

PROGRESS REPORT  
TRAJECTORY ANALYSIS OF THE ELECTRONS IN A  
MAGNETRON-TYPE GUN

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## ABSTRACT

In this report modification is introduced in the Herrmannsfeldt electron trajectory code to analyze the trajectories in magnetron injection guns with perveances  $\frac{I}{V^{3/2}}$  in the range of  $10^{-5}$  Perv. When tested on a particular magnetron gun, the computed perveance agreed well with experimental data for values  $\frac{V}{B^2} \lesssim 0.006$ , the flow was nearly laminar with  $\frac{v_{\perp}}{v_{\parallel}}$  values reaching 12%. For larger voltages, both experiment and calculations indicate a "turbulent" electron flow.

## I. Introduction

The analysis of the detailed motion of electrons emitted from a crossed-field gun is very difficult, especially if space charge effects are important. The magnetron gun has found application in strong beam-plasma discharge experiments and in gyrotron mm wave oscillators. In the first case, the operation is usually space charge limited and the perveance ( $I/V^{3/2}$ ) is in the range  $5 - 15 \times 10^{-6} \text{ A/V}^{3/2}$ , and the literature indicates little or no attention being paid to the ratio of perpendicular velocity to parallel velocity ( $v_{\perp}/v_{\parallel}$ ) of the emerging beam. In contrast gyrotron guns are operated temperature limited at fairly low perveance ( $\sim 10^{-6}$ ), and considerable care is taken to achieve a uniform and desired value of  $v_{\perp}/v_{\parallel}$ . The design of gyrotron guns is usually based on computer calculations of electron-trajectories. Various computer codes have been used, among them is the Herrmannsfeldt trajectory program. [1]

In the beam-plasma experimental literature several experimenters have noted that more efficient power transfer to the plasma occurs when the incident electron beam has a significant value of  $v_{\perp}/v_{\parallel}$  (about 1/2 or so). These values were achieved by tilting the gun axis to the magnetic field axis, or by using a magnetic lens.[2] However, in many other experiments where it is implicitly assumed that  $v_{\perp}/v_{\parallel} \approx 0$ , this may not have been true. An example is the experiment by Klinkowstein and Smullin [3] where the drift cyclotron instability was quenched by injecting an electron beam into a mirror confined plasma. A magnetron

injection gun was located far outside the mirror peak. If the initial beam had any significant amount of  $v_{\perp}/v_{\parallel}$  at the gun, this would have been multiplied several times by the time it reached the plasma where the field was some four times greater.

If electron beams are to be used for plasma heating or stabilization it is important to be able to design them so as to optimize the efficiency of the beam-plasma interaction. This was the motivation for undertaking these studies.

The Herrmannsfeld SLAC program is a computer code [1] that has been developed at Stanford Linear Accelerator (SLAC) to handle problems involving the design of electron guns and particle trajectories under static electric and magnetic fields.

In Constance I experiments [3] the electron gun was located in a region having a strong magnetic field and consequently the electrons emerging from the cathode start to bend due to Lorentz force several tens of microns away from the cathode surface. This fact which characterizes all electronic emission in magnetron guns is not handled properly in the SLAC program.

In most applications using this code the magnetic field was weak enough such that it was quite reasonable to assume that electrons come out almost perpendicular to the emitting surface and remain in this perpendicular direction for several mesh units, where the mesh unit is chosen to be almost  $\frac{1}{10}$  of the cathode radius.

Consequently, the simple Child's law used to calculate emission from the cathode surface, knowing the voltage at a surface drawn parallel to the cathode and at a small distance, should be modified to account

for the complicated trajectory behind that surface.

In the next section, the electronic space charge limited emission from a planar cathode under a strong magnetic field and electric field is discussed, in Section III the algorithm used to calculate the current of each ray being emitted from the cathode as well as the three velocity components of the ray is outlined; then in section IV the modifications we introduced into the code are listed as well as the location of the FORTRAN statements that are added to the program. Finally in Section V, a trajectory analysis of the electrons in our gun is shown and general results are drawn from the analysis.

## II. Electronic Space Charge Limited Emission from a Planar Cathode Under a Magnetic Field

Consider an emitting planar surface which emits electrons with zero velocity and let us then apply a magnetic field  $\bar{B}$  inclined to the planar surface by an angle  $\theta$ , and assume that an anode exists at a distance  $d$  from the cathode with an applied voltage  $V$  (Figure 1).

It is assumed that the end effects of both the cathode and anode can be neglected, so that the actual configuration approaches the infinite electrode picture; hence due to symmetry in the  $x$ - $z$  plane, space charge density, velocity and voltage depend on  $y$  only.

The force equations for a single electron are:

$$\frac{dv_x}{dt} = \frac{eB}{m} v_z \sin \theta \quad (1)$$

$$\frac{dv_y}{dt} = -\frac{eE(y)}{m} - \frac{eB}{m} v_z \cos \theta \quad (2)$$

$$\frac{dv_z}{dt} = -\frac{eB}{m} v_x \sin \theta + \frac{eB}{m} v_y \cos \theta \quad (3)$$

For a steady state flow the current density  $J$  is constant along  $y$  and consequently the space charge density  $\rho(y)$  is given by:

$$\rho(y) = \frac{J}{v_y(y)} \quad (4)$$

From Poisson's equation

$$\frac{dE(y)}{dy} = \frac{J}{\epsilon_0 v_y(y)} \quad (5)$$

Hence

$$\frac{dE}{dt} = \frac{dE}{dy} \cdot \frac{dy}{dt} = \frac{J}{\epsilon_0} \quad (6)$$

From equations (1), (2), (3), and (6) we can get the following set of equations

$$\frac{d^2 v_x}{dt^2} + \omega_c^2 \sin^2 \theta v_x - \omega_c^2 \sin \theta \cos \theta v_y = 0 \quad (7)$$

$$\frac{d^2 v_y}{dt^2} + \omega_c^2 \cos^2 \theta v_y - \omega_c^2 \sin \theta \cos \theta v_x = -\frac{eJ}{m\epsilon_0} \quad (8)$$

$$v_z = \frac{1}{\omega_c \sin \theta} \frac{dv_x}{dt} \quad (9)$$

where  $\omega_c = \frac{eB}{m}$ .

Assume that each electron is emitted at time  $t = 0$ , so the initial conditions of the above three equations are  $v_x(0) = v_y(0) = v_z(0) = 0$ . The solution is easily found to be:

$$v_x(t) = \mu \sin \theta \cos \omega_c t + \mu \sin \theta \left[ \frac{(\omega_c t)^2}{2} - 1 \right] \quad (10)$$

$$v_y(t) = -\mu \cos \theta \cos \omega_c t + \mu \cos \theta \left[ \frac{(\omega_c t)^2}{2} \tan^2 \theta + 1 \right] \quad (11)$$

$$v_z(t) = -\mu \sin \omega_c t + \mu(\omega_c t) \quad (12)$$

Let  $x(0) = x_0$ ,  $y(0) = 0$ ,  $z(0) = z_0$  at  $t = 0$ .

Hence the parametric equations describing the electron trajectories are:

$$\omega_c(x - x_0) = \mu \sin \theta \left[ \sin \omega_c t + \frac{(\omega_c t)^3}{6} - \omega_c t \right] \quad (13)$$

$$\omega_c y = \mu \cos \theta \left[ -\sin \omega_c t + \frac{(\omega_c t)^3}{6} \tan^2 \theta + \omega_c t \right] \quad (14)$$

$$\omega_c(z - z_0) = \mu \left[ \cos \omega_c t + \frac{(\omega_c t)^2}{2} \right] \quad (15)$$

where  $\mu = -\frac{eJ \cos \theta}{m\epsilon_0 \omega_c^2}$ .

Now let us derive the electric field and voltage at position  $y$  at time  $t$ . From equation (6)

$$E(t) = \frac{Jt}{\epsilon_0} \quad (16)$$

$$\frac{dV}{dy} = -E = -\frac{Jt}{\epsilon_0}$$

$$V = -\frac{J}{\epsilon_0} \int_0^t t' v_y(t') dt'$$

$$V(t) = \frac{\mu^2}{\eta} \left[ \frac{(\omega_c t)^4}{8} \tan^2 \theta + \frac{(\omega_c t)^2}{2} - (\omega_c t) \sin(\omega_c t) + 1 - \cos(\omega_c t) \right] \quad (17)$$



$\eta = \frac{e}{m}$ , and we assumed that the cathode voltage is zero.

The above set of equations are found in [5], but a more comprehensive discussion of the theory of the planar magnetron was originally given by Brillouin [6].

### III. Algorithm for the Starting Conditions of Each Ray

The SLAC program draws a starting equipotential surface parallel to the cathode; and the distance  $d$  from the cathode to the position where each ray emerges is computed for each ray. The starting surface is divided into small sections and each ray emerges from one section and is supposed to carry all the current emitted by it.

For each ray we have three available parameters:

- i.  $V$  the potential energy at the point where the ray emerges which is denoted by the FORTRAN matrix element  $PU(6)$ .
- ii. The normal separation of the starting surface from the cathode at this same point  $d$  which is denoted by  $XO(7,K)$  &  $PH1$ .
- iii. The width of every section  $DLR$ .

The first step to calculate the emission using the set of equations (10 through (17) is to calculate the inclination of magnetic field to the starting surface  $\theta$ .

Having determined  $\theta$ , equations (14) and (17) are used to solve for  $t$  and  $\mu$ . Let us put  $\omega_c t = \tau$ , hence from (14) we get

$$\omega_c d = \mu \cos \theta \left( \frac{\tau^3}{6} \tan^2 \theta + \tau - \sin \tau \right)$$

and hence

$$V_S - V_C = \frac{\omega_c^2 d^2 \left( \frac{\tau^4}{8} \tan^2 \theta + \frac{\tau^2}{2} - \tau \sin \tau + 1 - \cos \tau \right)}{\eta \cos^2 \theta \left( \frac{\tau^3}{6} \tan^2 \theta + \tau - \sin \tau \right)^2} \quad (18)$$

$V_C$  is the cathode voltage,  $V_S$  the starting surface voltage.

Equation (18) can be solved numerically to find  $\tau$  in terms of  $V_S - V_C$  and note that for the case of a tangential magnetic field ( $\theta = 0$ ) equation (18) has no solution for  $(V_S - V_C) < \frac{\omega_c d^2}{2\eta \cos^2 \theta}$ , but for  $\theta \neq 0$  there is always a solution. The physical reason we get no solution for the above conditions is that we get a Brillouin flow for small voltages and the rays may never reach the starting surface. In this case the starting surface should be chosen closer to the cathode.

Once equation (18) is solved for  $\tau$ , we can substitute in (14) to find  $\mu$  and the current density  $|J|$ ; also by substituting in equations (10), (11) and (12) the three components of velocity  $v_x$ ,  $v_y$ ,  $v_z$  are found. This algorithm is used for each ray provided that  $\tau \gg 1$  and this requires that

$$\frac{\mu}{\omega_c} = \frac{\eta |J|_{av}}{\epsilon_0 \omega_c^3} \gg \text{Mesh unit}$$

$|J|_{av}$  is put equal to the estimated maximum current density DENS.

If this condition is not satisfied the algorithm is still correct but in order to save computation time the original algorithm is used in

which Child's law of a planar diode is applied and the velocity of the emitted electron is assumed perpendicular to the starting surface.

#### IV. Modifications to the Original Program

The source of modifications are equations (10), (11), (12), (14) and (18). They are introduced into subroutine CHILDA and in a new subroutine that is called CHILDC. Let us state these modifications in some detail.

1. Subprogram CHILDA is used to evaluate the initial conditions of each ray using the given parameters: Voltage on the starting surface, and the separation of the point of emission of the ray from the cathode.

A transfer of control to our algorithm is introduced by satisfying the condition  $\frac{\eta \text{ DENS}}{\epsilon_0 \omega_c^3} \ll \text{Mesh Unit}$ , otherwise the calculations of the initial conditions proceed as usual. The new array element CR1(K) is used to determine for ray K whether we use CHILD's law or equation (18) to find the current;  $\text{CR1}(K) = \frac{\eta \text{ DENS}}{\epsilon_0 \omega_c^3 (\text{Mesh size})}$ .

2. A new subroutine CHILDC (BR, BZ, K, TAU, & 420) is used to calculate  $\text{SINE} = \sin \theta$  and  $\text{COSN} = \cos \theta$  using BR & BZ the components of the magnetic field along the radial and axial directions of the gun, and also the inclination of the starting surface which is contained in the direction cosine of the electric field PU(1), PU(2). Then using

the bisection method one can solve equation (18) to find  $\tau$ , and from  $\tau$ , the current density,  $v_x$ ,  $v_y$ ,  $v_z$  are calculated. The velocity components along the radial, axial directions are calculated by rotating the x-y coordinates by an angle  $\theta_1$  where  $\theta_1$  is the angle between the starting surface and the axial direction

$$X0(4,K) = v_y \cos \theta_1 - v_x \sin \theta_1$$

$$X0(5,K) = v_x \cos \theta_1 + v_y \sin \theta_1$$

$$X0(7,K) = v_z$$

This subroutine returns the values of  $X0(4,K)$ ,  $X0(5,K)$ ,  $X(6,K)$  as well as the current density  $J_c$  at the position where ray K emerges.

3. The amount of current emitted by the section per radian from which the ray K emerges is given by  $J_c \text{ DLR } X0(1,K)$ , where DLR is the width of the emitting section and  $X0(1,K)$  is the radial distance of that section.

The amount of space charge inserted behind the starting surface is kept untouched.

At the end of this report a listing of the added statements is given as well as the new subroutine CHILDC, new lines or modified lines are clear in the listing.

4. An external routine is stored on file to process the plotted output of the program on the Calcomp machine of IPS at MIT.

V. Results of the Trajectory Analysis of the Gun Used in Constance I Experiments

In figure 2 a sketch of the gun used in [3] is given, more details about the gun can be found in [7].

The perveance was found to vary for different  $\frac{V}{B^2}$  ratios.

The trajectories were almost laminar for  $V/B^2$  up to 0.00612 Volt/Gauss<sup>2</sup>.

At higher voltages the flow becomes nonlaminar and the program cannot handle this case with a moderate mesh size. In this situation the flow becomes very turbulent and this might be the source of large level of noise observed in [7] for this gun. The modification used in this program gives excellent results for very strong magnetic fields and the criterion we can use to find this condition is that the parameter  $\frac{H}{\omega_c}$  should be much smaller than the mesh size.

It is worth mentioning that in this code it is not advised to decrease the mesh size without limits since the cost of running the program will increase exponentially while the stability of the solution may deteriorate.

In our gun with a mesh size of 1 mm,  $\frac{H}{\omega_c}$  becomes larger than 1 mm for  $V/B^2$  higher than 0.00612 Volt/Gauss and the program doesn't converge in this region.

However for  $\frac{\text{anode voltages}}{B^2}$  less than  $0.00612 \text{ Volts}/(\text{Gauss})^2$  the solution is convergent and is characterized by the following features:

1. The perveance is  $12 \mu\text{P}$  at  $\frac{V}{B^2} = 0.00204 \text{ Volt}/(\text{Gauss})^2$  and increases to a maximum of  $18 \mu\text{P}$  at  $\frac{V}{B^2} = 0.00612 \text{ Volt}/(\text{Gauss})^2$  which is consistent with the experimental results found in [7].
2. The beam thickness increases with voltage.
3. The average velocity in the perpendicular direction is almost 4% of the axial velocity for  $\frac{V}{B^2} = 0.00204 \text{ Volt}/(\text{Gauss})^2$  and increases to 8% at  $\frac{V}{B^2} = 0.00408 \text{ Volts}/(\text{Gauss})^2$  to almost 12% at  $0.00612 \text{ Volts}/(\text{Gauss})^2$ , which shows that the perpendicular energy increases as the voltage is increased.
4. With increased voltage the spread in the perpendicular energy increases also especially near  $\frac{V_a}{B^2} = 0.00612 \text{ Volt}/(\text{Gauss})^2$  which is the limit beyond which the flow becomes turbulent.

It should be noted that in the experiments [3] the ratio of the magnetic field at the mirror center to the magnetic field at the gun  $\frac{B_m}{B_{\text{gun}}} \approx \frac{3000}{700} \approx 4$ . The ratio between the perpendicular energy of the electrons at the mirror center to their energy at the gun is given by the above magnetic field ratio.

$$\frac{E_{\perp m}}{E_{\perp \text{gun}}} = \frac{B_m}{B_{\text{gun}}} \approx 4.$$

Thus for  $V/B^2 \approx 0.006$ , the ratio  $\frac{v_{\perp}}{v_{\parallel}}$  is almost 24% at the center of the mirror in the above experiment.

The results were obtained for slightly higher ratios of  $V/B^2$  so that we conclude that the ratio 0.24 is only a rough estimate of the quantity  $\frac{v_{\perp}}{v_{\parallel}}$  in [3].

## REFERENCES

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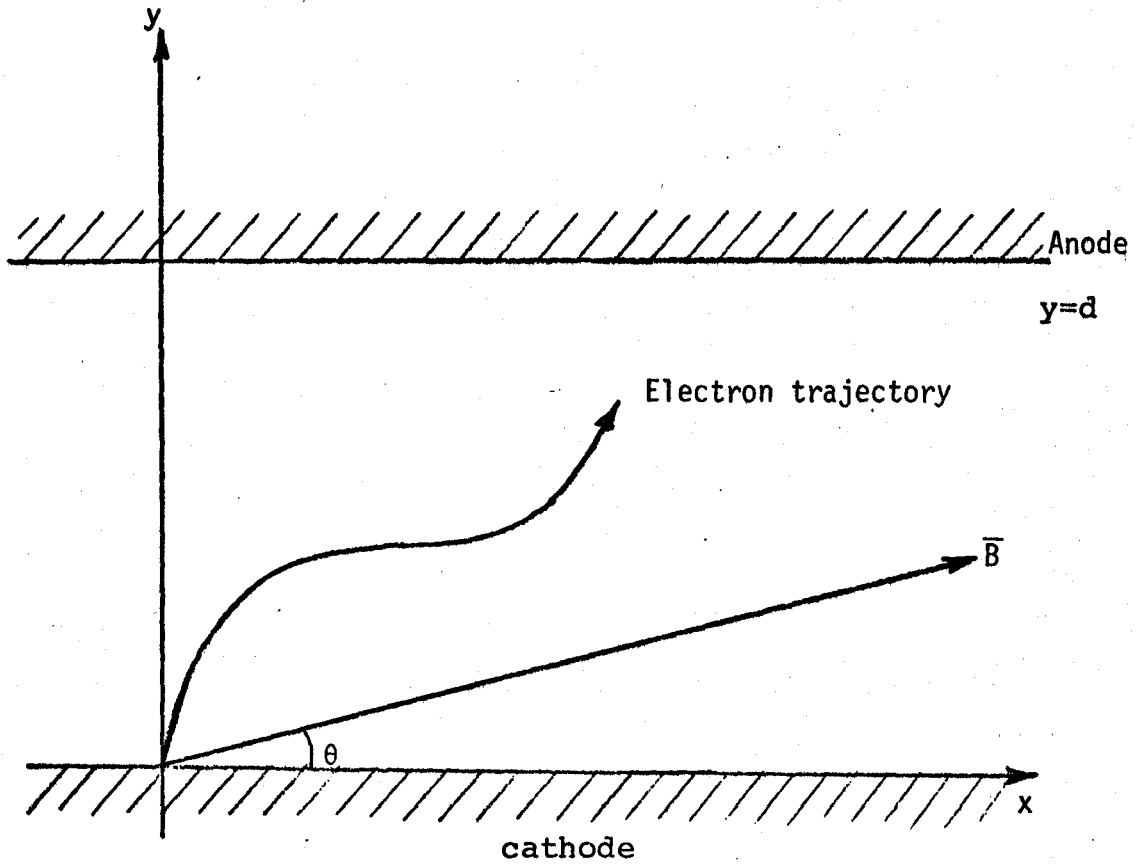


Figure 1: Planar Cathode Configuration

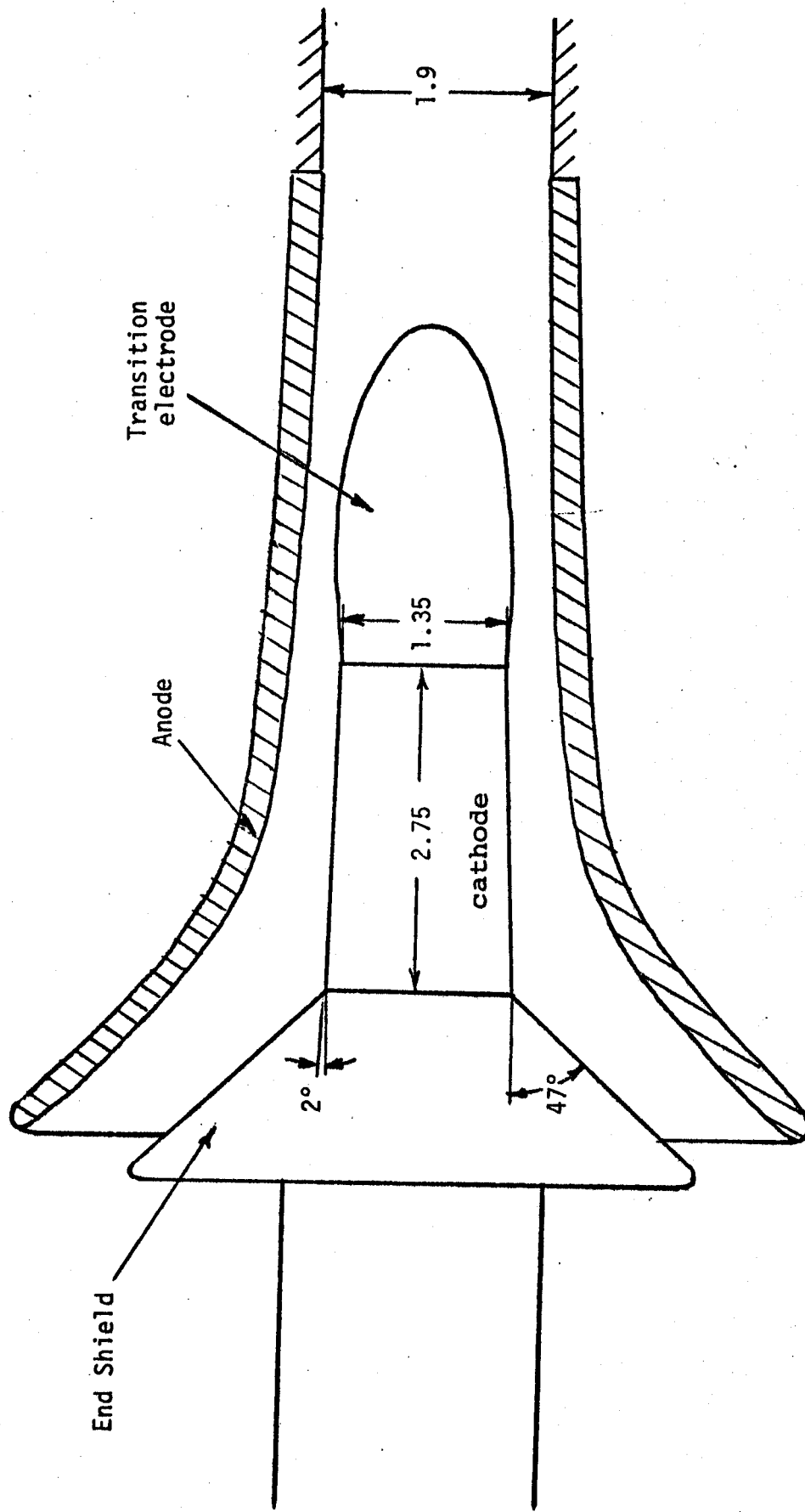


Figure 2: Conical Cathode Magnetron Gun Used in Constance I Experiments (all dimensions in centimeter)

MODIFICATIONS IN SUBROUTINE CHILDA

C \*\*\*\*\* CHILDA \*\*\*\*\*

```

      .
      .
180  DC 220 K = 1 , RAY,KRAY
      XO1 = XO (1,K)
      XO2 = XO (2,K)
      XC (3,K) = 0
      CALL PRTIAL (XO1,XO2,PU,&420)
      PHI = XO (7,K)
      DLR=XO (8,K)
      DEN = DENS * DLR
      EEV = PU (6) - POT (1)
      II (K)=0.0
      IF (EEV.LT.0.0) GO TO 220
      RHO = XO (1,K)
      ZETA=XC (2,K)
      IF (RECT) GO TO 185
      IF (.NOT.MAG) GO TO 185
      BORO2=0.0
      IF (MAG) CALL CHILMG (BORO2,&420)
      BT=DSQRT (BZ*BZ+BR*BR)
      CRI (K)=3.6511E-6*DENS/(BT*BT*BT)/(UNIT*UNIT*UNIT)
      IF (CRI (K).GT.1.0) GO TO 185
      CALL CHILIC (ER,BZ,K,TAU,&420)
      II (K)=DLR*II (K) *(RHO+IAX)
      GO TO 205
185  CRI (K)=2.0
      IF (XO (9,K) .NE. 0.0) GO TO 200
      IF (BETA2 .EQ. 0.0) GO TO 190
      IF (RECT) RHO = RAD + PHI
      BETA2 = DLOG (RHO/(RHO-PHI) )
      BETA2=(BETA2*(1.0-0.4*BETA2+0.0916667*BETA2*BETA2
1      -0.01424242*BETA2**3))**2
      .
      .
      DO 370 K = 1 , RAY,KRAY
      XO1 = XO (1,K)
      XO2 = XO (2,K)
      IF (PEEVL .NE. DABS (PERVO)) GO TO 290
      IO (K) = II (K) * PROPO
      GO TO 300
290  IO (K) =0.5*(IO (K)+II (K) )
300  CALL PRTIAL (XO1,XO2,PU,&420)
      IF (CRI (K).LT.1.0) GO TO 305
      EEV =DSQRT ( PU (1)*PU (1) + PU (2)*PU (2) )
      IF (EEV .EQ. 0.) GO TO 420
      SINE = PU (1) / EEV
      COSN = PU (2) / EEV

```

```

RHO = XO(1,K) - SINE*XO(7,K)
ZETA = XO(2,K) - COSN*XC(7,K)
BORC2=0.0
IF (MAG) CALL CHILMG(BORC2,&420)
305 RHO = XO(1,K)
DEN=DENS*XC(8,K)
IF (.NOT. RECT) GO TO 310
IO(K) = AMIN1(IO(K),DEN)
GO TO 320

C -----
310 TERM1=IO(K)
TERM5=DEN*(RHO+IAX)
IO(K) = DMIN1(TERM1,TERM5)
IF (CRI(K)-1.0) 315,320,320
315 XC4=XO(4,K)
XC5=XC(5,K)
SINE=XC4/ESQRT(XC4*XC4+XC5*XC5)
COSN=XC5/DSQRT(XO4*XC4+XO5*XO5)
GO TO 325

C -----
320 R = IDINT(RHC)
ZETA = XC(2,K)
Z=IDINT(ZETA)
BZR2=0.0
IF (MAG) CALL CHILMG(BZR2,&420)
C ** BJSCH'S THEOREM
XC(6,K) = UNIT*EGM*(BZR2-BORC2)/(2.0*(RHO+IAX))
C ** INCLUDE PE EV INITIAL ENERGY
325 EEV = PJ(6) - PCT(1)+PE
EEV = EEV - (XC(6,K)/SEOM)*(XC(6,K)/SEOM)
IF (EEV.LT.0.) GO TO 360
BETZ=EEV*(EEV+2.0*LAM)/(EEV+LAM)**2
.
.
.
END
C ***** END CHILDA *****

```

END OF SUBROUTINE CHILDA

SUBROUTINE CHILDC

```

C ***** CHILDC *****
C CHILDC IS USED TO CALCULATE THE EXACT ELECTRON TRAJECTORY BEHIND THE
C STARTING SURFACE
  SUBROUTINE CHILDC (BR, BZ, K, TAU, *)
  IMPLICIT REAL*8 (A-H, O-Z)
  REAL JC, II, XO, IC
  REAL*8 LL
  COMMON /ARRAY/ II (51), IO (51), II (51)
  COMMON /EOMKO/ UNIT, EEV, PHI, ALPH, AMPS, RAD, EOM, SECH,
C PE, PERVO, XO (9, 51)
  COMMON /RHZET/ RHC, ZETA, PU (6)
  EEV1=DSQRT (PU (1)*PU (1)+PU (2)*PU (2))
  SINE1=PU (2)/EEV1
  COSN1=PU (1)/EEV1
  BT=DSQRT (BZ*BZ+BR*BR)
  SINE2=BR/BT
  COSN2=BZ/BT
  SINE=SINE1*COSN2+SINE2*COSN1
  COSN=COSN1*COSN2-SINE1*SINE2
  IF (SINE.LT.0.005) SINE=0.5D-2
5  I=0
  TAU1=0.001D1
  TAU2=900.0D+1
10  IF (I.GT.100) RETURN 1
  I=I+1
  TAU=(TAU1+TAU2)/2.0
  JC=PHI/(3.654449D-6*CCSN*CCSN/(UNIT*UNIT*UNIT)*(TAU*TAU
1*TAU*SINE*SINE/(COSN*COSN)/6.0+TAU-DSIN (TAU))/(BT*BT*BT))
  F2=2.3494112D-8*CCSN*COSN*JC*JC/(BT*BT*BT*BT)/(UNIT*UNIT
1*UNIT*UNIT)*(TAU*TAU+TAU*TAU*SINE*SINE/(CCSN*COSN)/8.0+TAU*TAU/2.0
1-TAU*DSIN (TAU)+1.0-DCOS (TAU))-EEV+PE
C SOLVE FOR JC AND TAU USING THE BISECTION METHOD METHOD
  EPS=F2/EEV
  IF (DABS (EPS).LT.0.001) GO TO 18
  IF (F2) 16, 18, 17
16  TAU2=TAU
  GO TO 10
17  TAU1=TAU
  GO TO 10
18  CCNTINUE
  VX=0.64289069D+2*JC/(UNIT*UNIT)*SINE*CCSN*(TAU+TAU/2.0-1.0+DCOS (TA
1U))/(BT*BT)
  VY=0.64289069D+2*JC/(UNIT*UNIT)*CCSN*COSN*(TAU*TAU*SINE*SINE/(COSN
1*COSN)/2.0+1.0-DCOS (TAU))/(BT*BT)
  XO (6, K)=0.64289069D+2*JC/(BT*BT)*(TAU-DSIN (TAU))/(UNIT*UNIT)
  XO (4, K)=VY*CCSN1-VX*SINE1
  XO (5, K)=VY*SINE1+VX*COSN1
  IF (ABS (XO (5, K)/XO (4, K)).GT.0.05) GO TO 28
  EEV=0.9*EEV
  GO TO 5

```

28 II (K) =JC  
30 RETURN  
END

END OF CHILDC

PROGRAM FOR PLOTTING OUTPUTS ON THE CALCOMP PLOTTING MACHINE

```

REAL XFCT,YFCT,XPAGE,YPAGE
XFCT=1.0
YFCT=1.0
DIMENSION X(1000),XARRAY(1000),Y(1000),YARRAY(1000)
CALL PLOTS(ID, ID, 11)
WRITE(6,1)
XPAGE=0.0
YPAGE=0.5
IPEN=3
CALL PLOT(XPAGE,YPAGE,-IPEN)
1 FORMAT(1H1,'STARTING TO PLOT')
10 READ(1)I,L,A,B,C,D,(X(J),J=1,I),(Y(J),J=1,L)
IF(I.LT.0.OR.I.GT.8)GO TO 190
IF(I.EQ.4)GO TO 200
IF(I.EQ.3)GO TO 180
IF(I.EQ.0.OR.I.EQ.7.OR.I.EQ.8)GO TO 170
IF(I.EQ.5)GO TO 160
IF(I.EQ.1)GO TO 150
IF(I.EQ.2)GO TO 140
IF(I.EQ.6)GO TO 130
C SET SCALE FACTORS
130 XFCT=C/A
YFCT=D/B
CALL DFACT(XFCT,YFCT)
GO TO 10
C DRAW A Y AXIS
140 XPAGE=0.0
YPAGE=0.0
NCHAR=6
AXLEN=A
ANGLE=90.0
FIRSTV=D
DELTA=B
NBR=1
K=0
DIST=1.0
IF(M.EQ.0)GO TO 141
XPAGE=-1.0
CALL AXIS1(XPAGE,YPAGE,'B AXIS',NCHAR,AXLEN,ANGLE,FIRSTV,DELTA,NBR
*,K,DIST)
XPAGE=0.0
YPAGE=0.0
IPEN=3
CALL PLOT(XPAGE,YPAGE,-IPEN)
GO TO 5
141 M=1
CALL AXIS1(XPAGE,YPAGE,'R AXIS',NCHAR,AXLEN,ANGLE,FIRSTV,DELTA,NBR
*,K,DIST)
5 WRITE(6,2)
2 FORMAT(1H1,'Y AXIS')
GO TO 10

```

```

C   DRAW AN X AXIS
150 XPAGE=0.0
    YPAGE=0.0
    M=0
    NCHAR=6
    AXLEN=A
    SX=A
    ANGLE=0.0
    FIRSTV=D
    DELTA=B
    NBR=1
    K=0
    DIST=1.0
    CALL AXIS1(XPAGE,YPAGE,'Z AXIS',NCHAR,AXLEN,ANGLE,FIRSTV,DELTA,NBR
    *,K,DIST)
    WRITE(6,3)
3   FORMAT(1H1,'X AXIS')
    GO TO 10
C   PLOT FCINTS
160 CALL DIVIDE(A,B,L,X,Y,XARRAY,YARRAY)
    NBRPTS=-L
    SIZE=0
    CALL GRAPH(XARRAY,YARRAY,NBRPTS,SIZE,INTEQV)
    GO TO 10
C   PLOT A LINE
170 CALL DIVIDE(A,B,L,X,Y,XARRAY,YARRAY)
    NBRPTS=L
    SIZE=0
    CALL GRAPH(XARRAY,YARRAY,NBRPTS,SIZE,INTEQV)
    GO TO 10
C   GET A CLEAN AREA OF PAPER
180 XPAGE=XFCT*SX+3.0
    YPAGE=0.0
    IPEN=3
    CALL PLOT(XPAGE,YPAGE,-IPEN)
    WRITE(6,4)
4   FORMAT(1H1,'END OF GRAPH')
    GO TO 10
C   GET AN ERROR MESSAGE
190 WRITE(6,195)
195 FORMAT(1H1,'UNEXPECTED VALUE FOR I ,EXECUTION TERMINATED')
200 CONTINUE
    XPAGE=5.0
    YPAGE=0.0
    IPEN=3
    CALL PLOT(XPAGE,YPAGE,-IPEN)
    XPAGE=0.0
    YPAGE=0.0
    CALL ENDPLT(XPAGE,YPAGE,999)
    WRITE(6,210)
210 FORMAT(1H1,'END OF PLOT')
    END
    SUBROUTINE DIVIDE(A1,B1,L1,X1,Y1,X2,Y2)
    DIMENSION X1(1000),Y1(1000),X2(1000),Y2(1000)
    DO 25 J=1,L1

```



```
X2 (J) = X1 (J) /A1  
Y2 (J) = Y1 (J) /B1  
25 CONTINUE  
RETURN  
END
```

END OF PLOTTING