COIL AND SHELL CHARACTERISTICS
FOR PASSIVE STABILIZATION

by

R.J. Thome, R.D. Pillsbury, Jr., W.G. Langton,
and W.R. Mann

Massachusetts Institute of Technology
Plasma Fusion Center
Cambridge, Massachusetts 02139
FOREWORD

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1.0 INTRODUCTION AND SUMMARY

The vertical stabilization of an elongated plasma imposes severe requirements on the system since the stabilizing magnetic field increment must be initially imposed on the time scale of the plasma displacement. It is usually not feasible to accomplish this with an active coil system alone because of the excessive peak power which would be required. Consequently, stabilization using passive electrical coils, shells or shell segments has been under evaluation.

The general approach would be to allow the rapid plasma displacement to generate eddy currents in passive elements and produce an associated, induced magnetic field which would provide stabilization until the induced field could be augmented by an active control coil system. The latter would respond on a longer time scale to limit peak power requirements to reasonable levels. This imposes two basic requirements on the passive stabilization system: (1) its location and geometry must provide an induced field of sufficient magnitude and (2) the induced field must be applied for a sufficient time interval.

This report presents results of an investigation into the means for estimating the level of the induced fields and their associated time constants. In all cases, the plasma was represented by a single, circular current filament which could be displaced instantaneously either vertically or radially. The latter was considered for the sake of completeness even though the need for radial stabilizing elements is not anticipated at this time.

Section 2.0 considers the passive stabilizing elements to be two coils or discrete loops which are coaxial with the plasma filament and symmetrically located with respect to the \( z = 0 \) plane, the initial plane of the plasma. Instantaneous vertical or radial displacements are considered at \( t = 0 \) and the
induced currents and fields which result at $t = o^+$ are derived in normalized form and presented in the form of contour plots to allow their estimation for any specified loop location. These may be utilized with imposed restrictions on location for a specific system to determine the most effective locations and the level of interaction. The latter may then be compared with a stabilization requirement in terms of an induced field produced per unit displacement to determine if the magnitude of the interaction is sufficient.

Section 3.0 considers the effect of a complete passive shell around the plasma by presenting results for toroidal shells of circular section. Vertical and radial displacements were considered and induced current distributions are given for typical cases to illustrate the most effective portions of the shells. Induced fields are given in a normalized form which is directly comparable to those used in Section 2.0. Results indicate that complete shells can be about a factor of two more effective than discrete loops depending on shell aspect ratio.

The shells of circular cross-section in Section 3.0 are analytically tractable for the $t = o^+$ instant and useful for providing insight into the problem. However, shells for tokamaks such as INTOR, which utilize elongated plasmas, are unlikely to be circular. Hence, Section 4.0 presents the results for complete and partial shells of a more suitable geometry. Field results are again presented in a normalized form suitable for comparison with previous sections.

Section 5.0 contains examples using the results of the previous three sections to illustrate methodology and generate estimates based on INTOR (Phase 1) characteristics. Section 5.1 uses a simplified stability criterion to determine location for discrete passive, vertical stabilization, loops to provide
an interaction of sufficient strength. Results show that locations along cer-
tain sections of the first wall or blanket outer boundary are adequate, but
that no location as far out as the shield outer boundary is sufficient. Time
constants are also estimated and indicate that values of the order of several
seconds or lower are achievable depending on toroidal continuity, material cross-
section and electrical conductivity. Hence, considerable flexibility exists for
adjusting this time constant to be compatible with field penetration and excita-
tion times for active stabilization coils.

Discrete loops are compared with toroidal shells of circular section in
Section 5.3 which shows the shells to be substantially more effective. In addi-
tion, it should be noted that the field produced by a distributed element such
as a shell is likely to be much more uniform over a greater volume than that pro-
duced by discrete loops. This may be an advantage when more realistic plasma
characteristics are considered.

Noncircular complete and partial shell segments are related to INTOR in
Section 5.4 with shell locations within, but near the blanket outer boundary
and at the outer shield boundary. The former location is adequate and the
latter is not, when effectiveness is compared with the same simple requirement
utilized for loops in Section 5.1. Time constants are estimated and again show
that considerable flexibility is possible depending on selection of shell
thickness and material.

The models utilized in this work were selected to allow mathematical trac-
tability and to generate insight for this problem. Although they are simplified,
they indicate general trends and means for normalization to allow results to
be applied or scaled to other cases. Future work requires improved plasma
modeling to investigate dynamic effects.
Figure 2.1 - Illustration of Plasma Vertical Displacement Relative to a Pair of Passive Stabilizing Coils in Real and Normalized Space.
2.0 TWO DISCRETE LOOPS

This section will present estimates of the fields produced by induced currents in two co-axial, passive coils as a result of the instantaneous vertical or radial displacement of a plasma. The plasma is modeled as a single circular current filament. Curves are given which allow the estimation of the induced current magnitude as well as the radial field due to a vertical displacement of the plasma and the vertical field induced by a radial displacement. Time constants for induced current (or field) decay are given for both cases and are not equivalent unless the two passive loops are closely coupled.

2.1 Vertical Displacements

The model used in this section is shown in Fig. 2.1. The plasma is represented by a single circular current filament of radius \( r_0 \) which is located in the \( z = 0 \) plane for \( t < 0 \) and which instantaneously moves a distance \( \Delta z \) parallel to the \( z \)-axis at \( t = 0 \). Two coaxial, conducting loops of radius \( a \) are located with planes at \( z = \pm d \). The left sketch in Fig. 2.1 shows the dimensions of the model in real space whereas the right sketch illustrates the model in a space with dimensions normalized to the plasma radius.

For \( t < 0 \) the plasma is operating with a steady-state current \( I_p \) in the \( z = 0 \) plane and the current in the passive loops is zero. At \( t = 0 \) the plasma moves a distance, \( \Delta z \) vertically and induces currents of magnitude \( I \) in the loops. If it is assumed that \( I_p \) is constant, the induced current may be shown to be given by

\[
I = I_p \left( \frac{\Delta z}{r_0} \right) e^{-\tau} \sigma_1
\]  

(1)
where

\[ I_p = \text{plasma current} \]

\[ t = \text{time} \]

\[ \tau = t/\tau_0 \] \hspace{1cm} (2)

\[ \tau_0 = (L-M)/R \] \hspace{1cm} (3)

\[ L = \text{self inductance of one passive loop} \]

\[ M = \text{mutual inductance between passive loops} \]

\[ R = \text{resistance of one passive loop} \]

The induced current is proportional to the displacement and to the plasma current, and decays with a time constant, \( \tau_0 \). It is also dependent on \( G_1 \) which is primarily a function of the dimensionless coordinates which locate the passive loops, that is,

\[
G_1 = \frac{4\eta [(1 + \rho)^2 + \eta^2]^{-3/2} \left\{ -K/k^2 + \frac{E}{(1-K^2)} \left[ \frac{1}{k^2} - \frac{1}{2} \right] \right\}}{\ln \left( \frac{8a}{r_w} \right) - \frac{7}{4} - \frac{2}{k_o} \left[ (1 - k_o^2/2)K_o - E_0 \right]} \hspace{1cm} (4)
\]

where

\[ \eta = d/r_0 \]

\[ \rho = a/r_0 \]

\[ K, E = \text{Complete elliptic integrals with modulus, } k \]

\[ K_o, E_o = \text{Complete elliptic integrals with modulus, } k_o \]

\[ r_w = \text{Effective radius of cross-section of passive loop} \]

\[ k_o = \rho [\rho^2 + \eta^2]^{-1/2} \]

\[ k = 2\rho^{1/2} [(1 + \rho)^2 + \eta^2]^{-1/2} \]
The function $G_{1}$ may be considered to be a normalized current at $t = o^+$. Contours of constant $G_{1}$ are shown in Figs. 2.2a and 2.2b in ($a/r_{o}$, $d/r_{o}$) space. Passive loops located at $(a,d)$ may be located in this diagram once the plasma radius, $r_{o}$ is specified. The contour which passes through the point then gives the value of $G_{1}$ for use in (1) to determine the induced current in the passive loops. The diagram assumes that $(a/r_{w}) = 20$, that is, the loop cross-sectional dimension is small relative to the loop radius, $a$, and their ratio is the assumed constant regardless of location in the diagram.

The derivation of (1) assumes that the loops are continuous as shown in Fig. 2.1 and the left sketch in Fig. 2.3. However, if the current paths are formed from saddle coils so that the toroidal path is discontinuous as in the right sketch in Fig. 2.3 then (1) and Fig. 2.2 can still be used to estimate the induced current at $t = o^+$ provided the "legs" of adjacent saddle coils are close together. The time constant for current decay of the saddle coils will be substantially shorter than $T_{o}$ for the complete loops because of the added resistance of the legs. An example of this type is considered in Section 5.1.

The radial field at the plasma as a result of the induced currents given by (1) may be shown to be

$$B_{r} = \frac{\mu_{o} I_{p}}{r_{o}} \frac{\Delta z}{r_{o}} e^{-t} G_{2}$$  \hspace{1cm} (5)

where: \hspace{0.5cm} $\mu_{o} =$ permeability of free space = $4\pi \times 10^{-7}$ $[H/m]$
Figure 2.2a - Contours of normalized current induced in passive stabilizing coils located at \((a/r_0, d/r_0)\) as a result of a vertical plasma displacement. (See Eqn. 1). (Note: \(a/r_w = 20\)).
Figure 2.2b - Contours of normalized current induced in passive stabilizing coils located at \((a/r_0, d/r_0)\) as a result of a vertical plasma displacement. (See Eqn. 1). Note: \((a/r_w) = 20\).
Vertical Stabilizing Coils May Have Toroidal Continuity (i.e.-Complete Loops) or May be Segmented (i.e.-Saddle Coils).
Figure 2.4a - Contours of normalized radial field on the plasma due to currents induced in passive stabilizing coils located at \((a/r_0, d/r_0)\) as a result of vertical plasma displacement. (See Eqn. 5).

Note: \((a/r_w) = 20\).
The radial field is thus proportional to plasma current and displacement and also decays with the time constant, $\tau_0$. It is dependent on $G_2$, a function dependent primarily on the dimensionless location of the loops which is given by

$$G_2 = \frac{16 \rho \eta^2 [(1+\rho)^2 + \eta^2]^{-3} \left\{ \frac{-K/k^2 + \frac{E}{(1-k^2)} \left[ \frac{1}{k^2} - \frac{1}{2} \right]}{\ln \left( \frac{8a}{r_w} \right) - \frac{7}{4} - \frac{2}{k_0} \left[ (1 - \frac{k_0^2}{2}) K_0 - E_0 \right]} \right\}^2}{\pi}$$

The radial field provides a vertical restoring force on the plasma and the function $G_2$ may be considered to be a normalized radial field at $t = o^+$. Contours of constant $G_2$ are plotted in Figs. 2.4a and 2.4b. Loops with coordinate locations $(a,d)$ may be located in this diagram once the plasma radius is specified. In the diagram the plasma is located at $(0,1)$ and the contour value $G_2$ determines the strength of the field produced by the induced currents in the loops when used with (5). A constant $a/r_w = 20$ is again used.

The contour diagram may be used with an overlay which "deletes" areas in which passive loops cannot be located because of system constraints. The maximum contour values in "allowable" areas may then be used to determine the best location for passive loops. An estimation can then be made directly of the radial field which can be produced for a given displacement to determine if the interaction is of sufficient magnitude. This process is illustrated with an example in Section 5.1.
2.2 Radial Displacements

The model used in this section is shown in Fig. 2.5. The plasma is represented by a single circular current filament of radius $r_0$ which is located in the $z = 0$ plane and which instantaneously moves a radial increment, $\delta$, at $t = 0$. Two co-axial, conducting loops of radius "a" are again located with planes at $z = \pm d$. The left sketch in Fig. 2.5 shows the dimensions of the model in real space whereas the right sketch illustrates the model in a space with dimensions normalized to the plasma radius.

For $t < 0$ the plasma is operating with a steady-state current $I_p$ at a radius $r_o$ and the current in the passive loops is zero. At $t = 0$, the plasma moves radially to $(r_o + \delta)$, its current changes to $I'_p$, and it induces currents of magnitude $I$ in the loops. Note that $I$ circulates with the same sense in both loops in this case, whereas the currents were of opposite sense for vertical plasma displacements in the previous section. The induced current may be shown to be given by

$$I = I_p \frac{\delta}{r_o} e^{-\tau'} G_3$$

(7)

where:

$$\tau' = \frac{t}{\tau_0'}$$

$$\tau_0' = \frac{(L + M)}{R}$$

(8)

Note the difference in time constant for this case relative to the previous case (ie-compare (8) and (3)) because of the symmetric versus antisymmetric induced currents. The function, $G_3$, is primarily dependent on the dimensionless coordinates of the loops and is given by:
\[ \frac{2}{k} \rho^{1/2} \left[ -\frac{k}{k^2} + \frac{E}{(1-k^2)} \left[ \frac{1}{k^2} - \frac{1}{2} \right] \right] \left[ \frac{2\rho}{(1+\rho)^2 + \eta^2} \right] \left[ 1 - \frac{2\rho(1+\rho)}{(1+\rho)^2 + \eta^2} \right] - C_0 G_4 \]  

where: 

\[ G_4 = \left( \frac{2}{k} \right) \rho^{1/2} \left[ (1-k^2/2) K - E \right] \]  

\[ C_0 = \begin{cases} 
1/2 & \text{for constant flux expansion} \\
0 & \text{for constant energy expansion} \\
-1/2 & \text{for constant current expansion} 
\end{cases} \]  

The choice of the constant \( C_0 \) is dependent on whether the plasma is assumed to expand at constant flux, energy or current. For example, the flux produced by the plasma current filament may be assumed to be the same before and after the expansion, in which case \( C_0 = 1/2 \).

The function \( G_3 \) may be considered to be a normalized current at \( t = o^+ \). Contours of constant \( G_3 \) are shown in Figs. 2.6a and 2.6b in \((a/r_o, d/r_o)\) space for the case where \( C_0 = 1/2 \) (constant flux). Passive loops with coordinates \((a,d)\) may be located in this diagram once the plasma radius is specified. The contour which passes through the point then gives the value of \( G_3 \) for use in (7) to determine the induced current in the passive loops. The diagram is based on \((a/r_w) = 20\), that is, the loop cross-sectional dimension is small relative to the loop radius, \( a \), and their ratio is the assumed constant regardless of location in the diagram except as \( \eta = 0 \) where curves are extrapolated because of errors introduced by expressions for inductance in this range.

The presence of a \( G_3 = 0 \) contour should be noted in Figs. 2.6a and b. This implies that the direction of the induced current is dependent on loop location. This leads to the possibility of developing a destabilizing force instead of a stabilizing force and will become apparent in the figures which follow.
Figure 2.4b - Contours of normalized radial field on the plasma due to currents induced in passive stabilizing coils located at \((a/r_0, d/r_0)\) as a result of vertical plasma displacement. (See Eqn. 5). (Note: \(a/r_w = 20\)).
Fig. 2.5 - Illustration of plasma radial displacement from \( r_0 \) to \( (r_0 + \delta) \) relative to a pair of passive stabilizing coils in real and normalized space.
Figure 2.6a - Normalized current induced in passive stabilizing coils located at \((a/r_0, d/r_0)\) as a result of a radial displacement. Contours correspond to \(C_0 = 1/2\) (i.e. - constant flux produced by the plasma). (See Eqn. 7). (Note: \(a/r_w = 20\)).
Figure 2.6b - Normalized current induced in passive stabilizing coils located at \((a/r_0, d/r_0)\) as a result of a radial displacement. Contours correspond to \(C_0 = 1/2\) (i.e. constant flux produced by the plasma). (See Eqn. 7). (Note: \(a/r_w = 20\)).
Figure 2.7a - Normalized Z-directed field due to current induced in stabilizing coils located at \((a/r_0, d/r_0)\) as a result of a radial plasma displacement. Contours correspond to \(C_0 = 1/2\) (i.e. constant flux produced by the plasma). (See Eqn. 12). (Note: \(a/r_w = 20\)).
The $z$-directed field at the plasma as a result of the induced currents given by (7) may be shown to be:

$$B_z = \frac{\mu_0 I_p}{r_0} \left( \frac{\delta}{r_0} \right) e^{-\tau'} G_5 \tag{12}$$

where

$$G_5 = \frac{G_3}{\pi} \left\{ G_4/2 + \frac{4\rho^{3/2}}{k} \left[ -k^2 + E \left( \frac{1}{k^2} - \frac{1}{2} \right)/(1-k^2) \right] x \right\}$$

$$\left[ \frac{1}{[(1+\rho)^2 + \eta^2]} - \frac{2(1+\rho)}{[(1+\rho)^2 + \eta^2]^2} \right] \tag{13}$$

The induced $z$-field is thus proportional to plasma current and displacement and decays with the time constant $\tau'$. It is dependent on $G_5$, a function dependent primarily on the dimensionless coordinates of the loops and on $C_0$ (through $G_3$), the assumption governing the manner in which the plasma expands.

$G_5$ may be considered to be the normalized $z$-directed field at the plasma at $t = 0^+$. Contours of constant $G_5$ are plotted in Figs. 2.7a and 2.7b for a constant flux expansion. Loops with coordinates $(a,d)$ may be located in this diagram once the plasma radius, $r_0$, is specified. In the diagram, the plasma is located at $(0,1)$ and the contour value $G_5$ determines the strength of the field produced by the induced currents in the loops when used with (12). A constant $a/r_w = 20$ is again used in (13) to construct the contours except as $\eta = 0$ is approached where curves are again extrapolated.

A positive plasma current ($\varphi$-into the diagram) in Figs. 2.7a or 2.7b requires a negative $B_z$ for stabilization. This corresponds to a negative $G_5$. The diagrams show that loops may be located so as to be stabilizing ($G_5 < 0$) or destabilizing ($G_5 > 0$), hence, positioning passive coils in the latter region should be avoided.
The position of the destabilizing zone is strongly dependent on the assumption made concerning plasma action when displaced. The figures discussed thus far assume that the flux produced by the plasma current is the same before and after displacement ($C_0 = 1/2$). If, on the other hand, it is assumed that the plasma current remains constant then Figs. 2.8 and 2.9 can be generated for $G_3$, normalized current, and $G_c$, normalized z-field. The latter exhibits no destabilizing zone for passive coil location and illustrates the qualitative difference in results due to different assumptions regarding plasma character when the expansion take place.

The contour diagrams (eg-Figs. 2.7a and b) for $G_3$ may be used with an overlay which "deletes" areas in which passive loops cannot be located because of system constraints. The maximum stable contour values in "allowable" areas may then be used to determine the best location for passive loops and an estimation can be made of the vertical field which can be produced for a given displacement to determine if the interaction is of sufficient magnitude. This process is illustrated with an example in Section 5.2.

The derivation of (7) and (12) assumes that there are two loops as shown in Fig. 2.3 and that they are toroidally continuous. If a segmented design is desired for radial stabilization, the type of saddle coil arrangement shown in Fig. 2.3 for vertical stabilization cannot be used, because currents induced by radial plasma displacement are of like sense above and below the $z = 0$ plane, hence, no currents would flow. It is, therefore, necessary to base the coil segments for radial stabilization on a four loop system as shown in Fig. 2.10. The left sketch shows a four loop system which is continuous and the right sketch shows a segmented coil set. The currents induced in the
Figure 2.7b - Normalized Z-directed field due to currents induced in stabilizing coils located at \((a/r_0, d/r_0)\) as a result of a radial plasma displacement. Contours correspond to \(C_0 = 1/2\) (i.e. constant flux produced by the plasma). (See Eqn. 12). (Note: \(a/r_w = 20\)).
Figure 2.8 - Normalized current induced in passive stabilizing coils located at \((a/r_0, d/r_0)\) as a result of a radial plasma displacement. Contours correspond to \(C_0 = -1/2\) (i.e. - constant plasma current). (See Eqn. 12). (Note: \(a/r_w = 20\).)
Figure 2.9 - Normalized Z-directed field due to currents induced in stabilizing coils located at \((a/r_0, d/r_0)\) as a result of a radial plasma displacement. Contours correspond to \(C_0 = -1/2\) (i.e. constant plasma current). (See Eqn. 12). (Note: \(a/r_w = 20\)).
Figure 2.10 Passive Radial Stabilizing Coils may have toroidal continuity (i.e., complete loops) or may be formed from coil segments.
segmented coil set at \( t = 0^+ \) would be the same as in the four loop continuous set provided the radial connections of adjacent segments were close together, however, the currents would decay with faster time constants because of the added resistance of the radial connectors between loops.

The contour plots given in this section were based on a two loop system. They may be used as a first approximation to estimate the characteristics for a four loop set as shown in Fig. 2.10 by superposition of contour values for \( G_3 \) and \( G_5 \). However, this approach for 4 loops will neglect the inductive coupling between the inner and outer loops and may be expected to yield optimistic \( z \)-fields at \( t = 0^+ \). Decay time constants will also be altered.

### 2.3 Time Constants

The previous two sections derived induced currents and fields for two loop systems and showed that the time constant for decay of induced fields was not the same for the case of vertical vs radial plasma displacements. This is attributed to the different symmetry of the induced currents relative to the \( z = 0 \) plane which is manifested as mutually aiding or mutually opposing induced flux. The time constants are given by:

<table>
<thead>
<tr>
<th>DISPLACEMENT</th>
<th>DECAY TIME CONSTANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>( \tau_0 = (L-M)/R )</td>
</tr>
<tr>
<td>Radial</td>
<td>( \tau'_0 = (L+M)/R )</td>
</tr>
</tbody>
</table>

These time constants may be shown to be of the form

\[
\tau_0 = \mu_0 \sigma a^2 G_0(d/a, r_w/a)
\]
\[ \tau' = \mu_0 \sigma a^2 G_6' (d/a, r_w/a) \]  

(15)

where \[ \sigma = \text{conductivity of passive coil material} \]

The functions \( G_6 \) and \( G'_6 \) may be considered to be dimensionless time constants. They are plotted in Fig. 2.11 as a function of the ratio of loop coordinates. Curves in Fig. 2.11 are based on constant values of \( r_w/a = 0.05, 0.025 \) and \( 0.0125 \). There is also an implicit assumption that the induced current is uniformly distributed over the loop current carrying cross-section of radius, \( r_w \).

For large values of \( d/a \), the loops are far apart, hence, the mutual inductance is small and the time constants for both cases approach the same value. As \( d/a \to 0 \), the loops approach the \( z = 0 \) plane and \( M \to L \), thus the time constants diverge. For \( d/a = 0 \), the two loops merge into one and no current is induced by a small vertical plasma displacement whereas a current may still be induced by a radial plasma displacement.

If saddle coils are used for vertical stability as shown in Fig. 2.3, instead of two continuous loops, then (14) may still be used to estimate the time constant for current decay. However, \( \tau_0 \) is shortened by the added resistance of the saddle legs, hence, \( \tau_0 \) from (14) should be multiplied by

\[ C = \left(1 + \frac{l \ell_s r_w^2}{2a A_s}\right)^{-1} \]  

(16)

where:
- \( l = \text{number of saddle coils} \)
- \( \ell_s = \text{length of one saddle coil "vertical" leg} \)
- \( A_s = \text{effective cross-sectional area of one saddle coil leg for current flow} \)
Figure 2.11 - Time constants for toroidally continuous passive stabilization loops.
Equation (16) assumes that the saddle coil legs have the same conductivity as the loop material.

This section has derived time constants for induced current and field decay from passive loops following a single, step function in displacement of the plasma loop. Ultimately, the plasma must be dynamically stabilized. When the dynamic interaction is considered the time constants for growth or suppression of an instability may be expected to be related to, but somewhat different from the time constants in this section. However, this approach allows the relative effectiveness of different passive loops to be evaluated on an approximate basis and allows an insight to be gained into the important parameters.
3.0 TOROIDAL SHELLS WITH CIRCULAR CROSS SECTION ($t = o^+$)

Section 2.0 considered the passive stabilization effects of two discrete current loops symmetrically placed relative to the $z = o$ plane and developed contour plots which can be used to estimate the magnitude of the induced current and fields. However, passive stabilization coils which are more distributed in nature, for example, plates, shell segments or complete shells, may be more likely to be used if more uniform induced stabilization fields are desired.

Some insight into the characteristics of a complete shell problem can be gained by considering the $t = o^+$ induced current distribution and stabilizing field magnitude which results from plasma displacement within a toroidal shell of circular section. The results in this section assume that the effects of a small plasma displacement on the shell can be modeled by superposition of the simultaneous and instantaneous turn off of the steady-state plasma at its undisplaced position and turn on at its displaced position. The method of finding the induced current in the shell at $t = o^+$ due to turn off at $t = o$, may be found in Thome and Tarrh$^\dagger$; the currents induced by a turn on may be found in a similar fashion since they are the negative of those in a turn off problem. The superposition process is illustrated and described in Fig. 3.1. The induced current and fields at the plasma filament location following a displacement may be expressed as two infinite series involving toroidal coordinates and Legendre associated functions. Partial sums of these series were evaluated over a range of shell parameters for small vertical and radial plasma displacements. General results are given in this section and examples using these results are given in Section 5.3.

Fig. 3.1a - The plasma current, $I_p$, creates a steady-state magnetic field for $t < 0$ which links an electrically conducting shell.

Fig. 3.1b - At $t = 0$, $I_p$ is turned off instantaneously and induces a current sheet $K_p$ on the shell at $t = 0^+$ such that the field outside is unchanged from the $t < 0$ shape in Fig. 3.1a.

Fig. 3.1c - At $t = 0$, $I'_p$ is turned on instantaneously at a displaced position and induces a current sheet $K'_p$ at $t = 0^+$ which confines the field within the shell.

Fig. 3.1d - At $t = 0^+$, following a displacement of a steady-state plasma at $t = 0$, the field outside the shell is the same as in Fig. 3.1a or b; the total field inside is the superposition of the fields from the current sheets in Fig. 3.1b and Fig. 3.1c and the field from the displaced filament.
3.1 Vertical Displacements

The range of shell parameters and initial, undisplaced plasma positions which were evaluated using the series referred to in the previous section is illustrated in Fig. 3.2. The shell geometry is determined by the ratio of its major to its minor axes, $R/r_m$. The dimensionless initial plasma location is determined by the ratio of the plasma filament radius to the major axis of the shell, $r_o/R$. The scale of the system may then be found by specifying any one of the dimensions, $R$, $r_m$ or $r_o$.

A small vertical displacement of the plasma from the $z = 0$ plane induces currents in the shell which tend to have opposite signs above and below the $z = 0$ plane and thus produce a radial field tending to return the plasma to its initial position. The current sheet amplitude at $t = 0^+$ is of the form

$$K_Z = \left( \frac{I_p}{2\pi r_m} \right) \left( \frac{\Delta Z}{r_o} \right) G_7$$

(17)

where:

$$K_Z = \text{local current sheet amplitude (e g. - in [A/m])}$$

$$\Delta Z = \text{small vertical displacement}$$

$$G_7 = G_7 \left( R/r_m, r_o/R, \phi \right)$$

The function $G_7$ may be considered to be a normalized local current sheet amplitude and is plotted in Figs. 3.3. and 3.4 for shells with $R/r_m = 2$ and $R/r_m = 4$, respectively. Three curves are shown on each plot and correspond to initial plasma locations of $r_o/R = 0.9$, $1.0$ and $1.1$. Relative scale for the different cases may be visualized with the aid of Fig. 3.2. In each case, negative current is induced in the top of the torus and positive current in
Toroidal Conducting Shells with Circular Section; the plasma is initially located in the $z=0$ plane at radius $r_o$.

Figure 3.2
Fig. 3.3 - Induced Current Sheet for Vertical Plasma Displacement for a Shell with $R/r_m = 2$. 

Normalized Current Sheet Amplitude, $G_7$
Fig. 3.4 - Induced Current Sheet for Vertical Plasma Displacement for a Shell with $R/r_m = 4$. 
the bottom of the torus, thus producing a positive radial field which will tend
to push the plasma filament (positive current and positive $\Delta Z$) back toward the
$z = 0$ plane. Note that the area of current concentration is relatively localized
for each case, thus implying that a partial shell at that location would be
almost as effective as the complete shell. However, the area of concentration
is also relatively sensitive to initial plasma filament location which suggests
that a distributed plasma current model would produce induced currents which
were much less concentrated.

The induced radial field at the plasma location at $t = 0^+$ has the form:

$$B_r = \left( \frac{\mu_0 I_p}{r_o} \right) \frac{(\Delta Z)}{r_o} G_8$$

(18)

where:

$$G_8 = G_8 \left( \frac{R}{r_m}, \frac{r_o}{R} \right)$$

(19)

The function $G_8$ may be considered to be the normalized induced radial field
due to the small vertical displacement and is directly comparable to $G_2$ for
loops (e.g., see eqn.5). It is plotted in Fig. 3.5 as a function of $R/r_m$ for
selected values of $r_o/R$. When $R/r_m$ and $r_o/R$ are specified, $G_8$ may be found
and used with (18) to find $B_r$ following specifications of $I_p$, $r_o$, and $\Delta Z$.

3.2 Radial Displacements

A small radial displacement of the plasma filament in the $z = 0$ plane
induces currents in the shell which tend to have opposite signs on the outside
and inside extremes of the shell cross-section and thus produce a vertical
field which tends to return the plasma to its initial position. The current
sheet amplitude at $t = 0^+$ is of the form:
Figure 3.5 Normalized Radial Field vs. Shell Aspect Ratio for Three Cases of Plasma Filament Initial Position; see Eq. (18).
\[ K_r = \left( \frac{I_p}{2\pi r_m} \right) \left( \frac{\Delta r_o}{r_o} \right) G_9 \]  

(20)

where: \[ \Delta r_o = \text{small radial displacement} \]
\[ G_9 = G_9 \left( \frac{R}{r_m}, \frac{r_o}{R}, \phi \right) \]

The function \( G_9 \) may be considered to be a normalized local current sheet amplitude. It was generated assuming that the plasma produces the same amount of flux before and after expansion. \( G_9 \) is plotted in Figs. 3.6 and 3.7 for shells with \( R/r_m = 2 \) and \( R/r_m = 4 \), respectively. In each case, negative current is induced on the far outside of the toroidal shell and positive current on the far inside, thus producing a negative \( z \)-directed field which tends to push the plasma filament (positive current and positive \( \Delta r_o \)) back toward its initial position. Note that the current concentration is more localized toward the outside for \( r_o/R = 1.1 \) and more toward the inside for \( r_o/R = 0.9 \).

The induced \( z \)-directed field at the plasma location at \( t = 0^+ \) has the form
\[ B_z = \left( \frac{\mu_0 I_p}{r_o} \right) \left( \frac{\Delta r_o}{r_o} \right) G_{10} \]  

(21)

where:
\[ G_{10} = G_{10} \left( \frac{R}{r_m}, \frac{r_o}{R} \right) \]  

(22)

The function \( G_{10} \) may be considered to be the normalized induced \( z \)-directed field due to the small radial displacement, \( \Delta r_o \). It is plotted in Fig. 3.8 as a function of \( R/r_m \) for selected values of \( r_o/R \). When \( R/r_m \) and \( r_o/R \) are specified, \( G_{10} \) may be found from Fig. 3.8 and used with Eqn.(21) to find \( B_z \) following specification of \( I_p, r_o \) and \( \Delta r_o \).
Fig. 3.6 - Induced Current Sheet for Radial Plasma Displacement for a Shell with $R/r_m = 2$
Fig. 3.7  - Induced Current Sheet for Radial Plasma Displacement for a Shell with $R/r_m = 4$
Figure 3.8 Normalized Vertical Field vs. Shell Aspect Ratio for Three Cases of Plasma Filament Initial Position; see Eq. (21).
4.0 TOROIDAL SHELLS WITH NONCIRCULAR CROSS SECTIONS

The previous section presented results for toroidal shells with circular cross sections to gain insight into: 1) the regions where induced currents flow on complete shells when a plasma is displaced and 2) the difference in the magnitude of the induced fields relative to those achievable with the discrete loops of Section 2.0. The main advantage to the approach in Section 3.0 is that the model may be treated with standard analytical techniques. The disadvantages are that results are limited to the $t = o+$ instant, the shell is complete and the shell is of circular cross section.

In this section a finite element model* is used with toroidal shells that have complete and incomplete noncircular sections to more realistically represent likely configurations for passive stabilization coils or shell segments. Toroidal continuity is assumed throughout this section and only vertical plasma displacements are considered. Figure 4.1 shows the location of the shells considered and of the plasma which is modeled with a rectangular cross section of uniform current density. We will again assume that a small, rapid vertical displacement of the plasma may be adequately modeled by a rapid turn off of the plasma at its steady state position and simultaneous turn on at its displaced position. This may be shown to be exact in the limit of a very small displacement.

4.1 Complete and Incomplete Shells ($t = o+$)

Sections 2.0 and 3.0 showed that, for small displacements, the induced fields and currents were proportional to the plasma current, the displacement and a dimensionless function describing the geometry of the passive conducting material and plasma location. In Section 3.0 the dimensionless function

for a circular section shell was dependent on \( r_0/R \) (plasma location) and on \( R/r_m \) (shell geometry). One complete shell of the type shown in Figure 4.1 would require four dimensionless parameters to describe its geometry and one to describe initial plasma location if the plasma were modeled as a single filamental loop. A more complex plasma model would require additional parameters to describe its geometry. Furthermore, at \( t = o^+ \), the induced currents are generated on the shell surface so its thickness does not enter into the function. However, the shell thickness is important at later times because it affects the decay time constant.

In view of the above, we would expect to be able to normalize the induced radial field at \( t = o^+ \) for a shell (or shell segment) of the type shown in Fig. 4.1 to \( \mu_0 I_p / r_0 \) and to \( \Delta Z / r_0 \) as was the case in Sections 2.0 and 3.0. The normalized values based on calculations from the finite element model will constitute a function, \( G_{11} \). Radial fields at \( t = o^+ \) for other cases with different plasma currents, displacements or shell size may then be found by scaling from

\[
B_r |_{t=o^+} = \left( \frac{\mu_0 I_p}{r_0} \right) \left( \frac{\Delta Z}{r_0} \right) G_{11}
\]

provided the shell is geometrically similar to the shell used in obtaining \( G_{11} \). This is somewhat more restrictive than for the previous sections where the passive elements could be described by a single dimensionless ratio and the functions could be expressed analytically. In this case, the function cannot be found analytically, but can be found using the finite element method then scaled to other geometrically similar cases.

The function \( G_{11} \) is plotted in Fig. 4.2 for the inner shell and for the outer shell as shown in Fig. 4.1. The intersection of each curve with the vertical axis is the result for a complete shell. As a curve progresses from left to right, successive shell segments are removed beginning on the inboard
side. The angle subtended by the shell is the horizontal axis and is defined in the insert sketch. Points on the sketch are labeled to correspond to points on the curve. The curves are relatively flat for large subtended angles thus implying that the inboard segments are relatively ineffective in producing radial field. The region \( \theta \leq 30^\circ \) is also relatively ineffective. The relatively large difference between inner and outer shells is consistent with the trend shown in Fig. 3.5 for complete shells with circular sections with different aspect ratios, \( R/r_m \).

In general, one would expect to generate comparable radial field magnitudes at a point with a complete shell or an effectively placed shell segment. However, more complete shells would be expected to produce a more uniform radial field distribution over a larger region within the shell.

4.2 Time Constants

Selected cases based on Fig. 4.1 were computed to determine field decay in time. Results are plotted in Fig. 4.3. Four lines are shown, each corresponding to a copper shell or shell segment as indicated in the inset sketches.

The top curve represents a complete inner shell and the second curve is for an inner shell segment which occupies a relatively effective region. The latter case exhibits a relatively rapid initial-decay of the higher harmonic content of its induced current pattern, then decays at about the same time constant as the top curve. A curve fit to the top two curves to estimate a single time constant yields 3.13 sec for the top curve and 2.94 sec for the second.

The cases represented by the lower curves correspond to the outer shell and yield a lower level radial field because they are not as well coupled to the plasma. Part of the outer shell segment chosen for the lower curve is in a relatively ineffective region. A curve fit to the two lower curves to estimate
Fig. 4.1 - Two Shell Finite Element Model for Plasma Displacement Study; The plasma has a rectangular cross-section and uniform current density and produces the field lines shown in the steady state before plasma displacement.
Figure 4.2 - Normalized Radial Magnetic Field Due to a Plasma Vertical Displacement for Complete and Incomplete Shells based on Fig. 4.1.
Figure 4.3 - Normalized Radial Field at the Plasma Center due to a Vertical Plasma Displacement for copper shells 1.38 cm thick (based on Fig. 4.1).
a single time constant yields 1.39 sec for the third curve and 0.93 seconds for the fourth curve. These time constants may be scaled for other geometrically similar shells or segments by assuming that the time constant is proportional to shell thickness and inversely proportional to shell resistivity. The results assume toroidal continuity and all portions of a shell must be scaled by the same constant. If saddle coils are used, the impact on the time constants may be estimated using a method similar to that in Eq. (16).

Note that complex geometries cannot, strictly speaking, be described by a single time constant since they are, in effect, a collection of multiple coupled circuits. However, the current components with the faster time constants decay earlier; hence, a curve fit toward the end of a transient allows a single time constant to be estimated.
5.0 **EXAMPLES RELATED TO INTOR**

5.1 **Discrete Loops for Vertical Stabilization**

This section will illustrate the use of the general material developed in Section 2.1 to estimate the characteristics of two passive loops for stabilization in INTOR. Results show that the most effective location is on the first wall and that locations on the shield outer boundary are too far from the plasma to be effective. Maximum possible time constants for induced field decay are also estimated for continuous loops and for saddle coils (see Fig. 2.3).

Contours of dimensionless radial field induced by a vertical plasma displacement were developed in Section 2.1 and presented in Figs. 2.4a and b. Figure 5.1 is the same as Fig. 2.4a, but shows three boundaries corresponding to the location of the first wall, outer blanket boundary (assuming a 0.5 m thick blanket), and outer shield boundary (assuming a 1.0 m shield with 0.1 m gap between blanket and shield) for INTOR. Points A, B and C represent maximum values of the function $G_2$ on the inner boundary, blanket outer boundary, and shield outer boundary, respectively. They are, therefore, the most effective points for location of passive vertical stabilization coils on each of these boundaries. The contour values at these points are given in the third line in Table 5.1. If we now assume a plasma major radius of $r_o = 5.3$ m and current $I_p = 6.4$ MA, the baseline INTOR parameters, then Eq. (5) may be used with the $G_2$ values from Table 5.1 to estimate the maximum possible induced radial field per unit vertical displacement which can be generated by loops located on one of the three boundaries. Results are given in line 4 in Table 5.1. These are values at $t = 0$ and decay with time constant $\tau_o$ (See (5)).
### TABLE 5.1 - CHARACTERISTICS OF VERTICAL STABILIZATION COILS AT PTS A, B AND C IN FIG. 5.1

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Location</td>
<td>First Wall</td>
<td>Blanket Outer Boundary</td>
<td>Shield Outer Boundary</td>
</tr>
<tr>
<td>3. $G_2$ (Fig. 5.1)</td>
<td>0.72</td>
<td>0.38</td>
<td>0.13</td>
</tr>
<tr>
<td>4. $B_r/(\Delta z)$ [T/M]</td>
<td>0.20</td>
<td>0.11</td>
<td>0.036</td>
</tr>
<tr>
<td>5. Loop Radius, $a$ [m]</td>
<td>6.15</td>
<td>6.52</td>
<td>7.26</td>
</tr>
<tr>
<td>6. Loop z Coord, $d$ [m]</td>
<td>1.59</td>
<td>2.01</td>
<td>2.76</td>
</tr>
<tr>
<td>7. $d/a$</td>
<td>0.26</td>
<td>0.31</td>
<td>0.38</td>
</tr>
<tr>
<td>8. $G_6$ (Fig. 2.11)</td>
<td>$3.1 \times 10^{-3}$</td>
<td>$3.3 \times 10^{-3}$</td>
<td>$3.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>9. $\tau_0$ max for loops, [sec]</td>
<td>8.55</td>
<td>10.2</td>
<td>13.4</td>
</tr>
<tr>
<td>10. $\ell_s$, [m]</td>
<td>3.5</td>
<td>4.45</td>
<td>6.25</td>
</tr>
<tr>
<td>11. $\tau_{os}$ max for saddles*, [sec]</td>
<td>4.09</td>
<td>4.43</td>
<td>5.08</td>
</tr>
<tr>
<td>12. $A$ for $\tau_0$ max, $\tau_{os}$ max, [m²]</td>
<td>0.30</td>
<td>0.334</td>
<td>0.413</td>
</tr>
</tbody>
</table>

*based on 12 saddle coils
The value of $B_r/(\Delta Z)$ which is required may be estimated* by assuming that this induced field per unit displacement is equal in magnitude to the applied, steady-state value of $(\partial B_r/\partial z)$. Using the definition of the field index the requirement may be expressed as

$$
\left[\frac{B_r}{\Delta Z}\right]_{reqd} = \frac{B_{za}n}{r_o}
$$

where:

$B_{za}$ = applied steady-state vertical field

$n$ = field index

If we assume $n = -0.95$ and $B_{za} = -0.5$ T then $[B_r/(\Delta Z)]_{reqd} = 0.089$ T/m. Comparison of this requirement with line 4 in Table 5.1 indicates that it can be exceeded at points A or B, but cannot be achieved at point C. The criterion implies, therefore, that passive vertical stabilization by two loops cannot be achieved with locations on the shield outer boundary but could be achieved using locations at points A or B with considerable "excess" field production or by locating loops on any contour with $C_2 \geq 0.31$ which corresponds to $[B_r/(\Delta Z)]_{reqd} = 0.089$ T/m.

Maximum possible time constants for loops located at points A, B and C can be estimated using Fig. 2.11 as follows. Loop radius, $a$, z-coordinate, $d$, and the ratio $d/a$ for points A, B and C are given in lines 5, 6 and 7 in Table 5.1. Values of the normalized time constant, $G_6$, from Fig. 2.11 are given in line 8 assuming $(r_w/a) = 0.05$. If we now assume that the coils are copper which in turn, specifies the conductivity, $\sigma$, then Eq. (4) may be used to determine the time constant $\tau_o$ for points A, B and C. This is given in line 9 and has been labeled a maximum since materials with lower conductivity could be used and because line 9 corresponds to a large conductor cross-sectional area, $A$, for each loop (see line 12).

* J. Schmidt, PPPL, personal communication
The latter arises implicitly since \( A = \pi r_w^2 \) where \( r_w \) is the effective conductor radius and since the curves selected from Fig. 2.11 are for \( r_w/a = 0.05 \). Lower time constants than those in line 9 may therefore be achieved by using a poorer conductor and/or decreasing \( A \) by decreasing \( (r_w/a) \) and using a different curve from Fig. 2.11.

The time constant, \( T_0 \) in line 9 assumes that the loops are toroidally continuous as illustrated in the left sketch in Fig. 2.3. Maximum possible time constants for saddle type coils (right sketch in Fig. 2.3) may now be estimated using Eq. (5). These will be estimated by assuming that the saddle legs are also copper and that the leg conductor cross section is the same as that for the loop (i.e., line 12 in Table 5.1). Twelve saddle coils will be assumed and the length of a single leg of the saddle, \( Z_s \), is assumed to be twice the distance from the loop to the \( z = 0 \) plane along the boundaries shown in Fig. 5.1. The length, \( Z_s \), for each point is given in line 10 and the estimated maximum time constant for saddle coils, \( T_{os} \) in line 11 in Table 5.1. Results indicate that the maximum achievable time constants are relatively long and could be adjusted downward by using a conductor with higher resistivity and/or smaller conductor cross-sectional areas.

This section has estimated the characteristics of two loops for passive vertical stabilization for INTOR. Results indicate that considerable flexibility in loop location exists to develop a sufficient interaction based on a simplified stability criterion. Points along large sections of the first wall boundary and outer blanket boundary have sufficient capability, but points along the shield boundary do not. Estimates of maximum possible time constants indicate that either continuous loops or saddle type coils are suitable and that there is considerable flexibility available for adjustment of the time constants.
5.2 Discrete Loops for Radial Stabilization

Elongated plasmas such as those envisioned for INTOR are generally believed to be radially stable under usual operating conditions and, therefore, do not require discrete loops for radial stabilization as discussed in Section 2.2. However, this section will utilize INTOR dimensions to illustrate the use of the contours presented in Fig. 2.7.

Figure 5.2 is identical to Fig. 2.7a except that the INTOR dimensions have been used to outline the first wall, blanket outer boundary and shield outer boundary. $G_5$ contours represent normalized $z$-directed field at the plasma position due to currents induced in discrete loops located on a particular contour. Maximum field occurs with loops located on the maximum allowable contour. For the three boundaries, the maximum contour values occur at points A, B and C on the midplane of the machine although there is essentially no change in effectiveness if the loops are located slightly off the midplane. Contour values for points A, B and C are given in Table 5.2, line 3. These values may now be used with Eq. (12) to determine the induced field per unit radial displacement (line 4) which could then be compared with a suitable stability requirement to estimate whether the interaction with discrete loops was sufficiently strong. Time constants could be derived using (15) in a manner similar to that used in the previous section.
Figure 5.1 - INTOR boundaries superimposed on Normalized Radial Field Contours (see figures 2.4a and 2.4b); A, B and C show most effective locations on boundaries for passive loops for vertical stabilization.
Figure 5.2 - INTOR boundaries superimposed on Normalized \( z \)-directed Field Contours (see Figures 2.7a and 2.7b); A, B, and C show most effective locations on boundaries for passive loops for radial stabilization if required; Contours assume plasma expansion at constant flux \( (c_o = 1/2) \) see Equation (12).
TABLE 5.2 - CHARACTERISTICS OF RADIAL STABILIZATION COILS AT Pts A, B AND C IN Fig. 5.2

<table>
<thead>
<tr>
<th></th>
<th>Point</th>
<th>Location</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>First Wall</td>
<td>Blanket Outer Boundary</td>
<td>Shield Outer Boundary</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Location</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>G5 (Fig. 5.2)</td>
<td>0.87</td>
<td>0.49</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>Bz/δ [T/M]</td>
<td>0.25</td>
<td>0.14</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>Loop radius, a [m]</td>
<td>6.63</td>
<td>7.12</td>
<td>8.23</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>Loop z - coord, d [m]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
5.3 Discrete Loops vs Complete Shells of Circular Section

The toroidal shells of circular cross-section in Section 3.0 are of interest since the solutions illustrate the manner in which the induced fields will scale and Figs. 2.16 and 2.19 show the strength of the dependence of the normalized field functions on the aspect ratio of the shells. They are also useful for estimating the difference in the strength of the induced field from loops relative to that which could be obtained from a complete shell at the instant immediately following the plasma displacement. The time constants for decay of the induced fields would be expected to be different for the two cases.

Figure 5.3a is the same as Fig. 2.4a which showed contours of dimensionless radial field for discrete loops. Three complete toroidal shells with different aspect ratios are superimposed on the contours. The dimensionless radial field which can be produced by each of these shells is $G_8$ which may be found in Fig. 2.16. Values for $r_0/R = 1.0$ were used and are given in Table 5.3, line 2a. The corresponding function for discrete loops is $G_2$ and the maximum values for $G_2$ outside any one of the shells may be found in Fig. 5.3a as points A, B and C. These values are given in Table 5.3 in line 2b. Results indicate that a shell with aspect ratio $R/r_m = 3$, for example, can produce 1.5 times the radial field which can be produced by a pair of discrete loops at the most effective location outside the same shell envelope.

A similar approach was used in Fig. 5.3b which is the same as Fig. 2.7a for radial stabilization by discrete loops. Table 5.3 gives the dimensionless $z$-field function, $G_{10}$ for shells and the directly comparable maximum $G_5$ for loops on the shell boundaries. Results again show that the shells are substantially more effective.
### TABLE 5.3 - EFFECTIVENESS OF DISCRETE LOOPS VS TOROIDAL SHELLS OF CIRCULAR SECTION

1. Shell Geometry, $R/r_m$

<table>
<thead>
<tr>
<th>$R/r_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>2.5</td>
</tr>
<tr>
<td>3.0</td>
</tr>
</tbody>
</table>

2. Vertical Stabilization

<table>
<thead>
<tr>
<th>Description</th>
<th>2</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_8$ for shell</td>
<td>0.51</td>
<td>0.81</td>
<td>1.25</td>
</tr>
<tr>
<td>$G_{2 \text{ max}}$ for loops</td>
<td>0.28</td>
<td>0.5</td>
<td>0.88</td>
</tr>
<tr>
<td>Shell/loops</td>
<td>1.82</td>
<td>1.62</td>
<td>1.42</td>
</tr>
</tbody>
</table>

3. Radial Stabilization

<table>
<thead>
<tr>
<th>Description</th>
<th>2</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{10}$ for shell</td>
<td>-0.55</td>
<td>-0.85</td>
<td>-1.28</td>
</tr>
<tr>
<td>$G_5 \text{ max}$ for loops</td>
<td>-0.24</td>
<td>-0.37</td>
<td>-0.52</td>
</tr>
<tr>
<td>Shell/loops</td>
<td>2.29</td>
<td>2.30</td>
<td>2.46</td>
</tr>
</tbody>
</table>
Figure 5.3a - Toroidal shells of circular section superimposed on contours for radial field produced by discrete loops.

Figure 5.3b - Toroidal shells of circular section superimposed on contours for vertical field produced by discrete loops.
5.4 Noncircular Partial Shells and Time Constants

Section 4.0 presented some results for passive vertical stabilization using noncircular complete and partial shells based on computations using an axisymmetric finite element model. The inner and outer shells used in the model are shown in Fig. 5.4 together with three INTOR boundaries. The finite element shells were selected as possible locations for passive conductors then used to generate the information in Figs. 4.2 and 4.3 for complete and partial shells.

Table 5.4 gives results for the four cases treated in Fig. 4.3. Line 2 gives the normalized radial field function, $G_{11}$, for each case and line 3 gives the radial field generated per unit vertical displacement based on $G_{11}$ and the INTOR parameters ($I_p = 6.4$ MA and $r_o = 5.3$ m) using Eq. (23). Equation (24) was used in Section 5.1 to arrive at a criterion for stability with the result that $[B_T/\Delta z]_{reqd} = 0.089$ T/m. A comparison of this requirement with the results in Table 5.4 shows that the criterion can be satisfied with either the complete inner shell or partial inner shell but not with the complete outer shell or partial outer shell.

Characteristic time constants for the four cases were found for the computed curves in Fig. 4.3 and are given in Table 5.4, line 4a, 4b, and 4c, respectively, for copper, aluminum and stainless steel walls. This illustrates the relatively large range of possibilities for adjustment of this parameter. Time constants for other wall thicknesses or materials may be estimated by assuming these time constants to be proportional to the product of wall thickness and electrical conductivity.
Figure 5.4 - Illustration of finite element model shell locations relative to the INTOR inner boundary, outer blanket boundary (assumes 0.5 m blanket), and outer shield boundary (assumes 1 m shield + 0.1 m gap).
TABLE 5.4 - CHARACTERISTICS OF NON-CIRCULAR TOROIDAL SHELLS (SEE FIG. 4.3)

<table>
<thead>
<tr>
<th></th>
<th>Inner Shell</th>
<th>Inner Shell</th>
<th>Outer Shell</th>
<th>Outer Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Configuration</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. (G_{11} = G_8) Equivalent</td>
<td>0.78</td>
<td>0.68</td>
<td>0.28</td>
<td>0.17</td>
</tr>
<tr>
<td>3. (B_T/\Delta z, [T,M])</td>
<td>0.22</td>
<td>0.19</td>
<td>.078</td>
<td>.047</td>
</tr>
<tr>
<td>4. (\tau_{eq}, [sec])</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. 1.38 cm Cu</td>
<td>3.13</td>
<td>2.94</td>
<td>1.39</td>
<td>0.93</td>
</tr>
<tr>
<td>b. 1.38 cm Al</td>
<td>1.91</td>
<td>1.80</td>
<td>0.85</td>
<td>0.57</td>
</tr>
<tr>
<td>c. 1.38 cm SS</td>
<td>6.26x10^{-2}</td>
<td>5.88x10^{-2}</td>
<td>2.78x10^{-2}</td>
<td>1.86x10^{-2}</td>
</tr>
<tr>
<td>5. (\sigma t_s, [\Omega^{-1}])</td>
<td>8 x 10^5</td>
<td>8 x 10^5</td>
<td>8 x 10^5</td>
<td>8 x 10^5</td>
</tr>
</tbody>
</table>

Note: \(\tau_{eq} \propto \sigma t_s\)

\(\sigma = \) wall electrical conductivity

\(t_s = \) wall thickness
The model assumes toroidal continuity. If shells were sectored and gaps were relatively narrow, the values of normalized field function or, in turn, the radial field per unit displacement would be essentially the same immediately after an instantaneous displacement. The time constants, however, would be considerably decreased by a factor estimated at 2 to 2.5 as was the case for the discrete loops which were sectored in Table 5.1, lines 9 and 11.
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