Incorporating Property Characteristics and Capital Market Conditions in Optimizing Commercial Real Estate Portfolios

by

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Abstract

We all know for diversification purposes we cannot “put all our eggs in one basket.” Markowitz’s Modern Portfolio Theory leads us to diversify our portfolio to achieve the highest Sharp ratio. Fama-French’s Three-Factor Model links the asset’s characteristics to the risk-return profile and further advances the portfolio theory. However, in practice, due to uncertainty and lack of data, none of those theories get implemented in a way that can help construct a complex portfolio and generate portfolio optimization strategies. Especially for the Commercial Real Estate Industry, investors face challenges in long-term data collection and a tremendous amount of data processing.

In 2009, Michael W. Brandt, Pedro Santa-Clara, and Rossen Valkanov explored a new approach that fundamentally improves the portfolio optimization methodology. They modeled the portfolio weight in each asset as a function of the asset’s characteristics and the associated capital market conditions. The coefficients of this function are found by optimizing the investor’s average utility of the portfolio’s return over the sample period. This approach is computationally simple, and can be easily modified to include more asset characteristics and capital market variables. In a later study, Alberto Plazzi, Walter Torous, and Rossen Valkanov applied Brandt, Santa-Clara, and Valkanov’s methodology to optimize commercial real estate portfolios, and explored several techniques in commercial real estate portfolio management.

This thesis follows Plazzi, Torous and Valkanov’s research framework, applies the methodology to a specific real estate investment fund, and proposes several innovations to further explore the application of this theory in real estate fund management. First, I propose to rebalance the portfolio annually because real estate transactions are less frequent compared with other types of assets, such as stocks or bonds. Second, I construct sub-portfolios by property type and region because the sub-portfolio optimization can provide practical suggestions to specific asset managers in charge of a specific type of property or a specific region. Finally, I include capital market indicators, such as the Chicago Fed National Activity Index and Liquidity Metrics. These innovations use academic research to inform practice, thus providing asset managers practical suggestions to guide wealth allocation across different commercial properties, and to take advantage of movements in expected returns arising from the changing macroeconomic conditions.

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Chapter 1 Introduction

1.1 Portfolio Optimization Theory Review

Most people in the investment industry are familiar with the old saying “don’t put all your eggs in one basket.” To minimize the probability that the market volatility could result in the loss of all the eggs, it is important to invest in different levels of risk and potential returns. This basic yet important concept is known as diversification.

Diversification in portfolio management is the most important rule for portfolio optimization, and the purpose of diversification is to use distinct risk-return profiles of different assets to maximize portfolio return per unit of portfolio-level risk, or equivalently to minimize portfolio risk per unit of portfolio-level return. Diversification can lower the overall risk because different types of assets often change in value in opposite directions. In finance, this theory was first introduced as the Modern Portfolio Theory (MPT) by Harry Markowitz in 1952. He won a Nobel Memorial Prize in Economic Sciences for his pioneering work in this field.

MPT seeks to reduce the total variance of the portfolio return by combining different assets with returns that are not positively correlated. It assumes that investors are rational and risk-averse, and that the market is efficient. Here, risk is defined as the standard deviation of returns with the assumption that asset returns are normally jointly distributed. In theory, this is very solid, but in practice, it is rarely used by investment managers because: 1) some of the theory’s assumptions are unrealistic, 2) there is a need for a significant amount of data collection over a long period of time, and 3) data processing is too complicated. Even if we assume that the data collection is easy, the processes of modeling joint distribution of returns and solving corresponding optimal portfolio weights are highly complex, given the large number of assets, and can cause noisy and unstable returns.

In practice, most allocation decisions are made with incomplete information and a great deal of uncertainty about risk and return. Therefore, the allocation process is inevitably influenced by investment managers’ experience, personal intuition, and other behavioral issues. Some investment managers use the existing portfolio as a benchmark to make adjustments because
they view the current allocation as a safe base. However, the adjustment may not be optimal because it may result in only a minor improvement in return with a big increment of risk.

Later, Eugene Fama and Kenneth French explored the relationship between a stock’s expected risk-return profile and stock characteristics, including the market capitalization and book-to-market ratio. They successfully added two more variables, Small Minus Big (SMB) and High Minus Low (HML), to the traditional Capital Asset Pricing Model (CAPM) based on MPT. For the first time, the risk-return profile was associated with characteristics of stocks, and was later extended to other assets. In the last 25 years, numerous scholars have explored various methods to explain commercial real estate returns using CAPM and the Fama-French three-factor model, such as Geltner [1989], Liu et al. [1990], Chan, Hendershott, and Sanders [1990], Ling and Naranjo [1997], Breidenbach, Muelle, and Schulte [2006], and Pai and Geltner [2007].

However, extending Fama-French’s three factors model into portfolio optimization was extremely difficult, because it requires modeling the expected returns, variances, and covariance of all assets as a function of their characteristics. Brandt, Santa-Clara, and Valkanov [2009] explored a new approach that fundamentally improves the portfolio optimization methodology associated with assets’ characteristics. They modeled the portfolio weight in each asset as a function of the asset’s characteristics and the capital market conditions. The coefficients of this function are found by optimizing the investor’s average utility of the portfolio’s return over the sample period. This approach is computationally simple, and can be easily modified to include more property characteristics and capital market variables.

According to Brandt, Santa-Clara, and Valkanov [2009, p.3412], this approach first avoids the “auxiliary, yet very difficult step of modeling the joint distribution of returns and characteristics.” Instead, it focuses on the portfolio weights as a function of the asset characteristics. Second, “parameterizing the portfolio policy leads to a tremendous reduction in dimensionality.” Compared to the traditional Markowitz approach requiring “modeling N first and (N²+N)/2 second moments of returns,” this approach involves “modeling only N portfolio weights regardless of the investor’s preference and joint distribution of asset returns.” Therefore, this approach avoids the “common statistical problems of imprecise coefficient estimates and overfitting, while allowing us to solve very large-scale problems with arbitrary preferences.”
Third, this approach “captures implicitly the relation between the characteristics and expected returns, variances, covariances, and even higher-order moments of returns.” Fourth, this method can be easily modified and extended to include the use of different objective functions, “conditioning the portfolio policy on macroeconomic predictors,” and “use of different parameterizations of the portfolio policy to accommodate short-sale constraints.”

Based on Brandt, Santa-Clara, and Valkanov’s innovation [2009], Plazzi, Torous, and Valkanov [2011] extended the methodology to commercial real estate portfolio optimization using NCREIF property-level data. The next chapter of this thesis will review Plazzi, Torous, and Valkanov’s research framework and methodology in detail, and propose a number of possible extensions that this thesis aims to explore.

1.2 Research Motivations and Objectives

Because most asset allocation decisions in practice are highly influenced by qualitative thinking such as asset manager’s experience, intuition, and other behavioral issues, I deeply believe advancing quantitative thinking in this field is highly important. Given the existing methodology developed by Brandt, Santa-Clara, and Valkanov, and implemented by Plazzi, Torous, and Valkanov in commercial real estate portfolio optimization, I plan to further adapt this methodology to a specific real estate investment fund and explore a fine-tuned framework that can be helpful to real estate investment fund managers to optimize their portfolios.

In detail, this thesis aim to answer the following questions:

1. Under the assumption of annual fund rebalancing, how should investors alter portfolio composition to achieve the highest Sharp ratio? I choose annual rebalancing for aggregated national-level portfolios because real estate transactions are much less frequent compared with other types of assets such as stocks or bonds. The high transaction cost discourages the frequent transactions in the real estate market. Compared with the previous studies done in this field, with the assumption of quarterly rebalancing, I think annual rebalancing is better suited to the reality in the real estate market.

2. Which property type or region has the highest potential to be optimized? How should investors optimize those sub-portfolio classified by property type or region to achieve
the highest Sharp ratio? I create sub-portfolios by property type and region because a specific asset manager is usually in charge of a specific type of property or a specific region. Constructing sub-portfolios in those categories can help specific asset managers optimize their portfolio.

3. How should investors allocate their wealth across different commercial properties in different capital market conditions? How should investors alter the composition of their commercial real estate portfolios to take advantage of movements in expected returns arising from changing underlying macroeconomic conditions, such as liquidity, economic expansion, or contraction?

Answering those three questions requires a deep understanding of the methodology that Brandt, Santa-Clara, and Valkanov [2009] have developed and its implementation in commercial real estate portfolio optimization that Plazzi, Torous, and Valkanov [2010] have explored. This thesis is unique compared with previous research because of its close-to-reality assumption of annual rebalancing, its application in sub-portfolio management classified by property type and region, and its extension to other capital market conditions such as liquidity metrics.

In the following chapters, I will first introduce the methodology and the data collected from a specific investment fund, and then apply the methodology to the data to get some constructive suggestions for the optimization strategies that can be helpful for this specific fund’s management.
Chapter 2 Methodology

2.1 Basic Idea

We assume at each time (year/quarter/month) that an investor has \( N_t \) properties in the portfolio. Each property \( i \) has a return of \( r_{i,t+1} \) from date \( t \) to \( t+1 \) and a multi-dimensional vector of building characteristics \( x_{i,t} \) observed at time \( t \). The characteristics could be any combination of the cap rate, the vacancy rate, or the appraisal value of the property, etc. Our optimization objective is to calculate the optimized weights \( \omega_{i,t} \) (based on property value) to maximize the expected utility of the portfolio’s return \( r_{p,t+1} \) using the information contained in \( x_{i,t} \).

\[
\max_{\{\omega_{i,t}\}_{i=1}^{N_t}} E_t \left[ u(r_{p,t+1}) \right] = E_t \left[ u \left( \sum_{i=1}^{N_t} \omega_{i,t} r_{i,t+1} \right) \right]
\]

where:

- \( u \): a pre-specified utility function covered later in this chapter,
- \( \omega_{i,t} \): optimized property weight for property \( i \) at time \( t \),
- \( r_{p,t+1} \): a portfolio return from time \( t \) to \( t+1 \),
- \( r_{i,t+1} \): a specific property’s return from time \( t \) to \( t+1 \).

We can parameterize the optimal portfolio weights \( \omega_{i,t} \) as a linear function of the building’s characteristics,

\[
\omega_{i,t} = f(x_{i,t}; \theta) = \bar{\omega}_{i,t} + \frac{1}{N_t} \theta x_{i,t}.
\]

The portfolio return from time \( t \) to \( t+1 \) can be written as

\[
r_{p,t+1} = \sum_{i=1}^{N_t} f(x_{i,t}; \theta) r_{i,t+1} = \sum_{i=1}^{N_t} \left( \bar{\omega}_{i,t} + \frac{1}{N_t} \theta x_{i,t} \right) r_{i,t+1},
\]

where:

- \( \bar{\omega}_{i,t} \): the weight of property \( i \) at date \( t \) in a value-weighted benchmark portfolio,
\( \frac{1}{N_t} \): a normalization that allows the portfolio weight function to be applied to any arbitrary and time-varying number of properties,

\( \theta \): a multi-dimensional vector of a coefficient to be estimated. The coefficient is constant across properties and through time, and it implies that the portfolio weight in each property depends solely on the characteristics of the property and not on the property’s historic returns. For instance, two properties with the identical characteristics but with different returns have similar weights in the portfolio.

\( \mathbf{x}_{t,t} \): a multi-dimensional vector of building characteristics. This vector is standardized cross-sectionally to have a zero mean and unit standard deviation across properties at time \( t \). Therefore the second term in the second equation above sums to zero across properties at time \( t \) and it captures the deviation from the benchmark weight \( \omega_{t,t} \). Furthermore, the optimal portfolio weights always sum to one.

\( \theta \mathbf{x}_{t,t} \): a product of two vectors. For example, if \( \mathbf{x}_{t,t} \) denotes a 3-dimensional vector including cap rate, appraisal value, and vacancy for one property, it is a 1-by-3 matrix. To get one number after the multiplication, \( \theta \) has to be a 3-by-1 matrix.

The parameterizing process allows us to estimate weights as a single function of property characteristics that can apply to all properties over time instead of estimating one weight for each property at each point in time. For a certain pre-specified utility function, the optimization problem becomes:

\[
\max_{\theta} E[u(r_{p,t+1})] = \max_{\theta} \frac{1}{T} \sum_{t=0}^{T-1} u \left( \sum_{i=1}^{N_t} f(x_{i,t}; \theta) r_{i,t+1} \right) = \max_{\theta} \frac{1}{T} \sum_{t=0}^{T-1} u \left( \sum_{i=1}^{N_t} (\omega_{i,t} + \frac{1}{N_t} \theta \mathbf{x}_{i,t}) r_{i,t+1} \right)
\]

This optimization objective can be easily done in software like MATLAB using various algorithms, because for most common utility functions and the linearity of the equations in the coefficients \( \theta \), it is computationally trivial for MATLAB to derive the gradient and the Hessian of the optimization problem. The level of complexity in the optimization grows with the number of dimensions in the coefficient vector, not with the number of properties in the portfolio. This enables us to optimize portfolios with a large number of assets.
2.2 Objective Functions

The traditional Markowitz approach does not allow choice of objective function. In contrast, the optimization process I use in this paper can apply any objective function so long as the conditional expected utility is well specified with a unique solution. Several objective functions in decision theories include Hyperbolic Absolute Risk Aversion (HARA) with both constant relative risk aversion and constant absolute risk aversion, and some other behaviorally motivated utility functions focusing on ambiguity, loss, and disappointment. A practitioner can also set goals as the utility function such as beating or tracking a benchmark, maintaining a certain value at risk (VaR), or maximizing the Sharp ratios, etc. For simplicity, we use Constant Relative Risk Aversion (CRRA) as Brandt, Santa-Clara, and Valkanov [1996] did.

\[ u(r_{p,t+1}) = \frac{(1 + r_{p,t+1})^{1-\gamma}}{1 - \gamma} \]

Brandt, Santa-Clara, and Valkanov [1996] mentioned that this utility function "incorporates preference toward higher-order moments in a parsimonious manner" and "it is twice continuously differentiable," which allows a practitioner to "use more efficient numerical optimization algorithms."

2.3 Portfolio Weight Constraint - No-Short-Sale Constraint

One of the biggest differences between the commercial real estate market and the stock market is that the commercial real estate market is less liquid, so it won’t allow properties to be shorted. Up to this point, our methodology does not impose any constraint on the weight. Therefore it is possible that the optimized weight is negative. We need to modify the portfolio policy by imposing a No-Short-Sale Constraint. We use the following constraint to eliminate the negative weight and renormalize the portfolio weight to make sure the weights sum to one,

\[ \omega^+_{i,t} = \frac{\max[0, \omega_{i,t}]}{\sum_{j=1}^{N_t} \max[0, \omega_{j,t}]} \]
2.4 Macroeconomic Variation

Up to this point, our methodology relies on a critical assumption that the relationship between property characteristics and the joint distribution of returns is time-invariant, because the coefficients of the portfolio policy $\theta$ are constant through time. For simplicity in statistical calculation, it is a fair assumption; however, in reality, substantial evidence shows that the performance of commercial real estate is closely related to the business cycle.

To include possible time-varying macroeconomic factors in the coefficients of the portfolio policy, we can model the coefficients using the Kronecker product of building characteristics $\xi_{l,t}$ and the macroeconomic factor $z_t$ as suggested by Brandt, Santa-Clara, and Valkanov [2009],

$$\omega_{l,t} = \bar{\omega}_{l,t} + \frac{1}{N_t} \theta (z_t \otimes \xi_{l,t}).$$

$\otimes$ denotes the Kronecker product of two vectors. The Kronecker product is basically an operation on two matrices resulting in a block matrix. If A is an m-by-n matrix and B is a p-by-q matrix, then the Kronecker product $A \otimes B$ is an mp-by-nq block matrix. In our case, $z_t$ can be a 1-by-2 vector including two logical expressions for economic expansion or contraction. [1,0] indicates economic expansion, and [0,1] indicates economic contraction. $\xi_{l,t}$ can be a 1-by-3 vector including cap rate, vacancy, and appraisal value. $z_t \otimes \xi_{l,t}$ is a 1-by-6 vector, which means each dimension in vector $\xi_{l,t}$ will be associated with an economic expansion and an economic contraction. By estimating $\theta$, we can get to know an investor’s optimal preference for cap rate, vacancy, and appraisal value during different macroeconomic conditions.

In this paper, I use two different types of macroeconomic indicators, which will be explained in detail in the next section.

2.5 Variables for Building Characteristics and Macroeconomic Conditions

In this section, I will focus on choosing the important variables for vector $\xi_{l,t}$ and the macroeconomic vector $z_t$ to make my optimization robust.
2.5.1 Cap Rate

Cap rate is the reverse of P/E ratio in the stock market, and is one of the most important variables in the commercial real estate market. A number of researchers have proved that cap rate is correlated with returns for different types of properties. Plazzi, Torous, and Valkanov [2010] concluded that property’s cap rate predicts subsequent property returns. For each property’s cap rate, we use Net Operating Income (NOI) divided by the adjusted appraisal value.

For Cap Rate under the quarterly rebalanced scenario, I use quarterly NOI, adjusted appraisal value, and the following formula to calculate

\[ \text{Cap Rate}_{i,t} = 4 \times \frac{\text{NOI}_{i,t}}{\text{Appraisal Value}_{i,t}}. \]

For Cap Rate under the semi-annually rebalanced scenario, I use semi-annual NOI, average of adjusted appraisal values, and the following formula to calculate

\[ \text{Cap Rate}_{i,t} = 2 \times \frac{\sum_{t=1}^{2} \text{NOI}_{i,t}}{\sum_{t=1}^{2} \text{Appraisal Value}_{i,t}/2}. \]

For Cap Rate under the annually rebalanced scenario, I use annual NOI, average of adjusted appraisal values, and the following formula to calculate

\[ \text{Cap Rate}_{i,t} = \frac{\sum_{t=1}^{4} \text{NOI}_{i,t}}{\sum_{t=1}^{4} \text{Appraisal Value}_{i,t}/4}. \]

2.5.2 Size

Fama and French [1992] upgraded the traditional Capital Asset Pricing Model (CAPM) by adding two more variables, Small Minus Big (SMB) and High Minus Low (HML):

\[ r = R_f + \beta \times (R_m - R_f) + b_s \times SMB + b_v \times HML + \alpha. \]

Fama-French’s three-factor model proved that stocks with small market capitalization (small stocks) and low Price-to-Book ratio (value stocks) perform better than the market. Therefore, a
company’s size is correlated with the expected returns. To extend the CAPM and the three-factor model to the commercial real estate market, institutional investors and researchers in the real estate sector have tried to find a similarly effective model of the cross-section of real property returns. Geltner [1989], Liu et al. [1990], Chan, Hendershott, and Sanders [1990], Ling and Naranjo [1997], Breidenbach, Muelle, and Schulte [2006], and Pai and Geltner [2007] all explored ways to explain commercial real estate returns using CAPM and the three-factor model. Especially in Pai and Geltner [2007], the cross-section of property returns within the institutional real estate asset class was first explored. Pai and Geltner built a similar three-factor model to explore property size based on property value and tier classification by metropolitan statistical area (MSA). They found that their model captures the historical cross-section of the NCREIF property portfolio total returns quite well. Interestingly, they found the exact opposite of the Fama and French findings that larger properties based on property value and properties located in larger MSAs are riskier and therefore require a larger return premium. In this thesis, we also use appraisal value as an indicator for property size to explore the relationship of property size and property return.

2.5.3 Liquidity

Fisher, Geltner, and Pollakowski [2007] created the Transaction Based Liquidity Metric using the bid-ask spread by tracking the demand and supply sides of the market. The demand side index tracks the changes in prices that buyers are willing to pay. The supply-side index measures changes in the prices sellers are willing to accept. If a growth in demand side reservation price outpaces a growth from the supply side, the valuable gap between supply and demand shrinks, which leads to an increase in the trading volume and liquidity. Conversely, a combination of buyers’ lowering their willing-to-pay and sellers increasing their willing-to-accept prices will lower the trading volume and liquidity.
It is generally known that in the top 6 Metropolitan Statistical Areas, including New York (MSA code: 5600), Washington DC (MSA code: 8840), Boston (MSA code: 1123), Chicago (MSA code: 1600), San Francisco (MSA code: 7360), and Los Angeles (MSA code: 4480), transaction
volume in commercial real estate is much higher, and therefore the top 6 markets are more liquid. The simplest way to include liquidity information in our property characteristics is to use a dummy variable to show whether the property is located in one of the top 6 MSAs.

Meanwhile, I also include the liquidity information as one of the capital market conditions in the optimization formula using the Kronecker product. The liquidity metrics can be translated into dummy variables with 0 representing illiquid for a negative liquidity metric, and 1 representing liquid for a positive liquidity metric. The Kronecker product generates doubled coefficients for property characteristics that I can estimate for both liquid and illiquid capital market conditions.

**2.5.4 Chicago Fed National Activity Index**

Chicago Fed National Activity Index (CFNAI) is a monthly index indicating national economic activity and related inflationary pressure. It is generated from 85 monthly indicators and is standardized to have an average value of 0 and a standard deviation of 1 across continuous times. The 85 monthly indicators cover four categories of economic parameters, including production and income; employment, unemployment, and hours; personal consumption and housing; and sales, orders, and inventories. CFNAI is a weighted average of those four categories. A positive index reading represents an economic growth above the trend, and a negative index represents an economic growth below the trend. Our portfolio optimization model can include a dummy variable for economic expansion, which corresponds to a positive CFNAI, and economic contraction, which corresponds to a negative CFNAI.

![CFNAI](image)

**Figure 2.3 CFNAI**
Chapter 3 Empirical Data Summary

Plazzi, Torous, and Valkanov [2011] applied the advanced portfolio management theory in Brandt, Santa-Clara, and Valkanov [2009] to NCREIF property-level data and explored several meaningful techniques in commercial real estate portfolio management. In this thesis, I use a similar but smaller dataset from a real estate institutional investment fund, and further explore the application of this theory in fund management.

3.1 Aggregated National Level Portfolio Summary

From 1978 to 2013, this fund grew from $40 million to $14 billion and currently owns about 250 properties in the US across 5 different types, including retail, residential, hotel, industrial and apartment. As illustrated in Figure 3.1, the total asset value and number of properties in the portfolio fluctuated as the macroeconomic conditions changed. Especially during 1991-1994, the number of the properties dropped significantly, whereas the value of properties in the portfolio did not drop very much. In contrast, during the 2008-2009 crisis, the value of the properties in the portfolio dropped significantly, whereas the number of the properties stayed flat. After the crisis abated, the value of the portfolio and the number of properties in the portfolio skyrocketed. Investors invested in a significant number of devalued properties after the crisis.

![Aggregated Appraisal Value and Number of Properties in the Portfolio](image)

Figure 3.1 Aggregated Appraisal Value and Number of Properties in the portfolio
From Figure 3.2, we know that up to the end of 2013 the total property value from high to low of different property types is office, apartment, retail, industrial, and hotel. From Figure 3.3, we know that up to the end of 2013, the number of different property types from high to low is industrial, apartment, office, retail, and hotel.
Figures 3.4 and 3.5 reveal the volatility of returns at the aggregated portfolio level. We can easily relate the economic conditions with the portfolio return. The two negative yearly portfolio returns happened between 1990 and 1993, and between 2008 and 2009. There was another trough in 2001, but the portfolio return still stayed positive. During most of these years, the return fluctuated between 2% and 6%.
3.2 Sub-Portfolio Summary by Property Type

Below we look at each property type individually and examine its fluctuation in value and number of properties across different times. This will help us to understand the composition of this fund and each type of property’s change across different macroeconomic conditions.

Figure 3.6 Apartment Aggregated Appraisal Value and Number of Properties in the Portfolio

From Figure 3.6, for the apartment portfolio, we see that after 1994, both the value and the number of properties in the portfolio increased significantly until the 2008 crisis. During the recession, lots of properties were sold, but after the recession the value and number of apartment properties increased significantly. By the end of 2013, there were about 50 apartment properties worth $4 billion.

Figure 3.7 Industrial Aggregated Appraisal Value and Number of Properties in the Portfolio
Figure 3.7 reveals the changing trend of industrial properties in the portfolio. We notice that the number of properties only had one significant drop during 1990-1994. After that, the number of industrial properties in the fund grew moderately until after the 2008-2009 recession. After 2011, the number of properties jumped from 40 to more than 120. We also notice that during the 2008 recession, the fund did not dispose of very many industrial properties given the fact that many industrial properties’ values were greatly depreciated. This short period of patient holding and the large-scale acquisition led to the huge increase in property values after the recession.

![Graph showing the changing trend of industrial properties in the portfolio.](image)

Figure 3.8 Retail Aggregated Appraisal Value and Number of Properties in the Portfolio

Figure 3.8 shows the changing trend of retail property. Unlike other properties’ overall increase in the number of properties, the number of retail properties was never above 25, with the first peak at 1983 with 24 retail properties, and the second peak in 2013 with 25 properties. In between, the number of retail properties stayed around 5 for a long period of time. Despite the significant change in the number of retail properties, the value of the retail property changed relatively little between 1979 and 2000. After that, because of increasing property acquisition, the aggregated value of retail property increased significantly until a drop during the 2008 crisis. At the end of 2013, there were 25 retail properties with a $2.8 billion value in the portfolio.
The total value of office properties kept growing with a small drop in 1994 and a significant drop in the 2008 recession, as shown in Figure 3.9. We can also see that the acquisition and disposition activities are very correlated with the value change. Up to the end of 2013, there are 34 properties worth $5 billion in the portfolio.

Compared with other property types, hotel is not a largely held property type in this fund. The value and number of hotel properties is much less than the other property types. Currently there are 6 hotels worth $700 million.
Chapter 4 Optimization Results

Using MATLAB, I create multiple optimal portfolios using different input variables. These different portfolios allow me to examine the different investment strategies for different investment managers who are in charge of different types of properties or properties in different regions. I also include capital market conditions such as CFNAI and Liquidity Metrics using the Kronecker product, as mentioned in the previous chapter. The optimization results lead to the portfolio policies that help investment managers to increase the Sharp ratio of their portfolios.

4.1 Optimal Portfolios for All Property Types and $\gamma=5$

The first optimal model assumes the investor has utility function at $\gamma=5$ which is a median risk aversion level. I put different variables one by one into the model and explore the different weights the optimal portfolio places on different variables.

Figure 4.1:
Optimal Portfolio for All Property Types and $\gamma=5$
Optimal portfolio policy coefficient estimates for all properties under alternative specification with non-negative weight constraints. Assuming quarterly rebalancing and annual rebalancing.

<table>
<thead>
<tr>
<th>$\gamma=5$ Quarterly</th>
<th>Base Case</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: Cap Rate</td>
<td>0.0108</td>
<td>0.2013</td>
<td>0.1995</td>
<td>0.0567</td>
<td></td>
</tr>
<tr>
<td>0: Size</td>
<td></td>
<td>-1.8473</td>
<td>-1.8464</td>
<td>-1.8025</td>
<td></td>
</tr>
<tr>
<td>0: Top 6</td>
<td></td>
<td></td>
<td>0.0453</td>
<td>0.0390</td>
<td></td>
</tr>
<tr>
<td>0: Top6 x Size</td>
<td></td>
<td></td>
<td></td>
<td>-0.3787</td>
<td></td>
</tr>
<tr>
<td>0: East</td>
<td></td>
<td></td>
<td></td>
<td>0.9377</td>
<td></td>
</tr>
<tr>
<td>0: South</td>
<td></td>
<td></td>
<td></td>
<td>0.8916</td>
<td></td>
</tr>
<tr>
<td>0: West</td>
<td></td>
<td></td>
<td></td>
<td>1.2070</td>
<td></td>
</tr>
<tr>
<td>0: Midwest</td>
<td></td>
<td></td>
<td></td>
<td>0.9077</td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td>-0.2312</td>
<td>-0.2312</td>
<td>-0.2299</td>
<td>-0.2299</td>
<td>-0.2294</td>
</tr>
<tr>
<td>Avg. Return</td>
<td>0.0216</td>
<td>0.0216</td>
<td>0.0232</td>
<td>0.0232</td>
<td>0.0238</td>
</tr>
<tr>
<td>Rf</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.0266</td>
<td>0.0266</td>
<td>0.0283</td>
<td>0.0282</td>
<td>0.0287</td>
</tr>
<tr>
<td>SR</td>
<td>0.7970</td>
<td>0.7970</td>
<td>0.8057</td>
<td>0.8085</td>
<td>0.8153</td>
</tr>
</tbody>
</table>
From Figure 4.1, we know that all else being equal, the optimal portfolio tilts towards investing in properties with higher cap rate, tilts away from investing in properties with bigger size as measured by the appraisal value, and tilts towards investing in more liquid markets as indicated by the "Top6" MSAs, including New York, Boston, Washington DC, San Francisco, Los Angeles, and Chicago. The negative coefficient for "Top6 x Size" indicates the optimal portfolio places less weight on the properties with higher appraisal value located in Top 6 MSAs. However, combined with the positive coefficient for "Top6", the optimal portfolio tilts towards properties with lower appraisal value in the Top 6 MSAs. This strategy is the same for both annual and quarterly rebalancing. The coefficients in column IV for regional locations have the same signs, which indicates the portfolio is well diversified by locations across the country.

Holding the optimal portfolio has the benefit of an increased Sharp ratio from 79.7% to the highest 81.53% under the scenario of balancing quarterly, and from 88.99% to the highest 96.04% under the scenario of balancing annually. Under both rebalancing scenarios, the average returns of the portfolios are all increased.

<table>
<thead>
<tr>
<th>γ=5 Annually</th>
<th>Base Case</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ: Cap Rate</td>
<td>0.0068</td>
<td>0.1830</td>
<td>0.1830</td>
<td>0.0043</td>
<td></td>
</tr>
<tr>
<td>θ: Size</td>
<td>-1.8272</td>
<td>-1.8271</td>
<td>-1.7854</td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ: Top6</td>
<td>0.0042</td>
<td>0.1921</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ: Top6 x Size</td>
<td>-0.3106</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ: East</td>
<td>-1.0087</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ: South</td>
<td>-1.7167</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ: West</td>
<td>-0.6810</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ: Midwest</td>
<td>-1.7196</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td>-0.2308</td>
<td>-0.2308</td>
<td>-0.2293</td>
<td>-0.2293</td>
<td>-0.2273</td>
</tr>
<tr>
<td>Avg. Return</td>
<td>0.0215</td>
<td>0.0215</td>
<td>0.0231</td>
<td>0.0231</td>
<td>0.0261</td>
</tr>
<tr>
<td>Rf</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.0227</td>
<td>0.0227</td>
<td>0.0227</td>
<td>0.0227</td>
<td>0.0290</td>
</tr>
<tr>
<td>SR</td>
<td>0.8899</td>
<td>0.8899</td>
<td>0.9604</td>
<td>0.9604</td>
<td>0.8552</td>
</tr>
</tbody>
</table>
4.2 Optimal Portfolios for All Property Types and Varied $\gamma$

I further studied the optimal portfolio for different investors with different utility functions, including a low risk aversion with $\gamma=3$, a median-risk aversion with $\gamma=5$, and a high-risk aversion with $\gamma=9$.

Figure 4.2:
Optimal Portfolios for All Property Types and Varied $\gamma$
Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for quarterly rebalancing and annual rebalancing.

<table>
<thead>
<tr>
<th>Quarterly Rebalancing</th>
<th>Market Portfolio at $\gamma=3$</th>
<th>Optimal Portfolio at $\gamma=3$</th>
<th>Market Portfolio at $\gamma=5$</th>
<th>Optimal Portfolio at $\gamma=5$</th>
<th>Market Portfolio at $\gamma=9$</th>
<th>Optimal Portfolio at $\gamma=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: Cap Rate</td>
<td>-</td>
<td>0.1402</td>
<td>-</td>
<td>0.1638</td>
<td>-</td>
<td>0.1681</td>
</tr>
<tr>
<td>0: Size</td>
<td>-</td>
<td>-2.0119</td>
<td>-</td>
<td>-1.8989</td>
<td>-</td>
<td>-1.7777</td>
</tr>
<tr>
<td>0: Top6</td>
<td>-</td>
<td>0.944</td>
<td>-</td>
<td>0.834</td>
<td>-</td>
<td>0.7123</td>
</tr>
<tr>
<td>0: Top6 x Size</td>
<td>-</td>
<td>-1.5995</td>
<td>-</td>
<td>-1.4571</td>
<td>-</td>
<td>-1.3009</td>
</tr>
<tr>
<td>Utility</td>
<td>-0.4801</td>
<td>-0.4784</td>
<td>-0.2312</td>
<td>-0.2295</td>
<td>-0.1084</td>
<td>-0.1066</td>
</tr>
<tr>
<td>Avg. Return</td>
<td>0.0216</td>
<td>0.0235</td>
<td>0.0216</td>
<td>0.0234</td>
<td>0.0216</td>
<td>0.0234</td>
</tr>
<tr>
<td>Rf</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.0266</td>
<td>0.0269</td>
<td>0.0266</td>
<td>0.0265</td>
<td>0.0266</td>
<td>0.0262</td>
</tr>
<tr>
<td>SR</td>
<td>0.7970</td>
<td>0.8587</td>
<td>0.7970</td>
<td>0.8679</td>
<td>0.7970</td>
<td>0.8779</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Annual Rebalancing</th>
<th>Market Portfolio at $\gamma=3$</th>
<th>Optimal Portfolio at $\gamma=3$</th>
<th>Market Portfolio at $\gamma=5$</th>
<th>Optimal Portfolio at $\gamma=5$</th>
<th>Market Portfolio at $\gamma=9$</th>
<th>Optimal Portfolio at $\gamma=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: Cap Rate</td>
<td>-</td>
<td>0.1131</td>
<td>-</td>
<td>0.1457</td>
<td>-</td>
<td>0.1366</td>
</tr>
<tr>
<td>0: Size</td>
<td>-</td>
<td>-1.91</td>
<td>-</td>
<td>-1.9188</td>
<td>-</td>
<td>-1.8632</td>
</tr>
<tr>
<td>0: Top6</td>
<td>-</td>
<td>0.6329</td>
<td>-</td>
<td>0.8363</td>
<td>-</td>
<td>1.0565</td>
</tr>
<tr>
<td>0: Top6 x Size</td>
<td>-</td>
<td>-0.4451</td>
<td>-</td>
<td>-0.5583</td>
<td>-</td>
<td>-0.7001</td>
</tr>
<tr>
<td>Utility</td>
<td>-0.4799</td>
<td>-0.4782</td>
<td>-0.2308</td>
<td>-0.2291</td>
<td>-0.1075</td>
<td>-0.1057</td>
</tr>
<tr>
<td>Avg. Return</td>
<td>0.0215</td>
<td>0.0232</td>
<td>0.0215</td>
<td>0.0232</td>
<td>0.0215</td>
<td>0.0232</td>
</tr>
<tr>
<td>Rf</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.0227</td>
<td>0.0218</td>
<td>0.0227</td>
<td>0.0213</td>
<td>0.0227</td>
<td>0.0209</td>
</tr>
<tr>
<td>SR</td>
<td>0.8899</td>
<td>1.0046</td>
<td>0.8899</td>
<td>1.0282</td>
<td>0.8899</td>
<td>1.0478</td>
</tr>
</tbody>
</table>
Figure 4.2 shows that all the signs of the coefficients are consistent with the study in Section 4.1. The Sharp ratios all get increased. Those two observations further confirm that our optimization strategy works for all those three types of investors. I note that investors with high-risk aversion can implement those strategies to achieve the highest increase in Sharp ratio.

By comparing the coefficients for those three types of investors, I note the degree of tilting is different. For instance, investors with the highest risk aversion tilt most towards investing in properties with higher cap rate. This is like investing in a high-yield stock. Investors gain a higher cash flow return per unit of initial investment. Meanwhile, the higher cap rate will lower the purchase price, and therefore reduce the risk of investment. For investors with the highest risk aversion, they are more sensitive to the risk-return profile. For one unit of risk, they prefer to have more return. Therefore, the optimal portfolio for high risk-averse investors tilts most towards investing in higher cap rate properties.

Meanwhile, the optimal portfolios all tilt away from investing in properties with bigger size measured by appraisal value. This may be because the benchmark portfolio already includes many such properties. For diversification purposes, it suggests investing less in properties with higher appraisal value.

However, according to Pai and Geltner [2007], properties with larger size tend to be more risky, and therefore require substantial return premiums. Investors with the least risk aversion can capture this return premium by investing in larger size properties. I note that in my optimization, investors with the least risk aversion indeed tilt farther away from investing in larger size properties than the high risk-averse investors do. In theory, this is contradicted by Pai and Geltner’s paper, but in practice, for this specific portfolio management, an investor may be reluctant to have the optimal portfolio deviate from the benchmark portfolio for a sufficiently large relative-risk aversion. High-risk-averse investors may be more reluctant than low-risk-averse investors in tilting the property weight farther away from the benchmark portfolio that was perceived as a safe base.

The above observations are the same for rebalancing quarterly and annually. Besides that, I also observe the investors’ different preferences for market liquidity under different rebalancing
frequencies. If rebalancing quarterly, high-risk-averse investors care less about liquidity compared with other investors, because rebalancing is more frequent. However, for annual rebalancing, this trend gets reversed, because high-risk-averse investors care more about liquidity. For longer-term investment horizons, investors care more about how easily they can exit. Top 6 MSA markets have relatively high transaction volumes compared with non-Top 6 MSA markets, which can reduce the risk of illiquidity.

### 4.3 Optimal Portfolios for All Property Types and Varied $\gamma$ with Consideration of CFNAI

This study allows the effects of property characteristics on optimal portfolio weights to vary with the realization of CFNAI. I extend the optimization model first by incorporating the macroeconomic conditions of economic expansion and contraction. Using the Kronecker product, I create a doubled number of variables associated with these two macroeconomic conditions.

#### Figure 4.3:

Optimal Portfolios for All Property Types and Varied $\gamma$ with consideration of CFNAI

Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for annual rebalancing.

<table>
<thead>
<tr>
<th>Annual rebalancing for CFNAI</th>
<th>Market Portfolio at $\gamma=3$</th>
<th>Optimal Portfolio at $\gamma=3$</th>
<th>Market Portfolio at $\gamma=5$</th>
<th>Optimal Portfolio at $\gamma=5$</th>
<th>Market Portfolio at $\gamma=9$</th>
<th>Optimal Portfolio at $\gamma=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: Cap Rate +</td>
<td>-</td>
<td>0.0131</td>
<td>-</td>
<td>0.0144</td>
<td>-</td>
<td>0.0191</td>
</tr>
<tr>
<td>0: Cap Rate -</td>
<td>-</td>
<td>-0.2589</td>
<td>-</td>
<td>-0.2425</td>
<td>-</td>
<td>-0.1811</td>
</tr>
<tr>
<td>0: Size +</td>
<td>-</td>
<td>-0.4133</td>
<td>-</td>
<td>-0.4651</td>
<td>-</td>
<td>-0.5608</td>
</tr>
<tr>
<td>0: Size -</td>
<td>-</td>
<td>-1.9046</td>
<td>-</td>
<td>-1.9219</td>
<td>-</td>
<td>-1.9119</td>
</tr>
<tr>
<td>0: Top6 +</td>
<td>-</td>
<td>0.3251</td>
<td>-</td>
<td>0.3864</td>
<td>-</td>
<td>0.4774</td>
</tr>
<tr>
<td>0: Top6 -</td>
<td>-</td>
<td>0.185</td>
<td>-</td>
<td>0.4482</td>
<td>-</td>
<td>1.0762</td>
</tr>
<tr>
<td>0: Top6 x Size +</td>
<td>-</td>
<td>-1.0955</td>
<td>-</td>
<td>-1.1688</td>
<td>-</td>
<td>-1.272</td>
</tr>
<tr>
<td>0: Top6 x Size -</td>
<td>-</td>
<td>-0.2186</td>
<td>-</td>
<td>-0.3663</td>
<td>-</td>
<td>-0.7272</td>
</tr>
</tbody>
</table>

| Utility                    | -0.4799                        | -0.4769                        | -0.2308                        | -0.228                         | -0.1075                        | -0.1048                        |
| Avg. Return                | 0.0215                         | 0.0247                         | 0.0215                         | 0.0247                         | 0.0215                         | 0.0245                         |
| RF                         | 0.13%                          | 0.13%                          | 0.13%                          | 0.13%                          | 0.13%                          | 0.13%                          |
| Std.Dev                    | 0.0227                         | 0.0244                         | 0.0227                         | 0.0239                         | 0.0227                         | 0.0224                         |
| SR                         | 0.8899                         | 0.9590                         | 0.8899                         | 0.9791                         | 0.8899                         | 1.0357                         |
Under economic expansion, the signs of coefficients are the same as for the previous studies in Sections 4.1 and 4.2. The degree of tilting from the benchmark portfolio is relatively small for Cap Rate, Size, and Top 6, which indicates that when the economy is robust, there is less need to adjust the optimal portfolio. Conversely, the degree of tilting for Top6 x Size increases under economic expansion, which means the optimal portfolio places less weight on the properties with higher appraisal values in the Top 6 MSAs when the economy is robust. This is reasonable because properties with higher appraisal values in Top 6 MSAs are more stable and more liquid. During economic expansion, the risk of illiquidity is much lower.

Conversely, during economic contraction the optimal portfolio still tilts towards properties with smaller size and/or properties located in Top 6 MSAs; for cap rate, however, the optimal portfolio places more weight on lower cap rate. During recession, investors are concerned more about the depreciation of the value of their assets, because economic contraction may greatly impact the net operating income. Other things being equal, a property with lower cap rate has higher value than one with a higher cap rate. Therefore, investors tilt more towards lower cap rate properties, given the same NOI. I also note that during economic contraction optimal portfolios tilt farther away from properties with higher value. As I mentioned in the previous study, properties with higher value are more risky; therefore investors are more conservative in investing in those types of assets.

4.4 Optimal Portfolios for All Property Types and Varied $\gamma$ with Consideration of Liquidity

I further extend the optimization model by incorporating the Liquidity Metrics developed at the MIT Center for Real Estate. As liquidity is an important indicator for transaction volumes in the market, the study of different optimal portfolios under different liquidity conditions leads to different investment strategies in different capital market conditions.

Figure 4.4:
Optimal Portfolios for All Property Types and Varied $\gamma$ with Consideration of Liquidity
Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for annual rebalancing.
When the market is illiquid, with fewer transactions in the market, the optimal portfolio tilts more towards properties with higher cap rate to capture more income return instead of capital return. High-risk-averse investors tilt more towards higher cap rate than low-risk-averse investors.

As with cap rate, the optimized portfolio also tilts farther away from properties with high appraisal value when the market is illiquid, because those properties are risky and hard to dispose of. High-risk-averse investors tilt more away from properties with higher appraisal value than low-risk-averse investors.

When the market is liquid, the advantage of properties in Top 6 MSAs is not strong, compared with other factors. However, when the market is illiquid, the optimal portfolio places more weight on buildings in the Top 6 MSAs, because Top 6 MSAs are relatively more liquid than the other markets.

### 4.5 Optimal Portfolios for Different Property Types with $\gamma=5$

To optimize the allocation strategy for different property types and provide suggestions for asset managers in charge of specific property types, I classify sub-portfolios by 5 different types, and run the optimization for each of them. In this case, because the amount of data in each sub-

<table>
<thead>
<tr>
<th>Annual rebalancing for Liquidity</th>
<th>Market Portfolio at $\gamma=3$</th>
<th>Optimal Portfolio at $\gamma=3$</th>
<th>Market Portfolio at $\gamma=5$</th>
<th>Optimal Portfolio at $\gamma=5$</th>
<th>Market Portfolio at $\gamma=9$</th>
<th>Optimal Portfolio at $\gamma=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$: Cap Rate +</td>
<td>-0.4816</td>
<td>0.1781</td>
<td>-0.2325</td>
<td>-0.2311</td>
<td>-0.1092</td>
<td>-0.1075</td>
</tr>
<tr>
<td>$\theta$: Cap Rate -</td>
<td>0.0197</td>
<td>0.0209</td>
<td>0.0197</td>
<td>0.0209</td>
<td>0.0197</td>
<td>0.0209</td>
</tr>
<tr>
<td>$\theta$: Size +</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
</tr>
<tr>
<td>$\theta$: Size -</td>
<td>0.0239</td>
<td>0.0204</td>
<td>0.0239</td>
<td>0.0204</td>
<td>0.0239</td>
<td>0.0204</td>
</tr>
<tr>
<td>$\theta$: Top6 +</td>
<td>0.7699</td>
<td>0.9608</td>
<td>0.7699</td>
<td>0.9608</td>
<td>0.7699</td>
<td>0.9608</td>
</tr>
<tr>
<td>$\theta$: Top6 -</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
<td>0.13%</td>
</tr>
<tr>
<td>$\theta$: Size -</td>
<td>0.0239</td>
<td>0.0204</td>
<td>0.0239</td>
<td>0.0204</td>
<td>0.0239</td>
<td>0.0204</td>
</tr>
<tr>
<td>$\theta$: Top6 -</td>
<td>0.7699</td>
<td>0.9608</td>
<td>0.7699</td>
<td>0.9608</td>
<td>0.7699</td>
<td>0.9608</td>
</tr>
<tr>
<td>Utility</td>
<td>-0.4803</td>
<td>-0.4803</td>
<td>-0.2325</td>
<td>-0.2311</td>
<td>-0.1092</td>
<td>-0.1075</td>
</tr>
<tr>
<td>Avg. Return</td>
<td>0.0197</td>
<td>0.0209</td>
<td>0.0197</td>
<td>0.0209</td>
<td>0.0197</td>
<td>0.0209</td>
</tr>
<tr>
<td>Rf</td>
<td>1.675</td>
<td>-1.3616</td>
<td>-6.2725</td>
<td>-6.0995</td>
<td>-5.0995</td>
<td>-5.0995</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.0204</td>
<td>0.0204</td>
<td>0.0204</td>
<td>0.0204</td>
<td>0.0204</td>
<td>0.0204</td>
</tr>
<tr>
<td>SR</td>
<td>0.9608</td>
<td>0.9608</td>
<td>0.9608</td>
<td>0.9608</td>
<td>0.9608</td>
<td>0.9608</td>
</tr>
</tbody>
</table>
portfolio gets sparse if rebalanced annually, due to the consideration of accuracy, I decide to rebalance quarterly to include more data in the model.

Figure 4.5:
Optimal Portfolios for Different Property Types with $\gamma=5$

Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for quarterly rebalancing.

<table>
<thead>
<tr>
<th>$\gamma=5$ Quarterly rebalancing</th>
<th>Apartment</th>
<th>Hotel</th>
<th>Industrial</th>
<th>Office</th>
<th>Retail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cap Rate</td>
<td>0.069</td>
<td>0.912</td>
<td>0.256</td>
<td>0.321</td>
<td>0.011</td>
</tr>
<tr>
<td>Size</td>
<td>-0.727</td>
<td>0.452</td>
<td>-1.296</td>
<td>-1.527</td>
<td>0.096</td>
</tr>
<tr>
<td>Top6</td>
<td>1.701</td>
<td>-1.094</td>
<td>-0.019</td>
<td>-0.228</td>
<td>0.000</td>
</tr>
<tr>
<td>Utility</td>
<td>-0.231</td>
<td>-0.227</td>
<td>-0.231</td>
<td>-0.237</td>
<td>-0.231</td>
</tr>
<tr>
<td>Avg. Return</td>
<td>0.023</td>
<td>0.026</td>
<td>0.022</td>
<td>0.017</td>
<td>0.022</td>
</tr>
<tr>
<td>Rf</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.033</td>
<td>0.049</td>
<td>0.031</td>
<td>0.038</td>
<td>0.031</td>
</tr>
<tr>
<td>SR</td>
<td>0.683</td>
<td>0.512</td>
<td>0.704</td>
<td>0.435</td>
<td>0.688</td>
</tr>
</tbody>
</table>

The optimal portfolios for all the sub-portfolios are tilted towards properties with higher cap rate. For hotel and retail, optimal portfolios place more weight on properties with higher appraisal value, whereas for apartment, industrial, and office, optimized portfolios place more weight on lower appraisal value. Only the optimal apartment portfolio placed more weight on Top 6 MSAs.

The benefit of holding the optimal portfolio is evident across all property types if only looking at increased average return. By looking at the Sharp ratio, I note that holding the optimal apartment, hotel, and office portfolios gets a higher Sharp ratio, whereas the industrial and retail optimal portfolios' Sharp ratios indeed get decreased because of the increased standard deviation. Investors should choose whether to hold those optimal portfolios according to their risk appetite.
4.6 Optimal Portfolios for Different Property Types and $\gamma=5$ with Consideration of CFNAI

Figure 4.6:

Optimal Portfolios for Different Property Types and $\gamma=5$ with Consideration of CFNAI

Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for quarterly rebalancing.

<table>
<thead>
<tr>
<th>$\gamma=5$</th>
<th>Apartment Market Portfolio</th>
<th>Hotel Market Portfolio</th>
<th>Industrial Market Portfolio</th>
<th>Office Market Portfolio</th>
<th>Retail Market Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarterly rebalancing</td>
<td>0.069</td>
<td>0.912</td>
<td>0.256</td>
<td>0.321</td>
<td>0.011</td>
</tr>
<tr>
<td>$\theta$:Cap Rate+</td>
<td>-2.278</td>
<td>8.145</td>
<td>-1.57</td>
<td>-4.501</td>
<td>10.637</td>
</tr>
<tr>
<td>$\theta$:Size+</td>
<td>-0.730</td>
<td>0.229</td>
<td>-1.247</td>
<td>-1.514</td>
<td>0.094</td>
</tr>
<tr>
<td>$\theta$:Size-</td>
<td>3.239</td>
<td>2.602</td>
<td>-1.019</td>
<td>3.431</td>
<td>-13.319</td>
</tr>
<tr>
<td>$\theta$:Top6+</td>
<td>1.664</td>
<td>-4.888</td>
<td>-0.044</td>
<td>-0.230</td>
<td>-0.004</td>
</tr>
<tr>
<td>$\theta$:Top6-</td>
<td>15.523</td>
<td>-2.053</td>
<td>1.988</td>
<td>6.637</td>
<td>157.850</td>
</tr>
<tr>
<td>Utility</td>
<td>-0.231</td>
<td>-0.227</td>
<td>-0.225</td>
<td>-0.231</td>
<td>-0.234</td>
</tr>
<tr>
<td>Avg.Return</td>
<td>0.023</td>
<td>0.027</td>
<td>0.026</td>
<td>0.033</td>
<td>0.022</td>
</tr>
<tr>
<td>Rf</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.033</td>
<td>0.031</td>
<td>0.049</td>
<td>0.050</td>
<td>0.031</td>
</tr>
<tr>
<td>SR</td>
<td>0.683</td>
<td>0.847</td>
<td>0.512</td>
<td>0.649</td>
<td>0.704</td>
</tr>
</tbody>
</table>

During economic expansion, all the optimal portfolios are tilted more towards investing in high-cap-rate properties. During economic contraction, hotel and retail optimal portfolios still tilt more to the high-cap-rate properties, but apartment, industrial and office optimal portfolios deviate away from high-cap-rate properties.

We all know that during economic recessions, in a short-term horizon, hotel and retail properties can be largely impacted because the income returns for those types of properties are heavily dependent on sales. Therefore, the value of those properties will be depreciated. Due to a counter-cyclical investment policy, investors can leverage on the cyclical market behavior to buy cheap during economic contraction and sell high at economic expansion. In the other aspect, higher-cap-rate properties mean higher income return. During economic recessions, the stable cash flows for hotel and retail properties are significantly important. Therefore investors would tilt toward higher-cap-rate properties.
For apartment, industrial and office properties, due to the longer-term lease, those income returns may not be impacted severely during economic recession to the same extent as retail and hotel. A lower cap rate leads to a higher sale price; therefore, the optimal portfolios for those property types deviate towards a lower cap rate.

What is also interesting to notice is that during economic expansion, hotel and retail properties are consistent in deviating more towards larger properties, which is different from the other property types. During economic expansions, investors tend to capture more income by investing in properties with higher appraisal value. For retail and hotel, bigger size means more leasable area and more hotel rooms. The robust economy can help absorb those large properties and lead to a higher income return.

During economic expansion, apartment, industrial and office optimal portfolios are tilted away from high-appraisal-value properties. This may be because a robust economy may help to sell those types of properties to gain a higher capital return.

During economic recession, the optimal apartment, office, and hotel portfolios deviate towards high-appraisal-value properties. This may be because those properties are relatively cheap to acquire during economic recessions. In contrast, for industrial and retail properties, bigger-size and the intrinsically longer-term lease may add risk to the portfolio without bringing a reasonable amount of return. Therefore the optimal portfolio deviates away from higher-appraisal-value properties for those property types.

### 4.7 Optimal Portfolios for Different Property Types and $\gamma=5$ with Consideration of Liquidity

I further studied the impact of liquidity on the coefficients of cap rate and size for different property types.
Figure 4.7:
Optimal Portfolios for Different Property Types and $\gamma=5$ with Consideration of Liquidity
Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for quarterly rebalancing.

<table>
<thead>
<tr>
<th>$\gamma=5$</th>
<th>Apartment</th>
<th>Hotel</th>
<th>Industrial</th>
<th>Office</th>
<th>Retail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter rebalancing</td>
<td>Market Portfolio</td>
<td>Optimal Portfolio</td>
<td>Market Portfolio</td>
<td>Optimal Portfolio</td>
<td>Market Portfolio</td>
</tr>
<tr>
<td>0: Cap Rate+</td>
<td>-</td>
<td>0.142</td>
<td>-</td>
<td>0.080</td>
<td>-</td>
</tr>
<tr>
<td>0: Cap Rate-</td>
<td>-</td>
<td>7.382</td>
<td>-</td>
<td>4.396</td>
<td>-</td>
</tr>
<tr>
<td>0: Size+</td>
<td>-</td>
<td>0.469</td>
<td>-</td>
<td>0.195</td>
<td>-</td>
</tr>
<tr>
<td>0: Size-</td>
<td>-</td>
<td>9.886</td>
<td>-</td>
<td>3.702</td>
<td>-</td>
</tr>
<tr>
<td>Utility</td>
<td>-0.231</td>
<td>-0.228</td>
<td>-0.233</td>
<td>-0.232</td>
<td>-0.231</td>
</tr>
<tr>
<td>Avg.Return</td>
<td>0.023</td>
<td>0.026</td>
<td>0.023</td>
<td>0.033</td>
<td>0.022</td>
</tr>
<tr>
<td>Rf</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.033</td>
<td>0.030</td>
<td>0.048</td>
<td>0.050</td>
<td>0.031</td>
</tr>
<tr>
<td>SR</td>
<td>0.681</td>
<td>0.836</td>
<td>0.475</td>
<td>0.649</td>
<td>0.704</td>
</tr>
</tbody>
</table>

All the optimal portfolios deviate towards high-cap-rate properties in both liquid and illiquid macroeconomic conditions, with the only exception of retail’s slight deviation away from high-cap-rate properties under liquid macroeconomic conditions. I also note that all of the optimal portfolios deviate away from high-appraisal-value properties under liquid market conditions. During illiquid market conditions, apartment and office portfolios still deviate away from high-appraisal-value properties, whereas hotel, industrial, and retail optimal portfolios place more weight on high-appraisal-value properties. The average return and Sharp ratio of all the optimal portfolios get increased.

4.8 Optimal Portfolios for Different Regions and $\gamma=5$

As most investment managers focus only on certain regions, categorizing the portfolio by different regions is meaningful for those investment managers to optimize their individual investment strategies. Because the data get sparse if sorted by region, for the consideration of accuracy, I choose rebalancing quarterly and only focus on cap rate and size.
Figure 4.8:
Optimal Portfolios for Different Regions and Investors with $\gamma=5$
Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for quarterly rebalancing.

<table>
<thead>
<tr>
<th>$\gamma=5$</th>
<th>East Region</th>
<th>West Region</th>
<th>Midwest Region</th>
<th>South Region</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarterly Rebalancing</strong></td>
<td>Market Portfolio</td>
<td>Optimal Portfolio</td>
<td>Market Portfolio</td>
<td>Optimal Portfolio</td>
</tr>
<tr>
<td>0: Cap Rate</td>
<td>-</td>
<td>0.251</td>
<td>-</td>
<td>-0.620</td>
</tr>
<tr>
<td>0: Size</td>
<td>-</td>
<td>-0.956</td>
<td>-</td>
<td>-1.709</td>
</tr>
<tr>
<td>Utility</td>
<td>-0.230</td>
<td>-0.229</td>
<td>-0.230</td>
<td>-0.227</td>
</tr>
<tr>
<td>Avg.Return</td>
<td>0.024</td>
<td>0.025</td>
<td>0.024</td>
<td>0.030</td>
</tr>
<tr>
<td>Rf</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.031</td>
<td>0.029</td>
<td>0.033</td>
<td>0.048</td>
</tr>
<tr>
<td>SR</td>
<td>0.768</td>
<td>0.823</td>
<td>0.720</td>
<td>0.605</td>
</tr>
</tbody>
</table>

Similar to previous studies on the aggregated national level portfolio, most of the optimal portfolios place more weight on the high-cap-rate and small-size properties except the West Region. Unlike others, the West Region optimal portfolio tilts away from high-cap-rate portfolios. All the optimal portfolios have increased average returns. For the East and Midwest Regions, the Sharp ratios increase; whereas for the West and South Regions, the increased standard deviations decrease the Sharp ratio.
4.9 Optimal Portfolios for Different Regions and $\gamma=5$ with Consideration of CFNAI

Figure 4.9:

Optimal Portfolios for Different Regions and $\gamma=5$ with Consideration of CFNAI

Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for quarterly rebalancing.

<table>
<thead>
<tr>
<th>$\gamma=5$</th>
<th>Quarterly Rebalancing</th>
<th>East Region</th>
<th>West Region</th>
<th>Midwest Region</th>
<th>South Region</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Market Portfolio</td>
<td>Optimal Portfolio</td>
<td>Market Portfolio</td>
<td>Optimal Portfolio</td>
<td>Market Portfolio</td>
</tr>
<tr>
<td>0: Cap Rate+</td>
<td>0.195</td>
<td>-0.006</td>
<td>0.230</td>
<td>-0.228</td>
<td>-0.234</td>
</tr>
<tr>
<td>0: Cap Rate-</td>
<td>1.197</td>
<td>-12.921</td>
<td>2.504</td>
<td>6.892</td>
<td>3.492</td>
</tr>
<tr>
<td>0: Size+</td>
<td>-1.220</td>
<td>-1.478</td>
<td>-2.261</td>
<td>-2.213</td>
<td></td>
</tr>
<tr>
<td>0: Size-</td>
<td>-0.844</td>
<td>6.892</td>
<td>-3.492</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td>-0.230</td>
<td>-0.228</td>
<td>-0.234</td>
<td>-0.235</td>
<td>-0.234</td>
</tr>
<tr>
<td>Avg.Return</td>
<td>0.024</td>
<td>0.025</td>
<td>0.019</td>
<td>0.018</td>
<td>0.020</td>
</tr>
<tr>
<td>Rf</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.031</td>
<td>0.031</td>
<td>0.031</td>
<td>0.027</td>
<td>0.032</td>
</tr>
<tr>
<td>SR</td>
<td>0.768</td>
<td>0.783</td>
<td>0.547</td>
<td>0.671</td>
<td>0.660</td>
</tr>
</tbody>
</table>

The West Region optimal portfolio is the only portfolio that tilts away from high-cap-rate properties during economic expansions and contractions. Both the East and Midwest Regions’ optimal portfolios place more weight on high-cap-rate properties during economic expansions and contractions. For the South Region, the optimal portfolio tilts more towards high-cap-rate properties during economic expansions and tilts slightly away from high-cap-rate properties during economic contractions. I also see that all the optimal portfolios tilt away from high-appraisal-value properties during economic expansions. This result is the same as the previous optimization for aggregated national level portfolios. However, during economic contractions, the West and South Regions’ optimal portfolios both place more weight on the large size properties, whereas the East and Midwest Regions’ portfolios still tilt away from large-size properties.

All the optimal portfolios’ returns shown in Figure 4.9 increase. The East and Midwest Regions are the only two regions with increased Sharp ratios. The Sharp ratio increases most in the
Midwest Region. Although the optimal portfolio returns for the West and South Regions increase, their Sharp ratio decreases due to the increased standard deviation. My study of Figure 4.9 shows that the Midwest Region portfolio has the most potential to be optimized.

### 4.10 Optimal Portfolios for Different Regions and $\gamma=5$ with Consideration of Liquidity

Figure 4.10:

Optimal Portfolios for Different Regions and $\gamma=5$ with Consideration of Liquidity

Optimal portfolio policy coefficient estimates for all properties with non-negative weight constraints for quarterly rebalancing.

<table>
<thead>
<tr>
<th>$\gamma=5$</th>
<th>East Region</th>
<th>West Region</th>
<th>Midwest Region</th>
<th>South Region</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarterly Rebalancing</strong></td>
<td>Market Portfolio</td>
<td>Optimal Portfolio</td>
<td>Market Portfolio</td>
<td>Optimal Portfolio</td>
</tr>
<tr>
<td>0: Cap Rate+</td>
<td>-</td>
<td>0.248</td>
<td>-</td>
<td>-0.436</td>
</tr>
<tr>
<td>0: Cap Rate-</td>
<td>-</td>
<td>1.144</td>
<td>-</td>
<td>0.815</td>
</tr>
<tr>
<td>0: Size+</td>
<td>-</td>
<td>-1.590</td>
<td>-</td>
<td>-1.115</td>
</tr>
<tr>
<td>0: Size-</td>
<td>-</td>
<td>-0.874</td>
<td>-</td>
<td>-3.127</td>
</tr>
<tr>
<td>Utility</td>
<td>-0.232</td>
<td>-0.231</td>
<td>-0.232</td>
<td>-0.230</td>
</tr>
<tr>
<td>Avg.Return</td>
<td>0.022</td>
<td>0.024</td>
<td>0.021</td>
<td>0.024</td>
</tr>
<tr>
<td>Rf</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.034</td>
<td>0.033</td>
<td>0.032</td>
<td>0.031</td>
</tr>
<tr>
<td>SR</td>
<td>0.650</td>
<td>0.706</td>
<td>0.650</td>
<td>0.745</td>
</tr>
</tbody>
</table>

Extending the study by including Liquidity Metrics as the macroeconomic indicator, I realize that most of the optimal portfolios (except for the West Region) place more weight on high-cap-rate properties during liquid and illiquid situations. The West Region differs from the rest because the optimal portfolio tilts away from high-cap-rate properties during liquid times. Similar to cap rate, I see most of the optimal portfolios (except for the South Region) place more weight on small-size properties during economic expansions and contractions. The South Region portfolio is an exception because it tilts towards large-size properties during economic contractions.

My study of Figure 4.10 shows that all the portfolio returns and Sharp ratios increased. The Midwest Region has the most increased Sharp ratio, which indicates that it has great potential to be improved. This result is the same as the study of Figure 4.9.
Chapter 5 Conclusions

As we can see in the optimization models developed in the previous chapter, the parametric portfolio optimization approach of Brandt, Santa-Clara, and Valkanov [2009] can be effectively implemented in commercial real estate portfolio optimization. The research framework of Plazzi, Torous, and Valkanov [2011] on modeling the portfolio weight, based on cap rate, appraisal value, Top 6 and various capital market conditions, can lead to solid suggestions for investment managers regarding the future acquisition and disposition decisions that vary with prevailing macroeconomic conditions. The optimization policies generated from these models significantly improves the risk-adjusted performance of commercial real estate portfolios relative to their benchmark portfolios. This methodology avoids the very complicated steps of modeling the joint distribution of returns and building characteristics, and it is not constrained by the number of assets in the portfolio. Moreover, it can provide both qualitative and quantitative suggestions for the investment strategy.

This thesis further advances the framework of Plazzi, Torous, and Valkanov by classifying sub-portfolios by property type and region, and including more capital market variables, including CFNAI and Liquidity Metrics. This is practical because in the commercial real estate investment industry, the way to evaluate asset managers’ performance is based on their responsible portfolio’s performance compared with the benchmark portfolio. Most asset managers are usually responsible for a specific type of properties in a specific region. Classifying sub-portfolios by property type and region can generate practical portfolio optimization strategies for a specific asset manager, or provide a benchmark standard that can be used to evaluate asset managers’ performance. Moreover, this thesis implements the methodology with the assumption of rebalancing annually, instead of quarterly, as done with most existing models. This is close to practice because commercial real estate transactions are less frequent than with other asset types because of the high transaction costs.

This methodology also opens broad opportunities for future improvement. It can be used to test the importance of other property characteristics in commercial real estate portfolio management, and can be extended to include transaction costs and other market frictions. This methodology,
when practically applied, should lead to more innovations in future commercial real estate portfolio management.
Appendix: MATLAB Code

Section 1: Input data into MATLAB

%load data from excel file to MATLAB, point MATLAB to read different sheets in the excel file, read excel specific sheet into file dataset D.

filename='data_newest.xlsx';
sheetname='NPI_Returns per Prop';
sheetnameA='A';
sheetnameH='H';
sheetnameI='I';
sheetnameR='R';
sheetnameO='O';
sheetnameE='E';
sheetnameW='W';
sheetnameS='S';
sheetnameMW='MW';
D=xlsread(filename,sheetname);

%From dataset D, export specific columns and name it.

propid=D(:,5);
year=D(:,3);
quarter=D(:,4);
year_quarter=D(:,2);
retu=D(:,21);
sqrft=D(:,12);
caprate=D(:,23);
appval=D(:,16);

%Macroeconomic dummy variables

CFNAI=D(:,34);
CFNAI_pos=D(:,35);
CFNAI_neg=D(:,36);
Liquidity=D(:,40);
Liquidity_pos=D(:,41);
Liquidity_neg=D(:,42)

Location dummy variables

east_dummy=D(:,30);
midwest_dummy=D(:,32);
south_dummy=D(:,33);
w west dummy=D(:,31);
Top6 dummy=D(:,38);
| FirstTier=D(:,39);
| NonMSA=D(:,40);

Property type dummy variables

apt_dummy=D(:,24);
htl_dummy=D(:,25);
ind_dummy=D(:,26);
off dummy=D(:,27);
rtl dummy=D(:,28);

Section 2: Remove Bad Points

badpoints=find(sqrft<1000|abs(caprate)>0.2|retu>0.8|retu<-0.4);
propid(badpoints)=[];
year_quarter(badpoints)=[];
year(badpoints)=[];
quarter(badpoints)=[];
retu(badpoints)=[];
sqrft(badpoints)=[];
caprate(badpoints)=[];
Section 3: Group Data within the Same Quarter

diff_year_quarter=[year_quarter(2:end)-year_quarter(1:end-1)];
ind_switch=find(diff_year_quarter~=0);

cnt=0;
for yr_ind=2:length(ind_switch)
sprintf('For year-quarter %5.0d',year_quarter(ind_switch(yr_ind)));
cnt=cnt+1;
data(cnt).yyyyqq=year_quarter(ind_switch(yr_ind));
data(cnt).yr=floor(year_quarter(ind_switch(yr_ind))/10);
data(cnt).quarter=rem(year_quarter(ind_switch(yr_ind)),10);
data(cnt).ret=retu(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).N=length(data(cnt).ret);
data(cnt).propid=propid(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).CFNAI=CFNAI(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).Liquidity_pos=Liquidity_pos(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).Liquidity_neg=Liquidity_neg(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).CFNAI_pos=CFNAI_pos(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).CFNAI_neg=CFNAI_neg(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).cap=caprate(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).sqrft=sqrft(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).appval=appval(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).sumappval=sum(data(cnt).appval);
data(cnt).meanappval=mean(data(cnt).appval);
data(cnt).east_dummy=east_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).midwest_dummy=midwest_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).south_dummy=south_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).west_dummy=west_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).apt_dummy=apt_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).htl_dummy=htl_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).ind_dummy=ind_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).off_dummy=off_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).rtl_dummy=rtl_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).weight=appval(ind_switch(yr_ind-1)+1:ind_switch(yr_ind))./sum(appval(ind_switch(yr_ind-1)+1:ind_switch(yr_ind)));
data(cnt).Top6_dummy=Top6_dummy(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).FirstTier_dummy=FirstTier(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
data(cnt).NonMSA_dummy=FirstTier(ind_switch(yr_ind-1)+1:ind_switch(yr_ind));
end;
Section 4: Normalized Data and Set Up Matrices File for Optimization

T=size(data,2);
for t=1:T
    ret=data(t).ret;
    cap=data(t).cap;
    sqrf=t=data(t).sqrft;
    appval=data(t).appval;
    yy=data(t).yr;
    Top6=zscore(data(t).Top6_dummy);
    FirstTier=data(t).FirstTier_dummy;
    NonMSA=data(t).NonMSA_dummy;
    a1=zscore(cap);
    a2=zscore(sqrft);
    a3=zscore(appval);
    b1=zscore(data(t).apt_dummy);
    b2=zscore(data(t).off_dummy);
    b3=zscore(data(t).rtl_dummy);
    b4=zscore(data(t).ind_dummy);
    b5=zscore(data(t).htl_dummy);
    c1=zscore(data(t).east_dummy);
    c2=zscore(data(t).south_dummy);
    c3=zscore(data(t).west_dummy);
    c4=zscore(data(t).midwest_dummy);
    z1=data(t).CFNAI_pos;
    z2=data(t).CFNAI_neg;
    q1=data(t).Liquidity_pos;
    q2=data(t).Liquidity_neg;
    qq=Top6.*a3;

    %datastruct(t).x=[a1.*z1 a1.*z2 a3.*z1 a3.*z2];
    %datastruct(t).x=[a1.*q1 a1.*q2 a3.*q1 a3.*q2];
\%datastruct(t).x=[a1 a3 Top6 qq];
\%datastruct(t).x=[a1 a3];
\%datastruct(t).x=[a1.*z1 a1.*z2 a3.*z1 a3.*z2 Top6.*z1 Top6.*z2];
datastruct(t).x=[a1.*q1 a1.*q2 a3.*q1 a3.*q2 Top6.*q1 Top6.*q2 qq.*q1 qq.*q2];
\%datastruct(t).x=[a1.*z1 a1.*z2 a3.*z1 a3.*z2];
\%datastruct(t).x=[a1.*z1 a1.*z2 a3.*z1 a3.*z2 Top6.*z1 Top6.*z2 qq.*z1 qq.*z2];

datastruct(t).N=length(data(t).weight);
datastruct(t).X=datastruct(t).x ./datastruct(t).N;
datastruct(t).ret=ret;
datastruct(t).yy=yy;
datastruct(t).market_cap_lag=data(t).appval;
datastruct(t).weight_lag=data(t).weight;

end;

Section 5: Set Up Optimization Formula and Utility Function.

function f = calc(theta)
global datastructCopy
T=size(datastructCopy,2);
util=[];
for t=1:T
wbar=datastructCopy(t).weight_lag;
N=length(wbar);
X=datastructCopy(t).X;
ret=datastructCopy(t).ret;

\%Set up non negative weight constraint.

w=wbar+X*theta;
w=max(0,w);
DD=sum(w);
w=w./DD;

retp=w'*ret;
retpm=wbar'*ret;
portfolio(t).weight=w';
portfolio(t).weightstat=[mean(abs(w)) min(w) max(w) sum(w(w<0)) sum(w<=0)
sum(w<=0)/length(w)']';
portfolio(t).char=N*X'*w;
portfolio(t).ret=retp;
portfolio(t).retpmk=retm;
portfolio(t).retpmanaged=(X*theta)'*ret;
portfolio(t).N=N;

GAMMA = 5;
util(t) = (1+portfolio(t).ret)^(1-GAMMA)/(1-GAMMA);

end;
f=-mean(util);

end

Section 5: Set Up Optimization Algorithm, Constraint, and Find the Optimal Coefficients.

global datastructCopy
datastructCopy = datastruct;
theta0=ones(4,1);
A=-datastruct(t).X;
b=datastructCopy(t).weight_lag;
options=optimset('Algorithm','interior-point','TolFun',.0001);
[theta_opt,minvalue]=fmincon(@(calc,theta0,A,b,[],[],[],[],[],options)
Bibliography:


