Comments on Equilibrium Plasma Flows in the Limiter Shadow Region of Alcator C

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Abstract

Resistive MHD is used to examine bulk plasma flows in the limiter shadow region of Alcator C. Expressions for the Pfirsch-Schlüter perpendicular plasma flow velocities are obtained for a toroidal system in which the pressure profile depends on radius only. Data from Langmuir probe measurements in the shadow of the limiter is used to estimate the magnitude of this plasma convection.

Because of the short density scrape-off lengths in Alcator C edge plasma (~3 mm), the magnitude of the perpendicular flow velocity can lead to significant poloidal and/or toroidal flow velocity components. If the primary contribution to the perpendicular flow velocity is from the poloidal component, then the magnitude of the poloidal flow can easily exceed that of a radial flow velocity estimated from Bohm diffusion. On the other hand, if toroidal flow velocity dominates the total perpendicular flow, then the toroidal velocity can be a significant fraction of the sound speed. As a result, self-consistent plasma density and temperature profiles can exhibit a poloidal asymmetry. Such a poloidal asymmetry may explain the preferred location of the "marfe" phenomena observed in the Alcator C tokamak.

The scaling of the perpendicular flow velocity obtained is consistent with the observed combination of edge density, scrape-off length, and plasma current which precipitates a marfe. The poloidal component of the perpendicular flow velocity exhibits a stagnation point on the inside and outside midplane where this component changes sign. The direction of the poloidal flow is always towards the inside of the torus independent of toroidal field and plasma current directions.

Since this perpendicular equilibrium allows for an arbitrary radial ambipolar E-field to be present, an $E \times B$ plasma rotation can be superimposed on these flows. In this case, the inner stagnation point can move to the upper or lower inside position depending on the toroidal magnetic field direction. If such a poloidal rotation on the order suggested by CO$_2$ laser scattering data is included, the stagnation point coincides with the observed upper inside marfe position for the normal toroidal field direction and is consistent with preliminary observations of the marfe moving to the lower inside when the toroidal field is reversed.

Independent of the mechanism by which a marfe initiates, Pfirsch-Schlüter cross field convection may act to scrape-off the edge plasma boundary and transport energy into the radiating marfe region. In this way, a convection/radiation power balance can be maintained, forcing the marfe boundary to be toroidally symmetric and localized poloidally to the inside of the torus.
I. Introduction

There is an increasing interest in the edge plasma region of tokamaks particularly with the observation of low temperature, high density plasma regions near the edge in limited discharges [1-4] and low or high recycling regimes associated with diverted discharges [5-7]. The "marfe" phenomena in Alcator C appears as a strong poloidal asymmetry in plasma density and radiated power. Although the occurrence of a marfe is attributed to a thermal instability, the existence of a preferred poloidal position has not yet been explained satisfactorily.

Theoretical models of boundary plasma predicting local enhanced diffusion from shocks or increased turbulence have been studied in the past [8-11]. Rosen & Greene [12] discuss the velocity convection pattern for interior plasma in the Pfirsch-Schlüter regime and obtain a radial boundary layer model which matches the interior flows and satisfies edge boundary conditions. A weak shock is identified in this model. More recently, Motley [13] has emphasized the role that electric fields may have in edge plasma by discussing the electric potential mismatch between the outer plasma region and the limiter shadow region due to the presence of a conducting poloidal limiter. It is speculated that this non-axisymmetric boundary condition may be a source of free energy driving instabilities and an "active limiter" biased to the correct spatial variation of the electric potential might minimize this mismatch. Although turbulent transport due to instabilities and/or shocks may be operating to determine the rate of particle diffusion, $\mathbf{E} \times \mathbf{B}$ convective flows satisfying plasma equilibrium can be much larger than diffusive flows, approaching the sound speed in some cases. Thus, the steady state plasma pressure profile in the limiter shadow region may take on a poloidal asymmetry
as a result of perpendicular equilibrium alone.

This paper models the collisional limiter shadow plasma of Alcator C as a resistive MHD fluid. A scaling of perpendicular flow velocity with edge plasma density, density scrape-off length, plasma current, and edge electron temperature is obtained. The plasma pressure profile is modeled as an exponential in minor radius based on Langmuir probe measurements [14] and is assumed to be independent of poloidal or toroidal angle. Since convective flow velocities are shown to be large, a self consistent calculation including continuity and bulk flow momentum can yield a poloidally-asymmetric plasma pressure profile. Given that a marfe is a thermal instability, even a slight asymmetry in density and/or temperature could explain the marfe's preferred location of occurrence. For a small asymmetry, the magnitude of the asymmetry is expected to scale like the perpendicular flow velocity scaling since this is the asymmetric driving term. Also, the position of poloidal maxima and minima in density and/or temperature is expected to be related to the positions of maxima, minima, or stagnation points in the poloidal component of the flow.

Section II serves as a review and derivation of relevant MHD quantities needed to calculate Pfirsch-Schlüter plasma convection. First, the physics giving rise to a poloidally varying electric potential due to resistive MHD equilibrium in a toroidal geometry is reviewed. Using measurements of edge plasma density and temperature gradients in the limiter shadow region of Alcator C, it is shown that a radial E-field associated with the poloidally varying potential must exist and be oriented in such a way as to support $E \times B$ particle fluxes directed to the
stagnation point on inside midplane of the torus. With the superposition of an additional radial E-field arising from a poloidally symmetric ambipolar potential, the stagnation point is above or below the midplane on the inside edge of the plasma. This vertical position is reversed upon reversal of the toroidal field.

In Section III, a scenario is proposed in which bulk plasma flows might force a poloidal asymmetry in the edge plasma density and/or temperature and lead to the development of a marfe. A comparison of expected scaling with observations is made. A correlation is identified between the location of the inside stagnation point in the poloidal flow and the location of the marfe position. Once a marfe is initiated by whatever mechanism, perpendicular plasma convection may be responsible for transporting energy into the radiating marfe region thereby determining the poloidal location and extent of the toroidally symmetric marfe region.

Section IV provides some closing comments on the model's validity and describes work that is in progress.
II. Edge Plasma MHD Equilibrium

The starting point is to consider the requirement of steady state MHD equilibrium imposed on the limiter shadow plasma. A toroidal coordinate system is used (Figure 1). Equilibrium requires [15]

\[ \mathbf{J} \times \mathbf{B} = \nabla P \]  

(1)

For simplicity, it is assumed that only radial pressure gradients exist.

\[ \nabla P = \frac{3P}{2r} \hat{r} \]  

(2)

A self-consistent equilibrium may require that a poloidal variation in pressure exist. Thus, this calculation assumes that all poloidally asymmetric terms obtained work to perturb this poloidally symmetric pressure profile slightly.

Model B-fields assumed to be present in the edge plasma are

\[ \mathbf{B} = B_\phi \hat{\phi} + B_\theta \hat{\theta} \]  

(3)

\[ B_\phi = \frac{B_\phi \circ}{1 + \epsilon \cos \theta} \]  

(4)

\[ B_\theta = \frac{B_\theta \circ}{1 + \epsilon \cos \theta} \]  

(5)

Where \( \epsilon \) is the local inverse aspect ratio, \( B_\phi \circ \) is the toroidal field on axis and \( B_\theta \circ \) is the poloidal field at \( \theta = \pi/2 \) and some \( r \). The choice of the form of (4) and (5) allow \( \mathbf{B} \) to satisfy \( \nabla \cdot \mathbf{B} = 0 \) without any additional fields.
II.1 Plasma current

Taking $\mathbf{B} \times$ Equation (1) and using (3) one obtains the familiar expression for the "diamagnetic current",

$$ J_\perp = \frac{\mathbf{B} \times \nabla P}{B^2} = \frac{\partial P}{\partial r} \hat{\theta} - \frac{\partial P}{\partial r} \hat{\phi} $$

(6)

where $J_\perp$ is defined everywhere locally as the component of current flowing perpendicular to total $\mathbf{B}$. This MHD prescription for $J_\perp$ includes all contributions of current due to particle orbital motion including $\mathbf{V} |\mathbf{B}|$ and curvature drifts. The requirement of steady state also implies:

$$ \frac{\partial J}{\partial t} + \nabla \cdot J = 0 $$

(7)

resulting in

$$ \nabla \cdot J_\perp = - \nabla \cdot J_\perp $$

(8)

so that if $\nabla \cdot J_\perp \neq 0$ there must be currents flowing along magnetic field lines. For a tokamak, this is indeed the case and the resulting currents are referred to as the Pfirsch-Schlüter currents which flow parallel to total $\mathbf{B}$. Using vector identities, $\nabla \cdot J_\perp$ can be rewritten as

$$ \nabla \cdot J_\perp = 2
\mathbf{V}P \cdot \frac{\mathbf{B} \times \nabla |\mathbf{B}|}{B^3} $$

(9)

Now it can be easily seen that $\mathbf{B} \times \nabla |\mathbf{B}|$ particle drifts across a pressure gradient are responsible for the Pfirsch-Schlüter currents in a tokamak.
Using Equations (2 - 5), Equation (9) can be expressed for a tokamak as

\[ \nabla \cdot J_1 = -2 \frac{\partial P}{\partial r} \frac{B_{\phi}}{B^2} \epsilon \sin \theta \]  \hspace{1cm} (10)

retaining terms of order \( \epsilon \).

From \( \nabla \cdot \mathbf{v} \) given in Appendix A,

\[ \nabla \cdot J_1 = \frac{1}{r(1 + \epsilon \cos \theta)} \left[ \frac{\partial}{\partial r} \left( r(1 + \epsilon \cos \theta) J_1 r \right) + \frac{\partial}{\partial \theta} (1 + \epsilon \cos \theta) J_{\theta} + \frac{\partial}{\partial \phi} \epsilon J_{\phi} \right] \]  \hspace{1cm} (11)

and using Equations (8) and (10)

\[ \frac{\partial}{\partial \theta} (1 + \epsilon \cos \theta) J_{\theta} = 2 \frac{\partial P}{\partial r} \frac{B_{\phi}}{B^2} \epsilon \sin \theta (1 + \epsilon \cos \theta) \]

Now integrate in \( \theta \), again retaining terms of order \( \epsilon \)

\[ J_{\theta} = -2 \frac{\partial P}{\partial r} \frac{B_{\phi}}{B^2} \epsilon \cos \theta + \frac{f(r)}{(1 + \epsilon \cos \theta)} \]  \hspace{1cm} (12)
The toroidal component of $J_\| $ can be obtained by taking $J_\| \cdot (6)$. 

Then,

$$J_\| \theta \frac{\partial P}{\partial r} = J_\| \phi B_\phi \frac{\partial P}{\partial r}$$

or

$$J_\| \phi = \frac{B_\phi}{B_\theta} J_\| \theta$$

as expected.

The total parallel current is now determined for this pressure profile and magnetic field geometry. The first term in Equation (12) is the Pfirsch-Schl"{u}ter current contribution to $J_\|$. The second term contains an integration constant, $f(r)$, which must be independent of $\theta$. This constant is determined by parallel resistivity and the ohmic heating E-field and would take on a value such that the B-fields are self consistent in a more exact analysis.

II.2 E-fields

The next step is to consider the E-fields which must be present in order to drive these currents. For this, consider the following form of Ohm's law that neglects pressure gradient effects and the Hall current terms:

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \cdot \mathbf{J}$$

Looking at the component parallel to $\mathbf{B}$ one obtains

$$E_\| = \eta \| J_\| = \eta \| J_\| \theta \hat{\theta} + \eta \| J_\| \phi \hat{\phi}$$
and taking \((15) \times B\), using vector identities one obtains,

\[
\mathbf{v}_1 = \frac{\mathbf{E} \times B}{B^2} - \frac{n_1}{B^2} \mathbf{v}_P
\] (17)

\(E_1\) is easily obtained from (16) and the previous calculation of \(J_1\). If \(E_1\) can be calculated, the perpendicular plasma flow velocity due to \(E \times B\) can be obtained directly from the first term in Equation (17). The second term in Equation (17) is just classical diffusion due to the presence of the pressure gradient.

\(E_1\) will now be obtained by recognizing that there is an electrostatic \(E_1\) component driving the Pfirsch-Schlüter currents. As a result, there must be an electrostatic \(E_1\) such that \(\mathbf{v} \times \mathbf{E} = 0\). To examine this more clearly, divide the electric fields into a superposition of electrostatic and induced fields

\[
\mathbf{E} = \mathbf{E}_{es} + \mathbf{E}_{oh}
\] (18)

such that

\[
\mathbf{v} \times \mathbf{E}_{es} = 0 \quad \text{and} \quad \mathbf{v} \cdot \mathbf{E}_{oh} = 0
\]

Where superscripts "es" and "oh" refer to electrostatic and induced ohmic heating E-fields respectively. For a tokamak,

\[
\mathbf{E}_{oh} = \mathbf{E}_{oh} \hat{\phi}
\] (19)

while \(\mathbf{E}_{es}\) can contain all three components. In terms of parallel and perpendicular quantities,

\[
\mathbf{E}_{es} = \mathbf{E}_{r} \hat{r} + (\mathbf{E}_{10} + \mathbf{E}_{15}) \hat{\theta} + (\mathbf{E}_{q1} + \mathbf{E}_{q2}) \hat{\phi}
\] (20)
The radial direction is everywhere perpendicular to \( \mathbf{B} \) and so the \( r \) subscript is dropped for that component. For a toroidally symmetric system, the toroidal component of the electrostatic E-field must be zero. This implies

\[
E_{t\phi}^e = - E_{t\phi}^e
\]  
(21)

The ohmic heating E-field contains no \( \theta \) component similarly implying

\[
E_{t\theta}^o = - E_{t\theta}^o
\]  
(22)

The rotational transform relates the \((\hat{\theta}, \hat{\phi})\) coordinate system to the coordinates parallel and perpendicular to the magnetic field (Figure 2). By construction, the following relationships hold:

\[
\begin{align*}
E_{t\phi} & = - \frac{B_{\theta}}{B_{\phi}} \\
E_{t\theta} & = \frac{B_{\phi}}{B_{\theta}}
\end{align*}
\]  
(23)

Combining (22), (23), and (24), \( E_{t\theta}^o \) can be expressed as

\[
E_{t\theta}^o = \frac{B_{\phi} B_{\phi} E_{t\phi}^o}{B^2}
\]  
(25)

Now the integration constant, \( f(r) \), in Equation (12) must be

\[
f(r) = \frac{E_{t\theta}^o}{\eta_1} = \frac{B_{\phi} B_{\phi} E_{t\phi}^o}{\eta_1 B^2}
\]  
(26)
From Equation (16), (26) and the calculated parallel currents given by Equations (12) and (14), the total parallel E-fields can be expressed as

\[
E_{1\theta} = -2n_i \frac{\partial P}{\partial r} \frac{B_\phi}{B^2} \cos \theta + \frac{B_\theta B_\phi \rho_{oh}}{B^2 (1 + \epsilon \cos \theta)}
\]  
(27)

\[
E_{1\phi} = -2n_i \frac{\partial P}{\partial r} \frac{B_\phi^2}{B_\phi B^2} \cos \theta + \frac{B_\phi^2 \rho_{oh}}{B^2 (1 + \epsilon \cos \theta)}
\]  
(28)

Thus, the electrostatic fields are known.

Combining (21), (23), (24) with (20),

\[
E^{es}_{1\theta} = E^s_{1\theta} \frac{\partial}{\partial r} + (1 + \frac{B_\phi^2}{B_\phi B^2}) E^{es}_{1\theta} \frac{\partial}{\partial \theta}
\]  
(31)

Applying \( \nabla \times E^{es} \) defined in Appendix A and using Equation (31), the following three conditions result:

\[
\frac{\partial}{\partial \phi} \left[ (1 + \frac{B_\phi^2}{B_\phi B^2}) E^{es}_{1\theta} \right] = 0
\]  
(32)
Equations (32) and (33) are satisfied by toroidal symmetry. Equation (34) allows a calculation of \( E_{\text{es}}^r \), the final unknown electrostatic E-field component.

Using Equation (29) and (34) and integrating in \( \theta \),

\[
E_{\text{es}}^r = -\frac{3}{3} \left[ r \left(1 + \frac{B_\theta^2}{B_\phi^2}\right) 2 \eta_1 \frac{\partial P}{\partial r} \frac{B_\phi}{B^2} \epsilon \sin \theta \right] - \frac{3}{\partial r} [\phi(r)]
\]

where \( \phi(r) \) is a \( \theta \) independent integration constant. \( \phi(r) \) allows for a poloidally symmetric ambipolar potential to be present.

II.3 Bulk Flows

Now the perpendicular bulk plasma flow velocity given by Equation (17) can be calculated using the expressions for the E-fields.

Consider the r-component of (17) first. Using the expressions for \( E_{\text{es}}^r \) given by Equations (31) and (29) and including \( E_{\phi}^{\text{oh}} \),

\[
V_r = -2 \eta_1 \left(1 + \frac{B_\theta^2}{B_\phi^2}\right) \frac{B_\phi^2}{B^4} \frac{\partial P}{\partial r} \epsilon \cos \theta -
\]

\[
\frac{E_{\phi}^{\text{oh}} B_\theta}{B^2} - \frac{\eta_1}{B^2} \frac{\partial P}{\partial r}
\]

\[
(36)
\]
The first term looks similar to a diffusion term. After averaging $V_r$ over a flux surface, this first term combined with the diffusion term leads to the Pfirsch-Schlüter neoclassical correction factor of $1 + q^2$ times the classical diffusion coefficient. The second term is a $E \times B$ radial inward flow due to the toroidal $E$-field and the poloidal $B$-field. The magnitude of this term is on the order of 1 cm/sec in Alcator C and is always negligible in tokamaks.

Now consider the $\hat{\theta}$ component of Equation (17). Using the expression for $E_r$ given by Equation (35),

$$V_{\hat{\theta}} = \frac{B_\phi}{B^2} \frac{\partial}{\partial r} \left[ r(1 + \frac{B_\phi^2}{B_\theta^2}) 2n_\parallel \frac{\partial P}{\partial r} \frac{B_\phi}{B^2} \epsilon \sin \theta \right] +$$

$$\frac{B_\phi}{B^2} \frac{\partial \Phi(r)}{\partial r}$$

The $\hat{\phi}$ component is obtained by simply multiplying Equation (37) x $-B_\theta/B_\phi$

$$V_{\hat{\phi}} = \frac{-B_\theta}{B_\phi} V_{\hat{\theta}}$$

The perpendicular bulk plasma flow due to this pressure profile and magnetic field geometry is now contained in Equations (36)-(38). Equation (37) has a term modulated by $\sin \theta$ which can lead to stagnation points in the poloidal component of perpendicular flow velocity. Appendix B illustrates the origin of this perpendicular plasma convection.
The parallel flow velocity does not enter into the perpendicular equilibrium and therefore must either be specified or determined by solving an appropriate parallel momentum equation with parallel flow momentum included. An additional constraint arises in that \( V_\parallel \) must also be such that quantities are periodic in poloidal angle. Although we cannot simply determine the total flow velocity without solving for the parallel flow velocity in the parallel momentum equations, some simple cases illustrating the size and scaling of the bulk flow velocities can be considered.

Consider the case when \( V_\theta \) or \( V_\phi \) is completely specified. The total flow velocity can be written as:

\[
V = V_T \hat{r} + (V_{1\theta} + V_{\perp\theta}) \hat{\theta} + (V_{1\phi} + V_{\perp\phi}) \hat{\phi}
\]

with the understanding that \( V_{1\theta} \) and \( V_{1\phi} \) satisfy Equations (37) and (38) from perpendicular force balance. Specifying \( V_\theta \) or \( V_\phi \) completely determines \( V \) since we have only one free parameter. For the case of no toroidal flow,

\[
V_\phi(r, \theta) = 0 \quad \text{(case 1)}
\]

so that

\[
V_{1\phi} = -V_{\perp\phi}
\]

and using relations for \( V_1 \) and \( V_\perp \) similar to \( E_1 \) and \( E_\perp \) in equations (23) and (24), \( V_\theta \) can be expressed as

\[
V_\theta = \left(1 + \frac{B_\phi^2}{B_\theta^2}\right) V_{1\theta}
\]

(40)
For the case of no poloidal velocity,

\[ V_\theta(r, \theta) = 0 \quad \text{(case 2)} \]

so that

\[ V_{1\theta} = - V_{1\theta} \]

and \( V_\phi \) becomes

\[
V_\phi = - (1 + \frac{B_\phi^2}{B_\theta^2}) \frac{B_\theta}{B_\phi} V_{1\theta}
\]

(41)

The actual poloidal and toroidal velocities in a real system will probably be some complicated function of radius and poloidal angle, however, cases 1 & 2 will provide an estimate for the magnitude and scaling of the poloidal and toroidal flow velocities which must be present to satisfy perpendicular equilibrium.

Notice that in case 1, the poloidal flow velocity is about the same magnitude as the poloidal component of the perpendicular flow. This is expected since the perpendicular flow is mostly poloidal. Case 2, forcing the poloidal flow to be zero, requires the toroidal flow velocity to make up the necessary perpendicular flow and, as a result, the magnitude of the toroidal flow is factor of \( B_\phi/B_\theta \) larger.

In all cases, \( V_{1\theta} \) is identified as the relevant quantity that needs to be scaled in Alcator C edge plasma. The following section deals with sizing and scaling \( V_{1\theta} \) using Langmuir probe data.
II.4 **Calculation of $V_{18}$ from Measured Profiles**

The focus of this section is to estimate the magnitude of the first term in Equation (37) by applying measured profile data.

Divide Equation (37) into two terms.

$$V_{18} = V_{18}^I + V_{18}^{II}$$  \hspace{1cm} (42)

$$V_{18}^I = \frac{B_\phi}{B^2} \frac{\partial}{\partial r} \left[ r \left( 1 + \frac{B^2_\phi}{B^2_\theta} \right) \frac{B_\phi}{B^2} - \frac{2n_i}{3} \frac{\partial P}{\partial r} \right] \epsilon \sin \theta$$  \hspace{1cm} (43)

$$V_{18}^{II} = \frac{B_\phi}{B^2} \frac{\partial}{\partial r} \left[ \phi(r) \right]$$  \hspace{1cm} (44)

For $r > a$, ($a =$ limiter radius) the poloidal field at $\theta = \pi/2$ can be modeled as

$$B_{\phi0}(r) = \frac{a}{r} B_{\phi0}(a)$$  \hspace{1cm} (45)

Since $B^2_\phi/B^2_\theta \gg 1$ and $B_\phi/B_\theta = B_{\phi0}/B_{\theta0}$, approximate

$$1 + \frac{B^2_\phi}{B^2_\theta} = \frac{B^2_{\phi0}}{B^2_{\theta0}(a)} \frac{r^2}{a^2}$$  \hspace{1cm} (46)

Rewriting (43), and approximating $B_\phi = B$,

$$V_{18}^I = \frac{2\sin \theta}{B_{\phi0}(a)a^2R_0} \frac{\partial}{\partial r} \left[ n_i r^4 \frac{\partial P}{\partial r} \right]$$  \hspace{1cm} (47)
From probe measurements [14], the ion density profile in the shadow of the limiter is modelled well by

\[ n(r) = n(a) \exp \left( \frac{a-r}{\lambda_n} \right) \]  

(48)

Near the limiter radius the electron temperature profile in the limiter shadow plasma can be approximated by an exponential,

\[ T(r) = T(a) \exp \left( \frac{a-r}{\lambda_T} \right) \]  

(49)

so that the radial dependence of the pressure is calculated to be

\[ P(r) = 2A n(a) T(a) \exp \left( (a-r) \left( \frac{1}{\lambda_n} + \frac{1}{\lambda_T} \right) \right) \text{ (nt/m}^2 \text{)} \]  

(50)

\[ \frac{\partial P}{\partial r} = -2A n(a) T(a) \left( \frac{1}{\lambda_n} + \frac{1}{\lambda_T} \right) \exp \left( (a-r) \left( \frac{1}{\lambda_n} + \frac{1}{\lambda_T} \right) \right) \text{ (nt/m}^3 \text{)} \]  

(51)

with \( A = 1.6 \times 10^{-13}, n(a) \text{ in cm}^{-3}, T(a) \text{ in eV, } a, r, \lambda_n, \lambda_T, \text{ in meters.} \)

The Spitzer parallel resistivity \( \eta_1 \), can be expressed as:

\[ \eta_1 = \frac{B Z A}{T_e^{3/2}} \text{ (ohm-m)} \]  

(52)

with \( B = 5.2 \times 10^{-5} \text{ and } T_e \text{ in eV. } Z \text{ is the local plasma } Z_{\text{eff}} \text{ and } A \) is the coulomb logarithm.
Combining (51), (52) and (49),

\[
\eta_r r^4 \frac{\partial P}{\partial r} = \frac{-2 A B n(a) Z}{T(a)^{1/2}} \left( \frac{1}{\lambda_n} + \frac{1}{\lambda_T} \right) \Lambda r^4 \\
\exp((a-r)(\frac{1}{\lambda_n} - \frac{1}{2 \lambda_T}))
\]

(53)

Since \(\lambda_T = 2 \lambda_n = 6 \text{ mm}\), the exponential dominates the radial derivatives.

Equation (44), evaluated at the limiter radius, can be written as

\[
V_{1\theta} = \frac{4 A B \sin \theta n(a) Z a^2 \Lambda}{B_0(a) R_o T(a)^{1/2}} \left( \frac{1}{\lambda_n} + \frac{1}{\lambda_T} \right) \left( \frac{1}{\lambda_n} - \frac{1}{2 \lambda_T} \right) (\text{m/s})
\]

(54)

Now substituting

\[
B_0(a) = \frac{\mu_0 I}{2 \pi a}
\]

and combining the constants,

\[
V_{1\theta} = 8.3 \times 10^{-4} \frac{\sin \theta n(a) Z a^4 \Lambda}{I^2 R_o T(a)^{1/2}} \left( \frac{1}{\lambda_n} + \frac{1}{\lambda_T} \right) \left( \frac{1}{\lambda_n} - \frac{1}{2 \lambda_T} \right) (\text{m/s})
\]

(56)

with \(n(a)\) in \(\text{cm}^{-3}\), \(T(a)\) in eV and all other units MKS. Note that for \(\lambda_T > \lambda_n/2\), the direction of \(V_{1\theta}\) is to the inside midplane of the torus. Experimentally, \(\lambda_T >> \lambda_n/2\) for all cases.
Rewriting Equation (56) as

\[ \dot{V}_{l0} = \dot{V}_{l0}^{\text{MAX}} \sin \theta \]

\( \dot{V}_{l0}^{\text{MAX}} \) and the corresponding magnitudes of \( \dot{V}_{\theta} \) and \( \dot{V}_{\phi} \) in case 1 and case 2 can be evaluated for plasma parameters measured by probes in Alcator C edge plasma.

Table 1 compares measured edge parameters and calculated velocities for a discharge leading to a marfe event and a discharge which did not show such a poloidal asymmetry. The sound speed, Bohm diffusion coefficient, and radial diffusion velocity due to Bohm diffusion is included for reference. Note that although all flow velocities are under the sound speed, the toroidal flow can be a significant fraction of the sound speed and the poloidal flow can be at least a few times larger than the radial diffusive velocity. Thus, the simple picture of plasma diffusing radially out of a cylindrical plasma column clearly does not apply. Plasma flow in the limiter shadow region of Alcator C is dominated by flows perpendicular to \( \hat{r} \) satisfying perpendicular equilibrium and parallel momentum.
II.5 Including a Poloidal Rotation

The second term in Equation (37) contributing to the \( \hat{\theta} \) component of perpendicular plasma flow is

\[
V_{1\theta}^{I} = \frac{B_\phi}{B^2} \frac{\partial \Phi(r)}{\partial r} \tag{58}
\]

Where \( \Phi(r) \) is a poloidally symmetric electric potential determined by ambipolar plasma transport. The sign and magnitude of \( \Phi(r) \) cannot be simply calculated, however, again some simple observations can be made.

First, writing Equation (37) as

\[
V_{1\theta} = V_{1\theta}^{I\text{MAX}} \sin \theta + V_{1\theta}^{II} \tag{59}
\]

one can see that there are two values of \( \theta \) where \( V_{1\theta} = 0 \) as long as \( |V_{1\theta}^{II}| < V_{1\theta}^{I\text{MAX}} \). Figure 3 illustrates this assuming \( \partial \Phi / \partial r > 0 \) and \( B_\phi > 0 \). The sign of \( V_{1\theta}^{I} \) is independent of the sign of \( B_\phi \). However, the sign of \( V_{1\theta}^{II} \) changes upon reversal of \( B_\phi \) as seen by Equation (58).

Therefore, the result of a reversal of \( B_\phi \) is to change the zeros of \( V_{1\theta} \) from the range \( \pi, 2\pi \) to \( 0, \pi \). From the definition of the coordinate system in Figure 1, this is seen as a reflection of the zeros from above to below the midplane. Figure 4 illustrates this case. The sign of \( V_{\theta} \) when \( |V_{1\theta}^{II}| < V_{1\theta}^{I\text{MAX}} \) is always such that the poloidal component of perpendicular plasma flow points away from the zero on the outside and towards the zero on the inside of the torus.

If \( |V_{1\theta}^{II}| > V_{1\theta}^{I\text{MAX}} \), no stagnation point exits and an overall perpendicular plasma rotation occurs modulated by \( V_{1\theta}^{I\text{MAX}} \sin \theta \) (Figure 5). A minimum or maximum poloidal component of perpendicular flow velocity occurs at \( \theta = 3\pi/2 \) and \( \theta = \pi/2 \) depending on the sign of \( B_\phi \).
III. Correlation with Marfe Events

A poloidal asymmetry in edge density and/or temperature can be forced by perpendicular plasma convection particularly with the requirement that toroidal flow velocities become a significant fraction of the sound speed. In this case, locations of maxima and minima in poloidal pressure will be related to maxima and minima in perpendicular flow velocity. A mechanism which forces a toroidally symmetric poloidal asymmetry is a possible candidate for explaining the existence of a preferred marfe position in Alcator C.

A marfe is characterized by a poloidal asymmetry in plasma density and an associated strong local enhancement of low energy radiation emission [1,3,17]. The preferred position for this high density, radiating region is $\theta = 225^\circ$, $r = a$, and is toroidally symmetric. Marfes occur more often with high plasma density and/or low plasma current consistent with the scaling of $V_{1\theta}^I$ given by Equation (56). The density e-folding distances in the limiter shadow are short ($\approx 2.5$ mm) prior to the occurrence of a marfe while longer e-folding distances are measured ($\approx 4.0$ mm) when a marfe does not occur [14]. This is also consistent with the scaling suggested by Equation (56). Crossed beam CO$_2$ laser scattering data [18] indicate that prior to a marfe, fluctuations propagate ($V \approx 10^5$ cm/s) in the electron diamagnetic drift direction, $+\theta$, near the limiter radius. Figure 4 describes perpendicular plasma flows prior to a marfe consistent with a plasma rotation in the $+\theta$ direction. This would imply that $\partial \theta / \partial r$ is positive. If $V_{1\theta}^I$ is increased until $V_{1\theta}^{I\text{ MAX}} = 0.7 V_{1\theta}^{II}$, Figure 2 would result with the inside zero at $\theta = 225^\circ$. If the inside zero in perpendicular flow velocity corresponds
to a minimum in temperature and/or maximum in density, a preferred position for the marfe occurrence would coincide with the observed position of \( \theta = 225^\circ \).

Preliminary observations indicate that when \( B_\phi \) is reversed, the marfe develops at \( \theta = 135^\circ \) instead of \( \theta = 225^\circ \) consistent with Figure 3 [19]. Marfes have been observed near the inside midplane, \( \theta = 180^\circ \), and on rarer occasions, marfes have been seen to move from the usual \( \theta = 225^\circ \) position to \( \theta = 180^\circ \) and back in less than 2 msec [20]. In all cases the marfe occurs on the inside (\( 90^\circ < \theta < 270^\circ \)). When a marfe occurs, the radiating region is confined to a region near the inside midplane.

Independent of how a marfe is initiated, cross field convection may be the mechanism that supports the enhanced radiation loss thereby keeping the marfe region from spreading. Thus, the marfe region would be toroidally symmetric, as observed.
IV. Validity of Model - Future Work

The edge plasma in Alcator C is collisional enough to be treated as a single MHD fluid upon accepting the approximation that $T_e = T_i$. In the absence of poloidal and toroidal gradients, the equations presented above model the system well. Current work deals with obtaining a reduced set of MHD fluid equations appropriate for the gradient scale length ordering observed in Alcator C edge plasma. Poloidal and toroidal gradients need to be included particularly during a marfe event. The steady-state temperature and density profiles in conjunction with the bulk flow velocity profiles must satisfy the equations of continuity and momentum.

Future work includes obtaining numerically self-consistent temperature and density profiles in a 2 dimensional, axisymmetric system. In addition, an array of Langmuir probes is being built to directly measure density, temperature and their possible poloidal asymmetries in the limiter shadow region of Alcator C.

Acknowledgments

I would like to thank Bruce Lipschultz, Ian Hutchinson, Jim Terry, Steve Wolfe, Pete Politzer, Ron Parker, and the rest of the Alcator group for helpful comments, criticism and encouragement.
References


### TABLE 1 - EDGE PLASMA PARAMETERS AND CORRESPONDING $V_{\parallel \Phi \text{MAX}}, V_{\phi \text{MAX}},$ AND $V_{\theta \text{MAX}}$ FOR TWO DISCHARGES

<table>
<thead>
<tr>
<th>Measured Parameters</th>
<th>Before Marfe Occurred</th>
<th>In Discharge with no Marfe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n(a)$</td>
<td>$10^{14}\text{cm}^{-3}$</td>
<td>$10^{14}\text{cm}^{-3}$</td>
</tr>
<tr>
<td>$B_\Phi$</td>
<td>8 Tesla</td>
<td>8 Tesla</td>
</tr>
<tr>
<td>$I$</td>
<td>380 kA</td>
<td>450 kA</td>
</tr>
<tr>
<td>$T(a)$</td>
<td>20 eV</td>
<td>15 eV</td>
</tr>
<tr>
<td>$R_0$</td>
<td>.64 m</td>
<td>.64 m</td>
</tr>
<tr>
<td>$a$</td>
<td>.165 m</td>
<td>.165 m</td>
</tr>
<tr>
<td>$Z$</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>$\lambda_n$</td>
<td>.0025 m</td>
<td>.0048 m</td>
</tr>
<tr>
<td>$\lambda_T$</td>
<td>.0044 m</td>
<td>.014 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculated Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_S$</td>
<td>$4.5 \times 10^4\text{ m/sec}$</td>
</tr>
<tr>
<td>$D_{\text{BOHM}}$</td>
<td>.16 m²/sec</td>
</tr>
<tr>
<td>$V_{r\text{BOHM}}$</td>
<td>64 m/sec</td>
</tr>
<tr>
<td>$V_{\parallel\Phi\text{MAX}}$</td>
<td>285 m/sec</td>
</tr>
<tr>
<td>$V_{\phi\text{MAX}}$</td>
<td>285 m/sec</td>
</tr>
<tr>
<td>$V_{\theta\text{MAX}}$</td>
<td>$5.0 \times 10^3\text{ m/sec}$</td>
</tr>
</tbody>
</table>

It is interesting to note that although the flow velocities are under the sound speed, the toroidal flow can be a significant fraction of the sound speed and the poloidal flow is at least a few times larger than the radial diffusive velocity.
The right hand coordinate system \((r, \theta, \phi)\) used in this analysis is illustrated in Figure 1. \(r = 0\) defines the central axis of a torus and the angle \(\theta\) is measured from the outside midplane. A differential arc length in this coordinate system is defined as

\[
ds = dr \hat{r} + r d\theta \hat{\theta} + (R_0 + r \cos \theta) d\phi \hat{\phi}
\]

The following operators are thus defined:

\[
\nabla f = \hat{r} \frac{\partial f}{\partial r} + \hat{\theta} \frac{\partial f}{\partial \theta} + \hat{\phi} \frac{\partial f}{\partial \phi}
\]

\[
\nabla \cdot \mathbf{V} = \frac{1}{r(1 + \varepsilon \cos \theta)} \left\{ \frac{\partial}{\partial r} \left[ r(1 + \varepsilon \cos \theta) V_r \right] + \frac{\partial}{\partial \theta} \left[ (1 + \varepsilon \cos \theta) V_\theta \right] + \frac{\partial}{\partial \phi} [\varepsilon V_\phi] \right\}
\]

\[
\nabla \times \mathbf{V} = \frac{1}{r(1 + \varepsilon \cos \theta)} \left\{ \hat{r} \left[ \frac{\partial}{\partial \theta} (1 + \varepsilon \cos \theta) V_\phi - \frac{\partial}{\partial \phi} V_\theta \right] - \frac{\partial}{\partial r} \left[ (R_0 + r \cos \theta) V_\phi \right] - \frac{\partial}{\partial \phi} \left[ (1 + \varepsilon \cos \theta) \phi \left[ \frac{\partial}{\partial r} V_\theta - \frac{\partial}{\partial \theta} V_r \right] \right] \right\}
\]
Appendix B - Physical Picture of $V_{\perp B}$ Plasma Convection

The origin of a radial electric field giving rise to an $E \times B$ plasma flow to the inside of the torus can be seen in Figure 6. Due to the $1/R$ dependence of the magnetic field in a tokamak, particles drift in the $qB \times V|B|$ direction resulting in charge separation. The rotational transform provides a path along magnetic field lines for which currents can flow to short the charge imbalance out. These are the Pfirsch-Schlüter currents calculated in Equations (12) and (14). For edge plasma, $\eta_1$ can become large so that the charge separation is not cancelled completely. Parallel E-fields driving the Pfirsch-Schlüter currents are calculated in Equations (29) and (30). One can imagine a dipole field pattern resulting since charge separation is limited to the edge plasma region. A radial E-field must exist as a result of the charge configuration such that $V \times E = 0$. This E-field is calculated in Equation (35). From the figure, one can see that $E_r \times B$ drifts in the edge plasma will always be directed to the inside of the torus. Now, if an additional radial E-field arising from a poloidally symmetric ambipolar potential is super-imposed on this figure, the resulting $E \times B$ flows given by Equations (37) and (38) will result.
Figure Captions

Figure 1 - Toroidal Coordinate System.

Figure 2 - Relation Between ($\theta, \phi$) Coordinate System and Parallel and Perpendicular Directions.

Figure 3 - Poloidal Flow Velocity for $\partial \phi / \partial r > 0$, $|V_{1\theta}^{II}| < V_{1\theta}^{I\phi Max}$, $B_\phi > 0$.

Figure 4 - Poloidal Flow Velocity for $\partial \phi / \partial r > 0$, $|V_{1\theta}^{II}| < V_{1\theta}^{I\phi Max}$, $B_\phi < 0$.

Figure 5 - Poloidal Flow Velocity for $\partial \phi / \partial r > 0$, $|V_{1\theta}^{II}| > V_{1\theta}^{I\phi Max}$, $B_\phi > 0$.

Figure 6 - Physical Picture of $V_\perp$ Plasma Convection.
Figure 1 - Toroidal Coordinate System.
Figure 2 - Relation Between \( (\hat{\theta}, \hat{\phi}) \) Coordinate System and Parallel and Perpendicular Directions.
Figure 3

\[ \frac{\partial \phi}{\partial r} > 0 \]

\[ B \phi > 0 \]

\[ V_{\theta}^{\text{MAX}} > |V_\theta^\Pi| \]

Figure 4

\[ \frac{\partial \phi}{\partial r} > 0 \]

\[ B \phi < 0 \]

\[ V_{\theta}^{\text{MAX}} > |V_\theta^\Pi| \]

Figure 5

\[ \frac{\partial \phi}{\partial r} > 0 \]

\[ B \phi > 0 \]

\[ V_{\theta}^{\text{MAX}} < |V_\theta^\Pi| \]
Figure 6 - Physical Picture of $v_\perp$ Plasma Convection.