PFC/RR-90-13

A Synopsis of Collective Alpha Effects
and Implications for ITER

D. J. Sigmar

October 1990

Massachusetts Institute of Technology
Plasma Fusion Center
Cambridge, MA, USA

This work was supported by the Department of Energy D & T Grant No. DE-FG02-91ER-54110. Reproduction, translation, publication, use and disposal, in whole or part, by or for the United States government is permitted.
A Synopsis of Collective Alpha Effects and Implications for ITER

Contents

1. Introduction
2. Alpha Interaction with Toroidal Alfvén Eigenmodes
3. Alpha Interaction with Ballooning Modes
4. Alpha Interaction with Fishbone Oscillations
5. Implications for ITER
1 Introductory Remarks

So far, for the most part, ignited tokamak studies have assumed collisional (classical) alpha behavior. However, the history of tokamak plasma physics has shown classical transport mechanisms (such as neoclassical resistivity, bootstrap current, central impurity peaking and collisional NB ion slowing down) to belong to a (mostly welcome) minority class of effects but anomalous phenomena to represent the majority of transport mechanisms. History has also shown that in each transition to a new phase of plasma heating (from ohmic to NB to RF) new and unexpected effects arose (viz. saturation of $\tau_E$ with $n$, the density limit, degradation of $\tau_E$ with $P_{aux}$, the beta limit, but also positive discoveries such as the enhanced ohmic confinement, H-mode and sawtooth stabilization with ICRH.) Thus, we are preparing for additional surprises as we enter the super-Alfvénic and alpha power dominated regime ($Q \geq 5$), by theoretically delineating optimal operating regions for good confinement of the energetic alphas but also by exploring the foundations for a cautious optimism about the advantages of the intrinsically central alpha power deposition profile [1].

The collective oscillations produced by energetic alpha particles (with $v_{\parallel} > v_{Alfven}$ which holds even for high magnetic fields $B \geq 10$ T!) are in a special regime which is difficult to simulate in present tokamaks, particularly when the parallel wave particle resonance $v_{||\alpha} - (\omega/k||)_{Alfven}$ is involved in the Cherenkov excitation of the shear Alfvén spectrum. While this condition can be met using parallel NBI at degraded values of $B$, ICRH appears unsuitable because of its predominantly perpendicular energy spectrum of the energetic tail ions. Presently, these circumstances put the topic of alpha driven collective effects in a pre-paradigmatic stage, at least until the advent of D-T operation in the TFTR and JET experiments which will produce isotropic alpha particle distribution functions. Theoretical and numerical modelling efforts of the alpha driven TAE have recently surged worldwide, (cf. [2]) and specific ITER design information regarding collective alpha effects may become available by the mid 1990’s. Overall, particularly with regard to alpha driven collective effects, a well coordinated collaboration between theory and experiment will be essential in order to validate the intricacies of the linear and nonlinear theoretical predictions.
For the remainder, in subsections 2-4 we will specifically discuss the alpha driven toroidal Alfvén eigenmode (TAE), alpha driven ballooning modes, and alpha fishbones. Other important collective effects concerning the alpha driven components of the total fluctuation spectrum (which might increase anomalous bulk plasma transport but might also provide desired anomalous outward transport of the He-ash) are too little understood to be included here. In subsection 5, the possible impact of these effects on ITER design will be discussed together with new R & D needs for the next 2-3 year and 3-5 year periods.

2 Toroidal Alfvén Eigenmodes Destabilized by Super-Alfvénic Alphas

By the mid 1970's Mikhailovskii et al [3] and Rosenbluth et al [4] had discovered this fundamental low mode number instability mechanism (albeit for cylindrical Alfvén eigenmodes, worked out in detail in [26]) with a growth rate of $10^{-2}$ the Alfvén frequency. After the discovery of the toroidicity induced spectral gaps by Kieras et al [5] and C. Z. Cheng et al [6], Fu and Van Dam [7] demonstrated analytically an even faster growth rate of $2.5 \times 10^{-2} \times \omega_A$ in toroidal geometry, verified numerically by the Nova-K code of C. Z. Cheng [8]. Sigmar et al [9] then investigated the single alpha guiding center resonant motion given the exact radial mode $n = 1$ structure from Nova-K and assuming perturbed amplitudes $10^{-5} \leq \tilde{B}_r/B_o \leq 10^{-3}$. They also performed a Monte Carlo simulation of 5120 randomly distributed alphas. Resonant $(\omega - k_{\parallel}v_{\parallel\alpha} - k_{\perp}v_{D\alpha} = 0)$ losses of circulating alphas producing secular radial motion out of the system were observed for $\tilde{B}_r/B_o \geq 10^{-4}$, indicating possible fast alpha anomalous transport on a time scale comparable to the alpha slowing down time. However, this simulation, by assuming $\tilde{B}_r$ as given, was not self-consistent and did not answer questions about the wave particle resonance coherence time and stochastic transport in the fully developed multi-mode turbulence. A step in this direction is the work by Berk and Breizman [10], and further work by the authors of Ref. [9] on the diffusion in angle action space, with finite orbit and mode structure effects, is in progress. Also, still under investigation is the linear damping of the alpha driven TAE due to a stabilizing coupling effect of the main gap mode to continuum modes near the plasma edge [11]. A fully self-consistent computational effort evolving the finite
amplitude MHD waves nonlinearly with a fluid code and the alphas by a particle pushing algorithm has been started most recently [12].

In turn, this intense theoretical and numerical effort has stimulated major experimental efforts. Historically, NBI driven instabilities in conditions with \( v_{b\parallel} > v_A \) were suspected in ISX-B and JFT-2 and reported in T-11 by Leonov, Merezhikin, Mukhovatov et al [13] when \( \langle \beta_T \rangle \) exceeded 2% (at a highly peaked \( \beta_T(0) \approx 9\% \)). They observed periodic drops in the \( \beta_{pol} \) (diamagnetic) signal accompanied by a positive voltage spike. Recently, K. L. Wong has started a systematic effort in TFTR [14] at low values of \( B = 1 \) T, \( I_p \geq 400 \) kA, \( E_b = 95 - 110 \) keV. When \( \bar{v}_{\parallel \text{beam}}/v_A > .7 \) Alfvén frequency bursts (\( \leq 90 \) kHz) are observed at \( n = 3 \) but the mode number details and fast ion loss details (5% drop in neutron signal) have not yet been determined. An ITER relevant experimental-theoretical plan for the next four years of TFTR operation (through its D-T phase) has been outlined to compare the theoretical and experimental threshold conditions, and the effect of Alfvén turbulence on fast ion and alpha confinement. On DIII-D, Heidbrink et al [15] have conducted a series of experiments focussed on realizing the necessary conditions for the TAE instability, namely \( v_{\parallel \text{beam}} > v_A \), \( \beta_b > \beta_{b \text{ crit}} \) (a threshold value) and the beam density profile scale \( L_b \equiv \left( \frac{1}{n_b} \frac{dn_b}{dr} \right)^{-1} \) being short enough to allow \( \omega_{sb}/\omega_A > \frac{1}{2} \) which is a necessary instability condition. Toroidal mode numbers \( n = 3 - 9 \) and new features in the frequency spectrum around the Alfvén frequency (78 kHz for \( n = 4 \)) were observed when the beam beta exceeded \( \langle \beta_b \rangle = 2\% \), at \( \bar{v}_{\parallel \text{beam}}/v_A \geq .8 \). Here, \( \bar{v}_{\parallel \text{beam}} \) is the parallel beam velocity averaged over a slowing down distribution. Theoretically, for the DIII-D q-profile and parameters, the \( n = 4 \) TAE should indeed be unstable but the \( \langle \beta_b \rangle \) threshold observed to be necessary for instability is at \( \sim 2\% \), which is \( \approx 7 \) times larger than the theoretical one. This discrepancy would allow for the possibility that the “standard” TAE mode (with its theoretical \( \langle \beta_\alpha \rangle \) threshold of \( \approx .2\% \)) may be damped more than presently understood but a pressure driven variant may be triggered in high \( \beta_p \) discharges approaching the \( \beta \) limit. To relate this to alpha physics in a burning plasma, a \( \langle \beta_\alpha \rangle \simeq .2\% \) threshold could be realized at high central temperatures of \( T(o) \simeq 30 \) keV, see Figs. 1a and 1b computed by C. Z. Cheng [8] for ITER. (The figure caption provides further comments.) More discussion for ITER will be given in subsection 5.
3. Ballooning Modes Destabilized by Alphas

After early suggestions by Rosenbluth et al [16] to stabilize the low frequency ($\omega_r \sim \omega_{ci}$) branch of the mode using energetic trapped ions, Spong, Sigmar et al [17a, 17b] and Rewoldt [18] found destabilization of the ballooning mode due to trapped and circulating alphas to occur at higher values of $\omega_r$ (reaching up to the Alfvén frequency $\omega_A = k_B v_A$). In fact, in this range, the underlying MHD oscillations have been identified as high mode number TAE (gap) modes [19]. For trapped alphas the instability mechanism is the alpha banana precession frequency resonance with $\omega_r$, for the circulating alphas it is a transit resonance of the alphas near the circulating trapped boundary. Figure 2 from Ref. [19] shows the critical $\langle \beta \rangle$ as a function of minor plasma radius, for an ITER equilibrium. One notices a lowering of the $\langle \beta \rangle$ threshold (due to the intrinsic trapped alpha particle population). Reference [18] shows a similar effect obtained from an entirely different kinetic code which includes trapped and circulating alpha contributions. This lowering of the high mode number instability threshold at around $\beta_{Toyon}/4$ could affect the alphas by producing an anomalous alpha diffusion coefficient (which is estimated in [18] to be $D_\alpha \sim \chi_e \sim O(1)$ m$^2$/sec without however providing a self-consistent saturation calculation for this microturbulence). Simultaneously, the alpha stimulated ballooning mode spectrum could enhance the electron thermal conductivity $\chi_e$ of the bulk plasma thereby contributing to the $P_{heat}$ driven degradation of $\tau_E$. Concerning the anomalous diffusion of alphas due to higher mode number ballooning perturbations it has been shown [22] that stochastic transport is reduced inversely with $k_\perp \rho_\alpha > 1$ due to the "orbit averaging" effect. Thus only intermediate mode numbers ($n \lesssim 10$ such as observed in DIII-D [15]) may be contributing to transport.

So far, the ballooning mode instability and its transport in present day plasmas has not yet been clearly identified, although this mechanism (including resistive ballooning mode turbulence) was suspected to be responsible right from the first observations of $\tau_E$ degradation with increasing neutral beam power in ISX-B [20]. In recent experiments, e.g. in JET, there is an observation of an anomalously degraded fast ion energy distribution of the ICRH minority heated ions at lower plasma currents ($I_p = 2$ MA) [21] which is partially
but not totally explainable by large orbit effects of the energetic ions, presumably due to enhanced collisional slowing down on the colder electrons in the edge plasma. More detailed modelling is required here.

Returning to the above mentioned neutral beam driven instability in T-11 (Ref. [13]), the observed sharp periodic drops in $\beta_p$ (diamagnetic) may suggest a super-Alfvénic beam destabilization of the ballooning mode rather than of the low mode number TAE discussed in the previous subsection. With the exception of a transition from low toroidal mode numbers to higher ones, both of these modes are toroidal Alfvén gap modes destabilized by fast ions plus – in the case of alpha ballooning – the additional pressure drive of the bulk plasma. In fact, Heidbrink [15] mentions the possibility of a combination with pressure driven fluctuations.

We note that both experiments (T-11 and DIII-D) produced their instability with super-Alfvénic velocities mainly parallel to the magnetic field, i.e. without the trapped alpha component underlying Refs. [17] and [19]. However, in Ref. [18] it is clearly established that circulating alphas are very effective in destabilizing the ballooning mode. Thus, also from this point of view the $^3$He minority heating experiments in JET [21] may not constitute conclusive alpha simulations, notwithstanding their impressively large but perpendicular fast ion population.

4 Alpha Driven Fishbone Oscillations

Experimentally, NB driven fishboning was a strong effect in PDX which reduced the heating efficiency. Similarly, in a burning plasma, alpha particles can affect the $m = n = 1$ internal kink modes in the vicinity of the $q = 1$ surface. With a sufficient number density of hot trapped ions such as produced by ICRH or fusion burn the sawtooth oscillations can be stabilized [23, 24] but C. Z. Cheng has found (using the Nova-K code's capability to include all hot ion pitch angles and a finite aspect ratio, shaped plasma cross section equilibrium) that alphas near the passing trapped boundary may overwhelm the stabilizing effect of the trapped ions. Thus again, ICRH produced energetic trapped ions may not be
able to adequately simulate the collective alpha effects. Verification will have to await the combined ICRH NB heating scenarios of TFTR, JET and JT-60 U.

When the number of hot ions or alphas becomes too large the fishbone instability is triggered by the trapped alpha precession resonance with the mode frequency. Figure 3 shows the threshold condition, albeit for a CIT-like plasma [30], at a rather high temperature of around 20 keV. Depending on the shear at the $q = 1$ surface, this instability exists over a range of real frequencies ranging from $\omega_{\alpha}$ to $\langle \omega_D \rangle$, carried by a low frequency MHD oscillation of the background plasma [24] or undergoing a transition to a hot species oscillation [23] at high frequencies [23]. In either case, energetic alpha bananas may be ejected in bursts, periodically increasing the wall loading on the outboard side of the tokamak and reducing the energetic alpha energy coupling to the bulk plasma. The lower frequency fishbone branch may however produce a desirable loss of epithermal He-ash. (In this context, present He injection experiments to study the particle transport cannot be conclusive as long as the total turbulent fluctuation spectrum driving radial transport does not contain the Alfvén spectrum contributions characteristic of the burning plasma. For He-ash outward transport, these contributions could be beneficial.)

5 Implications for ITER

As presented above, the alpha driven Shear Alfvén turbulence (TAE) ballooning mode fluctuations and fishbone oscillations all indicate a potentially less than classical alpha confinement with implications for undesirable additional first wall loading by energetically charged particles and reduced coupling of $P_{\alpha}$ to the plasma. Here we discuss some of the detailed questions in this regard and indicate stable operating regions, as far as that is presently possible.

Before going into such detail, the effect of a reduced coupling of the alpha power at birth ($P_{\alpha 0} = S f E_{\alpha 0}$, $S f \sim \langle \sigma f v \rangle n_e^2/4$) to the background plasma can be modelled generically as follows. Given an anomalous $D_{\alpha}$ and ensuing fast alpha loss frequency

$$\nu_L = 4D_{\alpha}/a^2$$

(1)
it follows readily from alpha power balance that the coupling parameters \( \eta_\alpha \equiv \frac{P_\alpha}{P_{\alpha \alpha}} \) is given by

\[
\eta_\alpha = 1 - \nu_L \frac{n_\alpha E_\alpha}{S_f E_{\alpha \alpha}}
\]  

(2)

where in steady state \( n_\alpha/S_f = \tau_{SD} \), the \( \alpha \) slowing down time. (\( n_\alpha E_\alpha \) is the velocity distribution \( f_\alpha \) averaged alpha pressure including the effect of \( \nu_L \) on the kinetic equation \( f_\alpha \). If \( \nu_L \frac{E_\alpha}{S} > 1 \) the slowing down distribution would be strongly affected.) Given \( \eta_\alpha \), we follow a recent study of D. Cohn [27].

The bulk plasma power balance \( \frac{3nT}{\tau} \equiv P_{\text{loss}} = P_{\text{heat}} + \eta_\alpha P_\alpha \) and the definition \( Q = \frac{5P_\alpha}{P_{\text{loss}}} \) yields

\[
Q = \frac{5P_\alpha}{P_{\text{loss}}} \left[ 1 - \eta_\alpha \left( \frac{P_\alpha}{P_{\text{loss}}} \right) \right]^{-1}.
\]  

(3a)

Defining

\[
(n\tau)_* \equiv 12T/(\sigma_f v)E_{\alpha \alpha}, \quad \frac{P_\alpha}{P_{\text{loss}}} = \frac{n\tau}{(n\tau)_*}.
\]  

(3b)

Note that \( (n\tau)_* \) is a function of temperature and if \( \eta_\alpha = 1 \), achieving \( n\tau = (n\tau)_* \) describes ignition. We rewrite Eq. (3a) as

\[
Q = 5 \left( \frac{n\tau}{n\tau_*} \right) \left[ 1 - \eta_\alpha \frac{n\tau}{(n\tau)_*} \right]^{-1}
\]  

(3c)

and obtain Fig. 4, showing (i) how for a given normalized ignition margin \( n\tau T/(n\tau)_* T \), \( Q \) drops as \( \eta_\alpha \) decreases below 1, and (ii) how, for a desired \( Q \), the ignition margin has to be increased as \( \eta_\alpha \) drops. From Eqs. (2) and (3) it thus becomes apparent how collective alpha effects producing an anomalous \( D_\alpha \) could impact ITER performance. E.g., for the above mentioned \( \alpha \) ballooning mode transport, \( \eta_\alpha \simeq .9 \) has been inferred [30]. It must be emphasized strongly, however, that the state-of-the-art knowledge of \( D_\alpha \) is such that quantitative design guidelines are premature. In what follows we will discuss some of the known detailed processes producing a \( D_\alpha \) and mention ways to minimize or avoid the underlying mechanisms.
A. TAE

Nonlinearly, there are several competitive paths to saturation. Resonant passing alphas – while drifting out radially – can give up energy while conserving magnetic moment. Thus, \( \Delta \lambda = \mu B_0 \Delta (E^{-1}) \) can scatter them towards trapped alpha space thereby feeding them into the stochastic ripple domain. This synergistic loss will be reduced if the \( \omega_A - k_{||} v_{||} \) resonance is modified in velocity space [10], or if \( n_{\alpha}(r) \) flattens quasilinearly until \( (\omega_{*\alpha}/\omega - \frac{1}{2}) = 0 \), i.e. the local growth rate [7] vanishes:

\[
\frac{\gamma}{\omega_A} = \frac{9}{4} \left[ \beta_\alpha \left( \frac{\omega_{*\alpha}}{\omega_A} - \frac{1}{2} \right) \right] F \left( \frac{v_A}{\bar{v}_\alpha} \right) - \beta_e \frac{v_A}{v_{the}} = 0
\]

where \( \omega_{*\alpha} = m \rho_\alpha/L_A^2 \), \( \omega_A = v_A/2R_0 q \) (gap) and \( L_A \) is the \( n_{\alpha}(r) \) scale length. Noticing the sensitivity to \( L_A \) in Fig. 1a suggests solving (1) for the marginal value of \( L_A \) and to compare the ensuing \( n_{\alpha} \) profile to the alpha birth profile \( \propto (n_e^2 T^2 \sigma_v f)/T^2 \). Roughly, \( L_A \) (marginal) = .4 vs \( L_A \) (birth) = .25. More accurately, Eq. (4) should be replaced by Cheng's growth rate expression [8] using the alpha slowing down distribution instead of the Maxwellian implied in (4) and using the global solution of the Nova-K code in lieu of a local growth rate. More importantly, the additional damping of a single TAE on continuum modes near the plasma edge may alter Eq. (4) substantially. (This problem is under active investigation, cf. [11].) A broadening of the alpha density and thus alpha pressure profile in the central low shear region could affect the Troyon constant of the bulk plasma and in the alpha ballooning expression, to be discussed below.

Such broadening of \( n_{\alpha}(r) \) can also be expected if the interaction of several \( n/m \) gap modes produces alpha orbit stochasticity and a concomitant \( D_{\alpha} \) [26]. The self-consistent fluctuation amplitude will ultimately be determined experimentally but meanwhile, the linear marginal stability boundary can be explored in alpha simulation experiments and through D-T operation in TFTR and JET. Figure 1a shows, in conjunction with Fig. 1b, that the \( \langle \beta_\alpha \rangle \) threshold will always be exceeded but if necessary, ITER could consider working at lower values of \( \bar{v}_\alpha/v_{Alfven} \) (presently \( \bar{v}_\alpha/v_A \approx 1.5 \)).

9
B. Alpha Ballooning Modes

As can be seen from Figs. 2a,b, alpha ballooning driven \( D_\alpha \neq 0 \) values can be minimized by avoiding operation too close to the beta limit. Conversely, as discussed, the T-11 and DIII-D \( v_{\|b} > v_A \) experiments (which in order to lower the Alfvén speed work close to the \( \beta \) limit) gave indications of a pressure driven instability leading (e.g. for T-11) to periodic dumps of the plasma stored energy. Another possibility to avoid \( \beta \) limit activity would be operation at \( q(0) > 1 \) aiming for the second stability region. Recent work by Ramos [28] shows a relative ease of stabilizing the ballooning (high-\( n \)) mode in this way using very smooth \( j_\phi \) profiles vanishing at the edge. The second stability theory of kinetic ballooning modes including the alpha particle resonances has not yet been worked out but in present alpha simulation experiments it should not be too difficult to explore \( q(0) > 1 \) operation in conjunction with \( v_{\|b} > v_A \). Applied to the high poloidal beta technology phase of ITER these considerations suggest an operating regime stable against alpha ballooning driven fast alpha losses and, to the degree that the degradation of \( \tau_E \) with total power (including \( P_\alpha \)) is caused in part by high-\( n \) ballooning mode activity, operation at \( q(0) > 0 \) may remove the alpha ballooning driven part of the \( \tau_E \) degradation.

C. Alpha Fishbones

As can be inferred from Fig. 3 (for CIT) alpha fishbones require large values of \( n_\alpha \). To evaluate the alpha fishbone instability threshold for ITER we follow the work of R. White et al [23]. Fishbone oscillations set in when \( n_\alpha > n_{\alpha \text{ crit}} \), the normalized ideal kink growth rate \( \hat{\gamma}_I \) lies inside the interval

\[
(S_o/S)^{1/3} \hat{\gamma}_M < \hat{\gamma}_I < \hat{\gamma}_M
\]

where \( S/S_o \) is a normalized magnetic Reynolds number (\( \sim 5 \) for ITER) which must exceed 1 for instability, and \( \hat{\gamma}_I, \hat{\gamma}_M \) (which is the maximum of \( \hat{\gamma}_I \) vs \( \omega_\ast/\omega_d \)) can be found from Fig. 7 of Ref. [23]. The threshold fast alpha density follows from Eq. (40) of [23]:

\[
n_{\alpha \text{ crit}} = \left[ 20\beta_{hc} n_{i0}^{1/2} (r_q')_{r_1} \left( \frac{m_i}{m_p} \right)^{1/2} / R_o \right] \times 10^{11} \text{ cm}^{-3}
\]
where \( R_o \) is in meters. Taking a slowing down alpha distribution for which \( \hat{\beta}_{hc} = .4 \) (see [23]), and \( n_{20} = 1 \), \( (r q')_{r_1} \approx \frac{r_1}{R_o} = \frac{1}{8} \), one gets \( n_{\alpha \text{ crit}} = 2.6 \times 10^{10} \text{ cm}^{-3} \). Using Fig. 1b gives values \( n_{\alpha} \) greater than that for temperatures exceeding 15 keV. Thus, alpha fishbones may occur depending on the shear parameter at \( q = 1 \). This needs to be explored carefully for ITER, to ascertain and explore possible scaling differences between ITER and CIT (which may also be subject to alpha fishboning). Experimentally, it should be possible to perform alpha simulation experiments with a combination of NB and ICR minority heating to validate our theoretical expectations for ITER burning plasma parameters but the shear at the \( q = 1 \) location in a strongly burning plasma awaits exploration in the first burning plasma experiments.

Conclusion

Theoretical and experimental evidence for the alpha driven collective effects of resonant excitation of the shear Alfvén spectrum, for alpha ballooning (at medium range toroidal mode numbers \( 3 \leq n \leq 10 \)) and alpha fishboning is firm enough (and growing) to warrant R & D efforts to determine possible anomalous fast alpha diffusion due to these effects. Collaboration has begun between analytical and numerical physicists and experimentalists on several large tokamaks. The importance of producing a velocity distribution function approaching the isotropic one of the fusion born alphas is beginning to be recognized and will require additional experiments with a combination of strong NBI and ICRH minority heating during the next 2-3 year period of ITER R & D. Thereafter, D-T operation in TFTR and JET will provide the next set of answers. (Here, it is useful to keep Fig. 1b in mind which shows that fusion plasma-like values of \( \beta_{\alpha} \) and \( n_{\alpha} \) can be produced at high temperatures even in first generation burning plasma experiments with low values of \( Q \), cf. [29].)
References


Figure Captions

Fig. 1a: Global eigenvalue calculation [8] delineating the TAE stability boundary for standard ITER equilibrium. $L_\alpha/a$ is the ratio of alpha density gradient scale to minor radius. If typically, $\beta_{\text{thermal}} = 4\%$, $\bar{v}_\alpha/v_A \simeq 1.5$ and taking thermal D-T fusion one has at $T_e = 10$ keV, $\beta_\alpha = 2 \times 10^{-3}$; at $T_e = 20$ keV, $\beta_\alpha = 8 \times 10^{-3}$, see arrows.

Fig. 1b: Ratio $\beta_\alpha/\beta_{\text{thermal}}$ produced by thermalized D-T ions, as a function of plasma temperature ($T_e = T_i = T$). From Y. Sung, G. Bateman, PPPL, in [29].

Fig. 2a: Critical ($\beta$) for alpha destabilized ballooning mode, vs normalized plasma minor radius [19]. The beta limit is significantly reduced by the alphas. ITER parameters.

Fig. 2b: Linear growth rate of alpha destabilized ballooning mode vs $\beta$ for an older $R_o = 1.75$ m CIT design [18]. The beta limit is significantly reduced by the alphas.

Fig. 3: Approximate thresholds for high-n ballooning, fishbone and TAE for ITER, CIT and TFTR [29] and [8].

Fig. 4: Thermonuclear $Q$ vs normalized ignition margin for different values of power coupling parameter $\eta_\alpha = P_\alpha \text{ (effective)}/P_\alpha \text{ (at birth)}$ [27].
Figure 1a

\[ \frac{L_a}{a} = 0.25 \]
\[ T_{e0} = T_{i0} = 20 \text{keV} \]

Figure 1b

\[ \frac{\beta_a}{\beta_{Th}} \]
\[ \frac{n_a}{n_{Th}} \]

\[ T_i = T_e (\text{keV}) \]
\[ \kappa = 2.0, \delta = 0.4, \epsilon = 0.34 \]

Figure 2a

Figure 2b
Figure 3

Figure 4