Mode Conversion Current Drive
Experiments on Alcator C-Mod

Alexandre Parisot

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Mode Conversion Current Drive Experiments
on Alcator C-Mod
by
Alexandre Parisot
Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of
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Abstract

In tokamak plasmas with multiple ion species, fast magnetosonic waves (FW) in the Ion Cyclotron Range of Frequency can mode convert to shorter wavelength modes at the Ion-Ion hybrid layer, leading to localized electron heating and current drive. Due to $k ||$ upshifts associated with the poloidal magnetic field, only small net driven currents were predicted from mode converted Ion Bernstein Waves (IBW). As studied first by Perkins, and later confirmed experimentally with Phase Contrast Imaging measurements on Alcator C-Mod, poloidal field effects can also lead to mode conversion to Ion Cyclotron Waves (MCICW), on the low field side of the mode conversion layer. In this thesis, mode conversion current drive in the ICW-dominated regime is studied numerically and through experiments on Alcator C-Mod. Solving a dispersion relation for the mode converted waves in a slab geometry relevant to tokamak equilibria and in the finite Larmor radius limit, we find that mode conversion to Ion Cyclotron Waves is ubiquitous to high temperature conventional tokamaks, as a result of the central value for the safety factor $q_0 \sim 1$. MCICWs are identified as kinetically modified Ion Cyclotron Waves in the regime $\omega/k || \psi_{the} < 1$. Full wave simulations with the TORIC code predict net currents can be driven by MCICW as a result of up-down asymmetries in the mode conversion process. Initial estimates with the Ehst-Karney parametrization indicated up to $\sim 100 \text{ kA}$ could be driven for 3 MW input power in C-Mod plasmas. More accurate calculations, consistent with the polarization of MCICWs, were carried out by importing a quasilinear diffusion operator build from the TORIC fields in the Fokker-Planck code DKE, and predicted lower current drive efficiencies by a factor of $\sim 2$.

The TFTR discharges in 1996 where net MCCD currents were inferred experimentally from loop voltage differences were simulated with TORIC, which indicates mode conversion to ICW can account for the driven currents. Similar loop voltage experiments in $^3\text{He}$ plasmas were attempted on Alcator C-Mod, but did not yield conclusive current drive measurements. The lack of control over $Z_{\text{eff}}$ in C-Mod, which is illustrative of ICRF operation in tokamaks with metallic walls, makes reaching optimal plasma conditions for MCCD difficult, and limits the range of parameters in which MCCD can be useful as a net current drive tool in C-Mod. Solving the current diffusion equation in the cylindrical limit and with sawtooth reconnection models, the large sawtooth oscillations in C-Mod plasmas were also found to complicate current relaxation and hinder the loop voltage analysis for small central driven currents inside the $q = 1$ surface.
In separate experiments on Alcator C-Mod, sawtooth period changes were used to infer localized MCCD near the \( q = 1 \) surface. The mode conversion layer was swept outward through the \( q = 1 \) surface in \( D(\text{He}) \) plasmas, and the sawtooth period was found to vary from 3 to 12 ms, which is consistent with localized current drive and TORIC predictions. A similar evolution was found in heating and co-current drive phasing, which suggests net currents are driven with a symmetric antenna spectrum, as predicted by TORIC as a result of asymmetries in the mode conversion process. Simulations of the sawtooth cycle with the Porcelli trigger model indicate that TORIC currents can account for the sawtooth period evolution in heating phasing. Based on simulations of the sawtooth cycle with the Porcelli trigger model, localized electron heating, which could also explain the experimental results, was found not to be dominant compared to the current drive effect. The experimental results demonstrate that, while not optimal, MCCD can be used for sawtooth control.

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Notations

Special care is taken in the thesis about the consistency of notations and definitions. Tables below summarize the symbolic convention throughout the document. Some symbols are used to denote different quantities, in which case the context and dimensions should remove any ambiguity. Temporary use of symbols outside or beyond these conventions, when required by calculations or reference to original work, will be clearly indicated.

The International System of units (SI) is used in all formulas. Conventions for constants follow from the Plasma Physics Formulary [NRL 06].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning of subscript or symbol</th>
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<tr>
<td>$\alpha$</td>
<td>any plasma species</td>
</tr>
<tr>
<td>$e$</td>
<td>electron</td>
</tr>
<tr>
<td>$i$</td>
<td>any ion species</td>
</tr>
<tr>
<td>$p$</td>
<td>proton (or hydrogen ion)</td>
</tr>
<tr>
<td>$\cdot \times$</td>
<td>Vector, scalar product, vectorial product</td>
</tr>
<tr>
<td>$\perp, \parallel$</td>
<td>Components perpendicular and parallel to the equilibrium magnetic field</td>
</tr>
<tr>
<td>$r$, $\theta$, $\phi$</td>
<td>Radial, poloidal and toroidal components in tokamak geometry</td>
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<tr>
<th>Symbol</th>
<th>Name</th>
<th>Definition</th>
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<tr>
<td>$\omega$</td>
<td>Wave (angular) frequency</td>
<td></td>
</tr>
<tr>
<td>$\vec{k}$</td>
<td>Wavevector</td>
<td></td>
</tr>
<tr>
<td>$\vec{n}$</td>
<td>Wavenumber</td>
<td>$\vec{n} = \frac{c}{\omega} \vec{k}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
<td>$\frac{2\pi}{\vec{k}}$</td>
</tr>
<tr>
<td>$\vec{E}$</td>
<td>Electric field</td>
<td></td>
</tr>
<tr>
<td>$\vec{B}$</td>
<td>Magnetic field</td>
<td></td>
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### Symbols and Definitions

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<tr>
<th>Symbol</th>
<th>Name</th>
<th>Definition</th>
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<tr>
<td>( W, \mathcal{E} )</td>
<td>Energy</td>
<td></td>
</tr>
<tr>
<td>( P )</td>
<td>Power</td>
<td></td>
</tr>
<tr>
<td>( m_\alpha )</td>
<td>Mass of plasma species ( \alpha )</td>
<td>( m_i = A m_p )</td>
</tr>
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</table>
| \( q_\alpha \) | Charge of plasma species \( \alpha \) (signed) | \( q_i = Z e \)  
\( q_e = -e \) |
| \( T_{(\alpha)}, \ T_\alpha \) | Temperature and effective thermal energy | \( T_\alpha = k_B T_{(\alpha)} \)  
\((k_B \text{ is the Boltzmann constant})\) |
| \( n_\alpha \) | Density |  |
| \( v_{th\alpha} \) | Thermal velocity | \( \sqrt{2 T_{\alpha}/m_\alpha} \) |
| \( \Omega_{\omega\alpha} \) | Cyclotron frequency | \( \frac{q_\alpha B}{m_\alpha} \) |
| \( \omega_{p\alpha} \) | Plasma frequency | \( \frac{n_\alpha q_\alpha^2}{\epsilon_0 m_\alpha} \) |
| \( \rho_\alpha \) | Gyro- or Larmor radius | \( \frac{v_{th\alpha}}{\Omega_{\omega\alpha}} \) |

We acknowledge the widespread use of CGS units in the literature, and their convenience in electromagnetism and plasma physics. The following rules can be used to convert most formulas from SI to CGS in this thesis:

\[
\epsilon_0 \leftrightarrow \frac{1}{4\pi} \quad \mu_0 \leftrightarrow \frac{4\pi}{c^2} \quad \vec{B} \leftrightarrow \frac{1}{c} \vec{B} \tag{1}
\]

Finally, a convention must be taken for representing oscillating (electric and magnetic) fields in complex notations. In this work, we adopt the following form (sometimes called physics convention):

\[
\vec{E} = \text{Re} \left[ \vec{E}_0 \exp i(\omega t - \vec{k} \cdot \vec{r}) \right] \tag{2}
\]

The other convention (sometimes called electrical engineering convention) would have the opposite sign for the argument in the exponential. The most notable difference appears when considering the polarization of the electric field perpendicular to the static magnetic field (right-hand or left-handed), i.e. the sign of \( \epsilon_{xy} \) in the Stix frame.
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW</td>
<td>Alfven Wave</td>
</tr>
<tr>
<td>FLR</td>
<td>Finite Larmor Radius</td>
</tr>
<tr>
<td>ECCD</td>
<td>Electron Cyclotron Current Drive</td>
</tr>
<tr>
<td>ECH</td>
<td>Electron Cyclotron Heating</td>
</tr>
<tr>
<td>FW</td>
<td>Fast magnetosonic Wave</td>
</tr>
<tr>
<td>FWCD</td>
<td>Fast Wave Current Drive</td>
</tr>
<tr>
<td>IBW</td>
<td>Ion Bernstein Wave</td>
</tr>
<tr>
<td>ICCD</td>
<td>Ion Cyclotron Current Drive</td>
</tr>
<tr>
<td>ICRF</td>
<td>Ion Cyclotron Range of Frequencies</td>
</tr>
<tr>
<td>ICW</td>
<td>Ion Cyclotron Wave</td>
</tr>
<tr>
<td>IWL</td>
<td>Inner Wall Limited</td>
</tr>
<tr>
<td>ITER</td>
<td>International Thermonuclear</td>
</tr>
<tr>
<td></td>
<td>Experimental Reactor</td>
</tr>
<tr>
<td>LD</td>
<td>Landau Damping</td>
</tr>
<tr>
<td>MCCD</td>
<td>Mode Conversion Current Drive</td>
</tr>
<tr>
<td>MCEH</td>
<td>Mode Conversion Electron Heating</td>
</tr>
<tr>
<td>MHD</td>
<td>Magneto-Hydro-Dynamics</td>
</tr>
<tr>
<td>PCI</td>
<td>Phase Contrast Imaging</td>
</tr>
<tr>
<td>SCK</td>
<td>Swanson-Colestock-Kabusha</td>
</tr>
<tr>
<td>TFR</td>
<td>Tokamak de Fontenay aux Roses</td>
</tr>
<tr>
<td>TFTR</td>
<td>Tokamak Fusion Test Reactor</td>
</tr>
<tr>
<td>TTMP</td>
<td>Transit Time Magnetic Pumping</td>
</tr>
<tr>
<td>USN</td>
<td>Upper Single Null</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background

1.1.1 Fusion research and tokamaks

One of the most important scientific advances of the 20th century was the understanding of nuclear forces, which hold nuclei together, and its use for extracting vast amounts of energy from matter. In comparison with chemical reactions, which only involve electrostatic forces, nuclear reactions can typically release energies $10^5 - 10^6$ times larger for the same mass of fuel. From the dependence of the nuclear binding energy per nucleon as a function of atomic mass, shown on figure 1.1, two categories of exothermic reactions can be distinguished, involving respectively light and heavy elements.

For heavy elements, energy is released as the nucleus is broken, i.e. through a process of fission. Radioactive decay is a natural process, and much higher reaction rates can be obtained by subjecting the fuel to an influx of neutrons. The reaction products typically contain radioactive elements with very long half-lifes. While a significant fraction of the products can be retreated, the remainder must be stored for very long periods and may therefore constitute a serious environmental hazard. Fission reactions can produce neutrons, which allows the reaction to sustain itself but also makes the reacting fuel prone to explosive runaway situations. If the fission process is to be contained, the conditions in which the reaction occur must therefore be carefully controlled. Despite these inherent limitations, fission energy is now well mastered and has proved economically...
Figure 1-1: Binding energy per nucleon as a function of the atomic mass number A.

viable as a means to generate electricity. For its present use, only modest quantities of radioactive waste have been produced so far, and the fuel is available at acceptable cost despite limited overall resources.

Light elements like hydrogen or its isotopes, on the other hand, are available in much larger quantities. They can generate energy when they combine into heavier nuclei, i.e. through a process of fusion. Fusion reactions are inherently difficult to obtain, since the short range of the nuclear forces requires bringing the two positively charged nuclei at very short distances and thus overcoming the Coulomb repulsion between them. The reaction products are generally non-radioactive, and while their energy is sufficient to activate the surrounding materials, no long lasting waste is produced. Like in fission energy, the control of the environment in which the reactions occur is a central issue, however not in the same sense. In order to obtain sufficient reaction rates and thus energy output, the fuel must be held together as a dense volume without excessive loss of energy, at temperatures of 10 keV or more depending on the reactions. At these temperatures it is fully ionized and forms a plasma, which in the laboratory can only be confined inertially for a short duration or with a special apparatus using magnetic fields. In both cases, reaching these conditions requires a significant energy input. Accordingly, one of the primary constraints for a fusion power plant
will be to extract more power from the reacting fuel than that required to achieve fusion conditions and, if needed, sustain them. This constraint is usually quantified through the power amplification ratio \( Q = \frac{\text{Available fusion power}}{\text{Input power}} \), which must be substantially larger than unity for power generation.

While a fusion-based approach alleviates two of the major issues associated with fission energy - the fuel availability and the treatment or storage of the radioactive waste - and has therefore better long-term prospects for energy generation, it introduces another tremendous challenge, namely the confinement and control of fusion-grade plasmas on Earth. As a result, fusion energy has not yet found commercial and industrial applications, although as we will see below much progress has been accomplished to this end.

**Fusion energy research** - A primary goal of fusion research has been to develop knowledge relevant to this challenge, and more specifically to investigate optimal configurations and regimes given the economical and safety constraints associated with commercial energy generation. To this end, detailed models describing the behavior of fusion plasmas under various confinement schemes have been developed and tested experimentally over the last fifty years. The extrapolation of these models to fusion reactor designs shows encouraging prospects for a possible commercial use in the future. However, the exact specifications of a future fusion power plant, and therefore its economics, are still largely dependent on uncertainties in the models, some of them at a fundamental level, and on engineering issues like the choice of the reactor materials. In other words, fusion energy stands presently as a realizable scheme for power generation, but it is not yet clear how it will compare economically to other approaches. As a clean (little greenhouse gases or waste), renewable (fuel availability is not an issue) and safe (no chain reaction) technique, it possesses unique advantages to support long-term sustainable industrial development, and, as such, is certainly worth investigating. This thesis work contributes modestly to this effort.

Fusion research is also inherently tied to the more fundamental field of plasma physics, from which it borrows much of its formalism. The experiments and findings of fusion research help expand and strengthen our knowledge of plasma physics. While plasmas form over 99% of the matter in the known universe, and are thus always involved to some degree in astrophysical phenomena, stellar and interstellar plasmas are also hard to probe and study. The relevant plasma physics models are often better developed and tested in laboratory experiments. Given their scope, fusion experiments can access extreme plasma regimes and conditions, and are therefore an attrac-
tive environment for plasma physics research, even though the degree of control and reproductibility can sometimes be lesser than in smaller setups. While the present work is more directly placed in the context of fusion research, plasma physics will also be involved to a large extent.

- **Tokamaks and current drive** - The most promising avenue for controlled fusion is, at present, the tokamak configuration, with which this thesis work will be concerned exclusively. In this approach, reviewed extensively in [Wesson 96], the plasma is confined by strong magnetic fields in a doughnut shape, as illustrated on figure 1-2. At zero order, the charged particles in the plasma are held to the magnetic field lines, but additional forces can make them drift across them. If the field was purely toroidal, the centrifugal force would make ions and electrons drift in opposite directions, thus creating a vertical electric field. In turn, the field would induce an outwards $\vec{E} \times \vec{B}$ drift and result in a rapid loss of confinement. In order to obtain an equilibrium, additional fields are therefore required. Among them is the poloidal field, which makes field lines wrap into helical shapes and allow currents along field lines to compensate for the charge imbalance from the curvature drift. In a tokamak, it is generated by toroidal currents flowing in the plasma itself. In present experiments, the plasma current is driven by a toroidal electric field obtained
by ramping the currents in a solenoid inside the central column. This mode of operation, called *inductive* or *ohmic*, is incompatible with continuous operation, since the amount of flux stored in the central solenoid is limited. Therefore, a large amount of research has been devoted to developing alternative current drive techniques, called in contrast *non-inductive*. This thesis work focuses on a particular non-inductive scheme, referred as mode conversion current drive or MCCD, and we will examine its prospects to this end.

Beyond supplying the plasma current for steady-state operation, non-inductive schemes are also beneficial in that they can allow to control and optimize the profile of the toroidal current. The shape of the current profile is a critical factor in ensuring the stability of the plasma. In some situations, current profile control can also be used to reduce transport across field lines, and thus improve the confinement. For net current drive, the main requirement is to generate large net currents with the least input power, so as to avoid dropping Q. Control over the resulting current profile may be limited. At the other end of the spectrum, one can also seek a flexible tool for localized current drive and effective current profile control, even if the power demands are higher. These two requirements are mutually exclusive, and since optimal regimes may require both aspects, it is likely that the sustainment of the plasma current and control of its profile will be best achieved through a combination of several techniques. Hence, it is important to investigate possible current drive schemes in detail in order to understand their capabilities and limitations.

In terms of cross-section and required temperature, the most promising fusion reaction for a future reactor involves two heavy isotopes of hydrogen, deuterium and tritium:

\[
D + T \rightarrow ^4\text{He} [3.5\text{MeV}] + n [14\text{MeV}] \tag{1.1}
\]

The brackets show the kinetic energy of the products. A total of 17.5 MeV is released from the reaction, and large reaction rates can be achieved with plasma temperatures in the 5-20 keV range. A plasma containing 50% D and 50% T will generate enough fusion power for alpha particle heating to sustain its losses if the product of its electron density \(n\), ion temperature \(T\) and confinement time \(\tau_E\) exceeds a threshold value:

\[
nT\tau_E \geq 6 \times 10^{20} \text{m}^{-3}\text{keV s} \tag{1.2}
\]
If the triple product is below this limit, additional heating must be provided in order to maintain the plasma temperature. The plasma current provides significant ohmic heating to the plasma at low temperatures, however it becomes ineffective as the temperature is increased and the plasma resistivity $\eta \propto T^{-3/2}$ drops. The required power must then be injected through auxiliary heating systems, thus lowering the available fusion power.

As can be seen on figure 1-3, significant progress has been made towards ignition. The most advanced tokamak project is, at present, the International Thermonuclear Experimental Reactor (ITER), whose construction will begin shortly in France. It has the basic goal of achieving $Q \gtrsim 10$, and thus to explore the physics of burning plasmas in which the alpha power is the primary heating source.

**Alcator C-Mod** - The experiments reported here have been carried out on the Alcator C-Mod tokamak [Hutchinson 93, Greenwald 05, Marmar 07] at the Massachusetts Institute of Technology. C-Mod is the third tokamak of the Alcator series. It started operating in May 1993, and is a compact, high density (up to $10^{21} m^{-3}$), high field (up to 8 Tesla) and high power density device, thus exploring a unique regime in the world fusion program. Typical parameters for Alcator C-Mod discharges are listed in table 1.1. A drawing of the tokamak is shown on figure 1-4.
Figure 1-4: Cross section of the Alcator C-Mod tokamak.

Table 1.1: Typical parameters of the Alcator C-Mod tokamak

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Design</th>
<th>Typical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius</td>
<td>0.67 m</td>
<td>0.67 m</td>
</tr>
<tr>
<td>Minor radius</td>
<td>0.22 m</td>
<td>0.21 m</td>
</tr>
<tr>
<td>Toroidal field</td>
<td>max 9 T</td>
<td>5.4 T</td>
</tr>
<tr>
<td>Elongation</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Triangularity</td>
<td>.5</td>
<td></td>
</tr>
<tr>
<td>Plasma current</td>
<td>max 2 MA</td>
<td>0.6-1.2 MA</td>
</tr>
<tr>
<td>Average density</td>
<td>$1 - 10 \times 10^{20} m^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Central electron temperature</td>
<td>up to 7 keV</td>
<td></td>
</tr>
<tr>
<td>Pulse duration</td>
<td>Up to 5 s</td>
<td>2 s</td>
</tr>
</tbody>
</table>
1.1.2 ICRF heating and current drive systems

Wave propagation in magnetized plasmas is a well-developed, yet rich and vast field of study within plasma physics. Excellent introductory texts are available on the subject, like [Stix 92] and [Brambilla 98]. The theory is remarkably well established, and allows to develop sophisticated simulation tools. Yet, the variety of wave-related phenomena observed in astrophysics and in laboratory experiments is such that the area is still the object of very active research. Plasma waves have been found to be a useful tool for controlling fusion plasmas and optimizing their performance. Heating and current drive schemes exploit wave-particle interactions in plasmas, which allow local transfer power between the wave and the different species. Their implementation in fusion experiments gives unique opportunities to study these phenomena, and more generally, wave propagation in plasmas, however the use for fusion research comes generally with additional stringent constraints. For instance, the systems must be operated at high power without adverse effects on the overall reactor performance, like impurity or density production. The schemes should also be efficient, so as not to impact the power amplification factor Q, and robust. These requirements have limited the number of systems which can be practically installed in a tokamak reactor, and accordingly, in present day large-size experiments [ITER 99].

This thesis focuses on heating and current drive systems in the Ion Cyclotron Range of Frequencies (ICRF), i.e. at frequencies close to the ion cyclotron frequency of the main ion species, typically 10-100 MHz for 1-5 T magnetic fields. In this range, fast magnetosonic waves (see chapter 2) can be excited at the plasma periphery with large antenna structures. Figure [1-5] shows one such antenna [Wukitch 02, Wukitch 04] in the Alcator C-Mod tokamak. RF currents flow poloidally (vertical direction in the image) in the four copper-coated straps, directly in front of the plasma. The horizontal rods form a Faraday shield, which is generally found to improve antenna operation. The antenna is surrounded by a metallic limited box, which, together with the Faraday screen, limits plasma influx near the straps. The system routinely couples 3 MW to the plasma, with minimal impurity and density production. The straps are numbered 1 to 4 in the same toroidal direction as for the labeling of the horizontal ports, and are operated with set phasing relative to one another. This determines the toroidal spectrum of the coupled fast waves. In this thesis, the strap pairs 1-3 and 2-4 are fed with fixed phasing (0, π). The relative phasing between straps 1 and 2 (and thus 3 and 4) can be controlled at the source. This allows to change from three different
phasings \((0,\pi,\pi,0), \,(0,\pi/2,\pi,-\pi/2)\) and \((0,-\pi/2,\pi,\pi/2)\) between discharges. For the latter two phasings, the fast wave spectrum is toroidally asymmetric. The antenna is powered by two high power RF transmitters FMIT #3 and #4, delivering up to 2 MW source in the 40-80 MHz range. High power operation of this antenna at 50 MHz, 70 MHz and 78 MHz has been achieved to date in C-Mod, up to 3 MW. The power is carried to the antenna by coaxial transmission lines.

In addition to the antenna system described above, which is localed at the J horizontal port, two other antennas are installed at D and E horizontal ports. They are operated at 80.5 and 80 MHz respectively, up to 2 MW. An recent overview of the ICRF system in C-Mod can be found in [Bonoli 07].

The primary auxiliary heating scenario in Alcator C-Mod uses the fast wave antennas at 78-80.5 MHz for D(H) minority heating with toroidal fields in the 5-6 T range. As we will see in chapter 2, this approach allows efficient heating of the hydrogen ions at their fundamental cyclotron layer, and subsequently of the bulk plasma through collisions. Varying the toroidal field, plasma composition and antenna frequency, other scenarios can be investigated. Several of these schemes have been deemed suitable candidates for current drive applications: Ion Cyclotron Current Drive
(ICCD), from minority heating, Fast Wave Current Drive (FWCD), from direct fast damping on electrons, and Mode Conversion Current Drive (MCCD), in which electron heating results from mode conversion of fast waves to shorter wavelength modes at the ion-ion hybrid resonance layer (see chapter 2).

This thesis is an experimental and numerical study of this mode conversion process and of its potential use as a current drive tool. The physics of mode conversion and MCCD will be discussed in detail, based on the recent experimental and numerical results. The prospects of MCCD for net current drive and localized current control will be illustrated with experiments on Alcator C-Mod and simulations. Much of the motivation for this work results from prior work on the topic, which we will review in the following section.

1.2 Prior work on mode conversion and MCCD

1.2.1 Prior mode conversion results

- Mode conversion electron heating - From an experimental point of view, ICRF injection in the mode conversion regime is associated with strong electron heating in the vicinity of the mode conversion layer ($S = n_2^2$ in the Stix notations). This has been observed in several tokamak experiments, including Alcator C-Mod.

Efficient heating in the mode conversion regime was first demonstrated in the TFR tokamak using high field side antennas [Swanson 85]. The results were explained theoretically by solving a wave equation obtained for the fast wave in the cold plasma limit and in a one-dimensional geometry [Swanson 76, Jacquinot 77]. As we will see in chapter 2, the wave equation can be put in the form similar to that obtained by Budden [Budden 61] in his studies of wave propagation in ionospheric plasmas. When applied to mode conversion at the ion-ion hybrid layer, the Budden formalism predicts complete absorption of fast waves incident from the high field side.

For low field side incidence however, the Budden formalism predicts lower mode conversion efficiencies, with a maximum value of 25%. This is due to the presence of the left-hand cutoff ($L = n_2^2$ in the Stix notation) immediately on the low field side of the mode conversion layer. In the region between the two layers the fast wave is evanescent. The corresponding tunneling distance is the key parameter in this model. If it is too large, the fast wave cannot tunnel through
the evanescent region and is reflected towards the low field side. If it approaches zero, the wave is entirely transmitted and very little power is absorbed. The mode conversion efficiency peaks at 25% between these limits. This asymmetry in the Budden formalism was used as a possible explanation of the poorer antenna performance with low field side incidence compared to high field side in TFR [TFR group 84, Swanson 85].

Majeski et al. [Majeski 94] proposed an experimental scenario with low field side incidence with higher mode conversion efficiencies than the Budden prediction. The scenario relies on the right-hand cutoff layers \( R = n^2 \) in the Stix notation, usually at the plasma boundary, which makes the plasma interior act as a cavity for the fast wave. This property is a key aspect of the fast wave antenna coupling to plasmas, as the wave must tunnel from an outside antenna to the boundaries of this cavity. For mode conversion with low field side incidence, the high field side cutoff layer reflects the transmitted power back towards the mode conversion layer. This multipass situation can result in larger mode conversion efficiencies than for a single pass as in the Budden problem. The characteristic distances of the resonator and of the evanescent region associated with the mode conversion layer are largely dependent on \( k \). For large \( k \) values, the layers are in close proximity to each other and form a cutoff-resonance-cutoff triplet with small evanescent distance and an adjustable high field side resonant length. This is the scheme proposed in [Majeski 94].

Later studies recognized that the resonator must be studied in terms of wave constructive and destructive interference rather than a multipass picture. The internal resonator model proposed by Ram et al. [Ram 96] is based on solving the wave equation in the fast wave approximation (cold plasma model, zero electron mass) accounting for the cutoff-resonance-cutoff triplet. Mode conversion efficiencies of 100% can be achieved for particular parameters in this model. The global resonator model [Monakhov 99] follows the same approach but includes the low field side right-hand cutoff. The geometry is one-dimensional in these models.

A series of experiments in different tokamaks validated the principle of low field side Mode Conversion Electron Heating (MCEH) and showed that total power absorbed in excess of the Budden prediction can indeed be obtained. In these experiments, the electron deposition profiles are measured using modulation or switch-on/switch-off techniques.

In Tore Supra [Saoutic 96], strong electron heating could be obtained in H, ^{3}\text{He} plasmas and its location could be varied by changing the magnetic field, which rules out direct fast-wave electron
heating.

In TFTR [Majeski 95], with the mode conversion layer at the magnetic axis at 4.8 T in D(3He) plasmas, injecting 4 MW of ICRF power at 43 MHz resulted in central electron temperatures above 10 keV. This was the highest electron temperature achieved with RF alone in TFTR (minority heating experiments only achieved 8 keV). The density was $5 \times 10^{19} m^{-3}$ and the current 1.7 MA. For $^3$He concentrations higher than 15 %, up to 70-80 % of the coupled power was found in electron heating.

In ASDEX-Upgrade [Noterdaeme 96] [Noterdaeme 99], localized electron heating was clearly observed in H, $^3$He plasmas. Using a scan of $^3$He concentration, the transition from the mode conversion to the minority heating regime was investigated.

On Alcator C-Mod, highly localized electron heating was reported with on and off-axis mode conversion scenarios [Takase 97] [Bonoli 97]. In H-$^3$He and D-$^3$He plasmas, the electron heating efficiency was over 50 %, while it was considerably lower in D-H plasmas ($< 25 %$). The on-axis discharges were in H-$^3$He plasmas with a central magnetic field in the 6-6.5 T range. The off-axis discharges were in D-$^3$He plasmas at 7.9 T. The peak heating location could be controlled by changing the magnetic field and $^3$He concentration, in agreement with the expected location of the mode conversion layer. A comparison of the electron heating efficiency with the Budden, multipass and internal resonator predictions showed [Nelson-Melby 01] that the latter model was the most appropriate to study mode conversion heating in Alcator C-Mod.

- **Mode conversion current drive results** - The strong localized electron heating obtained in the mode conversion regime readily suggests possible current drive applications. If the $k_\parallel$ spectrum of the mode converted waves can be made asymmetric, the wave power would be transferred to the electron distribution function in an asymmetric manner, thus resulting in net driven currents. The underlying physics will be discussed in more detail in chapter 3.

Building upon their prior MCEH results, the TFTR group investigated mode conversion current drive in the 1995 TFTR campaign [Majeski 96b] [Majeski 96a]. Two-strap antennas were used to inject up to 4 MW at 43 MHz in -90 and +90 degrees phasings in D,$^4$He,$^3$He plasmas. Typical parameters were as follows. The central density was $n_e(0) = 4 \times 10^{19} m^{-3}$, the toroidal field was varied between 3.5 and 4.8 T, the plasma current was $\sim 1.4 - 1.7 MA$ and the species mix such that $n_{^3He}/n_e \sim 0.2$ and $n_{^4He}/n_e \sim 0.15$. Efficient mode conversion electron heating was obtained
(typically 60 % of the input power was found in electron heating). Clear changes in loop voltage between co and counter-current drive phasings were reported (see Figure 1-6), indicating driven currents up to 130 kA for 3.8 MW input power. These results are very encouraging, and constitute one of the primary motivations for this thesis work.

**Mode conversion to Ion Bernstein Waves** - The interpretation of mode conversion current drive results depends more critically on the nature of the mode conversion waves than for mode conversion electron heating. The dispersion relations used in the models mentioned above only include fast waves with exclusion of any other wave. No explicit absorption mechanism like cyclotron damping or Landau damping is considered. The mode conversion efficiency predictions are based only on resonant absorption at the ion-ion hybrid layer. Yet those models are used to determine experimental scenarios and interpret the results of mode conversion heating experiments. Since good agreement is generally obtained on the location of the electron heating and the mode conversion efficiency, one may conclude that the propagation and absorption characteristics of the mode converted waves are not critical in determining these quantities. For current drive applications however, it is essential to account for the propagation of the mode converted waves and the evolution of their \( k_{\parallel} \) spectrum, since, as we will see in chapter 3, the current drive efficiency depends critically on the parallel phase velocity \( \omega_{\parallel} / k_{\parallel} \).

Figure 1-6: Loop voltage traces and other plasma parameters in the TFTR MCCD experiments. From [Majeski 96b]
plasma model is insufficient and hot plasma corrections must be taken into account. We will discuss some of these corrections in chapter 2. In interpreting the mode conversion electron heating results, the most widely used paradigm has been models of mode conversion from fast waves to Ion Bernstein Waves (IBW). Bernstein predicted the existence of this hot plasma mode in [Bernstein 58], and its existence and propagation characteristics have since been widely confirmed in experiments (see, for example, [Ono 93] for a review).

An experimental confirmation of the FW → IBW mode conversion process was obtained using far-infrared laser scattering measurements. In Microtor [Lee 82], wave-induced density fluctuations were observed on the high field side of the mode conversion layer in D(H) plasmas. The fluctuations were attributed to Ion Bernstein Waves. The spatial dependence and magnitude of the radial wavenumber were in very good agreement with theoretical predictions from IBW dispersion curves. Fluctuations were also observed on the low field side of the mode conversion layer and tentatively attributed to a modified slow Alfvén wave. In TFR [TFR Group 82], fast and slow modes were observed in the vicinity of the mode conversion layer using similar far-infrared techniques and probe measurements in D(H) plasmas. A similar experiment was done in TNT-A [Ida 84] and yielded wavenumbers in very good agreement with the dispersion relation for the fast wave and Ion Bernstein Waves. Radial shifts in the location of the signals were in good agreement with the expected dependence of the mode conversion layer location with the magnetic field and plasma composition.

- **Current drive with mode converted IBWs** - As in MCEH experiments, the MCCD results in TFTR were also interpreted using FW → IBW mode conversion models. The analysis used the one-dimensional full wave code FELICE, which solves the wave equation in the Swanson-Colestock-Kabuska limit (see [Brambilla 99] and references therein) in a plane stratified geometry. The Ehst-Karney parametrization [Ehst 91], which we will present in chapter 3, was used to deduce the current drive efficiency associated with electron damping of the mode converted waves. This prediction was shown to be in good agreement with experimental values over a wide range of parameters, as shown in Figure 1-7.

The agreement between experiments and simulations in TFTR was very encouraging. However, when more complicated models attempted to treat the propagation of the mode converted waves in more detail, simulations predicted considerably less driven currents than in the one-dimensional
FW → IBW models. Ray tracing simulations by Ram and al. [Ram 91] showed that rapid upshifts in $k_\parallel$ occur in the presence of a poloidal field as the wave propagates away from the mode conversion layer outside the midplane. In toroidal geometry, $k_\parallel \approx B_\phi k_\phi + B_\theta k_\theta$, where $k_\phi, k_\theta$ are the wavenumbers in the toroidal and poloidal directions, and $B_\phi, B_\theta$ are the toroidal, poloidal component of the magnetic field $B$. As the IBW propagates, the intrinsic asymmetry in the poloidal field creates an asymmetry in $k_\parallel$ above and below the midplane, and therefore the initial $k_\parallel$ spectrum imparted by the fast wave antenna can be partly lost if $B_\theta/B_\phi$ is large enough. This situation was also encountered with direct IBW launch [Petrov 94], and ray tracing simulations predicted only negligible net current drive when the IBW launchers are poloidally symmetric with respect to the midplane. Large upshifts in $k_\parallel$ were also encountered in the context of alpha channeling [Valeo 94]. Simulations showed the possibility of a reversal in the sign of $k_\parallel$ ($k$-flip) as IBW propagates. This constitutes one of the key properties which could make, according to the authors, Ion Bernstein Waves usable for alpha channeling but makes the current drive applications more complicated. Following the TFTR results, Ram et al. [Ram 96] indicated that, with very strong electron absorption in the close vicinity of the mode conversion layer, current drive by MCIBW could still be achieved, since the wave would damp before the $k_\parallel$ reversal occurs. This scheme makes high $k_\parallel$ fast wave launch desirable, in accordance with the prescriptions in [Majeski 94]. More recent ray tracing studies by M. Brambilla [Brambilla 00] focused on diffraction effects for the mode converted IBWs. They showed that diffraction causes large upshifts in $k_\theta$ and thus spreads the $k_\parallel$ spectrum if the IBW wave field is symmetric above and below the midplane. The resulting net driven currents are again very small. The concurring
outcome of these studies is that net current drive from Ion Bernstein Waves (IBW) is unlikely, or
that at best the efficiency is strongly reduced by 2D wave propagation effects in the presence of a
poloidal field. This result is in apparent contradiction with experimental MCCD results in TFTR.

1.2.2 Recent results on Alcator C-Mod

The recent installation of a Phase Contrast Imaging (PCI) diagnostics [Mazurenko 01] on the
Alcator C-Mod tokamak has allowed a much more detailed study of mode conversion at the ion-ion
hybrid layer than in prior experiments. The PCI diagnostics allows a more complete mapping of the
mode conversion region than in far-infrared scattering, and better spatial and spectral resolution.

■ Mode conversion to Ion Cyclotron Waves - Mode conversion experiments in (H-D-\(^3\)He) plas-
mas were able to detect mode converted waves with the PCI setup [Nelson-Melby 03]. The plasma
composition was \(\sim 60\%\) H, \(\sim 4\%\) \(^3\)He and 33 % D. The central density was \(n_e = 2.0 \times 10^{20} m^{-3}\)
and \(T_e = 1.3\) keV. Strong wave-induced density fluctuations were observed on the low field side
of the mode conversion layer and could not be attributed to Ion Bernstein Waves since the wave
can only propagate on the high field side of it. The measured wave numbers were in the range
from 4 to 10 cm\(^{-1}\). Further analysis showed the results could be understood with an alternative
model of mode conversion by Perkins [Perkins 77]. The model predicts that another mode con-
version scheme can dominate in the presence of a poloidal field. In this scheme, the fast wave
can mode-convert to Ion Cyclotron Waves (ICW) propagating on the low field side of the mode
conversion layer. Poloidal field effects were taken into account in some subsequent publications
like [Brambilla 85, Faulconer 89], but were not used in the analysis of the mode conversion heat-
ing experiments mentioned above. The comparison between the Perkins model and experimental
observations indicated that the wave fluctuations observed by PCI can be attributed to mode con-
verted Ion Cyclotron Waves. This implies that ICRF mode conversion in tokamak plasmas is more
complicated than the FW \(\rightarrow\) IBW model would suggest. Since IBWs were not observed in these
plasmas, one may conclude that C-Mod can operate in a MCICW-dominated regime, however it
was not clear how these results extend to other tokamaks.

■ TORIC simulations and synthetic PCI - In parallel with the installation of the PCI on Al-
cator C-Mod, better numerical tools were developed to treat mode conversion in a 3D tokamak
(axisymmetric) geometry. The numerical code TORIC [Brambilla 99], presented in section 2.3,
is capable of solving the wave equation in the same limit as FELICE, but takes into account the full magnetic equilibrium and therefore poloidal field effects, like upshifts in $k ||$. Detailed comparison between the code predictions and experimental measurements on C-Mod were carried out and showed very good agreement. In the original experiment, the predicted electric fields agreed qualitatively with PCI observations \cite{Nelson-Melby 03, Nelson-Melby 01}, however only limited resolution could be used at that time in the simulations. Separate experiments in D(H) plasmas showed that experimentally measured deposition profiles matched well with TORIC predictions \cite{Lin 03}. The code has been recently parallelized \cite{Wright 04a}, and has been upgraded to import experimental magnetic equilibria. Subsequent PCI measurements in D($^3$He) plasmas were compared with TORIC simulations at higher resolution, in which the mode converted waves could be fully resolved \cite{Lin 05}. The simulations used synthetic PCI diagnostics routines to simulate the PCI measurements from the fields calculated by TORIC. The routines were originally used in \cite{Nelson-Melby 03, Nelson-Melby 01} and were reprogrammed for use with numerical equilibria and with the latest parallel versions of TORIC. Comparing the PCI results in experiments and simulations showed excellent agreement in the spatial structure and radial spectrum, both for off-axis scenarios at 5.4 T where MCICW dominate and for near-axis mode conversion scenarios at 8 T where IBWs dominate in C-Mod.

- **Initial MCCD calculations with TORIC** – Current drive calculations from the electric fields in TORIC were implemented in the late 1990s \cite{Bonoli 00} using the Ehst-Karney parametrization. The package was originally written to estimate the driven currents from direct fast wave electron damping and from mode converted Ion Bernstein Waves, with the Alfven Wave damping and Landau damping Ehst-Karney formulas respectively (see chapter 3). This approach can also be used to estimate the driven currents in TORIC simulations with mode converted Ion Cyclotron Waves, using the Ehst-Karney formula for the fast waves. With this procedure, a scenario with 100 kA predicted MCCD currents was found in simulations of Alcator C-Mod plasmas. The numerical results were reported in \cite{Wukitch 05}, and will be shown in chapter 4. This target scenario, along with the prior MCCD experiments in TFTR, suggests significant net currents can be driven by mode converted waves, and forms a good basis for MCCD experiments in Alcator C-Mod.
1.3 Scope and outline

The recent mode conversion results on Alcator C-Mod and improved numerical simulation tools constitute a strong motivation for further MCCD studies. This is the objective of this thesis. The emphasis here is on the regime where poloidal field effects determine the $k_\parallel$ spectrum evolution, and where mode conversion to Ion Cyclotron Waves dominates. The thesis will study both net and localized current drive in this regime, through experiments in C-Mod, simulations and basic theoretical arguments.

In chapter 2, we show how the proper treatment of poloidal field effects in models and simulations allows to draw a consistent picture of ICRF mode conversion in tokamaks. The original work of Perkins [Perkins 77] is revisited and extended. Dispersion relations for the mode converted Ion Bernstein and Ion Cyclotron waves are derived in a simple slab geometry relevant to the tokamak equilibrium, capturing the effect of $k_\parallel$ upshifts induced by the poloidal field. The transition from FW $\rightarrow$ IBW to FW $\rightarrow$ ICW mode conversion is studied, allowing to characterize the regimes when ICW-dominated mode conversion can be expected. This model is shown to be consistent with full wave TORIC simulations. The discussion is preceded by a short review of fast wave propagation in tokamak plasmas.

Chapter 3 focuses on the MCCD efficiency. The physics of current drive in tokamaks and several techniques for calculating the current drive efficiency are briefly reviewed, in the context of MCCD. We show how asymmetries in the TORIC wavefields can lead to significant net currents when estimated with the Ehst-Karney parametrization. These estimates are compared with more accurate calculations in which a quasilinear diffusion operator is build from the electric fields in TORIC and imported in a Fokker-Planck code. We determine if the assumptions on the wave polarization in the Ehst-Karney parametrization are applicable to MCICW. As an illustration, TORIC simulations for the TFTR MCCD experiments are shown and discussed.

Chapters 4 and 5 report on experiments aimed at measuring net MCCD from loop voltage differences on Alcator C-Mod. Despite the original 100 kA prediction and significant efforts to achieve optimal conditions in the experiments, no conclusive measurements of MCCD from loop voltage differences were obtained. This is attributed to the limited MCCD efficiency and the lack of control over $Z_{\text{eff}}$ in an all-metal machine. A model of current diffusion is also implemented to study the effect of sawtooth oscillations on the loop voltage.
Chapter 6 presents experiments on sawtooth control by MCCD. By sweeping the mode conversion through the $q = 1$ surface, significant changes in the sawtooth period were observed, in heating, co and counter current drive phasing. The evolution is shown to be consistent with localized MCCD. A similar behavior is obtained in the heating and co-current drive phasing, which can be explained either by the asymmetric MCCD profiles or by localized electron heating. In order to distinguish between the two effects, a sawtooth trigger model is introduced, and allows simulations of the sawtooth cycle with localized heating and current drive. The numerical results are shown to support the conclusion that the sawtooth period changes in heating phasing can be attributed to MCCD.

Finally, chapter 7 summarizes the results, contributions and conclusions from this work, and gives suggestions for future work.
Chapter 2

ICRF physics and mode conversion

2.1 Fast wave physics

2.1.1 Cold plasma formalism

The cold plasma model is a good starting point to study the propagation of ICRF waves in tokamak plasmas. The perpendicular wavelength is assumed to be much larger than the ion and electron gyroradii ($\lambda_\perp \ll \rho_e, \lambda_\perp \ll \rho_i$ or equivalently $k_\perp \rho_i \ll 1$, $k_\perp \rho_e \ll 1$), and the parallel phase velocity $\frac{\omega}{k||}$ is assumed to be large compared to the electron and ion thermal velocities. When these assumptions hold, the local dielectric tensor of the plasma can be approximated by the cold plasma dielectric tensor, written here in the Stix notation:

\[
S = \epsilon_{xx} = 1 - \sum_\alpha \frac{\omega_{pa}^2}{\omega^2 - \Omega_{ca}^2} \quad P = \epsilon_{zz} = 1 - \sum_\alpha \frac{\omega_{pa}^2}{\omega^2} \quad D = i \epsilon_{xy} = -i \epsilon_{yx} = \sum_\alpha \frac{\omega_{pa}^2}{\omega^2 - \Omega_{ca}^2} \frac{\Omega_{ca}}{\omega} \quad (2.1)
\]

\[
P = \epsilon_{xz} = 1 - \sum_\alpha \frac{\omega_{pa}^2}{\omega^2} \quad \epsilon_{xx} = \epsilon_{zz} = 0 \quad \epsilon_{yz} = -\epsilon_{zy} = 0 \quad (2.2)
\]

The x-y-z coordinate system refers to the Stix frame, for which z is along $\vec{B}$, x along $k_\perp$ and y is chosen to complete a right-hand orthonormal triplet. Note that the angular cyclotron frequency $\Omega_{ce}$ is negative in the notations adopted here. The wave equation for a uniform, infinite plasma takes
the following form:

\[
\begin{bmatrix}
S - n_\parallel^2 & -iD & n_\parallel n_\perp \\
iD & S - n_\parallel^2 - n_\perp^2 & 0 \\
n_\parallel n_\perp & 0 & P - n_\perp^2
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix} = 0
\] (2.3)

where \( n_\perp = \frac{k_i c}{\omega} \) and \( n_\parallel = \frac{k_e c}{\omega} \) are the perpendicular and parallel components of the index of refraction.

The form of the cold plasma dielectric tensor suggests the introduction of rotating components for the electric field \( E_+ = (E_x + iE_y) \) and \( E_- = (E_x - iE_y) \). The \( E_+ \) component corresponds to a left-handed circular motion for the tip of the electric field when looking at a fixed location in the direction of the static magnetic field. Conversely, the \( E_- \) component corresponds to a right-handed circular motion. In this frame the dielectric tensor becomes diagonal, and the components for \( E_- \) and \( E_+ \) are labeled \( R \) (for right-handed) and \( L \) (for left-handed), with \( R \) and \( L \) such that \( S = \frac{1}{2}(R + L) \) (hence the symbol \( S \) for sum) and \( D = \frac{1}{2}(R - L) \) (hence \( D \) for difference). The parallel component \( P \) is unchanged. Note that the \( E_+ \) component rotates in the same direction as the ions around their guiding center, and the \( E_- \) component rotates in the same direction as the electrons around their guiding center. The notations will therefore be useful when considering wave-particle resonances at the ion and electron cyclotron frequencies.

The corresponding dispersion is the same in both frames, and can be written in the following form:

\[
Sn_\perp^4 - [RL + PS - n_\parallel^2(P + S)] n_\perp^2 + P(n_\parallel^2 - R)(n_\parallel^2 - L) = 0
\] (2.4)

For the purpose of studying ICRF waves in tokamak plasmas, two further simplifications can be made to the cold plasma dielectric tensor.

\( \odot \) The electron to ion mass ratio \( \frac{m_e}{m_i} \ll 1 \) gives the following approximations \( \omega_{pe} \gg \omega_{pi} \) and \( |\Omega_{ce}| \gg \Omega_{ci} \gg \omega \). Tokamak plasmas usually operate in regimes such that \( \omega_{pe} \approx |\Omega_{ce}| \), which gives the ordering \( \frac{\omega_{pe}}{\omega} \sim \frac{\Omega_{ce}}{\omega} \sim \frac{m_e}{m_e} \gg 1 \).

\( \odot \) For typical plasma densities in the core of tokamak plasmas, \( \omega_{pi} \gg \Omega_{ci} \) is also a good
approximation. For a pure hydrogen plasma,

$$\frac{\omega_{pi}^2}{\Omega_{ci}^2} = \frac{n_i m_i}{e_0 B^2} \approx 756 \times \frac{5^2}{10^{20}} \frac{n_i}{B^2}$$  \hspace{1cm} (2.5)$$

with $B$ in the toroidal magnetic field in Telsa and $n_i$ the ion density in $m^{-3}$. $n_i \sim 10^{20}m^{-3}$ and $B \sim 5T$ is typical in medium scale tokamak plasmas, so that the prefactor suggests $\omega_{pi} \gg \Omega_{ci}$. This simplification is equivalent to $v_A \ll c$, where $v_A = c\frac{\Omega_{ci}}{\omega_{pi}}$ is the Alfvén velocity.

For a macroscopically neutral plasma containing a single ion species, the cold plasma dielectric in Stix notation can therefore be approximated as:

$$R = 1 - \frac{\omega_{pi}^2 + \omega_{pe}^2}{(\omega + \Omega_{ci})(\omega + \Omega_{ce})} \quad \rightarrow \quad R \approx \frac{\omega_{pi}^2}{\Omega_{ci}(\omega + \Omega_{ci})}$$

$$S \approx -\frac{\omega_{pi}^2}{\omega^2 - \Omega_{ci}^2}$$

$$L = 1 - \frac{\omega_{pi}^2 + \omega_{pe}^2}{(\omega - \Omega_{ce})(\omega - \Omega_{ce})} \quad \rightarrow \quad L \approx \frac{\omega_{pi}^2}{\Omega_{ci}(\omega - \Omega_{ce})}$$

$$D \approx \frac{\omega_{pi}^2}{\Omega_{ci}^2} \frac{\omega_{pe}^2}{\omega^2 - \Omega_{ci}^2} = -\frac{\omega_{pi}^2}{\Omega_{ci}} S$$

$$P = 1 - \frac{\omega_{pi}^2 + \omega_{pe}^2}{\omega^2} \quad \rightarrow \quad P \approx -\frac{\omega_{pe}^2}{\omega^2}$$

We made use of charge neutrality $\frac{\omega_{pe}^2}{\Omega_{ce}} = -\frac{\omega_{pi}^2}{\Omega_{ci}}$ in writing those approximated expressions. For plasma containing two or more ion species, similar approximations can be generally made, but the resulting expressions for the dielectric tensor elements are more complicated. In general, electrons terms will appear explicitly.

A final simplification consists in taking $P$ to be much larger than the other components, which is justified by $\frac{\omega_{pe}^2}{\omega^2} \sim \left(\frac{m_i}{m_e}\right)^2$. This can be symbolically written as $P \to \infty$ or equivalently $m_e \to 0$, and is therefore referred as zero electron mass approximation. Physically, the electrons in this limit have no perpendicular motion and shield instantaneously electric fields along the static magnetic field.

**MHD waves** - In order to identify the modes propagating in the Ion Cyclotron Range of Frequencies, it is useful to review briefly wave propagation in the cold plasma model for $\omega \ll \Omega_{ci}$, i.e. in MHD range. For a plasma containing a single ion species, to the lowest order in $\omega/\Omega_{ci}$,

$$R \approx L \approx S \approx \frac{\omega_{pi}^2}{\Omega_{ci}^2} \quad D \sim -\frac{\omega}{\Omega_{ci}} S \to 0 \quad P \sim -\frac{\omega_{pe}^2}{\omega^2} \to -\infty$$  \hspace{1cm} (2.6)
In this limit the dispersion relation can be written:

\[
\left(n_{\parallel}^2 - \frac{\omega_{pi}^2}{\Omega_{ci}^2}\right)n_\perp + \left(n_{\parallel}^2 - \frac{\omega_{pi}^2}{\Omega_{ci}^2}\right)^2 = 0
\] (2.7)

which gives:

\[
\left(\frac{\omega_{pi}^2}{\Omega_{ci}^2} - n_{\parallel}^2\right)\left(\frac{\omega_{pi}^2}{\Omega_{ci}^2} - n_{\parallel}^2 - n_\perp^2\right) = 0
\] (2.8)

Introducing the Alfvén speed \( v_A = \frac{c\omega_{pi}}{\Omega_{ci}} \), we recover the two modes from MHD theory in the limit of negligible sound speed \( c_s \rightarrow 0 \):

\[
\omega^2 = k^2 v_A^2 \quad \text{Compressional Alfvén wave}
\]

or fast magnetosonic/magnetoacoustic wave

\[
\omega^2 = k_\parallel^2 v_A^2 \quad \text{Torsional Alfvén wave}
\]

or shear Alfvén wave

When more than one ion species are present with \( \omega \ll \Omega_{ci} \), the dispersion relation can be written in a more general form:

\[
\frac{S}{P}n_\perp^4 - (S - n_{\parallel}^2)n_\perp^2 + (n_{\parallel}^2 - S)^2 = 0
\] (2.10)

which gives:

\[
(S - n_{\parallel}^2)(S - n_{\parallel}^2 - n_\perp^2) = 0
\] (2.11)

with two roots:

\[
S = n_{\parallel}^2 \quad \text{Compressional Alfvén wave}
\]

or fast magnetosonic/magnetoacoustic wave

\[
S = n_\perp^2 \quad \text{Torsional Alfvén wave}
\]

or shear Alfvén wave
ICRF waves - As $\omega$ increases and approaches $\omega_{ci}$ for the ion species, the S and L dielectric tensor elements increase rapidly in magnitude as the ions resonate with the left-handed perpendicular component of the wave electric field $E_+$. The low frequency MHD waves can be strongly modified by cyclotron effects.

(a) Fast magnetosonic wave - The fast magnetosonic branch in ideal MHD transitions continuously through the ICRF for $P \gg S$ (zero electron mass approximation) and $S \neq n_\|$. A dispersion relation taking cyclotron effects into account can be obtained in the zero electron mass limit ($P \to \infty$) [Brambilla 98, p. 222]:

$$n_\perp^2 = \frac{(L - n_\|)(R - n_\|)}{S - n_\|} \quad (2.13)$$

For $\omega \ll \Omega_{ci}$, $R = L = S$ and this reduces to the MHD limit $n_\perp^2 = S - n_\|$. At the fundamental cyclotron resonance ($\omega \to \Omega_{ci}$), $L \to \infty$, implying $S \to \infty$. The dispersion relation then becomes:

$$n_\perp^2 \approx 2(R - n_\|) \quad (2.14)$$

There is therefore no cold plasma resonance ($n_\perp^2 \to \infty$) for the fast magnetosonic wave at the ion cyclotron resonance. This can be understood in terms of the polarization of the wave electric field in the vicinity of the ion cyclotron layer:

$$\frac{E_x}{E_y} = i \frac{D}{S - n_\|} \to -i \quad \text{for} \quad \omega \to \Omega_{ci} \quad (2.15)$$

The wave is therefore right-handed and the electric field tip gyrates in the opposite direction to the ion gyromotion.

In general, the resonant term $L$ will be larger than the non-resonant term $R$, implying $n_\perp^2 \sim 2R$. This order of magnitude for $n_\perp^2$ allows us to rewrite $k_\perp \rho_i$ for a single ion species and $\omega \sim \Omega_{ci}$:

$$k_\perp^2 \rho_i^2 = \frac{\omega^2 n_\|^2 v_{thi}^2}{\Omega_{ci}^2} \sim 2 \frac{v_{thi}^2}{c^2} R \sim \frac{v_{thi}^2 \omega_{pi}^2}{\Omega_{ci}^2} = \beta_i \quad (2.16)$$

where $\beta_i$ is the ion beta, which is much smaller than 1 in tokamaks. Therefore, the condition $k_\perp \rho_i \ll 1$ will be usually satisfied for the fast wave as long as $L \sim S \neq n_\|$. On the other hand,
as we shall see in more details later, \( \frac{\omega}{k} \sim v_{the} \) is typical for the fast wave in present day ICRF scenarios. The kinetic electron response in the direction of the magnetic field, or parallel electron dispersion, must then be taken into account. In the ICRF, the single important modification will be in the parallel component \( P \). Even taking these corrections into account, \( P \) is still large and the zero electron mass approximation holds well, so that the fast wave dispersion relation in equation 2.13 can still be used.

(b) Alfven wave - In the zero electron mass limit \( |P| \gg |S| \), the dispersion relation for the slow wave \( n_1^2 = \frac{S}{P}(S - n_1^2) \) [Brambilla 98, p. 222] should still lead to the same dispersion relation as the Alfven wave as \( \omega \rightarrow \Omega_{ci} \). However, \( S \rightarrow \infty \) in the vicinity of the ion cyclotron resonance, therefore the approximation cannot hold. If parallel electron dispersion can be neglected \( \frac{\omega}{k} \sim v_{the} \), the cold plasma expression for the parallel component can be retained \( P \approx -\frac{\omega_p^2}{\omega^2} \). The slow branch of the cold plasma model in the vicinity of ion cyclotron resonance is called ion cyclotron wave.

If \( \frac{\omega}{k} \sim v_{the} \), the parallel electron response will lead to important corrections in the parallel component of the dielectric tensor, and the cold plasma approximation breaks down.

### 2.1.2 Fast wave propagation in tokamak plasmas

For typical tokamak plasmas, the fast magnetosonic mode remains well enough within the validity region of the cold plasma model to warrant its study in this limit. In this section, its propagation will be studied by solving the fast wave dispersion relation (equation 2.13). The local plasma and cyclotron frequencies across the plasma column, and the parameters of the external excitation determine all quantities in equation 2.13 except for \( n_1^2 \). In a low beta tokamak with large aspect radius:

1. The magnetic field can be taken as \( B \approx B_\phi \approx \frac{B_0 R_0}{R} \), where \( R_0 \) and \( B_0 \) are the major radius and magnetic field at the magnetic axis respectively. As a result, the local cyclotron frequency for the plasma ions decreases with increasing major radius. The contours for the cyclotron frequencies in the poloidal cross-section correspond to (nearly) vertical lines, which defines a major radius \( R_{ci} \) for the resonances:

\[
R_{ci} = \left( \frac{ZeB_0}{A m_p} \right) R_0 \quad \text{or} \quad \Omega_{ci} = \frac{R_{ci}}{R} \omega
\]
The relative location of the cyclotron layers for the different ions depends on their charge to mass ratios.

2. The plasma density and temperature peaks around the magnetic axis. In the midplane, the plasma frequency increases then decreases with increasing major radius. Its contours in the poloidal cross section map to the concentric flux surfaces.

3. The wavenumber in the toroidal direction is \( k_\phi = \frac{n_\phi R}{R} \), where the toroidal mode number \( n_\phi \) is conserved due to toroidal symmetry, and therefore specified by the antenna geometry and phasing. \( k_\parallel = k_\phi + k_0 \frac{B_0}{B} + k_r \frac{B_r}{B} \approx k_\phi \) if \( k_\perp \ll k_\phi \frac{B}{B_0} \), i.e. for modes with large perpendicular wavelengths. For shorter wavelength modes, \( k_\parallel \) can differ greatly from \( k_\phi \) and is not set by the antenna excitation any more.

In the following sections, we define \( R_c = R_{cH} \) and \( \omega_p = \frac{n_e e^2}{\epsilon_0 m_p} \) as reference constants for resonance radius and ion plasma frequencies. Then \( R_{ci} = \frac{Z}{\lambda} R_c \) and \( \omega_{pi} = \eta \frac{Z^2}{\lambda} \omega_p \), where \( \eta = \frac{n_e}{n_i} \). Note that quasi-neutrality implies \( \sum_{ions} \eta_i Z = 1 \), or \( \sum_{ions} \frac{\omega_{pi}^2}{\Omega_{ci}^2} = \frac{\omega_{pe}^2}{\Omega_{ce}^2} \).

**Single ion species** - We first consider the case of a plasma with a single ion species. The resonance condition for the fast wave \( S = n_\parallel^2 \) can be rewritten:

\[
\frac{\omega_{pi}^2}{\Omega_{ci}^2} = \left[ n_\parallel^2 - \left( 1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} \right) \right] \left( 1 - \frac{\omega^2}{\Omega_{ci}^2} \right) \tag{2.18}
\]

The cutoff conditions can be written, using the quasi-neutrality:

\[
R = n_\parallel^2 \quad \frac{\omega_{pi}^2}{\Omega_{ci}^2} = \left( n_\parallel^2 - 1 - \frac{\omega_{pe}^2}{\Omega_{ce}^2} \right) \left( 1 + \frac{\omega}{\Omega_{ci}} \right) \tag{2.19}
\]

\[
L = n_\parallel^2 \quad \frac{\omega_{pi}^2}{\Omega_{ci}^2} = \left( n_\parallel^2 - 1 - \frac{\omega_{pe}^2}{\Omega_{ce}^2} \right) \left( 1 - \frac{\omega}{\Omega_{ci}} \right) \tag{2.20}
\]

For the typical \( n_\phi \) radiated by dipole antennas, \( n_\parallel^2 \gg 1 \). The cutoff and resonance conditions define essentially critical densities, through \( \omega_{pi}^2 \propto n_i \). Since \( \omega_{pi} \gg \omega_{ci} \) in the plasma core, the corresponding layers will be usually situated at the plasma periphery. However, the critical densities can vary with \( n_\parallel^2 \) and the magnetic field, and these two quantities depend on the major radius. Assuming \( n_\parallel^2 \approx \left( \frac{n_\phi}{R} \right)^2 \left( \frac{e}{\omega} \right)^2 \gg 1 \), the major radius dependence for the right-hand cutoff density
The presence of the cutoff and resonance layers depends also on the location with respect to ion cyclotron resonance:

(a) **Low field side cutoff** On the low field side \((\omega > \Omega_{ci})\), there is no resonance or left-hand cutoff. The fast wave is evanescent in vacuum and starts propagating when the density exceeds the right-hand cutoff density defined by the \(R = n_R^2\) condition above.

(b) **High field side triplet** As the density increases on the high field side of the cyclotron resonance \((\omega < \Omega_{ci})\), a cutoff-resonance-cutoff triplet is encountered, typically at low to moderate densities, except for low \(n_\phi\) modes. Since \(S = (R + L)/2\), the usual sequence for increasing densities is L-cutoff / S-resonance / R-cutoff. For very low densities, it is possible that the right-hand cutoff condition is not met anywhere in the plasma. In this case, the L-cutoff / S-resonance doublet will still be present.

(c) **Ion Cyclotron Resonance** As mentioned above, there is no cold plasma resonance for the fast wave at the ion cyclotron resonance layer due to the unfavorable polarization.

We illustrate the discussion above with the particular case of a pure deuterium tokamak plasma on figure 2-1. The core density is high enough to be above the right-hand cutoff, and the ion cyclotron layer is present in the plasma. The high field triplet is observed in this case, with the left hand cutoff and the resonance layer close to each other. The right-hand cutoff defines a region of propagation \((k_\perp^2 > 0)\) in the core. The fast wave is not resonant at the ion cyclotron layer in the cold plasma limit.

- **Multiple ion species** - In a multi-ion mix like D-T, or a plasma with impurity(trace ions), additional resonance and cutoff layers are present. They appear between the ion cyclotron layers. A typical case is illustrated on figure 2-2 and can be used to appreciate the differences between single and multiple ion plasmas. The \(R\) component of the dielectric tensor is mostly unaffected. The \(L\) component of the dielectric has a pole at each cyclotron resonance: it goes to \(+\infty\) on the low field side, and \(-\infty\) on the high field side. This leads to the appearance of an additional resonance-cutoff doublet between adjacent ion cyclotron resonances, since \(L\) and \(S\) will change sign. The
In the two figures above, the three upper plots correspond to midplane cuts. In the bottom plot, the green color indicates where the fast wave propagates.
high field side triplet is present at the plasma periphery on the high field side of the innermost ion cyclotron layer.

The additional resonance-cutoff doublet can be interpreted physically as resulting from the out-of-phase resonant motion of two ion clouds [Buchsbaum 60]. For this reason, it is referred as ion-ion hybrid layer. The relative position and distance of the cutoff and resonance layer depends on the sign of \( R - n_0^2 \), i.e. on the density relative to the right-hand cutoff density. In the evanescent region at the plasma periphery, the ion-ion hybrid resonance layer is on the low field side of the left-hand cutoff. The arrangement is very similar to the high field side triplet. In the propagating region (for densities above the right-hand cutoff), the configuration is reversed and the ion-ion hybrid resonance layer is on the high field side of the left-hand cutoff. The parameters determining the location of the layer can be determined by rewriting resonance and cutoff conditions in terms of \( \frac{R}{R_c} \) and the charge to mass ratio \( \frac{Z}{A} \) of the ions species:

\[
S = n_0^2 - \frac{\omega_p^2}{\omega^2} \sum_{\text{ions}} (Z \eta_i) \frac{(Z A)}{(R/R_c)^2} = -n_0^2 R + 1 + \frac{\omega_p^2}{\Omega_e^2} \tag{2.23}
\]

\[
L = n_0^2 - \frac{\omega_p^2}{\omega^2} \sum_{\text{ions}} (Z \eta_i) \frac{(Z A)}{(R/R_c)^2} \left[ 1 + \frac{A R}{Z R_c} \right] = -n_0^2 R + 1 + \frac{\omega_p^2}{\Omega_e^2} \tag{2.24}
\]

In the plasma core, since \( \omega_p^2 \gg \omega^2 \approx \Omega_e^2 \), an approximate equation for the location of the ion-ion hybrid resonance layer can be obtained by ignoring the terms on the right side. In this limit, the electron density does not appear any more in the equations, and therefore the location of the layers depends mostly on the magnetic field and relative concentration of the ions species. If, then, only two ions species are present, with concentrations \( \eta_1 = \frac{n_1}{n_e} \) and \( \eta_2 = \frac{n_2}{n_e} \), we can write approximate expressions for the resonance \( (S = n_0^2) \) and \( (L = n_0^2) \) conditions [Brambilla 98]:

\[
S = n_0^2 \quad R = R_{res} = R_c \left[ \frac{Z_2 A_1 Z_1 \eta_1 + Z_1 A_2 Z_2 \eta_2}{Z_2 A_1 Z_1 \eta_1 + Z_1 A_2 Z_2 \eta_2} \right]^{1/2} \tag{2.25}
\]

\[
L = n_0^2 \quad R = R_{cut} = R_c \left[ \frac{Z_2 A_1 Z_1 \eta_1 + Z_1 A_2 Z_2 \eta_2}{Z_2 A_1 Z_1 \eta_1 + Z_1 A_2 Z_2 \eta_2} \right] \tag{2.26}
\]
where $\eta_1 = \frac{n_1}{n_e}$ and $\eta_2 = \frac{n_2}{n_e}$ are the relative concentrations for the two ions species, with indexes 1 and 2. Note that quasi-neutrality implies $Z_1\eta_1 + Z_2\eta_2 = 1$.

The relative position between the cutoff and resonance layer within these approximations can be obtained:

$$R_{\text{res}} = R_{\text{cut}} \left[1 + Z_1\eta_1 Z_2\eta_2 \left(\frac{A_2 Z_1}{Z_2 A_1} + \frac{A_1 Z_2}{Z_1 A_2} - 2\right)\right]^{-1/2}$$

(2.27)

Since for all $x = \frac{A_2 Z_1}{Z_2 A_1}$, $x + \frac{1}{x} \gtrsim 2$, this implies $R_{\text{res}} < R_{\text{cut}}$. The resonance layer in this context is always on the high field side of the cutoff. As illustrated in figure 2-2, this is only true when the plasma density is high enough so that the right hand side of equations 2.23 and 2.24 can be neglected. In the high density region, there is a small evanescent region between the cutoff and resonance layers, most easily seen on the fast wave dispersion curve. Since the fast wave must tunnel through it, it is important to characterize its width $\Delta R_{L-S}$:

$$\Delta R_{L-S} \equiv R_{\text{cut}} - R_{\text{res}} = R_c \left[1 - \left[1 + Z_1\eta_1 Z_2\eta_2 \left(\frac{A_2 Z_1}{Z_2 A_1} + \frac{A_1 Z_2}{Z_1 A_2} - 2\right)\right]\right]^{-1/2}$$

(2.28)

As we will see later, the case $Z_1\eta_1 \ll 1$ is particularly important for heating applications. The species 1 is then called minority, while the other is called majority. The resonance and cutoff layers are then very close to the resonance layer for the minority ions $R = R_{c1}$. Using $Z_1\eta_1 + Z_2\eta_2 = 1$, we can expand the expression above:

$$\Delta R_{L-S} \simeq \frac{R_{c1}}{2} \left(\frac{A_2 Z_1}{Z_2 A_1} + \frac{A_1 Z_2}{Z_1 A_2} - 2\right) Z_1\eta_1 + \ldots$$

(2.29)

The distance between the cutoff layer and the minority cyclotron layer is:

$$\Delta R_{L-c} \equiv R_{c1} - R_{\text{cut}} = R_{c1} \left(1 - \frac{A_1 Z_2}{Z_1 A_2}\right) Z_1\eta_1.$$  

(2.30)

A last important parameter in determining the location of the cutoff and resonance layers in the parallel index of refraction $n_||^2$. It appears on the right hand side of equation 2.23. Its effect, which is not immediately clear for the equation, can be understood by looking at the plots for S,L,R in figure 2-2. Since S and L are the monotonic functions of major radius between two cyclotron layers, we can see that increasing $n_||^2$ will move both layers $S = n_||^2$ and $L = n_||^2$ towards the low
field side cyclotron resonance. It will also decrease the evanescent distance. The high field side right-hand cutoff will also move towards the low field side. The evanescent layer will disappear for \( L = S = R = n_{\parallel}^2 \), i.e. when the resonance and cutoff layers merge. In this situation, the fast wave would be evanescent on the high field side of the ion-ion hybrid layer.

### 2.1.3 Fast wave absorption

In the regions where the fast wave dispersion relation holds, i.e. far from cold plasma resonance layers \( S = n_{\parallel}^2 \), the cold plasma dispersion relation can be used to determine \( n_{\perp}^2 \) and the wave polarization even at finite temperature and in the vicinity the ion cyclotron resonance layers. This allows linear calculations of the direct fast wave absorption on electrons and ions in cases when the damping is weak, so that the finite temperature and ion cyclotron resonance corrections in the full hot plasma tensor can be taken as perturbative terms in the cold plasma dielectric tensor.

Direct fast wave damping is relevant to this thesis work since the corresponding absorption mechanisms compete with mode conversion. We want to identify regimes and conditions where the competing mechanisms are weak, and it turns out linear theory is appropriate for this goal. When other absorption schemes dominate, the linear theory introduced here can become inadequate, and more complicated treatments must be used. Linear theory evaluates the strength of the absorption mechanisms through the idealized situation of a single transit of the wave across the absorbing region, or single pass. The fraction of the incident power absorbed in the layer \( P_{\text{abs}}/P_{\text{inc}} \) is usually quantified in terms of an optical depth \( 2\eta \):

\[
P_{\text{abs}}/P_{\text{inc}} = 1 - \exp(-2\eta). \tag{2.31}
\]

- **Single ions species** - Expressions for the optical depth corresponding to direct electron damping, ion cyclotron absorption by majority ions at their fundamental and second harmonics resonance are compiled in table 2.1 based on [Porkolab 93] and [Porkolab 98]. The table shows also numerical values for the different absorption processes for typical Alcator C-Mod parameters.

In the table, \( x_{0e} = \frac{\omega_{ce}}{k_{||}v_{te}} \), \( \beta_e = \frac{\omega_{pe}^2}{k_{||}^2 v_{te}^2 c^2} \), \( \beta_i = \frac{\omega_{pi}^2}{k_{||}^2 v_{ti}^2 c^2} \) and \( a \) is the minor radius of the plasma, \( R \) is its major radius. The numerical values correspond to a deuterium plasma, with:
Optical depth $2\eta \approx 0.03$

Direct electron damping
$$a \frac{\pi^{1/2}}{2} \frac{\omega_i}{\Omega_{ci}} e^2 \beta_e x_{0e} \exp(-x_{0e}^2)$$

Majority fundamental
$$\frac{\omega_i}{c} k^2 \rho_i^2 R$$

Majority 2nd harmonic
$$\frac{\pi \omega_i}{2} \frac{c}{R}$$

| Table 2.1: Single pass absorption for direct fast wave damping |
|-----------------|-----------------|-----------------|
| $R = 0.67 m$    | $a = 0.21 m$    | $\Omega_{ci} \sim \omega = 2\pi \times 80 MHz$ |
| $n_e(0) = 2 \times 10^{20} m^{-3}$ | $T_e(0) = 5 \text{ keV}$ | $T_i = 5 \text{ keV}$ |

Direct electron damping corresponds to the power transfer from the fast wave to bulk electrons via Landau damping when $\frac{\omega}{k_{\parallel}} \sim v_{th,e}$. The single pass absorption is evaluated by retaining the corresponding hot plasma dielectric tensor elements [Porkolab 98]. The function $x_{0e} \exp(-x_{0e}^2)$ has a maximum for $x_{0e}^2 = \frac{1}{2}$, which leads to the value quoted in table 2.1. Since $E_{\parallel}$ is small, this damping mechanism is usually weak.

From the fast wave dispersion relation, very little absorption is predicted at the fundamental cyclotron resonance due to the unfavorable electric field polarization. Even through the hot plasma corrections are important in this situation, similar cancellations occur and the actual single pass absorption with finite temperature is still very small. The absorption at higher harmonics comes from finite Larmor radius (FLR) effects, and is usually weak since $\beta_i \ll 1$.

- **Multiple ion species** - The weak linear absorption predicted in a plasma with a single ion species can be related to unfavorable electric field polarization for damping on electrons and on ions at the fundamental ion cyclotron frequency. In a plasma with multiple ion species, the fast wave polarization is determined by the contribution of the different ions depending on their relative concentrations, and should not be affected much by the presence of impurity or trace ions if their concentration is low enough. This gives rise to a situation where a significant $E_{\parallel}$ polarization may be present at the ion cyclotron resonance for these impurity/trace ions, and significant ion damping can occur. This scenario, called minority heating, was first observed experimentally in PLT during
second harmonic heating experiments in a Deuterium plasma. Strong heating was measured, in excess of the predictions for second harmonic damping, and was later found to be related to the presence of hydrogen impurities.

In order to quantify this effect, it is necessary to retain the hot plasma expression for the resonant minority ion terms in the dielectric tensor both in the damping calculation and the evaluation of the polarization. Indeed, in the cold plasma model, these terms go as $\frac{\omega}{\omega - \omega_{ci}}$ and their singular behavior will always imply $E_+ \to 0$ at $\omega = \omega_{ci}$ even if the concentration is very low. This behaviour is not encountered in the hot plasma dielectric tensor, since the corresponding term $\frac{\omega}{k_{||} v_{thi}} Z(\frac{\omega - \Omega_{ci}}{k_{||} v_{thi}})$, where $Z$ is the plasma dispersion function [Fried 61], is not singular as $\omega \to \Omega_{ci}$, and its absolute value remains below $\sqrt{\pi}$ (see figure 2-3). If the minority concentration is small, the minority ion contributions can be ignored when calculating the fast wave polarization at the minority ion cyclotron resonance. In general, this leads to a finite $E_+$ polarization, and therefore to significant damping. As the minority concentration is increased, the minority contribution reduce the $E_+$ component.

![Plasma dispersion function](image)

**Figure 2-3: Plasma dispersion function**

A calculation of the damping decrement for the minority ions, with subscript m, has been carried
out in [Porkolab 93]:

\[
2\eta = \frac{\pi}{2} \frac{\omega_{pm}^2}{c \Omega_{cm} |n_\perp|} R \left( \frac{E_+}{E_y} \right)^2
\]

(2.32)

where the integration was performed through the absorption layer assuming no spatial variation for \( n_\perp \) and \( \frac{E_+}{E_y} \). The perpendicular index of refraction and polarization are calculated at the minority ion cyclotron layer by separating the minority contributions from the non-resonant terms which are calculated in the cold plasma limit.

\[
S = S_M + i S_m \quad D = D_M + i D_m \quad S_m = -D_m = \frac{\omega_{pm}^2 \Omega_{cm} \sqrt{\pi}}{\Omega_{cm}^2 k_{\parallel} v_{thi}} \frac{1}{2}
\]

(2.33)

Assuming \( n_{\parallel}^2 \ll S, D \), this gives for the polarization:

\[
\frac{|E_+|^2}{|E_y|^2} \approx \frac{|D_M - i S_m + 1|^2}{|S_M + i S_m|^2} = \frac{(\frac{D_M}{S_M} + 1)^2}{1 + \frac{S_M}{S_M}}
\]

(2.34)

In particular case of a plasma with a single majority species, labeled M, we take \( n_\perp \approx \frac{n_m}{n_e} \) and simplify the expressions above, with \( \eta_m = \frac{n_m}{n_e} \):

\[
2\eta = \frac{\pi}{2} \frac{\omega_{pM}}{c} R \left( \frac{\Omega_{cm}}{\Omega_{cM}} - 1 \right)^2 \frac{Z_m \eta_m}{Z_M \eta_m} \frac{Z_m \eta_m}{Z_M \eta_m}
\]

(2.35)

with

\[
\varsigma = \frac{\sqrt{\pi}}{2} \frac{Z_m/A_m}{Z_M/A_M} k_{\parallel} v_{thi} \frac{\Omega_{cm}}{\Omega_{cM}} \left| \Omega_{cm} \right|^2 - 1
\]

(2.36)

The single pass absorption peaks therefore for \( \frac{Z_m \eta_m}{Z_M \eta_m} = \frac{1}{\varsigma} \). As we indicated, \( \frac{\omega}{v_{thi} k_{\parallel}} \sim 1 \) is typical for fast wave antenna spectra, which implies \( \frac{\Omega_{cm}}{k_{\parallel} v_{thi}} \sim \left( \frac{n_i}{n_e} \right)^{1/2} \approx 43 \). Therefore the absorption will peak for concentrations \( \frac{Z_m \eta_m}{Z_M \eta_m} \sim 1 - 10\% \). Above this concentration, the single pass absorption drops rapidly. The maximum value for \( 2\eta \) is:

\[
2\eta|_{max} = \frac{\sqrt{\pi}}{2} \frac{\omega_{pM}}{c} R \frac{\Omega_{cm}}{\Omega_{cM}} \left| \Omega_{cm} - \Omega_{cM} \right| k_{\parallel} v_{thi} \frac{\Omega_{cm}}{\Omega_{cM}}
\]

(2.37)

Table 2.2 shows the cold plasma polarization term, the critical concentration and maximum minority single pass absorption for two scenarios used frequently in C-Mod, assuming \( \frac{\omega}{v_{thi} k_{\parallel}} = 1 \) and \( n_e = 2 \times 10^{20} \text{m}^{-3} \). The polarization term for \(^3\text{He} \) minority in reactor relevant 50%-50% D-T
plasmas is also shown for comparison. The corresponding curves for $2\eta$ as a function of $\eta_m$ are showed on figure 2-4.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Polarization term</th>
<th>Optimal concentration</th>
<th>Optimal absorption</th>
</tr>
</thead>
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<tr>
<td>D(H)</td>
<td>$(\frac{\eta_{cm}}{\eta_{cM}} - 1)^2$ or $(\frac{D_{2M}}{S_{2M}} + 1)^2$</td>
<td>$\eta_c = \frac{Z_{M}^2}{Z_{m}^2}$</td>
<td>$2\eta</td>
</tr>
<tr>
<td>D($^3$He)</td>
<td>1</td>
<td>1.7 %</td>
<td>0.35</td>
</tr>
<tr>
<td>H($^3$He)</td>
<td>1/9</td>
<td>2.2 %</td>
<td>0.1</td>
</tr>
<tr>
<td>D-T($^3$He)</td>
<td>1/3.6</td>
<td>1.6 %</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2.2: Single pass absorption for minority heating

Figure 2-4: Optical depth for D(H) and D($^3$He).

Within the framework of linear theory, significant single pass absorption for minority ions is predicted. This approach has at least two obvious shortcomings, however both tend to make the absorption even stronger at low minority concentrations. The first is a consequence of the strong linear damping: since the absorption is significant, the distribution function of the minority ions will deviate from a Maxwellian and a fast minority ion population will be created. This results
in larger Doppler broadening and stronger absorption. The second shortcoming is our assumption that the dispersion relation will not be affected by the minority ion contributions for moderate minority concentrations \( (\eta_m \sim \eta_c) \) in the vicinity of the minority cyclotron layer. The strong linear damping indicates that these contributions are significant, and therefore we expect kinetic modes to be present. We also recall the presence of the ion-ion hybrid layer near the minority ion cyclotron layer for moderate minority concentrations, which indicates that the cold plasma formalism becomes invalid and corrections must be included. While it is difficult to determine which hot plasma modes will be excited as a result of these corrections, it is not unreasonable to expect that these modes will be strongly damped on the minority ions, which may also enhance the single pass absorption for the incident fast waves. The study of these effects is well beyond the scope of this thesis.

As the minority concentration is increased, the two shortcomings mentioned above tend to become less important: the linear damping drops significantly, and the distance between the minority ion cyclotron layer and the ion-ion hybrid layer increases so that the two layers become well separated.

**Edge absorption** - A last effect to consider is absorption or losses at the plasma edge. When the core damping is weak, the wave reaches the edge of the plasma with significant power and can be absorbed outside the core plasma. Since an evanescent region surrounds the plasma, the plasma will act as a cavity, with losses at the edge. Collisional absorption in the plasma edge is usually thought to be small. However there exists potentially strong absorption mechanisms associated with wave electric fields at the vessel walls (usually referred as far-field sheath [Myra 94]). Note that relating the wave electric fields at the walls to that in the plasma core is conceptually similar to the antenna-plasma coupling problem. A model of the dissipation is not yet available, but would be of great importance both for core calculations and for predictions of wall-induced impurity production. This issue is, again, beyond the scope of this discussion, and we will consider that the edge losses are weak enough compared to core absorption so that they can be ignored.

**Absorption and tunneling at the \( S = n^2 \) layer** - In a plasma with two or more ion species, a resonance-cutoff doublet associated with the ion-ion hybrid resonance can exist in the plasma core. Although finite temperature corrections can become important in this region and will modify the dispersion relation locally, retaining the fast wave dispersion relation allows one to obtain a
simple wave equation and study the tunneling and absorption associated with the resonance-cutoff
doublet. The obtained equation has the following form:

$$\frac{\partial^2 E}{\partial \xi^2} + Q(\xi)E = 0$$

where $E$ is the electric field, $\xi$ is a radial coordinate and $Q$ is a potential function approximat-
ing $n^2_\perp = \frac{(L-n^2_\parallel)(R-n^2_\parallel)}{S-n^2_\parallel}$ in the singular region. During his studies of propagation in ionospheric
plasmas, Budden [Budden 61] solved this equation for $Q \approx \frac{2}{\xi} + \beta$, which retains the resonance
and nearby cutoff. This solution was applied to the ion-ion hybrid case in [Swanson 76] and
[Jacquinot 77]. As mentioned in chapter 1, complete resonant absorption is predicted for high field
side incidence. For low field side incidence, the single pass absorption depends on the length of
the evanescent layer, and peaks at 25 %.

Absorption near the ion-ion hybrid layer can therefore be the dominant fast wave damping sce-
nario if minority heating is small, usually at large enough minority concentrations. This criterion
delimits the minority regime, for which minority absorption dominates, to the mode conversion
regime, for which absorption near the $S = n^2_\parallel$ dominates. At this stage, however, it is not clear
what is the “fate” of this absorbed power, since the fast wave dispersion relation becomes invalid
in this region. The relevant corrections are the object of the following section.
2.2 Dispersion curves in the mode conversion region

The $S = n_{||}^2$ condition defines a singular resonance layer in the fast wave dispersion relation, which, as stated in [Swanson 85], indicates that finite temperature corrections or other effects must be introduced in order to resolve the singularity. The corrections reflect the presence of additional modes which can be excited as the fast wave approximation breaks down. This excitation is referred as mode conversion. Note that in this extended formalism, the $S = n_{||}^2$ layer is no longer a resonance layer, and will therefore be more accurately referred as mode conversion layer.

2.2.1 Finite temperature corrections

In order to obtain dispersion relations near the ion-ion hybrid, it is necessary to include relevant finite temperature corrections in the cold plasma dispersion relation, with the goal of removing the singularity.

■ 1st order FLR approximation - The simplest physical description of mode conversion from fast magnetosonic waves can be obtained with the hot plasma dielectric tensor truncated at the first order in the Larmor radius expansion. The corresponding dielectric tensor in the Stix frame is [Brambilla 98]:

\[
\begin{align*}
\epsilon_{xx} &= \bar{\epsilon} - \sigma n_{||}^2 \\
\epsilon_{xz} &= \epsilon_{zx} = n_{||} n_{\perp} \eta \\
\epsilon_{yz} &= \epsilon_{zy} = i n_{||} n_{\perp} \xi \\
\epsilon_{yy} &= \bar{\epsilon} - (\sigma + 2\tau) n_{||}^2 \\
\epsilon_{zz} &= \bar{\epsilon} - \pi n_{||}^2 \\
\end{align*}
\]

\[
\begin{align*}
\bar{\epsilon}_k &= 1 - \sum_\alpha \frac{\omega_{p}\alpha^2}{\omega^2} \frac{-x_{0\alpha}}{2} [Z(x_{-1\alpha}) + Z(x_{+1\alpha})] \\
\bar{D}_k &= - \sum_\alpha \frac{\omega_{p}\alpha x_{0\alpha}}{\omega^2} \frac{1}{2} [Z(x_{-1\alpha}) - Z(x_{+1\alpha})] \\
\bar{P}_k &= - \sum_\alpha \frac{\omega_{p}\alpha x_{0\alpha}^2 Z'(x_{0\alpha})}{\omega^2} \left\{ \text{Zero order terms} \right\} (2.39)
\end{align*}
\]
\[
\begin{align*}
\sigma &= \frac{1}{2} \sum_{\alpha} \frac{\omega_{pa}^2 v_{th\alpha}^2 x_{0\alpha}}{\Omega_{ca}^2} \left( [Z(x_{-1\alpha}) + Z(x_{+1\alpha})] - [Z(x_{-2\alpha}) + Z(x_{+2\alpha})] \right) \\
\delta &= \frac{1}{2} \sum_{\alpha} \frac{\omega_{pa}^2 v_{th\alpha}^2 x_{0\alpha}}{\Omega_{ca}^2} \left( [Z(x_{-1\alpha}) - Z(x_{+1\alpha})] - \frac{1}{2} [Z(x_{-2\alpha}) - Z(x_{+2\alpha})] \right) \\
\tau &= \frac{1}{2} \sum_{\alpha} \frac{\omega_{pa}^2 v_{th\alpha}^2 x_{0\alpha}}{\Omega_{ca}^2} \left( -Z(x_{0\alpha}) + \frac{1}{2} [Z(x_{-1\alpha}) + Z(x_{+1\alpha})] \right) \\
\eta &= \frac{1}{4} \sum_{\alpha} \frac{\omega_{pa}^2 v_{th\alpha}^2 x_{0\alpha}^2}{\omega \Omega_{ca}^2} \left( Z'(x_{-1\alpha}) - Z'(x_{+1\alpha}) \right) \\
\xi &= -\frac{1}{2} \sum_{\alpha} \frac{\omega_{pa}^2 v_{th\alpha}^2 x_{0\alpha}^2}{\omega \Omega_{ca}^2} \left( Z'(x_{0\alpha}) - \frac{1}{2} [Z'(x_{-1\alpha}) + Z'(x_{+1\alpha})] \right) \\
\pi &= \frac{1}{2} \sum_{\alpha} \frac{\omega_{pa}^2 v_{th\alpha}^2 x_{0\alpha}}{\Omega_{ca}^2} \left( -x_{0\alpha} Z'(x_{0\alpha}) + \frac{1}{2} [x_{-1\alpha} Z'(x_{-1\alpha}) + x_{+1\alpha} Z'(x_{+1\alpha})] \right)
\end{align*}
\]

where \(Z\) is the plasma dispersion function and \(x_{na} = \frac{\omega - n \Omega_{ca}}{k_{\perp} v_{th\alpha}}\).

Order of dielectric tensor contributions - Restricting our discussion to the Ion Cyclotron Range of Frequencies (\(\omega \sim \Omega_{ci}\)), the dielectric tensor can be simplified based on the following orderings:

1. **Beta** The 1st order FLR terms have the following prefactor:

\[
\beta_e = \frac{\omega_{pe}^2 v_{th e}^2}{\Omega_{ce}^2 \Omega_{ci}^2} \sim \frac{T_e}{m_e c^2} \tag{2.40}
\]

\[
\beta_i = \frac{\omega_{pi}^2 v_{th i}^2}{\Omega_{ci}^2 \Omega_{ci}^2} \sim \frac{T_i}{m_e c^2} \tag{2.41}
\]

\(m_e c^2 = 511\) keV, to be compared with \(T_i \sim T_e \sim 5\) keV in present day tokamaks. This small parameter is used to justify the truncation at first order.

2. **Electron to ion mass ratio** \(\frac{n}{\omega} \sim \frac{n}{\Omega_{ci}} \sim \frac{m_i}{m_e} \gg 1\). This implies \(x_{ne} \sim n \frac{\Omega_{ci}}{\omega} \gg 1\). Therefore we can use the fluid approximation for the plasma dispersion function in electron terms where \(n \neq 0\). For \(n \sim 1\),

\[
\frac{\omega_{pe}^2 - x_{0e}}{\omega^2} \left[ Z(x_{-ne}) + Z(x_{+ne}) \right] \sim 1 \tag{2.42}
\]
\[
\frac{\omega_{pe}^2 - x_{0e}}{\omega^2} \left[ Z(x_{-ne}) - Z(x_{+ne}) \right] \sim \frac{\Omega_{ce}}{\omega} \sim \frac{m_i}{m_e} \tag{2.43}
\]

For ions, \( k_{//} v_{thi} \sim \omega \sqrt{\frac{m_e}{m_i}} \). Therefore, if \( \frac{|k_{//} v_{thi}|}{\omega} \gg 1 \), for \( n \sim 1 \),

\[
\frac{\omega_{pi}^2 - x_{0i}}{2} \left[ Z(x_{-ni}) + Z(x_{+ni}) \right] \sim \frac{\omega_{pi}^2}{\Omega_{ci}^2} \sim \frac{m_i}{m_e} \left( \frac{\omega}{\Omega_{ci}} \right)^2 \sim 1 \tag{2.44}
\]

\[
\frac{\omega_{pi}^2 - x_{0i}}{2} \left[ Z(x_{-ni}) - Z(x_{+ni}) \right] \sim \frac{m_i}{m_e} \left( \frac{\omega}{\Omega_{ci}} \right)^2 \sim 1 \tag{2.45}
\]

\[
\frac{\omega_{pi}^2}{\omega^2} x_{0i} Z'(x_{0i}) \sim \frac{\omega_{pi}^2}{\Omega_{ci}^2} \sim \frac{m_i}{m_e} \tag{2.46}
\]

**Cyclotron effects** If \( \frac{|k_{//} v_{thi}|}{\omega} \gg 1 \), the plasma dispersion function needs to be retained for the ions. This is required in the immediate vicinity of ion cyclotron resonances. The local value of \( k_{//} \) determines the Doppler broadening of the resonance layer.

**Parallel electron response** As \( x_{0e} \sim 1 \), the plasma dispersion function must be retained for electron terms involving \( Z(x_{0e}) \) and \( Z'(x_{0e}) \). We can take \( x_{0e} Z(x_{0e}) \sim 1 \) and \( x_{0e}^2 Z'(x_{0e}) \sim 1 \).

\[
\frac{\omega_{pe}^2}{\omega^2} x_{0e} Z'(x_{0e}) \sim \left( \frac{m_i}{m_e} \right)^2 \tag{2.47}
\]

**First order FLR terms** Similar expansions can be used as the zero order terms. Compared to zero order terms, the first order terms are smaller by:

\[
\beta_e \frac{\omega^2}{\omega_{pe}^2} \sim \frac{T_e}{m_e c^2} \left( \frac{m_e}{m_i} \right)^2 \tag{2.48}
\]

\[
\beta_i \frac{\omega^2}{\omega_{pi}^2} \sim \frac{T_i}{m_e c^2} \frac{m_e}{m_i} \tag{2.49}
\]

\[
\beta_e \frac{\omega_{ce}}{\omega_{pe}^2} \sim \frac{T_e}{m_e c^2} \frac{m_e}{m_i} \tag{2.50}
\]

\[
\beta_i \frac{\omega_{ci}}{\omega_{pi}^2} \sim \frac{T_i}{m_e c^2} \frac{m_e}{m_i} \tag{2.51}
\]
We see that four dimensionless ordering factors appear:

$$\beta = \frac{T}{m_e c^2} \sim 0.01 \ll 1 \quad \mu = \frac{m_i}{m_e} = 1836 \gg 1 \quad \Theta_{n=1,2} = \frac{1}{(\frac{\omega}{m_e})^2 - 1} \quad \text{or} \quad x_{ni} Z(x_{ni})$$

(2.52)

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<tr>
<th>Element</th>
<th>Electron term</th>
<th>n = 0 ion term</th>
<th>n = 1 ion term</th>
<th>n = 2 ion term</th>
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<tbody>
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<td>(\beta\Theta_1)</td>
<td>(\beta\Theta_1)</td>
</tr>
</tbody>
</table>

Table 2.4: Ordering of the zero and first order FLR terms near the Ion Ion hybrid layer

**Reduced FLR expansion** - The truncation of the hot plasma dielectric tensor elements at first order in the finite Larmor expansion is invalid in the immediate vicinity of cyclotron resonances or when \(k_{\perp} \rho_i \gtrsim 1\). In the mode conversion regime, where the minority concentration is large, the ion-ion hybrid layer is well separated from the resonance layers, and for the fast wave, we have typically \(k_{\perp} \rho_i < 1\). This suggests that the truncated dielectric tensor can be used in the near vicinity of the mode conversion layer, but does not guarantee that the mode converted waves will not violate the condition \(k_{\perp} \rho_i < 1\) as \(k_{\perp}\) increases or as the waves propagate away from the mode conversion region towards the nearby ion cyclotron resonances. If fact, as we will see later, the perpendicular wavelength of Ion Bernstein waves can increase rapidly and violate \(k_{\perp} \rho_i < 1\), while Ion Cyclotron Waves propagate towards the low field side Ion Cyclotron resonance. We will therefore examine the validity of this approximation, by comparing the 1st order FLR roots with the full hot plasma results.

While we will return to this issue when presenting numerical results for the dispersion curves in both models, we can already mention the analysis in [Brambilla 98], which observed pathological differences between the two models. These differences arise mainly from the divergent nature of both zero and first order ion terms corresponding to the fundamental resonance (n=1). The
approximate dispersion relation for the mode converted waves involves the ratio of such quantities, and thus the singularities cancel each other. This effect is not present in the hot plasma approach.

In order to obtain a better description for the mode converted waves, [Brambilla 98] proposed a modification to the truncation approach, called reduced FLR expansion. It consists of omitting the \( n = 1 \) resonant ion terms in the first order FLR terms. The main differences are in the \( \sigma \) and \( \delta \) elements:

\[
\sigma = \frac{1}{4} \sum_i \frac{\omega_{pi}^2 v_{thi}^2}{\Omega_{ci}^2} \left[ x_{0i} Z(x_{-2i}) - x_{0i} Z(x_{+2i}) \right] 
\tag{2.53}
\]

\[
\delta = \frac{1}{4} \sum_i \frac{\omega_{pi}^2 v_{thi}^2}{\Omega_{ci}^2} \left[ x_{0i} Z(x_{-2i}) + x_{0i} Z(x_{+2i}) \right] 
\tag{2.54}
\]

A numerical comparison between the full hot plasma solutions for the dispersion relation and the reduced FLR approximation for the Bernstein wave was carried out in [Brambilla 98]. Somewhat surprisingly, the hot plasma root for the IBW was found to agree very well with the reduced FLR root even as \( k \rho_i \leq 1 \). As a result, it is possible to use a single approximate dispersion relation over the entire tokamak cross-section, even in region where the first-order FLR expansion approach breaks down.

For the purpose of deriving dispersion relations for the mode converted waves, the differences between the full and reduced first-order FLR expansions are minimal, and we will able to carry out the algebra in both models at the same time.

### 2.2.2 Poloidal field effects

When studying fast wave propagation in tokamak plasmas, it is usually appropriate to ignore the poloidal field, i.e take \( k_\parallel \approx k_\phi = \frac{n_0}{R} \). It turns out that this approximation can be violated for the mode converted waves as \( k_\perp \) increases. In order to include the presence of a poloidal field in a slab geometry, we introduce the following model, largely inspired by [Perkins 77]:

\[ 1 \] The plasma is infinite and uniform along the \( \vec{z} \) and \( \vec{y} \) directions. Therefore \( k_y \) and \( k_z \) are constant and determined by the antenna excitation at \( x = x_0 \).
② In the poloidal cross section, the waves are assumed to propagate along the major radius direction, i.e. $k_y = 0$.

③ There is a static magnetic $\vec{B} = B_t \hat{z} + B_p (\cos \theta \hat{x} + \sin \theta \hat{y})$, where $\theta$ is the angle between the major radius direction and the poloidal field vector, and can vary in the x direction.

The geometry is illustrated on figure 2-5.

$k_y$ and $k_z$ being constant, the local dispersion relation $D(k ||, k_\perp) = 0$ can be used to determine $k_x(x)$, using:

$$k || = \frac{B_t}{B} k_z + \frac{B_p \cos \theta}{B} k_x$$  \hspace{1cm} (2.55)$$

$$k_\perp^2 = k_z^2 + k_x^2 - k ||^2$$  \hspace{1cm} (2.56)$$

Figure 2-5: One dimensional geometry

We define $b_p = \frac{B_p \cos \theta}{B}$ and $b_t = \frac{B_t}{B}$. Note that $b_t^2 + b_p^2 / \cos \theta^2 = 1$.

This geometry corresponds to a slab approximation of a toroidal geometry, obtained by taking an horizontal slice of the plasma at a given vertical position. In this case, the dependency on the variable $x$ (which would then be the major radius) could be such as to reproduce the toroidal and poloidal fields in strength and angle, and the density and temperature profiles. Note that however,
in toroidal geometry, \( k_z = n_\phi / R \) changes with major radius as the toroidal number \( n_\phi \) is conserved. The constraint \( k_z = \) constant in the model can be lifted within the WKB approximation, however since \( n_z^2 \) is small compared to the dielectric tensor elements for typical tokamak densities, the differences are minimal. A second difference with the toroidal geometry is that the flux surfaces form concentric surfaces with a \( 2\pi \) symmetry in the poloidal direction, therefore suggesting the use of a poloidal number \( m \). This second characteristic cannot be implemented in the 1-D model used here, however for in the mode conversion region, the model turns out to be a good representation of the tokamak magnetic geometry. The basic reason is that, except at the plasma periphery, the variation of the dielectric tensor elements is dominated by the change in the toroidal field \( B_\phi \) along the major radius direction. Therefore one can take the vertical wavevector component to be zero, in which case the \( \vec{x} \) direction can be taken as the major radius direction, \( \vec{y} \) is the vertical direction (thus \( k_y = 0 \)), and \( \vec{z} \) in the toroidal direction. At the midplane, the poloidal field \( B_\theta \) and the wavevector component in the poloidal cross section are perpendicular, i.e. \( \theta = \pm \pi / 2 \). The angle \( \theta \) is zero on the vertical layer crossing through the magnetic axis. For a tokamak with circular cross section, \( b_p = \frac{\epsilon}{q} \cos \theta \) where \( \epsilon \) is the local inverse aspect ratio and \( q \) the local safety factor. Present day tokamaks have \( q \sim 1 - 5 \) and \( \epsilon \sim 0.3 \) at the edge of the plasma, therefore \( 0 \lesssim b_p \lesssim 0.1 \) is typical. Figure 2-6 shows contours of \( b_p \) in a tokamak cross section with major and minor radii as in C-Mod, assuming a safety factor profile as shown on the left plot. The red curve on the right shows \( b_p \) along the vertical plane going through the magnetic axis.

### 2.2.3 Dispersion curves

- **Cold plasma model** - With \( k_y = 0 \), substituting \( n_\parallel^2 \) and \( n_\perp^2 \) for \( n_x^2 \) and \( n_z^2 \) according to equations 2.55 and 2.56 in the cold plasma dispersion relation (equation 2.4) gives the following dispersion relation, obtained with MAPLE:

\[
A_4 n_x^4 + A_3 n_x^3 + A_2 n_x^2 + A_1 n_x + A_0 = 0
\]  

\[\text{(2.57)}\]

\(^1\text{MAPLE is a mathematics software package, with capabilities for symbolic algebra. Detailed information is available at http://www.maplesoft.com/}\)
\[ A_4 = S + b_p^2(P - S) \]
\[ A_3 = 2b_p b_1 n_z (P - S) \]
\[ A_2 = D^2 - (S - b_p^2 n_z^2)(S + P) + 2S n_z^2 (1 - b_p^2) + b_p^2 ((n_z^2 - S)(P - S) - D^2) \]
\[ A_1 = 2b_p b_1 n_z [n_z^2 (P - S) + S^2 - D^2 - SP] \]
\[ A_0 = P((S - n_z^2 b_p^2)^2 - D^2) + (1 - b_p^2)n_z^2(b_p^2 n_z^2 P + S n_z^2 + D^2 - S^2 - SP) \]

In the limit \( b_p \to 0 \), the cold plasma dispersion relation without poloidal field is recovered. We note that, in this limit, the cold plasma ion-ion hybrid resonance \( A_4 = 0 \) occurs for \( S = 0 \), and not \( S = n_z^2 \) as suggested by the fast wave approximation. The slow wave root can propagate between the \( S = n_z^2 \) and the \( S = 0 \) layer, where \( n_z^2 \to \infty \). This distinction is actually somewhat irrelevant in practice in this limit, since (A) the distance between the mode conversion layer \( S = n_z^2 \) and the ion-ion hybrid layer proper \( S = 0 \) is very small for usual \( n_z \) values, and (B) the cold plasma approximation breaks down as \( n_z^2 \) increases.

When poloidal field effects are included, the resonance condition \( A_4 = 0 \) is modified and condition (A) does not hold any more. Indeed, for finite \( b_p \) of order \( \mu^{-1/2} = 0.023 \), the second term in \( A_4 = 0 \), i.e. \( b_p^2 P \), is non negligible comparable to the first term \( S \), as we can see for the

Figure 2-6: Typical values for \( b_p = \frac{1}{q(r)} \frac{a}{R} \) in a circular tokamak geometry, assuming a safety factor profile as shown on the left plot.
ordering between $P$ and $S$ in table 2.4. The resonance condition is met then for positive values of $S$. Referring the plots for $S$, $R$ and $L$ on figure 2.2 we see that the ion-ion hybrid layer will be shifted to the low field side of the mode conversion layer $S = n_z^2$. It will always be on the high field side of the minority ion cyclotron resonance, since $S \to \infty$ there.

This behavior can be illustrated by solving the dispersion relation numerically with Laguerre’s method [Press 92]. The dispersion curves in the region around the ion-ion hybrid layer are shown for a typical C-Mod D($^3$He) mode conversion scenario and with a small poloidal field component on figure 2.7. The presence of the poloidal field shifts the ion-ion hybrid resonance towards the low field side, extending the fast wave branch beyond the ion-ion hybrid layer for propagation towards the low field side. Since $P$ is large, this shift can be significant, as can be seen on figure 2.8 using values of $b_p$ as on figure 2.6.

Figure 2-7: Cold plasma dispersion curves around the ion-ion hybrid layer with a poloidal field component.

Figure 2-8: Propagation region for cold plasma ICWs.

- **Kinetic electrons, cold ions** - For finite electron temperature and negligible ion temperature, we can retain the cold plasma ion terms and ignore the first order FLR terms entirely (Electron terms in $\tau, \eta, \xi, \pi$ are nonzero but their contributions to the dispersion relation are negligible, while that in $\sigma$ and $\delta$ are down by a mass ratio and can therefore be ignored). However, since $k_\| \| \|$ can be upshifted and downshifted as $k_\perp$ changes to satisfy the dispersion relation, $x_{0e} = \frac{\omega}{k_\| V_\text{me}}$ can become close to unity and the kinetic corrections to the zero-order electron terms must be included. Figure
Figure 2-9: Contour plots of $\omega/|k||v_{\text{th}e}$ as a function of the parameters $b_p = \frac{B_{\text{pol}}}{B_{\text{tot}} \cos(\theta)}$ and $k_\perp$. The pink and blue stripes show the typical range for fast waves and Ion Cyclotron Waves according to PCI data and TORIC simulations for C-Mod parameters.

The most important modification associated with parallel electron dispersion is the change in the dielectric tensor element $P$, since the electron terms in $S$ and $D$ are negligible (cf table 2.4).
The element $P$ becomes:

$$\tilde{P} = -\frac{\omega_{pe}^2}{\omega^2} x_0^2 Z'(x_0e)$$

The function $x \rightarrow x^2 Z'(x)$ is plotted on figure 2-10. We can see that $|\tilde{P}|$ becomes smaller as $x \leq 1$ and $\text{Re}(\tilde{P})$ becomes positive. The fast wave root does not change much in this limit since $\tilde{P}$ does not appear in its dispersion relation. However the slow wave is strongly modified by parallel electron dispersion.

![Figure 2-10: $x \rightarrow x^2 Z'(x)$ function in the hot plasma dielectric tensor element $\tilde{P}$.](image)

In order to appreciate this, we write a dispersion relation for $n_x^2$ in the limit of cold ions but hot electrons, based on the general dispersion relation for $n_x^2$ in the finite Larmor radius approximation [Brambilla 98, p. 279]. For simplicity, we take $n_z = 0$ and substitute $n_\parallel^2$ and $n_\perp^2$ for $n_x^2$, using MAPLE. The obtained relation is simplified using the ordering $|\tilde{P}| \gg |S|, |D|$ in in table 2.4 and $b_p^2 \ll 1$:

$$(S + b_p^2 \tilde{P}) n_x^4 - \tilde{P} S n_x^2 + \tilde{P}(S^2 - D^2) = 0$$

(2.59)

While the dispersion relation has the same form as in the cold plasma limit, it is not a fourth-order polynomial equation in $n_x$ anymore as $\tilde{P}$ is a non-polynomial function of $n_x$. When the fast and
slow branches are well separated, the slow root can be approximated as:

\[ n_x^2 = \frac{S\tilde{P}}{S + \tilde{P}b_p^2} \]  

(2.60)

Away from the mode conversion layer, as \( S \gg b_p^2\tilde{P} \), this dispersion relation approximates that of Langmuir waves \( n_x^2 = \tilde{P} \). These waves can only propagate (albeit with strong damping) if \( \text{Re}(\tilde{P}) > 0 \), which, as we see on figure 2-10 corresponds to the acoustic range \( \frac{\omega}{k_v\nu_{th_e}} < 1 \) [Brambilla 98, p. 354]. We see that mode converted ion cyclotron waves can access this range due to the upshift in \( k_{||} \). This also suppress the ion-ion hybrid resonance \( S + Pb_p^2 \), since \( \tilde{P} \) becomes imaginary for \( \frac{\omega}{k_v\nu_{th_e}} \sim 1 \) and its real part changes sign as \( k_{||} \) increases further.

A typical dispersion curve can be seen on figure 2-11, which confirms the main features found in the analytical expression (equation 2.60). The parameters are \( \frac{\omega}{2\pi} = 50 \text{ MHz} \), \( n_e = 1 \times 10^{20} \text{ m}^{-3} \) and \( T_e = 5 \text{ keV} \), while \( b_p \) is constant at 0.05. As \( k_x \) increases, \( k_{||} \) is upshifted, and the two horizontal lines show the values of \( k_x \) for which \( \frac{\omega}{k_{||}v_{th_e}} \) reaches 1 and 1/3. The kinetic correction to the electron response allow mode converted (kinetic) ICW to propagate to the ion cyclotron layer, in contrast to their cold plasma counterparts, although the mode is strongly damped and is unlikely to reach the cyclotron layer with significant power. The colors show the imaginary part of \( k_x \), and we can see that the damping is significant. The ICW damps primarily on electrons near the mode conversion layer, but will damp strongly on ions as it propagates towards the ion cyclotron layer. As \( k_x \) increases, \( k_{||} \) is upshifted and the Doppler broadening of the ion cyclotron layer becomes more important. We can also see that the approximated FLR dispersion relation agrees well with the roots obtained from the full hot plasma formalism. This is not surprising, since we retained parallel electron dispersion in full and kinetic ion effects are negligible here, nonetheless it indicates that the physics of mode converted ICW can be well captured with an approximate dispersion relation using the hot plasma dielectric tensor truncated at first order in the FLR expansion.

As mentioned in the introduction, this mode conversion scenario was first studied by Perkins [Perkins 77], who derived a polynomial equation for \( n_x^2 \) in the kinetic limit for \( \tilde{P} \) ( \( \frac{\omega}{k_{||}v_{th_e}} \to 0 \)). The paper extended the terminology Ion Cyclotron Waves to this mode, which may be justified by the fact that the cold plasma ICW transitions to it continuously as the electron temperature is
Figure 2-11: Dispersion curves for finite electron temperature. The imaginary part is shown in color. The dotted line corresponds to the finite Larmor radius approximation, while the triangles are obtained by solving the full hot plasma dispersion relation.
increased. However, we must stress the kinetic character of this wave.

- **Warm ions, no poloidal field** - The limit $b_p \rightarrow 0$ corresponds to the slab model used to introduce fast wave propagation in the previous section, and therefore constitutes a natural starting point for studying temperature corrections around the mode conversion layer. Referring to table 2.4, the dominant corrections to the dispersion relation in the limit of warm ions and cold electrons correspond to an additional term at order $n_\perp^6$:

$$-\sigma n_\perp^6 + S n_\perp^4 - P(S - n_\parallel^2) n_\perp^2 + P(R - n_\parallel^2)(L - n_\parallel^2) = 0 \quad (2.61)$$

This dispersion relation is a polynomial equation in $n_\perp^2$, with an additional root $n_\perp^2 = \frac{S}{\sigma}$. This branch approximates the dispersion relation for the lowest order Ion Bernstein Wave [Bernstein 58, Ono 93]. Since $\sigma < 0$, the wave exists on the high field side of the mode conversion layer. Its perpendicular wavelength increase rapidly as its propagates away from it and $|S|$ increases. Therefore the assumption $k_\perp \rho_i < 1$ is expected to be violated and higher order terms in the FLR expansion can become important.

As discussed in the beginning of this section, it is possible to obtain a good representation of the IBW dispersion relation when $k_\perp \rho_i \approx 1$ by using a corrected expression for $\sigma$. As we can see on figure 2-12, the reduced FLR formulation yields results close to the hot plasma root.

- **Kinetic electrons, warm ions** - Poloidal field and ion temperature effects compete when simultaneously present. As $b_p$ increases, for example further away from the axis, the dispersion curves transition from the scenario in figure 2-12 (FW $\rightarrow$ IBW) to that in figure 2-11 (FW $\rightarrow$ ICW). This can be seen analytically by writing the dispersion relation in the limit $k_z = 0$ with finite $b_p$:

$$-\sigma n_x^6 + (S + \tilde{P}(b_p^2 + \sigma)) n_x^4 - \tilde{P} S n_x^2 + \tilde{P}(S^2 - D^2) = 0 \quad (2.62)$$

In this equation, we use the reduced FLR form for $\sigma$ (equation 2.53). We can identify the three modes mentioned above, in the limit when the roots are well separated:
Figure 2-12: Dispersion curves for finite ion temperature. The dotted line corresponds to the finite Larmor radius approximation, while the triangles are obtained by solving the full hot plasma dispersion relation.
\[ n_x^2 = \frac{S + \tilde{P}(b_p^2 + \sigma)}{\sigma} \quad \text{Ion Bernstein Wave} \]
\[ n_x^2 = \frac{\tilde{P}S}{S + \tilde{P}(b_p^2 + \sigma)} \quad \text{Ion Cyclotron Wave} \]
\[ n_x^2 = \frac{(S^2 - D^2)}{S} \quad \text{Fast magnetosonic wave} \]

Since \( b_p^2 \) and \( \sigma \) are respectively positive and negative, poloidal field effects and finite ion temperature corrections will compete in setting the sign of the term \((b_p^2 + \sigma)P\). This suggests the condition \( b_p^2 = -\sigma \) as a turning point between the two scenarios mentioned above. In order to verify this point numerically, we can explore the parameter space \((T_i, b_p)\) and for each value try to recognize dispersions curves of the type \( FW \rightarrow IBW \) or \( FW \rightarrow ICW \). A typical outcome of this parameter scan is shown on figure 2-13. We see that the transition region matches well with the condition \( b_p^2 = -\sigma \). A similar criterion for the transition IBW/ICW is much harder to find for the case \( k_z \neq 0 \). Numerically, we find that the transition occurs at lower values of \( b_p \) as \( k_z \) becomes more positive, and larger values of \( b_p \) for \( k_z \) more negative.

### 2.2.4 MC physics in tokamak conditions

From the dispersion relation studies in the previous subsection, we can draw a schematic of the predicted regions of propagation for the mode converted waves. This diagram is in figure 2-14. Parabolic profiles for the density and ion temperature are chosen, leading to the radial dependence of \( \sigma \) on the right-hand side plot. The condition \(-\sigma = b_p^2\) marks the transition from IBW to MCICW mode conversion, with \( b_p \) as on figure 2-6.

The green region around the cyclotron resonance layer indicates the extent of the local Doppler-broadening of the resonance, i.e. \( |\omega - \Omega_{ci, He}| < |k||v_{thi}|. \) The parallel wavevector is calculated from \( k_p = b_p k_R \), with \( k_R = 5\text{cm}^{-1} \) as a typical value. This simple estimate shows that the Doppler broadening can be significant as \( k_p \) is upshifted by poloidal field effects. Therefore, we expect significant ion damping from MCICWs far from the magnetic axis. Due to the combined effect of Landau damping on electrons and cyclotron damping on the minority ions, very little power will
Figure 2-13: Parameter scan showing mode conversion to IBW vs ICW.
Figure 2-14: Schematic representation of ICRF mode conversion in tokamak geometry. The peak density and ion temperature are $n_e0 = 1 \times 10^{20} m^{-3}$ and $T_i0 = 5$ keV.

remain in the MCICW as it reaches the cyclotron layer. Closer to the IBW/ICW transition point, electron damping should be dominant.

The effect of changes in the total plasma current $I_p$ is worth commenting based on this picture. Varying $I_p$ changes the safety factor at the edge but not in the center where sawtooth oscillations maintain $q \approx 1$. For higher currents, the parameter $b_p$ will increase at the edge of the plasma, resulting in a larger Doppler-broadening on the resonance layer and thus stronger ion damping. However, the changes in the ICW/IBW partition should be minimal.

### 2.3 Full wave TORIC simulations

The TORIC code is used extensively in this work for numerical simulations of wave propagation and damping in the ion cyclotron range of frequencies. The code was written by Marco Brambilla and is described in [Brambilla 99].
2.3.1 The TORIC code

The code TORIC solves the finite Larmor radius (FLR) wave equations in the Swanson, Colestock and Kabusha limit [Swanson 81, Colestock 83], augmented by appropriate FLR terms for electrons [Brambilla 89]. The SCK wave equations include all zero order terms in FLR expansion for ions and electrons but retain only the terms resonant at \( \omega = 2\Omega_{ci} \) in ion FLR current. These approximations to the dielectric tensor are equivalent to that used in the preceding section, including the adoption of the reduced FLR formalism. Following the Colestock and Kabusha approach, the wave equation is put into Galerkin’s weak-variational form by multiplying it with an arbitrary vector function satisfying the same boundary conditions as the electric field:

\[
\nabla \times \nabla \times \vec{E} = -\frac{\omega^2}{c^2} \vec{E} + \omega \mu_0 i (\vec{J} + \vec{J}_P)
\]

(2.63)

\[
\int dV \vec{F}_* \cdot [-\frac{c^2}{\omega^2} \nabla \times \nabla \times \vec{E} + \frac{c^2}{\omega} \mu_0 i (\vec{J} + \vec{J}_P)] = 0
\]

(2.64)

where \( \vec{F} \) is an arbitrary vector function satisfying the same boundary conditions as \( \vec{E} \).

The high frequency plasma current \( \vec{J}_P \) is reduced to the sum of the zero and first order contributions in the FLR expansion, calculated as integral operators. A spectral ansatz is used for the form of the electric field solution:

\[
\vec{E} = \sum_{n_\phi} e^{in_\phi \phi} \sum_{m_\theta} \vec{E}_{n_\phi,m_\theta}(\psi)e^{im_\theta \theta}
\]

(2.65)

Due to toroidal symmetry, the \( \{ \vec{E}_{n_\phi} \} \) functions are independent and can be solved for separately. An advantage of the spectral ansatz approach is that each \( (n_\phi, m_\theta) \) component has a well defined \( k_\parallel \) value:

\[
k_\parallel(n_\phi, m_\theta) = \frac{n_\phi B_\phi}{R} + \frac{m_\theta B_\theta}{r}
\]

(2.66)

Therefore a plasma dispersion function \( \tilde{Z} \) can in principle be used to treat parallel dispersion. The function is analogous to the Fried-Comte dispersion function \( Z \) used in the homogeneous case, but includes toroidal corrections. The deviations are important in the vicinity of the resonance layer and result in a broadening of the resonance domain. This is referred as toroidal broadening.
full treatment would be too computationally demanding, therefore in TORIC an approximate form for \( \tilde{Z} \) is used [Brambilla 99]. TORIC locates the radiating antenna and cavity in a vacuum volume surrounding the plasma. This region requires a different formulation in order to avoid numerical pollution. The code uses the Jiang, Wu and Povinelli scheme [Jiang 96] to this effect.

![Figure 2-15: Schematic of a tokamak plasma showing the \((\psi, \theta, \phi)\) and \((R, Z, \phi)\) coordinates used in TORIC to determine the location of an arbitrary point P in the plasma.](image)

- **Equilibrium model** - TORIC parametrizes an arbitrary MHD equilibrium using a \((\psi, \theta, \phi)\) coordinate system. The covariant metric of the magnetic coordinates is deduced through a change of variable form the \((R, Z, \phi)\) system with Jacobian \(J_p\):

\[
R = R_0 + X \quad X = X(\psi, \theta) \quad Z = Z(\psi, \theta) \quad g_{ij} = \begin{vmatrix} N_{\psi}^2 & G & 0 \\ G & N_{\tau}^2 & 0 \\ 0 & 0 & R^2 \end{vmatrix}
\]

\[
N_{\tau}^2 = \left( \frac{\partial X}{\partial \theta} \right)^2 + \left( \frac{\partial Z}{\partial \theta} \right)^2 \quad G = \left( \frac{\partial X}{\partial \psi} \right) \left( \frac{\partial X}{\partial \theta} \right) + \left( \frac{\partial Z}{\partial \psi} \right) \left( \frac{\partial Z}{\partial \theta} \right) \\
N_{\psi}^2 = \left( \frac{\partial X}{\partial \psi} \right)^2 + \left( \frac{\partial Z}{\partial \psi} \right)^2 \quad J_p = \left( \frac{\partial X}{\partial \tau} \right) \left( \frac{\partial Z}{\partial \theta} \right) - \left( \frac{\partial Z}{\partial \tau} \right) \left( \frac{\partial X}{\partial \theta} \right)
\]

Orthogonal unit vectors can be defined as:

\[
\vec{u}_\phi = \frac{J_p}{N_\tau} \nabla \psi \quad \vec{u}_\tau = N_\tau \left( \nabla \theta + \frac{G}{N_\tau^2} \nabla \psi \right) \quad \vec{u}_\phi = R \nabla \phi
\]
The static magnetic field has the following representation:

\[ \vec{B} = B_0 R_0 (a f(\theta) \nabla \phi \times \nabla \psi + g(\psi) \nabla \phi) = \frac{R_0}{R} B_0 (\sin \Theta \vec{u}_r + \cos \Theta \vec{u}_\phi) \quad (2.70) \]

where

\[ \tan \Theta = \frac{B_\tau}{B_\phi} = a \frac{N_{\tau} f(\psi)}{J_\rho g(\psi)} \quad (2.71) \]

While a tokamak equilibrium code (like EFIT) is most commonly used to specify the functions X, Z and Θ, an analytical equilibrium is also available:

\[ X(\psi, \theta) = \Delta(\psi) + a \psi \cos(\theta - \delta(\psi) \sin(\theta)) \quad Z(\psi, \theta) = a \kappa(\psi) \sin \theta \quad (2.72) \]

where \( \Delta(\psi) \) is the Shafranov shift, \( a \) is the minor radius, \( \delta(\psi) \) is the triangularity and \( \kappa(\psi) \) the ellipticity (or elongation). In this case, \( g(\psi) = 1 \), i.e. finite-\( \beta \) effects are ignored and \( f(\psi) \) is calculated from the current profile.

- **Computational resources** - The resolution of a TORIC run for a given toroidal mode number \( n_\phi \) is determined by the number of radial elements \( N_r \) and poloidal modes \( N_\theta \). As discussed in [Wright 04b], the discretization of the problem leads to a block tri-diagonal matrix with \( 3 \times N_r \) dense blocks of size \((6 \times N_\theta)^2\), which TORIC inverts. As a result, the memory and computation time demands are linear in \( N_r \), and respectively quadratic and cubic in \( N_\theta \). These requirements are too stringent for calculations on a single processor for mode conversion scenarios, but the operation can be distributed over multiple processors. A parallelized version of the code has been developed and implemented on the Marshall cluster at MIT [Wright 04b], allowing simulations with \( N_r = 500 \) and \( N_\theta = 255 \) in about one hour on 48 processors. The resolution is sufficient to resolve the mode converted waves, and convergence is reached.

### 2.3.2 Mode conversion simulations

- **Comparison with 1-D model** - Figures 2-16 to 2-19 show results from a TORIC simulations for a simple circular geometry and set of profiles. The intent here is to compare TORIC results with the conclusions from the preceding sections. The mode conversion scenario is D(\(^3\text{He})\) with 15 \% \(^3\text{He}\), \( n_\phi = 0 \), flat density and temperature profiles \((n_e = 8 \times 10^{19} m^{-3}, T_e = T_i = 3 \text{keV})\).
The values of $b_p$ are calculated assuming the wavevector component in the vertical direction is zero ($k_y = 0$).

Mode converted ICW are observed on the low field side of the mode conversion layer off the magnetic axis, for $b_p \lesssim 0.03$, while the IBW wavefronts are more clearly apparent on the $E_+$ two-dimensional plot. We see that the transition from IBW to ICW mode conversion occurs for values of $b_p$ close to that predicted from the dispersion relation in section 2.2 (compare for example with figure 2-13 with $T_i = 3keV$). The wavelengths for the two modes are also compatible with that predicted by the 1-D models. The ICW wavelength increases as $b_p$ increases away from the magnetic axis.

Figures 2-18 and 2-19 show two-dimensional absorption plots on ions and electrons on the same scale. We see that most of the power is damped on electrons in the region where ICWs propagate. Some ICW damping on ions occurs off-axis, consistent with the increased Doppler broadening of the Ion Cyclotron Layer for higher $k_\parallel$. The absorbed power density is highest in the IBW-ICW transition region, which is also the region where $E_\parallel$ is largest. From the dispersion curves in section 2.2, we expect the radial group velocity to be small in the region, resulting in large electric fields.

It is remarkable that these observations are qualitatively consistent with the conclusions of section 2.2. This indicates that many features of mode conversion in realistic tokamak conditions can still be determined and understood from the dispersion relation, even when complicated poloidal field effects come into play. However, there is a major difference. The TORIC predictions are not up-down symmetric, even with $n_\phi = 0$ as in this case. The effect is clearest in the electron deposition on figure 2-18, but is also present in the parallel electric field predictions. This was not expected from our analysis in the last section: the dielectric tensor is formally unchanged at symmetric points above and below the midplane, and for $n_\phi = 0$, the roots of the local dispersion relation should therefore be up-down symmetric. The predicted up-down asymmetry in the wave fields appears to be therefore a two-dimensional full-wave effect, i.e. resulting from the non-local nature of the wave equation in the spatially non-uniform plasma cross-section. While simplified models may be useful for understanding the wave physics, quantitative predictions for the two-dimensional power deposition in the mode conversion regime require solving the wave equation in the complete plasma cross-section.
Figure 2-16: Parallel electric field predicted by TORIC.

Figure 2-17: $E_+$ electric field component predicted by TORIC.

Figure 2-18: Absorption on electrons predicted by TORIC.

Figure 2-19: Absorption on $^3\text{He}$ ions predicted by TORIC.
Simulations in shaped plasmas and for $n_\phi \neq 0$ - TORIC runs for non-circular plasma shapes and realistic temperature and densities profiles yield the same qualitative picture as in the cases shown above. We will show here some examples of simulations for C-Mod plasmas and use them to highlight some further properties of mode conversion at the ion-ion hybrid layer. Figure 2-20 shows two simulations for the same plasma parameters but opposite toroidal mode numbers $n_\phi = \pm 7$, for an D($^3$He) C-Mod plasma. In the modeled experiment, the magnetic field was 5.6 T and toroidal current 600 kA. The density and temperature profiles were taken from experiment, with central values of 3.5 keV and $1.7 \times 10^{20} m^{-3}$ respectively. Qualitatively, varying $n_\phi$ moves the mode converted waves along in the vertical direction: in the cross-section, the mode converted ICW below the midplane start propagating closer to the magnetic axis for $n_\phi = 7$ than they do for $n_\phi = -7$. The ICW wavelength below the midplane is also shorter for $n_\phi = -7$ than for $n_\phi = 7$. The opposite behavior occurs above the midplane. The fields are also stronger close to the axis, which is consistent with the focusing of the incident fast waves.

![Figure 2-20: Comparison of the parallel electric fields predicted by TORIC for $n_\phi = \pm 7$ in a C-Mod plasma.](image)

These observations can be understood in terms of dependency of $k_\parallel$ on $n_\phi$ and the dispersion relation for MC ICW, which imposes $|k_\parallel| \sim (2 - 3) \frac{\omega}{v_{\phi e}}$. At a given location, a lower $m_\theta$ value and
therefore longer wavelength will be needed to upshift $k_{\parallel}$ in this range for $n_\phi > 0$, while a larger $m_\theta$ value and therefore shorter wavelength will be needed to upshift $k_{\parallel}$ in this range for $n_\phi < 0$. This partially explains the behavior observed for MC ICW below the midplane.

### 2.3.3 Scaling parameters

While the picture for mode conversion discussed above is systematically observed in simulations of typical C-Mod conditions, it may be interesting to discuss its validity to a broader range of parameters and to other tokamak experiments. While many plasma, RF and geometrical parameters are involved here, it is important to realize that some of these parameters are common or similar in most experiments, and that the chosen mode conversion scenario also determines many of them directly. For example, the inverse aspect ratio $\frac{R}{a}$ is very similar in most tokamak experiments. As pointed out in the preceding section, the safety factor profiles are also very similar, which implies that the poloidal field parameter $b_p$ will have the same radial variation in all tokamak experiments. The profiles of density, temperature are also very self-similar, but their peak values can vary by about an order of magnitude. The choice of an ICRF scenario determines the relative position of the ion cyclotron, mode conversion and cutoff layers in the plasma cross section. It also imposes either the wave frequency $\omega$ or the central magnetic field $B_{\phi,0}$.

- **IBW/ICW partition** - While the partition between electron damping from Ion Bernstein and Ion Cyclotron Waves can only be evaluated with full wave modeling, the condition $b_p^2 = -\sigma$ indicates that plasmas with larger ion pressures or with central safety factors significantly above unity will be more favourable to mode conversion to Ion Bernstein Waves. This can seen from $b_p \approx \frac{\epsilon}{q}$, where $\epsilon$ is the local inverse aspect ratio and $q$ the local safety factor, and $\sigma \sim \beta_i$ where $\beta_i$ is the ion beta. Note that formally, the ratio $\frac{\epsilon}{b_p^2}$ is of order of the local poloidal beta, evaluated with the local pressure and the local poloidal field. The condition $b_p^2 = -\sigma$ can thus be rewritten as a threshold on the local poloidal beta, which goes to infinity at the magnetic axis and vanishes at the edge as the pressure becomes small.

- **Effective plasma size** - Depending on their wavelength, ICRF waves see the plasma as an effectively larger or smaller volume. This determines the effective scalelengths for the density, temperature and poloidal field profiles, and for the different ICRF layers. Taking $n^2_\perp \approx 2R$ for the
fast wave:

\[ k_\perp a \approx \frac{\omega}{c} \frac{\omega_{pi}}{\Omega_{ci}} a \propto a \sqrt{n_e} \]  

(2.73)

since \( \frac{\omega}{\Omega_{ci}} \) and \( \frac{\omega_{pe}}{\omega_{pi}} \) are entirely determined by the chosen ICRF scenario. This scaling also holds for MCICWs based on its dispersion relation in section 2-2. The effective change in size affects the evanescent layer length and the size of the propagating region (cavity) for the fast waves. It will also determine the focusing of the fast waves in the plasma core. Finally, it will also determine the effective distance between the mode conversion layer and the ion cyclotron layer (hence decreasing direct ion damping for the mode converted waves in larger size plasmas).

For Ion Bernstein Waves, the dispersion relation \( n_{\perp}^2 \approx -\frac{\xi}{\eta} \) yields:

\[ k_{\perp,IBW} a \propto \frac{\omega}{v_{thi}} \propto \frac{a}{\rho_i} \]  

(2.74)

where the ion Larmor radius \( \rho_i \propto \frac{\sqrt{T_i}}{B_\phi} \) has no density dependence. The IBW wavelengths is usually the smallest scalelength to be resolved in the simulations.

- **Antenna spectrum** - To the extent that the antenna spectrum determines \( k_\parallel \), it enters in the parallel phase speed \( \frac{\omega}{k_\parallel} \) and can be compared to the electron thermal velocity. It also enters the fast wave cutoff condition \( R = n_{\parallel}^2 \). We can define two reference toroidal mode numbers, respectively capturing the \( n_\phi \) value for which \( \frac{\omega}{k_\parallel} = v_{the} \) at the plasma center,

\[ \frac{n_\phi}{R} = \frac{\omega}{v_{the}} \propto \frac{B_\phi}{\sqrt{T_e}} \]  

(2.75)

and the \( n_\phi \) value for which the fast wave is cutoff at the plasma center.

\[ \frac{n_\phi}{R} = \frac{\omega_{pi}}{\Omega_{ci}} \frac{\omega}{c} \left( 1 + \frac{\omega}{\Omega_{ci}} \right)^{-1/2} \propto \sqrt{n_e} \]  

(2.76)

again for a given ICRF scenario. Hence, schemes based on the internal resonator model \([\text{Ram 96}]\), in which the high field side cutoff is near the plasma core require to launch fast waves at higher \( n_\phi \) values in higher density plasmas.

- **Doppler broadening of the Ion Cyclotron layer** - Doppler-broadening of the ion cyclotron layer near the mode conversion can result in direct ion damping of the mode converted waves,
which in MCEH scenarios is considered parasitic. The width of the Doppler broadened ion cyclotron layer is determined by $k_{||}$, the ion thermal speed and the magnetic configuration (through $\Omega_{ci} \propto \frac{1}{R}$). For mode converted Ion Cyclotron Waves, we can take $k_{||} \sim 2\frac{\omega}{v_{thi}}$ and evaluate $\exp\left(-\frac{\omega-\Omega_{ci}}{k_{||}v_{thi}}\right)$ at the mode conversion layer as a figure of merit for ion cyclotron damping. The scenario (magnetic configuration and plasma composition) will determine the distance between the ion cyclotron layer and the mode conversion layer. Using $k_{||} \sim 2\frac{\omega}{v_{thi}}$,

$$\exp\left(-\frac{\omega-\Omega_{ci}}{k_{||}v_{thi}}\right) \propto \exp\left(-\sqrt{\frac{A}{T_i}}\right)$$

(2.77)

where A the ion mass number. Note that $k_{\perp}$ and the size of the tokamak are only involved here by determining the effective distance between the mode conversion layer and the ion cyclotron layer.
Chapter 3

Mode conversion current drive

3.1 Current drive physics

Mode conversion of ICRF waves at the ion-ion hybrid layer results in a strong interaction with the electron population around the mode conversion layer. In addition to localized heating, the subsequent distortion of the electron distribution function $f_e$ can give rise to a net parallel current density $j_{\parallel}$:

$$j_{\parallel} = \int v_{\parallel} f_e d^3v$$  \hspace{1cm} (3.1)

The relationship between this interaction, determined mainly by the wave propagation and damping characteristics, and the resulting driven currents constitute the central question underlying current drive in tokamaks. A large body of literature has been devoted to this issue, mainly in order to develop techniques for non-inductive current drive. Our aim here is to apply this formalism to the particular case of MCCD. Since the topic can in fact be approached in quite general terms, common to any current drive technique, we can be guided here by excellent review papers and reports like [Fisch 87], [Uckan 85] or [Kolesnichenko 86]. A key parameter for current drive is the efficiency, which corresponds to the ratio of the obtained current to the power required to drive it, either locally or for the entire plasma column.
3.1.1 Basic efficiency scalings

An elementary model can give some general scaling properties of the efficiency ratio $J/P$. We assume that the currents are being driven by electrons with a density $n_e$ and a relative velocity $v_{rel}$ with respect to the background Maxwellian electrons and ions. As we will see later, for current drive with plasma waves, $v_{rel}$ can be taken as the parallel phase velocity of the wave driving the currents. The resulting current density is:

$$J = -en_e v_{rel}$$

(3.2)

Coulomb collisions with the background electrons create a drag/friction force:

$$F = m_e n_e v_{p||} v_{rel}$$

(3.3)

where $v_{p||}$ is the collision frequency for parallel momentum. The power required to maintain the current and assumed to be supplied by the wave is then:

$$P = F v_{rel}$$

(3.4)

Based on these expressions, we can form the efficiency ratio $J/P$:

$$\frac{|J|}{P} = \frac{e}{m_e v_{p||} v_{rel}}$$

(3.5)

With the normalizations $\tilde{J} = \frac{-J}{n_e e v_{the}}$ and $\tilde{P} = \frac{P}{m_e n_e v_{the} v_0}$ where $v_{the}$ is the electron thermal velocity and $v_0 = \omega_{pe} ln\Lambda / 4\pi n_e v_{the}^3$,

$$\eta = \frac{|\tilde{J}|}{\tilde{P}} = \frac{4\pi^2 e^2 m_e v_{the}^2}{n_e e^3 v_{the}^3 \ln \Lambda} \frac{\tilde{J}}{\tilde{P}} = \frac{0.384}{\ln \Lambda} \frac{T_e}{n_{20}} \frac{\tilde{J}}{\tilde{P}}$$

(3.6)

where $T_e$ is in keV and $n_{20}$ is the electron density in units of $10^{20}m^{-3}$.

This elementary treatment suggests a normalization of the current drive efficiency $\tilde{J}/\tilde{P}$, which we replace by a normalized efficiency factor $\tilde{\eta} = \frac{\tilde{J}}{\tilde{P}}$. We can push the model further by looking at the dependence of the collision frequency $\nu_{p||}$ on $u = \frac{v_{rel}}{v_{the}}$. In the limit $u \ll 1$, the electrons driving the currents belong to the bulk of the electron distribution function and the collision frequency.
is practically independent of the relative velocity $v_{rel}$. This suggests $\tilde{\eta} \sim 1/u$. In the opposite limit, $u \gg 1$, the current is carried by high velocity electrons and the collision frequency depends strongly on the relative velocity, $\nu \sim 1/u^3$, which suggests $\tilde{\eta} \sim u^2$ in this case. A physical interpretation of these limits can be obtained from the relative changes in momentum and energy in the two cases, from which we might expect it is easier to accelerate slow electrons than fast electrons [Fisch 87]. Indeed, for a given change in velocity $\Delta v$, the obtained current can be written $\Delta J = q\Delta v_\parallel$ while the required energy is $\Delta E = m v_\parallel \Delta v_\parallel$. Therefore $\Delta J/\Delta E$ scales as $1/v_\parallel$.

While this is physically correct at low velocities compared to the thermal speed, higher velocity electrons become much less collisional and therefore experience less drag. This compensates for the higher cost in energy associated with accelerating high velocity electrons. Note that while this simple model can give physical intuition about the scaling properties for the efficiency parameter, it does not yield accurate estimates for its magnitude. For example, as illustrated in [Fisch 80], significant enhancements in the efficiency can appear at high velocities when considering distortions and collisional effects for the entire distribution function in velocity space. We will discuss this formalism in the following section.

The resonance condition for the considered damping mechanism determines $u$ from the wave characteristics. Mode converted waves have $\omega \ll \Omega_{ce}$ and the damping follows the Cerenkov condition with Doppler broadening $\omega - k_\parallel v_\parallel = 0$. This indicates that the wave will transfer momentum to electrons with parallel velocity $v_\parallel$ equal to its parallel phase speed $\omega / k_\parallel$. For the mode converted ion cyclotron waves, the dispersion relation imposes $\frac{\omega}{k_\parallel v_{the}} \lesssim 1$ so that we expect to be in the situation $w \lesssim 1$. MCCD with mode converted Ion Cyclotron Waves is therefore a low to intermediate parallel phase velocity current drive technique. Since the current are driven near the bulk of the distribution, we expect the efficiency to be limited.
3.1.2 Fokker-Planck and quasilinear treatments

More detailed calculations of the efficiency factor require predicting the evolution of the entire electron distribution function $f_e$ in response to the wave fields. The relevant physics is contained in the Fokker-Planck equation:

$$\frac{df_e}{dt} = C_{\text{col}}(f_e) - \frac{\partial}{\partial \tilde{v}} \cdot \tilde{S}_{RF}$$  \hspace{1cm} (3.7)

The collision operator $C_{\text{col}}(f_e)$ captures the effect of self-collisions and collisions with other plasma species. Self collisions tend to drive the distribution function $f_e$ to an isotropic Maxwellian with no net currents. Collisions with ions result mainly in pitch angle scattering and an exchange of relative momentum, since the mass ratio $m_e/m_i$ prevents significant energy exchange between ions and electrons and implies $v_{thi} \ll v_{the}$ for $T_i \sim T_e$.

The second term on the right side captures the effect of wave-particles interactions associated with the wave fields on a collisional timescale. It can be written as the divergence of a wave-induced flux $\tilde{S}_{RF}$ in velocity space, thus ensuring conservation for the number of particles. The quasilinear formalism describes $\tilde{S}_{RF}$ in terms of an operator $D_{QL}$ acting on the distribution function $f_e$:

$$\tilde{S}_{RF} = -D_{QL} \cdot \frac{\partial f_e}{\partial \tilde{v}}$$  \hspace{1cm} (3.8)

Physically, the quasilinear term gives rise to diffusion in velocity space, resulting in a local flattening of the distribution function. A quite general method for evaluating the quasilinear diffusion operator for given wave fields has been given by Kennel and Engelmann [Kennel 66]. This approach can be followed to derive a diffusion operator from the fields calculated by TORIC, and we will shortly discuss this derivation in the following section. Alternatively, the form of the operator can be anticipated based on simple considerations on the wave polarization and damping mechanism. For instance, for Landau damping of electrostatic waves, like Ion Bernstein Waves, the quasilinear diffusion operator will have the following form [Brambilla 98, eq. 44.2 p.545]:

$$D_{LD} = \frac{|E||^2}{8\epsilon_0\omega^2} \left( \frac{e}{m_e} \right)^2 \delta (\omega - k||v||) \tilde{u}_||\tilde{u}_||$$  \hspace{1cm} (3.9)
where the delta function corresponds to the condition for Cerenkov resonance, and \( u_\| \) is the unit vector in the parallel direction. For low frequency Alfven waves with \( \omega \lesssim \Omega_{ci} \ll \Omega_{ce} \), like fast magnetosonic waves, both the Landau damping and magnetic pumping forces are involved. The two forces are out of phase and their relative contributions can be related through the MHD frozen-in law \( \vec{E} + \vec{v} \times \vec{B} \), where \( \vec{v} \) is the fluid velocity. This suggests the following form for the quasilinear diffusion operator [Fisch 85]:

\[
D_{AW} = \frac{|E_\||^2}{8\varepsilon_0\omega^2} \left( \frac{e}{m_e} \right)^2 \delta \left( \omega - k_\| v_\| \right) (2v_{the}^2 - v_{\perp}^2)^2 \vec{u}_\| \vec{u}_\|
\]

The total derivative \( \frac{df}{dt} \) captures the explicit time dependence \( \frac{\partial f}{\partial t} \) and the effect of spatial gradients and of external forces. The gradient terms lead to spatial diffusion and transport, and will not be considered in this work. The electromagnetic forces can be separated in three contributions: the wave fields, the parallel electric field leading to the ohmic current and the static equilibrium magnetic field. The first contribution, from the wave fields, occurs on timescales much slower than considered in the Fokker-Planck equation, and its effect is captured in the quasilinear diffusion operator. The second term, related to the ohmic electric field, is usually very important for current drive schemes involving fast electrons, even the usual situation when only very few electrons runaway. It is not expected to be as important for schemes with low parallel phase velocities since the wave-particle interaction occurs in the bulk of the distribution function. The last contribution, from the static magnetic fields, leads to particle orbits in toroidal geometry, and it will be important if the collisions are too infrequent to prevent these orbits from completing. This will be the case for fusion-grade plasmas in tokamaks, and we will discuss the consequences of electron orbits in the following subsection.

If the effects of spatial diffusion, particle orbits and of the ohmic electric field are ignored, the Fokker-Planck equation for the steady state distribution function \( \frac{\partial f_{e1}}{\partial t} = 0 \) reduces to:

\[
C_{col}(f_{e1}) + \frac{\partial}{\partial \vec{u}} \cdot D_{QL} \frac{\partial f_{e1}}{\partial \vec{u}} = 0
\]

Physically, this equation captures the equilibration between wave-induced diffusion in velocity space and collisional relaxation towards a Maxwellian. The input of energy and momentum associated with the quasilinear diffusion is balanced by collisions with the background plasma. The
power transferred from the wave to the plasma or *quasilinear heating rate* can be obtained by considering only the change in energy associated with the quasilinear term for $f_{e1}$:

\[
P = \int \frac{1}{2} m v^2 \left( \frac{\partial f_{e1}}{\partial t} \right)_R d^3 v = - \int m v \cdot D_{QL} \frac{\partial f_{e1}}{\partial v} d^3 v
\]  

(3.12)

The right side expression is obtained through integration by parts. The driven current density is:

\[
\hat{j} = \int v_{\parallel} f_{e1} d^3 v
\]  

(3.13)

Since the Fokker-Planck equation is in general non-linear (through the collisional operator, not the quasilinear term) and too complicated for analytical treatments, it is usually necessary to solve for $f_{e1}$ numerically through iterative methods. When convergence is obtained, $P$ and $\hat{j}_{\parallel}$ can be calculated. This is the approach taken by *Fokker-Planck codes*, including the DKE code [Decker 05] which we will use in the following section to calculate the efficiency in MCCD scenarios. An alternative approach for calculating the efficiency is the *adjoint method* [Antonsen 82, Fisch 87], which only requires solving the Fokker-Planck equation once for a given equilibrium. It will be generally valid when the quasilinear diffusion induces only small distortions of $f_e$ from a Maxwellian.

The technique exploits the self-adjoint character of the linearized collision operator to introduce a response function $\chi$ such that:

\[
C_{col} (\chi (\vec{v})) = -qv_{\parallel}
\]  

(3.14)

\[
\hat{j}_{\parallel} = \int d^3 v' S_{RF} (\vec{v}') \cdot \frac{\partial}{\partial \vec{v}'} \chi (\vec{v}')
\]  

(3.15)

The quasilinear flux and absorbed power are calculated assuming $f_{e1}$ is close to a Maxwellian. This reduces the efficiency calculation to the determination of the response function $\chi$, which depends only on the magnetic equilibrium and is usually numerically determined. The Ehst-Karney parametrization is based on a numerical implementation of the adjoint method by Karney [Karney 89]. Figure 3-1 shows the predicted efficiency curve versus $u$ for the two forms of the diffusion operator discussed above.
Figure 3-1: Current drive efficiency, ignoring magnetic trapping. The Ehst-Karney parametrization is described in section 3.1.4.

Figure 3-2: Current drive efficiency, with effects from magnetic trapping. The Ehst-Karney parametrization is described in section 3.1.4.

3.1.3 Magnetic trapping

- Electron orbits in tokamaks - Particle orbits in toroidal geometry can be determined from three approximate invariants for the guiding centers:

\[
\begin{align*}
\text{Energy} & \quad \mathcal{E} = \frac{1}{2} mv^2 + Z e \Phi \\
\text{Magnetic moment} & \quad \mu = \frac{mv^2}{2B} \\
\text{Toroidal canonical angular momentum} & \quad p_\phi = m R v_\parallel \frac{B_\phi}{B} - Z e \frac{\psi_p}{2\pi}
\end{align*}
\]

where \( \Phi \) is the potential and \( \psi_p \) is the poloidal flux.

Particles staying on flux surfaces are subject to a stronger magnetic field as they move towards the inboard side of the torus. Assuming that the potential \( \Phi \) is constant along the orbit (implying \( v^2 = \text{constant} \)), the orbits can be classified according to the evolution \( v_\parallel^2 = 2(\mathcal{E} - Z e \Phi - \mu B)/m \).
If $v_{||}^2 > 0$, the particle is called *passing* or *circulating*, and the projection of its orbit on the poloidal plane will span the entire flux surface. If not, the particle can only move on the outboard side of the flux surface. It will be reflected by the mirror $F = -\mu \nabla_{||} B$ at the *turning* point, where $v_{||} = 0$. These particles are referred as *trapped*. In a low beta tokamak with circular cross section and $q \sim 1$, $B \approx \frac{R_0 B_0}{R}$ and $v_{||}^2 = v^2 \left[ 1 - \lambda (1 - \epsilon \cos \theta) \right]$, where $\epsilon = r/R_0$ is the local inverse ratio and $\lambda = \frac{2 \mu B_0}{m v^2}$. Considering again orbits not deviating too much from flux surfaces, particles are circulating for $0 < \lambda < \epsilon$ and trapped for $1 - \epsilon < \lambda < 1 + \epsilon$, i.e. $\frac{v_{||}}{v_{\perp}} < \left( \frac{2 \epsilon}{1+\epsilon} \right)^{1/2}$ at the outer midplane point of the flux surface. For an isotropic maxwellian, the trapped particle fraction is therefore $f_t = \left( \frac{2 \epsilon}{1+\epsilon} \right)^{1/2}$. For a large aspect ratio tokamak, $f_t \approx \sqrt{2 \epsilon}$.

Since $p_\phi \approx m R v_{||} - Z e \frac{\psi_0}{2 R}$, the trapped particles deviate from magnetic surfaces by $\delta r = \frac{\delta \psi_0}{2 \pi R B_0} \sim \epsilon^{1/2} \rho_p$ where $\rho_p$ is the poloidal gyroradius. The orbits are referred as *banana orbits*. The passing particles deviate by $\delta_r \sim \frac{q_0}{1+\epsilon}$, which is usually much smaller than the shift for trapped particles. The poloidal gyroradius for electrons is very small compared to the size of the plasma in most medium to large scale present day tokamaks, and we can therefore usually neglect the excursion of the electrons from their flux surfaces as they complete their orbits. This assumption will nonetheless fail near the magnetic axis, as the orbit width $\frac{q_0 \rho}{\epsilon^{1/2}}$ formally becomes comparable to the minor radius $r$. Accordingly, the trapped particle orbits near the axis can have significantly different shapes than that of banana orbits and are referred as *potato* orbits. In the potato-orbit region, defined by $r \sim r_p = \left( q^2 \rho^2 R \right)^{1/3}$, particle crossing the magnetic axis can be trapped if $\frac{v_{||}}{v_{\perp}} \lesssim \left( \frac{q_0 \rho}{R} \right)^{1/3}$. Therefore the trapped particle fraction does not go to zero at the magnetic axis, $f_t(r = 0) = \left( \frac{q_0 \rho}{R} \right)^{1/3}$. This region is small enough to be irrelevant for current drive applications, however potato orbits will play an important role when considering neoclassical conductivity, as we will do in chapter 5.

Since electron orbits do not deviate much from flux surfaces, for most applications it is appropriate to neglect their excursions entirely and treat each flux surface independently. Due to magnetic trapping, the distribution function will be different at each point, and will contain the entire electron population only at the outer midplane where the magnetic field has its minimum. At this point, the trapping condition written above can be visualized easily on a velocity space diagram and defines a *trapped region* (see for example figure 3-9).

- **Implications for current drive** - Particles in the trapped region complete closed periodic
orbits with $v_\parallel$ changing sign at the banana tips, and therefore they can be expected to carry no net parallel currents. Since this holds for any particle in the trapped region, it follows that imparting momentum to trapped electrons will not result in current drive. Therefore magnetic trapping will potentially reduce the current drive efficiency, as seen on figures 3-1 and 3-2. As the trapped particle fraction increases with radius, this reduction is also more severe for off-axis current drive.

The inclusion of magnetic trapping into the Fokker-Planck calculations for current drive in the preceding subsection is relatively simple as long as the characteristic timescale for completing trapped orbits is shorter than the collisional timescales involved in current drive. Trapped orbits are complete within a bounce time $\tau_B$ [Wesson 96]:

$$\tau_B = \int \frac{dl}{v_\parallel} \approx \frac{2\pi qR}{v_{\perp,0}} \left( \frac{2R}{r} \right)^{1/2}$$

(3.16)

where $v_{\perp,0}$ is the perpendicular velocity at the low field side point of the orbit.

This time must be compared to two other characteristic timescales associated with collision, the electron collision time $\tau_e$ and the effective collision time for detrapping $\tau_{\text{detrapp}} \approx \frac{2R}{R_e} \tau_e$, which corresponds to the time it takes an electron to diffuse out of the trapped region due to collisions. The collisionality ratio $\nu_e = \frac{\tau_B}{\tau_{\text{detrapp}}}$ must be less than one for trapped orbits to exist. $\nu_e \sim 0.01 - 0.1$ is typical in the core of C-Mod plasmas. Since the inverse aspect ratio $R/a$ is large, this also implies $\tau_B \ll \tau_e$.

One can therefore assume that the electrons will complete multiple trapped orbits within the collisional timescale involved in current drive. The Fokker-Planck equation is thus bounce-averaged, i.e. averaged over a bounce orbit:

$$\langle Q \rangle_b = \frac{1}{\tau_B} \int \frac{dl}{v_\parallel} Q$$

(3.17)

Although formally well defined, this procedure is by no means trivial, and constitutes an important step in determining the current drive efficiency from Fokker-Planck treatments when trapping is important. The adjoint method can be applied to the bounce-averaged Fokker-Planck equation: response functions need to be calculated on each flux surface given a specific magnetic equilibrium. After bounce-averaging of the quasilinear flux $S_{RF}$, the efficiency can be calculating using the same formulas as in the last subsection.
In general, magnetic trapping will significantly reduce the efficiency of low parallel phase velocity current drive techniques, since in this situation the quasilinear interaction occurs largely with trapped electrons. The effect of magnetic trapping may actually be more subtle than canceling the contribution of trapped electrons to the driven currents. Collisions permanently shuffle particles in and out of the trapped region near the trapped passing boundary. Trapped orbits symmetrize the distribution function around the $v_\perp = 0$ axis, resulting in an exchange of particles between the trapped-passing boundaries on both sides. As discussed first by Ohkawa [Ohkawa 76], this effect can in fact lead to current drive, if the quasilinear diffusion occurs asymmetrical with nearly trapped passing particles. This will push electrons into the trapped region and effectively populate the distribution on the other side of the $v_\perp = 0$ axis through detrapping. Therefore currents will be obtained in the opposite direction that the predicted Fisch-Boozer currents [Fisch 80] in absence of trapping.

### 3.1.4 Ehst-Karney parametrization

Analytic calculations of current drive efficiencies provide insight but are often only limited to simple geometries and problems, so that practical constraints cannot be easily studied. On the other hand, numerical calculations with Fokker-Planck codes and the quasilinear operator formalism can give very good predictions for experimental scenarios, but are not always well suited to explore a parameter space or to optimize the efficiency for a particular scheme. A good compromise is the use of parametrizations for the current drive efficiency, which are obtained by fitting the numerical results with analytical expressions.

A convenient parametrization is that obtained by Ehst and Karney [Ehst 91] in the late 1980’s. The RF driven current density on a given flux surface $\psi$ enclosing a volume $V$ with a field line length $L = \int dl$ is expressed as:

$$j_{RF}(\psi) = \frac{dV}{d\psi} \frac{< p_{RF} >_\psi}{L} \eta$$  \hspace{1cm} (3.18)

In this expression, $\frac{dV}{d\psi} < p_{RF} >_\psi$ is the power absorbed in a volume defined by a differential flux function $d\psi$. $L$ is then the ratio between differential volume and area. The efficiency parameter is
normalized:

\[ \eta = 38.4 \times 10^{18} \frac{T_e}{n_e \ln \Lambda} \tilde{\eta}(Z_{\text{eff}}, \epsilon, \theta, w = \frac{v_\parallel}{v_{\text{the}}}) \]  

(3.19)

The calculations are performed for two particular forms of the quasilinear diffusion operator, already discussed above. The parameter \( \lambda_t = 1 - B(\psi, \theta)/B(\psi, \theta = 0) \) determines the location on the flux surface. The analytical expressions obtained in both cases are reproduced below for reference.

- **Alfven wave (AW)** - \( D_{\text{AW}} = D_0 \delta (\omega - k_{||} v_\parallel) (2v_{\text{the}}^2 - v_\perp^2)^2 \tilde{u}_{||} \tilde{u}_{||} \)

\[
\tilde{\eta}_0 = \left[ \frac{11.91}{(0.678 + Z)} + \frac{4.13}{Z^{0.707}} + \frac{4w^2}{5 + Z} \right] \tilde{\eta}_0 \quad (3.20)
\]

\[
\tilde{\eta} = (1 - 0.0987 e^{-\left(\frac{1-\lambda_t^2 w^2}{\lambda_t^2 w^2}\right)^{1.48}}) \left(1 + 12.3 \left(\frac{\lambda_t}{w}\right)^{3.0} \right) \left[1 - \frac{e^{0.77 \sqrt{12.25 + w^2}}}{3.5 e^{0.77 + w}} \right] \tilde{\eta}_0 \quad (3.21)
\]

- **Landau damping (LD)** - \( D_{\text{LD}} = D_0 \delta (\omega - k_{||} v_\parallel) \tilde{u}_{||} \tilde{u}_{||} \)

\[
\tilde{\eta}_0 = \left[ \frac{3.0}{Z} + \frac{3.83}{Z^{0.707}} + \frac{4w^2}{5 + Z} \right] \tilde{\eta}_0 \quad (3.22)
\]

\[
\tilde{\eta} = (1 - 0.389 e^{-\left(\frac{1-\lambda_t^2 w^2}{\lambda_t^2 w^2}\right)^{1.38}}) \left[1 - \frac{e^{0.77 \sqrt{12.25 + w^2}}}{3.5 e^{0.77 + w}} \right] \tilde{\eta}_0 \quad (3.23)
\]

As we can see on figure 3-2, the Ehsst-Karney parametrization indicates a significant reduction of the current drive efficiency for low parallel phase velocities. Magnetic trapping is thus expected to reduce the MCCD efficiency significantly. The effect will be dependent on the damping location in the plasma cross section. In order to illustrate this point, we plot the predicted efficiency as a function of the damping location in a typical C-Mod plasma cross section in figures 3-3 and 3-4 for the LD and AW damping formulas respectively. In obtaining these plots, we assumed \( w = \frac{\omega}{k_{||} v_{\text{the}}} = \frac{1}{2} \), flat profiles of density and temperature, and we inferred \( \epsilon \) and \( \theta \) from an experimental C-Mod equilibrium. We see that the efficiency is strongly reduced by magnetic trapping, by a factor 2 for \( r/a > 0.3 \) compared to the center. The reduction is stronger on the low field side.
than on the high field side, as expected from the fact that most trapped particles orbit do not reach the high field side point of their flux surfaces. The effect of trapping is also more severe for the Alfvén wave polarization: the magnetic pumping contribution to the current drive efficiency has a $v_\perp^4$ dependence [Fisch 80], and therefore the quasilinear diffusion will be stronger at higher $v_\perp$ than for Landau damping. Therefore quasilinear diffusion with the Alfvén Wave polarization will more readily occur inside the trapped region.

### 3.2 MCCD simulations

#### 3.2.1 Simulations based on the Ehst-Karney parametrization

The electric field solution in TORIC provides all the necessary parameters to calculate the currents driven by ICRF waves as they damp on electrons. As we mentioned in the previous chapter, the spectral representation adopted in TORIC decomposes the fields in terms of modes with well defined $k_{||}$ values, and the current drive efficiency formalism developed in the preceding section
can be readily used. A current drive calculation using the Ehst-Karney parametrization was implemented in TORIC as a routine post-processing package by John Wright and Paul Bonoli in the late 1990s [Bonoli 00]. The Ehst-Karney Alfven Wave damping parametrization was found to yield accurate estimates for current drive from fast magnetosonic waves [Wright 98]. On the other hand, mode converted Ion Bernstein Waves have a more electrostatic character, and, as we mentioned earlier, the Landau damping parametrization should be more appropriate in this case. The IBW and fast wave contributions to the electron damping can be distinguished on the basis that the IBW dispersion relation involves first order FLR terms at order $n_\perp^6$. It is therefore possible to use the Landau damping formula to calculate the currents driven by IBW. On the other hand, no such distinction is possible between fast waves and mode converted Ion Cyclotron Waves, and therefore currents driven by mode converted ICWs have to be calculated with the same parametrization as for the fast wave, i.e. by default the Alfven Wave damping formula.

Figures 3-5 and 3-6 show examples of MCCD profiles calculated from TORIC in a circular cross section plasma. The parameters are inspired from an MCCD target on Alcator C-Mod. The scenario is D($^3$He), with 20\% $^3$He. The toroidal field is adjusted at 5.4 T to obtain mode conversion near the axis at 50 MHz. The current profile is chosen so that $q \approx 1$ in the central region, and $q$ is 4 at the edge. The temperature and electron density are 5 keV and $10^{20} m^{-3}$, with flat profiles, and $Z_{\text{eff}} = 1.5$. With a coupled power of 3 MW in these single toroidal mode number $n_\phi$ scenarios, we see that large currents can be driven. For $n_\phi = 7$, net currents in excess of 100 kA are predicted. Magnetic trapping reduces the currents by a factor of 3. For injection in the other direction, $n_\phi = -7$, the current profile is ambipolar, and the resulting net current is very small. The contour plots on the bottom show the location where the damping occurs in the 2-D plasma cross section. For $n_\phi = 7$, the peak absorption is located at the IBW-ICW confluence point below the midplane. For $n_\phi = -7$, the absorption takes place above and below the midplane, again at the IBW-ICW confluence points. Deposition above the midplane results in counter-current drive, while deposition below the midplane leads to co-current drive. This is consistent with $k_\parallel \approx \frac{m B_\phi}{T F}$, since the mode converted waves have their radial phase velocity towards the low field side, and therefore $m$ has opposite signs above and below the midplane. Hence, net MCCD can be directly associated with up-down asymmetries in the mode conversion process.

As this example illustrates, TORIC predicts significant net currents can be driven with mode
$n_f = 7$

Total power
3.0 MW

Fund. D 2.8 %
Fund. $^3$He 35.6 %
Electrons 61.6 %

Net currents
FW/ICW
2.1 kA

Without trapping:
87.8 kA

Figure 3-5: MCCD prediction with TORIC/Ehst-Karney, for $n_f = -7$.

$\phi = 7$

Total power
3.0 MW

Fund. D 2.9 %
Fund. $^3$He 41.3 %
Electrons 55.8 %

Net currents
FW/ICW
116.7 kA

Without trapping:
354.4 kA

Figure 3-6: MCCD prediction with TORIC/Ehst-Karney, for $n_f = 7$.
converted waves. The current drive effect is associated with the excitation of mode converted Ion Cyclotron Waves, and results from predicted up-down asymmetries in the electron damping. The physical reason for this asymmetry is not clear, and should be examined in future work, as discussed in chapter 7. In this thesis, we will treat the up-down asymmetry as a prediction from full wave modeling, and show that the resulting driven currents and driven current profiles are consistent with experimental observations. However, at this point, another potential shortcoming of this prediction will be examined. Indeed, it is not clear that the magnitude of the driven currents is properly estimated in figures 3-5 and 3-6. As discussed above, the use of the Alfven Wave damping Ehst-Karney parametrization for MCCD with ICW has not been rigorously justified. In addition, we note that the power and driven current density are very large in the simulations, which suggests that the electron distribution function $f_e$ will deviate significantly from a Maxwellian and that, therefore, the assumption of linear damping may fail. These two points can be improved upon through Fokker-Planck simulations, which is the object of the following section.

### 3.2.2 Fokker-Planck simulations

The fields calculated by TORIC can be used to construct a quasilinear diffusion operator, for use in a Fokker-Planck simulations. For $\omega \ll \omega_{ce}$, the quasilinear diffusion for electrons comes from Cerenkov resonances only. Following a similar derivation as in Kennel and Engelmann [Kennel 66], the quasilinear diffusion operator in $(\epsilon = \frac{v}{v_c}, \mu = \frac{v^2 \sin^2 \theta}{2B_0})$ coordinates is expressed in the following form [Bilato 02]:

$$D_{\epsilon, \mu} = \frac{e^2}{2m_e} R e \sum_n \sum_{m_1, m_2} \int R J_p \frac{B}{v} d\theta \exp(i(m_1 - m_2)\theta)$$  \hspace{1cm} (3.24)

$$\times \int_{-\infty}^{t} e^{i((m_1+qm)(t'-\theta)-\omega(t'-t))} \left( v \hat{E}_{m_1, n} - i \frac{\omega}{2\Omega_{ce}} \frac{v^2}{c} B_{m_1, n} \right)^* \left( v' \hat{E}_{m_2, n} - i \frac{\omega}{2\Omega_{ce}} \frac{v'^2}{c} B_{m_2, n} \right) dt$$

The first integral corresponds to the bounce-averaging, with $J_p$ the Jacobian of the transformation to the $(\psi, \theta, \phi)$ spatial coordinates. If the transit through resonance is short enough so that the orbit remains unaffected and if phase randomization is assumed, the phase factor in the second
integration (with respect to $t'$) is a delta function:

$$D_{e,e} = \pi \frac{e^2}{\omega' 2m_e^2} \text{Re} \sum_n \sum_{m_1,m_2} \left[ R J_p \frac{B}{v} \exp(i(m_1 - m_2)\theta) \mathcal{W}_{m_2,n} \right] (3.25)$$

$$\left( v' E^{m_2,n} - i \frac{\omega}{2\Omega_{ce} c} B^{m_2,n} \right) \ast \left( v E^{m_1,n} - i \frac{\omega}{2\Omega_{ce} c} B^{m_1,n} \right) \right]_{\theta = \theta_r}$$

where $\theta_r$ is obtained from the resonance condition:

$$k_{\parallel}^{m_2,n} = \omega v_{\parallel} \equiv \frac{m_2}{N_0} \sin \arctan \frac{B_{pol}}{B_{tor}} + \frac{n}{R} \cos \arctan \frac{B_{pol}}{B_{tor}} = \omega v_{\parallel} (3.26)$$

and the weights $\mathcal{W}_{m_2,n}$, formally proportional to the time spent by an electron in resonance, are determined so that the quasilinear heating rate agrees with the absorbed power in TORIC. The output of the calculation is a decomposition of the quasilinear diffusion operator in Landau damping (LD), mixed (MXD) and transit time magnetic pumping (TTMP) terms, which depend only on $v_{\parallel}$:

$$D_{e,e} = v_{\parallel}^2 (D_{LD}(v_{\parallel}) + D_{MX}(v_{\parallel}) v_{\perp}^2 + D_{TTM}(v_{\parallel}) v_{\perp}^4) (3.27)$$

This operator can be imported in a Fokker-Planck solver. In this work, we have used the DKE code, written by J. Decker and Y. Peysson. The code solves the 3-D relativistic bounce-averaged electron drift kinetic equation in arbitrary magnetic equilibrium. Details on the code and its capabilities can be found in [Decker 05].

Figure 3-7 shows a comparison of the electron deposition and driven current profiles predicted with the Ehst-Karney and the DKE code, for $n_{\phi} = 7$ case above. In order to ensure that the deviations from a Maxwellian are limited, the quasilinear operator was divided by $10^2$. The resulting absorbed power density and driven currents were scaled up again for comparison. This procedure guarantees that the assumptions of linearity in TORIC and in the adjoint technique are not violated. Even with this restriction, the DKE prediction differs significantly from that with the Alfven Wave damping Ehst-Karney parametrization, by a factor of $\sim 2 - 3$. This indicates that the default calculation for the driven currents in TORIC can noticeably overestimate the MCCD efficiency. The Ehst-Karney formula still captures the dependence of the efficiency on temperature and density, as can be seen on figure 3-8. The plot shows the non-normalized global efficiency $I/P$ predicted.
Figure 3-7: Comparison between Ehst-Karney and DKE predictions for the MCCD case on figure 3-6. The AW and LD correspond to the Ehst-Karney Alfven wave damping and Landau damping formula, while the DKE points are calculated using the DKE code. In all three cases, NT stands for no trapping, and indicates the effects of magnetic trapping are ignored.

by DKE and by the AW Ehst-Karney formula when running the same case as above with different temperature and densities. More precisely, the electron temperature and density were varied from 1 to 5 keV, and $0.5 \times 10^{20} \text{m}^{-3}$ to $4 \times 10^{20} \text{m}^{-3}$ respectively. The curve does not follow exactly the expected $T_e/n_e$ dependence for the non-normalized efficiency (equation 3.6), indicative of other direct effects of the temperature and density on wave propagation. However, albeit from the cor- rective factor of $\sim 2$ above, the Ehst-Karney formula captures the correct parametric dependence found in the Fokker-Planck calculations.

The discrepancy in the normalized efficiency confirms that the Alfven Wave damping Ehst-Karney formula is not appropriate for MCCD calculations. The form of the quasilinear operator it uses implies a significant TTMP contribution, more precisely $P_{\text{abs, TTMP}} = -P_{\text{abs, MX}} = \frac{1}{2} P_{\text{abs, LD}}$. This relation is indeed well satisfied for fast wave current drive, as expected from dispersion relation arguments [Porkolab 93] and confirmed numerically with TORIC and the quasilinear diffusion operator used in this work [Bilato 02]. For MCCD, however, the Landau damping contribution is found to dominate over the TTMP and mixed term contribution. This can be observed by comparing the absorption associated with the three terms or through the $v_\perp$ dependence of the quasilinear
Figure 3-8: Comparison between Ehst-Karney and DKE predictions for TORIC runs with different values of electron temperature and density. The efficiency is calculated based on the power absorbed on electrons $P_{abs,el}$.

diffusion operator. Figure 3-9 shows contours of the quasilinear diffusion operator for the peak driven current in figure 3-7. The quasilinear diffusion associated with the mode converted waves occurs at low values of $v_\parallel$, but without $v_\perp$ dependence. A small fast wave contribution is seen at higher values of $v_\parallel$ and at large values of $v_\perp$, which can be taken as a signature of the TTMP term and its $v_\perp^4$ dependence. Note that the code enforces the symmetrization of the quasilinear diffusion coefficient inside the trapped region.

Therefore, it appears that the use of the Landau damping Ehst-Karney formula would be more appropriate to calculate the MCCD currents. Figure 3-7 suggests this is indeed the case. For example, the ratio of the predicted driven currents with and without trapping in DKE agrees better with the Landau damping estimates. The Landau damping formula still overestimates the driven currents. The remaining discrepancy is compatible with the error in the Ehst-Karney parametrization itself at low parallel phase velocity, as illustrated in the original article (Figure 1 in [Ehst 91]).

**$Z_{eff}$ dependence** - Figure 3-10 shows the reduction in the MCCD efficiency with increasing effective charge $Z_{eff}$, for example from increased impurity concentrations. The absolute value of
the efficiency is only indicative here, and we are mainly interested in the parametric dependence with $Z_{\text{eff}}$. The Ehst-Karney parametrization predicts $\eta \propto \frac{1}{Z_{\text{eff}}}$ for low $v_{||}$ Landau damping. Fokker-Planck simulations with DKE in the case above predict a similar dependence, more accurately fitted with a slight offset in the denominator $\eta \propto \frac{1}{0.5 + Z_{\text{eff}}}$. The $\frac{1}{Z_{\text{eff}}}$ prediction still yields a reasonable estimate for $Z_{\text{eff}} \gtrsim 2$.

This scaling with $Z_{\text{eff}}$ is very unfavorable compared to that for high parallel phase velocities waves [Fisch 80], for which $\eta \propto \frac{4}{5 + Z_{\text{eff}}}$. As we will see in chapter 5, this has implications on the achievable efficiencies in experiments.

- **Power threshold for quasilinear broadening** - TORIC and the adjoint calculation on which the Ehst-Karney parametrization is based both assume that the deviation of the electron distribution function from a Maxwellian is small. For fast magnetosonic waves and FWCD, this linear assumption generally holds, since the weak damping on electrons results in broad power deposition profiles and only moderate absorbed power densities. Mode conversion electron heating, in contrast, is associated with strong localized electron damping, and therefore with a large power absorbed in a small volume. In this context, the distribution function may deviate significantly from a Maxwellian, which typically results in less damping than predicted by the linear theory and

Figure 3-9: Contours of the quasilinear diffusion operator at the peak of the current profile in figure 3-7. Red indicates a stronger magnitude for the quasilinear diffusion operator.
Figure 3-10: Comparison of the $Z_{\text{eff}}$ dependence for the current drive efficiency predicted by DKE and the Ehst-Karney parametrization in the low velocity limit ($1/Z_{\text{eff}}$) and high velocity limit ($1/(Z_{\text{eff}} + 5)$). The analytical curves were scaled to match the DKE points.

Therefore a quasilinear broadening of the deposition profiles.

Figure 3-11 shows a comparison between the absorbed power density predicted from linear theory by TORIC and that computed by the DKE from changes in the electron distribution function. Where linear theory holds, the two values should agree. As the deviation from a Maxwellian becomes significant, the quasilinear heating rate differs from the power absorbed predicted in the linear limit. For a density of $10^{20} \text{ m}^{-3}$, we see that the transition occurs for a power density of 5-10 MW/m$^3$. We can therefore expect broadening of the power deposition profiles for TORIC simulations predicting absorbed power density above this threshold. Clearly, this is the case for the simulations in figures 3-5 and 3-6. When the full antenna spectrum is considered, the power deposition profiles will be broader and the absorbed peak power densities will be reduced. As the quasilinear heating rate is reduced, the driven current density is also smaller and the current drive efficiency is initially not affected. The distortions of the distribution function at the highest power densities predicted by quasilinear theory may affect the propagation of the mode converted waves, and require iterating between the full wave solver and the Fokker-Planck code. This procedure is being implemented in TORIC, and we see here that it may be required for high power MCEH and
3.3 TFTR MCCD experiments

This section presents TORIC simulations for the MCCD experiments in TFTR [Majeski 96b, Majeski 96a].

3.3.1 Antenna and plasma parameters

The following antenna parameters describe two strap Bay M antenna used in the experiment:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strap width</td>
<td>10.2 cm</td>
</tr>
<tr>
<td>Strap center-center spacing</td>
<td>33 cm</td>
</tr>
<tr>
<td>Strap length</td>
<td>77.7 cm</td>
</tr>
<tr>
<td>Strap major radius</td>
<td>3.64 m</td>
</tr>
<tr>
<td>Minor radius or radius of curvature</td>
<td>104 cm</td>
</tr>
<tr>
<td>Minimum distance from the strap to the (flat) backplane</td>
<td>10 cm</td>
</tr>
</tbody>
</table>
Directional launched spectra were obtained by running the antenna in $(0,\pi/2)$ and $(0, -\pi/2)$ phasing. The strap to strap distance gives $n_\phi \approx \pm 16$ at the peak of the radiated spectrum. This is the value we will use in the simulations.

The experiments were done with a $^3$He,$^4$He, D plasma mix. We use the following plasma parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic axis</td>
<td>$R_0 = 2.72$ m</td>
</tr>
<tr>
<td>RF power (43 MHz)</td>
<td>$P_{RF} = 3.8$ MW</td>
</tr>
<tr>
<td>Toroidal field</td>
<td>$B_T \approx 4.8$ T</td>
</tr>
<tr>
<td>$^3$He concentration</td>
<td>$n_{^3\text{He}}/n_e \approx 0.2$</td>
</tr>
<tr>
<td>Effective charge</td>
<td>$Z_{eff} = 3.5$</td>
</tr>
<tr>
<td>Central temperature</td>
<td>$T_{e0} = 7$ keV</td>
</tr>
<tr>
<td>Neutral beam</td>
<td>$P_{NBI} = 2$ MW</td>
</tr>
<tr>
<td>Plasma current</td>
<td>$I_p = 1.4$ MA</td>
</tr>
<tr>
<td>$^4$He concentration</td>
<td>$n_{^4\text{He}}/n_e \approx 0.1$</td>
</tr>
<tr>
<td>Central density</td>
<td>$n_e(0) = 5.5 \times 10^{19}$ m$^{-3}$</td>
</tr>
</tbody>
</table>

In the experiments, the $^3$He concentration was estimated from puff duration. Plasmas were outboard limited, with the limiter at $R = 3.6$ m at the midplane. Since $Z/A = 1/2$ for $^4$He, D, C, we consider only $^3$He and D in the simulations.

### 3.3.2 TORIC simulations

Figure 3-12 shows the parallel electric field predicted by TORIC in the conditions above, for $n_\phi = -16$ and with the mode conversion layer tangent to $r/a \approx 0.07$ on the high field side. Mode conversion to Ion Cyclotron Waves is observed in the simulation, as expected from the dispersion relation studies in chapter 2. Comparing with figure 2-16, we see that the field structure is similar to that for C-Mod sized plasmas. However, we observe that the ICW wavelength is smaller relative to the plasma size. This is consistent with the scaling for $k_\perp a \propto \sqrt{n_e}a$ in section 2.3, since the density is smaller by a factor $\sim 2$ and the minor radius larger by a factor of $\sim 4$ between C-Mod and TFTR. Accordingly, the simulations generally predict more peaked power deposition profiles.

Using the Ehst-Karney post-processing routines in C-Mod, we can estimate the driven currents in these simulations. For the simulation parameter above, with $P_{RF} = 3.8$ MW and the mode conversion layer at the magnetic axis, TORIC predicts total driven currents in excess of 400 kA for $n_\phi = 16$. For $n_\phi = -16$, the predicted driven current profiles are bidirectional, with an integrated value of $\sim 50$ kA. Simulations with the Fokker-Planck code DKE in these plasmas indicate that the
driven currents are also reduced by a factor $\sim 2$. In these conditions, the predicted difference in the net driven currents between co and counter-current drive phasings is 170 kA, which is consistent with the $\sim 130$ kA value deduced from loop voltage differences. The driven currents predicted by TORIC in TFTR plasmas are higher than expected when scaling the C-Mod predictions with temperature, density, RF power and $Z_{\text{eff}}$. Due to the larger plasma size and lower density, the power fraction damped on electrons peak at 80 % in the TFTR simulations, and mode conversion to MCICW occurs closer the magnetic axis in terms of normalized minor radius, which lowers the effect of magnetic trapping. The latter aspect is associated with the lower plasma density, which reduces the FLR parameter $\sigma$, and higher electron temperature, which lowers $\frac{\omega_p}{k_{\parallel}v_{\text{the}}}$. 

Figure 3-12: Parallel electric field predicted by TORIC in TFTR.
Chapter 4

Scenarios and experiments on Alcator C-Mod

4.1 MCCD scenarios for C-Mod

A primary objective of this thesis work has been to investigate net current drive with MCCD from loop voltage comparisons on Alcator C-Mod. The MCCD experiments on TFTR, discussed in the introduction and analyzed with TORIC in chapter 3, constitute a solid proof of principle for these studies, and the objective has been to use the flexible ICRF system and additional diagnostics capabilities on C-Mod to obtain similar results and expand their scope. In this chapter, we determine optimal regimes for net MCCD on C-Mod, and discuss in more detail two important aspects of the experiments, the control of the plasma composition and principle for loop voltage measurements.

4.1.1 Mode conversion scenarios

Figure 4-1 shows the location of the Ion Cyclotron resonance layers as a function of the toroidal magnetic field for four ICRF frequencies used routinely on C-Mod. Deuterium is the most common majority species on Alcator C-Mod, although helium 4 and hydrogen plasmas can also be run. The primary minority heating scenario is D(H), for which central heating is obtained around 5.4 T at 78-80 MHz. Hydrogen tends to be trapped in significant quantities in the vessel walls,
and is present during all discharges. After up-to-air periods or operation with hydrogen plasmas, dedicated operation and wall conditioning has to be carried out in order to lower the hydrogen concentrations down to levels compatible with minority heating. Even after conditioning, a residual hydrogen concentration of $\sim 1\text{-}5\%$ is still present in C-Mod plasmas. This prohibits antenna operation at frequencies and magnetic fields for which the hydrogen cyclotron layer is inside the antenna.

![Diagram](image)

**Figure 4-1:** Fundamental (solid) and 2nd harmonic (dash) cyclotron resonances in C-Mod.

With the present setup, $^3\text{He}$ minority scenarios are available for central heating at 8 T and 78-80.5 MHz, or 5.4 T and 50 MHz for the J-port antenna. Based on single pass absorption arguments (see chapter 2), $\text{D}^3\text{He}$ and $\text{H}^3\text{He}$ are the most favorable scenarios for mode conversion experiments on C-Mod. $\text{H}^3\text{He}$ is also analogous to $\text{D}(\text{T})$ mode conversion, a regime readily accessible in D-T reactors. However, for the reason indicated above, operation with hydrogen plasmas usually conflicts with the rest of the C-Mod research program, which is heavily based on $\text{D}(\text{H})$ minority
Figure 4-2: Major radius location of the mode conversion layer in D,$^3$He plasmas, at 50 MHz.

heating. D($^3$He) at 8 T and 78 MHz is not optimal either, due to the more limited diagnostics set available at this high field and the slower discharge cycle compared to 5-6 T operation. The 50 MHz D($^3$He) scenario is most appropriate, since it leads to on-axis mode conversion in the 5-6 T range. Figure [4-2] shows the location the mode conversion layer as a function of magnetic field and $^3$He concentration for D($^3$He) and 50 MHz. The $^3$He ion cyclotron layer is at the magnetic axis at 5 T. From figure [4-1] we see that the hydrogen cyclotron layer is on axis at 5.2 T for 80/80.5 MHz, which allows to use D and E antennas for central D(H) minority heating.

<table>
<thead>
<tr>
<th>Antenna</th>
<th>J</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>50 MHz</td>
<td>80.5 MHz</td>
<td>80 MHz</td>
</tr>
<tr>
<td>Power</td>
<td>3.0 MW</td>
<td>1.5 MW</td>
<td>1.5 MW</td>
</tr>
<tr>
<td>Number of straps</td>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Strap width</td>
<td>8 cm</td>
<td>10 cm</td>
<td></td>
</tr>
<tr>
<td>Strap center-center spacing</td>
<td>26.5 cm</td>
<td>35.75</td>
<td></td>
</tr>
<tr>
<td>Strap length</td>
<td>48 cm</td>
<td>48 cm</td>
<td></td>
</tr>
<tr>
<td>Strap major radius</td>
<td>93.5 cm</td>
<td>93.5 cm</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Antenna parameters for the C-Mod antennas as used in TORIC

Phased operation with the four strap antenna at J-port (see fig. [4-3]) allows control of the toroidal spectrum coupled to the plasma. In the experiments reported in this thesis, the antenna was operated with (0,$\pi$,$\pi$,0) (heating or symmetric), (0, $\pi$/2, $\pi$, $3\pi$/2) (+90°, counter-current drive) and
Figure 4-3: Top view of the C-Mod tokamak showing the directions of the field and currents.

Figure 4-4: Antenna spectrum for J-port in co-counter drive phasing.
(0, −π/2, −π, −3π/2) (−90°, co-current drive) phasings. The antenna phasings are labeled according to the predicted driven current directions from the launched antenna spectrum, ignoring poloidal field effects. The typical directions of the toroidal magnetic field and ohmic plasma current are shown on figure 4-3. Note that for mode conversion current drive, the direction in which the currents are driven with respect to the plasma current is not obvious, since the $k_\parallel$ spectrum is strongly modified by poloidal field effects and thus only indirectly controlled by the antenna spectrum. In these conditions, it is a priori unclear whether net MCCD will be driven in the direction of the plasma current with co-current drive phasing, as labeled above. Full wave simulations for C-Mod indicate that this indeed the case, and we will see in chapter 6 that the sawtooth period response in C-Mod experiments is consistent this prediction.

Table 4.1 shows the relevant antenna dimensions for simulations of the launched spectrum. The radiated spectrum for J-port in co-current drive phasing is shown on figure 4-4, as calculated from the toroidal Fourier transform of the strap currents and from the one-dimensional full wave code FELICE. As can be seen by comparing the two curves, fast wave tunneling through the evanescent layer at the plasma periphery cuts out the high $n_\phi$ modes.

### 4.1.2 Simulated target discharge for net current drive experiments

Building upon the efficiency estimates in chapter 3, we can identify optimal scenarios for net current drive in C-Mod. The following figure of merit for the current drive efficiency can be used as a guide:

$$\eta \propto \frac{P_{rf} T_e}{Z_{\text{eff}} n_e}$$

with

\[\begin{align*}
P_{rf} & \quad \text{RF power} \\
T_e & \quad \text{Electron temperature} \\
Z_{\text{eff}} & \quad \text{Effective charge} \\
n_e & \quad \text{Electron density}
\end{align*}\]

(4.1)

As shown in chapter 3, this dependency is generally well satisfied when scaling these parameters for a given mode conversion scenario.

Based on achieved discharges on Alcator C-Mod, a target discharge with peak density and temperature $n_e = 1 \times 10^{20} \text{m}^{-3}$, $T_e = 5 \text{keV}$ was identified and simulated with TORIC as a basis for net current drive experiment in D(3He) plasmas. The equilibrium had a central field of 5.4 T, and was diverted with an upper single null (USN) shape. The plasma composition, 65 % D, 15 % $^3$He and 5 % H gives on-axis mode conversion at 50 MHz. The effective charge $Z_{\text{eff}}$ was 1.5 in the
Simulations.

In these conditions, as illustrated in figure 4-5, TORIC shows \( \sim 100 \text{ kA} \) can be driven for 3 MW injected power in co-current drive phasing.

This original prediction was taken as a reasonable basis for net current drive MCCD experiments on Alcator, using the same measurement technique with the loop voltage as in TFTR. The subsequent efficiency calculations discussed in chapter 3 show that the estimate was too optimistic, and that therefore the driven MCCD currents with the target conditions could be much lower, \( \sim 50 \text{ kA} \).

4.2 Control of the \(^3\text{He}\) concentration

4.2.1 \(^3\text{He}\) injection

Achieving optimal or desired plasma composition and maintaining these conditions are key experimental requirements for mode conversion heating and current drive scenarios. For low minority concentrations, ion cyclotron damping from the fast wave (minority heating) or from the mode converted waves can lower the power absorbed by electrons, and thus the overall current drive efficiency. While not so much of an issue in a small plasma like in C-Mod (see section 2.3), higher minority concentrations increase the width of the evanescent layer and tunneling can be problematic in larger tokamak plasmas. Additionally, the plasma composition determines the location of the mode conversion layer, and must therefore be controlled (or at least monitored) for applications where the heating and/or current drive location is important. This is obviously the case for localized current profile control, but also to a lesser extent for net current drive since the efficiency can be significant reduced for off-axis scenarios.

For D(\(^3\text{He}\)) mode conversion scenarios on C-Mod, the plasma breakdown and density control (feedback) are carried out with deuterium gas using the B-Top and C-Side gas valves. A prede-
terminated \(^3\)He gas puff from the B-side-upper valve is used in each single discharge to achieve the desired \(^3\)He concentration. For mode conversion scenarios in the current flat-top (usually 500 ms to 1500 ms after breakdown), the \(^3\)He puff is triggered at 200 ms to 300 ms in the discharge, for 100 to 200 ms.

4.2.2 Monitoring the \(^3\)He concentration

- **Break-in-slope analysis** - Temperature measurements can be used to deduce electron power absorption profiles associated with wave heating. The peak deposition can be taken as the location of the mode conversion layer, and be used to estimate the \(^3\)He concentration. C-Mod is equipped with several high temporal and spatial resolution Electron Cyclotron Emission (ECE) diagnostics: a 10-channel grating polychromator (GPC) \([\text{O'Shea 97}]\) and a 32 channel 2nd harmonic X-mode radiometer (FRC-ECE) \([\text{Heard 99}]\).

Deposition profiles are obtained using fast turn-offs in the source power (\(\leq 10\mu s\) for the ICRF system on C-Mod) and *break-in-slope* techniques \([\text{Gambier 90}]\). In this approach, the derivative of the electron temperature \(\frac{\partial T_e(r)}{\partial t}\) is determined immediately before and after the turn-off by fitting the slope of \(T_e(t)\) in a time interval \(\delta t\). From the difference, we can calculate the electron heating power density \(S(r, \delta t) \approx \frac{3}{2} n_e \Delta \frac{\partial T_e(r)}{\partial t}\) associated with the RF source at the location of the temperature measurement. Reducing \(\delta t\) allows to isolate the fastest processes contributing to \(S(r, \delta t)\). Ultimately, noise in the temperature measurements sets the smallest \(\delta t\) value which can be used, \(\sim 0.5\) ms in C-Mod. Direct electron heating from RF waves is reflected in \(S(r, \delta t)\) within an electron collision time (\(10\mu s\)). Indirect RF heating by deposition on ion and subsequent slowing down occurs on a slower timescale (\(\sim 10\) ms) and can therefore be neglected for \(\delta t < 1\) ms. Other processes contributing to \(S(r, \delta t)\) such as radiation, Ohmic heating or MHD activity are usually not directly affected by the RF turn-off and can therefore be ignored unless other independent events occur within \(\delta t\) of the turn-off. This applies to sawtooth crashes or other core-localized MHD activity, in which case the break-in-slope technique is inapplicable. Since the sawtooth period is usually around 5 ms in C-Mod, most turn-offs can be used for the analysis. For \(\delta t \sim 1\) ms, electron transport also broadens the electron deposition profile \([\text{Lin 03}]\).

Figure 4-6 shows an illustration for the break-in-slope technique. As the transmitter is turned off, a clear change in the temperature evolution occurs on the considered channel. The change
in slope is very prompt, indicating direct electron heating. Comparison between channels allows to locate the flux surface where the electron heating is largest. Taking this location to be that of the mode conversion layer, a $^3$He concentration estimate can be inferred. The radial resolution with the FRC-ECE diagnostics is sufficient to constrain the estimate within 1-2 percentage points. The actual uncertainty is larger, since the peak electron deposition does not coincide exactly with the mode conversion layer, and other plasma species may be present. The former aspect can be alleviated by running TORIC with several assumptions for the concentrations and comparing the predicted and experimental deposition profiles. From an operational point of view, the quick estimate is satisfactory and allows adjustments in the puff duration. The heating location is also the parameter of interest for MCEH and MCCD experiments, and is measured directly here.

4.2.3 Controlling the $^3$He concentration

Using the break-in-slope at several times during a given C-Mod discharge, we can evaluate the rate of change of the helium concentration after the initial injection. In typical conditions for mode conversion and MCCD experiments, we find that the concentration varies only by one or two percentage points for a 1 second flattop discharge. This is also consistent with Phase Contrast Imaging measurements [Mazurenko 01], in which the peak density fluctuations can be taken as an indication of the mode conversion layer position, and was found to move only slowly in experiments [Lin 05]. For mode conversion current drive experiments in C-Mod, this result indicates that no control of the $^3$He concentration after the initial injection is necessary.
4.3 Current drive measurements from the loop voltage differences

The MCCD experiments in TFTR showed an illustration of current drive estimates based on loop voltage differences. In this section we provide more background on this approach.

**Circuit model** - The basis for the measurement can be explained as follows [Hutchinson 02]. Writing Poynting’s theorem for the vessel volume $V$ and the corresponding surface $S$,

$$
\int_V \left( \vec{E} \cdot \vec{J} + \frac{1}{2\mu_0} \frac{\partial}{\partial t} B^2 \right) d^3x = -\frac{1}{\mu_0} \int_{\delta V} (\vec{E} \times \vec{B}) \cdot \vec{n} dS \quad (4.2)
$$

The input power by external circuits can be expressed in terms of the loop voltage $V_\phi$, which is the voltage created by the central solenoid or transformer along a toroidal loop enclosing the central column and outside the plasma. Assuming axisymmetry, this corresponds to a contribution to the Poynting vector equal to $V_\phi \int_{CS} B_\theta ds$ where CS is the wall contour in the poloidal cross-section. Using Ampere’s law, this power is equal to $V_\phi I_p$, where $I_p$ is the plasma current. If the toroidal field is not constant, there is an additional term corresponding to $\vec{E}_\theta \times \vec{B}_\phi \cdot \vec{n}$.

Defining the inductance $L = \frac{1}{\mu_0} \int_V B_\theta^2 d^3x$, we obtain for negligible $\frac{\partial}{\partial t} B_\phi$:

$$
P_{\text{Ohmic}} = \int_V \vec{E} \cdot \vec{J} d^3x = V_\phi I_p - \frac{\partial}{\partial t} \left( \frac{1}{2} L I_p^2 \right) \quad (4.3)
$$

Using Ohm’s law, $\vec{E} = \eta (\vec{J} - \vec{J}_{BS} - \vec{J}_{NH})$, where $\eta$ is the resistivity of the plasma, $\vec{J}_{BS}$ denotes the bootstrap current density and $\vec{J}_{NH}$ the non-inductive current density. Taking the flux surface average of those quantities:

$$
\int \eta(\psi) \vec{J} \cdot (\vec{J} - \vec{J}_{BS} - \vec{J}_{NH}) \frac{dV}{d\psi} d\psi = V_\phi I_p - \frac{I_p}{2} \left[ I_p \frac{\partial L}{\partial t} + L \frac{\partial I_p}{\partial t} \right] \quad (4.4)
$$

If one assumes flat profiles of current for the different contributions, it is possible to obtain a simple expression by introducing the plasma resistance $R$:

$$
V_\phi = R(I_p - I_{BS} - I_{NH}) + \frac{1}{2} \left[ I_p \frac{\partial L}{\partial t} + L \frac{\partial I_p}{\partial t} \right] \quad (4.5)
$$

$I_p$ and $V_\phi$ can be measured from external magnetics measurements. Therefore, if the plasma
resistance and inductance can be evaluated, it is possible to determine the non-Ohmic currents $I_{BS} + I_{NH}$ under the assumption of flat current profiles. However, the quantities $R$ and $L$ are difficult to measure. The inductance can in principle be derived from magnetics measurements, and the plasma resistance can be evaluated from electron temperature measurements and spectroscopic information on the plasma composition. In cases where the internal inductance of the plasma varies, it might also be difficult to distinguish between this effect and non-inductive current drive.

In most tokamak current drive experiments, the total current $I_p$ is maintained constant through a feedback on the central solenoid driving the inductive current. If steady state can be reached, the expression above reduces to:

$$V_\phi - RI_p = -R(I_{BS} + I_{NH})$$  \hspace{1cm} (4.6)

Comparing the loop voltage in two discharges with the similar conductivity profiles can therefore give a good estimate of the change in non-inductive current. The bootstrap current depends on the radial profiles of temperature and density and therefore similar profiles will lead to very similar bootstrap current profiles.

— **Current relaxation** — The characteristic time for current relaxation is related to the resistive diffusion time $\tau_\eta = \mu_0 \sigma a^2$, where $\sigma$ is the plasma conductivity and $a$ the minor radius of the plasma. While there is ambiguity in the conductivity value to be used in this formula (central, volume averaged, etc...), it gives an order of magnitude, namely $\tau_\eta \sim 1s$ for typical C-Mod conditions. Experimental results and modeling [Mikkelsen 89] suggest that the timescale $\tau_I$ for the loop voltage response can in fact be faster than this value by an order of magnitude. In cylindrical geometry, the response can be written in terms of Bessel functions with exponential amplitudes in time, and $\tau_I$ can be taken as the slowest argument:

$$\tau_I = 1.4 \frac{a^2 T_e^{3/2}}{Z_{\text{eff}}} \text{ s}$$  \hspace{1cm} (4.7)

with the minor radius $a$ in meters and the temperature $T_e$ in keV. For typical Alcator C-Mod conditions, $T_e = 5$ keV and $Z_{\text{eff}} = 2$, this gives $\tau_I \sim 300$ ms. This is shorter than the typical current flattop duration in C-Mod, which is 1 s.
Chapter 5

Loop voltage experiments on C-Mod

5.1 C-Mod loop voltage experiments

5.1.1 Comparisons in flattop

Discharges utilizing loop voltage measurements to study MCCD were run on Alcator C-Mod in July-August 2005 and June 2006. Figure 5-1 shows the first comparison discharge in the series. The RF power was 2.5 MW from the J-port antenna at 50 MHz, and 1.4 MW from E-port at 80 MHz for D(H) minority heating. The $^3$He concentration in these discharges was estimated at about 15%, well in the mode conversion regime, leading to strong electron heating. As a result, central temperatures above 5 keV were obtained. However, no clear differences between co- and counter-current drive phasings were observed on the loop voltage traces, which suggests insufficient net currents were driven in these conditions. While the central temperature and coupled power were close to their target values, the central density $n_{e0} \sim 1.7 \times 10^{20} m^{-3}$ and effective charge $Z_{\text{eff}} \sim 4$ were larger than their respective target values $1.0 \times 10^{20} m^{-3}$ and 1.5 in the modeled discharges. According to the figure of merit for MCCD in equation 4.1, these unfavorable parameters reduce the MCCD efficiency by a factor of 4 compared to the simulated 100 kA scenario.

In order to raise the current drive efficiency noticeably, the $n_e Z_{\text{eff}}$ product must be reduced, since only small relative improvements in the central electron temperature and coupled power (limited to 3 MW for J-port) can be expected experimentally. However, experimental observations on several tokamaks, including C-Mod [Greenwald 95], show that discharges in L-mode with reduced target...
Figure 5-1: Loop voltage comparison between co current phasing (red) and counter-current drive phasing (blue)
densities tend to have higher $Z_{\text{eff}}$. In fact, the correlation between the two parameters is such that, at low or moderate densities, the product $n_e Z_{\text{eff}}$ is almost constant. Figure 5-2 shows typical values of the product for a large set of discharges dedicated to MCCD experiments. For discharges in upper-single null (USN) and inner-wall limited (IWL) plasmas, no significant changes can be seen as the target density is varied.

The effective single method in reducing the $n_e Z_{\text{eff}}$ product is the boronization technique, which consists in depositing a thin layer of boron on the vessel walls. It belongs to a larger category of wall conditioning techniques [Winter 96], and is used frequently on C-Mod in order to improve the overall performance of the plasmas and access higher confinement regimes. Boronization lowers the impurity influx into the plasma and therefore maintains the radiated power at low levels. The positive effects of boronization can last for several discharges depending on the amount of boron deposited. In terms of MCCD experiments, overnight boronization drops the $n_e Z_{\text{eff}}$ by a factor of 2, as can be seen on figure 5-2. Unfortunately, boronization has also detrimental effects on the performance of ICRF antennas. Immediately after overnight boronization, the antennas cannot operate at high power levels, and several discharges are required before high power can be coupled effectively. This limits the number of discharges which can be used to set up optimal scenarios for MCCD. High power input is also required to achieve large central temperatures and maximize the efficiency, and its application appears to make the erosion of the boron layer faster, as illustrated in the 1060620 discharges in figure 5-2.

Figure 5-3 shows an example discharge obtained after boronization. For the same central density as in the previous discharges, the effective charge was reduced to $Z_{\text{eff}} \approx 2$. However, antenna power from E-antenna could not be coupled reliably for these plasmas, and as a result the central temperature was lower, $T_{e0} \approx 3$ keV. Very little changes in loop voltage was observed between co and counter current drive phasings, with the voltage in co current drive phasing slightly higher than in counter current drive. This trend may in fact be reinforced by the difference in conductivity associated with $Z_{\text{eff}}$, since the higher $Z_{\text{eff}}$ value in counter-CD phasing would lead to a lower loop voltage than measured. The loop voltage difference is in the opposite direction as expected from current drive, and therefore the experiment is again inconclusive.
Figure 5-2: Evolution of the density and effective charge in MCCD experiments
Figure 5-3: Loop voltage comparison between co current phasing (red) and counter-current drive phasing (blue)
5.1.2 Experiments in ramp-up

As discussed in the previous section, loop voltage measurements require relaxed radial conductivity and current profiles, which is not valid in these discharges due to the strong sawtooth activity. This may constitute an additional difficulty in obtaining loop voltage changes with MCCD in the flattop experiments. Sawteeth are present in nearly all plasmas in C-Mod. The only situation when sawtooth activity can be avoided is the current ramp-up phase of the discharges, when the central safety factor is still above unity. Therefore, MCCD experiments during current ramp-up were proposed.

Before these experiments, MCEH during current ramp-up had not been demonstrated on C-Mod. The principal challenge is to obtain the proper plasma composition, with \( \sim 15 - 25\% \) \(^3\)He early during the ramp-up. Discharges in C-Mod are initiated by breaking down deuterium gas at low pressure using the toroidal electric field set-up by the central solenoid. Immediately after break-down, the gas is injected in the vessel while the toroidal current is ramped-up. Careful adjustments are required to obtain break-down and avoid early disruptions or excessive runaway electrons during ramp-up. Therefore, it was not immediately clear if it was possible to inject \(^3\)He gas with concentrations up to \( \sim 15 - 25\% \) without detrimental effects on the startup reliability. Several discharges were devoted to this question, and showed that MCEH conditions could be achieved in ramp-up without much difficulty. We were able to inject \(^3\)He through a gas puff as early as 40 ms before break-down, and still be able to initiate the plasma discharge.

Ramp-up discharges also illustrate how \(^3\)He penetrates in fusion plasmas. A consistent observation in the flattop discharges was that, to achieve the same relative \(^3\)He concentration in the plasma core, a shorter puff duration was required if it occurred earlier in the discharges. This trend was confirmed in ramp-up experiments. In the case of the \(^3\)He puff starting 40 ms before break-down, a duration of 50 ms only was required to obtain a concentration of \( \sim 20\% \) at 200 ms, while a puff starting at 300 ms will have to last 200 ms or more in order to achieve the same concentration after 0.6 sec. In both cases, the central densities were very similar. A consistent interpretation is as follows. \(^3\)He ions penetrate less easily in denser plasmas or plasmas surrounding with denser scrape-off layers, including collisions with neutrals. Accordingly, the control of the \(^3\)He concentration may be difficult in scenarios where penetration through scrape-off layer is limited.

Very strong MCEH was generally obtained in these experiments, with central temperatures up
Electron temperature profile with MCEH

Figure 5-4: Temperature profile during ramp-up with MCEH. The blue points are from Thomson scattering measurements, while the red and black points correspond to Electron Cyclotron Emission (ECE) measurements.

to 7 keV, as can be seen on figure 5-4. This is the record central electron temperature on C-Mod to date. However, the MCCD comparison was also not conclusive, as can be seen on figure 5-5. While the temperature traces and sawteeth show different behavior between co- and counter-current drive phasings, no difference is observed on the loop voltage traces. Again, in these experiments, the \( Z_{\text{eff}} \) was high, above 5 in some discharges (figure 5-2). Despite the high central temperatures, which lead to long current diffusion times and usually freeze the current profile so that safety factor remains above unity, sawteeth appear very early in the discharge. This may be also due to high \( Z_{\text{eff}} \) values, which reduce the conductivity.

5.1.3 Prospects of MCCD for net current drive

These inconclusive MCCD experiments suggest that the MCCD efficiencies achievable on C-Mod are too low to obtain loop voltage measurements. Despite much effort in target development,
Figure 5-5: Comparison between co current phasing (red) and counter-current drive phasing (blue) during current ramp-up
we were not able to achieve favorable conditions for these measurements. Table 5.1 shows a comparison of the best parameters achieved in C-Mod with the simulated conditions in the 100 kA scenario, and with the conditions in the TFTR MCCD experiments. This indicates that the parameter range in which MCCD can be useful as a net current drive technique is very limited in C-Mod.

<table>
<thead>
<tr>
<th></th>
<th>C-Mod, fig [5-1]</th>
<th>C-Mod, fig [5-3]</th>
<th>C-Mod target</th>
<th>TFTR</th>
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<tr>
<td>Central temperature</td>
<td>5 keV</td>
<td>2-3 kev</td>
<td>5 keV</td>
<td>6-7 keV</td>
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<tr>
<td>Central density</td>
<td>(1.7 \times 10^{20} \text{ m}^{-3})</td>
<td>(1.5 \times 10^{20} \text{ m}^{-3})</td>
<td>(1.0 \times 10^{20} \text{ m}^{-3})</td>
<td>(5 \times 10^{19} \text{ m}^{-3})</td>
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<td>50-100 kA</td>
<td>(simulated)</td>
<td>130 kA (measured)</td>
</tr>
</tbody>
</table>

Table 5.1: Achieved and target conditions in C-Mod and TFTR MCCD experiments

As we saw in our discussion of these experimental results, the main issue in achieving favorable conditions is the difficulty in controlling \(Z_{\text{eff}}\) at low density. In TFTR, the vessel walls in TFTR were covered with carbon, and as a result \(Z_{\text{eff}}\) is maintained at low to moderate values. In contrast, the walls in C-Mod are covered with Molybdenum tiles. As a result, even moderate influx of metallic impurities can lead to high \(Z_{\text{eff}}\) values and reduce the MCCD efficiency significantly. Unfortunately, the use of carbon walls in future reactors is problematic, due to its high retention of hydrogenic species, hence of tritium. The total inventory of tritium in a reactor will have to remain within regulatory limits, for safety reasons. As a result, future reactors are expected to have metallic walls like in C-Mod, which have lower tritium retention and will allow longer operation periods [Lipschultz 05]. For net current drive, the efficiency parameter is a very critical parameter owing to the large required non-inductive currents. The low MCCD efficiency resulting from moderate \(Z_{\text{eff}}\) levels will rule out the technique in favor of more favorable high velocity current drive techniques. For lower hybrid current drive for example, the scaling \(\dot{I} \propto \frac{1}{5+Z_{\text{eff}}}\) makes the level of impurity levels less critical. For this reason, MCCD does not appear as a tool of choice for net current drive in future reactors. If the installed ICRF system is compatible with phased antenna operation, MCCD scenarios may be used as a small complement for central current drive in central MCEH scenarios, which will be readily available in D-T plasmas.

Further studies of MCCD as a tool for net current drive in C-Mod are therefore not worthwhile
given the present experimental setup and available current profile measurements on C-Mod. Our discussion of the experimental results will instead focus on a particular observation from these experiments. In figure 5.3 the loop voltage in counter-current drive is lower than that in co-current drive, in the opposite direction as expected from current drive in the circuit model. This indicates that the loop voltage analysis may be more complicated than suggested in this model. A better treatment of the loop voltage evolution can be obtained by solving a diffusion equation for the current profile. This approach will also be useful when analyzing sawtooth control experiments in chapter 6.

5.2 Model for the current profile evolution

5.2.1 Evolution of the loop voltage profile

The evolution of the toroidal electric field is usually easier to study than that of the internal inductance of the plasma. This comes in part from the fact that the toroidal loop voltage is constant throughout the plasma in steady state and the boundary value can be measured. Therefore the radial diffusion of the voltage can be easier to track than current diffusion. Combining Ampere’s and Faraday’s law:

\[
\nabla \times \nabla \times \vec{E} + \mu_0 \frac{\partial}{\partial t} (\sigma \vec{E} + \vec{J}_{cd}) + \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0
\]

The diffusion is slow compared to the speed of light, and we can neglect the displacement current,

\[
\nabla^2 \vec{E} - \mu_0 \frac{\partial}{\partial t} (\sigma \vec{E}) = \mu_0 \frac{\partial}{\partial t} \vec{J}_{cd}
\]

In this equation, the conductivity \(\sigma\) is a function of time and space. In general, it depends mostly on temperature and \(Z_{\text{eff}}\), as expressed by the Spitzer formula [Spitzer 53]:

\[
\sigma_{\text{Spitz}} = 1.9012 \times 10^4 \frac{T_e^{3/2}}{Z_{\text{eff}} N(Z_{\text{eff}}) \ln \Lambda_e [m^{-1} \Omega^{-1}]}
\]

\[
N(Z) \equiv 0.58 + \frac{0.74}{0.76 + Z} \quad \ln \Lambda_e = 31.3 - \ln\left(\frac{\sqrt{n_e}}{T_e}\right) \quad Z_{\text{eff}} = \frac{1}{n_e} \sum_i Z_i^2 n_i
\]
with $T_e$ in eV and $n_e$ in m$^{-3}$. The tokamak configuration changes the conductivity of the plasma and this expression must be usually corrected for magnetic trapping. Useful expressions can be found in [Sauter 99] and references therein.

If the temporal evolution of the conductivity profiles is known, it is then possible to relate the evolution of the electric field in the plasma and the driven current profile. In the lowest order in aspect ratio, the equation above leads to an inhomogeneous diffusion equation for the toroidal electric field component:

$$\frac{1}{\mu_0 r} \frac{\partial}{\partial r} \left( r \frac{\partial E}{\partial r} \right) - \frac{\sigma}{\partial t} - \frac{\partial E}{\partial t} = \frac{\partial J_{cd}}{\partial t}$$

(5.4)

5.2.2 Numerical model

A complete simulation of the sawtooth cycle can be done in cylindrical geometry using a set of simplifying assumptions about the boundary conditions at the plasma edge and about electron temperature losses and transport.

The two main equations in this simple model correspond to energy conservation for the electrons and the radial current diffusion.

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r q_e) = S_e$$

(5.5)

with $q_e = -n_e \chi_e \frac{\partial T_e}{\partial r}$ and $S_e = p_{\text{ohm}} + p_{\text{heat}}$ (5.6)

where the ohmic power is $p_{\text{ohm}} = E_{\|} j_{\text{ohm}}$ and the heating power $p_{\text{heat}}$ is specified as an input. Electron-ion coupling is neglected in this simple approach.

$$\frac{1}{\mu_0 r} \frac{\partial}{\partial r} \left( r \frac{\partial E_{\|}}{\partial r} \right) - \frac{\sigma}{\partial t} - \frac{\partial E_{\|}}{\partial t} = \frac{\partial J_{cd}}{\partial t}$$

(5.7)

Formulation of the problem for MATLAB - The partial differential equation solver in MAT-
LAB\[1\] is able to solve equations of the form:

\[
c \left( x, t, u, \frac{\partial u}{\partial x} \right) \frac{\partial u}{\partial t} = x^{-m} \frac{\partial}{\partial x} \left( x^m f \left( x, t, u, \frac{\partial u}{\partial x} \right) \right) + s \left( x, t, u, \frac{\partial u}{\partial x} \right)
\]

(5.8)

where \( f \) must depend on \( \frac{\partial u}{\partial x} \), and \( c \) must be a diagonal matrix. The initial condition is \( u(x, t_0) = u_0(x) \). At the boundary \( x = a \) or \( x = b \), \( u \) satisfies a boundary condition of the form:

\[
p(x, t, u) + q(x, t) f \left( x, t, u, \frac{\partial u}{\partial x} \right) = 0
\]

(5.9)

where \( q \) is diagonal.

The diffusion equations [5.5] and [5.7] are coupled through the conductivity \( \sigma \), and the source terms. In order to retain the coupling, one must advance the two equations together. The most stringent requirement here is that the matrix \( c \) must be diagonal. In order to satisfy this, one must use \( j_{\text{ohm}} = \sigma E_\parallel \) instead of \( E_\parallel \) in the system of equations.

Writing \( \sigma = \bar{\sigma} T_e^{3/2} \), with \( \bar{\sigma} \) capturing neoclassical corrections to the conductivity, the system of equation takes the following form:

\[
\frac{\partial}{\partial t} \begin{bmatrix} \frac{3}{2} n_e & 0 \\ 0 & \mu_0 \end{bmatrix} \begin{bmatrix} T_e \\ j_{\text{ohm}} \end{bmatrix} = \frac{1}{r} \frac{\partial}{\partial r} r \begin{bmatrix} n_e T_e \frac{\partial T_e}{\partial r} \\ \frac{1}{\bar{\sigma} T_e^{3/2}} \frac{\partial j_{\text{ohm}}}{\partial r} - \frac{j_{\text{ohm}}}{\bar{\sigma} T_e^{3/2}} \left( \frac{3}{2} \frac{\partial T_e}{\partial r} + \frac{\partial \bar{\sigma}}{\partial r} T_e \right) \end{bmatrix} + \begin{bmatrix} j_{\text{ohm}}^{2} \bar{\sigma} T_e^{3/2} \\ 0 \end{bmatrix}
\]

(5.10)

\( r \) is normalized as \( r = ax \), with \( x \) equally spaced from 0 to 1. We take \( n_e = \bar{n}_e n \) with \( \bar{n}_e = 10^{20} \text{m}^{-3} \), \( T_e = kT_{(0)} T \) where \( k \) is the Boltzmann constant and \( kT_{(0)} = 1.6022 \times 10^{-10} \text{ J} \) is the energy associated with 1 keV. \( j_{\text{ohm}} \) is in A/m, \( E_\parallel \) in V/m, \( \bar{\sigma} \) is such that \( \bar{\sigma} T \) in Sm\(^{-1}\). With these units, the equation becomes:

\[
\frac{\partial}{\partial t} \begin{bmatrix} \frac{3}{2} n_e & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T \\ j_{\text{ohm}} \end{bmatrix} = \frac{1}{x} \frac{\partial}{\partial x} x \begin{bmatrix} n_e \frac{\partial T}{\partial x} \\ \frac{1}{\mu_0 \alpha^2 T_e^{3/2}} \frac{\partial j_{\text{ohm}}}{\partial x} - \frac{j_{\text{ohm}}}{\mu_0 \alpha^2 \bar{\sigma} T_e^{3/2}} \left( \frac{3}{2} \frac{\partial T}{\partial x} + \frac{\partial \bar{\sigma}}{\partial x} T \right) \end{bmatrix} + \begin{bmatrix} \frac{1}{m_e kT_{(0)} \bar{\sigma} T_e^{3/2}} j_{\text{ohm}}^{2} \bar{\sigma} T_e^{3/2} \\ 0 \end{bmatrix}
\]

\( ^1 \)MATLAB is a numerical environment and programation language. More information is available at http://www.mathworks.com/
For simplicity, the equation is solved with a fixed edge temperature, regardless of the heat flux at the boundary. The total current is obtained through $J = J_{\text{ohm}} + J_{\text{cd}}$. The determination of the edge electric field will be described in details in the following section, which discusses the loop voltage evolution with current drive.

- **Electrical conductivity model** - Neoclassical corrections to Spitzer resistivity tend to peak the conductivity profiles and therefore the current profiles, which is an important effect when treating sawtooth oscillations. In this simplified model, we treat neoclassical effects by multiplying the classical formula with a correction factor $\tilde{\sigma}_{\text{neo}} = (1 - \epsilon^{1/2})^2$, as suggested in [Wesson 96]. Unfortunately, the corrective factor leads to strong peaking of the conductivity profile and thus current density profile at the magnetic axis, since $\frac{\partial \tilde{\sigma}_{\text{neo}}}{\partial \epsilon} = \frac{1}{\sqrt{\epsilon}} - 1 \to \infty$ as $\epsilon \to 0$. This singularity comes from taking the trapped particle fraction to go as $\sqrt{\epsilon}$ near this axis in the corrective factor, and can be resolved by including the existence of trapped orbits (such as potato orbits) at and near the axis. As a result, the trapped fraction does not go to zero at the axis, and the singularity disappears.

- **Thermal conductivity model** - Even for a simple model as that presented here, a minimal treatment of anomalous transport is required, and enters the system of equations through the choice of $\chi_e$. The goal here is to reproduce the overall behavior of the temperature profiles as observed in the experiment when strong localized heating is applied. In particular, we want to account for the experimentally observed stiffness or resilience of the profiles: in tokamak experiments (and C-Mod in particular [Greenwald 97]), the measured temperature (and thus conductivity) gradients tend to remain close to or below critical values, even as the heat flux increases. This property can be modeled with the heat diffusion equation above if $\chi_e$ is a function of the local temperature gradient $\nabla T_e$ and becomes very large for $|\nabla T_e| > |\nabla T_{e,\text{crit}}|$, thus preventing a further increase of $\nabla T_e$. Turbulence-induced transport is thought to provide the mechanism for the steep increase in $\chi_e$ above the threshold, and therefore profiles near the critical gradients are usually referred as marginally stable for these turbulent modes.

The expression for $\chi_e$ in the model presented here follows loosely from [Garbet 04]:

$$\chi_e = q^{3/2} T_e^{3/2} \left[ \chi_0 + \chi_s H (|\nabla T_e| - \kappa_e) (|\nabla T_e| - \kappa_e) \right]$$

(5.11)

where $H$ is a Heaviside step function. The prefactor captures a gyroBohm scaling $\chi_e \sim T_e \frac{e q}{e B H}$ and
the dependence of the confinement time on plasma current through $q^{3/2}$. The Heaviside function captures the steep increase of $\chi_e$ above the critical gradient $\kappa_c$. The threshold criterion here is on the temperature gradient and not the gradient scalelength $L_{Te} = \left(\frac{1}{T_e \frac{dT_e}{dr}}\right)^{-1}$. Models based on the critical gradient lengths, while more consistent with the results of turbulence simulations, predict a non-stiff edge, where $T_e$ is small. Since the description of the edge boundary in the model is very crude, it is preferable here to impose a stiff edge. The parameters $\chi_0(r), \chi_s(r)$ and $\kappa_c(r)$ are adjusted in order to match the behavior in the experiment and can be varied to obtain a more or less stiff transport model.

5.2.3 Loop voltage response

Since the plasma current is required for stability in tokamaks, it is common practice to control its value with the ohmic transformer during discharges. The total current is usually measured with Rogowski loops or another combination of magnetics measurements, and through a feedback mechanism, the transformer adjusts the toroidal electric field at the plasma edge so that changes in total current are compensated. In a cylinder, the change in net current can be related to the electric field gradient at the boundary:

$\frac{\partial}{\partial t} \int_0^a \rho_j(r) r dr = 2\pi \int \left( \frac{\partial}{\partial t} \rho_j(r) \right) r dr = 2\pi \int \left( \frac{1}{\mu_0 r} \frac{\partial}{\partial r} \left( r \frac{\partial E_\parallel}{\partial r} \right) \right) r dr$

Therefore, it is possible to maintain $\frac{\partial}{\partial t} I_p$ by adjusting $E_\parallel(a)$. The measured loop voltage $2\pi R E_\parallel(a)$ is thus an externally controlled parameter, and, since $I_p$ is held constant, it will show changes in the ohmic currents and therefore indicate the presence of driven currents. However, it is important to note that its time evolution will also depend on the details of the current relaxation when driven currents are applied.

Figure 5-6 shows a simulated current profile response for a circular cross section plasma with minor and major radii as in C-Mod. The total current is kept at 500 kA, and 100 kA is driven at $r/a = 0.5$ after 100 ms. The loop voltage response is shown in the center of the figure, while the four
Current drive turns on at $t = 100$ ms

Total plasma current: 500 kA

Ohmic only

$\sim 0.2$ V

100 kA driven

Figure 5-6: Simulated loop voltage response with 100 kA driven currents out of 500 kA.
plots at each corner show snapshots for the current density and electric field profiles. The central temperature is 3 keV, and goes down linearly as a function of radius. The initial state has a flat electric field profile and peak current density profile due to the temperature profile and neoclassical corrections to the conductivity. As the current turns on at 100 ms, we see that the immediate effect is a local reduction in the ohmic current density, which compensates the driven currents exactly. On very short timescales, the diffusion term \( \frac{1}{\mu_0 \sigma} \frac{\partial}{\partial r} \left( r \frac{\partial E}{\partial r} \right) \) can be neglected in the current diffusion equation, and the equation states that the ohmic current will change so as to conserve the total current. This results in a local change in the electric field, which will diffuse as a function of time. The plot on the lower right shows the profiles 3 ms after the currents were turned on. The currents have already diffused, resulting in an increased total current at the current drive location, and a local decrease of the total current on both sides of it. We recall that, according to equation 5.12, the total current cannot change until the electric field perturbation has reached the edge of the plasma. For this particular case, the loop voltage starts changing only 5 ms after the driven currents are turned on, and the transient evolution in the first 5 ms must keep the total current constant. After 70 ms, the electric field is relaxed in the outer half of the plasma, and the ohmic density profile is similar to that in the initial state, albeit with a smaller value. The excess currents have not diffused away from the core, which is consistent with the higher conductivity. The relaxation in the core is slower, and the loop voltage will continue to evolve until \( \sim 0.7 \) sec. The final loop voltage change, 0.2 V out of 1.0 V is consistent with a total driven of 100 kA out of 500 kA.

Note that the shape of the conductivity profile generally means that the loop voltage response will be faster for off-axis current drive. This also provides some ground for a sound definition of the edge loop voltage in realistic plasmas: while the plasma edge can be very inhomogeneous, it is also a region of low temperature and therefore conductivity, which means that fast electric field diffusion will guarantee that the location where the loop voltage is measured is not important.

We also note that the loop voltage response presented here is entirely linear as a function of the total driven current. In practice, current drive is almost always associated with localized heating, which modifies the conductivity profile. If the temperature changes, there will not be any reference loop voltage evolution like the green curve in the central plot for figure 5-6. In such cases, one can compare discharges with similar heating profiles, like co and counter-current drive. Another important aspect is the presence of MHD modes, which can modify the current density profile.
independently of equation 5.4. The C-Mod discharges above were strongly sawtoothing, and we will examine the resulting effects on the loop voltage response in the next section.

5.3 Effect of sawtooth events

5.3.1 Prescriptions for the post-crash profiles

A predictive simulation of the sawtooth cycle requires a model describing the changes in the current and temperature profiles as a result of a sawtooth crash. Unfortunately, the physics behind sawtooth reconnection is, at present, not fully understood (see for example [Wesson 96] for a discussion). In order to model the full sawtooth cycle, an alternative is to treat sawtooth events as instantaneous, compared to the preceding and subsequent sawtooth ramp phases. Simplified models can be used to prescribe the post-reconnection profiles. Three models have been considered in this thesis work:

- **Complete reconnection: Kadomstev model** - The complete reconnection model by Kadomstev [Kadomstev 75] is based on the following assumptions:
  1. The initial q profile is monotonic.
  2. Surfaces of equal helical flux $\Psi_s$ reconnect. In a cylinder, the helical flux is given by:

     \[
     \Psi_s(r) = \int_0^r B_\theta(\rho)(1 - q(\rho))d\rho
     \]  

     (5.13)

  3. The toroidal flux is conserved during reconnection, i.e. if surfaces with $r = r_a$ and $r = r_b > r_a$ reconnect (implying $\Psi_s(r_a) = \Psi_s(r_b)$), their final minor radius location $r_p$ will be such that $r_p^2 = r_b^2 - r_a^2$ and $\Psi_s(r_p) = \Psi_s(r_a) = \Psi_s(r_b)$.

For a monotonic profile with $q(\rho = 0) < 1$, the helical flux is zero on axis, increases up to the $q = 1$ radius then decreases. The radius $r > 0$ for which $\Psi_s(r) = 0$ is the mixing radius $r_{\text{mix}}$. It represents the outer flux surface involved in the full reconnection process. The Kadomstev prescription is illustrated in figure 5-7.

- **Incomplete reconnection: Porcelli model** - In the full reconnection model, the final safety factor on axis is always 1, and $q > 1$ everywhere in the profile. However, experiments in several tokamaks indicated that the central safety factor can remain well below unity during the entire
sawtooth cycle. In order to resolve this incompatibility, Porcelli [Porcelli 96] introduced an incomplete reconnection model, in which only a fraction of the mixing radius volume reconnects as in the Kadomstev model, while the remaining inner flux surfaces undergo a Taylor reconnection [Taylor 86, Gimblett 94]. The outer mixing radius will be smaller than in the Kadomstev model, thus the terminology incomplete but not partial, since the q profile is always changed from the magnetic axis outwards.

The prescription goes as follows [Porcelli 96, Bateman 06]:

1. The reconnection process starts as in the Kadomstev model, however it is interrupted when a critical width \( w \) is reached, defining two radii \( r_{\text{in}} \) and \( r_{\text{out}} \) such that \( \Psi_{\ast}(r_{\text{in}}) = \Psi_{\ast}(r_{\text{out}}) \) and \( r_{\text{out}} - r_{\text{in}} = \frac{w}{2} \).

2. The inner core \( r < r_{\text{in}} \) undergoes a Taylor relaxation, following which the final q profile is flat with:

\[
\frac{1}{q_f} = \frac{4}{r_1} \int_0^{r_{\text{in}}} r^3 dr \frac{1}{q} \tag{5.14}
\]
The annulus $r_{\text{in}} < r < r_{\text{out}}$ reconnects so that the final safety factor is 1.

Outside the mixing radius $r_{\text{mix}} = r_{\text{out}}$, the $q$ and helical flux profiles are unchanged.

In this model, two current sheets are generally formed at $r_{\text{in}}$ and $r_{\text{out}}$.

- **Partial reconnection: Parail-Pereversez-Pfeiffer model** - The assumption of a monotonic $q$ profile can be sometimes too restrictive to describe sawteeth reconnection when strong non-ohmic heating or current sources are present. Either the Kadomstev or Porcelli model can still be used when this assumption is violated if the helical flux profile increases monotonically then decreases monotonically with increasing major radius. When this condition is not met (for instance if multiple $q = 1$ surfaces are present, or $q_0 > 1$), neither the Kadomstev procedure nor the Porcelli prescription are well posed. A modified model proposed originally by Parail and Pereversev [Parail 80] can be used to form a prescription for such conditions [Pfeiffer 85]: the basic idea is that the instability is driven by the parts of the profiles where $q < 1$, and that the corresponding flux surfaces tend to move *outwards* and reconnect with flux surfaces with equal helical flux.

  1. The reconnection can occur for any region from $r_0$ to $r_1$ for which $\frac{\partial q}{\partial r} > 0$ (or $q < 1$). The outer mixing radius $r_{\text{mix, out}}$ is the smaller radius for which $\Psi_*(r_{\text{mix, out}}) = \Psi_*(r_0)$ and $r_{\text{mix, out}} > r_0$. The inner mixing radius $r_{\text{mix, in}}$ is the smallest radius for which $\Psi_*(r_{\text{mix, in}}) = \Psi_*(r_1)$ and $r_{\text{mix, in}} < r_1$. If no such radius exists, $r_{\text{mix, in}} = 0$.

  2. As in the Kadomstev model, surfaces of equal helical flux will reconnect.

The toroidal flux conservation in the general case is more subtle. For instance, if surfaces with equal helical flux $\Psi_* < 0$ are present, the final radii following the Kadomstev prescription do not extend up to the full reconnection region. In [Pfeiffer 85], the final safety factor profile in the reconnecting region seems such that $q = 1$ everywhere, although $\frac{\partial q}{\partial r} \neq 0$ suggests $q > 1$. The paper states: *Conservation of helical flux [...] is sufficient to determine the current density and safety factor profiles after mixing.* This condition alone does not give the final radii of the reconnected surfaces unless a final flat $q$ profile is assumed in the reconnected region. A more explicit prescription is as follows:

  ③ The toroidal flux is conserved, i.e. the sum of the area enclosed between adjacent reconnecting flux surfaces of equal helical flux determines the final toroidal flux of the reconnected surface. More specifically, for each $\Psi_{s_0}$ in the reconnecting region, we can define an increasing set of radii $r_i$, $1 < i < n$ in the reconnecting region for which $\Psi_*(r_i) = \Psi_{s_0}$. If $\Psi_{s_0} < 0$, we impose
\( r_1 = 0 \). Since \( \Psi_s(r) \) is continuous and \( \Psi_s(r = 0) = 0 \), the set is such that, excluding \( r_1 = 0 \), \( \frac{\partial}{\partial r} \Psi_s(r_{2i}) > 0 \) and \( \frac{\partial}{\partial r} \Psi_s(r_{2i+1}) < 0 \). By definition, the enclosed toroidal flux for surfaces of helical flux \( \Psi_{s0} \) will be proportional to the total enclosed area \( \pi \sum_{i>0} (r_{2i}^2 - r_{2i-1}^2) \). The conservation of flux gives \( \pi r_p^2 = \pi \sum_{i>0} (r_{2i}^2 - r_{2i-1}^2) \), where \( r_p \) is the radius of the reconnected flux surface with \( \Psi_s(r_p) = \Psi_{s0} \). This procedure is illustrated on figure 5-8. When done for the helical flux interval in the reconnecting region, this procedure leads to a monotonically decreased helical flux function covering the entire region.

In general, current sheets will be formed at \( r_{\text{mix,in}} \) and \( r_{\text{mix,out}} \). The volume inside \( r_{\text{mix,in}} \) will not be affected, hence this model describes a partial reconnection. As presented, however, the reconnection is complete. The prescription will give the same result as in the Kadmovstev model if \( r_{\text{mix,in}} = 0 \). It may be possible to make it incomplete by introducing a maximum island width \( w \).

### 5.3.2 Loop voltage measurements in sawtooothing plasmas

![Figure 5-9: Simulated loop voltage evolution with current drive in a sawtooothing plasma.](image)

Figure 5-9 shows loop voltage simulations with +50 kA and -50 kA driven currents in the mixing region of a plasma with 5 ms long sawteeth. The plasma conditions are similar to those in figure 5-6 and the Parail-Pereversez-Pfeiffer is used for the post-crash profiles. The equilibrium loop
voltage, 1.16 V, is larger than that in absence of sawteeth, as expected since sawteeth prevents the peaking of current in the core and therefore larger currents must be driven in the outer region. The loop voltage trace also shows oscillations, which reflect the rapid diffusion of the current sheet outside the mixing region, and subsequent relaxation. The driven currents are turned on at 100 ms, with Gaussian profiles at $r/a = 0.15$ and a normalized half-width of 0.05. The mixing radius in the simulation is at $r/a = 0.4$, so that the driven currents are entirely in the mixing region. We observe that, in spite of the driven currents, the loop voltage does not change significantly. After a transient phase of 50-100 ms, in which the loop voltage in co-current is higher than that in counter-current drive, the loop voltage traces reach back to the value they had before the currents were turned on. A loop voltage difference of 0.2 V would have been expected in the circuit model.

While the details of the sawtooth reconnection can be important for the simulations, the outstanding effect of sawtooth events is to short-circuit the plasma inside the mixing region and establish a flat current profile with $q \approx 1$. Intuitively, one can expect that the reconnection also re-establishes Maxwellian distributions inside the mixing region, and therefore the driven currents will be initially zeroed, however this distinction is irrelevant: the driven current profile will be quickly re-established after a sawtooth crash, and for the purpose of the simulations we can take them to be unaffected. Given driven current profiles inside the mixing radius, the requirement $q \approx 1$ completely determines the electric field profile in the mixing region, and each sawtooth event will reset the core electric field profile to these conditions. The driven currents can only affect the electric field profile outside the mixing region according to how they diffuse during the sawtooth ramp phase or how they have modified the sawtooth period or reconnection. In the simulations above, the period is fixed, and is short compared to the current diffusion time in the core, so that the edge loop voltage does not reflect the presence of the driven currents.

In the C-Mod MCCD experiments in section 5.1, clear differences are observed in the sawtooth cycle when the antenna phasing is varied. Figure 5-10 illustrates these differences by showing the central temperatures traces in a short time window for the discharges of figure 5-1. The period is longer in the co-current drive case, and the central temperature peaks at 5.3 keV instead of 4.5 keV in counter-current drive. This observation alone makes the use of the circuit model approach problematic, since the model would require matching electron temperatures. The current diffusion model discussed here shows that the presence of sawteeth greatly complicates the loop
Figure 5-10: Comparison of the sawteeth oscillations in the comparison discharges on figure 5-1

voltage analysis, and makes a direct measurement of the driven currents almost impossible. Instead, the driven currents change the loop voltage indirectly by modifying the sawtooth cycle, and the determination of their magnitude becomes impractical.
Chapter 6

Sawtooth period changes with MCCD

6.1 C-Mod experiments

While the presence of sawtooth oscillations can impede loop voltage measurements of net current drive, it provides at the same time a quite sensitive technique to detect localized current drive. Currents are driven close to \( q = 1 \) surface change the sawtooth period depending on their relative direction to the ohmic current density and relative position with respect to the \( q=1 \) surface. By scanning the current drive location through the \( q = 1 \) surface, the observed changes in the sawtooth period can therefore indicate localized current drive.

6.1.1 Experimental setup and results

For mode conversion current drive, it is possible to move the deposition location through the \( q = 1 \) surface by varying the toroidal field, since the minority concentration does not change much in the flattop. Two sets of discharges, referred as 1050729 and 1050802 have been dedicated to this experiment, and will be analyzed in detail here.

**Experimental setup** - The scenario is similar to that in the previous chapter, however a lower magnetic field is used in order to locate the mode conversion layer off-axis near the inversion radius at \( r/a \sim 0.3 \). The plasma mix was D, \(^3\)He and H, with a central electron density \( n_e \sim 1.5 \times 10^{-20} m^{-3} \). \(^3\)He gas was injected around 200 ms after breakdown in deuterium in order to obtain the desired \(^3\)He concentration in the flattop phase (0.5 - 1.5 sec here). A residual hydrogen content
Figure 6-1: Magnetic configuration and position of the mode conversion and cyclotron layers for two sets of discharges.

\[ \frac{n_H}{n_e} \sim 5 - 10\% \] due to wall recycling is estimated from the ratio of \( H_\alpha \) to \( D_\alpha \) emission at the plasma edge. At a toroidal field of 5.3T, a \(^3\)He concentration \( \frac{n_{\text{He}}}{n_e} = 20\% \) and H concentration \( \frac{n_H}{n_e} = 5\% \) creates conditions for mode conversion on the magnetic axis at 50 MHz. In this scenario, the ion cyclotron layers for \(^3\)He and deuterium are also present in the plasma. The small hydrogen concentration also allows D(H) minority heating on axis with the E antenna at 80 MHz. Figure 6-1 shows the location of the layers at 50 MHz and the plasma shape for the two sets of discharges 1050729 and 1050802, which will be analyzed in detail in this paper. The location of the mode conversion surface depends mostly on the magnetic field and the \(^3\)He concentration. Increasing the \(^3\)He concentration moves the mode conversion layer away from the \(^3\)He cyclotron layer towards the high field side, while increasing the magnetic field moves the cyclotron and mode conversion layers towards the low field side.
Figure 6-2: Evolution of the temperature in two groups of discharges with similar parameters but different phasings. The shaded regions indicate events like impurity injections or RF trips which hinder the comparison.

The requirement of a complete scan through the $q = 1$ surface is not compatible with the break-in-slope technique, and therefore it is not possible to measure power deposition profiles in this approach. The $^3$He concentration was estimated with the break-in-slope technique in the discharge immediately prior to the discharges dedicated to this experiment, as is assumed to remain comparable. The phase contrast imaging (PCI) system [Mazurenko 01] on C-Mod was not used in the experiment since the $q = 1$ surface is outside its observation window.

- **Experimental results** - The evolution of the toroidal field and electron temperature measured by the FRC-ECE X-mode radiometer is shown on Fig. 6-2 for the two sets of discharges 1050802 (a) and 1050729 (b). The temperature trace corresponds to a single ECE channel. Therefore the
Figure 6-3: Evolution of the temperature inside and outside the inversion radius between 0.65 and 0.75 sec in figure [6-2]. The transient increase outside \( r_{\text{inv}} \) gives a better indication of the sawtooth frequency.

The location of the temperature measurement changes as the toroidal field is varied, but it remains inside the \( q = 1 \) surface and allows a comparison of the sawtooth cycle and its period. In the two sets of discharges 1050729 and 1050802, the antenna phasing was the only control parameter varied. Clear differences in the sawtooth oscillations are observed as the phasing is changed. Their period changes noticeably for the different phasings.

A closer examination of the temperature traces shows noticeable MHD activity in the 1050802 discharges, especially in the early phase of the scan (figure [6-3]). Oscillations in the temperature traces can make the extraction of a sawtooth period difficult, however it is possible to discriminate between sawtooth crashes and other variations in temperature by inspecting the outer part of the mixing region, where sawtooth reconnections will increase the temperature transiently. This observation allows to extract the sawtooth period evolution for both sets as a function of time. The
Figure 6-4: Evolution of the sawtooth period in the first group of discharges on Fig. 6-2. The mode conversion layer location is deduced from the toroidal field and $^3\text{He}$ concentration estimate.

corresponding curves are shown on Figs. 6-4 and 6-6 for the 1050802 and 1050729 discharges respectively.

The curves show significant variations in the sawtooth period evolution in both sets of discharges. Comparing Figs. 6-4 and 6-6, it can be seen that as the mode conversion layer moves away from the magnetic axis, the sawtooth period shortens to $\sim 4$ ms in co-current drive then lengthens to $\sim 10$ ms. A reversed evolution is obtained for counter-current drive, although the period is not reduced as much. The sawtooth period lengthens to $\sim 11$ ms. The evolution in symmetric phasing is similar to that in co-current drive phasing until 0.9 s (see Fig. 6-4). Using concentration estimates from the break-in-slope analysis, it is possible to compare the location of the mode conversion surface with the inversion radius $r_{\text{inv}}$. In the 1050802 discharges, the inversion radius is around 75.5 cm. This is about 1 cm outside the mode conversion layer location for which the co and counter-current drive curves crossover. The position of the inversion radius $r_{\text{inv}}$ and mode conversion layer is also shown as a function of time on figure 6-5. In the 1050729 discharges, only
Figure 6-5: Evolution of the inversion radius corresponding to figure 6-4. The mode conversion layer location is deduced from the toroidal field and $^3$He concentration estimate.

Figure 6-6: Evolution of the sawteeth in the second group of discharges on Fig. 6-2. The mode conversion layer location is deduced from the toroidal field and $^3$He concentration estimate.
half of the evolution is obtained but the cross-over position is less obvious. The curves suggest it occurs when the mode conversion layer is around 75 cm. Since the inversion radius is around 76.5 cm in this discharge, we find that the relative positions are qualitatively consistent.

While the achieved discharges parameters are very similar, unexpected events like impurity injections or antenna faults can hinder the comparison. Such events are highlighted with shaded color in Fig. 6-2. Impurity injections induce a rapid increase in the radiated power and cool down the edge plasma. The core temperature is reduced and the sawtooth period is typically shortened. The effects typically last for a confinement time $\tau_E \sim 50$ ms, after which the plasma recovers. This typical evolution after an impurity event is illustrated on Fig. 6-6 between 0.95 - 1 s. Similar events occurred in the counter-current drive discharge of 1050802 around 0.75 s, and repeatedly in the heating discharge of 1050802 after 0.9 s. In addition to the transient effect on temperature, it is possible that the impurity influx changes the plasma composition and therefore the location of the mode conversion layer, although this effect is hard to evaluate.

- Initial discussion - The observed variations in the sawtooth period are similar to that obtained in Ion Cyclotron Current Drive (ICCD) or Electron Cyclotron Current Drive (ECCD) experiments when the deposition is scanned around the $q = 1$ surface. In [Bhatnagar 94], as the ion cyclotron layer was swept on JET through the inversion radius $r_{\text{inv}}$ on the inboard side, $+90^\circ$ injection inside $r_{\text{inv}}$ shortened the sawtooth period and lengthened it outside $r_{\text{inv}}$. The trend was reversed with $-90^\circ$ injection. On the outboard, the changes were in the opposite direction in both phasings. The observed sawtooth period evolution was attributed to local changes in the safety factor shear at the $q = 1$ surface due to localized ICCD. Recent experiments with ECCD on ASDEX-Upgrade [Mueck 05] also have provided evidence for this mechanism. As the ECCD deposition was scanned across the plasma radius, sawtooth oscillations were completely stabilized with co-ECCD outside the inversion radius or counter-ECCD inside the inversion radius. Complete stabilization is usually the equivalent of a long or infinite sawtooth period. The sawtooth period can also be decreased with co-ECCD inside the $r_{\text{inv}}$. In further experiments on ASDEX-Upgrade [Manini 05], the sawtooth period evolution was studied in a similar scan but without complete stabilization and using different current profile widths. Again, the sawtooth period was lengthened for co-current drive outside the inversion radius, and shortened inside. The evolution was reversed in counter-current drive. Note that in both ASDEX-Upgrade experiments, the plasmas were also heated with Neutral Beam
Injection (NBI). Careful analysis was required to isolate the ECCD and NBI-related contributions to the sawtooth period changes.

These results suggest that localized MCCD can account for the observed sawtooth period evolution in the experiments reported here. The location of the inversion radius and mode conversion layer is consistent and so is the evolution for the opposite phasings. However, it is essential to discuss other possible mechanisms which may affect the sawtooth period and their relevance to this experiment. Since the plasmas considered here are ICRF-heated with exclusion of other auxiliary heating systems, the main competing processes are related to fast wave ion cyclotron damping and fast ions populations.

The stabilizing effect of energetic ions inside the $q=1$ surface is discussed in [Porcelli 91] and has been obtained experimentally with ICRF minority heating on several tokamaks including C-Mod. Varying the antenna phasing can change the fast particle pressure profiles inside the $q=1$ surface due to an ICRF-induced particle pinch [Eriksson 98] and thus result in sawtooth period changes. In the experiment reported here, minority heating of $^3$He ions can occur in addition to MCEH, and while the large $^3$He concentration minimizes this mechanism, one can also exclude a contribution from an energetic $^3$He ion population based on the experimental data alone. A comparison between the ion cyclotron layer positions in the 1050729 and 1050802 discharges on Figs. 6-1 and 6-7 shows that at the cross-over point the $^3$He ion cyclotron layer was on axis and around the inversion radius respectively. In the 1050802 discharges, the $^3$He ion cyclotron layer is inside the inversion radius surface after 0.8 sec, yet the sawtooth period in the three phasings is comparable after 1.1 sec. The scan was continued until 1.4 sec, at which point the $^3$He ion cyclotron layer is on axis and the sawtooth period (not shown) is still unchanged as the antenna phasing is varied. The sawtooth period is also only slightly longer than during the ohmic phase (before .6 sec), indicating little fast particle stabilization. Therefore, a contribution from energetic $^3$He ions in this experiment is very unlikely.

Following the same approach, one can exclude ICCD as a dominant mechanism in this experiment for the 1050729 discharges, since from Fig. 6-7 the ion cyclotron layer is on axis as the sawtooth period changes are observed. In the 1050802 discharges however, the $^3$He resonance layer is close to the inversion radius. Therefore, a contribution from ICCD cannot be ruled out based on the layer position alone, but again, is very unlikely owing to the large $^3$He concentration.
in the discharges (above 20 %) and the low single pass absorption for ion cyclotron damping in these conditions [Porkolab 93]. This will be further supported by the TORIC simulations in the following section, which predict very little fast wave absorption on $^3$He ions. Ion Cyclotron Current Drive has also been investigated on Alcator C-Mod with D(H) minority heating but a similar deposition scan as discussed here (with the same input power) did not induce clear changes in the sawtooth period. As pointed out in [Bhatnagar 94], the current drive efficiency for D($^3$He) scenario is lower compared to D(H). Therefore, comparing these D(H) minority heating results with the D($^3$He) mode conversion regime here, one may conclude that ICCD is not the dominant contribution here.

Based on this initial discussion, it appears that localized current drive around the mode conversion surface is the most plausible mechanism to explain the observed sawtooth period evolution. To further support this conclusion, we will show in the next section that the driven current profiles predicted by full wave TORIC simulations are qualitatively consistent with the experimental results.
Figure 6-8: Power deposition and driven current profiles predicted by TORIC for the 1050802 discharges at 0.85 sec.

6.1.2 TORIC simulations

TORIC simulations for the 1050802 discharges at 0.85 sec are shown on Fig. 6-8 for the three phasings in the experiment. Fig. 6-8(a) shows the power deposition profiles for electrons and all ions species. Electron absorption dominates and accounts for $\sim 60\%$ of the coupled ICRF power. Over the entire antenna spectrum $(-20 \leq n_\phi \leq 20)$, the profiles show a two peak structure corresponding to $n_\phi = \pm 7$ at the maximum of the radiated spectrum. The co and counter-CD phasings favor positive or negative toroidal mode numbers respectively, while the symmetric heating phasing appears as a combination of the two. It appears that the power deposition is closer to the axis for positive $n_\phi$. This corresponds to a vertical shift in the deposition location along the mode conversion surface.

Fig. 6-8(b) shows an estimate of the current profiles for the 1050802 discharges based on the current profile reconstructed by EFIT and the driven currents computed by TORIC. For this off-axis mode conversion current drive scenario, the overall current drive efficiency is low. The net driven currents are less than 20 kA. While this value is small compared to the plasma current $I_p = 600kA$, the local change in the current density is nevertheless significant. The peak driven current density
for the co-current drive phasing is 4MA m\(^{-2}\). At the location of the mode conversion layer, the current density estimated by EFIT is \(\sim 10\)MA m\(^{-2}\). Therefore, we can expect significant changes in the q profile associated with these driven current profiles.

The shape of the driven current density profiles predicted by TORIC exhibits noticeable differences for the co, counter-CD and heating phasings. While the driven current density profile for co-current drive is peaked with a slight reversal outside, the profile in counter-current drive is bipolar, with currents in the counter-CD direction inside and in the co-direction outside. Note that the small difference in current density for \(r/a < 0.2\) corresponds to a weak FWCD effect.

With these simulation results, we can move forward in our interpretation of the experimental results. The driven currents predicted by TORIC are clearly large enough to change the safety profile locally. We can compare the overall shape of the driven current profiles predicted by TORIC with that in the other experiments mentioned above. In co-current drive phasing, the driven current profile is similar to that in ECCD experiments [Manini 05]. It increases the shear at the q=1 surface when deposited inside and decreases it when deposited outside, which results in an shorter sawtooth period inside and longer outside. The bipolar profile for counter-current drive phasing appears more similar to that in the ICCD experiments [Bhatnagar 94]. The driven currents tend to flatten the shear over a relatively large radius and thus lengthen the sawtooth periods. As in [Bhatnagar 94], the sawtooth period is not reduced during the sweep. In both cases, assuming that the q=1 surface does not move during the scan, the width of the predicted driven current profiles should also be consistent with the radial extent of the sawtooth period changes on Fig. 6-4. We can see that this is indeed the case in the co- and counter-current drive phasings.

Therefore, this qualitative comparison shows that the TORIC predictions and the experimental results are compatible. This supports the interpretation of the observed sawtooth period evolution in co- and counter-current drive phasing in terms of localized mode conversion current drive. We conclude that MCCD can be used for localized current profile control.

**Evolution in heating phasing** - As seen on figure 6-4 the sawtooth period evolution in heating and co-current drive phasing is similar for the 1050802 set of discharges. While impurity injections in the heating discharge after .9 sec can hamper the comparison, the similarly is very striking in the earlier part. A possible explanation for this observation is the effect of localized heating on the sawtooth period, which has been identified most clearly in Electron Cyclotron Heating
(ECH) experiments on TCV \cite{Angioni03}. In these experiments, an ECH beam was injected in the plasma so that $k_\parallel \approx 0$. As the deposition location was swept through the inversion radius, the sawtooth period increased from 4 ms to 8 ms then decreases again to 4 ms. This behavior is similar to what would be obtained with co-current drive. Therefore, since strong localized MCEH is also expected in the C-Mod experiments, this mechanism could account for the sawtooth period evolution observed in heating phasing.

An alternative interpretation is found in the TORIC predictions. As can be seen on figure 6-8 (b), net current drive is predicted in symmetric/heating phasing, as a result of up-down asymmetries in the mode conversion process, as discussed in chapter 3. The presence of net driven currents in symmetric phasing can also explain the similarity with the sawtooth period evolution in co-current drive phasing.

In order to estimate the relative strength of the two effects in the C-Mod experiments, a more quantitative model is required. In addition to the interpretation of the heating phasing results in C-Mod, this model will also allow us to understand in more details the physics behind sawtooth period variations with current drive.

6.2 Sawtooth trigger model

As we saw in the last section, the relationship between the observed sawtooth period evolution in C-Mod and localized current drive is not straightforward, although the overall behavior appears to be consistent when comparing with similar experimental results using ICCD and ECCD. A more quantitative and theoretical interpretation would require a complete model for the sawtooth cycle. In the preceding chapter, we treated sawtoothing plasma by ignoring MHD activity during the sawtooth ramp phase and using a phenomenological model for sawtooth events, with prescriptions for the relaxed temperature and current density profiles after a crash. Within these assumptions, the missing element for a full model of the cycle is a condition for the onset of a sawtooth event, i.e. a trigger model. Given current density, temperature and pressure profiles at each time step during the sawtooth ramp, this model will indicate whether a crash should occur.

Experimental observations suggest that sawtooth events are always preceded by the growth of an $m = 1$ internal kink instability. A reasonable trigger model can therefore follow from the linear
stability criteria for such modes, which has been studied extensively since the early observations in the ST tokamak. As will be illustrated in this section, the linear theory of \( m = 1 \) internal kink modes in tokamaks is complicated, since it involves details of the magnetic configuration, toroidal effects and non-ideal MHD effects like resistivity, viscosity and kinetic treatments. To obtain a stability criterion compatible with simulations of the sawtooth cycle, a simplified model must be developed. Luckily, a significant amount of work has also been devoted to this task. Indeed, sawtooth oscillations may have severe deleterious effects on ITER confinement and performance by creating seed islands for neoclassical tearing modes and thus inducing indirectly a loss of alpha particle confinement. There is therefore much interest in controlling sawtooth oscillations, either through complete stabilization or by making them small and frequent. The leading model used in these studies was developed by Porcelli [Porcelli 96].

It is important to realize that, contrary to these models, we are not interested in predicting the actual period of sawtooth oscillations for a particular plasma equilibrium and set of profiles. Instead, in order to interpret the C-Mod experiments presented in the preceding sections, we want to understand how localized heating and current drive can induce relative changes in the sawtooth period. Recent studies suggest that, in addition with the trigger model and the evolution of the profiles in the sawtooth ramp, the details of the sawtooth reconnection model can also affect the absolute magnitude of the sawtooth oscillations. One parameter involved in this model is the fraction of reconnected flux in each crash [Porcelli 96] [Bateman 06]. Experimental results on C-Mod cannot indicate whether the sawtooth reconnection is systematically partial (Porcelli model) or complete (Kadomstev model), nor is a model available to predict this behavior theoretically. In this context, we will assume that reconnections on C-Mod are systematically complete and will always occur according to Kadomstev/Parail prescriptions.

### 6.2.1 \( m = 1 \) internal kink modes

The structure and basic stability properties of internal kink modes can be introduced in a cylindrical plasma with length \( 2\pi R \) and periodic boundary conditions at its ends. To approximate a tokamak magnetic field configuration, we consider a general screw pinch and take a low \( \beta \) and low aspect ratio approximation. In this *straight tokamak geometry*, \( m = 1 \) internal kink modes are
perturbations from the equilibrium of the form:

$$\tilde{\zeta} = \tilde{\zeta}(r) \exp(-i\omega t + im\theta - ik_z z)$$

(6.1)

with \(m = 1\), \(k_z = 1/R\), and \(\xi(a) = 0\) where \(a\) is the (minor) radius of the column. Introducing the safety factor \(q = \frac{rB_z}{RB_g}\), this structure gives \(k || = (\frac{1}{q} - 1)\frac{B_z}{BR}\) and thus \(k || = 0\) at radii where the safety factor is unity. On such flux surfaces, the perturbation can develop without bending magnetic field lines.

The stability properties of the internal kink mode Ideal MHD theory indicates that a mode of this form is always unstable in a straight tokamak [Freidberg 87] if a \(q = 1\) surface exists in the plasma. By the usual minimization procedure in a screw pinch, the potential energy \(\delta W\) can be reduced to a term of order \(\epsilon^2 = \left(\frac{a}{R}\right)^2\):

$$\delta W(\xi) = \frac{1}{2} \int F \left(\frac{\partial \xi}{\partial r}\right)^2 d^3r \quad \text{with} \quad F = \left(\frac{r}{R}\right)^2 \frac{B^2}{\mu_0}(1 - \frac{1}{q})^2 = \left(\frac{r}{R}\right)^2 \frac{R^2}{\mu_0}(\vec{k} \cdot \vec{B})^2$$

(6.2)

This term captures the contribution of field line bending, and will always be strictly positive unless a \(q = 1\) surface exists in the plasma. In this particular case, the integrand is zero at the resonant surface and it is possible to construct a nonzero displacement function \(\xi\) which will make this term vanish, by taking \(\frac{\partial \xi}{\partial r} = 0\) except at the resonant layer. For example, if the safety factor is monotonically increasing, with \(q = 1\) at \(r = r_1\), one would choose displacement function of the form

$$\xi_{r0}(r) = \begin{cases} 
\xi_\infty & \text{for } r < r_1 \\
0 & \text{for } r > r_1 \end{cases}$$

(6.3)

This procedure can be extended to any situation where the safety factor profile goes through unity. The stability of the internal kink mode will then depend on higher order contributions to \(\delta W\). In a cylinder, the next contribution is of order \(\epsilon^4\), and can be written as a sum of destabilizing terms involving both the plasma pressure and parallel current. Therefore the mode will always be unstable if \(q = 1\) is satisfied inside the plasma, with a growth rate of order \(\epsilon^2\).

The cylindrical result does not extend to realistic tokamak conditions, since corrections due
toroidicity, shaping or non ideal MHD effects are of the same order in \( \epsilon \). We will discuss these corrections in the next section. However, the global structure of the perturbation minimizing \( \delta W \) remains similar, since the field line bending term is still the dominant contribution. After the minimization with respect to \( \xi \) is carried out, we can write \( \delta W \) in a general form, formally applicable both to the torus and cylinder:

\[
\delta W(\xi) = \frac{1}{2} \int F \left( \frac{\partial \xi_r}{\partial r} \right)^2 d^3r + \delta W_\epsilon \quad \text{where} \quad \delta W_\epsilon(\xi) = \frac{1}{2} \int \xi \cdot G(\xi) d^3r \quad (6.4)
\]

where \( G \) is of order \( \epsilon^3 \) or higher. Carrying the minimization of \( \delta W(\xi) \delta W \) can be expressed as a function of the radial component \( \xi_r \) only, and obtain the other components implicitly as a consequence of minimization arguments. The displacement function is written \( \xi_r(r) = \xi_{r0}(r) + \xi_{re}(r) \), with \( \xi_{r0} \) as above and \( |\xi_{re}| \ll \xi_\infty \). A normal mode equation can be obtained by applying variational methods to the total energy, yielding \( \delta E_{\text{kin}} + \delta W = 0 \). In evaluating the change in kinetic energy \( \delta E_{\text{kin}} \) and \( \delta W \), special care must be taken to be consistent with the minimization approach from which \( \delta W \) was obtained \[\text{Freidberg 87}\]. In particular, one must account for all three components of \( \xi \) even if \( \delta W(\xi) \) in equation \[6.4\] depends only on \( \xi_r \). For the \( m=1, n=1 \) mode, this procedure gives the following equation \[\text{Rosenbluth 73}\]:

\[
2\pi^2 R \frac{\partial}{\partial r} \left[ \left( \frac{\tilde{k} \cdot \tilde{B}}{\mu_0} \right)^2 + \rho \omega^2 \right] r^2 \frac{\partial \xi_r}{\partial r} - G(\xi_r) = 0 \quad (6.5)
\]

Recalling that the growth rate is of order \( \epsilon^2 \), the inertial term \( \rho \omega^2 \) will be negligible compared to the field line bending term \( \tilde{k} \cdot \tilde{B} \), except in the vicinity of the singular layer. Where it can be ignored, the solutions of the normal mode equation are, to first order in the expansion parameter:

\[
\frac{1}{\xi_\infty} \frac{d\xi_r}{dr} = \begin{cases} 
\frac{1}{2\pi^2 R} \int_0^r \xi_{r0} \cdot G(\xi_{r0}) d^3r - \frac{\mu_0}{r^3(\tilde{k} \cdot \tilde{B})^2} & \text{for } r < r_1 \\
\frac{1}{2\pi^2 R} \int_0^{r_1} \xi_{r0} \cdot G(\xi_{r0}) d^3r - \frac{\mu_0}{r^3(\tilde{k} \cdot \tilde{B})^2} & \text{for } r > r_1
\end{cases} \quad (6.6)
\]

We note that, to leading order, \( \delta W_{\text{min}} = \int_0^{r_1} \xi_{r0} \cdot G(\xi_{r0}) d^3r \) is the minimum of the potential energy for the perturbation in the non resonant region, also called external region in the litterature. The solutions diverge for \( (\tilde{k} \cdot \tilde{B})^2 \to 0 \), i.e. at the singular layer \( r = r_1 \).

To obtain the growth rate of the mode, the normal mode equation is solved inside the narrow
resonant layer by linearizing \((\vec{k} \cdot \vec{B})^2 = \frac{B^2}{R^2} (1 - \frac{k}{q})^2\), and assuming all other plasma quantities remain constant. We expand \(q = 1 + x s_1\), where \(x = \frac{(r-r_1)}{r_1}\) and \(s_1 = \frac{r_1}{q} \left. \frac{dq}{dr} \right|_{r_1}\) is the shear at the \(q = 1\) surface. This gives \((\vec{k} \cdot \vec{B})^2 = \frac{B^2}{R^2} x^2 s_1^2\) and

\[
2\pi^2 R \frac{\partial}{\partial x} \left[ \left( \frac{B^2}{\mu_0 R^2} x^2 s_1^2 + \rho_1 \omega^2 \right) r_1^3 \frac{\partial \xi_r}{\partial x} \right] - G(\xi_r) = 0 \tag{6.7}
\]

In the ideal MHD case, \(G(\xi_r)\) is negligible near the resonant layer and it is easy to show that \(\xi_r\) has inverse tangent solutions plus or minus a constant. We take a solution with \(\xi_r \to \xi_\infty\) as \(x \to -\infty\) and \(\xi_r \to 0\) for \(x \to \infty\). This imposes \(\xi = \frac{\xi_\infty}{\pi} \left( \frac{\pi}{2} - \arctan \left( \frac{x}{\lambda_H} \right) \right)\), and the remaining free parameter \(\lambda_H\) is obtained by matching the derivatives in equation \(6.6\) for \(x \to -\infty\), i.e. at the inner boundary of the resonant layer, for \(\delta W_{\text{min}} < 0\):

\[
\frac{\lambda_H}{\pi x^2} = \frac{1}{\xi_\infty} \frac{d\xi_r}{dx} \quad \lambda_H = -\frac{1}{2\pi} \frac{\delta W_{\text{min}}}{\xi_\infty} \frac{\mu_0 R}{r_1^2 B^2 s_1^2} \tag{6.8}
\]

The parameter \(\lambda_H\) is the normalized width of the resonant layer, and is related to the growth rate of the mode \(\gamma = -i\omega\):

\[
\gamma = \lambda_H \frac{s_1}{\tau_A} = -\frac{\tau_A}{2\pi} \frac{\delta W_{\text{min}}}{\rho_1 \xi_\infty^2 B^2 / \mu_0 s_1 r_1^2} \frac{R^2}{\delta W_{\text{min}}} \tag{6.9}
\]

where we introduce the Alfven time \(\tau_A = \frac{R}{v_A}\), with \(v_A = \frac{B}{\sqrt{\rho_1 \mu_0}}\). In the tearing mode formalism, another quantity of interest is the discontinuity is the logarithmic derivative of the perturbed radial magnetic flux \(\Delta'\), which is formally related to \(\lambda_H\) through \(\Delta' = -\pi / \lambda_H\) [Ara 78].

From this derivation, we see that growth rate of the mode is related to the quantity \(\delta W_{\text{min}} = \frac{1}{2} \int_0^{r_1} \xi_{r0} \cdot G(\xi_{r0}) d^3 r\) through the singular physics. \(\delta W_{\text{min}}\) is obtained from \(\delta W_r\) in equation \(6.4\) and the minimizing perturbation at zero order in equation \(6.3\). The procedure used here strictly speaking for an ideal MHD screw pinch plasma can be extended to obtain the growth rate and mode structure (particularly the singular layer width) in more general cases, including toroidal shaped plasmas where non ideal MHD effects are important. The corrections will be formally separated in two categories, depending on whether they affect the global stability of the mode \(\delta W_{\text{min}}\) or modify the normal mode equation only in the resonant layer.
6.2.2 Global corrections to $\delta W$

In writing the relevant corrections to $\delta W_{min}$, it is useful to introduce a normalized functional $\delta \hat{W}$ [Porcelli 96]:

$$\delta \hat{W} = \frac{1}{\pi s_1 \xi_2 \gamma_1^2 R \beta^2 / \mu_0} = \lambda_H s_1$$  \hspace{1cm} (6.10)

so that the ideal growth rate satisfies $\delta \hat{W}_{\text{core}} = \gamma \tau_A$. We will discuss here contributions associated with toroidal effects, non-circular cross sections and the magnetic trapping of thermal ions. Much effort has been devoted to evaluating these corrections, both analytically and numerically, however for implementation in a sawtooth trigger model, it is desirable to use simplified expressions. In [Porcelli 96], $\delta \hat{W}$ is expressed as the sum of the following contributions:

$$\delta \hat{W} = \delta \hat{W}_{\text{core}} + \delta \hat{W}_{\text{fast}} \quad \hat{W}_{\text{core}} = \delta \hat{W}_{\text{Bussac}} + \delta \hat{W}_{\text{el}} + \delta \hat{W}_{\text{KO}}$$  \hspace{1cm} (6.11)

- **Toroidal corrections** - As mentioned in the last subsection, the stability of $m = 1$ modes in a circular tokamak differs significantly from that in a cylinder. The original treatment by Bussac et al. [Bussac 75] showed that toroidal corrections cancel exactly the cylindrical corrections, and the additional contributions lead to stability for parabolic safety factor profiles if the poloidal beta inside the $q = 1$ surface was below a critical value $\beta_{p,\text{crit}}$. Physically, at lowest order, toroidal geometry couples the $m = 1$ modes to $m = 2$ perturbations, which are stable. The Porcelli model retains this particular case to evaluate the toroidal ideal MHD contribution to $\delta \hat{W}$:

$$\delta \hat{W}_{\text{Bussac}} = -\frac{9\pi}{s_1} (l_{i1} - \frac{1}{2}) \xi_1 (\beta_{p1}^2 - \beta_{pc}^2)$$  \hspace{1cm} (6.12)

with $\beta_{p1} = \frac{2\mu_0}{\rho_{p1}}(< p >_1 - p_1)$ and $\beta_{pc} = 0.3(1 - \frac{5\rho_1}{3a})$. In this expression, the subscript 1 indicates that quantities are evaluated at the $q = 1$ surface, $s_1$ is the shear, $l_{i1}$ the internal inductance and $p_1$ is the pressure. $a$ is the minor radius. In the poloidal beta parameter, $< >_1$ denotes volume averaging inside the $q = 1$ surface, so that $\beta_{p1} = 0$ for flat pressure profiles.

The Bussac formalism can be extended to non-monotonic profiles, or profiles with multiple $q = 1$ surfaces [Hastie 87]. The calculation involves solving the radial mode structure for $m = 2$ modes.
for the considered q profile shape:

\[
\frac{d}{dr} \left[ r^3 \left( \frac{1}{q} - \frac{1}{2} \right)^2 \frac{d\xi^{(2)}}{dr} \right] - 3r \left( \frac{1}{q} - \frac{1}{2} \right)^2 \xi^{(2)} = 0
\]  

(6.13)

After algebraic minimization of the potential energy contributions, analytic expressions can be obtained from the derivatives of \(\xi^{(2)}\) at the singular \(q = 1\) surfaces, and are derived for the case of a single \(q = 1\) surface and for a pair \([\text{Hastie 87}]\). This situation could in principle happen as a result of strong localized current drive.

A last notable correction \([\text{Glasser 75}]\) appears in the inertial term \(\rho_1 \omega^2\), which is higher by \((1 + 2q^2) \approx 3\) in toroidal geometry. The numerical factor can be included in the definition of \(\tau_A\), which we replace by a toroidal Alfven time \(\tau_A = \frac{\sqrt{\pi} c}{v_A}\).

- **Corrections due to shaping** - Non-circular cross sections also couple \(m = 1\) modes to perturbations with higher poloidal mode numbers. In the Porcelli mode \([\text{Porcelli 96}]\), only the effect of elongation is considered. In a plasma with elongation \(\kappa\), the Bussac term retains the same form but \(r_1, \epsilon_1\) and \(a\) need to be corrected. \(r_1\) is formally replaced by \(\bar{r}_1 = \kappa^{1/2} r_1\), \(\epsilon_1\) by \(\bar{\epsilon}_1 = \frac{\bar{r}_1}{\bar{r}}\) and \(a\) by \(\bar{a} = \kappa^{1/2} a\). In addition, a corrective elongation term is added to the ideal Bussac term:

\[
\delta \hat{W}_{\text{el}} = -\frac{18\pi}{s_1} (l_{i1} - \frac{1}{2})^3 (\kappa_1 - 1 - \frac{1}{2})^2
\]  

(6.14)

The effect of other shaping parameters is discussed in \([\text{Wahlberg 98, Martynov 05}]\).

- **Contribution from trapped thermal ions (Kruskal-Oberman term)** - As stated in \([\text{Antonsen 93}]\), this correction accounts for the fact that thermal trapped ions are unable to respond to flute-like perturbations which develop slowly compared the toroidal precession frequency timescale \(\frac{2\pi}{\omega_D}\), where \(\omega_D\) is the toroidal precession drift frequency \([\text{Eriksson 01}]\). This eliminates their contribution to the potential energy, which would be destabilizing since trapped particles experience mostly unfavorable curvature. Perturbations with timescales comparable to \(\frac{2\pi}{\omega_D}\) are not affected by this effect, and can still be unstable, as for example is the case for fishbone instabilities.

The correction to the potential energy associated with this effect can be estimated using the low frequency kinetic energy principle \([\text{Antonsen 93, Porcelli 96}]\).

\[
\delta \hat{W}_{\text{KO}} = \frac{0.6c_p}{s_1} \epsilon_1^{1/2} \beta_{i0}  \\
\text{with } c_p = \frac{5}{2} \int_0^{r_1} \frac{dr}{r_1} \left( \frac{r}{r_1} \right)^{3/2} \frac{p_i(r)}{p_i(r = 0)}
\]  

(6.15)
where $\beta_{i0}$ is the peak ion toroidal beta.

Fast particle stabilization - As briefly alluded to in our initial discussion of the experimental results, energetic ions can have a strong stabilizing effect on $m = 1$ modes. This can be understood [Porcelli 91] based on the conservation of the third adiabatic invariant $\Phi$ [Northrop 63] associated with the trapped fast ion toroidal precession. The invariance holds when the rate of change of the magnetic field is slow compared to the precession drift frequency $\omega_D$ [Eriksson 01, Porcelli 91]. For $m = 1$ modes, this rate can be taken as the ideal growth rate, which gives the condition:

$$-\delta \dot{W}_{\text{core}} < c_h T_A \omega_D$$  where $c_h$ is a constant, or order unity  \hfill (6.16)$$

When this condition is met, the invariance of $\Phi$ makes the fast ion population, if present, strongly stabilizing against perturbations of the magnetic field. This results in a large positive stabilizing contribution to $\delta \dot{W}$, usually larger than $\delta \dot{W}_{\text{core}}$ when the fast ion pressure inside the $q = 1$ surface is significant. This contribution $\delta \dot{W}_{\text{fast}}$ depends on the details of the fast ion pressure profile, and should in principle be evaluated using MHD codes. As mentioned in the last section, we do not expect fast particles to play an important role in the MCCD experiments discussed here, and therefore we will ignore this contribution here.

6.2.3 Corrections in the resonant layer

Local corrections to the ideal MHD model near $r_1$ will only be important when the mode is near marginal stability, in which case we expect the resonant layer to be narrow and behave essentially like a slab. The normal mode equation is extended to include two fluid resistive plasma effects, with appropriate kinetic corrections [Ara 78], and solved to calculate the normalized width of the resonant layer $\delta$ and the growth rate of the mode $\gamma$ given the boundary condition in equation 6.6. The two parameters are conveniently written as functions of the quantity $\lambda_H$ introduced above, which measures the contribution of the external region. It is also useful to introduce a normalized growth rate $\lambda = \frac{\gamma}{s_{1}}$, so that in the ideal MHD case we simply have $\delta = \lambda = \lambda_H$.

Like in the previous section, our goal is not to present a comprehensive review of all possible phenomena in the resonant layer, but rather to identify appropriate limits and approximation for typical plasma conditions in Alcator C-Mod. This will yield expressions for the growth rate and
help determine relevant trigger conditions for our analysis. The interested reader can consult a quite extensive review by Migliulo [Migliuolo 93]. As a general indicator of the relevant effects, one can come back to equation 6.7 and note that the field line bending term will always dominate over all other terms far away from the resonant layer. The dominant correction will start competing with this term at a larger distance from the q = 1 surface, which physically determines the actual extent of the layer, but can also be used to assess the relative importance of the corrective terms. We consider here two main destabilizing effects in the singular layer: resistivity and finite ion Larmor radius effects.

- **Resistivity** - Resistivity is one of the potentially important non ideal MHD effects in the resonant layer [Coppi 76, Ara 78, Porcelli 87]. For $\lambda_H = 0$, the width of the singular layer with finite resistivity is of order $r_1 \epsilon_1^{1/3}$ where $\epsilon_1 = \frac{1}{s_1 \mu \sigma_1 r_1^2}$ and $\sigma_1$ is the conductivity at the q = 1 surface. The growth rate of the mode can be evaluated analytically as a function of $\lambda_H$ [Porcelli 87], and is plotted on figure 6-9. We see that the resistive internal kink mode will always be unstable in this picture, albeit with a small growth rate. When $\lambda_H > \epsilon_1^{1/3}$, we find the ideal MHD limit, $\lambda \sim \lambda_H$. Physically, finite resistivity decouples the plasma motion from the field lines, and therefore facilitates the release of free energy.

- **FLR effects** - When the width of the singular layer (as set by other mechanisms, for instance resistivity) becomes smaller than the ion Larmor radius, kinetic effects come into play and dominate the layer physics [Pegoraro 89]. This regime is referred as the kinetic ion regime and is retained in the Porcelli model, owing to the low electrical resistivity and high ion temperatures anticipated for future burning plasma devices. Physically, since ions provide most of the plasma inertia, the kinetic ion response will modify the inertial term in the ideal MHD internal kink. The width of the layer in this regime is of order of the ion gyroradius $\rho_i$ and the growth rate at marginal stability $\lambda_H = 0$ is $\lambda = (\frac{4}{7})^{2/7} (\epsilon_1/\rho_i^3)^{1/7} \dot{\rho}_i$, with $\dot{\rho}_i = \rho_i/r_1$. This expression is obtained assuming $T_i = T_e$.

In order to determine if this limit is valid for C-Mod, we form the ratio of the resistive layer width to the ion gyroradius:

$$\frac{\rho_i}{r_1 \epsilon_1^{1/3}} = s_1^{1/3} \left( \frac{\mu \sigma_1 r_1^2}{\tau_A} \right)^{1/3} \frac{\rho_i}{r_1}$$  \hspace{1cm} (6.17)

Taking $r_1 = 0.07 m$, $B = 5.4 T$, $s_1 = 0.1$, $n_e = 1.5 \times 10^{20} m^{-3}$, $T_i = T_e$ and assuming Spitzer
conductivity,

\[ \frac{\rho_i}{r_{1\eta}^{1/3}} \approx 0.55 \frac{T_e}{(Z_{\text{eff}} N(Z_{\text{eff}}))^{1/3}} \quad \text{with } T_e \text{ in keV} \quad (6.18) \]

Therefore, for typical C-Mod plasmas, we find that the resistive layer width is of order of the ion gyroradius. In [Pegoraro 89], the expression for the growth rate in the ion kinetic regime is deemed appropriate for \( \epsilon_{\eta}^{1/3} \gtrsim \frac{1}{3} \frac{\rho_i}{r_i} \). For low temperatures \( T_e \lesssim 3 keV \), the use of this limit may therefore not be correct. The intermediate regime between the resistive and large gyroradius limits has been investigated in [Migliuolo 91a]. For \( \hat{\rho}_i \sim \epsilon_{\eta}^{1/3} \), the resistive growth rate in absence of FLR corrections can be used as an estimate for the actual growth rate of the mode, as increased ion viscosity is found to initially compensate for the other overall destabilizing FLR effects.

- **Stabilizing effects** - The main stabilizing mechanism for internal kink modes is provided by diamagnetic and drift wave effects, in the presence of large temperature and density gradients in the resonant layer [Migliuolo 91b]. These effects modify again the nature of the internal kink, and the stability analysis is better carried out as a limiting case of the drift-tearing mode formalism. Since the parameter \( \Delta' = -\pi/\lambda_H \) used in this formalism can be related to \( \lambda_H \) in the ideal
MHD stable case, a unified treatment is indeed possible. The stability boundaries are complicated \cite{Migliuolo91b}, and need to be simplified for use in a trigger model. We assume that the diamagnetic effects can be characterized by a single frequency $\omega_s$, which we normalize in the same manner as for the growth rates: $\Omega_s = \omega_s \frac{s_A}{s_1}$. To estimate the characteristic growth rate $\lambda_0$ of the resistive modes and layer width $\delta_0$, we take their respective values at marginal stability $\lambda_H = 0$ in absence of diamagnetic effects. For example, when FLR effects are not important, we can take the resistive limit $\lambda_0 = \delta_0 = \epsilon_i^{1/3}$. With these definitions, we can draw a simplified stability diagram as function of the two parameters $\lambda_H$ and $\Omega_s$ on figure 6.10 based on the following considerations:

1. Diamagnetic effects modify the ideal MHD stability boundary. The internal kink will be unstable for $\lambda_H > c_H \Omega_s$, where $c_H$ is a constant of order unity. This condition can be rewritten in the more familiar form $\delta \bar{W} > \omega_s \tau_A$.

2. If the diamagnetic/drift-wave frequency exceeds the characteristic growth rate of the resistive mode, the actual growth rate will be significantly reduced. The resistive internal kink is therefore found only a low values of $\Omega_s / \lambda_0$, for example $\Omega_s < c_s \lambda_0$, where $c_s$ is a constant of order unity. At higher values, the growth rate is not sufficient to trigger a sawtooth.

3. For large negative values of the ideal MHD parameter $\lambda_H$, or equivalently low values of $\Delta'$, the drift-tearing mode will be too localized to trigger a sawtooth. Therefore the resistive internal kink will only occur when $\lambda_H$ is large enough, for example $c_\delta \delta_0 < \lambda_H$, where $c_\delta$ is a constant of order unity.

This picture is also appropriate in the kinetic ion regime, with $\lambda_0 = (\frac{1}{\pi})^{2/7} \left( \epsilon_i \rho_i^{3/7} \right)^{1/7} \rho_i$ and $\delta = \rho_i / r_1$.

### 6.2.4 Trigger conditions

Based on the analysis above, a trigger model for sawtooth crashes can be proposed. Given an equilibrium and radial profiles, a sawtooth crash is predicted to occur when:

1. **The internal kink mode is ideal MHD unstable:** This condition is met if the ideal MHD growth rate $\delta \bar{W}_{core} / \tau_A$ exceeds the ion diamagnetic frequency:

$$-\delta \bar{W}_{core} - \delta \bar{W}_{fast} > 0.5 \omega_s \rho_{q1} \tau_A$$

(6.19)
The contribution $\delta \dot{W}_{\text{fast}}$ is stabilizing if the fast particle stabilization criterion is satisfied, i.e.:

$$-\delta \dot{W}_{\text{core}} < c_h \tau \gamma_D$$

where $c_h$ is a constant.

2. The resistive growth rate overcomes diamagnetic stabilization. This criterion depends on the regime in the resonant layer. If $\epsilon_{\eta}^{1/3} \lesssim 3 \frac{\omega}{\nu_1}$, the resistive kink will be unstable if:

$$-c_{\delta \eta} \epsilon_{\eta}^{1/3} < \lambda_H$$

and

$$\omega_{se} < c_{s \eta} \gamma_R$$

where $\gamma_R = \frac{\gamma_i}{\tau_{1D}}$ is the resistive growth rate and $c_{\delta \eta}$, $c_{s \eta}$ are constants of order unity.

If $\epsilon_{\eta}^{1/3} \gtrsim 3 \frac{\omega}{\nu_1}$, the mode will be unstable if:

$$-c_{\delta \rho} \frac{\rho_i}{\nu_1} < \lambda_H$$

and

$$\omega_{se} < c_{s \rho} \gamma_{\rho}$$

where $\gamma_{\rho} = \frac{\gamma_i}{\tau_{1D}} (\frac{4}{\pi})^{2/7} \left( \epsilon_{\eta} / \dot{\rho}_i^3 \right)^{1/7} \dot{\rho}_i$ is the growth rate in the kinetic ion regime and $c_{\delta \rho}$, $c_{s \rho}$ are constants of order unity.

In the experiments discussed here, we do not expect fast particles to play an important role.
In this case, condition 2 will always be valid before condition 1. This is consistent with the conclusions of Bombarda et al. [Bombarda 98], who found numerically that C-Mod plasmas are stable to ideal internal kink modes and that therefore the sawteeth have to be resistive in nature. One of the most important parameters involved in condition 2 is the magnetic shear at the q = 1 surface $s_1$:

$$\lambda_H \propto \frac{1}{s_1^2}, \quad \epsilon_\eta^{1/3} \propto \frac{1}{s_1^{1/3}}, \quad \gamma_R \propto s_1^{2/3}, \quad \gamma_\rho \propto s_1^{6/7} \quad (6.23)$$

In obtaining the scaling for $\lambda_H$, we note that, except for the fast particle term, all terms contributing to $\delta \dot{W}_{\text{core}}$ in section 6.2.2 have a $\frac{1}{s_1}$ prefactor. The dependence on $s_1$ will generally dominate over the effects of other plasma parameters in condition 2, so that the sawtooth trigger criteria are conveniently rewritten in terms of critical values for the magnetic shear. As we will see in the next section, this pertains particularly for the conditions of the growth rates $\omega_{se} < c_\eta \gamma_R$ (equation 6.21) and $\omega_{si} < c_\rho \gamma_\rho$ (equation 6.22), which we rewrite, respectively:

$$s_1 > s_{\text{crit},\eta} = c_\eta^{-3/2} (\tau_A \omega_{se})^{3/2} \left( \frac{\tau_\eta}{\tau_A} \right)^{1/2} \quad (6.24)$$

$$s_1 > s_{\text{crit},\rho} = c_\rho^{-7/6} (\tau_A \omega_{si})^{7/6} \left( \frac{\pi}{4} \right)^{2/6} \left( \frac{\tau_\eta}{\tau_A} \right)^{2/6} \rho_i^{-1} \quad (6.25)$$

where $\tau_\eta = \mu_0 \sigma r_1^2$.

### 6.3 Simulations

In this section, we revisit the experimental data presented in section 6.1 using the sawtooth trigger model presented in 6.2 and the models of current diffusion used in chapter 5. The goal here is twofold. First, this model will clarify our explanation of the current drive effects on the sawtooth period. Second, it will allow a more quantitative answer on the relative roles played by localized heating and by current drive in the experimental sawtooth period evolution for heating phasing.
6.3.1 Sawtooth cycle simulations

- **Ohmic cycle** - The sawtooth cycle with ohmic heating only is an obvious starting point to build intuition on the dynamics of the current profile in the sawtooth ramp phase. Figure 6-11 shows a time history of a simulated ohmic cycle in a plasma with circular cross-section. The plasma parameters are that of a typical C-Mod plasma, with a central toroidal field \( B_0 = 5.2T \), a central density \( n_{e0} = 1.5 \times 10^{20} m^{-3} \). \( Z_{eff} \) is set to unity in the simulations. The plasma current is 500 kA, which corresponds to an edge safety factor \( q_a \approx 3.5 \) in this geometry. After a transient evolution related to the initial conditions, a sawtooth cycle with a period of \( \sim 7 \) ms is established.

The curves in figure 6-11 are taken over 200 ms after the start of the simulation, at which point the cycle is entirely periodic. The predicted \( q = 1 \) radius \( r_1 \) at the crash is \( r/a \approx 0.35 \). The shear \( s_1 \) at the time of crash is slightly above 0.1. The time evolution of \( r_1 \) and \( s_1 \) show opposite trends: \( r_1 \) rises quickly after the sawtooth crashes then stabilizes, while \( s_1 \) remains initially low then rises.

The two bottom plots in figure 6-11 show the time evolution of several key quantities in the sawtooth trigger model. The resistivity parameter \( \epsilon_\eta^{1/3} \) is comparable to the normalized ion gyroradius \( \rho_i/r_1 \), so that the critical shear corresponds here to the resistive limit \( s_{crit,\eta} \). The ideal MHD parameters are initially larger than \( \epsilon_\eta^{1/3} \) but becomes negligibly small as the shear increases. This is associated with the \( \frac{1}{s_1} \) dependence noted above. As \( -\epsilon_\eta^{1/3} \ll \lambda_H \), the internal kink is near marginal stability and the critical shear condition determines the onset of a sawtooth event. The evolution of the critical shear \( s_{crit} \) is related to the growth of \( r_1 \).

The radial profiles after the sawtooth crash, during the sawtooth ramp and immediately before the following crash are shown respectively on figures 6-12, 6-13, 6-14. In the top right panel, the radial profile for the ohmic current density is shown against two reference profiles. \( j_{q1} = \frac{2B_0}{\mu_0 R} \) is the current density which would give a flat safety factor profile equal to unity, and is shown in dash green. Immediately after a sawtooth crash, \( j(r) \approx j_{q1} \) inside the mixing radius. The cyan curve is the current density \( \sigma E(r/a = 0.5) \) obtained by assuming a flat electric field profile with a reference value taken as the actual value at \( r/a = 0.5 \). The electric field profile (shown on the bottom left panel) is flat for \( r \gg 0.5 \), and the cyan curve overlays the actual ohmic current density profile in the outer half of the plasma. Inside the mixing radius, the curve indicates the deviation from an equilibrated profile in absence of sawteeth. The bottom right panel shows the radial profile of the shear and critical shear, and highlights their values at the \( q = 1 \) radius.
Figure 6-11: Simulated sawtooth cycle. The four panels from top to bottom show, as a function of time, the temperature, $q = 1$ radius, actual and critical shear, and three other relevant dimensionless quantities involved in the trigger model (the ideal MHD layer width $\lambda_H$, the resistive layer width $\ell_n^{1/3}$, and normalized ion Larmor radius. The purple lines corresponds to three times for which radial profiles are shown on figures 6-12, 6-13 and 6-14.
We observe that the shear remains low in the core, up to the mixing radius of the previous crash, at which point it increases rapidly. During the sawtooth ramp, the radius at which this increase occurs moves slowly inwards, covering approximately the distance between the mixing radius and $q = 1$ radius for the subsequent crash.

![Graphs showing Te [keV], Safety factor, Current density, E|| [v/m], and Shear](image)

Figure 6-12: Simulated radial profiles after a sawtooth crash (label A in figure 6-11).

This simulation suggests that the trigger conditions involve both the temporal evolution of the shear at a given location and that of the $q = 1$ radius $r_1$, since the shear will increase rapidly as a function of a radius close to the mixing radius of the previous cycle. It is instructive to obtain equations for the time derivative of these two quantities from the current diffusion equation in a cylinder.
Figure 6-13: Simulated radial profiles during the sawtooth ramp (label B in figure 6-11).
Figure 6-14: Simulated radial profiles prior to a sawtooth crash (label C in figure 6-11).
**Time evolution of the $q = 1$ radius** - In a cylinder, the $q = 1$ radius $r_1$ can be expressed as the zero of an integral function $Q(r_1, t)$:

$$Q(r, t) = \int_0^r (j_{\|}(\rho) - j_{q1}) \rho d\rho \quad \text{with} \quad j_{q1} = \frac{2B_{\phi}}{\mu_0 R}$$

(6.26)

$Q(r, t) = 0 \iff q = 1$ follows from the condition $B_\theta = \frac{r}{R} B_{\phi}$ and Ampere’s law. The time derivative of the $q = 1$ radius $r_1$ can be written:

$$\frac{dr_1}{dt} = -\frac{\partial Q}{\partial r} = \frac{\partial}{\partial t} \int_0^{r_1} \rho j_{\|}(\rho) d\rho$$

(6.27)

The numerator is the time derivative of the current enclosed in the $q = 1$ surface, as determined by current diffusion. The denominator is proportional to the shear at the $q = 1$ surface, as seen from Ampere’s law:

$$2\pi r B_\theta = \mu_0 I(r) \quad \Rightarrow \quad \frac{\partial}{\partial r} (B_\theta r) = \mu_0 r j_{\|}(r)$$

(6.28)

$$\frac{dq}{dr} = \frac{B_{\phi}}{R} \frac{d}{dr} \frac{r^2}{B_\theta r} = \frac{2q}{r} - \frac{\mu_0 j(r)}{B_\theta}$$

(6.29)

$$s = \frac{r \frac{\partial q}{\partial r}}{q \frac{\partial q}{\partial r}} = 2 - \frac{\mu_0 j(r)r}{B_\theta}$$

(6.30)

At the $q = 1$ radius, $B_\theta = \frac{r B_{\phi}}{R} = \frac{1}{2} \mu_0 j_{q1} r$, so that:

$$s_1 = \frac{2j_{q1} - j_{\|}(r_1)}{j_{q1}}$$

(6.31)

The numerator in equation 6.27 can be integrated using the current diffusion equation for the ohmic current $j_{\text{ohm}} = j_{\|} - j_{\text{RF}}$, in the case $\frac{\partial j_{\text{RF}}}{\partial t} = 0$:

$$\frac{\partial j_{\|}}{\partial t} = \frac{1}{\mu_0 r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \frac{j_{\|} - j_{\text{RF}}}{\sigma} \right)$$

(6.32)

Combining equations 6.31 and 6.32 in the expression for $\frac{dr_1}{dt}$ above (equation 6.27):

$$\frac{dr_1}{dt} = \frac{2}{\mu_0 s_1 j_{q1}} \left[ \frac{\partial}{\partial r} \frac{j_{\|}(r_1) - j_{\text{RF}}(r_1)}{\sigma(r_1)} \right]$$

(6.33)

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If the sawtooth period is short, we can expect \( j_\parallel \approx j_{q_1} \) and neglect the radial derivative of \( j_\parallel \) compared to that of the driven currents and conductivity:

\[
\frac{dr_1}{dt} \approx \frac{2}{\mu_0 s_1 j_{q_1} \sigma(r_1)} \left[ -\frac{j_{q_1} - j_{RF}(r_1)}{\sigma(r_1)} \frac{\partial}{\partial r} \sigma(r_1) - \frac{\partial}{\partial r} j_{RF}(r_1) \right]
\] (6.34)

If the radial coordinate is normalized to the minor radius, we recognize in the prefactor \( \frac{1}{\mu_0 \sigma_{1\alpha}} \), i.e. the inverse of the resistive diffusion time. The inverse dependence on the shear can be related to the intuition that the \( q = 1 \) radius will grow faster if the \( q \) profile is flatter, and we indeed recall it arises from the radial derivative of the \( Q(r, t) \) quantity used above. This is consistent with the ohmic cycle simulations, where the growth of the \( q = 1 \) surface slowed down as the shear increased. The two terms between the bracket relate the growth of the \( q = 1 \) surface with the conductivity and driven current profiles.

**Time evolution of the shear** - A second useful equation is that for the evolution of the shear at a particular radial location, i.e. for \( \frac{\partial s}{\partial t} \). To obtain this equation, we will need an equation for \( \frac{\partial q}{\partial t} \), which we obtain again from Ampere’s law and the current diffusion equation, assuming \( \frac{\partial j_{RF}}{\partial t} = 0 \):

\[
\frac{\partial (r B_\theta)}{\partial t} = \mu_0 \int_0^r \rho \frac{\partial}{\partial t} j_\parallel(\rho)d\rho = r \frac{\partial}{\partial r} \left( \frac{j_\parallel - j_{RF}}{\sigma} \right) \quad \Rightarrow \quad -\frac{1}{q^2} \frac{\partial q}{\partial t} = \frac{2}{\mu_0 j_{q_1} r} \frac{\partial}{\partial r} \left( \frac{j_\parallel - j_{RF}}{\sigma} \right)
\]

Expressing \( \frac{\partial s}{\partial t} = r \frac{\partial}{\partial r} \frac{\partial}{\partial r} \ln(q) = r \frac{\partial}{\partial r} \left( \frac{1}{q} \frac{\partial q}{\partial t} \right) \), one obtains:

\[
\frac{\partial s}{\partial t} = -\frac{2r}{\mu_0 j_1 r} \frac{\partial}{\partial r} \left[ \frac{q}{r} \frac{\partial}{\partial r} \left( \frac{j_\parallel - j_{RF}}{\sigma} \right) \right]
\] (6.35)

\[
\frac{\partial s}{\partial t} = \frac{2}{\mu_0 j_1 r} \left[ (q-s) \frac{\partial}{\partial r} \left( \frac{j_\parallel - j_{RF}}{\sigma} \right) - rq \frac{\partial^2}{\partial r^2} \left( \frac{j_\parallel - j_{RF}}{\sigma} \right) \right]
\] (6.36)

Inside the mixing radius, \( q \sim 1 \) and \( s \ll q \). Therefore, the first term is very similar to the right-hand side term in equation [6.34] substituting \( s_1 \) by \( r \) in the numerator. The second term corresponds to the second derivative of the parallel electric field \( E_\parallel = \frac{j_\parallel - j_{RF}}{\sigma} \), is not found to be negligible in the simulations. The expression for \( \frac{\partial s}{\partial t} \) derived in [Graves 05], which retains only the first term, is therefore incomplete. With \( s \ll q \), the full expression inside the mixing radius can be written in
terms of the electric field:

\[
\frac{\partial s}{\partial t} \approx 2q \frac{1}{\mu_0 j_1} \left[ \frac{\partial}{\partial r} E_r - \frac{\partial^2 E}{\partial^2 r} \right] \quad (6.37)
\]

Equation (6.36) is more complicated than its counterpart for the evolution of \( r_1 \) (equation (6.34)), since it involves second derivatives. Both are involved in the evolution of the shear at the \( q = 1 \) surface:

\[
\frac{ds_1}{dt} = \left. \frac{\partial s}{\partial t} \right|_{r=r_1} + \left. \frac{dr_1}{dt} \frac{\partial s}{\partial r} \right|_{r=r_1} \quad (6.38)
\]

Based on the simulated ohmic cycle and this expression, we can discuss how localized heating and current drive will affect the sawtooth cycle, based on the behavior observed with ohmic heating. As an illustration and basis for this discussion, figure 6-15 shows the predicted sawtooth period evolution in a similar scenario as in the C-Mod experiments. The parameters are the same as in the simulated ohmic cycle above. Gaussian driven current and power deposition profiles are added, with a width of \( r/a = 0.05 \) and peak values of 2 MA m\(^{-2}\) and 20 MW m\(^{-3}\) respectively at \( r/a = 0.25 \). The peak values are varied as \( \frac{1}{r} \) during the radial scan in order to maintain the integrated driven current and absorbed power constant. The peak values lead to a similar efficiency as in the TORIC simulations for the C-Mod experiments (figure 6-8). The driven currents are zero in the heating phasing curve in order to show the effect of localized heating only.

\begin{itemize}
  \item **Effect of localized current drive** - Figure 6-16 shows the contribution of the co-current driven current profile to \( \frac{\partial s}{\partial r} \), from equation (6.36). In obtaining this equation, we took \( \sigma \propto \tilde{\eta_0} T_e^{3/2} \) and \( T_e \propto (1 - r/a) \); this creates a small asymmetry in \( \frac{\partial s}{\partial r} \), however the effect is very limited. The derivatives of \( j_{RF} \) determine the contribution to \( j_{RF} \) and we can see that the second derivative term dominates. For the co-current driven profile, the magnetic shear will be reduced at the peak of the driven current profile, and increased on both sides of it. The intuition that one can predict the evolution of the shear by adding \( j_{RF} \) to the currents in the ohmic phase is not correct. It only applies to the relaxed state, which in the sawtooth cycle is never reached. Instead, during the transient phase, the driven currents can only modify the total current profile indirectly, through the diffusion of the electric field, according to equation (6.32). Hence, the effect of localized current can be more complicated than suggested in [Graves 05].

  For some deviations from \( q = 1 \), we can discuss the current drive effects by adding the shear
Figure 6-15: Simulated sawtooth period with current drive and localized heating.

Figure 6-16: Effect of a gaussian driven current profile on the magnetic shear.
perturbation \( \frac{\partial \eta}{\partial t} \) to the ohmic shear profile. The results are shown on figure 6-17. The magnetic shear profile has a steep increase at \( r/a = 0.3 \), and we use the driven current profile on figure 6-16 to perturb it. The peak locations for inside and outside current drive are at \( r/a = 0.25 \) and \( r/a = 0.35 \) respectively. We observe that the magnetic shear perturbation distorts the safety factor profile significantly, but is not enough to make the shear exceed the critical value \( s_{\text{crit}} \) at the peak location of the driven currents. The distorted safety factor profile will be such that a \( q = 1 \) surface will develop faster or slower in the region of high magnetic shear. This will trigger shorter sawtooth cycles for co-current drive inside or counter-current drive outside, and delay the onset of sawteeth for co-current drive outside or counter-current drive inside. This is consistent with the experimental observations and the simulations on figure 6-15. This is not consistent, however, with the intuition that the sawtooth period changes arise directly from changes in the shear, since we see that the determinant factor is how fast the \( q = 1 \) surface grows and reaches the region of high shear. This observation can also explains why the inversion radius does not change significantly in experiments, and why the current drive effects on the sawtooth period are strong only with deposition in its vicinity. The discussion above, of course, applies only to the case of localized and moderately strong current drive. For example, broader current profiles can stabilize sawteeth by keeping \( q \) above unity. Very strong localized current drive will violate our assumption \( j \approx j_{q1} \) in the core region, and there is then no alternative to simulating each particular case. The distortion of the safety factor profile may also be such that the effect of other MHD modes need to be considered.

**Effect of localized heating** - Localized heating results in a rapid local increase in \( T_e \) from its post-crash profile. If the resistivity profile evolves as \( \eta = \eta_{\text{post-crash}} + \Delta \eta(t) \), this can induce similar effects as for localized current drive. The contribution of \( \delta \eta \) to \( \frac{dr_{\perp}}{dt} \) and \( \frac{ds}{dt} \) is, from equations 6.34 and 6.36:

\[
\left. \frac{dr_{\perp}}{dt} \right|_{\Delta \eta} \approx \frac{2}{\mu_0 s_1 j_{q1}} \frac{\partial}{\partial r} \left( \Delta \eta j_{\parallel} \right) \tag{6.39}
\]

\[
\left. \frac{\partial s}{\partial t} \right|_{\Delta \eta} \approx \frac{2q}{\mu_0} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \Delta \eta j_{\parallel} \right) - \frac{\partial^2}{\partial r^2} \left( \Delta \eta j_{\parallel} \right) \right] \tag{6.40}
\]

We see that this contribution is formally equivalent to that with localized current drive. The important parameter is the perturbation to the electric field: \( \Delta \eta j_{\parallel} \) and \( -\frac{\dot{\eta} e}{\sigma} \) in the two cases. At the
Figure 6-17: Current drive effects on the safety factor and magnetic shear profiles, for gaussian driven current profiles with full max half width of $\Delta (\frac{q}{a}) = 0.05$, centered inside ($r/a = 0.25$) and outside ($r/a = 0.35$) the $q = 1$ radius ($r/a = 0.3$) for a typical q profile (dash line).
temperature increases, the resistivity drops, and $\Delta \eta$ is negative. Therefore, localized heating has the same effect of co-current drive through this mechanism. With strong heating, the changes in temperature can be large and rapid, however profile stiffness prohibits changes in the resistivity gradients. The rapid heat diffusion will broaden the $\Delta \eta$ profile, make its derivative smaller and thus limit the direct heating effects.

Changes in the sawtooth period can also result from global changes in the conductivity profile. For instance, off-axis heating tends to flatten the temperature profile in the core and lower the conductivity gradient. From equation [6.34] we see that this will slow down the growth of the $q = 1$ surface. Again, profile stiffness will limit this effect. Also, global changes in the conductivity profile shape are usually visible in the evolution of the inversion radius, since they result in a global redistribution of the current profile.

The strength of heating effects is therefore quite dependent on electron transport. The simulation code used to illustrate this discussion can be run with flexible assumptions for the transport model. Qualitatively, the changes in the sawtooth period with localized heating were found to be very sensitive with respect to the degree of stiffness in the model. In setting the parameters for the simulation on figure [6-15] the model was adjusted so that the evolution of the temperature profile is similar to that observed in the experiments. For this choice and using the current drive efficiency as predicted by TORIC, we observe that the sawtooth period changes due to localized current drive are stronger than those predicted from localized heating only. From this discussion, it appears that the key factor in assessing the strength of the localized effects in the C-Mod experiments is the evolution of the temperature profile. On C-Mod, the electron temperature is measured with high spatial and temporal resolution, therefore it would be much better to use this data as a basis. We will do so in the following section.

### 6.3.2 Heating effects in the C-Mod experiments

- **Temperature profile evolution** - Figure [6-18] shows the measured electron temperature profiles for three different times during the heating phasing scan in the 1050802 set of discharges. The data is shown immediately before a sawtooth crash. We see that the changes in the temperature gradients are very small, which is consistent with the strong profile resiliency observed in C-Mod L-mode plasmas [Greenwald 97].

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Figure 6-18: Temperature profiles at three times during the 105802 scan, in heating phasing.
The absence of large changes in the temperature gradients in the C-Mod experiments supports the conclusion that the current drive effects are dominant. The same conclusion is generally reached in other sawtooth control experiments, on ASDEX-Upgrade [Mueck 05] and TCV [Henderson 01]. The TCV experiments are particularly illustrative: small amounts of current driven by the ECH beam for injection with zero toroidal angle were sufficient to change the sawtooth period significantly, compared to the effect of localized heating. The ECH beam was swept through the entire plasma cross section and crossed the q = 1 surface both above and below the midplane. A different sawtooth period evolution was observed in the two crossings for off-axis deposition, which could not be explained by localized heating effects alone. Ray tracing simulations indicated that small currents, with a total of ~5kA out of a plasma current ~200kA, were driven with different polarity above and below the midplane due to poloidal field effects. This was confirmed by reversing the sign of the toroidal field and observing a mirrored behavior during the scan. For injection with zero toroidal angle, the current drive efficiency was very low, and even in this situation, the current drive effects dominate. In the TCV experiments reported above [Angioni 03], where the effect of localized heating were identified, special care had to be taken to minimize the driven currents associated with this effect.

**TRANSP simulations** - To further support the conclusion that a current drive effect was necessary to explain the evolution in heating phasing in the C-Mod experiments, TRANSP simulations were carried out and analyzed with the Porcelli model using an NTCC module written by G. Bateman [Bateman 06]. The simulations use the EFIT equilibrium and measured profiles, including the electron temperature. The current profile is evolved according the current diffusion equation, and the Kadomstev reconnection model is used to determine the post-crash profile at each time when a sawtooth event is observed in the experiment. This allows to calculate the time evolution of the magnetic shear, and compare it with the computed critical shear from the Porcelli model. If the simulation is consistent, the trigger condition will agree with the observed crash times, i.e. $c_p \gamma_p \approx \omega_{ci}$ when a sawtooth event occurs in the experiment. Figure 6-19 shows a comparison of the two quantities at each sawtooth crash time, under two assumptions: (1) no currents are driven, (2) currents are driven in the co-current direction, with a location and magnitude according to the predicted driven current profile from TORIC in the heating phasing. We observe that the simulated current profile evolution is not consistent with the trigger condition in the absence of current drive,
i.e. only with heating effects. When a co-current driven current profile consistent with TORIC is
used, the agreement is much better. In both cases, the irregular crashes are not modeled well.

Figure 6-19: TRANSP simulation for the heating phasing discharge in the 1050802 set
Chapter 7

Conclusions and future work

7.1 Summary of thesis results

This thesis presents experimental and numerical results on Mode Conversion Current Drive (MCCD) in tokamaks.

7.1.1 Physics of MCCD with Ion Cyclotron Waves

Poloidal field effects and mode conversion to Ion Cyclotron Waves - Building upon the work of Perkins [Perkins 77], dispersion relations for the mode converted Ion Bernstein and Ion Cyclotron waves are derived, in a slab geometry relevant to tokamak equilibria. Mode conversion to Ion Cyclotron Waves is found to be ubiquitous to all high temperature conventional tokamaks, as a result of the central value for the safety factor $q_0 \sim 1$. The mode converted Ion Cyclotron Waves observed on Alcator C-Mod and in the TORIC simulations are kinetic modifications of the cold plasma Ion Cyclotron Waves in the acoustic range $\frac{\omega}{k_{||} v_{th}} \lesssim 1$, which is accessed as $k_{||}$ is upshifted. The kinetic MCICW can propagate without encountering the ion-ion hybrid resonance between the mode conversion layer and the ion cyclotron layer, but are strong damped, primarily on electrons. The model is consistent with full wave TORIC simulations, which in addition predict the electric field and absorption peaks at the transition from FW $\rightarrow$ MCIBW to FW $\rightarrow$ MCICW mode conversion. The damping is also found to be up-down asymmetric, which results in net current drive even for toroidally symmetric wave excitation, since $k_{||} \approx \frac{m a B_0}{r B}$ reverses sign above
and below the midplane.

- **Applicability of the Ehst-Karney parametrization for MCCD** - More accurate calculations for the MCCD currents were implemented by importing a quasilinear diffusion operator built from TORIC fields into a Fokker-Planck solver. It is found that the polarization considered in the Ehst-Karney Alfven Wave damping parametrization is not applicable for mode converted Ion Cyclotron Waves. As a result, the Ehst-Karney calculations can overestimate the MCCD currents by a factor 2-3. The Landau damping formula is more consistent with the wave physics, but can be inaccurate in the regime $\frac{\omega}{k_{||}f_{\text{blue}}} \lesssim 1$ where MCICWs propagate. The TORIC estimates with the Ehst-Karney parametrization capture the changes in current drive efficiency with varying plasma parameters, and can therefore be used for parameter scans.

- **TORIC simulations for the TFTR MCCD experiments** - Discharges based on the TFTR MCCD experiments where net current drive was inferred from loop voltage differences were simulated with TORIC, and net currents consistent with the experimental estimates were predicted with the Fokker-Planck formalism. Mode conversion to Ion Cyclotron Waves is predicted in the TFTR plasmas, and accounts for the net driven currents. Hence, contrary to the FW $\rightarrow$ IBW models used so far to interpret the TFTR measurements, MCCD models including poloidal field effects can explain the experimental results consistently with the wave physics.

### 7.1.2 MCCD as a tool for net current drive

Experiments aimed at measuring net MCCD currents from loop voltage were conducted on Alcator C-Mod as part of this thesis work. Despite extensive discharge development, and significant mode conversion electron heating in the experiments, no conclusive current drive measurements could be obtained with this approach. Two explanations for this negative result are identified:

- **Lack of control over $Z_{\text{eff}}$ and unfavorable efficiency scaling** - As suggested by the Ehst-Karney parametrization, and confirmed with the quasilinear formalism, the following scaling can be used as a figure of merit for the plasma conditions in MCCD experiments:

$$\eta \propto \frac{P_{\text{rf}} T_e}{Z_{\text{eff}} n_e}$$

with

\[
\begin{align*}
P_{\text{rf}} & \text{ RF power} \\
T_e & \text{ Electron temperature} \\
Z_{\text{eff}} & \text{ Effective charge} \\
n_e & \text{ Electron density}
\end{align*}
\]
The $1/Z_{\text{eff}}$ scaling for the MCCD efficiency is unfavorable, since the product of the density and the effective charge cannot be varied significantly in high power RF experiments. Thus, conditions for the simulated 100 kA discharge could not be realized in the experiments. Boronization decreased the $n_e \times Z_{\text{eff}}$ product, but degraded the antenna performance in these experiments. Furthermore, the initial 100 kA prediction was made with an overestimated MCCD efficiency. We conclude that the range of parameters where MCCD could be useful in C-Mod for net current drive is very limited.

- Sawteeth oscillations complicate the loop voltage analysis - A model of current diffusion in cylindrical geometry was used to study how the loop voltage response to current drive is affected by sawtooth oscillations, which are present in nearly all plasmas in Alcator C-Mod. By periodically short-circuiting the core current profile, sawteeth are found to hinder the relaxation of the ohmic electric field profile and thus prevent direct estimates of current drive from loop voltage differences as in the traditional circuit model, when the currents are driven inside the mixing region.

While net current drive is predicted from mode converted ICWs, unlike MCIBW, we find that the prospects of the technique are limited. Mode converted Ion Cyclotron Waves interact with electrons near their thermal velocity, which is inherently less efficient than that with superthermal electrons (as in, for example, lower hybrid waves). Magnetic trapping reduces the efficiency for off-axis scenarios and makes MCCD practical only for near-axis current drive. Finally, as pointed out above, the efficiency follows an unfavorable $1/Z_{\text{eff}}$ scaling. These limitations are common to any low phase velocity current drive technique, with $k_{||}/f_{\text{the}} \lesssim 1$, which for MCICW is imposed by their dispersion relation. In addition, parasitic absorption on energetic minority ions and fusion-born deuterium-like alpha particles $^4$He could reduce the power fraction damped by the mode converted waves on electrons, and thus the global current drive efficiency as a function of the power coupled power by the ICRF antenna. In addition, the higher ion pressure in burning plasmas should be more favorable to mode conversion to Ion Bernstein Waves, and may therefore further lower the power deposited on electrons by Ion Cyclotron Waves. Despite these limitations, MCCD may be useful during the startup phase in reactors. Robust and efficient MCEH has been demonstrated in non-burning D-T plasmas. TORIC also predicts net currents can be obtained in heating phasing, thus without the need of phased ICRF antenna operation.
7.1.3 Sawtooth control experiments

Experiments with MCCD near the $q = 1$ surface aimed at modifying the sawtooth period were carried out and compared to TORIC predictions.

- **Significant sawtooth period changes with MCCD were observed** - As the mode conversion layer was swept outward through the $q = 1$ surface in D($^3$He) plasmas, the sawtooth period was found to increase and then decrease for counter-current drive phasing. For co-current drive and heating phasings, it was observed to decrease and then increase. With 2 MW ICRF power, the period varied from 3 to 12 ms. The observed evolution is consistent with localized current drive by mode converted waves in the vicinity of the $q=1$ surface.

- **Experiments are consistent with predicted up-down asymmetries** - The sawtooth period evolution in heating phasing was found experimentally to be similar to that with co-current drive phasing. This is consistent with TORIC predictions, which indicate net co-current drive in symmetric phasing as a result of the up-down asymmetry in the mode conversion process. Localized electron heating, which could also explain the experimental results, was found not to be dominant compared to the current drive effect based on simulations of the sawtooth cycle with the Porcelli trigger model.

These experiments demonstrate that MCCD can be used as a technique for sawtooth control in tokamaks. However, since current drive near the $q = 1$ surface is an off-axis scenario, the efficiency is low due to magnetic trapping. The current drive location varies with the mode conversion layer location and depends both on the plasma composition and toroidal magnetic field. While this was not necessary in C-Mod experiments, the control of the minority concentration may prove problematic and impact the ability to achieve optimal current drive scenarios. Accurate measurements of the deposition locations and concentrations are not easy to obtain, which would complicate a feedback scheme based on changes in the toroidal field to maintain optimal conditions. Electron cyclotron current drive, which has similar efficiencies as MCCD in present experiments, is much easier to control and has therefore better prospects.
7.2 Suggestions for future work

- *Experiments on Alcator C-Mod* - The results above suggest that practical uses of MCCD are limited on Alcator C-Mod. However, the technique is at present the only available tool for localized current profile control on the tokamak. Therefore, by default, it can be used for instance for carry out further sawtooth control experiments. Of interest is, for example, the possibility to shorten the period of sawteeth stabilized by energetic ion populations. In future reactors, fusion-born alpha particles are expected to induce long sawteeth and *monster* crashes, which can trigger neoclassical tearing modes or result in large density and heat fluxes on the divertor and first wall components. The ICRF system in C-Mod, with minority D(H) at 80 MHz and D($^3$He) MCCD at 50 MHz, is well suited for such experiments.

At present, the diagnostics capabilities for current profile measurements on C-Mod are limited, and the loop voltage approach was the only available technique. We have seen that given the present configuration on C-Mod, the loop voltage technique is not practical for MCCD measurements. Nonetheless, it would be useful to carry out further MCCD experiments if more ICRF power or better current profile measurements becomes available. Present plans for the Alcator C-Mod program include the replacement of the D and E antennas by a single four strap antenna at E-port, with a design and capabilities similar to that in J-port. Pitch angle measurements with the Motional Stark Effect (MSE) diagnostic may become available in future campaigns. Another possible improvement is the planned installation of a polarimetry system.

The recent successful implementation of a lower hybrid heating and current drive on Alcator C-Mod paves the way for experiments with combined LHCD and MCCD/MCEH. While sawtooth stabilization with LHCD suggests it could be use to achieve better conditions for MCCD experiments, it is likely that the synergistic effect of MCEH on the lower hybrid current drive will be the dominant effect in these conditions. It is not clear how MCCD could be used in such conditions, however the Fokker-Planck approach used in the thesis can be readily used to study the synergy between MCEH and LHCD.

- *Modeling* - A topic of interest on the simulation side is to implement self-consistent full wave simulations with non-Maxwellian distributions. This capability is being currently developed with the TORIC and AORSA codes, particularly in the context of minority heating simulations. For MCEH/MCCD, we have seen that the strong electron heating can start distorting the electron
distribution function in high power experiments, which would require computing the full wave solutions consistently with the modified distribution function. The treatment of non-Maxwellian distributions also opens the possibility of realistic simulations for the parasitic ion damping of mode converted waves in the Doppler-broadened ion cyclotron resonance layers, with either energetic minority ion or fusion-born alpha populations. In the limit of significant Doppler-broadening, it may be impossible to access the mode conversion regime. The simulation tools under development will allow quantitative studies of these effects.

- **Up-down asymmetries** - As discussed in chapter 3, net mode conversion current drive is predicted in TORIC as a consequence of up-down asymmetries in the mode conversion process and wave absorption. The physical reason for this asymmetry is not clear.

The fields are found to be up-down asymmetric even in TORIC simulations with $n_\phi = 0$, a circular plasma cross-section and flat profiles of density and temperature. In this case, given that the symmetry properties of the dielectric tensor (Onsager relations) and of the exciting RF currents, it is not clear how the asymmetry in the wavefields arises. Since $n_\phi = 0$ in the simulations, the topology with respect to the direction of the fields is entirely determined by the relative directions of the toroidal field and plasma current. The effect of reversing the directions of both the plasma current and the toroidal field can be trivially studied through the (mental) operation of turning the tokamak up-side down. The plasma current can be reversed alone by a planar symmetry with respect to an vertical plane going through the vertical axis of symmetry for the tokamak, in which case the radio-frequency fields should remain unchanged. Consistently, the toroidal field can be reversed by a planar symmetry with respect to the horizontal midplane, in which case the radio-frequency are also changed. In TORIC simulations where only the plasma current direction was reversed, the predicted electric fields were unchanged, as expected from the mental operations above. As a result, this exercise gives little indications as to the physical reason for the asymmetry.

The up-down asymmetry is observed consistently in TORIC simulations, and can be also seen in simulations with the all-order AORSA code [Jaeger 00] in comparison runs between the two codes. We have also shown in this thesis that the predicted MCCD currents and driven current profiles associated with the asymmetry are consistent with experimental results in TFTR and Alcator C-Mod. In this context, it would certainly be worthwhile to investigate the physics behind the observed asymmetry in more detail.
Bibliography


