A One-Dimensional Fluid Model for Transport in Divertor and Limiter Tokamak Scrape-off Layers

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Abstract

Single-fluid transport in the plasma scrape-off layer is modeled for poloidal divertor and mechanically limited discharges. This numerical model is one-dimensional along a field line and time-independent. Conductive and convective transport, as well as impurity and neutral source (sink) terms are included. A simple "shooting" method technique is used for obtaining solutions. Results are shown for the case of the proposed Alcator DCT tokamak.
Introduction

Recent experimental measurements at the plasma edge in both diverted \cite{1,2} and mechanically limited tokamaks \cite{3-5} indicate that the electron density and temperature are not constant along a field line. In addition large local radiation losses are found in regions of the edge called marfes \cite{5}. Understanding the mechanisms involved in these inhomogeneities could be used to control the power flowing into the scrape-off layer. If that power could be dissipated over a large area at the plasma edge through charge exchange and radiation processes, then the remaining sputtering and heat loads to limiter/divertor surfaces would be greatly reduced. Similar results occur when the high recycling condition, analogous to a marfe, is created in a divertor chamber.

Various models have been developed elsewhere to understand edge transport. These include 2-d \cite{6}, and 1-d \cite{7-10} computer codes. Variations in these models occur due to inclusion (exclusion) of impurity and neutral sources (sinks), viscosity terms and treatment of ions and electrons as two fluids or one.

Model Equations

The equations used in this model are time-independent and single fluid ($T_e = T_i$). They are solved in one dimension along a field line in the scrape-off layer. This computer model is applicable to either mechanically (fig. 1a) or magnetically (fig. 1b) limited plasma geometries by
simply changing the distribution of the source terms involving perpendicular transport out of the main plasma. Basically one end \(x = 0\) is defined to be the symmetry point between limiter/divertor plates. The other end, \(x = L\), is then located at the limiter/plate surface. The equations are:

\[
\frac{\partial}{\partial x} (nV) = S_N \tag{1}
\]

\[
\frac{\partial}{\partial x} (2nT + M_4 nV^2) = S_p \tag{2}
\]

\[
\frac{\partial}{\partial x} (5nTV + 1/2 M_4 nV^3 - \kappa_e T^{5/2}/VT) = S_E \tag{3}
\]

\(\kappa_e\) is the usual Braginskii electron thermal conduction coefficient [11]. \(S_N, S_p\) and \(S_E\) are respectively the particle, momentum and energy sources:

\[
S_N = n_e [n_o \langle ov \rangle_{\text{ionization}} - n_i \langle ov \rangle_{\text{recombination}}] + S_{\text{perp}}
\]

\[
S_p = -M_4 V [\langle ov \rangle_{\text{recombination}} + \langle ov \rangle_{\text{cx}}] \tag{5}
\]

\[
S_E = Q_{\text{perp}} - \langle ov \rangle_{\text{recombination}} [13.6 \text{ eV} + 3 \text{ kT} + 1/2 M_4 V^2]
- \langle ov \rangle_{\text{cx}} [3/2 \text{ kT} + 1/2 M_4 V^2]
- n_1 n_e L_{\text{OXY}} (T) \tag{6}
\]

These terms and others can be found with a more detailed description in Morgan [9]. \(S_{\text{perp}}\) and \(Q_{\text{perp}}\) are the particle and energy sources originating from the main plasma. \(N_I\) and \(N_o\) are the impurity (oxygen) and neutral densities respectively. \(L_{\text{OXY}} (T)\) is the radiative cooling rate for the ex-
amined impurity (oxygen) and \( V \) the bulk fluid velocity along a field line.

**Boundary conditions**

The end of the computational grid (figure 1), defined by \( x = 0 \), is a symmetry point. The boundary conditions there are determined accordingly:

\[
\frac{\partial T}{\partial x} \bigg|_{x=0} = 0
\]

At the target plate or limiter surface \( (x = L) \) the boundary conditions are determined by the Bohm sheath criterion. The form used here is that derived by Emmert [12]. Equating the power flow along the field line at the sheath edge to the power flow through the sheath using this criterion we have:

\[
\left[ 5nTV + \frac{1}{2} M_i v^3 - \kappa_e T^{5/2} v T \right]_{x=L} = \Gamma x (W_i + W_e)
\]

\[
= \left[ \beta \left( \frac{\kappa T}{2\pi M_i} \right)^{1/2} \right] \times [(\mu kT) + (\phi_{\text{sheath}} - \phi_{\text{wall}} + 2kT)]
\]

\( \Gamma \) and \( \omega_i,e \) are particle flux and energy respectively. \( \beta \) and \( \mu \) are defined in reference 12.

**Numerical method and assumptions**

The code uses a "shooting" method moving from \( x = 0 \) to \( x = L \). The first four points are determined using a Runge-Kutta extrapolation. The remaining grid points are calculated with a Milne predictor-corrector [13].
Neutral density along the field line is set to:

\[ n_0(x) = n_{\text{plate}} \left( \frac{x-L}{\lambda_n} \right) + n_{\text{background}} \]  

(10)

Typical values for \( n_{\text{plate}} \) and \( n_{\text{background}} \) are \( 1 \times 10^{12} \) and \( 1 \times 10^{10} \) particles/cm\(^3\) respectively. The background impurity is chosen to be oxygen, but any other low-z impurity would produce similar results. The impurity density is set to be a fixed fraction of the electron density everywhere. Similar to Shimada [8], the cooling rate \( L(T) \) used is noncoronal. \( Q_{\text{perp}} \) and \( S_{\text{perp}} \) are assumed uniform along the length of field line within the main plasma chamber. The electron density at \( x = 0 \) is chosen as an input. \( T \) at the same point is guessed, followed by calculations at the remaining grid points. Successive guesses of \( T(x = 0) \) are made until the sheath criterion is satisfied at \( x = L \). With 200 equally spaced grid points the cpu time required on a VAX 11/780 ranged from 8 to 60 seconds depending on initial conditions.

**Results**

This code has primarily been used to model the scrapeoff layer of a diverted discharge in the proposed Alcator DCT tokamak [14]. The basic parameters of Alcator DCT used in this model are provided in Table 1. A typical profile of various quantities along a field line can be seen in figure 2. A divertor discharge with medium edge plasma density in the main chamber (\( x = 0 \)) is used. Temperature at the plate drops rapidly as the edge density (\( x = 0 \)) is raised. The opposite dependence on edge density is exhibited by the density at the plate (figure 3). This strong dependence of the divertor chamber parameters on the main plasma edge density is suggestive of the marfe phenomenon. Basically, the inclusion
of impurities in divertor simulations allows achievement of the high recycling condition at lower edge densities. Illustration of the high recycling condition and its effects can be found in figure 4. As the edge density is raised the flux multiplication factor, defined as the ratio of particle flux at the plate to that at the divertor throat, also rises. The average energy per incident ion drops quickly as well. A reduction of the sputtering coefficient follows from this. Power to the plate will be reduced due to the inclusion of charge exchange neutral losses. Radiation from oxygen impurities also figure heavily in reducing the heat flow to the plate. These effects are illustrated in figure 5.

Future Work

One possible next step is to apply this program to modeling of the marfe. This requires $Q_{\text{perp}}$, and perhaps $S_{\text{perp}}$ as well, to be nonuniform along a field line [5]. Other improvements to the code that should be made are: proper impurity and neutral transport algorithms; generalization of the single fluid description to two fluids; and relating the neutral pressure at the plate/limiter to the particle flux and recycling coefficient ($R$) there. In a steady state plasma the $R$ of the plate itself must be 1. But inclusion of processes such as pumping or loss of neutrals to the main plasma effectively reduces $R$ to less than 1. Relating plate neutral pressure to the incident charged particle flux will obviate the need to specify $n_e(x = 0)$. That edge plasma density will be determined by the transport of particles out of the main plasma ($S_{\text{perp}}$) and edge physics.

An area of improvement that would not be a modification of the existing program would be the creation of a completely new code. A problem of the shooting method used here is that there is strong control over the
boundary conditions only at one end of the computational grid. Furthermore, impurity and neutral transport terms cannot be added easily to the present numerical algorithm. A matrix inversion solution of the equations is the obvious choice. Time dependent equations will be used because of the ability to model experimental data.

**Summary**

A one dimensional, single fluid, edge transport code has been developed. It includes conductive and convective transport as well as neutral and impurity sources. Previous divertor modelling results have been qualitatively reproduced with a reduced expenditure in computer cpu time.

**Table 1**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma major radius, $R_0$</td>
<td>2.0 m</td>
</tr>
<tr>
<td>Plasma minor radius</td>
<td>$0.40 \times 0.56$ m</td>
</tr>
<tr>
<td>Vacuum vessel bore</td>
<td>$1.1 \times 1.56$ m</td>
</tr>
<tr>
<td>Toroidal magnetic field ($R = R_0$)</td>
<td>7.0 T</td>
</tr>
<tr>
<td>Toroidal field ripple on axis</td>
<td>0.04%</td>
</tr>
<tr>
<td>Maximum field in conductor</td>
<td>10.0 T</td>
</tr>
<tr>
<td>TF conductor</td>
<td>Nb$_3$Sn</td>
</tr>
<tr>
<td>Poloidal field conductor</td>
<td>NbTi</td>
</tr>
<tr>
<td>Total flux swing ($R = R_0$)</td>
<td>35 Wb</td>
</tr>
<tr>
<td>Plasma current ($q = 3$)</td>
<td>1.0 MA</td>
</tr>
<tr>
<td>Heating</td>
<td>LH 4 MW(cw)</td>
</tr>
<tr>
<td></td>
<td>ICRF 5 MW(cw)</td>
</tr>
<tr>
<td></td>
<td>8 MW(30 sec)</td>
</tr>
<tr>
<td>Maximum discharge duration (minutes)</td>
<td>1.5 - 5, w/ohmic</td>
</tr>
<tr>
<td></td>
<td>$\infty$, w/LH current drive</td>
</tr>
</tbody>
</table>
Acknowledgements

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References


Figure Captions

Figure 1: Field line geometry for plasmas that are a) mechanically limited and magnetically limited plasmas. x = 0 is the symmetry point between limiter/divertor plate surfaces (x = L).

Figure 2: Typical profiles of n_e, T_e, P_{TOT}, P_{cons}, and P_{con v} along a field line in divertor geometry.

Figure 3: Ratio of plate (n_p) to plasma edge (n_e) density vs. plasma edge density n_e.

Figure 4: Flux multiplication (-x-x-) defined as \( \frac{\Gamma_{\text{divertor plate}}}{\Gamma_{\text{divertor entrance}}} \) and E_{AVE} per incident ion (0-0-0) vs. n_e at the plasma edge.

Figure 5: Total (-x-x-) and charge exchange power (0-0-0) losses versus n_e at the plasma edge.
MAGNETIC FIELD

UNIFORM SOURCE OF ENERGY+PLASMA PARTICLES

LIMITER

SHEATH

$\begin{align*}
  x = 0 & \quad v_{\|} = 0 \\
  x = \frac{1}{2} L_1 & \quad v_{\|} = C_s
\end{align*}$

ENTRANCE TO DIVERTOR CHAMBER

DIVERTOR PLATE

Figure 1
Figure 2
Figure 3

- $n_{\text{oxy}}/n_e = 0.05$
- $n_{\text{oxy}}/n_e = 0.005$
\[ n_{\text{oxy}}/n_e = 0.01 \]
\[ P_{\text{edge}} = 10 \text{ Mwatts} \]
$n_{\text{oxy}}/n_e = 0.01$

$P_{\text{edge}} = 10 \text{ Mwatts}$

**Figure 5**

$N_e$ AT SYMMETRY POINT ($\times 10^{13} \text{ cm}^{-3}$)