Fabrication of Chip-Scale Radio Frequency Inductors

by

Joshua C. Nation
B.S. Mechanical Engineering
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Signature of Author……………………………………………………………………………………………………
Joshua C. Nation
Department of Mechanical Engineering
May 2014

Certified by………………………………………………………………………………………………………………
Martin L. Culpepper
Professor of Mechanical Engineering
Thesis Supervisor

Accepted by……………………………………………………………………………………………………………………
David E. Hardt
Professor of Mechanical Engineering
Graduate Officer
Fabrication of Chip-Scale Radio Frequency Inductors

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Joshua C. Nation

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ABSTRACT

The purpose of this research was to learn the relationship between force and deformation in forming of micro-scale inductor coils. This was accomplished by applying large-deflection beam bending to the case of planar wire deformation and through experimental validation. Generating this knowledge is important because it establishes fabrication limits for wire-based chip-scale inductors. There are many potentially viable methods for fabricating planar inductor coils. Without an understanding of the relevant physics, it is impossible to know which of these techniques is most appropriate or even feasible. The analysis presented in this thesis directly led to the stencil-and-guide inductor fabrication concept, the details of which were specified using an analytic electrical model. The process utilizes a wire conductor, is compatible with any desired substrate, and features the ability to exactly control spiral properties. Multiple inductors were fabricated using this process. These inductors demonstrate performance specifications predicted by the model, including inductances ranging from 2 – 4 nH, quality factors in excess of 100, and self-resonant frequencies beyond 10 GHz. Furthermore, the area of the inductors is less than 1.5 mm² and the entire device thickness is only 260 μm. The inductors are most readily applied to increasingly small communication devices, which require thin and efficient electrical components to boost the performance of the radio frequency transceiver. Accordingly, these inductors offer the potential for substantial improvement in signal quality and reception.

Thesis Supervisor: Martin L. Culpepper
Title: Professor of Mechanical Engineering
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This chapter provides motivation for fabricating chip-scale radio frequency (RF) inductors. Section 1.1 highlights the purpose of the research, its importance to the scientific community, and its potential impact on society. Section 1.2 presents target applications for the inductors and defines the inductor quality factor. Section 1.3 prescribes the inductor functional requirements and offers a comprehensive discussion of prior art that includes both commercial and academic devices. Section 1.4 outlines the organization of the thesis.

1.1 Research Overview

The purpose of this research was to learn the relationship between force and deformation in forming of micro-scale inductor coils, one of which is pictured in Figure 1.1. This was accomplished by applying large-deflection beam bending to the case of planar wire deformation and through experimental validation. Figure 1.2 shows a free body diagram of a deformed length of wire [1]. Elliptic integrals were used to solve the large-deflection problem and produce Figure 1.3, a plot of the applied force (F) required to achieve 90° deflection for three common copper wire diameters.

Figure 1.1: Fabricated inductor
Figure 1.2: Free body diagram for large deflection wire bending

Figure 1.3: Relationship between force and deflection for copper wire

Generating this knowledge is important because it establishes fabrication limits for wire-based chip-scale inductors. There are many potentially viable methods for fabricating planar
inductor coils. Without an understanding of the relevant physics, it is impossible to know which of these techniques is most appropriate or even feasible.

The analysis presented in this thesis directly led to the stencil-and-guide inductor fabrication concept, the details of which were specified using an analytic electrical model. The process is summarized in Figure 1.4 and shown in detail in Figure 4.10. A three-dimensional channel is machined into a thin polyimide layer to define the inductor shape. Next, this layer is bonded to any desired chip substrate, such as glass. A wire guide tool is used to both thread the wire into the channel and also hold it in place until assembly is complete. Once the wire is inside the spiral channel, the leads are secured and an additional cover adhesive may be applied.

![Figure 1.4: Stencil-and-guide inductor fabrication process](image)

Multiple inductors were fabricated using this process. These inductors demonstrate performance specifications predicted by the model, including inductances ranging from 2 – 4 nH, quality factors in excess of 100, and self-resonant frequencies beyond 10 GHz. Furthermore, the area of the inductors is less than 1.5 mm² and the entire device thickness is only 260 μm.

Wire-based chip-scale inductors are most readily applied to increasingly small communication devices, which require thin and efficient electrical components to boost the performance of the radio frequency transceiver. Accordingly, these inductors offer the potential for substantial improvement in signal quality and reception.
1.2 RF Inductor Applications

Inductors are essential components for radio frequency communication devices. A typical device such as a cell phone may utilize more than 20 inductors [2]. In 2012 alone, over 1.7 billion mobile phones were sold [3], and that number is only expected to increase. With the evolution of technology, there is an ever-increasing demand for communication devices with improved signal quality and reception, reduced power consumption, and smaller packaging. To accomplish these goals, it is critical to integrate high-quality passive components, but unfortunately, available on-chip inductors traditionally exhibit poor performance. Designers are thus faced with suffering the low quality of on-chip devices, or they must resort to larger off-chip inductors that come with an increase in power consumption, cost, and area [4].

1.2.1 Defining the Frequency Spectrum

Radio frequency engineering is an essential part of everyday life. RF engineering concerns the design and fabrication of any device that operates within the radio frequency band. Technically, the radio frequency band refers to operational frequencies as low as 3 kHz and as high as 300 GHz. For applications relevant to this thesis, such as mobile phones and wireless networks, typical operational frequencies range from 1 – 10 GHz. As shown in Figure 1.5, this target range is contained within the ultra-high frequency and super-high frequency bands.

![Radio Frequency Spectrum](image)

**Figure 1.5:** Target range (1 – 10 GHz) is contained within the UHF and SHF bands
1.2.2 RF Transceivers

A radio frequency transceiver is found in nearly every RF communication device. As the name implies, a transceiver consists of both a transmitter and receiver that share a common packaging. The circuitry located between the antenna and the first intermediate frequency stage is commonly referred to as the RF front end. It usually consists of impedance-matching circuits, low-noise amplifiers, local oscillators, and filters [5]. Inductors are crucial components in all of these circuits, many of which are useful for both generating and processing radio signals.

1.2.3 LC Circuit

![LC Circuit Diagram]

Figure 1.6: The tank circuit is fundamental to more complex circuits

The parallel LC circuit, commonly referred to as the tank or resonant circuit, is the building block for the more complex circuits found in the RF front end. The performance of the tank strongly influences the performance of the entire circuit. The quality of the inductor, which is typically the limiting factor, dominates the performance of the tank. For example, noise rejection, power consumption, and tuning range of an oscillator are all dependent on the inductor [6].

1.2.4 Quality Factor and Self-Resonance

The quality factor $Q$ of an inductor is a measure of its efficiency. An ideal inductor would have an infinite $Q$. Conventionally, $Q$ is defined as:
The ideal behavior of an inductor is storage of *magnetic*, not *electric*, energy. To account for losses caused by parasitic capacitance in the windings and electrostatic coupling with the substrate, \( Q \) is further defined as [7]:

\[
Q = 2\pi \frac{|\text{Peak magnetic energy} - \text{peak electric energy}|}{\text{Energy dissipated per cycle}} = 2\pi \frac{|E_M - E_E|}{E_{\text{diss}}} \quad (1.2)
\]

When the electric energy of the inductor matches the magnetic energy, the quality factor becomes zero. The frequency at which this occurs is called the *self-resonant frequency* (SRF). When the device is operated at this frequency, the effective inductance is zero because it is completely negated by unwanted capacitance. For an inductor to retain its functionality, it must be operated well below the self-resonant frequency.

On-chip inductors integrated on silicon substrates suffer from notoriously low quality factors caused by generated eddy currents that oppose the magnetic field. Replacing the silicon substrate with gallium arsenide has been shown to raise \( Q \) into the range of 20 – 40 [6], but even this is a modest improvement. In the following section, prior art with respect to improving inductor performance is analyzed. Both commercial offerings and laboratory achievements are recognized and evaluated based on a common set of inductor functional requirements.

### 1.3 Prior Art

Substantial progress has been made toward improving inductor performance over the past decade. Inductors serve a variety of purposes; accordingly, the demands placed on the inductor vary depending on the application. These demands heavily influence the nature of the research. For example, one group in academia may only be concerned with attaining the maximum possible quality factor, while another group in industry may demonstrate especially tight control over inductance.

In order to judge the merits of prior artwork discussed herein, it was necessary to establish functional requirements for the desired RF inductor. All surveyed inductors are evaluated relative to these requirements. In the following sections, justification for the functional
requirements is provided. Then, notable achievements in both industry and academia are highlighted and summarized. Finally, it is shown that inductors fabricated using the proposed stencil-and-guide process are particularly well-suited to meet and exceed the functional requirements.

### 1.3.1 Functional Requirements

Table 1.1 shows the functional requirements for surveyed inductors. In general, it is difficult to influence one inductor performance parameter without affecting another. Increasing the length of the conductor increases the magnetic field and improves inductance. This comes at the penalty of increased resistance and parasitic capacitance, which lowers the quality factor and self-resonant frequency of the inductor. Increasing the spacing between spiral turns helps mitigate parasitic capacitance and resistance caused by the proximity effect, improving $Q$ and SRF, but this comes with a sacrifice to inductor area. Capping the thickness of the inductor is often required by packaging constraints, but this may prevent coiling the inductor out of plane, where substantial gains in the magnetic field strength can be made. Thus, improving overall inductor performance is a complex task, and the application of the inductor must be kept in mind when assigning values for functional requirements.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductance</td>
<td>&gt; 1 nH</td>
</tr>
<tr>
<td>Quality Factor</td>
<td>&gt; 100</td>
</tr>
<tr>
<td>Self-Resonant Frequency</td>
<td>&gt; 5 GHz</td>
</tr>
<tr>
<td>Area</td>
<td>&lt; 2.5 mm$^2$</td>
</tr>
<tr>
<td>Thickness</td>
<td>&lt; 300 μm</td>
</tr>
</tbody>
</table>

Standard RF circuitry requires inductors in the range of 1-10 nH [8]; thus, the inductor must have an inductance > 1 nH to be useful. The target frequency range was previously defined as 1 – 10 GHz. Setting the minimum self-resonant frequency to 5 GHz ensures that the inductor at least retains its functionality at low GHz frequencies. The size requirements for the inductor are based on thin-chip dimensions. The thickness is most critical: inductors with a thickness > 300 μm cannot be directly integrated alongside their much thinner capacitive counterparts. This
threshold is also significant because it is a typical device thickness set by some commercial inductor manufacturers, including TDK and AVX [9], [10]. (The following analysis shows that these inductors fail to meet other functional requirements.) Finally, the requirement for the quality factor to be $> 100$ is derived from the performance of the best off-chip inductors. It is the high $Q$ of the inductor that substantially improves the ability of a communication device to transmit and receive a clear radio signal.

1.3.2 Commercial Inductors

Commercial inductor manufacturers have a strong offering of radio frequency inductors. Off-the-shelf inductors are typically low-cost and easy to integrate. Furthermore, many inductors feature exceptionally high quality factors and tightly-controlled inductance values. Despite these features, many of these high-performing inductors do not meet the size requirements necessary for thin-chip integration.

Nearly all commercially-available chip inductors are categorized by the footprint of their casing. For example, the 1005 (0402 English) case has approximate overall length and width dimensions of 1.00 mm x 0.50 mm (0.04” x 0.02”). This standardization makes it easy for designers to integrate the inductors into circuits. The thickness of the inductor casing varies by vendor and is not directly indicated in the case size. Additionally, the mix of SI and English case sizes may quickly cause confusion. For example, as shown in Table 1.2, an 0603 inductor can refer to two different inductor footprints, depending on the system of measurement. For simplicity, unless explicitly stated, all inductor case sizes in this thesis are reported in SI units.

<table>
<thead>
<tr>
<th>Case Size (SI)</th>
<th>Case Size (English)</th>
<th>Dimensions (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0402</td>
<td>--</td>
<td>0.40 x 0.20</td>
</tr>
<tr>
<td>0603</td>
<td>0201</td>
<td>0.60 x 0.30</td>
</tr>
<tr>
<td>1005</td>
<td>0402</td>
<td>1.00 x 0.50</td>
</tr>
<tr>
<td>1608</td>
<td>0603</td>
<td>1.60 x 0.80</td>
</tr>
</tbody>
</table>
Coilcraft, AVX, and TDK are three major manufacturers of radio frequency chip inductors. Figure 1.7 summarizes the inductance and quality factor for a collection of devices offered by all three corporations [9]–[15]. All of these inductors have a self-resonant frequency of at least 5 GHz and an inductance of at least 1 nH. These inductors do not necessarily meet quality factor or size requirements. In general, if multiple modules report similar $L$, then the one with greater $Q$ is utilized in this collection. Thickness of these devices ranges from 300 μm (smallest available) to 610 μm.

**Figure 1.7: Collection of available commercial inductors**

There are a number of commercial inductors that are able to satisfy the inductance ($>1$ nH) and quality factor ($>100$) requirements. Unfortunately, none of the devices meeting those performance specifications is less than 300 μm thick. Figure 1.8 shows the same plot as Figure 1.7, only any device not meeting size requirements has been removed.
The best offering is the TDK 0603, which has $L = 2.3 \text{ nH}$ and $Q = 71$. Coilcraft does not offer any inductors meeting the thickness requirement. The smallest available Coilcraft inductor is the 0603 device, which has a thickness of $450 \text{ μm}$ [13].

While some commercial inductors certainly offer impressive inductance and quality factor, they fail to combine these specifications with thin-chip size requirements. Integrating inductors directly on-chip is crucial for savings in power consumption, cost, and circuit area [4].

### 1.3.3 Academic Inductors

Many efforts have been undertaken in academia to improve the size and performance characteristics of integrated inductors. A wide variety of methods have been used, but most of these can be grouped into three major categories, presented in Table 1.3.
Table 1.3: Inductor Improvement Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substrate Removal</td>
<td>Minimize losses resulting from eddy currents in the substrate by creating a cavity beneath the inductor [16]–[20].</td>
</tr>
<tr>
<td>Free-Standing Structures</td>
<td>Minimize losses resulting from eddy currents in the substrate by creating 3D inductor structures above the substrate [21]–[26].</td>
</tr>
<tr>
<td>Magnetic Material</td>
<td>Apply a thin film of magnetic material below and/or above the inductor to improve its conductive properties [27]–[34].</td>
</tr>
</tbody>
</table>

A successful usage of substrate removal is demonstrated in [19]. Rais-Zadeh utilizes 30:1 aspect ratio trenches in a silicon substrate to minimize eddy current losses. The trenches are then bridged over or refilled with a low-loss dielectric for stability, and a thick layer of electroplated copper is used to improve conductivity. An inductance of 0.8 nH and a quality factor of 71 at 8.75 GHz are reported.

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Figure 1.9: Silicon trenches to minimize eddy current losses
Rather than removing the substrate to reduce eddy current losses, another common effort is to create a free-standing structure that is suspended over the substrate. In [24], Pinel shows a micromachined inductor suspended over an RF ceramic substrate. $Q = 80$ is obtained for a 2.7-nH inductor, and a $Q > 100$ is demonstrated for a 0.8-nH inductor. In [26], Weon uses 3D processing techniques such as stressed metal technology to form a vertical inductor on top of a silicon substrate. Although this inductor does not meet the size requirements established in this thesis, it does reach a record-high quality factor of 140 at 12 GHz.

![Figure 1.10: Suspended inductors (a) and stressed-metal inductors (b)](image)

Finally, application of magnetic material attempts to improve inductor performance by significantly increasing inductance rather than reducing losses. These inductors easily meet size requirements by adding thin films that are usually only a few microns thick. In [28], Gardner presents an astounding inductance density of 1700 nH/mm$^2$. Despite this high density, the inductors are generally fabricated directly onto a silicon substrate, which results in quality factors that are much lower than other devices ($< 10$).
Figure 1.11: Magnetic material surrounds conductor to improve inductance

Figure 1.12 summarizes the $L$ and $Q$ performance results for all surveyed academic inductors meeting size requirements. While creative laboratory techniques have yielded inductors with high quality factors, there are still no available devices that meet both the inductance and quality requirements.

Figure 1.12: Academic inductors with thickness < 300 μm

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1.3.4 Inherent Limitations of Surveyed Inductors

Fabricating a small, thin inductor for RF applications with high inductance and quality factor is no trivial task. Commercial inductor manufacturers generally use round wire for the conductor, which offers significant performance benefits over micromachining techniques such as electrodeposition. These benefits have allowed the commercial inductors to obtain remarkable $L$ and $Q$. The fabrication process, however, requires three-dimensional winding around a magnetic core, which has inherent limitations in size. For example, Coilcraft’s smallest chip inductor has a thickness of 450 μm [13]. Figure 1.13 shows a typical device schematic. No commercial modules meeting performance requirements also have a thickness less than 300 μm.

![Top View](image1.png) ![Side View](image2.png)

**Figure 1.13: High-performance commercial inductors have inherent size limitations**

In academia, researchers are pushing the limits on inductor size and inductance density. With the exception of some free-standing structures, the vast majority of laboratory inductors meet size requirements. These thin inductors, however, are typically fabricated using micromachining techniques that have fundamental performance limitations caused by the surface roughness and rectangular cross-section of the conductor. As a result, these inductors are unable to obtain both high inductance and high quality factor.

1.3.5 New Inductors Meet Functional Requirements

In summary, this research fills an important performance/size gap for available thin-chip inductors. The utilization of round copper wire enables the inductors to obtain both high $L$ and high $Q$. Unlike the commercial inductors, however, the stencil-and-guide process allows formation of *planar* coils that meet the demanding size specifications required of thin-chip devices.
Figure 1.14: Stencil-and-guide process fills performance/size gap for thin-chip inductors

Figure 1.15 shows a complete summary of available thin-chip inductors meeting size requirements: devices from both industry and academia are included. Three new inductors fabricated using the stencil-and-guide fabrication process are shown on the plot as well. Inductors 2 and 3 demonstrate $Q > 100$ for inductance values of 2.5 nH and 4 nH, respectively. The first inductor was intentionally designed for high inductance and demonstrates a substantial increase in $Q$ compared to available inductors with similar $L$. Performance and size specifications for all three inductors are provided in Table 1.4.
Table 1.4: New Inductors Compared to Functional Requirements

<table>
<thead>
<tr>
<th></th>
<th>L (nH)</th>
<th>Peak Q</th>
<th>SRF (GHz)</th>
<th>A (mm²)</th>
<th>t (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required</td>
<td>&gt; 1</td>
<td>&gt; 100</td>
<td>&gt; 5</td>
<td>&lt; 2.5</td>
<td>&lt; 300</td>
</tr>
<tr>
<td>Inductor 1</td>
<td>7.5</td>
<td>80</td>
<td>7.2</td>
<td>2.4</td>
<td>260</td>
</tr>
<tr>
<td>Inductor 2</td>
<td>2.5</td>
<td>107</td>
<td>17.5</td>
<td>0.83</td>
<td>260</td>
</tr>
<tr>
<td>Inductor 3</td>
<td>4.0</td>
<td>111</td>
<td>11.9</td>
<td>1.2</td>
<td>260</td>
</tr>
</tbody>
</table>

1.4 Thesis Organization

Chapter 1 of this thesis established functional requirements based on the demands of desired applications. Prior art in the field of radio frequency inductors was quantified and inherent limitations were discussed. Chapter 2 considers loss mechanisms associated with operating at the target frequency range, and it is shown how these challenges dictate a set of design parameters necessary for meeting the functional requirements. Chapter 3 uses a large-deflection beam bending analysis to define the relationship between force and deformation in forming of micro-scale inductor coils. Then, this knowledge is used to select the stencil-and-guide process as the best fabrication concept of those considered. Chapter 4 expands the electrical model and uses it to specify the details of the stencil-and-guide concept. Chapter 5 discusses measurement results with respect to the original functional requirements.

Figure 1.16: Organization of the thesis
This chapter provides the theoretical framework for modeling radio frequency inductors. Section 2.1 establishes the role of the inductor. Section 2.2 defines the quality factor of the inductor by systematically quantifying various inductor loss mechanisms. Section 2.3 specifically considers skin and proximity effects. Section 2.4 uses the loss mechanisms to dictate a set of design parameters that are critical for any viable inductor fabrication strategy.

2.1 Role of the Inductor

An inductor is a passive electrical component that resists changes in current passing through it. A mechanical analog for an inductor is a heavy waterwheel in a narrow channel [35]. Imagine that the water is not flowing initially, but then it begins moving through the channel. The waterwheel inhibits the flow of the water until it comes up to speed with the water. Now imagine that the water stops flowing. The spinning waterwheel preserves the movement of the water until the wheel stops.

Current flowing through the conductor causes the inductor to exhibit this same effect. When current begins flowing through the conductor, the inductor initially provides resistance to the current while the magnetic field builds up around the coil. Later, when the current stops, energy stored in the magnetic field sustains the current flow until the field collapses.

This effect may also be explained using the constitutive relation of the inductor. When the current flowing through an inductor changes, the time-varying magnetic field induces an electromotive force (EMF) in the conductor that opposes the change. Thus, inductance is a measure of the amount of voltage generated per unit change in current.

\[
\nu = L \frac{di}{dt} \quad (2.1)
\]
2.2 Inductor Quality

The quality factor of the inductor is a critical metric. The performance of the inductor dominates the performance of the LC tank circuit, which is a crucial building block for many higher level circuits (see section 1.2.3). The quality factor quantitatively captures the efficiency of the inductor, providing an effective tool for comparing inductors with similar inductance values. The next three sections show how to model an inductor and determine its quality factor. Each successive model provides a more accurate depiction of the inductor during operation.

2.2.1 Modeling the Ideal Inductor

In section 1.2.4, the following definition was provided for quality factor:

\[
Q = 2\pi \frac{|\text{Peak magnetic energy} - \text{peak electric energy}|}{\text{Energy dissipated per cycle}} = 2\pi \frac{|E_M - E_E|}{E_{\text{diss}}}
\]  

(1.2)

This definition accounts for capacitive as well as resistive losses and is discussed in greater detail in the following. Let us first begin with an ideal inductor circuit model, one that only considers the series resistance in the conductor, as shown in Figure 2.1.

![Figure 2.1: Ideal inductor circuit model only considers series resistance](image)

In this model, the only loss in the inductor comes from the resistance in the conductor (wire or deposited layer). This series resistance is given by

\[
R_s = \frac{\rho l}{A},
\]  

(2.2)

where \(\rho\) is the resistivity, \(l\) is the length, and \(A\) is the cross-section of the conductor. The resistance and inductance increase with the length of the conductor, which causes a trade-off between the two. Accurate formulas for predicting inductance are generally based on the geometry of the inductor and empirical constants; the well-known Wheeler formula is presented in section 4.1.1. The cross-section of the conductor is typically circular (for a wire) or
rectangular (for a deposited layer). Cross-section has a significant impact on the series resistance of the inductor and is considered in section 2.4.1. Finally, resistivity is a measure of how strongly a material opposes the flow of electric current. It is the inverse of conductivity and does not vary with frequency (although it may change with temperature). The resistivity of several common conductor materials are provided in Table 2.1 [36].

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity $\rho$ at 20 °C (Ω⋅μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>0.0159</td>
</tr>
<tr>
<td>Copper</td>
<td>0.0168</td>
</tr>
<tr>
<td>Gold</td>
<td>0.0244</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.0282</td>
</tr>
</tbody>
</table>

A sensitivity analysis illustrating how resistivity affects quality factor and other performance parameters is provided in section 4.2.1.

Since we are momentarily disregarding the electric energy of the inductor, the quality factor is simply:

$$Q = 2\pi \frac{E_M}{E_{diss}}$$

(2.3)

The peak magnetic energy is calculated as

$$E_M = \frac{1}{2} LI^2,$$

(2.4)

and the energy dissipated per cycle is calculated as:

$$E_{diss} = P_{avg}T = \frac{1}{2} R_s I^2 \left(\frac{2\pi}{\omega}\right)$$

(2.5)

Substituting (2.4) and (2.5) into (2.3) yields

$$Q = \frac{\omega L}{R_s},$$

(2.6)
which is the classic definition of quality factor for an inductor ($\omega$ is the frequency in radians). This equation is valid at frequencies well below self-resonance, where capacitive effects are not yet significant.

### 2.2.2 Modeling Parasitic Capacitance

Inductors commonly suffer from parasitic capacitance at high frequencies, as shown in Figure 2.2. If the inductor consists of more than one turn, the conductor acts like a continuous capacitor and causes unwanted electric energy. The problem is exacerbated if the inductor is three-dimensional: the stacked conductor layers also store electric energy. The stored electric energy gradually consumes a greater percentage of the inductor’s reactance as frequency increases, thus detracting from the efficiency of the inductor. Figure 2.3 provides the updated equivalent circuit model that includes parasitic capacitance.

![Figure 2.2: Parasitic capacitance acts between conductor turns (a) and layers (b)](image)

![Figure 2.3: Inductor circuit model including parasitic capacitance](image)
We now return to the definition of quality factor provided in (1.2). The peak electric energy from parasitic capacitance is calculated as:

\[ E_E = \frac{1}{2} C_s V_0^2 \]  

(2.7)

The expressions for peak magnetic energy and energy dissipated per cycle are both reduced using the impedance \( Z \) of the circuit shown in Figure 2.3:

\[ I = \frac{V_0}{Z} = \frac{V_0}{\sqrt{(\omega L)^2 + R_s^2}} \]  

(2.8)

\[ E_M = \frac{1}{2} L I^2 = \frac{L V_0^2}{2 \left[ (\omega L)^2 + R_s^2 \right]} \]  

(2.9)

\[ E_{diss} = \frac{1}{2} R_s I^2 \left( \frac{2\pi}{\omega} \right) = \frac{R_s V_0^2}{2 \left[ (\omega L)^2 + R_s^2 \right]} \left( \frac{2\pi}{\omega} \right) \]  

(2.10)

Substituting (2.7), (2.9), and (2.10) into (1.2) and rearranging yields a new definition of quality factor that considers both series resistance and parasitic capacitance [7]:

\[ Q = \frac{\omega L}{R_s} \left( 1 - \frac{R_s^2 C_s}{L} - \omega^2 L C_s \right) \]  

(2.11)

Note that this definition is arranged so that the attenuation to \( Q \) resulting from parasitic capacitance is clear. As various spiral parameters are adjusted, such as the spacing between turns, inductor performance may be substantially affected. The sensitivity of \( Q \) to design changes is examined in section 4.2.

2.2.3 Modeling Substrate Losses

The final step in accurately modeling actual inductor performance is the inclusion of losses in the substrate. First, there is electrostatic coupling between the inductor and the substrate just as there exists parasitic capacitance between spiral turns and conductor layers. This effect is quite pronounced if conductive or even semi-conductive substrate materials such as silicon are
utilized. Second, at high frequencies, the inductor’s magnetic field penetrates into the substrate and generates eddy currents that oppose the actual current flow. These currents cause additional ohmic losses and also reduce the strength of the magnetic field of the inductor [37]. Augmenting our model with these new losses results in the following circuit diagram shown in Figure 2.4:

\[ \begin{align*}
   I(t) & + \\
   L & C_s \\
   V_o & R_s \\
   - & R_p C_p \\
\end{align*} \]

**Figure 2.4: Complete circuit model with parasitic capacitance and substrate losses**

The electrostatic coupling between the inductor and the substrate \((C_p)\) combines with the parasitic capacitance \((C_s)\) to increase the overall capacitance of the inductor:

\[ C_o = C_s + C_p \quad (2.12) \]

The electric energy must then be redefined to include this overall capacitance:

\[ E_E = \frac{1}{2} C_o V_o^2 \quad (2.13) \]

The eddy current losses in the substrate contribute to the energy dissipated per cycle. The resulting energy magnitude may be calculated using the substrate resistance, which effectively decreases at high frequencies as the eddy current strength increases. Hence, \(R_p\) is in the denominator:
\[ E_{\text{diss}} = \frac{1}{2} \left( R_s l^2 + \frac{V_0^2}{R_p} \right) \frac{2\pi}{\omega} = \frac{V_0^2}{2} \left( \frac{R_s}{(\omega L)^2 + R_s^2} + \frac{1}{R_p} \right) \frac{2\pi}{\omega} \]  

Substituting (2.9), (2.13), and (2.14) into (1.2) and rearranging yields the complete definition of quality factor that includes series resistance, parasitic capacitance, and substrate losses [7]:

\[ Q = \frac{\omega L}{R_s} \left( 1 - \frac{R_s^2 C_o}{L} - \omega^2 L C_o \right) \left( R_p \left( \frac{R_p}{R_p + [(\omega L/R_s)^2 + 1]R_s} \right) \right) \]

Once again, this equation is presented so that contributions from the various physical effects are clear. The first term is the classic definition of inductor quality factor that is valid at low frequencies. The second term accounts for the capacitive effects of the inductor. Note that \( C_o \) is used, which includes both parasitic capacitance and the electrostatic coupling with the substrate. The final term considers losses in the substrate resulting from eddy currents generated by the inductor’s magnetic field.

### 2.2.4 Self-Resonance

At a certain frequency, the parasitic capacitance and electrostatic coupling with the substrate become so developed that the inductor actually stores equal amounts of electric and magnetic energy, thus rendering it useless. This frequency is called the *self-resonant frequency* (SRF), and from (1.2), the quality factor goes to zero. The inductor must be operated well below the SRF in order to be effective.

### 2.3 Skin and Proximity Effects

The *skin effect* refers to the phenomenon in which current flows along the outside of a conductor (along the “skin”) at high frequencies. The skin effect reduces the effective cross-sectional area of the conductor, which causes the series resistance to increase and the quality factor of the inductor to decrease. A basic knowledge of electromagnetic theory is necessary to understand why skin effect occurs and how it should be mitigated.
2.3.1 Electromagnetic Theory

In section 2.1, it was established that when current flowing through an inductor changes, the time-varying magnetic field induces an electromotive force (EMF) in the conductor that opposes the change. This effect may be explained by Maxwell’s equations, two of which are presented here.

The Maxwell-Ampere equation relates the current density \( J \) inside a conductor to the generated magnetic field \( B \):

\[
\nabla \times \vec{B} = \left( \mu \vec{J} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t} \right)
\]

(2.16)

Note that \( \mu \) is the magnetic permeability of the material and \( \varepsilon \) is the electric permittivity.

Next, the Maxwell-Faraday equation states that a time-varying magnetic field is always accompanied by a spatially-varying electric field \( E \):

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]

(2.17)

According to (2.16), current flow within a conductor generates a magnetic field lying in the plane orthogonal to that of the current flow. Then, by (2.17), the magnetic field induces an electric field that lies in the same plane as the original current flow and opposes the magnetic field itself and, in turn, the original current [38].

2.3.2 Skin Effect

All conductors with current flow create this self-induced, opposing electric field. The field is highest at the center of the conductor, which causes the current to naturally crowd radially outward toward the surface of the conductor. As frequency increases, the strength of the electric field also grows, and so the current is forced into an increasingly thin layer along the conductor’s surface. See Figure 2.5.
The current decays exponentially from the surface of the conductor to the center. The thickness of the layer of current is called the skin depth $\delta$, and it is measured from the outer surface to the point at which the current density falls to $1/e$ of its initial value.

\[
J = J_0 e^{-\frac{x}{\delta}}
\]  

(2.18)

\[
\delta = \sqrt{\frac{2\rho}{\omega\mu}}
\]  

(2.19)

The skin depth only depends on the conductor resistivity $\rho$, magnetic permeability $\mu$, and frequency $\omega$, and thus it is a known (frequency-dependent) value for various materials. The skin depth may be used to approximate the effective cross-section of the conductor at high frequencies, which determines the series resistance of the inductor. At low frequencies, where the skin depth is large and current fills the entire cross-section, the series resistance is referred to as DC Resistance (DCR). At frequencies where the skin effect is noticeable, the AC Resistance (ACR) is typically reported for a given frequency.

### 2.3.3 Proximity Effect

The proximity effect is similar to the skin effect, but it is more difficult to quantify. While the skin effect is present for any conductor, the proximity effect occurs when current is applied to neighboring conductors, which is the exact scenario for a spiral inductor. The self-induced electric field generated by one conductor not only opposes its own flow of current, but it also opposes the flow of current inside conductors of close “proximity”. Since the electric field
diminishes spatially away from the original conductor, the proximity effect diminishes as neighboring conductors are spaced farther apart.

In general, the proximity effect is most pronounced in the center of the spiral, where the magnetic field intensity is highest. Furthermore, the short length of the inner windings results in reduced contribution to the overall inductance. Therefore, it is generally more beneficial to design inductors with a large inner diameter to minimize the proximity effect [37]. The larger diameter comes with trade-offs to overall inductor area and parasitic capacitance, and thus the optimal dimension must be carefully determined. The sensitivity of electrical performance parameters such as $L$ and $Q$ to inner diameter is discussed in section 4.2.2.

2.4 Improving Inductor Performance

The preceding sections established the role of the inductor, evaluation of its efficiency using quality factor, and the influence of the skin and proximity effects on performance. Several inductor loss mechanisms were discussed throughout these sections, and these are summarized in Table 2.2.

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series Resistance</td>
<td>Dependent on conductor resistivity, length, and cross-section.</td>
</tr>
<tr>
<td>Parasitic Capacitance</td>
<td>Coupling between neighboring conductors increases capacitance. Increases with smaller spacing between spiral turns.</td>
</tr>
<tr>
<td>Substrate Capacitance</td>
<td>Coupling between spiral and substrate increases capacitance. Increases with substrate permittivity.</td>
</tr>
<tr>
<td>Substrate Eddy Currents</td>
<td>Currents cause ohmic losses and reduce magnetic field strength. Increase with substrate conductivity.</td>
</tr>
<tr>
<td>Skin Effect</td>
<td>Opposing electric field reduces conductor cross-sectional area. Skin depth dependent on frequency and material properties.</td>
</tr>
<tr>
<td>Proximity Effect</td>
<td>Electric field from neighboring conductors reduces inductance. Increases with smaller spacing between spiral turns.</td>
</tr>
</tbody>
</table>
These losses are present for any integrated inductor operating at the target frequency range of 1-10 GHz. The inductor functional requirements presented in Table 1.1 dictate both high inductance \((L > 1 \text{ nH})\) and high quality factor \((Q > 100)\) for a small, thin inductor package. Inductors found in the literature and in industry do not meet the functional requirements because of the formidable challenge of overcoming the loss mechanisms.

The following set of design parameters help mitigate the losses described in Table 2.2. These parameters, presented in Table 2.3, are specific guidelines for any viable inductor fabrication strategy.

**Table 2.3: Inductor Design Parameters Dictated by Loss Mechanisms**

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Effect on Loss Mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire Conductor</td>
<td>The use of round wire rather than micromachining techniques reduces series resistance and eliminates the corner effect.</td>
</tr>
<tr>
<td>Spiral Property Control</td>
<td>The ability to exactly determine the spiral inner diameter and turn spacing reduces parasitic capacitance and proximity effect.</td>
</tr>
<tr>
<td>Desired Substrate</td>
<td>The freedom to use a desired substrate, especially an insulator, reduces substrate capacitance and eddy current formation.</td>
</tr>
</tbody>
</table>

Current inductor fabrication strategies typically do not follow these guidelines. For example, commercial inductor manufacturers normally wrap wire around a magnetic core, as described in section 1.3.4. This technique obviously meets the first design parameter (wire conductor), but it does not allow exact control over spiral properties such as turn spacing. On the other hand, many fabrication techniques found in academia do allow spiral property control and use of a desired substrate, but they rarely utilize drawn wire.

These three design parameters are crucial for developing a fabrication strategy capable of producing high-performing inductors. The following sections provide detailed justification for each parameter.

### 2.4.1 Wire Conductor

As illustrated in the discussion of academic inductors in section 1.3.3, there are many options available for creating spiral-shaped conductors. Apart from wire, conductors are also
formed using common micromachining techniques such as electroplating, sputtering, and etching. Table 2.4 provides descriptions for these processes:

<table>
<thead>
<tr>
<th>Process</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electroplating</td>
<td>A metal surface is coated with a thin layer of another metal through electrolysis.</td>
</tr>
<tr>
<td>Sputtering</td>
<td>Energetic noble gas particles bombard a metal target, releasing atoms and forming a thin film.</td>
</tr>
<tr>
<td>Etching</td>
<td>Metal is selectively removed using photolithography, chemicals (wet etching), or ion bombardment (dry etching).</td>
</tr>
</tbody>
</table>

*Deposition* refers to any process, such as electroplating or sputtering, in which a thin metal layer is formed on a substrate. *Micromachining* refers to the overall process of depositing and then etching to form a microstructure. While these methods certainly have advantages—particularly the ability to create thin conductors—there are two prominent disadvantages inherent to nearly all micromachining techniques. First, the resulting conductor suffers from increased resistance simply because of its rectangular cross-section. Based on the previous discussion of skin effect in section 2.3.2, it might be expected that the current distribution at high frequencies is uniform around the perimeter, as shown in Figure 2.6:

![Figure 2.6: Inaccurate depiction of skin effect for a rectangular cross-section](image)

In actuality, the current distribution is *not* uniform for rectangular cross-sections because of a phenomenon known as the *corner effect*. The current does flow in a thin strip on the top and bottom of the conductor, but there is also additional crowding in the corners that increases the
overall resistance. Because of the complexity of the magnetic fields, it is difficult to quantify this effect analytically; numerical methods are almost exclusively used in practice.

Figure 2.7 illustrates skin and corner effects. COMSOL Multiphysics® (version 4.2) was used to simulate the current density inside three spiral copper conductors at 1 GHz. Each of the three cross-sections is of equal area, but different shape: circular, square, and rectangular with 5:1 aspect ratio. Excess current crowding in the corners is noticeable for both the square and rectangular conductors. Resistive losses of the square and rectangular conductors are 15% and 55% greater than the circular conductor, respectively. Circular cross-sections minimize the ratio of perimeter to area, and thus they always feature the most uniform current distribution.

![Image of current density simulation](image)

**Figure 2.7: The corner effect causes increased resistance for rectangular cross-sections**

The second disadvantage to using surface micromachining techniques is the resulting surface finish of the copper layer. Not only is a fine surface finish difficult to obtain for deposited layers, but often times, the surface is left intentionally coarse to aid with bonding to the
substrate. For example, typical treatments suitable for use with FR-4 dielectrics (a common RF substrate material) produce surface heights of 6 – 18 μm [39]. In a study conducted at McGill University in 2008, Timoshevskii et. al. showed that even atomic-scale surface roughness may lead to a 30 – 40% reduction of the conductance of a copper thin film [40]. The problems associated with poor surface finish are worsened by the skin and corner effects previously described. Since the current primarily travels along the surface of the conductor at high frequencies, surface roughness extends the path the current must travel, increasing series resistance. See Figure 2.8.

While the same concept still applies to wires, the problem is much less severe. Drawn wires typically feature a sub-micron surface roughness [41]. Additionally, the fabrication process presented in this thesis does not rely on bonding of the wire to the substrate, thus eliminating any desire for a coarse finish.

Figure 2.8: Rough surface finish extends current path and increases resistance

2.4.2 Spiral Property Control

Spiral inductors are defined by five properties: spiral shape, inner diameter, number of turns, turn spacing, and wire diameter (conductor cross-section). Two additional properties, conductor length and outer diameter, are determined from the other properties.

The shape of the spiral is typically either square or circular, but it may take other forms as well. In general, circular spirals are preferred because they achieve similar inductance to other geometries with substantially lower DC resistance [38]. Figure 2.9 shows a square and circular spiral and also illustrates the other defining properties:
In section 1.3.1, trade-offs between various inductor performance parameters were highlighted. For example, increasing inner diameter and turn spacing increases the total area of the inductor, but it comes with the benefit of reducing losses from parasitic capacitance and the proximity effect. Often, electrical performance is highly sensitive to the geometric properties of the spiral. Figure 2.10 shows a plot of parasitic capacitance versus turn spacing (given in units of wire diameter). Notice that in this case, if the spiral designer unwittingly chose a turn spacing of 1 wire diameter rather than 1.5, the inductor would suffer from more than quadruple the parasitic capacitance. Thus, it is imperative for the inductor fabrication strategy to allow strict control over spiral properties. Section 4.2.2 provides a detailed analysis of the sensitivity of inductor performance parameters to spiral properties such as turn spacing.
Figure 2.10: Turn spacing substantially affects parasitic capacitance

2.4.3 Desired Substrate

In section 1.3.3, successes of academic efforts to improve inductor performance were highlighted. Many laboratory techniques are concentrated on mitigating losses caused by conductive substrates, and rightfully so: As discussed in section 2.2.3, substrate losses play a significant role in degrading quality factor.

The third and final design parameter necessary for a high-performing fabrication strategy is the freedom to utilize any desired substrate, especially an insulator. Fabrication processes requiring silicon or metallic substrates are severely handicapped compared to alternative strategies. For the fabrication concept presented in this thesis, borosilicate glass was selected as the desired substrate.

Dielectric constant (κ) and coefficient of thermal expansion (CTE) are two critical properties for choosing a substrate. The dielectric constant, or relative permittivity, is the ratio of the material’s permittivity to that of free space. High-permittivity materials are more susceptible to electrostatic coupling and are generally not preferred. Section 4.2.1 describes in detail the impact of the dielectric constant on inductor performance. CTE is also significant: Since a variety of electrical components are placed on a single chip, it is necessary for them to react...
similarly to thermal changes to prevent mechanical failure. Many electrical components other than inductors are fabricated on silicon as well, and so choosing a substrate with a close CTE-match with silicon is desirable.

Table 2.5 shows some common substrate material choices. Notice that borosilicate glass features a low dielectric constant as well as a CTE that is comparable to silicon.

<table>
<thead>
<tr>
<th>Table 2.5: Properties of Common Substrate Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon (Si)</td>
</tr>
<tr>
<td>Dielectric Constant (κ)</td>
</tr>
<tr>
<td>CTE (μm/m°C)</td>
</tr>
</tbody>
</table>
This chapter explains how the stencil-and-guide inductor fabrication concept was selected. Section 3.1 uses the design parameters of Table 2.3 to choose an overall fabrication strategy of planar winding. Section 3.2 applies a large-deflection beam bending analysis to planar inductors. Through this analysis, the relationship between force and deformation in forming of micro-scale coils is defined. Section 3.3 uses this information to eliminate common fabrication techniques that cannot compensate for the magnitude of forces acting on the wire. As a result, the stencil-and-guide concept is developed, a process that ensures complete control over inductor design and reliable fabrication.

### 3.1 Selecting a Fabrication Strategy

There are two strategies for inductor fabrication under consideration: spindle winding and planar winding. In the following sections, the design parameters of Table 2.3 are used to evaluate both of these strategies. Note that all of these parameters can be met in some way by both strategies; the purpose of this analysis is to reveal which strategy is **better** suited to satisfy the parameters. Other strategies unable to meet all parameters are not presented. One example is surface micromachining, which cannot utilize a round wire conductor.

#### 3.1.1 Spindle Winding

The first fabrication strategy is *spindle winding*, whereby magnet wire is unspooled onto an inductor core fixed to a rotating spindle. This is the classic method of fabricating wire-based inductors, and it is usually accomplished using a coil winding machine. Refer to the diagram shown in Figure 3.1. Spindle winding is particularly suitable for larger inductors requiring many
turns, but it can be adapted to thin coils as well. This method is also used for coiling other materials such as ropes and fibers.

**Figure 3.1: Coil winding machine unspools wire onto a rotating spindle**

Four sample coils were fabricated using a coil winding machine to quickly determine the feasibility of this strategy. An appropriate fixture was created for winding the coils, pictured in Figure 3.2. Two Teflon discs control the thickness of the coils. The discs are separated using shim stock and a 2-56 set screw on the top disc. A 1.59-mm steel dowel pin serves as the (removable) inductor core, which is clamped into the spindle. An anti-stick coating is applied to the pin prior to winding.

**Figure 3.2: Coil winding machine fixture for sample coils**
41 AWG magnet wire with self-bonding adhesive was used for the coils. First, one end of the wire was secured using a machine screw on the bottom disc. Then, the wire was wound around the dowel pin by rotating the spindle. The magnet wire’s adhesive was activated by spraying isopropyl alcohol onto the wire during winding. After winding was complete, the wire was cut and secured to the second machine screw on the bottom disc. The adhesive was then allowed to cure for approximately six hours. After curing, the coil was removed from the fixture. The wire adhesive maintained the coil’s shape, while the anti-stick coating on the dowel pin and the Teflon disc material prevented the coil from adhering to the fixture. The fabricated inductor coils are pictured in Figure 3.3:

![Sample inductors fabricated using coil winding machine](image)

**Figure 3.3: Sample inductors fabricated using coil winding machine**

There are inherent limitations to producing coils using the spindle winding strategy. First, the wire is kept in tension throughout the winding process, which results in bunching of the wire. Second, the only way to set the planar turn spacing is by changing the thickness of the wire insulation, but even this does not solve the problem. As shown by the second coil, the spiral turns are not concentric and the spacing is irregular. Third, the inner diameter of the coil is completely determined by the diameter of the inductor core, which forces a lower limit on the spiral property (the core must be sufficiently large to be clamped by the spindle and withstand wire forces). Finally, the ability to use a desired substrate requires transfer from the fixture to the substrate and then additional post-processing steps to attach the wire to the substrate.

### 3.1.2 Planar Winding

*Planar winding* is a unique approach developed by this research. Rather than coiling wire around a rotating spindle, this strategy proposes to lay the wire directly onto the desired substrate. This enables complete control of spiral properties, particularly turn spacing and inner
diameter, with no limitations around wire coatings or dowel pin sizes. Any spiral shape can be formed, not just circular inductors. Furthermore, no transfer process or complex packaging steps are required after winding.

Figure 3.4: Planar winding strategy enables complete control over spiral properties

3.1.3 Strategy Selection

A Pugh chart summarizing both strategies with respect to the design parameters is presented in Table 3.1. While details for the planar winding strategy are still ambiguous, it is clear that it offers worthwhile advantages over spindle winding.

<table>
<thead>
<tr>
<th></th>
<th>Wire Conductor</th>
<th>Spiral Property Control</th>
<th>Desired Substrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar Winding</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Spindle Winding</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: Both strategies readily utilize round wire for the conductor. Spindle winding has limited ability to set turn spacing and inner diameter. Spindle winding requires transfer to desired substrate.
3.2 Understanding Wire Forces

The challenge of the planar winding strategy is determining how to form and maintain the wire in a desired spiral shape. Once deflected, the elasticity of the wire forces it to return to its previous position. There are many concepts that could be employed to control the wire; ascertaining which of these is most appropriate, particularly for the given size scale, requires an understanding of the forces acting on the wire during deflection.

A constrained wire may be represented in two dimensions as a cantilever beam with circular cross section, as shown in Figure 3.5. The wire has modulus of elasticity $E$, length $l$, and area moment of inertia $I$, given in (3.1). The free end of the wire is deflected by a force $F$.

$$I = \frac{\pi D^4}{64}$$  \hspace{1cm} (3.1)

Figure 3.5: Constrained wire is represented as a cantilever beam

The fixed endpoint condition may be replaced by a reaction force and moment, the magnitudes of which are easily determined using a basic force balance.

Figure 3.6: Free body diagram of cantilever beam with endpoint force
Any technique used to constrain the wire during bending must be able to generate a holding force equal to the applied bending force as well as support the accompanying moment.

The next step is to determine the magnitude of the bending force, which may be calculated from the deflection of the wire. The deflection may be specified by either the change in endpoint location or the change in beam slope, as shown in Figure 3.7. Bernoulli-Euler beam theory provides relationships between beam deflection and bending force, given in (3.4) and (3.5).

\[ F = \frac{3EI}{l^3} \delta_{max} \]  

\[ F = \frac{2EI}{l^2} \theta \]  

![Figure 3.7: Endpoint deflection of cantilever beam](image)

(3.4) and (3.5) are valid for small beam deflections. Unfortunately, the act of bending wire into a suitable inductor geometry often requires large deformations, with beam slopes approaching 90°. In [1], Larry Howell provides multiple solutions to the large deflection problem. Section 3.2.1 offers a derivation of the Bernoulli-Euler equations that is solved using elliptic integrals. Section 3.2.2 presents a pseudo-rigid-body approximation that is useful for visualizing the deflection and checking the results of integration. As further validation, section 3.2.3 uses a finite element analysis to simulate the beam deflection and verify the calculated force vectors.
3.2.1 Bernoulli-Euler Solution to Large Deflection

The deflected beam of Figure 3.7 is replaced by a beam undergoing large deflection. Figure 1.2 was originally presented in section 1.1 and is provided again here for reference. The force acting on the end of the wire is no longer assumed to be purely vertical. For convenience, the angle \( \phi \) is defined by a parameter \( n \), which may be positive or negative.

\[
\begin{align*}
\phi &= \tan^{-1}(1/n) \text{ for } n < 0 \\
\phi &= \tan^{-1}(1/n) + \pi \text{ for } n \geq 0
\end{align*}
\]

(3.6)

![Figure 1.2: Free body diagram for large deflection wire bending](image)

The Bernoulli-Euler equation states that bending moment is proportional to beam curvature:

\[
\kappa = \frac{d\theta}{ds} = \frac{M}{EI}
\]

(3.7)

Using the notation of Figure 1.2, the internal moment at any point along the beam is given by:

\[
M = P(a - x) + nP(b - y)
\]

(3.8)

Substituting (3.8) into (3.7) and differentiating with respect to \( s \) yields:
\[
\frac{d\kappa}{ds} = \frac{d^2 \theta}{ds^2} = \frac{P}{EI} \left( -\frac{dx}{ds} - n \frac{dy}{ds} \right)
\]  
(3.9)

Since \(dx\), \(dy\), and \(ds\) are infinitesimal, this may also be written as:

\[
\frac{d\kappa}{ds} = \frac{d^2 \theta}{ds^2} = -\frac{P}{EI} (n\sin \theta + \cos \theta)
\]  
(3.10)

Rewriting the second derivative of \(\theta\) using the definition of \(\kappa\) and the chain rule, separating variables, and integrating produces:

\[
\frac{\kappa^2}{2} = \frac{P}{EI} (n\cos \theta - \sin \theta) + C_1
\]  
(3.11)

Substituting for \(C_1\) and rearranging into non-dimensionalized form yields:

\[
\kappa = \frac{d\theta}{ds} = \sqrt{2} \frac{\alpha}{\sqrt{\lambda}} \frac{\left( \lambda - \sin \theta + n\cos \theta \right)}{\sqrt{\lambda - \sin \theta + n\cos \theta}}
\]  
(3.12)

Finally, separating variables and integrating gives:

\[
\alpha = \frac{1}{\sqrt{2}} \int_{0}^{\theta_0} \frac{d\theta}{\sqrt{\lambda - \sin \theta + n\cos \theta}}
\]  
(3.13)

This may be solved using numerical integration, or perhaps more readily, using elliptic integrals. Using the elliptic integral tables of Byrd and Friedman [42] results in

\[
\alpha = \frac{1}{\sqrt{\eta}} [F(t) - F(g, t)],
\]  
(3.14)

where \(F(t)\) and \(F(g, t)\) are the complete and incomplete integrals of the first kind, respectively, and
\[ \eta = \sqrt{1 + n^2} \quad (3.15) \]

\[ t = \sqrt{\frac{\eta + \lambda}{2\eta}} \quad (3.16) \]

\[ g = \sin^{-1} \left( \frac{\eta - n}{\sqrt{\eta + \lambda}} \right) \quad (3.17) \]

\[ \lambda = \sin \theta_0 - n \cos \theta_0 \quad (3.18) \]

Once \( \alpha \) is known, then the forces acting at the end of the beam for a desired large angular deflection \( \theta_0 \) are given by:

\[ P = \frac{\alpha^2 EI}{l^2} \quad (3.19) \]

\[ F = \eta P \quad (3.20) \]

So, the procedure for finding the magnitude of the applied force is as follows: First set the force angle \( \phi \), which determines the parameter \( n \) through (3.6). Then set the beam deflection angle \( \theta_0 \). Once these are both known, (3.14) is solved to find \( \alpha \), which is used along with the beam properties \( E, I \), and \( l \) to find the forces using (3.19) and (3.20).

In section 1.1, Figure 1.3 was presented for \( \theta_0 = 90^\circ \) and \( \phi = 135^\circ \). A 90° deflection angle is representative of one quarter of a spiral turn. The 135° applied force angle corresponds to \( n = 1 \) and assumes that the force acts equally in the horizontal and vertical directions. Wire lengths as small as 1 mm are certainly possible considering the small coil area.

An even worse scenario is one in which the applied force is not distributed equally between the horizontal and vertical directions. Figure 3.8 shows the force required for a nearly vertical force, \( \phi = 96^\circ \) (\( n = 0.1 \)). In this plot, the magnitude of the required force is approximately triple the magnitude in the previous case. These values serve as an adequate upper limit on the applied force magnitude.
Figure 1.3: Relationship between force and deflection for copper wire

Figure 3.8: Magnitude triples for near-vertical applied force angle
3.2.2 Pseudo-Rigid-Body Approximation for Large Deflection

In order to verify the results of integration, the large-deflection beam problem is represented as a pseudo-rigid body [1], as shown in Figure 3.9. This model has a torsional spring with constant $K$ and stiffness coefficient $K_\theta$ that is located using a characteristic radius factor $\gamma$. The pseudo-rigid-body angle $\Theta$ is defined using a parametric angle coefficient $c_\theta$:

$$\theta_0 = c_\theta \Theta$$  \hspace{1cm} (3.21)

$K_\theta$, $\gamma$, and $c_\theta$ are all related to $\phi$ (and $n$) and are provided for various angles in [1]. $K_\theta$ and $\gamma$ are valid up to a maximum value of $\Theta$, which is also given for each $\phi$. The desired $\theta_0$ and $c_\theta$ is used with (3.21) to find $\Theta$ and check the validity conditions. Once $K_\theta$ and $\gamma$ are verified, the spring constant $K$ is calculated as:

$$K = \frac{\gamma K_\theta EI}{l}$$  \hspace{1cm} (3.22)

Then, the tangential component of the applied force is found by:
\[ F_t = \frac{K\theta}{\gamma l} \]  

(3.23)

Next, the vertical component of the force is determined:

\[ P = \frac{F_t}{\eta \sin(\phi - \theta)} \]  

(3.24)

Finally, the applied force is calculated using (3.15) and (3.20).

Following this procedure, the plot in Figure 1.3 is recreated using the pseudo-rigid-body calculations. The plot in Figure 3.10 shows excellent agreement with the previous one: the approximated solutions are correct to within 4% of the Bernoulli-Euler solutions.

![Figure 3.10: Pseudo-rigid-body approximation of applied force](image)

**3.2.3 Model Validation with Finite Element Analysis**

The Bernoulli-Euler solution to large deflection and the pseudo-rigid-body approximation are in excellent agreement. As a final validation, the solutions were confirmed using finite element analysis. CAD models of beams of varying length and diameter were created using SolidWorks. The simulation tool was used to apply the forces predicted from analysis and verify
the resulting angular deflection of the beam. The large displacement required a nonlinear study, and a two-dimensional beam simplification was used to decrease computation time.

Figure 3.11 shows one simulation of a 2-mm length of copper wire having a 75-μm diameter. The Bernoulli-Euler solution predicts that an applied endpoint force of 175 mN \((F)\) at an angle of 135° \((\phi)\) is required to deflect the beam 90° \((\theta_0)\). Applying this force to the beam results in an angular deflection within 1% of 90°.

---

**Figure 3.11: Simulated wire deflects 90° as predicted by the Bernoulli-Euler solution**

### 3.3 Selecting a Fabrication Concept

In section 3.1.3, planar winding was chosen as the fabrication strategy because of its ability to satisfy the design parameters. In this section, four concepts for executing the planar winding strategy are presented. The purpose of this section is not to consider every possible concept for planar winding, only to evaluate those ideas that are common and often applied to similar problems. In section 3.2, large-deflection beam models were applied to planar wire bending. This analysis provided relationships between force, deformation, and wire properties, as shown in Figure 1.3 and Figure 3.8. As illustrated in the following sections, this information is critical for selecting a fabrication concept.
3.3.1 Capacitive Zipper

One concept is to use the electrostatic attraction between a conductive spiral trace and the wire to “zip” the wire down onto the substrate, as shown in Figure 3.12:

![Capacitive Zipper Diagram](image)

**Figure 3.12: Capacitive force causes wire to “zip” down onto substrate**

To decide if this is a feasible concept, we must utilize the force analysis presented in section 3.2. The capacitive force between the wire and trace acts normal to the substrate. This force must be able to generate friction forces and moment that are high enough to balance the applied forces during bending (see Figure 3.6). Friction force is defined as:

\[
F_f = \mu N
\]  
(3.25)
Here, the normal force $N$ is the required capacitive force $F_{req}$. The magnitude of the friction force $F_f$ must be equal to the magnitude of the applied force $F_{app}$ for equilibrium. Using Figure 3.8, a lenient estimate of the applied force is 250 mN. The coefficient of friction between glass and metal is generously estimated as 0.7 [43].

$$F_{req} = \frac{F_{app}}{\mu_{g-m}} = \frac{250 \text{ mN}}{0.7} \approx 360 \text{ mN} \quad (3.26)$$

The final step is to determine if the capacitance between the wire and the metal trace can generate the required force of 360 mN. The capacitance of a wire parallel to a wall is calculated as [44]

$$C_{wire-wall} = \frac{2\pi \varepsilon l}{\cosh^{-1}(2x/D_{wire})}, \quad (3.27)$$

where $\varepsilon$ is the dielectric permittivity, $l$ is the wire length, $x$ is measured from the center of the wire to the surface of the wall, and $D_{wire}$ is the wire diameter. Then, capacitive force is given by:

$$F = \frac{1}{2} V^2 \frac{dC}{dx} \quad (3.28)$$

Figure 3.13 shows the generated capacitive force for a 75-μm-diameter wire (< 2% difference in force for 50-μm and 25-μm wire sizes). Even for a liberal length of 5 mm and a 1-μm gap, the force is only ~0.5 mN, which is much less than the required 360 mN. Using a capacitive zipper to form and hold the wire is not a feasible concept. At this scale, the forces due to large-deflection bending of the wire are far too great compared to the capacitive force.
3.3.2 Lorentz Actuation

This concept is similar to the capacitive zipper, only magnetic force is substituted for electrostatic actuation. Magnets are configured in a spiral pattern beneath the substrate to attract the current-carrying wire. Figure 3.14 shows a cross section of the setup.

![Magnets generate Lorentz force to secure wire](image)

**Figure 3.14:** Magnets generate Lorentz force to secure wire

Unfortunately, a first order approximation of the magnitude of the Lorentz force reveals that it, too, is much too small. The following calculation is for a current of 5 A, a wire length of 5 mm, and an exceptional rare earth magnet having a field strength of 1.5 T:
\[
F = iLB = (5 \text{ A})(5 \text{ mm})(1.5 \text{ T}) = 37.5 \text{ mN}
\] (3.29)

### 3.3.3 Peg Board

The third concept is to coil the wire around an array of pegs placed on top of the substrate, as shown in Figure 3.15.

![Figure 3.15: Wire is wrapped around pegs to form desired spiral shape](image)

To determine if this concept is feasible, it is necessary to examine the force acting on a peg by the wire:

![Figure 3.16: Wire force causes peg bending](image)

This diagram represents the best case scenario of the wire being situated at the base of the peg. The bending stress on the peg is given by:

\[
\sigma = \frac{My}{I} = \frac{(Fr)R}{ \pi R^4 / 4}
\] (3.30)

The area moment of inertia \(I\) is taken to be that of a circle (assuming cylindrical pegs). Rearranging and substituting bending stress for the yield strength \(S_y\) produces:
\[ R = \frac{3 \sqrt[3]{4Fr}}{\pi S_y} \]  

Figure 3.17 shows a plot of required peg diameter versus yield strength for an applied wire force \( F \) of 250 mN. Unfortunately, the required peg diameters are too large for practical purposes. For example, if using 25-μm wire, the peg diameter must be more than twice the wire diameter for all yield strengths considered. For plastics, which typically have \( S_y < 50 \text{ MPa} \), the required peg diameters are exceedingly large for all wire sizes. These restrictions severely limit the ability of the designer to control the geometry of the spiral.

![Figure 3.17: Required peg diameters restrict spiral geometry](image)

### 3.3.4 Stencil and Guide

The final concept under consideration, and the one selected, is the stencil and guide, a unique process developed by this research. A spiral channel is formed in a machinable layer, referred to as the stencil, which is adhered to the desired substrate. The small inductor size and hair-like wire diameters prompted the design of a wire guide tool used to thread the wire into the trench. Two through-holes allow the wire leads to be secured to the bottom of the substrate.
This concept is not dependent on the forces acting on the wire. The machinable stencil layer allows for desired insulating substrates such as glass to be utilized. Furthermore, the designer has total freedom over the spiral geometry, which is critical for inductor performance.
This chapter utilizes an electrical model to detail the stencil-and-guide fabrication concept. Section 4.1 expands upon the theoretical framework presented in Chapter 2 by providing methods for calculating the inductance, quality factor, and self-resonant frequency of planar inductor coils. Section 4.2 determines the sensitivity of these parameters relative to material and spiral properties. Section 4.3 examines the relationship between the performance parameters themselves. Section 4.4 uses the information from the previous sections to set inductor design rules and to specify the details of the stencil-and-guide process, including material choices, fabrication techniques, and tool functionality.

4.1 Predicting Inductor Performance

The next three sections show how to predict the inductance, quality factor, and self-resonant frequency for a planar wire coil. Many of the formulas presented in this section rely on the spiral properties defined in section 2.4.2: inner diameter (ID), number of turns (N), turn spacing (sp), and wire diameter (Dwire).

4.1.1 Predicting Inductance

The classic formula for predicting the inductance of planar wire inductors is given by Harold Wheeler [45]:

\[
L = 31.33 \mu_0 \frac{r_{avg}^2 N^2}{8r_{avg} + 11r_{diff}} \text{ [H]}
\]

In this equation, \( \mu_0 \) is the permeability of free space, and \( r_{avg} \) and \( r_{diff} \) are based on the inner and outer radius of the spiral:
\[ r_{avg} = \frac{r_{inner} + r_{outer}}{2} \]  
\[ r_{diff} = r_{outer} - r_{inner} \]  
\[ r_{inner} = \frac{ID}{2} \]  
\[ r_{outer} = r_{inner} + N(sp) \]

Wheeler’s formula is simple to use and widely accepted, with a reported accuracy of 5% [45]. Many other methods of calculating inductance have been developed and utilized, but for the purpose of the following sensitivity analysis, Wheeler’s formula is sufficient.

### 4.1.2 Predicting Quality Factor

In section 2.2.3, the complete definition of inductor quality factor was presented, which includes losses from series resistance, parasitic capacitance, and eddy currents in the substrate [7]:

\[
Q = \frac{\omega L}{R_s} \left(1 - \frac{R_s^2 C_o}{L} - \omega^2 L C_o \right) \left(\frac{R_p}{R_p + [\left(\omega L/R_s\right)^2 + 1] R_s}\right)
\]

Calculation of the inductance \( L \) has already been considered. Next is the series resistance \( R_s \). The following definition of series resistance was provided in section 2.2.1:

\[
R_s = \frac{\rho l}{A}
\]

In section 2.3.2, it was shown that the skin effect reduces the effective cross-sectional area \( A \) of the conductor. The skin depth, a frequency-dependent material property, was also defined:

\[
\delta = \sqrt{\frac{2 \rho}{\omega \mu}}
\]
The skin depth is used to estimate the increase in series resistance. For conductors having a circular cross-section, the AC series resistance (resistance at high frequencies) is given by:

\[
R_s = \frac{\rho l}{\pi(D_{wire} - \delta)\delta}
\]  

(4.6)

The length \( l \) of the spiral conductor is closely approximated by:

\[
l = 2\pi r_{avg}N = \pi(r_{inner} + r_{outer})N
\]

(4.7)

The next parameter in the quality factor definition is the overall capacitance \( C_o \), which considers both parasitic capacitance between the inductor turns (\( C_s \)) and electrostatic coupling with the substrate (\( C_p \)), as given in section 2.2.3:

\[
C_o = C_s + C_p
\]

(2.12)

For the stencil-and-guide process, any desired substrate may be directly utilized. If an insulating substrate such as glass is used, then

\[
C_s \gg C_p
\]

(4.8)

and accordingly:

\[
C_o \approx C_s
\]

(4.9)

\( C_s \) captures the parasitic capacitance between successive spiral turns and is calculated using the capacitance between parallel wires [44]:

\[
C_s = \frac{\pi \varepsilon l_{cap}}{\cosh^{-1}(sp/D_{wire})}
\]

(4.10)

Note that for a planar spiral, the length between parallel wires is approximated as:

\[
l_{cap} = \pi(r_{inner} + r_{outer})(N - 1)
\]

(4.11)
Thus, by (4.10), capacitance is dependent on material as well as spiral properties.

Finally, \( Q \) is also dependent on \( R_p \), the effective resistance of the substrate. As discussed in section 2.2.3, \( R_p \) is a frequency-dependent parameter that captures energy dissipation resulting from eddy currents in the substrate. Analytic expressions for \( R_p \) are not readily available. Instead, an exponential curve is tuned using experimental data:

\[
R_p = Ae^{-Bf}
\]

(4.12)

For the purpose of the sensitivity analysis, \( R_p \) is assigned the following parameters, which are typical for a wire-based RF inductor on the size scale under consideration [7]:

\[
\begin{align*}
A &= 2 \times 10^4 \\
B &= 0.3 \times 10^{-9}
\end{align*}
\]

(4.13)

Figure 4.1 shows the corresponding curve over the target frequency range:

**Figure 4.1: Eddy current losses reduce effective substrate resistance at high frequencies**

Notice that the effective resistance diminishes at high frequencies. In this range, the generated eddy currents in the substrate cause substantial losses in the inductor.
4.1.3 Predicting Self-Resonant Frequency

The self-resonant frequency corresponds to when the inductor stores equal amounts of desirable magnetic energy and unwanted electric energy. The quality factor equals zero at the SRF. See sections 1.2.4 and 2.2.4 for a more detailed discussion.

The expression for $Q$ developed in the previous section is plotted across the frequency spectrum to forecast the SRF. Figure 4.2 shows an example:

![Figure 4.2: The self-resonant frequency corresponds with zero quality factor](image)

4.2 Evaluating Performance Sensitivity

The previous section established methods for calculating each of the three performance parameters ($L$, $Q$, and SRF). The following sections discuss the sensitivity of these parameters to changes in material and spiral properties. A theoretical inductor having the following properties is used as a reference point:
Table 4.1: Reference Inductor Material and Spiral Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductor Resistivity (Ω·μm)</td>
<td>( \rho )</td>
<td>0.0168 (Cu)</td>
</tr>
<tr>
<td>Dielectric Constant</td>
<td>( \kappa )</td>
<td>4</td>
</tr>
<tr>
<td>Wire Diameter (μm)</td>
<td>( D_{\text{wire}} )</td>
<td>50</td>
</tr>
<tr>
<td>Number of Turns</td>
<td>( N )</td>
<td>2</td>
</tr>
<tr>
<td>Inner Diameter (μm)</td>
<td>( ID )</td>
<td>300</td>
</tr>
<tr>
<td>Turn Spacing (μm)</td>
<td>( sp )</td>
<td>100</td>
</tr>
</tbody>
</table>

The influence of these properties on performance are illustrated by reporting the resulting change in \( L \), \( Q \), and SRF and showing the effect on the quality factor curve (which also captures the self-resonant frequency). The reference properties produce the following curve and performance specifications:

![Reference Quality Factor Curve](image)

**Figure 4.3: \( L \), \( Q \), and SRF for Reference Inductor**
4.2.1 Sensitivity to Material Properties

*Resistivity* is a measure of how strongly a material opposes the flow of electric current and primarily affects series resistance. It is also a factor in the skin depth calculation, which further increases the series resistance at high frequencies. Resistivity was first introduced in section 2.2.1.

Figure 4.4 and Table 4.2 illustrate the effects of changing the conductor resistivity. Four common materials are considered for practical application. Since $L$ is only dependent on spiral properties, it does not change. Similarly, SRF remains the same because it only accounts for electric energy storage. Potential improvement in $Q$ is substantial.

![Figure 4.4: Increasing resistivity reduces Q](image)

**Table 4.2: Performance Sensitivity to Resistivity**

<table>
<thead>
<tr>
<th>$\rho$ (Ω·μm)</th>
<th>$L$ (nH)</th>
<th>Peak $Q$</th>
<th>SRF (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0159 (Ag)</td>
<td>2.3</td>
<td>91</td>
<td>8.9</td>
</tr>
<tr>
<td>0.0168 (Cu)</td>
<td>2.3</td>
<td>89</td>
<td>8.9</td>
</tr>
<tr>
<td>0.0244 (Au)</td>
<td>2.3</td>
<td>76</td>
<td>8.9</td>
</tr>
<tr>
<td>0.0282 (Al)</td>
<td>2.3</td>
<td>72</td>
<td>8.9</td>
</tr>
</tbody>
</table>
A material’s *dielectric constant*, or *relative permittivity*, is the ratio of that material’s permittivity to the permittivity of free space. The dielectric constant affects parasitic capacitance—high-permittivity materials are more susceptible to electrostatic coupling. The dielectric constant was introduced in section 2.4.3 with reference to utilizing a desired substrate, and Table 2.5 provides the dielectric constant for several possible substrate materials.

Figure 4.5 and Table 4.3 demonstrate the effects of changing the dielectric constant of the substrate. Once again, $L$ is not dependent on material properties, so it remains the same. Increasing the permittivity reduces $Q$ and substantially decreases the SRF.

![Figure 4.5: Increasing the dielectric constant reduces $Q$ and SRF](image)

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$L$ (nH)</th>
<th>Peak $Q$</th>
<th>SRF (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.3</td>
<td>91</td>
<td>12.6</td>
</tr>
<tr>
<td>4</td>
<td>2.3</td>
<td>89</td>
<td>8.9</td>
</tr>
<tr>
<td>6</td>
<td>2.3</td>
<td>87</td>
<td>7.4</td>
</tr>
</tbody>
</table>
4.2.2 Sensitivity to Spiral Properties

The first spiral property to be considered is wire diameter. The cross-sectional area of the conductor affects both series resistance and parasitic capacitance. A smaller area is similar to a crowded hallway: the restricted current flow increases resistance. At the same time, a smaller cross-section also reduces parasitic capacitance because less conductor area is exposed.

The interaction between these effects is not easily determined. Figure 4.6 and Table 4.4 show that despite a reduction in SRF, increasing wire diameter tremendously increases $Q$. Furthermore, increasing wire diameter does not substantially increase the overall area of the inductor.

![Q Sensitivity to Wire Diameter](image)

**Figure 4.6:** Increasing wire diameter improves $Q$ but reduces SRF

<table>
<thead>
<tr>
<th>$D_{\text{wire}}$ (μm)</th>
<th>$L$ (nH)</th>
<th>Peak $Q$</th>
<th>SRF (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>2.3</td>
<td>51</td>
<td>11.1</td>
</tr>
<tr>
<td>50</td>
<td>2.3</td>
<td>89</td>
<td>8.9</td>
</tr>
<tr>
<td>75</td>
<td>2.3</td>
<td>121</td>
<td>6.9</td>
</tr>
</tbody>
</table>
The final three spiral properties (number of turns, inner diameter, and turn spacing) determine the size and layout of the spiral, which sets the inductance. By (4.7) and (4.11), they also define the length of the conductor and the capacitive length, which impacts series resistance and parasitic capacitance, respectively.

Once again, the relative magnitude of these effects is not readily apparent. Figure 4.7 and Table 4.5 show that while $Q$ remains relatively the same, increasing the number of turns produces large gains in $L$, but severely reduces the SRF. Furthermore, adding even a single turn comes with a significant cost to inductor area.

![Q Sensitivity to Number of Turns](image)

**Figure 4.7: Increasing number of turns improves $L$ but reduces SRF**

**Table 4.5: Performance Sensitivity to Number of Turns**

<table>
<thead>
<tr>
<th>$N$</th>
<th>$L$ (nH)</th>
<th>Peak $Q$</th>
<th>SRF (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.58</td>
<td>87</td>
<td>$&gt; 20$</td>
</tr>
<tr>
<td>2</td>
<td>2.3</td>
<td>89</td>
<td>8.9</td>
</tr>
<tr>
<td>3</td>
<td>5.6</td>
<td>84</td>
<td>3.7</td>
</tr>
</tbody>
</table>
Figure 4.8 and Table 4.6 illustrate the sensitivity to inner diameter. As expected, the performance parameters trend in the same directions as with the number of turns, but the effects are slightly less pronounced. In section 2.3.3, inner diameter was mentioned in reference to the proximity effect, which is most pronounced at the center of the spiral. Here it is shown that increasing inner diameter (and thus spiral size) does improve $L$. The extra conductor length also results in additional resistance and capacitance, however, which reduces the SRF. $Q$ remains essentially constant—the positive increase in $L$ is negated by the negative effects for inductors of the size considered.

![Q Sensitivity to Inner Diameter](image)

**Figure 4.8: Increasing inner diameter improves $L$ but reduces SRF**

<table>
<thead>
<tr>
<th>$ID$ (μm)</th>
<th>$L$ (nH)</th>
<th>Peak $Q$</th>
<th>SRF (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>1.3</td>
<td>88</td>
<td>14.1</td>
</tr>
<tr>
<td>300</td>
<td>2.3</td>
<td>89</td>
<td>8.9</td>
</tr>
<tr>
<td>450</td>
<td>3.5</td>
<td>88</td>
<td>6.4</td>
</tr>
</tbody>
</table>
Finally, Figure 4.9 and Table 4.7 demonstrate the effects of changing turn spacing. In section 2.4.2, it was shown that parasitic capacitance significantly decreases when the turn spacing is increased beyond 1 wire diameter (50 μm). That point is confirmed here: notice the leap in SRF when the spacing is increased to 100 μm. It is also interesting that $Q$ decreases as turn spacing increases, indicating that the additional conductor length causes resistance to dominate the quality.

![Q Sensitivity to Turn Spacing](image)

**Figure 4.9: Increasing turn spacing improves SRF but reduces $Q$**

<table>
<thead>
<tr>
<th>$sp$ (μm)</th>
<th>$L$ (nH)</th>
<th>Peak $Q$</th>
<th>SRF (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>2.3</td>
<td>93</td>
<td>3.9</td>
</tr>
<tr>
<td>100</td>
<td>2.3</td>
<td>89</td>
<td>8.9</td>
</tr>
<tr>
<td>150</td>
<td>2.5</td>
<td>80</td>
<td>9.1</td>
</tr>
</tbody>
</table>
4.3 Relating Performance Parameters

The previous section examined the sensitivity of the performance parameters ($L$, $Q$, and SRF) to changes in material and spiral properties. It is also important to determine if there is a relationship between the performance parameters themselves. During the evaluation of prior art in section 1.3, inductors were presented on charts showing $Q$ versus $L$. For example, Figure 1.15 presents the complete collection of surveyed inductors meeting size requirements, including devices from both industry and academia. The trend of data points along the axes of that chart implies that there may be a fundamental limit on quality factor for a given inductance.

The definition of quality factor for an ideal inductor was first presented in section 2.2.1:

$$Q = \frac{\omega L}{R_s}$$  \hspace{1cm} (2.6)

The series resistance $R_s$ was also defined:

$$R_s = \frac{\rho l}{A}$$  \hspace{1cm} (2.2)

Substituting the approximation for the length $l$ given by (4.7), the resistance is rewritten as:

$$R_s = \frac{2\pi \rho r_{avg} N}{A}$$  \hspace{1cm} (4.14)

In section 4.1.1, inductance was calculated using (4.1). For the purpose of scaling, the Wheeler formula is reasonably estimated by [46]:

$$L \approx \mu_0 r_{avg} N$$  \hspace{1cm} (4.15)

Substituting (4.14) and (4.15) into (2.6) yields:

$$Q \approx \frac{\omega \mu_0 A}{2\pi \rho}$$  \hspace{1cm} (4.16)
For the ideal inductor, \( Q \) is only dependent on material properties (permeability of free space \( \mu_0 \) and resistivity \( \rho \)), the frequency \( \omega \), and the cross-sectional area of the conductor \( A \). Figure 4.6 illustrates the tremendous impact of wire diameter on the quality factor, and that point is confirmed here: the maximum attainable quality factor is affected by the wire size. As discussed in section 2.3.2, the skin effect is also critical because it reduces the effective cross-sectional area.

By the definition of quality factor presented in (4.16), it appears that the inductance has been removed, which is contrary to the previous notion of a fundamental limit on \( Q \) for a given \( L \). The new \( Q \) definition still contains the frequency \( \omega \), however, which is also related to the inductance:

\[
\omega \propto \frac{1}{\sqrt{L}} \tag{4.17}
\]

Substituting (4.17) into (4.16) yields:

\[
Q \propto \frac{1}{\sqrt{L}} \frac{\mu_0 A}{2\pi \rho} \tag{4.18}
\]

This relationship between \( Q \) and \( L \) produces the expected trend along the axes shown by the data in Figure 1.15.

### 4.4 Detailing the Concept

Conclusions drawn from the preceding sections are summarized into design rules for fabricating planar inductor coils, presented in Table 4.8. These design rules inform specific decisions for the stencil-and-guide fabrication concept. An overview of the process was first presented in section 1.1 and then again in section 3.3.4 during concept selection. The following sections specify the details of the process, including material choices, fabrication techniques, and tool functionality.
Table 4.8: Inductor Design Rules Dictated by Sensitivity Analysis

<table>
<thead>
<tr>
<th>Design Rule</th>
<th>Motivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use Ag or Cu conductor.</td>
<td>Silver/copper significantly improves $Q$ with no cost to $L$ or SRF.</td>
</tr>
<tr>
<td>Use low-$\kappa$ substrates.</td>
<td>Low-permittivity substrates improve $Q$ and SRF with no cost to $L$.</td>
</tr>
<tr>
<td>Use largest possible $D_{\text{wire}}$.</td>
<td>Large wire sizes tremendously improve $Q$ with relatively small cost to SRF and inductor area.</td>
</tr>
<tr>
<td>Optimize $N$, $ID$, $sp$.</td>
<td>First set $N$, which has the most impact on performance and size. Then tune $ID$ and $sp$ for desired $L$, $Q$, SRF, and area.</td>
</tr>
</tbody>
</table>

4.4.1 Selecting Materials

Copper wire is the preferred conductor, as recommended by the first design rule. While silver has slightly lower resistivity, copper’s low cost and availability make it the more attractive choice and the industry standard for inductor fabrication.

The best material for the machinable layer is not as apparent. One of the original design parameters of Table 2.3 is the ability to utilize a desired insulating substrate to minimize electrostatic coupling. This notion is confirmed by the sensitivity analysis, which shows that low-dielectric substrates improve the quality factor and substantially increase the self-resonant frequency of the inductor. Consequently, the machinable layer to be adhered to the substrate must also have low permittivity.

DuPont™ Kapton® polyimide film [47] is utilized for several reasons. First, it has a low dielectric constant (3.5) that is similar to glass (4.1), the chosen insulating substrate. In addition, it features excellent dimensional stability and adhesion, and its properties are stable over a wide range of temperatures (-269 – 400 °C). Perhaps most importantly, it is easily machinable: In industry, Kapton is most commonly etched with a laser, but it can also be mechanically milled. Furthermore, Kapton is readily available in thin sizes (25 – 125 μm), with optional adhesive already integrated on the sheet. It is commonly used on flexible printed circuits and is widely accepted by the electrical community.
4.4.2 Creating the Stencil

The next decision is to choose the technique for forming the spiral channel in the polyimide layer. The trench must have a width that is at least equal to the diameter of the desired wire. The sensitivity analysis reveals that larger wire diameters substantially improve the quality of the inductor. Since the inductor area requirements readily accommodate 75-μm wire sizes, this dimension is taken as the smallest required channel width.

Two techniques are considered for forming the spiral trench: laser cutting and mechanical milling. Cutting with a short-wavelength laser (~200 nm) enables formation of very precise channels [48]. In the absence of a laser with this capability, setup expenses may cause this process to be costly (> $1,000). Mechanical milling, by comparison, still offers fine feature sizes—end mills as small as 25 μm are available for ~$50, and prices are substantially cheaper for 50- and 75-μm tools [49]. Since the smallest required channel widths are as large as 75 μm, these tools are more than sufficient for creating the desired spiral channel.

Furthermore, it is possible to intentionally produce a coarse finish when using a cutting tool. Although usually not desirable, in this case, the roughness on the inner surface of the spiral trench creates additional friction with the wire and may prevent slipping during threading.

Table 4.9 compares both techniques and summarizes the advantages offered by milling:

<table>
<thead>
<tr>
<th></th>
<th>Trench Quality</th>
<th>Application Suitability</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser Cutting</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mechanical Milling</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Notes:
- Both techniques can be used to create the desired channel.
- Cutting tools enable the ability to control surface finish.
- Expensive setup costs for laser cutting make it impractical.
4.4.3 Using the Wire Guide Tool

The wire guide tool enables threading of the wire into the spiral channel while keeping it in place. The tool is constructed from acrylic because of the material’s transparency, machinability, and low friction with polyimide. Figure 4.10 shows a detailed process flow for utilizing the tool. Note that there are two through-holes for the inductor leads, located at the center and outside of the spiral. First, the bottom hole of the wire guide tool is aligned with the center through-hole. At the start, the tool simply rests on top and is free to slide in the planar directions. Second, an oversized length of wire is unspooled and threaded through all three pieces, and one end of the wire is secured to the bottom of the substrate. Third, the wire is threaded into the channel by sliding the guide tool over the spiral, starting from the center and continuing past the outer through-hole. Finally, the wire is cut, and the free end is pushed through the outer through-hole and secured to the bottom of the substrate. With the wire now in tension, the tool is discarded and an optional adhesive may be applied to the top of the device.
2. Wire guide tool (on top)
Bottom of glass substrate (transparent)

3. Push wire through here
The sensitivity to turn spacing influenced the design and function of the wire guide tool. For example, consider 75-μm wire being threaded into an oversized channel having a width of 80 μm. In the worst case, sequential wire turns are resting against opposite walls of the channel, which would result in a spacing shift of 5 μm. By Figure 4.9, however, adjustments in spacing on the order of 50 μm are required for substantial changes in performance. Consequently, the purpose of the wire guide tool is only to ensure that the wire is threaded into the channel—it does not need to determine wire placement within the trench.
CHAPTER

5

RESULTS AND CONCLUSION

This chapter evaluates three inductors created using the stencil-and-guide fabrication process. Section 5.1 discusses the design of each inductor and provides predicted performance parameters based on the electrical model. Section 5.2 describes the measurement setup and the process for extracting the performance parameters. Section 5.3 compares the measurement results to the predicted values and shows that the new inductors meet the original functional requirements. Section 5.4 summarizes this research, and section 5.5 offers insight for future work.

5.1 Fabricated Inductors

Three inductor prototypes were fabricated using the stencil-and-guide fabrication process, pictured in Figure 5.1. As determined in section 4.4.1, all three inductors utilize copper wire threaded into a Kapton polyimide stencil. The polyimide layer is 150 μm thick and is bonded to a 110-μm-thick glass substrate, for a total device thickness of 260 μm. A micro milling machine (Microlution 363-S) with a 75-μm square end mill was used to cut the spiral channel to a depth of 100 μm. A drill press (100-μm bit) with a microscope alongside for easy viewing was used to create the through-holes at the center and outside of the spiral.

![Figure 5.1: Three fabricated inductor prototypes](image)

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Table 5.1: Spiral Properties of Fabricated Inductors

<table>
<thead>
<tr>
<th>Spiral Property</th>
<th>Inductor 1</th>
<th>Inductor 2</th>
<th>Inductor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire Diameter (μm)</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Number of Turns</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Inner Diameter (μm)</td>
<td>250</td>
<td>250</td>
<td>500</td>
</tr>
<tr>
<td>Turn Spacing (μm)</td>
<td>252</td>
<td>195</td>
<td>195</td>
</tr>
<tr>
<td>Outer Diameter (μm)</td>
<td>1,760</td>
<td>1,030</td>
<td>1,280</td>
</tr>
<tr>
<td>Wire Length (μm)</td>
<td>9,481</td>
<td>4,021</td>
<td>5,592</td>
</tr>
</tbody>
</table>

Figure 5.2: Quality factor curves generated by model for fabricated inductors

Table 5.2: Predicted Performance of Fabricated Inductors

<table>
<thead>
<tr>
<th></th>
<th>$L$ (nH)</th>
<th>Peak $Q$</th>
<th>SRF (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductor 1</td>
<td>7.3</td>
<td>93</td>
<td>6.1</td>
</tr>
<tr>
<td>Inductor 2</td>
<td>2.4</td>
<td>104</td>
<td>17.6</td>
</tr>
<tr>
<td>Inductor 3</td>
<td>4.0</td>
<td>105</td>
<td>11.5</td>
</tr>
</tbody>
</table>
Inductor 1 is a three-turn inductor with a correspondingly high predicted inductance and relatively low self-resonant frequency. Inductor 2 and Inductor 3 are identical except for inner diameter, which was doubled for Inductor 3. According to Figure 4.8 of the sensitivity analysis, this has the effect of improving $L$ and maintaining $Q$ at the expense of SRF, an acceptable concession if the operating frequency is sufficiently low. The predicted curves and performance parameters confirm the expected trends.

All three inductors were fabricated using 75-μm wire, a large size that still accommodates spacing and overall area requirements. As established by the sensitivity analysis and stated by the third design rule in Table 4.8, large wire diameters should be used because of their tremendous impact on quality factor (see Figure 4.6). It is primarily for this reason that the predicted peak $Q$ for both Inductors 2 and 3 is beyond 100.

The sensitivity analysis shows that there is a minimum threshold for turn spacing: below this value, the parasitic capacitance becomes substantial and the SRF plummets (see Figure 2.10 and Figure 4.9). The turn spacing for the fabricated inductors was set beyond this value and then tuned for desired size and performance.

Outer diameter and wire length are dependent properties and are provided in Table 5.1 for reference. The outer diameter is calculated using (4.4) and (4.5), and the wire length is closely approximated using (4.7).

### 5.2 Measurement Setup

The performance of the inductors was measured using a Cascade Microtech Microwave R&D Probe Station (Model 44) and an Agilent Microwave Network Analyzer (N5241A PNA-X), similar to the one pictured in Figure 5.3.

![Agilent microwave network analyzer](image)
One-port scattering parameters (s-parameters) were collected using Picoprobe Ground-Signal RF wafer probes. Prior to every measurement, short-open-load calibration was completed. Test equipment details are presented in Table 5.3:

### Table 5.3: Probe Size and Calibration Substrate for Measured Inductors

<table>
<thead>
<tr>
<th>Spiral Property</th>
<th>Inductor 1</th>
<th>Inductor 2</th>
<th>Inductor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probe Pitch (μm)</td>
<td>755</td>
<td>390</td>
<td>390</td>
</tr>
<tr>
<td>Calibration Substrate</td>
<td>CS-11</td>
<td>CS-14</td>
<td>CS-14</td>
</tr>
</tbody>
</table>

The inductors were measured on the *bottom* of each device, where the leads are secured after threading (the leads were trimmed to eliminate losses due to extra wire). To take the measurements, the probe tips were placed in contact with the leads where they exited the through-holes, as shown in Figure 5.4:

![Figure 5.4: Inductors measured on the bottom of each device](image)

The s-parameters were measured by stepping through 801 points over a frequency range of 0.01 – 13.5 GHz. A high averaging factor (50) was used to reduce noise and improve dynamic range. During measurement, the analyzer exponentially weights the data, which allows the averaging to proceed without termination even after the desired factor has been reached [50]. Since s-parameters are complex numbers, the data is averaged vectorially.

Once the scattering parameters are collected, they are converted to impedance parameters using the following relationships [51]:
\[
\begin{aligned}
    R &= \frac{1 - Re^2 - Im^2}{[(1 - Re)^2 + Im^2]Z_0} \\
    X &= \frac{2Im}{[(1 - Re)^2 + Im^2]Z_0}
\end{aligned}
\] (5.1)

In these equations, \( Z_0 \) is the calibrated load, which was 50 Ω. \( Re \) and \( Im \) are the real and imaginary components of the scattering parameters. \( R \) is the resistance of the device, and \( X \) is the reactance of the device: these are the real and imaginary components of the impedance.

Once the impedance parameters at every frequency are known, then the measured quality factor is simply the ratio of the reactance to the resistance:

\[
    Q_{meas} = \frac{X}{R}
\] (5.2)

As stated in section 2.2.4, the self-resonant frequency of the inductor occurs when the quality factor goes to zero:

\[
    \text{SRF}_{meas} = f \text{ at } (Q_{meas} = 0)
\] (5.3)

The reactance \( X \) contains both the inductance and capacitance of the device. As discussed in sections 2.2.2 and 2.2.3, parasitic capacitance between the spiral turns and electrostatic coupling with the substrate becomes more prominent as frequency increases. At operating frequencies well-below self-resonance, the unwanted electric energy is not yet developed and the reactance consists only of desirable magnetic energy. Consequently, the inductance is extracted as

\[
    L_{meas} = \frac{X}{\omega} \text{ for } f \ll \text{SRF}_{meas},
\] (5.4)

where \( \omega \) is the frequency in radians.

## 5.3 Measurement Results

Figure 5.5 presents the measured performance results for the three fabricated inductors. \( L, Q, \) and SRF have been extracted from the impedance and are presented on the charts.
(a) Measured $Q$, $L$, and SRF for Inductor 1

$Q = 80$ @ 0.7 GHz

$L = 7.5$ nH

SRF = 7.2 GHz

(b) Measured $Q$, $L$, and SRF for Inductor 2

$Q = 107$ @ 1.7 GHz

$L = 2.5$ nH

SRF = > 12 GHz
5.3.1 Comparison to Predicted Performance

Table 5.4 compares the predicted $L$, peak $Q$, and SRF to the extracted measured values. Figure 5.6 shows the predicted quality factor curves laid over the measured curves.

Table 5.4: Measured vs Predicted Performance Parameters

<table>
<thead>
<tr>
<th></th>
<th>$L$ (nH)</th>
<th>Peak $Q$</th>
<th>SRF (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductor 1 Predicted</td>
<td>7.3</td>
<td>93</td>
<td>6.1</td>
</tr>
<tr>
<td>Inductor 1 Measured</td>
<td>7.5</td>
<td>80</td>
<td>7.2</td>
</tr>
<tr>
<td>Inductor 2 Predicted</td>
<td>2.4</td>
<td>104</td>
<td>17.6</td>
</tr>
<tr>
<td>Inductor 2 Measured</td>
<td>2.5</td>
<td>107</td>
<td>17.5*</td>
</tr>
<tr>
<td>Inductor 3 Predicted</td>
<td>4.0</td>
<td>105</td>
<td>11.5</td>
</tr>
<tr>
<td>Inductor 3 Measured</td>
<td>4.0</td>
<td>111</td>
<td>11.9</td>
</tr>
</tbody>
</table>

*Projected measured SRF, since frequencies beyond 13.5 GHz were not measured.
Figure 5.6: Measured vs predicted $Q$ for Inductor 1 (a), Inductor 2 (b), and Inductor 3 (c)

The measured inductance values do not deviate from the predicted values by more than 0.2 nH. This agreement confirms the applicability of the Wheeler formula (section 4.1.1) to inductors fabricated using the stencil-and-guide process.

The model also demonstrates its ability to forecast $Q$ and SRF, especially for Inductors 2 and 3, the two-turn inductors. In fact, the measured performance parameters for Inductors 2 and 3 shift in nearly the exact manner and magnitude predicted by the model.

The discrepancy in the predicted and measured peak quality factor for Inductor 1 is likely caused by the proximity effect (section 2.3.3), which is sensitive to the number of turns. Since the measured self-resonant frequency in higher than expected, it may safely be assumed that electric energy losses are accounted for by the model. On the other hand, losses resulting from the proximity effect deteriorate the magnetic field strength, which would reduce the quality factor without substantially affecting the SRF.
5.3.2 Comparison to Functional Requirements

Table 1.4, first presented in section 1.3.5, identifies the original functional requirements and summarizes the size specifications and measured performance parameters for all three fabricated inductors. The area of the inductor is a conservative over-estimate: it was calculated using the greatest outer radius of the spiral—see Figure 2.9 along with (4.4) and (4.5). All three inductors have a total device thickness of 260 μm, as stated during the description of the fabrication process in section 5.1.

<table>
<thead>
<tr>
<th></th>
<th>L (nH)</th>
<th>Peak Q</th>
<th>SRF (GHz)</th>
<th>A (mm²)</th>
<th>t (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required</td>
<td>&gt; 1</td>
<td>&gt; 100</td>
<td>&gt; 5</td>
<td>&lt; 2.5</td>
<td>&lt; 300</td>
</tr>
<tr>
<td>Inductor 1</td>
<td>7.5</td>
<td>80</td>
<td>7.2</td>
<td>2.4</td>
<td>260</td>
</tr>
<tr>
<td>Inductor 2</td>
<td>2.5</td>
<td>107</td>
<td>17.5</td>
<td>0.83</td>
<td>260</td>
</tr>
<tr>
<td>Inductor 3</td>
<td>4.0</td>
<td>111</td>
<td>11.9</td>
<td>1.2</td>
<td>260</td>
</tr>
</tbody>
</table>

This table shows that Inductors 2 and 3 meet all functional requirements. As indicated in section 1.3.5, Inductor 1 was intentionally designed for high inductance, and its extra turn comes with an expected quality factor below 100 (see Figure 5.2). Figure 1.14 and Figure 1.15 are also shown again for reference. Figure 1.14 illustrates the usefulness of inductors fabricated using the stencil-and-guide process relative to other strategies. Figure 1.15 shows a complete summary of available thin-chip inductors meeting size requirements, including devices from both industry and academia. All devices shown have a self-resonant frequency beyond 5 GHz.

**Figure 1.14: Stencil-and-guide process fills performance/size gap for thin-chip inductors**
The purpose of this research was to learn the relationship between force and deformation in forming of micro-scale inductor coils. This was accomplished by applying large-deflection beam bending to the case of planar wire deformation and through experimental validation. Generating this knowledge is important because it establishes fabrication limits for wire-based chip-scale inductors. There are many potentially viable methods for fabricating planar inductor coils. Without an understanding of the relevant physics, it is impossible to know which of these techniques is most appropriate or even feasible.

The analysis presented in this thesis directly led to the stencil-and-guide inductor fabrication concept, the details of which were specified using an analytic electrical model. The process utilizes a wire conductor, is compatible with any desired substrate, and features the ability to exactly control spiral properties.
Multiple inductors were fabricated using this process. These inductors demonstrate performance specifications predicted by the model, including inductances ranging from 2 – 4 nH, quality factors in excess of 100, and self-resonant frequencies beyond 10 GHz. Furthermore, the area of the inductors is less than 1.5 mm$^2$ and the entire device thickness is only 260 μm. The inductors are most readily applied to increasingly small communication devices, which require thin and efficient electrical components to boost the performance of the radio frequency transceiver. Accordingly, these inductors offer the potential for substantial improvement in signal quality and reception.

5.5 Future Work

This research was completed for the purpose of fabricating inductor prototypes. If volume production is desired, the stencil-and-guide process could be automated by controlling the wire guide tool with a precision motion stage and dispensing the wire through a nozzle. Another path for future work is to improve the electrical model to better account for the proximity effect, a loss factor that is difficult to quantify but may have a significant effect on performance. Additionally, there are other potential applications beyond inductors for the stencil-and-guide fabrication process that should be investigated. Other electrical components such as antennas could likely benefit from the performance boost offered by a wire conductor, property control, and freedom to use a desired substrate. Outside of radio frequency electronics, the large deflection beam bending analysis applied to micro-scale inductor coils could also be applied to optical fibers or miniature cables.
REFERENCES


[47] DuPont, “Kapton FPC Polyimide Film.”


