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**The Reissner-Sagoci problem for a transversely isotropic half-space**

by M. Rahimian, A.K. Ghorbani-Tanha and M. Eskandari-Ghadi,

IJNAMG 30 (11), 1063-1074, 2006

Discussion by

Eduardo Kausel

Professor, Dept. of Civil Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139

The writers of this paper revisited the classical problem of torsional oscillations of a rigid, circular footing welded to an elastic half-space, and provided an extension to a medium with transverse isotropy, but restricted to purely static loads. As it turns out, not only is this limitation unnecessary, but the complete derivation can be fully reduced into the same form as that of the classical problem. Thus, a dynamic solution to the transverse isotropy problem follows directly from that of the isotropic problem. It suffices for this purpose to scale appropriately the spatial coordinates.

Using the same notation of the writers' paper, the dynamic counterpart to the static equation (4) for torsional oscillations in a transversely isotropic solid is

$$\rho \frac{\partial^2 u_\theta}{\partial t^2} - \mu \left[ \frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} \right] - \mu' \frac{\partial^2 u_\theta}{\partial z^2} = b(r, z, t) \quad (25)$$

which is subjected to the same boundary conditions as in the paper. In this expression,  $\rho$  is the mass density,  $t$  is the time,  $b$  are the body loads, and all other quantities are as in the original paper. While the body loads do not exist for the problem at hand, we include them here to illustrate the effects of the revised formulation on such loads. We define next the scaled coordinates  $r = \lambda_r \tilde{r}$  and  $z = \lambda_z \tilde{z}$ , in which the  $\lambda_r, \lambda_z$  are stretching factors to be determined. Introducing these into equation (25), and multiplying by  $\lambda_r \lambda_z$ , we obtain

$$\lambda_r \lambda_z \rho \frac{\partial^2 u_\theta}{\partial t^2} - \mu \frac{\lambda_z}{\lambda_r} \left[ \frac{\partial^2 u_\theta}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial u_\theta}{\partial \tilde{r}} - \frac{u_\theta}{\tilde{r}^2} \right] - \mu' \frac{\lambda_r}{\lambda_z} \frac{\partial^2 u_\theta}{\partial \tilde{z}^2} = \lambda_r \lambda_z b(\tilde{r}, \tilde{z}, t) \quad (27)$$

We define the equivalent isotropic shear modulus  $\tilde{\mu}$

$$\tilde{\mu} = \mu \frac{\lambda_z}{\lambda_r} = \mu' \frac{\lambda_r}{\lambda_z} \quad (28)$$

This implies

$$\frac{\lambda_z}{\lambda_r} = \sqrt{\frac{\mu'}{\mu}} \quad (29a)$$

$$\tilde{\mu} = \sqrt{\mu \mu'} \quad (29b)$$

On the other hand, the two shearing stresses involved in this problem are

$$\begin{aligned}\sigma_{r\theta} &= \mu \left( \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right) = \frac{\mu}{\lambda_r} \left( \frac{\partial u_\theta}{\partial \tilde{r}} - \frac{u_\theta}{\tilde{r}} \right) = \frac{1}{\lambda_r} \sqrt{\frac{\mu}{\mu'}} \tilde{\mu} \left( \frac{\partial u_\theta}{\partial \tilde{r}} - \frac{u_\theta}{\tilde{r}} \right) \\ &= \frac{1}{\lambda_r} \sqrt{\frac{\mu}{\mu'}} \tilde{\sigma}_{r\theta}\end{aligned}\quad (30a)$$

$$\begin{aligned}\sigma_{z\theta} &= \mu' \frac{\partial u_\theta}{\partial z} = \frac{\mu'}{\lambda_z} \frac{\partial u_\theta}{\partial \tilde{z}} = \frac{1}{\lambda_r} \tilde{\mu} \frac{\partial u_\theta}{\partial \tilde{z}} \\ &= \frac{1}{\lambda_r} \tilde{\sigma}_{z\theta}\end{aligned}\quad (30b)$$

At this point, we choose arbitrarily  $\lambda_r = 1$  (i.e. no scaling of the radial coordinate), a choice that in the presence of layered soils (which is not the case herein) would guarantee the physical compatibility of any layer interfaces. Hence, we reduce the problem into the fully isotropic form

$$\tilde{\rho} \frac{\partial^2 u_\theta}{\partial t^2} - \tilde{\mu} \left[ \frac{\partial^2 u_\theta}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial u_\theta}{\partial \tilde{r}} - \frac{u_\theta}{\tilde{r}^2} - \frac{\partial^2 u_\theta}{\partial \tilde{z}^2} \right] = \tilde{b}(\tilde{r}, \tilde{z}, t) \quad (31a)$$

$$\sigma_{r\theta} = \sqrt{\frac{\mu}{\mu'}} \tilde{\sigma}_{r\theta} \quad (31b)$$

$$\sigma_{z\theta} = \tilde{\sigma}_{z\theta} \quad (31c)$$

$$\tilde{\rho} = \sqrt{\frac{\mu'}{\mu}} \rho \quad (31d)$$

$$\tilde{b} = \sqrt{\frac{\mu'}{\mu}} b \quad (= 0) \quad (31e)$$

Furthermore, both the scaled shearing stresses below the rigid footing at the surface  $\tilde{\sigma}_{z\theta}(\tilde{r}, \tilde{0}, t) \equiv \sigma_{z\theta}(r, 0, t)$  and the displacements  $u_\theta(\tilde{r}, \tilde{0}, t) \equiv u_\theta(r, 0, t)$  are identical to those in the original problem, and the scaled radius of the footing remains unchanged i.e.  $\tilde{a} \equiv a$ . It follows that the entire problem of torsional oscillations of a disk welded to an elastic, transversely isotropic half-space emanates directly from the original Reissner-Sagoci formulation for an isotropic medium. It suffices to use the equivalent shear modulus of eq. 29b, and the equivalent mass density in equation 31d. These imply in turn an equivalent shear wave velocity

$$\tilde{\beta} = \sqrt{\frac{\tilde{\mu}}{\tilde{\rho}}} = \beta_r \quad (32)$$

Finally, to locate points and evaluate stresses or displacements within the half-space, we must use the mapping of scaled to physical coordinates, namely  $\tilde{r} = r$  and  $\tilde{z} = z\sqrt{\mu/\mu'}$ .