LARGE MAGNET SYSTEM SAFETY AND PROTECTION STUDIES - FY1987 REPORT

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2.0 Filamentary Plasmas with Decoupled Current Decay &amp; Motion</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Constant Current Plasma Moving Toward One Wall</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Constant Current Plasma Moving Parallel to One Wall</td>
<td>11</td>
</tr>
<tr>
<td>2.3 Stationary Plasma Decaying Near One Wall</td>
<td>15</td>
</tr>
<tr>
<td>2.4 Stationary Plasma Decaying Between Two Walls</td>
<td>18</td>
</tr>
<tr>
<td>3.0 Distributed Current Plasma with Decoupled Current Decay &amp; Motion</td>
<td>22</td>
</tr>
<tr>
<td>3.1 Simplified Distributed Current Plasma Model</td>
<td>22</td>
</tr>
<tr>
<td>3.2 Currents and Pressures Induced in a Conducting Wall</td>
<td>28</td>
</tr>
<tr>
<td>4.0 Filamentary Plasmas with Coupled Current Decay &amp; Motion</td>
<td>31</td>
</tr>
<tr>
<td>4.1 Plasma Equilibrium Model</td>
<td>33</td>
</tr>
<tr>
<td>4.2 Typical Induced Current &amp; Pressure Profiles for CIT</td>
<td>35</td>
</tr>
<tr>
<td>4.3 Typical Induced Current &amp; Pressure Profiles for CMOD</td>
<td>38</td>
</tr>
</tbody>
</table>
1.0 Introduction

One of the important problems in the design of a Tokamak is the prediction of the electromagnetic loads on the vacuum vessel due to a plasma disruption. The collapse of the magnetic field associated with the decay of the plasma current and/or the change in the magnetic field due to the plasma motion, induces currents in nearby conductors. The induced currents then interact with the magnetic fields from the plasma, the poloidal field coil system and the toroidal field coil system to create Lorentz forces which can be structurally difficult to support.

Significant strides have been made in the availability of a code which can consistently treat the dynamic character of the plasma coupled with its interaction with PF, TF and fields due to currents induced in passive conductors. The Tokamak Simulation Code [TSC, Jardin1] is particularly noteworthy in this area. However, the accuracy of such a model is necessarily coupled with complexity and the need for extended runs on large machines such as those available on the MFENET.

The MIT Safety & Protection Program in FY87 focused on simple models of plasma disruption induced currents and forces to try to gain insight into their magnitude & distribution.

In all cases in this report, the time scale for a disruption will be assumed to be fast relative to the time scale for field diffusion through nearby conducting walls. As a result, a high magnetic Reynolds number approximation will be applied, in which the induced currents "freeze" the original field on the far side of the wall(s) from the plasma. The field on the plasma side collapses on the time scale of the disruption to a distribution consistent with the induced currents on the surface of the walls. In the simple cases which follow, solutions for the instant following disruption were found by applying the method of images and satisfying boundary conditions for the continuity of the normal component of magnetic flux density and discontinuity of the tangential magnetic field intensity. The latter is related to the magnitude of the induced current sheet at the discontinuity.

Section 2.0 of this report considers cases in which the plasma consists of an infinitely long filamentary current near a plane conducting wall, but with the restriction that plasma current and motion are uncoupled (this is relaxed in Sections 3.0 and 4.0).

The first case involves an infinitely long current filament (plasma) which is parallel to a conducting wall and which suddenly moves toward the wall while maintaining constant current. The second case involves the filament moving parallel to the wall. The third case considers the plasma filament to be stationary, but to discharge completely to zero. The fourth case involves a current filament located midway between parallel conducting walls. The filament is stationary and suddenly experiences a disruption to zero current.

The derivations associated with the simple models are done in a dimensionless form which then naturally implies the way induced currents and pressures scale with size and plasma current. Results are also given in "de-normalized" form for parameters characteristic of CIT and CMOD. In summary, the models in Section 2.0 show that:

1. The current decay of a stationary plasma near a single conducting wall creates more severe pressure profiles than a comparable case between two walls;

2. A stationary decay to zero current gives more severe normal loads than small motions at constant current, but considerably less severe normal loading conditions than large motions at constant current. In fact, the results imply that large radial motions must be controlled and avoided. However, the loads are overestimated because the large motion would lead to current decay through scrape-off and an induced current of opposite (beneficial) sign in the wall toward which the plasma was moving; and

3. The maximum tangential pressure on the wall is worst for the stationary decay for the range of situations considered, but is, in general, less severe in magnitude than the normal pressure.

The worst case considered in Section 2.0 was a constant current, large displacement (60% of the minor radius) toward a wall. Since it is unrealistic to expect a displacement of this magnitude without a current decay, Section 3.0 considers a simple model combining the effects of motion and current decay. Plasmas were assumed to have an exponential current density profile in space and a motion, $\delta$, was assumed to scrape-off the amount of current on an outside shell of that thickness. The results indicate that broad profiles lead to loads which are probably manageable from the structural standpoint, but a highly peaked profile leads to loads which are probably not manageable if the displacement, $\delta$, is large. Presumably, the very highly peaked profiles are not realistic, but are useful for comparison with Section 2.0 analyses based on a filamentary plasma. The model is somewhat optimistic in the amount of current which can be scraped-off for a given arbitrary displacement, and is, therefore, unsatisfactory.
In Section 4.0 another simple, but more realistic, plasma model was implemented. The plasma was assumed to be maintained in radial equilibrium by the vertical field $B_y$. At time zero, $\beta$ (poloidal) was assumed to drop to zero suddenly at constant current. The applied field $B_y$ cannot change rapidly so the plasma moves and induces currents in a nearby wall which provides a vertical field increment which, together with $B_y$, holds the plasma in a new equilibrium position. For parameters typical of CIT and CMOD, the model implies that the $\beta$ change results in moderate displacements (ie—16 & 10 per cent of the initial minor radii, respectively). After $\beta$ decays to zero, the plasma current decays to zero. Each increment of the current decay causes the plasma to move again and to induce currents and fields which provide equilibrium at the new position. The assumed equilibrium condition allows the displacement to be found as a function of the current.

Results of the calculations in Section 4.0 are summarized in Table 1.1. The first two columns give the fraction of plasma current relative to the initial current and the corresponding dimensionless distance moved toward the wall. The distance moved is normalized to the initial minor radius. These are equilibrium positions for the plasma as its current decays. They were found with the simplified model and are about the same for CIT and for CMOD when in dimensionless form, particularly after the current decays below 50% of its initial value.

The next two columns give the maximum value of the pressure on the wall in directions perpendicular to the wall and tangential to the wall (typical distributions along the wall can be found in Sections 4.2 & 4.3). These two columns are dimensionless with normalization factors of $\mu*Ip^2/a^2$ and are essentially the same for both machines for large displacements. They have been translated into pressure in MPa in the last two columns. Note that the perpendicular pressure is particularly severe in that it reaches about 40 MPa for CMOD and about 54 MPa for CIT. Note also that the value rises as the plasma moves toward the wall and that it tends to reach a maximum value. The latter value is about 20 to 40 times the value for the initial displacement which results from the decrease of poloidal beta at constant current.

Column 5 gives the dimensionless perpendicular force. This is the integral of the perpendicular pressure from the midplane to a distance along the wall equal to twice the initial minor radius. The force has been made dimensionless by normalizing to $\mu*Ip^2/a$. Note that, in dimensionless form, it is similar for the two machines, as are the dimensionless pressures. The results show that the force also increases and saturates at a given value as the wall is approached by the plasma, but that the large displacement value is about 7 to 10 times the initial displacement value. This implies that the pressure, which grew to a higher ratio, is becoming more localized. This is also evident in the plots of the pressure distributions which are given in Section 4.2.

The models derived in this report were not intended to replace the calculations possible (and necessary) with more sophisticated methods. However, the analytical forms are simple and are useful for gaining insight into the geometrical character of the interactions and in estimating their magnitude.
Table 1.1 a- Maximum Normal and Tangential Pressure Estimates for CIT

<table>
<thead>
<tr>
<th>Plasma Current Movement</th>
<th>Dimensionless Pressure</th>
<th>Dimensionless Force</th>
<th>CIT Pressure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>max perp</td>
<td>max tang</td>
<td>perp</td>
</tr>
<tr>
<td>1</td>
<td>0.16</td>
<td>0.008</td>
<td>0.002</td>
</tr>
<tr>
<td>0.9</td>
<td>0.295</td>
<td>0.014</td>
<td>-0.002</td>
</tr>
<tr>
<td>0.7</td>
<td>0.531</td>
<td>0.028</td>
<td>-0.006</td>
</tr>
<tr>
<td>0.5</td>
<td>0.714</td>
<td>0.05</td>
<td>-0.01</td>
</tr>
<tr>
<td>0.3</td>
<td>0.853</td>
<td>0.08</td>
<td>-0.014</td>
</tr>
<tr>
<td>0.2</td>
<td>0.91</td>
<td>0.115</td>
<td>-0.015</td>
</tr>
<tr>
<td>0.1</td>
<td>0.958</td>
<td>0.16</td>
<td>-0.016</td>
</tr>
<tr>
<td>0.05</td>
<td>0.98</td>
<td>0.16</td>
<td>-0.016</td>
</tr>
</tbody>
</table>

Table 1.1 b- Maximum Normal and Tangential Pressure Estimates for CMOD

<table>
<thead>
<tr>
<th>Plasma Current Movement</th>
<th>Dimensionless Pressure</th>
<th>Dimensionless Force</th>
<th>CMOD Pressure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>max perp</td>
<td>max tang</td>
<td>perp</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>0.9</td>
<td>0.25</td>
<td>0.008</td>
<td>-0.002</td>
</tr>
<tr>
<td>0.7</td>
<td>0.51</td>
<td>0.023</td>
<td>-0.006</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
<td>0.044</td>
<td>-0.01</td>
</tr>
<tr>
<td>0.3</td>
<td>0.85</td>
<td>0.083</td>
<td>-0.014</td>
</tr>
<tr>
<td>0.2</td>
<td>0.91</td>
<td>0.085</td>
<td>-0.015</td>
</tr>
<tr>
<td>0.1</td>
<td>0.96</td>
<td>0.16</td>
<td>-0.016</td>
</tr>
<tr>
<td>0.05</td>
<td>0.98</td>
<td>0.16</td>
<td>-0.016</td>
</tr>
</tbody>
</table>

Notes: Assumptions

- CIT
- CMOD
- Plasma current
- Major radius
- Minor radius
- Vert field at wall
- Rad field at wall

Dimensionless pressure is pressure/(4e-7*pi*I*p^2/a^2)
Dimensionless force is force/(4e-7*pi*I*p^2/a)
Force is integral of perpendicular pressure from y=0 to 2*a
2.0 Filamentary Plasmas with Decoupled Current Decay & Motion

The time scale for a disruption will be assumed to be fast relative to the time scale for field diffusion through nearby conducting walls. As a result, a high magnetic Reynolds number approximation will be applied, in which the induced currents 'freeze' the original field on the far side of the wall from the conductor. The field on the plasma side collapses on the time scale of the disruption to a distribution consistent with the induced currents on the surface of the walls. In the simple cases which follow, solutions for the instant following disruption can be found by applying the method of images and satisfying boundary conditions for the continuity of the normal component of magnetic flux density and discontinuity of the tangential magnetic field intensity. The latter is related to the magnitude of the induced current sheet at the discontinuity.

The first case involves an infinitely long current filament (plasma) which is parallel to a conducting wall and which suddenly moves toward the wall while maintaining constant current. The second case involves the filament moving parallel to the wall. The third case considers the plasma filament to be stationary, but to discharge completely to zero.

The fourth case involves a current filament located midway between parallel conducting walls. The filament is stationary and suddenly experiences a disruption to zero current.

The induced current and pressure distributions can be determined in a more consistent manner with more complicated models which combine the plasma motion and current decay. In this section, the intent is to estimate the order of magnitude of the loads and to estimate the relative effects of motion perpendicular to the wall, motion parallel to the wall, stationary filament current decay, and multiple walls. Results are summarized in Table 2.1, which shows the maximum dimensionless pressures under several conditions (cases treated in this section). Columns are also given with estimates for the maximum pressures in CIT and CMOD vacuum vessel walls under the different conditions. The dimensionless vertical fields, \( \beta \) (see analysis for definition), for the two machines differed by only 12%, so a common value of \(-5.5e-2\) was used for the table and graphs which follow.

The first section of the table compares several cases involving a single conducting wall whereas the last line involves a stationary current decay between two conducting walls. Comparing the first and last cases implies that multiple walls tend to reduce the peak loads substantially, as would be expected. Cases 2 to 5 involve relatively small (ie - 20% of the distance from the wall) and relatively large (ie - 60% of the distance) motions, either toward ("to") or parallel ("//") to the wall. The implication of the results is that a stationary decay to zero current gives more severe normal loads than small motions at constant current, but considerably less severe normal loading conditions than large motions at constant current. In fact, the results imply that large radial motions must be controlled and avoided. However, the loads are overestimated because the large motion would lead to current decay through scrape-off and an induced current of opposite (beneficial) sign in the wall toward which the filament was moving. A simple model including this effect will be the subject of the next section.

The maximum tangential pressures in the table are worst under the stationary decay for the range of situations considered, but are, in
Table 2.1 - Maximum Normal and Tangential Pressure Estimates for CIT and CMOD

<table>
<thead>
<tr>
<th>Case</th>
<th>Single Wall</th>
<th>Double Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dimensionless Pres</td>
<td>CIT Pres (MPa)</td>
</tr>
<tr>
<td></td>
<td>max perp</td>
<td>max tang</td>
</tr>
<tr>
<td>1</td>
<td>no move, turn-off</td>
<td>0.018</td>
</tr>
<tr>
<td>2</td>
<td>move 20% to wall</td>
<td>0.012</td>
</tr>
<tr>
<td>3</td>
<td>move 60% to wall</td>
<td>0.015</td>
</tr>
<tr>
<td>4</td>
<td>move 20% // wall</td>
<td>0.003</td>
</tr>
<tr>
<td>5</td>
<td>move 60% // wall</td>
<td>0.011</td>
</tr>
<tr>
<td>6</td>
<td>no move, turn-off</td>
<td>0.011</td>
</tr>
</tbody>
</table>

---

Note: assumptions

<table>
<thead>
<tr>
<th>CIT</th>
<th>CMOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ip, MA</td>
<td>9</td>
</tr>
<tr>
<td>a, m</td>
<td>0.55</td>
</tr>
<tr>
<td>By, T</td>
<td>-1.2</td>
</tr>
<tr>
<td>Bx, T</td>
<td>0</td>
</tr>
</tbody>
</table>

plasma current

minor radius

vert field at wall

rad field at wall
general, less severe in magnitude than the normal pressures.

This table and these models provide an indicator of load levels, but are oversimplified because the motion and current decay cannot be decoupled in the actual machines. The value of the following models lies in their ability to provide insight into the relative magnitude of effects.

Equations and graphs are given in normalized form in the following section to define the simple models, show typical current and pressure distributions, and gain insight into the interactions.

2.1 Constant Current Plasma Moving Toward One Wall

Figure 2.1 shows an infinitely long filamentary plasma initially located at a distance $r_2$ from a conducting wall. The nomenclature in the equations is defined below.

- $\mu = \pi \times 10^{-7}$
- $I_p =$ initial plasma current
- $r_2, r_1 =$ initial, final dist from filament to wall
- $\delta =$ normalized distance moved toward wall
- $\delta : 0.2, 0.4, \ldots, 0.6$
- $k : 0, 1, \ldots, 25$ integer range for plotting purposes only
- $y_{\text{max}} : 2.0$ max normalized distance along wall
- $k$ distance along wall
- $y : y_{\text{max}}$ normalized to $r_2$

If the filament moves suddenly from $r_2$ to $r_1$ in a time $\tau$, then the field is frozen on the far side of the conducting wall. The induced current sheet can be found by turning off the current filament at $r_2$ and turning on a current filament at $r_1$ in a time $\tau$. The result is given by:

$$Kn(k, \delta) := \frac{1}{\pi} \left[ \frac{1}{2} - \frac{1 - \delta}{y + 1} \right] k$$

$I$ is time normalized to $\tau$

$Kn$ is local current sheet magnitude normalized to $(I_p/r_2)$

The normal pressure on the plate is due to the interaction of the induced current with the tangential component of the applied field, $B_\parallel$, and the tangential component of the field at the wall from the filaments. In the plots which follow we will assume $\tau=1$ and typical values for the normalized tangential and parallel applied fields at the plate.
Fig. 2.1 Infinitely long Plasma (Filamentary) Moving Toward a Conduction Wall, Dimensions are Normalized to Plasma Minor Radius, $r_2$

$y = \frac{y}{r_2}$

Conducting Wall

Normalized Space
\[
tot = 1.0 \quad \beta_{ye} := -5.5 \quad \beta_{xe} := 0.0
\]

\[
pxn(k, \delta) := -Kn(k, \delta) \left( \beta_{ye} + \frac{1}{2\pi} \left( \frac{1 - \tot}{y + 1} + \frac{1}{y + (1 - \delta)} \right) \right)
\]

where: 
\[
\beta_{ye} = \frac{Bye}{(\mu Ip/r^2)} = \text{norm external y field component}
\]
\[
\beta_{xe} = \frac{Bxe}{\mu Ip/r^2} = \text{norm external x field component}
\]
\[
pxn = \text{dimensionless normal pressure} = \frac{\text{normal pressure}}{(\mu Ip^2/r^2^2)}
\]

The tangential pressure on the plate is due to the interaction of the induced current with the normal component of the applied field, \( B_{xe} \), and the normal component of the field from the filaments at the wall.

\[
pyn(k, \delta) := Kn(k, \delta) \left[ \beta_{xe} - \frac{2}{2\pi} \frac{1}{y + 1} \right]
\]

Consider a typical case with the plasma moving toward the inboard vessel wall in a time \( \tau \) and with the previously given dimensionless field components at the wall. The normalized induced currents and the pressures on the wall as a function of distance along the wall are given in the following figures at the instant when the filament current reaches zero and for values of \( \delta = 0.2, 0.4, \) and 0.6.
The induced current pattern is independent of the applied fields, which are time invariant. The maximum value of the induced current sheet is at \( y=0 \), in the plane of the motion. Note also that the induced currents are negative to repel the motion of the filament.

The maximum value of the normal pressure is also in the \( y=0 \) plane. Its magnitude, but not its location is dependent on the value of \( \beta y e \) which is the same for all pressure plots in this memo.
Note that the magnitude of the tangential pressure is substantially smaller than the normal pressure. The distribution and the point near the plane of motion at which the tangential pressure passes through zero is dependent on the value of $\beta x_e$.

2.2 Constant Current Plasma Moving Parallel to One Wall

In this case the filament is carrying a constant current, $I_p$, and is a distance $r_2$ from the conducting wall as illustrated in Fig 2.5. Suddenly it moves parallel to the wall from $y=0$ to $y=b$ in a time $\tau$ which is fast compared to the time constant of the wall.

The normalized induced current sheet is given by:

$$K_p(k, \xi) := \frac{-1}{\pi} \left[ \frac{1}{1 + \left| \frac{y - \xi}{k} \right|^2} \right] - \left[ \frac{1}{1 + \left| \frac{y + \xi}{k} \right|^2} \right]$$

tot

where: the current sheet amplitude is again normalized to $I_p/r_2$; the distance along the wall, $y$, is normalized to $r_2$, such that $y_k = y/r_2$ and the distance the filament moves, $b$, is normalized to $r_2$, such that $\xi = b/r_2$.

The dimensionless normal pressure is then given by:
Fig. 2.5 Filamentary Plasma Moving Parallel to a Conducting Wall; Dimensions Normalized to Plasma Minor Radius, $r_2$
where the pressure has been made dimensionless by dividing by \( \mu \rho^2 / r^2 \).

The dimensionless tangential pressure is given by:

\[
pxp(k, \epsilon) := -Kp(k, \epsilon) \left( \frac{1}{2 \pi} \frac{1 - \cot \epsilon}{1 + \frac{y}{k} \cot \epsilon} + \frac{\cot \epsilon}{1 + \frac{y}{k}} \right)
\]

The following graphs use the same values of \( \beta_y \) and \( \beta_x \) as those in the previous examples. In addition, the dimensionless distance moved has the same magnitude, but is now parallel to the wall.

\[
pyp(k, \epsilon) := Kp(k, \epsilon) \left( \frac{1}{2 \pi} \frac{1 - \cot \epsilon}{1 + \frac{y}{k} \cot \epsilon} + \frac{\cot \epsilon}{1 + \frac{y}{k}} \right)
\]

\( \epsilon = 0.2, 0.4, 0.6 \)

Note that a positive \( \epsilon \) for the filament induces a negative current sheet in the wall ahead (approximately) of the filament which tends to push it back and a positive current sheet in the wall behind (approximately) the filament which tends to pull it back to its initial position.
The normal pressure induced by the motion parallel to the wall has a tendency to rotate the wall by pushing on it ahead of the filament and pulling on it behind the filament.
2.3 Stationary Plasma Decaying Near One Wall

In this case the filament is stationary at a distance of \( r_2 \) from the wall as shown in Fig 2.9. It has an initial current of \( I_0 \) which decays to zero in a time \( \tau \). The dimensionless current sheet amplitude is given by the following. Note that the current sheet is normalized to \( I_0/r_2 \) and that the result is then independent of \( r_2 \).

\[
K_s(k) := \frac{\tau_{\text{tot}}}{\pi \left( \frac{2}{y + 1} \right)}
\]

The dimensionless normal pressure on the wall is, then:

\[
p_{xs}(k) := -K_s(k) \left[ \beta ye + (1 - \tau_{\text{tot}}) \frac{1}{2 \pi} \left( \frac{2}{y + 1} \right) \right]
\]

The dimensionless tangential pressure on the wall is:

\[
p_{ys}(k) := K_s(k) \left[ \beta xe - \frac{1}{(2 \pi) \left( \frac{1 + y}{k} \right)} \right]
\]
Normalized Space

Fig. 2.9 Stationary Plasma with Current Decay to Zero Near a Conducting Wall
Fig 2.10 DIMENSIONLESS CURRENT SHEET MAGNITUDE vs
NORMALIZED DISTANCE ALONG WALL

Fig 2.11 DIMENSIONLESS NORMAL PRESSURE
vs NORMALIZED DISTANCE ALONG WALL
2.4 Stationary Plasma Decaying Between Two Walls

In this case, the filament is located midway between two parallel conducting walls as shown in Figure 2.13. The distance between walls is $2r_2$ and all quantities are normalized as in the previous cases. If the current decays rapidly to zero in a time, $\tau$, then the currents induced in the walls will 'freeze' the field to the right of the right wall and to the left of the left wall. The tangential field component 'frozen' on right side of the right wall is:

$$
\beta y f(k) := \frac{1}{2\pi \left( \frac{2}{y+1} \right)^{\frac{1}{k}}}
$$

The normal component of field is also 'frozen' on the right side of the wall, but is continuous through the wall.

$$
\beta x f(k) := \frac{-y}{k} \left( \frac{2}{y+1} \right)^{\frac{1}{k}}
$$
Fig. 2.13 Stationary Plasma with Current Decay to Zero
While Midway Between 2 Conducting Walls
The disrupting plasma produces induced currents on the surface of the walls which 'freeze' the fields outside the walls. The fields produced between the walls by the induced currents are the same as those produced by replacing the walls with an infinite series of images of the plasma which reflect across a boundary at the location of each wall interface. The series of images produce a normal component of field at the right wall (inside), $\beta_{xd}$ (dimensionless), such that:

$$\beta_{xd}(k) := \beta_{xf}(k)$$

The dimensionless tangential component of field produced at the inside of the right wall by the images, $\beta_{yd}$, is given by:

$$\beta_{yd}(k) := \frac{1}{2 \pi} \sum_{n=1}^{n=30} \frac{(-1)^{2n-1}}{2} \frac{1}{n} y + \frac{(2n-1)}{2} k$$

Note that the use of the dimensionless formulation leads to a general field distribution which is not explicitly dependent on the distance between plates. Selected tests on the above expression indicate that the series converges well for plotting purposes if 30 terms are used. Therefore,

$$n = 1 \ldots 30$$

The dimensionless current sheet induced on the right wall is:

$$K_z(k) := (\beta_{xf}(k) - \beta_{yd}(k))$$

Fig 2.14 DIMENSIONLESS CURRENT SHEET MAGNITUDE vs NORMALIZED DISTANCE ALONG WALL
The dimensionless normal pressure on right wall is:

\[ px(k) := -Kz(k) \left[ \frac{\beta y_f(k) + \beta y_d(k)}{2} \right] \]

![Fig 2.15 DIMENSIONLESS NORMAL PRESSURE vs NORMALIZED DISTANCE ALONG WALL](image)

The tangential pressure on the right wall is:

\[ py(k) := Kz(k) (\beta x_f(k) + \beta x_e) \]

![Fig 2.16 DIMENSIONLESS TANGENTIAL PRESSURE vs NORMALIZED DISTANCE ALONG WALL](image)
3.0 Distributed Current Plasma with Decoupled Current Decay & Motion

In Section 1, we considered several cases where the plasma either moved toward a conducting wall while maintaining constant current or which decayed to zero current while motionless. The worst case considered was a constant current, large displacement (60% of the minor radius) toward a wall. Since it is unrealistic to expect a displacement of this magnitude without a current decay, this section considers a simple model combining the effects of motion and current decay.

First we describe the model for the plasma current profile and the manner in which it is assumed to 'scrape-off' with displacement. This is then combined with a model to estimate the normal and tangential pressure distributions on a conducting wall as the plasma displaces and decays. Finally, results are given for typical dimensionless profiles along the wall for parameters based on CIT and CMOD.

Table 3.1 summarizes the results of this section by giving the maximum dimensionless, normal and tangential pressures for different current density profiles (exponential with parameter, $lmb$) as illustrated later in this section. The pressures are then de-normalized by factors appropriate for CIT and CMOD to gain insight into the magnitude of the loads to be expected as a function of displacement and current profile in the plasma. The results indicate that the broad profiles ($lmb\approx$ about 6) lead to loads which are probably manageable from the structural standpoint, but the highly peaked ($lmb=20$) profile leads to loads which are probably not manageable if the displacement, $\delta$, is large. Presumably, the very highly peaked profiles are not realistic, but are useful for comparison with earlier analyses based on a filamentary plasma (see Section 2). The model is somewhat optimistic in the amount of current which can be scraped-off for a given displacement.

3.1 Simplified Distributed Current Plasma Model

We assume that the plasma is circular with an exponential profile, as illustrated in Figure 3.1 where:

\[ J=\text{normalized current density}=\frac{j}{j_0} \]
\[ j=\text{current density at radius } r \]
\[ j_0=\text{current density at } r=0 \text{ (i.e- at the major radius)} \]

\[ r_2=\text{minor radius}\text{=initial distance to wall} \]
\[ \delta=\text{distance moved}/r_2 \]
\[ lmb,a,c=\text{constants which define profile} \]
\[ \sigma=r/r_2=\text{dimensionless radius} \]
\[ k=\text{integer for plotting purposes only} \]
Table 3.1- Maximum Normal and Tangential Pressure Estimates for CIT and CMOD

Note: Plasma Moves and Loses Current

<table>
<thead>
<tr>
<th>Norm Disp Plas Prof Param</th>
<th>Dimensionless Pres</th>
<th>CIT Pres (MPa)</th>
<th>CMOD Pres (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta</td>
<td>max perp</td>
<td>max tang</td>
<td>max perp</td>
</tr>
<tr>
<td>-20 flat</td>
<td>0.018</td>
<td>-0.016</td>
<td>6.06</td>
</tr>
<tr>
<td>2</td>
<td>0.014</td>
<td>-0.016</td>
<td>4.71</td>
</tr>
<tr>
<td>4</td>
<td>0.008</td>
<td>-0.014</td>
<td>2.63</td>
</tr>
<tr>
<td>6</td>
<td>0.006</td>
<td>-0.012</td>
<td>2.02</td>
</tr>
<tr>
<td>10</td>
<td>0.006</td>
<td>-0.01</td>
<td>2.02</td>
</tr>
<tr>
<td>20 peaked</td>
<td>0.157</td>
<td>0.01</td>
<td>52.83</td>
</tr>
<tr>
<td>-20 flat</td>
<td>0.016</td>
<td>-0.016</td>
<td>5.38</td>
</tr>
<tr>
<td>-2</td>
<td>-0.004</td>
<td>-0.01</td>
<td>-1.35</td>
</tr>
<tr>
<td>2</td>
<td>-0.004</td>
<td>-0.0047</td>
<td>-1.35</td>
</tr>
<tr>
<td>4</td>
<td>0.0003</td>
<td>-0.0026</td>
<td>0.10</td>
</tr>
<tr>
<td>6</td>
<td>0.006</td>
<td>-0.0015</td>
<td>2.02</td>
</tr>
<tr>
<td>10</td>
<td>0.011</td>
<td>0.0022</td>
<td>3.70</td>
</tr>
<tr>
<td>20 peaked</td>
<td>0.012</td>
<td>0.003</td>
<td>4.04</td>
</tr>
</tbody>
</table>

Note: assumptions

<table>
<thead>
<tr>
<th>CIT</th>
<th>CMOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>lp, MA</td>
<td>9</td>
</tr>
<tr>
<td>a, m</td>
<td>0.55</td>
</tr>
<tr>
<td>By, T</td>
<td>-1.2</td>
</tr>
<tr>
<td>Bx, T</td>
<td>0</td>
</tr>
</tbody>
</table>

plasma current
minor radius
vert field at wall
rad field at wall
Fig. 3.1  Normalized Plasma Current Density Profile Before and After Displacement $\delta$ Toward Wall
To define the shape of the profile the first step is to choose \( lmb \), then:

\[
\begin{align*}
\text{eg- } & \quad n := 1, 2, \ldots, 5 \\
\text{and } & \quad lmb := -20 \quad lmb := -6 \quad lmb := -2 \\
& \quad lmb := 2 \quad lmb := 20
\end{align*}
\]

The constants \( a_n \) and \( c_n \) to normalize the profile are:

\[
\begin{align*}
a & := 1 - e^{-lmb} \\
c & := 1 - a
\end{align*}
\]

The normalized profile will be assumed to be:

\[
J(\sigma, lmb) := a e^{-lmb \sigma} + c
\]

Typical normalized profiles (before displacement and scrape-off) for selected values of \( lmb \) are shown in the following plot. Note that large negative \( lmb \)'s yield very flat or uniform current density profiles whereas large positive \( lmb \)'s produce very peaked profiles.

Fig 3.2 Typical Normalized Profiles (Assumed)
Before Displacement and Scrape-off
The integral of \( j \) over the plasma cross-section yields the plasma current, \( I_p \), a quantity which we would like to keep constant when comparing results for different profiles later in this section. The procedure will be to specify \( \lambda m b \) as above then choose \( r_2 \) and adjust \( j_0 \) to give the desired value of \( I_p \). This requires \( j_0 = G_n I_p / (\pi r_2^2) \), where the dimensionless function, \( G_n \), is given by:

\[
G := \begin{bmatrix}
     a & -\lambda m b \\
     n & n
\end{bmatrix}
\begin{bmatrix}
     e & -1 \\
     1 + \lambda m b & n
\end{bmatrix}
\begin{bmatrix}
     -1 \\
     -\lambda m b & n
\end{bmatrix}
\begin{bmatrix}
     1 \\
     n
\end{bmatrix}
\]

Next, we specify the dimensionless distance for the plasma to move, \( \delta \), and assume that the current in an annular shell of width \( \delta \) around the outside surface is lost through scrape-off as shown in Fig. 3.3. At the same time, we will assume that the profile collapses with the same shape such that the current density at the newly formed outside edge remains zero. This is probably optimistic since the plasma will react to the attempted current loss with a partial gain to conserve flux which is not trapped on the other side of the wall by the wall currents.

If we define:

\[
\alpha := 1 - \delta \\
J_{\text{del}} := a e - \lambda m b - \frac{1}{n}
\]

\[
f := a e - \alpha e - \lambda m b - \frac{1}{n}
\]

then the dimensionless drop in current due to the motion and scrape-off is given by \( \delta I_p n = (\text{current drop}) / I_p \), where:

\[
\delta I_p := 2 G \frac{J_{\text{del}} + f + \frac{2}{n} \begin{bmatrix}
     a \\
     1 - \alpha
\end{bmatrix}}{n}
\]

and the normalized plasma current, \( I_n \), at the displaced position is given by:

\[
I_n := \begin{bmatrix}
     1 - \delta I_p \\
     n
\end{bmatrix}
\]
Fig. 3.3 A Plasma Moves a Normalized Distance \( \delta \) Toward a Conducting Wall and Simultaneously "Loses" Current in an outer annulus of width, \( \delta \).
3.2 Currents and Pressures Induced in a Conducting Wall

The relationships in this section can be derived directly from those in Section 2. The dimensionless induced current and pressure distributions for the case of motion toward the wall with current decay as specified in the previous section are given by the following.

\[ m := 0, 1 \ldots 25 \quad \text{for plotting purposes only} \]
\[ ymax := 2 \quad \text{max norm distance along wall}=2*r^2/r^2 \]
\[ y := y_{\text{max}} \quad \text{norm distance along wall} \]
\[ m \quad 25 \]

The dimensionless induced current sheet on the wall is given by:

\[ K(n,m) := -1 \]
\[ \frac{1}{\pi^2} \frac{1}{1 - \alpha} \begin{vmatrix} I \\ n \end{vmatrix} \]
\[ \frac{2}{1 + \frac{y}{\alpha + \frac{m}{y}}} \]
\[ \text{where: } \tau = \text{time}/\tau \]
\[ K = \text{current sheet normalized to } I_p/r^2 \]

The \( y \)-directed applied field at the wall is \( \beta y \) which, when normalized to \( \mu I_p/r^2 \), becomes \( \beta y \) and the dimensionless normal pressure on the wall is \( \beta x \), which is the actual pressure normalized to \( \mu I_p^2/r^2 \). Similarly the \( x \)-directed applied field at the wall is \( \beta x \), which is normalized in the same way to \( \beta x \) for use in determining the dimensionless tangential pressure, \( \beta y \).

\[ p_x(n,m) := -K(n,m) \left[ \beta y + \frac{1}{2 \pi} \left( \frac{1 - \tau}{\tau} + \alpha \right) \frac{I}{2} \right] \]
\[ \frac{2}{1 + \frac{y}{\alpha + \frac{m}{y}}} \]
\[ \frac{y}{m} \]
\[ \left( \frac{2 \pi}{1 + \frac{y}{m}} \right) \]

\[ p_y(n,m) := K(n,m) \left[ \beta x - \frac{2}{\tau} \right] \]
\[ \frac{y}{m} \]

Next we apply the formulae derived above to cases using parameters characteristic of CIT and CMOD to illustrate typical dimensionless current and pressure profiles along the walls for the two cases. The normalized \( y \)-directed field for CIT and CMOD differ by only about 12% so we shall use the same value of \( \beta y = -5.5e^{-2} \) in both cases. We shall also assume that the \( x \)-directed applied field is zero in these cases, ie- \( \beta x = 0 \), and evaluate the expressions at the end of the motion, ie- \( \tau = 1 \).
A relatively large normalized displacement of $\delta = 0.6$ will be used in these examples. Note the qualitatively different response for the very peaked ($\lambda = 20$) profile primarily because it essentially loses no current in this model even with this relatively large displacement.

$$\beta_{ye} = -5.5 \times 10^{-2} \quad \beta_{xe} = 0.0 \quad \text{tot} = 1.0 \quad \delta = 0.6$$

**Fig 3.4 Dimensionless Current Sheet Magnitude vs Distance Along the Wall**

The same plot for $p_x$ is given on two different scales to aid in seeing the different results for the different plasma current profiles.

**Fig 3.5 Dimensionless Normal Pressure vs Distance Along the Wall**
Fig 3.6 Dimensionless Normal Pressure vs Distance Along the Wall (note: scale change)

\[ px(1,m), px(2,m), px(3,m), px(4,m), px(5,m) \]

Fig 3.7 Dimensionless Tangential Pressure vs Distance Along Wall

\[ py(1,m), py(2,m), py(3,m), py(4,m), py(5,m) \]
4.0 Filamentary Plasmas with Coupled Current Decay & Motion

In Section 2.0 of this report, induced currents and pressure distributions on a conducting wall were estimated for a stationary plasma filament decaying to zero current on a rapid time scale or for a plasma moving rapidly toward or parallel to a wall while maintaining constant current. Large motions produced the worst case, but it was clear that this could not realistically occur and still maintain constant current. Section 3.0 attempted to account for current decay with motion by peeling off successive layers of current as displacement occurred--this was arbitrary and basically unsatisfactory, although the resulting pressure distributions were more favorable.

In this section, another simple model has been implemented. The plasma is assumed to be maintained in radial equilibrium by the vertical field Bye. At time zero, $\beta$ (poloidal) is assumed to drop to zero suddenly at constant current. The applied field Bye cannot change rapidly so the plasma moves and induces currents which provide a vertical field increment which, together with Bye, holds the plasma in a new equilibrium position. For parameters typical of CIT and CMOD, the model implies that the $\beta$ change results in moderate displacements (i.e., 16 & 10 percent of the minor radii, respectively).

After $\beta$ decays to zero, the plasma current decays to zero. Each increment of the current decay causes the plasma to move again and to induce currents and fields which provide equilibrium at the new position. The assumed equilibrium condition allows the displacement to be found as a function of the current. This model is discussed in Section 4.1 and typical induced current and pressure profiles are given in Section 4.2 for CIT & Section 4.3 for CMOD.

Results of the calculations are summarized in Table 4.1. The first two columns give the fraction of plasma current relative to the initial current and the corresponding dimensionless distance moved toward the wall. The distance moved is normalized to the initial minor radius. These are equilibrium positions for the plasma as its current decays as found with the simplified model and are about the same for CIT and for C-MOD when in dimensionless form, particularly after the current decays below 50% of its initial value.

The next two columns give the maximum value of the pressure on the wall in directions perpendicular to the wall and tangential to the wall (typical distributions along the wall can be found in sections 4.2 & 4.3). These two columns are dimensionless with normalization factors of $\mu*Ip^2/a^2$ & are essentially the same for both machines for large displacements. They have been translated into pressure in MPa in the last two columns. Note that the perpendicular pressure is particularly severe in that it reaches about 40 MPa for C-MOD and about 54 MPa for CIT. Note also that the value rises as the plasma moves toward the wall and that it tends to reach a maximum value. The latter value is about 20 to 40 times the value for the initial displacements which result from the decrease of poloidal beta at constant current.

Column 5 gives the dimensionless perpendicular force. This is the integral of the perpendicular pressure from the midplane to a distance along the wall equal to twice the initial minor radius. The force has been made dimensionless by normalizing to $\mu*Ip^2/a$. Note that, in dimensionless form, it is similar for the two machines, as are the dimensionless pressures. The results show that the force also increases and saturates at a given value.

-31-
Table 4.1a- Maximum Normal and Tangential Pressure Estimates for CIT

<table>
<thead>
<tr>
<th>Plasma Current</th>
<th>Dimensionless Movement</th>
<th>Dimensionless Pressure max perp</th>
<th>Dimensionless Force max tang</th>
<th>CIT Pres (MPa) max perp</th>
<th>CMOD Pres (MPa) max tang</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16</td>
<td>0.008</td>
<td>0.002</td>
<td>3.0E-03</td>
<td>2.69</td>
</tr>
<tr>
<td>0.9</td>
<td>0.295</td>
<td>0.014</td>
<td>-0.002</td>
<td>4.0E-03</td>
<td>4.71</td>
</tr>
<tr>
<td>0.7</td>
<td>0.531</td>
<td>0.028</td>
<td>-0.006</td>
<td>7.0E-03</td>
<td>9.42</td>
</tr>
<tr>
<td>0.5</td>
<td>0.714</td>
<td>0.05</td>
<td>-0.01</td>
<td>1.1E-02</td>
<td>16.82</td>
</tr>
<tr>
<td>0.3</td>
<td>0.853</td>
<td>0.08</td>
<td>-0.014</td>
<td>1.6E-02</td>
<td>26.92</td>
</tr>
<tr>
<td>0.2</td>
<td>0.91</td>
<td>0.15</td>
<td>-0.015</td>
<td>1.9E-02</td>
<td>39.89</td>
</tr>
<tr>
<td>0.1</td>
<td>0.958</td>
<td>0.16</td>
<td>-0.016</td>
<td>2.1E-02</td>
<td>53.84</td>
</tr>
<tr>
<td>0.05</td>
<td>0.98</td>
<td>0.16</td>
<td>-0.016</td>
<td>2.1E-02</td>
<td>53.84</td>
</tr>
</tbody>
</table>

Table 4.1b- Maximum Normal and Tangential Pressure Estimates for CMOD

<table>
<thead>
<tr>
<th>Plasma Current</th>
<th>Dimensionless Movement</th>
<th>Dimensionless Pressure max perp</th>
<th>Dimensionless Force max tang</th>
<th>CMOD Pres (MPa) max perp</th>
<th>CMOD Pres (MPa) max tang</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.004</td>
<td>0.001</td>
<td>2.0E-03</td>
<td>1.00</td>
</tr>
<tr>
<td>0.9</td>
<td>0.25</td>
<td>0.008</td>
<td>-0.002</td>
<td>3.0E-03</td>
<td>1.99</td>
</tr>
<tr>
<td>0.7</td>
<td>0.51</td>
<td>0.023</td>
<td>-0.006</td>
<td>6.0E-03</td>
<td>5.73</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
<td>0.044</td>
<td>-0.01</td>
<td>1.0E-02</td>
<td>10.97</td>
</tr>
<tr>
<td>0.3</td>
<td>0.85</td>
<td>0.083</td>
<td>-0.014</td>
<td>1.6E-02</td>
<td>20.69</td>
</tr>
<tr>
<td>0.2</td>
<td>0.91</td>
<td>0.085</td>
<td>-0.015</td>
<td>1.8E-02</td>
<td>21.19</td>
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<tr>
<td>0.1</td>
<td>0.96</td>
<td>0.16</td>
<td>-0.016</td>
<td>2.1E-02</td>
<td>39.89</td>
</tr>
<tr>
<td>0.05</td>
<td>0.98</td>
<td>0.16</td>
<td>-0.016</td>
<td>2.1E-02</td>
<td>39.89</td>
</tr>
</tbody>
</table>

Notes: assumptions: CIT, CMOD

- Ip, MA: plasma current
- R, m.: major radius
- a, m.: minor radius
- By, T: vertical field at wall
- Bx, T: radial field at wall

Dimensionless pressure is pressure/((4e-7*π*Ip^2/a^2))
Dimensionless force is force/((4e-7*π*Ip^2/a))
Force is integral of perpendicular pressure from y=0 to 2*a
as the wall is approached by the plasma, but that the large displacement value is about 7 to 10 times the initial displacement value. This implies that the pressure, which grew to a higher ratio, is becoming more localized. This is also evident in the plots of the pressure distributions.

4.1 Plasma Equilibrium Model

\[ \mu := \pi \cdot 4 \cdot 10 \]

\[ \text{Ip}= \text{plasma initial current} \]

\[ R= \text{major radius} \]

\[ r_2= \text{minor radius before displacement} \]

\[ r_1= \text{distance to wall after displacement} \]

The externally applied field required for equilibrium is given by:

\[ \text{Bye} := -\ln \left( \frac{8}{r_2} \right) + \beta \frac{\mu \cdot \text{Ip}}{4 \pi R} \]

If \( \beta \) becomes zero and the current changes to \( \text{Ip}_n \) then the required vertical field must change accordingly.

However, the PF coils cannot change the vertical field on the time scale of the \( \beta \) change, so the plasma moves until the field it induces can produce the required change and, in turn, a new equilibrium position. The plasma motion is shown in normalized space in Fig 4.1. The induced currents in the conducting wall produce an induced field which is given by:

\[ \text{Bi} := \left[ \frac{\text{Ip}_n - \text{Ip}}{2 \cdot r_1} \right] \frac{\mu}{r_1 + r_2} \]

If we now set \( \text{Bi}=\text{Bv} \) and define:

\[ \delta = 1 - \frac{r_1}{r_2} = \text{dimensionless displacement} \]

\[ \text{Ip}_n = \text{plasma current at new equilibrium position} \]

\[ \text{In} = \text{normalized current}=\text{Ip}_n/\text{Ip} \]

\[ s = \frac{r_2}{R} = \text{inverse aspect ratio} \]

The relationship which must be satisfied at the new equilibrium position is:

\[ \frac{\text{In} \cdot (2 - \delta) - 2 \cdot (1 - \delta)}{(1 - \delta) \cdot (2 - \delta)} := -\text{In} \cdot s \cdot \ln \left( \frac{8}{s \cdot (1 - \delta)} \right) + s \cdot \frac{1 - \text{In}}{4} + \beta + \ln \left( \frac{8}{s} \right) \]

For a given \( \beta \) and inverse aspect ratio, \( s \), the previous equation can be solved for the displacement \( \delta \) in terms of the current fraction \( \text{In} \). For both CIT and C-MOD these dimensionless results are given in the following table.
Fig. 4.1 Plasma with initial current $I_p$ undergoes a poloidal $\beta$ change to zero and subsequent current decrease to $I_{pn}$ which leads to a new equilibrium position.
The table shows that, in dimensionless terms, the initial displacement following the change in beta poloidal to zero at constant current is about 10% of the minor radius for CMOD and about 16% of the minor radius for CIT. As the current decays the dimensionless displacements for the two cases approach one another.

4.2 Typical Induced Current & Pressure Profiles for CIT

The relationships in this section can be derived directly from those in Section 2.0 of this report. The dimensionless induced current and pressure distributions for the case of motion toward the wall with current decay as specified in the previous section are given by the following.

\[ m = 0, 1 \ldots 100 \] for plotting purposes only

\[ y_{\text{max}} = 2 \] max norm distance along wall = \(2r_2/r_2\)

\[ y = y_{\text{max}} \frac{m}{100} \] norm distance along wall

\[ n = 1.2 \ldots 5 \]

\[ \delta = 0.16 \quad \delta = 0.30 \quad \delta = 0.53 \quad \delta = 0.71 \quad \delta = 0.85 \]

\[ I = 1.0 \quad I = 0.9 \quad I = 0.7 \quad I = 0.5 \quad I = 0.3 \]
\[
\alpha := 1 - \frac{\delta}{n}
\]

The dimensionless induced current sheet on the wall is given by:

\[
K(n, m) := \pi \left[ \begin{array}{ccc}
1 - \alpha & I_n & 1 \text{ tor} \\
2 & n & 2 \\
1 + y & \alpha + y & m & n
\end{array} \right]
\]

where: \( \text{tor} = \text{time}/r \)

The \( y \)-directed applied field at the wall is \( B_y \) which, when normalized to \( \mu Ip/r^2 \), becomes \( \beta_y \) and the dimensionless normal pressure on the wall is \( p_x \), which is the actual pressure normalized to \( \mu Ip^2/r^2 \). Similarly the \( x \)-directed applied field at the wall is \( B_x \), which is normalized in the same way to \( \beta_x \) for use in determining the dimensionless tangential pressure, \( p_y \).

\[
px(n, m) := -K(n, m) \left[ \begin{array}{ccc}
\beta_y & 1 & 1 - \text{tor} \\
2 & 2 & n \\
1 + y & \alpha + y & m & n
\end{array} \right]
\]

\[
py(n, m) := K(n, m) \left[ \begin{array}{ccc}
\beta_x & y \\
(2 \pi) & 1 + y & m
\end{array} \right]
\]

In this section we apply the formulae derived in the previous section to cases using parameters characteristic of CIT to illustrate typical dimensionless current and pressure profiles along the walls for this case. The normalized \( y \)-directed field for CIT and CMOD differ by only about 10% so we shall use the same value of \( \beta_y = -5.5 \times 10^{-2} \) in both cases. We shall also assume that the \( x \)-directed applied field is zero in these cases, ie- \( \beta_x = 0 \), and evaluate the expressions at the end of the motion, ie- \( \text{tor} = 1 \).

\[
\beta_y = -5.5 \times 10^{-2} \quad \beta_x = 0.0 \quad \text{tor} = 1.0
\]
NORMALIZED CURRENT SHEET AMPLITUDE

The same plot for px is given on two different scales to aid in seeing the different results for the different plasma current profiles.

NORMALIZED PRESSURE PERPENDICULAR TO WALL
NORMALIZED PRESSURE PERPENDICULAR TO WALL

\[ px(1,m), px(2,m), px(3,m), px(4,m), px(5,m) \]

NORMALIZED TANGENTIAL PRESSURE ALONG WALL:

\[ py(1,m), py(2,m), py(3,m), py(4,m), py(5,m) \]

4.3 Typical Induced Current & Pressure Profiles for CMOD

The equations in this section are identical to those in the previous section but are repeated here for plotting purposes only.

\[ m := 0, 1 \ldots 100 \]
\[ y_{\text{max}} := 2 \quad \text{max norm distance along wall}=2*r2/r2 \]
\[ y := y_{\text{max}} \quad \text{norm distance along wall} \]
\[ m \]
\[ n := 1, 2 \ldots 5 \]
\[ \delta := 0.85 \quad \delta := 0.70 \quad \delta := 0.51 \quad \delta := 0.25 \quad \delta := 0.1 \]
\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \]
\[ I := 0.3 \quad I := 0.5 \quad I := 0.7 \quad I := 0.9 \quad I := 1.0 \]
\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \]
\( \alpha := 1 - \frac{\delta}{n} \)

The dimensionless induced current sheet on the wall is given by:

\[
K(n,m) := \pi^{-1} \left( \begin{array}{c} 1 \\ 1 + y \end{array} \right) \frac{I_n}{n} \frac{t \tau}{\gamma} = \left( \begin{array}{c} 1 \\ 2 \end{array} \right) \frac{\alpha}{2} \left( \begin{array}{c} 1 + y \\ 1 + y \end{array} \right) \frac{\alpha + y}{m} \]

where: \( t \tau := \text{time}/\gamma \)

\( K := \text{current sheet normalized to } I_p/r^2 \)

\[
px(n,m) := -K(n,m) \left[ \beta y e + \frac{1}{2 \pi} \left( \begin{array}{c} 1 - t \tau \end{array} \right) + \alpha \frac{I_n}{n} \frac{t \tau}{\gamma} \right] \\
py(n,m) := K(n,m) \left[ \beta x e - \frac{1}{(2 \pi)} \left( \begin{array}{c} 2 \\ 1 + y \end{array} \right) \right]
\]

In this section we apply the formulae derived in the previous section to cases using parameters characteristic of CMOD to illustrate typical dimensionless current and pressure profiles along the walls for this case. The normalized y-directed field for CIT and CMOD differ by only about 10% so we shall use the same value of \( \beta y e = -5.5 \times 10^{-2} \) in both cases. We shall also assume that the x-directed applied field is zero in these cases, ie- \( \beta x e = 0.0 \), and evaluate the expressions at the end of the motion, ie- \( t \tau = 1.0 \).

\( \beta y e = -5.5 \times 10^{-2} \quad \beta x e = 0.0 \quad t \tau = 1.0 \)
The same plot for px is given on two different scales to aid in seeing the different results for the different plasma current profiles.
NORMALIZED PRESSURE PERPENDICULAR TO WALL

\[ px(1,m), px(2,m), px(3,m), px(4,m), px(5,m) \]

NORMALIZED TANGENTIAL PRESSURE ALONG WALL:

\[ py(1,m), py(2,m), py(3,m), py(4,m), py(5,m) \]

Dimensionless normal force perpendicular to wall:

\[ F_n = \frac{y_{\text{max}}}{100} \sum_{m} px(n,m) \]

\[ \begin{array}{c}
F_n \\
0.016 \\
0.01 \\
0.006 \\
0.003 \\
0.002
\end{array} \]