EVIDENCE FOR MAGNETIC FLUCTUATIONS AS THE HEAT LOSS MECHANISM IN ALCATOR

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ABSTRACT

It is argued that resonant magnetic fluctuations are the mechanism of anomalous electron heat loss in Tokamaks. The experimental and theoretical support for this assertion are given.
While anomalous electron heat loss is clearly one of the major physics problems in toroidal confinement, the process or processes responsible for it are still largely unknown. Traditionally, anomalous losses have been attributed to drift waves of various kinds\(^1\), but the experimental evidence for this assumption is ambiguous at best. More recent measurements\(^2\) seem to rule out drift waves, for densities \(> 10^{14}\), at least in the usual quasilinear sense. However, even at fluctuation levels below those required to produce significant quasilinear transport, the inherently non-linear effect of magnetic surface destruction can lead to sizeable transport. These points were emphasized recently by Callen\(^3\). There are basically two forms this magnetic surface destruction can take. In the first\(^3\), the so-called "magnetic flutter" model, one has essentially coherent island structures most of the time. These grow up out of noise to an amplitude at which islands overlap. Stochasticity of the field lines ensues and presumably damps the underlying drift waves, allowing the original equilibrium to reform, wherein the process repeats itself. Alternatively one can have some quasi-steady saturated turbulence level in which "stochasticity" prevails\(^4\). Either form can lead to substantial enhancements of thermal transport.

In this letter we make a connection between these magnetic fluctuations and an anomaly in the soft X-ray spectrum that has been found persistently in Alcator\(^5\). This anomaly cannot be explained by classical processes. We show that magnetic fluctuations give rise to an enhanced suprathermal tail in the electron distribution function which is related to the thermal flux. The tail and energy flux are both gauged by the same parameter, \(\tau_E^*\), the energy confinement time. By fitting the X-ray spectrum we determine a \(\tau_E^*\). The \(\tau_E^*\) so obtained agrees
with the bulk energy confinement measurements both in absolute magnitude and scaling with the plasma density.

Soft X-ray spectra between 1 and 7 keV have been collected from the Alcator tokamak using a Si(Li) crystal and pulse-height analysis system. The X-ray spectrum from a non-isotropic electron distribution function, \( f(\vec{v}) \), may be calculated from the convolution

\[
\frac{d\nu}{d\nu} = \int_{-\infty}^{\infty} \nu^2 d\nu \int_{0}^{2\pi} \int_{0}^{\pi} \phi \sin \theta d\phi d\theta \left[ \frac{\nu N^2}{h} f(\vec{v}) \sum I_i(h\nu, \vec{v}) \right]
\]

where \( d\nu/d\nu \) is the observed power spectrum, \( \nu \) is the photon energy, \( f(\vec{v}) \) is the normalized electron distribution function (\( \int \nu^2 dv d\Omega f(\vec{v}) = 1 \)), and \( I_i(h\nu, \vec{v}) \) is proportional to the X-ray production cross-section for a given polarization. For a Maxwellian distribution (\( f_{\text{Max}} \)), Eq.1 yields a nearly exponential spectrum \( d\nu/d\nu \sim e^{-\nu/T} \).

Fig.1 shows a typical non-thermal spectrum taken during the steady state portion of a 145 kA discharge with a peak density of \( 3 \times 10^{14} \text{ cm}^{-3} \), a toroidal magnetic field of 60 kG and \( Z_{\text{eff}} \sim 1 \). The feature at 2.7 keV is due to L line radiation from the Molybdenum introduced into the plasma by interaction with the limiter. The open circles were taken through a .005 cm Beryllium filter to enhance the high energy region. A temperature of 850 eV is deduced by fitting the points between 1.3 and 2.1 keV and accounting for density and temperature profiles. The portion of the spectrum between 3.5 and 6.25 keV is best described by a temperature of 1500 eV. During a thermal discharge (with higher density) the temperatures obtained in these two regions are the same. The lower curve in Fig. 1
represents the spectrum from a temperature of 850 eV. This non-thermal behaviour occurs at all radii in the plasma.

This high energy component cannot be due to pulse pileup because it persists at low counting rates (< 10 kHz) with a short (1/2 μs) shaping time constant in the amplifier. A Be filter is used to attenuate the low energy events which at high counting rates would combine to form false high energy counts. The shape of this spectrum is not characteristic of a piled up spectrum which would have only one "temperature" and not a tail. Furthermore this tail is seen to disappear as the density and subsequent counting rate are increased.

The main classical modifications of the Maxwellian distribution are a consequence of the applied electric field, E. The Spitzer-Härm distribution\(^{(6)}\) is

\[
f(\tilde{v}) = f_{\text{max}} \left(1 + \frac{E}{E_R} \mu \frac{D(\tilde{v})}{A}\right).
\]

Here \(E_R\) is the runaway electric field = \(\frac{4\pi e^3 n}{m} \ln A\), \(\mu\) is the cosine of the angle between the incoming electron velocity vector and \(E\), and \(D(\tilde{v})/A\) is roughly equal to \(\frac{1}{6} (\frac{\tilde{v}}{v_e})^4\), 

\(v_e = \sqrt{\frac{8kT_e}{m}}\). Due to the dipole nature of bremsstrahlung, this perturbation makes no contribution to the spectrum when inserted in Eq.(1) because it is odd in \(\mu\). The distribution function of neo-classical theory\(^{(7)}\) is also odd in \(\mu\) and likewise does not change a thermal spectrum. The bulk-runaway transition region in the distribution function of Kruskal and Bernstein\(^{(8)}\) does produce a tail on a thermal spectrum since it has terms even in \(\mu\),

\[
f(\tilde{v}) = f_{\text{max}} \left\{ 1 + \frac{1}{6} \mu \frac{E}{E_R} (\frac{\tilde{v}}{v_e})^4 + \left[ \frac{1}{336} + \frac{1}{12\pi} \mu^2 \right] (\frac{E}{E_R})^2 (\frac{\tilde{v}}{v_e})^8 + \ldots \right\}.
\]
However, this contribution is too small (by several orders of magnitude) due to the size of the coefficients of the second order term in $\frac{E}{E_R}$, (which is itself small - $\frac{E}{E_R}$ in Fig.1 is .003). Runaway electrons ($1/2 \, m v^2 > 100$ keV) are too few to account for the distortion in these high density, low $E/E_r$ discharges, and even if sufficiently numerous would produce the wrong spectral shape. In summary, the tail on the X-ray spectrum cannot be a result of the classical electron distribution function.

The quasilinear effects of drift waves cannot account for the spectrum for two reasons. First, drift waves are resonant with slow electrons, $\omega/k_n v_e << 1$, (in the sheared field case, only a small portion of the eigenmode has $\omega/k_n v_e > 1$). Second, if they were resonant at high phase velocities, the density fluctuations in the center on Alcator[2] ($\bar{n}/n < 10^{-2}$) are known to be too small to account for the energy losses\(^{(3)}\). It follows from what we are about to show that they then cannot explain the X-rays, either.

In the following, for definiteness, we consider the consequences of the "stochasticity" limit of magnetic fluctuations. It will then become apparent which aspects of the model are necessary, allowing us to generalize the conclusions and make a comparison with "magnetic flutter" or other possible models.

Several authors\(^{(4,9)}\) have considered the diffusion of test particles in a stochastic magnetic field. Estimates of heat transport are made by equating the diffusion coefficient to the thermal conductivity. Of course to be more accurate one must use the appropriate electron kinetic equation. Its solution gives distortions (from Maxwellian) associated with the energy flow.
From the test particle diffusion picture one is led to add a term 
\[ \frac{\partial}{\partial x} \frac{\partial}{\partial x} f_e \] to the electron kinetic equation (x, here, is the radial variable). However, such a procedure omits the possible ambipolar potential and gives a kinetic equation which does not conserve particles locally, (except in the trivial case \( \frac{\partial f_e}{\partial x} = 0 \)). The radial ambipolar field, \( E_A \), will cause electrons to lose energy as they step out radially (and vice versa). Thus the random walk occurs not in space but along paths in the energy-radius \((w,x)\) plane given by \( \frac{dx}{ds} = 1 \); \( \frac{dw}{ds} = - \frac{eE_A}{m} \). More generally, then, stochastic diffusion gives rise to a term \( \frac{\partial}{\partial x} \frac{\partial}{\partial x} f_e \), where \( \frac{\partial}{\partial x} = \frac{\partial}{\partial x} - \frac{eE_A}{m} \frac{\partial}{\partial w} \), and the appropriate kinetic equation is therefore,

\[ \frac{d f_e}{d t} = \frac{e}{m} E^T \frac{1}{\partial w} f_e + C(f_e) + \frac{\partial}{\partial x} \frac{\partial}{\partial x} f_e, \quad (2) \]

where \( E^T \) is the applied electric field, \( C \) is the collision operator, and \( w = v^2 \) has the dimensions of velocity squared. In the limit most appropriate to Tokamaks, where the collisional mean free path is longer than the stochastic mixing length, \( L_c = \pi R / \ln(\pi S/2), (\text{less than } \pi R, \text{ when } S >> 1 \text{ as required for stochasticity}) \), and the diffusion coefficient is given by\[^{[4]}\]

\[ D = \frac{\pi R}{\sqrt{n_m}} \sum_{n,n} \left| \frac{B_{mn}}{B} \right|^2 \delta (n - \frac{m}{q}). \quad (3) \]

For an explanation of the notation and some of the background, we refer the reader to Ref.[4].

A more rigorous derivation of eq.(2) can be given by solving the drift kinetic equation directly for the slow evolution of the smooth distribution function\[^{[10]}\]. The derivation shows that a) stochasticity of the lines is necessary
to obtain eqs. (2) & (3); and b) the δ-function in (3) is actually a resonance function of sufficient width to make several modes contribute to \( D \) at each spatial point (\( D \) is then smooth as it would not be from a strict application of eq. (3)).

Equation (2) is elliptic and thus, technically requires boundary data, i.e. the velocity space distribution function on bounding spatial surfaces. Since, in the present case, \( D \) is small (specifically \( D \ll v_e a^2 \)) this mathematical requirement manifests itself only in thin boundary layers. For our present purpose of calculating distortions of a nearly Maxwellian \( f_e \) these boundary requirements may be neglected.

We thus seek a solution to (2) in the steady state by an expansion in powers of \( E/E_R \), \( f = f^{(0)} + f^{(1)} + f^{(2)} + \cdots \), regarding \( D \) as second order. The ordering is suggested by both the size of \( D \) and the fact that the diffusive loss process must balance ohmic heating, which is second order in \( E/E_R \).

To lowest order we have \( C(f_e) = 0 \), which says that \( f_e \) is a local Maxwellian but gives no information on the density and temperature. This is provided by the transport equations implied by integrability conditions on the higher order equations.

The first order equation is

\[
0 = C(f') - \frac{e}{m} E \cdot \frac{\partial}{\partial x} f^0
\]

or the Spitzer-Härm problem. The solubility conditions on (4) permit the inversion of the \( C \) operator to obtain \( f^{(1)} \). These are obtained by applying annihilators
of C to equation (4). In the present case, these are the particle and energy moments. (C allows momentum transfer to the ions). This order gives no information on \( n \) and \( T_e \), since both terms in (4) annihilate, but provides \( f^{(1)} = f_{SH} \), and the electrical conductivity.

At second order,

\[
O = C(f^2) + \mathcal{L} D f^0 - \frac{e}{m} E \cdot \frac{\partial}{\partial x} f. \tag{5}
\]

The particle moment of (5) requires no particle fluxes on the electron time scale, or an ambipolarity constraint,

\[
O = \frac{\partial^2}{\partial x^2} n \nu_e + \frac{e E^A}{2T_e} \nu_e^2 \frac{\nu_e}{\nu_e} + \frac{\partial}{\partial x} \frac{e E^A}{T_e} n \nu_e
\]

\[
+ n \nu_e \left( \frac{e E^A}{2T_e} \right)^2. \tag{6}
\]

which will be recognized as a Riccati equation for \( E^A \). From the energy moment, we find, using (6), the heat transport equation

\[
O = J \cdot E + \frac{2}{\nu_e} D \left[ \nu \frac{\partial^2 T_e}{\partial x^2} + \frac{\nu_e}{\nu_e} \left( \frac{\partial n \nu_e}{\partial x} + \frac{e E^A}{2T_e} n \nu_e \right) \frac{\partial T_e}{\partial x} \right]. \tag{7}
\]

where \( D = D(\nu_e) \). For simplicity in writing (7) we have neglected heat transfer to the ions (a process involving the low energy part of \( f_e \) which is not of interest here).

The distribution function distortion associated with heat transport is obtained by solving eq.(5) for \( f^{(2)} \), once the solubility conditions (6) & (7) are
satisfied. The dominant contribution, for $u = v/v_e \gg 1$, comes from the spatial diffusion term,

$$f^{(2)} = - C^{-1} \left[ \frac{\partial}{\partial x} D \frac{\partial}{\partial x} f^0 \right]$$  \hspace{1cm} (8)

where $C^{-1}$ denotes the inverse of the collision operator. For $u^2 \gg 1$, the $C$ operator is,

$$C(f_e) \approx \frac{\gamma}{u^3} \frac{1}{\delta \mu} (1 - \mu^2) \frac{\partial}{\delta \mu} f_e + \frac{\gamma \mu}{u^2} \frac{1}{\delta \mu} \left[ f_e + \frac{1}{u} \frac{\partial f_e}{\partial \mu} \right]$$  \hspace{1cm} (9)

By writing $f_e = e^{-u^2/2} p$, the second term of (9) becomes $\frac{\gamma}{u^2} \left[ -e^{-u^2/2} \partial p/\partial u + O(u^{-2}) \right]$, and it follows that $C$ can be inverted by a Legendre polynomial expansion in $u$ and direct integration for the $u$ dependent coefficients. The result, to order $u^{-2}$, is

$$f^{(2)} = \frac{D_0}{\nu} \left| \frac{d \ln T_e}{dx} \right| f_{\text{MAX}} \sum_m P_m(\mu) \frac{a_m}{4(8 + m(m+1))} \left( \frac{\nu}{v_e} \right)^B$$  \hspace{1cm} (10)

where $a_m = \frac{2m + 1}{m + 2} (P_{m-2}(0) + P_m(0))$. Defining the local energy confinement time,

$$\tau^*_E = 1/D_0 \left| d \ln T_e/dx \right|$$  \hspace{1cm} (10)

so that the perturbation to the Maxwellian measures $\tau^*_E$.

The x-ray spectrum can now be found by using this distribution function in eq. (1). The fitting procedure was to first find the bulk temperature by a fit
to the low energy ($h\nu<2.1$ keV) points. This gives the lower curve in Fig. (1). The parameter $T^*$ is then varied to fit the tail, with the overall spectra shown by the upper curve in Fig. (1). Although $f(2)$ is of order $f(0)$ as a result and perturbation theory is not a priori valid, the series it generates is correct, as can be shown by a proper asymptotic procedure paralleling region II of Ref. (9). (The spectra of Fig. (1) includes the next term of this series.)

A sequence of discharges of varying density but constant current and toroidal field (145 KA and 60 kG) were similarly analyzed. The values for the confinement times are shown as open circles in Fig. (2). Shown for comparison are confinement times, $\tau_\text{E}$, obtained by the standard method (11).

A comparison such as Fig. (2) has some obvious limitations, notably the discrepancy between the local values of $\tau_\text{E}^* = 1/D_0|d\ln \tau_\text{E}/dx|^2$ and the global parameter, $\tau_\text{E}$. In fact, we have treated $\tau_\text{E}^*$ as a constant, while retaining the radial dependence of the remaining parameters in $f_\text{E}$. For a more quantitative comparison one needs profiles of spectra (which are obtainable in practice) to determine $D_0$ as a function of radius and then explicit transport code calculations of $\tau_\text{E}$. In view of these limitations the agreement between the two $\tau_\text{E}$ determinations is rather remarkable, particularly since the slopes of the two curves are virtually identical. Without including correct profiles better agreement in magnitude is not meaningful.

At this point what we have shown is that when a kinetic equation of the form (2) is valid, the amount of $D$ required to account for the soft x-rays also accounts for the observed anomalous losses, and vice versa. The evidence, then, suggests that eq. (2) is appropriate for Tokamak plasmas. Magnetic fluctuations are the only known process that can lead to an equation of the form (2), consistent with other observations. Thus, while it is conceivable that some very unusual drift wave could be resonant with high energy electrons and give an equation like (2) in
quasilinear theory, the D implied by observed density fluctuations (2) is too small to explain either the X-rays or the energy transport. Our tentative conclusion is that resonant magnetic fluctuations are responsible for anomalous electron heat loss in Alcator and presumably similar tokamaks as well.

A definitive conclusion should be possible through measurement of the profiles, establishing a consistent fitting procedure, and possibly extending the spectra to higher energy. The latter measurement also has a bearing on which of the two competing models, "magnetic flutter" or "stochasticity", is correct. It has been suggested\(^{(12)}\) that magnetic flutter will also give an equation like (2), since it is essentially a diffusion process in which the step size and time are the average island width and lifetime. The diffusion coefficient is then independent of velocity and leads to a perturbation of the form \( f(2) \sim \frac{1}{\sqrt{T_E}} \left( \frac{v}{v_e} \right)^7 f_{\text{max}} \) as compared to eq. (11) for stochasticity. These differences are indistinguishable in the present analysis but ultimately lead to different asymptotic behavior at high \( v/v_e \) which should be measureable.
To show that quasilinear effects are not sufficient one can use the following simple estimate of the thermal conductivity:

\[
X_{QL} \sim \sum \gamma_k \frac{k^2}{\omega^*} \left| \frac{cT e}{\epsilon B} \right|^2 \left| \frac{e \phi}{T_e} \right| \sim a^2 \sum \gamma_k \tilde{n}^2/n^2
\]

The largest possible value of \(X_{QL}\) is obtained by taking \(\gamma \sim \omega_e \sim 3 \times 10^5\) and the upper limit of \(\tilde{n}/n < 10^{-2}\) in the center implying a minimum confinement time \(\tau_{QL} \sim a^2/X_{QL} > 30\) msec which is larger than the observed confinement time.
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Figure Captions

Fig. 1 An x-ray spectrum for a typical non-thermal Alcator discharge. The lower curve is from an 850 eV Maxwellian distribution, the upper curve includes the perturbation induced by magnetic fluctuations.

Fig. 2 $\tau_E$ vs. $n_0$. The open circles are $\tau_E^*$ deduced from fitting the tail of the x-ray spectrum; uncertainties are about a factor of 2. Solid points are $\tau_E$ determined from energy content divided by the input power. The dashed line is from Ref. 11. The solid line is the best linear fit to the points.
$n_0 = 3.0 \times 10^{14} \text{ cm}^{-3}$
$I = 145 \text{ KA}$
$B_T = 60 \text{ KG}$
$T_0 = 850 \text{ eV}$

**Fig. 4**
\[ I = 145 \text{ KA} \]
\[ B_T = 60 \text{ KG} \]

\[ \tau_E (\text{ms}) \]

\[ n_0 (10^{14}/\text{cm}^3) \]

\text{Fig 2}