Forecasting the System-Level Impact of Technology Infusion on Conventional Submarine Design

by

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ABSTRACT

A strategic technology development plan has a main objective to identify the technology areas that can maximize the company's return on investment. Usually the resources for R&D technology development are allocated based on past performance, in the specific area, rather than the impact on the quality of the vessel. If a complex system such as a submarine is designed with the use of current technologies, when it is produced there is a significant risk that the system will not be able to perform well enough to meet the threat. New technologies must be considered at the initial phases of the design concept exploration, since the impact of adding technologies later on the design process will significantly decrease the probability of attaining desired cost schedule and system performance goals. Therefore the decision maker/designer must have some means of predicting how the new technologies will impact the final product.

In addition to showing how the technology strategy can be performed on complex systems where emergent properties drive the metrics for decision making, this method is applied to conventional submarine concept exploration. This thesis will examine a new methodology, which can aid the decision maker in projecting the performance of future vessel concepts, and in allocating the resources for R&D technology development in an optimum way.

The impact of technology will be assessed through the use of technology k-factors. These factors will be introduced into the mathematical synthesis model and they will modify technical characteristics of the design. The modification will result in changes of the technical metrics, to simulate the hypothetical improvement or degradation associated with the new technology. The parametric mapping will be done with the use of Response Surface Equations based on
regression analysis using the results from the synthesis model. The Response Surface Method approach allows the decision maker to perform efficiently trade-off studies and evaluate the cost and benefit of new technologies without spending the time and money to mature it.

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1 Introduction

Following the end of the Cold War there have been significant changes in the needs for naval missions. The focus of undersea warfare has shifted from traditional “blue water” missions to littoral operations. This new strategic environment is a key driver in shaping future naval vessels.

Powerful nuclear submarines with unlimited underwater endurance are well suited to the task of sea control in the open ocean, and are able to transit at high speeds while submerged to a distant patrol area or to escort surface shipping. However, a modern submarine’s role in littoral warfare is likely to be one of access denial to opposing forces. While this is not a new mission, small conventionally powered submarines remain suitable for littoral operations because of their low acoustic, magnetic, and thermal signatures. Highly capable conventional submarines now form a key part of more than 66 nations’ order of battle [25].

The Falklands Conflict of 1982 can be used to illustrate the impact a conventional submarine can have in littoral operations. At the time of the conflict, the Argentinian Navy possessed four diesel electric submarines, two modern German built Type 209s and two older submarines. Of these four boats, only one of the Type 209s was capable of active patrol during the conflict, the San Luis [26].

The San Luis, which “operated 800 nautical miles from its base and made two attacks on British warships...demonstrated considerable proficiency... when it eluded the best ASW efforts of the Royal Navy, [further,] over 200 items of ASW ordnance were employed against this one submarine, mostly against false contacts” (Challenge, 2002). Following the war, it was determined that the torpedoes failed to hit their targets due to faulty fire control maintenance, and the San Luis’ commander related:

"There was no effective counterattack. I don’t think they knew we were there
until they heard our torpedoes running, and then the erratic nature of those weapons' behavior apparently prevented them from tracing the torpedoes back to our position. We were never under direct attack” [26].

From the attempts to hunt this one submarine, it can be seen that fighting conventional submarines in littoral environments can be time consuming and expensive undertaking.

The increasing importance of the conventional submarine in many nations’ order of battle [25] and demands for improved capabilities drive the need for new technologies. Even though the existing designs are very successful, the competitive global market leads submarine producers to invest in several technology areas in order to diversify their products and increase their market share. The development of a technology strategy is the goal of this study, as it is explained in the following paragraph.

1.1 Motivation

The improved underwater sensors and surface radars, and the introduction of the off-board sensors as part of a netted force, require new more advanced submarine capabilities, which are not always technically feasible with the current stage of technology. In addition, if a complex system such as a submarine is designed with the use of current technologies, when it is produced there is a significant risk that the system will not be able to perform well enough to meet the threat. New technologies must be considered at the initial phases of the design concept exploration, since the impact of adding technologies later on the design process will significantly decrease the probability of attaining desired cost schedule and system performance goals.

Historically, the more promising technologies are funded through a variety of sources, leading to experiments or prototypes. The best technologies are selected and introduced in the design. However, the current environment of reduced funding for Research and Development (R&D), and increased competition, requires very careful selection of the R&D projects that the company is going to undertake. A strategic technology development plan has as main objective the identification of the technology areas that can maximize the company’s return on investment.
The technologists have an understanding of what is currently being developed, what is just over the horizon and they have an idea of what opportunities technologies may provide in the far term. In all cases, the true impact of the technology could not be completely assessed without consideration of its impacts on the entire system. This assessment is needed to be done to the whole system level, and it should integrate the new technology with the rest of the design. Therefore the decision maker/designer must have some means of predicting how the new technologies will impact the final product.

This thesis will examine a new methodology, which can aid the decision maker in projecting the performance of future vessel concepts, and in allocating the resources for R&D technology development in an optimum way. The impact of technology will be assessed through the use of technology k-factors. These factors will be introduced into the mathematical synthesis model and they will modify technical characteristics or cost parameters of the design. The modification will result in changes of the technical metrics, to simulate the hypothetical improvement or degradation associated with the new technology. The parametric mapping will be done with the use of Response Surface Equations based on regression analysis using the results from the synthesis model. The Response Surface Method allows the decision maker to perform efficiently trade-off studies and evaluate the cost and benefit of new technologies without spending the time and money to mature it.

1.2 Overview of Conventional Submarine Current Technologies

This section provides an overview of technologies that impact the design of conventionally powered submarines. This is followed by a selection of technologies that forms the basic of the thesis study.

The hull shape of the modern submarines is optimized for their underwater performance. The body of revolution or “Albacore” form with length to diameter ratio in the range of four to six \[1\] has been proven as the optimum hull shape. However due to draft and other limitations, modern submarines have a length to diameter ratio between 8 and 10. Even though the body of revolution is adopted by most of the designs some variations of this shape are proposed by
different studies. A study conducted by Taylor, McHugh, and Warren at MIT [27] proposed a full stern submarine that provides flexibility in the machinery arrangements, adequate buoyancy at the aft sections that counteracts the effect of the heavy machinery equipment, and good resistance characteristics when coupled with an integrated propulsor.

In addition, different configurations of the stern-fins are implemented in some of the modern submarines. The X-stern rudders were developed to improve the emergency measures in the event of stern-plane jam, and to allow for the installation of planes with larger span, since the span of the cruciform stern planes is limited by the distance between the hull line and the maximum block dimension of the submarine.

In addition to the requirement for lower resistance and reduced hydrodynamic noise, which are the drivers for the hull shape, the requirements for higher diving depth, lower signatures, and lower life cycle cost require investigation of alternative structural materials. The United Kingdom's Defense Evaluation and Research Agency (DERA) has been pursuing a special research program on novel submarine structures investigating the use of composites as structural materials for secondary structures, such as rudders and sail. The theoretical capability developed up to now would allow the basic design of composite free flood structures. This would reduce the signatures, weight, and life cycle cost of the vessel. However, two questions remain to be answered: how resistant are these structures to underwater explosions and how will they be attached to the pressure hull [21].

Apart from the hull shape and the structures, the main field for technology development is the submarine propulsion. The propulsion plant consists of the propulsor, the propulsion motor, the storage battery, and the energy converters. Submarines have traditionally fixed pitch propellers for their propulsion. The need for high propulsive efficiency requires a large diameter, low rpm propeller with high torque. Other types of propulsors such as ducted propellers, or contra-rotating propellers have been proposed but have not yet been successful.

The propellers of the submarines are usually driven by direct current motors that are directly coupled to the propeller shaft. The motors are sized to meet the high power requirements
of the submarine at its top speed. However, conventional submarines usually operate at very low speeds and therefore the motor is required to deliver high torque at low rpm. That requirement calls for a large diameter motor [5], which is usually heavy and consumes a large volume. The advances in control technology and motor design have expanded the types and sizes of electric machines that can be used for submarine propulsion. Some of the available choices are: induction, permanent magnet, and superconducting motors. The permanent magnet excited synchronous motor is based on a proven technology and can lead to great savings in weight and volume. The weight savings can be up to 50% and the volume savings up to 40% [4]. Permanent magnet motors are more expensive than conventional DC motors, but the additional expense can be compensated for by the savings from the improved efficiency. Superconducting motors can develop the same torque and horsepower having nearly one third the size of a conventional motor. It is also estimated that the active length of a superconducting motor will be on the order of one quarter of the length of a permanent magnet motor [22].

During submerged operations of a conventional diesel electric submarine, the required power to drive the propulsion motor is provided by the battery. When new battery technologies that show considerable advantages in non marine industries enter the market, and “scale up”, they will significantly improve the submarine operating profile. Higher energy densities and specific energies introduced by advanced technology batteries could translate into longer submerged endurance, and increased submerged speeds. Currently the most common battery type is the lead acid battery, which is a proven technology. Although it is easy to operate and has a long cell life, it requires frequent monitoring and it evolves hydrogen while charging. Other proposed types of batteries [4] are the nickel-cadmium, silver-zinc, and lithium-aluminum/iron sulfide (LAIS). All of those types are not yet mature technologies but can bring significant changes to the operating profile and the design of the submarine. A nickel cadmium battery capable of 800 kW of delivered power could have 54% less weight, and 25% less volume than a lead acid battery with the same power [4]. Silver-zinc batteries have power densities three times greater than the lead acid battery [4], and have been used for special purpose submarines, such as the Deep Submergence Rescue Vehicles (DSRVs), and as a backup power source in nuclear submarines. The LAIS is the most promising of the battery technologies because it has the higher energy density, and it has a short charging time. However, the high operating temperature is the
main disadvantage of LAIS batteries.

The major developments of the last few years have been in the area of Air Independent Propulsion (AIP), as the requirement for extended underwater endurance drove manufacturers to depart from the traditional diesel engine. It is clear that nuclear propulsion is the optimum AIP solution for an open-ocean submarine. It provides unlimited air-independent energy to support lengthy open ocean submarine missions. However, the size and weight of the nuclear power plant increases the displacement, which becomes too large to operate well in the shallow water of littorals. In addition, very few nations worldwide have the financial resources to build and operate nuclear submarines. Therefore, the development of new types of AIP systems is the primary option for reducing conventional submarine vulnerability.

The performance factors of the AIP system that affect the vulnerability of the submarine are the AIP endurance and the balance speed. AIP endurance is the period of time that a submarine can stay submerged without the need to use its diesel engines in order to charge the batteries. Balance speed is the speed at which the maximum AIP power is equal to the submarine power requirements for hotel load and propulsion. Above the balance speed it is better to run both the AIP system and the storage battery, since a lightly loaded battery has a larger effective capacity. Typical advertised values of AIP endurance for some modern submarines are 12 to 14 days at a balance speed of four to six knots.

Using a mathematical model developed for this study, the underwater range, underwater endurance, and the indiscretion ratio of a notional submarine were estimated, as a function of speed, in two cases. First the submarine is operating solely on the battery and then using the battery and the AIP system. The submerged displacement of the notional submarine is 1,480 tons, and it has a 163 kW Stirling engine AIP system. Figure 1 shows the comparison of the underwater endurance (in hours) as a function of speed. The notional submarine developed for this comparison had a balance speed of four knots. For speeds above the balance speed the endurance decreases rapidly. It should be noted that the discharge fraction used for the calculations of battery endurance was 30%.
The submarine modeled has an AIP endurance of 14 days at 4 knots; hence the maximum underwater range is 1,344 nm as it is shown in Figure 2. The maximum underwater range of the
same submarine operating solely on battery is 420 nm, which clearly shows the advantage of the Air Independent Propulsion.

Another way to verify the advantages of the AIP submarine is to examine the indiscretion ratio. Snorkel period is usually referred to as the “indiscreet” period of the submarine operations, and the ratio of snorkel to the total mission duration is described as the indiscretion ratio. Since the required power varies with speed, the indiscretion ratio varies also with speed. From Figure 3 becomes clear that the indiscretion ratio, and hence the probability of detection, of a submarine having AIP system is much lower than that of a conventional submarine.

![Graph showing indiscretion ratio as a function of submarine speed](image)

**Figure 3:** Indiscretion ratio as a function of submarine speed

The most common AIP types, that are proven technologies tested or installed in submarines are:

1. Proton Exchange Membrane Fuel Cells (PEM FC).
2. Stirling Engines.
3. Closed Cycle Diesel Engines (CCD).
4. MESMA (Module Energie Sous-Marin Autonome) AIP system.

A Proton Exchange Membrane Fuel Cell (PEMFC) consists of a fuel cell stack, which converts the hydrogen and oxygen to DC electricity and water. The fuel cell stack consists of a number of single cells assembled in a filter press arrangement. Each cell is separated by a bipolar plate. Between each bipolar plate there is a membrane and electrode assembly consisting of a proton exchange membrane on either side of which is coated a platinum based electro-catalyst [10].

Hydrogen is input at the anode, where the catalyst forces the release of electrons. Hydrogen ions then pass through the polymer material to the cathode where they combine with oxygen and free electrons to form the water. The electrical circuit is formed by insulating the anode and cathode electrically [4]. The electrons are led through the circuit and transit from the anode to the cathode. A diagram of the PEMFC is presented in Figure 4. The reactions are the following:

Anode: \[ \text{H}_2 \rightarrow 2\text{H}^+ + 2e^- \]

Cathode: \[ \frac{1}{2} \text{O}_2 + 2\text{H}^+ + 2e^- \rightarrow \text{H}_2\text{O} \]

Advantages for the use of PEMFC are the flexibility concerning dimensions and power, the low operation temperature, and the short start-up time. The fuel cell operates at about 80 °C, which means that it warms up quickly. At the same time the electrolyte is solid and does not require any monitoring system. PEMFC needs very few systems to support its operation, which is an advantage for the use in submarine power plants.

A significant issue about PEMFC is the fuel source selection, since the solid polymer electrolyte membrane is susceptible to contamination by impurities in the fuel gas [4], especially carbon monoxide. The most popular fuel sources for PEMFC are:

1. Pure hydrogen, stored in metal hydrides and,
2. Methanol, which provides hydrogen after a reformation process.

The PEMFC is a low temperature unit, and therefore the high heat required to reform diesel fuel makes it unacceptable as a hydrogen source, even though it is the most advantageous in terms of logistics. Storage of hydrogen as a gas requires ten times larger volume than for a liquid fuel with the same energy content [10]. The volume impact is reduced if the hydrogen is stored as a liquid or bound chemically to a metal alloy.

From the available hydrogen storage methods, only the metal hydride storage has been at sea. Metal hydride storage is based on the fact that when a metal matrix of some form is saturated with hydrogen gas, the hydrogen bonds itself to the matrix. The amount of hydrogen absorbed depends on the temperature, pressure and varies with the type of matrix [4]. During loading, the metal hydride cylinders need to be cooled by a land based cooling device. During the operation of the Fuel Cell the cylinders are heated by cycling the fuel cell cooling water to free the hydrogen [13]. The maximum possible mass flow of hydrogen depends on the heat transfer to the hydride.

Metal hydride storage is compact and overcomes most of the hydrogen safety problems. The limiting factor for this method of storage is weight. Low temperature metal hydrides, such as TiFe or TiMn alloys can absorb up to 2% in weight hydrogen [13]. Therefore, the metal hydride tanks are placed externally and close to the keel of the submarine. Some of the weight penalty
associated with the use of metal hydride can be offset by using it in place of the stability lead. The metal hydride cylinders do not need to be replaced over the 30 year lifetime of the submarine. Their lifetime is limited only by the impurities in the hydrogen that is used.

The requirement for very long underwater endurance leads to weight critical submarine designs because of the heavy hydrogen storage. In the long term two alternative methods might solve this problem [19]. The two potential methods are: hydrogen storage in carbon-nanofibers (CNF); and storage of liquid hydrocarbons and generation of the hydrogen required.

![Figure 5: PEMFC with metal hydride storage of hydrogen [14]](image_url)

In the last few years, optimistic results about the potential storage of hydrogen in CNF were published. A considerable effort is taken around the world, but the technology is not yet mature. The other alternative, and more mature, hydrogen storage method is the reformation of methanol. A reformer converts hydrocarbon or alcohol fuels into hydrogen, which is then fed to the Fuel Cell. Methanol produces more hydrogen gas per mole of fuel, compared to the other hydrocarbons, and minimizes the production of carbon dioxide [4]. Methanol is a synthetic fuel that is easy to supply and transport. However it is immiscible in water, which means that it needs to be stored in its own tank, or in seawater compensated tanks with bladders separating fuel and water [4].

Different methods can be applied to process methanol and provide hydrogen for the operation of a PEMFC. The basic chemical reactions using methanol are as follows [10]:
Steam reforming: \[ \text{CH}_3\text{OH} + \text{H}_2\text{O} \text{ (steam)} + \text{Heat} \rightarrow 3\text{H}_2 + \text{CO}_2 \]

Partial oxidation: \[ 2\text{CH}_3\text{OH} + \text{O}_2 \text{ (air)} \rightarrow 3\text{H}_2 + \text{CO} + \text{H}_2\text{O} + \text{Heat} \]

Auto thermal reforming: A combination of the two above methods, such that the heat release from the second method balances the heat required for the first method.

![Diagram](image)

Figure 6: PEMFC with methanol reformer AIP system [14].

Apart from hydrogen, oxygen is also required to complete the process in the PEMFC. It can be stored onboard using many different methods:

1. Oxygen cryogenic storage: The oxygen is stored in double insulated tanks, with a typical storage temperature of -183 °C. The boil off would be approximately one per cent of the volume per day [4]. The vaporized oxygen can be used as breathing oxygen for the crew.

2. Oxygen gaseous storage: The oxygen can be stored in high pressure flasks. The method is inefficient in terms of volume and weight.

3. Oxygen chemical reformation: Oxygen can be produced as a by-product of a chemical reaction.

Cryogenic storage of oxygen is considered as the best option since it is the most efficient,
and it is already used for transportation tanks. The tanks are placed inside or outside the pressure hull. Storage inside the pressure hull provides flexibility and reduced cost. In addition to the storage of oxygen, the safety issues of hydrogen storage must be considered. Hydrogen has a very low flammability limit in air (4%), and hence any leaks can have catastrophic consequences. Hydrogen is also a low molecular weight gas and leaks are very hard to prevent. Metal hydrides are safer than the high-pressure hydrogen gas tanks. The German submarines have the metal hydride cylinders outside the pressure hull with a minimum number of internal pipes. As an additional safety measure, double piping is used with nitrogen between the two pipes to prevent hydrogen forming an explosive mixture [20].

In addition to fuel cells some submarine producers invest on the Stirling engine technology. Stirling engines are energy conversion devices that operate over a closed, regenerative thermodynamic cycle. The power pistons operate in a closed helium (or hydrogen) working gas system and heat is continuously transferred to the cycle via a heat exchanger. The Stirling system has been installed to the Swedish submarines Nacken and Gotland produced by Kockums. Kockums has its own Stirling system, which is also available for retrofit of different submarine types.

![Diagram of Stirling AIP system](image)

**Figure 7:** Stirling AIP system [14].

The Stirling engine consists of four interconnected cylinders. Each cylinder is split into two volumes by the piston, the hot volume above the piston and the cold volume below. The two volumes, the hot from one cylinder and the cold from the other cylinder are connected to form a closed system containing the working gas. The external combustion system continuously
transfers heat to the hot volume of the system [14]. Oxygen and hydrocarbon are burnt in the external combustion chamber of the Stirling engine. Since the combustion chamber is external and separated from the working gas in the Stirling cycle it is possible to select the pressure of the combustion chamber [12]. A relatively high combustion pressure allows the exhaust products to be discharged overboard at depth without the use of additional equipment. The fresh water cooling system is closed circulating and it is used to cool the Stirling engine and supply heat to the oxygen evaporator. The exhaust products are carbon dioxide, water vapor, and small amount of oxygen. The water vapor is condensed in a condenser and escapes to the sea. The other gases are dispersed in the seawater cooling system through a special mixing unit where the carbon dioxide is dissolved.

The main advantage of the Stirling AIP system is that it is a mature technology, proven in operational service. It is practically vibration free, silent, and its infrared signature is very low. The initial capital investment and the life cycle cost of the system are low. The main disadvantage is that it has low power density, and it requires clean fuel to prevent fouling of heat transfer surfaces.

![Diagram](image)

**Figure 8**: Closed Cycle Diesel AIP system [14].

The Closed Cycle Diesel (CCD) system uses a standard diesel engine, which can be operated in the open cycle (surface or snorkeling operations), and in the closed cycle (submerged operations). The function of the CCD is described in the following Figure 8. The system consists of a standard diesel engine in which the inert gas part is enriched with oxygen and led back to the input side of the engine to feed a new combustion cycle. To reproduce the characteristics of
ambient air, besides adding oxygen to the recycled gas, it is necessary to include also monatomic inert gas. The exhaust gas is cooled down and led into an absorber, where the carbon dioxide is dissolved into the water.

In closed cycle mode, the produced exhaust gas leaves the diesel engine outlet with a temperature of approximately $350 - 400 \, ^\circ C$ and a pressure of about 3 bars. The exhaust gas consists of carbon dioxide, nitrogen, water vapor, and a small amount of unburned oxygen. After been cooled by a spray cooling system to approximately $80 - 100 \, ^\circ C$, the gas is fed into the absorber. Usually the absorber is a rotating scrubbing system, consisting of a rotor which mixes the exhaust gas with sea water. The required sea water is supplied in a way which enables the CCD to operate without specific depth restrictions [11].

The main advantages of the CCD are that it is the lower cost option, has good power density and good efficiency. It is based on proven technology and can be easily implemented. The main disadvantages are that it has higher oxygen consumption, and with the current technologies the CO$_2$ removal system is large.

![Figure 9: MESMA AIP system](image)

The MESMA system is the air independent propulsion system that DCN of France developed, mainly for export purposes. The operation of the system is based on a closed Rankine cycle engine. Liquid oxygen is stored at $-185^\circ C$ and it is pumped into a vaporizer, where it becomes gaseous. Then it is lead into the combustion chamber where it mixes with ethanol and produces a thermal output of $700^\circ C$, and 60 bar pressure to heat the secondary cycle. The high
pressure of the exhaust gasses allows for operation of the system at any diving depth without the need for additional equipment.

The secondary circuit is a steam-driven Rankine cycle turbine which drives a high speed generator. The water is vaporized and overheated, and passes through a turbine coupled with generator that produce the electric power for the submarine hotel power and propulsion.

### 1.3 Review of Conventional Submarine Market

The specific design features of the diesel electric submarines depend upon the geographic location of the owner nation and the related requirements. A country with a strategic role that requires a vessel capable for long open-ocean offensive and defensive missions needs a submarine with different design characteristics than a country that needs vessel capable for short duration coastal missions. The geographical and political situations of countries like Australia, Japan, South Korea and India drives the need for larger conventional submarines (SSKs). Typical examples are the Collins and Kilo class submarines. However, the market of the conventional submarines is focused on vessels in the range of 1,100 to 1,800 tons. Compared to the past there is a trend to increase displacement, mainly due to the increase of the required reloads carried on-board, and the addition of air independent propulsion systems in most of the designs. A summary of the most popular current designs is presented in Table 1.

One of the most successful modern diesel electric submarines is the Type 209 class submarine designed by Ingenieurkonto Lubeck GmbH in corporation with the Howaldtswerke-Deutsche (HDW) shipyards. The first 209 submarine, “GLAFKOS”, was delivered in 1971 to the Hellenic Navy and since then more 55 vessels were delivered to 13 different nations. The very good submerged range, the high submerged speed and the good handling aspects are still today the most important features of the 209 class submarines. Depending on the various customer requirements the size increased from the original 1,100 tons displacement, in some cases as much as 50%. The latest and the most capable platform is the Type 209/1400 Mod, ordered in
December 1999 by South Africa. An AIP propulsion module is also available for retrofit to the existing ships. The submarine will be lengthened by the addition of an extra 6m section, aft or the bridge fin. The fuel cell system will consist of two 120 kW fuel cell modules, a liquid oxygen tank placed inside the pressure hull, and all the necessary pipes and electrical equipment. The hydrogen is stored in metal hydride cylinders installed in the keel of the submarine for trim and stability reasons, since this storage method adds significant weight to the submarine. With the addition of the AIP system the submerged endurance of the 209 submarine will be increased approximately by a factor of five, as compared with the baseline diesel electric version.

Table 1: Summary of AIP/Conventional Submarines

<table>
<thead>
<tr>
<th>Submarine Type</th>
<th>HDW</th>
<th>HDW</th>
<th>HDW</th>
<th>DCN/IZAR</th>
<th>Kockums</th>
<th>Kockums</th>
<th>Kockums</th>
<th>DCN</th>
<th>Admiralty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer</td>
<td>Germany</td>
<td>Germany</td>
<td>Germany</td>
<td>France/Spain</td>
<td>Sweden</td>
<td>Sweden</td>
<td>Sweden</td>
<td>France</td>
<td>Russia</td>
</tr>
<tr>
<td>Year in Service</td>
<td>1,285</td>
<td>1,830</td>
<td>1,980</td>
<td>1,700</td>
<td>1,494</td>
<td>3,353</td>
<td>1,760</td>
<td>3,076</td>
<td></td>
</tr>
<tr>
<td>Submerged Displacement (tons)</td>
<td>1,100</td>
<td>1,450</td>
<td>1,700</td>
<td>1,450</td>
<td>1,240</td>
<td>3,051</td>
<td>1,510</td>
<td>2,325</td>
<td></td>
</tr>
<tr>
<td>Suraced Displacement (tons)</td>
<td>181</td>
<td>183</td>
<td>213</td>
<td>218</td>
<td>198</td>
<td>255</td>
<td>222</td>
<td>242</td>
<td></td>
</tr>
<tr>
<td>Length (ft)</td>
<td>20.3</td>
<td>23</td>
<td>20.7</td>
<td>20.3</td>
<td>20.4</td>
<td>25.6</td>
<td>22.3</td>
<td>32.5</td>
<td></td>
</tr>
<tr>
<td>Diameter (ft)</td>
<td>820</td>
<td>Unknown</td>
<td>&gt;1300</td>
<td>984</td>
<td>Unknown</td>
<td>Unknown</td>
<td>1050</td>
<td>985</td>
<td></td>
</tr>
<tr>
<td>Diving Depth (ft)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>Submerged Speed (knt)</td>
<td>50</td>
<td>306</td>
<td>240</td>
<td>No</td>
<td>164</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>AIP Capacity (Kw)</td>
<td>PEMFC</td>
<td>PEMFC</td>
<td>Stirling</td>
<td>50</td>
<td>84</td>
<td>50</td>
<td>Unknown</td>
<td>70</td>
<td>68</td>
</tr>
<tr>
<td>AIP Type</td>
<td>50</td>
<td>Unknown</td>
<td>84</td>
<td>8</td>
<td>31</td>
<td>25</td>
<td>42</td>
<td>36</td>
<td>52</td>
</tr>
<tr>
<td>Mission Duration (days)</td>
<td>31</td>
<td>27</td>
<td>27</td>
<td>31</td>
<td>25</td>
<td>42</td>
<td>36</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>Complement</td>
<td>7,500@ 8 knots</td>
<td>8,000@ 8 knots</td>
<td>12,000</td>
<td>6,500@ 8 knots</td>
<td>6,500@ 8 knots</td>
<td>11,500@ 10 knots</td>
<td>10,000</td>
<td>7,500@ 7 knots</td>
<td></td>
</tr>
<tr>
<td>Suraced Range (nm)</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torpedo Tubes</td>
<td>Total Weapons</td>
<td>14</td>
<td>12</td>
<td>16</td>
<td>18</td>
<td>16</td>
<td>22</td>
<td>16</td>
<td>18</td>
</tr>
</tbody>
</table>
A more powerful AIP system is fitted in the new class of submarines that HDW and Thyssen Nordseewerke GmbH (TNSW) are jointly developing. The first 212 submarine will be commissioned in 2004, and it will be the first worldwide equipped with a fuel cell plant. The Italian shipyard Fincantieri is also going to build two 212 submarines for the Italian Navy, with the first expected to be commissioned in 2005. The propulsion plant of the 212 combines a conventional system consisting of a diesel engine and a lead acid battery, with an AIP system used for silent slow cruising. The AIP system consists of nine proton exchange membrane fuel cells (PEMFC) providing energy between 30 and 50 kW each. The oxidant is liquid oxygen stored in tanks outside the pressure hull, and the fuel is hydrogen which is stored in metal hydrides in tanks located at the lower part of the submarine, outside the pressure hull. The submarine is designed with a partial double hull which has a larger diameter forward. This is joined to the after end by a conical section which houses the fuel cell plant. The two liquid oxygen cylinders and the hydrogen cylinders are carried around the circumference of the smaller hull section.

The latest design of HDW which combines the strong points of the proven 209 with the advanced technology of 212, is the 214. The first is expected to be delivered to the Hellenic Navy in 2005. The 214 will have an increased diving depth due to improvements in the pressure hull materials. The propulsion system consists of two diesel generators, a lead acid battery, an AIP system, and a propulsion motor coupled directly to the seven-bladed propeller. The AIP system consists of two Siemens PEM fuel cell modules which produce 120 kW each, and give to the submarine an AIP endurance of two weeks. The oxidant is liquid oxygen, stored inside the pressure hull, and the fuel is hydrogen, stored in metal hydride cylinders outside the pressure hull. Storage of the oxygen inside the pressure hull has been used in order to reduce cost. The safety measures have been cleared with the German classification society, Germanischer Lloyd [13]

The Scorpene submarine has been jointly developed by DCN of France and Izar of Spain. The first submarine is expected to be delivered in 2003 to the Chilean Navy. The design goal of Scorpene was to achieve an extremely quiet platform. The shape of the envelope was designed to reduce the hydrodynamic noise. The designers worked very hard to reduce the boats complement
to just 31, with a standard watch team of only nine. The propulsion system of Scorpene is
different in the two existing variants [18]. The first variant, the CM-2000, has a conventional
propulsion system which has two diesel generator sets providing 1,250 kW of power. The second
variant, the AM-2000 is equipped with a MESMA AIP system. The MESMA system is modular
and can be installed during construction or at any stage of the life cycle. It consists of a turbine
receiving high-pressure steam from a combustion chamber, burning a gaseous mixture of ethanol
and liquid oxygen [18].

Another conventional submarine design of DCN is the Agosta submarine. Agosta
submarines are currently in service in the French, Spanish, and Pakistan Navy. The first of the
improved version of the submarine, the Agosta 90B, was delivered to the Pakistan Navy in 1999.
The second vessel is now at the final stage of completion, and the third is planned to be delivered
to Pakistan in 2005. It will be fitted with a MESMA air independent propulsion system which
will be retrofitted to the first two.

The Swedish company Kockums is another major player in the conventional submarine
market. Kockums produced three submarines of Gotland class, with the first been commissioned
in 1996. The unique design feature of this submarine is its propulsion system. Gotland is
equipped with two MTU diesel engines, and two Kockums Stirling Air independent propulsion
units which provide up to 75 kW [18] each, and give to the submarine an AIP endurance of two
weeks at a speed of 5 knots. The oxidant of the AIP system is liquid oxygen which is stored
inside the pressure hull, and the fuel is diesel. Stirling AIP system is equally well suited for
modernization of existing submarines and for integration into new submarine designs.

Another design product of Kockums is the Collins class submarine for the Australian
Navy. The six Collins submarines are being built by the Australian Submarine Corporation of
Adelaide, South Australia, which is owned jointly by Kochums and two Australian companies.
The design of the submarine was driven by the requirements for a long range, multi-mission
patrol submarine. The submarine is expected to be capable of short-duration coastal missions and
longer open-ocean offensive and defensive missions with duration of up to 70 days. Collins has
comfortable accommodation for the crew, and carries 22 missiles or torpedoes and up to 44
mines in place of torpedoes. The propulsion system consists of three diesel engines, a storage battery, one main propulsion motor driving a skew back propeller, and an emergency retractable hydraulic motor.

Another very successful diesel electric submarine is the Kilo, designed by the Russian Rubin Construction Bureau. The first entered service in the early 1980’s. The development of the baseline has lead to the current production versions, the 877EKM and the Type 636. A successor of 636, the Amur, which incorporates an air independent propulsion system, is being developed. The AIP system could also be available for retrofit to the other versions. Twelve Kilo submarines remain operational with the Russian fleet and over 15 have been exported to six different countries [17]. One of the most important features of the design is the double hull. It has high buoyancy reserve, which is over 32%, and makes the submarine extremely survivable. The hull is covered by anti-sonar protection tiles to reduce risk of detection. The propulsion system consists of two diesel generators, one main motor, one fuel economic motor, two storage batteries, and a single shaft driving a seven bladed fixed pitch propeller [18].

1.4 Thesis Study

The characteristics of a technology strategy for the development of a more capable submarine with regard to system level mission parameters will be studied. Specifics of the propulsion plant will be modified to simulate improvement or degradation associated with new technologies. The impact of technology will be assessed through the use of technology k-factors. These factors will be introduced into the mathematical synthesis model and they will modify the technical characteristics of the design. The parametric mapping will be done with the use of Response Surface Equations based on regression analysis using the results from the synthesis model.

This study is structured in the following format: Chapter Two will discuss the mathematical model used to generate the data for this thesis. Chapter Three will describe the methodology of forecasting the system-level impact of technology infusion on conventional submarine design. Chapter Four will show the effect of uncertainty of the
future/forecasted technologies on submarine performance and construction cost. Chapter Five will then summarize the application of the methodology and provide direction for further work.
2 Submarine Synthesis Model

In order to apply the technology strategy method a synthesis model must be used. The characteristics that the synthesis model should have are the following [23]:

1. It must have parametric inputs, in order to facilitate the use of Response Surface Methods.

2. It should be physics based, in order to be able to analyze the impact of the new technologies. A model based on regression analysis of previous designs will not be able to capture the impact of new technologies.

3. It needs to include disciplinary technical metric impact factors, in order to simulate the impact of the new technologies. These factors will be referred to as k-factors, and it should be easy for the user to change their value.

4. The responses should be quantifiable, in order to relate the responses to the variation of inputs.

The mathematical model for this study was developed using the software program MathCAD by MathSoft and it is presented in Appendix 1. Using this software package the designer directly inputs the mathematical equations into the document. The ease of use and the ability to quickly change the equations are advantages of using MathCAD. The model of reference [3] was used as a starting point.

2.1 The Design Process

The concept exploration is the part of the design process where the designer specifies the main characteristics of the product. The objective of the concept design phase is to determine the
size, weight, and geometric configuration within which the detailed studies can take place [5]. To achieve a design solution, an iterative procedure needs to be applied, which starts with the definition of requirements.

With the requirements stated, the process of determining the characteristics of the submarine can begin. The flowchart of the model is presented on Figure 10:

![Design Flowchart](image)

**Figure 10: Design Flowchart**
For conventional submarines, the volume occupied by the payload is approximately 30% of the total pressure hull volume [5]. Based on this payload volume requirement a preliminary estimate of the pressure hull volume can be made and the envelope volume can be calculated. Next, the shape and dimensions of the submarine can be iterated to design a hull with the required envelope volume.

The selected shape and dimensions provide the ability to calculate the wetted surface and the resistance of the submarine. Based on that preliminary estimate of the resistance the propulsion motor can be sized to meet the speed requirements. After specifying the required power at different speeds, battery size can be determined based on the required underwater endurance at loiter speed, or the required time that the submarine needs to sustain maximum speed.

The sizing of the diesel generator plant is based on the submarine’s desired operational profile during snorkeling operations. The limiting factor for the power of the diesel engines is the maximum current limitation on charging the batteries. Having determined the power of the engines, and knowing the required endurance, the necessary amount of fuel can be calculated.

In addition to designing a diesel electric submarine, the model developed for this study has the ability to design a “hybrid” submarine, which retains the diesel electric capability and adds an AIP system. In the case of the “hybrid” submarine, the size of the AIP system and the necessary amount of fuel and oxidant are based on the required balance speed and underwater endurance.

Based on the size estimates of the individual components presented above, a preliminary required size of the pressure hull is determined. This volume is fed back to the beginning of the model, and a new iteration of the above calculations begins, leading to a new pressure hull volume, new dimensions, new power requirements, and new sizes for the pressure hull components. The iterative process of the volume balance stops when the difference between the required and the available volume of the pressure hull is less than 1%.
The weights and centers of gravity of the submarine’s systems are derived from the physical dimensions of the equipment or from regression equations. For every submarine, a balance between weight and buoyancy is necessary; however, there are many ways to achieve balance. In this model, the displacement that corresponds to the everbuoyant volume is compared to the surfaced displacement of the submarine. Everbuoyant volume is the sum of the pressure hull and volume of outboard items.

In the case that the everbuoyant volume is less than the total weight of the submarine, the submarine is a weight limited design. Due to uncertainty in the conceptual design stage, the designer does not generally have the luxury of saving weight. For small adjustments some of the lead ballast can be removed; however, the fraction of lead to the normal surfaced displacement should not be reduced below 5%. If greater adjustment is necessary, buoyancy must be added by increasing the length over diameter ratio, which adds length to the parallel mid-body of the submarine. Then, the iterative process of volume balancing should start again.

In the case that the everbuoyant volume is greater than the total weight of the submarine, the submarine is a volume limited design. Due to the uncertainty at this level of design the volume requirement cannot be reduced. Therefore, fixed ballast must be added in order to balance buoyancy and weight.

In both the weight limited and the volume limited case, weight balance is assumed when the difference of the displacement that corresponds to the everbuoyant volume with the surfaced displacement of the submarine is less than 1%.

Having obtained the volume and weight balance of the design the longitudinal balance must be obtained. The center of gravity of the submerged submarine is required to be at the same vertical position as the center of buoyancy. The center of buoyancy is calculated based on the geometric shape of the submarine, and the center of gravity is estimated from the centers of gravity of the individual weight groups. In addition to the requirement for submerged longitudinal balance, the submarine must be balanced in the surfaced condition as well. The longitudinal location of the center of gravity must be in the same vertical plane as the surfaced
center of buoyancy. This can be achieved by proper placement of the ballast tanks. In order to ensure that the center of gravity is in the same vertical plane as the surfaced and submerged center of buoyancy, it may be necessary to adjust the location of the submarine's center of gravity. This can be done by adjusting the longitudinal location of the lead ballast.

Submerged stability requires that the center of gravity be below the center of buoyancy. The magnitude of their distance determines the restoring moment of the submarine. The vertical location of the lead ballast's center of gravity is iterated until the vertical distance between the center of gravity and the center of buoyancy of the submarine is at least 1 foot.

In addition to stability and longitudinal balance requirements, the submarine should be able to maintain neutral buoyancy and level trim in all conditions. Any loading condition must be able to be compensated by the trim and compensating system. In order to ensure that the submarine can operate in all loading conditions, the equilibrium polygon must be checked. If any loading conditions fall outside the enclosure of the polygon, the submarine cannot be properly ballasted with the use of the trim and compensating system. Therefore, the system must be resized, or the fixed ballast must be rearranged.

Having achieved a balanced design, the model estimates the performance parameters to make sure that it achieves the owner requirements. The performance module calculates the maximum surfaced range, the maximum submerged range at different speeds of advance (SOA), and the IRs that correspond to those speeds. It also calculates the Overall Measure of Effectiveness (OMOE) of the design, based on relative weights that can be specified by the user.

2.2 Hull Envelope

The naval architecture of submarines is the same as that of surface ships. The design of surface ships is optimized for surface performance and modern submarines are optimized for their underwater performance. Therefore the body of revolution or "Albacore" form has been
adopted as the hull shape of the model.

In the conceptual design phase the outer hull shape can be described using mathematical models. Jackson [1], [2] introduces a super-ellipse which describes separately the longitudinal contour of the entrance, middle and run section. The entrance has a length, \( L_f \), of 2.4 diameters, and the run or after end has a length, \( L_a \), of 3.6 diameters. The entrance is represented by an ellipsoid of revolution and the run by a paraboloid of revolution.

Entrance: \[ Y_f = \frac{D}{2} \left[ 1 - \left( \frac{X_f}{L_f} \right)^{\frac{1}{n_f}} \right]^{\frac{1}{n_f}} \]

Run: \[ X_f = \frac{D}{2} \left[ 1 - \left( \frac{X_a}{L_a} \right)^{n_a} \right], \]

where \( X_f, X_a \) are distances from the maximum diameter of the hull shape.

The displacement of the submarine can be increased by adjusting the \( n_a \) and \( n_f \) which describe the “fullness” of the body, or by adding a parallel middle body (PMB) of cylindrical shape, which can have the maximum diameter, \( D \), and a length less than 6D [1]. If \( C_{pf} \) and \( C_{pa} \) are the prismatic coefficients of the entrance and the run, and \( C_{wf} \) and \( C_{wa} \) are the wetted surface coefficients, the total volume and the wetted surface of the envelope can be calculated using the following expressions:

Envelope Volume: \[ V = \frac{\pi D^3}{4} \left( 3.6C_{pa} + \frac{L_f}{D} - 6 + 2.4C_{pf} \right) \]

Wetted Surface: \[ WS = \pi D^2 \left( \frac{L_f}{D} - K_2 \right), \text{ where } K_2 = 6 - 2.4C_{wf} - 3.6C_{wa} \]

2.3 Resistance and Powering

In order to achieve the target requirements, a balance between the resistance and the available power at the different operating conditions, needs to be achieved.
2.3.1 Resistance

The resistance of a submarine has similar components as that of a surface ship. Froude hypothesis is assumed, and the residuary and frictional resistances are calculated separately. The wavemaking resistance is negligible when a submarine is submerged and it is at a depth greater than three to four hull diameters. The modern submarines are optimized for their underwater performance.

2.3.1.1 Submerged Resistance

The two main categories of the submarine resistance are the hull, and the appendages resistance. The main components of the hull resistance are the frictional resistance, the residual resistance, and the correlation allowance. The non-dimensional frictional resistance coefficient can be calculated by the formula agreed at the International Towing Tank Conference:

\[ C_f = \frac{0.075}{(\log Re - 2)^2} \], where Re is the Reynolds Number.

The residual resistance accounts for the pressure difference along the hull while the submarine is moving [1]. The following formula developed by Horner calculates the increase of the drag coefficient of the hull due to the residuary resistance [3].

\[ \frac{C_r + C_f}{C_f} = 1 + 1.5 \left( \frac{D}{L} \right)^{2/3} + 7 \left( \frac{D}{L} \right)^{3/2} \]

The drag is increased due to the flow separation caused by the decrease of hull diameter at the aft end of the submarine.

Correlation allowance, \( C_a \), represents an adjustment between resistance data obtained by model testing and data obtained from full size ship tests. For surface ships the typical value is 0.004 and for submarines between 0.0015 and 0.002 [3].
The two main components of appendage resistance are the Sail (Bridge) resistance, and the resistance of the various control surfaces. The Sail can be modeled as a foil with drag coefficient $C_{DSail}$ and area $A_{Sail}$.

**Drag Coefficient:**  
$$C_{DSail} = 0.009 \text{ [4]}$$

**Sail Area:**  
$$A_{Sail} = 0.0805 \cdot WS \text{ [5]}$$

**Sail Drag:**  
$$D_{Sail} = C_{DSail} \cdot A_{Sail}$$

The resistance of the remaining appendages can be approximated by the following formula [4]:

$$D_{Appendage} = \frac{L \cdot D}{1000}, \text{ where } L \text{ and } D \text{ are the length and diameter of the body of revolution.}$$

Hence the effective horsepower required to move the submarine, when it is submerged, as a function of submerged speed, is given by the equation:

$$EHP = 0.00872 \cdot (\text{Speed})^3 \left[ WS \left( C_f \left( \frac{C_r + C_f}{C_f} \right) + C_a \right) + D_{Sail} + D_{Appendage} \right]$$

The effective horsepower is translated into shaft horsepower through the propulsive coefficient (PC).

**Shaft Horsepower:**  
$$\text{SHP} = \frac{EHP}{PC}$$

**Propulsive Coefficient:**  
$$PC = \eta_0 \eta_h \eta_{rr}$$

Where $\eta_0$ is the open water efficiency of the propeller, $\eta_h$ is the hull efficiency, and $\eta_{rr}$ is
the relative rotating efficiency.

2.3.1.2 Snorkeling Resistance

When the submarine operates near the free surface, it generates gravity waves. Therefore, the resistance of the submarine is increased. The increase in the required shaft horsepower is not significant, unless the submarine is operating at Froude numbers greater than 0.6 [7]. Reference [7] lists a chart and provides a methodology for determining the added resistance coefficient, $C_w$, when operating close to the surface. The coefficient is given as a function of Froude number, length to diameter ratio, and submergence ratio (operating depth divided by the overall length). The calculations are as follows [7]:

Added Resistance Coefficient:

$$C_w = \frac{(Ch\#)}{4 \cdot \left[\frac{L}{D}\right] - 1.3606 \cdot \left(\frac{L}{D}\right)^2}$$

Added Shaft Horsepower:

$$SHP_w = 0.0087 \cdot \left(\text{Speed}\right)^3 \cdot WS \cdot C_w$$

Where (Ch#) is the number obtained from the chart presented in reference [7].

2.3.1.3 Surfaced Resistance

The surfaced resistance of a submarine has the same components as the resistance of a surface ship. Comparing with the submerged resistance there is an additional component, which is the wave resistance that dominates at higher speeds. Assuming that the propulsive coefficient (PC) surfaced is the same as the PC submerged [3], the surfaced shaft horsepower can be calculated using the following formula:

$$SHP_{\text{surfaced}} = 1.25 \cdot PC \cdot 0.00872 \cdot C_T(\text{Speed}) \cdot WS \cdot (\text{Speed})^3,$$

where the $C_T(\text{Speed})$ is the total surfaced resistance coefficient, which is a function of Froude
number and it is estimated from a graph contained in the model.

2.3.2 Propulsion Plant

2.3.2.1 Propeller

Submarines have traditionally fixed pitch propellers. Jackson [1] provided data on the wake fraction, \( w \), the thrust deduction fraction, \( t \), and the hull efficiency, \( \eta_h = (1-t)/(1-w) \). The hull-propeller interaction parameters are functions of the length to diameter ratio of the hull, the hull shape, and the ratio of the propeller diameter to the maximum diameter of the hull. The need for high propulsive efficiency requires a large diameter, low rpm propeller with high torque. The estimate of the propeller diameter in \( \text{ft} \) is done based on the following formula [3]:

\[
D_p = 50 \cdot \text{SHP} \cdot \left( \frac{V}{N} \right)^{0.6}
\]

where \( V \) is the speed of the submarine (knots), and \( N \) is the rotational speed of the shaft (rpm).

2.3.2.2 Propulsion Motor

The propulsion motors of submarines are direct current motors and usually the rotor is directly coupled to the propeller shaft. The motor must be sized in order to meet the high power requirements of the submarine at its top speed. Since the motor is directly coupled to the propeller, it needs to have a rotational speed equal to the speed at which the propeller has been designed to deliver full thrust. However, conventional submarines usually operate at very low speeds and therefore the motor is required to deliver high torque at low rpm.

The weight and volume estimate of the propulsion motor in the model is based on a study conducted by Kirtley [9]. The equations are the following:
\[ Weight = Const_1 \cdot RPM^{-0.44} \cdot Power^{0.79}, \text{ where } Const_1 = 43.13 \]

\[ Volume = Const_2 \cdot RPM^{-0.44} \cdot Power^{0.56}, \text{ where } Const_2 = 0.304 \]

\( RPM \) represents the revolutions of the motor rotor, per minute, at loiter speed. Since the motor is directly coupled to the shaft, \( RPM \) represents the revolutions of the shaft, at loiter speed.

\( Power \) is the power of the motor in HP at loiter speed, which is estimated based on the resistance of the submarine at loiter speed, and the efficiencies of the motor and the shaft bearings.

\( Weight \) is the weight of the motor in tons, and \( Volume \) is the volume of the motor in ft\(^3\). The values of the two constants are selected in order to model the known weight and volume of the Permasyn IFR6943 motor [3].

2.3.2.3 Battery

During the submerged condition the power requirements of a conventional non-AIP submarine, are satisfied by the battery. For the AIP submarine, part of the power is provided by the battery and part by the AIP system, depending on the submerged speed and the operational requirements.

In the model, the hotel load is assumed to be proportional to the pressure hull volume. It is estimated by an equation found in reference [5]:

\[ HL_{SUBMARINE} = 0.75 \cdot HL_{PAYLOAD} + 0.075 \cdot VOLUME_{PH} \]

The main demand, which calls for a large number of battery cells is that of propulsion [5]. Even a large number of cells will be drained in a very short period of time at top speed. Therefore in order to size the battery we have to take into account both the maximum speed power requirements and the loiter power requirements. The propulsive power requirement is
proportional to the cube of speed. Therefore, the propulsive power is the greatest part of the total power at maximum speed. At loiter speed the hotel load represents the higher portion of the total required power.

In the model the battery was sized according to the battery data provided in reference [3], which uses an ASB 49C Lead Acid battery. This baseline battery has 126 cells and the dimensions, weight, voltage and current data are included in the model. Estimates for the battery capacity at burst (1 hour rate) and creep (48+ hour rate) speed are determined as follows:

Table 2: Estimates of Baseline Battery Capacity

<table>
<thead>
<tr>
<th>Burst Capacity (1 hour rate)</th>
<th>1092.4 kWh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance Capacity (48+ hour rate)</td>
<td>2647 kWh</td>
</tr>
</tbody>
</table>

The required battery energy to meet the burst speed power requirements is calculated based on the required maximum speed and the required time at maximum speed. Similarly, the endurance speed power requirements are calculated based on the required loiter speed and the required time at loiter speed. The maximum of those two values is selected as the required capacity of the battery. Knowing the required battery capacity, the total number of cells, the total weight, and volume of the battery, can be calculated.

Usually the battery is divided in two half-batteries that are located in two different compartments at the lower level of the pressure hull. This arrangement is used in the development of this model.

2.3.2.4 Diesel Engines

The primary role of the main diesel engines is to charge the batteries when the submarine is surfaced or during snorkeling. The battery charging power is determined by the battery capacity and the maximum charging current that the cells can accept. Hence the time to charge the batteries is determined by the batteries and not by the diesels [5]. Knowing that, the total
power of the diesel engines in the model, is calculated using the peak charging power of the battery.

High speed diesel engines are used to minimize the effect on volume and weight of the submarine. The engines are assumed to be placed in acoustic enclosures and mounted on resilient supports. The volume and the weight of the engine are multiplied by suitable factors [3], in order to model the additional weight and volume of the engine auxiliaries, the enclosure, and the electric generator.

If the possibility that the submarine will leave the base with the batteries fully charged and it will arrive with the batteries discharged is ignored, the diesel fuel for the patrol can be calculated based on the hotel load and the propulsive energy of the patrol. Of course the patrol will be conducted at the submerged condition operating on batteries, but the energy to charge the batteries will have been provided by the diesel engines. Consequently, in the model, the total fuel is calculated as the maximum of the fuel required for the surface range requirement, and the fuel that corresponds to the snorkeling range requirement.

2.3.2.5 **Air Independent Propulsion Plant**

Diesel electric submarines are dependent on air, in order to operate the diesel engines and charge the batteries during snorkeling operations. The dependency on air limits the usefulness of the submarine. Therefore, several different approaches to Air Independent Propulsion (AIP) are being pursued, in order to reduce the vulnerability of the submarine. In the mathematical synthesis model, developed for this study, the following AIP options are examined:

1. Proton Exchange Membrane Fuel Cells (PEM FC) with metal hydride storage of hydrogen.

2. Proton Exchange Membrane Fuel Cells (PEM FC) with the hydrogen being produced by reforming methanol.

3. Stirling Engines.

In this study, all systems are assumed to use Liquid Oxygen as the source of oxidant. Despite some of the systems being capable of operating in snort mode to recharge the batteries during transit, the design of the mathematical model assumes that the additional submarine AIP plant is used to provide slow speed submerged operations at the patrol area. Hence, the submarine design is hybrid.

2.4 Volume Requirements

2.4.1 Pressure Hull Volume

The required volume of the pressure hull is estimated, so that the submarine will meet all the requirements. The pressure hull is divided into the following compartments starting from the stern of the submarine:

1. Engine Room
2. AIP System Room
3. Machinery Control Room
4. Operations Compartment
5. Torpedo Room

The total volume of the pressure hull is the sum of the individual groups described above. The components of those groups are described in the following paragraphs.

2.4.1.1 Engine Room Volume

The main components of the engine room are the propulsion motor with its auxiliaries, the main diesel engines, two thirds of the diesel fuel, and the aft trim tanks. The estimate of the
propulsion motor volume in the model was based on a study conducted by Kirtley, found in reference [9], and was presented in section 2.3.2.2. The volume of the motor was multiplied by a factor to account for the motor auxiliaries.

The volume of the main diesel engines is calculated in the model using the average of the specific volume (ft³/kW) of different diesel engines. The diesel fuel volume is calculated based on the endurance requirements. The volume of the aft trim tanks is calculated as a percentage of the total volume of the trim and compensating system, which is estimated by the following equation found in reference [5]:

\[ T \& C \_Volume = \left( \frac{V \cdot PH \cdot (\rho_{\text{MAX}} - \rho_{\text{MIN}})}{\rho_{\text{SW}}} + \frac{\text{Weight}_{\text{STOBE}}}{\rho_{\text{SW}}} \right) \frac{1}{0.98}, \]

where \((\rho_{\text{MAX}} - \rho_{\text{MIN}})\) is the required range of sea-water density that the submarine is required to operate, \(\rho_{\text{SW}}\) is the density of sea water, and 0.98 is the utility factor of the tanks.

The total volume of the engine room is estimated by multiplying the sum of the above components by a packing factor that was derived based on calculations from figures contained in reference [15].

2.4.1.2 AIP System Volume

In this mathematical model we assume that all the required equipment, consumables, and compensating systems for the operation of the AIP plant are contained in the AIP compartment. The only exception is the PEMFC system with storage of hydrogen in metal hydrides. For this option the fuel is assumed to be stored outside the pressure hull and near the keel of the submarine.

When the AIP plant and its consumables are integrated in the submarine many different constrains limit the volumetric and gravimetric efficiency of the design. The necessary key drivers that we need to consider according to reference [14] are the following:

2. Plant Integration: Power density and capability of the plant, structure required, mounting and signature reduction requirements.

3. Submarine Power Requirements: Types and proportions of power required.

The impact of those integration drivers on the volume of the submarine for each of the used AIP options is shown in Table 3. When the model is used to design a conventional submarine without the AIP option, the volume of the AIP compartment is automatically set to zero.

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>FC Metal Hydride</th>
<th>FC Methanol Reformer</th>
<th>CCD</th>
<th>Stirling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Consumption (l/kWh)</td>
<td>0.634</td>
<td>0.519</td>
<td>0.329</td>
<td>0.37</td>
</tr>
<tr>
<td>Fuel Storage (l/kWh)</td>
<td>0.250</td>
<td>0.2224</td>
<td>0.017</td>
<td>0.0195</td>
</tr>
<tr>
<td>Oxygen Consumption (l/kWh)</td>
<td>0.390</td>
<td>0.539</td>
<td>0.735</td>
<td>0.895</td>
</tr>
<tr>
<td>Oxygen Storage (l/kWh)</td>
<td>0.058</td>
<td>0.081</td>
<td>0.1304</td>
<td>0.1579</td>
</tr>
<tr>
<td>Ar/N2/He Consumption (l/kWh)</td>
<td>-</td>
<td>0.005</td>
<td>0.0212</td>
<td>0.0067</td>
</tr>
<tr>
<td>Ar/N2/He Storage (l/kWh)</td>
<td>-</td>
<td>0.0011</td>
<td>0.0037</td>
<td>0.0011</td>
</tr>
<tr>
<td>Compensation (l/kWh)</td>
<td>0.434</td>
<td>0.64</td>
<td>0.773</td>
<td>0.940</td>
</tr>
<tr>
<td>Total (l/kWh)</td>
<td>1.766</td>
<td>2.0075</td>
<td>2.0093</td>
<td>2.3902</td>
</tr>
</tbody>
</table>

| Plant Size                             | 64               | 115                  | 89     | 83       |

2.4.1.3 Machinery Control Room

The machinery control room is located forward of the AIP room, if an AIP submarine is being designed, or forward of the Engine Room, if a diesel electric submarine is being designed. It contains the equipment for control of the propulsion system, half of the battery cells, and half
of the fresh water. The final volume of the compartment is estimated by multiplying the sum of
the above components by a packing factor that was derived based on calculations from figures
contained in reference [15].

The volume of the fresh water tanks is calculated by multiplying the estimate of the
volume of the fresh water that a man needs per day, by the duration of the mission, and the total
number of the crew.

2.4.1.4 Operations Compartment

The main components of this compartment are: the Combat Information Center, the
berthing and meshing of the crew, the stores for the mission, the weapon reloads, half of the
battery cells, one third of the diesel fuel, one half of the fresh water, the sanitary tank, and the
compensating tank.

The volume of the Combat Information Center is an input in the payload module of the
model. The volume of the berthing and meshing is calculated by multiplying an estimate of the
volume required per crew member, by the number of crew. The number of torpedo reloads and
the volume of each torpedo is a user input. The volume of stores is based on commercial ship
standards, and it is calculated by multiplying the volume of dry and reefer stores per man per
day, by the crew number, and the mission duration.

2.4.1.5 Torpedo Room

The main components of the torpedo room are the torpedo tubes and the torpedo
compensating tanks. The number of the torpedo tubes and the volume of each one are inputs in
the payload section of the model. The volume of the torpedo compensating tanks is estimated in
order to accommodate for the weight of the torpedoes that are not taken on board or are used
during the mission. The packing factor used to calculate the total volume of the torpedo room
was estimated based on the figures contained in reference [16].
2.4.2 Other Volumes

An estimate of the remaining volume between the pressure hull and the hull envelope is necessary in order to calculate the total volume of the hull envelope. The outboard volume accounts for all the items outside the pressure hull, which will not flood with water when the vessel submerges. Some of these items are the high pressure air bottles, the metal hydride tanks of hydrogen for the case of AIP submarine with PEMFC, and the structural members. A standard estimate of the outboard volume for the typical submarines is 6.2% of the pressure hull volume [3]. In the case of PEMFC with metal hydride storage of hydrogen the volume of the metal hydride tanks is added to the outboard volume. The sum of the pressure hull volume and the outboard volume is termed everbuoyant volume. The free flood volume is the volume inside the envelope which floods with water when the vessel submerges and it is estimated as a percentage of the envelope volume [3].

The remaining volume is allocated to main ballast tanks, which corresponds to the reserve of buoyancy of the submarine. The volume of the ballast tanks is based on a percentage of the everbuoyant volume, nominally 10 – 15% [4]. The submerged volume is the sum of the main ballast tanks volume and the everbuoyant volume.

\[ V_{\text{outboard}} = \text{Const} \cdot V_{PH} \]

\[ V_{\text{everbuoyant}} = V_{\text{outboard}} + V_{PH} \]

\[ V_{\text{free\_flood}} = \text{Const} \cdot V_{\text{envelope}} \]

\[ V_{\text{MBT}} = \text{Const} \cdot V_{\text{everbuoyant}} \]

\[ V_{\text{submerged}} = V_{\text{everbuoyant}} + V_{\text{MBT}} \]

\[ V_{\text{envelope}} = V_{\text{submerged}} + V_{\text{free\_flood}} \]
2.5 Weight Estimates

The weights of a submarine are similar to the weights of a surface ship, with the exception that loads and lead ballast are accounted separately. The sum of the weights of the weight groups of a submarine is the condition A-1. The weight groups are:

Group 1: Hull Structure

Group 2: Propulsion Plant

Group 3: Electric Plant

Group 4: Command and Surveillance

Group 5: Auxiliaries

Group 6: Outfit and Furnishings

Group 7: Weapon Systems

The submerged stability of the submarine requires that the center of gravity is below the center of buoyancy, and they are at the same vertical plane. A typical value for the vertical separation of the center of gravity and center of buoyancy is 1ft. In order to achieve this separation usually it is necessary to place a significant amount of stability lead close to the keel of the submarine. Also an additional amount of ballast is required to account for future growth in the submarine weight, and for inaccuracies in the weight estimates. This additional lead is called margin lead. Since the weight growth is most likely to happen at the forward sections of the submarine, we place the margin lead forward to the center of buoyancy. In the mathematical model the total lead of the submarine is assumed to be a percentage of the surface displacement. The value used is 8.54% and it is varied to achieve the weight and buoyancy balance of the submarine. Normally the total lead is greater than 5% of the surface displacement [4].

The sum of the weights of condition A-1 and lead is condition A:
\[ Condition \, A = Condition \, A-I + Lead \]

The variable loads, such as stores, weapons, etc. are accounted as a separate item. When added to condition A the result is the Normal Surface Condition (NSC). In a balanced design the NSC equals the everbuoyant volume.

\[ NSC = Condition \, A + Variable \, Loads \]

When the submarine submerges the Main Ballast Tanks (MBT) are vented and allowed to fill with sea water. The sum of the NSC and the MBT is the submerged displacement. The volume of the MBT is divided aft and fwd in a way that keeps the center of gravity and center of buoyancy of the submarine at the same vertical plane, in both the submerged and the surfaced condition.

\[ Submerged \, Displacement = NSC + MBT \]

When the submerged displacement is added to the free flood we get the envelope displacement of the submarine:

\[ Envelope \, Displacement = Submerged \, Displacement + Free \, Flood \, Displacement \]

2.5.1 Weight Group 1

Weight group 1 represents the weight of the pressure hull, as well as the scantlings necessary to provide the required hull stiffness. Therefore, the structural weight should be proportional to the submarines size, and to the diving depth. In this model it is calculated as a fraction of NSC. The fraction is given by the following formula found in reference [3]:

\[ Group1_{FRACTION} = (0.0005 \cdot D_D \cdot 0.3048 + 0.15) + Const, \text{ where } D_D \text{ is the diving depth.} \]
2.5.2 Weight Groups 2 and 3

Weight groups 2 and 3 represent the weight of the propulsion and the electrical plant respectively. The total weight represents the mobility weight, which in this model is calculated as the sum of the individual components. The components accounted are: the propulsion motor, the battery, the diesel engines, the diesel fuel, and the AIP plant weight. The sum of those components is multiplied by a constant greater than one, to account for the auxiliary systems.

The weight estimate of propulsion motor, in the model, was based on a study conducted by Kirtley, found in reference [9]. The equation is presented in 2.3.2.2 paragraph of this study. The battery weight is calculated by multiplying the number of cells with the weight of each cell. The baseline battery is ABS 49C, and its characteristics were found in reference [3]. The weight of the diesel engines is calculated by multiplying the specific weight (lb/kW), calculated as average of surveyed diesel engines, by the power of the engines. Two factors are applied, found in reference [3], to account for the electric generator weight and the enclosure module weight.

The AIP plant weight is calculated based on values found in reference [14], which are presented in Table 4. The weight of the AIP consumables is accounted in the variable load weights.

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>FC Metal Hydride</th>
<th>FC Methanol Reformer</th>
<th>CCD</th>
<th>Stirling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant Weight (kg/kW)</td>
<td>19</td>
<td>29</td>
<td>34</td>
<td>30</td>
</tr>
</tbody>
</table>

2.5.3 Weight Group 4

This weight group represents the Command and Surveillance weight and it is calculated based on the following formula, presented in reference [7]:

\[ W_4 = 0.00836 \text{ Volume}_{CIC} \]
The parameter $\text{Volume}_{\text{CIC}}$ represents the command and surveillance volume and it is a user input.

2.5.4 Weight Groups 5 and 6

The weight of groups 5 and 6 represent the auxiliary systems and the outfitting furnishing. Their weight is calculated as a percentage (4.1%) of the NSC.

2.5.5 Weight Group 7

The weight of the weapons is based on the number of torpedo tubes, and the number of torpedo reloads. The formula used for the calculation of weight in Ltons [7] is:

$$W_7 = 0.002 \text{Volume}_{\text{Torpedo Reloads}} + 6 \text{Number}_{\text{Torpedo Tubes}} + 5$$

2.5.6 Variable Weight

The variable weight is consisted of the variable load items and the variable ballast. Variable load items are items of the ship that can vary during the patrol, and variable ballast is the amount of the water in the variable ballast (trim and compensating) tanks. A three tank system [5] is used in the model. The forward and after trim tanks, are isolated from the sea, and are used only for longitudinal balance adjustments. The compensating tank is located close to the center of gravity and it is used to adjust the weight of the submarine.

The components of the variable weight are: the fresh water weight, the lube oil weight, the AIP oxidant weight, and the AIP fuel weight. The burnt diesel fuel is compensated by the seawater, and therefore it is accounted in the mobility weight. The impact of the integration on the weight of the submarine for each of the used AIP options is shown in Table 5.
2.6 Stability and Ballast

Since the modern submarines are optimized for their submerged performance, we can expect them to have poor surfaced stability performance. The waterplane area is insufficient to cause the metacenter to be very high. The position of the metacenter will depend on the geometry of the outer envelope. Although the center of gravity might effectively rise from submerged to surfaced condition, it usually settles down to the location of the submerged center of gravity.

Table 5: Weight Effect of Integration [14]

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>FC Metal Hydride</th>
<th>FC Methanol Reformer</th>
<th>CCD</th>
<th>Stirling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Consumption (l/kWh)</td>
<td>3.49</td>
<td>0.41</td>
<td>0.227</td>
<td>0.239</td>
</tr>
<tr>
<td>Fuel Storage (l/kWh)</td>
<td>0.874</td>
<td>0.074</td>
<td>0.034</td>
<td>0.039</td>
</tr>
<tr>
<td>Oxygen Consumption (l/kWh)</td>
<td>0.44</td>
<td>0.648</td>
<td>0.84</td>
<td>1.022</td>
</tr>
<tr>
<td>Oxygen Storage (l/kWh)</td>
<td>0.165</td>
<td>0.225</td>
<td>0.317</td>
<td>0.338</td>
</tr>
<tr>
<td>Ar/N2/He Consumption (l/kWh)</td>
<td>-</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Ar/N2/He Storage (l/kWh)</td>
<td>-</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Total (l/kWh)</td>
<td>4.969</td>
<td>1.368</td>
<td>1.5</td>
<td>1.703</td>
</tr>
</tbody>
</table>

Submerged stability requires that the center of gravity should be below the center of buoyancy. The magnitude of their distance determines the restoring moment when the submarine experiences an angle of heel, or an angle of pitch.

The vertical location of the lead ballast’s center of gravity is determined, so that the vertical distance between the center of gravity and the center of buoyancy of the submarine (BG) is 1 ft.

The lead ballast is divided into two categories for the purpose of this study, the stability lead (S) and the margin lead (M). Stability lead is required to preserve adequate transverse and longitudinal stability and its vertical center of gravity is estimated to be 2 ft above the keel. Margin lead is carried to preserve a margin for design and construction uncertainties, and for future growth. Margin lead is assumed to be added vertically at D/2, and longitudinally forward of amidships, since typically weights are added forward over the ships service life. However,
having the longitudinal center of margin lead too far from the center of buoyancy should be avoided.

The lead solution in the model is determined by simultaneous equations, which calculate the amount of lead of each type [6]:

**Weight equation:** \( S + M = \text{Lead} \)

**Vertical moment equation:** \( 2 \cdot S + \left( \frac{D}{2} \right) \cdot M = (\text{Lead}) \cdot (VCG_{LEAD}) \)

### 2.7 Equilibrium Polygon

In the submerged condition the submarine must have neutral buoyancy, and level trim. The equilibrium polygon is a design tool used to ensure that the submarine can maintain neutral buoyancy and level trim in all the operational conditions. Any loading condition must be able to be compensated by the submarine's trim and compensating system.

The polygon is a diagram of weight versus moment, as shown in Figure 11. The boundaries of the diagram are determined by plotting the weight and the moment progress as the forward trim tank is filled, then the compensating tank is filled, then the aft trim tank is filled, and then the tanks are emptied in the same order.

If some extreme operating conditions can be plotted inside the polygon, then the submarine will be able to dive safely for any loading condition, and for every water density. The limiting conditions used in this study are the following:

1. **Heavy No. 1:** At the end of a short, fast patrol with no ammunition or torpedoes expended but all the fuel consumed. The submarine is diving in light density water. We have to note that as the fuel is burnt it is replaced by seawater, which is heavier.
2. Heavy No. 2: Same as the previous but only half of the fuel is burnt.

3. Heavy Forward: At the end of a moderate patrol with the torpedo reloads moved in the torpedo tubes. All the fuel oil is spent from the fwd fuel tanks.

4. Heavy Aft: At the end of a moderate patrol with the torpedoes in the torpedo tubes expended and the fuel consumed only from the aft fuel tanks.

5. Light No. 1: At the end of a very short patrol with all the weapons expended but no fuel oil consumed. 50% of the fresh water and 25% of the stores and the lube oil is consumed. The submarine is diving in heavy density water.

6. Light No. 2: Same as the Light No. 1, but all the stores, and 50% of the lube oil are consumed.

7. Condition N: Is the condition in which the submarine leaves the port carrying all the stores and liquids for a full mission.

Figure 11: Equilibrium Polygon
If any of the points falls outside the enclosure of the polygon, the submarine cannot be properly ballasted with the use of the trim and compensating system. Consequently, adjustments of the weight and the location of the fixed ballast are needed.

2.8 Performance Assessment

The first step of the design process is the definition of the requirements. The “owner” specifies a range of acceptable values, from a “goal” or optimum value for that characteristic, to a “threshold” or minimum acceptable value. A ship that does not at least meet the “threshold” values specified by the owner is not considered an acceptable design.

In order to compare all the acceptable designs, a performance assessment module was added to the mathematical model. The assessment module had two level parameters presented in the following Table 6.

<table>
<thead>
<tr>
<th>Level I</th>
<th>Level II</th>
<th>Goal</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobility</td>
<td>Max. Submerged Speed (knt)</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Days of Stores</td>
<td>30</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>Time at Max. Speed (hr)</td>
<td>0.5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>AIP Balance Speed (knt)</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Diving Depth (ft)</td>
<td>600</td>
<td>1,600</td>
</tr>
<tr>
<td>Endurance</td>
<td>AIP Endurance at Mission Speed (days)</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>Maximum Submerged Range (nm)</td>
<td>2,000</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td>Maximum Surfaced Range (nm)</td>
<td>4,000</td>
<td>14,000</td>
</tr>
<tr>
<td>Mission</td>
<td>Number of Torpedoes</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Capability</td>
<td>Total Number of Weapons</td>
<td>10</td>
<td>25</td>
</tr>
</tbody>
</table>

All platforms are constrained to meet the threshold level as a minimum requirement. Then, using a linear scale, the performance of the platform with respect to a specific level II system parameter is scored between 0 and 1. Attaining the threshold gives a 0 to the platform, while attaining the goal gives a 1. No additional points are assigned if the platform exceeds the goal level of performance.

The numerical output of the performance module is the mission measure of effectiveness.
(MOE). The levels of performance are compared to the level II goals and thresholds, and a score between 0 and 1 is assigned to each parameter. The scores for each level II parameter are combined and multiplied by the weights of the associated level I parameters. The mission MOE is then the sum of the level I scores. The performance module also contains calculations of the maximum submerged range, the maximum surfaced range, the speeds of advance (SOA), and the indiscretion ratios (IR).
3 Forecasting the System Level Impact of New Technologies

The impact of technology will be assessed through the use of technology k-factors. These factors are introduced into the mathematical synthesis model to modify technical parameters of the design. The modification will result in changes of the technical metrics to simulate improvement or degradation associated with the new technology. The parametric mapping will be done with the use of Response Surface Equations (RSE) based on regression analysis using the results from the synthesis model. With the use of Response Surface Method (RSM), an n-dimensional surface is developed, using a group of techniques in the empirical study of relationships between one or more measured responses (the output variables) and a number of factors (the input variables). The surface represents all the feasible balanced designs.

In order to develop the RSE a finite number of point designs must be developed. The combinations of k-factors are determined with the use of the Design of Experiments (DOE) method, by which the inputs of a process are varied in a way that will allow the designer to better understand the process and determine how the inputs can affect the outputs. The DOE provides a combination of the k-factors that will be statistically efficient in generating the required data for the development of response surface equations.

3.1 Overview of the Methodology

A flowchart of the methodology is presented in Figure 12. The first step is the definition of target requirements that the future concept has to fulfill. In order to develop the designs and show the effect of the new technologies on the system-level of the submarine, a baseline submarine has to be selected. The baseline submarine will be the base for the variation studies that will follow.

Following the selection of the baseline submarine, the technology areas that will be examined need to be selected. The technology areas are translated into k-factors that are
introduced into the mathematical synthesis model. For the purpose of this study, a submarine synthesis model was developed, which was presented in the previous chapter.

With the k-factors being introduced into the synthesis model, a finite number of point designs are developed. The k-factors are varied according to the DOE table that is generated with the use of the statistical software package “JMP” by SAS Institute Inc.. The k-factors that are statistically significant to the response are identified with the use of the Pareto plots and the prediction profiler, which are generated by JMP. The process is called a screening experiment, and identifies the k-factors that will be used for the development of the response surface equations.

After the selection of the k-factors that are statistically significant to the response, new point designs are generated. The new designs are utilized in a multidimensional curve fit, for the generation of the response surface equations that will be used for the tradeoff studies.

![Figure 12: Flow Chart of Methodology](image-url)
3.2 Definition of Target Requirements – Baseline Selection

For this study, an incremental increase in capability, compared to the capability of current designs, was chosen for the future submarine. The target requirements are summarized in the following Table 7:

<table>
<thead>
<tr>
<th>Target Requirements</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AIP Endurance</td>
<td>17 Days</td>
</tr>
<tr>
<td>Balance Speed</td>
<td>5.5 knots</td>
</tr>
<tr>
<td>IR @ 8 knots (with AIP)</td>
<td>8%</td>
</tr>
<tr>
<td>Diving Depth</td>
<td>950 ft</td>
</tr>
<tr>
<td>OMOE</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Given the target requirements, the next step in applying the method is to model a baseline submarine to serve as a departure point for the variation studies. The baseline submarine was modeled with the use of the mathematical synthesis model developed for this study; therefore, the performance of the submarine is based on estimates and is not intended to accurately model any actual existing design. The comparison of the baseline performance and the target requirements is presented in Table 8:

<table>
<thead>
<tr>
<th>Baseline Performance and Target Requirements</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AIP Endurance</td>
<td>14 Days</td>
</tr>
<tr>
<td>Balance Speed</td>
<td>4 knots</td>
</tr>
<tr>
<td>IR @ 8 knots (with AIP)</td>
<td>10%</td>
</tr>
<tr>
<td>Diving Depth</td>
<td>950 ft</td>
</tr>
<tr>
<td>OMOE</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The submerged displacement of the baseline submarine is 1,480 tons, and it has a 163kW Stirling AIP system. Figure 13 shows the comparison of the underwater endurance (in hours) as a function of speed. The typical advertised submerged endurance for some of the modern AIP submarines is 12 to 14 days. The Baseline submarine has an AIP endurance of 14 days at 4 knots and the Future Concept 17 days at 5.5 knots.

Closely related to endurance is a submarine’s range. As it is shown in Figure 14 the Baseline submarine and the Future Concept have maximum underwater ranges 1,344 nm and
2,370 nm respectively. These results demonstrate that the Future Concept can stay submerged for much longer than the Baseline submarine.

![Endurance diagram](image)

**Figure 13: Endurance as a Function of Speed for Baseline and Future Concept Submarine**

Figure 15 shows that the indiscretion ratio (IR) of the Future Concept is decreased compared to the baseline submarine. Especially at speeds between 4 and 5.5 knots, the Future Concept has zero IR.

![Range diagram](image)

**Figure 14: Range as a Function of Speed for Baseline and Future Concept Submarine**
3.3 Screening Experiment

Prior to the development of the RSEs, a "screening" experiment should be performed. The screening experiment identifies which design factors are statistically significant to the response, and provides a metric showing the relative importance of each k-factor on system level outcome. The designer can examine numerous k-factors and quantitatively understand the effect that each of these factors has on the overall design.

Given a set of $p$ factors to the overall design problem, a small set of designs is developed by linearly selecting two factor values over a significant range of each factor's value. The result is a set of $n$ designs determined as the following equation:

$$n = 2^p$$

After these designs are developed, the designer can use statistical techniques to determine the individual and interactive effects each factor has on the overall design [8]. Thus the designer can determine a smaller set $m$ of the $p$ factors that have the greatest impact on the design.
For this study nine k-factors were used to perform the screening experiment. The range of the k-factors reflects both benefits and penalties to the various subsystems in order to simulate the consequences of the use of new technologies. The selected k-factors for the screening experiment and their ranges are the following:

1. \( k_{\text{bat_w}} \) (Range: 60% to 110%): Used to modify the weight of the baseline storage battery (Lead-Acid battery, described in the paragraph 2.3.2.3), in order to model future batteries. This k-factor is multiplied with the total weight of the baseline battery.

2. \( k_{\text{bat_v}} \) (Range: 60%-110%): Used to modify the volume of the baseline storage battery. This k-factor is multiplied with the total volume of the baseline battery.

3. \( k_{\text{struct_w}} \) (Range: 95%-100%): Used to modify the weight of the structures, in order to model new structural materials. This k-factor is multiplied with the total structural weight.

4. \( k_{\text{aip_w}} \) (lb/kW) (Range: 50%-113%): Used to modify the weight per kW of the AIP plant, in order to model AIP systems different than the baseline. The baseline AIP system is a 163 kW Stirling plant. This k-factor is multiplied with the weight per kW of the baseline plant.

5. \( k_{\text{aip_v}} \) (ft\(^3\)/kW) (Range: 50%-138%): Used to modify the volume per kW of the AIP plant. It is multiplied with the volume per kW of the baseline plant.

6. \( k_{\text{aip_fc_w}} \) (lb/kWhr) (Range: 50%-140%): Used to modify the weight impact of AIP fuel consumption on the submarine, and it is multiplied with the weight impact of AIP fuel consumption of the baseline submarine.

7. \( k_{\text{aip_fc_v}} \) (ft\(^3\)/kWhr) (Range: 50%-140%): Used to modify the volume impact of AIP fuel consumption on the submarine, and it is multiplied with the volume impact of AIP fuel consumption of the baseline submarine.
8. $k_{aip\_oc\_w}$ (lb/kWhr) (Range: 50%-110%): Used to modify the weight impact of AIP oxidant consumption on the submarine, and it is multiplied with the weight impact of AIP oxidant consumption of the baseline submarine.

9. $k_{aip\_oc\_v}$ (ft³/kWhr) (Range: 50%-110%): Used to modify the volume impact of AIP oxidant consumption on the submarine, and it is multiplied with the volume impact of AIP oxidant consumption of the baseline submarine.

The ranges of the k-factors were selected in order to model benefits and penalties of new technologies. In the real case, the limits of the k-factors must be selected based on projections of technical feasibility and compatibility of known technologies.

![Figure 16: Prediction Profiler](image)

For the purpose of this study, the dimensions of the submarine were kept constant and the variation of sizes and weights of the different components, caused by the variation of k-factors, was reflected by improving or degrading the performance of the submarine.
The statistical software package “JMP” was used to analyze the results of the experiment. A prediction profiler was created to show the effect of each k-factor on the performance metrics as shown in Figure 16.

The magnitude and the direction of the slopes show the influence of the k-factors on the responses. For example increasing the k_bat_w decreases the diving depth that a balanced submarine can achieve. This is expected, since a heavier battery will require reduction of the structural weight in order to achieve the buoyancy and weight balance. Hence the structural design cannot be as robust as before and the diving depth that the submarine will be able to achieve will be decreased.

If a parameter does not contribute significantly to the response at the point selected by the analyst or designer, the slope is zero. More accurate results about the contribution of each factor to the final response can be found by investigating Pareto Plots, which illustrate the absolute values of the scaled estimates, showing their composition relative to the sum of absolute values [28].

The responses used to identify the k-factors that have the greater influence on the design are:

1. The OMOE, since it reflects the overall performance of the submarine, and

2. The Balance Speed, since it affects the underwater range and the indiscretion ratio of the submarine.

The Pareto plot of the OMOE is presented in Figure 17. The bars indicate the contribution of the k-factor to the overall change in the response metric of interest, and the line indicates the cumulative effect of the k-factor impacts. It is clear that 80% of the variability is due to the factors in the box. Therefore the screening experiment tells us that the k-factors that affect OMOE the most are: k_bat_w, k_bat_v, k_aip_oc_w, and k_aip_fc_w. Next, the Pareto plot of the Balance Speed is presented in Figure 18. It is also clear here that 80% of the
variability is due to: k\_bat\_v, k\_bat\_w, k\_aip\_oc\_w, and k\_aip\_fe\_v.

![Figure 17: OMOE Pareto Plot](image)

Therefore, the $m$ k-factors ($m=5$ in this example) that are most important, and will be used for the response surface study are: k\_bat\_v, k\_bat\_w, k\_aip\_oc\_w, k\_aip\_fe\_v, and k\_aip\_fe\_w.

![Figure 18: Balance Speed Pareto Plot](image)

### 3.4 Response Surface Methods

Response Surface Methods concentrate on the $m$ factors identified by the screening experiment that have the greatest impact on the overall ship design. To develop the Response Surface Equations, similar to the screening experiment, the values of the $m$ factors are linearly
varied; however, at least three values of each are generally used.

There are several existing templates for choosing the point designs that will be used for the development of the Response Surfaces. Two of them are Box-Behnken and Central Composite. Both of these methods are tailored towards creating quadratic response surfaces. Figure 19 shows the location of points in each design space for a three-factor design. The boxes represent the design space that it is believed an optimal solution lies in. Since the Box-Behnken does not have these extreme points, the surfaces will be probably less accurate in the corner regions. It is a very useful method, however, when the extreme points are not feasible [29]. The Central Composite Design method is the most common response surface design, and is accurate throughout the entire range of all factors due to the extreme points at the vertices.

After the designs are developed and the appropriate model is populated, JMP is used to develop the response surfaces. The “response surface” is essentially a multi-dimensional surface fit to the model by JMP. The response surface is defined by the following equation:

\[ y = b_0 + \sum_{i=1}^{k} b_i k_i + \sum_{i=1}^{k} b_{ii} k_i^2 + \sum_{i=1}^{k} \sum_{j=i+1}^{k} b_{ij} k_i k_j + \epsilon \]

Figure 19: Box-Behnken and Central Composite Design Spaces [29]
Where the \( b_0, b_{ii}, b_{iii} \) terms represent constants of regression, \( \varepsilon \) represents error, and the summations represent linear, quadratic, and interaction terms respectively [8]. The \( k_i \) and \( k_j \) are the \( k \)-factors (inputs) and \( y \) is the response (output). This equation defines the response surface and, if it is determined to have a statistically accurate fit, represents all feasible concept designs.

Table 9: Factors of Point Designs

<table>
<thead>
<tr>
<th>Variant</th>
<th>Pattern</th>
<th>( k ) bat w</th>
<th>( k ) bat v</th>
<th>( k ) aip fe w</th>
<th>( k ) aip fe w</th>
<th>( k ) aip oc w</th>
</tr>
</thead>
<tbody>
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<td>1.1</td>
<td>0.6</td>
<td>0.323</td>
<td>0.01832</td>
<td>1.1266</td>
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<td>0.323</td>
<td>0.00653</td>
<td>1.1266</td>
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<td>0.904</td>
<td>0.00653</td>
<td>2.4784</td>
</tr>
<tr>
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<td>0.6</td>
<td>0.85</td>
<td>0.6135</td>
<td>0.012425</td>
<td>1.8025</td>
</tr>
<tr>
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<td>1.8025</td>
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<tr>
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<td>1.1</td>
<td>0.323</td>
<td>0.00653</td>
<td>2.4784</td>
</tr>
<tr>
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<td>0.00653</td>
<td>2.4784</td>
</tr>
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<td>0.00653</td>
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</tr>
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</tr>
<tr>
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<td>0.85</td>
<td>0.6135</td>
<td>0.01832</td>
<td>1.8025</td>
</tr>
</tbody>
</table>

A Central Composite design of experiments was performed using the five presented \( k \)-factors. Twenty-seven balanced designs were required according to the model and the responses calculated by the submarine synthesis model were used by JMP to generate the Response Surface Equations. The required variants are listed in Table 9, and the recorded responses in Table 10.
This section outlined the basic concepts of RSM required to understand the studies presented in this report. There are several references available for a more detailed understanding of RSM. Reference [8] provides an overview and application of RSM to submarine concept design and [30] is an excellent text on the underlying concepts behind RSM.

Table 10: Responses of Point Designs

<table>
<thead>
<tr>
<th>Variant</th>
<th>Pattern</th>
<th>AIP Endurance (Days)</th>
<th>IR @ 8 knots</th>
<th>OMOE</th>
<th>Balance Speed (knots)</th>
<th>Diving Depth (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+--+-</td>
<td>18</td>
<td>0.07890</td>
<td>0.476</td>
<td>5.5</td>
<td>820</td>
</tr>
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<td>2</td>
<td>+++-</td>
<td>13.5</td>
<td>0.10612</td>
<td>0.443</td>
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<td>--+--</td>
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<td>1400</td>
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<td>1400</td>
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<td>950</td>
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<td>15</td>
<td>0.09444</td>
<td>0.463</td>
<td>5</td>
<td>950</td>
</tr>
</tbody>
</table>

The pattern column indicates which value is used for each factor, with “+” or “A” representing the upper limit, “-” or “a” representing the lower limit, and “0” representing the midpoint.
At this point, JMP's graphical interfaces can be used to assess all feasible design variants. Thus, with the addition of statistical modeling to the concept exploration process, a finite number of point designs can be used to examine an infinite number of k-factor variations.

3.5 Test of Curve Fit

In addition to providing the equations for each response surface, JMP also provides statistical information about the curve fit.

The Actual by Predicted Plot in Figure 20 shows how the values predicted by the response surface equations compare to the actual OMOE values. If the curve fit is perfect, each design point would fall exactly on the line with a slope of one. The dashed lines on the plot represent the 95% confidence interval, which in this case is very close to the line representing a perfect model. Therefore, this plot shows that the OMOE response surface equation is very accurate, with an R-squared value of 1.00. Similar are the plots of the other four responses, representing very accurate models.

Another very important result of JMP is the analysis of variance, shown in Table 11, which provides information about the fit of the response surface:
Table 11: OMOE Analysis of Variance Table

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Ratio</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
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<td>Model</td>
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<td>0.011015</td>
<td>0.000551</td>
<td>207.2725</td>
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<tr>
<td>Error</td>
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<td>0.000003</td>
<td></td>
<td>Prob&gt;F</td>
</tr>
<tr>
<td>C. Total</td>
<td>26</td>
<td>0.00001594</td>
<td>0.000003</td>
<td>0.0001</td>
<td></td>
</tr>
</tbody>
</table>

The sum of squares and mean square of the model, quantify the expected error from the curve fit of data. The unexplained error is quantified in the error sum of squares and mean square.

The F Ratio represents the ratio of the mean square of the model and the mean square of the error. "Prob > F" represents the probability that the F Ratio would be greater due to parameters in the synthesis model other than the factors. A very low "Prob > F" (on the order of 0.001) indicates that the main source of error is from the curve fit of the factors. A high "Prob > F" (on the order of 0.05 or greater), on the other hand, indicates that there is a great deal of error coming from other sources. This means that the difference in OMOE for two submarines designed using the same k-factor combination would be greater than the difference between this design and a submarine designed using different k-factor combination. In cases like this, the designer must try to improve the fit of the model by reconsidering the choice of factors, or determining other potential sources of error and holding them constant throughout all of the designs.

In the case of the OMOE of the submarine, the "Prob > F" combined with the R squared value of 1.00 indicates an excellent model [28]. Similar results are found for the other four responses.

3.6 Design Space Visualization & Tradeoff Studies

Given the RSEs, trade off studies can be conducted easily, and visualized in the interactive environment of JMP. Figure 21 presents a set of response surfaces as it is presented in the program.
Along this surface the designer can find the optimal solution and effectively perform trade-off studies. Two of the ways that a designer/decision maker can take advantage of this methodology are the following:

First Approach: Defining Technology Areas

Using the methodology, the decision maker will know the technology areas that have significant effect on the capabilities of the vessel, and he will be able to select the combination of k-factors that will give the submarine a target performance. Then the decision maker will know the technology areas that require further investigation, and will allocate the R&D resources towards that direction. New technologies that give the k-factor projections might need to be identified.

Second Approach: Selecting Technologies

The decision maker can predict the impact of a known technology on the performance of the submarine, or he can compare two candidate technologies, or select the optimum mix of technologies. The only thing the decision maker will have to do is set the k-factors at the levels that reflect the technologies of interest, and the response surface equations will predict the
responses. It is important to include positive and negative effects of the k-factors during the development of the response surfaces, since technologies can have positive effects in one metric and negative ones in another.

3.6.1 Defining Technology Areas

To facilitate the first approach, the decision maker can use the interactive environment of JMP in several ways. With the use of the prediction profiler, the user can understand the impact of the k-factors on the selected responses. Figure 22 shows how the variation of the selected k-factors affects the responses. It is clear that the variation of battery weight and volume affects all the responses significantly. This is expected because the battery weight and volume are significant fractions of the displacement and the pressure hull volume, of conventional submarines.

![Figure 22: Prediction Profiler](image)

The designer can also introduce desirability functions in order to select the optimum solution. For this study, as it was defined by the target requirements, it is desired to maximize the
AIP endurance, Balance Speed, and Diving Depth, while minimizing the Indiscretion Ratio. This is reflected in the far right column of Figure 23.

The desirability functions of the metrics that have to be maximized have positive slopes, and the function of the metric that needs to be minimized has negative slope. With the use of the interactive environment of JMP, the designer can choose to maximize desirability and the program will calculate the optimum combination of k-factors. It is important to note that the OMOE was not included in the calculation of maximum desirability in order to avoid double counting of metrics.

![Figure 23: Prediction Profiler with Desirability Functions](image)

The generated response surface equations allow the designer to visualize the entire design space and determine what regions are feasible based on different sets of constraints. The contour plot in Figure 24 shows the contours of the AIP endurance, OMOE, IR at 8 knots, Balance Speed, and Diving Depth in the k_bat_w- k_bat_v plane.
In this figure, the other three k-factors are fixed, and the AIP endurance curve represents all the combinations of $k_{bat\_w}$ and $k_{bat\_v}$ that yield an AIP endurance of 17 days. Since the target value of AIP endurance is 17 days, we can conclude that the feasible design space is the part of the plot that is not shaded.

The second target requirement stated that the indiscretion ratio at 8 knots should be less than 8%. Figure 25 shows the feasible design space, in order to meet this requirement.

The third target requirement is that the OMOE of the submarine should be greater than 0.49. Figure 26 represents the feasible design space in order to meet that constraint.
The fourth target requirement is that the AIP balance speed should be greater than 5.5 knots. Figure 27 shows the feasible design space, for the lower limit of 5.5 knots.

Finally, Figure 28 shows the feasible design space in order to meet the constraint of a diving depth greater than 950 ft.
Superimposing the previous five figures creates Figure 29, which represents the feasible design space in order to meet all the target requirements. Now, the decision maker is able to select the combination of k-factors that will allow the design to meet the requirements.

The following Figure 30 shows the design space in the k_aip_fc_w, k_aip_fc_v plane with the same constraints imposed to the model.
After the k-factor combinations are determined the decision maker can allocate the available R&D resources in order to develop the technology areas associated with the selected k-factors. New targets can be set and the scientists and contractors can work towards a direction that will improve the final product.

In addition, the decision maker has the ability to explore how the design space will change with the variation of the target requirements. Figure 31 presents the contours of 14, 15, 16, 17, and 18 days of AIP endurance. Using this Figure the decision maker can determine the required combinations of k-factors for different target requirements, and will be able to decide if...
the goals of the future are set too high and the requirements need to be relaxed. The decision maker will be able to tradeoff capability and risk associated with the candidate technologies.

3.6.2 Selecting Technologies

Given the Response Surface Equations, the decision maker can predict the impact that any technology has on the performance of the submarine. The only thing the user will have to do is set the k-factors at the levels that reflect the technologies of interest. Then the Response Surface Equations will predict the responses, and the decision maker will be able to know the impacts of the new technologies on the system level of the submarine, or compare candidate technologies.

For example, in order to illustrate the methodology, we can assume that a decision maker has to select between two candidate technologies. He can be either working for a shipyard, or for the acquisition community. Limitations in the R&D resources allow him to fund only one of the two technologies, and obviously he should choose the technology that will result in a submarine with the best performance.

Technology 1 is a new battery concept that will have 70% of the weight and 75% of the volume of a Lead Acid battery with the same capacity. Technology 2 is a new Air Independent Propulsion system, with reduced weight and volume impact of fuel consumption, and reduced weight impact of oxidant consumption. The values of those three characteristics are 50% of the values that represent the baseline Stirling AIP System. The k-factor levels that should be used in the Response Surface Equations in order to model the technologies are presented in Table 12.

<table>
<thead>
<tr>
<th>Area of Improvement</th>
<th>Technology 1</th>
<th>Technology 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery Weight</td>
<td>70%</td>
<td>100%</td>
</tr>
<tr>
<td>Battery Volume</td>
<td>75%</td>
<td>100%</td>
</tr>
<tr>
<td>Weight Effect of AIP Fuel Consumption</td>
<td>100%</td>
<td>50%</td>
</tr>
<tr>
<td>Volume Effect of AIP Fuel Consumption</td>
<td>100%</td>
<td>50%</td>
</tr>
<tr>
<td>Weight Effect of AIP Oxygen Consumption</td>
<td>100%</td>
<td>50%</td>
</tr>
</tbody>
</table>
With the use of the Response Surface Equations, and without spending the time to balance actual submarine designs, the technology that can be more advantageous for the final design, in terms of capabilities, can be determined. The results derived from the Response Surface Equations, using the interactive environment of JMP are presented in Table 13.

Table 13: Performance of the Submarine with the Application of New Technologies

<table>
<thead>
<tr>
<th>Responses</th>
<th>Technology 1</th>
<th>Technology 2</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIP Endurance (Days)</td>
<td>16.9</td>
<td>14.6</td>
<td>14</td>
</tr>
<tr>
<td>IR @ 8 knots</td>
<td>8.79%</td>
<td>10.07%</td>
<td>10.6%</td>
</tr>
<tr>
<td>OMOE</td>
<td>0.481</td>
<td>0.457</td>
<td>0.446</td>
</tr>
<tr>
<td>Balance Speed (knots)</td>
<td>5.25</td>
<td>4.5</td>
<td>4</td>
</tr>
<tr>
<td>Diving Depth (ft)</td>
<td>1031</td>
<td>985</td>
<td>950</td>
</tr>
</tbody>
</table>

Table 13 shows that Technology 1, provides improved capabilities, compared to Technology 2. Although the numbers of Technology 2 seem to be more promising, the system level impact is what the decision makers have to take into account when they make the decisions. In all cases, the true impact of the technology could not be completely assessed without consideration of its impacts on the entire system. This assessment is needed to be done to the whole system level, and it should integrate the new technology with the rest of the design. However, there are also other aspects that need to be considered in the final selection.

Uncertainty associated with new technologies, can be a significant issue, and can change the results of the comparison. Use of alternative techniques, for example Monte Carlo simulations, will give a better understanding of the uncertainty and the associated risk.
4 Uncertainty Associated with New Technologies

Producers must respond to the challenges of new requirements, and competition, by optimizing their R&D environment, creating value through strategic decisions. Forecasting and planning are key components in the prioritization of the long term R&D. Forecasting exercises the systematic identification of the areas of research that are more likely to yield the greatest economic benefits. Whether the R&D investment is wise depends on the commercial gain relative to the risk.

The risk of a project is associated with the uncertainty of the future. Uncertainty in the design process will significantly decrease the probability of attaining desired cost schedule and system performance goals. Therefore, it should be taken into account at the initial stages of the design process.

Developing a new piece of technology for incorporation into an existing complex product, such as a submarine, consists of three stages [24]: Stage one examines the feasibility of developing the technology; stage two is the full scale development of the R&D project; and stage three is the integration of the new technology in the product. Stage one is the point where the company needs to have flexibility and plan on various scenarios. The decision maker needs to recognize the risk areas of new technologies, and account for them in the decision process.

The decision-making methodology, presented in the previous chapters of this thesis, aids the decision maker in identifying the technology areas that can improve significantly the performance of the product. Uncertainty in terms of forecasted levels of technology affects the performance that a future concept can achieve and should be integrated in the decision process.

In addition, the presented methodology helps the decision maker in selecting between
candidate technologies. This selection will be explicit only if performance and cost uncertainty are parameters of the decision. The risk assessment presented in the following paragraphs is based on simulations and it should be used as a part of the decision process.

4.1 Impact of Uncertainty on the Design Space

Risk management can be broken into three stages: Identification; the assessment; and control or mitigation of risk [31]. The identification of the risk areas is based on the experience of the decision maker. There are several methods of assessing risk. Some representative types [31] are: Simple adjustments and sensitivity analysis, analytical methods, numerical methods, and simulation. The most widely used method is simulation, such as Monte Carlo simulation.

Simulation according to Rubinstein [34] is:

"...a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical models that describe the behavior of a business or economic system (or some component thereof) over extended periods of real time"

The presence of uncertainty in the prediction of future technologies results in an inability to predict the exact performance of the system. A system subject to uncertainty cannot be expressed in terms of a single solution, but it should be expressed as a probability distribution. Monte Carlo simulation is a random number generator that creates values for each of the uncertain variables. Values are selected within a specified range, and with a frequency which depends on the shape of the probability distribution of the variable.

The steps of a Monte Carlo simulation [33] are the following:

1. Define the probability distributions of the uncertain variables.
2. For each of the uncertain variables, randomly select a value from the distribution function.

3. Combine all the values of all the uncertain variables, and calculate the result based on the given mathematical relationships.

4. Repeat the above procedure \( n \)-times. Each cycle produces an output value, based on the given relationships.

5. Develop a frequency distribution of the output value, based on the \( n \) calculated outputs.

Usually 1,000 to 10,000 cases are necessary for a good representation of the probability distribution [32].

This task is very difficult without the use of Response Surface Equations. The Monte Carlo simulation would have had to be built around the design code, and for each of the thousand cases the designer would have had to produce a balanced design. With the use of Response Surface Methods, the Monte Carlo simulation can be built around the Response Surface Equation. Hence, the task of assessing the impact of uncertainty becomes easier, and can be performed with the use of forecasting and risk analysis programs.

The Monte Carlo simulation, for the purpose of this study, is performed with the aid of software called Crystal Ball, by Decisioneering Inc., which randomly generates numbers for the uncertain variables, based on user-defined probability distributions, and computes the probability distribution of the response. Crystal Ball extends the capabilities of a simple spreadsheet model and provides information necessary for the decision maker.

Each of the \( k \)-factors, used in the previous chapters for the development of the Response Surface Equations, was assigned a probability distribution function over the range used for the Response Surfaces. There are many choices of distributions, such as normal, triangular, beta,
lognormal etc. The type and shape of distributions used for this study were picked randomly and are not intended to represent reality. Triangular distributions were assigned to $k_{bat_w}$, $k_{bat_v}$, $k_{aip_{fc}_v}$, and $k_{aip_{oc}_w}$, and a uniform distribution was assigned to $k_{aip_{fc}}$, as it is presented in Figure 32.

![Figure 32: Probability Distributions of k-factors](image)

In a uniform distribution, all the values between the maximum and the minimum occur with the same likelihood. The triangular distribution describes a situation in which the designer knows the maximum, minimum, and most likely value of a parameter. Triangular distribution is suitable when there is a lack of detailed knowledge about the future state of technology. Even with limited knowledge someone can forecast the three necessary values. The values used, as fraction of the baseline values, are presented in Table 14.

### Table 14: Limits and most likely value of k-factors as fraction of the baseline component

<table>
<thead>
<tr>
<th>Technology Areas</th>
<th>Minimum</th>
<th>Most Likely</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery Weight</td>
<td>0.6</td>
<td>0.7</td>
<td>1.1</td>
</tr>
<tr>
<td>Battery Volume</td>
<td>0.6</td>
<td>0.65</td>
<td>1.1</td>
</tr>
<tr>
<td>Weight Effect of AIP Fuel Consumption</td>
<td>0.5</td>
<td>-</td>
<td>1.4</td>
</tr>
<tr>
<td>Volume Effect of AIP Fuel Consumption</td>
<td>0.5</td>
<td>0.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Weight Effect of AIP Oxygen Consumption</td>
<td>0.5</td>
<td>0.9</td>
<td>1.1</td>
</tr>
</tbody>
</table>
As mentioned earlier, the design is subject to the uncertainty of k-factors. The next step is to use Monte Carlo simulation to show the effect of this uncertainty on the responses (performance parameters), which were analyzed based on the cumulative and the reverse cumulative distributions obtained by running 10,000 cases. In order to perform this task without the use of the Response Surface Equations, the designer would have had to balance 10,000 submarine designs using the mathematical synthesis model. This is a very time-consuming and practically impossible task. On the other hand, by using the Response Surface Equations, a very simple Excel spreadsheet, and the software Crystal Ball, this task becomes very easy. In addition, Crystal Ball automatically produces meaningful plots, which can aid the decision maker in performing tradeoff studies.

Figure 33: Cumulative Distributions of Responses
The reverse cumulative distribution function is a plot of the frequency of a certain response used to calculate the probability of that response being greater than or equal to any given value on the x-axis. In this context, there is 0% chance of having AIP endurance greater than the upper limit and 100% chance of having an AIP Endurance greater than the lower limit. The reverse cumulative distributions of AIP Endurance, Balance Speed, and Diving Depth are presented in Figure 33. The Indiscretion Ratio response must be handled a little differently, because it is more desirable to minimize the Indiscretion Ratio, which is best understood in the context of a cumulative chart, shown in Figure 33, which displays the probability of achieving an Indiscretion Ratio lower than or equal to any given value on the x-axis.

The cumulative and reverse cumulative distributions presented in Figure 33, can aid the decision maker in predicting the certainty levels of the performance parameters of future designs, based on the forecasts of the technologies. For example, it is 60% probable that the future submarine, with the same dimensions as the baseline, will have more than 16.2 days of AIP Endurance, more than 5.1 knots of Balance Speed, a Diving Depth greater than 950 ft, and an Indiscretion ratio lower than 9%.

Table 15: Percentiles for Balance Speed Response

<table>
<thead>
<tr>
<th>Percentile</th>
<th>knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>4.00</td>
</tr>
<tr>
<td>10%</td>
<td>4.66</td>
</tr>
<tr>
<td>20%</td>
<td>4.85</td>
</tr>
<tr>
<td>30%</td>
<td>5.00</td>
</tr>
<tr>
<td>40%</td>
<td>5.12</td>
</tr>
<tr>
<td>50%</td>
<td>5.21</td>
</tr>
<tr>
<td>60%</td>
<td>5.31</td>
</tr>
<tr>
<td>70%</td>
<td>5.40</td>
</tr>
<tr>
<td>80%</td>
<td>5.50</td>
</tr>
<tr>
<td>90%</td>
<td>5.61</td>
</tr>
<tr>
<td>100%</td>
<td>5.93</td>
</tr>
</tbody>
</table>

In the previous chapter, each of the responses was represented by a multidimensional surface plot. Instead of representing the response with a single response surface, it can now be represented by a series of surfaces that indicate different probabilities of the performance metric being less than, or equal to a given value. Table 15 breaks the cumulative chart of Balance Speed
into 10% intervals and provides a series of values that describe cumulative distribution. For example, Table 15 shows that there is a 90% certainty that the Balance Speed will be lower than 5.61 knots, or 10% that it will be greater than 5.61 knots. Similar tables exist for all the responses.

Another way to show the impact of uncertainty on the design space is Figure 34, which presents the certainty levels of achieving different values of Balance Speed. For example, based on the forecasted technologies, there is a 96% certainty that the future submarine will have a balance speed of 4.5 knots, a 70% certainty that it will have a Balance Speed of 5 knots, and 19% certainty that it will have a balance speed of 5.5 knots.

Figure 34: Different Certainty Levels of the Design Space of Balance Speed

In addition to the above, Crystal Ball provides frequency distribution graphs, which show the number (frequency) of values occurring in the given interval, and fits the results of the frequency distribution to a known, standard distribution. Figure 35 then shows the frequency distribution of Balance Speed and the fitted distribution. The highest value on the probability scale is the probability for the mode (the value that occurs most frequently in the set of values) [32]. In the case of Balance Speed, the distribution that most closely reflects the nature of the response is the Beta Distribution with Alpha=29.08 and Beta=4.94.

The Beta distribution is a very flexible distribution commonly used to represent the variability over a fixed range [32]. The shape of a Beta distribution changes by varying the two
parameters: Alpha and Beta.

![Graph](image)

Figure 35: Frequency Distribution and Fitted Distribution of Balance Speed

A random variable $X$ is said to have a beta distribution, if the probability density function is given by the following conditional equation [33]:

$$f(x) = \frac{1}{b-a} \frac{\Gamma(a + \beta)}{\Gamma(a)\Gamma(\beta)} \left(\frac{x-a}{b-a}\right)^{a-1} \left(\frac{b-x}{b-a}\right)^{\beta-1} \quad \text{when } a < x < b.$$  

$$f(x) = 0 \quad \text{otherwise}.$$

In addition to the above Crystal Ball provides additional statistical information for the selected response. Table 16 lists forecast statistics of Balance Speed. The mean of the response is 5.17 knots. The median (the middle value of a set of sorted values) is 5.21, the variance (the measure of the dispersion of spread) is 0.13, and the Standard Deviation (the square root of the variance—measure of dispersion around the mean) is 0.36.

A distribution of values is said to be “skewed” if it is not symmetrical. The distribution of Balance Speed is negatively skewed, which means that the values on the right of the chart have higher frequencies, and it is moderately skewed since skewness is less than 1 and greater than -1 [32]. Kurtosis represents how steep is the peak of the distribution [32]. A fairly flat peak has a
kurtosis if two, and a peaked distribution has a kurtosis of 4. The distribution of Balance Speed has a kurtosis of 2.65, and can be considered fairly flat. The coefficient of variability provides the measurement of how much the forecast values vary relative to the mean value. Range minimum is the smallest number in the set, and the range maximum is the largest number. This range is the difference between the maximum and the minimum numbers in the set of values.

Table 16: Forecast Statistics of Balance Speed

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trials</td>
<td>10,000</td>
</tr>
<tr>
<td>Mean</td>
<td>5.17</td>
</tr>
<tr>
<td>Median</td>
<td>5.21</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.36</td>
</tr>
<tr>
<td>Variance</td>
<td>0.13</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.47</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.65</td>
</tr>
<tr>
<td>Coeff. of Variability</td>
<td>0.07</td>
</tr>
<tr>
<td>Range Minimum</td>
<td>4.00</td>
</tr>
<tr>
<td>Range Maximum</td>
<td>5.93</td>
</tr>
<tr>
<td>Range Width</td>
<td>1.93</td>
</tr>
</tbody>
</table>

As mentioned earlier, the level of the forecasted technology is uncertain. By using uncertainty analysis and Monte Carlo simulation, the decision maker can better explore the design space and understand the performance improvement that can be achieved by the future designs.

In addition, uncertainty must be taken into account for the selection between candidate technologies. The different certainty levels of performance of new technologies can affect the identification of the most suitable technology.

4.2 Impact of Uncertainty on Technology Selection

The selection between candidate technologies and the allocation of R&D resources will be optimum only if performance and cost uncertainty are parameters of the decision. Neither characteristics nor the cost impact of the technologies can be identified as single values but must reflect all the possible scenarios.


4.2.1 Performance Uncertainty

In the example presented in paragraph 3.6.2, a decision maker had to select between two candidate technologies. Limitations in the R&D resources allowed him to fund only one of the two technologies. **Technology 1** is a new battery concept that will have 70% of the weight and 75% of the volume of a Lead Acid battery with the same capacity. **Technology 2** is a new Air Independent Propulsion System, with reduced weight and volume impact of fuel consumption, and reduced weight impact of oxidant consumption. The values of those three characteristics are 50% of the values that represent the baseline Stirling AIP System. As was presented in the previous chapter, the Technology that was more advantageous for the final design was **Technology 1** which provided the performance presented in Table 13.

![Probability Distributions of k-factors](image)

Figure 36: Probability Distributions of k-factors

The uncertainty of the final design characteristics of components that will be produced based on new technologies, requires the identification of those characteristics as probability distributions and not as single values. Figure 36 shows the probability distributions of the k-factors used in the presented example. The triangular shape was again selected because very little is known about the true shape of the probability distribution except the minimum, maximum, and
The following Table 17 shows the minimum, most likely and maximum values, as a fraction of the baseline values.

Table 17: Limits and most likely value of k-factors as fraction of the baseline component

<table>
<thead>
<tr>
<th>Technology Areas</th>
<th>Minimum</th>
<th>Most Likely</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery Weight</td>
<td>0.7</td>
<td>0.95</td>
<td>1.1</td>
</tr>
<tr>
<td>Battery Volume</td>
<td>0.75</td>
<td>0.95</td>
<td>1.1</td>
</tr>
<tr>
<td>Weight Effect of AIP Fuel Cons.</td>
<td>0.5</td>
<td>0.55</td>
<td>0.7</td>
</tr>
<tr>
<td>Volume Effect of AIP Fuel Cons.</td>
<td>0.5</td>
<td>0.55</td>
<td>0.7</td>
</tr>
<tr>
<td>Weight Effect of AIP Oxygen Cons.</td>
<td>0.5</td>
<td>0.55</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The above distributions were used as inputs to the software Crystal Ball. Two Monte Carlo Simulations, one for each of the technologies, were run. In each of the simulations only the variables of the Response Surface Equation related to the new technologies were varied, and the other variables were held constant. After running 10,000 cases for each of the simulations, four overlay reverse cumulative charts of the responses of the two technologies were produced, as shown in Figure 37.

The selection of technology changes depending on the risk that the decision maker is ready to take. As is shown in the same figure, the two reverse cumulative distributions intersect. If the decision makers are risk averse they will choose Technology 2, which provides only incremental improvement, but with very low risk. A decision maker, who is willing to take some more risk, and potentially gain more advantages, will select Technology 1.

For example, if the decision makers want 80% certainty that the future submarine will have an AIP Endurance of at least 14.5 days, they will select Technology 2. If they accept a 20% certainty that the future submarine will have an AIP Endurance of at least 15 days, they will select Technology 1. Similar tradeoff studies can be performed for the other responses.

Table 18: Limits and most likely value of k-factors for Technology 1

<table>
<thead>
<tr>
<th>Technology Areas</th>
<th>Minimum</th>
<th>Most Likely</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery Weight</td>
<td>0.6</td>
<td>0.7</td>
<td>1.1</td>
</tr>
<tr>
<td>Battery Volume</td>
<td>0.6</td>
<td>0.65</td>
<td>1.1</td>
</tr>
</tbody>
</table>
If the probability distributions of the k-factors of Technology 1 change and have the shape presented in Figure 38, and the maximum, most likely and minimum values presented in
Table 18, the selection will change. Figure 39: shows the new overlay reverse cumulative distribution charts of the four responses. As is obvious, Technology 1 is more advantageous than Technology 2, since it provides improved performance with the same level of certainty.

![Probability Distributions of k-factors for Technology 1](image)

Figure 38: Probability Distributions of k-factors for Technology 1

The decision is clearer than before; it should be Technology 1. The reverse cumulative distributions presented in Figure 39 can aid the decision maker in predicting the certainty levels of the performance parameters that the selected technology can provide. However, in today’s environment of reduced funding, performance is not the only parameter that the decision maker has to take into account. Cost and the uncertainty associated with cost estimating techniques should be considered during the selection of the projects that will be funded.

4.2.2 Cost Uncertainty

Cost is always a significant decision factor in the development of complex systems. The decision maker wants to know the chance of the cost of a project exceeding a specified amount, by how much this amount might be exceeded, and the sources of cost uncertainty. While the cost of a design at the concept level is always expected to have a great deal of uncertainty, this uncertainty does not come only from the synthesis model and simply propagates through the cost model, but also from the cost model itself.

Concept exploration is the part of the design process in which the designer specifies the
main characteristics of the product. The objective of the concept design phase is to determine the size, weight, and geometric configuration within which detailed studies can take place [5]. Since the level of detail of a conceptual design is limited, there is great uncertainty associated with the weight and volume estimates. Given the way that most of the cost estimating methods are structured, the uncertainty of the weight and volume estimates propagates and alters the estimated cost of the design.

Figure 39: Overlay Reverse Cumulative Distribution Chart of the two Technologies
The cost estimates for the purpose of this study are based on a simplified weight-based cost model developed in an Excel format and is presented in Appendix 2. Inputs needed are the weights of the weight groups, and the output is the Total Lead Ship Acquisition Cost. The Cost Estimating Ratios used were found in reference [3]. Cost is divided into Labor Cost and Material Cost. Labor Cost is calculated based on historical data of the total man-hours necessary for a Long ton of each weight group and the average man-hour rate. Material Cost is estimated based on historical data of the cost per Long ton of each weight group. Material Cost of the Air Independent Propulsion system is calculated as a percentage of the material cost of Groups 2&3. A 4% Integration Factor and a 24% Assembly Factor are added to the Material Cost in order to calculate the total Direct Costs. A Profit Margin of 10% is added in order to calculate the Total Acquisition Cost of the Submarine. The base year for all the cost estimates is 1993, and they are adjusted for FY03 with a 2.25% annual inflation rate.

The uncertainty associated with the cost model comes from various sources. Cost Estimating Ratios are based on historical data and do not always reflect reality. Total man-hours required for the production of a system do not depend only on the weight of the system, but also on the accessibility of the working area and the complexity of the system. New production techniques can make the production line more efficient and reduce the total man-hours required. On the other hand, a new system can be much more difficult to manufacture, and can require more man-hours per unit weight.

The cost per Long ton of a Weight Group does not only vary depending on the inflation rate. It also depends on the accessibility of the manufacturing area, on the distance of the manufacturing area from the material resources, on the supply and demand relation in the particular period, on the political situation, etc. In addition, the man-hour rates are also potential source of cost estimating uncertainty. The man-hour rates do not depend only on the inflation rate. Changes in society, as well as demographic or political landscapes, can cause variations which can significantly change the final cost.

All the above sources of variability are not captured in the usual cost estimating models, where the Cost Estimating Ratios and the Labor Rates are determined as point estimates.
most reasonable way to capture the variability and be able to answer all the “what if” scenarios, is to present all the uncertain cost elements as probability distributions [33]. Historical data or expert’s judgment can be used for the identification of the shape and the limits of the probability distributions.

The probability distributions used for the cost estimation of the baseline submarine are presented in Figure 40. The first column contains the probability distributions of the weights of the weight groups. A 10% variability compared to the estimate given by the mathematical synthesis model was assumed for the weights of the weight groups. A triangular distribution was...
used, with limits of 110% and 90% and the most probable value was 100% of the weight forecasted by the synthesis model.

Lognormal distributions with Mean equal to one and Standard Deviation equal to 0.2 were used for the Material Costs of the various weight groups. Lognormal is a probability distribution with broad applicability in engineering, economics, and cost analysis [33]. It is used for values that can have a lower limit, but can increase without bounds. A random variable \( X \) is said to be lognormally distributed if its probability density function is given by:

\[
f(x) = \frac{1}{\sqrt{2\pi}\sigma_x x} e^{-\frac{1}{2} \left( \frac{\ln x - \mu_x}{\sigma_x} \right)^2}, \quad 0 < x < \infty, \quad \mu_x = E(\ln X), \quad \sigma_x^2 = Var(\ln X)
\]

The man-hours per Long ton of each of the weight groups are represented by triangular distributions shown in the third column of Figure 40. The minimum man-hours used is 80% of the required by the baseline model (came from reference [3]), the maximum is 110%, and the most likely value is 100%. In addition to that a Lognormal distribution with a Mean of $29.4, and a Standard Deviation of $2.94 was assumed for the Man-hour Rate.

The uncertainty associated with the complex system's work breakdown structure (SWBS) cost model presented in the previous paragraphs can be assessed very efficiently with the use of the Monte Carlo simulation, performed with the use of the software Crystal Ball. The frequency
probability distribution of the Total Acquisition Cost is presented in Figure 42, and the cumulative distribution in Figure 43.

![Figure 42: Frequency Distribution of Total Acquisition Cost](image_url)

The frequency distribution shows that the highest frequency of the calculated cost occurs at a Total Acquisition Cost of $425 million. Due to the lack of published data concerning the production cost of Air Independent Propulsion submarines, the calculated cost might not be realistic. However, it supports the presentation of the methodology.

![Figure 43: Cumulative Distribution of Total Acquisition Cost](image_url)

The cumulative distribution chart shows the likelihood of achieving the cost schedule.
For example, if the cost constraint was $425 million, there is a 60% chance that the Total Acquisition Cost will be within the limits.

4.2.2.1 Cost Uncertainty and Technology Selection

The impact of cost uncertainty in technology selection will be illustrated with the example presented in the previous paragraphs 3.6.2 and 4.2.1. The weights of the baseline submarine are used as inputs to the cost model. For this example all the cost variables are held constant. An additional element was added to the cost model. The new element is the Research and Development cost of the new technologies.

It is assumed that the two technologies have different Research and Development cost distributions, which are presented in Figure 44. The Research and Development cost of Technology 1 is a fraction of the material cost of Groups 2&3, and has a triangular distribution with minimum, maximum, and most likely values: 5%, 8%, and 10% respectively. The Research and Development cost of Technology 2 has also a triangular distribution and minimum, maximum, and most likely values: 4%, 6%, and 8% respectively.

![Figure 44: Probability Distributions of R&D Cost](image)

The above distributions were used as inputs to the software Crystal Ball. Two Monte Carlo Simulations, one for each of the technologies, were run. In each of the simulations only the Research and Development cost was varied. After running 10,000 cases for each of the simulations, one overlay cumulative chart was produced, as shown in Figure 45.
Figure 45: Overlay Cumulative Distributions of Acquisition Costs

Figure 45 shows that if for example the cost constraint is assumed to be $425 million, Technology 2 is more likely to be within the cost constraints. There is 100% certainty that the total acquisition cost of the submarine that incorporates Technology 2 will be below the cost constraint, and 70% certainty that the cost of the submarine that uses Technology 1 will be below the same constraint. From the above it is clear that Technology 2 is less risky than Technology 1.

As it was presented, the decision maker has to take into account all the elements of risk in order to make the final decision. The technical or performance risk, and the cost risk have to be considered in order to make educated decisions with respect to the technology strategy. Examining only one of the two risk elements might lead to wrong decisions that can result in the production of a vessel that is not capable to meet the requirements but meets the cost constraints or a vessel that can meet the requirements but it is not affordable.
5 Conclusions and Recommendations

New technologies must be considered at the initial phases of the design concept exploration, since the impact of adding technologies later on the design process will significantly decrease the probability of attaining desired cost schedule and system performance goals. The ability of a system to meet the performance and cost goals is also affected by the uncertainty associated with new technologies. Therefore, performance and cost uncertainty should be also taken into account at the initial stages of the design and it should be integrated in the decision process.

5.1 Conclusions

The presented methodology allows the decision maker/designer to predict how new technologies affect the final product. The impact of technologies is examined to the whole system level, integrating the new technology with the rest of the design.

Although the methodology can be applied to any complex system, in this study it is demonstrated by the example of conventional submarine design. The impact of technology on modern Air Independent Propulsion submarines was assessed through the use of technology k-factors. These factors were introduced into the mathematical synthesis model, which was developed for this study, and they modified technical characteristics of the design. The modifications resulted in changes of the technical metrics, to simulate the hypothetical improvement or degradation associated with the new technology. The parametric mapping was done with the use of Response Surface Equations based on regression analysis using the results from the synthesis model.

Given the Response Surface Equations, the decision maker can predict the impact that any technology has on the performance of the submarine. The only thing the user will have to do is set the k-factors at the levels that reflect the technologies of interest. Then the Response
Surface Equations will predict the responses, and the decision maker will be able to know the impacts of the new technologies on the system level of the submarine, easily and without the need to develop new designs.

This is expected to be very useful for both the shipyard and the acquisition community, especially during the development of new concepts that need to have improved performance and incorporate the latest technology achievements. In this case the method can be used in two ways:

1. The methodology will allow the decision maker to know the technology areas that have significant effect on the capabilities of the vessel, and he will be able to select the combination of k-factors that will give the submarine a target performance. If the k-factor levels correspond to technology levels that exceed the predicted, the decision maker can choose to relax the requirements, or seek the target performance by developing new technologies. Then the decision maker will know the technology areas that require further investigation, and will allocate the R&D resources towards that direction.

2. The decision maker can predict the impact of a known technology on the performance of the submarine, or he can compare two candidate technologies, or select the optimum mix of technologies. In many cases, technologies that seem to be promising do not improve significantly the performance of the final product, and others that seem to offer only incremental improvement provide excellent results. It is the system level impact that matters.

As a result this method aids decision makers and designers in performing tradeoff studies very efficiently, during meetings or discussions, by using the interactive environment of JMP. In addition the designers can use the Response Surface Equations to investigate the impact of uncertainty on the performance metrics of the design.

The presence of uncertainty in the prediction of future technologies results in an inability to predict the exact performance of the system. A system subject to uncertainty cannot be
expressed in terms of a single solution, but it should be expressed as a probability distribution. Monte Carlo simulation shows the effect of uncertainty on the responses, which can be analyzed based on the cumulative and the reverse cumulative distributions obtained by running 10,000 cases. In order to perform this task without the use of the Response Surface Equations, the designer would have had to balance 10,000 submarine designs using the mathematical synthesis model. This is a very time-consuming and practically impossible task. On the other hand, by using the Response Surface Equations, a very simple Excel spreadsheet, and the software Crystal Ball, this task becomes very easy. Furthermore, Crystal Ball automatically produces meaningful plots, which can aid the decision maker in performing tradeoff studies.

In addition to the performance uncertainty, cost uncertainty assessment is an integrated part of this decision process. Cost estimates are subject to uncertainty associated with the components of cost or the results of the synthesis model. Variation of material cost, man-hour rates, production techniques etc. can alter the final cost, which can exceed the available budget. By using Crystal Ball and Monte Carlo Simulation this thesis presents a way to assess cost uncertainty.

To conclude, the methodology presented in this study, provides a technology selection strategy, which shows the system level impacts of new technologies on the final product and integrates the performance and cost uncertainty in the decision making process. As a result, the decision making process becomes more robust, and the tradeoff studies more efficient.

5.2 Recommendations for Future Study

This study highlighted several areas that require further study in order to make the technology selection strategy more effective and meaningful.

First and foremost, a great deal of effort needs to go into developing a more detailed submarine synthesis model. The ideal synthesis model must be physics based, and it should
include disciplinary technical metric impact factors, in order to be able to analyze the impact of the new technologies. It should be much more detailed than the model used for this study, in order to be able to simulate new technologies associated with more design areas.

One good way to accomplish the goal of developing a synthesis model that can be used for concept exploration through detailed design is to make it modular, using object oriented architecture. The connection of appropriate modules for detailed analysis of uncertain areas could be accomplished easily and allow better technology evaluation in the system concept.

Furthermore, the synthesis model should be able to converge on system level parameters without the need for manual designer input in between feasible solutions, and it should extract the data in a JMP format, in order to make the process more efficient. A method that verifies the technology compatibility [23] should be also added to the process, in order to ensure that the technologies used to produce the design are compatible.

In addition, the k-factor limits must be selected based on projections of technical feasibility of known technologies. The projections can be identified by industry experts, and scientists. More realistic data are also necessary for the cost estimating model. The simplified model that was used for this study is based on data available in reference [3] and is not very accurate.

Finally, it should be noted that, for the purpose of this study, the dimensions of the submarine were kept constant and the variation of sizes and weights of the different components caused by the variation of k-factors was reflected by improving or degrading the performance of the submarine. Another way to solve the problem is keep the performance of the submarine fixed to a target value, and vary the dimensions depending on the technology levels.
List of References


[16] Jane’s Weapon Systems


[18] www.naval-technology.com


[34] Rubinstein, R. Y., Simulation and the Monte Carlo Method, New York: John Wiley & Sons, Inc.
Appendix 1

"Submarine Synthesis Model"
CONVENTIONAL SUBMARINE SYNTHESIS MODEL

Ship: Baseline AIP Submarine

I. TECHNOLOGY \( k \) FACTORS

This section lists the \( k \)-factors used to modify the technical parameters of the design to simulate improvement or degradation associated with new technologies.

<table>
<thead>
<tr>
<th>Selected Limits</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
</table>

**Battery**

- \( k_{bat_w} := 1 \)
  - 0.6
  - 1.1
  (Fraction of the baseline battery weight. The baseline battery is the ASB 49C lead acid battery)
- \( k_{bat_v} := 1 \)
  - 0.6
  - 1.1
  (Fraction of the baseline battery volume)

**Motor**

- \( k_{motor_w} := 1 \)
- \( k_{motor_v} := 1 \)
  (Fraction of the weight calculated by the regression of existing motors)
  (Fraction of the volume calculated by the regression of existing motors)

**Diesel Engines**

- \( k_{de_w} := 1 \)
- \( k_{de_v} := 1 \)
- \( k_{de_sfc} := 1 \)
  (Fraction of the specific weight (lb/kW))
  (Fraction of the specific volume (ft^3/kW))
  (Fraction of the sfc compared to the sfc=0.4587 lb/kWhr)

**Structures**

- \( k_{struct_w} := 1 \)
  - 0.95
  - 1
  (Fraction of the structural weight compared to the baseline which comes from regression equations and represents HY-80)

**AIP Plant**

In order to calculate the impact of new AIP technology on the submarine we need to select the \( K_{F_AIP} \) equal to 1, and if we want to model an AIP system with metal hydride storage of Hydrogen we need to select the option=1.

\[ K_{F_AIP} := 1 \]
<table>
<thead>
<tr>
<th>Baseline Values</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>PEMFCmhb</th>
<th>PEMFCm meth</th>
<th>CCD</th>
<th>Stirling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{aip_w}$ := 66.138690 lb/kW</td>
<td>33.069345 (50%)</td>
<td>74.957182 (113%)</td>
<td>41.887837</td>
<td>63.934067</td>
<td>74.957182</td>
<td>66.138690</td>
</tr>
<tr>
<td>$k_{aip_v}$ := 2.931118 ft$^3$/kW</td>
<td>1.46555 (50%)</td>
<td>4.06118705 (138%)</td>
<td>2.2601389</td>
<td>4.06118705</td>
<td>3.14300563</td>
<td>2.931118</td>
</tr>
<tr>
<td>$k_{aip_fc_w}$ := 0.645955 lb/kW-hr</td>
<td>0.322975 (50%)</td>
<td>0.903895 (140%)</td>
<td>7.694134</td>
<td>0.903895</td>
<td>0.610681</td>
<td>0.645955</td>
</tr>
<tr>
<td>$k_{aip_fc_v}$ := 0.013066 ft$^3$/kW-hr</td>
<td>0.006533 (50%)</td>
<td>0.01832 (140%)</td>
<td>0.0223</td>
<td>0.01832</td>
<td>0.011618</td>
<td>0.013066</td>
</tr>
<tr>
<td>$k_{aip_stor_w}$ := 0.085980 lb/kW-hr</td>
<td>1.926841</td>
<td>0.163142</td>
<td>0.074957</td>
<td>0.085980</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{aip_stor_v}$ := 0.000689 ft$^3$/kW-hr</td>
<td>0.0088287</td>
<td>0.0078539</td>
<td>0.000600349</td>
<td>0.000689</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{aip_oc_w}$ := 2.253125 lb/kW-hr</td>
<td>1.1265625 (50%)</td>
<td>2.4784375 (110%)</td>
<td>0.970034</td>
<td>1.48596</td>
<td>1.851883</td>
<td>2.253125</td>
</tr>
<tr>
<td>$k_{aip_oc_v}$ := 0.031607 ft$^3$/kW-hr</td>
<td>0.0158035 (50%)</td>
<td>0.0347677 (110%)</td>
<td>0.0137727</td>
<td>0.019034607</td>
<td>0.02595628</td>
<td>0.031607</td>
</tr>
<tr>
<td>$k_{aip_ostor_w}$ := 0.745163 lb/kW-hr</td>
<td>0.363763</td>
<td>0.496040</td>
<td>0.698865</td>
<td>0.745163</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{aip_ostor_v}$ := 0.005576 ft$^3$/kW-hr</td>
<td>0.0020483</td>
<td>0.002860488</td>
<td>0.004605033</td>
<td>0.005576</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{aip_ArN2He_cons_w}$ := 0.022046 lb/kW-hr</td>
<td>0</td>
<td>0.022046</td>
<td>0.066139</td>
<td>0.022046</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{aip_ArN2He_cons_v}$ := 0.000237 ft$^3$/kW-hr</td>
<td>0</td>
<td>0.000176573</td>
<td>0.000748671</td>
<td>0.000237</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{aip_ArN2He_stor_w}$ := 0.002205 lb/kW-hr</td>
<td>0</td>
<td>0.002205</td>
<td>0.004409</td>
<td>0.002205</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{aip_ArN2He_stor_v}$ := 0.000038 ft$^3$/kW-hr</td>
<td>0</td>
<td>0.0000388461</td>
<td>0.000130664</td>
<td>0.0000388</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{aip_comp_v}$ := 0.033196 ft$^3$/kW-hr</td>
<td>0.0153266</td>
<td>0.022601389</td>
<td>0.02729824</td>
<td>0.033196</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
II. OWNER REQUIREMENTS

Owner Requirements

\[ I_{ton} = 2240 \text{ lb} \quad \text{NM} = 2000 \text{ yd} \quad \text{knt} = \frac{2000 \text{ yd}}{1 \text{ hr}} \quad \text{Design Number} = 1 \]

**Propulsion Plant Choice**

If the owner requires to have a Diesel Electric Submarine, you have to enter "choice=0". If he requires an AIP submarine, you have to enter "choice=1".

The AIP options are four: 1 is Fuel Cells with Metal Hydride storage of Hydrogen 2 is Fuel Cells with Methanol Reformer 3 is Closed Cycle Diesel 4 is Stirling Engine

**Operational Profile**

Range at surface transit: \[ \text{Surf. range} = 6500 \text{ NM} \]
Max. surface speed: \[ V_{\text{max surfaced}} = 11 \text{knt} \]
Range submerged transit: \[ \text{Snort. range} = 6500 \text{ NM} \]
AIP Endurance (days): \[ \text{AIP endur} = 14 \]
Submerged endurance (on battery): \[ \text{Time endur. bat} = 59 \text{hr} \]
Endurance (days): \[ E = 70 \]
Burst Speed: \[ V_{\text{max}} = 20 \text{knt} \quad \text{for} \quad \text{Time burst} > 1 \text{hr} \]

**Crew Size:**

\[ N_{\text{crew officer}} = 5 \]
\[ N_{\text{crew CPO enlisted}} = 20 \]
\[ N_T = N_{\text{crew officer}} + N_{\text{crew CPO enlisted}} \]
\[ N_T = 25 \]

**Diving Depth (ft):**

\[ D_D = 950 \text{ ft} \]

**Payload**

Number of Torpedo Tubes: \[ T_T = 6 \]
Torpedo Reloads: \[ R_L = 10 \]
Torpedo Volume (ft\(^3\)): \[ T_{\text{vol}} = 198 \pi \text{(1.75 ft)}^2 \]
Torpedo Weight (lb): \[ T_{\text{weight}} = 38431 \text{lb} \]
Torpedo Tube Volume (in PH) (ft\(^3\)): \[ T_T_{\text{vol}} = 211 \pi \text{(2.3 ft)}^2 \]
Torpedo Tube weight (lb): \[ T_T_{\text{weight}} = 2171 \text{ton} \]
CIC Volume: \[ CIC_{\text{volume}} = 2650 \text{ ft}^3 \]
Electrical Load of Payload: \[ HLPayload = 45 \text{kW} \]
III. VOLUME ESTIMATION

For the Diesel submarines the volume occupied by the payload is approximately 30% of the volume inside the PH. Based on that it is possible to make a preliminary estimate of the internal volume of the pressure hull (Reference [5] p.252). Therefore we need first to do an estimate of the payload based on the owner requirements.

The Torpedo tubes compensating tanks can be estimated in order to accomodate for the lost weight when all the weapons are not taken on board or are used:

$$TT_{comp\_tank\_Vol} = \frac{(RL + TT) \cdot T\_weight}{1035 \cdot \frac{kg}{m^3}}$$

$$TT_{comp\_tank\_Vol} = 852.6 \text{ ft}^3$$

The multipliers 1.8 and 1.2 in the following equation are estimates based on the figures from Janes Weapon Systems:

$$\text{Weapons volume} = 1.8 \cdot TT - \frac{TT_{comp\_tank\_Vol}}{ft^3} + 1.2 \cdot RL \cdot \frac{T\_vol}{ft^3} \text{ ft}^3$$

$$\text{Weapons volume} = 6815.4 \text{ ft}^3$$

$$\text{Payload volume} = \text{Weapons volume} + \text{CIC volume}$$

$$\text{Payload volume} = 9465.4 \text{ ft}^3$$

A. PH Volume Estimate:

$$V_{PH} = \frac{\text{Payload volume}}{0.3}$$

$$V_{PH} = 31551.3 \text{ ft}^3$$

The following is the iteration value. The evaluation should be disabled at the first run. After each run it should be reset to the Vph1 that comes out at the end of the Volume Requirements module.

$$V_{PH} = 42879.8 \text{ ft}^3$$

B. Outboard Volume Estimate:

$$V_{ob\_Main\_Hull} = 0.08 \cdot V_{PH}$$

.08\% volume of the pressure hull is a standard estimate of the outboard volume for typical submarines. In the case of FC AIP system with metal hydride storage of hydrogen, additional volume must be added to account for the fuel storage.

C. Everbouyait Volume:

$$V_{eb} = V_{PH} + V_{ob\_Main\_Hull}$$

$$V_{eb} = 46310.2 \text{ ft}^3$$

$$\Delta_{eb} = V_{eb} \cdot 64 \cdot \frac{lb}{ft^3}$$

$$\Delta_{eb} = 1223.1 \text{ lton}$$

D. Main Ballast Tank Volume:

$$V_{bt} = 0.125 \cdot V_{eb}$$

$$V_{bt} = 5788.8 \text{ ft}^3$$

$$W_{MBT} = V_{bt} \cdot 64 \cdot \frac{lb}{ft^3}$$

$$W_{MBT} = 165.4 \text{ lton}$$
E. Submerged Volume:

\[ V_s = V_{eb} + V_{bt} \]

\[ \Delta_{at} = \frac{V_s - \Delta_{at}}{\rho} \]

\[ V_s = 52099 \text{ ft}^3 \]

\[ \Delta_{at} = 1488.5 \text{ ton} \]

F. Envelope Volume:

\[ \rho := \frac{V_s}{1 - p} \]

\[ \Delta_{env} := (V_{env}) \frac{64 \cdot \text{lb}}{\rho} \]

\[ V_{env} = 54898.8 \text{ ft}^3 \]

\[ \Delta_{env} = 1568.5 \text{ ton} \]

G. Free Flood Volume:

\[ V_{ff} := \rho V_{env} \]

\[ V_{ff} = 2799.8 \text{ ft}^3 \]

IV. POWER ESTIMATION

In order to estimate the required power we need to estimate the hull drag. Therefore we can use the following procedure:

A. Spin a Hull:

We select forward & aft shape factors in order to shape a basic hull.

**Entrance:**

\[ \eta_f := 3.25 \]

\[ \eta_a := 3.35 \]

**Insert values for LOD and D in order to get the Total Volume equal to the Envelope Volume (from above):**

We have to keep in mind that the optimum value of L over D in terms of resistance is between 6.5 and 7.5. However the average L over D for the existing diesel electric and AIP submarines is 9.3.

\[ D := 20.529 \text{ ft} \quad \text{LOD} := 9.681 \quad L := 198.7 \text{ ft} \]

\[ L_f := 2.4 \times D \quad L_f = 49.3 \text{ ft} \] (Length of forward part)

\[ L_a := 3.6 \times D \quad L_a = 73.9 \text{ ft} \] (Length of aft part)

\[ L_{pmb} := (\text{LOD} - 6) \times D \quad L_{pmb} = 75.6 \text{ ft} \] (Length of parallel mid body)

B. Wetted Surface Calculations

1. Entrance:

\[ L_f = 49.3 \text{ ft} \quad x_1 := 0, 1, \text{ up to } L_f + L_{pmb} \]

\[ y_f(x_1) = \left[ 1 - \left( \frac{x_1 - L_f}{L_f} \right)^{\eta_f} \right] \frac{D}{2} \]

\[ off(x_1) = \text{if}(x_1 < L_f, y_f(x_1), \frac{D}{2}) \]

2. Run:

\[ L_a = 73.9 \text{ ft} \quad x_1 := 0, 1, \text{ up to } L \]

\[ y_a(x_1) = \left[ 1 - \left( \frac{x_1 - (L_f + L_{pmb})}{L_a} \right)^{\eta_a} \right] \frac{D}{2} \]

3. Total Ship:

\[ off(x_1) = \text{if}(x_1 \leq L_f + L_{pmb}, \text{off}(x_1), y_a(x_1)) \]
4. Total Ship Volume

\[ V_{\text{tot}} = \int_{0}^{L} \text{offt}(x) \cdot \pi \, dx \]

\[ V_{\text{tot}} = 54898.9 \, \text{ft}^3 \]

5. Tail Cone angle (measured from the axis of rotation to the tangent at the stern). Greater than 18 degrees is probably considered a full stem.

\[ \sin \left[ \frac{D}{L_a} \right] = \frac{1}{\sqrt{1 + \left( \frac{D}{L_a} \right)^2}} \approx 15.8 \, \text{deg} \]

6. Total Prismatic Coefficient

\[ C_p := \frac{V_{\text{tot}}}{\pi \left( \frac{D}{2} \right)^2 \cdot L} \]

\[ C_p = 0.835 \]

7. Forward Prismatic and Wetted Surface Area Coefficients:

\[ C_{pf} := \int_{0}^{2.4-D} \text{offt}(x) \cdot \pi \, dx \]

\[ C_{pf} = 0.8274 \]

\[ C_{wsf} := \int_{0}^{2.4-D} 2 \cdot \text{offt}(x) \cdot \pi \, dx \]

8. After Prismatic and Wetted Surface Area Coefficients:

\[ C_{pa} := \int_{(L-3.6-D)}^{L} \text{offt}(x) \cdot \pi \, dx \]

\[ C_{pa} = 0.6701 \]

\[ C_{wsa} := \int_{(L-3.6-D)}^{L} 2 \cdot \text{offt}(x) \cdot \pi \, dx \]

9. Available Envelope Displacement and Wetted Surface Area:

\[ K_1 := 6 - 2.4 \cdot C_{pf} - 3.6 \cdot C_{pa} \]

\[ K_1 = 1.6 \]

\[ K_2 := 6 - 2.4 \cdot C_{wsf} - 3.6 \cdot C_{wsa} \]

\[ K_2 = 1.1 \]
\[ \Delta_{enva} = \frac{\pi D^3}{4} - 64 \text{ lb ft}^3 (LOD - K1) \quad \Delta_{enva} = 1568.5 \text{ ton} \]
\[ WS := \pi D^2 (LOD - K2) \quad WS = 11395.6 \text{ ft}^2 \]

C. Speed at different conditions:

1. Speeds for a snorkel/submerge transit profile:  \[ V_{\text{snorkel}} = 8 \text{knt} \]
\[ V_{\text{submerged}} = \begin{cases} 2 \text{knt} \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ 14 \end{cases} \]
\[ i := 1..14 \]

2. Speed for a surfaced transit profile:  \[ V_{\text{surfaced}} = \begin{cases} 2 \text{knt} \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ 14 \end{cases} \]
\[ i := 1..14 \]

3. Patrol speed when using AIP propulsion:  \[ V_{\text{ballance}} = 4 \text{knt} \]

D. Effective Horsepower:  \[ V := 0, 1, 20 \]

1. Resistance calculation parameters:
\[ T_{SW} := 39 \quad \rho_{SW} := 1.9905 \text{ lb sec}^2 \text{ ft}^{-4} \quad V_{SW} := 1.2817 \times 10^{-5} \text{ ft}^5 \text{ sec}^{-5} \quad R_N(V) := L \frac{V}{V_{SW}} \]

Wetted Surface (previously calculated):  \[ WS = 11395.6 \text{ ft}^2 \]

Correlation Allowance:  \[ C_a := .02 \] For Surface Ships, this is typically 0.0004. Typical values for submarines are 0.0015-0.002.

2. Frictional resistance calculation:
\[ C_f(V) := \frac{0.575}{(\log(R_N(V)) - 2)^2} \]

3. \( C_r \) calculation: The following equation for \[ \frac{C_f + C_r}{C_f} \] was developed by Hoerner using the fact that the after end of the submarine has a large effect of the form coefficient:
\[ C_{fr} := 1 + 1.5 \left( \frac{D}{L_a} \right)^{1.5} + 7 \left( \frac{D}{L_a} \right)^3 + .002 (C_p - 6) \]
\[ C_{fr} = 1.37 \]

4. Appendage drag (including sail) calculation:
   a. The Surface area of the sail is estimated using the formula from Reference [5] (page 108). The drag coefficient is set as follows:
\[ C_{D_s} := .009 \quad A_s := 0.0805 \cdot WS \quad A_s = 917.3 \text{ ft}^2 \quad A_s C_{D_s} = 8.3 \text{ ft}^2 \]
b. For the remaining appendages, we use the expression: \( L^*D/1000 \) which is an initial estimate of the appendage drag. If we want to do more accurate calculations we should use 0.006 as drag coefficient for the various appendages.

\[
App = \frac{L^*D}{1000} \quad App = 4.08 \text{ ft}^2
\]

5. EHP submerged

\[
EHP_{\text{submerged}}(V) := \frac{0.00872 \cdot V^2 \left[ WS \cdot (C_{fr} V) \cdot C_{fr} + C_{a} \right] + (A_s \cdot C_{ps}) + App}{ft^2}
\]

6. SHP estimation: We need a SHP estimate in order to do the calculations for the propeller diameter. Later in the model, more accurate calculations of the SHP will be done.

\[
PC_{\text{estimate}} := 0.75-0.98 \\
\text{SHP}_{\text{estimate}}(V) := \frac{EHP_{\text{submerged}}(V)}{PC_{\text{estimate}}}
\]

E. Shaft Horsepower submerged:

1. Propeller Selection: Use series or other method to determine Propeller Open Water Efficiency 

\( \eta_o := 0.66 \)

2. Hull Efficiency:

a. A good starting point for the Propeller diameter is \( D/2 \). We can assume that the revolutions of the motor are proportional to the submarine's speed and that we want to design the propeller for the loiter speed:

\[
N = 9 \frac{V_{\text{loiter battery}}}{\text{knt}} \frac{1}{\text{min}} \quad N = 36 \text{min}^{-1}
\]

\[
D_p := 50 \left[ \text{SHP}_{\text{estimate}} \left( \frac{V_{\text{loiter battery}}}{\text{knt}} \right) \right] \left( \frac{1}{N} \right)^{\frac{1}{3}} \left( \frac{1}{\text{min}} \right)^{\frac{1}{5}} \quad D_p = 12.8 \text{ ft} \quad D_p \text{over} \frac{D}{D} = 0.62
\]

b. Wake Fraction 1-w \( (w_1) \) and Thrust Deduction 1-t \( (t_1) \):

\[
C_w := \frac{WS}{\pi \cdot L \cdot D} \quad w_1 := 0.74 + 1.7151 \cdot \left( \frac{D_p^2}{D} \right) \quad t_1 := 0.632 + 1.3756 \cdot \left( \frac{D_p}{D} \right)
\]

\[
C_w = 0.889 \quad w_1 = 0.74 \quad t_1 = 0.92
\]

c. Hull Efficiency: 

\[
\eta_h := \frac{t_1}{w_1} \quad \eta_h = 1.257
\]

3. Relative Rotative efficiency:

\( \eta_{rr} := 1.02 \)

4. Propulsive Coefficient:

\[
PC := \eta_o \cdot \eta_h \cdot \eta_{rr}
\]

\[
PC = 0.85
\]

If PC is above .88 it is unrealistically high for this stage of the design (i.e. you'll have insufficient powering margin), especially since we have not characterized or analyzed the propeller hull interaction.
5. Submerged SHP:  
\[ \text{SHP}_{\text{submerged}}(V) := \frac{\text{EHP}_{\text{submerged}}(V)}{\text{PC}} \]

F. Shaft Horsepower Snorkeling:  
\[ D_{\text{snorkel}} \text{ is the snorkel depth.} \]
\[ D_{\text{snorkel}} = 1.4 \times D + 4 \text{ ft} \]
\[ D_{\text{snorkel}} = 32.7 \text{ ft} \]
\[ V_{\text{snorkel}} = \frac{\text{Fr}_{\text{snorkel}}}{(g \times L)^5} \]
\[ \text{Fr}_{\text{snorkel}} = 0.167 \]
\[ \text{C}_{\text{Dw}} = 0.8 \]

1. Determine \( C_W \):
\[ C_W = \frac{C_{\text{Dw}}}{4\left(\frac{L}{D} - 1.3608\right)\left(\frac{L}{D}\right)^2} \]
\[ C_W = 0.0002568 \]

2. Additional SHP required due to snorkeling:
\[ \text{SHP}_{\text{added}}(V) = \frac{0.03872 \times \text{C}_W \times \text{WS} \times V^3}{\text{PC} \times \text{R}^2} \]

3. Snorkeling Shaft Horsepower:
\[ \text{SHP}_{\text{snorkel}}(V) := \text{SHP}_{\text{submerged}}(V) + \text{SHP}_{\text{added}}(V) \]

E. Surface Shaft Horsepower Requirements

1. \( C_T \) is the total surface resistance coefficient. \text{NOTE: DO NOT USE THE CSPLINE FUNCTION BEYOND A FROUDE NUMBER OF .32!!!! THE CSPLINE FUNCTION IS NOT ACCURATE OUTSIDE OF THE DATA RANGE BELOW!!!!}

\[ a := 1.20 \]

\[ \begin{array}{c|c|c|c}
\text{Froude} & \text{CT} & \text{Froude}(v) & \text{C}(v) \\
.04 & 5.8 & 0.10 & \text{interp}(\text{Ispline}(\text{Froude}C_T, \text{Froude}, \text{CT}), \text{Froude}, \text{CT}, \text{x}) \\
.06 & 5.9 & 0.81 & \\
.08 & 5.8 & 0.66 & \\
.1 & 5.5 & 0.73 & \\
.125 & 5.5 & 0.69 & \\
.175 & 5.6 & 0.71 & \\
.19 & 5.9 & 0.83 & \\
.2 & 6.8 & 0.86 & \\
.21 & 7.1 & 0.87 & \\
.22 & 7.6 & 0.9 & \\
.23 & 8.1 & 0.9 & \\
.24 & 8.6 & 0.9 & \\
.25 & 9.0 & 0.9 & \\
.26 & 8.8 & 0.9 & \\
.27 & 8.7 & 0.9 & \\
.28 & 9.8 & 0.9 & \\
.29 & 11.7 & 0.9 & \\
.3 & 12.6 & 0.9 & \\
.31 & 13.2 & 0.9 & \\
.32 & 13.6 & 0.9 & \\
\end{array} \]

\[ C_T(x) = \text{interp}(\text{Ispline}(\text{Froude}, \text{CT}), \text{Froude}, \text{CT}, x) \]

\[ \text{SHP}_{\text{surface}}(v) := 1.25 \times \text{PC} \times 0.03872 \times C_T(v) \times \frac{\text{WS} \times V^3}{\text{R}^2} \]

\[ f := 10 \]

\[ \text{Surface SpeedMax Accuracy} := \text{root}(\text{Froude}(f) - 0.32, f) \times \text{knt} \]

\[ v := 1.1, 1.1, \ldots \text{knt} \]

\[ \begin{array}{c}
\text{CT} \times \times \times \\
\text{Froude}, \text{Froude}(v) \\
\end{array} \]
F. Comparison Calculated Horsepowers:

![Shaft Horsepower Comparison Graph]

G. Hotel Load:

The hotel load will be the power required by the payload in normal patrol state plus the power required to run the auxiliary machinery and the ventilation and air conditioning. For estimating purposes the hotel load can be assumed to be proportional to the PH volume (Reference [5], Page 256):

\[ HL := 0.75 H_{\text{Payload}} + 0.075 \frac{V_{PH}}{3.2808^2 \text{ ft}^3} \]

\[ HL = 124.8 \text{ kW} \]

V. PROPULSION PLANT

1. Propulsion Motor

1. Propulsion Motor Power Required: \( \eta_{\text{motor}} \) and \( \eta_s \) are the efficiencies of the motor and the shaft bearings. The power calculated below is the power for the maximum submerged speed. The PMF is the power margin factor.

\[ \eta_{\text{motor}} = 0.95 \quad \eta_s = 0.99 \quad \text{PMF} = 0.03 \]

\[ P_{\text{Motor\_max\_submerged}} = \left( \frac{\text{SHP}_{\text{submerged}} (\frac{V_{\text{max}}}{\text{knt}})}{\eta_{\text{motor}} \eta_s} \right)(1 + \text{PMF}) \quad P_{\text{Motor\_max\_submerged}} = 1131.7 \text{ kW} \]

\[ P_{\text{Motor\_snorkel}} = \frac{\text{SHP}_{\text{snorkel}} (\frac{V_{\text{snorkel}}}{\text{knt}})}{\eta_{\text{motor}} \eta_s} (1 + \text{PMF}) \quad P_{\text{Motor\_snorkel}} = 92.4 \text{ kW} \]
PMotor-submerged := \( \frac{\text{SHP}_{\text{submerged}} (\frac{V_{\text{submerged}}}{\text{knot}}) \text{hp}}{\eta_{\text{motor}} \text{\eta_s}} \cdot (1 + \text{PMF}) \)

PMotor-loiter := \( \frac{\text{SHP}_{\text{submerged}} (\frac{V_{\text{balance}}}{\text{knot}}) \text{hp}}{\eta_{\text{motor}} \text{\eta_s}} \cdot (1 + \text{PMF}) \)

PMotor-surfaced := \( \frac{\text{SHP}_{\text{surfaced}} (\frac{V_{\text{surfaced}}}{\text{knot}}) \text{hp}}{\eta_{\text{motor}} \text{\eta_s}} \cdot (1 + \text{PMF}) \)

PMotor_max_surfaced := \( \frac{\text{SHP}_{\text{surfaced}} (\frac{V_{\text{max surfaced}}}{\text{knot}}) \text{hp}}{\eta_{\text{motor}} \text{\eta_s}} \cdot (1 + \text{PMF}) \)

PMotor := \text{max}(\text{PMotor}_{\text{max submerged}}, \text{PMotor}_{\text{snorkel}}, \text{PMotor}_{\text{loiter}}, \text{PMotor}_{\text{surfaced}}) \)

\( \text{PMotor} = 4113.7 \text{kW} \)

In order to calculate the weight and the volume of the motor we are using the evaluation of the electric motors done by Kirtley. The constants are selected in order to get the weight and the volume of the Permasyn 1FR6943. The Permasyn 1FR6943 has the following characteristics:

- Motor baseline Weight := 53 \times 10^2 \text{kg}
- Motor baseline Volume := (1950 \text{mm})^2 \times 2480 \text{mm}
- Motor baseline Length := 1950 \text{mm}
- Motor baseline Power := 3700 \text{kW}

Weight and Volume calculations:

\( \text{Constant}_V := 43.13 \)

\( \text{Constant}_W := 0.304 \)

\( \text{Motor Volume} := k_{\text{motor v}} \cdot \text{Constant}_V \left( \frac{N}{\text{min}^{-1}} \right)^{-0.44} \left( \frac{\text{PMotor}}{\text{hp}} \right)^{0.56} \cdot \text{ft}^3 \)

\( \text{Motor Weight} := k_{\text{motor w}} \cdot \text{Constant}_W \left( \frac{N}{\text{min}^{-1}} \right)^{-0.44} \left( \frac{\text{PMotor}}{\text{hp}} \right)^{0.79} \cdot \text{ton} \)
MotorVolume = 1101.1 ft³

Volume_Fraction_Motor_auxiliaries := .35

Motor_and_auxVolume := MotorVolume \left(1 + \text{Volume Fraction Motor auxiliaries}\right)

Motor_and_aux_weight := MotorWeight \left(1 + \text{Weight Fraction Motor auxiliaries}\right)

Motor_and_auxVolume = 1498.6 ft³

MotorWeight = 56.81 ton

Weight_Fraction_Motor_auxiliaries := 0.15

Motor_and_auxWeight = 65.3 ton

2. Battery Sizing

Based on the values for a lead acid battery (ASB-49C) we have the following:

has a burst capacity (at 1 hour rate): \(\text{Cap baseline bat burst} := 1092.4\text{kW-hr}\)

and an endurance capacity (48+ hour rate): \(\text{Cap baseline bat endurance} := 2647\text{kW-hr}\)

The battery will be sized based on the maximum energy required for the burst or endurance condition:

\[
\text{Energy burst} := \text{Time burst} \left(\text{SHP submerged} \left(\frac{\text{Vmax}}{\text{knt}}\right) \cdot \text{hp} + \text{HL}\right)
\]

\[
\text{Energy burst} = 3881.1\text{kW-hr}
\]

\[
\text{Energy end} := \text{Time endurance bat} \left(\text{SHP submerged} \left(\frac{\text{Vloiter battery}}{\text{knt}}\right) \cdot \text{hp} + \text{HL}\right)
\]

\[
\text{Energy end} = 9322.7\text{kW-hr}
\]

\[
\text{N batt burst} := \frac{\text{Energy burst}}{\text{Cap baseline bat burst}}
\]

\[
\text{N batt burst} = 3.6
\]

\[
\text{N batt endur} := \frac{\text{Energy end}}{\text{Cap baseline bat endurance}}
\]

\[
\text{N batt endur} = 3.5
\]

The total number of batteries will be given by the following:

\[
\text{N batt} := \max(\text{N batt burst, N batt endur})
\]

\[
\text{N batt} = 3.6
\]
1. Input the desired battery information from the above table:

   \[
   \text{BattType} \leftarrow 1, \quad \text{Wcell} \leftarrow \text{BatteryChar2}, \quad \text{Lcell} \leftarrow \text{BatteryChar1}, \quad \text{Hcell} \leftarrow \text{BatteryChar3}, \quad \text{Cells_per_battery} \leftarrow \text{BatteryChar4}
   \]

2. Total Battery cells

   \[
   N_{\text{cell}} = \text{ceil}(N_{\text{batt}} \cdot \text{Cells_per_battery}) = 448
   \]

3. Battery configuration and volume calculations

   The Batt_Space variable is the minimum spacing allowed between the top of the cell and the ceiling above. These volume calculations use the deck height again, so make sure that this is the correct value.

   \[
   \begin{align*}
   \text{Batt_Space} & = 2.5 - \text{ft} \\
   \text{Max_Batt_Space} & = 2 - \text{ft} \\
   \text{Deck Height} & = \frac{D}{2}\text{ft}
   \end{align*}
   \]

   \[
   b = \frac{D}{2} - \left(\text{Deck Height} - \text{Hcell} - \text{Batt Space}\right)
   \]

   \[
   \text{Number cells wide} = \text{floor}\left(\frac{(D - c) - \text{b}^2}{-\text{Wcell}}\right)
   \]

   \[
   \text{Number cells long} = \text{ceil}\left(\frac{N_{\text{cell}}}{\text{Number cells wide}}\right)
   \]

   \[
   \text{Batterywidth} = \frac{\text{Number cells wide} \cdot \text{Wcell}}{2}
   \]

   \[
   \text{Batterylength} = \frac{\text{Number cells long} \cdot \text{Lcell}}{2}
   \]

   \[
   c = \frac{D}{2} - \left(\text{Deck Height} - \text{Hcell} - \text{Max Batt Space}\right)
   \]

   \[
   \text{Cross section area}_{\text{Tank Under Battery}} = \frac{D}{2}\text{ft} - \text{Cross section area}_{\text{Battery length}}
   \]

   \[
   \text{Cross section area} = \frac{D}{2}\text{ft} \cdot \text{Cross section area}_{\text{Battery length}}
   \]

   \[
   \text{VBAT} = k_{\text{bat w}} \cdot \text{Cross section area}_{\text{Battery length}}
   \]

   \[
   \text{VTank Under Battery} = \text{Cross section area}_{\text{Tank Under Battery}} \cdot \text{Battery length}
   \]

   \[
   \text{Weight battery} = k_{\text{bat w}} \cdot \text{Weightcell} \cdot N_{\text{cell}}
   \]

   Usually for safety and redundancy the battery is placed in the submarine as two half batteries:

   \[
   \text{Weight half battery} = \frac{\text{Weight battery}}{2}
   \]

   \[
   \text{VBAT} = \frac{\text{Cross section area}_{\text{Tank Under Battery}}}{2}
   \]

   \[
   \text{VTank Under Half Battery} = \frac{\text{VTank Under Battery}}{2}
   \]

   \[
   \text{Cross section area}_{\text{Tank Under Half Battery}} = \frac{3056.3}{2}\text{ft}^3
   \]

   \[
   \text{Weight half battery} = 241.4\text{ton}
   \]

   \[
   \text{Half Batterylength} = \frac{\text{Batterylength}}{2}
   \]

   \[
   \text{Half Batterywidth} = \frac{\text{Batterywidth}}{2}
   \]

   \[
   \text{Half Batterylength} = 22.2\text{ft}
   \]

   \[
   \text{Half Batterywidth} = 14\text{ft}
   \]

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3. Battery Charging Power Required: the variables below are read from the battery spreadsheet. \( N_{\text{charge}} \) indicates the number of batteries that will be charged simultaneously. The charge profile is that of an ASB-49 equalizing charge for a 100% discharged battery and does not vary with the battery type selected.

\[
N_{\text{charge}} = N_{\text{batt}} \quad \text{Discharge fraction} = 0.3
\]

\[
I_{\text{charge}} = \text{submatrix}(\text{ChargeChar}, 1, 17, 1, 1) \text{ hr}
\]

\[
V_{\text{charge}} = \text{submatrix}(\text{ChargeChar}, 1, 17, 3, 3) \text{ volt}
\]

\[
\text{Power}(x) = \text{interp}(I_{\text{charge}}, P_{\text{charge}}, x)
\]

\[
aa = 2/\text{hr} \quad \text{(Initial Estimate for the equation)}
\]

\[
\text{tsnorkel} = \int_0^{11} \text{Discharge fraction} \cdot \text{Power}(x) dx - \int_0^{\text{tsnorkel}} \text{Power}(x) dx, \text{aa}
\]

\[
\text{tsnorkel} = 1.1 \text{ hr}
\]

\[
\text{PAverage}_{\text{Batt}} = \frac{\text{tsnorkel}}{N_{\text{charge}}} \quad \text{PPeak}_{\text{Batt}} = \max(P_{\text{charge}}) \cdot N_{\text{charge}}
\]

\[
\text{PAverage}_{\text{Batt}} = 2426.4 \text{ kW} \quad \text{PPeak}_{\text{Batt}} = 2558 \text{ kW}
\]

3. Power Conversion Units

A. Diesel Engines:

a. Power Calculations:

Average Diesel BHP required while snorkeling and battery power required submerged: \( \eta_{\text{bus}} \) is the efficiency of the power plant bus work, \( \eta_{\text{gen}} \) is the efficiency of the generator. \( P_{\text{trans}_{\text{snorkel}}} \) Power at output of the DSL, \( P_{\text{peak}_{\text{snorkel}}} \) Power at output of the DSL Generator. \( P_{\text{trans}_{\text{submerged}}} \) power output at the battery.

\[
\eta_{\text{bus}} = 0.99 \quad \eta_{\text{gen}} = 0.97
\]

\[
P_{\text{trans}_{\text{snorkel}}} = \frac{\text{HL} + P_{\text{Motor}_{\text{snorkel}}} + P_{\text{Average}_{\text{Batt}}}}{\eta_{\text{bus}} \cdot \eta_{\text{gen}}} \quad P_{\text{trans}_{\text{snorkel}}} = 2959.1 \text{ kW}
\]

\[
P_{\text{peak}_{\text{snorkel}}} = \frac{\text{HL} + P_{\text{Motor}_{\text{snorkel}}} + P_{\text{Peak}_{\text{Batt}}}}{\eta_{\text{bus}} \cdot \eta_{\text{gen}}} \quad P_{\text{peak}_{\text{snorkel}}} = 3096.2 \text{ kW}
\]
\[ P_{\text{Trans submerged}} = \frac{(HL + PMotor_{\text{submerged}})}{\eta_{bus}} \]
\[ P_{\text{Loiter submerged}} = \frac{(HL + PMotor_{\text{loiter}})}{\eta_{bus}} \]
\[ P_{\text{Max surfaced}} = \frac{(HL + PMotor_{\text{max surfaced}})}{\eta_{bus}} \]
\[ P_{\text{Trans surfaced}} = \frac{(HL + PMotor_{\text{surfaced}})}{\eta_{bus}} \]

\[ P_{\text{Trans submerged}} = 278.6 \text{ kW} \]
\[ P_{\text{Loiter submerged}} = 16 \text{ kW} \]
\[ P_{\text{Max surfaced}} = 1056.9 \text{ kW} \]
\[ P_{\text{Trans surfaced}} = 376.1 \text{ kW} \]

\[ \text{Power}_{\text{diesel}} = \text{PPeak snorkel} = 3096.2 \text{ kW} \]

**b. Diesel Engines Volume and Weight Calculations:**

\[ \text{Specific Weight}_{\text{diesel}} := k_{\text{de-w}} \times 7.095 \frac{\text{lb}}{\text{kW}} \]
\[ \text{Specific Volume}_{\text{diesel}} := k_{\text{de-v}} \times 0.18877 \frac{\text{ft}^3}{\text{kW}} \]

\[ \text{Average Length}_{\text{diesel}} := 9 \text{ ft} \]
\[ \text{SFC}_{DE} := k_{\text{de-sfc}} \times 0.4587 \frac{\text{lb}}{\text{kW-hr}} \]

**Factors for the auxiliary, generator and enclosure (The factors used at the 13.414 design of 1999):**

\[ \text{Diesel aux weight factor} := 0.2 \]
\[ \text{Diesel generator weight factor} := 0.27 \]
\[ \text{Diesel enclosure weight factor} := 0.35 \]
\[ \text{Total Diesel W factor} := \text{Diesel aux weight factor} + \text{Diesel generator weight factor} + \text{Diesel enclosure weight factor} \]
\[ \text{Total Diesel V factor} := \text{Diesel aux volumet factor} + \text{Diesel generator volumet factor} + \text{Diesel enclosure volumet factor} \]

\[ \text{Volume}_{\text{diesel}} := \text{Specific Volume}_{\text{diesel}} \times (1 + \text{Total Diesel V factor}) \times \text{Power}_{\text{diesel}} \]

\[ \text{Volume}_{\text{diesel}} = 1244.9 \text{ ft}^3 \]

\[ \text{Weight}_{\text{diesel}} := \text{Specific Weight}_{\text{diesel}} \times (1 + \text{Total Diesel W factor}) \times \text{Power}_{\text{diesel}} \]

\[ \text{Weight}_{\text{diesel}} = 17.8 \text{ t} \]
c. Fuel Estimation for the Required Ranges:

If the possibility of the submarine leaving the harbor fully charged and returning with fully discharged battery is ignored, then the fuel storage has to meet the energy requirements of the patrol. The diesel fuel for the patrol will be sized by the hotel load (assumed constant for all the different conditions) plus the total propulsive energy of the patrol. The calculation can be made by assessing the time and speed surfaced, time at snorting and the time and speed pattern submerged. Now we will do the estimate based on the maximum value between the fuel to achieve the maximum surface range and the fuel to achieve the maximum snorkeling (transit) range:

\[
\text{fuel}_{\text{allowance}} := 0.99 \quad V_{\text{DFM}} := 43.93 \text{ ton} \\
\text{Diesel Fuel Lower Heating Value:} \quad DF_{\text{heat val}} := 43.2 \times 10^6 \text{ joule/kg}
\]

Transmission Efficiency: \( \eta_s := 0.97 \)

\[
V_{\text{DFM}} := k_{\text{de sfc-max}}(V_{\text{DFM snort}}, V_{\text{DFM surf}})
\]

\[
W_{\text{DFM}} := k_{\text{de sfc-max}}(\text{Fuel snort}, \text{Fuel surf})
\]

d. Fuel Requirements for snorkeling operation: SFC and \( V_{\text{DFM}} \) are based on typical Diesel parameters. \( t_{\text{snorkel}} \) is based on the time required to charge all batteries to the desired capacity. \( V_{\text{fuel cycle}} \) is the amount of fuel burned during one snorkel evolution.

\[
V_{\text{fuel cycle}} := P_{\text{Trans snorkel}} \cdot SFC_{\text{DE}} \cdot V_{\text{DFM}} \\
V_{\text{fuel cycle}} = 220.5 \text{ gal}
\]

B. AIP System:

a. AIP Plant Power Calculations:

The speed corresponding to the maximum rating of the AIP system keeping its batteries full is called balance speed. Based on the owners requirement on balance speed we can size the AIP plant.

\[
\text{Power}_{\text{AIP}} := \frac{\left( S.HP_{\text{submerged}} \cdot V_{\text{Balance}} \right)_{\text{hp + lbh HL}}}{\eta_s} \\
\text{Power}_{\text{AIP}} = 162.9 \text{ kW}
\]

The power of the AIP system has to be zero if the owner requires to have Diesel Electric Submarine.

\[
\text{Power}_{\text{AIP}} := \text{if}(\text{choice} = 1, \text{Power}_{\text{AIP}}, 0 \text{ kW})
\]

\[
\text{Power}_{\text{AIP}} = 162.9 \text{ kW}
\]
b. AIP Volume Calculations:

Select the secondary power plant option:

The specific volume information comes from the paper "AIP selection: the importance in Submarine integration and other non-headline discriminators", WARSHIP 99. The fuel and the oxidant in all the situations is assumed to be stored internally, except for the fuel cell with metal hydride storage of hydrogen, where the hydrogen is stored externally.

<table>
<thead>
<tr>
<th>Source parameters volume :=</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Volume Effect of Integration</th>
<th>Fuel Cell</th>
<th>FC Methanol</th>
<th>Closed Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hydride</td>
<td>Reformer</td>
<td>Diesel</td>
</tr>
<tr>
<td>Fuel Consumption (ft^3/kWh)</td>
<td>0.0223895</td>
<td>0.018328314</td>
<td>0.011618528</td>
</tr>
<tr>
<td>Fuel Storage (ft^3/kWh)</td>
<td>0.0088287</td>
<td>0.007853983</td>
<td>0.00600349</td>
</tr>
<tr>
<td>Oxygen Consumption (ft^3/kWh)</td>
<td>0.0137727</td>
<td>0.019034607</td>
<td>0.025962828</td>
</tr>
<tr>
<td>Oxygen Storage (ft^3/kWh)</td>
<td>0.0020483</td>
<td>0.002860488</td>
<td>0.004605033</td>
</tr>
<tr>
<td>Ar/ N2/ He consumption (ft^3/kWh)</td>
<td>0</td>
<td>0.000176573</td>
<td>0.000748671</td>
</tr>
<tr>
<td>Ar/ N2/ He storage (ft^3/kWh)</td>
<td>0</td>
<td>3.88461E-05</td>
<td>0.000130664</td>
</tr>
<tr>
<td>Compensation (ft^3/kWh)</td>
<td>0.0153266</td>
<td>0.022501389</td>
<td>0.02729824</td>
</tr>
<tr>
<td>Total (ft^3/kWh) within PH</td>
<td>0.0311475</td>
<td>0.0708942</td>
<td>0.07346317</td>
</tr>
<tr>
<td>Plant volume (ft^3/kWh)</td>
<td>2.2601389</td>
<td>4.06118705</td>
<td>3.14300563</td>
</tr>
</tbody>
</table>

\[
AIP_{Volume} := \text{Source parameters volume}_9, \text{Option}_{secondary} \frac{\text{ft}^3}{\text{kW} \cdot \text{Power}_{AIP}} \quad AIP_{Volume} = 13.5 \text{ m}^3
\]

\[
AIP_{Volume} := \text{if}(K_{F\_AIP} = 1, \text{Power}_{AIP} \cdot k_{aip\_v}, AIP_{Volume}) \quad AIP_{Volume} = 13.5 \text{ m}^3
\]

\[
AIP_{ConsumablesVolume} := \text{Source parameters volume}_8, \text{Option}_{secondary} \frac{\text{ft}^3}{\text{kW} \cdot \text{Power}_{AIP} \cdot \text{AIP endur-24hr}} \quad AIP_{ConsumablesVolume} = 130.8 \text{ m}^3
\]

The following a1 and b1 are used to shorten the equation of the AIP_ConsumablesVolume.

\[
a1 := k_{aip\_fc\_v} + k_{aip\_fstor\_v} + k_{aip\_oc\_v} \quad b1 := k_{aip\_ostor\_v} + k_{aip\_ArN2He\_cons\_v} + k_{aip\_ArN2He\_stor\_v} + k_{aip\_comp\_v}
\]

\[
AIP_{ConsumablesVolume} := \text{if}(K_{F\_AIP} = 1, \text{Power}_{AIP} \cdot (AIP endur-24hr) \cdot (a1 + b1), AIP_{ConsumablesVolume})
\]

\[
AIP_{ConsumablesVolume} = 130.8 \text{ m}^3
\]

At this point we have to note that the case of FC with hydride storage we assume that the H2 is stored outside the PH and it is not considered at the PH volume calculation. Therefore we need to take it into account for the outboard volume calculations:

\[
a2 := \text{Source parameters volume}_1, \text{Option}_{secondary} + \text{Source parameters volume}_2, \text{Option}_{secondary}
\]

\[
V_{H2\_outPH} := \left[ \text{Option}_{secondary} = 1, \left[ \left( a2 \left( \frac{\text{ft}^3}{\text{kW} \cdot \text{Power}_{AIP} \cdot \text{AIP endur-24hr}} \right) \right) \cdot 0 \right] \right] \quad V_{H2\_outPH} = 0 \text{ ft}^3
\]

\[
V_{H2\_outPH} := 0 \text{ ft}^3
\]

\[
AIP_{Total\_Vol} := AIP_{Volume} + AIP_{ConsumablesVolume} + V_{H2\_outPH} \quad AIP_{Total\_Vol} = 144.3 \text{ m}^3
\]

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c. AIP Weight Calculations:

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>Fuel Cell</th>
<th>FC Methanol</th>
<th>Closed Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hydride</td>
<td>Reformer</td>
<td>Diesel</td>
</tr>
<tr>
<td>Fuel Consumption (lb/kWh)</td>
<td>7.694134</td>
<td>0.903895</td>
<td>0.610681</td>
</tr>
<tr>
<td>Fuel Storage (lb/kWh)</td>
<td>1.926841</td>
<td>0.163142</td>
<td>0.074957</td>
</tr>
<tr>
<td>Oxygen Consumption (lb/kWh)</td>
<td>0.970034</td>
<td>1.428596</td>
<td>1.851883</td>
</tr>
<tr>
<td>Oxygen Storage (lb/kWh)</td>
<td>0.363763</td>
<td>0.496040</td>
<td>0.698865</td>
</tr>
<tr>
<td>Ar/ N2/ He consumption (lb/kWh)</td>
<td>0.000000</td>
<td>0.022046</td>
<td>0.066139</td>
</tr>
<tr>
<td>Ar/ N2/ He storage (lb/kWh)</td>
<td>0.000000</td>
<td>0.002205</td>
<td>0.004409</td>
</tr>
<tr>
<td>Total (lb/kWh)</td>
<td>10.954772</td>
<td>3.015924</td>
<td>3.308935</td>
</tr>
</tbody>
</table>

Plant weight (lb/kW) = 41.887837 63.934067 74.957182

**AIPWeight** := Sourceparameters Weight, Option secondary kW

**AIPWeight** := if(K_F_AIP = 1, PowerAIP.k_aip_w, AIPWeight)

If we choose option 1 the stored Hydrogen in the metal Hydride is placed externally closed to the keel of the submarine. That weight is going to be:

\[ W_{\text{Hydride}} := (\text{Sourceparameters Weight}_1 + \text{Sourceparameters Weight}_2) \text{ lb/kW-hr PowerAIP (AIP endur-24hr)} \]

\[ W_{\text{Hydride}} := \text{if(Option secondary = 1, W_Hydride, 0ton)} \]

\[ W_{\text{Hydride}} := \text{if(K_F_AIP = 1, PowerAIP (AIP endur-24hr) - (k_aipfcw + k_aipfstor_w), W_Hydride)} \]

\[ AIP_{\text{ConsumablesWeight}} := \text{Sourceparameters Weight}_7, Option secondary kW \text{ hr PowerAIP (AIP endur-24hr)} \]

The following a1 and b1 are used to shorten the equation of the AIP_Consumables Weight.

\[ a3 := k_{aipfcw} + k_{aipfstor_w} + k_{aipocw} + k_{aipostor_w} \]

\[ b3 := k_{aip_arN2He_cons_w} + k_{aip_arN2He_stor_w} \]

\[ AIP_{\text{ConsumablesWeight}} := \text{if(K_F_AIP = 1, PowerAIP (AIP endur-24hr) - (a3 + b3), AIP_ConsumablesWeight)} \]

\[ AIP_{\text{ConsumablesWeight}} = 91.7 \text{ ton} \]

\[ AIP_{\text{Total Weight}} := AIPWeight + AIP_{\text{ConsumablesWeight}} \]

\[ AIP_{\text{Oxygen Weight}} := \text{Sourceparameters Weight}_3, Option secondary kW-hr PowerAIP (AIP endur-24hr) \]

\[ AIP_{\text{Fuel Weight}} := \text{Sourceparameters Weight}_1, Option secondary kW-hr PowerAIP (AIP endur-24hr) \]

\[ AIP_{\text{Fuel Weight}} = 15.8 \text{ ton} \]

\[ AIP_{\text{Oxygen Weight}} = 55.1 \text{ ton} \]

\[ AIP_{\text{Total Weight}} = 96.5 \text{ ton} \]
VI. TANKS

A. Trim and Compensating Tanks:

The required range of sea water density that the ship is required to operate is:

\[ \rho_{\text{range}} := 40 \text{ kg/m}^3 \]

The sea water density is:

\[ \rho_{\text{sw}} := 1025 \text{ kg/m}^3 \]

The weight and volume of the stores is proportional to the number of crew and the total patrol endurance. The merchant marine reefer and dry stores allowances per man per day are the following:

\[ W_{\text{reefer\ per\ man\ day}} := 4.339 \text{ lb} \]
\[ V_{\text{reefer\ per\ man\ day}} := 0.32 \text{ ft}^3 \]
\[ W_{\text{dry\ per\ man\ day}} := 3.167 \text{ lb} \]
\[ V_{\text{dry\ per\ man\ day}} := 0.27 \text{ ft}^3 \]

Weight stores = \( (W_{\text{reefer\ per\ man\ day}} + W_{\text{dry\ per\ man\ day}}) - NT\cdot E \)

\[ \text{Weight stores} = 13135.5 \text{ lb} \]

For the trim and compensating tanks we will use the three tank system. It has a central tank (the compensating tank) as well a Fwd and Aft trim tanks. Only the central tank is connected to the sea and the two trim tanks are isolated and are used only for adjustments to longitudinal balance. Using the formula from the "Concepts in submarine design" we can estimate the total volume of the tanks. Then assuming that the trim tanks are 30% (15% each) of the calculated volume we will calculate the volume of each trim tank:

\[ \text{Trim}_{\text{Compens\ Vol}} := 1.05 \left( \frac{VPH_{\text{rho\ range}} + \text{Weight\ stores}}{\rho_{\text{sw}}} \right) \cdot \frac{1}{0.98} \]

\[ \text{Trim}_{\text{Compens\ Vol}} = 2012.8 \text{ ft}^3 \]

\[ \text{Vol}_{\text{trim\ fwd}} := 0.15 \cdot \text{Trim}_{\text{Compens\ Vol}} \]
\[ \text{Vol}_{\text{trim\ fwd}} = 301.9 \text{ ft}^3 \]
\[ \text{Weight}_{\text{trim\ fwd}} := \text{Vol}_{\text{trim\ fwd}} \cdot 1025 \text{ kg/m}^3 \]

\[ \text{Vol}_{\text{trim\ aft}} := 0.15 \cdot \text{Trim}_{\text{Compens\ Vol}} \]
\[ \text{Vol}_{\text{trim\ aft}} = 301.9 \text{ ft}^3 \]
\[ \text{Weight}_{\text{trim\ aft}} := \text{Vol}_{\text{trim\ aft}} \cdot 1025 \text{ kg/m}^3 \]

\[ \text{Weight\ trim\ fwd} = 8.6 \text{ Iton} \]
\[ \text{Weight\ trim\ aft} = 8.6 \text{ Iton} \]

\[ \text{Vol}_{\text{compensating}} := \text{Trim}_{\text{Compens\ Vol}} - \text{Vol}_{\text{trim\ fwd}} - \text{Vol}_{\text{trim\ aft}} \]
\[ \text{Vol}_{\text{compensating}} = 1409 \text{ ft}^3 \]

\[ \text{Weight}_{\text{compensating}} := \text{Vol}_{\text{compensating}} \cdot 1025 \text{ kg/m}^3 \]
\[ \text{Weight}_{\text{compensating}} = 40.2 \text{ Iton} \]

B. Fresh Water Tanks:

Assuming that each man needs 14 liter of fresh water per day we will size the fresh water tanks of the submarine:

\[ \text{Vol}_{\text{FW\ per\ man\ day}} := 14 \text{ liter} \]
\[ \text{Vol}_{\text{FW}} := \text{Vol}_{\text{FW\ per\ man\ day}} \cdot \text{(NT\cdot E)} \]
\[ \text{Vol}_{\text{FW}} = 865.2 \text{ ft}^3 \]
\[ \text{Weight}_{\text{FW}} := \text{Vol}_{\text{FW}} \cdot 1000 \text{ kg/m}^3 \]
\[ \text{Weight}_{\text{FW}} = 24.1 \text{ Iton} \]

Assuming that half of the fresh water is placed at the lower level of the MCR and the other half at the lower level of the OPS, we have the following:

\[ \text{Fw\ tank\ CIC} := \frac{\text{Vol}_{\text{FW}}}{2} \]
\[ \text{Fw\ tank\ CIC} = 432.6 \text{ ft}^3 \]
\[ \text{Weight}_{\text{Fw\ CIC}} := \text{Fw\ tank\ CIC} \cdot 1000 \text{ kg/m}^3 \]
\[ \text{Weight}_{\text{Fw\ CIC}} = 12.1 \text{ Iton} \]

\[ \text{Fw\ tank\ MCR} := \frac{\text{Vol}_{\text{FW}}}{2} \]
\[ \text{Fw\ tank\ MCR} = 432.6 \text{ ft}^3 \]
\[ \text{Weight}_{\text{Fw\ MCR}} := \text{Fw\ tank\ MCR} \cdot 1000 \text{ kg/m}^3 \]
\[ \text{Weight}_{\text{Fw\ MCR}} = 12.1 \text{ Iton} \]
C. Sanitary Tank:
Assuming that the sanitary tank is 10% of the fresh water tank we have the following:

\[ \text{Vol}_{\text{Sanitary}} = 0.1 \times \text{Vol}_{\text{FW}} \]
\[ \text{Weight}_{\text{Sanitary}} = \text{Vol}_{\text{Sanitary}} \times 1025 \text{ kg/m}^3 \]
\[ \text{Weight}_{\text{Sanitary}} = 2.5 \text{ ton} \]

D. Fuel Tanks:
Assuming that 2/3 of the fuel is placed at the lower level of the engineroom and 1/3 at the lower level of the berthing we have the following:

\[ V_{\text{DFM, ER}} = \frac{2}{3} \times V_{\text{DFM}} \]
\[ V_{\text{DFM, ER}} = 1939.6 \text{ ft}^3 \]
\[ V_{\text{DFM, OPS}} = \frac{1}{3} \times V_{\text{DFM}} \]
\[ V_{\text{DFM, OPS}} = 969.8 \text{ ft}^3 \]

D. Lube Oil Tank:
Using the formula from ASSET to calculate the lube oil of a diesel system we have the following:

\[ \text{Weight}_{\text{L.oil}} = 0.94 \times \frac{\text{Power}_{\text{diesel}}}{2.240 \text{hp}} \times \text{kg} \]
\[ \text{Weight}_{\text{L.oil}} = 1.7 \text{ ton} \]
\[ \text{Vol}_{\text{L.oil}} = \frac{\text{Weight}_{\text{L.oil}}}{910 \text{ kg/m}^3} \]
\[ \text{Vol}_{\text{L.oil}} = 67.6 \text{ ft}^3 \]

VII. VOLUME REQUIREMENTS
In order to determine the required volume of the different compartments we divide the PH into the following compartments:

- Engine room
- Machinery control room/Swichboard Room
- AIP room
- OPS (CIC & Berthing)
- Torpedo room

For each of the rooms we assign packing factors, and the required volume comes from the multiplication of the packing factor with the volume required by the main components of the room.

A. Engineroom Volume:
The main components of the engineroom are: The Propulsion Motor with its auxiliaries, the Diesel Engines, 2/3 of the Diesels Fuel, the AFT Trim Tanks.

\[ \text{Motor and aux Volume} = 1498.6 \text{ ft}^3 \]
\[ V_{\text{DFM, ER}} = 1939.6 \text{ ft}^3 \]
\[ \text{Volume}_{\text{diesel}} = 1244.9 \text{ ft}^3 \]
\[ \text{Vol}_{\text{trim aft}} = 301.9 \text{ ft}^3 \]
\[ \text{factor}_{\text{ER}} = 1.8 \]
\[ \text{ER VOL} = \text{factor}_{\text{ER}} \times \left( \text{Motor and aux Volume} + V_{\text{DFM, ER}} + \text{Volume}_{\text{diesel}} + \text{Vol}_{\text{trim aft}} + \text{Vol}_{\text{L.oil}} \right) \]
\[ \text{ER VOL} = 9094.7 \text{ ft}^3 \]

B. AIP System Room:
Based on the pictures of AIP systems we can derive the packing factor of the AIP system

\[ \text{AIP Total Vol} = 5097.6 \text{ ft}^3 \]
\[ \text{AIP VOL} = \text{AIP Total Vol} \]
\[ \text{AIP VOL} = 5097.6 \text{ ft}^3 \]
C. Machinery Control Room:

The main components of the lower level of the machinery control room are: The Aft battery, the Two aft FW tanks:

\[ V_{HALF\_BAT} = 3056.3 \text{ ft}^3 \quad V_{Tank\_Under\_Half\_Battery} = 893.2 \text{ ft}^3 \quad \text{Fw\_tank\_MCR} = 432.6 \text{ ft}^3 \]

Based on the above values we will size the MCR compartment:

\[ \text{factor\_MCR} := 2 \]

\[ \text{MCR\_VOL} := \text{factor\_MCR} \times (V_{HALF\_BAT} + V_{Tank\_Under\_Half\_Battery} + \text{Fw\_tank\_MCR}) \]

\[ \text{MCR\_VOL} = 8764.1 \text{ ft}^3 \]

D. OPS Compartment Volume

The main components of the OPS compartment are:

1. CIC:
   
   The CIC volume is defined by the owners requirements:
   \[ \text{CIC\_volume} = 2650 \text{ ft}^3 \]

2. Berth & Mess:
   
   If we assume that for each CPO and Enlisted we need:
   \[ V_{\_berth\_mess\_CPO\_Enlisted} = 100 \text{ ft}^3 \]
   
   and for every Officer we need:
   \[ V_{\_berth\_mess\_Officer} = 110 \text{ ft}^3 \]
   
   We have that the total volume is:
   \[ V_{\_berth\_mess} := V_{\_berth\_mess\_CPO\_Enlisted} \times N_{\_crew\_CPO\_Enlisted} + V_{\_berth\_mess\_Officer} \times N_{\_crew\_Officer} \]
   \[ V_{\_berth\_mess} = 2550 \text{ ft}^3 \]

3. Stores:

   \[ V_{\_reefer\_per\_man\_day} = 0.32 \text{ ft}^3 \]
   \[ V_{\_dry\_per\_man\_day} = 0.27 \text{ ft}^3 \]
   
   We can assume that half of the stores are placed on the lower level and half at the upper level:
   \[ V_{\_stores\_upper} := \frac{V_{\_stores}}{2} \]
   \[ V_{\_stores\_upper} = 516.3 \text{ ft}^3 \]

4. Lower Level:

   The main components of the lower level are: Fw fresh water tanks, Fw battery, sanitary tank, Fw fuel tanks, compensating tank, torpedo reloads, storeroom.

   \[ V_{\_stores\_lower} := \frac{V_{\_stores}}{2} \]
   \[ V_{\_stores\_lower} = 516.3 \text{ ft}^3 \]

   \[ \text{T\_Reloads\_vol} := 1.2 \times \text{RL\_T\_vol} \]
   \[ \text{Fw\_tank\_CIC} = 432.6 \text{ ft}^3 \]
   \[ V_{\_HALF\_BAT} = 3056.3 \text{ ft}^3 \]
   \[ V_{\_Tank\_Under\_Half\_Battery} = 893.2 \text{ ft}^3 \]
   \[ \text{Vol\_Sanitary} = 86.5 \text{ ft}^3 \]
   \[ V_{\_DFM\_OPS} = 969.8 \text{ ft}^3 \]
   \[ \text{Vol\_compensating} = 1409 \text{ ft}^3 \]
   \[ \text{T\_Reloads\_vol} = 2193.6 \text{ ft}^3 \]

   \[ \text{OPS\_VOL1} := \text{CIC\_volume} + V_{\_berth\_mess} + V_{\_stores\_upper} + V_{\_stores\_lower} + \text{Fw\_tank\_CIC} + V_{\_HALF\_BAT} \]
   \[ \text{OPS\_VOL2} := V_{\_Tank\_Under\_Half\_Battery} + \text{Vol\_Sanitary} + V_{\_DFM\_OPS} + \text{Vol\_compensating} + \text{T\_Reloads\_vol} \]

   \[ \text{OPS\_VOL} := \text{OPS\_VOL1} + \text{OPS\_VOL2} \]
   \[ \text{OPS\_VOL} = 15273.5 \text{ ft}^3 \]
E. Torpedo Room:

The main components of the torpedo room are: the torpedo tubes and the torpedoes compensating tanks:

The volume that includes the torpedo tubes, based on the picture from Janes Weapon Systems can be calculated as:

\[
\text{TR}_\text{VOL} := \text{Vol}_{\text{TT Space}} + \text{TT comp tank Vol}
\]

\[
\text{Vol}_{\text{TT Space}} = 1.8\text{TT-rem vol}
\]

\[
\text{TT comp tank Vol} = 852.6 \text{ ft}^3
\]

\[
\text{TR}_\text{VOL} = 4621.8 \text{ ft}^3
\]

F. PH Volume:

The following value is the value calculated by the above components:

\[
\text{Volume}_{\text{PH iteration}} := \text{ER VOL} + \text{MCR VOL} + \text{OPS VOL} + \text{TR VOL} + \text{AIP VOL}
\]

\[
\text{Volume}_{\text{PH}} = 42879.8 \text{ ft}^3
\]

\[
\text{Volume}_{\text{PH iteration}} = 42851.7 \text{ ft}^3
\]

\[
\text{dif} := \frac{\text{Volume}_{\text{PH iteration}} - \text{Volume}_{\text{PH}}}{\text{Volume}_{\text{PH iteration}}} \times 100
\]

\[
\text{dif} = -0.066 \%
\]

We assume that the program converges when the difference between the PH volumes of two runs is less than 1%.

VIII. INITIAL INTERNAL LAYOUT

\[
D_{\text{fw}} := D
\]

\[
\text{BT}_f := 0.511
\]

The above value is the fraction of the Fw Ballast Tank against all the total volume of the ballast tanks.

It will be an iteration value for the longitudinal balance of the submarine!

\[
\text{OB}_\text{mud} := 0.1
\]

\[
\text{FF}_\text{mud} := .01
\]

\[
\text{BT}_a := 1 - \text{BT}_f
\]

\[
\text{OB}_\text{fnb} := \frac{(1 - \text{OB}_\text{mud})}{2}
\]

\[
\text{VI MOD frac} := .426
\]

\[
\text{FF}_\text{amb} := \text{FF}_\text{fnb}
\]

\[
\text{OB}_\text{amb} := \text{OB}_\text{fnb}
\]

Check Area (all should total to 1!!)

\[
\text{BT}_f + \text{BT}_a = 1 \quad \text{OB}_\text{fnb} + \text{OB}_\text{amb} + \text{OB}_\text{mud} = 1 \quad \text{FF}_\text{fnb} + \text{FF}_\text{amb} + \text{FF}_\text{mud} = 1
\]

A. Aft FMBT Bulkhead:

Enter initial guess: \( \text{FMBT}_{\text{aft}} = 24.1 \text{ ft} \)

\[
\text{FMBT}_{\text{aft}} = \sqrt{\frac{\text{FMBT}_{\text{aft}}}{0}}
\]

\[
\text{offl(x2)} := \text{dx2} - \text{BT}_f \text{ Vbt - OB}_\text{fnb} \text{ Vol Main Hull} - \text{FF}_\text{fnb} \text{ Vff, FMBT}_{\text{aft}}
\]

\[
\text{FMBT}_{\text{aft}} = 25.992 \text{ ft}
\]
B. Fwd AMBT Bulkhead (Aft ER Bulkhead): Enter initial guess: \( ER_{\text{aft}} = 135 \text{-ft} \)

\[
ER_{\text{aft}} := \sqrt{L - \int_{ER_{\text{aft}}}^{L} \left(\frac{\text{off}(x)^2}{2} \pi \text{ dx} - \left(B_{\text{a}} V_{\text{bt}} + F_{\text{ambt}} V_{\text{ff}} + O_{\text{ambt}} V_{\text{ob Main Hull}}\right)\right)}
\]

\( ER_{\text{aft}} = 157.97 \text{-ft} \)

L - \( ER_{\text{aft}} = 40.77 \text{-ft} \)

C. Fwd After ER Bulkhead:

Enter initial guess: \( ER_{\text{fwd}} = 120 \text{-ft} \)

\[
ER_{\text{fwd}} := \sqrt{\int_{ER_{\text{fwd}}}^{ER_{\text{aft}}} \left(\frac{\text{off}(x)^2}{2} \pi \text{ dx} - \left(ER_{\text{b}} V_{\text{b}}, ER_{\text{fwd}}\right)\right)}
\]

\( ER_{\text{fwd}} = 129.48 \text{-ft} \)

\( ER_{\text{aft}} - ER_{\text{fwd}} = 28.49 \text{-ft} \)

\( \text{Thrust Bearing} = 5 \text{-ft} \quad \text{Motor Length} = 16 \text{-ft} \)

\( \text{Stack Length}_{ER_{\text{aft}}} = (\text{Motor Length} + \text{Thrust Bearing}) \times 1.25 \quad \text{Stack Length}_{ER_{\text{fwd}}} = 26.3 \text{-ft} \)

D. Fwd AIP System Section Bulkhead:

Enter initial guess: \( AIP_{\text{fwd}} = 110 \text{-ft} \)

\[
AIP_{\text{fwd}} := \sqrt{\int_{AIP_{\text{fwd}}}^{ER_{\text{fwd}}} \left(\frac{\text{off}(x)^2}{2} \pi \text{ dx} - \left(AIP_{\text{b}} V_{\text{b}}, AIP_{\text{fwd}}\right)\right)}
\]

\( AIP_{\text{fwd}} = 114.08 \text{-ft} \)

\( ER_{\text{fwd}} - AIP_{\text{fwd}} = 15.4 \text{-ft} \)

E. Fwd MCR Section Bulkhead:

Enter initial guess: \( MCR_{\text{fwd}} = 80 \text{-ft} \)

\[
MCR_{\text{fwd}} := \sqrt{\int_{MCR_{\text{fwd}}}^{AIP_{\text{fwd}}} \left(\frac{\text{off}(x)^2}{2} \pi \text{ dx} - \left(MCR_{\text{b}} V_{\text{b}}, MCR_{\text{fwd}}\right)\right)}
\]

\( MCR_{\text{fwd}} = 87.602 \text{-ft} \)

\( \text{Length MCR} := AIP_{\text{fwd}} - MCR_{\text{fwd}} \)

\( \text{Stack Length}_{MCR} := \text{Half Battery Length} \quad \text{Stack Length}_{MCR} = 22.2 \text{-ft} \)

\( MCR_{\text{fwd}} := \text{if}(\text{Stack Length}_{MCR} > \text{Length MCR}, AIP_{\text{fwd}} - \text{Stack Length}_{MCR}, MCR_{\text{fwd}}) \)

\( MCR_{\text{fwd}} = 87.6 \text{-ft} \)

F. Fwd OPS Bulkhead:

Enter initial guess: \( OPS_{\text{fwd}} = 40 \text{-ft} \)

\[
OPS_{\text{fwd}} := \sqrt{\int_{OPS_{\text{fwd}}}^{MCR_{\text{fwd}}} \left(\frac{\text{off}(x)^2}{2} \pi \text{ dx} - \left(OPS_{\text{b}} V_{\text{b}}, OPS_{\text{fwd}}\right)\right)}
\]

\( OPS_{\text{fwd}} = 41.46 \text{-ft} \)

G. Fwd Torpedo Room Bulkhead:

Enter initial guess: \( TR_{1\text{fwd}} = 25 \text{-ft} \)

\[
TR_{1\text{fwd}} := \sqrt{\int_{TR_{1\text{fwd}}}^{OPS_{\text{fwd}}} \left(\frac{\text{off}(x)^2}{2} \pi \text{ dx} - \left(TR_{1\text{b}} V_{\text{b}}, TR_{1\text{fwd}}\right)\right)}
\]

\( TR_{1\text{fwd}} = 27.26 \text{-ft} \)

Should be zero or very, very close!!!

\( TR_{1\text{fwd}} - FMB_{\text{aft}} = 1.27 \text{-ft} \)
IX. WEIGHT ESTIMATION

A. Weight Group Estimates

1. Weight Groups 1

The factor is added to give a consistent result with the breakdown of the known designs. It represents the
average of the structural weight of an MIT 13.414 Design Project, a UCL project, and my calculations for a notional
submarine. It is a percentage of the surfaced displacement.

\[ W_{1\text{frac}} : \approx k_{\text{struct}} \left( 0.0055 \frac{D_D}{\text{ft}} - 0.3048 + 0.13 \right) + 0.093 \]

\[ W_{1\text{frac}} = 0.402 \]

2. Weight Groups 2, 3

The factor 1.12 is inserted to account for all the auxiliary equipment of the weight groups 2 and 3. It comes from
the derivation of the total weight of 2 and 3 groups of the notional submarine over the same components.

(a5 is used to make the W23 equation shorter)

\[ a_5 : = \text{Motor and aux weight} + \text{Weight battery} + \text{Weight diesel} + \text{Weight DFM} \]

\[ W_{23} : = 1.12a_5 + (AIP_{\text{Total Weight}} - AIP_{\text{Oxygen Weight}} - AIP_{\text{Fuel Weight}}) \]

\[ W_{23_{\text{aux}}} = 0.12a_5 \]

\[ W_{23} = 465 \text{ ton} \]

Percentage of submerged displacement: \[ W_{23_{\text{per sub}}} = \frac{W_{23}}{\Delta_{sr}} \cdot 100 \]

\[ W_{23_{\text{per sub}}} = 31.2 \% \]

3. Weight Group 4

For the weight estimation of group 4 I used the formula from Stenard's thesis. The result is very close to the
estimate I had for the notional submarine.

\[ W_4 : = \left( 0.00836 \frac{\text{CIC volume}}{\text{ft}^3} \right) \cdot \text{ton} \]

\[ W_4 = 22.2 \text{ ton} \]

Percentage of submerged displacement: \[ W_{4_{\text{per sub}}} = \frac{W_4}{\Delta_{sr}} \cdot 100 \]

\[ W_{4_{\text{per sub}}} = 1.5 \% \]

4. Weight Groups 5, 6

In the calculation we will use a percentage of the Surfaced displacement based on the percentage of the notional
submarine.

\[ W_{56_{\text{fraction}}} : = 0.04 \]

5. Weight Group 7

Using Stenard's thesis and Conventional Submarine parametrics:

\[ W_7 : = \left( 0.002 \frac{T_{\text{Reloads vol}}}{\text{ft}^3} + 6 \cdot \text{TT} + 5 \right) \cdot \text{ton} \]

\[ W_7 = 45.4 \text{ ton} \]

Percentage of submerged displacement: \[ W_{7_{\text{per sub}}} = \frac{W_7}{\Delta_{sr}} \cdot 100 \]

\[ W_{7_{\text{per sub}}} = 3 \% \]

6. Weight of Pb: The following is the fraction of the lead over the surface displacement.

\[ W_{\text{Pb frac}} : = 0.0845 \]
7. Variable Weight

In this calculation of the variable weight we will not include the weight of the fuel since it is included to the weight of the Groups 2, 3. We assume that half of the trim tanks are full and also that the compensating tanks are half full. We are adding also 0.15 Iton per crew member to account for the weight of the people and their luggage. Hence the following items are the VLI:

- Weight_{trim fwd} = 8.6 Iton
- Weight_{trim aft} = 8.6 Iton
- Weight_{compensating} = 40.2 Iton
- Weight_{FW} = 24.1 Iton
- Weight_{FW CIC} = 12.1 Iton
- Weight_{FW MCR} = 12.1 Iton
- Weight_{Sanitary} = 2.5 Iton
- Weight_{stores} = 5.9 Iton
- Weight_{L oil} = 1.7 Iton
- AIP_{Fuel Weight} = 15.8 Iton
- AIP_{Oxygen Weight} = 55.1 Iton

W_{VL} := \frac{(\text{Trim Compens Vol})-641b}{20^3} + \text{Weight}_{FW} + \text{Weight}_{stores} + \text{Crew weight} + \text{Weight}_{L oil} + \text{AIP}_{Fuel Weight} + \text{AIP}_{Oxygen Weight}

W_{VL} = 135 Iton

Calculations

Add the equations for W1, W23, W4, W56, W7, Wpb and Wvl together and solve for NSC:

\[ \Delta_{surf} := \frac{W_{23} + W_{7} + W_{4} + W_{VL}}{1 - (W1Frac + WPBFrac + W56_fraction)} \]

W1 := \Delta_{surf} \cdot W1Frac

W56 := \Delta_{surf} \cdot W56_fraction

Wpb := WPBFrac \cdot \Delta_{surf}

\[ \Delta A1 := W1 + W23 + W4 + W56 + W7 \]

\[ \Delta A := \Delta A1 + Wpb \]

\[ \Delta_{surf CHK} := \Delta A + WVL \]

WEIGHT SUMMARY:

- W1 = 537.3 Iton
- W23 = 465 Iton
- W4 = 22.2 Iton
- W56 = 53.4 Iton
- W7 = 45.4 Iton
- \Delta A1 = 1123.2 Iton
- Wpb = 77.5 Iton
- \Delta A = 1200.7 Iton
- WVL = 135.04 Iton

###CHK-should be the same

\[ \Delta_{surf CHK} = 1335.7 Iton \]

\[ \Delta_{surf} = 1335.7 Iton \]
X. BALANCE WEIGHT VS BOUYANT VOLUME

This is the first opportunity to bring Weight and Volume calculations together. There are many different ways to compare the two, we will use everbuoyant volume \( V_{eb} \) and NSC \( \Delta_{surf} \) weights.

**NOTE:** All use of "volumes" in this section will be expressed in terms of Itons.

\[
V_{eb} = V_{eb\text{Main}} + V_{H2\text{outPH}} \\
V_{eb\text{Main}} = 0 \text{ Itons}
\]

\[
\Delta_{eba} = \frac{V_{PH} + V_{eb}}{35 \text{ ft}^3} = 1323.1 \text{ ton} \\
\Delta_{surf} = 1335.7 \text{ ton} \\
\text{Err} = \frac{\Delta_{eba} - \Delta_{surf}}{\Delta_{eba}} = -0.00952
\]

The tolerance at this point is \(-0.01 < \text{Err} < 0.01\). If out of tolerance, continue reading.

**A. WEIGHT LIMITED CASE \( (V_{eb} < \Delta_{surf}) \):** Due to the uncertainty in your design at this point, you do not have the luxury of shaving weight. For small differences we can reduce the lead fraction but not less than 0.08. For greater differences we need to add buoyancy. Return to Section III. and change the length (adding parallel mid-body) or \( D \) as appropriate for the magnitude of your imbalance. Recognize that your volume available will be greater than the previously calculated required volume and at the same time higher power will be required for the submarine to have the same performance.

**B. VOLUME LIMITED CASE \( (V_{eb} > \Delta_{surf}) \):** Due to the uncertainty in your design you do not have the luxury of "tightening up the design." At this point add lead (fixed ballast) you will have what is termed as a "lead mine." The best, and most consistent approach would be to increase \( W_{PB\text{fix}} \) until the Err above is within tolerance. Alternatively you could choose to be less structurally efficient or a higher specific propulsion weight.

**C. \( \Delta_{eb} = \Delta_{surf} \):** After a couple iteration you should reach this condition.

XI. LONGITUDINAL BALANCE:

**A. ENVELOPE LCB CALCULATION.** Use the previously calculated parameters to determine the LCB of the envelope:

\[
\text{LCB}_{\Delta e} = \frac{\int_0^L \text{offh(x)}^2 \cdot x \cdot dx}{\int_0^L \text{offh(x)}^2 \cdot dx} \quad \text{LCB}_{\Delta e} = 91.8 \text{ ft}
\]

**B. SUBMERGED LCB CALCULATION.**

\[
\text{LCB}_{\text{fmbt}} = \frac{\text{FMBT}_{\text{ft}}}{2} \\
\text{LCB}_{\text{fambt}} = \frac{2}{3} \left(L - \text{EAFT}_{\text{ft}}\right) + \text{EAFT}_{\text{ft}}
\]

If we assume that:

\[
\text{LCB}_{\Delta e} = \text{LCB}_{\Delta e} \quad \text{and} \quad \text{LCB}_{ff} = \text{LCB}_{\text{fmbt}} - \text{FF}_{\text{mbt}} + \text{LCB}_{\text{fambt}} - \text{FF}_{\text{ambt}}
\]

\[
\Delta_{v} = \frac{V_{f} - 64}{\text{ft}^3} \quad \Delta_{ff} = \frac{V_{ff} - 64}{\text{ft}^3} \quad \Delta_{ff} = 80 \text{ ton} \quad \Delta_{v} = 1488.5 \text{ ton}
\]

\[
\text{LCB}_{ff} = 98.1 \text{ ft} \quad \text{LCB}_{\Delta e} = 91.8 \text{ ft} \quad \text{LCB}_{\Delta e} = 91.8 \text{ ft}
\]

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C. NSC LCB CALCULATION. Since the draft at NSC is not known, LCB must be determined from a plot of LCE vs. draft and displacement vs. draft. Therefore, we must calculate curves of form:

1. Calculate displacement and LCB at draft=3/4D. 

\[ y_{3q}(x_l) = \frac{D}{4} \]

Find x location for forward and aft three quarter D waterline:

Initial guess forward: \( x_2 := 0.1 \text{ ft} \)
\( x_{fwd} := \sqrt{\text{offt}(x_2) - \frac{D}{4} \cdot x_2} \) \( x_{fwd} = 1.7 \text{ ft} \)

Initial guess aft: \( x_3 := 200 \text{ ft} \)
\( x_{aft} := \sqrt{\text{offt}(x_3) - \frac{D}{4} \cdot x_3} \) \( x_{aft} = 184.9 \text{ ft} \)

Calculate the area below the waterline:

\[ \Theta(x_l) := \pi + 2 \arcsin \left( \frac{D}{4 \cdot \text{offt}(x_l)} \right) \]
\[ V_{3q} := \int_{0 \text{ ft}}^{x_{fwd}} \pi \cdot \text{offt}(x_l)^2 \, dx_l + \int_{x_{fwd}}^{x_{aft}} \frac{\text{offt}(x_l)^2}{2} (\Theta(x_l) - \sin(\Theta(x_l))) \, dx_l + \int_{x_{aft}}^L \pi \cdot \text{offt}(x_l)^2 \, dx_l \]

\[ \Delta_{3q} := \frac{V_{3q} \cdot 64}{\text{ft}^3} \]

\[ A_{3q} := \int_{0 \text{ ft}}^{x_{fwd}} x_1 \pi \cdot \text{offt}(x_l)^2 \, dx_l + \int_{x_{fwd}}^{x_{aft}} x_1 \left[ \frac{\text{offt}(x_l)^2}{2} (\Theta(x_l) - \sin(\Theta(x_l))) \right] \, dx_l + \int_{x_{aft}}^L x_1 \pi \cdot \text{offt}(x_l)^2 \, dx_l \]

\[ LCB_{3q} := \frac{A_{3q}}{V_{3q}} \]

\[ V_{3q} = 44939.7 \text{ ft}^3 \]
\[ \Delta_{3q} = 1284 \text{ ton} \]
\[ LCB_{3q} = 92.2 \text{ ft} \]

Correct \( T = 0.75D \) (3Q) for freefloods which are still submerged - input % & LCB:

Input fraction of freefloods still submerged: \( \text{ff}_{3qfrac} := 0.75 \)
Input LCB of freefloods still submerged: \( LCB_{3qff} = LCB_{eff} \)

\[ \Delta_{3qcorr} := (\Delta_{3q} - \text{ff}_{3qfrac} \cdot \Delta_{ff}) \]
\[ LCB_{3qcorr} := \frac{LCB_{3q} \cdot \Delta_{3q} - LCB_{3qff} \cdot (\text{ff}_{3qfrac} \cdot \Delta_{ff})}{\Delta_{3qcorr}} \]

\[ LCB_{3qcorr} = 91.9 \text{ ft} \]
2. Calculate displacement and LCB at draft=1/4 D

\[
\begin{align*}
V_{1q} &= \int_{xaft}^{xfwd} \pi \cdot \text{off}(x_1)^2 \, dx_1 - \int_{xfwd}^{xaft} \frac{\text{off}(x_1)^2}{2} \left( \Theta(x_1) - \sin(\Theta(x_1)) \right) \, dx_1 \\
\Delta_1q &= \frac{V_{1q} \cdot 64}{ft^3} \\
\Delta_1q &= 637401.8 \text{lb}
\end{align*}
\]

\[
\begin{align*}
\text{LCB}_{1q} &= \frac{\int_{xaft}^{xfwd} \pi \cdot \text{off}(x_1)^2 \cdot x_1 \, dx_1 - \int_{xfwd}^{xaft} x_1 \left[ \frac{\text{off}(x_1)^2}{2} \left( \Theta(x_1) - \sin(\Theta(x_1)) \right) \right] \, dx_1}{V_{1q}} \\
\text{LCB}_{1q} &= 89.9 \text{ft}
\end{align*}
\]

\[
\begin{align*}
\text{ffl}_{qfrac} &= .1 \\
\Delta_1qcorr &= \left( \Delta_1q - \text{ffl}_{qfrac} \cdot \Delta f \right) \\
\text{LCB}_{1qcorr} &= \text{LCB}_{1q} \\
\Delta_1qcorr &= 61942.8 \text{lb}
\end{align*}
\]

\[
\begin{align*}
\text{LCB}_{1qcorr} &= \frac{\text{LCB}_{1q} \cdot \Delta_1q - \text{LCB}_{1q} \cdot \left( \text{ffl}_{qfrac} \cdot \Delta f \right)}{\Delta_1qcorr} \\
\text{LCB}_{1qcorr} &= 89.7 \text{ft}
\end{align*}
\]

3. Plot Construction: Fill in arrays for Draft (vt), Displacement (vd) and LCB (vl). This will be done at drafts of 0, 0.25D, 0.5D, 0.75D, D.

\[
t = 0 \text{ ft}, 1 \text{ ft} \ldots D
\]

\[
\begin{align*}
\text{vd} &= \text{Re} \begin{bmatrix} 0 \text{ ft} \\ \Delta_1qcorr \\ \Delta_{enva} \end{bmatrix} \\
\text{vt} &= \text{Re} \begin{bmatrix} 0 \text{ ft} \\ 10.3 \text{ ft} \\ 15.4 \text{ ft} \\ 20.5 \text{ ft} \end{bmatrix} \\
\text{vl} &= \text{Re} \begin{bmatrix} 87.1 \text{ ft} \\ 89.7 \text{ ft} \\ 91.9 \text{ ft} \\ 91.8 \text{ ft} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\text{Ds} &= \text{cspline(vt, vd)} \\
f(t) &= \text{interp(Ds, vt, vd, t)} \\
\text{Ls} &= \text{lspline(vt, vl)} \\
f(t) &= \text{interp(Ls, vt, vl, t)}
\end{align*}
\]

Disp vs. T

LCB vs. T
Recall: \( \Delta_{eba} = 1323.1 \) ton \( \Delta_{surf} = 1335.7 \) ton

Enter initial guess for draft at NSC: 
\[ t_1 = 18 \text{ ft} \]
\[ \text{Draft} = 16.8 \text{ ft} \]
\[ \text{LCB}_{nsc} = 91.819 \text{ ft} \]

D. MBT LCB Calculations:

1. Desired MBT LCB:

\[
\text{LCB}_{MBT} := \frac{\Delta_{as} \Delta_s - \Delta_{eba} \Delta_{eba}}{\Delta_s - \Delta_{eba}} \]
\[ \text{LCB}_{MBT} = 91.2 \text{ ft} \]

2. Actual MBT LCB:

\[
V_{fmbt} := BT_f V_{ft} \quad V_{fmbt} = 2958.1 \text{ ft}^3
\]
\[
L_{fmbt} := \int_0^{FMBA_\text{ft}} \frac{\text{off}(x_1)^2 \pi x_1 \, dx_1}{\text{off}(x_1)^2 \pi x_1 \, dx_1}
\]
\[ L_{fmbt} = 91.2 \text{ ft} \]
\[
V_{ambt} := BT_a V_{at} \quad V_{ambt} = 2830.7 \text{ ft}^3
\]
\[
L_{ambt} := \int_0^{L} \frac{\text{off}(x_1)^2 \pi x_1 \, dx_1}{\text{EF}_{aft}}
\]
\[ L_{ambt} = 91.2 \text{ ft} \]
\[
\text{LCB}_{mbt} := \frac{L_{fmbt} V_{fmbt} + L_{ambt} V_{ambt}}{V_{ambt} + V_{fmbt}}
\]
\[ \Delta_{mbt} = 165.4 \text{ ton} \]

Placing the MBT LCB at the desired location will give you a zero surfaced trim. However, it is more imperative that the submerged LCG match the submerged LCB. Optimally, both conditions should be achieved, but realistically, ensure that the submerged LCG matches the submerged LCB and ensure that the sub is trimmed by the stern on the surface. This will be achieved by placing the stability lead at the appropriate position.

The longitudinal center of buoyancy of the Ballast tanks is now:
\[ \text{LCB}_{mbt} = 91.25 \text{ ft} \quad \text{It should be:} \quad \text{LCB}_{MBT} = 91.23 \text{ ft} \]
\[ \text{LCB}_{MBT} := \text{LCB}_{mbt} \]

Iteration point: change the percentage of the FW Ballast tank at the begining of VIII section in order to satisfy the above condition.
XII. VCB Estimate for Surfaced Stability Calculations.

Estimates Sub as a right cylinder floating out of the water with a draft corresponding to the NSC condition.

\[
\text{Area}_{\text{Above WL}} := \int_0^{\text{Draft}} \left( \frac{D}{2} \right)^2 - x^2 1 \, dy \, dx \quad \text{ft}^2
\]

\[
\text{Area}_{\text{Submerged}} := \left( \frac{D}{2} \right)^2 \pi - \text{Area}_{\text{Above WL}}
\]

\[
\text{VCBArea}_{\text{Above WL}} := \frac{\int_0^{\text{Draft}} \left( \frac{D}{2} \right)^2 - x^2 y \, dy \, dx}{\text{Draft} \, \frac{D}{2} \pi} \quad \frac{D^3}{2} \quad \text{Area}_{\text{Submerged}}
\]

\[
\text{VCBNSC} := \frac{\left( \frac{D}{2} \right)^2 \pi - \text{Area}_{\text{Above WL}} \, \text{VCBArea}_{\text{Above WL}}}{\text{Area}_{\text{Submerged}}} \quad \text{VCBNSC} = 9.7 \, \text{ft}
\]

XIII. CENTERS OF GRAVITY CALCULATIONS

A. VL LCG:

Variable Loads are divided into two groups Variable Load Items (VL1) and Variable Ballast Tanks (VB).

The components of the variable load are the following:

- Weight trim fwd = 8.6 ton
- Weight trim aft = 8.6 ton
- Weight compensating = 40.2 ton
- Weight stores = 5.9 ton
- Weight FW CIC = 12.1 ton
- Weight FW MCR = 12.1 ton
- Weight Sanitary = 2.5 ton
- Crew weight = 0.15 \( \left[ \text{NT} \right] \) ton
- Weight L oil = 1.7 ton
- AIP Fuel Weight = 15.8 ton
- AIP Oxygen Weight = 55.1 ton

a. Longitudinal Center of Gravity

The trim tanks are placed at the two ends of the ship:

- \( \text{LC}_{\text{trim fwd}} := \text{TR}_{\text{fwd}} + 3 \, \text{ft} \)
- \( \text{LC}_{\text{trim aft}} := \text{ER}_{\text{aft}} - 3 \, \text{ft} \)
- \( \text{LC}_{\text{trim fwd}} = 30.3 \, \text{ft} \)
- \( \text{LC}_{\text{trim aft}} = 155 \, \text{ft} \)

The compensating tank is placed at the center of buoyancy for the submerged submarine:

- \( \text{LC}_{\text{comp}} := \text{MCR}_{\text{fwd}} - 4 \, \text{ft} \)
- \( \text{LC}_{\text{comp}} = 83.6 \, \text{ft} \)

Half of the fresh water as it was stated before it is placed to the CIC and half to the MCR:

- \( \text{LC}_{\text{FW CIC}} := \frac{\text{OPS}_{\text{fwd}} + \text{MCR}_{\text{fwd}} - \text{OPS}_{\text{fwd}}}{2} \)
- \( \text{LC}_{\text{FW MCR}} := \frac{\text{MCR}_{\text{fwd}} - \text{AIP}_{\text{fwd}}}{2} \)
For all the fresh water we have: \[ \text{Weight}_{FW} = 24.1 \text{ ton} \]
\[ \text{LC}_{FW} = \frac{\text{Weight}_{FW\_CIC} + \text{Weight}_{FW\_MCR} + \text{Weight}_{FW\_MCR}}{\text{LC}_{FW}} \]
\[ \text{LC}_{FW} = 69.4 \text{ ft} \]

The weight of the crew can be assumed to be near the center of the OPS compartment:
\[ \text{LC}_{\text{crew}} := \text{OPS}_{fwd} + \frac{3\text{MCR}_{fwd} - \text{OPS}_{fwd}}{5} \]

The stores are placed at the OPS compartment:
\[ \text{LC}_{\text{Stores}} := \text{OPS}_{fwd} + \frac{\text{MCR}_{fwd} - \text{OPS}_{fwd}}{2} \]

The sanitary tank is placed at the OPS compartment:
\[ \text{LC}_{\text{Sanitary}} := \text{OPS}_{fwd} + 3 \frac{\text{MCR}_{fwd} - \text{OPS}_{fwd}}{5} \]

The AIP Oxygen is placed at the lower part of the AIP compartment:
\[ \text{LC}_{\text{ox}} := \text{AIP}_{fwd} + \frac{(\text{AIP}_{fwd} - \text{ER}_{fwd})}{2} \]

The Lube Oil can be assumed to be placed at the close to the center of the Engine Room:
\[ \text{LC}_{\text{L-oil}} := \text{ER}_{fwd} + \frac{4(\text{ER}_{aft} - \text{ER}_{fwd})}{5} \]

The AIP fuel is placed at the AIP compartment for all the conditions except for the Fuel Cell system with metal hydride storage of Hydrogen, where it is placed near the keel of the submarine. However we will assume that the longitudinal center of the fuel is at the center of the AIP compartment:
\[ \text{LC}_{\text{AIP\_fuel}} := \text{AIP}_{fwd} + \frac{\text{AIP}_{fwd} - \text{ER}_{fwd}}{2} \]

In order to make the equation LCGvl shorter we are using \( a_4, b_4, c_4, d_4 \):
\[ a_4 := \frac{\text{Weight}_{trim\_aft} + \text{Weight}_{trim\_fwd} + \text{Weight}_{comp\_weight\_compensating}}{2} \]
\[ b_4 := \frac{\text{Weight}_{FW\_CIC} + \text{Weight}_{FW\_MCR} + \text{Crew\_weight} + \text{Weight}_{\text{Sanitary}}}{2} \]
\[ c_4 := \frac{\text{AIP\_Oxygen\_Weight}}{2} \]
\[ d_4 := \frac{\text{Weight}_{\text{L-oil}} + \text{Weight}_{\text{Stores}}}{2} \]

Hence the longitudinal center of the variable load is given by:
\[ \text{LC}_{\text{VL}} := \frac{a_4 + b_4 + c_4 + d_4}{WVL} \]
\[ \text{LC}_{\text{VL}} = 95 \text{ ft} \]

b. Vertical Center of Gravity

The vertical centers of gravity can be assumed to be the following:
\[ \text{VC}_{\text{trim\_fwd}} := 0.5D \]
\[ \text{VC}_{\text{trim\_aft}} := 0.5D \]
\[ \text{VC}_{\text{Stores}} := 0.5D \]
\[ \text{VC}_{\text{FW\_CIC}} := 0.4D \]
\[ \text{VC}_{\text{FW\_MCR}} := 0.4D \]
\[ \text{VC}_{\text{Sanitary}} := 0.25D \]
\[ \text{VC}_{\text{crew}} := 0.7D \]
\[ \text{VC}_{\text{ox}} := 0.7D \]
\[ \text{VC}_{\text{L-oil}} := 0.35D \]
For the vertical center of gravity of the AIP fuel we have:

\[ VC_{AIP\_fuel} := 0.35D \]

\[ VC_{AIP\_fuel} := 0.25D \quad \text{if} (\text{Option\_secondary} = 1, 18, 0.25D) \]

In order to make the equation \( VCGvi \) shorter we are using \( a5, b5 \):

\[ a5 := VC_{FW\_CIC-Weight\_FW\_CIC} + VC_{FW\_MCR-Weight\_FW\_MCR} + VC_{crew\_Weight\_crew} + VC_{Sanitary\_Weight\_Sanitary} \]

\[ b5 := VC_{ex-AIP\_Oxygen\_Weight} + \text{AIP\_Fuel\_Weight} - VC_{AIP\_fuel} + \text{Weight\_L-oil} - VC_{L\_oil} + VC_{Stores\_Weight\_stores} \]

\[ VCGvi := \frac{a5 + b5}{WVL} \]

\[ VCGvi = 9\text{ft} \]

B. Standard Displacement (A) LCG:

\[ LCG_A := \frac{A_{surf\_LCB} - WVL\cdot LCGVL}{A_A} \]

\[ LCG_A = 91.5\text{ft} \]

C. A-1 LCG: \( LCG_{A1} \) is determined by determining the center of weight groups 1 through 7

Based on the arrangements of the baseline submarine we can derive the center of gravity of the different weight groups:

1. Weight Group 1

We assume that the longitudinal center of gravity is:

\[ LC_{1} := 0.43L \]

For the vertical center of gravity for the weight group 1 I am using the same assumption with the 13.414 project of 1999:

\[ VC_{1} := 0.52D \]

2. Weight Groups 2,3

a. Longitudinal Center of Gravity

Based on the arrangements of the baseline submarine we have the following:

\[ LC_{motor} := \frac{4\cdot (ER_{aft} - ER_{fwd})}{5} \]

\[ LC_{dieisel} := ER_{fwd} + \frac{1\cdot (ER_{aft} - ER_{fwd})}{3} \]

\[ LC_{AIP} := ER_{fwd} + \frac{ER_{fwd} - AIP_{fwd}}{2} \]

\[ LC_{aft\_bat} := AIP_{fwd} + \frac{MCR_{fwd} - MCR_{fwd}}{2} \]

\[ LC_{fwd\_bat} := OPS_{fwd} + \frac{2\cdot MCR_{fwd} - OPS_{fwd}}{5} \]

\[ LC_{23\_OPS} := OPS_{fwd} + \frac{2\cdot MCR_{fwd} - OPS_{fwd}}{5} \]

\[ LC_{23\_aux} := 0.6L \]

In order to make the equation \( LC_{23} \) shorter we are using \( a6, b6 \):

\[ a6 := \text{Motor\_and\_aux\_Weight}\_LC_{motor} + \text{Weight\_fwd}\_LC_{dieisel} + \frac{2}{3}\cdot \text{Weight\_DFM}\_LC_{DFM\_ER} + \text{AIP\_Weight}\_LC_{AIP} \]

\[ b6 := \text{Whalf\_battery}\_LC_{aft\_bat} + \text{Whalf\_battery}\_LC_{fwd\_bat} + \frac{1}{3}\cdot \text{Weight\_DFM}\_LC_{DFM\_OPS} + W_{23\_aux}\_LC_{23\_aux} \]

\[ LC_{23} := \frac{a6 + b6}{W_{23}} \]

\[ LC_{23} = 105\text{ft} \]
b. Vertical Center of Gravity

Based on the arrangements of the baseline submarine we have the following:

\[
\begin{align*}
VC_{motor} &= \frac{2D}{5} \\
VC_{diesel} &= \frac{D}{2} \\
VC_{DFM\_ER} &= \frac{D}{4} \\
VC_{aft\_bat} &= \frac{D}{4} \\
VC_{fw\_bat} &= 0.5D \\
VC_{DFM\_OPS} &= \frac{D}{4} \\
VC_{23\_aux} &= 0.5D
\end{align*}
\]

We can assume that the vertical center of gravity of the system without hydride is:

\[
VC_{AIP\_no\_hydride} = 0.4D
\]

For the AIP system we have to take into account the situation where we choose the Fuel Cell with the Metal Hydride storage of the Hydrogen. At this case we have a significant amount of weight near the keel.

\[
VC_{AIP} = \frac{W_{Hydride} + (AIP\_Total\_Weight - W_{Hydride}) - VC_{AIP\_no\_hydride}}{AIP\_Total\_Weight}
\]

(If we have a different AIP option the formula gives the correct result, since the \(W_{Hydride}\) will be zero)

In order to make the equation \(VC_{23}\) shorter we are using \(a7, b7:\)

\[
\begin{align*}
a7 &= Motor\_and\_auxweight \cdot VC_{motor} + Weight\_diesel \cdot VC_{diesel} + \frac{2}{3} \cdot Weight\_DFM \cdot VC_{DFM\_ER} + AIP\_Total\_Weight \cdot VC_{AIP} \\
b7 &= W_{half\_battery} \cdot VC_{aft\_bat} + W_{half\_battery} \cdot VC_{fw\_bat} + \frac{1}{3} \cdot Weight\_DFM \cdot VC_{DFM\_OPS} + W_{23\_aux} \cdot VC_{23\_aux}
\end{align*}
\]

\[
VC_{23} = \frac{a7 + b7}{W_{23}}
\]

3. Weight Group 4

Based on the arrangements of the baseline submarine:

\[
LC_{4} = \frac{MCR_{fwd} + \frac{W_{OPS_{fwd}} - W_{OPS_{fwd}}}{5}}{W_{7}}
\]

For the vertical center of gravity for the weight group 4 we are using the same assumption with the 13.414 project of 1999:

\[
VC_{4} = 0.65D
\]

4. Weight Groups 5, 6

We assume that the center of gravity is: \(LC_{5,6} = 0.31\).

For the vertical center of gravity for the weight groups 5, 6 we are using the same assumption with the 13.414 project of 1999:

\[
VC_{5,6} = 0.65D
\]

5. Weight Group 7

Based on the arrangements of the baseline submarine:

\[
LC_{7} = \frac{T_{weight\_RL} \cdot \left(\frac{W_{OPS_{fwd}} + \frac{MCR_{fwd} - W_{OPS_{fwd}}}{5}}{W_{7}}\right) + \left(W_{7} - T_{weight\_RL}\right) \cdot \left(\frac{TR_{fwd} + \frac{W_{OPS_{fwd}} - TR_{fwd}}{2}}{W_{7}}\right)}{W_{7}}
\]

For the vertical center of gravity for the weight group 7 we are using the same assumption with the 13.414 project of 1999:

\[
VC_{7} = 0.46D
\]
To summarize we have the following:

\[
\begin{array}{cccc}
\text{W}_i & \text{W}_i \text{ton} & \text{LCG}_i & \text{VCG}_i \\
537.3 & 85.5 & \text{VCG}_1 & 10.7 \\
485 & 105 & \text{VCG}_2 & 7.4 \\
22.2 & 69.1 & \text{VCG}_4 & 13.3 \\
53.4 & 59.6 & \text{VCG}_5 & 13.3 \\
45.4 & 39.9 & \text{VCG}_7 & 9.4 \\
\end{array}
\]

Sum up the A-1 weights and determine their centers:

\[
\begin{align*}
\sum \text{W}_i \cdot \text{LCG}_i & = \text{LCG}_{A1} = 90.1 \text{ ft} \\
\sum \text{W}_i \cdot \text{VCG}_i & = \text{VCG}_{A1} = 9.4 \text{ ft}
\end{align*}
\]

D. LEAD LCG: \( \text{LCG}_{\text{lead}} \) is determined by making \( A \) & \( A-1 \) consistent.

\[
\text{LCG}_{\text{lead}} = \frac{\Delta A \cdot \text{LCG}_A - \Delta A_1 \cdot \text{LCG}_A}{W_{pb}}
\]

Check if the lead is inside the Boat!

**XIV. STABILITY:**

The following is a process which steps through the methodology suggested by section 6-5. Lead (or Fixed Ballast) is divided into Stability (\( \text{LEAD}_s \)) and Margin (\( \text{LEAD}_m \)). Section 6-5 suggests 1 weight equation and two moment equations. The first moment equation was used in VI.B.4. above. Thus we are left with 2 equations and 5 unknowns:

\[
\text{LEAD}_s + \text{LEAD}_m = \text{LEAD}
\]

\[
\text{LEAD}_s \cdot \text{VCG}_{\text{LEAD}_s} + \text{LEAD}_m \cdot \text{VCG}_{\text{LEAD}_m} = \text{LEAD} \cdot \text{VCG}_{\text{lead}}
\]

However, by applying some basic assumptions, we can reduce the number of unknowns.

First, we will set \( BG = 1 \). We can also state that, due to our shape being a body of revolution, \( \text{VCB}_{\text{sub}} = D/2 \).

Therefore, \( \text{VCB}_{\text{sub}} = D/2 - 1 \). Similarly, \( \text{VCB}_{\text{MBT}} \) can be assumed to be \( D/2 \). Now we can determine \( \text{VCG}_{\text{NSC}} \):

A. NSC VCG:

For the BG we will use the following value: \( BG := 1 \text{ ft} \)

\[
\text{VCG}_{\text{NSC}} = \frac{\left( \frac{D}{2} - BG \right) \Delta_5 - \frac{D}{2} (\Delta_5 - \Delta_{\text{surf}})}{\Delta_{\text{surf}}}
\]

\( \text{VCG}_{\text{NSC}} = 9.2 \text{ ft} \) \( \text{VCB}_{\text{NSC}} = 9.7 \text{ ft} \)
B. Condition A VCG:
1. Calculate VCG for VL: \( VCG_{VL} = 9 \text{ ft} \)
2. Use a moment equation of NSC-VL to yield A
   \[
   \frac{\Delta A}{\Delta A} = \frac{\Delta A_{VCG_{NSC}} - W_{VL} VCG_{VL}}{W_{VL}}
   \]
   \( VCG_A = 9.2 \text{ ft} \)

C. A-1 VCG:
Recall the VCG calculated above:
   \( VCG_{A1} = 9.4 \text{ ft} \)

D. Finally we can solve for VCG LEAD:
   \[
   VCG_{LEAD} = \frac{\Delta A \cdot VCG_A - \Delta A_{1} \cdot VCG_{A1}}{W_{p}}
   \]
   \( VCG_{LEAD} = 5.2 \text{ ft} \)

E. Stability and Margin ballast weights:

We will assume that Stability ballast will be placed \( S_b \) above the baseline and the Margin ballast will be placed at \( M_b \) above the baseline.

   \[ S_b := 2 \text{ ft}, \quad M_b := \frac{D}{2} \]
   \[ W_{pbs} := \left( \frac{L.D. - 1 \cdot S_b}{S_b} \right), \quad W_{p} = 47.2 \text{ ton} \]
   \[ W_{pbm} := \left( \frac{L.D. - 1 \cdot S_b}{M_b} \right), \quad W_{pbm} = 30.3 \text{ ton} \]

   \textbf{Check if both are positive!}

We might need to iterate and change the position of the margin lead in order to have positive numbers.

F. Stability and Margin ballast LCG:

It's time to revisit LCGs to place stability and margin lead longitudinally. Usually the margin ballast is added at the forward half of the boat, since it is the part that it is more likely to have an increase in weight.

   \[ LCG_{pbm} := 0.4 \text{ ft} \]

We choose the value in such a way that the locations below are within the envelope. We might need to iterate to get reasonable numbers.

   \[ LCG_{LEAD} := \frac{W_{pb} \cdot LCG_{LEAD} - W_{pbm} \cdot LCG_{pbm}}{W_{pbs}}, \quad LCG_{LEAD} = 130.67 \text{ ft}, \quad LCG_{pbm} = 79.5 \text{ ft}, \quad LCG_{LEAD} = 110.7 \text{ ft} \]

G. LAST CHECK!!! Are the lead piles inside the envelope?

The least impact fix is internal re-arrangements. The next best is to use some margin lead for trim lead. One approach is to assign a reasonable center to stability and trim lead and solve for trim lead:

   \[
   \text{Initial guess:} \quad LCG_{pbt} := 0 \text{ ft}, \quad LCG_{pbs} := 73.73 \text{ ft}, \quad W_{pbt} := 0 \text{ ton},
   \]
   \[
   \frac{W_{pbs} \cdot LCG_{pbs} + \left( W_{pbm} - W_{pbt} \right) \cdot LCG_{pbm} + W_{pbt} \cdot LCG_{pbt}}{W_{pb}} = 76 \text{ ft}, \quad W_{pbml} := W_{pb} - W_{pbt} - W_{pbs}
   \]
H. Our total lead solution:

\[ W_{pm} = 0 \text{ton} \]
\[ LCG_{pm} = 0 \text{ft} \]
\[ W_{pbs} = 47.2 \text{ton} \]
\[ LCG_{pbs} = 73.7 \text{ft} \]
\[ W_{pbn} = 30.3 \text{ton} \]
\[ LCG_{pbn} = 79.5 \text{ft} \]
\[ W_{ph} = 77.5 \text{ton} \]
\[ LCG_{lead} = 110.7 \text{ft} \]

XV. PERFORMANCE

A. Battery:

1. Battery Characteristics: the characteristics below are for the selected battery. \( \text{Discharge fraction} \) indicates the percentage of the battery capacity that will be utilized before another charge evolution is required.

\[
\text{t}_{\text{Discharge}} = \text{submatrix(DischargeChar, 1, 9, Batt_Type, Batt_Type)} \cdot \text{hr}
\]
\[
\text{t}_{\text{Discharge}} = \text{submatrix(DischargeChar, 10, 18, Batt_Type, Batt_Type)} \cdot \text{amp}
\]
\[
V_{\text{Discharge}} = \text{submatrix(DischargeChar, 19, 27, Batt_Type, Batt_Type)} \cdot \text{volt} \cdot \text{Cells per battery}
\]
\[
p = \left( \text{t}_{\text{Discharge}} \cdot V_{\text{Discharge}} \right)
\]
\[
t_{\text{submerged}}(x) = \left( \text{interp}(p, \text{t}_{\text{Discharge}}, x) \right) \cdot \left( N_{\text{batt}} \cdot \text{Discharge fraction} \right)
\]

x is power

B. Performance Based only on battery (For the diesel electric submarines)

\[ V_{\text{DFM}} = 82.4 \text{m}^3 \]

\[
\text{Range}_{\text{snorkeling}} = V_{\text{snorkel}} \cdot t_{\text{snorkel}}
\]

\[
\text{Range}_{\text{submerged}} = \left( V_{\text{submerged}} \cdot t_{\text{submerged}}(P_{\text{Trans submerged}}) \right)
\]

\[
\text{Range}_{\text{cycle}} = \left( \text{Range}_{\text{snorkeling}} + \text{Range}_{\text{submerged}} \right)
\]

\[
\text{Cycles available} = \frac{\text{fuel allowance} \cdot V_{\text{DFM}}}{V_{\text{fuel cycle}}}
\]

\[
\text{Cycles available} = 97.73
\]

\[
\text{Range}_{\text{transit}} = \text{Range}_{\text{cycle}} \cdot \text{Cycles available}
\]

\[
\text{SOA}_{\text{transit}} = \frac{V_{\text{snorkel}} \cdot t_{\text{snorkel}} + \left( V_{\text{submerged}} \cdot t_{\text{submerged}}(P_{\text{Trans submerged}}) \right)}{t_{\text{snorkel}} + t_{\text{submerged}}(P_{\text{Trans submerged}})}
\]

\[
IR = \frac{t_{\text{snorkel}}}{t_{\text{snorkel}} + t_{\text{submerged}}(P_{\text{Trans submerged}})}
\]
C. Performance based on battery and AIP system (for hybridic submarines):

\[
P_{\text{Trans,submerged}} = \begin{pmatrix}
130.9 \\
141.9 \\
162.8 \\
177.9 \\
196.7 \\
219.5 \\
246.7 \\
278.6 \\
315.7 \\
406.7 \\
666.7 \\
1050.2 \\
1580.3 \\
2280.3
\end{pmatrix}
\text{kW}
\]

Even when the required power is greater than the power provided by the AIP system, part of the power can be provided by the AIP system. Hence the battery will be more lightly loaded and the underwater endurance will be extended.
If we assume that the submarine uses the AIP system until it runs out of fuel, and after that it uses the battery we have the following calculations:

As we had before: \( V_{DFM} = 82.4 \text{ m}^3 \)

\[ \text{Range}_{snorkeling} := V_{snorkel}/snorkel \]

\[ \text{Range}_{cycle} := (\text{Range}_{snorkeling} + \text{Range}_{submerged}) \]

\[ \text{Cycle}_{available} = 97.73 \]

\[ \text{Range}_{transit \_zz} := \text{Range}_{cycle \_zz} - \text{Cycle}_{available} \]

\[ t_{loiterAIP} := \text{if } [P_{Trans \_submerged} \leq \text{Power\_AIP}, \begin{bmatrix} \text{AIP\_Oxygen\_Weight} \\ P_{Trans \_submerged} \end{bmatrix}^{\text{lb}}/\text{kWh}, 0] \]

\[ t_{loiterAIP} = \begin{bmatrix} 418.1 \\ 385.8 \\ 336.2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ hr} \]

\[ t_{submerged}(P_{from\_bat}) = \begin{bmatrix} 105 \\ 105 \\ 105 \\ 90.3 \\ 72 \\ 49.8 \\ 33.3 \end{bmatrix} \text{ hr} \]

\[ \text{Range}_{submerged} := \frac{(V_{submerged} - \text{submerged}(P_{from\_bat}))}{\text{fuel\_allowance} \cdot V_{DFM}} \]

\[ \text{Cycles}_{available} := \frac{V_{fuel\_cycle}}{\text{cycle}_{available}} \]

\[ i := 1 \ldots 14 \]

\[ \text{Range}_{cycle} := (\text{Range}_{snorkeling} + \text{Range}_{submerged}) \]

\[ \text{Range}_{transit} := \text{Range}_{cycle} - \text{Cycle}_{available} \]

\[ i := 1 \ldots 14 \]

\[ t_{loiterAIP} = \begin{bmatrix} 105 \\ 105 \\ 105 \\ 90.3 \\ 72 \\ 49.8 \\ 33.3 \end{bmatrix} \text{ hr} \]

\[ t_{submerged}(P_{from\_bat}) = \begin{bmatrix} 22.1 \\ 17.4 \\ 9.5 \\ 3.9 \\ 1.9 \\ 0.7 \end{bmatrix} \text{ hr} \]

\[ \text{Range}_{submerged} = \begin{bmatrix} 142.5 \\ 122.1 \\ 76.2 \\ 38.7 \\ 22.7 \\ 10.2 \end{bmatrix} \text{ NM} \]

\[ t_{submerged} := t_{submerged}(P_{from\_bat}) \]

\[ t_{submerged} := \text{if } [P_{Trans \_submerged} \leq \text{Power\_AIP}, \begin{bmatrix} t_{loiterAIP} \\ t_{submerged} \end{bmatrix} \begin{bmatrix} t_{submerged} \end{bmatrix}] \]

\[ t_{submerged} = \begin{bmatrix} 22.1 \\ 17.4 \\ 9.5 \\ 3.9 \\ 1.9 \\ 0.7 \end{bmatrix} \text{ hr} \]
\[ \text{Rangesubmerged}_AIP := \left( V_{\text{submerged}} - t_{\text{submerged}} \right) \]

<table>
<thead>
<tr>
<th>NM</th>
<th>Range_{submerged} AIP =</th>
</tr>
</thead>
<tbody>
<tr>
<td>836.2</td>
<td>210.1</td>
</tr>
<tr>
<td>1157.3</td>
<td>315.1</td>
</tr>
<tr>
<td>1344.8</td>
<td>420.1</td>
</tr>
<tr>
<td>406.5</td>
<td>406.5</td>
</tr>
<tr>
<td>360</td>
<td>360</td>
</tr>
<tr>
<td>273.7</td>
<td>273.7</td>
</tr>
<tr>
<td>198.3</td>
<td>198.3</td>
</tr>
<tr>
<td>143.5</td>
<td>143.5</td>
</tr>
<tr>
<td>122.1</td>
<td>122.1</td>
</tr>
<tr>
<td>76.2</td>
<td>76.2</td>
</tr>
<tr>
<td>38.7</td>
<td>38.7</td>
</tr>
<tr>
<td>22.7</td>
<td>22.7</td>
</tr>
<tr>
<td>10.2</td>
<td>10.2</td>
</tr>
<tr>
<td>-0.1</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

SO_{\text{AIP}} = \left( V_{\text{snorkel}} + t_{\text{snorkel}} + (V_{\text{submerged}} - t_{\text{submerged}}) \right)

| SO_{\text{transit AIP}} = | \begin{array}{c}
| 2.016 | 0.0027 |
| 3.015 | 0.00292 |
| 4.013 | 0.00335 |
| 4.543 | 0.01237 |
| 5.046 | 0.01546 |
| 5.556 | 0.02223 |
| 6.066 | 0.03309 |
| 6.573 | 0.04873 |
| 7.061 | 0.06088 |
| 8 | 0.10613 |
| 9.548 | 0.22508 |
| 10.503 | 0.37425 |
| 10.346 | 0.60899 |
| 7.962 | 1.00475 |
\end{array} |

IR_{\text{AIP}} = \left( t_{\text{snorkel}} + V_{\text{submerged}} \right)^{-1}

| IR_{\text{AIP}} = | 147 |
XVI. OMOE RESULTS:

In order to calculate the OMOE for an AIP and a Diesel Submarine:

\[
V_{\text{Ballance}} := \begin{cases} V_{\text{max}} \quad \text{knt} \\ E \quad \text{hr} \\ V_{\text{Ballance}} \quad \text{knt} \\ D_{\text{D}} \quad \text{fi} \\ \text{AIP} \text{ endur} \\ \text{Snort range} \quad \text{NM} \\ \text{Surf range} \quad \text{NM} \\ \text{TT} \quad \text{Total weapons} \end{cases}
\]

\[
\text{OMOE}_{\text{in}} := \frac{\text{OMOE}_{\text{out}}}{\text{OMOE}_{\text{out}}} = 0.4466
\]

| Maximum Submerged Speed (knt) | 20 |
| Stores for mission (days)     | 70 |
| Time at burst speed            | 1  |
| AIP balance speed (knt)        | 4  |
| Diving depth (ft)              | 950|
| AIP Endurance at mission speed (days) | 14 |
| Maximum Submerged Range (NM)   | 6500|
| Maximum Surfaced Range (NM)    | 6500|

\[
\text{OMOE}_{\text{in}} = \frac{\text{OMOE}_{\text{out}}}{0.4466}
\]

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Final Design Summary

Payload

\[ \text{TT} = 6 \quad \text{RL} = 10 \quad \text{CIC}_{\text{volume}} = 2650 \text{ft}^3 \quad \text{HL}_{\text{Payload}} = 45 \text{kW} \]

External dimensions and arrangements

\[ L = 198.7 \text{ft} \quad D = 20.5 \text{ft} \quad \text{Draft} = 16.8 \text{ft} \]

\[ \text{LOD} = 9.7 \quad \rho_{\text{range}} = 2.5 \text{lb/ft}^3 \quad \text{Weight stores} = 5.9 \text{ton} \]

The lead is placed in order to have the LCB at the same position with the LCG

\[ \text{LCGLEAD} = 110.7 \text{ft} \]

\[ \text{LCGLEADs} = \frac{\text{W}_{\text{pb}} - \text{LCGLEAD} - \text{W}_{\text{pbbm}}}{\text{W}_{\text{pbs}}} \]

\[ \text{LCGLEADs} = 130.67 \text{ft} \quad \text{LCG}_{\text{pbbm}} = 79.5 \text{ft} \quad \text{LCGLEAD} = 110.7 \text{ft} \]

Total lead solution

\[ \text{W}_{\text{pb}} = 0 \text{ton} \quad \text{LCG}_{\text{pbb}} = 0 \text{ft} \]

\[ \text{W}_{\text{pbs}} = 47.2 \text{ton} \quad \text{LCG}_{\text{pbs}} = 73.7 \text{ft} \]

\[ \text{W}_{\text{pbbm}} = 30.3 \text{ton} \quad \text{LCG}_{\text{pbbm}} = 79.5 \text{ft} \]

\[ \text{W}_{\text{pb}} = 77.5 \text{ton} \quad \text{LCGLEAD} = 110.7 \text{ft} \]
**Propulsion Components**

- $D_P = 12.8\text{ ft}$  \quad PC = 0.85
- $HL = 124.8\text{ kW}$
- $P_{Motor} = 4113.7\text{ kW}$  \quad $N_{cell} = 448$
- $Power_{diesel} = 3096.2\text{ kW}$  \quad $Volume_{diesel} = 1244.9\text{ ft}^3$  \quad $Weight_{diesel} = 17.8\text{ ton}$
- $Weight_{DFM} = 67.7\text{ ton}$
- $Power_{AIP} = 162.9\text{ kW}$  \quad $AIP\_Total\_Vol = 5097.6\text{ ft}^3$  \quad $AIP\_Total\_Weight = 96.5\text{ ton}$

**Tanks**

- $ROV := \frac{V_{bt}}{V_{PH}}$  \quad $ROV = 0.135$
- $Vol\_trim\_fwd = 301.9\text{ ft}^3$  \quad $Vol\_trim\_aft = 301.9\text{ ft}^3$
- $Vol\_compensating = 1409\text{ ft}^3$
- $Vol\_FW = 865.2\text{ ft}^3$
- $Vol\_Sanitary = 86.5\text{ ft}^3$

**Displacements**

- $\Delta_{eba} = 1323.1\text{ ton}$
- $\Delta_{surf} = 1335.7\text{ ton}$
- $\varepsilon := \frac{\Delta_{eba} - \Delta_{surf}}{\Delta_{eba}}$  \quad $\varepsilon = -0.0095$
- $\Delta_{AF} = 1488.5\text{ ton}$

**Weight Summary:**

- $W_1 = 537.3\text{ ton}$
- $W_{23} = 465\text{ ton}$
- $W_4 = 22.2\text{ ton}$
- $W_{56} = 53.4\text{ ton}$
- $W_7 = 45.4\text{ ton}$
- $\Delta A_1 = 1123.2\text{ ton}$
- $W_{PB} = 77.5\text{ ton}$
- $\Delta A = 1200.7\text{ ton}$
- $W_{VL} = 135.04\text{ ton}$
### Performance

For a diesel electric submarine

<table>
<thead>
<tr>
<th>SOAtransit (knt)</th>
<th>IR</th>
<th>Rangesubmerged (NM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.323</td>
<td>0.054</td>
<td>210.1</td>
</tr>
<tr>
<td>3.286</td>
<td>0.057</td>
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<tr>
<td>4.259</td>
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<tr>
<td>4.751</td>
<td>0.072</td>
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</tr>
<tr>
<td>5.248</td>
<td>0.083</td>
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<td>5.744</td>
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<td>6.214</td>
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<td>6.682</td>
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<td>10.2</td>
</tr>
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<td>6.55</td>
<td>1.184</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

For an AIP submarine

<table>
<thead>
<tr>
<th>IR_AIP</th>
<th>SOAtransit AIP (knt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0027</td>
<td>0.0029</td>
</tr>
<tr>
<td>0.0034</td>
<td>0.0124</td>
</tr>
<tr>
<td>0.0155</td>
<td>0.0222</td>
</tr>
<tr>
<td>0.0331</td>
<td>0.0487</td>
</tr>
<tr>
<td>0.0609</td>
<td>0.1061</td>
</tr>
<tr>
<td>0.2261</td>
<td>0.3743</td>
</tr>
<tr>
<td>0.609</td>
<td>1.0047</td>
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Rangesubmerged AIP (NM)

<table>
<thead>
<tr>
<th>Rangesubmerged AIP (NM)</th>
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<tbody>
<tr>
<td>836.2</td>
</tr>
<tr>
<td>1157.3</td>
</tr>
<tr>
<td>1344.8</td>
</tr>
<tr>
<td>406.5</td>
</tr>
<tr>
<td>360</td>
</tr>
<tr>
<td>273.7</td>
</tr>
<tr>
<td>198.3</td>
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<tr>
<td>143.5</td>
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<tr>
<td>122.1</td>
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<tr>
<td>76.2</td>
</tr>
<tr>
<td>38.7</td>
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<td>22.7</td>
</tr>
<tr>
<td>10.2</td>
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<td>-0.1</td>
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</table>
Appendix 2

“Simplified Cost Model”
Simplified Submarine Cost Model

Variant Summary

<table>
<thead>
<tr>
<th>Variant</th>
<th>Cost (million)</th>
</tr>
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<tbody>
<tr>
<td>W₁ (Itons)</td>
<td>537.32</td>
</tr>
<tr>
<td>W₂₃ (Itons)</td>
<td>464.95</td>
</tr>
<tr>
<td>W₄ (Itons)</td>
<td>22.15</td>
</tr>
<tr>
<td>W₅₆ (Itons)</td>
<td>53.43</td>
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<tr>
<td>W₇ (Itons)</td>
<td>45.39</td>
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Cost Assumptions

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Desired Fiscal Year</td>
<td>2003</td>
</tr>
<tr>
<td>Base Year</td>
<td>1993</td>
</tr>
<tr>
<td>Assumed Inflation Rate</td>
<td>2.25%</td>
</tr>
<tr>
<td>Inflation factor</td>
<td>1.25</td>
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Man hour rate $26.90

LeadShip Costs

Labor Cost Factors

<table>
<thead>
<tr>
<th>SWBS Group</th>
<th>hrs / LTON</th>
<th>Cost (million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>486</td>
<td>$8.78</td>
</tr>
<tr>
<td>2 &amp; 3</td>
<td>1,838</td>
<td>$28.72</td>
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<tr>
<td>4</td>
<td>3,066</td>
<td>$2.28</td>
</tr>
<tr>
<td>5 &amp; 6</td>
<td>1,374</td>
<td>$2.47</td>
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<tr>
<td>7</td>
<td>810</td>
<td>$1.24</td>
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<tr>
<td>Integration Factor</td>
<td>16.40%</td>
<td>$7.13</td>
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<tr>
<td>Assembly Factor</td>
<td>65.80%</td>
<td>$28.61</td>
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</tbody>
</table>

Material Cost Factors

<table>
<thead>
<tr>
<th>SWBS Group</th>
<th>$ / LTON</th>
<th>AIP (factor of Group 2&amp;3 - AIP cost)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$10,634</td>
<td>5.0%</td>
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<tr>
<td>2 &amp; 3</td>
<td>$124,121</td>
<td>5.0%</td>
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<tr>
<td>4</td>
<td>$97,725</td>
<td>10.0%</td>
</tr>
<tr>
<td>5 &amp; 6</td>
<td>$145,657</td>
<td>10.0%</td>
</tr>
<tr>
<td>7</td>
<td>$10,644</td>
<td>10.0%</td>
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<tr>
<td>Integration Factor</td>
<td>4.00%</td>
<td>$3.30</td>
</tr>
<tr>
<td>Assembly Factor</td>
<td>24.00%</td>
<td>$19.80</td>
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</tbody>
</table>

Direct Costs $184.83

Overhead Cost Factor 1.104 $204.05
Profit Margin 10.00% $38.89

Total Lead Ship Acquisition Cost $427.77