A STUDY OF PARAMETRIC INSTABILITIES DURING 
THE ALCATOR C LOWER HYBRID WAVE HEATING EXPERIMENTS

Yuichi Takase

Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, MA 02139

October 1983

This work was supported by the U.S. Department of Energy Contract No. DE-AC02-78ET51013. Reproduction, translation, publication, use and disposal, in whole or in part by or for the United States government is permitted.
A STUDY OF PARAMETRIC INSTABILITIES
DURING THE ALCATOR C
LOWER HYBRID WAVE HEATING EXPERIMENTS

by
YUICHI TAKASE
S.B., UNIVERSITY OF TOKYO
(1978)

SUBMITTED TO THE DEPARTMENT OF PHYSICS
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE
DEGREE OF
DOCTOR OF SCIENCE IN PHYSICS

at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 1983

© Massachusetts Institute of Technology 1983

Signature of Author

Certified by

Accepted by

Prof. Miklos Porkolab
Thesis Supervisor

Prof. G.F. Koster
Chairman, Graduate Committee
A STUDY OF PARAMETRIC INSTABILITIES 
DURING THE ALCATOR C 
LOWER HYBRID HEATING EXPERIMENTS 
by 
YUICHI TAKASE 

Submitted to the Department of Physics on August 12, 1983 
in partial fulfillment of the requirements for 
the Degree of Doctor of Science in Physics 

ABSTRACT 

Parametric excitation of ion-cyclotron quasi-modes ($\omega_R \approx n\omega_{ci}$) and ion-sound quasi-modes ($\omega_R \approx k_\parallel v_{ti}$) during lower hybrid wave heating of tokamak plasmas have been studied in detail. Such instabilities may significantly modify the incident wavenumber spectrum near the plasma edge. Convective losses for these instabilities are high if well-defined resonance cones exist, but they are significantly reduced if the resonance cones spread and fill the plasma volume (or some region of it). These instabilities preferentially excite lower hybrid waves with larger values of $n_\parallel$ than themselves possess, and the new waves tend to be absorbed near the outer layers of the plasma. 

Parametric instabilities during lower hybrid heating of Alcator C plasmas have been investigated using rf probes (to study $\vec{\phi}$ and $\vec{n}_e$) and CO$_2$ scattering technique (to study $\vec{n}_e$). At lower densities ($\vec{n}_e \lesssim 0.5 \times 10^{14}$cm$^{-3}$) where waves observed in the plasma interior using CO$_2$ scattering appear to be localized, parametric decay is very weak. Both ion-sound and ion-cyclotron parametric decay processes have been observed at higher densities ($\vec{n}_e \gtrsim 1.5 \times 10^{14}$cm$^{-3}$) where waves appear to be unlocalized. Finally, at still higher densities ($\vec{n}_e \gtrsim 2 \times 10^{14}$cm$^{-3}$) pump depletion has been observed. Above these densities heating and current drive efficiencies are expected to degrade significantly. 

Thesis Supervisor: Prof. Miklos Porkolab 
Title: Professor, Department of Physics
ACKNOWLEDGMENTS

The Alcator project is supported by many dedicated individuals. I would like to acknowledge all the support staff, technicians, staff scientists, and fellow graduate students who have helped me in pursuing this thesis research. I would like to mention a few of them in particular.

Professor Miklos Porkolab, my thesis supervisor, has guided me throughout the whole thesis research. He gave me numerous useful advices, suggested ideas that were new to me, and was always willing to discuss all the questions that I had brought him despite his extremely busy schedule. Dr. Jack Schuss, with whom I have worked ever since I came to MIT, has taught me all the details of the rf experiment, and was always helpful when I needed some advice.

\text{CO}_2\text{ scattering, which is the subject of Chap. V of this thesis, was carried out in collaboration with Dr. Reich Watterson, and Drs. Dick Slusher and Cliff Surko of Bell Laboratories. They let me use their equipment which needed only a few minor modifications in order to take data that were necessary for this thesis. Especially, Dr. Watterson has taught me all about laser scattering, reminded me of the things that I did not foresee, and helped me align the laser early in the morning whenever I needed to get my data.}

Dave Griffin has always maintained the rf equipment in good condition. Everything was ready to go whenever I needed to do an rf experiment. When a problem arises he would solve it in a matter of minutes. Frank Silva and Joe Daigle made the rf control systems work reliably. Ken Rice was helpful in getting me the components I needed for the experiment and for calibrations.

Drs. Dave Gwinn, Bruce Lipschultz, Brian Lloyd, and Professor Ron Parker
operated the tokamak for a large number of shots needed to get the data. In addition, Brian also operated the rf system when I could not operate it myself. Dave Gwinn, Frank Silva, Charlie Park, and the technicians always did an extremely good job in keeping the tokamak operate reliably.

George Chihoski and Bob Childs have helped me design the mechanical part and the vacuum part of the probe drives, respectively. Ed Thibeault and his technicians always did excellent machining and welding on a short notice, as did Francis Woodworth (Woody) of the Nuclear Reactor machine shop.

I would like to thank the staff physicists and graduate students, especially Professor Ron Parker, for raising insightful questions during the weekly staff meeting where preliminary data were presented to the group. I also benefitted from discussions with Steve Knowlton of the Versator group who was doing similar work. I would also like to thank Dr. Bob Granetz for his advice in computer programming and also for teaching me how to use \texttt{\LaTeX}.

I would also like to thank Professor George Bekefi and Professor Bruno Coppi, who were members of my thesis committee, and Dr. Reich Watterson for reading this thesis and making valuable suggestions.

The person who is most happy about completion of this thesis is my wife Emi (and the children, Ken and Jo). She has put up with my "thesis writing schedule" of more than 80 hours/week for nearly 8 months without any complaint. Without her support I would not have survived. Finally, I would like to thank my parents for their support and encouragement.

This work was supported by the U.S. Department of Energy, Contract No. DE-AC02-78ET51013.
To Emi, Ken and Jo
TABLE OF CONTENTS

ABSTRACT .................................................. 2

ACKNOWLEDGMENTS ............................................ 3

DEDICATION .................................................. 5

TABLE OF CONTENTS ........................................... 6

I. INTRODUCTION .............................................. 10
   1. MOTIVATION FOR WORK .................................. 10
   2. ORGANIZATION OF THESIS .............................. 12
   3. THE ALCATOR C TOKAMAK .............................. 13
   4. THE RF SYSTEM ......................................... 18

II. REVIEW OF LOWER HYBRID WAVES ....................... 21
   1. COLD PLASMA DISPERSION RELATION
      AND ACCESSIBILITY ................................... 21
   2. ELECTROSTATIC APPROXIMATION AND
      RESONANCE CONES ..................................... 27
   3. EFFECTS OF TOROIDICITY ON WAVE PROPAGATION .... 32
   4. SCATTERING FROM DENSITY FLUCTUATIONS ............ 35
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>STUDY OF THE EDGE PLASMA</td>
<td>128</td>
</tr>
<tr>
<td>A.</td>
<td>DENSITY AND TEMPERATURE PROFILES</td>
<td>128</td>
</tr>
<tr>
<td>B.</td>
<td>DENSITY FLUCTUATIONS</td>
<td>134</td>
</tr>
<tr>
<td>3.</td>
<td>RF SPECTRA</td>
<td>139</td>
</tr>
<tr>
<td>A.</td>
<td>ION-SOUND QUASI-MODES</td>
<td>139</td>
</tr>
<tr>
<td>B.</td>
<td>ION-CYCLOTRON QUASI-MODES</td>
<td>157</td>
</tr>
<tr>
<td>4.</td>
<td>SUMMARY OF PROBE DATA</td>
<td>168</td>
</tr>
<tr>
<td>V.</td>
<td>STUDY OF LOWER HYBRID WAVES USING</td>
<td>171</td>
</tr>
<tr>
<td></td>
<td>CO₂ LASER SCATTERING</td>
<td>171</td>
</tr>
<tr>
<td>1.</td>
<td>SCATTERING OF CO₂ LASER RADIATION FROM</td>
<td>177</td>
</tr>
<tr>
<td></td>
<td>DENSITY FLUCTUATIONS IN PLASMAS</td>
<td>171</td>
</tr>
<tr>
<td>2.</td>
<td>EXPERIMENTAL SETUP AND DATA ACQUISITION</td>
<td>177</td>
</tr>
<tr>
<td>3.</td>
<td>PUMP BROADENING</td>
<td>184</td>
</tr>
<tr>
<td>4.</td>
<td>LOW FREQUENCY DENSITY FLUCTUATIONS</td>
<td>191</td>
</tr>
<tr>
<td>5.</td>
<td>ION-CYCLOTRON SIDEBANDS</td>
<td>197</td>
</tr>
<tr>
<td>VI.</td>
<td>SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK</td>
<td>208</td>
</tr>
<tr>
<td>1.</td>
<td>SUMMARY AND CONCLUSIONS</td>
<td>208</td>
</tr>
<tr>
<td>2.</td>
<td>RECOMMENDATIONS FOR FUTURE WORK</td>
<td>212</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

I-1. MOTIVATION FOR WORK

Rf heating experiments in the lower hybrid frequency range have been carried out in a number of tokamaks including ATC, Wega, Petula, JFT-II, JIPPT-II, Doublet II-A, Alcator A, Versator and PLT. This scheme has the advantage that waveguides external to the vacuum vessel can be used to couple rf power into the plasma. In addition, MW level microwave power sources in the GHz range are available. Depending on the plasma parameters and on $n_{||}$ (the component of the refractive index parallel to the confining magnetic field), either electron or ion heating can be obtained. Furthermore, by launching a traveling wave (asymmetric $n_{||}$ spectrum), a toroidal current can be driven inside the plasma. Since rf current drive offers the possibility of steady state operation of tokamaks, extensive experimental work is being carried out in this field.

Parametric instabilities are often observed during lower hybrid heating experiments. In ATC, ion tail generation was well correlated with the appearance of strong parametric instability which was identified as decay into ion-cyclotron quasi-modes occurring near the plasma center. On the other hand, in JFT-2, both parametric decay into resonant ion-cyclotron waves and that into nonresonant quasi-modes have been observed, and importance of preventing parametric decay occurring in the boundary plasma region was stressed. Parametric excitation of nonresonant quasi-modes was first observed experimentally on a linear machine. In ACT-1 (a steady state toroidal machine), decay into ion-cyclotron quasi-modes was observed. In PLT it has been suggested that parametric instability may be the cause of their density limit in current drive. In Alcator A, the pump wave was observed to be broadened and downshifted in frequency both with rf probes and with
Moreover, enhanced level of low frequency fluctuations were observed on rf probes.\textsuperscript{15} Another curious result obtained during this experiment was that the heating appeared to be independent of the relative phasing of the two-waveguide array.\textsuperscript{15} While scattering of lower hybrid waves by existing low frequency fluctuations can explain some features of these results,\textsuperscript{34} it cannot easily explain the downshift and the enhancement of the low frequency fluctuations. In addition, nearly a hundred scattering events must take place in order to achieve the observed broadening of up to 8MHz FWHM from the density fluctuations characterized by $f \lesssim 200$kHz observed by CO\textsubscript{2} laser scattering.\textsuperscript{35,36} Parametric processes may have been responsible, at least in part, for these observed phenomena. The parallel index of refraction, $n_\parallel$, is an important parameter governing propagation and absorption of lower hybrid waves in plasmas. Since $n_\parallel$ can be easily changed by parametric instabilities, waves can damp near the plasma edge and decrease the heating efficiency. On the other hand, for two-waveguide launchers accessibility can be achieved due to parametric process. In order to apply lower hybrid wave heating and current drive to reactor-grade plasmas, parametric processes must clearly be understood.

In the present thesis parametric instabilities during lower hybrid heating experiments on the Alcator C tokamak will be discussed. Decay into both ion-cyclotron quasi-modes ($\omega_R \approx n \omega_{ci}$) and ion-sound quasi-modes ($\omega_R \approx k_\parallel v_{ti}$)\textsuperscript{37} are studied. The theory presented in this thesis is also consistent with the results of Alcator A lower hybrid heating experiments. Although current drive experiments have been carried out quite extensively on Alcator C, and they are very important part of the Alcator lower hybrid program, they will not be discussed in any detail in this thesis since parametric instabilities at these densities ($n_e \lesssim 1 \times 10^{14}$cm\textsuperscript{-3}) are usually very weak.
I-2. ORGANIZATION OF THESIS

Description of the Alcator C tokamak and its lower hybrid heating (and current drive) system are given in the remaining of this chapter. In the next chapter, a review of lower hybrid waves relevant to the present work, including accessibility, mode conversion, toroidal effects, scattering from density fluctuations, $n_{||}$ spectrum and damping mechanisms, is presented. In Chap. III, results of analytic calculations and numerical studies of the frequencies and growth rates of ion-sound quasi-modes and ion-cyclotron quasi-modes are presented. Various thresholds due to inhomogeneities and finite extent of the wave launcher are also calculated. Rf probe measurements are presented and probe data are compared with predictions of theory in Chap. IV. In Chap. V CO$_2$ laser scattering data are presented and are compared with probe data and theory. Finally, in Chap. VI, summary and conclusions as well as recommendations for future work are given.
I-3. THE ALCATOR C TOKAMAK

Alcator C is a high field tokamak of major radius $R = 64\text{cm}$ and minor radius (limiter radius) $a = 16.5\text{cm}$. Due to its strong toroidal field, it operates at higher densities than other tokamaks. Alcator C has operated in a wide range of parameters: $1 \times 10^{13}\text{cm}^{-3} \lesssim n_e \lesssim 8 \times 10^{14}\text{cm}^{-3}$, $1\text{keV} \lesssim T_e \lesssim 3\text{keV}$, $0.6\text{keV} \lesssim T_i \lesssim 1.5\text{keV}$, $I_p \lesssim 750\text{kA}$, and $B_T \lesssim 13\text{T}$. The plasma ion species is usually hydrogen or deuterium, and the discharge duration is typically $0.5\text{ sec}$. The density profile, measured by a five-chord methyl alcohol laser interferometer system, has the form $n_e(r) = n_{e0}(1 - r^2/a_n^2)\alpha$ where $a_n$ is a characteristic radius slightly greater than the limiter radius $a$ and the exponent lies the range $0.5 \lesssim \alpha \lesssim 1$.\textsuperscript{38} The electron temperature profile, measured by electron cyclotron emission, has the form $T_e(r) = T_{e0}\exp(r^2/a_T^2)$ where $a_T^2 = (3/2)(q_0/q_a)a^2$, and $q_0$ and $q_a$ are the values of the safety factor $q$ at the center of the plasma and at the limiter, respectively.\textsuperscript{39}

Figure I-1 shows the Alcator C tokamak viewed from the top. The lower hybrid waveguide arrays are located at the C-side (MW1) and F-side (MW2) ports, the rf probes are located at the C-top, D-top and F-top ports, and the CO$_2$ scattering has its input window at the E-bottom port and output window at the E-top port. The main limiters (Molybdenum, graphite, or Silicon carbide coated graphite) are located at the B and E ports. The vacuum chamber walls are made of stainless steel bellows with an inner radius of $r = 19\text{cm}$ at the port flanges. In order to protect the walls and the welds, in addition to the main limiters there are two stainless steel virtual limiters (secondary limiters) on either side of every diagnostic port. They protrude $1\text{cm}$ in from the vacuum vessel wall and have the inner radius of $r = 18\text{cm}$. The toroidal magnetic field is in the counter-clockwise direction (as viewed from the top of the machine) whereas the plasma current is in the clockwise direction during the usual mode of operation. Figure I-2 shows a cross sectional
view of the tokamak. A typical shot in the electron heating regime is shown in Fig. I-3.
FIGURE I-1 — The Alcator C tokamak viewed from the top.
FIGURE 1-2 — A cross sectional view of Alcator C.
FIGURE I-3 — A typical shot in the electron heating regime. Hydrogen plasma, $B = 9$T, $I_p = 400$kA, $\bar{n}_e = 1.5 \times 10^{14}$cm$^{-3}$. 
I-4. THE RF SYSTEM

The lower hybrid system\textsuperscript{40,41} consists of four independent 1MW units. Each unit consists of 4 klystrons of 250kW each and a 16-waveguide array of 4 rows by 4 columns. As of April 1983, two units (referred to as MW1 and MW2) have been installed and used in experiments on Alcator C. A block diagram of one such unit is shown in Fig. I-4. The master oscillator produces a stable 4.600GHz signal for the experiment and 4.601GHz and 1MHz signals for the phase measurements of the incident and reflected waves in each waveguide. The 4.600GHz signal is amplified by a TWT amplifier and is split into 4 ways, each of which will be amplified by 4 different klystrons. Each output of the klystrons is divided into 4 ways again and are fed into the 4 waveguides belonging to the same vertical column. The drive signals are adjusted before the klystrons so that the powers and phases of each klystron output are roughly equal at the waveguide mouth. Once this is done, power can be varied by changing the voltage on the klystrons and phases between different columns can be varied by switching the electronic phase shifters. The pulse length is controlled by the rf gate applied on the diode switch. A square pulse is used for the heating experiments, but during the current drive experiments the rf power was ramped up gradually with a time constant of about 10 msec. This was necessary to prevent sudden changes in plasma current (and hence equilibrium) which would result in sudden changes of plasma position. The phases of the four waveguides within the same column are fixed and are nearly the same. The powers and phases of incident and reflected rf signals in all 16 waveguides are monitored at the waveguide array. The powers are measured with calibrated crystals and the phases are measured at the IF frequency of 1MHz by comparing with the 1MHz phase reference signal. The protect circuits protect the system from high reflections and arcs. The protect circuit at the waveguide array shuts off the rf power whenever
the reflectivity in any waveguide exceeds a preset value which is usually around 50\%. The protect circuit at the klystron output shuts the rf pulse off whenever it detects a visible arc at the klystron output window or when the reflected power into the klystron exceeds a certain value. The waveguides are filled with dry nitrogen up to the BeO vacuum window which is located approximately 10cm from the waveguide mouth so that the $\omega = \omega_{ce}$ layer is located in the pressurized section of the waveguide. The power throughput achieved without any breakdown is roughly 9kW/cm$^2$ for an incident power of $P_{inc} \approx 700\text{kW}$. The coupling of the incident waves into the plasma is optimized by adjusting the waveguide position relative to the plasma, or by moving the plasma position with respect to the waveguide. The overall reflectivity is usually less than 10\%.
FIGURE I-4 — A block diagram of the Alcator C rf system.
II. REVIEW OF LOWER HYBRID WAVES

II-1. COLD PLASMA DISPERSION RELATION AND ACCESSIBILITY

From Maxwell's equations

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}$$
$$\nabla \times B = \frac{1}{c} \frac{\partial E}{\partial t} + \frac{4\pi}{c} J$$

the wave equation can be derived after Fourier analyzing in space and time: \(^4\)

$$n(n \cdot E) - n^2 E + \mathbf{K} \cdot E = 0 \quad (1)$$

where \(n = ck/\omega\) and \(\mathbf{K} = 1 + \mathbf{K}_e + \mathbf{K}_i\) is the dielectric tensor. The terms \(\mathbf{K}_e\) and \(\mathbf{K}_i\) are the contributions due to electrons and ions, respectively. The dielectric tensor \(\mathbf{K}\) is defined by the relationship

$$\mathbf{K} \cdot E = E + \frac{4\pi i}{\omega} J.$$

The dispersion relation is given by setting the determinant of Eq. (1) to zero.

$$D(k, \omega) = \begin{vmatrix} K_{xx} - n^2 & K_{xy} & K_{xz} + n || n \perp \\ K_{yx} & K_{yy} - n^2 & K_{yz} \\ K_{zx} + n || n \perp & K_{zy} & K_{zz} - n^2 \end{vmatrix} = 0.$$  

For a cold plasma \((k_\perp \rho_e \ll k_\perp \rho_i \ll 1, |\omega - n \omega_{ce}/k_\parallel v_{ti} | \gg 1),\) and for \(\omega_{ci}^2 \ll \omega^2 \ll \omega_{ce}^2,\) the dielectric tensor has a particularly simple form:

$$K_{xx} = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2} = K_{yy}$$
$$K_{xy} = -i \frac{\omega_{pe}}{\omega_{ce}} = -K_{yz}$$
$$K_{zz} = 1 - \frac{\omega_{pe}^2}{\omega^2}$$
with all other elements equal to zero. Here, a constant magnetic field in the $z$-direction is assumed and field quantities are assumed to vary as $\exp(i k_z x + i k_z z - i \omega t)$. The dispersion relation for the cold plasma approximation can be written as

$$A_2 n_x^4 + A_1 n_x^2 + A_0 = 0$$  \hspace{1cm} (2)

where

$$A_2 = K_{zz}$$

$$A_1 = (K_{zz} + K_{zz})(n_{||}^2 - K_{xx}) - K_{zy}^2$$

$$A_0 = K_{zz}[n_{||}^2 - K_{xx}^2 + K_{zy}^2].$$

Equation (2) has two roots

$$n_x^2 = \frac{-A_1 \pm \sqrt{A_1^2 - 4 A_2 A_0}}{2 A_2}.$$  \hspace{1cm} (3)

The plus sign corresponds to the slow wave (lower hybrid wave) and the minus sign corresponds to the fast wave (whistler wave).

Near the critical density (defined by $K_{zz} = 0$, i.e., $\omega_{pe} = \omega$), $K_{zz} \approx 1$, $|K_{zy}| \approx 0$ and $K_{zz} \approx 1 - \omega_{pe}^2/\omega^2$ so the wave equation Eq. (1) can be factored to give

$$(1 - n_{||}^2 - n_x^2)E_y = 0$$

which corresponds to the fast wave ($n_x^2 = 1 - n_{||}^2$) which is polarized in the $y$-direction, and

$$\begin{pmatrix} 1 - n_{||}^2 & n_{||} n_x \\ n_{||} n_x & K_{zz} - n_x^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0$$
which corresponds to the slow wave \([n_x^2 = (1 - \omega_{pe}^2/\omega^2)(1 - n_\parallel^2)]\) which is polarized in the \(x-z\) plane. It is clear that in order to have a propagating slow wave in the density range \(\omega_{pe} > \omega, |n_\parallel|\) must be greater than 1.

In the density range \(\omega^2 \ll \omega_{pe}^2\), the slow wave dispersion relation is given by

\[
n_x^2 = \frac{K_{xz}^2}{K_{zz}^2} (K_{zz} - n_\parallel^2) \\
\simeq \frac{\omega_{pe}^2}{\omega^2} (n_\parallel^2 - 1)
\]

whereas the fast wave dispersion relation is

\[
n_x^2 = (K_{zz} - n_\parallel^2) + \frac{K_{zr}^2}{K_{zz}^2 - n_\parallel^2} \\
\simeq \frac{\omega_{pe}^2}{\omega^2 \omega_{ce}^2} \frac{1}{(n_\parallel^2 - 1)} - (n_\parallel^2 - 1).
\]

Here, \(n_\parallel^2 > K_{zz}\) is assumed, and in going from the first line to the second line \(\omega_{pe}^2 \ll \omega_{ce}^2\) is also assumed. The dispersion relations given by Eqs. (4) and (5) are shown in Fig. II-1 with the dotted lines. The solid line is the exact solution of the cold plasma dispersion relation given by Eq. (3). These dispersion relations (4) and (5) are not valid near the fast wave-slow wave mode conversion layer where \(A_1^2 - 4A_2A_0 = 0\) and the two modes coalesce. This happens at a density given by

\[
\frac{\omega_{pi}}{\omega_0} = n_\parallel y - \sqrt{1 + n_\parallel^2 (y^2 - 1)}
\]

where \(y \equiv \omega_0/\sqrt{\omega_{ce}\omega_{ci}}\). This density is indicated by \(n_{LH-Wh}\) in Fig. II-1. If this density exists between the waveguide array and the plasma center, the incoming lower hybrid wave will mode convert into the fast wave and propagate back toward the edge of the plasma column. The mode-conversion density \(n_{LH-Wh}\) is plotted

23
against $n_\parallel$ for given ion species and magnetic field in Fig. II-2. In general, the waves have better accessibility (i.e., $n_{LH-Wh}$ is larger) for higher fields and for hydrogen plasma. In order for the lower hybrid wave to be accessible to the center of the plasma column, this mode conversion must be avoided. For a given plasma density and magnetic field, only the waves with $n_\parallel > n_{\parallel ac}$ can reach the plasma center, where

$$n_{\parallel ac} = \frac{\omega_{pe}(0)}{\omega_{ce}} + \sqrt{K_{xx}(0)}$$

where $\omega_{pe}$ and $K_{xx}$ are evaluated at the plasma center.
FIGURE II-1 — The approximate dispersion relations given by Eqs. (4) and (5) are shown by dotted lines whereas the exact cold plasma electromagnetic dispersion relation given by Eq. (3) is shown by the solid line. The lower hybrid-whistler wave mode conversion occurs at the density $n_{LH-Wh}$. Parameters used are: deuterium plasma, $B = 8T$, $n_{||} = 2$. 
FIGURE II-2 — The lower hybrid--whistler wave mode conversion density \( n_{LH-Wh} \) given by Eq. (6) is plotted against \( n_\parallel \). The waves having with values of \( n_\parallel \) can propagate to higher density.
II-2. ELECTROSTATIC APPROXIMATION AND RESONANCE CONES

If $|k \times E| \ll |k \cdot E|$, the wave is mainly electrostatic. In this case the electric field can be given by $E = -\nabla \phi$ where $\phi$ is the electrostatic potential, and the dispersion relation is further simplified:

$$K_{zx} n_x^2 + K_{zx} n_z^2 = 0. \quad (7)$$

The electrostatic approximation is valid when

$$\|K\| \ll n^2$$

where $\|K\| \equiv \sqrt{\sum_{i,j} |K_{i,j}|^2}$ is the norm of the matrix $K$. This condition is satisfied for lower hybrid waves with $n_\parallel \geq 2$.

The cold plasma electrostatic dispersion relation for lower hybrid waves in the density range $\omega_{pe}^2 \gg \omega^2$ is given by

$$\omega^2 = \omega_{LH}^2 \left(1 + \frac{k_\parallel^2}{k^2} \frac{m_i}{m_i} \right) \quad (8)$$

where $\omega_{LH} \equiv \omega_{pe}(1 + \omega_{pe}^2/\omega_{ce}^2)^{-1/2}$ is the lower hybrid frequency. This result can also be obtained from the dispersion relation Eq. (4) by neglecting $K_{xx}$ compared to $n_\parallel^2$. The group velocity can be calculated by differentiating Eq. (8) by $k$. With the approximation $k_\parallel^2 \ll k_\perp^2 \approx k^2$ (this follows from $\omega_{pe}^2 \gg \omega^2$), the parallel and the perpendicular components of the group velocity are given by

$$v_{g\parallel} \equiv \frac{\partial \omega}{\partial k_\parallel} = \frac{\omega}{k_\parallel} \left(1 - \frac{\omega_{LH}^2}{\omega^2}\right),$$

$$v_{g\perp} \equiv \frac{\partial \omega}{\partial k_\perp} = \frac{\omega}{k} \left(1 - \frac{\omega_{LH}^2}{\omega^2}\right).$$
The propagation angle $\theta$ with respect to the magnetic field is thus given by

$$\tan \theta \equiv \frac{v_{\perp}}{v_{\parallel}} \approx \frac{k_{\parallel}}{k}$$

$$= \sqrt{\frac{m_e}{m_i} \left( \frac{\omega^2}{\omega_{LH}^2} - 1 \right).}$$

It can be seen that $\theta$ is independent of $n_{\parallel}$, and therefore, waves with different values of $n_{\parallel}$ propagate along a single ray. Furthermore, electrostatic waves within the cold plasma approximation that are generated by a source of finite extent such as a waveguide array, will propagate inside a region called resonance cone.\textsuperscript{44-47}

It is important to note that this is not true for waves with $n_{\parallel} \leq 2$ that do not satisfy the electrostatic approximation. These waves generally propagate at angles that depend on $n_{\parallel}$ and may even turn around (mode convert into a fast wave) if accessibility is not satisfied. The group velocity for the slow wave can be calculated from Eq. (4)

$$v_{\parallel} = \frac{\omega}{k_{\parallel}} = \frac{c}{n_{\parallel}}$$

$$v_{\perp} = -v_{\parallel} \frac{\omega}{\omega_{pe}} \sqrt{1 - \frac{1}{n_{\parallel}^2}}$$

where $\omega_{pe}^2 / c^2 \ll \omega_{ce}^2$ is assumed since waves with $n_{\parallel} < n_{\parallel ac}$ cannot propagate to the plasma center and tend to stay on the outer layers of the plasma column. This expression reduces to that for the electrostatic case in the limit $n_{\parallel}^2 \gg 1$. With the same approximation, the group velocity for the fast wave is given by

$$v_{\parallel} = c \frac{n_{\parallel}^2 - 1)^2 - K_{zy}^2}{(n_{\parallel}^2 - 1)^2 - n_{\parallel}^2 K_{zy}^2}$$

$$v_{\perp} = c(n_{\parallel}^2 - 1)^{3/2} \sqrt{-K_{zy}^2 - (n_{\parallel}^2 - 1)^2} \frac{1}{(n_{\parallel}^2 - 1)^2 - n_{\parallel}^2 K_{zy}^2}.$$
and the propagation angle for the fast wave also depends on \( n_\parallel \).

The local electric field inside the resonance cone can be calculated using the WKB theory and the cold electrostatic dispersion relation

\[
\frac{\partial}{\partial x} K_{zz} \frac{\partial}{\partial x} \phi - k_x^2 K_{zz} \phi = 0.
\]

The slab geometry with \( x, y \) and \( z \) axes chosen in the radial, poloidal and toroidal directions, respectively, is used. The electric field at position \( x \) is given in terms of the electric field at the waveguide mouth located at \( x = x_{WG} \) as: \(^{46,47}\)

\[
|E_z(x)| = |E_z(x_{WG})| \left[ \frac{K_{zz}(x)K_{zz}(x)}{K_{zz}(x_{WG})K_{zz}(x_{WG})} \right]^{-\frac{1}{4}}
\]

\[
|E_x(x)| = \frac{|k_x(x)|}{|k_\parallel|} |E_z(x)|.
\]

The parallel and perpendicular components of the electric field calculated from Eq. (9) are shown in Fig. II–3 as a function of local density. The parallel component decreases while the perpendicular component increases as the wave propagates toward the interior of the plasma. The electric field at the waveguide mouth (on the plasma side) is calculated from the net rf power \( P_{rf} \) by requiring the conservation of power flux across the waveguide-plasma interface

\[
\xi_{ac} P_{rf} = W_T(x_{WG}) v_{g\parallel}(x_{WG}) L_y L_z
\]

where \( \xi_{ac} \) is the fraction of power in the accessible range of \( n_\parallel \) (i.e., the fraction of power that will end up inside the resonance cone), \( W_T = (E^2/16\pi)\omega(\partial\epsilon/\partial\omega) \) is the total wave energy density and \( L_y L_z \) is the total area of the waveguide array. In deriving this result it was assumed that there is only one resonance cone. If the waveguide array is phased so that it is launching a symmetric spectrum of \( n_\parallel \), as is the usual case during the heating experiments, \( P_{rf} \) must be replaced by \( P_{rf}/2 \)
since there will be two resonance cones propagating in the opposite directions. It should be noted that the WKB solution also conserves the power flux along the resonance cone. The WKB approximation may be violated near the critical layer \( \omega_{pe} = \omega \), since here \( k_x^{-1}(dk_x/dx) \gtrsim k_x \). However, typically the plasma is overdense \( (\omega_{pe} > \omega) \) at the waveguide mouth in order to optimize coupling\(^{48}\) and the WKB theory usually remains valid all the way back to the waveguide mouth. In cylindrical geometry the electric field is enhanced by the focusing factor \( \sqrt{a/r} \).\(^{49}\) In toroidal geometry it becomes more complicated since \( k_y \) is no longer conserved and the electric field can acquire finite values of \( E_y \).\(^{34,50-52}\) Furthermore, scattering from density fluctuations may also play an important role in wave propagation and tend to distort resonance cones.\(^{34,53,54}\)
FIGURE II-3 — The parallel and the perpendicular components of the electric field normalized to $E_{||WG}$ along the resonance cone calculated using the WKB theory Eq. (9) are plotted against the local density. Alcator C parameters with deuterium plasma, $B = 10T$, $n_{WG} = 5 \times 10^{12} \text{cm}^{-3}$, $n_{||} = 3$ are used. The arrow indicates the location of the waveguide mouth (where $E_{||}/E_{||WG} = 1$).
II-3. EFFECTS OF TOROIDICITY ON WAVE PROPAGATION

The trajectory of a wave packet that satisfies the local dispersion relation
\[ D(r, k, \omega) = 0 \]
is given by the ray equations\(^{34}\)

\[
\begin{align*}
\frac{dr}{dt} &= -\frac{\partial D}{\partial k_r}, \\
\frac{d\theta}{dt} &= -\frac{\partial D}{\partial m}, \\
\frac{d\phi}{dt} &= -\frac{\partial D}{\partial n}, \\
\frac{dk_r}{dt} &= -\frac{\partial D}{\partial r}, \\
\frac{dm}{dt} &= \frac{\partial D}{\partial r}, \\
\frac{dn}{dt} &= \frac{\partial D}{\partial \omega}.
\end{align*}
\]  

(10)

where \( r \) is expressed in terms of the toroidal coordinates shown in Fig. II-4. The variables \((k_r, m, n)\) are canonically conjugate to the toroidal coordinates \((r, \theta, \phi)\). The usual components of the wavenumber are related to the poloidal and toroidal wavenumbers \( m \) and \( n \) by \( k_\theta = m/r \) and \( k_\phi = n/R \) where \( R \) is the major radius of the torus. For a toroidally symmetric system, \( \partial D/\partial \phi = 0 \) and the toroidal mode number \( n \) is conserved. Similarly, in a cylindrically symmetric system, \( \partial D/\partial \theta = 0 \) and the poloidal mode number \( m \) is conserved. Since \( \theta \) is no longer a cyclic coordinate in a torus, the poloidal mode number \( m \) is not conserved. In the presence of rotational transform, the component of \( k \) parallel to the magnetic field is \( k_{||} \approx (n/R) + (B_\theta/B)(m/r) \) and \( n_{||} = c k_{||}/\omega \) is not conserved, either. Therefore, both \( m \) and \( n_{||} \) will undergo large variations as the wave propagates into the plasma interior.

For the electrostatic case \( D = K_{zz}k_{\perp}^2 + K_{xz}k_{||}^2 = 0 \), the variation in \( m \) can be calculated using the ray equations Eq. (10)\(^ {34}\)
\[
\frac{dm}{d\theta} = -\frac{\partial D}{\partial \theta} \approx -k_{\parallel} R_0 q(r) \left( 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2 K_{zz}} \right) \frac{r}{R_0} \frac{\sin \theta}{1 + \frac{r}{R_0} \cos \theta}
\]

where \( q(r) = r B_0 / R_0 B_\theta (r) \). For Alcator C, \( B_\theta > 0 \) and \( B_\phi < 0 \), and therefore \( q(r) < 0 \) for this particular choice of coordinate system. It can easily be seen that \( dm/d\theta \) is initially small for the waves launched at \( \theta = 0 \) (outside of the torus). For the waves propagating against the plasma current (i.e., \( n < 0 \)) \( \sin \theta \) is positive as the waves propagate from \( \theta = 0 \) through \( \theta = \pi/2 \) to \( \theta = \pi \), \( dm/d\theta \) is negative, and therefore \( m \) (and hence, \( n_{\parallel} \)) decreases. Conversely, in going from \( \theta = \pi \) to \( \theta = 2\pi \) through \( \theta = 3\pi/2 \), \( m \) increases. For the waves propagating in the direction of the plasma current, \( n > 0 \) and the waves propagate in the negative \( \theta \) direction. Again, \( m \) decreases during the first half revolution in the poloidal direction (\( 0 > \theta > -\pi \)), and increases during the second half (\( -\pi > \theta > -2\pi \)). Bonoli has found by using the surface of section method that for small values of the inverse aspect ratio \( \epsilon \equiv a/R \) such that \( \epsilon \lesssim 0.1 \), an additional invariant \( \tilde{m}(r, \theta; k_r, m) \) existed and the results for accessibility and energy deposition were similar to those of slab and cylindrical models. However, for \( \epsilon \gtrsim 0.2 \), which applies to most recent tokamaks, the rays exhibited ergodic behavior and toroidal effects could significantly affect accessibility and energy deposition. \(^{34}\) In addition, inclusion of scattering may significantly modify accessibility and energy deposition (at least those waves which are initially inaccessible). \(^{34}\)
FIGURE II-4 — The toroidal coordinate system used in Sec. II-3.
II-4. SCATTERING FROM DENSITY FLUCTUATIONS

Large levels ($\delta n/n \approx O(0.5)$) of density fluctuations have been observed near the limiter radius on both Alcator A and Alcator C. Such density fluctuations can significantly scatter the incoming lower hybrid wave near the plasma periphery. Since $k_{\parallel}$ of these fluctuations are much smaller than $k_{0\parallel}$ of the incident lower hybrid wave, the main effect of the scattering is to rotate $k_{0\perp}$ in the poloidal plane (or it can scatter into the fast mode). However, due to toroidal effects and shear $k_{\parallel}$ of the lower hybrid wave can change subsequently. Since the wavevector is rotated in the poloidal plane, waves are not expected to be confined in a well-defined resonance cone. In addition, the scattered lower hybrid wave will be shifted in frequency by the frequency of the fluctuation it scattered from. This will cause frequency broadening of the pump wave. The frequency half width $\Delta \omega$ of the broadened pump wave would be

$$\frac{\Delta \omega}{\Omega_0} \approx 2\sqrt{2}\left(\frac{k_0 l}{\xi_0 l_s}\right)$$

(11)

where the density fluctuation was assumed to have the spectral shape

$$\left\langle \left( \frac{\delta n(k, \omega)}{n} \right)^2 \right\rangle \approx \frac{1}{\pi^{3/2} \xi_0^2 \Omega_0^2} \left\langle \left( \frac{\delta n}{n} \right)^2 \right\rangle \exp \left( -\frac{k_1^2}{\xi_0^2} - \frac{\omega^2}{\Omega_0^2} \right),$$

(12)

and $l \gg l_s$ was assumed. Here $l$ is the width of the turbulent region, and $l_s$ is the $90^\circ$ scattering length given by

$$l_s^{-1} = \frac{3}{16} \sqrt{\pi} \left\langle \left( \frac{\delta n}{n} \right)^2 \right\rangle \frac{\xi_0^2}{k_0^2} \frac{\omega_{pe}^4}{\omega_0^2 \omega_{ce}^2} \xi_0$$

(13)

where the angular brackets denote averaging and the $E \times B$ term was assumed to be the dominant term. Note the increasing trend of $l/l_s$ and $\Delta \omega$ with plasma density. This result predicts frequency width as much as a few MHz for Alcator parameters which is comparable to the frequencies of ion-sound quasi-modes, and may affect parametric excitation of these modes.
II-5. FINITE TEMPERATURE EFFECTS AND DAMPING

The cold plasma dispersion relations Eq. (4) and Eq. (7) predict a resonance \( k_{\perp} \to \infty \) at the lower hybrid layer \( \omega_{LH} = \omega \) \( (K_{xx} = 0) \). However, as the lower hybrid wave approaches this resonance layer \( k_{\perp} \) becomes large and finite temperature effects must be considered as \( k_{\perp} \rho_i \) approaches (or even exceeds) unity.\(^{57}\) The lower hybrid dispersion relation including electromagnetic effects and finite temperature effects is given by

\[
A_3 n_x^6 + A_2 n_x^4 + A_1 n_x^2 + A_0 = 0, \tag{14}
\]

where

\[
A_3 = \left[ \frac{3 \omega_{pi}^2}{2 \omega^2 c^2} \left( \frac{v_{ei}^2 + 3 \omega_{pe}^2 v_{te}^2}{8 \omega_{ce}^2 c^2} \right) \right]
\]

and other coefficients were defined earlier. Equation (14) has three roots corresponding to the lower hybrid wave (slow wave), the fast wave (whistler wave), and in addition, the ion plasma wave. In the electrostatic approximation Eq. (14) reduces to

\[
A_3 n_x^4 + K_{xx} n_x^2 + K_{zz} n_z^2 = 0, \tag{15}
\]

The two roots of Eq. (15) correspond to the lower hybrid wave and the ion plasma wave. As the cold lower hybrid wave propagates inward, it will mode convert to a warm ion plasma mode at the density given by \( K_{xx}^2 - 4 A_3 K_{xx} n_z^2 = 0 \):\(^{43}\)

\[
\frac{\omega_{pi}^2}{\omega^2} = \left[ 1 - y^2 + n_z \frac{v_{te}}{c} \left( \frac{T_i}{T_e} + \frac{3}{2} y^4 \right)^{\frac{1}{2}} \right]^{-1}
\]

where \( y^2 = \omega^2 / \omega_{ce} \omega_{ci} \), and propagate back out before it reaches the cold plasma resonance layer. Figures II-5(a)-(d) show the variation of \( n_{\perp}^2 \) as a function of
radius for several Alcator C parameters. The square-root-of-parabola density profile, typical of Alcator C discharges, is assumed with \( n_e(a)/n_{e0} = 0.1 \). The mode conversion from the lower hybrid wave to the ion plasma wave is observed in Fig. II–5(a). Figure II–5(b) is the case when neither of the mode conversion layers are present in the plasma. Figures II–5(c) and (d) show the cases when the lower hybrid wave mode converts into the whistler wave. Note that this mode conversion layer moves outward as the density is increased. The lower hybrid–ion plasma wave mode conversion density \( n_{LH-IP} \) is plotted against \( n_{||} \) for several different cases in Fig. II–6. \( T_e = 1 \)keV and \( T_i = 0.8 \)keV are used. Waves with larger values of \( n_{||} \) mode convert at lower densities, and lower values of \( n_{LH-IP} \) are realized for higher fields and for smaller ion mass.

The ion plasma wave

\[
\omega^2 = \omega_{LH}^2 \left[ 1 + \frac{3 k_\perp^2 v_{ti}^2}{2 \omega^2} \left( 1 + \frac{1}{4} y^2 \frac{T_e}{T_i} \right) \right]
\]

has \( \omega/k_\perp v_{ti} \approx 1 \) and is strongly damped near the mode conversion layer either by harmonic ion-cyclotron damping (assuming magnetized ions) or by perpendicular ion Landau damping (assuming magnetized ions with decorrelation mechanisms or assuming unmagnetized ions).\textsuperscript{58–60} Note, however, that waves with sufficiently large values of \( n_{||} \) will Landau damp on electrons before they can reach the ion plasma wave mode conversion layer. The condition for avoiding electron Landau damping is \( c/n_{||} v_{te} \gtrsim 3 \). Utilizing this inequality a lower bound can be placed on the lower hybrid–ion plasma wave mode conversion density given by Eq. (16). For \( T_e = T_i \), hydrogen plasma, \( B = 9 \)T, and \( f = 4.6 \)GHz \( (y^2 = 0.61) \), the lower bound on mode conversion density is given by \( n_e \gtrsim 3.8 \times 10^{14} \text{cm}^{-3} \). At the mode conversion density
\[ n_\perp^2 \approx \frac{K_{zz}}{2 A_3} = 2n_{\parallel} \frac{c}{v_{te}} \frac{\kappa_1}{m_i} \left( \frac{3}{2} y^4 + 6 \frac{T_i}{T_e} \right)^{-1/2} \]

and hence

\[ \left( \frac{\omega}{k_\perp v_{ti}} \right)^2 \approx \frac{1}{2} \frac{T_e}{T_i} \frac{c}{n_{\parallel} v_{te}} \left( \frac{3}{2} y^4 + 6 \frac{T_i}{T_e} \right)^{1/2}. \]

Using \( c/n_{\parallel} v_{te} \gtrsim 3 \) gives a lower bound of \( \omega/k_\perp v_{ti} \gtrsim 1.9 \) at the mode conversion layer which indicates the possibility of strong ion Landau damping near this layer. The lower bound on the ion plasma wave mode conversion density given above is not necessarily the lower bound on density for ion tail formation due to ion Landau damping. As soon as \( \omega/k_\perp v_{ti} < 3 \) appreciable ion Landau damping can occur. In addition, parametric decay into ion plasma wave (which is readily damped) can occur before this mode conversion density.\(^{49}\) If harmonic ion-cyclotron damping is applicable instead of ion Landau damping the wave power can be absorbed below the mode conversion density. Provided that \( c/n_{\parallel} v_{te} \gtrsim 3 \) at the mode conversion layer it can be shown that ion Landau damping dominates over electron Landau damping since

\[ \left( \frac{\omega}{k_\perp v_{ti}} \right)^2 = \frac{1}{2} \frac{T_e}{T_i} \frac{n_{\parallel} v_{te}}{c} \left( \frac{3}{2} y^4 + 6 \frac{T_i}{T_e} \right)^{1/2} \]

\[ \leq \frac{\sqrt{7.5}}{6} < 1 \]

where \( y^2 < 1 \) and \( T_e = T_i \) have been assumed. Assuming magnetized ions, it can be shown that the warm ion plasma mode further mode converts into an ion Bernstein mode (with finite \( k_{\parallel} \)) and is absorbed by harmonic ion-cyclotron damping.\(^{58}\)

The wave absorption by perpendicular ion Landau damping creates ion tails in the perpendicular direction. For ion heating to be effective, these high energy ions

38
must be confined until they thermalize on the background ions. One loss mechanism for these energetic ions is the banana loss. In the case of Alcator tokamaks, the toroidal field has a significant ripple near the diagnostic ports and the ripple loss could also be significant. If the mode conversion density for a particular \( n_\parallel \) is located near the plasma surface, the high energy ions will be rapidly lost to the walls since they are poorly confined.

If the electron temperature is high enough, or if \( n_\parallel \) is large enough so that \( \omega/k_\parallel v_{te} < 3 \), electron heating due to electron Landau damping becomes important. For an initial electron temperature of \( T_e = 1 \text{keV} \), waves with \( n_\parallel \gtrsim 5 \) satisfy this condition. With this mechanism, plateau electrons are formed which then thermalize on the bulk electrons. Bulk ion heating can also be obtained by electron-ion collisions. The electron-ion thermal equilibration time for deuterium plasma with \( T_e = 1 \text{keV} \) and \( n_e = 2 \times 10^{14} \text{cm}^{-3} \) is about 7 msec. Waves with large values of \( n_\parallel \) will be Landau damped near the edge where the local electron temperature is low, and these electrons may rapidly be lost.

Another damping mechanism is that due to electron-ion collisions. Collisional damping is favored near the plasma surface where the temperature is low but the density may still be high. This damping can be significant if the wave spends a long time in the edge region. Since the wave energy absorbed near the surface is rapidly lost, collisional damping in this region should be minimized. Hence, plasmas with high edge electron temperatures and low densities are preferred for efficient lower hybrid wave penetration.
FIGURE II-5(a) — The warm plasma dispersion relation Eq. (14) solved for a square-root-of-parabola density profile. $n_\perp^2$ is plotted against minor radius $r/a$ for hydrogen plasma, $B = 12\, T$, $n_{e0} = 5 \times 10^{14}\, \text{cm}^{-3}$, $T_{e0} = T_{i0} = 1\, \text{keV}$, and $n_\parallel = 3$. 
FIGURE II-5(b) — Same as Fig. II-5(a) for deuterium, \( B = 8 \text{T} \), \( n_{e0} = 3 \times 10^{14} \text{cm}^{-3} \), \( T_{e0} = 1 \text{keV} \), and \( n_{||} = 3 \).
FIGURE II-5(c) — Same as Fig. II-5(b) except $n_{\parallel} = 1.5$. 
FIGURE II-5(d) — Same as Fig. II-5(c) except $n_{e0} = 2 \times 10^{14} \text{cm}^{-3}$
FIGURE II-6 — Lower hybrid–ion plasma wave mode conversion density \( n_{\text{LH-IP}} \) as given by Eq. (16) versus \( n_{\parallel} \). Here, \( T_e = 1 \text{keV} \) and \( T_i = 0.8 \text{keV} \) were taken. Waves having with values of \( n_{\parallel} \) mode convert at lower densities.
II-6. WAVEGUIDE-PLASMA COUPLING AND $n_{||}$ SPECTRUM

As seen in Sec. II-1, the slow wave can propagate in the density range $\omega_{pe} > \omega$ only if $n_{||} > 1$. In addition, in order to be able to propagate to the plasma center, the accessibility condition $n_{||} > n_{||ac} > 1$ must be satisfied. Waves with $n_{||} > 1$ can be generated by a slow wave structure such as a phased waveguide array. Typically, a waveguide array consists of rectangular waveguides placed side by side in the toroidal direction. Phasing each waveguide in a periodic fashion imposes a periodicity shorter than the vacuum wavelength in the toroidal direction. The short dimension of the waveguide is oriented along the toroidal direction and TE$_{10}$ mode is excited in order to couple to the slow wave that is polarized in the $x$-$z$ plane. In general, more waveguides in the toroidal direction produces better defined $n_{||}$ spectrum. In Alcator C, since the access ports are only about 4cm wide there are only 4 waveguides in the toroidal direction. However, 4 rows of waveguides are stacked on top of each other to maximize the power input to the plasma.

Waveguide-plasma coupling and the launched $n_{||}$ spectrum can be studied by matching the waveguide fields to the fields in the plasma at the waveguide-plasma interface. Assuming an overdense plasma at the waveguide mouth, it can be shown that optimum coupling can be obtained if the density is overdense by $\omega_{pe}^2/\omega^2 \approx n_{||}^2$.

The theoretical $n_{||}$ spectra for the Alcator C waveguide array for different phasings are shown in Fig. II-7. The $n_{||}$ spectrum for 180° phasing (or 0-π-0-π phasing) is peaked at $|n_{||}| \simeq 3$ and has most of the power between 2 and 4. Since this is a symmetric phasing, the $n_{||}$ spectrum is symmetric in the positive and the negative directions. The positive direction is defined so that majority of the power is going opposite to the plasma current (i.e., in the direction of Ohmic electron drift). Thus, in case of +90° phasing most of the power (actually only about 2/3)
is concentrated in the range $+1 < n_\parallel < +2.5$ extending to $n_\parallel \simeq +2.5$. The remaining $1/3$ of the power is located around $n_\parallel \simeq -4.5$.

Note, however, that the $n_\parallel$ spectrum generated by the waveguide array may undergo significant changes either by toroidal effects, scattering from density fluctuations, parametric decay, reflection from corrugated walls (such as in Alcator tokamaks), or combination of the above.
FIGURE II-7 — The $n_\parallel$ spectra for the Alcator C waveguide array for different phasings $\Delta \phi$ between adjacent waveguides (after Ref. 66): positive $n_\parallel$ (top) and negative $n_\parallel$ (bottom).
III. THEORY OF PARAMETRIC INSTABILITIES

III-1. PARAMETRIC DISPERSION RELATION

The parametric dispersion relation derived from the Vlasov equation with the 0th order pump electric field of the form

\[ E_0 = (E_{0\|} + E_{0\perp}) \cos(k_0 \cdot \mathbf{z} - \omega_0 t) \]

is given by: 68

\[
1 + \frac{1}{\chi_i} = J_0^2(\mu) \frac{\chi_e}{1 + \chi_e} + J_1^2(\mu) \left( \frac{\chi_e^+}{1 + \chi_e^+} + \frac{\chi_e^-}{1 + \chi_e^-} \right) \\
\quad + \frac{J_0^2(\mu) J_1^2(\mu) \left[ \frac{\chi_e}{1 + \chi_e} - \frac{\chi_e^+}{1 + \chi_e^+} \right]}{1 + \frac{1}{\chi_i^+} - J_0^2(\mu) \frac{\chi_e^+}{1 + \chi_e^+} - J_1^2(\mu) \frac{\chi_e}{1 + \chi_e}} \\
\quad + \frac{J_0^2(\mu) J_1^2(\mu) \left[ \frac{\chi_e}{1 + \chi_e} - \frac{\chi_e^-}{1 + \chi_e^-} \right]}{1 + \frac{1}{\chi_i^-} - J_0^2(\mu) \frac{\chi_e^-}{1 + \chi_e^-} - J_1^2(\mu) \frac{\chi_e}{1 + \chi_e}} \tag{17}
\]

Here, \( j = i, e \) is the species index, \( \chi_j \equiv \chi_j(\omega, \mathbf{k}) \) and \( \chi_j^{\pm} \equiv \chi_j(\omega^{\pm}, \mathbf{k}^{\pm}) \), are the linear susceptibilities at the low frequency \((\omega, \mathbf{k})\) and at the sidebands \((\omega^{\pm} \equiv \omega \pm \omega_0, \mathbf{k}^{\pm} \equiv \mathbf{k} \pm \mathbf{k}_0)\) respectively, \( J \) 's are the Bessel functions, and

\[
\mu = \frac{e}{m_e} \left[ \left( \frac{k_{\|} E_{0\|}}{\omega_0^2} + \frac{k_{\perp} \cdot E_{0\perp}}{\omega_0^2 - \omega_{ce}^2} \right)^2 + \left( \frac{(k_{\perp} \times E_{0\perp}) \omega_{ce}}{(\omega_0^2 - \omega_{ce}^2) \omega_0} \right) \right]^{1/2} \tag{18}
\]

is the coupling constant. In deriving this result, dipole approximation \((k_0 \ll \mathbf{k})\) was used. The dominant terms for \( \omega_{ce}^2 \gg \omega_0^2 \) are the parallel drift and the \( \mathbf{E} \times \mathbf{B} \) drift.
of the electrons and Eq. (18) is simplified:

$$\mu = \frac{e}{m_e} \left[ \frac{k_{||}^2 E_{0||}^2}{\omega_0^2} + \frac{(k_{\perp} \times E_{0\perp})^2}{\omega_0^2 \omega_{ce}^2} \right]^{1/2}. \quad (19)$$

The susceptibilities including the effects of density gradient (through the drift frequencies $\omega_{*,j}$) and collisions (through the Krook model) are given by:

$$\chi_j = \frac{1}{k^2 \lambda_{Dj}^2} \frac{1 + \frac{\omega + i\nu_j - \omega_{*,j}}{k_{||}v_{tj}} \sum_{n=-\infty}^{\infty} e^{-b_j I_n(b_j)} Z \left( \frac{\omega + i\nu_j - \omega_{*,j}}{k_{||}v_{tj}} \right)}{1 + \frac{i\nu_j}{k_{||}v_{tj}} \sum_{n=-\infty}^{\infty} e^{-b_j I_n(b_j)} Z \left( \frac{\omega + i\nu_j - \omega_{*,j}}{k_{||}v_{tj}} \right)} \quad (20)$$

where $\lambda_{Dj}^2 \equiv T_j/4\pi n^2$, $\omega_{*,j} \equiv k_{||}v_{tj}^2/2\Omega_j L_n$, $v_{tj}^2 = 2T_j/m_j$, $\Omega_{i,e} = \pm \omega_{i,e}$, $\nu_j$ is the effective collision frequency, $b_j \equiv k_{||}^2 \rho_j^2$, $\rho_j^2 \equiv v_{tj}^2/2\Omega_j^2$, $I$'s are the modified Bessel functions and $Z$ is the Fried-Conte plasma dispersion function. The usual slab geometry with $x$, $y$ and $z$ corresponding to the radial, poloidal and toroidal directions, respectively, is used.

The effect of finite $k_0$ can be studied using the ponderomotive potential.\textsuperscript{70} The pump electric field is assumed to be given by $(E_0 + E_0^*)/2$ where the asterisk denotes complex conjugate and

$$\tilde{E}_0 = (E_{0||} + E_{0\perp}) \exp(iK_0 \cdot x - i\omega_0 t)$$

is the complex pump electric field. The 0\textsuperscript{th} order electron equation of motion

$$\frac{\partial \tilde{u}_{D0}}{\partial t} = -\frac{e}{m_e} \left( \tilde{E}_0 + \frac{\tilde{u}_{D0} \times B_0}{c} \right)$$

can be solved to give the complex electron drift velocity due to the pump electric field

$$\tilde{u}_{D0} = \frac{i\omega_0 K_e}{4\pi n_0 e} \cdot \tilde{E}_0$$

$$= -\frac{e}{m_e} \left[ \frac{i\omega_0 E_{0\perp}}{\omega_0^2 - \omega_{ce}^2} + \frac{\Omega_e \times \tilde{E}_0 \perp}{\omega_0^2 - \omega_{ce}^2} + \frac{iE_{0||}}{\omega_0} \right]$$

49
and the complex displacement $\tilde{x}_{D0} = \tilde{u}_{D0}/(-i\omega_0)$ where $\Omega_e = -eB_0/m_ec$ and $K_e$ is the contribution from electrons to the cold plasma dielectric tensor $K$. Assuming a low frequency density fluctuation of the form $\tilde{n}_e = n_{e1} \exp(ik \cdot x - i\omega t)$, the continuity equation

$$\frac{\partial \tilde{n}_e^\pm}{\partial t} + \nabla \cdot (\tilde{n}_e \tilde{u}_{D0}) = 0$$

yields the perturbed density at the beat frequencies $\omega \pm \omega_0$

$$n_{e1}^+ = \frac{in_{e1}}{4\pi n_{e0}e} k^+ \cdot K_e \cdot E_0$$

$$n_{e1}^- = \frac{in_{e1}}{4\pi n_{e0}e} k^- \cdot K_e^* \cdot E_0^*$$

where the approximation $\omega \ll \omega_0$ has been made. Substituting these into Poisson's equation

$$(k^\pm)^2 e^\pm \phi^\pm = -4\pi e \tilde{n}_e^\pm$$

gives the expressions for the electrostatic potentials at the sidebands:

$$\phi^+ = -i \frac{n_{e1}}{n_{e0} (k^+)^2 e^+} \frac{1}{k^+ \cdot K_e \cdot E_0}$$

$$\phi^- = -i \frac{n_{e1}}{n_{e0} (k^-)^2 e^-} \frac{1}{k^- \cdot K_e^* \cdot E_0^*}$$

The ponderomotive potential at the low frequency $\omega$ is given by the $\omega$ frequency component of

$$\left( \frac{\tilde{F} + \tilde{F}^*}{2} \cdot \nabla \right) \left( \frac{\tilde{E} + \tilde{E}^*}{2} \right)$$

where both $\tilde{F}$ and $\tilde{E}$ contain the frequency components $\omega_0$ and $\omega \pm \omega_0$. The resulting ponderomotive potential at frequency $\omega$ is

$$\tilde{\phi}_p = -\pi e \tilde{n}_e \left[ \frac{|k^+ \cdot \tilde{x}_{D0}|^2}{(k^+)^2 e^+} + \frac{|k^- \cdot \tilde{x}_{D0}|^2}{(k^-)^2 e^-} \right]$$

(21)
The Vlasov equation for electrons, including the ponderomotive potential
\[
\frac{\partial f_e}{\partial t} + v \cdot \nabla f_e - \frac{e}{m_e} \left[ -\nabla (\phi + \phi_p) + \frac{v \times B}{c} \right] \cdot \frac{\partial f_e}{\partial v} = 0
\]
is solved to give the electron density perturbation at the low frequency \( \omega \):
\[
\tilde{n}_e = \frac{k^2 \chi_e(k, \omega)}{4\pi e} \left[ \tilde{\phi} + \tilde{\phi}_p \right]
\]
where \( \chi_e \) is the linear electron susceptibility. Since due to its large mass, the ion response to the ponderomotive force can be ignored, the perturbed ion density is simply
\[
\tilde{n}_i = -\frac{k^2 \chi_i(k, \omega)}{4\pi e} \tilde{\phi}_p.
\]
Substituting into Poisson's equation gives
\[
\varepsilon(\tilde{\phi} + \tilde{\phi}_p) = (1 + \chi_i)\tilde{\phi}_p,
\]
and by using Eqs. (21) and (22) the dispersion relation is obtained:
\[
1 + \frac{1}{4} \frac{\chi_e(1 + \chi_i)}{\varepsilon} \left[ \frac{\mu^+}{\varepsilon^+} + \frac{\mu^-}{\varepsilon^-} \right] = 0 \quad (23)
\]
where
\[
\mu^\pm = \frac{k^2}{(k^\pm)^2} |k^\pm \cdot \vec{D}_0|^2
\]
\[
\approx \frac{e}{m_e} k^\pm \left[ \frac{k_{\parallel}^\pm E_{0\parallel}^2}{\omega_0^2} + \frac{(k_{\perp}^\pm \times E_{0\perp})^2}{\omega_0^2\omega_{ce}^2} \right]^{1/2}.
\]
This dispersion relation is derived for finite \( k_0 \) but it is valid only for low pump powers \( \mu \ll 1 \), whereas Eq. (17) assumes \( k_0 = 0 \) but is valid even when \( \mu \lesssim 1 \). In
the limit $k_0 \to 0$ Eq. (23) reduces to the familiar dispersion relation

$$1 + \frac{\mu^2}{4} \frac{\chi_e(1 + \chi_i)}{\epsilon} \left[ \frac{1}{\epsilon^+} + \frac{1}{\epsilon^-} \right] = 0. \quad (24)$$

In the limit $\mu \ll 1$, and for $|\chi_i^\pm| \ll |\chi_i|$ and $|\chi_e^\pm| \ll |\chi_e|$, the dispersion relation Eq. (17) can be simplified as

$$1 + \frac{\mu^2}{4} \frac{\chi_e \chi_i}{\epsilon} \left[ \frac{1}{\epsilon^+} + \frac{1}{\epsilon^-} \right] = 0 \quad (25)$$

which agrees with Eq. (24) in the same limit.

Assuming a resonant lower hybrid wave at the lower sideband so that $\epsilon_R(\omega_2) = 0$, the dielectric function at the lower sideband can be expanded around $\omega_2$ to give

$$\epsilon^- \approx -\frac{\partial \epsilon_R}{\partial \omega_2} [(\omega_R - \delta) + i(\gamma + \Gamma_2)]$$

where $\delta \equiv \omega_0 - \omega_2$ and $\Gamma_2 \equiv \epsilon_I(\omega_2)/(\partial \epsilon_R/\partial \omega_2)$ is the linear damping rate at the lower sideband which includes collisional damping, electron Landau damping and harmonic ion-cyclotron damping. If the upper sideband is off resonance, the $1/\epsilon^+$ term in Eq. (25) can be neglected, and the expression for the parametric growth rate can be derived from the imaginary part of Eq. (25):\(^{49}\)

$$\gamma + \Gamma_2 = \frac{\mu^2}{4} \left( \frac{\partial \epsilon_R}{\partial \omega_2} \right)^{-1} \text{Im} \left( \frac{\chi_e \chi_i}{\epsilon} \right)$$

$$= \frac{\mu^2}{4} \frac{|\chi_i|}{\epsilon} \chi_i \frac{\partial \epsilon_R}{\partial \omega_2} |\epsilon|^2 \quad (26)$$

where $\text{Im}$ denotes the imaginary part, and the homogeneous plasma threshold can be obtained by setting $\gamma = 0$. Near the lower hybrid–ion plasma wave mode conversion layer the warm plasma dielectric function must be used instead of the cold plasma
In the following two sections, calculations concerning two kinds of parametric decay in a homogeneous plasma will be presented. In Sec. III–2 decay into lower hybrid wave and ion-sound quasi-mode \((\omega_R \simeq k_\parallel v_{ti})\) will be discussed, and in Sec. III–3 decay into lower hybrid wave and ion-cyclotron quasi-mode \((\omega_R \simeq n \omega_{ci})\) will be discussed.
For $T_e \gg T_i$ resonant decay into ion sound (acoustic) wave is possible. However, for $T_e \approx T_i$ such as the case in Alcator or near the edge of a tokamak plasma, the ion sound mode is strongly Landau damped by ions and becomes the ion-sound (or ion-acoustic) quasi-mode $\omega_R \approx k_\parallel v_{ti}$. Typically, $|\chi_\varepsilon R| \gg |\chi_i R|$, $|\chi_i l| \gg |\chi_\varepsilon l|$, and $|\chi i l| \ll |\chi_\varepsilon R|$ for this case and the growth rate is given by

$$\gamma + \Gamma_2 = \frac{\mu^2}{4} \frac{\chi_{ii} \chi_{eR}}{\partial_\omega (\chi_{eR}^2 + \chi_{ii}^2)}$$

$$\approx \frac{\mu^2}{8} \frac{F}{k^2 \lambda_{Di}^2} \left( 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right)^{-1} \omega_2$$

(28)

where

$$F \equiv \frac{\sqrt{\pi} \zeta_0 \exp(-\zeta_0^2)}{1 + \left( \frac{T_e}{T_i} \right)^2 \pi \zeta_0^2 \exp(-2 \zeta_0^2)}; \quad \zeta_0 \equiv \frac{\omega}{k_\parallel v_{ti}}$$

and has the value $F \lesssim 0.48$.

It is found that this decay process is important near the edge of the plasma and is driven mostly by the parallel drift of the electrons. It can also be driven by the $E \times B$ drift motion of the electrons near the limiter radius. The ratio of the $E_\parallel$ coupling term to the $E \times B$ coupling term in $\mu^2$ is $\omega_{pe}^2 \omega_{ce}^2 / \omega_{pe}^4$ so that the $E_\parallel$ coupling dominates for densities $\omega_{pe}^2 < \omega_0 \omega_{ce}$ and the $E \times B$ coupling dominates in the opposite case. When the $E_\parallel$ coupling dominates the growth rate is

$$\gamma + \Gamma_2 \approx \frac{F v_{D\parallel}^2}{4 v_{ce}^2 T_i} \omega_0$$

(29)

where $v_{D\parallel} \equiv e E_{0\parallel}/m_e \omega_0$ is the parallel drift velocity of the electrons, and the approximations $\omega_{pe}^2 \ll \omega_{ce}^2$ and $\omega_{pi}^2 \ll \omega_0^2$ were used. The uniform plasma threshold,
obtained by setting $\gamma = 0$ in Eq. (29) reduces to

$$E_{0\parallel}^2 \simeq \frac{m_e^2 \omega_0^2}{e^2} \frac{4 T_i \nu_e}{F m_e \omega_0}$$

(30)

Similarly, when the $E \times B$ coupling dominates, the growth rate is

$$\gamma + \Gamma_2 \simeq \frac{F}{4} \frac{U^2}{v_{ti}^2} \left( \frac{k_0 k_2}{k_0 k_2} \right)^2 \frac{\omega_{LH}^2 \omega_0}{\omega_0^2}$$

(31)

where $U \equiv c E_{0\perp}/B$ is the $E \times B$ drift velocity of the electrons. The uniform plasma threshold becomes

$$E_{0\perp}^2 \simeq \frac{B^2}{e^2} \frac{\omega_0^2}{\omega_{LH}^2} \frac{4 T_i \nu_e}{F m_i \omega_0} \left( 1 - \frac{\omega_{LH}^2}{\omega_0^2} + \frac{\omega_{LH}^2}{\omega_{ce} \omega_{ci}} \left( \frac{k_0 k_2}{k_0 k_2} \right) \right)^2$$

(32)

Typical threshold values given by Eq. (30) are given in Table III–1 of Sec. III–4–A. It can be seen that the thresholds in a uniform plasma are low compared with the experimental values.

Since this process takes place in the plasma edge region it could modify the $n_{||}$ spectrum of the incoming lower hybrid wave. It was shown in Sec. II that low $n_{||}$ lower hybrid waves cannot satisfy the accessibility condition while high $n_{||}$ waves damp their energy near the plasma surface either by electron Landau damping or when they approach the lower hybrid–ion plasma wave mode conversion layer, which is then rapidly lost from the plasma. Therefore, this process is undesirable if the waveguide array is launching a good spectrum. However, if the initial $n_{||}$ spectrum is concentrated near $n_{||} = 1$, as was the case for Alcator A, this process may help upshift the $n_{||}$ of the incoming waves.

Equation (17) was solved numerically for $\omega_R/\omega_0$ and $\gamma/\omega_0$. The following
forms of susceptibilities were used:

\[
\chi_e(\omega, k) = \frac{1}{k^2 \lambda_D^2} \left[ 1 + \frac{\omega - \omega_e + i\nu_e I_0(b_e)e^{-b_e}Z(\frac{\omega + i\nu_e}{k||vte})}{1 + \frac{i\nu_e}{k||vte} I_0(b_e)e^{-b_e}Z(\frac{\omega + i\nu_e}{k||vte})} \right],
\]

\[
\chi_i(\omega, k) = \frac{1}{k^2 \lambda_D^2} \left[ 1 + \frac{\omega - \omega_{ei}}{k||v_{ti}} \sum_{n=-100}^{100} I_n(b_i)e^{-b_i}Z(\frac{\omega - n\Omega_i}{k||v_{ti}}) \right].
\]

In a given numerical search, \( k_\perp \) was varied while specific values of \( n_e, T_e, T_i, B_T, L_n, P_{rf}, k_{0||}, \) and \( k|| \) were kept constant. \( E_0 \) was calculated from \( P_{rf} \) using the WKB theory discussed in Sec. II.

A typical result of the numerical calculation is shown in Fig. III-1(a). The parameters used are: \( n_e = 4 \times 10^{11} \text{cm}^{-3} \), \( T_e = T_i = 3 \text{eV}, B = 5 \text{T} \) (\( B = 5 \text{T} \) at the outer edge corresponds to \( B = 6 \text{T} \) at the plasma center), \( f_0 = 2.45 \text{GHz} \), deuterium plasma, \( n_{0||} = 2 \) and \( n_{||} = 5 \), which represent typical experimental conditions during the Alcator A lower hybrid heating experiments. The rf power was taken to be \( P_{rf} = 10 \text{kW} \). The real part of the frequency \( \omega_R/\omega_0 \) and the growth rate \( \gamma/\omega_0 \) of the quasi-mode normalized to the pump frequency are plotted against \( k\lambda_D \), where \( k \) is the magnitude of wavenumber of the quasi-mode. A plot of \( \gamma/\omega_0 \) vs. \( \omega_R/\omega_0 \) is shown in Fig. III-1(b). For \( P = 10 \text{kW} \) the growth rates are comparable with the real frequencies of the quasi-modes and that \( \gamma \) is maximum for \( \omega_R/\omega_0 \approx 10^{-3} \).

In Alcator A where \( f_0 = 2.45 \text{GHz} \), this would predict \( f \approx 2.5 \text{MHz} \) for the most unstable quasi-mode at the particular density of \( n_e = 4 \times 10^{11} \text{cm}^{-3} \) near the edge. It is important to note that the frequency width of this quasi-mode is of the order of the frequency at the maximum growth rate. The results are quite insensitive to the relative orientation of \( k_{0\perp} \) and \( k_\perp \). Hence, \( k_{0\perp} \parallel k_\perp \) is always assumed in all the calculations. If \( \mu^- \) were used instead of \( \mu \), the difference in the maximum growth rate would be typically of the order of 10%.
Figure III-2(a) shows the radial variation of $\omega_R/\omega_0$ and $\gamma/\omega_0$ (where $\omega_R \equiv \omega_{R_{\text{max}}}$ is the value of $\omega_R$ at maximum growth rate and $\gamma \equiv \gamma_{\text{max}}$) for the case of Alcator A with $a = 10\, \text{cm}$, $n_{0||} = 2$, $n_{||} = 5$, $P_{rf} = 10\, \text{kW}$, deuterium plasma and $B = 5\, \text{T}$. The assumed temperature and density profiles are shown in the inset. The density at the waveguide mouth was assumed to be $1.5 \times 10^{11}\, \text{cm}^{-3}$. Although the WKB theory is not strictly valid in region A of Fig. III-2(a), the results from this region is also included for the sake of comparison. The growth rate is large at the edge ($\gamma/\omega_R \approx 2$ at $r > 12\, \text{cm}$ for $P_{rf} = 10\, \text{kW}$) and decreases significantly as the waves propagate inward. The dotted lines show solutions with the $E \times B$ coupling term neglected while the solid lines give the results when both the $E \times B$ and the $E_||$ terms are retained. The $E \times B$ coupling term starts to dominate for $n_e \gtrsim 10^{13}\, \text{cm}^{-3}$ ($\omega_{pe}^2 \gtrsim \omega_0 \omega_{ce}$) in agreement with theory. Figure III-2(b) shows a similar result for Alcator C using the measured density and temperature at the waveguide mouth ($r = 17.8\, \text{cm}$ or $r/a = 1.08$), namely $n_e \approx 5 \times 10^{12}\, \text{cm}^{-3}$ and $T_e \approx T_i \approx 5\, \text{eV}$, respectively (see Chap. IV). Other parameters are $n_{0||} = 3$, $n_{||} = 5$, $P_{rf} = 40\, \text{kW}$, hydrogen plasma and $B = 8\, \text{T}$ ($B = 10\, \text{T}$ at the plasma center). The growth rates are again large for $r/a \gtrsim 1.08$ (behind the waveguide mouth).

Figure III-3 shows the variation of $\omega_R/\omega_0$ and $\gamma/\omega_0$ as $k_||$ of the quasi-mode is varied for the same conditions as Fig. III-2(b) and $n_e = 5 \times 10^{12}\, \text{cm}^{-3}$, $T_e = T_i = 5\, \text{eV}$. The parallel wavenumber of the lower sideband, $k_{||}^-$, also increases as $k_||$ is increased since $k_{0||}$ is kept constant and $k_{||}^- = k_|| - k_{0||}$. For higher $k_||$, $\omega_R \approx k_{||} v_{te}$ is larger and so is the value of $\gamma/\omega_0$. But these high $k_{||}^-$ lower hybrid waves will be strongly damped shortly beyond the plasma edge and will not propagate to the plasma center.

Figure III-4 shows the frequencies and the growth rates vs. applied rf power
for the same conditions as Fig. III–3 and \( n_{||} = 5 \). The homogeneous threshold for this case at \( r = 17.8 \text{cm} \) is \( P \lesssim 4 \text{kW} \). And near the threshold \( \omega_R \simeq 1.2 k_{||} u_{ti} \). The homogeneous thresholds obtained numerically agree quite well with the theoretical predictions Eqs. (30) and (32). However, the growth rate well above threshold increases only like \( E_0^{2/3} \) instead of \( E_0^2 \).

For powers well above threshold such that \( \gamma \gtrsim \omega_R \gg k_{||} u_{ti} \), \( \chi_i \) can be expanded as

\[
\chi_i = \frac{1}{k^2 \lambda_{Di}^2} \left[ 1 - \left( 1 + \frac{1}{2 \xi_{0i}^2} + \frac{3}{4 \xi_{0i}^4} + \cdots \right) \right]
\]

\[
\simeq - \frac{k_{||}^2 \omega_{pi}^2}{k^2 \omega^2} \left[ 1 + \frac{3 k_{||}^2 u_{ti}^2}{2 \omega^2} + \cdots \right]
\]

where the perpendicular component of the ion susceptibility has been neglected since it is negligibly small in the outer plasma layers. For sufficiently large \( \omega_R / k_{||} u_{ti} \) and \( \gamma / k_{||} u_{ti} \) the approximations \( |\chi_{eR}| \gg |\chi_{iR}| \) and \( |\chi_{eI}| \gg |\chi_{iI}| \) hold, in which case Eq. (25) can be rewritten as

\[
\frac{\partial \epsilon_R}{\partial \omega_2} [(\omega_R - \delta) + i(\gamma + \Gamma_2)] = \frac{(\mu^--)^2}{4} \chi_i
\]

\( \delta \equiv \omega_0 - \omega_2 \). Solving the real and imaginary parts simultaneously and maximizing the growth rate with respect to \( \delta \) gives

\[
\frac{\omega_R}{\omega_0} \simeq \frac{1}{4} \left( \frac{(\mu^-)^2}{4} \frac{k_{||}^2 \omega_{L,H}^2}{k^2 \omega_0^2} \right)^{1/3}
\]

\[
\frac{\gamma}{\omega_0} \simeq \frac{\sqrt{3}}{4} \left( \frac{(\mu^-)^2 k_{||}^2 \omega_{L,H}^2}{k^2 \omega_0^2} \right)^{1/3} = \frac{\sqrt{3} \omega_R}{\omega_0}
\]

and \( \delta = 0 \), where the approximation \( \Gamma_2 \ll \gamma \) has been used. We see that the growth
rate increases only like $\gamma \sim E_0^{2/3}$. For the $E_{\parallel}$ dominated case

$$\frac{\gamma}{\omega_0} = \frac{\sqrt{3}}{4} \left[ \frac{m_e}{m_i} \frac{k^2 \nu^2_{D_{\parallel}}}{\omega_0^2} \right]^{1/3}$$

and for $E \times B$ dominated case

$$\frac{\gamma}{\omega_0} = \frac{\sqrt{3}}{4} \left[ \left( \frac{k_{0_{\perp}} \times k_{2_{\perp}}}{k_0^2 k_2^2} \right)^2 \frac{\omega_{LH}^2 \kappa_{\parallel}^2 U^2}{\omega_0^2 \omega_0^2} \right]^{1/3}$$

and the growth rate increases only like $E_0^{2/3}$. This is similar to the fluid quasi-mode discussed by Porkolab. This will be referred to as the reactive quasi-mode (since the imaginary part of the susceptibility is due to parametric growth rate $\gamma$) and that described by Eq. (28) as the dissipative quasi-mode in this thesis.

The comparison between the analytic scaling given by Eq. (29) (solid curves) and the numerical solutions of Eq. (17) (circles and triangles) is shown in Fig. III–5(a) for powers slightly above threshold. The scaling given by Eq. (29) is valid only near the threshold. The comparison between the analytic scaling for powers well above threshold given by Eq. (33) (solid curves) and the numerical solutions of Eq. (17) (circles and triangles) is shown in Fig. III–5(b). The term $\delta$ in Eq. (33) was neglected compared to $\gamma$. The agreement between the two techniques is good. However, the ion-sound quasi-mode decay tends to get overshadowed by the ion-cyclotron quasi-mode decay at high pump powers when $\gamma/\omega_{ci}$ approaches 1.

The transition from decay into dissipative quasi-modes to that into reactive quasi-modes occurs for $\gamma \gtrsim \omega_R \gtrsim 2k_{\parallel}v_{ti}$ as can be seen from Fig. III–6. Here, a contour plot of $\log_{10} |\chi_{it}/\chi_{el}|$ as a function of $\omega_R/k_{\parallel}v_{ti}$ and $\gamma/k_{\parallel}v_{ti}$ is shown. The following parameters were used for this plot: Alcator A, deuterium plasma, $B = 5T$, $n_e = 1 \times 10^{12} \text{cm}^{-3}$, $T_e = T_i = 3eV$, $ck_{0_{\parallel}}/\omega_0 = 2$ and $ck_{\parallel}/\omega_0 = 7$. 59
FIGURE III-1(a) — A typical numerical solution of Eq. (17) for Alcator A edge parameters: deuterium plasma, $n_e = 4 \times 10^{11} \text{cm}^{-3}$, $T_e = T_i = 3 \text{eV}$, $B = 5 \text{T}$, $P_{rf} = 10 \text{kW}$, $n_{0\parallel} = 2$ and $n_{\parallel\perp} = 5$. The real part of the frequency $\omega_R/\omega_0$ and the growth rate $\gamma/\omega_0$ of the quasi-mode normalized to the pump frequency $\omega_0$ are plotted against $k\lambda_{De} \times 10^2$ where $k$ is the magnitude of wavenumber of the quasi-mode.
FIGURE III-1(b) — $\gamma/\omega_0$ vs. $\omega_R/\omega_0$ for the same parameters as Fig. III-1(a).
FIGURE III-2(a) — The radial variation of $\omega_R/\omega_0$ and $\gamma/\omega_0$ for Alcator A, deuterium plasma, $B = 5T$, $P_{rf} = 10kW$, $n_{0\parallel} = 2$ and $n_{\|} = 5$. In region A, the WKB approximation does not hold. The broken lines show the case when the $E \times B$ coupling term is neglected. Assumed density and temperature profiles are shown in the inset.
FIGURE III-2(b) — The radial variation of $\omega_R/\omega_0$ and $\gamma/\omega_0$ for Alcator C, hydrogen plasma, $B = 8\text{T}$, $P_{rf} = 40\text{kW}$, $n_{0\parallel} = 3$ and $n_{\parallel}^{-} = 5$. The broken lines show the case when the $E \times B$ coupling term is neglected. Assumed density and temperature profiles are shown in the inset.
FIGURE III-3 — Variations of $\omega_R/\omega_0$ and $\gamma/\omega_0$ with $k_\parallel$ of the quasi-mode for the same conditions as Fig. III-2(b) and $n_e = 5 \times 10^{12}\text{cm}^{-3}$, $T_e = T_i = 5\text{eV}$. 
FIGURE III-4 — Power dependences of $\omega_R/\omega_0$ and $\gamma/\omega_0$ for the same conditions as Fig. III-3 and $n_{||}^- = 5$. The threshold is $P \lesssim 4$ kW and $\omega_R \approx 1.2 k_{||} v_{ti}$ at threshold.
FIGURE III-5(a) — Power dependences of $\omega_R/k_{\parallel}v_{ti}$ and $\gamma/k_{\parallel}v_{ti}$ for powers slightly above threshold. Alcator C, deuterium plasma, $n_e = 1 \times 10^{12}\text{cm}^{-3}$, $T_e = T_i = 3\text{eV}$. The solid line is the analytic scaling given by Eq. (29).
FIGURE III-5(b) — Power dependences of $\omega_R/k_v$ and $\gamma/k_v$ for powers well above threshold for the same parameters as Fig. III-5(a). The analytic power scaling given by Eq. (33) is shown in solid curves. The numerical solutions of Eq. (17) are shown by circles and triangles.
FIGURE III-6 — A contour plot of $\log_{10} |\chi_{\|}/\chi_{\perp}|$ as functions of $\omega_R/k_{\|}v_{ts}$ and $\gamma/k_{\|}v_{ts}$. The following parameters were used for this plot: Alcator A, deuterium plasma, $B = 5T$, $n_e = 1 \times 10^{12}$ cm$^{-3}$, $T_e = T_i = 3$ eV, $ck_{\|}/\omega_0 = 2$ and $ck_{\perp}/\omega_0 = 7$. 
III-3. DECAY INTO ION-CYCLotron QUASI-MODES

This is the parametric decay process that is more widely known. Porkolab has studied this process in great detail. The same numerical analysis as described in the previous section was used to study the modes in the frequency range $\omega \approx n\omega_{ci}$. These modes have growth rates peaking at $\omega \approx n\omega_{ci}$ due to the periodic variation in $\chi_i$. This decay process is driven mostly by the $E \times B$ motion of the electrons and is important near the limiter radius if convective losses are not effective. However, if convective effects play an important role it is most unstable in the plasma interior close to the lower hybrid-ion plasma wave mode conversion layer where nonresonant decay into ion plasma wave becomes possible. Depending on which terms are dominant in Eq. (26) Porkolab has classified the decay processes into three different regimes.

For resonant decay the low frequency mode is on resonance at $\omega_1$ so that $\epsilon_R \approx 0$ and $\epsilon_I \approx (\gamma + \Gamma_1)(\partial \epsilon_R / \partial \omega_1)$ where $\Gamma_1 \equiv \epsilon_I(\omega_1)/(\partial \epsilon_R / \partial \omega_1)$. The growth rate given by Eq. (26) can be rewritten as

$$\gamma + \Gamma_1)(\gamma + \Gamma_2) = -\frac{\mu^2 \text{Re}(\chi \chi)}{4} \frac{\partial \epsilon_R \partial \epsilon_R}{\partial \omega_1 \partial \omega_2}$$

where $\text{Re}$ denotes the real part. The low frequency dielectric function for $\omega \approx \omega_{ci}$ is given by

$$\epsilon_R = 1 + \frac{1}{k^2 \lambda_D^2} - \frac{1}{k^2 \lambda_D^2} \frac{\omega_{ci}^2}{\omega^2 - \omega_{ci}^2} - \frac{2I_1(b_i) \exp(-b_i)}{k \omega_{pi}^2}$$

and $\epsilon_R \approx 0$ gives the electrostatic ion-cyclotron wave

$$\omega_1^2 = \omega_{ci}^2 \left[ 1 + 2 \frac{T_e}{T_i} I_1(b_i) \exp(-b_i) \right].$$

(33)
The growth rate for resonant decay into (fundamental) ion-cyclotron wave is given by

\[
\frac{(\gamma + \Gamma_1)(\gamma + \Gamma_2)}{\omega_0^2} = \frac{1}{16} \frac{U^2 \omega_{iH}^2 \omega_{ci}^2 \omega_2}{c_s^2 \omega_0^2 \omega_1} T_e I_1(b_i) \exp(-b_i) \tag{34}
\]

where \(U = c(E_0 \times B_0)\) is the \(E \times B\) drift velocity and \(c_s = \sqrt{T_e/m_i}\) is the ion sound velocity. Equation (33) predicts a real frequency slightly greater than the ion cyclotron frequency.

When the low frequency mode is strongly damped by electron Landau damping and ion-cyclotron damping so that \(|\epsilon_I| \gg |\epsilon_R|\) and \(|\epsilon_I| \gg |\gamma(\partial\epsilon_R/\partial \omega_1)|\), the growth rate can be approximated as

\[
(\gamma + \Gamma_2) = -\frac{\mu^2}{4} \frac{\text{Re}(\chi e \chi_i)}{\partial \epsilon_R/\partial \omega_2} \epsilon_I
\]

For \(\omega \approx \omega_{ci}\) and for \(|\chi e R||\chi_i R| \gg |\chi e I||\chi_i I|\), and using the warm plasma dielectric function Eq. (27) at the lower sideband, it reduces to

\[
\frac{\gamma + \Gamma_2}{\omega_0} \approx \frac{1}{8} \frac{U^2 \omega_{iH}^2 \omega_2}{c_s^2 \omega_0^2 \omega_1} \frac{\left(\frac{\chi_i R}{\epsilon_I}\right)}{1 + \frac{3}{2} \frac{\omega_{iH}^2 k^2 v_{ti}^2}{\omega_2^2}} \tag{35}
\]

where

\[
\epsilon_I = \frac{\sqrt{\pi}}{k^2 \chi_{De}^2} \left[ \frac{\omega}{k||v_{te}} \exp \left( -\frac{\omega^2}{k^2 v_{te}^2} \right) + \frac{T_e}{I_t} \frac{\omega}{k||v_{ti}} I_1(b_i) \exp(-b_i) \exp \left( -\frac{(\omega - \omega_{ci})^2}{k^2 v_{ti}^2} \right) \right].
\]

Finally, in the limit \(|\epsilon_R| \gg |\epsilon_I|\) the low frequency mode is strongly nonresonant and can exist only in the presence of the pump wave. The growth rate maximizes for \(\omega_R \approx k||v_{te}\) (i.e., when the low frequency mode is resonant with electrons) and
has the form

$$\gamma + \Gamma_2 \simeq \frac{\mu^2}{4} \frac{\chi_{ei}}{\partial \varepsilon_R} (1 - \Delta)$$

where $\Delta$ is a hot-ion correction term and has the numerical value $\Delta \gtrsim 0.5$. Since

$$\frac{\gamma + \Gamma_2}{\omega_0} \approx \frac{1}{8} \frac{U^2}{c_s^2} \frac{\omega_{ci}^2}{\omega_0^2} \frac{\omega_2}{\omega_0} \frac{1 - \Delta}{1 + \frac{3}{2} \frac{\omega_{L_{LH}}^2}{\omega_0^2} \frac{k_{\perp}^2 \nu_{ti}^2}{\omega_2^2}}$$

which is comparable with the growth rate for ion-cyclotron quasi-mode decay. This mode can couple directly with the short wavelength ion plasma mode (or ion Bernstein mode). Due to their short wavelengths, nonresonant decay into ion plasma waves and quasi-modes has the lowest convective threshold.

For Alcator C parameters, a clear classification of modes is rather difficult. Numerical solutions of Eq. (17) near the ion-cyclotron harmonics usually indicate

$$|\chi_{ei}| \approx |\chi_{i\ell}|$$

when the growth rate is maximized with respect to $k_\parallel$ and $k_\perp$ of the low frequency mode, and $|\varepsilon_R|$ can be either smaller than or greater than $|\varepsilon_\ell|$. Since all terms contribute almost equally, and different terms dominate in different situations, analytic simplification of Eq. (26) will not be made. The growth rate should be evaluated numerically. At powers well above threshold, the imaginary part of the susceptibilities are mainly due to parametric growth rate and the mode becomes reactive in nature as in the case of ion-sound quasi-modes. In this thesis the (low frequency) modes in the ion-cyclotron frequency range will be referred to as ion-cyclotron quasi-modes and distinction between "ion-cyclotron quasi-mode" and "nonresonant quasi-mode" will not be made.

In Fig. III-7(a) radial variations of $\gamma/\omega_{ci}$ and $\omega_R/\omega_{ci}$ for $P_{rf} = 40kW$, $n_{0\parallel} = 3$, and $n_{\parallel} = 3$ are shown. These results were obtained by numerically solving
Eq. (17) for typical Alcator C parameters of hydrogen plasma, $B = 10T$, $n_{e0} = 2.5 \times 10^{14} \text{cm}^{-3}$, and $T_{e0} = 1.3 \text{keV}$. It can be seen that the growth rate has a maximum at or slightly inside the limiter radius. There was a corresponding peak in the growth rate for the case of ion-sound quasi-mode due to the $E \times B$ coupling term [see Fig. III-2(b)]. However, the peak due to the $E_\parallel$ coupling term near the plasma edge is insignificant for the present case. Similar result is shown for $n_\parallel = 5$ in Fig. III-7(b). Since the lower sideband waves are strongly Landau damped on the electrons near the plasma center for this value of $n_\parallel$, the growth rate is positive only in the outer half of the plasma column. Finally, in Fig. III-7(c) similar result for $n_\parallel = 5$ and $n_{e0} = 5 \times 10^{14} \text{cm}^{-3}$ is shown. The peak in the growth rate is now larger due to larger WKB enhancement factor. However, the lower hybrid-ion plasma wave mode conversion layer for the lower sideband appears near $r/a = 0.5$ for these values of central electron density and $n_\parallel$, and the growth rate decreases very rapidly as the waves approach this mode conversion layer. In all three cases, the density at the limiter radius was assumed to be $1 \times 10^{14} \text{cm}^{-3}$, and the same density and temperature profiles in the shadow of the limiter ($r/a > 1$) were used. Another feature of these calculations is that the peak of the growth rate shifts from $n = 1$ at the edge to $n = 3$ at $r/a \simeq 0.8$ as the local electron temperature increases. This can be more clearly seen in Fig. III-8 where $\gamma/\omega_{ci}$ and $\omega_R/\omega_{ci}$ are plotted as a function of $v_{te}/c$ for fixed density and fixed $k_\parallel$. The electron temperatures used for these plots were from 5eV to 500eV. The real part of the frequency at maximum growth rate follows the scaling $\omega_R \simeq 0.4k_\parallel v_{te}$, and the growth rate decreases with increasing $v_{te}$ (i.e., increasing temperature). However, as shown in Fig. III-9, the growth rate increases while the real part of the frequency remains unchanged as the density is increased. This is mainly due to the WKB enhancement of $E_\perp$ and $k_\perp$ which results in higher values of the parametric coupling constant $\mu$. The peak in
the growth rate in Fig. III-7 is determined by competition between higher density and lower temperature. The variations of $\omega_R/\omega_{ci}$ and $\gamma/\omega_{ci}$ with $k_\parallel$ of the low frequency mode are shown in Figs. III-10(a)-(c) for three different values of $n_0 ||$, the pump value ($n_0 || = 3, 5, \text{and} 1.5$). The parameters used are: hydrogen plasma, $B = 10T, n_e = 1 \times 10^{14}\text{cm}^{-3}, T_e = T_i = 50\text{eV},$ and $P_{rf} = 400\text{kW}.$ The growth rate increases with $k_\parallel$ until $k_\parallel^-$ becomes so large that Landau damping at the lower sideband becomes appreciable. This happens when $|\omega^-/k_\parallel^- v_{te}| \simeq 3$. Consequently, in all three cases there is a peak in the growth rate when $|\omega^-/k_\parallel^- v_{te}| \simeq 4$. The real part of the frequency, $\omega_R$ again follows the scaling $\omega_R \simeq 0.4k_\parallel v_{te}$ until it bends over at $ck_\parallel/\omega_0 \simeq 20$. The power variations of $\omega_R/\omega_{ci}$ and $\gamma/\omega_{ci}$ for the same parameters as Fig. III-10(a) and $ck_\parallel/\omega_0 = 8$ are shown in Fig. III-11. The growth rate is found to increase roughly like $P_{rf}^{0.6}$ (i.e., like $E_0^{1.2}$) for powers well above threshold. Note that the simplified analytical formulae for ion-cyclotron quasi-mode decay and for nonresonant quasi-mode decay [Eq. (35) and Eq. (36)] both predicted the growth rate to increase like $E_0^2$. Thus, the numerical growth rate, rather than Eq. (35) or Eq. (36), should be used in calculating the non-uniform plasma or finite pump extent thresholds.

Convective losses due to the finite extent of the waveguide array and of the resonance cones (see Sec. III-4-A) is rather large in the outer plasma layers. However, in reality waves may not be localized inside resonance cones due to scattering from turbulent density fluctuations existing in the outer layers of the plasma. In such a case, parametric decay into ion-cyclotron quasi-modes (or resonant ion-cyclotron waves) may be important in the vicinity of the limiter radius.

There are two classes of lower sideband lower hybrid waves that are observable by rf probes: (i) waves which come from the plasma interior, including those which were generated near the plasma center and also those which were generated near the
edge and have subsequently propagated to the plasma center; (ii) waves which were generated near the plasma edge and have propagated out directly. The first class of these waves must have relatively low $n_\parallel$ since they are strongly Landau damped near the center if $|\omega^-/k_\parallel v_{te}| \lesssim 3$. The upper limit on $n_\parallel$ is given by $|n_\parallel| \lesssim 4$ for the central electron temperature of $T_e = 1$keV (requiring $|\omega^-/k_\parallel v_{te}| \gtrsim 4$). This also puts an upper limit on $\Delta \omega/\omega_0$. Using $\omega_R \simeq 0.4k_\parallel v_{te}$, $\Delta \omega/\omega_0 \lesssim 0.18$ is obtained for the central electron temperature of $T_e = 1$keV. The second class of waves can have a relatively large value of $n_\parallel$ limited only by the local electron temperature. In Figs. III-12(a) and (b) typical ion-cyclotron quasi-mode frequency spectra for parameters at the waveguide mouth are shown for two different values of $k_\parallel$. Hydrogen plasma, $B = 10$T, $T_e = T_i = 5$eV, $n_e = 1 \times 10^{13}$cm$^{-3}$ ($\omega_0/\omega_{LH} = 7.0$), $P_{rf} = 100$kW, and $ck_0||/\omega_0 = 3$ are assumed for both cases. In Figs. III-13(a) and (b) frequency spectra near the limiter radius for two different values of $k_\parallel$ are shown. Hydrogen plasma, $B = 10$T, $T_e = T_i = 20$eV, $n_e = 1 \times 10^{14}$cm$^{-3}$ ($\omega_0/\omega_{LH} = 2.3$), $P_{rf} = 100$kW, and $ck_0||/\omega_0 = 3$ are assumed for both cases. It can be seen that only a few ($n \lesssim 3$) cyclotron harmonics are excited near the edge [see also Fig. III-10(a)]. Typical frequency spectra near the plasma center (higher temperatures) at two different densities are shown in Figs. 14(a) and (b). Hydrogen plasma, $B = 10$T, $T_e = 1.2T_i = 1$keV, $ck_0||/\omega_0 = 3$ and $ck_\|/\omega_0 = 7$ are assumed for both cases. Higher cyclotron harmonics, peaking around $\Delta \omega/\omega_0 \simeq 0.15$, are excited near the plasma center. As $\omega_0/\omega_{LH}$ approaches 1 decay into nonresonant quasi-mode and ion plasma wave become possible for shorter wavelengths (larger values of $k_\perp$) and higher harmonics start to disappear [Fig. III-14(b)]. This has been noted previously by Porkolab.\textsuperscript{49} The rf power for the case of Figs. III-14 was taken to be a factor of 10 higher than for the case of Figs. III-12 and 13 to account for the $a/r$ cylindrical focusing factor. The small peak below the first ion-cyclotron
peak is due to the ion-sound quasi-mode decay which was not shown in Figs. III–12 and 13.
FIGURE III-7(a) — Radial variations of $\omega_R/\omega_{ci}$ and $\gamma/\omega_{ci}$ obtained by numerically solving Eq. (17) for $P_{rf} = 40kW$, $n_{0\parallel} = 3$, $n_{\perp} = 3$, hydrogen plasma, $B = 10T$, $n_{e0} = 2.5 \times 10^{14} \text{cm}^{-3}$, and $T_{e0} = 1.3 \text{keV}$. 
FIGURE III-7(b).—Radial variations of $\omega_R/\omega_{ci}$ and $\gamma/\omega_{ci}$ obtained by numerically solving Eq. (17) for $P_{rf} = 40\text{kW}, n_{0||} = 3, n_{0\perp} = 5$, hydrogen plasma, $B = 10\text{T}, n_{e0} = 2.5 \times 10^{14}\text{cm}^{-3}$, and $T_{e0} = 1.3\text{keV}$.
FIGURE III-7(c) — Radial variations of $\omega_R/\omega_{ci}$ and $\gamma/\omega_{ci}$ obtained by numerically solving Eq. (17) for $P_{rf} = 40\,\text{kW}$, $n_{0\parallel} = 3$, $n_{\perp} = 5$, hydrogen plasma, $B = 10\,\text{T}$, $n_{e0} = 5 \times 10^{14}\,\text{cm}^{-3}$, and $T_{e0} = 1.3\,\text{keV}$. 

mode conversion layer
FIGURE III-8 — The variations of $\omega_R/\omega_{ci}$ and $\gamma/\omega_{ci}$ as electron temperature is varied. Hydrogen plasma, $B = 10T$, $n_e = 1 \times 10^{14} \text{cm}^{-3}$ ($\omega_0/\omega_{LH} = 2.3$), $P_{rf} = 100kW$, $ck_0/\omega_0 = 3$ and $ck_0/\omega_0 = 8$. 

$\omega_R = 0.4 k_0 v_{te}$
FIGURE III-9 — The variations of $\omega_R/\omega_{ci}$ and $\gamma/\omega_{ci}$, as the electron density is varied. Hydrogen plasma, $B = 10T$, $T_e = T_i = 20eV$, $P_{rf} = 100kW$, $ck_0/\omega_0 = 3$ and $ck/\omega_0 = 8$. 
FIGURE III-10(a) — The variations of $\omega_R/\omega_{ci}$ (dots) and $\gamma/\omega_{ci}$ (crosses) as $k_{||}$ of the low frequency mode is varied. Hydrogen plasma, $B = 10\text{T}$, $n_e = 1 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{LP} = 2.3$), $T_e = T_i = 50\text{eV}$, $ck_{||}/\omega_0 = 3$ and $P_{rf} = 400\text{kW}$.
FIGURE III-10(b) — The variations of $\omega_R/\omega_{ci}$ (dots) and $\gamma/\omega_{ci}$ (crosses) as $k_{||}$ of the low frequency mode is varied. Same parameters as Fig. 10(a) except $ck_{0||}/\omega_0 = 5$. 
FIGURE III–10(c) — The variations of $\omega_R/\omega_{ci}$ (dots) and $\gamma/\omega_{ci}$ (crosses) as $k_{||}$ of the low frequency mode is varied. Same parameters as Fig. 10(a) except $ck_{0||}/\omega_0 = 1.5$. 
FIGURE III-11 — The power variation of $\gamma/\omega_{ci}$ for the same parameters as Fig. III-10(a), $ck_{||}/\omega_0 = 8$, and $\omega/\omega_{ci} \approx 1$. 
FIGURE III-12(a) — Typical ion-cyclotron quasi-mode frequency spectra near the waveguide mouth for hydrogen plasma, $B = 10\, \text{T}$, $n_e = 1 \times 10^{13}\, \text{cm}^{-3}$ ($\omega_0/\omega_{LH} = 7.0$), $T_e = 5\, \text{eV}$, $P_{rf} = 100\, \text{kW}$ ($E_0_{\perp} = 2.9\, \text{kV/cm}$), $c k_{\parallel}/\omega_0 = 3$ and $c k_{\parallel}/\omega_0 = 7$. 

$\omega_R = \omega_{ci}$
FIGURE III-12(b) — Same as Fig. 12(a) except $ck\parallel/\omega_0 = 53$. 

\[ \omega_R = \omega_{ci} \]
FIGURE III-13(a) — Typical ion-cyclotron quasi-mode frequency spectra near the limiter radius for hydrogen plasma, $B = 10\, \text{T}$, $n_e = 1 \times 10^{14}\, \text{cm}^{-3}$ ($\omega_0/\omega_{LH} = 2.3$), $T_e = 20\, \text{eV}$, $P_{rf} = 100\, \text{kW}$ ($E_{0\perp} = 5.6\, \text{kV/cm}$), $ck_{0\parallel}/\omega_0 = 3$ and $ck_{\parallel}/\omega_0 = 7$. 
FIGURE III-13(b) — Same as Fig. 13(a) except $c k_\parallel /\omega_0 = 28$. 
\[
\frac{\omega_R}{\omega_0} = 10^3 \frac{\gamma}{\omega_0}
\]

FIGURE III-14(a) — Typical ion-cyclotron quasi-mode frequency spectra near the plasma center for hydrogen plasma, \(B = 10\, \text{T}, n_e = 1 \times 10^{14}\, \text{cm}^{-3} \left(\omega_0/\omega_{LH} = 2.3\right), T_e = 1\, \text{keV}, P_{rf} = 1000\, \text{kW} \left(E_{0 \perp} = 18\, \text{keV/cm}\right), n_{\parallel} = 3\) and \(n_{\|} = 4\). The small peak below the fundamental ion-cyclotron frequency is the ion-sound quasi-mode.
FIGURE III-14(b) — Same as Fig. 14(a) except $n_e = 2.5 \times 10^{14}$ cm$^{-3}$ ($\omega_0/\omega_{\text{LH}} = 1.6$). The local electric field is $E_{0\perp} = 2.9\text{kV/cm}$.
III-4. INHOMOGENEOUS THRESHOLDS

The thresholds calculated in the previous sections may have limited applicability to actual experiments. If the pump wave were localized inside resonance cones, decay waves could grow while they were still inside the resonance cone of the pump wave. However, since decay waves have lower frequencies than the pump wave, in order to satisfy the dispersion relation they must propagate at an angle to the pump resonance cone. Therefore, to be of interest the decay waves must be amplified by a sufficient amount (a factor of exp(2π) amplification in power is usually considered to be "sufficient") before they convect out of the pump resonance cone. This effect will be examined in Sec. III-4-A. If the resonance cones are destroyed and the pump wave fills some portion of the plasma column, convective losses become ineffective. However, the pump electric field would be reduced for this case since the pump wave has now spread out over a larger volume. Attempts to estimate the threshold rf power injected at the waveguide array will be made in Sec. III-4-B. In Sec. III-4-C, the effect of density and temperature gradients will be examined. In order to satisfy the dispersion relation at both the low frequency and the lower sideband, the selection rules in frequency and wavenumber may be violated which would result in a reduction of the parametric growth rate. All of these effects raise the threshold above the homogeneous threshold.

Note also that applying the calculations presented in this section to actual experiments assumes a pump wave with a narrow bandwidth. If the surface of the plasma is turbulent, the pump wave frequency spectrum would broaden so that Δω/ω₀ ≃ γ/ω₀ (as might be the case for ion-sound quasi-mode decay), and then there is some question in applying these thresholds.71-77 In this thesis the effects of such low frequency density fluctuations are not considered.
III-4-A. CONVECTIVE THRESHOLDS

Finite extent of the resonance cones produced by a wave launcher of finite size ($L_y$ in the $y$-direction and $L_z$ in the $z$-direction) introduces the finite pump extent (or convective) threshold even for a homogeneous plasma since the decay waves cannot grow further once they convect out of the resonance cone of the pump wave.\textsuperscript{49, 78} For simplicity a spatially homogeneous plasma is assumed. The pump wave is assumed to be accessible and electrostatic so that a well-defined pump resonance cone is developed. The geometry is shown in Fig. III-15. In region A a standing wave is formed in the $z$-direction.

If the coupling is mainly due to the $E_\parallel$ term, the decay wave can propagate almost parallel to the pump wave. However, since the decay wave has slightly lower frequency than the pump wave, it has to propagate in a slightly different direction and will eventually convect out of the pump resonance cone. In order to minimize the convective loss, here the decay wave is assumed to propagate in the same plane as the pump wave (i.e., the $x$-$z$ plane). The convective threshold for this case is given by\textsuperscript{79}

$$
\frac{\gamma \Delta x}{|v_{g2x}|} \equiv \frac{\gamma L_z}{v_{g2z} \left( \frac{v_{g2z}}{v_g} - \frac{v_{g0z}}{v_g} \right)} = \pi
$$

where $\gamma$ is the homogeneous growth rate and $\Delta x$ is the maximum distance the decay wave can travel in the $x$-direction before it convects out of the pump resonance cone (Fig. III-16). In order to get this threshold, an $\exp(2\pi)$ growth of the decay wave power was assumed. The $E_\parallel$ driven decay is important for ion-sound quasi-mode decay in the low density plasma edge region (see Sec. III-2). Near the edge, $\omega_2 \approx \omega_{pe}(k_{2z}/k_2)$, so $v_{g2z} \equiv \partial \omega_2/\partial k_{2z} = (\omega_2/k_{2z}) (k_2^2/k_{2z}^2)$, $v_{g2z} \equiv \partial \omega_2/\partial k_{2z} = -(\omega_2/k_{2z}) (k_2^2/k_{2z}^2)$, where $k_{2\perp} = k_{2z}$, $k_{2y} = 0$, and for the $E_\parallel$ coupling regime...
\( \omega_{pe}^2 \ll \omega_{ce}^2 \) and \( \omega_{pi}^2 \ll \omega_2^2 \). In this sub-section \( \omega_2 \) will be used for the frequency of the lower sideband lower hybrid wave so that \( \omega_2 = \omega_0 - \omega_R \). The threshold is given by

\[
P_{0||}^2 \simeq \frac{4}{F} \frac{m_e^2 \omega_0^2}{e^2} \frac{T_i}{m_e} \left[ \frac{\nu_e}{\omega_0} + \left( \frac{k_{2z}}{k_2} \right)^3 \frac{2\pi}{k_{2z} L_z} \frac{\omega}{\omega_0} \right] \tag{38}
\]

where \( \omega \) is used for \( \omega_R \) in this sub-section and Eq. (29) was used since \( \gamma_{th}/\omega_0 < k_{\|} v_{ti}/\omega_0 \) where \( \gamma_{th} \) is the homogeneous growth rate needed for the \( \exp(2\pi) \) convective threshold. The second term in the bracket can be of the order of \( \nu_e/\omega_0 \) or less, in which case the threshold is essentially the same as the collisional threshold. However, to obtain this low threshold \( \Delta x \) may become larger than the plasma minor radius and Eq. (38) is no longer valid. Furthermore, effects of density and temperature gradients must be considered, including large variations in \( \gamma \) due to the changing plasma parameters.

If the coupling term is mainly due to the \( E \times B \) term, the decay wave has to travel at a relatively large angle with respect to the pump resonance cone since coupling tends to vanish as \( |k_{2y}| \) approaches zero. The geometry, projected on the \( x-y \) plane, is shown in Fig. III-17. In general, if \( |k_{2y}/k_{2\perp}| \) is large, the \( E \times B \) driving term is large (and hence, the temporal growth rate \( \gamma \) is large) but the convective loss is also large. On the other hand, if \( |k_{2y}/k_{2\perp}| \) is small, the convective loss is small but the \( E \times B \) driving term also becomes small. Therefore, the angle of propagation \( \phi \) that \( k_{2\perp} \) makes with the \( x \)-direction has a finite optimum value for convective growth. The convective threshold in the \( x-z \) plane given by Eq. (37) can be rewritten as

\[
\frac{\gamma}{\omega_2} \left( 1 - \frac{\omega_{L,H}^2}{\omega_2^2} \right)^{-1} \frac{k_{2z} L_z}{|k_{2z}|} \frac{k_{2z}^2}{k_{2z}^2} \frac{k_{0z}^2}{k_{0z}^2} = \pi, \tag{39}
\]
where \( v_{g2z} = (\omega_2/k_2)(1 - \omega_{LH}^2/\omega_2^2) \), \( v_{g2x} = -(k_{2x}\omega_2/k_2^2)(1 - \omega_{LH}^2/\omega_2^2) \), were used. The optimum value of \( \phi \) can be found analytically if the rf power scaling of the growth rate is known.

Since for \( E \times B \) driven ion-sound quasi-mode decay well above threshold the convective growth in the \( x-z \) plane \( \gamma \Delta x / |v_{g2z}| \) is proportional to \((1 - \cos^2 \phi)^{1/3} / (A - \cos \phi)\), where \( \cos \phi = k_{2z}/k_2 \) and \( A \equiv (k_2^2/k_2^2 \| k_2^2 \perp) (k_0^2/k_0^2 \perp) > 1 \), it maximizes for \( \cos \phi = -A + \sqrt{A^2 + 3} \approx 1 - (1/2)(\omega/\omega_0) \). Here, the approximations \( \omega \ll \omega_0 \) and \( \omega_{LH}^2 \ll \omega_0^2 \) have been used so that \( A \approx 1 + (\omega/\omega_0) \). For \( \omega/\omega_0 \approx 10^{-3} \), \( \cos \phi \approx 1 \) (i.e., \( |k_{2y}| \ll |k_{2z}| \)) for the optimum convective growth. The convective threshold in the \( y \)-direction is given by

\[
\frac{\gamma L_y}{|v_{g2y}|} \approx \frac{\gamma}{\omega_2} \left(1 - \frac{\omega_{LH}^2}{\omega_2^2}\right)^{-1} k_2 \frac{|k_2 L_y|}{|k_2^2|} = \pi \tag{40}
\]

where \( v_{g2y} = -(k_{2y}\omega_2/k_2)^2(1 - \omega_{LH}^2/\omega_2^2) \) was used. The convective growth \( \gamma L_y / |v_{g2y}| \) decreases with \( |k_{2y}| / k_2 \) since \( \gamma \) is proportional to \( |k_{2y}| / k_2^2 \) whereas \( |v_{g2y}| \) is proportional to \( |k_{2y}| / k_2 \). So the lowest convective threshold in the \( y \)-direction is also achieved when \( |k_{2y}| \ll |k_{2z}| \). The convective threshold is determined by the higher of the thresholds given by Eqs. (40) and (39). However, the homogeneous growth rate \( \gamma \) is reduced by a factor \((1 - \cos^2 \phi)^{1/3} \approx (\omega/\omega_0)^{1/3} \approx 10^{-1} \) (which corresponds to a factor of \( 10^3 \) in rf power) from the optimum coupling case \( |k_{2y}| = k_{2x} \perp \) in order to get this low convective loss.

In Sec. III-3 it was found that the ion-cyclotron quasi-mode decay is mainly driven by the \( E \times B \) coupling term and the growth rate maximizes for \( r/a \approx 1 \). For parameters appropriate for this region of maximum growth rate, numerically the growth rate was found to scale roughly like \( B^{1.2} \). Using this power scaling, \( \gamma \Delta x / |v_{g2z}| \) is found to maximize for \( \cos \phi \approx 0.95 \) for \( \omega/\omega_0 = 0.033 \) (corresponding to \( n = 1 \), hydrogen plasma, \( B = 10 \)T). The convective growth in the \( y \)-direction
\( \gamma L_y / |v_{g2y}| \) increases with \( |k_{2y} / k_2| \) for this case since the growth rate \( \gamma \), which is proportional to \( |k_{2y} / k_2|^1 \), increases more rapidly with \( |k_{2y} / k_2| \) than the group velocity in the \( y \)-direction \( v_{g2y} \) (which is proportional to \( |k_{2y} / k_2| \)).

Here, the case of optimum \( E \times B \) coupling (i.e., \( k_{2x} = 0, |k_{2y}| = k_{2\perp} \)) will be considered. The convective thresholds in the \( y \)- and \( z \)-directions, respectively, are:

\[
\frac{\gamma L_y}{|v_{g2y}|} \simeq \frac{\gamma L_y |k_{2y}|}{\omega_2 \left( 1 - \frac{\omega_{LH}^2}{\omega_0^2} \right)} = \pi (41)
\]

\[
\frac{\gamma L_z}{|v_{g2z}|} \simeq \frac{\gamma L_z |k_{2z}|}{\omega_2 \left( 1 - \frac{\omega_{LH}^2}{\omega_0^2} \right)} = \pi (42)
\]

where \( |k_{2y}| \simeq k_2 \) was used. There is no convective loss in the \( x \)-direction because \( k_{2x} = 0 \). Convective loss in the \( z \)-direction dominates because \( L_z < L_y \) and \( |k_{2z}| < |k_{2y}| \simeq |k_{2\perp}| \). For typical Alcator C edge parameters with \( \omega_0 / \omega_{LH} \gtrsim 2 \), Eq. (42) gives \( \gamma_{\text{th}} / \omega_0 \gtrsim 0.1 \). Such large values of \( \gamma / \omega_0 \) can never be achieved for ion-sound quasi-mode decay. Even for ion-cyclotron quasi-mode decay, \( \gamma / \omega_0 \lesssim \omega_{ci} / \omega_0 \approx 0.03 \) (for hydrogen) for the injected rf power of 1MW (see Fig. III-11). Therefore, the \( E \times B \) coupling in the outer plasma layers would not be expected to play a significant role either for ion-sound quasi-mode decay or for ion-cyclotron quasi-mode decay if the pump waves were localized inside resonance cones. This result has been noted by Porkolab,\(^{49}\) and more recently by Liu et al.\(^{80}\)

Next, the possibility of \( E_\parallel \) driven ion-sound quasi-mode decay within a finite radial extent will be considered. As shown in Sec. III-2, the homogeneous growth rate associated with the \( E_\parallel \) driven ion-sound quasi-mode decay is strongly peaked near the edge. Furthermore, unless \( |k_{2y} / k_2| \ll 1 \) (in which case the coupling is lost) the convective loss is very large for the \( E \times B \) driven decay. Therefore, the
relevant threshold is that associated with the $E_\parallel$ coupling, but the growth has to occur within a radial distance $L_x \ll a$ from the waveguide mouth where the growth rate is large (where $a$ is the minor radius of the plasma) [see Figs. III-2(a) and (b)]. Requiring a factor of $\exp(2\pi)$ growth in the decay wave power within this distance gives the threshold due to finite radial growth region:

$$\int \frac{\gamma(x)}{|v_{g2z}(x)|} dx = \frac{\gamma L_x}{|v_{g2z}|} = \left[ \frac{\gamma}{\omega_2} \left( \frac{k_2}{k_{2z}} \right)^2 |k_{2z}| L_x \right]_{x=x_{wa}} = \pi.$$  \hspace{1cm} (43)

The convective threshold in the $y$-direction and that in the $x$-$y$ plane are still given by Eq. (40) and Eq. (39), respectively. The highest of the thresholds given by Eqs. (43), (40) and (39) determines the threshold. Note that the threshold (43) can be made smaller than the threshold (40) for

$$\frac{|k_{2z}|}{k_{2y}} < \frac{L_x}{L_y} \ll 1.$$  \hspace{1cm} (44)

If this is true, the threshold Eq. (43) reduces to

$$\frac{\gamma L_x}{|v_{g2z}|} = \pi$$  \hspace{1cm} (45)

which is just the convective threshold in the $z$-direction. The convective threshold in the $z$-direction is always higher than that in the $y$-direction under the assumed conditions since $(L_y/L_z)(\omega_{pe}/\omega_0) > 1$. However, the convective threshold in the $z$-direction may not exist in the standing wave region A of Fig. III-15. A lower limit can be placed on this threshold by taking the optimum condition for this decay process. The decay is assumed to occur in the standing wave region A and Eq. (40) is taken to be the relevant threshold. The situation shown in Fig. III-18, i.e., $k_{0z} \simeq k_{1z} \gg k_{2z}$, $k_{1y} \simeq -k_{2y} \gg k_{0y}$ is chosen so that Eq. (44) is satisfied. In
this case the threshold becomes

\[
\frac{\gamma_{th}}{\omega_0} = \frac{\pi}{|k_{2z}|L_y} \begin{vmatrix} k_{2y} & k_{2z} \\ k_2 & k_2 \end{vmatrix}.
\] (46)

For Alcator C with \( n_e = 5 \times 10^{12}\text{cm}^{-3} \) and \( n_{\parallel} = 5 \), Eq. (46) will predict a threshold power of \( P_{th} \approx 0.7\text{MW} \). Examples of threshold electric fields calculated using Eq. (46) are shown in Table III–1.
TABLE III-1 — Examples of thresholds due to finite growth region in the z-direction for accessible \( n_{0||} \) for ion-sound quasi-modes discussed in Sec. III-4-A. The convective threshold electric fields \( E_{0||}^{\text{conv}} \) calculated from Eq. (46) are shown. The homogeneous (collisional) thresholds \( E_{0||}^{\text{hom}} \) given by Eq. (30) are also shown for comparison. \( T_i \) was assumed to be equal to \( T_e \).

| \( n_e (\text{cm}^{-3}) \) | \( T_e (\text{eV}) \) | \( n_{0||} \) | \( n_{||} \) | \( E_{0||}^{\text{hom}} (\text{kV/cm}) \) | \( E_{0||}^{\text{conv}} (\text{kV/cm}) \) |
|-----------------|--------------|--------|--------|-----------------|-----------------|
| Alcator A\(^a\) | \( 4 \times 10^{11} \) | 3 | 2 | 5 | 0.03 | >10 |
|                 |              | 2 | 20   |     |     | 4   |
| Alcator C\(^b\) | \( 5 \times 10^{12} \) | 5 | 3 | 5 | 0.12 | 4   |
|                 |              | 3 | 20   |     |     | 0.6 |
| 800 MHz expt.\(^c\) | \( 2 \times 10^{11} \) | 10 | 5 | 7 | 0.015 | 0.6 |
|                 |              | 5 | 20   |     |     | 0.15 |

\(^a\)P_{rf} = 1\text{ kW} \text{ corresponds to } E_{WG} = 0.33\text{kV/cm. } L_z = 2.6\text{cm, } L_y = 8.1\text{cm.}\n
\(^b\)P_{rf} = 1\text{ kW} \text{ corresponds to } E_{WG} = 0.15\text{kV/cm. } L_z = 3.8\text{cm, } L_y = 23\text{cm.}\n
\(^c\)P_{rf} = 1\text{ kW} \text{ corresponds to } E_{WG} = 0.09\text{kV/cm. } L_z = 12\text{cm, } L_y = 25\text{cm.}\n
98
FIGURE III-15 — The geometry considered in Sec. III-4-A. A standing wave is formed in the $z$-direction in region A for 180° phasing. The distance $L_x$ is the effective growth distance in the $x$-direction where the growth rate for the ion-sound quasi-mode decay is large [see Figs. III-2(a) and (b)].
FIGURE III-16 — The geometry considered in Secs. III-4-A and III-5-A when \( k_{0y} = k_{2y} = 0 \). The region \( |z - C_0 x| \leq L_z/2 \) is the pump resonance cone. The sideband lower hybrid waves grow while they are in this region but they stop growing as they travel parallel to the resonance cone \( |z - C_2 x| \leq L_z/2 \) and get out of this region. \( C_0 \equiv v_{g0z}/v_{g0x} \) and \( C_2 \equiv v_{g2z}/v_{g2x} \).
FIGURE III-17 — The geometry considered in Secs. III-4-A and III-5-B when $k_{2y} \neq 0$. The projection on the $x$-$y$ plane is shown. The $z$-direction points out of the paper. $C_0 \equiv v_{g0y}/v_{g0x}$ and $C_2 \equiv v_{g2y}/v_{g2x}$. 

101
FIGURE III-18 — The relative magnitudes of the wavevector components assumed to obtain the finite $L_z$ convective threshold.
III-4-B. THRESHOLDS WITH NO RESONANCE CONES

As discussed in Secs. II-1 and II-2, the components of the pump electric field in the inaccessible part of the \( n_{0||} \) spectrum (namely, \( 1 \leq n_{0||} \leq n_{||ac} \)) will be confined to the outer plasma shell of radial extent \( \delta r \) between the slow wave cut-off layer (\( \omega = \omega_{pe} \)) and the slow wave-whistler wave mode conversion layer given in Sec. II-1. The fast wave cut-off layer is between these critical densities, and thus it is also contained within \( \delta r \). The pump waves tend to fill up this volume more or less uniformly because of different ray trajectories for different \( n_{||} \)'s. Toroidal effects and scattering from density fluctuations\(^{34} \) further accentuate this behavior. The pump wave can acquire large values of the poloidal wavenumber due to toroidal effects and scattering from small \( k_{||} \) density fluctuations also tend to scatter the pump wave in the poloidal plane.

For simplicity, a shell of thickness \( \delta r \) (which is typically a small fraction of the plasma minor radius \( a \)) and circumference \( 2\pi a \) (i.e., a volume \( (2\pi R)(2\pi a)\delta r \) where \( R \) is the plasma major radius) is assumed to be filled uniformly with the pump wave. If the parametrically excited lower hybrid waves propagate mainly in the \( y-z \) plane (i.e., the poloidal-toroidal plane), these waves will stay inside the region filled with the pump wave and the case of the homogeneous threshold of Sec. III-2 is recovered. The decay waves will keep amplifying until they convect out of this region by acquiring finite \( k_x \) through toroidal effects or scattering. Note that these parametrically excited waves tend to have larger values of \( n_{||} \) and can satisfy the accessibility condition (\( n_{||} > n_{||ac} \)). In order to explain the enhanced low-frequency fluctuations observed on the probes located at different toroidal locations from the waveguide array by this mechanism, the decay had to occur at the location of the probe. If the decay occurred only in front of the waveguide, the quasi-mode would damp as soon as it would leave the growth region and these probes would not detect.
the enhanced fluctuations.

The threshold electric field for this case is given by Eq. (30). The rf threshold power at the waveguide corresponding to this electric field can be estimated as follows: assume that $\xi$ is the fraction of rf power in the inaccessible region of $n_{0||}$, and that this power fills the cylindrical volume $(2\pi a\delta r)(2\pi R)$. Conservation of power flux requires

$$Q\xi P_{rf} = \langle W_T u_{g0} \rangle \cdot \mathcal{A} \tag{47}$$

where $W_T$ is the total wave energy density defined in Sec. II–2, $u_{g0}$ is the group velocity of the pump wave, and $\mathcal{A}$ is the effective area through which the pump wave propagates. The angular brackets denote averaging over the area $\mathcal{A}$ since $W_T$ and $u_{g0}$ are functions of radial position. The quantity $Q$ is the enhancement factor due to the fact that the pump wave may propagate around the torus several times before it loses its energy by collisions and other damping mechanisms, and is defined as $Q \equiv (1 - P_1/P_0)^{-1}$ where $P_1/P_0 \equiv \exp(-2\pi R \Gamma / u_{g0||})$ is the fraction of the pump wave power left after one toroidal pass ($\Gamma$ is the "effective" temporal damping rate averaged over the area $\mathcal{A}$). Since $\partial \epsilon / \partial \omega_0 \simeq (2/\omega_0)(1 + \omega^2/\omega^2_e) \simeq 2/\omega_0$ near the edge,

$$W_T \simeq \frac{E^2_{0\perp}}{8\pi} \simeq \frac{E^2_{0\parallel}}{8\pi} \left(\frac{k_{0\perp}}{k_{0||}}\right)^2 \simeq \frac{E^2_{0\parallel} \omega^2_{pe}}{8\pi} \frac{\omega^2_e}{\omega_0^2}. \tag{48}$$

Taking the effective area to be the cross sectional area of the pump wave shell at one poloidal plane so that $\mathcal{A} = 2\pi a\delta r \hat{\mathcal{A}}$, Eqs. (47) and (48) give

$$P_{th} \simeq \frac{\langle E^2_{0\perp} \rangle}{8\pi} \frac{c}{n_{0||}} \frac{2\pi a(\delta r)}{Q\xi}. \tag{49}$$

In the case of ion-sound quasi-mode decay in Alcator A, $\omega^2_{pe}/\omega^2_e \simeq 5$, $n_{0||} = 1.5$, $Q \simeq 14$, $\xi \simeq 0.4$, $a = 10\text{cm}$, and $\delta r \simeq 2\text{cm}$, the threshold power predicted...
by Eq. (49) is \( P_{th} \approx 90\text{W} \) for \( E_{0||} = 0.03\text{kV} \) which should be compared with the injected power of \( P_{rf} = 80\text{kW} \). For Alcator C, \( \omega_{pe}/\omega_0^2 \approx 5 \), \( n_{0||} = 1.5 \), \( Q \approx 3 \), \( \xi \approx 0.2 \), \( a = 16.5\text{cm} \), and \( \delta r \approx 3\text{cm} \), the threshold power is \( P_{th} \approx 4\text{kW} \) for \( E_{0||} = 0.04\text{kV/cm} \) (homogeneous threshold electric field for \( \omega_{pe}^2/\omega_0^2 \approx 5 \) and \( T_e = T_i = 3\text{eV} \)). The \( P_{rf} \approx 650\text{kW} \) power injected into this device (from each array) can easily exceed this threshold. The calculated threshold of \( 4\text{kW} \) is in fair agreement with the experimental value of \( \approx 10\text{kW} \). However, this model may not give a good estimate for the threshold power for ion-cyclotron quasi-mode decay which occurs further inside the plasma (around \( r/a \approx 1 \)).

In addition to the electromagnetic effects discussed above, the resonance cone could broaden due to poloidal scattering of the pump wave when the density is high so that the pump wave is severely scattered by density fluctuations (corresponding to the case \( l/l_s \gg 1 \), see Sec. II-4). The resonance cone spreads in the \(-x\)-direction and \( \pm y\)-directions but not in the \(+x\)-direction (see Fig. III-16 for the coordinate system). In the case of maximum spread the pump wave fills the volume between the unperturbed resonance cone and the vacuum chamber wall. The maximum spread in the \( y\)-direction would be given by the resonance cone half angle \( \theta_{a0} = v_{g0\perp}/v_{g0||} \approx \omega_0/\omega_{pe} \). Note that since the lower sideband wave has a lower frequency than the pump wave, the half angle of its resonance cone would be smaller than that of the pump wave. Therefore, the lower sideband wave cannot convect out of the region filled with the pump wave and convective losses need not be considered. However, the electric field becomes weaker as the pump wave propagates away from the waveguide array, and effects of changing plasma parameters along the sideband wave trajectory must also be considered. Note that this threshold depends on how strong the scattering from density fluctuations is \( (i.e., \text{on } l/l_s) \), which depends on the electron density \( n_e \) near the plasma edge where the density fluctuation is large.
(Sec. II–4) rather than the value of $\omega_0/\omega_{LH}$ near the plasma center (which was the relevant parameter for convective threshold assuming well-defined resonance cone\textsuperscript{49}). Requiring an $\exp(2\pi)$ amplification within a distance $L \simeq 2\pi R/4 \simeq 1\text{m}$ from the waveguide array (based on the CO$_2$ scattering data to be presented in Sec. V–5), the threshold for ion-cyclotron quasi-mode decay would be of the order of a few hundred kilowatts. This threshold is still higher than the experimentally observed value $P_{th} \simeq 10$–$100\text{kW}$. It is noted that the theoretical threshold could be reduced if the spatial broadening of the pump wave were assumed to be less than the maximum spread case considered here (but enough to reduce the convective threshold).
III–4-C. THRESHOLDS DUE TO DENSITY AND TEMPERATURE GRADIENTS

In the parametric decay process under consideration the following selection rules must be satisfied:

\[ \omega_2 = \omega_0 - \omega \]  \hspace{1cm} (50)

\[ k_2 = k_0 - k \]  \hspace{1cm} (51)

where

\[ \omega_2 \simeq \omega_{LH} \sqrt{1 + \frac{m_i}{m_c} \left(\frac{k_{2||}}{k_2}\right)^2}; \quad \omega_0 \simeq \omega_{LH} \sqrt{1 + \frac{m_i}{m_c} \left(\frac{k_{0||}}{k_0}\right)^2} \]  \hspace{1cm} (52)

and

\[ \omega \simeq k_{||} v_{it,e} \]  \hspace{1cm} (53)

for the case of ion-sound and ion-cyclotron quasi-modes, respectively.

First, consider the case of inhomogeneous density, but spatially uniform temperatures. For a given \( T_{i,e} \) and \( k_{||} \), \( \omega \) is fixed through Eq. (53). Equation (50) can be satisfied with the same value of \( \omega_2 \). As the waves propagate radially inward to a region of higher density, \( k_{0||} \) and \( k_{2\perp} \) change in order to satisfy the dispersion relationship Eq. (52). Since there is no restriction on \( k_{\perp} \), it can always be chosen so as to satisfy Eq. (51). Therefore, density gradients do not introduce new thresholds.

Now consider the case when both the density and temperature vary in the radial direction. In this case, \( \omega \) changes as the waves propagate inward to a region of higher temperature. Equation (51) can still be satisfied as in the case of density gradients only, but Eq. (50) can no longer be satisfied with the same value of \( \omega_2 \) and frequency mismatch occurs. But as seen in Section III–2 the frequency spectrum...
of the quasi-mode is quite broad (except at powers very close to threshold) and $\Delta \omega$ (frequency width of the unstable quasi-modes) is typically of the order of $\omega_R$. Therefore, the effect of mismatch is not so important unless $L_x \gtrsim L_T$ where $L_T \equiv T[dt/dx]^{-1}$ and $L_x$ was defined in Sec. III-4-A. Thus $L_x$ is taken to be less than $L_T$ and the thresholds obtained in Sec. III-2 remain valid.
III-5. PUMP DEPLETION

If the decay waves remain small in amplitude, the pump wave power can be considered as essentially constant along its trajectory. However, if the decay waves are amplified to such an extent that significant fraction of the pump power is transferred to the decay waves, the power contained in the pump wave decays as it propagates towards the plasma center. The case of decay into ion-sound quasi-modes in particular will be discussed.

The spatial evolution of the pump and the sideband powers after reaching a steady state is described by the following coupled equations

\[
\begin{align*}
\psi \cdot \nabla I_0 & = -\alpha I_0 I_2 \\
\psi \cdot \nabla I_2 & = +\alpha I_0 I_2
\end{align*}
\]

where \( I_0(x, y, z) = E_0^2(x, y, z)/\omega_0 \) and \( I_2(x, y, z) = E_2^2(x, y, z)/\omega_2 \) are the action variables, \( \psi \) and \( \psi' \) are the group velocities of the pump wave and the sideband lower hybrid wave, respectively, and

\[
\alpha \equiv \frac{\mu^2 \omega_0^2}{4 E_{0WG}^2} \left( 1 + \frac{\omega_p^2}{\omega_{ce}^2} \right)^{-1} \text{Im} \left( \frac{\chi_{e}X_{i}}{\epsilon} \right)
\]

Here \( E_{0WG} \) is the pump electric field at the waveguide mouth and \( \gamma \) is the linear growth rate. It can be seen from Eq. (54) that \( \nabla \cdot (\psi \psi_0 I_0 + \psi \psi I_2) = 0 \) and the action flux is conserved. In the limit \( \omega \ll \omega_2 \approx \omega_0 \), the power flux is conserved among the pump wave \( \omega_0 \) and the lower sideband \( \omega_2 \) (there is negligible power going into the low frequency mode \( \omega \)).

In this section two cases discussed in Secs. III-4-A and B will be presented: the case when a well-defined resonance cone exists (Sec. III-5-A), and the case when
a uniformly filled pump wave shell exists in the outer layers of the plasma (Sec. III-5-B).

III-5-A. CASE WITH WELL-DEFINED RESONANCE CONE

First, consider the case when \( v_{2y} = 0 \) (i.e., \( k_{2y} = 0 \)). The geometry is shown in Fig. III-16. The pump wave is assumed to propagate in the \( x-z \) plane so that \( v_{g0y} = 0 \). Equation (54) can be written in a two-dimensional form

\[
\frac{\partial G_0}{\partial x} + C_0 \frac{\partial G_0}{\partial z} = -G_0 G_2
\]

\[
\frac{\partial G_2}{\partial x} + C_2 \frac{\partial G_2}{\partial z} = +G_0 G_2
\]

where \( C_0 \equiv v_{g0x}/v_{g0z} \), \( C_2 \equiv v_{g2x}/v_{g2z} \) and \( G_0 \equiv \alpha I_0/v_{g2z} \), \( G_2 \equiv \alpha I_2/v_{g0z} \). \( G_0 \) and \( G_2 \) are proportional to the power flux in the radial direction of the pump wave and the decay wave, respectively.

The boundary condition for the case of uniform finite extent pump wave and a uniform initial noise level (which may be enhanced over the thermal noise) for the lower sideband can be written as

\[
G_0 = \begin{cases} 
A_0 & \text{for } -\frac{L_y}{2} < y < \frac{L_y}{2}, \ -\frac{L_z}{2} < z < \frac{L_z}{2} \\
0 & \text{otherwise}
\end{cases}
\]

and \( G_2 = A_2 \) (everywhere) on the plane \( x = 0 \). The solutions of Eq. (55) with the boundary condition Eq. (56) in region A of Fig. III-16 [i.e., \( |z - C_0 x| < L_z/2 \), \( |z - C_2 x| < L_z/2 \), and \(-L_y/2 < y < (L_y/2)\)] are given by

\[
G_0 = \frac{A_0}{A_0 + \frac{A_2}{A_t} \exp(A_t x)}
\]

\[
G_2 = \frac{A_2 \exp(A_t x)}{A_0 + \frac{A_2}{A_t} \exp(A_t x)}
\]
and in region B [i.e., \(|z - C_0x| < L_z/2, z - C_2x < -L_z/2\) and \(-L_y/2 < y < (L_y/2)\)] by

\[
G_0 = \frac{A_0}{\frac{A_0}{A_t} + \exp(A_2x)\exp[A_0\left(\tau - \frac{L_z}{2V}\right)]} - \frac{A_0}{A_t}\exp[A_t\left(\tau - \frac{L_z}{2V}\right)]
\]

\[
G_2 = \frac{A_2\exp(A_2x)\exp[A_0\left(\tau - \frac{L_z}{2V}\right)]}{\frac{A_0}{A_t} + \exp(A_2x)\exp[A_0\left(\tau - \frac{L_z}{2V}\right)]} - \frac{A_2}{A_t}\exp[A_t\left(\tau - \frac{L_z}{2V}\right)].
\]

Here, \(A_t \equiv A_0 + A_2\), \(V \equiv C_0 - C_2\) and \(\tau \equiv -(z - C_0x)/V\). It can be seen from Eq. (57) that the characteristic scale length for the decay wave growth in the \(x\)-direction is \(A_t^{-1}\). This agrees with the result obtained in Sec. III-4-A since Eq. (57) also predicts an \(\exp(2\pi)\) growth in the decay wave power at the point C in Fig. III-16 for

\[
A_t\Delta x = \frac{2\gamma\Delta x}{v_{g2x}} \approx 2\pi
\]

where \(A_2/A_0 \ll \exp(2\pi)\) is assumed and \(\Delta x \equiv L_z/|V|\) is the same \(\Delta x\) defined in Sec. III-4-A. It is clear that half of the pump power will be depleted at point C in Fig. III-16 when \(A_t\Delta x = \ln(A_0/A_2)\) which is a few times above the \(\exp(2\pi)\) convective threshold.

If \(k_{2y} \neq 0\), the \(v_{g2y}(\partial I_2/\partial y)\) term in Eq. (54) must be retained. However, after performing the shear coordinate transformation:

\[
x' = x
\]

\[
y' = y
\]

\[
z' = z - ax - by
\]

where \(a \equiv (v_{g0x}v_{g2y} - v_{g0y}v_{g2x})/(v_{g0x}v_{g2y} - v_{g0y}v_{g2x})\) and \(b \equiv (v_{g0x}v_{g2y} - v_{g0y}v_{g2x})/(v_{g0x}v_{g2y} - v_{g0y}v_{g2x})\), the problem can be reduced to that in 2-dimensions.
The transformed equations are:

\[
\begin{align*}
\frac{\partial G_0}{\partial x'} + C_0 \frac{\partial G_0}{\partial y'} &= -G_0 G_2 \\
\frac{\partial G_2}{\partial x'} + C_2 \frac{\partial G_2}{\partial y'} &= +G_0 G_2 
\end{align*}
\]

where \( C_0 \equiv v_{g0y}/v_{g0z} \), \( C_2 \equiv v_{g2y}/v_{g2z} \) and \( G_0 \equiv \alpha I_0/v_{g2z}, \) \( G_2 \equiv \alpha I_2/v_{g0z}. \) The transformed equations are independent of \( z' \) which is now a parameter that specifies a plane parallel to the plane that is spanned by the two vectors \( v_{g0} \) and \( v_{g2}. \) On a given plane specified by \( z' \), this case reduces to the two dimensional case discussed above. For a finite extent uniform pump field given by Eq. (56), the transformed boundary condition on the plane \( z' = 0 \) becomes

\[
G_0 = \begin{cases} 
A_0 & \text{for } -\frac{L_y}{2} < y' < \frac{L_y}{2}, -\frac{L_z}{2} - z' < b'y' < \frac{L_z}{2} - z' \\
0 & \text{otherwise}
\end{cases}
\]

and the background fluctuation level is again given by \( G_2 = A_2 \) (everywhere). The geometry, projected on the \( x\-y \) plane, is shown in Fig. III-17.

For simplicity, consider the extreme case that \( k_{2z} = 0 \) and take the plane \( z' = 0. \) In this case \( |b| = |v_{g2z}|/v_{g2y} | \ll \omega_{pe}/\omega_2 > 1 > L_z/L_y \) so that the boundary condition for \( G_0 \) becomes \( G_0 = A_0 \) for \( -L_z/2|b| < y' < L_z/2|b| \). \( G_0/A_0 \) and \( G_2/A_0 \) for the case \( C_0 = 0, C_2 = 10, A_0(L_z/2|b|) = 150 \) (corresponding to \( \gamma/\omega_0 \approx 2 \)) and \( A_2/A_0 = 10^{-4} \) are shown in Figs. III-19(a) and 19(b) respectively. In this case the decay lower hybrid waves were assumed to travel in the \(+y\)-direction, but there may also be waves traveling in the \(-y\)-direction. The fraction of the power remaining in the pump is plotted against the radial distance in Fig. III-20. The pump does not get completely depleted in this case because of the assumption that the decay wave travels only in the \(+y\)-direction so that the depletion is not efficient near \( y = -L_z/2|b|. \)
FIGURE III-19(a) — Pump wave power as a function of $x$ and $y$ on the plane $z' = 0$. $C_0 = 0$, $C_2 = 10$, $A_0(L_x/2|b|) = 150$ (corresponding to $\gamma/\omega_0 \approx 2$) and $A_2/A_0 = 10^{-4}$. 

113
FIGURE III-19(b) — Decay wave (lower sideband) power as a function of $x$ and $y$ on the plane $z' = 0$. Same parameters as Fig. III-19(a).
FIGURE III-20 — Pump wave power integrated over $y$ as a function of $x$. 
III-5-B. CASE WITH UNIFORM PUMP WAVE SHELL

Since the pump wave is assumed to fill the outer shell region of the plasma, there are no convective losses in the y- and z-directions and Eq. (54) reduces to

\[
\frac{\partial G_0}{\partial x} = -G_0 G_2 \\
\frac{\partial G_2}{\partial x} = +G_0 G_2.
\]

The pump wave is now assumed to be homogeneous in the shell region of thickness \( L_x \) so the boundary conditions become

\[
G_0 = A_0 \\
G_2 = A_2
\]

at \( x = 0 \). Solving for \( G_0 \) and \( G_2 \) within the shell region gives

\[
G_0 = \frac{A_0}{A_0 + \frac{A_2}{A_t} \exp(A_t x)} \\
G_2 = \frac{A_2 \exp(A_t x)}{A_0 + \frac{A_2}{A_t} \exp(A_t x)}
\]

Half of the pump wave power will be depleted within the \( x \)-distance \( L_x \) if

\[
A_t L_x = \frac{2\gamma L_x}{|v_{y2x}|} > \ln \left( \frac{A_0}{A_2} \right)
\]

so that appreciable pump depletion is not expected unless \( |v_{y2x}| \ll |v_{y2y}| \), i.e., when the decay wave propagates mainly in the \( y \)-direction (and therefore, can spend a long time inside the pump wave shell). In reality, the situation is more complex. The pump wave fills up the shell region by undergoing many reflections from the lower hybrid–whistler wave mode conversion layer and the plasma wave cut-off layer and the location of the mode conversion layer is different for different \( n_1 j \)'s. Moreover, the source of the pump wave is localized both toroidally and poloidally. However, the analysis given in the present sub-section remains valid in an approximate sense.
In this section, the relationships between fluctuation levels in electrostatic potential and ion and electron densities at both the low frequency quasi-mode and the lower sideband lower hybrid wave are derived. The potential fluctuations can be measured with rf probes at both low frequency and high frequency, the ion density fluctuations can be measured at low frequency by using rf probes as Langmuir probes, and the electron density fluctuations can be measured at both low frequency and high frequency using CO₂ laser scattering.

In the oscillating frame of reference, the fluctuating potential and density are related as follows:

\[ \rho_j = -\frac{k^2}{4\pi} [J_0(\mu_j) \chi_j \phi + J_1(\mu_j) \chi_j \phi^+ e^{-i\beta} + J_{-1}(\mu_j) \chi_j \phi^- e^{+i\beta}] \]  \hspace{1cm} (61)

\[ \phi = \frac{4\pi}{k^2} \sum_j [J_0(-\mu_j) \rho_j + J_1(-\mu_j) \rho^+_j e^{-i\beta} + J_{-1}(-\mu_j) \rho^-_j e^{+i\beta}] \]  \hspace{1cm} (62)

where \( \chi^{n\pm}, \phi^{n\pm}, \) and \( \rho^{n\pm} (n = 0, \pm 1, \pm 2, \cdots) \) are evaluated at \( (\omega \pm n\omega_0, k \pm n\kappa_0) \), \( j \) is the species index, and \( \beta \) is the phase which drops out in the final result. The equations for \( \rho^{n\pm}_j \) and \( \phi^{n\pm} \) can be obtained from Eqs. (61) and (62) by permuting arguments \( (\omega \rightarrow \omega \pm n\omega_0) \).

Since \( \mu_i \ll \mu_e \) and \( \mu_e \) is typically less than 1, terms of order \( \mu_i^2 \) are neglected (which is equivalent to setting \( \mu_i = 0 \)). In this approximation, Eq. (61) reduces to

\[ \rho_i = -\frac{k^2}{4\pi} \chi_i \phi \]

which can be rewritten as

\[ \frac{n_i(\omega)}{n_i} = -k^2 \gamma_{Di}^2 (\chi_i(\omega) \frac{\phi(\omega)}{T_i}). \]  \hspace{1cm} (63)
The relationship between \( \bar{n}_i^{\pm} \) and \( \bar{\phi}^{\pm} \) can be obtained by permuting arguments
\( (\omega \to \omega \pm \omega_0) \):
\[
\frac{\bar{n}_i^{\pm}}{n_i} = -k^2 \chi_D^2 \chi_i^{\pm} \frac{e^{i \phi^{\pm}}}{T_i}. \tag{64}
\]

For the electrons, \( \mu_e \) (the subscript \( e \) will be omitted from now on) will be kept finite and terms of order \( \mu^2 \) will be retained. Terms containing \( \phi^{3\pm} \) and higher order will be neglected compared to \( \phi \) since they are of order \( \mu^3 \) or higher. Substituting equations for \( \rho^{\pm}, \rho^{2\pm} \), and \( \rho^{3\pm} \) into equation for \( \phi^{2\pm} \) and solving for \( \phi^{2\pm} \) gives \( \phi^{3\pm} \) in terms of \( \phi^{\pm} \) and \( \phi \):
\[
\phi^{2\pm} = \pm \frac{J_0 J_1 (\chi_e^{2\pm} - \chi_e^{\pm}) \phi^{\pm} e^{\pm i \beta} + J_1^2 \chi_e^{\pm} \phi e^{2i \beta}}{D^{2\pm}} \tag{65}
\]
where
\[
D^{2\pm} \equiv 1 + J_0^2 \chi_e^{2\pm} + J_1^2 (\chi_e^{3\pm} + \chi_e^{\pm}) + \chi_i^{2\pm} \tag{66}
\]
and the argument for the Bessel function (\( \mu \)) has been suppressed. Substituting equations for \( \rho^{\pm} \) and \( \rho^{2\pm} \) into equation for \( \phi^{\pm} \) and utilizing Eq. (65) gives \( \phi^{\pm} \) in terms of \( \rho_e \) and \( \phi \):
\[
\phi^{\pm} = \pm \frac{4\pi D^{2\pm}}{D^{\pm}} \rho_e e^{\pm i \beta} \pm J_0 J_1 \chi_e^{\pm} \frac{D^{2\pm} + J_1^2 (\chi_e^{2\pm} - \chi_e^{\pm}) \phi e^{2i \beta}}{D^{2\pm}} \tag{67}
\]
where
\[
D^{\pm} = D^{2\pm}(1 + J_0^2 \chi_e^{2\pm} + J_1^2 \chi_e^{2\pm} + \chi_i^{\pm}) - J_0^2 J_1^2 (\chi_e^{2\pm} - \chi_e^{\pm})^2. \tag{68}
\]
Finally, substituting Eq. (67) into equation for \( \rho_e \) and solving for \( \rho_e \) gives
\[
\rho_e = -\frac{k^2}{4\pi \chi e \phi} \frac{J_0 \left[ 1 + J_1^2 \left( \chi_e^{2\pm} \frac{D^{2\pm}}{D^{\pm}} + \chi_e^{\pm} \frac{D^{2\pm}}{D^{\pm}} \right) \right]}{\left[ 1 + J_1^2 \left( \chi_e^{2\pm} \frac{D^{2\pm}}{D^{\pm}} + \chi_e^{\pm} \frac{D^{2\pm}}{D^{\pm}} \right) \right]}.
\]

118
Substituting Eqs. (68) and (66), expanding the Bessel functions \( J_0 \simeq 1 - \mu^2/4 \), \( J_1 \simeq \mu/2 \), and keeping terms to order \( \mu^2 \) yields

\[
\rho_e = -\frac{k^2}{4\pi \chi_e \phi} \frac{F_1}{F_2}
\]  

(69)

where

\[
F_1 = 1 + \frac{\mu^2}{4} \left[ \frac{\chi_e^2 - \chi_e^-}{\epsilon^{-}} + \frac{\chi_e^3 - 2\chi_e^2 - + \chi_e^-}{\epsilon^{-}} - \frac{\left(\chi_e^2 - \chi_e^-\right)^2}{\epsilon^{-}\epsilon^{-2}} \right.
\]

\[
+ \frac{\chi_e^2 + - \chi_e^+}{\epsilon^{+}} + \frac{\chi_e^3 - 2\chi_e^2 + + \chi_e^+}{\epsilon^{+}} - \frac{\left(\chi_e^2 + - \chi_e^+\right)^2}{\epsilon^{+}\epsilon^{+2}} - \frac{1}{1}
\]

and

\[
F_2 = 1 + \frac{\mu^2}{4} \left[ \frac{\chi_e^2 - 2\chi_e^- + \chi_e}{\epsilon^{-}} + \frac{\chi_e^3 - 2\chi_e^2 - + \chi_e^-}{\epsilon^{-}} - \frac{\left(\chi_e^2 - \chi_e^-\right)^2}{\epsilon^{-}\epsilon^{-2}} \right.
\]

\[
+ \frac{\chi_e^2 + - 2\chi_e^+ + \chi_e}{\epsilon^{+}} + \frac{\chi_e^3 - 2\chi_e^2 + + \chi_e^+}{\epsilon^{+}} - \frac{\left(\chi_e^2 + - \chi_e^+\right)^2}{\epsilon^{+}\epsilon^{+2}} \right]
\]

where \( \epsilon^{\pm} = 1 + \chi_e^{n\pm}^{e\pm} \). Typically, \( |\epsilon^{-}| \ll 1, \, |\chi_e| \gg 1 \), and other terms are of order 1 and Eq. (69) can be reduced to

\[
\rho_e \simeq -\frac{k^2}{4\pi \chi_e \phi} \frac{1 + \frac{\mu^2}{4} \left(\chi_e^2 - \chi_e^-\right)\left(1 + \chi_e^- + \chi_e^2\right)}{\frac{\epsilon^{-}\epsilon^{-2}}{1 + \frac{\mu^2}{4} \frac{\chi_e}{\epsilon^{-}}}}.
\]

(70)

For powers well above threshold, the second term in the numerator can be neglected, and in the denominator the second term dominates so that

\[
\rho_e \simeq -\frac{k^2 \epsilon^{-}}{\pi \mu^2 \phi}
\]

or

\[
\frac{n_e(\omega)}{n_e} \simeq \frac{4}{\mu^2} \epsilon^{-} k^2 \lambda_D e^\phi(\omega) \frac{T_e}{T_e}.
\]

(71)
It is important to note that this equation predicts much smaller fluctuation level in electron density for a given fluctuation level in potential compared to the case with no parametric coupling (i.e., \( \mu = 0 \)).

Similarly, under the same assumptions \( \bar{n}_e \) at the lower sideband can be obtained as

\[
\rho_e^- \simeq -\frac{k^2}{4\pi} \chi_e^- \phi^- \frac{1 + \frac{\mu^2}{4} \frac{\chi_e (1 + \chi_i)}{\epsilon^+ \epsilon}}{1 + \frac{\mu^2}{4} \frac{\chi_e (1 + \chi_i)}{\epsilon^+ \epsilon} + O(1)}
\]

or

\[
\frac{\bar{n}_e^-}{n_e} \simeq k^2 \chi_e^- D_e \frac{e\phi^-}{T_e}
\]

which is the same as that without the pump electric field.

In summary, the ion density fluctuation is related to the potential fluctuation by Eq. (63) which is the same relationship with and without application of rf power. However, if the value of \( |\chi_i| \) is reduced in the presence of rf power, the enhancement in \( \bar{n}_i \) due to rf injection may be smaller than the enhancement in \( \beta \). The enhancement in electron density fluctuation \( \bar{n}_e \) at the low frequency is usually small compared to the enhancement in \( \phi \) for quasi-mode decay since \( 4|\epsilon^-|/\mu^2 \ll |\chi_e| \). In contrast, the enhancement in \( \bar{n}_e \) at the lower sideband should be comparable to the enhancement in \( \phi \) since \( \chi_e^- \) is not greatly affected in the presence of rf power. A comparison of these results with experimental measurements will be given in Sec. V-3.
IV. RF PROBE MEASUREMENTS

IV-1. PROBE CONSTRUCTION

Several coaxial electrostatic probes were used to study rf frequency spectra. The plasma density and the electron temperature in the shadow of the main limiter were also measured using the same probes. A schematic of the probe used in Alcator C is shown in Fig. IV-1. The probe was designed so that the perturbation to the plasma due to its presence is minimized. A 0.038cm (0.015 inch) diameter Molybdenum wire was used as the center conductor of the coaxial probe tip. The probe tip is covered by a floating stainless steel sheath so that the outer conductor of the coaxial probe tip, which is grounded to the machine, cannot collect current from the plasma. This sheath also ensures that no ceramic insulators are directly in contact with the plasma. All the metal parts exposed to the vacuum were made of Type 304 stainless steel (except for the center conductor which was made of Molybdenum) and Alumina was used for the insulating parts. The vacuum seal is made at the ceramic feedthrough and the signal is transmitted through a semi-rigid coaxial cable.

The air gaps between the stainless parts and the ceramic parts are calculated so that each section of the probe is 50Ω for optimum rf transmission. The probe response was measured with the setup shown in Fig. IV-2. The signal was applied from the tip of the probe (as in the actual experimental situation). The center pin of an SMA female connector was used to make electrical contact with the probe tip while the shield of the signal cable, which protrudes from the center pin, was pushed against the shield of the probe using an x-y-z stage. The result of this measurement is shown in Fig. IV-3. Here, the power transmission coefficient $T$ relative to the value at 1GHz is plotted as a function of frequency. The transmission coefficient
near \( f = 4.6\text{GHz} \) is degraded from that at 1GHz by less than 30\%. In the frequency range of interest (i.e., \( 3 \leq f \leq 5\text{GHz} \)), the probe response is flat to within ±5\% which is certainly adequate for measuring rf frequency spectra.

A double probe was used at the F-top-center port whereas single probes were used on D-top-center, F-top-outside, C-top-inside and C-top-outside ports. Figure IV-4 shows a schematic of the double probe. The double probe consists of two identical and independent rf probes that are separated in the poloidal direction by approximately 1cm. Both of these probes can be moved simultaneously by means of the larger bellows and one probe (referred to as the F2 probe) can be moved with respect to the other one (referred to as the F1 probe) by means of the smaller bellows assembly which is identical to the probe drives used for single probes. The poloidal locations of the two probes are fixed by a stainless steel block that can slide in the keyhole (access way from the port to the plasma chamber). The F2 probe is electrically isolated from the block to avoid a ground loop. This double probe permits simultaneous measurement of the rf frequency spectrum, ion density and low frequency density fluctuation at the same location.

The locations of rf probes were shown in Figs. I-1 and I-2. The probes located at the top-center ports enter the plasma at a poloidal angle of 90° (i.e., directly above the center line of the plasma). The F-top-outside probe is designed so that the probe is located right in front of the upper tip of the MW2 waveguide array. The same probe was also used at the C-top-outside port. In addition, another probe (named “marfe” probe since it was designed to study density and temperature profiles in the “marfe” region\(^{83}\)) located at the C-top-inside port was also used. The local ion cyclotron frequency at the C-top-inside probe is a factor of 1.5 times higher than that at the C-top-outside probe. Information concerning the toroidal and poloidal distribution of the waves can be obtained by using rf probes at different locations.
During the initial phase of the Alcator C lower hybrid experiments, a 4-waveguide array located at the C port was used. This waveguide array had two movable (in-out, up-down and side-to-side) rf probes. Using these probes, the rf frequency spectrum in front of the waveguide mouth was also measured. The top-outside probes on the C and F ports were also located very close to the top of the waveguide mouth.
FIGURE IV-1 — A schematic of the rf probe used on Alcator C. Shaded regions are insulators.
FIGURE IV-2 — A setup for measuring probe response.
FIGURE IV-3 — Measured power transmission coefficient relative to the value at $f = 1\text{GHz}$ as a function of frequency.
FIGURE IV-4 — A schematic of the double probe drive. The single probe drive moves one probe (the one on the left) relative to the other probe.
IV-2. STUDY OF THE EDGE PLASMA

IV-2-A. DENSITY AND TEMPERATURE PROFILES

In order to understand the edge plasma, the rf probes were used as Langmuir probes to obtain the ion density (equal to the electron density for a $Z = 1$ plasma) and the electron temperature profiles in the shadow of the main limiter. A schematic of the measuring circuitry is shown in Fig. IV-5. The biasing power supply was allowed to float and the probe current was measured with a Tektronix current probe. The voltage on the probe was swept from $-35V$ to $+5V$ every 20msec and sampled the regions of the ion saturation and the tail of the transition region. A typical probe characteristics is shown in Fig. IV-6. The electron temperature is deduced from the least squares fitted exponential of the form $I(V) = I_{ss} + I_{e0} \exp(V/T_e)$ (where $V$ is measured in V and $T_e$ is measured in eV). To get the ion density, the 1-dimensional plane probe theory was used since the Larmor radii and the Debye lengths are small compared to the smallest dimension of the probe (0.038cm). For $n_e \approx 1 \times 10^{13} cm^{-3}$, $T_e \approx T_i \approx 5eV$, hydrogen plasma, and $B_T = 8T$, $\lambda_{De} \approx 0.0005cm$ and $\rho_i \approx 0.003cm$, and therefore, the correction due to finite values of $\lambda_{De}$ and $\rho_i$ is of the order of 10%. It was assumed that all the ions that enter the sheath strike the probe and that the ions are Maxwellian outside the sheath so that the probe is collecting the random ion current when the probe is biased in the ion saturation regime. The ion density is deduced from the ion saturation current using the following relationship:

$$I_{ss} = \frac{1}{2} n_i \frac{v_{ti}}{\sqrt{\pi}} e A_p$$

where the ion velocity was averaged over the Maxwellian distribution to get this coefficient. $A_p$ is the total cross-sectional probe area (front side plus back side) and
\[ v_{ti} \equiv \sqrt{2T_i/m_i} \] is obtained from the measured electron temperature by assuming \( T_i = T_e \).

Typical ion density and electron temperature profiles in the shadow of the main limiter are shown in Figs. IV-7 and 8. The ion density is quite high, of the order of \( 10^{13}\text{cm}^{-3} \), even at the virtual limiter radius and has a fairly steep gradient with \( 1/e \) length of approximately 3mm. The waveguide arrays are usually located near the virtual limiter radius \( r = 18\text{cm} \) where optimum coupling can be obtained. The theoretically predicted value of optimum coupling density \( n_{||} \) for \( 180^\circ \) phasing is \( n_e \approx 2.5 \times 10^{12}\text{cm}^{-3} \), which is in fairly good agreement (within factors of 2) with the measured value. It can be seen that the electron temperature at the virtual limiter radius is slightly below 5eV and increases slowly towards the main limiter. It is important to note that there exists a region of high density (\( \approx 10^{14}\text{cm}^{-3} \)) and low temperature (\( \approx 10\text{eV} \)) in the shadow of the main limiter where collisional damping of lower hybrid waves may be significant. For the above parameters \( \nu_{ei}/\omega_0 \approx 0.005 \) where \( \nu_{ei} \) is the electron-ion collision frequency and \( \omega_0/2\pi = 4.6\text{GHz} \) is the pump wave frequency. The neutral (atomic and molecular) hydrogen (or deuterium) density in the plasma edge region is also estimated to be large (of the order of \( 10^{13}\text{cm}^{-3} \) at non-limiter, non-gas ports, and as much as \( 10^{15}\text{cm}^{-3} \) at the limiter ports) based on H\( _\alpha \) measurements, accounting for particle inventory.\(^{86}\) Thus, electron-neutral collisional damping may also be significant. For a neutral density of \( 10^{14}\text{cm}^{-3} \) and an electron temperature of \( 10\text{eV} \), \( \nu_{e0}/\omega_0 \approx 0.002 \) where \( \nu_{e0} \) is the electron-neutral collision frequency.
FIGURE IV-5 — The biasing circuit used during Langmuir probe measurements.
FIGURE IV-6 — A typical Langmuir probe characteristics.
FIGURE IV-7 — Typical ion density profile in the shadow of the main limiter.
FIGURE IV-8 — Typical electron temperature profile in the shadow of the main limiter.
IV-2-B. DENSITY FLUCTUATIONS

A large level of fluctuation is always observed on the ion saturation current. In Fig. IV-9, the ion saturation current during a typical discharge is shown. This fluctuation can be interpreted as a fluctuation in ion density. It can be seen that $|\delta n/n|$ is of order 0.5 where $|\delta n|$ is the amplitude of the fluctuating component of the density.

By using a spectrum analyzer the frequency spectrum of the fluctuations in the ion saturation current can be obtained. The ion saturation current was monitored simultaneously and the spectra were taken when the slowly varying component of the ion saturation current was in a steady state. The spectra are usually well fitted with an exponential function with the $1/e$ (e-folding) width of the order of 100kHz. Figs. IV-10(a) and (b) show the variation of this e-folding width with density for different ion species. In Fig. IV-10 results for two different magnetic fields are plotted. It can be seen that the e-folding width is fairly independent of density over a wide range and has a value of approximately 150kHz, and that it does not seem to vary strongly either with the ion mass or the magnetic field.

Figure IV-11 show the frequency spectra of the ion saturation current before and during the rf pulse in hydrogen plasma at $\bar{n}_e = 2 \times 10^{14} \text{cm}^{-3}$ (corresponding to $\omega_0/\omega_{LH0} = 1.6$ where $\omega_{LH0}$ is the value of $\omega_{LH}$ at the plasma center). There is no appreciable change either in the shape of the spectrum or in amplitude. Strong ion-sound quasi-mode parametric decay is observed on the potential fluctuations under these conditions as will be discussed in the next section.
FIGURE IV-9 — A typical time evolution of the ion saturation current. The probe was biased at $V = -25V$. A large level of density fluctuation is observed.
FIGURE IV-10(a) — The $1/e$ width of the density fluctuation as a function of density in hydrogen, $B = 8T$. 
FIGURE IV-10(b) — The 1/e width of the density fluctuation as a function of density in deuterium, $B = 8T$ (circles) and $B = 10T$ (triangles).
FIGURE IV-11 — The frequency spectra of the density fluctuation before (top) and during (bottom) the rf pulse. Hydrogen plasma, $B = 8T$, $\bar{n}_e = 2.0 \times 10^{14} \text{cm}^{-3}$ ($\omega_0/\omega_{LH0} = 1.6$).
IV-3. RF SPECTRA

IV-3-A. ION-SOUND QUASI-MODES

Rf frequency spectra above 10MHz were taken with the setup shown in Fig. IV-12. The frequency ranges measured this way include the 4.6GHz pump and its broadening, the parametrically excited ion-cyclotron harmonic sidebands near the pump frequency, and the low frequency ion-cyclotron harmonics near 0Hz. With this setup, $\omega_{pe} \cos \theta$ modes excited during the current drive experiments could also be monitored. These modes provide important information on the mechanism of current drive, but they will not be discussed in the present thesis. A 1MHz high-pass filter was used in place of the DC block to look at frequencies below 10MHz. The low frequency fluctuations observed this way are mainly potential fluctuations, whereas the low frequency fluctuations observed with the setup of Sec. IV-2-B (i.e., with the probe biased to collect ion saturation current) are mainly ion density fluctuations.

Figures IV-13(a)-(c) show the rf spectra near the pump frequency of $f = 4.6$GHz observed on probes at different toroidal locations in deuterium plasma at $n_e = 3.4 \times 10^{14}$cm$^{-3}$ which corresponds to $\omega_0/\omega_{LH0} = 1.9$. The central electron density is deduced from the measured line-averaged electron density assuming $n_{e0}/\bar{n}_e = 1.3$. A four waveguide array located at the C-side port was used to inject rf power. The probe at the waveguide (C-side) port can directly pick up the waves launched by the waveguide array and the spectrum is naturally dominated by the narrow-band rf pump wave. However, there is a broad-band feature (typically a few MHz wide) which is about 40dB lower in amplitude. The spectrum obtained at the D-top port, which is 60° away from the waveguide array toroidally, looks quite similar. This probe samples both the waves that stay on the plasma surface (inside the shadow of the main limiter) and the waves that first propagate into the
plasma and then subsequently propagate back out to the probe; however, it does not detect the waves that propagate toward the plasma center and damp there. The narrow-band component at $f = 4.6\text{GHz}$, which represent the low $n_{\parallel}$ inaccessible wave component for this case, is reduced from that at the waveguide mouth.

The F-top-center probe is different in that it can only sample the waves that have once propagated in past the main limiter radius and propagated out to the probe location (see Fig. I–1). The waves that stay in the shadow of the main limiter cannot reach this probe since they will be reflected off the main limiters at the B and E ports. The narrow-band "unscattered" pump wave is absent and only the broad-band feature is observed. Note that the waves observable by this probe, i.e., the waves that have once propagated past the main limiter radius, must go through a region where a large level of fluctuations is present, and should be broadened.

Near the limiter radius, a large level ($|\delta n/n| \approx 0.5$) of density fluctuations with the characteristic frequencies in the range $f \leq 200\text{kHz}$ is observed by both the CO$_2$ scattering and the probes (Sec. IV–2–B). In addition, a large level of low frequency potential fluctuations is also observed with the rf (floating) probes. The frequency spectrum of the potential fluctuations in the absence of rf power is almost identical to that of the density fluctuations. This is to be expected since without rf power the density fluctuations should be proportional to potential fluctuations as seen in Sec. III–6:

$$\frac{\bar{n}_e}{n_e} = k^2 \lambda_{De}^2 X_e \frac{e \phi}{T_e}$$

$$\frac{\bar{n}_i}{n_i} = -k^2 \lambda_{Di}^2 \chi_i \frac{e \phi}{T_i}.$$  

However, with the application of rf power the density fluctuations do not change appreciably, while the potential fluctuations in the frequency range $f \geq 1\text{MHz}$ are greatly enhanced in the density range $\bar{n}_e \gtrsim 1 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{LH0} \lesssim 2$
in hydrogen and $\omega_0/\omega_{LH0} \lesssim 3$ in deuterium). The frequency spectra of the low frequency potential fluctuations before and during the rf pulse in deuterium at $\bar{\pi}_e = 1.4 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{LH0} = 2.6$) are shown in Fig. IV-14. More than 20dB enhancement in fluctuation level is observed with the rf power of 140kW. In addition, the broadened pump frequency spectrum is very closely a mirror image of this low frequency spectrum and is slightly downshifted indicating the presence of parametric decay process (ion-sound quasi-mode). An example of such a downshifted frequency spectrum is shown in Fig. IV-15. This spectrum was taken in deuterium plasma, $B = 8\text{T}$, and $\bar{\pi}_e = 2.3 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{LH0} = 2.2$). The sharp peak at the pump frequency is due to the surface components and serves as the pump frequency "marker". The asymmetry in the spectrum is evident. The observed enhancement of the low frequency potential fluctuations and the downshift of the pump frequency spectrum cannot be explained easily by scattering of the pump wave from density fluctuations. Absence of appreciable enhancement in density fluctuations is also consistent with the theory presented in Sec. III-6. This will be further discussed in Sec. V-4.

The density dependences of the frequency width of the broadened frequency spectrum near the pump frequency, observed on the F-top probe for deuterium and hydrogen plasmas and for different magnetic fields, are shown in Figs. IV-16(a) and (b). Only the MW1 waveguide array was used to inject rf power so that surface waves are not detected by the probe. Here, full width at half maximum (FWHM) is plotted against $\bar{\pi}_e$. The frequency width increases as the density is increased and at higher densities it is higher for hydrogen by about a factor of 2. The theory described in Sec. III-2 would predict that for a given value of $\omega_{LH}/\omega_0$, $\omega_R \approx k_{||}v_i$ is inversely proportional to the square root of the ion mass. The dependence of the frequency width on the magnetic field is also found to be slight, if any, in agreement with
theory. The increase in the frequency width with density is not predicted by theory if $T_i$ and $k_{||}$ (of the quasi-mode) stayed fixed as the density is increased. However, it is likely that as the density is increased the $E \times B$ driven decay becomes stronger at regions where the local temperature is higher (the $E_{||}$ driven decay, on the other hand, becomes slightly weaker as the density is increased), and higher frequency quasi-modes can be excited (since $\omega_R \approx k_{||}v_i$). If the frequency broadening of the pump wave is entirely due to scattering of the incoming pump wave from pre-existing low frequency density fluctuations the broadening of the pump wave should increase with density (see Sec. II-4), but it would be difficult to explain the difference between hydrogen and deuterium cases. At lower densities $n_e \lesssim 0.5 \times 10^{14} \text{cm}^{-3}$ ($\omega_0/\omega_{LH0} \gtrsim 3$ in hydrogen and $\omega_0/\omega_{LH0} \gtrsim 4$ in deuterium) the frequency width approaches that of the previously existing density fluctuations ($\lesssim 200 \text{kHz}$). It should be noted that the low frequency ($\approx$ a few MHz) potential fluctuations are not enhanced with power at these densities, and the broadening is attributed to scattering from density fluctuations.

The low frequency spectra before and during rf injection in deuterium at $n_e = 0.4 \times 10^{14} \text{cm}^{-3}$ ($\omega_0/\omega_{LH0} = 4.4$) are shown in Fig. IV-17. The two spectra are virtually indistinguishable even with the rf power of 420kW. This should be compared with the higher density results shown in Fig. IV-14. The high frequency spectrum for the same parameters as Fig. IV-17 is shown in Fig. IV-18. The frequency spectrum is very narrow as compared to Fig. IV-15 and symmetric about the pump frequency, which is expected for passive scattering from density fluctuations. The sharp peak at the pump frequency is real in this case since the probe was located on the other side of the limiters from the waveguide port. These observations suggest that parametric excitation of ion-sound quasi-modes plays a role in the broadening of the pump wave spectrum at higher densities only.
In Figs. IV-19(a) and (b) frequency spectra of the broadened pump wave at two different powers are shown, and in Fig. IV-20 the frequency width (HWHM) is plotted as a function of rf power. These data were taken in hydrogen plasma, $B = 8\text{T}$, and $\bar{n}_e = 2.1 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{LH_0} = 1.6$). There is a slight increasing trend in frequency width as the rf power is increased. This increase in the frequency width is hardly visible when spectra were taken on a logarithmic scale. The width of the low frequency fluctuations $\Delta f_{1/2}$ for the same shots are plotted as a function of rf power in Fig. IV-21. The width $\Delta f_{1/2}$ is taken from the slope of the frequency spectrum above 2MHz. The solid circles and the open circles represent the frequency widths before and during rf injection, respectively. For $P_{rf} \lesssim 10\text{kHz}$ the low frequency potential fluctuations are not significantly enhanced during injection of rf power and the slope is the same before and during rf injection. The broadening of $\Delta f$ (HWHM)$\approx 5\text{MHz}$ shown in Fig. IV-20 for these power levels (below threshold for ion-sound quasi-mode decay) is attributed to scattering from density fluctuations. Above the threshold for parametric decay the low frequency $\Delta f_{1/2}$ during rf injection also increases with power, and at highest powers the low frequency half width (4-5MHz) is comparable to the half width at the high frequency. This is another evidence that ion-sound quasi-mode decay is playing a role at least at high power levels (and high density). The increase in the frequency widths with rf power is consistent with the numerical results of Sec. III-2 which showed the increase of $\omega_R$ with rf power. From the data shown in Figs. IV-20 and IV-21, the experimental threshold of $P_{th} \approx 10\text{kHz}$ is obtained. In Sec. III-4-B the threshold for ion-sound quasi-mode decay was estimated to be $P_{th} \approx 4\text{kHz}$ by assuming a uniformly filled pump wave shell. Considering the uncertainties in the theoretical estimates, the agreement between experiment and theory is considered to be satisfactory.

Since the waves are strongly scattered even in the absence of parametric decay
(as evidenced by the broadening of HWHM~ 2.5MHz in Fig. IV-20 at low rf powers), it is reasonable to assume that the resonance cone propagation is destroyed at higher densities ($\bar{n}_e \gtrsim 1 \times 10^{14} \text{cm}^{-3}$) when pump broadening is significant. It should also be remembered that significant parametric decay near the plasma surface cannot occur if well-defined resonance cones exist (see Sec. III-4-A). At higher rf powers, parametric decay becomes important and the waves tend to be scattered even more through the parametric process. As a consequence, the waves tend to spread over the whole plasma volume. This is consistent with the observation that the wide-band sidebands, which have undergone intensive scattering, look similar on all three probes. Furthermore, the amplitude should be much smaller than the narrow-band pump wave since only a small fraction of the total power can reach the probes.
FIGURE IV-12 — A setup for measuring rf frequency spectra. Frequencies above 10MHz can be studied with this setup. The DC block is replaced by a 1MHz high-pass filter to study the frequency range $1 \leq f \leq 10$ MHz.
FIGURE IV-13 — Rf frequency spectra on probes at different toroidal locations. The 4-waveguide array located at the C-side port was used. Spectra obtained with the probes located at (a) C-side port, (b) D-top port, and (c) F-top port are shown. The main limiters with inner radii of 16.5cm were located at B and E ports. Deuterium, $B = 8$T, $n_e = 3.4 \times 10^{14} \text{cm}^{-3}$ ($\omega_0/\omega_{LH0} = 1.9$), $P_{rf} = 40$kW.
FIGURE IV-14 — The low frequency \((1 \leq f \leq 10\text{MHz})\) spectrum before and during the rf pulse in deuterium, \(B = 8\text{T}, \bar{n}_e = 1.4 \times 10^{14}\text{cm}^{-3}\) \((\omega_0/\omega_{LH0} = 2.6)\).
FIGURE IV-15 — Downshifted frequency spectrum. Deuterium, \( B = 8 \text{T}, \)
\( \bar{n}_e = 2.3 \times 10^{14} \text{cm}^{-3} \) (\( \omega_0/\omega_{LH0} = 2.3 \)), \( P_{rf} = 680\text{kW} \).
FIGURE IV-16(a) — Frequency width (FWHM) of the broadband component near the pump frequency as a function of density in hydrogen, $B = 8T$. 

$\frac{\omega_0}{\omega_{LH0}} = 2$ and $\frac{\omega_0}{\omega_{LH0}} = 1.5$
FIGURE IV-16(b) — Frequency width (FWHM) of the broadband component near the pump frequency as a function of density in deuterium, $B = 8T$ (circles) and $B = 10T$ (triangles).
FIGURE IV-17 — The low frequency \((1 \leq f \leq 10\text{MHz})\) spectrum before and during the rf pulse in deuterium plasma, \(B = 10\text{T}, \bar{n}_e = 0.4 \times 10^{14}\text{cm}^{-3} (\omega_0/\omega_{LH0} = 4.4)\).
FIGURE IV-18 — The high frequency spectrum for the same parameters as Fig. IV-17.
FIGURE IV–19(a) — Frequency spectra (linear in power) at $P_{rf} = 400$ kW. Hydrogen plasma, $B = 8$ T, $\bar{n}_e = 2.1 \times 10^{14}$ cm$^{-3}$ ($\omega_0/\omega_{LH0} = 1.6$).
FIGURE IV-19(b) — Frequency spectra (linear in power) at $P_{rf} = 2.5$ kW. Same parameters as Fig. IV-19(a).
FIGURE IV–20 — Frequency width (HWHM) as a function of power for hydrogen plasma, $B = 8\,\text{T}$, $n_e = 2.1 \times 10^{14}\,\text{cm}^{-3}$ ($\omega_0/\omega_{LH0} = 1.6$).
FIGURE IV-21 — The width of the low frequency fluctuations $\Delta f_{1/2}$ for the same shots as Fig. IV-20 are plotted as a function of rf power. The width $\Delta f_{1/2}$ is taken from the slope of the frequency spectrum above 2MHz. The solid circles represent the frequency widths just before the rf power is applied and the open circles represent those during application of the rf power.
IV-3-B. ION-CYCLOTRON QUASI-MODES

Strong parametric decay into ion-cyclotron harmonic modes is observed for densities such that $\bar{n}_e \gtrsim 1.5 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{LH0} \lesssim 1.8$ for hydrogen and $\omega_0/\omega_{LH0} \lesssim 2.5$ for deuterium). The low frequency modes are probably ion-cyclotron quasi-modes rather than resonant ion-cyclotron waves since the numerical results presented in Sec. III-3 usually indicate that $\epsilon_R \gg 1$. At these densities the lower sidebands are much more intense than the upper sidebands, which is characteristic of parametric decay instability with weakly damped sidebands.

Figures IV-22(a)-(c) show examples of the high frequency spectra in hydrogen plasma at three different densities obtained with the F-top probe with lower hybrid waves launched at the C-port. The power in the ion-cyclotron sidebands relative to the pump power, as well as the number of observable ion-cyclotron harmonic peaks, generally increase as the density is increased. The frequency integrated pump power (including the ion-sound quasi-mode sidebands) and the frequency integrated ion-cyclotron sideband power (integrated over all other lower and upper sidebands) in deuterium plasmas are plotted as functions of density in Fig. IV-23. Ion-cyclotron sidebands are still observable down to densities $\bar{n}_e \lesssim 0.5 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{LH0} \gtrsim 3$ for hydrogen and $\omega_0/\omega_{LH0} \gtrsim 4$ for deuterium). However, the sidebands observed at these densities may not be due to parametric decay, since the low frequency ion-cyclotron modes are unstable even without application of rf power at these low densities as will be discussed shortly. The real parametric "density threshold" may be in the range $\bar{n}_e \simeq 1.5 \times 10^{14}\text{cm}^{-3}$. A second peak around $\Delta \omega/\omega_0 \simeq 0.15$ becomes observable for densities $\bar{n}_e \gtrsim 1.5 \times 10^{14}\text{cm}^{-3}$. This feature is somewhat more pronounced in hydrogen plasmas than in deuterium plasmas. The frequency spectrum observed at these densities consists of two components: the first few ($\leq 3$) harmonics which are generated in the low temperature edge region, and higher
harmonics centered around \( \Delta \omega/\omega_0 \approx 0.15 \) which are generated in the plasma interior where the temperature is higher (see the numerical results presented in Sec. III–3). At densities \( n_e \gtrsim 2 \times 10^{14} \text{cm}^{-3} \) the frequency integrated sideband power becomes comparable to the frequency integrated pump power (pump depletion). At these densities (which are far below the mode conversion density into ion plasma waves) production of an ion tail is observed by the charge exchange analyzer.\(^{87}\)

Similar behavior (i.e., the sharp rise of the frequency integrated sideband power for \( n_e \gtrsim 1.5 \times 10^{14} \text{cm}^{-3} \) and pump depletion for \( n_e \gtrsim 2 \times 10^{14} \text{cm}^{-3} \)) is observed in hydrogen plasmas, and the ion tail production is even more pronounced than in deuterium plasmas.

In general, the pump power (and hence, the total power since usually \( P_{\text{tot}} \approx P_{\text{pump}} \)) decreases drastically when the waves are absorbed by the plasma. The decrease of the pump power for \( n_e \lesssim 1 \times 10^{14} \text{cm}^{-3} \) is probably due to increased damping on the electrons (due to higher initial electron temperatures at these densities) whereas the decrease for \( n_e \gtrsim 2 \times 10^{14} \text{cm}^{-3} \) is presumably due to absorption via parametric processes and/or absorption by the ion tail that is observed. Note the correlation between decrease in the pump power and increase in the sideband power for \( n_e \gtrsim 1.5 \times 10^{14} \text{cm}^{-3} \). There is also a good correlation between this pump depletion and ion tail formation. Thus, the pump wave power (or the total power) observed on the probes (which must be insensitive to the surface waves) is a measure of wave absorption by the plasma.

In Fig. IV–24 power dependences of frequency integrated pump power and frequency integrated ion-cyclotron sideband power are shown for hydrogen, \( B = 8\text{T}, \ n_e \approx 2.6 \times 10^{14} \text{cm}^{-3} \ (\omega_0/\omega_{LH0} = 1.5) \). The frequency integrated pump power increases linearly with injected power up to at least \( P_{\text{rf}} \approx 400\text{kW} \) and no significant deviation from the linear power dependence is observed at these den-
sities. The threshold for this parametric decay is $P_f \approx 10$ kW and the power in the ion-cyclotron sidebands increases to $\approx 30\%$ of the pump power at the highest injected power of $P_{RF} \approx 400$ kW. This experimental threshold lies between the homogeneous threshold calculated in Sec. III-3 and the inhomogeneous threshold calculated in Sec. III-4-B. In contrast, Fig. IV-25 shows power dependences of frequency integrated pump power and frequency integrated ion-cyclotron sideband power for deuterium, $B = 8$ T, $\bar{n}_e \approx 0.7 \times 10^{14}$ cm$^{-3}$ ($\omega_0/\omega_{LH0} = 3.5$). The pump power does not increase linearly with the injected rf power, perhaps indicating formation of electron tail and increased absorption of the waves at higher powers inside the plasma (the probe samples what propagates out from the plasma interior; CO$_2$ scattering experiments$^{88}$ established that the pump wave near the plasma center is proportional to the injected power even at these low densities). Parametric instabilities do not seem to play an important role in this nonlinear power dependence.

The parametrically excited low frequency ion-cyclotron modes are observed whenever the high frequency ion-cyclotron sidebands are observed near the pump frequency. Typical low frequency ion-cyclotron harmonic spectrum is shown in Fig. IV-26(a) together with the high frequency ion-cyclotron harmonic sidebands. These spectra were taken in hydrogen plasma, $B = 8$ T, and $\bar{n}_e = 2.1 \times 10^{14}$ cm$^{-3}$ ($\omega_0/\omega_{LH0} = 1.6$). The low frequency ion-cyclotron modes and the high frequency ion-cyclotron lower sidebands are mirror images of each other. In addition, the enhanced spectrum in the frequency range $f \lesssim 50$ MHz is also a mirror image of the broadened pump frequency spectrum, confirming that these modes are also excited parametrically. The frequencies of these modes correspond to the ion-cyclotron frequency (and its harmonics) corresponding to the magnetic field at the outside (larger major radius side) edge of the plasma, suggesting that these modes are excited on the outside edge of the plasma. More direct evidence that they are
excited on the outside edge will be provided by CO\textsubscript{2} scattering data (Sec. V-5).

At densities $\bar{n}_e \lesssim 1.5 \times 10^{14}$ cm\textsuperscript{-3} the first few low frequency ion-cyclotron modes are observed even in the absence of rf power. These modes have narrower bandwidth than the parametrically excited ion-cyclotron modes, but the frequencies are roughly the same indicating they also are generated near the outside edge. These modes are amplified and broadened in frequency in the presence of rf power. The high frequency spectra in this low density regime are always dominated by the pump wave which has a narrower frequency width compared to the case discussed above. Typically, the power in the pump wave exceeds the power in the ion-cyclotron sidebands by about 40dB and the upper and the lower sidebands are roughly equal in amplitude and almost symmetric about the pump frequency. This suggests that these sidebands are produced by beating between the unstable low frequency mode and the pump wave. The low frequency and high frequency spectra in this density range (hydrogen, $B = 8$ T, $\bar{n}_e = 1.1 \times 10^{14}$ cm\textsuperscript{-3}, $\omega_0/\omega_{LH0} = 2.0$) are shown in Fig. IV-26(b).

At still lower densities ($\bar{n}_e \lesssim 0.5 \times 10^{14}$ cm\textsuperscript{-3}), the $\omega_{pe} \cos \theta$ modes become observable in the frequency range $\omega_{pi0} \lesssim \omega \lesssim \omega_{pe0}$ where $\omega_{pi0}$ and $\omega_{pe0}$ are the ion and electron plasma frequencies at the center of the plasma column. In Fig. IV-27 a typical frequency spectra before and during current drive are shown. Before the rf power is turned on, unstable ion-cyclotron modes are again observed, but this time they are separated by the ion-cyclotron frequency at the plasma center. There is also rf emission with a broad range of frequencies around $\omega_{pi0}$. When the rf power is on, the peak at $\omega_{pi0}$ is enhanced and a continuum above this frequency is also excited. These are probably due to Čerenkov resonance rather than anomalous Doppler resonance which tend to excite modes near $\omega_{pe0}$.\textsuperscript{99-92} A similar spectrum has been observed in slideaway discharges.\textsuperscript{93} A weak parametric decay is observable in the
frequency range of a few ion-cyclotron harmonics. The sensitivity of the spectrum analyzer had to be turned up to maximum level to observe these features. Thus, these processes are rather weak as compared with the parametric decay discussed before.
FIGURE IV-22 — The frequency spectra of the ion-cyclotron sidebands at three different densities in hydrogen plasma and $B = 8$T. (a) $1.0 \times 10^{14}$cm$^{-3}$ ($\omega_0/\omega_{LH0} = 2.1$), (b) $1.8 \times 10^{14}$cm$^{-3}$ ($\omega_0/\omega_{LH0} = 1.7$), and (c) $2.3 \times 10^{14}$cm$^{-3}$ ($\omega_0/\omega_{LH0} = 1.5$).
FIGURE IV-23 — The frequency integrated pump power (solid circles) and the frequency integrated ion-cyclotron sideband power (triangles) vs. density for deuterium plasma, $B = 8T$, $P_{rf} \approx 280kW$, $\Delta \phi = 180^\circ$. 
FIGURE IV-24 — Power dependences of frequency integrated pump power (solid circles) and frequency integrated ion-cyclotron sideband power (triangles) for hydrogen plasma, $B = 8T$, $\bar{n}_e = 2.6 \times 10^{14} \text{cm}^{-3}$ ($\omega_0/\omega_{LH0} = 1.5$), $\Delta \phi = 180^\circ$. 
FIGURE IV-25 — Power dependences of frequency integrated pump power (solid circles) and frequency integrated ion-cyclotron sideband power (triangles) for deuterium plasma, \( B = 8 \text{T}, \bar{n}_e = 0.7 \times 10^{14} \text{cm}^{-3} \) \((\omega_0/\omega_{LH0} = 3.5), \Delta \phi = +90^\circ \).
FIGURE IV-26 — Typical spectra of low frequency ion-cyclotron modes (left) and high frequency ion-cyclotron sidebands (right). Hydrogen plasma, $B = 8$ T, (a) $n_e = 2.1 \times 10^{14}$ cm$^{-3}$ ($\omega_0/\omega_{LH0} = 1.6$) and (b) $n_e = 1.1 \times 10^{14}$ cm$^{-3}$ ($\omega_0/\omega_{LH0} = 2.0$).
FIGURE IV-27 — A typical spectrum (a) before and (b) during current drive.
Hydrogen plasma, $B = 8\text{T}$, $\bar{n}_e \simeq 5 \times 10^{13}\text{cm}^{-3}$ ($\omega_0/\omega_{LH_0} = 2.8$).
The rf probe results can be summarized as follows: Density fluctuations of $|\delta n/n| \lesssim 0.5$ with the $1/e$ width $\Delta f_{1/e} \lesssim 200\text{kHz}$ is present over a wide range of plasma parameters. At low densities ($n_e \lesssim 1 \times 10^{14}\text{cm}^{-3}$), the pump wave width is comparable to the frequency width of the low frequency density fluctuations, symmetric about the pump frequency, and no enhancement of low frequency potential fluctuations is observed. These results indicate absence of parametric decay into ion-sound quasi-modes and relatively weak scattering of the pump wave by the low frequency density fluctuations. The pump wave resonance cones are probably not completely destroyed at these densities. Due to large convective losses parametric decay (either into ion-sound quasi-modes or ion-cyclotron quasi-modes) is not observed. However, since the low frequency ion-cyclotron modes are already unstable even without rf power, beating between these modes and the pump wave produces the symmetric sidebands, just as the beating of the low frequency ($\lesssim 200\text{kHz}$) density fluctuations and the pump wave produced the symmetric broadening of the pump wave.

As the density is increased ($n_e \gtrsim 1.5 \times 10^{14}\text{cm}^{-3}$), scattering from density fluctuations become more and more intense as the $E \times B$ coupling increases (see Sec. II–4) and the resonance cones get destroyed. Strong scattering is also evidenced by much larger broadening of the pump wave observed by rf probes located on the other side of the limiters. The sharp peak observed on the probes located on the same side of the main limiters as the waveguide port is due to “surface waves” with $n_\parallel$ close to 1. In general, these waves do not propagate past the limiter radius, but rather bounce back and forth between the two limiters where effect of scattering from density fluctuations is relatively weak. For higher densities
$n_e \gtrsim 1 \times 10^{14} \text{cm}^{-3}$, the waves that have passed through the region of turbulence beyond the limiter radius (where the density is higher, and thus scattering is more severe) undergo strong scattering from density fluctuations and from parametric decay and are broadened in frequency and are spread out in space. Therefore, the probes located on the other side of the main limiters (and the CO$_2$ scattering as will be presented in Sec. V-3) see broadened spectra regardless of the exact observation point. Since convective losses are reduced and at the same time the $E \times B$ coupling for parametric decay also increases, parametric decay into ion-cyclotron quasi-modes becomes possible. This mechanism (destruction of resonance cones due to scattering) predicts a density threshold which is insensitive to the ion mass in contrast to the case when the convective loss out of the pump resonance cone determines the threshold. Since the low frequency ion-cyclotron modes are most unstable on the outside edge of the plasma column (in the absence of rf power), and since the electric field is probably highest on the outside edge just in front of the waveguide array where the pump wave has not spread out significantly (this is confirmed by CO$_2$ scattering data presented in Sec. V-5), this region is most susceptible to parametric instability. Consequently, the sidebands are separated by multiples of the ion-cyclotron frequency corresponding to the magnetic field at the outside edge of the torus. Parametric decay into ion-sound quasi-modes also becomes possible. The pump is downshifted in frequency, has a much broader frequency width, and the low frequency potential fluctuations are greatly enhanced.

Since parametric growth rate is largest near the limiter radius $r/a \approx 1$ where $n_e \approx 1 \times 10^{14} \text{cm}^{-3}$ and $T_e \approx T_i \lesssim 50 \text{eV}$, initially only a few lowest harmonics ($n = 1$ in particular) become unstable on the outside edge (Sec. III-3). As the density is further increased, higher harmonics (peaked around $\Delta \omega/\omega_0 \approx 0.15$) are also excited further inside the plasma even though here the local temperature is
higher. The low frequency spectra, on the other hand, never show this second peak. This is to be expected since if these low frequency modes were quasi-modes, they must have been generated locally at the location of the probe where $T_e \approx T_i \lesssim 10$eV. As the density is further increased, the probe signal decreases because more power is absorbed near the plasma edge through excitation of parametric decay processes.
V. STUDY OF LOWER HYBRID WAVES USING CO\textsubscript{2} LASER SCATTERING

V-1. SCATTERING OF CO\textsubscript{2} LASER RADIATION FROM DENSITY FLUCTUATIONS IN PLASMAS

CO\textsubscript{2} scattering has recently been used extensively for measuring density fluctuations in plasmas. This technique has advantages over rf probes in that it is nonperturbative, density fluctuations in the plasma interior can be studied, and different wavenumbers can be resolved. Scattering of electromagnetic radiation from plasmas is reviewed by Sheffield\textsuperscript{94} and scattering using CO\textsubscript{2} laser is described in detail by Slusher and Surko.\textsuperscript{95} Scattering from lower hybrid waves have been carried out on Alcator A by Slusher and Surko\textsuperscript{93} and on ACT-1 by Wurden.\textsuperscript{96}

The choice of CO\textsubscript{2} laser ($\lambda \textsubscript{i} = 10.6\mu$m, the subscript \textit{i} denotes incident radiation) as the incident radiation source is made so that refraction, reflection, absorption and emission by the plasma do not obscure the measurement since $\omega \gg \omega_{pe}, \omega_{ce}$. The range of wavenumbers of the density fluctuation that can be studied is $k \lesssim \lambda_{De}^{-1}$ (or $\lambda \gtrsim 2\pi\lambda_{De}$) where the scattering is due to collective motions of the electrons.\textsuperscript{94}

In scattering of electromagnetic waves from density fluctuations the following conservation relationships are satisfied when Compton scattering can be neglected:\textsuperscript{94}

$$k_i + k = k_s \quad (75)$$

$$\omega_i + \omega = \omega_s \quad (76)$$

where the subscript \textit{s} denotes scattered radiation. This is shown schematically in Fig. V-1. Since $\omega \ll \omega_i$ (because $\omega_i \gg \omega_{pe}, \omega_{ce}$ was chosen), $\omega_s \approx \omega_i$ and $k_s \approx k_i$ (since $k \approx \omega/c$ for incident and scattered radiation). Furthermore, $k \ll k_i$ since for a plasma of $n_i = 1 \times 10^{14}\text{cm}^{-3}$ and $T_i = 1\text{keV}$, $2\pi\lambda_{De} = 0.15\text{mm}$ whereas $\lambda_i =$
0.0106mm for a CO₂ laser beam. To satisfy the wavenumber conservation Eq. (75), under the condition \( k \ll k_i \simeq k_s, \), \( k \) must be almost perpendicular to \( k_i \) and the scattering angle is quite small \((\phi_s \lesssim 6^\circ)\) even for the largest wavenumber allowed for collective scattering \((k \simeq \lambda D_e^{-1})\). This is an important advantage when angular access to the plasma is limited because this technique requires only a straight small diameter path through the plasma. The penalty one pays is limitation in the spatial resolution of the scattering volume.

Scattering of the incident radiation is mainly due to electrons since acceleration of ions due to the incident electric field is down by the mass ratio from that of electrons. A single axial, lowest order transverse mode (TEM₀₀) incident laser beam of waist radius \( w_0 \) is assumed to propagate along the \( y \)-direction (vertically in this experiment), linearly polarized in the \( x \)-direction (along the major radius), and focused at \( z = 0 \) (center of the scattering volume) so that

\[
E_i(\mathbf{r}, t) = \hat{\mathbf{e}} \cdot E_i \exp[-(z^2 + x^2)/w^2(y)] \cos[k_i y - \omega_i t + \psi(y)]
\]

where \( \psi(y) = \tan^{-1}(y/z_R) \), \( w^2(y) = w_0^2[1 + (y/z_R)^2] \) and \( z_R \equiv \pi w_0^2/\lambda_i \). The scattered electric field \( E_s \) at position \( \mathbf{R} \) from the electron density fluctuation \( n_e(\mathbf{r}, t) \) located around \( \mathbf{z} = 0 \) in a scattering volume \( V \) is given by\(^95 \)

\[
E_s(\mathbf{R}, t) = \left[ \hat{\mathbf{R}} \times (\hat{\mathbf{R}} \times \hat{\mathbf{z}}) \right] \frac{w_0^2 r_0}{32\pi^3 R^3} E_i \int \frac{d\omega d\mathbf{k} d\mathbf{y}}{4} \exp \left[ - \frac{(k_z - k_{sz})^2 w_0^2}{4} \right] \exp \left[ - \frac{(k_z - k_{sz})^2 w_0^2}{4} \right] \exp[i(k_y + k_i - k_{sy})y] \exp \left[ -i(\omega + \omega_i)(t - R/c) \right] n_e(k, \omega) + \text{c. c.}
\]

172
Since $\omega_i \gg \omega_{ce}, \omega_{pe}$ the Born approximation was used in deriving this result and relativistic effects were ignored ($v_e/c \ll 1$). It is also assumed that $R \gg z_R$ and that $l_y \leq z_R$ where $l_y$ is the extent of $V$ in the $y$-direction. It is evident from the exponential terms in this expression that scattering is appreciable only when the frequency and the wavenumber selection rules are approximately satisfied.

The electron density perturbation in the presence of a lower hybrid wave can be derived in the following manner. The electron density perturbation is assumed to be of the form $n_e = n_{e0} + \tilde{n}_e$ where $\tilde{n}_e = \delta n_e \exp(ik \cdot z - i\omega t)$. Using the continuity equation

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0$$

and the solution of the equation of motion

$$\mathbf{v}_e = \frac{i\omega}{4\pi n_{e0} c} \mathbf{K}_e \cdot \mathbf{E},$$

where $\mathbf{K}_e$ is the electron part of the cold plasma dielectric tensor, $\tilde{n}_e$ can be expressed in terms of the complex wave electric field $\mathbf{E}$:

$$\tilde{n}_e = \frac{i}{4\pi e} k \cdot \mathbf{K}_e \cdot \mathbf{E}$$

$$= \frac{i}{4\pi e} \left[ \frac{\omega_{pe}^2}{\omega_{ce}^2} k_x E_x - i \frac{\omega_{pe}^2}{\omega_{ce} \omega_{ce}} k_x E_y - \frac{\omega_{pe}^2}{\omega^2} k_z E_z \right]$$

(77)

where the $z$-direction was chosen along the static magnetic field and the $x$-direction in the direction of $\mathbf{n}_\perp \equiv ck_\perp/\omega$ so that $k_y = 0$. The first term is identified as the polarization drift term, the second term as the $\mathbf{E} \times \mathbf{B}$ drift term, and the last term as the parallel drift term. Using the cold plasma electromagnetic dispersion relation

$$\mathbf{n} \times (\mathbf{n} \times \mathbf{E}) + \mathbf{K} \cdot \mathbf{E} = 0$$

173
the components of electric field \( \vec{E}_y \) and \( \vec{E}_z \) can be expressed in terms of \( \vec{E}_x \equiv n_{\perp} \cdot \vec{E}/n_{\perp} \):

\[
\vec{E}_y = \frac{K_{xy}}{K_{xx} - n_z^2} \vec{E}_x,
\]

\[
\vec{E}_z = \frac{n_z n_x}{n_z^2 - K_{zz}} \vec{E}_x. \tag{78}
\]

Combining Eqs. (77) and (78) gives

\[
\vec{n}_e = \frac{i}{4\pi e} (k_{\perp} \cdot \vec{E}) \left[ \frac{\omega_{pe}^2}{\omega_{ce}^2} \left( \frac{1}{\omega_{ce}^2} \frac{n_{\parallel}^2}{n_{\perp}^2 - K_{zz}} \right) - \frac{\omega_{pe}^2}{\omega^2} \frac{n_{\parallel}^2}{n_{\perp}^2 - K_{zz}} \right] \tag{79}
\]

where \( \vec{E} \) is now the real electric field amplitude. In the electrostatic limit \( (n_{\parallel}^2 \gg 1) \) this relationship can be simplified as

\[
\vec{n}_e = \frac{i}{4\pi e} (k_{\perp} \cdot \vec{E}) \frac{\omega_{pe}^2}{\omega_{ce}^2} \left( 1 - \frac{\omega_{pe}^2}{\omega_{ce}^2} \frac{n_{\parallel}^2}{n_{\perp}^2} \right) \tag{80}
\]

where the cold lower hybrid wave dispersion relation was used in going from the first line to the second line. The plasma was assumed to be cold in this analysis. Near the lower hybrid mode conversion layer \( (\omega_0/\omega_{LH} \lesssim 2) \) finite temperature effects become important and must be included in the dielectric tensor. However, the Alcator C lower hybrid experiments are usually carried out at densities below the mode conversion density \( (\omega_0/\omega_{LH} \gtrsim 2) \) and therefore finite temperature effects can be ignored in calculating \( \vec{n}_e \). Furthermore, the CO\(_2\) scattering signal becomes undetectable for \( \vec{n}_e \gtrsim 2.5 \times 10^{14}\text{cm}^{-3} \), and therefore the finite temperature effects can be neglected for all practical purposes.

The \( n_{\parallel} \) power spectrum \( P(n_{\parallel}) \) can be deduced from scattered signals obtained during a \( k_{\perp} \) scan. A given value of \( k_{\perp} \) corresponds to a certain value of \( n_{\parallel} \) through
the lower hybrid dispersion relation. In the electrostatic limit \( P(n_{||}) \) is proportional to \( E^2_{\perp}/k_{\perp} \) integrated over \( k_{\perp} \), and the scattered power \( P_{sc} \) is proportional to \( k^2_{\perp} E^2_{\perp} \). Therefore, \( P(n_{||}) \) is proportional to \( P_{sc}/k^2_{\perp} \) if \( k_{\perp} \) of the lower hybrid wave is assumed to be uniformly oriented in the poloidal plane.\(^{33}\)
FIGURE V-1 — Conservation of wavevector.
V-2. EXPERIMENTAL SETUP AND DATA ACQUISITION

The experimental apparatus for the scattering experiments from lower hybrid waves is shown in Fig. V-2. This is the same experimental configuration originally set up by Drs. Slusher, Surko, and Watterson. A Coherent Radiation model 41 cw CO$_2$ laser ($\lambda = 10.6\mu$m) operating at power levels $\approx 100$W is used as the radiation source for the scattering experiments. By tuning the laser, it can be made to operate stably in the lowest order transverse mode (TEM$_{00}$ mode) which has a Gaussian transverse beam profile that was assumed in the theory presented in the previous sub-section. The stripping aperture SA placed in the focal plane of the output window (lens) OL of the laser eliminates the higher order non-Gaussian transverse structure caused by imperfections. The beam is collimated at the lens L1, and is split at the beam splitter BS into the main (or the incident) beam I (with about 98% of the power) and the local oscillator beam LO (with the remaining 2% of the power). The main beam and the LO beam are indicated by the solid line and the broken line, respectively. Both the main beam and the LO beam are focused inside the plasma at the focus of the lens L2 and are made parallel again with the lens L3. The intersection of the two beams defines the scattering volume $V$. The main beam is stopped at the beam dump BD whereas the LO beam, together with the scattered main beam $S$ propagating along the LO beam, is focused onto the detector by the lens L4 (this is shown by the dash-dot-dash line). The optimum LO power level on the detector was typically 50$mW$ and was adjusted with the attenuator AT. Different scattering angles $\phi_s$ (and hence, the wavenumber of the lower hybrid wave being observed) can be selected by changing the separation between the two beams by moving the mirror M1 vertically. The vertical location of the scattering volume $V$ can be adjusted by moving the lenses L2 and L3 vertically, keeping the relative distance between the two lenses constant. Similarly, the horizontal position can be
altered by moving the lens-mirror assemblies L2-M2 and L3-M3 horizontally.

The waist radius \( w_0 \), measured by inserting apertures of different diameters at the focal plane of the lens L2, was about 1mm. In the small angle limit \( \phi_s \ll 1 \) the spatial resolution in the major radius direction (and also in the toroidal direction) is determined by the waist radius: \( \Delta R \simeq w_0 \). The spatial resolution in the vertical direction, determined by the extent of the scattering volume in the vertical direction, is \( \Delta z \simeq 2w_0/\phi_s \simeq 15\text{cm} \) for \( k_\perp = 80 \text{cm}^{-1} \) and 7cm for \( k_\perp = 180 \text{cm}^{-1} \). The wavenumber resolution is \( \Delta k \simeq 2/w_0 = 20 \text{cm}^{-1} \).

The detector (Eaton Corporation model 971 infrared heterodyne receiver) was placed in a carefully rf shielded box to prevent any rf power leakage into the detection electronics. A block diagram of the detection circuitry inside the rf shielded box is shown in Fig. V-3. The LO beam and the scattered beam enters through a cut-off waveguide. A copper wire mesh was placed at the input end of the waveguide for further rf shielding. The scattered beam is optically mixed with the LO beam and is detected by a liquid He cooled Ge:Cu photoconductive detector counter-doped with Sb (heterodyne detection). The laser beams incident on the detector excite electrons into the impurity levels in the band gap, thus decreasing the resistance of the detector. The resistance of the photoconductor is proportional to

\[
\int_{V_s} dR |E_{LO}(R, t) + E_s(R, t) + E_i(R, t)|^2.
\]

Since \( \omega_{LO} = \omega_i \) and \( |E_{LO}| \gg |E_i|, |E_s| \), the resistance can be written as

\[
R_d^{-1} = 2B(E_{LO} \cdot E_s) + C|E_{LO}|^2
\]

where \( B \) and \( C \) are constants. The first term is the mixing signal and the second term is the broadband LO shot noise. The total power in the heterodyne signal is
given by \(95\)

\[
\langle P^2_n(k_s) \rangle \Omega_c = \frac{\pi^2 r_0^2 P_i P_{LO}}{16\pi^3 k_i^2 V T} \int d\omega dk |n(k, \omega)|^2 \delta(k_s) \exp \left[ -\frac{(k - k_s)^2 \omega_0^2}{4} \right]
\]

where \(P_i\) and \(P_{LO}\) are the incident and the LO powers, \(|n(k, \omega)|\) is obtained by averaging over the volume \(V\) (scattering volume) and the time \(T\) (integration time), and the heterodyne power is integrated over the coherent solid angle \(\Omega_c = \lambda_i^2 / \pi w_0^2\).

The noise equivalent power of the detector (the incident signal power which will produce a signal equal to the rms background noise power), is less than \(1 \times 10^{-18} W/Hz\) in the frequency range \(4 \lesssim f \lesssim 4.9\)GHz. The photo-mixed signal is amplified by an rf pre-amp, mixed with an rf local oscillator signal, amplified by an IF amplifier, and is sent out of the rf shielded box. The IF signal was sent out through a 3GHz low-pass filter. The IF signal was divided by a power splitter and analyzed using a spectrum analyzer and a Lassen Research filter bank (using 15 1MHz adjacent bands centered around the pump frequency). The spectrum analyzer has the advantage that different frequency bands can be selected easily and has a much better frequency resolution and wider dynamic range than the filter bank. However, the frequency is swept in time to get a frequency spectrum, and therefore, time resolution is limited by the sweep duration (typically 10–50msec). On the other hand, the filter bank can follow the time evolution of each frequency channel continuously and has the time response determined by the integration time (typically 1msec). The channel-to-channel sensitivity (and bandwidth) calibration is also more troublesome than calibrating a spectrum analyzer. However, for scattering from lower hybrid waves, the use of spectrum analyzer turned out to be difficult since the signal was small compared to the amplifier noise and the steady state level of the local oscillator shot noise. The signal had to be extracted by subtracting the time averaged noise from the time averaged signal using a filter bank.
For the scattering experiments from low frequency density fluctuations a different setup is used. This is shown schematically in Fig. V-4. The LO beam did not pass through the plasma for this case. The main beam was not focused inside the plasma. The scattering volume for this case is defined by the intersection of the main beam path and the field of view of the detector, which is typically across the whole plasma vertically. The scattering angle \( \phi_s \) can be selected by tilting the mirror M2, and the horizontal location of the scattering volume can be selected by moving the mirrors M1 and M2 horizontally. With this setup scattering angles down to \( \phi_s = 0 \) (corresponding to \( k_\perp \approx 0 \)) can be studied. A similar, but different detector was used to detect the scattered signal from the low frequency fluctuations, which generally have the frequencies \(< 1\text{MHz}\). The detected signal was analyzed using a spectrum analyzer since the scattered signal was much larger than the amplifier noise and the LO shot noise for the low frequency case.
FIGURE V-2 — The experimental setup for CO₂ laser scattering experiments from lower hybrid waves.
FIGURE V-3 — A block diagram of the heterodyne detection system.
FIGURE V-4 — The experimental setup for CO₂ laser scattering from low frequency fluctuations.
V-3. PUMP BROADENING

The broadening of the lower hybrid wave injected at the C-port (MW1) and the F-port (MW2) was measured with CO$_2$ laser scattering located at the E-port (see Fig. I-1). At densities $\bar{n}_e \gtrsim 1 \times 10^{14}$ cm$^{-3}$ the frequency spectrum observed is broad with the frequency width $\Delta f$ (HWHM) $\gtrsim 1$ MHz. At densities $\bar{n}_e \lesssim 1 \times 10^{14}$ cm$^{-3}$ the pump frequency width is relatively narrow. These results are very similar to the results observed on the rf probes located on the other side of the limiters from the waveguide array (see Sec. IV-3-A). A typical frequency spectrum obtained near the plasma center (5 cm outside on the midplane) using a 16-channel filter bank (1 broadband channel and 15 narrowband channels with 1 MHz passband) in hydrogen plasma at $\bar{n}_e = 1.5 \times 10^{14}$ cm$^{-3}$ is shown in Fig. V-5. A downshift of about 1 MHz and a width of about 1.5 MHz HWHM (consistent with the rf probe data) can be seen. At lower densities ($\bar{n}_e \lesssim 1 \times 10^{14}$ cm$^{-3}$) the broadening becomes narrower, and narrower bandwidth filters must be used to resolve the broadening. A spectrum obtained using 300 kHz filters in hydrogen plasma at $\bar{n}_e = 1 \times 10^{14}$ cm$^{-3}$ is shown in Fig. V-6. For comparison an rf probe spectrum on the same shot is also shown. No appreciable downshift can be seen at these low densities. Generally, the CO$_2$ scattering spectrum is very similar to the rf probe spectrum in spite of the fact that they are sensitive to different components of the wavenumber spectrum. This further strengthens the belief that rf probe signals indeed carry information about the waves that have propagated out from the plasma interior.

The rms frequency width of the broadened pump wave (which corresponds to the half width) is plotted as a function of density in Fig. V-7. The frequency widths (HWHM) obtained from rf probes are also plotted on the same graph for comparison. Again, the agreement between the two techniques is surprisingly good. The rms frequency width is defined as
\[
\Delta f_{\text{rms}}^2 = \frac{\sum_{i=1}^{15} (f_i - f_M)^2 P(f_i)}{\sum_{i=1}^{15} P(f_i)}
\]

where

\[
f_M = \frac{\sum_{i=1}^{15} f_i P(f_i)}{\sum_{i=1}^{15} P(f_i)}
\]

is the mean frequency, \(f_i\) is the center frequency of the \(i^{th}\) filter, and \(P(f_i)\) is the power detected on the \(i^{th}\) channel.

In addition, the waves appeared not to be localized inside resonance cones at higher densities \((\bar{n}_e \geq 1 \times 10^{14} \text{cm}^{-3})\). The waves seem to be spread out more or less uniformly over the whole poloidal cross section of the plasma. Furthermore, the scattered signal is usually proportional to the rf power.\(^{97,88}\) The scattered power varies linearly with total rf power during a shot-to-shot power scan at fixed plasma parameters. When the rf power is constant in time, the scattered signal is also constant in time. When the rf power is ramping up, the scattered power also follows the time evolution of the rf power. However, at lower densities \((\bar{n}_e \leq 0.5 \times 10^{14} \text{cm}^{-3})\) this is usually not the case. The scattered power often varies significantly as a function of time during an rf pulse. In addition, the scattered CO\(_2\) powers do not always add linearly when the second unit is turned on.\(^{98}\) These results indicate that at these densities the waves are not spread out evenly over the whole plasma volume like they were at higher densities. If the waves were spatially localized into resonance cones, one would expect that the scattered signal would significantly change as the waves swept through the scattering volume. At
these low densities the scattered signal was strongly dependent on the waveguide phasing whereas at higher densities it was relatively insensitive to the waveguide phasing,\(^9^8\) which also indicate much stronger scattering from density fluctuations at higher densities. On the other hand, at highest densities \(n_e \gtrsim 2 \times 10^{14}\text{cm}^{-3}\) much less signal is observed near the plasma center than near the outside edge, perhaps indicating pump depletion due to parametric decay or absorption by the ion tail discussed before.

Summarizing this section, the CO\(_2\) scattering data presented in this section are consistent with the rf probe data presented in Sec. IV–3–A. The following overall picture concerning the pump wave broadening and excitation of ion-sound quasi-modes emerges from these experimental observations: The injected lower hybrid wave packet originally has a narrow frequency bandwidth which is determined by the rf source (\(< 1\text{kHz}\) as observed by rf probes located at the waveguide mouth). Due to the dependence of the scattering length \(l_s\) on the density \(n_e\) (Sec. II–4), scattering from a large level of density fluctuations near the plasma edge is more severe at higher densities. This is evidenced by the increasing broadening of the pump wave frequency spectrum as the density is increased, and also by the increasing tendency for the waves to spread out over the whole plasma volume as observed by CO\(_2\) scattering.

Relatively narrow broadening of the pump wave observed at low densities (\(n_e \lesssim 1 \times 10^{14}\text{cm}^{-3}\)) is probably due to scattering from the turbulent density fluctuations, since no sign of parametric decay has been observed at these densities. As the density is increased, resonance cones are destroyed and the convective losses due to the finite extent of the pump wave are greatly reduced. Together with the increasing \(E \times B\) parametric coupling, parametric excitation of ion-sound quasi-
modes (and ion-cyclotron quasi-modes) become feasible at higher densities \( \bar{n}_e \gtrsim 1.5 \times 10^{14} \text{cm}^{-3} \). At these densities the frequency width of the pump wave is significantly broadened, and the frequency spectrum is downshifted. In addition, according to the rf probe measurements potential fluctuations in the range of a few MHz is greatly enhanced, and the low frequency spectrum is closely a mirror image of the high frequency spectrum. These are indications of parametric excitation of ion-sound quasi-modes. The effect on the frequency spectrum near the pump frequency is not as dramatic as one might expect because frequency broadening due to scattering from low frequency density fluctuations (Sec. II–4 and Sec. IV–3–A) is comparable to the frequencies of ion-sound quasi-modes (Sec. III–2).
FIGURE V-5 — A typical frequency spectrum of the broadened pump wave observed near the plasma center (5cm outside on the midplane) using CO$_2$ laser scattering. Hydrogen plasma, $B = 9$T, $\bar{n}_e = 1.5 \times 10^{14}$cm$^{-3}$, $k_\perp = 80$cm$^{-1}$. 1MHz bandwidth filters were used.
FIGURE V-6 — A frequency spectrum of the pump wave observed near the plasma center (5 cm outside on the midplane) using CO₂ laser scattering at a lower density. Hydrogen plasma, $B = 10 \text{T}$, $n_e = 1.0 \times 10^{14} \text{cm}^{-3}$, $k_\perp = 80 \text{cm}^{-1}$. 300 kHz bandwidth filters were used. Rf probe spectra taken on the same shot is also shown for comparison.
FIGURE V-7 — The rms frequency width of the 4.6GHz pump wave obtained with CO$_2$ scattering near the plasma center (5cm outside on the midplane) plotted as a function of density (solid circles). Probe data (HWHM) are also shown for comparison (open circles). Deuterium plasma, $B = 9T$, $k_\perp = 80\text{cm}^{-1}$. Probe data are for $B = 8T$ and $B = 10T$. Scattering data were reproduced from Ref. 98.
V-4. LOW FREQUENCY DENSITY FLUCTUATIONS

Rf probe data presented in Chap. IV showed that the potential fluctuations in the range of a few MHz were observed to be greatly enhanced during rf injection while the ion density fluctuations were not significantly enhanced. The electron density fluctuations in the same frequency range were studied using CO₂ scattering. The ion density fluctuation observed on the ion saturation current was also monitored simultaneously to compare the probe data with the scattering data. The scattering geometry was such that the scattering volume was a vertical chord across the entire plasma cross section (see Sec. V-2). These density fluctuations have been measured to be localized near the plasma edge. The frequency spectra obtained using CO₂ scattering are similar in spectral shape to those obtained using rf probes. However, frequency spectra for larger wavenumbers are broader than those for smaller wavenumbers (see Fig. V-8; also Ref. 55). Since rf probes do not resolve wavenumber, direct comparison between the two techniques is rather difficult. In Fig. V-8 the frequency spectra taken along the chord passing near the plasma center (5cm outside from the plasma center) for \( k = 65\text{cm}^{-1} \) and \( k = 130\text{cm}^{-1} \) are shown, both before and during the rf pulse. Similar spectra near the plasma edge (14cm outside) for \( k = 30\text{cm}^{-1} \) and \( k = 130\text{cm}^{-1} \) are shown in Fig. V-9. The scattered signal (which is proportional to \( n_e^2 \)) shows no significant enhancement of the fluctuation level at any \( k (\leq 200\text{cm}^{-1}) \) and any \( \omega (\leq 5\text{MHz}) \). This result is similar to what was observed on the ion saturation current (which is proportional to \( \tilde{n}_i \)).

Experimental results from rf probes and CO₂ laser scattering concerning pump broadening and enhancement of low frequency fluctuations are summarized in Table V-1. These results will be compared with the prediction of the theory presented in Sec. III-6. The relationship between \( \tilde{n}_e \) and \( \tilde{\phi} \) at the high frequency (\( \simeq \omega_0 \)) are
not affected by finite values of \( \mu \) and should be proportional to each other. This is consistent with the remarkably close spectral shape observed on the probe (\( \Phi \)) and on CO\(_2\) scattering (\( \tilde{n}_e \)). At low frequencies with \( \mu \) finite, \( \tilde{n}_i \) is related to \( \Phi \) by the relation

\[
\frac{\tilde{n}_i(\mu)}{n_i} = k^2 \lambda^2 D_i \chi_i(\mu) \frac{e\Phi(\mu)}{T_i} \tag{82}
\]

and without rf power (\( \mu = 0 \))

\[
\frac{\tilde{n}_i(\mu = 0)}{n_i} = k^2 \lambda^2 D_i \chi_i(\mu = 0) \frac{e\Phi(\mu = 0)}{T_i}. \tag{83}
\]

Dividing Eq. (82) by Eq. (83) gives the relationship between the enhancement (due to rf power) in \( \tilde{n}_i \) and the enhancement in \( \Phi \):

\[
\frac{\tilde{n}_i^2(\mu)}{\tilde{n}_i^2(\mu = 0)} = \left( \frac{\chi_i(\mu)}{\chi_i(\mu = 0)} \right)^2 \frac{\Phi^2(\mu)}{\Phi^2(\mu = 0)}. \tag{84}
\]

For \( P_{rf} = 100\text{kW} \) at \( n_e = 5 \times 10^{12}\text{cm}^{-3} \) and \( T_e = T_i = 5\text{eV} \) the value of \( \chi_i(\mu)/\chi_i(\mu = 0) \) is 0.05, and for \( P_{rf} = 400\text{kW} \) this ratio is 0.03. For a 20dB enhancement in \( \Phi \) (i.e., \( \Phi^2(\mu)/\Phi^2(\mu = 0) \)), the enhancement in \( \tilde{n}_i \) would be

\[
\frac{\tilde{n}_i^2(\mu)}{\tilde{n}_i^2(\mu = 0)} \lesssim 0.25
\]

which would correspond to a 1dB enhancement if the fluctuations added as \( \tilde{n}_i^2(\mu) + \tilde{n}_i^2(\mu = 0) \). Therefore, the enhancement in ion density fluctuations (as measured by biasing the probe to collect ion saturation current) would be hard to detect. Similarly, at low frequencies, with \( \mu \) finite \( \tilde{n}_e \) is related to \( \Phi \) by the relationship

\[
\frac{\tilde{n}_e(\mu)}{n_e} = k^2 \lambda^2 D_e \frac{4}{\mu^2} \varepsilon(\mu) \frac{e\Phi(\mu)}{T_e}. \tag{85}
\]

192
In the absence of rf power ($\mu = 0$) the relationship becomes

$$\frac{\tilde{n}_e(\mu = 0)}{n_e} = k^2 \lambda D_e \chi_e(\mu = 0) \frac{\epsilon \tilde{\phi}(\mu = 0)}{T_e} \sim \frac{\epsilon \tilde{\phi}(\mu = 0)}{T_e}. \quad (86)$$

Dividing Eq. (85) by Eq. (86) gives the enhancement in $\tilde{n}_e$ in terms of enhancement in $\tilde{\phi}$:

$$\frac{\tilde{n}_e^2(\mu)}{\tilde{n}_e^2(\mu = 0)} = \left( \frac{4}{\mu^2 \chi_e(\mu = 0)} \right)^2 \frac{\tilde{\phi}^2(\mu)}{\tilde{\phi}^2(\mu = 0)}. \quad (87)$$

The value of $4\epsilon^-/\mu^2 \chi_e(\mu = 0)$ for the same parameters as above is 0.02 for $P_{rf} = 100\text{kW}$ and 0.06 for $P_{rf} = 400\text{kW}$. For a 20dB enhancement in $\tilde{\phi}$, the enhancement in $\tilde{n}_e$ would be

$$\frac{\tilde{n}_e^2(\mu)}{\tilde{n}_e^2(\mu = 0)} \lesssim 0.36$$

and hence the enhancement over the existing electron density fluctuation level is again small. If the enhancement in $\tilde{\phi}$ were due to destabilization of these low frequency modes due to the presence of the 4.6GHz rf fields (by changing the density gradients, for example), the density fluctuations should also be enhanced proportionally to the enhancement in potential fluctuations, which would be in contradiction with the experimental observations.

Based on these results, the enhancement in potential fluctuations and the lack of enhancement in electron and ion density fluctuations in the frequency range of a few MHz during rf injection observed by rf probes and CO$_2$ scattering are interpreted as manifestations of parametric excitation of ion-sound quasi-modes.
TABLE V-1 — Summary of experimental results on pump broadening and enhancement of the low frequency density fluctuations obtained using rf probes and CO₂ scattering.

<table>
<thead>
<tr>
<th></th>
<th>( \tilde{n}_e (\text{CO}_2) )</th>
<th>( \tilde{n}_i ) (probe)</th>
<th>( \tilde{\phi} ) (probe)</th>
</tr>
</thead>
<tbody>
<tr>
<td>high freq.</td>
<td>broadened</td>
<td>not measured</td>
<td>broadened</td>
</tr>
<tr>
<td>((\omega_0 - \omega))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low freq.</td>
<td>not enhanced</td>
<td>not enhanced</td>
<td>greatly enhanced</td>
</tr>
<tr>
<td>((\omega))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIGURE V-8 — The CO$_2$ scattered low frequency spectra 5cm outside the plasma center before (left) and during (right) the rf pulse for (a) $k = 65\text{cm}^{-1}$ and (b) $k = 130\text{cm}^{-1}$. Deuterium plasma, $B = 8T$, $\bar{n}_e = 2.7 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{LH0} = 2.1$).
FIGURE V.9 — The CO$_2$ scattered low frequency spectra 14cm outside the plasma center before (left) and during (right) the rf pulse for (a) $k = 30$cm$^{-1}$ and (b) $k = 130$cm$^{-1}$. 
V-5. ION-CYCLOTRON SIDEBANDS

The optics and detection system shown in Figs. V-2 and 3 were used to detect parametrically excited ion-cyclotron sideband lower hybrid waves in the plasma interior. Another set of local oscillator, mixer, and filter bank was used in place of the spectrum analyzer in Fig. V-3. By adjusting the frequency of this local oscillator, different ion-cyclotron peaks can be mixed down to the passband of the filter bank (which has a 14 MHz bandwidth centered at 22MHz). This is the first time that in a tokamak plasma parametrically excited waves have ever been observed by means other than rf probes.

In this thesis only the first ion-cyclotron sideband wave has been studied. The frequency width of the ion-cyclotron peak is broader than the frequency width at the pump frequency. At $k_\perp = 180 \text{cm}^{-1}$, the frequency width (HWHM) at the first ion-cyclotron peak is 3MHz whereas the frequency width at the pump frequency is only 1.5MHz. Similar frequency widths were obtained on the rf probes.

In Fig. V-10(a) the scattered power $P_{sc}$ from the sideband waves obtained during a $k_\perp$ scan in deuterium plasma at $B = 8T$, $I_p = 360kA$, $\bar{n}_e = 2.4(\pm0.1) \times 10^{14} \text{cm}^{-3}$ (corresponding to $\omega_0/\omega_{LH0} = 2.15$), and $P_{rf} = 260kW$ (MW2 only) is plotted as a function of $k_\perp$. The scattering volume was located 12cm outside the plasma center ($R - R_0 = 12 \text{cm}$) on the midplane. The detected signal in a 14MHz band centered around $\omega \simeq \omega_0 - \omega_{ci}$ (where $\omega_{ci}$ is the value corresponding to the magnetic field at the outside edge of the torus) was time averaged for 15msec during the rf pulse. For comparison the scattered power from the pump wave, which was obtained in parallel with the above data, is also shown. The range of wavenumbers that could be studied was limited by the optics to $80 \lesssim k_\perp \lesssim 180 \text{cm}^{-1}$. The $n_{\parallel}$ power spectra $P(n_{\parallel})$ for both the pump wave and the first ion-cyclotron harmonic sideband wave, deduced from the data plotted in Fig. V-10(a) using the procedure
described in Sec. V-1, are shown in Fig. V-10(b). The ion-cyclotron sideband has a fairly broad \( n_\parallel \) spectrum with maximum around \( n_\parallel \approx 6.5 \). These high \( n_\parallel \) sideband waves would be damped before they could propagate to the plasma center. This should be compared with the wavenumber spectrum at the pump frequency which is peaked around \( n_\parallel \approx 3 \). Note that the scattered power for the pump wave is reduced by a factor 10 in order to fit the two spectra on the same graph.

The density dependence of the scattered power from the ion-cyclotron sideband waves in deuterium plasma, \( B = 8T, P_{rf} = 300kW, R - R_0 = 14cm, \) and \( k_\perp = 140cm^{-1} \) is shown in Fig. V-11. Parametric excitation of the ion-cyclotron sideband waves with this value of \( k_\perp \) observed at this location has a sharp density threshold around \( \bar{n}_e = 1.8 \times 10^{14}cm^{-3} \). Above \( \bar{n}_e = 2.3 \times 10^{14}cm^{-3} (\omega_0/\omega_{LH0} = 2.2) \) the scattered power decreases again. This is qualitatively similar to the rf probe results of Sec. IV-3-B, but there are some differences as will be discussed later in this section.

The optics was modified so that higher \( k_\perp \) (i.e., higher \( n_\parallel \)) lower sideband waves could be studied. The \( n_\parallel \) spectra of the sideband wave obtained during two \( k_\perp \) scans are shown in Fig. V-12. Both spectra were taken in deuterium plasma, \( B = 8T, I_p = 325kA, \) and at \( R - R_0 = 12cm \). The triangles were taken at \( \bar{n}_e = 1.9 \times 10^{14}cm^{-3} \) with \( P_{rf} = 240kW \) and circles were taken at \( \bar{n}_e = 1.8 \times 10^{14}cm^{-3} \) with \( P_{rf} = 230kW \) (both with MW2 only). Even though the parameters were quite similar to the case of Figs. V-10(a) and (b), the density window for parametric decay waves has shifted down slightly from the case shown in Fig. V-11. The peak in the \( n_\parallel \) spectrum is more pronounced than in the case of Fig. V-10(b). This peak in the \( n_\parallel \) spectrum is expected since waves with large values of \( n_\parallel \) would be Landau damped on electrons. In Sec. III-3 numerical results for ion-cyclotron quasi-mode decay were presented which showed that there is a peak in the growth rate at
\[ \frac{c}{n_{||}v_{te}} \approx 4 \] since electron Landau damping becomes important for \( \frac{c}{n_{||}v_{te}} \lesssim 3. \)

Taking \( T_e \approx 300 \text{eV} \) at \( r = 12 \text{cm} \) this condition translates to \( n_{||} \approx 7.5 \) which agrees quite well with the experimental data.

The rf probe spectra were also monitored during this run. Some differences between the CO\(_2\) scattering data and the rf probe data were observed. At densities where strong parametric decay (> 10% of the pump wave power) was observed on the rf probe located at the C port \( (\bar{n}_e \gtrsim 2.1 \times 10^{14} \text{cm}^{-3}) \) no significant decay wave signal was observed by CO\(_2\) scattering. On the other hand, only a very weak parametric decay wave signal (30dB down from the pump) was observed on the rf probe at densities where the decay wave signals \( (\gtrsim 10\% \text{ of the pump signal at } k_\perp = 180 \text{cm}^{-1}) \) were observed by CO\(_2\) scattering \( (\bar{n}_e \lesssim 1.9 \times 10^{14} \text{cm}^{-3}) \). At these densities the intensity of the first harmonic sideband observed by the rf probe was smaller than that observed at higher densities by at least 20dB. The large value of \( n_{||} \) of the decay waves may be responsible for the differences observed between the scattering and the probe. The decay waves that were observed by CO\(_2\) scattering are highly damped and may not be able to propagate to the probe location like the pump wave can. It should also be mentioned here that when strong parametric decay is observed on rf probes located at the MW1 port (C top-inside port), the sideband wave power due to MW1 is roughly 10dB higher than that due to MW2. This observation also suggests that the sideband waves are highly damped and cannot propagate around the torus easily. The absence of CO\(_2\) scattering signal from the decay wave at higher densities is probably due to the sensitivity of the measurement rather than absence of the decay wave since the pump wave intensity also decreases strongly as a function of density. (In general, the decay wave signal is not observed by CO\(_2\) scattering unless large pump wave signals are observed.) Care must be taken when interpreting parametric decay data obtained with rf probes. In
contrast to this, the pump wave signal behaves similarly on both the rf probe and CO$_2$ scattering.

Rf power scaling of both the pump wave and the ion-cyclotron sideband wave in a deuterium plasma at $B = 8$T, $\bar{n}_e = 2.1 \times 10^{14}$cm$^{-3}$, $R - R_0 = 12$cm, and $k_\perp = 150$cm$^{-1}$ are shown in Fig. V-13. The scattered power for the pump wave varies linearly with the injected rf power, whereas the scattered power for the sideband wave has a threshold around $P_{rf} \approx 100$kW. This threshold is closer to the inhomogeneous threshold of Sec. III-4-B than the homogeneous threshold of Sec. III-3.

The scattering volume was scanned up-down (from $z = -16$cm to $z = +16$cm) at $R - R_0 = 0$. The pump waves observed along this chord at any vertical location were significantly smaller in amplitude than at $R - R_0 = +12$cm, and often could not be detected. The ion-cyclotron sideband were also much weaker and could not be detected. When both MW1 and MW2 waveguide arrays were used to inject rf power (at different time intervals during a shot), large scattered signals were observed when MW2 was on but no significant signals were detected when MW1 was on by itself. This was also true for the pump waves (i.e., much larger signals were observed at this location from MW2 than from the MW1 waveguide array). Since both waveguide arrays were phased 0-π-0-π so that they were launching symmetric spectra, the only major difference is the relative distance between the waveguide array and the CO$_2$ scattering port. The MW2 waveguide array is closer to the CO$_2$ scattering port by a factor of 2 ($30^\circ$ away instead of $60^\circ$ away toroidally). These observations may suggest that parametric decay into ion-cyclotron quasi-modes and ion-cyclotron sideband lower hybrid waves is mainly occurring near the outside edge of the torus near the waveguide array with toroidal extent of the order of $45^\circ$. This is consistent with the observations made with both rf probes and CO$_2$ scattering that
the frequency of the ion-cyclotron sideband is displaced from the frequency of the pump wave by the ion-cyclotron frequency which corresponds to the magnetic field at the outside edge of the torus (which is roughly 20% lower than that corresponding to the magnetic field at the plasma center).

Summarizing this section, qualitatively the CO$_2$ scattering data from the first ion-cyclotron sideband peak agree well with the probe data presented in Sec. IV-3-B. However, the parametric sideband ($< 10\%$ of the pump wave power, integrated over $n_\parallel$) was observed by CO$_2$ scattering at $R - R_0 = 12$cm, 60° away toroidally from the waveguide array only within a narrow density band. Within this density band the sideband power observed on the rf probe located 180° away toroidally from the waveguide array was typically 30dB down from the pump. Strong parametric decay was observed on rf probes at a slightly higher density, but no significant signal was observed by CO$_2$ scattering at these densities. This is probably due to sensitivity of the measurement because the pump wave signal also becomes barely detectable at these high densities. Furthermore, there is important new information that could not be obtained by rf probes. The wavenumber spectrum has been measured which indicate that the $k_\perp$ (hence, $n_\parallel$) power spectrum of the sideband waves is upshifted from that of the pump wave. A peak in the $n_\parallel$ spectrum was observed near $n_\parallel \approx 7$. This is in agreement with the theory presented in Sec. III-3 which predicted that there is a peak in the parametric growth rate when $c/n_\parallel v_{te} \approx 4$ (assuming the local temperature to be $T_e \approx 300$eV). These high $n_\parallel$ lower sideband waves would damp in the outer plasma layers, and would not propagate to the plasma center. Thus, this process provides a loss channel for the injected lower hybrid waves.

In addition, the spatial distribution of the lower sideband waves inside the plasma has also been measured (although more complete survey should still be performed). Within the given range of densities and in the specified range of $k_\perp$
space, the lower sideband waves have not been detected along the central chord ($R - R_0 = 0$). This result indicates strong damping of the sideband waves which is expected from the large values of $n_\parallel$ that they possess. At $R - R_0 = 12$ cm more signal was observed from MW2 than from MW1 for both the pump wave and the sideband wave. The pump wave signals detected near the plasma center at these densities ($\bar{n}_e \gtrsim 2 \times 10^{14}$ cm$^{-3}$) were also much smaller than the signals detected at $R - R_0 = 12$ cm, and were often not detected at all. This may be due to pump depletion due to parametric excitation of ion-cyclotron quasi-modes, increased collisional damping near the surface when toroidal ray propagation is considered, or increased wave absorption due to the presence of ion tail created through parametric excitation of ion-cyclotron quasi-modes.
FIGURE V-10(a) — Scattered power from the ion-cyclotron sideband wave \((\omega_0 - \omega_{ci})\) and the pump wave \((\omega_0)\) obtained in deuterium plasma, \(B = 8\text{T}, \bar{n}_e = 2.4 \times 10^{14}\text{cm}^{-3}\) (corresponding to \(\omega_0/\omega_{LH0} = 2.15\)), and \(P_{rf} = 260\text{kW}\) (MW2 only), plotted as a function of \(k_\perp\). The scattering volume was located on the torus midplane 12cm outside of the plasma center \((R - R_0 = 12\text{cm})\).
FIGURE V-10(b) — The $n_{\|}$ power spectra of the ion-cyclotron sideband wave ($\omega_0 - \omega_{ci}$) and the pump wave ($\omega_0$) obtained from the data plotted in Fig. V-10(a). Deuterium plasma, $B = 8$ T, $n_e = 2.4 \times 10^{14}$ cm$^{-3}$ ($\omega_0/\omega_{LH0} = 2.15$), and $P_f = 260$ kW (MW2 only).

\[\text{PUMP (÷10)}\]

\[\text{SIDEBAND}\]

\[P(n_{\|})(\text{arb. units})\]

\[n_{\|}\]
FIGURE V-11 — Scattered power from the ion-cyclotron sideband wave as a function of density. Deuterium plasma, $B = 8T$, $P_{rf} = 300kW$, $R - R_0 = 14cm$, $k_\perp = 140cm^{-1}$. 

\[ P_{SC} \text{ (arb. units)} \]

$\bar{n}_e \left(10^{14} \text{cm}^{-3}\right)$
FIGURE V-12 — The $n_{\parallel}$ power spectra of the ion-cyclotron sideband wave ($\omega_0 - \omega_{ci}$) at $\bar{n}_e = 1.9 \times 10^{14}\text{cm}^{-3}$ and $P_{rf} = 240\text{kW}$ (triangles) and at $\bar{n}_e = 1.8 \times 10^{14}\text{cm}^{-3}$ with $P_{rf} = 230\text{kW}$ (circles). Both spectra were taken in deuterium plasma, $B = 8\text{T}$, and at $R - R_0 = +12\text{cm}$.
FIGURE V-13 — Rf power dependence of the pump wave and the ion-cyclotron sideband wave powers in deuterium plasma, $B = 8T$, $\bar{n}_e = 2.1 \times 10^{14}\text{cm}^{-3}$, $R - R_0 = 12\text{cm}$, and $k_\perp = 150\text{cm}^{-1}$. 
VI. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

VI-1. SUMMARY AND CONCLUSIONS

Parametric decay instabilities during lower hybrid wave heating experiments on the Alcator C tokamak plasmas have been investigated both theoretically and experimentally. Both parametric decay of a lower hybrid wave into another lower hybrid wave and an ion-sound quasi-mode \( \omega_R \approx k_{||} v_{ei} \) and decay into a lower hybrid wave and a (nonresonant) ion-cyclotron quasi-mode \( \omega_R \approx n \omega_{ci} \) with maximum growth rate around \( \omega_R \approx k_{||} v_{te} \) have been studied in detail.

The lower hybrid waves were excited inside the plasma by injection of microwave power through phased waveguide arrays located near the plasma edge. These waves were detected using rf probes located in the shadow of the main limiters at several toroidal and poloidal locations. In addition, CO\(_2\) laser scattering technique was used to observe lower hybrid waves in the plasma interior. The low frequency fluctuations were also studied with both rf probes and scattering. CO\(_2\) laser scattering detects fluctuations in electron density \( \langle \tilde{n}_e \rangle \) while rf probes are mainly sensitive to fluctuations in potential \( \langle \tilde{\phi} \rangle \). Low frequency ion density fluctuations \( \langle \tilde{n}_i \rangle \) can also be detected by biasing the probes to collect ion saturation current. The rf probes located on the same side of the main limiters as the waveguide port were dominated by a strong narrow-band peak at the source frequency which is due to low \( n_{||} \) surface waves. However, the frequency spectra obtained by probes on the other side of the limiters, and spectra obtained by CO\(_2\) scattering in the plasma interior did not have this narrow-band peak, but rather a broadened \( (\gtrsim 1\text{MHz}) \) pump spectra were observed for densities \( \bar{n}_e \gtrsim 1 \times 10^{14}\text{cm}^{-3} \) \( \omega_0/\omega_{LH0} \gtrsim 3 \) in deuterium, \( \omega_0/\omega_{LH0} \gtrsim 2.1 \) in hydrogen, \( B = 8T \). The peak of the spectrum was slightly downshifted \((\lesssim 1\text{MHz})\) at higher densities. A slight increase in the pump frequency width and a large
increase in the low frequency potential fluctuations were observed for $P_{\text{rf}} \gtrsim 10\text{kW}$ which indicate parametric excitation of ion-sound quasi-modes. In agreement with theoretical estimates, neither the measured electron density fluctuations nor the ion density fluctuations were enhanced significantly. The threshold pump power is in agreement with theoretical estimates if a uniformly distributed pump wave in the outer plasma region is assumed. At these densities the lower hybrid waves appeared to be uniformly distributed over the whole plasma volume. (At highest densities $\bar{n}_e \gtrsim 2 \times 10^{14}\text{cm}^{-3}$ more wave power appeared to be concentrated near the outside plasma edge than near the plasma center.) At low densities ($\bar{n}_e \lesssim 1 \times 10^{14}\text{cm}^{-3}$), enhancement of low frequency potential fluctuations has not been observed. This correlated well with the narrow frequency width of the pump wave (comparable to the characteristic frequencies of the turbulent density fluctuations which exist in the absence of rf power) and the appearance of time varying features on the scattered signal.

At densities $\bar{n}_e \gtrsim 1.5 \times 10^{14}\text{cm}^{-3}$ ($\omega_0/\omega_{\text{LH0}} \lesssim 2.5$ in deuterium, $\omega_0/\omega_{\text{LH0}} \lesssim 1.8$ in hydrogen, $B = 8\text{T}$), in addition to the broadening of the pump wave parametric excitation of ion-cyclotron quasi-modes was clearly observed on both the high frequency and the low frequency spectra obtained by rf probes. In addition, the parametrically excited ion-cyclotron sideband lower hybrid waves have been detected for the first time using the CO$_2$ scattering technique. The wavenumber spectrum at the first ion-cyclotron sideband peak, measured at $r/a = 0.73$, had a maximum at significantly higher values of $n_\parallel (n_\parallel \approx 7)$ than the pump wave ($n_\parallel \lesssim 3$). The ion-cyclotron peaks were separated from the pump wave by multiples of the ion-cyclotron frequency corresponding to the magnetic fields near the outside edge of the plasma. As the density increases the number of cyclotron peaks increases and the intensity of the lower sidebands relative to the pump intensity increases. As the
density further increases, higher harmonics peaking around $\Delta \omega/\omega_0 \approx 0.15$ appear. This is somewhat more pronounced in hydrogen plasmas than in deuterium plasmas. The frequency integrated ion-cyclotron sideband power observed on rf probes, which is always dominated by the first harmonic peak, becomes comparable to the frequency integrated pump power (including the ion-sound quasi-mode sidebands) at $\bar{n}_e \gtrsim 2 \times 10^{14} \text{cm}^{-3}$ ($\omega_0/\omega_{LH0} \lesssim 2.3$ in deuterium, $\omega_0/\omega_{LH0} \lesssim 1.6$ in hydrogen, $B = 8\text{T}$). The frequency integrated pump power decreases rapidly for $\bar{n}_e \gtrsim 1.5 \times 10^{14} \text{cm}^{-3}$ when parametric excitation of ion-cyclotron quasi-modes becomes important. The CO$_2$ scattering detects the first harmonic sideband only within a narrow density band which is slightly below the density where the frequency integrated sideband power obtained by rf probes attains the maximum value. At densities $\bar{n}_e \gtrsim 2 \times 10^{14} \text{cm}^{-3}$ the CO$_2$ scattered power from the pump wave near the plasma center is significantly smaller than that originating near the plasma periphery. The sideband waves have not been observed near the plasma center.

These experimental observations can be explained in the following way. If the incident lower hybrid wave propagates inside a well-defined resonance cone, the finite pump extent (convective) threshold for parametric decay process is very large due to large convective losses. However, at higher densities $\bar{n}_e > 1 \times 10^{14} \text{cm}^{-3}$ ($\omega_0/\omega_{LH0} \lesssim 3$ in deuterium, $\omega_0/\omega_{LH0} \lesssim 2.1$ in hydrogen, $B = 8\text{T}$) the pump wave is severely scattered from density fluctuations near the plasma edge due to the large $\mathbf{E} \times \mathbf{B}$ coupling term and the resonance cones are destroyed. Furthermore, electromagnetic effects also broaden the resonance cone. Since convective losses are thus eliminated, parametric decay may occur above this density. Since these modes are most unstable near the limiter radius, and since the pump electric field is strongest near the waveguide array, modes that first become unstable are the ones excited on the outside edge. Only a few harmonics separated by the ion-cyclotron
frequency at the outside edge can be observed at densities $n_e \lesssim 1.5 \times 10^{14} \text{cm}^{-3}$. As the density is increased further, higher harmonics can be excited further inside, creating a second peak in the spectrum around $\Delta \omega/\omega_0 \approx 0.15$. At the same time, the intensity of the lower sideband increases due to the increasing $E \times B$ parametric coupling. As parametric decay becomes important at higher densities, more and more pump power is transferred into higher $k_\parallel$ lower sideband lower hybrid waves (or ion plasma waves) which are readily absorbed by the plasma before they can propagate to the plasma center. If the density becomes high enough, the pump will get depleted near the plasma edge and higher harmonics, which are mainly generated in the plasma interior, start to disappear. Even if the pump wave is not depleted by parametric decay, the ion tail created by absorption of wave energy through parametric decay process may increase damping of the pump wave. Another possibility is that the lower hybrid pump wave is scattered by the decay waves so that in the presence of toroidal effects the wave cannot penetrate into the plasma center.

In conclusion, excitation of both ion-sound quasi-modes and ion-cyclotron quasi-modes are observed with rf probes and CO$\text{}_2$ scattering. Frequency spectra and wavenumber spectra are in agreement with theoretical estimates. Since the $n_\parallel$ spectrum of the incident lower hybrid waves can be raised by this mechanism which leads to unwanted surface absorption, prevention of such processes may be essential in obtaining a good heating and current drive efficiency at high densities ($\omega_0/\omega_{LH} \lesssim 2$).
VI-2. RECOMMENDATIONS FOR FUTURE WORK

Thresholds and growth rates for both ion-sound quasi-modes and ion-cyclotron quasi-modes have been studied extensively. It was concluded that assuming well-defined resonance cones would predict much higher threshold powers compared to the experimental values. CO₂ laser scattering measurements also indicate that well-defined resonance cones do not exist at the densities where parametric decay processes are observed. Alternative ways to estimate the threshold rf power have been proposed, but improvements in these calculations may still be possible.

Since rf probe spectra are not toroidally uniform, and there may also be poloidal variations in the strength of parametric excitation, several probes located at different toroidal and poloidal positions should be used to study lower hybrid waves in a tokamak. If the waves that propagate out from the plasma interior are to be studied with rf probes, care should be taken to prevent detection of surface waves (by installing a pair of local limiters on either side of the probe, for example). Care should be taken when studying parametric decay waves using rf probes. A relatively strong parametric decay observed by CO₂ scattering in the plasma interior was not detected by rf probes located in the shadow of the limiters.

Much more work remains to be done in the area of CO₂ scattering, especially scattering from parametrically excited sideband lower hybrid waves and from low frequency quasi-modes. Only the first harmonic ion-cyclotron lower sideband (ω₀ - ω_ci) has been studied in this thesis. Higher harmonics are probably more difficult to see since their amplitudes are theoretically predicted to be lower, and observed to be lower by rf probes. It would be interesting to look for higher harmonics around \( \Delta \omega/\omega_0 \gtrsim 0.1 \) where the rf probe spectrum shows a second peak. Unlike the lower harmonics, these higher harmonics are theoretically expected to be generated further inside the plasma. More complete mapping of spatial distribution of parametric
decay waves should be pursued. The limited port access available on Alcator C made it difficult to carry out a detailed spatial mapping of both the pump wave and the sideband waves. The low frequency ion-cyclotron quasi-modes \((\omega \simeq n\omega_{ci})\) should also be studied. These low frequency quasi-modes would be more useful than the sideband lower hybrid waves in localizing where parametric decay is occurring, since quasi-modes exist only where they are excited whereas the sideband waves can propagate away.

Based on the results presented in this thesis, the following suggestions can be made with regard to using lower hybrid waves to heat plasmas to thermonuclear temperatures. Care should be taken in using lower hybrid waves to directly heat ions via the ion plasma wave mode conversion process and perpendicular ion Landau damping which typically requires \(\omega_0/\omega_{LH} < 2\) (for \(y^2 \equiv \omega_0^2/\omega_{ce}\omega_{ci}\) not too close to 1). This method of heating is not recommended unless an effective way to prevent parametric decay processes near the plasma edge is found. This may be accomplished by heating the edge plasma using ECH power (or neutral beam heating as was demonstrated on JFT-II\(^{28}\)). Impurity generation must also be controlled in case of electron heating and current drive at high power levels when energetic electrons are produced.\(^9^9\) Electron heating by Landau damping and subsequent ion heating through electron-ion equilibration in the density regime \(\omega_0/\omega_{LH} \gtrsim 2\) (to avoid strong parametric decay processes) may be a more efficient way to heat plasmas. Current drive efficiency at high plasma densities \((\omega_0/\omega_{LH} \lesssim 2)\) may also degrade significantly due to excitation of parametric decay processes. The driving frequency \(\omega_0\) should be chosen high enough \((\omega_0/\omega_{LH} \gtrsim 2)\) to avoid parametric decay processes. However, it cannot be chosen arbitrarily high since accessibility becomes worse as the driving frequency is increased. The accessibility consideration is particularly important for current drive since low \(n_{\parallel}\) waves play a major role
in these experiments. Therefore, a compromise must be made between improving accessibility and preventing parametric decay processes. This would argue for a higher magnetic field machine for extending the operating regime to higher densities.
REFERENCES


J. Terry, private communication.


