Characterization of low-frequency density fluctuations in dipole-confined laboratory plasmas

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June 2010

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CHARACTERIZATION OF LOW-FREQUENCY DENSITY FLUCTUATIONS IN DIPOLE-CONFINED LABORATORY PLASMAS

By

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SUBMITTED TO THE DEPARTMENT OF NUCLEAR SCIENCE AND ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN NUCLEAR SCIENCE AND ENGINEERING AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2010

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Submitted to the Department of Nuclear Science and Engineering on May 21, 2010, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Nuclear Science and Engineering

Abstract

Low-frequency fluctuations of plasma density, floating potential, ion saturation current, visible light intensity, and edge magnetic field are routinely observed in the Levitated Dipole Experiment (LDX). For the purposes of this thesis, we define low-frequency as $\omega << \omega_{be}$ where $\omega_{be}$ is the electron bounce frequency. These fluctuations in a laboratory dipole confined plasma lead to turbulence mixing that maintain peaked density profiles. The relationship between the different types of low-frequency fluctuations and plasma density transport is considered.

Two 16-channel photodiode arrays were designed and constructed to study the spatial and temporal structure of these fluctuations as part of this dissertation. In addition to the photodiode arrays, a four-channel microwave interferometer is used to estimate the density profile and to measure density fluctuations in the plasma. Several electrostatic probes, including a 24-channel floating probe array, measure fluctuations at the plasma edge and eight Mirnov coils measure magnetic fluctuations.

The fluctuations fall into three general categories: broadband, turbulent fluctuations observed during nearly all plasma conditions; quasi-coherent fluctuations with low azimuthal mode numbers and peak frequencies on the order 1 kHz observed during discharges with low neutral pressure; and coherent fluctuations with zero toroidal mode number and peak frequencies on the order of 100 Hz, observed during discharges with moderate neutral fueling. The relationship between time-averaged fluctuation characteristics and plasma parameters are explored. The spatial structure of the fluctuations for several representative shots are discussed.

The turbulent fluctuations and concurrent density profiles are compared to quasilinear diffusion of interchange mixing in dipolar plasmas for cases where the fluctuations are random. I show that the quasilinear diffusion equation agrees well with the experimental observations of random fluctuations, supporting the conclusion that interchange mixing is causing inward transport that results in peaked density profiles. For other cases, where nonlinear effects appear to dominate the plasma dynamics, the saturated fluctuation amplitudes are compared to the plasma density profiles.

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Acknowledgments

I would especially like to thank my thesis advisers Jay Kesner and Darren Garnier, whose support and guidance has meant so much to me; and to the current and former members of the LDX team for their encouragement, advice, and hard work. Jay is always willing to answer a question, discuss an idea, or read a draft. Darren has advised me on diagnostic design, data collection, and data analysis techniques. I am also grateful to him for introducing me to sailboat racing. Mike Mauel has inspired me with his seemingly endless enthusiasm and we have had many helpful discussions about data analysis and plasma physics. I thank Ishtak Karim, Eugenio Ortiz, Alex Boxer, Brian Kardon, Matt Davis, Emmanuel Mimoun, and Ryan Bergmann, who were each fellow students with me at one time. John O’Brien and Susannah Brown both worked with me as UROP students and provided valuable assistance. I thank Rick Lations, Don Strahan, Alex Zhukovsky and Phil Michael for their time and expertise keeping the LDX experiment running smoothly.

I would also like to thank my thesis committee Ron Parker, Anne White, and Jeff Freidberg for their time and for their reviews of my work.

I am indebted to many other people who have assisted me during my time at MIT. Thank you to Vincent Tang and Natalia Krashenikova, with whom I shared an office when I first arrived, for making me feel welcome. I thank Rachael McDermott for loaning me the lamps for the spectrometer calibration and for showing me how to use them. Catherine Fiore and Bill McCarthy have provided valuable assistance with safety issues. I am grateful to Andrea Schmitt for her friendship.

On a personal level, I would especially like to thank my husband, Michael Hohensee for his love and support. I thank my son, Friedrich for being so agreeable about my research commitments and for not coloring in my lab notebooks. I would like to thank my mother for her unwavering confidence in me and for the hundreds of hours of free babysitting she put in so that I could write and collect data.
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Skewness and kurtosis of the PDFs of floating potential measured near the magnetic separatrix indicate whether the plasma fluctuations are random as a function of the plasma diamagnetism. The data from shots 90312025-044 and 90313001-023 are time-averaged between $8 \, s < t < 8.5 \, s$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.

The average azimuthal electric field and the quasilinear diffusion coefficient increase with diamagnetism. They are also shown in relation to the density peakedness. The data from shots 90312025-044 and 90313001-023 are time-averaged between $5 \, s < t < 9 \, s$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.

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Chapter 1

Introduction

Planetary magnetospheres are confined by dipolar magnetic fields and gravity. During magnetic storms, electric fields accelerate particles inward and increase the ring current [2]. The centrifugal interchange instability is important for transport, and both inward and outward transport are observed [3]. An explanation given for this transport is that convection electric fields transport particles inwards [4].

The Levitated Dipole Experiment (LDX) was designed to study the physics of dipole plasma confinement. The relationship between plasma fluctuations and transport is an important component of this physics. The plasmas created in LDX are confined by an internal, superconducting coil. Levitating the coil eliminates the end losses to the supports and enhances confinement [5] and importantly, allows radial transport to be studied.

A dipole confined plasma can be modeled as flux tubes, where each “tube” encloses an equal amount of magnetic flux and the magnetic field lines at the surface of the flux tube are parallel to the surface of the flux tube. During a low frequency interchange instability, flux tubes move adiabatically to change places. As flux tubes move inwards, towards the dipole, the plasma is heated and compressed, while the plasma in flux tubes that move outwards expands and cools. The interchange can lead to mixing in the plasma. This instability is analogous to the Rayleigh-Taylor instability that occurs when a heavy fluid rests atop a light fluid.

In the low-frequency range ($\omega << \omega_b$, where $\omega_b$ is the bounce frequency), dipole-confined plasmas can be unstable to MHD modes [6] and entropy modes [7], all of which are interchange type (or flute-like) where the flux tubes move without bending or changing shape.
The frequencies of these instabilities are either zero or the low ordered diamagnetic and curvature drift frequencies, but the linear growth rates are different. The MHD modes are driven by pressure gradients or the centrifugal force, while the entropy modes are driven by density and temperature gradients. Theoretical studies [8] indicate that transport due to MHD fluctuations can be described by a quasilinear diffusion coefficient acting on the number density $N$ in flux space, where $N$ is the number of particles per unit flux. Whereas collisions will act to remove density gradients (particles per unit volume), fluctuations will act to remove gradients in $N$. Non-linear simulations have shown that these instabilities can lead to the formation of convective cells [9] and to turbulent transport [10]. In addition to instabilities, external drives such as electron cyclotron resonance heating (ECRH) can also lead to oscillatory behavior.

The idea that fluctuations can lead to peaked density profiles may seem counter-intuitive. Imagine that you have put a small pile of sand on a sheet of paper. Then you shake the paper gently back and forth. The sand will spread out, and after some time, the sand will be uniformly distributed over the surface of the paper. If you shook the paper and the sand all piled up in the middle, it would be surprising, but a similar peaking of density and pressure profiles is observed in magnetospheres and is explained by interchange mixing leading to an equal number of particles per flux tube. This density profile is called the invariant density profile and is the stationary solution to the quasi-linear diffusion equations [11]. The volume per unit flux increases with radius in a dipole plasma so in real space, the invariant density profile is highly peaked.

One would expect to see these turbulent fluctuations resulting from unstable profiles in LDX. Although linear theory predicts that the plasma will be completely stable if the pressure and density profiles are invariant to interchange motion, it is unlikely that we can create these exact profiles in the laboratory and although transport is predicted to drive the plasma towards invariant profiles, these profiles are asymptotically driven so the turbulent fluctuations would remain even when the plasma profiles are close to the invariant profiles.

Thus, unsurprisingly, low-frequency fluctuations were observed during the first plasmas created in LDX using ion saturation probes at the edge, a single interferometer channel with a viewing chord that passed through the central region of the plasma, and an uncollimated photodiode that viewed a large portion of the plasma. In order to study these fluctuations further, I constructed a set of two photodiode array diagnostics with the purpose of
characterizing the low-frequency fluctuation activity observed in LDX and investigating the relationship between these fluctuations, external plasma controls of fueling and heating, and the later observed density profiles. In this thesis, the fluctuations are measured using two sixteen-channel photodiode arrays that have nearly identical radial views, but view from different toroidal locations, a twenty-four channel floating probe array [12], and a four-channel microwave interferometer [13]. These are the first measurements of the characteristics of low-frequency fluctuations in a levitated dipole plasmas. This is also the first time that these fluctuations have been parameterized with respect to experimental controls.

The observed fluctuations can be separated in three categories: broadband, turbulent fluctuations, quasi-coherent fluctuations that have $f/\Delta f \sim 5$, and coherent fluctuations that have $f/\Delta f \sim 100$. Experiments were performed to investigate the plasma conditions under which each of these types of fluctuations were observed, and how the characteristics of the fluctuation spectra varied with plasma conditions. The toroidal and radial structure of the fluctuations were measured. The relationship between the observed fluctuations and the observed density profiles is discussed.

In LDX, broadband, turbulent fluctuations are usually observed throughout the plasma. Peaked density profiles were recently observed for plasmas created with the internal coil levitated, but when the internal coil was supported (so that the field lines intersected the supports, resulting in end losses), the plasma density profiles were not peaked [14]. It was inferred that a density pinch was sustained by inward turbulent transport resulting from interchange fluctuations in the plasma. The quasilinear diffusion coefficient was calculated from floating probe measurements at the plasma edge and rate of formation of the density profile was consistent with the rate predicted by the quasilinear diffusion equation. This result is extended in this thesis to consider the variation of the quasilinear diffusion coefficient (measured from an edge floating probe array) with plasma conditions.

Quasi-coherent fluctuations are observed in both the plasma core and edge during discharges with low levels of neutral fueling. These fluctuations have low, non-zero toroidal mode numbers and frequencies on the order of $10^3$ Hz. In plasmas that have density profiles that are steeper than the invariant density profile, radial variation of the fluctuation spectrum is observed. This radial variation may be evidence of rotational shear in the plasma. The quasilinear diffusion coefficients measured for plasmas with quasi-coherent fluctuations are not consistent with the observed density profiles. The probability distribution functions
of floating potential at the plasma edge indicate that these fluctuations are not random so the assumptions of quasilinear theory are not met, thus explaining the discrepancy. Non-linear effects appear to play an important role in the quasi-coherent fluctuations and they do not appear to cause transport.

Coherent fluctuations are observed in the plasma core during discharges that have moderate neutral fueling. These fluctuations occur at very low frequencies, on the order of $10^2$ Hz. The measured toroidal mode number is zero. The frequency of the fluctuations does not vary radially. This fluctuation may be a breathing mode related to ECRH accessibility in the plasma.

In Chapter 2, I introduce the Levitated Dipole Experiment (LDX) and summarize what is known about LDX plasmas. In chapter 3, I show measurements of the spectrum of visible light emission from a survey spectrometer. I then use the ratios of emission from two neutral helium lines to calculate the plasma temperature. This measurement can be used in combination with an edge temperature measured from a swept probe to estimate the temperature profile in LDX. In Chapter 4, I discuss the diagnostics used to measure low frequency fluctuations in the plasma, including information on the design and calibration of the two photodiode arrays, and I also explain the mathematical methods that are used to analyze the data.

In Chapter 5, I present observations of properties of low-frequency fluctuations in the plasma including examples of broadband fluctuations, quasi-coherent fluctuations and coherent fluctuations. I show the plasma condition during which each of the fluctuations are observed and present measurements of the toroidal and radial structure of the fluctuations. Most of the measurements that I show are average characteristics of fluctuations, so I also show an example of a gas puff experiment in which the plasma conditions are changing during the discharge and show that the behavior of the fluctuations in steady state agrees well with the behavior of the fluctuations when the plasma conditions are changing. In Chapter 6, I compare the low frequency fluctuations to the observed density profiles and the quasilinear diffusion coefficient measured from the magnitude and correlation time of the edge floating potential fluctuations. I also present observations of rotational shear in the plasma by looking at the cross-correlations of toroidally separated photodiode array channels and toroidally separated probes. I summarize my conclusions in Chapter 7.
Chapter 2

The Levitated Dipole Experiment

2.1 The dipole fusion concept

The idea that plasma confined by a dipole like magnetic field might be ideal for a fusion reactor was put forth by Akira Hasegawa in 1987 [15] after doing research on planetary magnetospheres which are confined by dipolar magnetic fields. Inward diffusion and heating are observed in planetary magnetospheres during active times [16, 2]. Hasegawa’s idea was to externally drive inward diffusive transport in a laboratory dipole plasma to achieve a high density in the core of the plasma.

The dipole magnetic geometry may offer some advantages as a fusion concept. The system can inherently be operated in steady state which is preferred to pulsed operation for a power plant. The internal magnets allow for excellent magnetic field utilization. Non-interlocking coils allow the floating coil to be replaced easily for servicing while maintaining high plant availability.

In addition, it may be possible to decouple the energy confinement time $\tau_E$ from the particle confinement time $\tau_p^*$ which would allow the use of “advanced” fuels; in particular a $^3$He-catalysed D-D [17, 18] fusion design has been proposed [19] which requires $\tau_p^*/\tau_E < 2$ [20]. In tokamaks [21], the particle confinement time and the energy confinement time are dominated by turbulent processes so the two times are strongly coupled [22]. In a levitated dipole device, the magnetic field is purely poloidal and the field lines are closed. Because there is no magnetic shear, field lines can interchange without breaking, making a two dimensional system where convective cells may form. At marginal stability, convective cells may transport particles without transporting any energy, thus decoupling the energy and
particle confinement times. This would be very important for a fusion reactor because it would mean that tritium and ash could be transported out of the plasma without degrading the energy confinement time.

2.2 Experimental configuration

The Levitated Dipole Experiment (LDX) has been designed to test the theory of dipole confinement and explore the dipole fusion concept. The magnets and the vacuum chamber have been designed for plasmas with marginally stable pressure profiles and densities and temperatures that are nearer to fusion grade plasmas than space plasmas. This requires a strong dipole magnet and a large vacuum chamber. The major goals of the LDX are to evaluate the confinement properties and stability of dipole confined plasmas. Plasmas in planetary magnetospheres are confined by dipolar magnetic fields so the study of plasmas confined by dipolar magnetic fields in the laboratory has applications to space physics, in addition to enhancing our knowledge of basic plasma physics and furthering the pursuit fusion energy.

The operational goals can be broken down into three phases of research. The first phase began in August 2004 when the experiment first became operational. During the first phase, hot electron plasmas were studied in plasmas created around the floating coil while it is suspended from three thin supports. During this phase of operation, we learned to operate the magnets safely and reliably while studying hot electron physics. In the second phase of operation, which began in November 2007, the floating coil has been levitated. The confinement and stability of warm, low density plasmas have been studied. The third phase of operation has not yet begun. During this phase, an ion heating source will be added so that thermal plasmas with fusion relevant densities may be studied.

2.2.1 Magnetic configuration

Plasma in the Levitated Dipole Experiment [23] is confined by a superconducting coil situated in the center of a large vacuum vessel. A superconducting internal coil, or floating-coil, creates a dipole-like magnetic field that confines the plasma. The floating-coil is charged inductively in the bore of the charging-coil, then lifted to the center of the vacuum vessel with a hoist. When the floating-coil is levitated, the force of gravity is balanced by magnetic
Figure 2-1: Schematic view of LDX. The floating-coil confines the plasma and the levitation-coil provides the upward force to counteract gravity. The black lines (solid and dashed) illustrate the flux surfaces for the vacuum field. The mechanical lifting fixture transports the floating-coil from the charging station (not shown) to the center of the vacuum vessel, then is retracted outside the closed field line region (indicated by the shaded area). The resonance locations for the 2.45 GHz and 6.4 GHz ECRH sources are also shown.

attraction to a third magnet, the levitation-coil located outside the vacuum vessel directly above the floating-coil. A schematic drawing of the experiment, Fig. 2-1, shows the location of these three magnets and the hoist. Plasmas are created by injecting neutral gas into the vacuum vessel and then ionizing and heating those particles using multi-frequency electron cyclotron resonance heating (ECRH).

The floating-coil must be levitated in order to obtain a closed field line configuration, which improves confinement by removing the losses of plasma and energy along field lines to the supports. A steady state levitating experiment is only possible with the use of a superconducting floating-coil so that no external power supply is necessary to combat resistive power losses. The coil must operate without any external instrumentation, power lines, cooling lines, or external quench detection and protection. In addition the system requires real time feedback control to maintain levitation.

Levitation is achieved using a magnet which is placed above the floating coil so that an upward attractive force between the two coils will balance the downward gravitation forces.
Levitation is inherently an unstable process. A system that is levitated using an attractive force from above (as is the case for LDX) is unstable in the vertical direction. As long as the upward force is applied from far enough away, the system is stable to tilt and translation. On the other hand, a system levitated from below stable to vertical displacements, but unstable to tilts and translations. The levitation system uses eight lasers to measure the position of the floating-coil and uses a real time feedback system to adjust the current in the levitation-coil, keeping the vertical position of the floating-coil constant to within three millimeters. The limiting factor in the time the floating-coil can be levitated is the length of time that the floating-coil will remain cold enough to be superconducting: two hours if the coil will be re-cooled and re-lifted, or two and one-half hours if the coil will be discharged after the lift.

The floating-coil is made of Nb$_3$Sn and has a maximum of current of 1.2 MA-turns [23]. When the levitation-coil is turned on, there is a slight vertical asymmetry which creates an upper ring null, as shown in Fig. 2-1. The flux surface that passes through this null point is a separatrix which divides the region of closed field lines from the region of open field lines. The separatrix passes through the magnetic mid-plane near $R = 175$ cm for the vacuum field, where $R$ is the major radius measured along the magnetic mid-plane from the center of the vacuum vessel. On the inboard side, the closed field line region begins at $R = 75$cm. The vacuum shell of the floating-coil extends to a maximum of $R = 57.9$cm, but the vacuum shell is not shaped exactly the same as the flux surfaces, so some field lines close to coil intersect vacuum shell.

Magnetic field strength is not adjustable when the floating-coil is levitated. When the floating-coil is fully charged, the current is nearly the maximum value. If the current were reduced, then the current in the levitation-coil would need to be increased in order to support the weight of the floating-coil, but the levitation-coil is already operating at maximum current. Magnetic field strength can be varied when the floating-coil is mechanically supported. In addition, a pair of Helmholtz coils (shown in Fig. 2-1) are installed on the vessel to provide a vertical field. These were not operational when the data for this thesis was collected.

Initial experiments in LDX were conducted with the floating-coil supported by three thin supports. In preparation for operation with the floating-coil fully levitated, a new launching fixture was installed in the vacuum vessel (Fig. 2-1). The floating-coil rests on a lifting
fixture and is mechanically pushed to the center of the vacuum vessel prior to magnetic levitation. Plasmas can be created while the floating-coil rests on the lifting fixture, but the fixture that allowed the floating-coil to be suspended from three thin supports is not part of the new launcher. When the levitation-coil is turned on, the magnetic field structure is nearly identical to the magnetic field while the floating-coil is levitated except for the effect of the lifting fixture. For levitated operation, the lifting fixture is retracted to a position outside of the last closed flux surface.

For the purpose of this thesis, we will refer to the original configuration as suspended operation (although it was historically called supported operation) in order to distinguish it from supported operation in which the floating-coil rests on the lifting fixture. In both suspended operation and supported operation, the main source of plasma loss is to the supports, but in suspended operation, the supports intersect only a portion of the field lines, whereas in supported operation, the lifting fixture intersects all field lines. Both suspended and supported operation are essentially magnetic mirror [24] confinement schemes with the consequent particle loss cones and anisotropic plasma distribution functions. When the floating-coil is levitated (during levitated operation), the end losses are eliminated. For the most part, this thesis discusses data that was collected during levitated operation, but in some cases comparisons will be made to supported mode plasmas.

2.2.2 Heating sources

There are essentially two “knobs” for controlling the plasma parameters in LDX: electron cyclotron resonance heating (ECRH) and neutral fueling. LDX uses multi-frequency ECRH. Both the power level of each of the sources and the combination of sources that are in use are adjustable. We can choose the fuel gas which essentially determines the ion mass, and we can also adjust the neutral fueling level. Increasing the neutral fueling level has been shown to increase the plasma electron density.

Plasmas are created using multi-frequency electron cyclotron resonance heating (ECRH). A combined maximum of 16 kW at three frequencies was available at the time of these experiments. Two 2.45 GHz sources have maximum output power of 3 kW and 1.7 kW, respectively. The 6.4 GHz source has a maximum output power of 3 kW and the 10.5 GHz source has a maximum output power of 10 kW. The resonance locations for each frequency depend on the magnetic field strength which varies as $R^{-3}$, where $R$ is the mid-plane radius,
Figure 2-2: Equilibrium magnetic flux contours and ECRH resonance locations. The fundamental (thick) and second (thin) harmonics are shown for each of the heating frequencies, including 2.45 GHz (orange), 6.4 GHz (red), 10.5 GHz (orange). The upper hybrid resonance (blue) overlaps the fundamental harmonic 6.4 GHz ECRH source and the second harmonic of the upper hybrid overlaps the fundamental harmonic of the 2.45 GHz ECRH source.

so each frequency will heat the plasma at a different radial location.

Figure 2-2 illustrates the resonance locations for the fundamental and second harmonics of each of the three heating frequencies. Higher frequency sources resonate closer to the floating-coil because the magnetic field is stronger there. The upper hybrid resonance occurs in the same region as the heating resonances. The equilibrium flux contours are also shown. Notice the presence of a separatrix that divides the closed field line region from the open field line region.

Only the 2.45 GHz source has fundamental a resonance location at the mid-plane, within the closed field line region. The fundamental resonances of higher frequency sources resonate at the mid-plane on field lines that intersect parts of the floating-coil. In warm plasmas, we expect heating from the higher harmonics and broad heating profiles.

The power of each heating frequency can be varied. The combination of sources used for heating and the times at which each source turns on and off are also programmable. Each RF source can be operated for the full length of the plasma discharge. The maximum
heating duration is limited to fourteen seconds by the data system. Increased heating power results in increased plasma density.

LDX uses a cavity heating scheme for ECRH in which there is little absorption during the first pass of the waves through the vessel, but the waves reflect of the walls many times. Most of the plasma heating occurs during these subsequent passes through the plasma so the heating is as axisymmetric as possible even though the antennas are localized. The 10.5 GHz antenna is located at the bottom of the vacuum vessel and the waves are launched towards the center of the vessel. The 2.45 GHz and 6.4 GHz antennas are launched in X-mode from the side of the vacuum vessel at the north-east port [25, 26].

2.2.3 Fueling sources

Plasmas are fueled by neutral gas that is injected into the vacuum chamber through a piezoelectric valve on the south-west port. The amount of injected gas is controlled by varying the length of time that the valve is open. The fueling sequence for plasma shots has two parts, a large pre-puff that is injected before the heating is turned on and a steady low-level fueling during the discharge generated by opening the valve for short periods of time with a frequency of 100 Hz.

Both the type of gas that is injected, and the amount of fuel are adjustable. Plasmas are generally created with deuterium ($H_2^+$) gas because that is the fuel that would be used in a dipole fusion reactor if the concept is shown to be feasible and advantageous. Hydrogen ($H^+$), helium, and argon gas are also available for fueling. Different fuel gasses can be used to evaluate the dependance of ion mass on plasma phenomena.

2.2.4 Plasma diagnostics

LDX has a basic set of diagnostics to measure plasma density profiles, pressure profiles, edge characteristics and fluctuation properties. A brief overview the diagnostic systems is given here. Detailed discussion diagnostics used to measure plasma fluctuations is given in Chapter 4, while information about the survey spectrometer is given in Chapter 3. The survey spectrometer gives us an idea of the spectrum of visible light emitted from the plasma and the impurity content of the plasma. This diagnostic was also used to measure the electron temperature from line ratio measurements. Using this temperature measurement and an additional temperature measurement from an edge swept probe, I present the only mea-
surement of temperature profiles in LDX to date. A four-channel microwave interferometer is used to measure electron density profiles and fluctuations [13] (§4.3). A single channel reflectometer is currently being developed, but data from that diagnostic is not presented here. The neutral pressure is measured by a vacuum ion gauge. Visible light emission is strongly dependent on the neutral pressure. The plasmas in LDX are fairly low density, most of the visible light emission is the result of electron impact ionization. In addition, many other plasma parameters have been shown to trend with neutral pressure.

Magnetic pick-up coils and flux loops are used to reconstruct the equilibrium plasma pressure [27]. A total of eighteen pickup coils measure the magnetic field in the directions normal and tangential to the vacuum vessel surface at nine locations. A total of 14 flux loops are located inside and outside the vacuum vessel to measure magnetic flux in the vertical direction [28]. When data for plasma diamagnetism is presented in this thesis, it is measured by flux loop 5 which surrounds the outside of the vacuum vessel slightly above the magnetic mid-plane (z = 23cm).

Moveable electrostatic probes are used to diagnose the edge of the plasma near the magnetic separatrix (§4.4). A 24-channel floating probe array is used to measure the toroidal structure of edge fluctuations. Two ion-saturation probes can be used as a mach probe to measure plasma flow. A swept probe is used to measure edge density and temperature. To measure magnetic fluctuations, eight Mirnov coils are arrayed toroidally on the vacuum vessel at the mid-plane (§4.5).

The visible light diagnostics include two linear photodiode array cameras (§4.1) and Phantom fast cameras (§4.2). The photodiode array diagnostic is a custom design while the Phantom fast cameras are a commercial product. An H-α filtered, Sony camera collects standard monochrome video of the plasma. This camera has a larger field of view than the arrays which are only one-dimensional and the Phantom cameras which are often operated with a reduced view in favor of increased speed.

A survey spectrometer measures the intensity of light emission between 330 nm (ultra-violet) and 580 nm (yellow) with an average resolution of 1.2 angstroms. A series of spectra are collected during a plasma discharge. This diagnostic can be used to discover the impurity species in the plasma. We can use line ratio information to determine the time resolved density and temperature of the cool background plasma along a single chord, although the resolution of this spectrometer is not high enough to provide information about individual
X-rays are emitted from the plasma as a result of bremsstrahlung radiation that occurs when fast electrons collide with ions or electrons, although the electron-electron emission is much weaker than the electron-ion emission. A single channel pulse height analyzer measures the energy spectrum of x-rays in the 1-20 keV range. This information can be used to deduce an effective line-averaged electron temperature. In addition, x-ray intensity is measured along a single chord by a sodium iodide detector, which is sensitive to x-ray with energies up to 1 MeV. The x-ray intensity detector views the floating-coil so it is also sensitive to x-rays that are emitted when fast electrons collide with the floating-coil vacuum shell. This is useful for identifying when instabilities such as hot electron interchanges occur.

Two single-channel radiometers measure the electron cyclotron emission from the plasma. These provide information about the fast electron population. Initial measurements of temperature suggest the peak fast electron temperature is on the order of 100 keV for plasmas created with 15 kW of power at three frequencies [29].

2.3 Comparison of LDX to other laboratory confined dipole plasma experiments

Plasmas in LDX are stabilized by compressibility. This is very different from the tokamak confinement scheme. LDX has been designed with a large vacuum chamber so there can be a lot of flux expansion. In this way it’s very different from other current and past dipole and multipole experiments.

The Collisionless Terrella Experiment (CTX) [30] at Columbia University uses an internal supported copper magnet to study hot electron plasmas confined by a dipole-like magnetic field. In such a configuration the predominant loss mechanism in the plasma is to the supports. Supported operation in LDX can be compared to the configuration of the CTX, as that experiment also uses a support which intersects all field lines. Even so, LDX is a much larger experiment than CTX. The floating-coil in LDX produces a larger magnetic field than the internal coil in CTX. The LDX floating-coil carries eight time more current than the CTX internal coil and the LDX vacuum chamber is about three times larger than the CTX vacuum chamber. CTX investigates plasmas that are relevant to space astrophysics and fusion plasmas. Important results include studies of the hot electron
interchange mode [31, 32] and characterization of turbulent fluctuations [33].

There is also a supported double dipole experiment, Magnetor, in Russia [34]. In this experiment an external magnetic field is applied to reduce the plasma volume. They find there is a good agreement between the theoretical density profile and the measured density profile. Unlike the plasmas created in LDX in levitated mode, the plasmas created in Magnetor have flat temperature profiles.

Several ring trap (RT) experiments including proto-RT, mini-RT, and RT-1 are currently underway in Japan. The largest experiment, RT-1 is about one-fifth the size of LDX. The internal coil is a high temperature superconductor that carries 250 kA of current and their vacuum chamber is 2 m in diameter [35]. The experiment was designed to investigate a self-organized state observed in Jupiter’s magnetosphere during which the hydrodynamic pressure from a fast plasma flow balances the plasma’s thermal pressure [36]. In addition, they are interested in studying the confinement of non-neutral plasmas in the device and have successfully obtained long confinement times [37]. All of these experiments have a smaller flux expansion than LDX.

2.4 What is known about LDX plasmas

We will briefly review the major experimental results from LDX and the outstanding questions. One important goal of the LDX program is to evaluate the stability and dynamics of high-beta plasmas with energetic particles confined by dipole magnetic fields. High beta plasmas have been confined and sustained in the LDX device and peak beta of over 20% has been observed [28]. The pressure profiles of plasmas created with the floating-coil in suspended mode were highly anisotropic and the pressure is contained mostly in the fast electrons which are a small fraction of the total electron population [27]. In contrast, the pressure profiles of plasmas confined by a levitated internal coil were much more isotropic [38] and the pressure was stored in both the fast electrons and the warm electrons, which are a much larger fraction of the electron population [5].

The biggest threat to stability observed to date is the hot electron interchange mode (HEI) which can lead to total plasma loss [39], but has been shown to be stabilized by sufficiently low hot electron fraction [40], which is easily obtained with levitated operation with its improved bulk plasma confinement. Most of the work on hot electron interchange
modes in LDX has been done on suspended mode plasmas, but HEIs have also been observed in levitated mode during extreme low density operation with low gas fueling and ideal wall conditions.

LDX is able to confine high (electron) beta plasmas for many confinement times. Levitation of the internal floating-coil greatly enhances plasma confinement. Plasma diamagnetism increases by a factor of two when the magnet is levitated and line-averaged plasma density increases by a factor of nearly three for similar fueling conditions. In addition, the discharges created during levitated operation are characterized by steep, centrally peaked density profiles [14].

MHD predicts that linear stability is determined by pressure profiles. Non-linear MHD predicts stationary pressure and density profiles. Kinetic theory predicts that plasmas that are marginally stable to interchange modes will be stable when they have an equal number of particles confined within each volume of equal magnetic flux or flux tube defined as \( \oint \frac{d\mathbf{l}}{B} \) [11]. These density profiles have been observed in planetary magnetospheres and are called “invariant profiles” because the density profile is invariant to interchange motion [41]. They have also been called “stationary profiles” because this density profile is the time-independent solution to the quasi-linear transport equation [11]. In LDX, these profiles are also referred to as “natural” profiles because the plasma assumes these profiles without external profile control. In LDX, the invariant profiles are of the form \( n \propto r^{-4} \). Peak densities during levitated operation are typically on the order of \( 10^{12} \text{ cm}^{-3} \).

A. C. Boxer used a four-channel microwave interferometer to reconstruct the plasma density profiles in LDX under a variety of conditions. He found that when the floating-coil was levitated, density profiles were often highly peaked and approached the situation where the number of particles contained within equal volumes of magnetic flux was constant [13]. In contrast, when the floating-coil was supported, the density profiles were fairly flat. In addition, the plasma density profile has been observed to relax to the natural profile during some discharges at times when there were no changes made to the external controls [42].

We assume that these highly peaked density profiles are sustained by inward turbulent transport because the reconstructed profiles of visible light emission indicate that there is no emission near the internal coil. The visible light emission is proportional to the neutral density so we assume that there is no source of neutral particles to peak up the density profile. The motivation for this thesis is to investigate the low frequency, turbulent
fluctuations that drive this transport.

Several diagnostics are used to estimate the effective temperatures of the electron components. The radiometers provide information about the hot electrons; the pulse height analyzer provides information about the warm electrons; and the spectrometer provides information about the cool electrons. If we allow each component to have an effective temperature for the purpose of gaining a conceptual picture of the plasma, then it is estimated that the cool electrons have a temperature on the order of 10 eV, the warm electrons have a temperature on the order of 500 eV and the hot electrons (or fast electrons) have a temperature on the order of $10^5$ eV.

The plasma pressure is carried almost entirely by the electrons, $T_e >> T_i$. The electrons are heated by ECRH, while the ions are heated only through collisions and Franck-Condon processes. The ion temperature is not measured, but we estimate the ions have energies on the order of 1 eV. The electron distribution function is not measured, but it is clearly non-Maxwellian. When the heating is turned off, the stored energy in the plasma decays with two characteristic time scales indicating that the electrons can be thought of as a multi-component fluid with warm and hot components [5]. There is likely a cool electron component as well.

The decay rates can be used to estimate the fraction of pressure that is carried by the hot electrons $\frac{P_{hot}}{P_{tot}}$, where $P_{tot}$ is the total plasma pressure. In supported mode, the ratio is nearly one, meaning that all of the pressure is carried by hot electrons. The hot electron fraction, which is the ratio of the number of hot electrons to the total number of electrons $\frac{n_{hot}}{n_e}$ is small. The density ratio is not measured directly, but we know that if the hot electron fraction gets too large, then the hot electron interchange mode is excited and this mode is observable with the current diagnostics. The hot electron interchange mode can lead to a dramatic plasma instability and subsequent loss of plasma beta and density [39]. In levitated mode, the fraction of the pressure carried by the hot electrons varies from 0.2 to 0.9 depending on the plasma conditions, so in most cases a significant amount of the plasma pressure is coming from the bulk plasma [5].

Electrons are heated only in the direction perpendicular to the magnetic field so some degree of anisotropy in the pressure may be expected. Plasmas created in supported configuration have a significant degree of anisotropy because electrons pitch angle scatter into the loss cone (created by the supports) which enhances the anisotropy [28, 27]. In contrast,
during levitated operation, pitch angle scattering does not cause particle losses and the data is best fit by an isotropic pressure profile [38].

Plasma density, diamagnetism and neutral pressure are used as measures of the plasma to make comparisons between different discharges. We typically use the line-averaged value of the plasma density because additional error is introduced when the density is inverted to find the peak value. The diamagnetism \( \Psi \) is roughly proportional to the stored energy in the plasma. The plasma stored energy has been measured for some discharges using equilibrium reconstruction of the pressure profile from magnetic measurements, but this analysis is not yet performed routinely so we use the diamagnetism as a measure of the plasma stored energy. The conversion from diamagnetism to stored energy depends on the pressure profile. For the range of expected pressure profiles (which are consistent with the magnetic reconstructions), \( W_E/\Psi \approx 80 - 100 \text{ J/mV·s} \) [43]. The neutral pressure is measured by the vessel ion gauge and is a measure of how strongly the plasma was fueled.

Plasma density, diamagnetism and neutral pressure are not independent. Plasma density tends to increase with neutral pressure until an optimum density is reached (Fig. 2-3), while diamagnetism is largest at low neutral pressures and tends to decrease with neutral pressure (Fig. 2-4). In most cases, when investigating how things scale with plasma density, neutral pressure, and diamagnetism, it is sufficient to plot just one of these three quantities.

Increased neutral gas fueling leads to increased neutral gas pressure as measured by the ion gauge. The electron density reaches an optimum value (Fig. 2-3). The value of the optimum density and the neutral pressure at which the optimum density is reached both vary with ECRH power, ECRH frequency, neutral gas species, and whether or not the floating-coil is levitated [42]. With the current power levels, the plasma density is limited by available power rather than by other plasma processes [42].

Plasma diamagnetism is inversely related to neutral pressure. It decreases as neutral pressure and line-averaged density increase. This is probably because the diamagnetism is coming mostly from the fast electron population. As the neutral pressure increases, more heating power goes into ionization and less power goes towards heating the fast electrons.

Several experiments have looked at helium plasmas and the results show higher density and higher beta than in deuterium plasmas. An optimum density has not yet been observed for helium plasmas. The density increases with neutral pressure for all of the observed neutral pressures. The higher density may indicate a slower flow of particles into the wall.
Figure 2-3: Line-integrated plasma density measured along three interferometer channels versus neutral pressure for deuterium plasmas with 15 kW of multi-frequency ECRH including shots 90312025-044 and 90313001-023.

Figure 2-4: Plasma diamagnetism versus neutral pressure for deuterium plasmas with 15 kW of multi-frequency ECRH including shots 90312025-044 and 90313001-023.
2.4.1 Theoretical predictions of low-frequency instabilities in dipolar plasmas

Dipole confined plasmas can be unstable to MHD-like interchange modes and to drift/entropy waves. In the dipole field line geometry, interchange modes and drift waves are both electrostatic fluctuations and have flutelike characteristics. The main difference is the growth rate and the instability drive. Interchange modes are driven by pressure gradients, while drift waves are driven by temperature and density gradients. The stability of interchange modes and drift waves in dipoles has been investigated both theoretically [44, 7, 45, 46, 11] and in simulation [47, 10].

Interchange modes are a form of the Raleigh-Taylor instability (RTI), which occurs at the interface of the two fluids and results in large scale mixing. In a plasma, hot dense plasma can interchange with cool diffuse plasma and that interchange becomes unstable in regions where the curvature of the magnetic field has the same direction as the pressure gradient [48]. In LDX, the curvature of the magnetic field is always negative, but magnetic compressibility stabilizes the interchange mode as long as the pressure gradient is sufficiently small [49].

The fluid-like MHD theory predicts that dipole confined plasmas will be marginally stable to interchange modes when

\[ PV^\gamma = \text{Constant} \]  \hspace{1cm} (2.1)

where \( P \) is the plasma pressure, \( V \equiv \oint \frac{\partial \ell}{p} \) is the volume per unit flux and \( \gamma \) is the ratio of specific heats [49]. We typically assume \( \gamma = \frac{5}{3} \). In a dipole, \( B \sim R^{-3} \) and \( \oint \partial \ell \sim R \) so \( V \sim R^4 \), where \( R \) is the distance from the current carrying ring. Then at marginal stability, the maximum pressure gradient is \( P \sim V^{-\gamma} \sim R^{-4\gamma} \sim R^{-\frac{20}{3}} \). The location of the pressure peak depends strongly on plasma edge parameters. The most stable operating point occurs when \( \eta = \frac{\partial \ln T}{\partial \ln n} = \frac{2}{3} \). At this value, adiabatic interchange of flux tubes can occur without changing the density and temperature profiles [49, 7].

Drift waves are driven by density and temperature gradients. They can be driven unstable by resistivity, finite electron larmor radius, and Landau damping. Unlike interchange
modes which have \( k_{||} = 0 \), drift waves usually have finite \( k_{||} \) to become unstable. Here the designation of parallel is in reference to the magnetic field.

Entropy modes or temperature gradient modes, as they have also been called, are a special type of drift wave that occur in dipolar plasmas and have \( k_{||} \sim 0 \). These modes perturb the entropy of the system. Temperature and density fluctuate, but to leading order the pressure is left unchanged [50].

The stability regions for the MHD like instabilities and entropy modes have been predicted by linear theory for thermal plasmas as functions of the pressure gradient \( d = \frac{d \ln P}{d \ln V} \), where \( V = \int dl / B \) is the volume per unit flux and the ratio of the density and temperature gradients \( \eta = \frac{d \ln T}{d \ln n} \). At present, the only profile measurements available for LDX plasmas are density profile measurements. The plasma is unstable to either MHD modes or entropy modes in all cases where the density profile is steeper than the invariant profile which has an equal number of particles confined within each flux tube. The plasma can also be driven unstable for density profiles that are shallower than the invariant profile, depending on the value of \( \eta \). Fig. 2-5 shows the stability regions for MHD modes and entropy modes as a function of the density gradient \( d \ln n_e / d \ln V \) and \( \eta \).

In a dipolar plasma, the diamagnetic drift frequency \( \omega_d \) is the same order of magnitude as the curvature drift \( \omega_d \). Both of these are much smaller than the bounce frequency.
The ordering of the collision frequency $\nu_j$ where $j$ is particle species depends on the collisionality of the plasma. In LDX, we expect that the electrons are collisional, but have long mean free paths so $\omega_{be} > \nu_e > \omega_{de}, \Omega_e$ [7]. Low frequency fluctuations (including the MHD-like instabilities and drift waves) have frequencies that are low with respect to the bounce frequency and the same order as the drift frequencies.

Kouznetsov et al. investigated the effects of velocity shear resulting from azimuthal flows in a hard-core Z pinch on the stability of interchange modes. Depending on the velocity profile, velocity shear can either enhance or degrade stability in comparison to the case with no shear. They found that a constant velocity shear enhanced stability, while a counter-streaming profile degraded stability, but profiles with peaked velocity profiles had very little effect on the stability boundary [46].

In practice, non-linear effects are very important. Kobayashi et al. used non-linear simulations to look at turbulent transport resulting from entropy modes in a dipole geometry [10]. The results are roughly consistent with linear theory. Density gradients are destabilizing and temperature gradients are stabilizing. In addition, transport increases when the electron temperature is larger than the ion temperature [10], as is the case in the discharges described in this thesis.

2.4.2 Low frequency fluctuations are observed in LDX

Broadband, low frequency fluctuations were observed in LDX during the first plasma discharges in 2004. The plasma diagnostic set was fairly small during the first campaign, but an ion saturation probe in the plasma edge showed large, broadband fluctuations.

As the diagnostic set grew, a low frequency, quasi-coherent mode was observed on the interferometer, a photodiode that viewed the entire plasma, the edge electrostatic probes, and the Mirnov coils. Although the theory of these low-frequency plasma fluctuations predicts electrostatic modes, even electrostatic modes can have a magnetic component in high beta plasmas [51].
Chapter 3

Visible Spectrometer

Measurements

When LDX first began operating, a color video camera filmed the plasma. The color of deuterium plasmas created in LDX was a deep blue, rather than the expected pink. Blue plasmas continued to be observed in subsequent campaigns. Sometimes, shots taken at the end of a campaign were more purplish-pinkish than blue. When a large puff of D2 gas was injected during a shot, the plasma would turn pink briefly and then blue again. One explanation was that an impurity population was causing the blue color of the plasmas. Potential impurity species included $D_2^+$, helium, nitrogen, oxygen, iron, nickel, manganese, and water.

An AvaSpec USB spectrometer (AvaSpec-2048-SPU) was installed on the machine in 2006 to investigate the composition of the plasmas and determine the reason for the blue color. The survey spectrometer can also be used to measure electron temperature, which will be discussed in §3.3. In the future, the spectrometer may also be used to measure electron density as well as to identify specific portions of the spectrum that would be well suited for measurement with a higher resolution spectrometer.

This survey spectrometer measures the intensity of light emission between 340 nm (ultraviolet) and 550 nm (yellow) using a 1200 lines/mm grating and 2048 pixels. Strong lines from each of the suspected impurity species are found within this range of wavelengths. The slit size is 25 $\mu$m and the corresponding resolution is 0.3 nm.

The spectrometer is located outside the vacuum chamber. Visible radiation emitted
from the plasma passes out of the vacuum vessel through a Corning 7056 glass viewport. The transmission is greater than 90% for the wavelength range of interest. A quartz lens with a transmission range from 200–1100 nm focuses light from the plasma onto a 2 m long, 600 μm UV/VIS fiber optic that is connected to the spectrometer. A series of spectra are collected during a plasma discharge so I can evaluate the time evolution of the light emission spectrum. The integration time is programmable. Typical integration times range from 0.125 – 0.25 seconds. There is no practical limit on the number of spectra that can be collected.

The spectrometer has not yet been integrated into the data system so there is some ambiguity in the time synchronization. The key time in the shot, including the turn on and turn off of the ECRH are easily identified by comparing the total counts collected during each integration time to the data from a channel of the photodiode array with a comparable view. This is sufficient to associate times with each of the spectra because there are generally only four to eight spectra collected per second. In the future, the spectrometer should be integrated into the data system so that the times the spectra are collected can be synchronized with the rest of the data that is collected.

Both a wavelength calibration and a relative brightness calibration have been performed. For the wavelength calibration, the spectra of hydrogen, deuterium, helium, and neon lamps were measured. In the resulting data, a Lorentzian was fit to each of the lines and the bin where the line was centered was identified. Recall that this is a survey spectrometer, so it does not have enough resolution to provide detailed information on most line shapes or shifts. A fourth order polynomial was fit to the wavelength versus bin number data, based on the specifications for the spectrometer. For the brightness calibration, a lamp of known brightness was imaged. The fiber response was not measured independently so the calibration results give only the relative brightness of each channel, not the absolute brightness. This is adequate for line ratio comparisons. Additional details about the calibrations can be found in Appendix A.

3.1 Typical Spectra

The spectra reveal that the primary impurities in the plasma are oxygen and helium. Carbon is also observed. The oxygen is responsible for the bright blue color observed in the plasmas.
Figure 3-1: Average visible light emission for a typical deuterium plasma. The average of the time-averaged spectra of eleven discharges from the July 2006 run. Hydrogen Balmer series line are identified. Helium, carbon, and oxygen impurities are present. The line ratio of two neutral helium lines at 471 nm and 504 nm will be used to measure electron temperature.

The oxygen levels in the plasma decrease as the number of discharges produced during a run campaign increases. After the oxygen level is diminished, a helium impurity remains. That impurity causes the plasma to appear purple in color.

The source of the oxygen impurity is primarily water. Good vacuum conditioning techniques have helped to reduce the amount of water in the vacuum vessel. This has reduced the oxygen levels.

There are several sources of helium. Liquid helium is used to cool the floating-coil and charging-coil. Sometimes the helium leaks into the vacuum vessel when the vacuum feedthroughs that supply helium to the floating-coil are removed, just before the floating-coil is lifted to the center of the vacuum vessel for plasma operation. Steps have been taken to mitigate this problem and leaks are now rare.

Another source of helium is that helium is used for glow discharge cleaning (which will be discussed in the next section) and also as a fueling gas for plasmas. Residual helium can remain in the vacuum vessel.

The presence and absence of lines can provide some clues about the temperature of the
plasma. In the example spectrum shown is Fig. 3-1, there are no lines of higher order charge states, particularly of carbon. This suggests that this particular discharge was fairly cool in the region where the spectrometer views. In §3.3, I use the ratio of intensity of two helium lines to measure the plasma temperature.

### 3.2 Vacuum Conditioning

Vacuum conditioning techniques are used to reduce the impurities that accumulate on the surfaces in the vacuum chamber. Glow discharge cleaning (GDC) is the primary method used in LDX for vacuum conditioning. An electrode is inserted into the vacuum chamber and an arc discharge is struck between the electrode and vacuum chamber walls. After a vacuum break, GDC with He is typically performed for 24-72 hours. Ideally, the GDC is performed two weeks before the run campaign to allow adequate time for the hydrogen gas that is created by breaking down water to pump out of the vessel.

The content of the vacuum is measured using a residual gas analyzer. The impurity levels are measured before and after glow discharge cleaning. The GDC does an excellent job of reducing the water levels in the vacuum vessel. The partial pressures of hydrogen and helium are typically higher after GDC, but these gasses pump out over the course of several days.

Purging the vacuum vessel with dry gas is another method for reducing water. In LDX, the vacuum vessel can be purged using dry nitrogen gas that boils off from the liquid nitrogen (LN2) tank. This technique is used in combination with the glow discharge cleaning.

Other techniques for vacuum conditioning include baking the vacuum vessel, striking repeated RF discharges, and titanium gettering. Baking the vacuum vessel involves heating it up so that the particles trapped in the metal will outgas. LDX is not set up to have the vacuum vessel baked for two reasons. The cost of heaters and insulators to bake a vacuum vessel of such a large size is quite high. Also, heating the vessel would cause it to expand by 2 cm and the vacuum vessel was not designed with enough margin in strength to be heated while under vacuum. RF discharge cleaning is another possible method for cleaning vacuum chambers and it may be possible to use this method on LDX, but a magnetic field is required. Operating the superconducting magnets for vacuum vessel cleaning is prohibitively expensive. It is possible to create RF discharges using the copper levitation-
coil, but the cleaning may not be uniform. Titanium gettering is also possible, but is being avoided until it is deemed absolutely necessary.

### 3.3 Electron temperature measurements from helium line ratios

I use the spectrometer data to compute temperatures from the line ratio of emission from two neutral helium lines. The spectrometer view has a tangency radius of approximately 125 cm. The measured temperature is the result of emission along the chord of the spectrometer, but the light emission and the plasma temperature both decrease with radius so most of the emission is assumed to be coming from the region near the tangency radius. As a result, I assume that the measured temperature is a local temperature.

The probability that light will be emitted at a given line depends on the plasma density and temperature. Particular pairs of lines can be chosen so that the ratio of the probability of emission at these two lines has a monotonic dependence on temperature at a known density. I chose to use the ratio of two neutral helium lines because for these pairs, the line ratio is a function of temperature, but does not depend on density, for low density plasmas. This means that I do not need an independent measure of the plasma density to measure the temperature.

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Line ratios are one of a number of techniques that can be used to get density and temperature information about ions and electrons in the plasma from light emission. Most of these techniques require a higher resolution spectrometer than the one that is installed on LDX. Visible bremsstrahlung radiation is observed on the spectrometer. Ratios of visible bremsstrahlung intensities have been used to measure electron temperatures on other experiments. This technique could be used on LDX in the future.

Line ratio measurements are often made using two lines from different charge states of the same elements, such as CII and CIII. The 426.73 nm CII line and the 464.74 nm CIII line are in range of wavelengths that the spectrometer measures, but these lines were not emitting strongly in the plasmas that were measured. The ratio of two neutral helium lines, a singlet transition $4^1S \rightarrow 2^1P$ at 504.8 nm and a triplet transition $4^3S \rightarrow 2^3P$ at 471.3 nm, often emit strongly enough to make a ratio measurement so these lines have been selected.

Visible light is emitted from an ion or a neutral particle in an excited state when that
particle is de-excited. The probability that an individual particle will relax to a less excited state is given by the total cross-section \( \sigma(v) \) for that transition, where \( v \) is the particle velocity. This total cross-section is the sum of individual cross-sections for each of the mechanisms that can cause a particular transition.

In the plasma, there are many particles. The visible light emitted from a population of particles (ions or neutrals in an excited state) in the plasma is proportional to the cross-section averaged over the electron distribution function,

\[
< \sigma v > = \frac{\int \sigma(v) v f(v) d^3 v}{\int f(v) d^3 v}
\]

(3.1)

where \( v \) is the velocity and \( f(v) \) is the electron distribution function. If the distribution function is Maxwellian, then

\[
< \sigma v > = \left( \frac{m}{2 \pi k T} \right)^{3/2} \frac{4 \pi}{m^3} \int_0^\infty \sigma(v) \exp \left( \frac{-m v^2}{2 k T} \right) v^3 dv
\]

(3.2)

where \( m \) is the mass, \( k \) is Boltzmann’s constant, and \( T \) is the electron temperature.

There are many mechanisms for excitation and de-excitation of particles in the plasma. An appropriate model must be chosen to relate the rates of excitation and relaxation in the plasma and to identify which mechanisms for excitation and relaxation are dominant. The plasma density, temperature and ionization fraction are all helpful in selecting an appropriate model.

The Coronal equilibrium model [52] was selected because it is a simple model and has been shown to be accurate for low density plasmas [53] and the density along the viewing chord of the spectrometer is in this range. Only electron impact ionization is considered, because the ions are cold. In this model, the rate of collisional excitation is balanced by the rate of spontaneous relaxation. Boivin et. al. [53] showed that in low density plasmas where \( n_e < 10^{11} \text{cm}^{-3} \), the steady-state corona model can be to determine the electron temperature in RF heated plasmas. The plasma density measured by the interferometer in LDX indicates that the plasma density remains below this value in the region where the spectrometer views.

The electron impact cross-section is

\[
\sigma_{ij} = \frac{\Omega_{ij}(X)}{\omega_i E[Ry]} \frac{1}{\pi a_i^2}.
\]

(3.3)
where $\Omega_{ij}(X)$ is the collision strength, $X = \frac{E}{\Delta E_{ij}}$ is the collision energy in threshold units. The excitation energy $\Delta E_{ij}$ is given in table 3.1 for each of the transitions. $a_0$ is the Bohr radius and $\omega_i$ is the statistical weight of the initial state. $E[Ry]$ is the incident electron energy in Rydberg units, where $1Ry = 13.6$ eV.

The collision strengths for the transitions between the ground state and the first metastable states of neutral helium lines are given by Kato and Janev [1] as analytic fits to experimental data. The collision strengths for the singlet and triplet transitions have the form

$$1^1S \rightarrow 1s4^1S :$$

$$\Omega(X) = \left( A_1 + \frac{A_2}{X} + \frac{A_3}{X^2} + \frac{A_4}{X^3} \right) \left( \frac{X - 1}{X} \right)^{A_5}$$

(3.4a)

$$1^1S \rightarrow 1s4^3S :$$

$$\Omega(X) = \frac{1}{X} \left( A_1 + \frac{A_2}{X} + \frac{A_3}{X^2} + \frac{A_4}{X^3} \right).$$

(3.4b)

The values of the fitting parameters $A_1, ..., A_5$ are given in table 3.1.

Table 3.1: The excitation energies $\Delta E_{ij}$ and fitting parameters $A_1, ..., A_5$ for the a singlet transition and a triplet transition of neutral helium [1].

<table>
<thead>
<tr>
<th>$\Delta E_{ij}$</th>
<th>$1^1S \rightarrow 1s4^1S$</th>
<th>$1^1S \rightarrow 1s4^3S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$23.67$ eV</td>
<td>$23.59$ eV</td>
<td></td>
</tr>
<tr>
<td>$A_1$</td>
<td>$1.5387e-2$</td>
<td>$3.7705e-5$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$-4.6305e-2$</td>
<td>$4.6867e-2$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$4.7398e-2$</td>
<td>$-5.9202e-2$</td>
</tr>
<tr>
<td>$A_4$</td>
<td>$-3.9669e-2$</td>
<td>$1.7539e-2$</td>
</tr>
<tr>
<td>$A_5$</td>
<td>$1.7198e-1$</td>
<td></td>
</tr>
</tbody>
</table>

The line ratio can be constructed by assuming the distribution function is Maxwellian and substituting Eq. 3.3 into Eq. 3.2

$$\frac{< \sigma v >_{ij}}{< \sigma v >_{kl}} = \left( \frac{\omega_k}{\omega_i} \right) \int_0^\infty \Omega_{ij}(X) \exp \left( -\frac{\Delta E_{ij}}{kT} X \right) X \, dX \int_0^\infty \Omega_{kl}(X) \exp \left( -\frac{\Delta E_{kl}}{kT} X \right) X \, dX.$$ 

(3.5)

A constant term $C = 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} \left( \frac{2\pi a_0^3 E_{Ry}}{m^2} \right)$ multiplies both the numerator and the denominator so it cancels out. The collision strength depends only on temperature, so Eq. 3.5
Figure 3-2: Computed helium line ratios as a function electron temperature for plasma in Coronal equilibrium.

also depends only on temperature.

The integrals in Eq. 3.5 can be performed analytically. Mathematica was used to compute the line ratios for temperatures ranging from 2–100 eV (Fig. 3-2). The line ratio increases monotonically with temperature and there is a one-to-one correspondence between line ratio and temperature.

The line ratios measured by fitting a four parameter Gaussian to the He singlet line and triplet line in the measured spectrum, from which the background has been subtracted. The area under the curve is then computed from the fit parameters. A relative calibration of the spectrometer is necessary to get an accurate measurement of the line ratio. This calibration has been performed and the details are presented in Appendix A.

Ideally, this would give us a time-resolved measure of electron temperature because time resolved spectra are available. In practice, there is a fair amount of scatter in the data, so time-averaging the data is necessary. Figure ?? shows that the line ratio measurement is fairly insensitive to temperature so there is a large error associated with the measurement, even with the averaging.

For shot 90312011, the average temperature along a chord passing through a minimum
radius of approximately 125 cm, during the time when all the ECRH sources (~15 kW heating power) were on, is 53 eV and the standard deviation is 19 eV. The field of view is narrow with a spot size of approximately 3 cm in diameter. Technically, this temperature is not a local measurements, but rather some average of the temperatures along the chord. However, the average is not evenly weighted because more light is emitted from the inside of the plasma than from the outside and more light is collected from the region where the viewing chord is tangent to the plasma.

The chord-averaged density measured by the interferometer chord passing through a minimum radius of 96 cm was 5 \cdot 10^{16} m^{-3}. The 125 cm chord of the interferometer was not functioning properly during this shot. If the density profile had an equal number of particles confined within each flux tube, then the chord-averaged density along the 125 cm chord would be about a factor of 3 smaller than the chord-averaged density along the 96 cm chord. The density is within the range over which the Coronal equilibrium assumption is valid.

The edge probes typically measure a temperatures of 15 - 25 eV near the magnetic separatrix at \( R = 171 \) cm. If I assume that the temperature profile has the form \( T_e \propto r^x \) then I can use these two measurements to estimate \( p \). The temperature in the core of the plasma can then be calculated from the assumed profile

\[
T(R) = T_p \left( \frac{R}{R_p} \right)^x
\]

\[
x = \ln \left( \frac{T_s}{T_p} \right) / \ln \left( \frac{R_s}{R_p} \right)
\]

where \( T_p \) and \( T_s \) are the temperatures measured by the probe and the spectrometer, respectively and \( R_p = 171cm \) and \( R_s = 125cm \) are their radial locations. Using an edge temperature of 15 eV and the measured spectrometer temperature of 53 eV, this equation predicts the core temperature at 77 cm is 373 eV. If the edge temperature is higher, then the core temperature would be lower. Using an edge temperature of 25 eV predicts the core temperature at 77 cm is 169 eV. In this temperature range, I would expect to observe CII and CIII lines because the plasma temperature is high enough to ionize the carbon. Some CII lines were shown in the example spectrum (Fig. 3-1), but CIII was not identified. It is possible that the carbon impurity is small so the the CII lines are not intense enough to be evident in the spectrum.
3.4 Summary

A visible spectrometer has been installed on LDX. The diagnostic views visible light between 330 and 580 nm along a viewing chord passing through a minimum radius of 125 cm. The diagnostic has been calibrated for both wavelength and relative intensity. Time resolved spectra of the plasmas is measured.

The dominant impurities in LDX plasmas are oxygen and helium. The amount of impurities is reduced as the number of discharges created during a campaign increases. Improved vacuum conditioning techniques have also been able to reduce the impurity content of the plasmas.

The plasma temperature can be measured using helium line ratios. An average temperature of 53 ±19 eV was obtained using this method.
Chapter 4

Fluctuation Diagnostics and Methods

In this chapter, I discuss the details of the diagnostics that are used to measure low frequency plasma fluctuations, both at the edge of the plasma and in the core. These include the photodiode arrays and fast framing cameras, a four channel microwave interferometer that measures plasma density, a twenty-four channel probe array that measure edge potential fluctuations, additional electrostatic probes that measure ion saturation current, and Mirnov coils that measure edge magnetic fluctuations.

The views for the photodiode array diagnostic and the four chord interferometer and the location of the probe array are illustrated in Fig. 4-1. This is a top down view of the LDX vacuum chamber.

I then discuss the methods that are used to investigate these fluctuations. Methods for spectral analysis including the short-time Fourier transform which transforms time series data to the time-frequency domain, spectral bicoherence which measures the coherence between data collected simultaneously at different locations in the plasma, and the two point method for estimating wave numbers from the time series data of two spatially separated diagnostics. Probability distribution functions and higher order moments are used to look at how random the fluctuations are. Biorthogonal decomposition is a technique that can be used to separate the signal into contributions from different modes. The Abel inversion is used to assess the local behavior of the plasma from chord integrated diagnostics. In addition, a synthetic diagnostic has been developed to simulate the data that would be
Figure 4-1: The views for the fluctuation diagnostics are shown in a top down view. The views for the two photodiode arrays are shown in green and blue. The interferometer views are shown in black. The probe locations for probe array are shown mapped to the mid-plane. The magnetic separatrix is indicated by a dashed line.
collected by the photodiode array from a known fluctuation model.

4.1 The photodiode array diagnostic

Plasmas naturally radiate in the visible spectrum. When the plasma is optically thin, the radiation leaves the plasma and can be measured. An advantage to using light emission to study fluctuations in optically thin plasmas is that it is a non-perturbative measure of the plasma. Fast framing cameras have been used on many plasma experiments to measure the structure of plasma turbulence [54, 55, 56]. LDX has both fast framing cameras and linear photodiode arrays that are essentially one-dimensional fast cameras.

Collection and measurement of visible light from the plasma is an inexpensive way to measure fluctuations in the core of an optically thin plasma. Numerous processes cause plasmas to radiate in the visible spectrum. These include line radiation resulting from atomic de-excitation or recombination, collisions of free electrons with neutral particles, and visible bremsstrahlung resulting from collisions of free electrons with other charged particles in the plasma. For low density plasmas \( n_e < 10^{12} \text{ cm}^{-3} \) such as those created in LDX, visible light emission is proportional to the electron density, so measurements of visible light can be used to investigate density fluctuations. Comparison with other plasma diagnostics supports this assumption.

Two one-dimensional fast cameras have been constructed to image the radial structure and toroidal variation of visible light emission from the plasma. These cameras rely on a 16-channel, linear, photodiode array sensor and I will refer to them as the photodiode arrays. They are sensitive to visible light from 400–1100 nm. The signals are digitized at 50 kHz and 20 seconds of data can be collected with the current digitizers. The bandwidth of the array is limited by the preamplifier electronics to 15 kHz. The design, construction and placement of these diagnostics on LDX is discussed in detail in the third section of this chapter.

4.1.1 Specifications

A photodiode array diagnostic was first installed on the LDX by Emanuel Mimoun in 2005 [57]. The diagnostic used a 16 element linear photodiode array mounted in the film plane of an SLR camera. The signal from the photodiodes travelled through a 6' cable to
a trans-impedance amplifier box and then was digitized. The currents generated by each element of the photodiode array are on the order of 100 nA so the 6' cable introduced an unacceptable amount of noise into the measurement. Mimoun managed to get the signal to noise level up to acceptable levels, but when the diagnostic was later moved to a better viewport, those low noise levels could not be reproduced.

The original photodiode array diagnostic demonstrated that a diagnostic of this type was able to capture the low frequency fluctuations observed on other diagnostics in LDX such as the microwave interferometer and the electrostatic probes. I was interested in studying these fluctuations further in order to better understand what they are and how they affect the plasma. The toroidal and radial mode numbers and structures of these fluctuations are of particular interest because previous observations detected radial variation of the fluctuations. To measure these, it was necessary to build a second array and to design appropriate view ports.

The photodiode array diagnostic has been redesigned and two identical systems were built using the new design. The system still relies on a linear photodiode array chip and the SLR camera, but the photodiode array chip is now soldered directly to the redesigned trans-impedance amplifier which is mounted on the back of the camera. A photograph of the redesigned array is shown Fig. 4-2(a) and a simplified schematic of the electronics is shown in Fig. 4-2(b).

The two arrays view the plasma through identical viewports separated toroidally by ninety degrees. Additional viewports are available depending on the nature of the experiment (§ 4.1.2). The views have been calibrated using an in-vessel calibration and the two arrays are aligned to within half a channel (§ C).

Although two dimensional fast cameras have much better spatial resolution than these one dimensional arrays, the photodiode array diagnostic has many advantages. The biggest advantage is that the diagnostic can collect data for the entire plasma shot. This is important in an experiment that has a very limited operating time. The long data records are also very helpful for statistical analyses. The photodiode array diagnostic can also operate at higher speed for lower cost.

The light sensor is a PDB-216, is a blue enhanced, silicon photodiode, sixteen element linear array with a common cathode made by Advanced Photonix, Inc. The chip had been selected for the first version of the diagnostic because it was inexpensive, in-stock, and had
(a) Photograph of the photodiode array sensor in place on the film door of the SLR camera. The box containing the preamplifier is attached to the film door.

(b) Schematic view of the electronics for a single channel of the photodiode array diagnostic.

Figure 4-2: Photodiode array diagnostic design.
enough elements to provide some information about the spatial structure of the plasma. The chip worked well and has a larger sensor area than other similar products on the market so I continue to use it in the redesigned diagnostic. Each sensor has a 2.24 mm\(^2\) active area: 1.22 mm wide by 1.84 mm tall. The total width of the array is 25.07 mm so it can be used with a standard 35 mm format camera although the field of view is reduced because the sensor is not as large as 35 mm film.

The spectral response varies as a function of frequency, increasing from 0.1 A/W at 350 nm to 0.4 A/W at 600 nm to 0.7 A/W at 1000 nm and then falling rapidly back to 0.1 A/W at 1100 nm. Calculating the total power in the light emitted from the plasma then requires some knowledge of the wavelength of the light emitted. A survey spectrometer is available to look at the wavelength spectrum of light with wavelengths less than 580 nm, but little is known about how light with wavelengths greater than 580 nm is distributed in LDX. The spectrometer does show that the average spectra are quite similar for similar discharges. I will show data from the photodiode array in volts rather than watts because the precise distribution of the wavelengths of light emitted from the plasma cannot be determined with the current diagnostic set.

The photodiode array is mounted in the film plane of a Nikon FG single lens reflex (SLR) camera body. Although the Nikon FG camera bodies are no longer being manufactured, used ones are readily available at low cost. The use of the camera body simplifies aligning the optics and eliminates the need for fiber optic cables or a custom enclosure with a lens mount. A Nikkor zoom lens with adjustable focal lengths from 35 mm - 105 mm and an adjustable f-stop f/22-3.5 is attached to the camera. This range of focal lengths makes it possible to view the full radial extent of the plasma or to zoom in on a smaller region of plasma with more spatial resolution.

No interference filters are placed in front of the lens for these experiments so the arrays image the full spectrum of light emitted from the plasma. Low signal levels constrain the use of filters with fast cameras and photodiode arrays. Gas puff imaging has been used to enhance the light levels in other experiments imaging the tokamak edge so that filters can be used with fast cameras [58]. That approach could be considered for future experiments on LDX.

The low light levels emitted from the plasma and the small active area of each photodiode mean the nominal current emitted from each element is on the order of 100 nA. Significant
amplification is required to bring this signal into a useful range for measurement. The array chip is soldered directly to a single stage trans-impedance preamplifier with buffered outputs. The output from the preamplifier is sent to a dual stage amplifier and then to a digitizer. A simplified schematic view of the electronics for a single channel of the diagnostic is shown in 4-2(b).

The trans-impedance preamplifier has been designed especially for this diagnostic. A schematic of the circuit for a single channel is shown in figure 4-3. Each preamplifier board has sixteen channels and the photodiode array chip mounts directly on the board. The full circuit schematic and the board layout diagram be found in Appendix B.

The photodiodes in the array have a common cathode which is biased at positive 10 volts. This improves the response time of the sensor by more than a factor of 10. Biasing the photodiode array theoretically decreases the signal to noise ratio compared to operating it the photodiode with no bias. Biasing the photodiodes also reduces their capacitance which makes the circuit more stable. The response time is typically 13 nS when the bias voltage is 10 V and dark current is typically 5 nA.
The preamplifier requires a high gain bandwidth and very low input bias current op-amp chip. The photodiode array has a common cathode an amplifier that is powered by both a positive and negative input voltage was required. The OPA657D chip has a 1.6 GHz gain bandwidth product and an input bias current of 2 pA so it meets all these requirements. Theoretically the preamplifier should be able to operate with a gain of $1 \cdot 10^6$ and a frequency response of 20 kHz, but in practice, this was not the case. The preamplifier has been configured with a gain of $5 \cdot 10^5$ and a frequency response of 8 kHz for the data presented in this thesis. Data from electrostatic probes with a frequency response of 160 kHz shows that most of the low frequency fluctuation activity in the plasmas is occurring below 15 kHz.

The signals from the preamplifier are sent to an amplifier with a gain of 100. This amplifier is located in the rack with the digitizer and the cable run to the rack is approximately 50'. This long cable run likely introduces some additional noise to the system. The amplifier uses the Alcator C-Mod “Quad Amp 2" boards. These are differential input, dual stage amplifiers. Each amplifier stage has a gain of ten, for a combined gain of one hundred. The output signals are typically one the order of 1 Volt. The amplifiers have an input filter with $f_{3dB} = 20$ kHz and an output filter of $f_{3dB} =15$ kHz. These filtering values are below the Nyquist frequency for the digitization rate of 50 kHz so aliasing should not be a problem. The gain is constant to within 5% for frequencies up to 5 kHz.

The signals are digitized at 50 kHz by a 32-channel single-ended input digitizer.

If the electronics are updated in the future, it would be wise to use multiple stages of amplification on the preamplifier so that the frequency response could be increased and the additional amplifier that is currently being used in unnecessary. This may also result in decreased noise because any noise introduced in the long cable run to the digitizers will not be amplified.

### 4.1.2 Diagnostic Views

**Viewport Configurations**

There are four viewports available for the two photodiode arrays. This flexibility allows us to configure the cameras in different ways for different experiments. Two side views are separated by 90 degrees toroidally. The views are otherwise identical. In addition
two bottom views are separated by 90 degrees toroidally but are otherwise identical. The cameras can both view from the side, both view from the bottom, or one camera can view from the side and one can view from the bottom.

I will refer to these three configurations as 'side view configuration', 'bottom view configuration', and 'toroidal slice configuration', respectively. The three configuration are illustrated in Fig. 4-4. In the side view configuration and bottom view configuration, I can use the data to compute radial and toroidal mode numbers. If the assumption of axial symmetry hold (for example with \( m=0 \) modes) I can use the toroidal slice configuration to collect more detailed information about a toroidal slice of the plasma.

For the experiments described in this thesis, the lens was set to a focal length of 105 mm which provides maximum spatial resolution in the core region of the plasma. The f-stop is full open to \( 1/3.5 \) which allows in the most light. For minimum overlap between channels, the f-stop should be made as small as possible, but this would let in the least amount of light. There is a small amount of overlap between adjacent channels.

**Position Calibration of PHDA**

An in-vessel calibration was performed to confirm the photodiode array views in the side view configuration. These viewing chords are illustrated in a top down view of the vacuum vessel in Fig. 4-1. The calibration is described in detail in Appendix C. The results are summarized below.

The viewing chords for the chord averaged diagnostics on LDX are labelled by the tangency radius \( R_\perp \) of the chord, defined as the radius of a circle centered in the vacuum
vessel that contains a point tangent to the viewing chord. The tangency radius of a chord is also the minimum radius through which the chord passes. The arrays view from 50.7 cm to 118.6 cm and the width of each view at the mid-plane ranges from 3 to 6 cm. The sensor is taller than it is wide by about 50% so the vertical extent of the view at the mid-plane is nearly 6.5 cm. The views of the arrays are about where they were designed to be.

Channels 15 and 16 view the internal ring. Channels 13–14 view the inner scrape off layer of the plasma, the region where the field lines intersect the internal coil. Channels 1–12 of both arrays view plasma the closed field line region of the plasma at the core. Of course all channels are chord integrated so that all view the open field line region at the outer edge of the plasma.

The calibration shows that the two arrays are misaligned by about half a channel. The average offset is 0.36 channels. The inner channels appear to be better aligned than the outer channels.

4.1.3 Visible Light Intensity Profiles

The photodiode arrays were designed to measure plasma fluctuations, but they also measure DC visible light emission. In this section, the DC behavior of the photodiode arrays is compared to other visible light cameras that have slower time response in order to confirm that the behavior of the diagnostic is consistent with other diagnostics. The visible light intensity profile is then compared to the plasma density profile. This comparison shows that the visible light emission is often burnt out in the core of the plasma.

The visible light emission measured by the photodiode arrays can be compared to the visible light emission measured by a Sony monochrome CCD camera that has a very similar view. In Fig. 4-5, the time-averaged, chord-integrated visible light intensity measured by both photodiode arrays and the Sony camera is shown for a deuterium plasma with 15 kW ECRH and created with floating-coil levitated. The visible light emission measured by the photodiode arrays agrees well with the Sony camera data for plasmas in which a quasi-coherent mode is not observed, but there is less agreement when the quasi-coherent mode is observed. The presence of a coherent mode with toroidal mode number n=0 does not affect the agreement, so the data agree well for cases when the coherent mode is observed,

\footnote{Data are from shots 90313007 and 90313018, which are part of the neutral pressure scan experiment described in § 5.2.1}
Figure 4-5: Comparison of the time-averaged, chord-integrated visible light intensity measured by the photodiode arrays 1 (blue diamonds) and 2 (green triangles) and a monochrome video camera with a similar view (solid line). The measurements agree well in plasmas where the n=1 mode is not observed.

but the quasi-coherent mode is not and for cases when neither the coherent mode, nor the quasi-coherent mode are observed. However, the data do not agree well for the cases where both the quasi-coherent mode and the coherent mode are observed, or for the cases where the quasi-coherent mode is observed, but the coherent mode is not.

When an H-α filter is placed in front of the video camera, the measured light intensity profiles are very similar to those measured without the filter so I infer that most of the light measured by the sony camera is H-α light, specifically D-α because these are deuterium discharges.

Now, I compare the time-averaged, local light emission profile\(^2\) to the time-averaged, local plasma density profile for two plasmas with similar neutral pressures and identical RF heating, Fig. 4-6. One plasma was created in supported configuration and the other in levitated configuration. The supported mode shot has an average neutral pressure of 3.2 μTorr during the two-second time window, while the levitated mode shot has an average neutral pressure of 3.4 μTorr. The levitated shot has both a higher density and a more peaked density profile. The levitated shot also has a larger visible light intensity.

In the levitated shot, the visible light emission burns out in the core of the plasma. This probably occurs because the temperature is high enough that all the neutrals have been ionized. The local light emission profile is flatter than the density profile for the levitated

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\(^2\)This data was collected before the photodiode array was calibrated for position and it is possible that the array position was not replicated exactly for the Dec. run and the March run. However, the array was not moved between the two shots that are being compared.
Figure 4-6: Comparison of plasma density profile (dashed) and visible light emission profile (solid) for the cases where the floating-coil is supported (shot 81218012) and levitated (shot 81217017). Both shots have identical RF heating and similar neutral pressures. The profiles have been reconstructed from time-averaged interferometer and photodiode array data during a time with 15 kW heating.
4.1.4 The relationship between visible light intensity and chord-averaged plasma density

Low-frequency density fluctuations have been observed by the interferometer and by visible light diagnostics. I now consider the assumptions that the visible light emission from the plasma is proportional to the plasma density. For low-density plasmas, I assume that the light intensity measured by the photodiode arrays and the fast cameras is proportional to the plasma density and further that fluctuations of visible light intensity are representative of fluctuations in plasma density. This assumption is made for other RF heated, low-density plasmas [54] and is supported by theory [59]. The visible light emission averaged over all wavelengths for a plasma with effective charge, $Z_{\text{eff}} = 1$,

$$I \propto n_e n_0 f(T_e), \quad (4.1)$$

where $n_e$ is the electron density, $n_0$ is the neutral density and $f(T_e)$ is a weakly varying function of electron temperature. The effective charge is not measured in LDX so I assume a value of one because these are deuterium plasmas. In the last section, I showed that the visible light begins to burn out for $R < 85$ cm where the plasma density is highest, but I will show that these assumptions are reasonable for viewing chords with $R > 85$ cm for cases where $n_e n_0$ is not too high.

The visible light intensity can be compared to the product of the chord-integrated density from the interferometer and the neutral pressure measured by the ion gauge to confirm that the assumption is valid. The visible light intensity increases as the product of the neutral pressure and line-integrated density increases for all three pairs of interferometer and photodiode array chords (Fig. 4-7). The data from the 96 cm chords is fit linearly by

$$I_{\text{vis}} L_3 = 0.034 P_0 n_e L_3 + 0.1821 \quad (4.2)$$

with $R^2 = 0.87$, where $I_{\text{vis}} L_3$ is the visible light intensity measured along chord 3 ($R_\perp = 97\text{cm}$), $P_0$ is the neutral pressure measured by the ion gauges, and $n_0 L_3$ is the chord-averaged plasma density measured by the third chord of the interferometer. For chords
Figure 4-7: Visible light intensity versus the product of the neutral pressure and the line-integrated density for deuterium discharges for deuterium discharges. The data from shots 90312025-043, 90313001-024, and 90722037-048 have been time averaged over half-second intervals when the light intensity, density and neutral pressure were steady. The ECRH configuration is held constant during the half second interval, but the data points were collected under a variety of ECRH configurations.
where the product of neutral pressure and chord-integrated density exceeds \(65 \times 10^{13} \text{ cm}^{-2} \mu \text{Torr}\), the linear relationship no longer holds. In these cases, the temperature is likely high enough that the visible light begins to burn out, as shown in Fig. 4-6.

There is a fair amount of scatter in the data, but the three sets of chords look quite similar. Figure 4-7 includes data from a number of different ECRH configurations and power levels. The varying ECRH power levels and frequency combinations may account for some of the scatter in the data. The neutral pressure is essentially independent of the RF power. Recall that the ECRH configuration and the fueling levels are the two user-controlled parameters in the experiment. The chord-averaged density generally increases with increased ECRH power, but a wide range of chord-averaged density are observed with each ECRH power configuration.

Figure 4-8 shows visible light emission versus the product of chord-averaged density and neutral pressure for 77 cm view with each ECRH configuration as a separate data series. For each configuration, the light intensity increases with \(P_0n_eL_1\). The rolloff above \(P_0n_eL_1 > 50 \times 10^{13} \text{ cm}^{-2} \mu \text{Torr}\) is visible for several different ECRH configurations with higher power levels.

In conclusion, the data is well fit by the model in Eq. 4.1 when the product of the neutral pressure and the chord-averaged density is low enough. The assumption that the visible light emission is proportional to the plasma density is reasonable for chords with \(R > 85 \text{ cm}\) during levitated operation. For the channels inside of 85 cm, the visible light emission begins to burn out, probably because the neutrals cannot penetrate the higher density plasma. Analyses of fluctuations measured by the photodiode array diagnostic should focus on data outside of 86 cm.

### 4.1.5 The assumption of axi-symmetry

Another assumption that I make is that the plasma is axi-symmetric. The visible light diagnostics are the only ones that view the plasma at multiple toroidal and radial locations. Fig. 4-9 shows the time-averaged, line-integrated light intensity measured by each channel of arrays 1 and 2 during shots with 15.5 kW of multi-frequency ECRH and different neutral fueling levels (as in Fig. 4-8). The light emission from the two arrays is very similar, but array 2 often measures a lower light intensity than array 1, especially for chords with minimum radii inside the peak, and when the pressure is higher than the optimum value.
Figure 4-8: Visible light intensity versus the product of the neutral pressure and the line-integrated density for deuterium plasmas with different ECRH configurations. The data from shots 90312025-043, 90313001-024, and 90722037-048 have been time averaged over half-second intervals when the light intensity, density and neutral pressure were steady.
Figure 4-9: The line-integrated light intensity measured by photodiode arrays 1 (solid) and 2 (dashed) for 15.5 kW ECRH and different neutral pressures. The data from shots 90312037, 90313018, 90313005, 90313006, 90313012, 90313011, and 90313009 (listed in order of increasing neutral pressure) were averaged between $8 \, \text{s} < t < 8.5 \, \text{s}$. 
I can also compare cases where the heating configuration was varied while the neutral pressure remained close to the optimum value for that heating configuration (Fig. 4-10). The optimum value of the neutral pressure is the value which maximizes the plasma density. The ECRH configuration has a large affect on both the light intensity levels and the profile shape.

Array 1 views from the Northeast port towards the Northwest port while array 2 views from the Southeast port towards the Northeast port (Fig. 4-1). The gas fueling is puffed in from the Southwest port towards the center of the vessel. The RF heating antennas for 2.45 and 6.4 are located on the Southeast port. Either or both of these may be causing the discrepancy. The deviation at 108 cm is systematic. Channel 04 of array 2, which has a minimum radius of 109 cm, was not working when this data was collected. No data has been plotted for that point.

### 4.2 Fast framing cameras

Two commercial Phantom fast cameras (v.7.1) record a two-dimensional monochrome image of the visible light emission from the plasma. The speed of the data acquisition is adjustable up to 10 kHz and the size of the image is adjustable up to 640x480 pixels. The low light levels produced by the plasma limit the bandwidth of the fast cameras to an effective bandwidth between 2 kHz and 6 kHz, depending on the neutral fueling level for the plasma shot. Shots with higher neutral fueling have larger total light emission. The cameras are used without interference filters because filters would further reduce the signal levels, and thus reduce the bandwidth.

The two cameras are setup in a master/slave configuration so that the data collection is synchronized between the two cameras. The cameras are triggered to start collecting data using an external trigger that is synchronized with the data system clock. After the initial trigger, the cameras use the internal clock from the master camera.

Due to the large image size, these cameras are only set to record for a limited time during the discharge. Typical time windows are 0.25 seconds in duration. In contrast, the one-dimensional photodiode array diagnostic can record the entire plasma shot. The time window is limited both by the amount of available memory to store video and the time it takes to store the data. The storage space could certainly be increased in the future.
Figure 4-10: The line-integrated light intensity measured by photodiode arrays 1 (solid) and 2 (dashed) averaged over half second intervals for different ECRH configurations and optimal neutral pressures.
The fast cameras can be placed on a number of viewports. Typically, the cameras are placed so that they have nearly identical views that are separated azimuthally by \( \pi / 2 \). Primarily horizontal views are available at the outboard mid-plane and primarily vertical views are available at the bottom of the vessel. It is also possible to configure the cameras so that one camera has a horizontal view and the other has a vertical view which overlaps the horizontal view. This configuration has been used to look at fluctuations that are azimuthally symmetric.

The views from the two bottom ports and the two side ports that have been used were calibrated by collecting a video of a ruler placed inside the vacuum vessel.

4.3 Microwave interferometer to measure electron density

A four-channel microwave interferometer measures the core density. An interferometer works by comparing the phase shift of a beam passing through a medium to the phase shift of a beam traversing a similar distance in vacuum. Then the difference in phase shifts can be related to the density of the medium.

The LDX interferometer is a heterodyne system that uses two free running oscillators and operates at 60 GHz [13]. The beam passes through the plasma at the mid-plane where the magnetic field is almost entirely in the vertical direction, so it is generally valid to assume that the beam propagates in O-mode with the wave vector of the beam perpendicular to the magnetic field and the electric field of the beam parallel to the magnetic field. In this configuration, there is a linear relationship between the phase shift of the microwave beam and the line-averaged plasma density [13]. The interferometer chords 1, 2, 3, and 4 have radii of tangency 77 cm, 86 cm, 96 cm and 125 cm, respectively. These are illustrated in Fig. 4-1.

The uncertainty in the phase measurement is approximately 5° for each channel of the interferometer [42]. The measured phase shifts can be as large as 5\( \pi \) so the instrumental uncertainty is a few percent [13]. However, the assumption that the phase is linearly proportional to the line-averaged density breaks down when refraction of the microwave beam is significant [13]. The outermost channel of the interferometer rarely provides valid data. The reason for this may be refraction or it may be that there is something else wrong with that channel. I present data from only the first 3 channels, in most cases.
The radial density profile can be determined from inversion using the data from the four interferometer chords and the density measured by an edge probe. In addition, for plasma density profiles that are decreasing monotonically, the steepness of the density profile can be estimated by comparing the ratio of the line-averaged density measured along two chords.

In addition to identifying trends with the magnitude of the plasma density, I would like to see if the low frequency fluctuations depend on the shape of the density profile, particularly the steepness. To quantify the steepness of the density profile, I define the peakedness ratio as the ratio line-averaged density measured by two chords of the interferometer,

\[ P_{23} = \frac{n_{12}}{n_{13}}, \]

where \( n_{12} \) and \( n_{13} \) are the line-averaged density measured by the second and third channels of the interferometer, respectively. These chords have tangency radii of 86 cm and 96 cm.

The peakedness ratio of a density profile with an equal number of particles confined within each flux tube is calculated from the equilibrium flux contours to be \( P_{23} = 1.5116 \). If the ratio is larger than \( P_{23} \), then the density profile is more peaked than the invariant profile, but if the ratio is lower than \( P_{23} \), then the density profile is less peaked than the invariant profile. The peakedness ratio of a flat density profile \( (n(r) = n_0) \) is one.

The interpretation of the ratio of two interferometer chords as a measure of the density steepness assumes that the density profile is decreasing monotonically with radius. That assumption is only valid for radii that are larger than the radius of the density peak. Under some circumstances, hollow density profiles are observed with a density peak between 77 cm \( \lesssim R \lesssim 86 \) cm (particularly when in discharges with a high level of neutral fueling), although in most cases, the density is maximum at or inside of 77 cm [42]. Figure 4-6 shows an example of the latter density profile. There are no density measurements inside of 77 cm so that portion of the density profile has not been characterized. Recall that the closed field line region extends between 125 cm \( \lesssim R \lesssim 171 \) cm, so the interferometer has good coverage of that region. For this reason, the innermost chord of the interferometer is not a good choice for the peakedness ratio and the second and third chords are used instead.
4.4 Electrostatic probes and the floating probe array

Moveable electrostatic probes are used to diagnose the edge of the plasma near the magnetic separatrix. The probes cannot be inserted much beyond this point or they are damaged by the plasma.

Floating probes measure the floating potential which is related to the plasma potential. In some situations, one can assume that the fluctuations measured by the floating probes are really fluctuations of the plasma potential. Then these signals can be used to measure the fluctuating electric field ($\tilde{E} \sim \nabla \tilde{\Phi}_p$). The fluctuating electric field is also used to compute the quasi-linear diffusion coefficient (§ 6.2).

A moveable floating probe array is used to measure the toroidal structure of edge fluctuations [12]. The array looks like a candelabra and contains 24 cylindrical, tungsten-tipped Langmuir probes [12]. This is the configuration of the probe array that was used when the data included in this thesis was collected.

After the initial installation of the probe array, two floating probes were swapped out for ion-saturation probes. Ion saturation probes are Langmuir probes that are biased with a negative voltage so that electrons are repelled and only ions are collected. The ion saturation current is proportional to density and the square root of temperature, $I_{\text{sat}} \propto n \sqrt{T}$. When temperature fluctuations can be ignored, then the fluctuations in ion saturation current are directly proportional to the density fluctuations. The combined information from the floating probes and ion saturation probes can then be used to calculate the particle fluxes at the edge of the plasma. This is an exciting upgrade for future work.

The probes are evenly spaced in the azimuthal direction covering a span of $\pi/2$ radians, which corresponds to a resolution of 14 cm. Each probe has the same radial position and $z$ position. The $z$ position of the probes is adjustable, which places the probes on different flux surfaces and corresponds to a radial shift when mapped to the mid-plane. The probes have a frequency response of 160 kHz [12].

In addition to the probe array, there are three additional moveable probes on the machine and one fixed location probe. These Langmuir probes have been operated as swept probes, ion saturation probes, and floating probes, depending on the run campaign.
4.5  Mirnov coils measure magnetic fluctuations

To measure magnetic fluctuations, eight Mirnov coils are arrayed toroidally on the vacuum vessel wall at the mid-plane. The coils are equally spaced over 2π radians, so the separation between adjacent coils is π/4.

The voltage measured from the Mirnov coils is proportional to the time rate of change of the fluctuating magnetic field,

\[ V = N A \frac{dB}{dt}, \]  

where \( N \) is the number of turns and \( A \) is the cross-sectional area of the coil. The Mirnov coils were made with 200 turns of 30 AWG magnet wire wrapped around a boron nitride core that is 0.75” in diameter and 1.25” in length. The cross-sectional area is \( A = 3 \text{cm}^2 \). Each coil is surrounded by a 0.01” thick stainless steel shielding to prevent electrical pickup on the leads [28].

Each of the Mirnov coils is connected to dual-stage voltage amplifier with a gain of 1.3 \( \cdot 10^3 \) and a frequency response of 30 kHz. The amplified signals are digitized at 50 kHz. The Mirnov coils are more sensitive to higher frequencies than they are to lower frequencies. The frequency response of the amplified Mirnov coils was measured for the installed system using an electromagnet that was driven by an oscillating voltage and then scanning the oscillation frequency. The response is fairly constant for frequencies between 1 kHz and 30 kHz, but decreases for frequencies outside that range.

4.6  Fluctuation Analysis

The work presented in Chapters 5 and 6 employs a number of mathematical methods and data analysis tools. The details of these methods are provided in this section.

4.6.1  Short-time Fourier transform

The frequency content of a signal often contains important information about fluctuations. A Short-Term Fourier Transform (STFT) is used to assess how the frequency content of a signal evolves in time. The discrete version of the transform is analogous to the continuous
form, which is defined

\[ STFT\{x(t)\} \equiv X(\tau, \omega) = \int_{-\infty}^{\infty} x(t)w(t - \tau)e^{-i\omega t}dt, \]  

(4.5)

where \(X(\tau, \omega)\). The data \(x(t)\) is partitioned in many shorter, possibly overlapping time segments by taking the product of the data and a set of window functions \(w(t - \tau)\). The window functions are non-zero during one time segment only. The results presented in this chapter use a Hanning window,

\[ w(n) = 0.5 \left(1 - \cos \left(\frac{2\pi n}{N}\right)\right). \]  

(4.6)

and a time overlap that is equal to half the time window.

The results are displayed as Time–Frequency Domain plots or “spectrograms”, \(|X(\tau, \omega)|^2\), with time on the horizontal axis and frequency on the vertical axis. Fluctuation power is represented by a linear color scale. The spectrogram can be time averaged over a time range of interest \(\tau_k=1,2,...,K\)

\[ <X(\omega)> = \frac{1}{K} \sum_{k=1}^{K} |X_{\tau_k, \omega}|^2. \]  

(4.7)

Averaging reduces noise in the frequency domain.

The STFT can also be used to compute the time evolution of the frequency spectrum of the cross-correlation and the cross-phase of a signal. The correlation function for two arbitrary functions \(f(t)\) and \(g(t)\) is

\[ (f \ast g)(t) = \int_{-\infty}^{\infty} f^*(\tau)g(t + \tau)d\tau. \]  

(4.8)

Fourier transforming both sides gives the cross-power

\[ H(\omega) = \mathcal{F}\{f \ast g\} = (\mathcal{F}\{f\})^* \cdot \mathcal{F}\{g\}. \]  

(4.9)

Then the cross-correlation \(C_{fg}\) is the scaled value of the cross-power

\[ C_{fg} = \frac{(\mathcal{F}\{f\})^* \cdot \mathcal{F}\{g\}}{|\mathcal{F}\{f\}| \cdot |\mathcal{F}\{g\}|}. \]  

(4.10)

The average cross-correlation is determined by computing the average cross-power and
dividing by the square root of the product of the average power spectra.

4.6.2 Two-point method

The two-point method provides an estimate of the local spectral density of fluctuations as a function of both frequency and wavenumber $S(k, \omega)$ using the data collected from two spatially separated probes [60]. The local frequency spectrum is calculated by taking a fast Fourier transform of the data from each probe

$$S(x, \omega) = \mathcal{F}\{\phi(x, t)\}, \quad (4.11)$$

where $\phi(x, t)$ is the time trace of the data measured by a probe at location $x$. The cross-spectrum is then computed by taking convolving the frequency spectra from the two probes according to Eq. 4.9

$$H(\Delta x, \omega) = \mathcal{F}\{\phi(x_1, t) \star \phi(x_2, t)\} = C(\omega) + iQ(\omega). \quad (4.12)$$

The cross-spectrum is imaginary and the cross-phase is the phase of the cross-spectrum. The local wavenumber as a function of frequency is calculated by dividing the cross-phase by the distance that separates the two probes $\Delta x$.

$$K(\omega) = \frac{1}{\Delta x} \tan^{-1} \left[ \frac{Q(\omega)}{C(\omega)} \right] \quad (4.13)$$

The wavenumber is restricted to values between $-\pi/\Delta x$, and $\pi/\Delta x$. Finally, the local spectral density $S_l(K, \omega)$ is computed summing the average local frequency spectrum at a fixed frequency for the points $j$ that have a local wavelength in a given range

$$S_l(K, \omega) = \frac{1}{M} \sum_{j=1}^{M} I_{[0,\Delta K]}(K - K^{(j)}(\omega)) \cdot \frac{1}{2} [S^{(j)}(x_1, \omega) + S^{(j)}(x_2, \omega)] \quad (4.14)$$

where the superscripts indicate individual elements of the discrete data set that is being summed and $I$ is an indicator function defined as

$$I_{[0,\Delta K]}(x) = \begin{cases} 1, & 0 \leq x \Delta K \\ 0, & \text{elsewhere} \end{cases} \quad (4.15)$$
4.6.3 Probability distribution functions and higher order moments

The probability distribution function (PDF) of a diagnostic gives the probability that the diagnostic will measure a particular value. The probability distribution function is generated by creating a histogram of the data measured by a diagnostic during the time period of interest. The probability distribution function of a fluctuating quantity $X$ is $P(X)$. It is also possible to use a moving time window and generate a time series of probability distribution function for a sufficiently long data series.

In order to compare multiple probability distribution functions from different situations, it is useful to normalize the probability distribution function by the standard deviation. I define the function

$$F_X(Y) = \sigma_X P(X),$$

where $\langle X \rangle$ and $\sigma(X)$ are the mean and standard deviation of $X$, respectively and $Y = (X - \langle X \rangle)/\sigma_X$ [61].

If the diagnostic is measuring random fluctuations, then the probability distribution function will approach a normal distribution.

The higher order moments skewness and kurtosis are a measure of how the probability distribution function deviates from a normal distribution. Skewness is a measure of whether the data is to the right of the mean (positive) or to the left of the mean (negative). Kurtosis is a measure of whether the data is steeper than the normal distribution (positive) or shallower than the normal distribution (negative). In this analysis, I have followed the convention of setting the kurtosis of a normal distribution to zero.

Probability distribution functions and higher order moments are used to analyze turbulent fluctuations. Non-zero higher order moments can indicate that there is intermittency in the turbulence.

4.6.4 Biorthogonal decomposition

Biorthogonal decomposition (BD) is a singular value decomposition (SVD) technique that decomposes data from multichannel diagnostics into a set of modes that are orthogonal in space and time [62]. This technique can be applied to local measurements and chord average measurements, and has been generalized for complex and vectorial measurements. Biorthogonal decomposition was first used to investigate plasma fluctuations by T. Dudok.
de Wit [63]. I adopt his notation in this paper.

Consider an $N \times M$ array $Y$ containing the data at $M$ spatial positions for $N$ times. The array can be expanded into discrete orthogonal modes by transforming the data into the frame where the cross-correlation matrix is diagonalized.

$$ (Y)_{ij} = \sum_{k=1}^{K} A_k \phi_k(x_j) \psi_k(t_i) $$

(4.17)

where $K$ is the smaller of $N$ and $M$, $A_k$ are the weights, and $\phi_k$ and $\psi_k$ are orthogonal modes such that,

$$ \sum_{i=1}^{N} \psi_k(t_i) \psi_l(t_i) = \sum_{j=1}^{M} \phi_k(x_j) \phi_l(x_j) = \delta_{kl}. $$

(4.18)

Convention dictates that the right hand modes $\phi_k$ are called Topos and the left hand modes $\psi_k$ are called Chronos. The Topos and Chronos are determined entirely by the data set. This is unlike a Fourier transform where the basis functions are prescribed.

This technique can be very useful for improving signal to noise ratios. After signal has been deconstructed, it is then reconstructed using a truncated series,

$$ (\tilde{Y}_L)_{ij} = \sum_{k=1}^{L} A_k \phi_k(x_j) \psi_k(t_i) $$

(4.19)

for any $L < K$ where $K$ is the smaller of $M$ and $N$. The criteria for selecting the number of dominate modes to use in the truncated series are discussed in detail by de Wit [64]. Biorthogonal decomposition has been successfully used in the past as a noise reduction technique to look at broadband turbulent structures in plasmas [65]. This method has also been used to study quasi-coherent turbulent fluctuations in a supported laboratory dipole experiment [33]. Once the data has been reconstructed in a truncated series, it is still possible to perform tomographic reconstructions, Fourier transforms and other analysis techniques.

### 4.6.5 Chordal Abel inversion

The photodiode array measurements and other visible camera measurements are integrated along the line of sight of the diagnostic. In some cases, local information can be obtained by inverting the chord-integrated data. In cases where inversion is not valid, a synthetic...
diagnostic has been designed to simulate the data that would be collected by the chord-integrated diagnostics from a known plasma model. The next two sections discuss the inversion techniques and the synthetic diagnostic and the circumstances during which each are applicable.

If the assumption of axial symmetry hold, then an Abel inversion can be used to reconstruct the local profile $f(r)$ from the line integrated data $F(x)$,

$$f(r) = -\frac{1}{\pi} \int_{r}^{\infty} \frac{dF}{dx} \frac{dx}{\sqrt{x^2 - r^2}}, \quad (4.20)$$

where $x$ is the location of the viewing chord. The Abel inversion is defined for parallel chords and the viewing chords for the photodiode array and other cameras on LDX are not parallel. However, using the axial symmetry assumption, it can be shown that the actual geometry of the view chords is equivalent to the case of parallel chords with $x = r_\perp$. The derivative term in the Abel inversion enhances noise, so it's helpful to have as many data points as possible. For this reason, I invert the Sony camera data rather than the photodiode array data for the local emissivity profile.

Inversion techniques can be used to reconstruct the density and visible light emission profiles. The time-average density profile is estimated with an Abel inversion using the four chords of the interferometer and the edge density measured by a Langmuir probe. The inversion assumes that the density is axisymmetric, and that the plasma density is piecewise continuous and varies linearly within the four annular regions defined by the chords of the interferometer [43]. The time-average light emission profiles are estimated using Abel inversion of the camera data.

The fluctuations that the photodiode array diagnostic measures are also chord integrated. H. Weisen [66] discussed the problem of obtaining local fluctuation information from a chord integrated diagnostic with parallel chords. If the fluctuations are both homogenous and isotropic, then the local correlations spectrum and the correlation lengths can be calculated from the measured correlations. An Abel inversion of the frequency spectrum $S(x, f)$ gives,

$$S(x, f) = S(r, f)l_z(r, f)L \quad (4.21)$$
where

\[ l_z(f) = \frac{\int \langle \hat{n}^*(x, z, f)\hat{n}(x, z + \Delta z, f) d\Delta z \rangle}{\langle |\hat{n}^*(x, z, f)|^2 \rangle} \]  

(4.22)

is the local correlation length defined as the variance along the line of sight and \( L \) is the length of the fluctuating plasma. For non-homogenous fluctuations that are isotropic and have known spatial symmetries, the local variance and the correlation length can only be treated separately if an independent measurement of the local correlation length is available.

### 4.6.6 Synthetic diagnostic for the photodiode array

I am interested in the behavior of the plasma locally, but the data from the diagnostics that view the core of the plasma are chord integrated. A simple Abel inversion is not always sufficient to reconstruct the local fluctuation behavior in the plasma (see § 4.6.5), so a synthetic diagnostic has been developed to simulate the photodiode array signals for a user supplied fluctuation model. The model results can be compared to the actual data to see which local fluctuation models are consistent with the data. Although the results are not unique, this technique is useful to gain a better understanding of the plasma behavior.

The synthetic diagnostic uses a three dimensional grid and incorporates the equations for the viewing chords of each of the thirty-two photodiode channels as specified in Eq. C.9. The signal value at each grid point \((r, \theta, z, t)\) is computed according to the user specified model and the data are integrated along each chord for each time point. The diagnostic views aren’t really individual chords, but rather cones. If the variation in the signal doesn’t vary significantly over the cone, as is the case for fluctuations with low \( k \) numbers that I am considering, then the signal can be approximately by the chord integral multiplied by the étendue of the system. The étendue is approximately the same for each channel so this just acts as a multiplier.

The synthetic diagnostic requires a model of the fluctuations. I’ve chosen a fairly simple model to investigate the idea that the quasi-coherent mode may be radially localized and in some cases, may consist of a set of a radially localized modes with different frequencies. The model is a set of functions that are zero-valued everywhere except within an annular region defined by a minimum and maximum radii. For these simulations I used six, non-overlapping annular regions that span from the edge of the floating-coil (60 cm) to the edge of the vacuum vessel (250 cm). The minimum and maximum radii are defined such
that either i) each annulus has an equal radial extent \((r_{\text{max}} - r_{\text{min}} = \text{constant})\) or ii) each annulus covers a region of equal flux \((\psi(r_{\text{max}}) - \psi(r_{\text{min}}) = \text{constant})\). The fluctuations within each annulus are sinusoidal varying in time with frequency \(\omega\) and in space in the azimuthal mode number \(n\),

\[
\tilde{f}_j(t, \theta) = A_j \sin (\omega_j t - n_j \theta),
\]

(4.23)

where \(j\) indexes the annulus and each annular region can have a distinct amplitude, frequency and toroidal mode number.

An example of the results from the synthetic diagnostic using this model is shown in Fig. 4-11. For this case, the six regions are determined so that each covers an equal area of flux. The toroidal mode number is one for all regions, but the amplitude and frequency vary such that both are increasing as the radius increases. The exact model parameters are given in table 4-11(b). These results suggest that a model with two frequencies and two spatially localized regions may be able to reproduce the data from shot 90312028.
Figure 4-11: Example of results from the synthetic diagnostic using six annular regions of constant flux.
Chapter 5

Low frequency fluctuations of the bulk plasma

5.1 Introduction

Plasmas created in LDX exhibit a lot of low frequency fluctuation activity. Large amplitude, broadband fluctuations of ion saturation current were observed on some of the first discharges created in LDX. The diagnostic set has improved so better measurements of these low frequency fluctuations are available. This chapter will discuss the observations of low frequency fluctuations using the diagnostics and methods described in the previous chapter.

5.1.1 Three classes of fluctuations

The fluctuations have been divided into three categories: quasi-coherent modes, coherent modes, and broadband fluctuations. The fluctuations that have quality factor $Q = f / \Delta f$ between one and ten are called quasi-coherent modes because they are coherent for only a few periods. Fluctuations that have $Q$ values on the order of 100 are called coherent modes because they are coherent for many periods. The broadband fluctuations typically have $Q < 1$.

The quasi-coherent fluctuations typically have frequencies on the order of 1 kHz and low, non-zero toroidal mode numbers. The coherent fluctuations typically have frequencies on the order of 100 Hz and toroidal mode numbers of zero. Examples of each of the three
classes of fluctuations will be shown.

The quasi-coherent fluctuation is observed on many of the diagnostics including the interferometer, visible light diagnostics, edge Langmuir probes and edge Mirnov coils. The coherence times are typically a few periods \((1 < Q < 10)\) and the frequencies are typically on the order of 1 kHz. Fig. 5-1(a) shows an example of a quasi-coherent fluctuations observed on a floating potential probe located near the magnetic separatrix for shot 90312041. The RF heating power is increased in steps at \(t = 0\) sec, \(t = 2\) sec and \(t = 4\) sec. Quasi-coherent fluctuations are present until the heating power is turned off at \(t = 10\) sec, then the frequency slowly decreases to zero. The frequencies are steady between 5 and 10 seconds at 750 Hz, 1.5 kHz and 3 kHz.

The toroidal (or azimuthal) mode number \(n\) can be measured independently by the floating probe array and by the Mirnov coils and the results from two diagnostics agree. A two-point plot of the data (Fig. 5-1(b)) shows the fluctuations at the frequencies 750 Hz, 1.5 kHz and 3 kHz correspond to toroidal mode numbers of 1, 2, and 4, respectively.

Coherent fluctuations are observed by the interferometer and the visible light diagnostics under certain conditions. The coherence length of the fluctuations is typically on the order of 100 periods \((Q \sim 100)\) and the frequency is on the order of 100 Hz. Fig. 5-2(a) shows an example of the coherent mode measured by the interferometer during shot 90313014. The ECRH is turned on in steps, as it was in the previous example. A coherent fluctuation appears at four-seconds, when the 10.5 GHz power is turned on. The frequency of the fluctuation is steady between six and nine seconds.

The toroidal mode number of the coherent fluctuation is nearly zero. The interferometer channels all originate at the same toroidal location, so they cannot be used to measure toroidal mode number, but the two photodiode arrays are located at different toroidal locations and they also observe the coherent fluctuation. Fig. 5-2(b) shows a two-point plot from two toroidally separated photodiode array channels. The phase difference between the two signals is nearly zero at the frequency of the coherent fluctuation.

Broadband, turbulent fluctuations are almost always observed. An example of the spectrum of these fluctuations in shown in Fig. 5-3. The fluctuations are fairly constant for frequencies up to about 1 kHz and then have a power law dependance above 1 kHz. A power law dependance for low frequency turbulence is typical of toroidally confined plasmas [67], but in this spectrum two different decay rates are visible (neither of which can
(a) Spectrogram of floating potential fluctuations show steady state quasi-coherent fluctuations at 750 Hz, 1500 Hz, and 2250 Hz after the 10.5 GHz heating source is turned on at four seconds. The average power spectrum between six and nine seconds is also shown.

(b) Local fluctuation spectrum as a function of frequency and toroidal mode number using data calculated from adjacent probes using the two-point method during the time period $6 < t < 9$ s. Mode numbers 1, 2, 3 are visible.

Figure 5-1: An example of quasi-coherent fluctuations measured during shot 90312041 by the floating probe array located at the edge of the plasma near the magnetic separatrix.
(a) Spectrogram of the line-averaged plasma density measured by the 86 cm channel of the interferometer shows a very low frequency, coherent fluctuation at $1 \cdot 10^2$ Hz.

(b) Two-point plot of the visible light intensity measured by two azimuthally separated photodiode array channels with $r_1 116$cm show the coherent fluctuation has a toroidal mode number of zero.

Figure 5-2: An example of a coherent, $n=0$ fluctuation observed during shot 90313014.
be attributed to the amplifier response). These two decay rates typical of two dimensional turbulence because of the dual cascade of energy to lower wave numbers and enstrophy to higher wave numbers. These are similar to spectra observed in the collisionless terrella experiment (CTX), although in CTX the floating potential fluctuations went as $f^{-5}$, while the edge density fluctuations went as $f^{-3}$ [68], but CTX is confined by a supported dipole magnet while these measurements were taken with a levitated dipole magnet. Turbulence theory predicts that the energy of fluctuations as a function of the wave number $E(k)$ will have a power law dependence with two slopes. The shallower slope goes like $E(k) \sim k^{-\frac{5}{3}}$ and the steeper slope goes like $E(k) \sim k^{-3}$. Measurements in CTX found a linear relationship between $f$ and $k$ for interchange turbulence in a dipole [68]. These broadband fluctuations are generally observed in all plasmas, although I have observed that the fluctuations are suppressed or masked when large puffs of gas are injected into the plasma.
Figure 5-4: This one-dimensional plot shows the neutral pressures during which the n=0, coherent modes and n=1, quasi-coherent modes are observed during deuterium fueled discharges with 15 kW of microwave heating. An additional column notes the values at which both modes were observed. The neutral pressure scan includes data from shots 90312025-044 and 90313001-023.

5.1.2 Conditions during which each class of fluctuations is observed

While almost all shots exhibit broadband turbulent fluctuations, the coherent and quasi-coherent fluctuations only appear under certain plasma conditions. Fig. 5-4 shows the neutral pressures at which the different classes of modes are observed in deuterium plasmas with 15 kW of microwave heating power.

The n=1, quasi-coherent modes are observed only when the neutral pressure is low. The n=0, coherent modes are observed over a wider range of neutral pressures from moderately low to moderate. At moderately low neutral pressure, both the n=0 and n=1 modes can be observed concurrently. For discharges with high neutral pressure, only broadband fluctuations are observed.

Recall that the neutral pressure, the line-averaged plasma density and the plasma diamagnetism are not independent, so it is generally sufficient to plot relationships against only one of these parameters and the behavior with respect to the others with follow the established trend. The data in this thesis are presented as a function of the neutral pressure because that is the parameter that we have external control over. For this heating configuration, the line-averaged plasma density increases with increasing neutral pressure until an optimum density is reached in the range of 5–7 μTorr (Fig. 2-3). Low density discharges have high diamagnetism and are created with low neutral pressure. Plasmas have densities near the optimum values and moderate diamagnetism when the fueling levels is moderate, while plasmas created with high neutral pressure have the lowest diamagnetism.
and densities which are slightly below the optimum value.

The diamagnetism is slightly better for predicting which modes will be observed in the plasma because the range in which both the quasi-coherent and coherent modes are observed better separated a function of the diamagnetism. Quasi-coherent modes are observed when the diamagnetism is high $\Phi_d > 2.5$ mWb. Coherent modes are observed when the diamagnetism is moderate $2 \leq \Phi_d \leq 3.5$ mWb. Both the coherent and quasi-coherent mode are observed in discharges with moderately-high diamagnetism $2.5 \leq \Phi_d \leq 3.5$ mWb.

The observations of the properties of each of these three classes of fluctuations are discussed in the following sections.

5.2 Properties of the quasi-coherent fluctuations

The detailed behavior of the quasi-coherent mode is explored in this section. A neutral pressure scan experiment was performed to study when each type of fluctuation was observed (Fig. 5-4). In addition, this experiment is used to study dependance of quasi-coherent mode frequency on neutral pressure (as well as plasma density and stored energy). The statistical properties of the fluctuating quantities are discuss as well. A gas puff experiment was performed to study how the dynamics of how the fluctuations respond to changing neutral pressure. Finally, the radial variation of the quasi-coherent mode frequency is presented.

5.2.1 Neutral pressure scan experiment

In order to investigate how the properties of the low frequency fluctuations depend on plasma parameters, I use a set of shots in which the neutral fueling level was scanned. Varying the neutral pressure has the additional effect of scanning the plasma density and the total stored energy in the plasma. The timing of the neutral fueling, RF heating power, and the timing of the RF heating are identical for these discharges. Only the fueling levels during the shots were varied. The shots were collected during two lifts of the floating-coil on subsequent days and include shots 90312025 through 90312044 and 90313001 through 90313024. I begin by discussing a typical shot from the data set and then present the results of the experiment.

The time evolution of a typical shot from this data set is shown in Fig. 5-5. The first panel shows the total measured forward power from the RF sources in kW. The RF sources
Figure 5-5: Time traces of total ECRH power, neutral gas pressure, diamagnetic flux and line averaged density for three channels of the interferometer minimum radii, 77 cm (black solid), 86 cm (blue dashed), and 96 cm (green dotted) for shot 90313005, a representative shot from the neutral pressure scan experiment. The plasma is in a steady state from time $t \approx 6$ s until the ECRH is turned off.
are turned on sequentially: first, the 2.45 GHz source turns on at time $t = 0$; then the 6.4 GHz source turns on at time $t = 2$ s; lastly the 10.5 GHz source is turned on $t = 4$ s. The RF sources are turned off simultaneously at time $t = 10$ s. Each source is operated at full power.

The second panel shows the neutral pressure measured by the vessel ion gauge. The working gas for these discharges is deuterium so the vessel ion gauge measurements have been calibrated accordingly. A puff of gas is injected before the heating turns on at $t = -0.3$ s. This level is the same for each of the shots in the data set. During the time when the RF heating is on, the plasma is fueled by much smaller puffs of gas injected at a frequency of 100 Hz. In order to keep the neutral pressure as constant as possible, two fueling levels are defined for each shot, the first level is used during times $0 < t < 4$ s and the second level, which is typically lower, is used during times $4$ s $< t < 10$ s. These two fueling levels were varied from shot to shot.

The mid-plane flux, which is shown in the third panel, is roughly proportional plasma diamagnetism. The flux increases with increasing heating power. The plasma takes about six-seconds to reach a steady state.

The fourth panel shows the chord-averaged plasma density for three channels of the interferometer. For each channel, the chord-averaged density increases with increased heating power. The ratio of the chord-averaged density from two channels is indicative of the steepness of the density profile. The ratios also change with increased heating power. Both the chord-averaged densities and their ratios reach a steady state about a half a second after the last RF heating source is turned on.

The fluctuation spectrum is also fairly steady during the time period between five and ten seconds so there ample data for statistically analysis. This was shown in the examples in the introduction, figs. 5-1(a) and 5-2(a). The data collected between 6 and 9 seconds is used to compute the average fluctuation behavior. During this time all three sources of ECRH are on at full power.

I focus on the fluctuations observed on the 100 cm channel of photodiode array 1. The toroidal mode number can be estimated by examining the cross-correlation of the 100 cm channels of photodiode arrays 1 and 2. The data I present is all for fluctuations with toroidal mode number $n = 1$.

The average fluctuation spectrum of the visible light is computed from Eq. 4.7 using
10,000 frequency bins and 200 time windows. The 60 Hz and 360 Hz noise from the power supply of the 10.5 GHz source are removed using a notch filter. Then a six-parameter gaussian fit to the frequency peak is performed to determine the mode frequency, amplitude, and frequency spread. These can then be compared to average plasma parameters during the same time period.

Quasi-coherent fluctuations are observed when the neutral fueling pressure is low ($P_0 < 5\mu$ Torr) and the plasma diamagnetism is high ($\Phi_d > 2.5$ mWb). Furthermore, when the neutral pressure is at the high end of the range and the diamagnetism is at the low end of the range ($2.5 < \Phi_d < 3.4$ mWb), a coherent fluctuation at lower frequency is observed in addition to the quasi-coherent fluctuation (Fig. 5-4).

The frequency of the quasi-coherent mode decreases with increasing neutral pressure and line-averaged density and increases with increasing diamagnetism (Fig. 5-6(a)). The frequency is often dramatically reduced in discharges where the quasi-coherent mode is observed concurrently with the coherent $n = 0$ mode, particularly for plasmas with neutral pressure larger than 3.8 $\mu$ Torr (corresponding to $\Phi_d < 3.0$ mWb).

For the shots during which only the quasi-coherent mode is observed, the mode frequency increases as the plasma density profile peakedness increases (Fig. 5-6(b)). The ratio of the line-averaged density measured by interferometer chords 2 and 3 with minimum radii of 86 cm and 96 cm, respectively, were used to estimate the peakedness of the density profile $P_{23}$. Many of the discharges have peakedness which is significantly higher than the value for an invariant profile $P_{23} = 1.51$. The implications of this will be discussed in the next chapter. The peakedness is close to invariant value for all of the discharges in which both the $n=0$ and $n=1$ modes are observed, regardless of the frequency of the quasi-coherent mode.

Both the mode amplitude and the spectral width of the mode decrease as the neutral pressure increases when both data series are considered (figs. 5-6(c) and 5-6(d)). The trends are much less clear if I consider only the cases in which only quasi-coherent fluctuations are observed separately from the cases in which both coherent and quasi-coherent fluctuations are observed.

5.2.2 Statistical properties of edge floating potential

I now consider the statistical properties of the fluctuating quantities by looking at the probability distribution functions of edge floating potential from shots in the neutral pressure
Figure 5-6: Results of the neutral pressure scan experiment. The frequency, amplitude and standard deviation of the quasi-coherent n=1 mode are shown for discharges during which only the n=1 mode was observed (solid diamonds) and discharges during which both the n=0 and n=1 modes were observed concurrently (open diamonds). The data from shots 90312025-044 and 90313001-023 are time-averaged between $5 \text{s} < t < 9 \text{s}$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.
scan experiment. The probe measurements are used because they are local measurements, rather than chord integrated like the density and visible light measurements so the results are easier to interpret. Comparing the probability distribution functions to Gaussian distributions can provide insights about how random the fluctuations are. One reason why probability distribution functions sometimes exhibit non-Gaussian characteristics is intermittency of fluctuations.

The probability distribution functions of the floating potential measured near the magnetic separatrix show strong, non-Gaussian properties for many shots during which the quasi-coherent fluctuations are observed (Fig. 5-7). This distribution is skewed slightly negative, meaning that the values are more likely to be lower than the mean value than higher. The probability distribution functions have negative skewness for most of the discharges in which a quasi-coherent fluctuation was observed. The distribution in this example is significantly less peaked than the Gaussian distribution, but this is not always the case.

Figure 5-7: Normalized probability distribution function of floating potential near the magnetic separatrix (solid) for shot 90313031, which has a quasi-coherent fluctuation. A Gaussian distribution with the same mean and standard deviation is shown for comparison (dashed).
5.2.3 Gas puff experiment

Thus far, the time-averaged properties of the quasi-coherent fluctuations have been presented. The gas puff experiment was designed to investigate the temporal evolution of the fluctuations when the neutral pressure is varied during a single shot.

The time traces of key plasma parameters for the gas puff experiment are shown in Fig. 5-8(a). During the first two-seconds of the shot, the RF power is turned on in steps until the combined heating power of 15.5 kW is reached using sources at three frequencies.

A quasi-coherent mode with frequency of 1.75 kHz appears within 20 ms after the 10.5 GHz power is turned on (5-8(b)). It is difficult to estimate this timescale with great precision because there is a trade-off between time resolution and frequency resolution when using the STFT. During this time the plasma density and the stored energy are increasing on a time scale of about 200 ms. The density profile, which was relatively flat before the 10.5 GHz source turned on, now becomes peaked.

The plasma takes about two seconds to reach a steady state. During this time the $P_{23}$ continues to increase to a supercritical value and then decreases to reach a steady state value that is close to the value for an invariant density profile. The plasma stored energy rises slowly (compared to the initial increase) as the density adjusts. The quasi-coherent mode frequency and frequency remain fairly steady during this time.

During the steady state period between four and six-seconds, the average neutral fueling pressure 2.9 $\mu$Torr, the average diamagnetic flux measured by flux loop 5 is 4.3 mWb and $P_{23}$ is very close to the invariant value. All of these values are consistent with the steady state behavior in the neutral pressure scan experiment.

A large gas puff is injected between 7 and 8 seconds. During the time the neutral pressure rises steadily. The diamagnetic flux drop from 4.3 mWb to 2.8 mWb, and the line-averaged electron density increases by a factor of two on all three channels of the interferometer, while $P_{23}$ remains steady. Fluctuations are suppressed. The quasi-coherent mode is no longer observed on either the photodiodes or the probe array and the broadband fluctuations are also suppressed. This is particularly apparent on the floating potential spectrograph (Fig. 5-8(c)). A coherent mode appears is briefly observed when the gas puff begins, but it is also suppressed as the neutral pressure continues to rise\textsuperscript{1}.

\textsuperscript{1} The 60 Hz and 360 Hz noise from the 10.5 GHz power supply have not been filtered out. This noise is visible 7 and 9 seconds.
Figure 5-8: A large puff of D2 gas is injected into a D2 plasma between $t = 7$ and 8 seconds during shot 90313045.
After the gas puff, the neutral pressure decreases and levels off to a 3.4 \( \mu \text{Torr} \). The flux increases throughout the rest of the shot from a 2.7 mWb at the end of the gas puff to 3.7 mWb when the heating sources are turned off. The line averaged density also decreases and the density profile appears to undergo a rearrangement between 8 and 14 seconds, flattening, then steepening, then flattening and steepening again. The plasma does not reach a steady state before the ECRH is turned off at 14 seconds.

The visible light shows a burst of broadband fluctuation activity at \( t = 8 \) seconds and a strong coherent mode appears. The frequency of the coherent mode increases from 80 Hz at \( t = 8 \) seconds to 200 Hz at \( t = 12 \) seconds as the density and neutral pressure decrease. This behavior is consistent with the steady states from the neutral pressure scan, which are presented in the next section.

A quasi-coherent mode appears around \( t = 9 \) seconds on the floating probe. A quasi-coherent also become visible on the photodiode at this time. The mode increases in frequency as the neutral pressure and density decrease, but then appears to level off to a steady value when the density levels out at around \( t = 12 \) seconds. The results from the neutral pressure scan experiment show a jump in the quasi-coherent frequency when neutral pressure is in the range of 3.8 \( \mu \text{Torr} \) and the neutral pressure is in this range when the quasi-coherent mode frequency is increasing steadily.

### 5.2.4 Radial variation of the quasi-coherent mode frequency

The quasi-coherent mode is observed on both the probe array and the Mirnov coils, both of which are toroidally spaced.

The frequency of the quasi-coherent mode is often the same on all channels of the spatial diagnostics and the edge probes, as is the case in shot 90312025. Sometimes, the frequency of the quasi-coherent mode varies radially over the plasma. Cases of radial variation have been observed during both suspended operation and levitated operation.

I compare two shots from the data set above, 90312025 and 90312028. These shots have identical heating power and timing and identical neutral fueling. The time traces are important plasma parameters are shown in Fig. 5-9. Shot 90312025 has slightly higher neutral pressure and slightly lower diamagnetism. The line-averaged densities measured by the innermost chord of the interferometer very similar, but the line-averaged densities measured by channels 2 and 3 are much lower for shot 90312028 than for shot 90312025.
Shot 90312028 also has a much higher value of $P_{23}$ than shot 90312025, although both shots have values of $P_{23}$ that are super-critical.

Radial variation in the quasi-coherent mode frequency is observed in 90312028, but not in shot 90312025. The average fluctuation spectra on each channel of the photodiode arrays for the two shots are shown in Fig. 5-10. These scans were created using 4 seconds of data collected between 5 and 9 seconds, split into 195 time bins and 2048 frequency bins. The data is presented as the line-integrated, average fluctuation spectra versus the tangency radius of the chord. The data have not been inverted.

The time-frequency domain plots of line-averaged density for shot 90312028 show a similar fluctuation spectrum to the photodiode arrays qualitatively. The inner channels show fluctuation spectra that peak around 1.75 kHz (the 77 cm and 86 cm chords of the interferometer) and the outer channels show fluctuations that peak around 1 kHz (the 96 cm chord). The photodiode array channels with tangency radii of 77 cm and 87 cm have fluctuation spectra that peak near 1.75 kHz. For outer photodiode array channels, with tangency radii between 108 and 120 cm, the fluctuations peak at around 1 kHz. So both diagnostics are seeing a higher frequency mode that occurs further into the plasma.

This 1.75 kHz mode observed on the photodiode arrays and the interferometer for shot 90312028 is not present at the plasma edge. Electrostatic probes provide local measurements of the fluctuations at the plasma edge. The spectra of the edge probes show a peak at 1 kHz and then a second peak with a wider frequency spread and about half the power at 2.5 kHz.

In addition to the Fourier spectra of the signals, I can also examine the cross-power and the cross-correlations of the signals. The data between arrays can be correlated for each channel of the two arrays, so array 1, channel 1 is cross-correlated with array 2, channel 1 and so forth. This provides information about the toroidal cross-correlation. Or the channels of an individual array can be correlated against a reference channel within the array. This may provide information about the radial structure of the mode. For both shots 90312025 and 90312028, the mode is well correlated between the two arrays (Fig. 5-11).

In Fig. 5-12(a), the cross-correlation between each channel of photodiode array 2 and the outermost channel of array 2. The x-correlation is the average value during the time.
Figure 5-9: A comparison of the time traces of total ECRH power, neutral pressure, diamagnetism, line-averaged density, and density profile peakedness for shots 90312025 (red solid) and 90312028 (black dot-dash). The three interferometer channels have minimum radii of 77 cm (solid line), 86 cm (dotted line) and 96 cm (dashed line).
5-10: The frequency of the quasi-coherent mode varies with radius in shot 90312028, but not in shot 90312025. Chord-integrated fluctuation spectrum measured by each of the photodiode arrays for times between 5 and 9 seconds during shots 90312025 and 90312028. Photodiode array channels are indicated by the minimum radius of the viewing chord. The fluctuation power is represented by a linear color scale.
Figure 5-11: Average toroidal cross-correlation between each channel of photodiode arrays 1 and 2 for shots 90312025 and 90312028 also show that the quasi-coherent mode frequency varies radially during shot 90312028. The cross-correlation data has been time-averaged between 5 and 9 seconds.

period from five to nine-seconds. There is a strong correlation at 1 kHz on channels on tangency radii as low as 80 cm. This suggests that the 1 kHz mode is radially localized to the edge of the plasma.

In Fig. 5-12(b), the cross-correlation between each channel of photodiode array 2 and channel 8 with minimum radius of 93 cm is shown. The x-correlation is the average value during the time period from five to nine-seconds. A strong correlation is apparent at both 1 kHz and 1.75 kHz. This suggests that there is a second quasi-coherent mode with frequency 1.75 kHz radially localized in the closed field line region of the plasma.

The probability distribution functions of floating potential measured near the separatrix are similar for both shots (Fig. 5-13). Although these probability distribution functions are normalized, the mean and variance of the signals have similar magnitudes for these two shots. The skewness and the kurtosis are negative for both shots. Although the radial structure of the quasi-coherent mode in the core of the plasma is very different, the behavior of the plasma edge appears to be similar for both discharges.

A biorthogonal decomposition of the photodiode array data for the two shots shows that shot 90312028 has additional radial structure that is not present in shot 90312025 (Fig. 5-14). This technique decomposes the data in two sets of biorthogonal modes. The spatial or topological modes are called topos and the temporal or chronological modes are called chronos. The modes are ordered based on their relative power so the mode with the most
(a) Radial cross-correlation between each channel of photodiode array 2 and the channel with $R_L = 119$ cm.

(b) Radial cross-correlation coefficient between each channel of photodiode array 2 and the channel with $R_L = 93$ cm.

Figure 5-12: Radial cross-correlations of photodiode array data for shot 90312028 during $5 \leq t \leq 9$ seconds.

(a) Shot 90312025

(b) Shot 90312028

Figure 5-13: Normalized probability distribution functions of floating potential measured near the magnetic separatrix are very similar for shots 90312025 and 90312028.
power is represented by chrono 1 and topo 1, and so forth. For the data in the figure, the DC component was subtracted off and the data were normalized by the standard deviation between the channels. This biorthogonal decomposition was performed on data collected between 5 and 9 seconds during each of the two shots.

Topos 1 and 2, the spatial or topological modes, are similar for both shots. Most of the fluctuation power is contained in topo 1. This mode peaks in the core of the plasma and does not vary toroidally. Chrono 1, the temporal or chronological mode, does not show peaking at any particular frequency. This mode probably results from slow changes in the DC light levels during the four seconds. Topo 2 exhibits no significant radial variation and has a toroidal mode number n=1. This is evident because array 2 behaves in the opposite way to array 1. The FFT of chronos 2 peaks near 1.5 kHz. Topo 3 in shot 90312025 doesn’t show much of anything, but topo 3 in shot 90312028 shows an n=1 mode that varies radially. The FFT of chronos 3 for shot 90312028 has a peak near 2.25 kHz. Topo 4 doesn’t show much of anything for either shot and neither do the additional topos.

The radial structure of the fluctuations in shot 90312025 are consistent with an n=1 mode of a single frequency that extends across the plasma. The simulation of this case is found in Fig. 5-15(a). The radial structure of the fluctuations in shot 90312028 are consistent with two n=1 modes of different frequencies, each with a large radial extend, but not extending over the entire plasma. The simulation of this case is found in Fig. 5-15(b).

5.3 Properties of the coherent, \( n=0 \) fluctuations

Now, I will discuss the properties of the coherent fluctuations. Many of the same analyses that were applied to the quasi-coherent fluctuations will be applied to the coherent fluctuations in this section. The radial and toroidal mode structure are discussed. The neutral pressure scan experiment is used to determine when the modes are observed and the dependance of the coherent mode frequency on neutral pressure. The statistical properties of the floating potential are presented. The radial and toroidal mode structure are discussed. Then, the coherent fluctuations are compared to a previously observed 500 Hz coherent fluctuation that is still not well understood.

The coherent mode typically has \( Q \) on the order of 100 and frequencies on the order of 100 Hz. This fluctuation is observed on the chord integrated diagnostics that view the
Figure 5-14: Biorthogonal decompositions of the photodiode array data for shots 90312025 (right) and 90312028 (left) show that shot 28 has additional radial structure that is apparent in topo 3. The topos are plotted for array 1 (solid) and array 2(dashed). The chronos have been Fourier transformed.
Model parameters

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<td>250</td>
<td>0</td>
<td>-</td>
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</tr>
</tbody>
</table>

(a) Simulation on a single frequency mode at 1.2 (b) Simulation of two modes with 1.8 kHz in the core and 1.2 kHz at the edge, respectively.

Figure 5-15: Time frequency domain plots of simulated photodiode array data from the synthetic diagnostic confirm that the observed spectra in shot 90312028 is consistent with a plasma that has two spatially localized quasi-coherent modes at different frequencies, while the observed spectra in shot 90312025 is consistent with a single frequency mode that extends across the plasma.
plasma core, but is not observed on the edge electrostatic probes. These very low frequency fluctuations cannot be detected by the Mirnov coils for instrumental reasons.

When the coherent mode is observed, it is the dominant frequency on the channels that see it, meaning that the peak frequency of coherent fluctuation has the largest Fourier coefficient (Fig. 5-2(a)). The fluctuation amplitude is usually a few percent of the signal. The time trace of 100 cm channel of photodiode array 1 is shown in Fig. 5-16 for shot 90313017 from \( t = 7 \) s to \( t = 7.05 \) s. The mean value of the intensity, 0.326 V, has been subtracted from the signal. An oscillation at \( 185 \) Hz is clearly visible. The shape of the oscillation is approximately sinusoidal.

5.3.1 Neutral pressure scan experiment

The results of the neutral pressure scan experiment for the coherent fluctuations are now presented. The data set was discussed in §5.2.1. Recall that this experiment uses a set of shots in which the neutral fueling level is varied from shot to shot. The timing of the neutral fueling and the RF heating are identical for all shots. The average fluctuation properties are invested during the time when all three RF sources are turned on and the plasma is in
a steady state.

Again, I focus on the fluctuations observed by the 100 cm channel of photodiode array 1. The average fluctuation spectrum between six and nine-seconds is computed using Eq. 4.7. 10,000 frequency bins and 200 time windows are used. The 60 Hz and 360 Hz noise from the 10.5 GHz RF source power supply are removed using a notch filter. Then a six-parameter gaussian fit to the frequency peak is performed to determine the mode characteristics.

The coherent mode is observed when the plasma diamagnetism is moderate, $2 \text{mWb} \leq \phi_d \leq 3.5 \text{mWb}$. Both the n=0 and n=1 modes are observed during shots where $2.5 \text{mWb} \leq \phi_d \leq 3.5 \text{mWb}$. These fluctuations have frequencies on the order of 100 Hz (Fig. 5-2(a)) and azimuthal mode number $n = 0$ (Fig. 5-2(b)). The fluctuations are coherent for many periods. Coherence times on the order of 100 periods are routinely observed.

The frequency of the coherent fluctuation is typically on the order of 100 Hz. For the shots in this data set, the frequencies range from 80 Hz to 200 Hz. The frequency decreases with increasing neutral pressure (Fig. 5-17(a)) and line-averaged density and increases with increasing diamagnetism. These trends are consistent for discharges during which only the coherent mode is observed and discharges during which both the coherent and quasi-coherent modes are observed.

Discharges that have a coherent mode have values of plasma peakedness clustered near $P_{23} = 1.51$, which is the value that is consistent with an invariant density profile. The values of the peakedness for these shots with the coherent mode all fall between 1.4 to 1.7 (Fig. 5-17(b)). If just the shots that have only a coherent fluctuation are considered, then the frequency peaks when $P_{23} = 1.48$, but when the shots in which both the coherent and quasi-coherent modes are observed are also considered, there is no clear trend of frequency with density peakedness.

The width of the coherent fluctuation and the amplitude both decrease with increasing neutral pressure (figs. 5-17(c) and (d)). Similarly, they decrease with increasing line-averaged density and increase with increasing diamagnetism. These trends are consistent for both the cases where only the coherent mode is observed and the cases where both the coherent and quasi-coherent fluctuations are observed. One explanation for this may be that the plasma stored energy (that is proportional to the diamagnetism) is driving the coherent fluctuation and when the stored energy increases, the strength of the fluctuation increases as well.
Figure 5-17: Results of the neutral pressure scan experiment. The frequency, amplitude and standard deviation of the coherent n=0 mode are shown for discharges during which only the n=0 mode was observed (solid diamonds) and discharges during which both the n=0 and n=1 modes were observed concurrently (open diamonds). The data from shots 90312025-044 and 90313001-023 are time-averaged between 5 s < t < 9 s. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.
The trends of frequency, amplitude and standard deviation with neutral pressure for the coherent mode are all in the same direction as the trends for the quasi-coherent mode. The relationship between frequency and plasma peakedness is different and the quasi-coherent mode is observed over a much wider range of $P_{23}$ values. The average properties of the coherent mode presented in this section are consistent with the observations from the gas puff experiment present in §5.2.3 during which the neutral pressure was varied during a single shot.

5.3.2 Statistical properties of edge floating potential

The statistical properties of the fluctuating floating potential are investigated by looking at the normalized probability distribution functions. The coherent fluctuations are not observed on the edge floating probes, but it is still interesting to see what the distribution of the floating potential looks like.

The probability distribution functions of edge floating potential for shots in which coherent fluctuations are observed are typically close to Gaussian (Fig. 5-18). This indicated that the fluctuations at the plasma edge are random.
Figure 5-19: Time-averaged fluctuation amplitude as a percentage of the visible light intensity for each channel of array 1 between times $6 \, s < t < 9 \, s$ during shot 90313017.

5.3.3 Radial and toroidal structure

Toroidally, the coherent mode appears to have $n=0$. Measurements made by pairs of photodiode array channels separated toroidally show phase shifts near zero (5-2(b)).

Unlike the quasi-coherent mode, radial variation of the coherent mode frequency has not been observed.

The fluctuation amplitude as a percentage of the visible light intensity is shown for each channel of array 1 in Fig. 5-19. The data from array 2 are similar. Recall that both the fluctuation power and the signal level are chord integrated so the local fluctuation percentage may vary. The mode amplitude is very small for channels with radii less than 80 cm because the visible light emission is burnt out in the plasma core. The data from the interferometer are valid in this region and the coherent mode is observed on all three channels of the interferometer, including the innermost channel that passes through 77 cm.

The coherent fluctuations appear to be occurring in the plasma core, but not at the plasma edge. The mode does not appear to vary toroidally (Fig. 5-2(b)). The coherent fluctuations measured by two toroidally separated photodiode arrays are in phase.

A biorthogonal decomposition has been performed on the photodiode array data for a shot 90312042 from the neutral pressure scan experiment data set. A coherent fluctuation at $173 \, Hz$ was observed in the average fluctuation spectrum, but a quasi-coherent fluctuation was not observed. Most of the fluctuation power is contained in the first mode, as was the case with the biorthogonal decompositions shown in §5.2.4. Topo 1 from this shot
looks similar to both of the topo 1 in Fig. 5-14, but unlike decompositions in other shots, the frequency spectrum of chronos 1 for shot 90312042 is peaked at the coherent mode frequency.

Topo 2 is an n=1 mode with no significant radial variation. The frequency spectrum of chronos 2 has a broad peak near 1 kHz. This looks a very low amplitude quasi-coherent mode. This biorthogonal decomposition is representative of all of the shots which were classified as having a coherent fluctuation, but not a quasi-coherent fluctuation. They all show an n=1 mode as Topo 1. However, this n=1 mode is not large enough to be seen above the noise in the spectrograms or two-point plots of the photodiode array data.

The 3rd through 6th modes are all associated with the 173 Hz fluctuation. The topos show that the toroidal structure is approximately n=0.

5.3.4 Relationship between the coherent mode and a previously observed fluctuation at 500 Hz

Boxer [42] observed that the plasma sometimes underwent “spontaneous” density transitions. After these transitions, the plasma density on the 77 cm channel increased and the
plasma density profile was close to the invariant profile. A 500 Hz density fluctuation appears at the same time as the density transition. It is not known if the 500 Hz fluctuation results from change in the density profile or if the 500 Hz fluctuation causes the density profile to change. This transition was only observed during discharges that were heated by 3 kW of 2.45 GHz ECRH and 3 kW of 6.4 GHz ECRH. The 6.4 GHz source was stepped on and off during the discharge. The density transition and the 500 Hz mode were only observed during time when the sole heating source was 2.45 GHz.

This transition was observed repeatedly under similar plasma conditions during two run campaigns, but we have not been able to reproduce either the transition or the 500 Hz mode since the photodiode arrays, fast cameras and probe array diagnostics were installed. However, I can compare the properties of the 500 Hz fluctuation to the other low frequency fluctuations that are observed in LDX using the diagnostics the data from the interferometer and the edge ion saturation and floating probes. Unfortunately, the toroidal mode number of the 500 Hz mode can not be measured by the interferometer alone.

Neither the 500 Hz mode nor the coherent mode are observed on the edge diagnostics. The 500 Hz mode was observed during 2.45 GHz only heating when the peakedness of the density profile was near the value for an invariant profile. The coherent mode is observed on the photodiode arrays during configuration when all heating frequencies are on, but has been observed during other heating configurations on the interferometer. The frequency of the coherent mode is typically lower than 500 Hz.

5.4 Broadband, turbulent fluctuations

Broadband fluctuations are observed on nearly every shot. In some cases, broadband turbulence appears to be suppressed or masked by more coherent fluctuations.

For the case with broadband fluctuations only, the distribution has a positive skewness and positive kurtosis. This is true for all of the cases that have only broadband turbulence (Fig. 6-4). Positive skewness is also typical of edge turbulence in the scrape-off layer of tokamaks [67].

The edge floating probes measure a real rotation in the plasma. The rotation rate is determined by computing the cross-correlation between the floating potential measured by the probe array and then fitting an exponential to the lag time of the maximum cross-
Figure 5-21: Plasma rotation frequency at the edge versus neutral pressure. The dashed line is a power law fit to the data. The data from shots 90312025-044 and 90313001-023 are time-averaged between $5 \, s < t < 9 \, s$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.

correlation between each of the probes. The rotation rate decreases as the neutral pressure increases (Fig. 5-21). A power law fit to the data give the relationship

$$\frac{\omega}{2\pi} = 3.23P_0^{-1.12} \quad \text{(kHz)}$$

where $P_0$ is the neutral pressure measured by the vessel ion gauge. Because of the relationship between neutral pressure, density and diamagnetism, the edge rotation frequency will decrease with increasing line-averaged plasma density and increase with increasing plasma diamagnetism.

In the next chapter, I discuss the relationship between the low-frequency fluctuations described here and radial transport in the plasma.
Chapter 6

On the relationship of low frequency turbulence and transport

6.1 Introduction

The processes governing magnetic confinement of plasma can be modeled by a feedback loop where profiles provide an instability drive that leads to turbulence that causes transport that changes the profiles. This feedback loop (Fig. 6-1) is non-linear. In this thesis, I will look only at the turbulence to profiles portion of the loop. The instability drive is not considered.

Nonlinear effects are important in each link of the feedback loop. However, in some cases, many of these nonlinear effects can be neglected and that simplifies the analysis.

Figure 6-1: The closed system that reaches a steady state during magnetic confinement.
Quasilinear theory describes this feedback loop in the framework of weak turbulence.

Low frequency fluctuations conserve the dipole moment \( \mu = m v_\perp^2 / 2B \) and the longitudinal invariant \( J = m \oint v_\parallel ds \). The resulting flux tube mixing leads to radial transport. As the flux tubes mix due to \( E \times B \) motion, they conserve particles. The density thus increases as tubes of flux move inwards and compress and the plasma is heated. Flux tubes at the edge with lots of particles mix with flux tubes in the core with fewer particles. The result is a randomization of flux that leads to an equal number of particles confined within each flux tube. In a dipole-confined plasma, that means the density is centrally peaked. Indeed, a strong density pinch is sometimes observed in LDX [14].

The pinch can be characterized as flux-space diffusion with a fixed diffusion constant. This assumes that random fluctuations result from interchange-like modes and that particles move with the flux tubes. The plasma is fueled at the edge so inward, radial transport of particles is the source for the density pinch.

Low-frequency fluctuation activity is observed in both the plasma core and the plasma edge during this time. Broadband fluctuations are observed almost all the time. Some discharges also exhibit quasi-coherent fluctuations. The frequency of these quasi-coherent fluctuations sometimes varies radially. The amplitude of the fluctuations measured locally at the edge is higher than those measured by chord integrated diagnostics passing through the core. However, the fluctuation amplitude measured by chords passing through different radii do not vary dramatically.

### 6.1.1 Plasma density profiles and stability

In LDX the density is measured by a four-chord interferometer which was described in §4.3. The ratio of the line-averaged densities measured by different interferometer chords provides information about how steep the density profile is. If the plasma density profile is the invariant profile then the ratio of chord 2 \((R_2 = 86 \text{ cm})\) to chord 3 \((R_3 = 96 \text{ cm})\) would be \( P_{23} = \frac{n_2}{n_3} \sim 1.5 \). The ratio \( P_{23} \) is used as a measure of the closeness of the density profile to the invariant profile.

Recall that the stability of the plasma depends on the temperature profile as well as the density profile. Density, temperature, and pressure gradients can all drive the plasma unstable so the density profile is just one piece of a larger picture. However, when the density profile is sufficiently steep, the plasma will always be unstable to either MHD interchange.
Figure 6-2: Location of the time-averaged observed density profile gradient for shot 90312011 between $t = 6$ and 9 seconds. Stability regions for MDH like modes and entropy modes are identified as a function of plasma density gradient $\zeta = -d \ln n_e / d \ln V$ and the ratio of the temperature and density gradients $\eta = d \ln T / d \ln n$.

Flux tube mixing leads to density profiles that increase dramatically as the plasma radius decreases. However, the density does not peak next to the floating-coil, but rather peaks in the plasma core and then decreases with radius between the density peak and the floating-coil. One possible cause for this decrease in density is that neutral particles (which are released from the floating-coil when it is hit by ions) escape the plasma through the thin flux bundle on the inside of the floating-coil by scraping off on the inside of floating-coil without re-ionizing. This could cause a reduction of the line-averaged density measured along the innermost interferometer chord ($R_{||} = 77$ cm). This is the reason that the ratio of the second and third interferometer chords are used to estimate the steepness of the density profile, rather than the first and second chords. The $P_{23}$ does not provide information about whether the density profile has the shape of the invariant profile inside of 86 cm, but rather give a local estimate of the density profile steepness in the region where the photodiode
Figure 6-3: The ratio $P_{23}$ of line-averaged density from interferometer chords 2 and 3 indicates that locally, the plasma density profile is close to the invariant density profile for plasmas when the plasma diamagnetism is less than about 4 mWb. The density profile steepness increases with diamagnetism for plasmas in which the diamagnetism is high. Recall that the plasma can be unstable to either MHD modes or entropy modes even when the density profile is invariant, depending on the gradient of the temperature profile. The data from shots 90312025-044 and 90313001-023 are time-averaged between $5 \, \text{s} < t < 9 \, \text{s}$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.

Boxer [42] used an Abel inversion to unfold the density profile and found it was somewhat shallower than the invariant profile, but I find that a direct comparison of the second and third interferometer chords often reveals a region where the density profile has a similar steepness to the invariant profile or even exceeds it. Figure 6-3 shows the variation of $P_{23}$ with diamagnetism for the neutral pressure scan shots described in §5.2.1. The local density profile is close the invariant density profile ($P_{23} \sim 1.5$) for plasmas with low to moderate diamagnetism ($\Psi_D < 3.5 \, \text{mWb}$). The profiles observed when the diamagnetism is high are steeper than the invariant profile will be discussed later in §6.4. The location of a typical density profile is shown on a plot of the instability regions for drift modes and interchange modes (Fig. 6-2). However, I do not know for certain that the fluctuations discussed in Chapter 5 are caused by either MHD interchange modes or entropy modes.

The temperature profiles are not measured and magnetic reconstruction of the equilibrium pressure profiles are available only for a limited number of shots. The only mea-
measurements of temperature profiles come from comparing the temperature measured by the edge probes to the temperature measured from the line ratios of neutral helium emission described §3.3 so \( \eta = \frac{d\ln T}{d\ln n} \) has been assigned large error bars. Depending on the value of edge temperature that is used, the temperature profile calculated using the line ratio data correspond \( \eta = 1 \) or \( \eta = 1.7 \), assuming an invariant density profile based on \( P_{23} = 1.5 \) for this shot.

When the pressure profile is marginally stable to interchange modes and the density profile is invariant to interchange modes, then \( \eta = \frac{2}{3} \) for all radii outside of the pressure and density peaks. If the temperature profile is uniform in space \( T(r) = T_0 \), then \( \eta = 0 \) for all radii, regardless of the density profile. There is no evidence to support the idea that the temperature at the edge is higher than the temperature in the core, so I assume that \( \eta \) is positive. If the density profile is steeper than the invariant density profile and the pressure profile is near marginal stability, then the temperature profile will be shallower and \( \eta \) will be positive, but less than \( \frac{2}{3} \). Similarly, if the density profile is shallower than the invariant profile and the pressure profile is near marginal stability, then \( \eta \) will be larger than \( \frac{2}{3} \).

I have assumed that the pressure profiles are near marginal stability for interchange modes (Eq. 2.1). Magnetic reconstruction of the equilibrium pressure profiles supports this assumption [27, 43], but the error bars for \( \eta \) are large. The error in the phase measured by the interferometer is approximately \( \pm 5\% \) [13]. Additional error is introduced when the line-integrated density is inverted to get the density profile.

Whenever the density profile is not stationary, the value of \( \zeta = -\frac{d\ln n_e}{d\ln V} \) will vary radially. However, when the density profile is close to the stationary profile, that radial variation is small. In the example shown, \( \zeta = 1.4 \) at 89 cm with error bars that are approximately \( \pm 0.4 \). The value at 89 cm (using the data from interferometer chords 2 and 3) is shown in Fig. 6-2.

## 6.1.2 Overview

Quasilinear diffusion can be used to model the inward, turbulent transport that results in peaked plasma density profiles in some of the discharges that were presented in the

\[ \text{The error for } \zeta \text{ at 89 cm was computed by adjusting the measured phases for the 86 cm and 96 cm channels up and down by known phase error of the interferometer 5% and then reconstructing the density profiles using a matrix inversion. This results in a minimum value of } \zeta = 0.9 \text{ and a maximum value of } \zeta = 1.8 \]
previous chapter. For these cases, the quasilinear diffusion coefficient at the plasma edge is computed from the measured fluctuations in floating potential using a model developed for magnetospheric physics that requires only floating probe measurements and not ion saturation current measurements. This measurement assumes that the floating potential fluctuations are representative of the plasma potential fluctuations. Assuming a constant diffusion coefficient equal to the measured quasi linear edge coefficient, the rate of formation of the stationary profiles can be predicted by the diffusion equation and compared to the observed formation of the profiles. The saturated amplitudes of the turbulent fluctuations and the resulting plasma density profiles are measured. For a constant diffusion coefficient, the time-independent diffusion equation predicts \( nV = \text{constant} \).

At other times, particularly those times when the quasi-coherent mode is present, non-linear effects dominate the dynamics of the plasma and quasilinear theory cannot be properly applied. For these cases, the observed turbulence levels are compared to the observed profiles, but comparisons to theory are not made.

### 6.2 Quasilinear diffusion

Quasilinear theory describes the dynamics of transport for plasmas in the framework of weak turbulence. This framework assumes that the perturbation amplitudes of the turbulence are small. When the imaginary component of the growth rate, the portion due to non-linear coupling between modes, is much less than the real component of the growth rate then the turbulence can be modeled by a set of real frequency eigenmodes with amplitudes that change slowly in time [69]. Weak turbulence further assumes that the wave spectrum is dense enough that the coherence between modes is destroyed by phase mixing. In addition to the assumptions of weak turbulence, quasilinear theory assumes that the phase distribution between the initial particle motions and the electromagnetic field is randomly distributed. All non-linear mode coupling is neglected.

#### 6.2.1 Applicability of quasilinear diffusion to LDX plasmas

Quasilinear diffusion applies to plasmas that have random fluctuations. At high mode numbers, the random phase approximation holds, so there are many modes. The modes saturate and lead to transport. In quasilinear theory, the modes saturate because of a
relaxation to equilibrium profiles. Saturation of the modes may be determined by non-linear effects but a broadband spectrum of saturated turbulence will nevertheless diffuse the background plasma as formulated by quasilinear theory. (The quasilinear diffusion coefficient contains the mode amplitude which is determined from non-linear effects).

The probability distribution function (PDF) of a plasma with random fluctuations has a gaussian character. The higher order moments, skewness and kurtosis, are useful for identifying when plasmas have probability distribution functions that are near gaussian and when plasmas have probability distribution functions that deviate strongly from Gaussian distributions, indicating that the fluctuations are not random (see §4.6.3).

Fig. 6-4 shows the time-averaged skewness and kurtosis of the floating potential measured near the magnetic separatrix versus the plasma diamagnetism for deuterium discharges with 15 kW of multi-frequency ECRH. The floating potential is the measurement that will be used to compute the diffusion coefficient so this is the most relevant statistical quantity to consider. The skewness shows a clear trend with diamagnetism. The skewness decreases as the diamagnetism increases for all classes of fluctuations. The discharges which exhibit the coherent, n=0 fluctuations have skewness which is closest to zero, even if the quasi-coherent mode is also present. Discharges with high diamagnetism exhibit only the quasi-coherent fluctuations and have very negative skewness.

Looking at the kurtosis versus the diamagnetism, there appear to be different trends for shots during which only the quasi-coherent fluctuations are observed and all other categories of shots. In the first case, the kurtosis increases as the diamagnetism increases, while the opposite relationship is true in the latter case. Although shots with high diamagnetism have kurtosis which is near zero, they still have highly negative skewness so these fluctuations are not gaussian like. On the other hand, the shots that have moderate diamagnetism (which are mostly those shots in which the coherent fluctuation is observed) have values of skewness and kurtosis which are both near zero. The discharges with very low diamagnetism, during which neither the coherent or quasi-coherent modes are observed have positive skewness and positive kurtosis.

The skewness and kurtosis of the probability distribution functions of the chord-integrated plasma density and visible light intensity have also been examined. The skewness and kurtosis of the plasma density probability distribution functions have values that are scattered near zero regardless of the plasma diamagnetism and the types of fluctuations present.
Figure 6-4: Skewness and kurtosis of the PDFs of floating potential measured near the magnetic separatrix indicate whether the plasma fluctuations are random as a function of the plasma diamagnetism. The data from shots 90312025-044 and 90313001-023 are time-averaged between $8 \, \text{s} < t < 8.5 \, \text{s}$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.
In plasmas where only broadband fluctuations are observed at the plasma edge and a very low frequency coherent mode is observed by the chord integrated diagnostics in addition to broadband turbulent fluctuations, the probability distribution functions of floating potential have a gaussian character indicating that the fluctuations at the edge are random. Quasilinear theory can be applied to these discharges. The probability distribution functions of floating potential deviate from Gaussian distributions for plasmas where quasi-coherent fluctuations are observed at the edge. For these discharges, it is likely that nonlinear effects play an important role in the dynamics.

6.2.2 The quasilinear diffusion equations for dipolar plasma

Kouznetsov et al. [11] calculated the quasilinear diffusion coefficient for a toroidal dipole-confined plasma that is weakly unstable to interchange motion. The time invariant solution to these quasilinear diffusion equation is the density profile that has an equal number of particles per flux tube and a pressure profile that is marginally stable to interchange motion. Both of these predictions are roughly consistent with the experimental observations made thus far in LDX.

The result [70] is a set of quasilinear diffusion equations for the density \( n \) and a function related to entropy per unit volume \( s = p/n^{\gamma-1} \)

\[
\frac{\partial}{\partial t} (nV) = \frac{\partial}{\partial \psi} \left[ D \frac{\partial}{\partial \psi} (nV) \right] \tag{6.1}
\]

\[
\frac{\partial}{\partial t} (sV) = \frac{\partial}{\partial \psi} \left[ D \frac{\partial}{\partial \psi} (sV) \right] \tag{6.2}
\]

where \( \psi \) is the magnetic flux, \( V = \oint dl/B \) is the volume per unit flux. The quasilinear diffusion coefficient \( D \) is defined as

\[
D = \sum_n \gamma_n |X_n|^2 \tag{6.3}
\]

where \( \gamma_n \) is the MHD growth rate of the nth mode and \( X_n \) is the radial component of the normalized displacement vector of the nth mode such that \( \nabla \psi \cdot \xi_n(\psi, l) \approx X_n(\psi) + O(2) \).

Analyses have also been performed to calculate quasilinear diffusion in magnetospheric plasmas [4]. In these cases, assumptions were made about the form of the electric potential that allow the diffusion coefficient to be measured directly. If there are no losses along the
field line, then the diffusion of the number of particles per flux tube, \( N(\psi, t) = \langle n \rangle \delta V \) can be written

\[
\frac{\partial N}{\partial t} = \langle S \rangle + \frac{\partial}{\partial \psi} D \frac{\partial N}{\partial \psi} .
\]

(6.4)

Brackets indicate that a quantity is flux-tube averaged. The diffusion coefficient,

\[
D = R^2 \langle E_{\phi}^2 \rangle \tau_{\text{corr}} \left[ \frac{(V \cdot s)^2}{s} \right]
\]

(6.5)

is proportional to the mean squared fluctuations of the azimuthal electric field \( E_{\phi} \) and the correlation time of the fluctuations \( \tau_{\text{corr}} \), where \( R \) is the radial location of a floating probe along the magnetic mid-plane [43].

The solution of equation 6.4 with \( \langle S \rangle = 0 \) in the closed field-line region, a fixed plasma density at the outer edge and an assumption of rapid loss to the dipole magnet at the inner edge will reproduce the observed density build-up [14] for this value of \( D \). In steady state, the large value of \( D \) (0.047 \( V^2 \cdot s \)) implies a small gradient in \( N \). Therefore \( D \partial N / \partial \psi \) is both small and constant in the outer plasma that lies beyond the pressure peak. In the inner plasma, located between the floating coil and the pressure peak, I expect that \( D \) is small; permitting a large \( N \) gradient so as to match the low \( N \) boundary condition at the floating-coil.

This model was developed for the earth’s magnetosphere, but the assumptions are also true for plasmas in the LDX experiment. The ratio of the gyro-radius to the length scale of the diffusion is much less than one. The fluctuations of the magnetic field are slow compared to the autocorrelation time of the floating potential. In the case of the magnetosphere, these fluctuations are caused by the rotation of the earth. In the case of the LDX, there are some small fluctuations that are induced by the modulation of the levitation-coil current that is necessary to stabilize the vertical position of the floating-coil. Birmingham assumes that the autocorrelation time has the form \( \langle \delta \phi(t - \tau) \delta \phi(t) \rangle = \phi_m \exp(-\tau^2 / \tau_{\text{corr}}^2) \) [4], where \( \phi_m \) is the electric potential mapped to the magnetic mid-plane.

The diffusion coefficient derived by Birmingham is used in the following analysis because it can be related directly to the floating potential measurements.

The azimuthal electric field and the quasilinear diffusion coefficient computed from Eq. 6.5 are shown versus diamagnetism and plasma peakedness in Fig. 6-5. Recall, that the statistical data suggested that quasilinear diffusion was really only applicable to the region
Figure 6-5: The average azimuthal electric field and the quasilinear diffusion coefficient increase with diamagnetism. They are also shown in relation to the density peakedness. The data from shots 90312025-044 and 90313001-023 are time-averaged between $5 \text{s} < t < 9 \text{s}$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.
where the diamagnetism is between one and three mWb. In this region, the diffusion coefficient increases with increasing diamagnetism. All of the data in this region have $P_{23} \sim 1.51$ so there is no apparent trend with respect to plasma peakedness.

Although quasilinear diffusion is not applicable everywhere, the azimuthal electric field is measured directly by the probes so it is valid for all values of diamagnetism. The electric field is roughly constant at 50 V/m for values of diamagnetism less than 2 mWb. The electric field increases between $2 < \Phi_d < 4$ mWb. The quasi-coherent begins to appear when the diamagnetism is 2 mWb, which is where the azimuthal electric field and the correlation time both begin increasing. Both the coherent mode and the quasi-coherent mode are observed until the diamagnetism reaches 3.4 mWb. At this point, the coherence time increases by an order of magnitude and remains high.

If I assume that the diffusion coefficient is constant across the plasma, then the particle confinement time $\tau$ can be estimated

$$\frac{1}{\tau} = D A^2$$

(6.6)

where the step size $A = \frac{1}{N} \frac{\partial N}{\partial \nu}$ is determined using the interferometer data and the vacuum magnetic field. Fig. 6-6 shows the estimated confinement times as a function of the plasma diamagnetism. The plasmas in which quasilinear theory was shown to be a reasonable assumption have estimated confinement times that vary by four orders of magnitude from 1 msec to 2 sec. This is not very realistic because the plasmas are similar. Energy confinement times of 50 - 100 ms [5] have been measured for similar plasmas using the equilibrium reconstruction of the pressure profile and the input power. Theoretically, particle confinement time in dipolar plasmas should be much less than energy confinement time. The assumption that the diffusion coefficient is constant throughout the plasma may not be correct. In addition, I've assumed that step size at 87 cm is representative. That may also be incorrect.

Now consider the relationship between the edge rotation frequency and the quasi-coherent mode frequency and the edge azimuthal electric field (Fig. 6-7). A linear relationship is evident when the edge electric field is less than 100 V/m. The electric field value begins to exceed 100 for diamagnetism around 3 mWb. This is also the same time that the density profile steepness exceed the invariant density profile (Fig. ??).
The confinement time estimated from Eq. 6.6 are shown as a function of diamagnetism. The red squares highlight the plasmas to which quasilinear theory can be applied. The data from shots 90312025-044 and 90313001-023 are time-averaged between $5 \, \text{s} < t < 9 \, \text{s}$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.

The frequency of edge rotation increases with edge azimuthal electric field, but the relationship is not linear for $E_\phi > 100 \, \text{V/m}$. The quasi-coherent mode frequency measured by the 100 cm channel of photodiode array 1 has a similar trend to the edge rotation frequency. The data from shots 90312025-044 and 90313001-023 are time-averaged between $5 \, \text{s} < t < 9 \, \text{s}$. During this time period, the plasma is heated with 15 kW of RF power from three frequencies and the plasma behavior is fairly steady state.
6.3 Rotational shear

In this section, I will discuss rotational shear in LDX plasmas. Recall that LDX does not have magnetic shear because the magnetic field lines are closed. Although magnetic shear is often important for plasma stability in toroidal plasma confinement devices, stability in LDX is provided by plasma compressibility.

For the cases discussed in the previous section, where the fluctuations are random and result from weak turbulence, I can further assume that the fluctuations are both homogeneous and isotropic. Although, these high order modes are not truly homogenous, they are approximately homogenous given the resolution of our diagnostics. This allows us to invert the fluctuations measured by the chord-integrated diagnostics to get a azimuthal profile of the fluctuations. In weak turbulence, the fluctuations result from saturated modes with high toroidal mode numbers. However, the inversion introduces additional noise and the signal to noise ratio of the inverted data is too small in most cases to do the correlation analysis which give the rotation rates.

For discharges in which the quasi-coherent mode is observed, the fluctuations are not random and the frequency spectrum of the fluctuations is dominated by an $n = 1$ mode. In these cases, I can not assume that the plasma is homogenous and isotropic, nor do I have a local measurement of the correlations length of the fluctuations. However, I do observe some radial variation in the frequency of the fluctuations.

In discharges that have a strong quasi-coherent mode, the rotation rates for the inverted data are nearly the same as the rotation rates for the non-inverted data for $R > 85cm$. The line-integrated diagnostics do show a strong, coherent mode at very low frequencies. However, the rotation speed is much larger than the coherent mode frequency so it is not necessary to low-pass filter the data\(^2\).

Figure 6-8 shows the rotation profiles computed from photodiode array data and the edge probes for plasmas with different fluctuation spectra. The granularity in the data comes from the temporal resolution of the analysis. The minimum frequency spacing increases as the frequency increases and for rotation frequencies in the range of 1 kHz, the minimum spacing is about 200 Hz. Plasma discharges that have only broadband fluctuations and very low values of diamagnetism did not have strong enough signal to noise ratios to give valid

\(^2\)The computed rotation profile for the inverted photodiode array data is the same when the data is high pass filtered at 500 Hz and when no high pass filter is applied
Figure 6-8: Radial profiles of azimuthal rotation rates calculated from photodiode array and probe array data.
results using this technique.

Fig. 6-8(a) shows the rotation profiles of three plasmas that have a coherent mode, but no quasi-coherent mode. These plasmas have diamagnetic fluxes near 3 mWb. The rotation frequencies for these shots are typically lower than those for shots that have quasi-coherent modes. The rotation rate is typically larger on the outermost photodiode array channel than at the plasma edge. For some shots, the rotation rate measured by the photodiode array is roughly constant, but in other shots the rate decreases again the radius decreases (as it does in shot 90313002).

Fig. 6-8(b) shows the rotation profiles of three plasmas that have a quasi-coherent mode, but no coherent mode. This class includes shot 90312028 which was discussed in §5.2.4. Most of the profile from plasmas in this category show evidence of rotational shear. The rotation frequency increases as you move towards the core of the plasma. The frequency at 87 cm is typically double the frequency at the edge.

Fig. 6-8(c) shows the rotation profiles of three plasmas that have both a quasi-coherent mode and a coherent mode. This class includes shot 90312025 which was discussed in §5.2.4. In this shot, the rotation rate measured by the photodiode array does not vary radially, but the rotation rate measured by the probes is approximately 50% lower than the rate measured by the photodiode array. In the plasmas that had quasi-coherent modes with frequencies less than 500 Hz, such as 90313004, the rotation profile is approximately constant across the plasma and the rotation rates measured by the photodiode array agree well with the rotation rates measured by the probes.

6.4 Locally, density profiles can be steeper than the invariant profile when the quasi-coherent mode is observed

Now I will revisit Fig. 6-3 to consider the effect that these fluctuations have on the plasma profiles. The peakedness of the plasma density profile, $P_{23}$, is estimated using the ratio of the line-averaged densities measured along the 86 cm and 96 cm chords (§4.3). Recall that the value of plasma peakedness for an invariant profile is $P_{23} \approx 1.51$. When the density peakedness is compared to the plasma diamagnetism, it’s clear that the density profiles are steeper than the invariant profile for many of the discharges during which the quasi-coherent mode is observed. These profiles are unstable to either MHD modes or both entropy modes.
regardless of the density and temperature profiles.

Just because the peakedness factor has a value that is consistent with an invariant profile does not guarantee that the profile is indeed invariant. This factor is computed using the measurement from two chords. If instead the plasma got smaller, that could also lead peakedness factors near $P'_{23} = 1.51$. However, the appearance of these quasi-coherent fluctuations when the peakedness factor exceed $P'_{23}$ are compelling.

On possible explanation that the plasmas created when the neutral pressure is low, have low enough densities initially that the neutrals can penetrate past the second interferometer chord and create a highly peaked profile without the need for a particle pinch. The peak densities for plasmas with low neutral pressures are about a factor of two lower than the the plasmas created with optimum neutral fueling. Figure 4-6 showed visible light emission profile compared to the density profile for a plasma with an approximately invariant density profile created in levitated mode. The visible light emission dropped to zero next to floating coil, indicating that there was no neutral particle source near the coil. In the plasmas that also have a quasi-coherent observed, the visible light emission profile does not drop to zero in the region between the edge of the floating coil and the density peak. This indicates that there may be a source of neutrals in that region. The RF resonance locations are primarily located inside of the first interferometer chord so there is also a power source in this region. The quasi-coherent modes may cause circulation, but not transport.

6.5 Locally, the plasma density exceeds the cutoff density for 2.45 GHz heating when the coherent mode is observed

The data make it clear that the coherent mode is not AC line noise, which is observed at 60 Hz on the photodiodes. The frequency of the coherent fluctuation clearly changes with plasma conditions. This was shown in the gas puff experiment (Fig. 5-8). Also, AC line noise can be observed when there is no plasma present, but the fluctuation identified as the coherent mode has only been observed when the plasma is present.

If the RF power fluctuates, that is also detectable by the photodiodes. The 10.5 GHz system power supply has a 360 Hz oscillation that is often detected by the photodiodes. This noise is very coherent and the frequency does not change with plasma conditions. Additional experiments were performed during which the ECRH sources were intentionally modulated.
These fluctuations were detected by the photodiode arrays, but were not included as part of this thesis project.

Another possibility that I considered was that the coherent mode was related to the frequency at which the gas puff valve opened to fuel the plasma. The gas puff frequency was originally set to 100 Hz. The fueling level is set by modifying the length of time that the puff valve is open, but not the frequency at which it opens. However, the puff valve frequency does not change with plasma conditions and does not vary with time.

It is possible that the coherent mode is a limit cycle related to the motion through particle transport of the critical density layer for ECRH accessibility. The frequency The local density profile was reconstructed from the chordal interferometer measurements using a matrix inversion. In Fig. 6-9, the local density at three location in the core is plotted as a function of the plasma diamagnetism and the cutoff densities for 2.45 GHz and 6.4 GHz are indicated. The local density exceeds the 2.45 cutoff density at all three of the locations shown during times when the coherent mode is observed. The fundamental heating resonance location for 2.45 GHz heating occurs at $R \approx 80$ cm in the levitated configuration.
The densities also exceed the cutoff for plasmas that have very low diamagnetism. These plasmas also have high neutral pressure, meaning that the neutral fueling rate is larger than is needed to achieve the densities. The presence of excess neutrals may be the reason that the coherent mode is not observed in these discharges.
Chapter 7

Conclusions

Low-frequency fluctuations have been observed throughout the plasma during every plasma shot, but the characteristics of the fluctuations vary depending on the plasma conditions. These fluctuations were measured using toroidally spaced photodiode arrays, a four-chord interferometer and an edge floating probe array. Coherent and quasi-coherent structures were observed in addition to broadband turbulence.

Quasi-coherent fluctuations with $f/\Delta f \sim 5$ and $f \sim 1$ kHz were observed when the neutral pressure is low and the plasma diamagnetism is high. The fluctuation frequency decreased as the neutral pressure increased and increased as the density profile steepened. Low, non-zero toroidal mode numbers were measured using the twopoint method [60] applied to toroidally separated edge floating probes. Radial variation of the quasi-coherent fluctuation frequencies was observed in plasmas where the steepness of the density profile exceeded the steepness of an invariant density profile. This may indicate that a sheared rotation profile exists, which is inhibiting transport in these low density, centrally fueled discharges.

Locally, the plasma density profiles were observed to be steeper than the invariant density profiles during times when the quasi-coherent mode was observed (Fig. 6-3). The quasilinear diffusion coefficients calculated from the magnitude and correlation time of the edge floating probe fluctuations are very large, implying that the density profile should be close to the invariant density profiles. The observation of density profiles that are steeper than the invariant density profile show that the large amplitude quasi-coherent fluctuations do not result in turbulent mixing. Indeed, the probability distribution functions of the edge
floating potential for plasmas with quasi-coherent modes deviate from normal distributions so we concluded that the quasi-coherent fluctuations were not random. This violates the assumptions of the simple equation for quasilinear diffusion and therefore reasonably explains the discrepancy. The combined observations of these non-random fluctuations and peaked density profiles suggest that the quasi-coherent mode is not causing transport in the plasma and may indicate the presence of toroidally rotating closed convective cells.

Coherent fluctuations with $f/\Delta f \sim 100$ and $f \sim 100$ Hz were observed when the neutral pressure was low to moderate and the diamagnetism was moderate and only occurs in discharges where the steepness of the observed density profiles is close to the steepness of the invariant density profile. A toroidal mode numbers of zero was measured for the coherent fluctuations. These fluctuations were only observed on the diagnostics that view the core and are consistent with a single mode within the core. The coherent mode may be a limit cycle related to the motion through particle transport of the critical density layer for ECRH accessibility.

Broadband, turbulent fluctuations were observed throughout the plasma in all plasmas. During a gas puff experiment, during which a large puff of gas was injected into the plasma in the middle of the shot, the turbulent fluctuations were briefly suppressed or masked by the gas puff, but then resumed when the gas puff ended implying that high neutral pressure can suppress the turbulent mixing. The spectrum of a plasma with broadband fluctuations and no quasi-coherent fluctuations is approximately flat for frequencies less than approximately 1 kHz, then decays with a power law dependence at higher frequencies. This power law decay is typical of edge turbulence in dipole confined plasmas [33] and toroidal confinement devices [67]. Unlike the edge turbulence in toroidal confinement devices, the power spectrum that was shown had a power law dependence with two slopes. The magnetic geometry in LDX is approximately two-dimensional and two decay rates may be an indication of the dual cascade of energy and enstrophy that occurs in two-dimensional turbulence. While previously shown in supported dipole experiments [33], this is the first observation of this aspect of the dual cascade in a closed field line geometry.

In order to extend the results of Boxer et. al. [14] that indicated the observation of a strong pinch, the quasilinear diffusion coefficient was calculated from edge probe measurements for a variety of plasma conditions. This diffusion coefficient was then compared to the steepness of the observed density profile. The probability distribution of the edge floating...
potential was used to assess the randomness of the fluctuations and thus the applicability of the quasilinear diffusion coefficient. For discharges with random broadband fluctuations, the computed quasilinear diffusivity is consistent with the observed formation time of the peaked density profile. These discharges also had density profiles that were close to invariant density profiles. These measurements provide new evidence in support of the theories that indicate the broadband, turbulent fluctuations in dipole plasmas are leading to inward particle pinches and the observed density profiles.

In summary, low-frequency fluctuations were observed during the first plasmas created in LDX using ion saturation probes at the edge, a single interferometer channel with a viewing chord that passed through the central region of the plasma, and an uncollimated photodiode that viewed a large portion of the plasma. In order to study these fluctuations further, I constructed a set of two photodiode array diagnostics with the purpose of characterizing the low-frequency fluctuation activity observed in LDX and investigating the relationship between these fluctuations, external plasma controls of fueling and heating, and the later observed density profiles. In this thesis, the fluctuations are measured using two sixteen-channel photodiode arrays that have nearly identical radial views, but view from different toroidal locations, a twenty-four channel floating probe array [12], and a four-channel microwave interferometer [13]. These are the first measurements of the characteristics of low-frequency fluctuations in a levitated dipole plasmas. This is also the first time that these fluctuations have been parameterized with respect to experimental controls.
Appendix A

Spectrometer Calibration Data

Table A.1: Selected emission lines from various lamps used for the visible spectrometer wavelength calibration and the spectrometer bin number at which the lines are centered.

<table>
<thead>
<tr>
<th>Bin</th>
<th>(\lambda) (nm)</th>
<th>Lamp</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0015</td>
<td>336.99</td>
<td>Ne</td>
<td></td>
</tr>
<tr>
<td>0219</td>
<td>365.86</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>0222</td>
<td>366.34</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>0396</td>
<td>388.87</td>
<td>He</td>
<td></td>
</tr>
<tr>
<td>0453</td>
<td>396.47</td>
<td>He</td>
<td></td>
</tr>
<tr>
<td>0457</td>
<td>397.01</td>
<td>H</td>
<td>Balmer (\delta)</td>
</tr>
<tr>
<td>0500</td>
<td>402.62</td>
<td>He</td>
<td></td>
</tr>
<tr>
<td>0515</td>
<td>405.49</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>0539</td>
<td>408.60</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>0557</td>
<td>410.17</td>
<td>H</td>
<td>Balmer (\gamma)</td>
</tr>
<tr>
<td>0747</td>
<td>434.05</td>
<td>H</td>
<td>Balmer (\beta)</td>
</tr>
<tr>
<td>0755</td>
<td>435.56</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>0755</td>
<td>436.65</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>0844</td>
<td>447.15</td>
<td>He</td>
<td></td>
</tr>
<tr>
<td>1037</td>
<td>471.31</td>
<td>He</td>
<td></td>
</tr>
<tr>
<td>1204</td>
<td>492.37</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>1209</td>
<td>492.19</td>
<td>He</td>
<td></td>
</tr>
<tr>
<td>1287</td>
<td>501.57</td>
<td>He</td>
<td></td>
</tr>
<tr>
<td>1314</td>
<td>504.77</td>
<td>He</td>
<td></td>
</tr>
<tr>
<td>1672</td>
<td>546.75</td>
<td>Hg</td>
<td></td>
</tr>
</tbody>
</table>

The brightness calibration was performed using a lab sphere with known spectral radiance as a function of wavelength, as given in Table A.3. The spectral radiance for the wavelength of each bin is determined using a spline fit to the known data. A conversion
Figure A-1: The wavelength calibration data is fit by a fourth order polynomial.

Table A.2: The coefficients for the fourth order polynomial that relates spectrometer bin to wavelength are shown for the factory calibration and the our LDX calibration.

<table>
<thead>
<tr>
<th>Order</th>
<th>Factory Calibration</th>
<th>LDX Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^0$</td>
<td>335</td>
<td>335.2</td>
</tr>
<tr>
<td>$x^1$</td>
<td>0.1376</td>
<td>0.1397</td>
</tr>
<tr>
<td>$x^2$</td>
<td>$4.663 \cdot 10^{-6}$</td>
<td>$-8.257 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>$x^3$</td>
<td>$-1.366 \cdot 10^{-9}$</td>
<td>$1.407 \cdot 10^{-10}$</td>
</tr>
<tr>
<td>$x^4$</td>
<td>0</td>
<td>$4.320 \cdot 10^{-14}$</td>
</tr>
</tbody>
</table>
factor $B(\lambda)$ relates the measured spectrum of the lab sphere $L(\lambda)$ to the known spectral radiance $C(\lambda)$ such that

$$C(\lambda) = B(\lambda) \cdot \frac{L(\lambda)}{\Delta t}$$

(A.1)

where $\Delta t$ is the integration time for the spectrum and $C(\lambda)$ is given in units of counts/s. Eq. A.1 is solved for $B(\lambda)$. The measured spectra of the plasma $S_{\text{meas}}(\lambda)$ are then convolved with the conversion factor to yield the actual spectra of the plasma $S(\lambda)$ as follows,

$$S(\lambda) = \int \frac{S_{\text{meas}}(\lambda)B(\lambda)}{\Delta t} d\lambda.$$  

(A.2)
Table A.3: The spectral radiance as a function of wavelength for the labsphere used in the spectrometer calibration.

<table>
<thead>
<tr>
<th>( \lambda ) (( \mu \text{m} ))</th>
<th>Spectral Radiance ( \frac{W}{\text{cm}^2 \text{sr} \mu\text{m}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.320</td>
<td>( 6.80 \cdot 10^{-3} )</td>
</tr>
<tr>
<td>0.340</td>
<td>( 3.37 \cdot 10^{-2} )</td>
</tr>
<tr>
<td>0.360</td>
<td>( 9.00 \cdot 10^{-2} )</td>
</tr>
<tr>
<td>0.380</td>
<td>( 1.83 \cdot 10^{-1} )</td>
</tr>
<tr>
<td>0.400</td>
<td>( 3.51 \cdot 10^{-1} )</td>
</tr>
<tr>
<td>0.420</td>
<td>( 5.61 \cdot 10^{-1} )</td>
</tr>
<tr>
<td>0.440</td>
<td>( 8.32 \cdot 10^{-1} )</td>
</tr>
<tr>
<td>0.460</td>
<td>( 1.16 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.480</td>
<td>( 1.54 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.500</td>
<td>( 1.95 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.520</td>
<td>( 2.41 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.536</td>
<td>( 2.79 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.575</td>
<td>( 3.73 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.600</td>
<td>( 4.33 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.655</td>
<td>( 5.55 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.700</td>
<td>( 7.41 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.750</td>
<td>( 7.71 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.800</td>
<td>( 7.71 \cdot 10^{0} )</td>
</tr>
<tr>
<td>0.900</td>
<td>( 7.35 \cdot 10^{0} )</td>
</tr>
<tr>
<td>1.060</td>
<td>( 7.60 \cdot 10^{0} )</td>
</tr>
<tr>
<td>1.100</td>
<td>( 7.43 \cdot 10^{0} )</td>
</tr>
<tr>
<td>1.200</td>
<td>( 6.04 \cdot 10^{0} )</td>
</tr>
<tr>
<td>1.300</td>
<td>( 5.52 \cdot 10^{0} )</td>
</tr>
<tr>
<td>1.500</td>
<td>( 2.16 \cdot 10^{0} )</td>
</tr>
<tr>
<td>1.750</td>
<td>( 1.58 \cdot 10^{0} )</td>
</tr>
<tr>
<td>1.850</td>
<td>( 1.14 \cdot 10^{0} )</td>
</tr>
</tbody>
</table>
Table A.4: Polynomial fit coefficients to the brightness as a function of bin number.

<table>
<thead>
<tr>
<th>Order</th>
<th>Calibrated Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^0$</td>
<td>$-2.5957458 \cdot 10^{+08}$</td>
</tr>
<tr>
<td>$x^1$</td>
<td>$4.4438365 \cdot 10^{+06}$</td>
</tr>
<tr>
<td>$x^2$</td>
<td>$-2.4138855 \cdot 10^{+04}$</td>
</tr>
<tr>
<td>$x^3$</td>
<td>$6.6422943 \cdot 10^{+01}$</td>
</tr>
<tr>
<td>$x^4$</td>
<td>$-1.0701286 \cdot 10^{-01}$</td>
</tr>
<tr>
<td>$x^5$</td>
<td>$1.0703188 \cdot 10^{-04}$</td>
</tr>
<tr>
<td>$x^6$</td>
<td>$-6.7358478 \cdot 10^{-08}$</td>
</tr>
<tr>
<td>$x^7$</td>
<td>$2.5963801 \cdot 10^{-11}$</td>
</tr>
<tr>
<td>$x^8$</td>
<td>$-5.6019460 \cdot 10^{-15}$</td>
</tr>
<tr>
<td>$x^9$</td>
<td>$5.1838179 \cdot 10^{-19}$</td>
</tr>
</tbody>
</table>
Appendix B

Photodiode Array Diagnostic
Preamplifier Board

A custom preamplifier was designed for the photodiode array diagnostic. The photodiode array chip is soldered directly to the preamplifier board to minimize noise by reducing the path length of the leads to the preamplifier. The circuit board is small enough to be mounted to the back of the camera, in a small box. The circuit could likely be reduced in size even more, but this size was small enough for the design constraints.

The schematic for the photodiode array board is shown in Fig. A-1. The board is a double-sided, four-layer printed circuit board. The top of the board is shown in Fig. A-2 and the bottom is shown in Fig. A-3. The bill of materials can be found in table A.1.

The board consists of sixteen identical preamplifier circuits that use an OPA657 chip (Burr Brown). The outputs can be buffered if desired. The buffers were used on the photodiode array diagnostic, as built. If the buffer is left out of the circuit then a zero Ohm resistor should be used in the R3 (and equivalent) position. If the buffer is used, then R3 should be omitted.

The photodiode array chip is soldered directly to the backside of the board. The photodiode array bias voltage is included on the board. A bias voltage of 10 V was provided by a voltage regulator for the photodiode array diagnostic, as built. There is also an option to supply the photodiode array bias using a voltage regulating diode (D17, R19).

The board requires external DC power which is suppled through connector X1. The amplified signals leave the board through a 37 pin D-Subminiature connector. This is the
type of connector that the digitizers use and was chosen so that the outputs could be connected directly to the digitizer.

In hindsight, there are several design changes that would benefit this board. The first is to add a 20 pF feedback capacitor to the operation amplifier in parallel to R1. This capacitor is shown in Fig. 4-3 as capacitor C5, but it is not shown in Fig. B-1. Note that the component numbering in Fig. 4-3 was modified for simplicity and is not the same as the numbering on the board. The numbering on the full schematic (Fig. B-1). Although, there is no pad for this capacitor, it was possible to add it to the circuit by soldering it to the feedback resistor R1.

The second modification is to add a second stage of amplification to the circuit. This second stage would replace the additional amplifiers that are currently used, making the entire diagnostic more compact. More importantly, there may be some noise reduction by placing the second amplifier stage on the same board and the preamplifier because the cable run between the camera and the digitizer is quite long. This separation is necessary because of the strong magnetic fields produced by the charging-coil. As built, the component values were selected to maximize the gain of the first amplification stage, at the expense of the frequency response of the preamplifier. This gain was necessary for good noise performance because of the long cable run between the two stages of amplification. By placing both stages of amplification on the same board, the gain of the first stage could be reduced by a small factor and the frequency response increased by that factor.
Figure B-1: Photodiode array preamplifier board schematic.
Figure B-2: Top side of preamplifier circuit board.
Figure B-3: Bottom side of preamplifier circuit board.
<table>
<thead>
<tr>
<th>Destination</th>
<th>Description</th>
<th>Package</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Resistor, 1 MΩ, 1/4 W, 1%</td>
<td>1206</td>
<td>SMD</td>
</tr>
<tr>
<td>R2</td>
<td>Resistor, 100 Ω, 1/4 W, 1%</td>
<td>1206</td>
<td>SMD</td>
</tr>
<tr>
<td>R3</td>
<td>Resistor, 0 Ω, (NOT USED)</td>
<td>1206</td>
<td>SMD</td>
</tr>
<tr>
<td>C3 – C6</td>
<td>Ceramic Capacitor, 0.1 μF, 50V</td>
<td>1206</td>
<td>SMD</td>
</tr>
<tr>
<td>U1</td>
<td>Operation Amplifier OPA657UB</td>
<td>8SOIC</td>
<td>SMT</td>
</tr>
<tr>
<td>D1</td>
<td>Closed-Loop Buffer BUF602</td>
<td>8SOIC</td>
<td>SMT</td>
</tr>
<tr>
<td>U17</td>
<td>Photodiode Array PDB216</td>
<td>PCB-18</td>
<td>THRU</td>
</tr>
<tr>
<td>C1, C2</td>
<td>Tantalum Capacitor, 6.8 μF, 25V, 10%</td>
<td>3528-21 (EIA)</td>
<td>SMD</td>
</tr>
<tr>
<td>IC1</td>
<td>Voltage Regulator, 10V, 1A</td>
<td>TO-220</td>
<td>THRU</td>
</tr>
<tr>
<td>IC 2</td>
<td>Voltage Regulator, 5V, 1A</td>
<td>TO-220</td>
<td>THRU</td>
</tr>
<tr>
<td>IC 3</td>
<td>Voltage Regulator, Neg 5V, 1A</td>
<td>TO-220</td>
<td>THRU</td>
</tr>
<tr>
<td>X1</td>
<td>Header Conn., 6 Pos., 1.25 mm Vertical</td>
<td>53398-06</td>
<td>SMT</td>
</tr>
<tr>
<td>X3</td>
<td>D-Subminiature, 37 Pos. Conn.</td>
<td>F37HP</td>
<td>THRU</td>
</tr>
</tbody>
</table>

Table B.1: Bill of materials. The component names for the repeated elements (R1 – R3, C3 – C6, U1, D1) have been listed only for the first circuit, but are identical for all sixteen. In addition to the listed components, a 20 pF capacitor has been placed in parallel with R1.
Appendix C

Position Calibration of PHDA

C.1 Methods

An in-vessel calibration was performed to confirm the photodiode array views. A yardstick was positioned in the vacuum vessel perpendicular to the port where each array is installed. A laser was positioned at different locations along the yardstick and images were collected by the photodiode array. This is illustrated in Fig. C-1. The viewing chords for the chord averaged diagnostics on LDX are labelled by the tangency radius $R_\perp$ of the chord, defined as the radius of a circle centered in the vacuum vessel that contains a point tangent to the viewing chord. The tangency radius of a chord is also the minimum radius through which the chord passes.

Given two points along the chord, we can define both the equation of the line that defines the chord and the tangency radius. The two known points along each chord for the in-vessel calibration are aperture of photodiode array diagnostic, point A in Fig. C-1, and the laser location, point B in Fig. C-1. The position of A is the same for all viewing chords for the photodiode array because the chords not parallel, but instead all pass through the same lens.

We define the center of the vacuum vessel, point C, as the origin. A circle with center at C that is tangent to the viewing chord has been drawn. The point where the circle is tangent to the chord is labelled H and the tangency radius is h. Referring again to Fig. C-1, we define the length of line segment $\overline{AC}$ as $b$, the length of line segment $\overline{BC}$ as $a$ and the angle between $\overline{AC}$ and $\overline{BC}$ as $\gamma$. If points A and B are known, then $a$, $b$, and $\gamma$ are also known. The length of line segment $\overline{AB}$, defined as $c$, can be written in terms of the known
Figure C-1: Photodiode array calibration geometry

quantities using the law of cosines,

\[ c^2 = a^2 + b^2 - 2ab \cos \gamma \]  \hspace{1cm} (C.1)

Because, \( \overline{AB} \) is tangent to the circle at point H, \( \overline{CH} \) forms a right angle with \( \overline{AB} \) so the triangles formed by \( \overline{ACH} \) and \( \overline{BCH} \) are both right triangles. We can write a system of equations for the height of each of these triangles, \( h \), and the relationship between line segment \( \overline{AH} \) and \( \overline{BH} \),

\[ a^2 = h^2 + \overline{BH}^2 \]  \hspace{1cm} (C.2a)

\[ b^2 = h^2 + \overline{AH}^2 \]  \hspace{1cm} (C.2b)

\[ c = \overline{AH} + \overline{BH}. \]  \hspace{1cm} (C.2c)

The physical solution to this system of equations is

\[ h = \sqrt{a^2 - \frac{(a^2 + c^2 - b^2)^2}{4c^2}} \]  \hspace{1cm} (C.3)

\[ = \frac{ab \sin \gamma}{\sqrt{a^2 + b^2 - 2ab \cos \gamma}}. \]  \hspace{1cm} (C.4)
The set of tangency radii from chords that pass through A lie on a circle centered at the midpoint of line \( \overline{AC} \) with radius \( \frac{b}{2} \). This circle is indicated by a dashed line in Fig. C-1. The equation for this circle can be written,

\[
(x - A_x)^2 + (y - A_y)^2 = \frac{b^2}{2},
\]

where \( A_x \) and \( A_y \) are the x and y coordinates of point A, respectively. Rewriting the equation using polar coordinates and substituting for \( b \) in terms of \( A \) gives

\[
r = b \cos \phi
\]

where \( \phi = \theta - \gamma - B_\theta \), and \( B_\theta \) is the polar angle of the location of point B.

The aperture position, \( A \), was determined from a combination of measurements and technical drawings of the vacuum vessel and the multi-port flanges that contain the view ports. The distance from the center of the vacuum vessel to the inside edge of the port is 256 cm. The viewports on the side flange are offset from the flange center by 6.68 cm horizontally and 8.38 cm vertically. The center of the multi-port flange is 30.48 cm below the mid-plane. The viewports are angled at 68.5° toroidally so that the center of the view is tangent to the mid-plane at \( R = 90 \) cm. The viewports are also angled up at 5° so that the viewing chord crosses the mid-plane at the tangency radius. The coordinates of A for each of the two arrays are listed in Table C.1.

Table C.1: Aperture locations for the photodiode array diagnostics are given relative to the center of the vacuum vessel. North is 0° and up is in the positive z-direction.

<table>
<thead>
<tr>
<th>Array</th>
<th>( R )</th>
<th>( \theta )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>279.3 cm</td>
<td>47.8°</td>
<td>-24.1 cm</td>
</tr>
<tr>
<td>2</td>
<td>279.3 cm</td>
<td>137.8°</td>
<td>-24.1 cm</td>
</tr>
</tbody>
</table>

For the position calibration, an 8' measuring stick was suspended at the vessel mid-plane and measured the distance from the vacuum vessel wall to a laser. The measuring stick was aligned perpendicular to the multi-port flange for each array. For array 1, the measuring stick was placed at -45° from north and for array 2 the measuring stick was placed at 45° from north. A laser was placed on the measuring stick oriented perpendicular to the measuring stick in the direction of the photodiode array. In order to balance the
<table>
<thead>
<tr>
<th>$R_w - a$ (feet)</th>
<th>$R_\perp$ (cm)</th>
<th>Array 1 (Element #)</th>
<th>Array 2 (Element #)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00</td>
<td>118.5</td>
<td>01 (LSL)</td>
<td>01 (LSL)</td>
</tr>
<tr>
<td>4.25</td>
<td>112.5</td>
<td>02, 03</td>
<td>03</td>
</tr>
<tr>
<td>4.50</td>
<td>106.3</td>
<td>04</td>
<td>N/A</td>
</tr>
<tr>
<td>4.75</td>
<td>100.0</td>
<td>05, 06</td>
<td>05, 06</td>
</tr>
<tr>
<td>5.00</td>
<td>93.4</td>
<td>07</td>
<td>07</td>
</tr>
<tr>
<td>5.25</td>
<td>86.8</td>
<td>08</td>
<td>09</td>
</tr>
<tr>
<td>5.50</td>
<td>80.0</td>
<td>10</td>
<td>10, 11</td>
</tr>
<tr>
<td>5.75</td>
<td>73.0</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>6.00</td>
<td>65.9</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>6.25</td>
<td>58.7</td>
<td>14 (LSL)</td>
<td>15 (LSL)</td>
</tr>
<tr>
<td>6.50</td>
<td>51.4</td>
<td>16 (LSL)</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table C.2: The results of the in-vessel calibration of the photodiode array. The first column gives the measured distance from the laser to the vacuum vessel wall. The second column is the corresponding tangency radius for the viewing chord in centimeters. The last two columns give the array elements that responded to the laser for array 1 and array 2, respectively.

Laser on the measuring stick (which was necessary so it would not wobble) the front of the laser needed to protrude beyond the measuring stick by approximately 2”. This has been taken into account in the calculations.

In each position, the laser was blocked then unblocked 3 times. In many cases only one channel of the array sees the laser. In some cases, the laser is in the view of 2 channels. In most of these cases, it does not appear that cross-talk is the culprit so we assume the laser was positioned half way in between the two channels. Channel 4 of array 2 had a bad amplifier and this does seem to result in a small amount of cross-talk on array 2, particularly in channels 5-8.

### C.2 Measurements

Table C.2 shows the measured distance between the laser and the vacuum vessel wall $R_w = 250$ cm. The tangency radius calculated from the measured laser position using Equation C.3, is shown in the second column. The error in the mapped tangency radii is estimated to be ±0.25 cm. The channel or channels that responded to the laser are shown in the last column. In some cases, a channel responded but there was a low signal level (LSL). This is indicated in the table.
C.3 Results

The data should all lie along the curve derived in Eq. C.6, but that equation gives \( r(\theta) \). The calibration data are \( R(x) \) where \( x \) is then channel number. We write \( \theta(x) \) as a polynomial

\[
\theta = \theta_c + \Delta \theta (x - x_c) + \ldots
\]

(C.7)

where \( x \) is the channel number, \( x_c = 8.5 \) is the central channel, and \( \theta_c \) is the angle that defines the center of the photodiode array view. Combining Equations C.6 and C.7 gives

\[
R(x, [\theta_c, \Delta \theta]) = b \cos(\theta_c + \Delta \theta (x - x_c))
\]

(C.8)

Figure C.3 shows the data for arrays 1 and 2 with a three parameter fit to Eq. C.8 over-plotted as a solid line.

There are a finite number of elements in the array so we are limited by the digitization error of \( 1/\sqrt{N} \). In this case \( N = 16 \) so the minimum error in the channel number is a quarter channel. This is indicated by the horizontal error bars. In cases where the channel responded by the signal level was low (as indicated by LSL in Table C.2), the error bars were set to \( \pm 0.5 \). The tangency radii for each channel of each array are listed in table C.4.

Table C.3: The values of \( b, \theta_c \) and \( \Delta \theta \) resulting from fitting Eq. C.8 to the calibration data are given for each photodiode array.

<table>
<thead>
<tr>
<th>Array</th>
<th>( b ) (cm)</th>
<th>( \theta_c ) (radians)</th>
<th>( \Delta \theta ) (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>139.65</td>
<td>0.8758</td>
<td>4.2997 \cdot 10^{-2}</td>
</tr>
<tr>
<td>2</td>
<td>139.65</td>
<td>0.8601</td>
<td>4.2088 \cdot 10^{-2}</td>
</tr>
</tbody>
</table>

The calibration shows that the two arrays are misaligned by about half a channel. The average offset is 0.36 channels. The inner channels appear to be better aligned than the outer channels. The arrays view from 50.7 cm to 118.6 cm and the width of each view at the mid-plane ranges from 3 to 6 cm. The sensor is taller than it is wide by about 50% so the vertical extent of the view at the mid-plane is nearly 6.5 cm. The views of the arrays are about where they were designed to be. Channels 15 and 16 view the internal ring. Channels 13–14 view the inner scrape off layer of the plasma, the region where the field
Figure C-2: In-vessel calibration of photodiode array views. The radius of tangency that corresponds to the location of the laser is plotted versus the channel on which the laser light is observed for array 1 and array 2. A solid line shows the least squares fit to the combined data.
lines intersect the internal coil. Channels 1–12 of both arrays view plasma the closed field line region of the plasma at the core. Of course all channels are chord integrated so that all view the open field line region at the out edge of the plasma. These viewing chords are illustrated in a top down view of the vacuum vessel in Fig. 4-1.

<table>
<thead>
<tr>
<th>Element #</th>
<th>Array 1 $R_\perp$ (cm)</th>
<th>Array 2 $R_\perp$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>118.8</td>
<td>119.4</td>
</tr>
<tr>
<td>02</td>
<td>115.5</td>
<td>116.3</td>
</tr>
<tr>
<td>03</td>
<td>112.0</td>
<td>112.9</td>
</tr>
<tr>
<td>04</td>
<td>108.3</td>
<td>109.3</td>
</tr>
<tr>
<td>05</td>
<td>104.4</td>
<td>105.6</td>
</tr>
<tr>
<td>06</td>
<td>100.4</td>
<td>101.7</td>
</tr>
<tr>
<td>07</td>
<td>96.1</td>
<td>97.5</td>
</tr>
<tr>
<td>08</td>
<td>91.7</td>
<td>93.3</td>
</tr>
<tr>
<td>09</td>
<td>87.1</td>
<td>88.8</td>
</tr>
<tr>
<td>10</td>
<td>82.3</td>
<td>84.2</td>
</tr>
<tr>
<td>11</td>
<td>77.4</td>
<td>79.4</td>
</tr>
<tr>
<td>12</td>
<td>72.3</td>
<td>74.5</td>
</tr>
<tr>
<td>13</td>
<td>67.1</td>
<td>69.4</td>
</tr>
<tr>
<td>14</td>
<td>61.8</td>
<td>64.3</td>
</tr>
<tr>
<td>15</td>
<td>56.3</td>
<td>59.1</td>
</tr>
<tr>
<td>16</td>
<td>50.8</td>
<td>53.7</td>
</tr>
</tbody>
</table>

Table C.4: In-vessel photodiode array calibration results.

### C.4 Chordal Intersections

The chords from array 2 intersect the chords from array 1. By looking at the portions of the signals from the intersecting chords that are highly correlated and in-phase, we may be able to get some information about the local characteristics of the plasma in the region where the chords intersect. The equation for a line that defines each viewing chord in cylindrical coordinates with the origin at the center of the vacuum vessel can be written,

$$r(\theta) = r_\perp \csc(\theta - \phi_\perp) \quad \text{(C.9a)}$$

$$z(r, \theta) = \sin\left(\frac{\pi}{36}\right) \sqrt{\left(r_\perp^2 + A_r^2 - 2r_\perp A_r \cos(A_\theta - \theta)\right)} \quad \text{(C.9b)}$$
where $r_\perp$ is the radius of tangency for the chord and $\phi_\perp$ is the polar angle that defines the tangency point given by $\phi_\perp = A_\theta - \cos^{-1} \left( \frac{r_\perp}{A_r} \right)$ where $A_r$ and $A_\theta$ define the location of the array in polar coordinates (see point A in Fig. C-1) given in §4.1.2. Two chords (from different arrays) will intersect at

$$\theta = \tan^{-1} \left( \frac{r_{\perp 1} \cos \phi_{\perp 1} - r_{\perp 2} \cos \phi_{\perp 2}}{r_{\perp 2} \sin \phi_{\perp 1} - r_{\perp 1} \sin \phi_{\perp 2}} \right)$$  \hspace{1cm} (C.10)

where the subscripts 1 and 2 indicate the chosen from arrays 1 and 2, respectively.

The vertical positions of the viewing chords will not intersect, but the vertical extent of the views is nearly 6.5 cm. With the appropriate choice of chord from array 2, it is possible to find intersection points for all 16 channels on array 1 that are separated by less than 6 cm vertically.
Bibliography


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