Interaction Of Cylinders In Proximity
Under Flow-Induced Vibration

by

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B.Eng., National University of Singapore (2010)

Submitted to the Department of Mechanical Engineering and
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Abstract

This study examines the influence of a stationary cylinder that is placed in proximity to a flexibly mounted cylinder in the side-by-side arrangement. The problem is investigated with an immersed-boundary formulation of a spectral/hp element based (Nektar-SPM) fluid solver. The numerical method and its implementation is validated with benchmark test cases of the flow past an isolated cylinder in both the stationary and flexibly mounted configurations.

The study examines a parametric space spanning 6 center-to-center spacing configurations in the range 1.5D-4D and 13 equispaced reduced velocities in the range 3.0-9.0. The simulations are performed in two-dimensional space and the Reynolds number is held at 100. The response characteristics of the moving cylinder are classified into regimes based on the shape of the response curve and the variation of the r.m.s. lift coefficient. It is shown that the moving cylinder influences the lift and drag force characteristics on the stationary cylinder and the frequency composition in the wake.

A detailed look at the frequencies and the relative strengths of the frequencies indicates a diminishing influence of the moving cylinder on the stationary cylinder, both with increasing separation and smaller amplitudes. By examining the wake patterns and monitoring the frequencies in the wake of each cylinder, the interference level is qualified and explained to be the basis of the different families of response.

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Chapter 1

Introduction

1.1 Background & Motivation

Vortex induced vibration (VIV) is the vibratory response observed in a bluff body that is subject to a cross flow. A bluff body, such as a cylinder, interacting with the oncoming flow generates vortices on either side, which in turn exerts a force on the body. When such bodies are free to move, these alternating forces cause a vibration which is termed VIV.

VIV is an important class of flow-induced vibrations that is of critical importance to many fields. The study of the VIV of cylindrical structures, in particular, finds applications in the Ocean Engineering domain due to the abundance of structures that have a cylindrical shape such as oil pipes, marine cables and mooring lines. VIV is an important source for fatigue failure of such structures and hence is of great interest to industry stakeholders.

The behavior of an isolated cylinder is a well studied problem that has held the interest of researchers for several decades. Theoretical [5], experimental and numerical work have been carried out to study the various parameters that define this problem, and an in-depth review of this subject can be found in the works of Bearman [4] and Williamson [21]. In more recent times, researchers have also looked at the behavior of long flexible cylinders under various flow profiles, which has helped move research in this area to practical scenarios and applications.
Ocean structures, however, do not occur in isolation and it becomes important to study the influence of bodies in proximity on their fluid-structure interaction behavior. The study of interaction of cylinders dates back to the work of Zdravkovich [23, 24] where he systematically investigated the effect of proximity on the response characteristics. He classified interference regions as proximity region, wake-interference region and no-interference region as shown in Figure 1-1. Furthermore, he classified the flow patterns observed as shown in Figure 1-2.

Figure 1-1: Figure adapted from [24] showing the various interference regimes.

Several recent studies that have examined the interaction of cylinders have investigated arrangements that include the tandem configuration [1, 2, 13, 25], the side-by-side configuration [9, 13, 15, 25] and skewed configuration [25]. Recent experimental work by Huera-Huarte et al. [8, 9, 10] have also looked at these configurations
Figure 1-2: Figure adapted from [24] showing the various flow patterns for the side-by-side arrangement of stationary cylinders.

with flexible cylinders.

While there have been some studies investigating the side-by-side configuration of a pair of cylinders, most involve cylinders with symmetric properties, i.e., they are either both stationary, both forced to move at the same frequency or both free to move under the same natural frequency. The author is aware of only one study that breaks the symmetry of the problem by holding one of these cylinders rigid. This study by Huera-Huarte & Gharib [9] looked at effect of this symmetry-breaking configuration on the response of both degrees of freedom, showing that cross flow motion is diminished while in-line motion is amplified. The study, however, does not have a thorough analysis of the response characteristics and a qualification of the interference effect.

The present work aims to look at this symmetry-breaking problem at greater depth, by characterizing the response based on the separation, the wake patterns observed and the frequency spectra in the wake.
1.2 Thesis Organization

The thesis is organized into the following chapters.

In Chapter 2, the numerical method used by Nektar-SPM is outlined, followed by the validation of this numerical method, some comments on mesh selection and computational costs.

In Chapter 3, the central problem that is the subject of this thesis is addressed. The chapter describes the simulation configuration, the response characteristics and looks at wake patterns and frequency spectra behind each cylinder.

The thesis closes with Chapter 4, where the major conclusions from this study and recommendations for future work are listed.
Chapter 2

Numerical Method & Validation

2.1 Smoothed Profile Method Implementation

The numerical code employed for this study is the smoothed-profile-method (SPM) implementation of Nektar, a spectral/hp element based direct-numerical-simulation solver. While the original code has been in use for several years [19], the SPM implementation is relatively new. This numerical method is the basis of the thesis work of Luo [11, 12], where a detailed description of this method, error quantification and validation can be found. Here, a concise description of the method is first presented, followed by the validation studies that were performed.

2.1.1 Particle Representation

The smoothed-profile-method represents bodies with an indicator function, which is unity inside the solid domain, zero in the fluid domain and varies smoothly between these values along the interface of solid and the fluid. This representation of bodies gives a grid-independent representation where a body can be defined on any grid simply by the value of the indicator function. The indicator function over the whole domain is constructed by calculating the distance of a given point from the surface of the body.
We use the following general form to represent bodies:

\[
\phi_i(x, t) = \frac{1}{2} \left[ \tanh \left( \frac{-d_i(x, t)}{\xi_i} \right) + 1 \right],
\]  

(2.1)

where each body \(i\) has an indicator function field \(\phi_i(x, t)\), defined once the signed distance \(d_i(x, t)\) is known everywhere in the domain and the value for the interpolation thickness \(\xi_i\) is defined. The distance is defined to be positive for points outside the body and negative for those inside. For simple geometries, like the cylinder that is used in this study, analytical expressions for this distance function can be obtained, leading to a straightforward computation of the indicator function. Multiple bodies (that do not overlap) are handled by separately computing the indicator function field of each and summing them up to get a global indicator function field.

The smooth concentration field, for a domain consisting of \(N\) particles, is constructed next:

\[
\phi(x, t) = \sum_{i=1}^{N} \phi_i(x, t).
\]  

(2.2)
Based on this total indicator field and knowing the particle velocity $V_i$ at time $t$ for each of $N$ particles in the domain, the particle velocity field $u_p(x,t)$ is constructed:

$$\phi(x,t) u_p(x,t) = \sum_{i=1}^{N} \{V_i(t)\} \phi_i(x,t).$$  \hspace{1cm} (2.3)$$

The total velocity field is defined as the combination of the particle velocity field ($u_p$) and the fluid velocity field ($u_f$):

$$u(x,t) = \phi(x,t) u_p(x,t) + (1 - \phi(x,t)) u_f(x,t).$$ \hspace{1cm} (2.4)$$

This total velocity field gives the particle velocity in the particle domain ($u = u_p$ when $\phi = 1$) and the fluid velocity in the fluid domain ($u = u_f$ when $\phi = 0$).

### 2.1.2 Solution Methodology

SPM solves the Navier-stokes equations:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u + f_s,$$  \hspace{1cm} (2.5)$$

$$\nabla \cdot u = 0,$$ \hspace{1cm} (2.6)$$

where $\rho$ is the density of the fluid, $p$ is the pressure field, $\nu$ the kinematic viscosity of the fluid and $f_s$ is the body force term that represents the interactions between the particles and the fluid.

A two-step semi-discrete form is used to solve the velocity and pressure fields [14]. First, SPM solves for an intermediate velocity and pressure fields $u^*, p^*$ from the previous step solution $u^n$, by integrating the advection and viscous stress using forward Euler integration:
\[ u^* = u^n + \int_{t^n}^{t^{n+1}} dt[-(u \cdot \nabla)u - \frac{1}{\rho} \nabla p + \nu \nabla^2 u], \quad (2.7) \]

\[ u^* = u^n + \Delta t[-(u \cdot \nabla)u - \frac{1}{\rho} \nabla p + \nu \nabla^2 u]. \quad (2.8) \]

This is solved in conjunction with the incompressibility constraint on \( u^* \):

\[ \nabla \cdot u^* = 0. \quad (2.9) \]

Then SPM updates the total velocity and pressure fields from \( u^* \), \( p^* \):

\[ u^{n+1} = u^* \int_{t^n}^{t^{n+1}} dt[f_s - \nabla p_p] = u^* + [\phi(u_p - u^*) - \frac{\Delta t}{\rho} \nabla p_p], \quad (2.10) \]

\[ \int_{t^n}^{t^{n+1}} f_s dt = \phi(u_p - u^*). \quad (2.11) \]

The total pressure is given by the sum of the intermediate pressure \( p^* \) and the extra-pressure term \( p_p \) that can be computed as follows:

\[ \frac{\Delta t}{\rho} \nabla^2 p_p = \nabla \cdot [\phi(u_p - u^*)]. \quad (2.12) \]

SPM benefits from both finite element method techniques and spectral methods. Complex geometry can be dealt with by increasing the number of elements (h-refinement) – error scales algebraically for this. Interpolation order within each element can be increased (p-refinement) – this gives exponential decay of errors.
2.2 Non Dimensional Parameters Used

This section defines some non-dimensional parameters that will be used in this study.

The primary non-dimensional number that is of significance to fluid mechanics is the Reynolds number, defined as follows:

$$Re = \frac{U D}{\nu},$$  \hspace{1cm} (2.13)

where $U$ is the flow velocity, $D$ is the characteristic length and $\nu$ is the kinematic viscosity of the fluid. In this study the characteristic length is the diameter $D$. This non-dimensional number measures the relative importance of the inertial forces as opposed to the viscous forces. This study is concerned with flows at a Reynolds number value of 100 where the flow is still two-dimensional for an isolated cylinder.

The lift force ($F_{\text{Lift}}$) and drag force ($F_{\text{Drag}}$) are non-dimensionalized to give the list and drag coefficients, $C_L$ and $C_D$:

$$C_D = \frac{F_{\text{Drag}}}{\frac{1}{2}\rho U^2 A},$$  \hspace{1cm} (2.14)

$$C_L = \frac{F_{\text{Lift}}}{\frac{1}{2}\rho U^2 A},$$  \hspace{1cm} (2.15)

where $\rho$ is the fluid density and $A$ is the projected area of the body in the direction of the flow.

Finally, the motion of cylinders described in this thesis is controlled by two parameters, the reduced velocity and the mass ratio. Ignoring the effect of damping, the equation of motion of a cylinder of mass $m$ moving only in the cross-flow direction $y$ can be written as:
\[ m\ddot{y} + ky = F_L. \]  

(2.16)

Dividing throughout by mass \( m \), we get:

\[ \ddot{y} + \frac{k}{m} y = \frac{F_L}{m}. \]  

(2.17)

Non-dimensionalizing the equation with the problems parameters (velocity \( U \) and length \( D \)) and using the expression for natural frequency \( f \):

\[ \omega = 2\pi f = \sqrt{\frac{k}{m}}, \]  

(2.18)

we get the following non-dimensionalized form:

\[ (\ddot{y}^*) + \left(\frac{2\pi}{U_R}\right)^2 y^* = \frac{F_L}{M_R}, \]  

(2.19)

where \( U_R \) is the reduced velocity and \( M_R \) is the mass ratio as defined below:

\[ U_R = \frac{UD}{f}, \]  

(2.20)

\[ M_R = \frac{m}{\rho_f D^2}. \]  

(2.21)

The mass ratio used in this study is held at a value 10, while other parameters are varied. The choice of this value is based on the higher stability of fluid-structure interaction simulations for high-mass ratio bodies.

A cylinder is said to be in lock-in condition when the vortex shedding frequency is close to its natural frequency \( (f_{cyl,natural} = 1/U_R) \). The vortex shedding frequency corresponds to the peak frequency of the lift coefficient time trace.
2.3 Validation of Numerical Method

To validate the numerical solver, we perform tests with benchmark cases involving cylinders. Additionally, the code's capability to handle bodies with sharp edges, like a square, is also tested.

The following tests were performed:

1. Flow past a 2D rigid cylinder at Re=100.

2. Flow past a 2D flexibly mounted cylinder at Re=100 at a range of reduced velocities.

3. Flow past a 2D rigid square at subcritical Reynolds numbers (Re=20-30).

The first validation study is the flow past a rigid cylinder at Re=100. $C_{L,RMS}$, $C_{D,Mean}$ and the Strouhal number computed with Nektar-SPM compares very well against several published references as listed in Table 2.1.

Table 2.1: Comparison of results for static cylinder at Re=100

<table>
<thead>
<tr>
<th></th>
<th>$C_{L,RMS}$</th>
<th>$C_{D,Mean}$</th>
<th>St</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>0.248</td>
<td>1.37</td>
<td>0.167</td>
</tr>
<tr>
<td>Singh &amp; Mittal</td>
<td>0.25</td>
<td>1.35</td>
<td>0.161</td>
</tr>
<tr>
<td>Shields [16]</td>
<td>0.30</td>
<td>1.33</td>
<td>0.167</td>
</tr>
<tr>
<td>Henderson [7]</td>
<td>-</td>
<td>1.38</td>
<td>0.170</td>
</tr>
<tr>
<td>Nektar ALE [12]</td>
<td>0.24</td>
<td>1.38</td>
<td>0.170</td>
</tr>
</tbody>
</table>

To validate the capability of the code to handle moving bodies, simulations of the flow past a flexibly mounted cylinder at Re=100 is run. $C_{L,RMS}$, $C_{D,Mean}$ and Strouhal number for various reduced velocity cases are compared against values published by Bourguet & Jacono [6] in Figures 2-2 and 2-3.
Figure 2-2: Comparison of SPM-generated values with data from [6] (a) maximum amplitude and (b) Strouhal number.

Figure 2-3: Comparison of SPM-generated values with data from [6] (a) mean drag coefficient and (b) r.m.s value of lift coefficient.

The validation studies indicate excellent agreement with [6]. As a grid convergence test, the case corresponding to the maximum amplitude ($U_R=5$) is run at higher polynomial orders (p-refinement) of N=4, 5, 6 to show the convergence of the maximum-amplitude and Strouhal frequency.

As a final test of validation, simulations of the flow past a square cylinder for various angles of incidence at sub-critical Reynolds numbers (Re=20-30) are run. The streamline pattern generated from an SPM computation for two cases are compared against that published by Yoon et.al. [22] in Figure 2-4. A comparison of the separa-
tion bubble sizes in shown in Figure 2-5. This demonstrates an excellent agreement of the results obtained with Nektar-SPM.

This completes the validation process of the numerical method and its computational implementation.

Figure 2-4: A comparison of the streamline pattern published by Yoon et.al. [22] on the left, with those generated with SPM on the right, for the flow past a stationary square cylinder at a sub-critical Reynolds number of 20.

Figure 2-5: Comparison of separation bubble length for selected cases. The measured length from SPM simulations is indicated on this figure adapted from [22].
2.4 Mesh Selection

The mesh used in this study is a structured mesh consisting of hexahedral elements. The refinement in the central region is fine (dx=0.05) to ensure that the particle geometry is adequately captured and the flow features are well resolved. Outside of the region where the particle is expected to move, the mesh is gradually coarsened and near the extremities the element sizes are as large as 5 diameters.

A very thorough selection process was employed to converge on the right mesh to use, by taking into account the various parameters that could be varied in the mesh generation phase. The parameters that were considered are the global domain size, such as the upstream, downstream and side domain extremities, the extent of the fine region and the parameters which control the rate of coarsening of the mesh. These are indicated in Figures 2-6 and 2-7.

It was determined that the sides are to be approximately 50 diameters from the particle to accurately compute the lift coefficient value. Similarly a downstream domain size of 50 diameters was required for an accurate computation of the drag coefficient.

For the studies described in this thesis, the moving cylinder was positioned at the origin (0,0) of the mesh, while the static cylinder was moved to a negative y-axis position for the required spacing configuration. The mesh selection process was done by placing a flexibly mounted isolated cylinder at the (0,0) position to ensure that the asymmetry of this mesh does not affect the simulation results.

Finally, the order of polynomial chosen for all studies is set at N=3 based on accuracy of the validation tests and computational cost. While N=4 gives a slightly better result for some of the validation cases, the improvement could not justify the additional computational cost involved.
Figure 2-6: Image of the mesh used, extending 25 diameters ahead of the cylinder location and 50 diameters on all other sides.

Figure 2-7: Zoomed in view of the mesh showing the fine grid in the vicinity of the cylinder and the gradually increasing mesh element size.
2.5 Computational Resources Used

Nektar-SPM is a parallelized solver and to take advantage of this feature, computer clusters were employed for all simulations. The machine that was used for this project is a supercomputer of the Cray-XE6 type architecture. This machine has several thousand nodes available for processing, with each node split into two sets of 16 cores each.

Processor scaling tests are used to study the scaling properties of Nektar-SPM across different processor configurations to determine one that would make best use of the available resources. The tests are normalized by choosing one representative case and running the simulation for a total of 1000 time-steps.

An efficiency metric can be defined by computing the total compute time consumed to simulate one time step in each configuration. Based on the processing time comparison and the requirements of this study, it was determined that the N=128 configuration made best use of the available resources while generating results within an acceptable time-frame.

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Chapter 3

Simulation Cases

3.1 Interaction of Side-By-Side Cylinders

3.1.1 Problem Description

This study investigates the influence of a stationary cylinder on the response characteristics of a flexibly mounted cylinder when they are placed in a side-by-side arrangement. The center-to-center spacing between the two cylinders is varied to study the effect of proximity on the response of the moving cylinder. All simulations are two-dimensional and run at a constant Reynolds number of 100.

Figure 3-1 shows the simulation configuration and the various parameters of interest. The spacing values chosen are in the range 1.6-4D, while the reduced velocities are in the range 3.0-9.0. The mesh used is the same as the one described in Section 2.3 of this thesis. The central fine region, as shown in Figure 2-7, encompasses the region within which both cylinders exist.

The time step chosen is $t=0.005$ or $0.008$ and the corresponding interpolation thickness used is $\xi = 0.01176$ or $0.0093$, in line with the recommendation given in [11]. The order of polynomial is set to 3. A summary of all the SPM parameters used and the test matrix of cases are listed in Table 3.1.
The parameters that are analyzed are the amplitude response of the moving cylinder, the lift and drag force coefficients of the two cylinders and the frequency composition of each of these quantities. The following sub-sections discuss each of these parameters in detail.
3.1.2 Amplitude Response

The maximum amplitude of an isolated cylinder under VIV at Re=100 was described in Section 2.3 of this thesis and will form the comparison case for the side-by-side simulation studies.

Unlike the isolated cylinder, the moving cylinder in the current simulations does not, in general, exhibit a periodic oscillation. Two representative cases, from the separation configuration of 1.6D, are shown in Figure 3-2. The $U_R=5.0$ trajectory shows a largely periodic oscillation where the maximum amplitude does not vary much. The $U_R=9.0$ case on the other hand shows a highly aperiodic amplitude response.

![Figure 3-2: Representative cylinder trajectories: (left) periodic oscillation observed $U_R=5.0$ and (right) aperiodic oscillation observed for $U_R=9.0$, both for the configuration where the spacing is 1.6D.](image)

To better quantify the amplitude response the root mean square value of the amplitude response, computed over a long interval, is used to characterize the response. Figure 3-3 shows both the maximum amplitude and the r.m.s amplitude of the various cases that were examined. This contour map shows the peak ridge to be around $U_R=5.0$. With increase in separation the width of the high-amplitude region ($A_{RMS} \geq 0.1$) first decreases, reaches a minimum at around separation of 2.5D and increases beyond that.
Figure 3-3: On the left are contour plots of the peak of the amplitude response (top) and the rms of the amplitude response (bottom) for the various separation and reduced velocity cases. On the right are two measures of periodicity of the cylinder trajectory: (top) first metric indicates those cases where the period-to-period amplitude does not vary by more than 10% (indicated with filled green circles) and the second metric (bottom) indicates those cases where the amplitude rms is within 10% of that expected for a sine-type curve for a sine curve ($A_{RMS} = A_{MAX}/\sqrt{2}$).

Also shown on the right-panel of Figure 3-3 are two metric of the aperiodic nature of the response. The top sub-figure indicates periodic cases as determined by the closeness of amplitude peaks in the response (a 10% criterion). The bottom sub-figure indicates all cases where the trajectory deviates from a sinusoidal curve, by measuring the deviation of the ratio of the peak amplitude to the rms-amplitude from that expected for a sinusoidal curve, i.e., the deviation from the value of $\sqrt{2}$.

We next look at the r.m.s. amplitude reponse curves of the various separation cases in detail.
Figure 3-4: $A_{RMS}$ plot of all separation cases plotted against the reduced velocity. The response curve of the isolated cylinder is also plotted to form a baseline.

At first glance, no clear trend is evident from Figure 3-4; each of the amplitude response has a low amplitude for reduced velocities smaller than 4, a peak near $U_R = 5$ and a subsequent decline. To identify trends, it is necessary to look at the response in terms of their original configuration. We cluster the response curves for spacing configurations less than 2 under small separation cases and those above 2.5 as large separation cases as shown in Figures 3-5 and 3-6.

Small separation cases are characterized by a short, narrow peak in the $U_R = 5-5.5$ range and a wide lock-in region in the $U_R = 4.5-9$ range, where the amplitude is significant. The lock-in width extends to about $U_R = 12$ (not shown here). The maximum amplitude in this range is very close to that observed for an isolated cylinder case. Closer investigation of the amplitude traces show that many of the small separation cases have an irregular amplitude response curve with the peak of each cycle varying widely over the runtime of the simulation.
Figure 3-5: $A_{RMS}$ for small separation plotted against reduced velocity. The response curve of the isolated cylinder is also plotted to form a baseline.

Figure 3-6: $A_{RMS}$ for large separation plotted against reduced velocity. The response curve of the isolated cylinder is also plotted to form a baseline.
Large separation cases are characterized by a bell-shaped curve, reminiscent of the response curve that characterizes the VIV-response of an isolated cylinder. The various curves under this classification are geometrically similar and differ in the size of the region covered, with the furthest separation case being closest to that of the isolated cylinder. The response peaks around $U_R = 5$ and falls gradually to below $0.05D$ by $U_R = 8$. The width of the high-amplitude zone ($A_{RMS} \geq 0.1$) is least for $S=2.5$, which falls under this classification.

![RMS Amplitude vs Reduced Velocity (Different Regimes)](image)

Figure 3-7: $A_{RMS}$ for all three regimes and the isolated cylinder baseline case.

The configuration corresponding to $S=2D$ is different from either classification, where the response has a smaller peak than the other two regimes and a lock-in width that is intermediate between them. We will limit our discussion to the small and large separation regimes.
3.1.3 Lift and Drag Force Behavior

Figure 3-8: $C_{D,Mean}$ plots from both regimes: (left) small separation and (right) large separation. The curve for the isolated cylinder is plotted as a baseline case.

Figure 3-9: $C_{L,RMS}$ plots from both regimes: (left) small separation and (right) large separation. The curve for the isolated cylinder is plotted as a baseline case.

Figure 3-8 shows the variation of the mean drag coefficient of the moving cylinder with reduced velocity. The points for all separation cases closely follow the trend of the isolated cylinder. The mean drag increases to a maximum at the highest amplitude cases and then drops off at higher reduced velocities.

Next, we look at the variation of rms-value of the lift coefficient of the moving cylinder, which are shown in Figure 3-9. Again the figure is split in two to show how the curves compare against the isolated cylinder case and the distinction in their
shapes. The large separation cases all have the same general shape that compares closely with the isolated cylinder: they rise at the beginning of the corresponding lock-in region to a value close to 0.9, followed by a steep drop to a nearly zero value and a subsequent recovery to a steady value of approximately 0.2 at the higher reduced velocities. The small-separation cases are different from the isolated cylinder case in two ways. First, the region where $C_{L,RMS}$ is large is wider and corresponds to the peaking amplitude response zone of this regime. Second the drop is steeper and abruptly settles near the value of 0.15, that is carried into the high reduced velocity region.

3.1.4 Three Oscillator System

The earlier sections described how the lift and drag coefficient behavior is modified by proximity and distinct response regimes were identified. To better understand the underlying physical mechanism, we next examine the spectral composition of several quantities: the trajectory of the moving cylinder, the lift and drag coefficients of both cylinders and the cross-flow velocity component of monitor points in the wake.

Figure 3-10: Schematic image showing the two cylinders in a side-by-side arrangement, and the three monitor points; the cylinder trajectory (□), the moving wake (○) and the stationary wake (∗).

The monitor points are chosen to be spaced 1.5D downstream and 1.5D along the cross flow direction away from the neighboring cylinder. These points are chosen to
capture the dynamics of the wake associated with each cylinder, thereby giving rise to a three-oscillator system: the moving cylinder, the wake associated with the moving cylinder (hereafter referred to as the moving wake) and the wake associated with the stationary cylinder (hereafter referred to as the stationary wake).

For each spacing configuration, the data gathered is organized in the following manner: the primary frequency of the cylinder trajectory is plotted, followed by the primary and secondary peaks associated with each of the wakes. Within each plot, the region corresponding to high amplitude \((A_{RMS} \geq 0.1)\) is marked with a green box and within each green box, the cases corresponding to periodic oscillations is marked with a yellow box. These plots are shown in Figures 3-11 and 3-12.

For small separations, the characteristic feature is that there are multiple dominant frequencies in the wake. The trajectory, however, has a clear peak within the range of reduced velocities examined, and lies on or close to the natural frequency. The region with significant amplitude \((A_{RMS} \geq 0.1)\) spans the entire range displayed, while a small-subset of cases exhibit a periodic oscillation. For all cases exhibiting a periodic oscillation, the primary frequency of the trajectory is locked to one or more harmonics of both the wakes and the maximum amplitude case always corresponds to a periodic oscillation case.

For the large separation cases, the characteristic feature is the existence of one dominant frequency for each oscillator. It is interesting to note that the moving cylinder and moving wake are locked for all periodic oscillation cases and most high amplitude cases. The stationary wake’s primary frequency is not always locked to the moving cylinder; however, at least the second frequency is, with diminishing effect as the separation is increased.

To show the diminishing effect on the stationary cylinder, we pick two reduced velocity cases \((U_R=5.0, 7.0)\) and track the evolution of the primary frequency of the stationary cylinder. Due to the proximity of the moving cylinder to the stationary one in the small separation cases, we would expect the frequency characteristics to be primarily driven by the moving cylinder.
Dominant Frequencies of 3 Oscillators (S=1.5D)

Reduced Velocity (U*)

Dominant Frequencies of 3 Oscillators (S=1.6D)

Reduced Velocity (U*)

Dominant Frequencies of 3 Oscillators (S=2D)

Reduced Velocity (U*)

Figure 3-11: Spectral composition plot for 3 configurations (top to bottom): S = 1.5D, S=1.6D and S=2D. Legend: 1st peak of trajectory (□), 1st peak of moving wake (+), 2nd peak of moving wake (○), 1st peak of stationary wake (×), 2nd peak of stationary wake (▽), high amplitude cases (---), periodic oscillation cases (---), maximum amplitude case (○).
Figure 3-12: Spectral composition plot for 3 configurations (top to bottom): $S = 2.5D$, $S=3D$ and $S=4D$. Legend: $1^{st}$ peak of trajectory ($\square$), $1^{st}$ peak of moving wake (+), $2^{nd}$ peak of moving wake ($\bigcirc$), $1^{st}$ peak of stationary wake ($\times$), $2^{nd}$ peak of stationary wake ($\triangledown$), high amplitude cases (---), periodic oscillation cases (---), maximum amplitude case (○).
Figure 3-13: Plot showing the primary trajectory frequency and the primary and secondary frequencies in the lift coefficient time-trace of the moving and stationary cylinder for $U_R=5.0$.

Figure 3-13 clearly shows how the peak frequency of the lift coefficient of the stationary cylinder is locked to that of the moving cylinder trajectory. At separation of 2.5D, the stationary cylinder $C_L$ deviates away and as separation is increased to 4D it settles near the value of an isolated cylinder.

The trend is less straightforward for $U_R=7.0$. In this case the stationary cylinder lift coefficient is locked to the cylinder trajectory up to a separation of 2.5D, after which it deviates and eventually settles near the natural frequency of the isolated cylinder (0.166).
Figure 3-14: Plot showing the primary trajectory frequency and the primary and secondary frequencies in the lift coefficient time-trace of the moving and stationary cylinder for $U_R=7.0$. 
3.1.5 Wake Visualization

Small separation cases generate an unstructured wake for most reduced velocities due to the effect of the gap flow and the competing influences of the vortex shedding frequency of the stationary cylinder (\(\approx 0.17\)) and the moving cylinder (\(\sim 1/UR\)). The proximity of the cylinder gives rise to an intermixing of these two primary frequencies.

The first panel in Figure 3-15 shows the visualization of the wake of the stationary configuration. The flow is biased (upwards) and shows the interaction of vortices in the near wake region. Panels 2, 4 and 5 corresponding to reduced velocities \(UR=4\), 6 and 7 show the unstructured nature of the wakes.

The only exception to this rule is the \(UR=5.0\) case, which exhibits a pair of 2S wakes behind each cylinder that converge and coalesce downstream. The reasoning behind this exception is that the high amplitude of oscillation of the moving cylinder sets the vortex shedding frequency of the stationary one, which thereby leads to a single primary frequency in the wake leading to an organized wake.

Large separation cases are characterized by a structured wake for most reduced velocities. The relatively larger distance between the two cylinders allows for each wake to form and evolve separately in the near wake region. The representative case chosen from this regime is the configuration with a separation of 4, and images are shown in Figure 3-16. For the purposes of comparison, a similar set of vorticity snapshots for the isolated cylinder case is shown in Figure 3-17.

The \(UR=5.0\) case clearly demonstrates how two distinct wake patterns can coexist without significant intermingling up to about 10-15 diameters downstream. The P+S mode of the moving cylinder and the 2S mode of the stationary cylinder can be clearly seen in the visualization of the wake. The two cases \(UR=4.0\) and \(UR=7.0\), which both have a comparable amplitude response, exhibit a pattern where each cylinder generates a wake of the 2S mode at slightly different frequencies. The wakes co-exist to further show the limited effect of proximity at a separation of 4D.

Wake visualization for several cases, including that of an isolated cylinder, with snapshots taken across one time-period is included in Appendix A.
Figure 3-15: Panels of instantaneous vorticity fields for the $S=1.6D$ configuration under the following conditions (from top to bottom): rigid, $U_R=4.0$, $U_R=5.0$, $U_R=6.0$ and $U_R=7.0$. The vorticity value ranges between -1.5 and 1.5.
Figure 3-16: Panels of instantaneous vorticity fields for the S=4D configuration under the following conditions (from top to bottom): rigid, $U_R=4.0$, $U_R=5.0$, $U_R=6.0$ and $U_R=7.0$. The vorticity value ranges between -1.5 and 1.5.
Figure 3-17: Panels of instantaneous vorticity fields for an isolated cylinder under the following conditions (from top to bottom): rigid, $U_R=4.0$, $U_R=5.0$, $U_R=6.0$ and $U_R=7.0$. The vorticity value ranges between -1.5 and 1.5.
3.2 Concluding Remarks

The study of the side-by-side arrangement of cylinders with one being able to move is an important symmetry-breaking study. The flexibility of one cylinder acts as an additional frequency contributor to the wake frequencies. The proximity of the cylinders determines the shape of the response curve and the relative strengths of the frequency components in the response of the moving cylinder. Examining the problem as a three oscillator model reveals the influence of the moving cylinder on its wake and also the wake associated with the stationary cylinder. A detailed look at the frequencies and the relative strengths of the frequencies indicates a diminishing influence of the moving cylinder on the stationary cylinder both with increasing separation and smaller amplitudes.
Chapter 4

Conclusions & Future Work

The major conclusions of the study on side-by-side cylinders are:

1. The amplitude response can be classified into two regimes: small and large separation, each of which has distinguishing properties in terms of the shape of the response curve and the $C_{L,RMS}$ curve.

   (a) Small separation cases have a short, narrow peak near $U_R = 5$ and a wide lock-in range of $U_R = 4.5-12$ (beyond the parametric space discussed in this thesis).

   (b) Large separation cases have a bell-shaped curve with a peak near $U_R = 5$ and a narrower lock-in range of $U_R = 4.5-8$. The lock-in width is largest for the largest separation case, with the bell-curve approaching that of the response of an isolated cylinder.

2. The trajectory of the moving cylinder is observed to be periodic only in cases where the amplitude is large ($A_{RMS} > 0.1$) and when the 1st and/or 2nd harmonic of the moving wake are locked with it, corresponding to a situation where all three oscillators are locked. The maximum amplitude for each spacing configuration occurs at this point where a three-way synchronization occurs.
3. The stationary wake is strongly influenced by the moving cylinder in the small separation configurations as evidenced by the frequency composition derived from the monitor points in the wake. For the large separation configurations, this influence is limited to those cases where the amplitude response is high, when the moving cylinder is in lock-in condition.

4. The wake patterns show a clear distinction between the two regimes; small separation cases are characterized by unstructured and chaotic wake patterns while the large separation cases are characterized by structured wake patterns. The coexistence of different types of wakes behind the two cylinders for a large separation configuration point to a reduced degree of interference.

5. An examination of the monitor point spectra can reveal which regime the system is in:

   (a) For the small separation configuration, the stationary wake monitor point either exhibits a strongly multi-frequency response (for low amplitude cases) or a peak frequency away from the Strouhal frequency of a stationary cylinder \( f_{\text{Stat,Cyl}} \approx 0.17 \).

   (b) For the large separation configuration, the stationary cylinder wake is strongly mono-frequency, and this frequency is very close to the Strouhal frequency of a stationary cylinder \( f_{\text{Stat,Cyl}} \approx 0.17 \). In addition, monitor points near the moving cylinder is strongly mono-frequency and is locked to cylinder’s natural frequency.

On the basis of this work, the following new directions of work can be recommended:

1. The side-by-side studies, in close proximity revealed the influence of competing frequencies of the response of the moving cylinder. A more thorough analysis of the interference of competing frequencies can be studied by allowing both cylinders to move, with each one being set to a different natural frequency.
2. The limit of the closeness of the initial configuration is around 1.5D, where some reduced velocity cases lead to contact of the two cylinders. By implementing a collision condition, the effect of closer separations on amplitude response can be studied.
Bibliography


Appendix A

Wake Visualization

A.1 Small Separation Configuration (S=1.6D)

Table A.1: Period length (in non-dimensional time) for various cases (S=1.6D)

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Separation = 1.6D Rigid Cylinders

Figure A-1: Vorticity snapshots over one cycle based on the lift-coefficient variation. The vorticity value ranges between -1.5 and 1.5.
Separation = 1.6D $U_R=4.0$

Figure A-2: Vorticity snapshots over one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.

Separation = 1.6D $U_R=5.0$

Figure A-3: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.
Separation = 1.6D $U_R=6.0$

Figure A-4: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.

Separation = 1.6D $U_R=7.0$

Figure A-5: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.
A.2 Intermediate Separation Configuration (S=2D)

Table A.2: Period length (in non-dimensional time) for various cases (S=2D)

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Separation = 2D Rigid Cylinders

Figure A-6: Vorticity snapshots over one cycle based on the lift-coefficient variation. The vorticity value ranges between -1.5 and 1.5.
Separation = 2D $U_R=4.0$

Figure A-7: Vorticity snapshots over one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.

Separation = 2D $U_R=5.0$

Figure A-8: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.
Separation = 2D $U_R=6.0$

Figure A-9: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.

Separation = 2D $U_R=7.0$

Figure A-10: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.
A.3 Large Separation Configuration (S=4D)

Table A.3: Period length (in non-dimensional time) for various cases (S=4D)

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Figure A-11: Vorticity snapshots over one cycle based on the lift-coefficient variation. The vorticity value ranges between -1.5 and 1.5.
Separation $= 4D \, U_R=4.0$

Figure A-12: Vorticity snapshots over one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.

Separation $= 4D \, U_R=5.0$

Figure A-13: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.
Separation = 4D $U_R$=6.0

Figure A-14: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.

Separation = 4D $U_R$=7.0

Figure A-15: Vorticity snapshots from one cycle of oscillation of the moving cylinder. The vorticity value ranges between -1.5 and 1.5.
Appendix B

Tandem Cylinders

B.1 Wake Stiffness Effect

This section summarizes the results of simulations involving stationary cylinders of unequal diameters in staggered arrangements.

Assi et.al. [3] showed that for stationary cylinders of equal diameter, the mean lift force on the downstream cylinder at staggered positions varies linearly with its distance from the inline axis, for points within the wake interference region \( \frac{y}{D} \leq 1.0 \). Their experimental study shows this linear dependence for a range of Reynolds numbers (Re=9600-19200) for an inline separation of 4 diameters, as shown in Figure B-1. Furthermore, the slope of each of these curves, within the linear range was determined to be approximately 0.65, independent of the Reynolds number.

We perform simulations at Re=100 to examine if this phenomenon exists at low Reynolds numbers. Additionally, we have cylinders of unequal diameters, with the upstream cylinder diameter set to 2.5 times the diameter of the downstream one. Furthermore, the inline separation (T) range is chosen to be in the range 3-5 \( D_{UC} \), where \( D_{UC} \) is the diameter of the upstream cylinder. Taking advantage of the symmetry of the problem, we survey only the positive direction of the crossflow axis \( 0 \leq y/D_{UC} \leq 0.75 \).

Figure B-2 shows the mean lift coefficient at different staggered positions. For each
Figure B-1: Figure adapted from [3] show the mean lift and drag forces on the downstream cylinder at different cross-flow positions corresponding to a tandem configuration of separation 4D.

inline separation, a linear trendline that is plotted affirms the linear behavior of the mean lift coefficient in the wake interference region, similar to the one demonstrated by Assi et.al. The slope of trendline decreases with an increase in the inline separation.

Figure B-3 shows the mean drag coefficient variation, with a trend similar to that shown by Assi et.al. The tandem configuration, where the downstream cylinder is directly behind the upstream cylinder, exhibits the minimum drag coefficient value. The mean drag coefficient gradually increases as the downstream cylinder moves away from the inline axis. This is consistent with the diminishing influence of the wake of the upstream cylinder on the force characteristics of downstream cylinder, as it approaches the boundary of the wake interference region.
Figure B-2: Variation of the mean lift coefficient $C_{L,\text{Mean}}$ on the downstream cylinder at different cross-flow positions $y/D_{UC}$, corresponding to various inline separations $T$.

Figure B-3: Variation of the mean drag coefficient $C_{D,\text{Mean}}$ on the downstream cylinder at different cross-flow positions $y/D_{UC}$, corresponding to various inline separations $T$. 

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