STATUS REPORT

"FOCUSING OF INTENSE ION BEAMS"

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A. Critical Problem Areas

In the Winterberg scheme, a charge neutralized intense high-Z ion beam is axially injected into a mirror magnetic field, where it undergoes isothermal compression without significant loss of forward beam velocity. During compression, an azimuthal current is induced in the beam which then leads to ohmic heating of the beam. Simultaneously, the beam density increases to a collision-dominated regime. Thus, during this process, energy balance between ohmic heating, radiation loss (Bremsstrahlung, line emission, and recombination), and kinetic energy increase due to the magnetic compression becomes important for determination of the final beam configuration. Winterberg's calculation anticipates a final beam configuration of about $10^{18}$ ions/cm$^3$ and a radius of 0.1 cm. A critical temperature $T_c$ is identified such that for $T > T_c$, the radiation cooling time is less than the ohmic heating time.

In order to assess the feasibility and determine the operating conditions, we have identified the following critical problem areas as essential for understanding the Winterberg scheme.

1. Development of a self-consistent theory giving the evolution of such beam variables as density, parallel and perpendicular temperatures, beam velocity and current during compression by the magnetic field. Such a theory may be obtainable by extending presently available theories of ion ring compression schemes to include collisional heating and radiation. In this regard, the relative heating and cooling rates depend critically on the above mentioned beam variables.

2. In Winterberg's work, the radiation rate is calculated on the basis of an approximate radiation formula including Bremsstrahlung, line emission
and recombination radiation (cf. Artsimovich, 1964). However, this formula is only an approximate estimate for a plasma in steady-state lasting significantly longer than the confinement and compression time scales given by Winterberg. Correction to this formula or a new formula could affect the radiation rate, thus the overall energy relation critically.

3. The radiation cooling mechanisms are effective only if the beam is collision-dominated. If the beam is not sufficiently collisional except at the end of the compression stage, then the bulk of the cooling occurs after compression. In this regard, it is important to understand the dynamic evolution of an intense high-Z ion beam. Another question of interest is the stability of the intense high-Z ion beam during and subsequent to compression. Due to the curvature of the magnetic field lines, stability properties against gross MHD modes may be favorable.

We will report on progress in Items #1 and #3 in the next status report.
B. Equilibrium Properties of an Intense Ion Beam Propagating Through a Background Plasma

Under the auspices of the present grant we have investigated the equilibrium properties of a rotating, nonrelativistic, cylindrically symmetric ion beam propagating parallel to a uniform applied magnetic field $B_0 \hat{z}$ with constant axial velocity $V_z \hat{z}$. Equilibrium properties are calculated for the specific choice of ion beam distribution function corresponding to thermal equilibrium,

$$f_0^b (H, P_\theta, P_z) = \text{const.} \times \exp \left[ -(H - \omega_b P_\theta - V_z P_z)/T \right],$$

where $H$ is the energy, $P_\theta$ is the canonical angular momentum, $P_z$ is the axial canonical momentum, $T = \text{const.}$ is the temperature, $V_z = \text{const.}$ is the mean axial velocity of the ion beam, and $-\omega_b = \text{const.}$ is the angular velocity of mean rotation. In the present analysis, we assume that the net current carried by the background plasma electrons and ions is equal to zero, so that the magnetic self fields, $B_z^s(r)$ and $B_\theta^s(r)$, are generated entirely by the mean rotational and axial motion of the ion beam. However, we do allow for the possibility that the background plasma electrons and ions do not exactly neutralize the ion beam charge. In particular, it is assumed that the equilibrium plasma electron density $n_e(r)$, plasma ion density $n_i(r)$, and ion beam density $n_b(r)$ are related by $n_e(r) - n_i(r) = f n_b(r)$, where $f = \text{const.} = \text{fractional charge neutralization}$. Within the context of these assumptions, we have investigated a broad range of thermal equilibrium properties of the choice of beam distribution function in Eq. (1). These investigations include: (a) exact analytical solutions of the equilibrium Maxwell equations in the special limiting cases $\omega_b = 0$ and $\beta_z^2 = 1-f$, (b) properties of the general solutions when $\omega_b \neq 0$ and $\beta_z^2 \neq 1-f$, (c) conditions (constraints on $\omega_b$, $\hat{\omega}_b$, etc.) for
existence of radially confined equilibria with $n_b(r^\to\infty) = 0$, and (d) numerical solutions in regimes where solutions are not accessible analytically. The details of these investigations are summarized in a technical report previously forwarded to the Department of the Army.
C. Filamentation Stability Properties of an Intense Ion Beam

Propagating Through a Background Plasma

One of the most important instabilities that characterizes a relativistic electron beam propagating through a background plasma is the filamentation instability, which is an ordinary-mode electromagnetic instability \((\delta E \parallel B_0,\delta B \perp B_0\) and \(k \perp B_0\)) driven by the beam-plasma energy anisotropy. Under the auspices of the present grant, we have investigated the filamentation stability properties of an intense ion beam \((j=b)\) propagating through a plasma background \((j=e,i)\).

We consider the inner region of a beam-plasma system with constant interior density

\[
n_j(r) = n_j = \text{const.} \quad (j=b,e,i). \tag{2}
\]

and uniform axial velocity

\[
v_zj(r) = v_j = \text{const.} \quad (j=b,e,i). \tag{3}
\]

In a long cylindrical system, the equilibrium magnetic field can be expressed as \(B_0 \hat{z} + B^S(r) \hat{r}\), and the equilibrium electric field can be expressed as \(E^S(r) \hat{r}\), where \(B_0 = \text{const.}\) is the applied axial magnetic field, and the self magnetic and electric fields are given by

\[
B^S_0(r) = 2\pi \sum_k \hat{\beta}_k e_k \beta_0 \alpha_k \hat{\beta}_k \hat{r}, \tag{4}
\]

\[
E^S_r(r) = 2\pi \sum_k \hat{\beta}_k e_k \beta_0 \alpha_k \hat{\beta}_k \hat{r}, \tag{5}
\]

where \(\beta_k = \hat{v}_k/c\), and \(\sum_k\) denotes summation over \(k = b, e, i\). For present purposes we assume that the electrons \((j=e)\) and plasma ions \((j=i)\) are cold with \(T_j = 0\), and that the ion beam distribution function \(f_b^0\) is an isotropic Maxwellian in a frame of reference moving axially with the beam, i.e., in a frame of reference
moving with axial velocity $\hat{V}_b = \text{const}$. After some careful algebraic manipulation that incorporates the important influence of equilibrium self fields on stability behavior, the dispersion relation for ordinary-mode electromagnetic perturbations with $\delta E \parallel B_0 \hat{z}$, $\delta B \perp B_0 \hat{z}$ and $k \perp B_0 \hat{z}$ can be expressed as

$$
\omega^2 = c^2 k^2 + \sum_{p,j} \omega_{pj}^2 + \sum_{j=e,i,b} \frac{\omega_{pj}^2}{\omega^2 - (\omega_+ - \omega_-)^2 c^2 k_B^2}
$$

$$
+ \omega_{pj}^2 \sum_{n=-\infty}^{\infty} \frac{n^2 (\omega_+ - \omega_-)^2}{\omega - n^2 (\omega_+ - \omega_-)^2} \left(1 + \frac{m_b c^2 \beta_b^2}{T_b}\right) I_n(\lambda_b) \exp(-\lambda_b),
$$

(6)

where $\omega_{pj}^2 = 4\pi \bar{n}_j e_j^2 / m_j$ is the plasma frequency-squared, $T_b$ is the beam temperature, $\lambda_b$ is defined by $\lambda_b = k_T / m_b (\omega_+ - \omega_-)^2$, $k_\perp$ is the perpendicular wavenumber, $\omega$ is the complex eigenfrequency, $I_n(x)$ is the modified Bessel function, and $(\omega_+ - \omega_-)^2$ is defined by

$$
(\omega_+ - \omega_-)^2 = \omega_{cj}^2 \left\{ 1 + \frac{8\pi e_j}{m_j \omega_{cj}^2} \sum_k \hat{n}_k e_k (1 - \beta_k \beta_j) \right\}.
$$

(7)

In Eq. (7), $\omega_{cj} = e_j B_0 / m_j c$ is the cyclotron frequency, and $e_j$ is the particle charge for species $j$. Comparing Eqs. (4), (5) and (7), we note that the definition of $(\omega_+ - \omega_-)^2$ includes the influence of equilibrium self fields. Moreover, $(\omega_+ - \omega_-)^2 = \omega_{cj}^2$ is the limit of exact charge neutrality ($\sum \hat{n}_k e_k = 0$) and current neutrality ($\sum \hat{n}_k e_k \beta_k = 0$).

The dispersion relation in Eq. (6) has been extensively investigated for slow-wave perturbations satisfying

$$
\left| \frac{\omega^2}{c^2 k_\perp^2} \right| \ll 1.
$$

(8)

Assuming that the background plasma ions carry zero axial current ($\beta_i = 0$) and that the plasma electrons satisfy $(\omega_{pe}^2 / \omega_{ce}^2) \beta_e^2 \ll 1$, it is found that the most
interesting instability regime occurs for perturbations in which the beam ions are "unmagnetized" with

$$|k_{\perp}v_b|, |\omega| > |\omega^- - \omega^+|$$

(9)

where $v_b = (2T_b/m_b)^{1/2}$ is the thermal speed of the beam ions. A detailed analysis of the resulting approximate dispersion relation shows that the necessary and sufficient condition for filamentation instability is given by

$$\frac{\omega^2}{v_b^2} - \frac{2V^2}{2v_b^2} > \frac{\omega^2}{v_b^2}.$$

(10)

Moreover, whenever Eq. (10) is satisfied, the corresponding range of unstable $k_{\perp}$-values is given by

$$0 < k_{\perp}^2 < k_0^2 \equiv \frac{\omega^2}{v_b^2} - \frac{2V^2}{2v_b^2} - \frac{\omega^2}{v_b^2}.$$

(11)

In Eqs. (10) and (11), $V_b$ is the directed axial velocity of the ion beam, and $v_b = (2T_b/m_b)^{1/2}$ is the thermal speed.

The most striking feature of Eq. (10) is that an extremely small value of beam energy spread will completely stabilize the filamentation instability. In particular, instability is absent whenever the effective ion beam temperature satisfies $T_b/m_b V_b^2 > m_e/m_i$, a condition that is not difficult to satisfy experimentally.
D. Tearing-Mode Stability Properties

Our recent tearing-mode stability analysis for an intense ion beam described by the thermal equilibrium distribution in Eq. (1) [J. Chen and R. C. Davidson, Bull. Am. Phys. Soc. 24, 1070 (1979)] indicates that the system is stable except for sufficiently large rotation velocity $\omega_b$ that the axial magnetic field reverses on axis, i.e., $B_z^0(r=0) < 0$. Evidently, this instability should not occur for system parameters of interest in the Winterberg scheme.