase in resistance as the distance between the demi-hulls grows, is caused due to a favorable interaction between the demi-hulls. The *Kelvin* wave pattern, exerted from one demi-hull, 'wets' the transom stern of the adjacent demi-hull and vice versa. The induced, adverse drag force is decreasing, since less area of the stern is dry, hence the overall ideal fluid resistance is smaller.

The catamaran hull forms developed in this paper has a superior ideal fluid resistance characteristic over a monohull with a similar mission (they have lower values of ideal
fluid resistance). The catamarans wave making resistance is also lower relative to the separate demi-hull wave making resistance, due to favorable interaction between the hulls. A synergy is obtained when joining the demi-hulls into a catamaran hull since the catamaran wave making resistance values are better then the values of the separate hulls.

The seakeeping features of the hulls were evaluated in chapter five. The vessels were evaluated at six different speeds, at four different wave headings: bow waves (waves that advance towards the bow of the ship with an angle of 180°, as defined by SWAN-2, with respect to the negative x-axis), oblique waves (waves that approach the ship from the first quarter, towards the port or starboard sides of the ship, with an angle of 120° and 150° with respect to the negative x-axis) and beam waves (waves that approach from the side with an angle of 90° with respect to the direction of the ship advance). The ships speed were at the semi-displacement range (Froude Numbers = 0.5-1.0).

The catamarans were evaluated at the sea states elaborated above with and without lifting foils. Eight alternatives of lifting foils were examined, all foil in the same location, but differ in their area and aspect ratio.

Several summary conclusions were gathered during the analysis of the hydrodynamic features of a catamaran examined here. These conclusions are outlined below.

1. The heave motion of the ship at bow waves includes one peak. For all catamarans, this peak occurs at the same frequency, where the s/L = 0.3 catamaran has the highest peak amplitude, and the s/L = 0.5 catamaran has the lowest peak amplitude. Theoretically they should have had the same response for all frequencies, yet the growing distance between the demi-hulls damps the response.

2. The interaction between the demi-hulls is obvious when examining the pitch RAO's for beam and oblique waves. The response at all speeds has two distinct natural frequencies. As the distance between the demi-hulls grows, it is easier to observe these two frequencies (also in heave). When the angle of approach of the waves gets closer to 90°, it is also easier to observe two separate natural frequencies.
3. The heave response variance decays as the distance between the demi-hulls grows.

4. The monohull has better response amplitudes in pitch than the catamarans at beam waves, at all speeds. At roll the catamarans has a superior seakeeping, better then monohull roll.

5. The lifting foils attached to the hull suppressed the heave and pitch motion amplitude. The foils, in some cases, also reduced the number of peak natural frequencies of the response. For example, the pitch response at 120° wave heading, for separation ratios of s/L=0.4 and s/L=0.5 at all speeds included three distinct peaks without the foils and two pecks after attaching them. In some cases the peak amplitude is suppressed completely as for the heave response for wave headings of 120°, at Fn = 0.66, for separation ratio of s/L=0.4.

6. A combination of foils, where the rear foils has a smaller area then the bow foils, achieved the best motion suppression in heave and pitch. It is suggested that hydrofoil catamarans would be designed with a large foil at the bow and a smaller, narrower foil at the stern. The unique features of each foil (angle of attack, camber, thickness, etc.) should be designed separately.

The motions response standard deviation was calculated for each separation ratio, for each speed at all wave headings. Polynomial expressions were developed to represent the heave and pitch standard deviation of these particular catamarans.

Wave induced loads were evaluated in chapter six, for bow waves and wave headings of 120°, at three Froude Numbers, for all catamarans. The shear force, bending moment and torsion moment in five stations along the center-line of each catamaran were measured. SWAN-2 calculates these forces by direct integration of the pressure over the wetted surface of the ship. The significant one third highest non-dimensional forces and moments were calculated at each point. At bow waves, for higher speeds, larger shear forces and bending moments developed along the hull center line. The magnitudes of the significant force and moment, at each station, were similar for all three separation ratios. At wave heading of 120°, some differences are observed between the magnitudes of the forces, although the trend of the curves is similar. It is also noted at wave headings of 120° that the
torsion moment values increase in orders of magnitude, as the distance between the demi-hulls grows.

### 7.2 Recommended Future Work

The next step following this paper should include validation of results by a model testing. Resistance, seakeeping and body forces should be measured for the appropriate ship model, using calm water runs in a tow tank and runs under waves using a wavemaker. The results should then be compared with those obtained under this paper. A few model tests were conducted so far for catamarans and even less experiments were done for semi-displacement catamarans; hence their importance in validating the results is significant.

This paper checked one combination of displacement / length of a catamaran. It is recommended to verify the results and extent the findings to other catamarans with the same displacement / length ratio. The research can then be expanded for catamarans with different displacement / length ratios.

Checking following wave headings and their influence on a catamaran behavior at sea, was not in the scope of this paper. A complete analysis of catamaran in following waves (the origin of the waves are at the stern of the vessel) and oblique following waves is recommended. This work is important in order to complete mapping the catamaran response.

It is recommended to find the optimized shape and contour of the foils that would reduce the vessel response in the best way. This paper was using flat plate foils without emphasizing other foil alternatives (cambered foils, angle of attack etc.). It is also recommended to test the affect of an active control mechanism over the hulls response.
Chapter 8: Nomenclature.

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<td>[Kg]; [N]</td>
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Chapter 8. Nomenclature.

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<tr>
<td>( \eta(x,y,t) )</td>
<td>Wave elevation</td>
<td>[m]</td>
</tr>
<tr>
<td>( \eta_0 )</td>
<td>Basis-flow wave elevation</td>
<td>[m]</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Wave length</td>
<td>[m]</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Damping parameter strength</td>
<td>[Ns/m]</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Fluid kinematic viscosity</td>
<td>[m²/s]</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Ship waves distribution angle</td>
<td>[rad]; [deg]</td>
</tr>
<tr>
<td>( \Gamma; \Gamma_{2D} )</td>
<td>Three dimensional and two dimensional foil circulation</td>
<td>[-]</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Sea water density</td>
<td>[Kg/m³]</td>
</tr>
</tbody>
</table>
Chapter 8. Nomenclature.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta_i )</td>
<td>Ship response motion (translational motions)</td>
<td>[m]</td>
</tr>
<tr>
<td>( \alpha(t) )</td>
<td>Vessels rotational motions</td>
<td>[m/rad]</td>
</tr>
<tr>
<td>( \sigma_R )</td>
<td>Variance of ship response</td>
<td>[m^2]; [deg^2]</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Reduced frequency</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>( \psi(\bar{x}) )</td>
<td>Impulsive potential</td>
<td>[-]</td>
</tr>
<tr>
<td>( \phi(\bar{x},t) )</td>
<td>Residual wave potential</td>
<td>[-]</td>
</tr>
</tbody>
</table>
Chapter 9: References & Bibliography

10) ‘Rudder winglets on Sailing Yachts’, P.D. Sclavounos, Y. Huang, Massachusetts Institute of technology.


Annex A: Ship characteristics

A generic, 'numerical' hull forms were used to analyze the resistance, seakeeping and structural loads of the variable separation ratio catamarans and the monohull. The catamaran tested had a slender demi-hull shape and a deep transom stern. The following figure and table describes the main characteristics of the hull.

Figure A.1: Qualitative description of the examined catamaran
### Table A.1: Ship Characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Monohull</th>
<th>Catamaran s/L=0.3</th>
<th>Catamaran s/L=0.4</th>
<th>Catamaran s/L=0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (length over all)</td>
<td>100 [m]</td>
<td>100 [m]</td>
<td>100 [m]</td>
<td>100 [m]</td>
</tr>
<tr>
<td>b (beam of demi-hull)</td>
<td>---</td>
<td>11.75 [m]</td>
<td>11.75 [m]</td>
<td>11.75 [m]</td>
</tr>
<tr>
<td>B (over all beam)</td>
<td>22.5 [m]</td>
<td>41.75 [m]</td>
<td>51.75 [m]</td>
<td>61.75 [m]</td>
</tr>
<tr>
<td>D (depth)</td>
<td>9 [m]</td>
<td>9 [m]</td>
<td>9 [m]</td>
<td>9 [m]</td>
</tr>
<tr>
<td>T (draft)</td>
<td>5.625 [m]</td>
<td>5.75 [m]</td>
<td>5.75 [m]</td>
<td>5.75 [m]</td>
</tr>
<tr>
<td>Longitudinal Center of Buoyancy</td>
<td>-7.19 [m]</td>
<td>-10.19 [m]</td>
<td>-10.19 [m]</td>
<td>-10.19 [m]</td>
</tr>
<tr>
<td>Buoyancy (relative to midship)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water-line area</td>
<td>1800 [m²]</td>
<td>1644 [m²]</td>
<td>1644 [m²]</td>
<td>1644 [m²]</td>
</tr>
<tr>
<td>Wetted-surface area (average)</td>
<td>2250 [m²]</td>
<td>3000 [m²]</td>
<td>3000 [m²]</td>
<td>3000 [m²]</td>
</tr>
<tr>
<td>Longitudinal Center of Floatation</td>
<td>-6.43 [m²]</td>
<td>-9.00 [m²]</td>
<td>-9.00 [m²]</td>
<td>-9.00 [m²]</td>
</tr>
<tr>
<td>Floatation (relative to midship)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roll Radius of Gyration</td>
<td>11.25 [m]</td>
<td>15 [m]</td>
<td>20 [m]</td>
<td>25 [m]</td>
</tr>
<tr>
<td>Pitch Radius of Gyration</td>
<td>26.06 [m]</td>
<td>24.87 [m]</td>
<td>24.87 [m]</td>
<td>24.87 [m]</td>
</tr>
</tbody>
</table>

The parallel mid-body of all vessels starts 3 [m] before midship and ends at the transom stern. The draft and width of the transom stern equals the draft and width at midship. The cut-off ratio of the hulls is 1, hence no reduction in the hulls cross section at the aft of the vessel for shafts or propellers.
Annex B: Seakeeping Results.

Annex B include seakeeping results of the three variable separation ratio catamarans and the monohull chosen for comparison purposes. The seakeeping results developed were the heave and pitch RAO's of the vessels. Some roll motion responses were analyzed too. Roll motions evaluation was not in the scope of this paper, but it was interesting to investigate the relations between the separation ratio of the catamaran and its roll response. The monohull roll responses were evaluated too, in order to emphasize the superiority in roll of a catamaran over a monohull, especially in beam waves. The following figure presents a s/L=0.4 catamaran, advancing at 45 knots, responding to beam waves, with wave period of 7 [sec] and wave height of 2 [m].

Figure B.1: Catamaran advancing forward in beam waves

The next figures present the heave and pitch motion responses for the following Froude Numbers: 0.57, 0.66, 0.74, 0.82 and 0.90. The responses for Fn = 0.99 were presented in
Annex B. Seakeeping Results

Chapter 5. All different speeds were evaluated at four wave headings from 180° (bow waves) to 90° (beam waves). The roll motion responses are also attached, for all vessels, at the same Froude Numbers and also for Fn = 0.99, and for all wave headings.

Figure B.2: Roll motion response at Fn = 0.99
Annex B. Seakeeping Results

Figure B.3: Heave and Pitch motion response at Fn = 0.90
Annex B. Seakeeping Results

Figure B.4: Roll motion response at Fn = 0.90
Figure B.5: Heave and Pitch motions response at $F_n = 0.82$
Annex B. Seakeeping Results

Figure B.6: Roll motion response at Fn = 0.82
Figure B.7: Heave and Pitch motion response at Fn = 0.74
Annex B. Seakeeping Results

Figure B.8: Roll motion response at \( F_n = 0.74 \)
Annex B. Seakeeping Results

Figure B.9: Heave and Pitch motion response at \( F_n = 0.66 \)
Annex B. Seakeeping Results

Figure B.10: Roll motion response at $F_n = 0.66$
Figure B.11: Heave and Pitch motion response at Fn = 0.57
Annex B. Seakeeping Results

Roll response - 150 deg. wave heading - $F_n = 0.57$

Roll response - 120 deg. wave heading - $F_n = 0.57$

Roll response - 90 deg. wave heading - $F_n = 0.57$

Figure B.12: Roll motion response at $F_n = 0.57$
Annex C: Loading Results

In the following annex, the forces and moments RAO graphs are presented. As elaborated in chapter six, the exciting forces were calculated for all variable separation ratio catamarans, at three different advancing speeds, at two wave headings. The abscissa in the graphs is the non-dimensional value of $\lambda/L$, the y-axis is the non-dimensional RAO as explained in chapter six. The forces and moment were evaluated at five different stations along the center-line of the catamaran.
Figure C.1: Shear force and bending moment RAO for 180° wave heading at $F_n = 0.66$
Annex C: Loading Results

Figure C.2: Shear force, torsion moment and bending moment RAO for 180° wave heading, at Fn = 0.66
Figure C.3: Shear force and bending moment RAO for 180° wave heading.

at $Fn = 0.82$
Figure C.4: Shear force, torsion moment and bending moment RAO for 180° wave heading, at Fn = 0.82
Annex C: Loading Results

Figure C.5: Shear force and bending moment RAO for 180° wave heading,
at Fn = 0.99
Figure C.6: Shear force, torsion moment and bending moment RAO for 180° wave heading, at Fn = 0.99