ITER MAGNETICS PROGRAM

CS Model Coil
Fracture Mechanics Analysis
of Incoloy Alloy 908 Conduits

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Abstract

Estimates have been made of the maximum allowable crack size during bending at room temperature and of fatigue life during operation at 4 K for Incoloy alloy 908 conduits. Both the model coil and ITER magnets are considered. The study focuses on surface and corner cracks. Applied theories of fracture mechanics, fatigue crack growth and uncertainty analysis are discussed and sample calculations are included.
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1. Introduction and problem statement

The maximum flaw dimensions for final inspection of the ITER CS and TF conduits were estimated using specifications for navy nuclear piping as a guide. Flaws at or above the specified sizes can be found using standard NDE techniques. However, it is necessary to demonstrate that cracks or embedded defects of the maximum size do not result in failure of the conduit either during magnet fabrication or operation. This has been done by using a fracture mechanics analysis that focuses on surface and corner cracks.

The first problem considered is room temperature bending of the CS and TF conduits. Assuming a defect depth equal to 10% of the wall, what are the odds of fracture when the conduit is bent? Are the maximum defect plan areas that were specified reasonable? The accompanying analysis answers these questions and shows that the odds of failure in bending are extremely small.

The second problem considered is fatigue of the CS and TF conduits at 4 K. If the conduits survive fabrication, do they survive cyclic operation? Fatigue life has been estimated by numerical integration of the Paris equation. It is shown that safety factors in fatigue are more than adequate.

2. Results

Using specifications for navy nuclear piping as a guide, maximum flaw dimensions for the CS and TF conduits have been specified as

- Defect depth (CS and TF) \( \leq 10\% \) wall thickness
- Plan area of notch (CS) \( \leq 5 \text{ mm}^2 \)
- Plan area of notch (TF) \( \leq 2.5 \text{ mm}^2 \)

A surface crack of depth \( b \), length \( 2a \), and plan area \( \pi ab/2 \) is shown below. Refer to this figure when reading the discussion below.

![Fig. 2.1 Illustration of a two-dimensional surface crack.](image)

2.1 Bending at room temperature

Defect-depth specification

Consider a surface crack with an aspect ratio of \( b/a = 0.3 \). With a defect depth equal to 10% of the wall thickness, the odds against failure during room-temperature bending are approximately 10,000 to 1 for either the CS or TF conduits.
Plan-area specification

For surface cracks, the critical plan areas (notch areas) for failure during room-temperature bending are at least 16 times larger for the CS conduit and 5 times larger for the TF conduit than the plan areas used for inspection (given above).

2.2 Fatigue at 4 K

CS model coil

Based on a worst-case scenario (0% cold work, 10% wall-thickness surface crack, and 618 MPa peak stress), the fatigue life of the CS conduit in the model coil is about 8,700 cycles. Assuming a realistic scenario of 100 operating cycles, the safety factor is approximately 87.

TF model coil

Based on Incoloy conduit in stainless steel plates and a worst-case scenario (0% cold work, and 150 MPa peak stress), the fatigue life of the TF conduit in the model coil is about 4,000,000 cycles. Assuming a realistic scenario of 100 operating cycles, the safety factor is approximately 40,000.

ITER central solenoid

Based on a worst-case scenario (0% cold work, 10% wall-thickness surface crack, zero stress ratio, and b/a = 0.3), the fatigue life of the ITER central solenoid is infinite for compressive loads (normal operation). Assuming that quenches produce tension loads of 200 MPa, the fatigue life is approximately 450,000 cycles (quenches).

ITER TF coils

Assume Incoloy alloy 908 conduits in 316LN shear plates. Based on a worst case scenario (0% cold work, 10% wall-thickness surface crack, local tension of 150 MPa, zero stress ratio, and b/a = 0.3), the fatigue life of an ITER toroidal field coil is about 4,000,000 cycles.

3. ITER conduit

3.1 Material

Incoloy alloy 908 is a nickel-iron base superalloy developed for use in Nb3Sn superconducting magnets. The chemical composition is optimized for low thermal expansion, good mechanical properties, phase stability, workability and weldability. Alloy 908 shows high strength and fracture toughness from 298 to 4 K. Strengthening is achieved by precipitation of Ni3(Al,Ti,Nb) during heat-treatment. Its low thermal expansion coefficient minimizes compressive strain and therefore maximizes critical current in the Nb3Sn superconductor.[1]

3.2 Description of conduits

Cross sections of the CS1 and TF1 conduits are shown in Fig. 3.1. They are manufactured in 7 to 11 m lengths, joined by butt-welding and bent to form coils.
Fig. 3.1 Cross sections of the CS1 and TF1 conduits analyzed in this report. Dimensions are in millimeters.

4. Brief review of applied theories

4.1 Linear-elastic fracture mechanics

When a tensile load is applied perpendicular to a crack, stress at the crack tip is much higher than the average stress. Stress near the crack tip $\sigma$ is proportional to a constant $K$ and is a function of the coordinates relative to the tip. It is expressed in polar coordinates $(r, \theta)$ for a two-dimensional crack as in [2]:

$$\sigma(r, \theta) = \frac{K}{\sqrt{2\pi r}} f(\theta)$$

(4.1)

The constant $K$ is defined as the stress intensity factor. For mode I cracks, $K \rightarrow K_1$.

Fig. 4.1 Mode I crack showing tensile displacement perpendicular to crack tip.

For example, in the case of a two-dimensional crack with half-crack length $a$ in an infinite plate subject to a tensile stress $\sigma$, the stress intensity factor is
However, the stress intensity factor $K_1$ is different for different crack configurations. It is generally normalized as

$$Y = \frac{K_1}{K_{10}},$$  \hspace{1cm} (4.3)

or

$$K_1 = YK_{10} = Y\sigma\sqrt{\pi a}$$  \hspace{1cm} (4.4)

where the geometry factor $Y$ is a function of crack configuration. If $K_1$ exceeds the fracture toughness $K_{1c}$ (critical stress intensity factor), fast fracture occurs.

$$K_{1c} = Y\sigma\sqrt{\pi a_c},$$  \hspace{1cm} (4.5)

In other words, a component will fracture if crack length exceeds a critical length $a_c$, which is defined by:

$$a_c = \frac{1}{\pi} \sigma^2 K_{1c}^2 \frac{1}{Y^2}$$  \hspace{1cm} (4.6)

For a three-dimensional crack such as a surface crack, the length $a_c$ is often replaced by crack depth $b_c$, where $b_c$ is related to $a_c$ by the crack aspect ratio $b/a$. Therefore, Eq. 4.6 becomes

$$b_c = \frac{1}{\pi} \sigma^2 K_{1c}^2 \frac{1}{Y^2}$$  \hspace{1cm} (4.7)

The above analysis is for linear-elastic fracture mechanics. It is valid only for small scale yielding where the plastic zone size $r_p$ at the crack tip is much smaller than the crack length $a$, typically, $(r_p / a) < 1/50$. For most cases with the ITER conduits, this analysis gives good approximations.

4.2 Uncertainty analysis (error propagation) [3-8]

Uncertainty analysis is used to estimate allowable crack depths at arbitrarily chosen confidence limits. The word "allowable" means that there is reason to be confident that cracks up to the allowable size will not result in conduit fracture during room-temperature bending. Confidence is expressed as odds against failure with the following caveat. The reader must understand that the uncertainties in parameters such as $\sigma$, $K_{1c}$, or $Y$ exist and that these uncertainties are due to the combined effects of small errors or variations in measurements, material properties, and other factors. The true statistical distributions of the parameters are unknown. For example, measurements of fracture toughness are subject to variations in material batches, variations in heat treatment, variations in specimen preparation, variations in test procedures (who ran the test), and variations in test instrumentation (level of calibration, accuracy, precision, etc.). It is assumed in the uncertainty analysis that (1) the statistical distributions (probability density functions) are normal, that (2) the stated values of $\sigma$, $K_{1c}$, and $Y$ are independent and equal to statistical
mean values, and that (3) relative standard deviations are known independently of direct observation (a priori by observation of similar metals). These assumptions allow an educated guess of the maximum allowable crack depths of the CS and TF conduits.

As noted in reference [3], a mathematical theorem states that if $y$ is a linear function of independent variables $x_i$,

$$y = c_0 + c_1x_1 + c_2x_2 + c_3x_3$$  \hspace{1cm} (4.8)

then the mean value of $y$ is determined by the mean values of $x_i$ (mean is denoted by subscript 0).

$$y_0 = c_0 + c_1x_{01} + c_2x_{02} + c_3x_{03}$$  \hspace{1cm} (4.9)

Likewise, the standard deviation of $y$ is determined by the square root of the sum of the squares of the standard deviations.

$$s_y = \left( (c_1s_1)^2 + (c_2s_2)^2 + (c_3s_3)^2 \right)^{1/2}$$  \hspace{1cm} (4.10)

It is assumed that the independent variables are statistical in nature and normally distributed. Any given value of $x_i$ will most likely lie in an interval about the mean characterized by an uncertainty $u_i$. The size of the uncertainty depends on the level of confidence (odds) desired that the next value of $x_i$ will lie inside the uncertainty interval. The uncertainty interval about the central value is

$$x_{ui} = x_{0i} \pm u_i$$  \hspace{1cm} (4.11)

and

$$x_{0i} - u_i < x_i < x_{0i} + u_i$$  \hspace{1cm} (4.12)

The uncertainties are multiplies of the standard deviations and, in terms of statistics, may be thought of as one sigma, two sigma, three sigma, etc. levels. The effect of the uncertainty of each independent variable on the result is estimated using the maximum and minimum limits expressed in Eq. 4.11 while holding all other independent variables at their central values $x_{0(i \neq i)}$:

$$\Delta y_i = y_{ui} - y_o = y(x_{ui} - x_{0(i \neq i)}) - y_o$$  \hspace{1cm} (4.13)

The root-square uncertainty is estimated as the square root of the sum of $\Delta y_i$ squared, where all products are treated as positive.

$$u_{rs} = \left[ \sum_i \Delta y_i^2 \right]^{1/2}$$  \hspace{1cm} (4.14)

Equations 4.8 - 4.14 are only exact for a linear function. The goal is to analyze Eq. 4.7, which is nonlinear. However, it can be turned into a linear relation by taking logarithms of both sides.

$$\ln(b_r) = \ln\left(\frac{1}{\pi}\right) - 2\ln(\sigma) + 2\ln(K_{1c}) - 2\ln(Y)$$  \hspace{1cm} (4.15)
Comparing Eq. 4.15 to Eq. 4.8 gives

\[ y = \ln(b) \]
\[ x_1 = \ln(\sigma) \]
\[ x_2 = \ln(K_{1c}) \]
\[ x_3 = \ln(Y) \]

Eqs. (4.16)

\[ b = c_0 + c_1 x_1 + c_2 x_2 + c_3 x_3 \]

The mean value of log crack depth is obtained from Eq. 4.9

\[ y_0 = c_0 + c_1 x_0 + c_2 x_0^2 + c_3 x_0^3 \]

which yields:

\[ \ln(b_{c0}) = \ln\left(\frac{1}{\pi}\right) - 2 \ln(\sigma_0) + 2 \ln(K_{1c0}) - 2 \ln(Y_0) \]

Referring to Eq. 4.13, the effect of the uncertainty of each independent variable on the result can be estimated. This is tricky because linearity was obtained with logarithms and it must be remembered that the crack depth equation is nonlinear. First, equation 4.11 should be modified to remind the reader that the uncertainty interval may not be symmetric as would be expected in a linear problem.

\[ x_{mi}^{mx} = x_{0i} + u_i^{mx} \]
\[ x_{mi}^{mn} = x_{0i} - u_i^{mn} \]

where the superscripts mx and mn stand for maximum and minimum respectively. The maximum and minimum uncertainties in \( y \) are estimated for each independent variable in turn. For example, \( \Delta y_1^{mx} \) is obtained as

\[ \Delta y_1^{mx} = c_0 + c_1 (x_1 + u_1^{mx}) + c_2 x_2 + c_3 x_3 - y_0 \]

The resulting six uncertainties are:

\[ \Delta y_1^{mx} = c_1 u_1^{mx} \]
\[ \Delta y_2^{mx} = c_2 u_2^{mx} \]
\[ \Delta y_3^{mx} = c_3 u_3^{mx} \]
\[ \Delta y_1^{mn} = c_1 u_1^{mn} \]
\[ \Delta y_2^{mn} = c_2 u_2^{mn} \]
\[ \Delta y_3^{mn} = c_3 u_3^{mn} \]

The uncertainties \( u_i^{mx, mn} \) in 4.18 and 4.20 deserve special attention because they are uncertainties in log values. This means, for example, that uncertainty in the critical stress intensity factor, a number that will be called \( u_K \), translates into uncertainty in the log of \( K_{1c} \) in the following manner, which can be visualized in the plot of \( \ln(K_{1c}) \) versus \( K_{1c} \) shown in Figure 4.2.

\[ (K_{1c} + u_K) - K_{1c} \rightarrow \ln(K_{1c} + u_K) - \ln K_{1c} = u_2^{mx} \]
\[ (K_{1c} - u_K) - K_{1c} \rightarrow \ln(K_{1c} - u_K) - \ln K_{1c} = u_2^{mn} \]
One reason for spelling out the steps is to prevent confusion between real parameter values (e.g., $u_K$) and parameter values that are disguised by a log operator (e.g., $u_{\log}^{\max}$). That said, for the crack depth problem, the uncertainties in each independent variable are estimated as:

\[
\begin{align*}
    u_1^{\sigma} &= \ln(\sigma_0 + u_\sigma) - \ln(\sigma_0) \\
    u_1^{\sigma} &= \ln(\sigma_0 - u_\sigma) - \ln(\sigma_0) \\
    u_2^{\sigma} &= \ln(K_{1,c} + u_K) - \ln(K_{1,c}) \\
    u_2^{\sigma} &= \ln(K_{1,c} - u_K) - \ln(K_{1,c}) \\
    u_3^{\sigma} &= \ln(Y + u_Y) - \ln(Y_0) \\
    u_3^{\sigma} &= \ln(Y_0 - u_Y) - \ln(Y_0)
\end{align*}
\]

where

$u_\sigma$, $u_{\sigma,\log}^{max}$ = uncertainty in stress and $\ln$ (stress) respectively

$u_K$, $u_{K,\log}^{max}$ = uncertainty in fracture toughness and $\ln$ (fracture toughness) respectively

$u_Y$, $u_{Y,\log}^{max}$ = uncertainty in $Y$ factor and $\ln(Y)$ respectively

Fig. 4.2 Plot of the logarithm of $K_{1,c}$ versus $K_{1,c}$ that illustrates the meaning of $u_{K,\log}^{\max}$. The uncertainty in fracture toughness is $u_K$.

The root-square uncertainty in the log parameters follows as the square root of the sum of the products of the maximum and minimum values.
\[ u_{rs} = \sqrt{\Delta y_1^{mn} \Delta y_1^{mn} + \Delta y_2^{mn} \Delta y_2^{mn} + \Delta y_3^{mn} \Delta y_3^{mn}}. \]  

(4.23)

The allowable value of log crack depth is equal to the mean value minus the uncertainty:

\[ y_{all} = y_0 - u_{rs}. \]  

(4.24)

Therefore, the true allowable crack depth is equal to:

\[ b_{all} = e^{y_{all}}. \]  

(4.25)

### 4.3 Fatigue with constant stress amplitude

The fatigue-crack-growth rate with constant stress amplitude is expressed by the Paris equation, which is valid only for small-scale yielding at the crack tip (linear-elastic fracture mechanics),\[9\]

\[ \frac{da}{dN} = c(\Delta K)^n \]  

(4.26)

where \( \Delta K = K_{max} - K_{min} \) and \( c \) and \( n \) are Paris parameters.

Increasing the mean stress ((\( \sigma_{max} + \sigma_{min} \)/2) for an applied stress range (\( \sigma_{max} - \sigma_{min} \)) generally shortens fatigue life. The effect of mean stress is often expressed by an effective stress intensity factor range: \[10\]

\[ \Delta K_{ef} = K_{max}(1 - R)^m = \Delta K(1 - R)^{m-1} \]  

(4.27)

where \( R \) is the stress ratio (\( \sigma_{min} / \sigma_{max} \)) and \( m \) is the Walker exponent. Combining Eqs. 4.26 and 4.27 gives:

\[ \frac{da}{dN} = c(\Delta K)^n(1 - R)^{(m-1)n} \]  

(4.28)

where

\[ \Delta K = Y\Delta\sigma/\sqrt{\pi a} . \]

Equation 4.28 may be written

\[ \frac{da}{dN} = c(K_{max})^n(1 - R)^{mn} \]  

(4.29)

where

\[ K_{max} = Y\sigma_{max}/\sqrt{\pi a} . \]

Integration of Eqs. 4.28 or 4.29 gives the fatigue life (the number of cycles to failure):
\[ N_f = \sigma_{\max}^{-\alpha}(1 - R)^{-\beta\gamma} \xi \]  

(4.30)

where

\[ \xi = \frac{1}{c} \int\limits_{a_i}^{a_f} \frac{da}{Y^n(\pi a)^{n/2}} \]

5. Maximum allowable crack size for bending at room temperature

Here, (1) the critical crack size and (2) the maximum allowable crack size are estimated at odds of 10,000 to 1 against fracture (four standard deviations) in the CS and TF conduits subject to bending during the winding operation.

5.1 Assumptions

(a) Linear-elastic fracture mechanics is applied.

(b) Analysis includes surface cracks, single-edge notch cracks, corner cracks and embedded cracks for CS coils, as well as circumferential surface cracks for the TF coils. The model is a crack-in-a-plate that is subject to a uniform tensile load perpendicular to the crack (mode I).

(c) Failure occurs when the crack depth (or length) reaches either the wall thickness or the critical crack depth (or length) determined by the fracture toughness.

(d) Uncertainty of the crack depth at given odds against fracture is a function of three independent variables: bending stress, fracture toughness and the Y factor. All three variables follow a normal statistical distribution.

(e) Parameters needed to estimate critical and maximum allowable crack size in CS1 and TF conduits are listed in Table 5.1.

Table 5.1 Parameters to estimate critical and maximum allowable crack sizes for CS1 and TF conduits

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic bending strain</td>
<td>3 %</td>
<td>Ref. 11</td>
</tr>
<tr>
<td>Bending stress ( \sigma_b ) for base metal</td>
<td>760 MPa</td>
<td>Ref. 11</td>
</tr>
<tr>
<td>Bending stress ( \sigma_b ) for the welds</td>
<td>590 MPa</td>
<td>Ref. 11</td>
</tr>
<tr>
<td>Fracture toughness ( K_{1c} ) for base metal</td>
<td>160 MPa( \sqrt{m} )</td>
<td>Ref. 12</td>
</tr>
<tr>
<td>Fracture toughness ( K_{1c} ) for the welds</td>
<td>106 MPa( \sqrt{m} )</td>
<td>Ref. 12</td>
</tr>
<tr>
<td>relative SD of bending stress</td>
<td>10%</td>
<td>lore</td>
</tr>
<tr>
<td>relative SD of fracture toughness</td>
<td>15%</td>
<td>lore</td>
</tr>
<tr>
<td>relative SD of Y factor</td>
<td>5%</td>
<td>lore</td>
</tr>
</tbody>
</table>

Note: SD = standard deviation

(f) Inspection criteria for the CS and TF conduits are listed in Table 5.2
Table 5.2 Inspection criteria for the conduits

<table>
<thead>
<tr>
<th></th>
<th>crack depth</th>
<th>crack notch plan area</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS conduit</td>
<td>1/10 wall thickness</td>
<td>5 mm$^2$</td>
</tr>
<tr>
<td>TF conduit</td>
<td>1/10 wall thickness</td>
<td>2.5 mm$^2$</td>
</tr>
</tbody>
</table>

5.2 Results for CS1 conduit

The conduit material is Incoloy alloy 908, which has been solution-annealed before welding and given 5% cold work after welding. The conduit is then bent with about 3% plastic strain to make a coil. The estimated bending stress is 760 MPa for the base metal and 590 MPa for the welds.

(a) Critical and maximum allowable crack depth

Assume that a crack exists in a conduit during bending. The stress raiser at the crack tip is dominated by the stress intensity factor $K$. The relative stress intensity factor $Y$ and the stress intensity factor $K$ (see Appendix 1) as functions of crack depth are shown in Figs. 5.1 to 5.8 for a surface crack, single-edge-notch crack, corner crack, and embedded crack. The $Y$ factor is calculated from material given in references [13,14].

The fracture toughness is 160 MPa$\sqrt{m}$ for the base metal and 106 MPa$\sqrt{m}$ for the weld metal. If the stress intensity factor $K$ exceeds the fracture toughness, failure occurs. The corresponding crack depth is defined as the critical crack depth, which can be obtained either by solving Eq. 4.4 at given stress and fracture toughness, or graphically from Figs. 5.2, 5.4, 5.6 and 5.8.

Uncertainty methods have been used to analyze the maximum allowable crack depth at odds of 10,000 to 1 against failure. The critical crack depth and maximum allowable crack depth for a surface crack, single-edge-notch crack, corner crack and embedded crack are listed in Table 5.3. A sample calculation of the uncertainty is given in Section 7.2.

Table 5.3 Critical and maximum allowable crack depths of CS1 conduit in bending for the base and weld metals of alloy 908

<table>
<thead>
<tr>
<th>crack type</th>
<th>critical crack depth $b_c$ (mm)</th>
<th>$b_c$ wall_thickness</th>
<th>maximum allowable crack depth $b_m$ (mm) at odds of 10,000:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>surface crack (SC)</td>
<td>4.15 (3.95)</td>
<td>0.66 (0.63)</td>
<td>0.83 (0.79)</td>
</tr>
<tr>
<td>single-edge-notch crack (SEN)</td>
<td>2.7 (2.4)</td>
<td>0.43 (0.38)</td>
<td>0.54 (0.49)</td>
</tr>
<tr>
<td>corner crack (CC)</td>
<td>7.7 (6.4)</td>
<td>1.23 (1.02)</td>
<td>1.55 (1.29)</td>
</tr>
<tr>
<td>embedded crack (EC)</td>
<td>&gt;4.15 (&gt;3.95)</td>
<td>&gt;0.66 (&gt;0.63)</td>
<td>&gt;0.83 (&gt;0.79)</td>
</tr>
</tbody>
</table>

Note: parentheses ( ) contains results for welds
(b) Critical plan areas

Figures 5.9 to 5.12 show stress intensity factor as a function of crack-notch plan area for the four crack types. Table 5.4 lists the critical crack-notch plan areas for a surface crack with aspect ratios of 0.1 and 0.3 respectively in the CS1 conduit.

Table 5.4 Critical notch plan area of CS1 conduit in bending for a surface crack

<table>
<thead>
<tr>
<th>crack aspect ratio b/a</th>
<th>critical notch plan area (mm²)</th>
<th>critical_area allowed_area</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>112 (98)</td>
<td>22 (20)</td>
</tr>
<tr>
<td>0.3</td>
<td>90 (81)</td>
<td>18 (16)</td>
</tr>
</tbody>
</table>

Note: parentheses ( ) contains results for welds

(c) Effect of crack types

Different types of cracks give different stress intensity factors for the same crack depth or notch area.

Figures 5.13 and 5.14 show stress intensity factor as a function of crack depth and crack-notch plan area for the four crack types. It is found that the single-edge notch and surface cracks have the highest stress intensity factor at the crack tip based on crack depth. However, corner and surface cracks turn out to be the most highly stressed, based on a plan area of about 5 mm².

Table 5.5 Effect of crack types on K values (b/a=0.3)

<table>
<thead>
<tr>
<th>based on crack depth</th>
<th>highest K</th>
<th>high K</th>
<th>low K</th>
<th>lowest K</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEN</td>
<td>SC</td>
<td>CC</td>
<td>EC</td>
<td>SEN</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>based on crack notch plan area</th>
<th>highest K</th>
<th>high K</th>
<th>low K</th>
<th>lowest K</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>SC</td>
<td>EC</td>
<td>SEN</td>
<td></td>
</tr>
</tbody>
</table>

(d) Impact of crack aspect ratio b/a

It is also observed (Figs. 5.2, 5.4, 5.6 and 5.8) that for a given crack depth, the smaller the aspect ratio, the higher the K value. A surface crack approaches a single edge notch crack as the aspect ratio becomes smaller and smaller and its K value gets larger and larger. However, for a given crack-notch plan area, the results are different. Table 5.6 lists aspect ratios for three different crack types from high to low K at a crack-notch plan area of about 5 mm². Aspect ratios of 0.3 and 0.5 give the highest K values for a surface crack.
Table 5.6 Impact of Crack Aspect Ratios on K Value
(crack notch-notch plan area = 5 mm²)

<table>
<thead>
<tr>
<th></th>
<th>highest K</th>
<th>high K</th>
<th>low K</th>
<th>lowest K</th>
</tr>
</thead>
<tbody>
<tr>
<td>surface crack</td>
<td>0.3, 0.5</td>
<td></td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>corner crack</td>
<td>0.5</td>
<td>0.3</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>embedded crack</td>
<td>0.5</td>
<td>0.3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

5.3 Results for TF conduit

The TF conduit is a thin-walled tube of thickness 1 mm. It has been solution-annealed
before welding and receives 5% cold work after welding. The plastic strain during coil
winding is about 3%. The bending stress is 760 MPa for the base metal and 590 MPa for
the welds. Only a circumferential surface crack (CSC) is significant and considered. If
there is any circumferential through crack, the conduit will not pass inspection.

Figures 5.15 to 5.17 show Y factors and stress intensity factors for a circumferential
surface crack[13-15]. The Y factors are obtained by modeling CSC as a surface crack on
1 mm thick plate. These Y factors are about 5 - 10% higher than those values listed in Ref.
15 for a true circumferential surface crack.

The critical and maximum allowable depths for circumferential surface cracks with
aspect ratios of 0.05 and 0.1 are listed in Table 5.7. The critical crack-notch plan areas are
listed in Table 5.8.

Table 5.7 Critical and maximum allowable crack depths of TF conduit
in bending for the base and weld metals of alloy 908

<table>
<thead>
<tr>
<th>crack aspect ratio</th>
<th>critical crack depth $b_c$ (mm)</th>
<th>$b_c / \text{wall thickness}$</th>
<th>maximum allowable crack depth $b_m$ (mm) at odds of 10,000:1</th>
<th>$b_m / \text{wall thickness}$ at odds of 10,000:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.7 (0.67)</td>
<td>0.7 (0.67)</td>
<td>0.14 (0.135)</td>
<td>0.14 (0.135)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.93 (0.87)</td>
<td>0.93 (0.87)</td>
<td>0.188 (0.175)</td>
<td>0.188 (0.175)</td>
</tr>
</tbody>
</table>

Table 5.8 Critical notch plan area of TF conduit
in bending for a surface crack

<table>
<thead>
<tr>
<th>crack aspect ratio b/a</th>
<th>critical notch area (mm²)</th>
<th>critical_area / allowed_area</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>15.4 (14.1)</td>
<td>6.16 (5.64)</td>
</tr>
<tr>
<td>0.1</td>
<td>13.6 (11.9)</td>
<td>5.44 (4.76)</td>
</tr>
</tbody>
</table>

Note: parentheses () contains results for welds
Fig. 5.1  Y factor as a function of crack depth for a surface crack in CS1 conduit made of Incoloy 908, crack aspect ratio b/a = 0.1, 0.3, 0.5 and 1.
Fig. 5.2 Stress intensity factor as a function of crack depth for a surface crack in CS1 conduit made of Incoloy 908, crack aspect ratio b/a = 0.1, 0.3, 0.5 and 1, applied tension stress $\sigma = 760$ MPa.
Fig. 5.3 Y factor as a function of crack depth for a single edge notch crack in CS1 conduit made of Incoloy 908.
SEN, CS1, 908, BENDING AT RT, 760 MPa

Fig. 5.4 Stress intensity factor as a function of crack depth for a single edge notch crack in CS1 conduit made of Incoloy 908, applied tension stress $\sigma = 760$ MPa.
Fig. 5.5 Y factor as a function of crack depth for a corner crack in CS1 conduit made of Incoloy 908, crack aspect ratio b/a = 0.1, 0.3, 0.5 and 1.
Fig. 5.6 Stress intensity factor as a function of crack depth for a corner crack in CS1 conduit made of Incoloy 908, crack aspect ratio $b/a = 0.1, 0.3, 0.5$ and 1, applied tension stress $\sigma = 760$ MPa.
Fig. 5.7 Y factor as a function of crack depth for an embedded crack in CS1 conduit made of Incoloy 908, crack aspect ratio b/a = 0.3, 0.5 and 1.
Fig. 5.8 Stress intensity factor as a function of crack depth for an embedded crack in CS1 conduit made of Incoloy 908, crack aspect ratio b/a = 0.3, 0.5 and 1, applied tension stress $\sigma = 760$ MPa.
SC, CS1, 908, BENDING AT RT, 760 MPa

Fig. 5.9 Stress intensity factor as a function of crack notch plan area for a surface crack in CS1 conduit made of Incoloy 908, crack aspect ratio b/a = 0.1, 0.3, 0.5 and 1, applied tension stress $\sigma = 760$ MPa.
Fig. 5.10 Stress intensity factor as a function of crack notch plan area for a single edge notch crack in CS1 conduit made of Incoloy 908, applied tension stress $\sigma = 760$ MPa.
Fig. 5.11 Stress intensity factor as a function of crack notch plan area for a corner crack in CS1 conduit made of Incoloy 908. crack aspect ratio b/a = 0.1, 0.3, 0.5 and 1, applied tension stress \( \sigma = 760 \text{ MPa} \).
Fig. 5.12  Stress intensity factor as a function of crack notch plan area for an embedded crack in CS1 conduit made of Incoloy 908, crack aspect ratio $b/a = 0.3, 0.5$ and 1, applied tension stress $\sigma = 760$ MPa.
Fig. 5.13 Stress intensity factor as a function of crack depth for four types of crack in CS1 conduit made of Incoloy 908, applied stress $\sigma=760 \text{ MPa}$, $b/a=0.3$;
Fig. 5.14 Stress intensity factor as a function of crack notch plan area for four types of crack in CS1 conduit made of Incoloy 908, σ=760 MPa, b/a=0.3; 1: surface crack, 2:single edge notch crack, 3:corner crack, 4:embedded crack.
Fig. 5.15  Y factor as a function of crack depth for a surface crack in TF conduit made of Incoloy 908, crack aspect ratio b/a = 0.05, 0.1 and 0.3.
Fig. 5.16 Stress intensity factor as a function of crack depth for a surface crack in TF conduit made of Incoloy 908, crack aspect ratio b/a = 0.05, 0.1 and 0.3, applied tension stress $\sigma = 760$ MPa.
Fig. 5.17 Stress intensity factor as a function of crack notch plan area for a surface crack in TF conduit made of Incoloy 908, crack aspect ratio b/a = 0.05, 0.1 and 0.3, applied tension stress $\sigma = 760$ MPa.
It is found, from Figs. 5.16 and 5.17, that a surface crack with aspect ratio of 0.3 will leak before fracture, and a crack with b/a=0.3 has the highest K at a given crack notch plan area.

6. Fatigue life of conduits for model and ITER coils at 4 K

The CS1 conduit of the ITER coil experiences compressive loads during normal operation. A local tension of less than 200 MPa might occur only during quenches. The maximum local tension stress for the TF coils of model and ITER TF coils is less than 150 MPa.

Stresses for the CS1 conduit of the model coil are listed in Table 6.1.

Table 6.1 Maximum stresses for the CS1 conduit of model coil [16,17]

<table>
<thead>
<tr>
<th>stress structure</th>
<th>hoop (MPa)</th>
<th>Tresca (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>self support</td>
<td>466</td>
<td>618</td>
</tr>
<tr>
<td>mutual support</td>
<td>382</td>
<td>531</td>
</tr>
</tbody>
</table>

6.1 Assumptions

(a) The Paris law applies for fatigue crack growth, \( \frac{da}{dN} = c(\Delta K)^n \), where \( \Delta K \) is the stress intensity factor range at the crack tip. The Paris parameters of the welds at 4 K are not available. However, the welds of alloy 908 show slower crack growth than base metal [18]. Therefore, Paris parameters for base metal are used in this analysis as a conservative option.

Table 6.2 Paris parameters for alloy 908 at 4 K (R=0.1) [12]

<table>
<thead>
<tr>
<th>cold work</th>
<th>c (m/cycle)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%CW</td>
<td>1.5 x10^{-12}</td>
<td>3.41</td>
</tr>
<tr>
<td>20%CW</td>
<td>7.03 x10^{-14}</td>
<td>4.06</td>
</tr>
</tbody>
</table>

(b) Failure criteria: either the crack depth reaches the wall thickness (conduit leaks), i.e., \( b \geq \) wall thickness, or the crack depth equals (or exceeds the critical crack depth, \( b_c \), i.e., \( b \geq b_c \). The latter means the stress intensity factor at the crack tip is equal to or greater than the fracture toughness, i.e., \( K_\delta \geq K_{ic} \). The fracture toughness of alloy 908 welds at 4 K is \( K_{ic} = 105 \text{ MPa} \sqrt{m} \) (based on alloy 908 as filler metal) [12,18].

(c) The Walker exponent \( m \) is estimated to be 0.81 for alloy 908 below 10 K based on data from Ref. 19.

(d) Possible cracks include a surface crack (SC), single-edge-notch crack (SEN), corner crack (CC) and embedded crack (EC).

(e) For the real ITER coil, the CS conduit is modeled as a plate with 5.5 mm thickness. The TF conduit is always modeled as a sheet with 1 mm thickness.
A crack will grow in an elliptical shape whose size is governed by growth rates at two points: the point at depth "b" and the point at edge "a". The axis lengths of the ellipse are obtained by separate numerical integrations at these two points.

6.2 Results for model coils

Fatigue lives are estimated for various stresses and initial crack sizes by numerical integration of the Paris equation using a finite-difference method. Lives are listed in Table 6.3 for several typical cases of a surface or corner crack in the CS1 conduit, where \( b_i \) is the initial crack depth and \( (b/a)_i = 0.3 \) is the initial aspect ratio. Fatigue life as a function of initial crack depth is shown in Fig. 6.1.

![Table 6.3](image)

<table>
<thead>
<tr>
<th>stress \ cold work</th>
<th>0% CW</th>
<th>20% CW</th>
</tr>
</thead>
<tbody>
<tr>
<td>self support hoop: 466 MPa</td>
<td>22,840 cycles (26,590)</td>
<td>55,400 cycles (60,430)</td>
</tr>
<tr>
<td>self support Tresca: 618 MPa</td>
<td>8,710 cycles (9,920)</td>
<td>17,590 cycles (18,980)</td>
</tr>
<tr>
<td>mutual support hoop: 382 MPa</td>
<td>44,990 cycles (52,520)</td>
<td>124,170 cycles (135,580)</td>
</tr>
<tr>
<td>mutual support Tresca: 531 MPa</td>
<td>14,640 cycles (16,890)</td>
<td>32,610 cycles (35,410)</td>
</tr>
</tbody>
</table>

The fatigue life estimate for the TF conduit of the model coil based on upper and lower stress limits is listed in Table 6.4. Fatigue life as a function of initial crack depth is shown in Fig. 6.2.

![Table 6.4](image)

| Surface crack, R = 0, 4 K, \( b_i = 0.1 \text{ mm (1/10 wall thickness)}, (b/a)_i = 0.3 \) |
|-------------------------------|-----------------|-----------------|
| local tension                | 0% CW           | 20% CW          |
| > 100 MPa                     | < 1.6 x 10^7 cycles | < 2.3 x 10^8 cycles |
| < 150 MPa                     | > 4.0 x 10^6 cycles  | > 3.8 x 10^7 cycles |

6.3 Results for ITER coils

During operation at 4 K, the ITER CS conduits experience compressive load. Therefore, the fatigue life is infinite as shown in Table 6.5, which also includes life based on quench loads. For TF conduits, the normal load is compressive, but possible local tension loads (<150 MPa) may exist. The life estimate for TF conduits is listed in Table 6.6.
Table 6.5 Fatigue life estimate for ITER CS conduit
Data given as: surface crack (corner crack)

<table>
<thead>
<tr>
<th>load condition</th>
<th>average fatigue life</th>
<th>fatigue life after safety factor of 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal load:</td>
<td>∞ cycles</td>
<td>∞ cycles</td>
</tr>
<tr>
<td>compression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>quench load:</td>
<td>4.5 x 10^5 cycles</td>
<td>1.1 x 10^5 cycles</td>
</tr>
<tr>
<td>≈ 200 MPa</td>
<td>(5.3 x 10^5)</td>
<td>(1.3 x 10^5)</td>
</tr>
</tbody>
</table>

Table 6.6 Fatigue life estimate for ITER TF conduits (surface cracks)

<table>
<thead>
<tr>
<th>load condition</th>
<th>average fatigue life</th>
<th>fatigue life after safety factor of 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal load:</td>
<td>∞ cycles</td>
<td>∞ cycles</td>
</tr>
<tr>
<td>compression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>local tension:</td>
<td>&gt; 4.0 x 10^6 cycles</td>
<td>&gt; 1.0 x 10^6 cycles</td>
</tr>
<tr>
<td>&lt; 150 MPa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.4 Analysis and formulation

6.4.1 Analysis

The expression for $\xi$ can also be obtained in terms of crack depth $b$, instead of the semi-crack length $a$, based on the known crack aspect ratio of $b/a$. Note that the $Y$ expression must be modified accordingly:

$$\xi = \frac{1}{c} \int_{b_i}^{b_f} \frac{db}{Y^n (\pi b)^{n/2}}$$

where $b_i$ and $b_f$ are initial and final crack depths respectively.

In most cases, the upper integration limit $b_f$ is the wall thickness of the conduit. Therefore, $\xi$ is independent of applied stress and only a function of the Paris parameters ($c, n$), the initial crack size and conduit geometry. $\xi$ can be obtained by a numerical integration using a finite difference method:

$$\xi = \frac{1}{c} \sum_j \frac{\Delta b}{Y_j^n (\pi b_j)^{n/2}}$$

where

$$b_j = b_i, b_i + \Delta b, b_i + 2\Delta b, \ldots, b_i + j\Delta b, \ldots, b_f.$$
Taking logarithms of both sides of Eq. 4.27 gives

\[ \log(N_f) = \log(\xi) - mn \log(1 - R) - n \log(\sigma_{\text{max}}) \]  

(6.3)

or

\[ \log(\sigma_{\text{max}}) = \frac{1}{n} \log(\xi) - m \log(1 - R) - \frac{1}{n} \log(N_f) \]  

(6.4)

Eqs. 6.3 and 6.4 show linear-log relations between the fatigue life and applied maximum stress (S-N curves for crack propagation). These linear relations are generally true for all thin plates. If the stress range is used instead of maximum stress in the above derivation, Eqs. 6.3 and 6.4 become

\[ \log(N_f) = \log(\xi) + n(1 - m) \log(1 - R) - n \log(\Delta \sigma) \]  

(6.5)

or

\[ \log(\Delta \sigma) = \frac{1}{n} \log(\xi) + (1 - m) \log(1 - R) - \frac{1}{n} \log(N_f) \]  

(6.6)

6.4.2 Formulation

Linear-log relations between the fatigue life and the maximum applied stress for surface cracks with b/a=0.3 for alloy 908 welds at 4 K are shown in Figs. 6.3 and 6.4 for the model coil CS 1 and TF conduits. Initial crack depths are 1/10th of the wall thickness (i.e., 0.625 mm for CS 1 and 0.1 mm for TF conduits respectively). These relations are obtained by a least squares regression method:

\[ \log(N_f) = 13.48176 - mn \log(1 - R) - 3.419 \log(\sigma_{\text{max}}) \]  

for CS 1 conduit,  

(6.7)

\[ \log(N_f) = 14.06595 - mn \log(1 - R) - 3.425 \log(\sigma_{\text{max}}) \]  

for TF conduit,  

(6.8)

where m=0.81, n=3.41 and R is the load ratio.

6.5 Discussion

6.5.1 Crack-growth models

The Paris law was obtained from 2-D crack-growth data. There have also been several models for 3-D cracks. Among them, the 2-point-crack-growth model [20] seems to best describe the physics of propagation and also provides conservative results. In this model, growth rates at two typical points (crack depth and surface edge) are examined. Crack dimensions at both points are then calculated. It has been found in this study that for alloy 908 the aspect ratio b/a approaches a stable number of about 0.7, at which the stress intensity factors at both points are about equal.
6.5.2 Cracks in compression

A crack only opens under tensile stress (mode I). If a shear stress is applied at a crack tip (modes II and III), the crack may grow, but will grow very slowly. A compressive stress at the crack tip closes the crack. It may result in a shear stress in certain orientations of the tip, but its effect on crack growth can be neglected. Therefore, a compressive load will lead to infinite life if there is no local tension at the crack tip.

6.5.3 Short cracks

Short-crack behavior may occur if the crack length is smaller than 1 mm or is about a grain size. Short cracks grow faster than long cracks in the region below the threshold. As a common practice, the Paris equation is extrapolated to below the threshold region, and generally gives conservative results in comparison with true short-crack behavior.

6.5.4 Cold work

It can be seen from Fig. 6.5 that the case with 0% cold work (CW) has less life than that with 20% CW. Therefore, the Paris parameters with 0% CW were used to estimate the fatigue life.

6.5.5 Crack type

Fatigue life varies with different types of cracks. The single-edge notch crack (SEN) has the shortest life and the embedded crack (EC) has the longest life at a given maximum applied stress, as shown in Fig. 6.6. Fortunately, a SEN crack is so large that it can be detected easily by routine inspection methods. Therefore, SEN cracks were not considered in this study.

6.5.6 Initial aspect ratio

The fatigue life is also a function of the initial crack aspect ratio b/a as shown in Fig. 6.7. A smaller aspect ratio at a given crack depth means a larger crack length and larger crack cross section, thus a higher crack growth rate and shorter fatigue life. However, b/a = 0.3 is assumed to be typical. Any deviations from the typical case are included in the safety factor or uncertainty analysis.

6.5.7 Fast fracture

It is assumed that a crack will undergo fast fracture as the maximum stress intensity reaches the fracture toughness. However, fracture toughness is measured for plane strain and given specimen dimensions. The true coil components may deviate from such ideal conditions.

It has been observed that most of the fatigue specimens suffer fast fracture earlier than predicted based on fracture toughness. For example, steel fractures at about 70% of the fracture toughness.[21] The error in fatigue life estimation due to this effect is included in the safety factor.

All the above analyses are based on linear-elastic fracture mechanics. However, as a crack approaches the critical size, severe plastic deformation may occur. Therefore, it is a wise practice, in parallel with the LEFM, to test a plastic-fracture criterion -- that is, net stress is equal to or greater than tensile strength.[21]
Fig. 6.1 Fatigue life as a function of initial crack length for a surface crack with b/a=0.3 in CS1 conduit of model coil, Incoloy alloy 908 (welds), 0%CW, stress = 382, 466, 531 and 618 MPa, R=0, T=4K.
Fig. 6.2 Fatigue life as a function of initial crack length for a surface crack with b/a=0.3 in TF conduit of model coil, Incoloy alloy 908 (welds), 0%CW, stress = 100 and 150 MPa, R=0, T=4K.
Fig. 6.3 Fatigue life as a function of maximum stress for a surface crack with $b/a=0.3$ and initial crack depth $=0.625$ mm in CS1 conduit of model coil, Incoloy alloy 908 (welds), 0%CW, $R=0$, 0.2, 0.4 and 0.6, $T=4$K. Linear log relation between fatigue life and stress is found.
Fig. 6.4 Fatigue life as a function of maximum stress for a surface crack with $b/a=0.3$ and initial crack depth = 0.1 mm in TF conduit of model coil, Incoloy alloy 908 (welds), 0\%CW, $R=0, 0.2, 0.4$ and $0.6, T=4K$. Linear log relation between fatigue life and stress is found.
Fig. 6.5 Fatigue life as a function of initial crack length for a surface crack with b/a=0.3 in CS1 conduit of model coil, Incoloy alloy 908 (welds), with cold work of 0%CW and 20%, stress = 618 MPa, R=0, T=4K.
Fig. 6.6 Fatigue life as a function of initial crack length for four types of crack with b/a=0.3 in CS1 conduit of model coil, Incoloy alloy 908 (welds), 0%CW, stress = 618 MPa, R=0, T=4K. 1: surface crack, 2: single edge notch crack, 3: corner crack, 4: embedded crack.
CS1, SC, b/a0 = 0.3, 908W, 4K, 0%CW, 618MPa

Fig. 6.7 Fatigue life as a function of initial crack length for a surface crack with different crack aspect ratios (b/a = 0.1, 0.3, 0.5 and 1) in CS1 conduit of model coil, Incoloy alloy 908 (welds), 0%CW, stress = 618 MPa, R=0, T=4K.
7. Sample Calculations

Sample calculations are provided below to help readers to better understand all the calculation procedures. Included are the estimations of critical size during bending (Section 4.1 and Chapter 5), the uncertainty analysis of cracks during bending (Section 4.2 and Chapter 5), the fatigue life estimation (Section 4.3 and Chapter 6), and the bending stress and strain (Chapter 5).

7.1 Estimation of critical crack depth during bending

The critical crack depth \( b_c \) is defined, based on linear elastic fracture mechanics in mode I by

\[
K_{ic} = Y\sigma\sqrt{\pi b_c},
\]

where crack depth \( b_c \) is related to semi-crack length \( a_c \) by a crack aspect ratio \( b_c/a_c \). \( K_{ic} \) is a fracture toughness, \( \sigma \) is an applied stress normal to the crack in tension, and \( Y \) is a crack geometry factor. A bending stress is not uniform; it is in maximum tension at the outside edge of the curvature and gradually decreases to zero at about the center and then turns to compression. However, the maximum tensile bending stress is used in Eq. 4.5 as a conservative approximation. The derivation of the maximum bending stress \( \sigma \) is given in Sec. 7.4.

\( Y \) is a constant as the crack is very small in comparison to the plate size, e.g., for a single edge notched crack with a crack depth of \( b \) in a plate of width \( t \), if \( b \ll t \), then \( Y = 1.12 \). Its critical crack depth is obtained for \( \sigma = 760 \text{ MPa} \) and \( K_{ic} = 160 \text{ MPa}\sqrt{m} \) (for base metal of alloy 908, see Table 5.1):

\[
b_c = \frac{1}{\pi} \sigma^2 K_{ic}^2 \frac{1}{Y^2} = \frac{1}{3.1416} \times 760^{-2} \times 160^2 \times \frac{1}{1.12^2} = 0.011 \text{ (mm)}.
\]

For an elliptical surface crack, the stress intensity factor and \( Y \) factor vary along the crack periphery. In general, if \( b < 0.7a \), \( K \) and \( Y \) are the largest at the deepest point, if \( b > 0.7a \), \( K \) and \( Y \) become maximum at the surface points. For a semi-circular surface crack (i.e., \( b = a \)) in a semi-infinite plate, \( Y \) is 0.662 at the deepest point and 0.732 at the surface points by neglecting both the crack closure and plastic zone effects. For the first approximation, let us take an average of \( Y = 0.7 \). Then the critical crack depth is:

\[
b_c = \frac{1}{\pi} \sigma^2 K_{ic}^2 \frac{1}{Y^2} = \frac{1}{3.1416} \times 760^{-2} \times 160^2 \times \frac{1}{0.7^2} = 0.029 \text{ (mm)}.
\]

As expected, the critical size of a single edge notched crack is smaller than that of a surface crack. However, as the \( b/a \) becomes smaller and smaller, the surface crack approaches the single edge notched crack, then \( K \) and \( Y \) of a surface crack approach those of a single edge notched crack.

For the ITER conduits, comparing wall thicknesses of 1 mm for the TF and 6.25 mm for the CS conduits, a crack with size about 1/10th of the wall thickness is small. However, it is no longer small as the crack grows to approach the wall thickness. \( K \) and \( Y \) may become much larger than the initial constant value of a very small crack. This can be observed in Figs. 5.1 to 5.17. Generally, the \( Y \) factor in Eq. 4.7 is a function of crack...
size. The critical crack size cannot be obtained by simply calculating the left side of Eq. 4.7, but only by solving the equation 4.7. This task can be performed by a numerical approach, which is used in this paper to determine the results (see Appendix 2). However, a graphical solution can be obtained from Figs. 5.2, 5.4, 5.6, 5.8 and 5.16 for SC, SEN, CC, EC, and CSC respectively. For example, Fig. 5.2 shows K vs. crack depth for a surface crack with different aspect ratios in the CS1 conduit. Let us focus on the line with aspect ratio of b/a=0.3. As crack depth increases, K also increases and eventually reaches the fracture toughness of 160 MPa√m for the base metal of alloy 908 at a crack depth of 4.15 mm, which is the critical crack depth. This number is identical to that in Table 5.3 obtained by the numerical approach. Note that the critical crack depth in the thin plate (4.15 mm) is much smaller than that in a semi-infinite plate (29 mm). Y at the critical crack size is obtained by substituting \( K_{ic} \) and \( b_c \) into Eq. 4.7:

\[
Y = \frac{K_{ic}}{\sigma\sqrt{\pi b_c}} = \frac{160}{760 \times \sqrt{3.1416 \times 0.00415}} = 1.844
\]

7.2 Uncertainty analysis for a surface crack in CS1 conduit during bending

A sample calculation of the uncertainty analysis is given for a surface crack in the CS1 conduit with an aspect ratio of 0.3. The parameters are listed in Table 7.1.

Table 7.1 Parameters of estimating maximum allowable crack size for a surface crack in CS1 conduit with an aspect ratio of 0.3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mean value</th>
<th>Relative uncertainty of 1 SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>applied stress (MPa)</td>
<td>760</td>
<td>10%</td>
</tr>
<tr>
<td>fracture toughness (MPa√m)</td>
<td>160</td>
<td>15%</td>
</tr>
<tr>
<td>Y factor at critical size</td>
<td>1.844</td>
<td>5%</td>
</tr>
</tbody>
</table>

Let us first look at the basic relationship between the applied stress, the fracture toughness and the Y factor:

\[
b_c = \frac{1}{\pi} \sigma^2 K_{1c} \frac{1}{Y^2}
\]

(4.7)

Taking the log of both sides of Eq. 4.7 gives:

\[
\ln(b_c) = \ln\left(\frac{1}{\pi}\right) - 2\ln(\sigma) + 2\ln(K_{1c}) - 2\ln(Y)
\]

(4.15)

Write Eq. 4.15 into the expression of a general equation

\[
y = c_0 + c_1x_1 + c_2x_2 + c_3x_3
\]

(4.8)

where
\[ y = \ln(b) \]
\[ x_1 = \ln(\sigma) \]
\[ x_2 = \ln(K_{1e}) \]
\[ x_3 = \ln(Y) \]

\[ c_0 = -\ln \pi \]
\[ c_1 = -2 \]
\[ c_2 = 2 \]
\[ c_3 = -2 \]

The mean value of \( y \) is obtained as:

\[ y_0 = c_0 + c_1 x_{01} + c_2 x_{02} + c_3 x_{03} \]
\[ = \ln\left(\frac{1}{\pi}\right) - 2 \ln(\sigma_0) + 2 \ln(K_{1e0}) - 2 \ln(Y_0) \]
\[ = \ln\left(\frac{1}{3.1416}\right) - 2 \ln(760) + 2 \ln(160) - 2 \ln(1.844) = -5.484 \]

For a normal distribution, one standard deviation (SD) gives 2:1 odds for two-sided intervals. The uncertainty is then scaled up to 4 SD to get an upper limit of allowable crack depth at odds of 10,000:1. The actual factor is 3.7 for a normal distribution and about 5 for a third kind extreme-value distribution. Uncertainties of log(stress), log(fracture toughness) and log(Y factor) at 4 SD, according to Eq. 4.22, are respectively:

\[ u_{1\sigma}^m = \ln(\sigma_0 + \sigma_a) - \ln(\sigma_0) = \ln\left(1 + \frac{\sigma_a}{\sigma_0}\right) = \ln(1 + 4 \times 0.1) = 0.3365 \]
\[ u_{1\sigma}^m = \ln(\sigma_0 - \sigma_a) - \ln(\sigma_0) = \ln\left(1 - \frac{\sigma_a}{\sigma_0}\right) = \ln(1 - 4 \times 0.1) = -0.5108 \]
\[ u_{2K}^m = \ln(K_{1e0} + K) - \ln(K_{1e0}) = \ln\left(1 + \frac{K}{K_{1e0}}\right) = \ln(1 + 4 \times 0.15) = 0.4700 \]
\[ u_{2K}^m = \ln(K_{1e0} - K) - \ln(K_{1e0}) = \ln\left(1 - \frac{K}{K_{1e0}}\right) = \ln(1 - 4 \times 0.15) = -0.9163 \]
\[ u_{3Y}^m = \ln(Y_0 + Y_f) - \ln(Y_0) = \ln\left(1 + \frac{Y_f}{Y_0}\right) = \ln(1 + 4 \times 0.05) = 0.1823 \]
\[ u_{3Y}^m = \ln(Y_0 - Y_f) - \ln(Y_0) = \ln\left(1 - \frac{Y_f}{Y_0}\right) = \ln(1 - 4 \times 0.05) = -0.2231 \]

Uncertainties of \( y \) due to stress, fracture toughness and Y factors are, according to Eqs. 4.20:

\[ \Delta y_{1\sigma}^m = c_1 u_{1\sigma}^m = -2 \times 0.3365 = -0.6729 \]
\[ \Delta y_{1\sigma}^m = c_1 u_{1\sigma}^m = -2 \times (-0.5108) = 1.0216 \]
\[ \Delta y_{2K}^m = c_2 u_{2K}^m = 2 \times (0.4700) = 0.9400 \]
\[ \Delta y_{2K}^m = c_2 u_{2K}^m = 2 \times (-0.9163) = -1.8326 \]
\[ \Delta y_{3Y}^m = c_3 u_{3Y}^m = -2 \times (0.1823) = 0.3646 \]
\[ \Delta y_{3Y}^m = c_3 u_{3Y}^m = -2 \times (-0.2231) = -0.4463 \]
The total uncertainty of $y$ is the square root of the sum of the component products taking always the positive values:

$$u_y = \sqrt{\Delta y_1^{max} \Delta y_1^{min} + \Delta y_2^{max} \Delta y_2^{min} + \Delta y_3^{max} \Delta y_3^{min}}$$

$$= \sqrt{0.6729 \times 1.0216 + 0.94 \times 1.8326 + 0.3646 \times 0.4463} = 1.604 \quad (4.23)$$

The allowable value of $y$ at odds of 10,000 to 1 is equal to the mean value minus the uncertainty:

$$y_{alt} = y_0 - u_y = -5.484 - 1.604 = -7.088 \quad (4.24)$$

Therefore, the allowable crack depth is equal to

$$b_{alt} = e^{y_{alt}} = \exp(-7.088) = 0.00083 \text{ m} = 0.83 \text{ mm} \quad (4.25)$$

This result is identical to that listed in Table 5.3.

7.3 Fatigue life estimation for a surface crack in CS1 conduit

Fatigue crack growth follows the Paris equation in LEFM:

$$\frac{da}{dN} = c(\Delta K)^m$$ \quad (4.26)

Integration of Eq. 4.26 combined with Eq. 4.27 gives fatigue life:

$$N_f = \frac{\sigma_{max}^{-n} (1 - R)^{n+1} \xi}{c (n-2) \pi^{n/2} (1-R)^{n+1} \sigma_{max}^n} \left[ \frac{1}{a_i^{(n-1)/2}} - \frac{1}{a_f^{(n-1)/2}} \right] \quad (4.30)$$

where

$$\xi = \frac{1}{c} \int_{a_i}^{\sigma_f} \frac{da}{y^n \sigma_{max}^n} \left[ \frac{1}{a_i^{(n-1)/2}} - \frac{1}{a_f^{(n-1)/2}} \right]$$

For a crack in an infinite plate, $Y$ is a constant, and Eq. 3.12 becomes

$$N_f = \frac{2}{(n-2) \pi^{n/2} (1-R)^{n+1} \sigma_{max}^n} \left[ \frac{1}{a_i^{(n-1)/2}} - \frac{1}{a_f^{(n-1)/2}} \right] \quad (7.1)$$

For example, for a semi-circular surface crack in a semi-infinite plate made of alloy 908, $Y=0.7$, $c=1.5 \times 10^{-12}$, $n=3.41$, $m=0.8$, $R=0.1$, $\sigma=618$ MPa. The fatigue life for a crack growing from 0.625 mm to 6.25 mm at room temperature is

$$N_f = \frac{2}{(3.41 - 2) \times 1.5 \times 10^{-12} \pi^{3.41/2} (1 - 0.1)^{0.8 \times 3.41} \times 0.7^{3.41} \times 618^{3.41}} \times \left[ \frac{1}{0.000625^{(3.41/2-1)}} - \frac{1}{0.00625^{(3.41/2-1)}} \right] = 13,374 \text{ (cycle)}$$
Unfortunately, most of cracks are not small as compared to the plate size. $Y$ is not constant, but becomes larger and larger as the crack grows to approach the wall thickness. Therefore, fatigue life cannot be obtained by a simple analytical integration, but must rely on numerical approaches. For example, in Eq. 4.30, the integration factor, in terms of crack depth $b$, becomes

$$\xi = \frac{1}{c} \sum_{j} \frac{\Delta b}{y_j^{\nu} (\pi b_j)^{\nu/2}} .$$  \hspace{1cm} (7.2)$$

Eq. 7.2 can be rewritten by assuming a $k$-fold integration:

$$\xi = \frac{\Delta b}{c \pi^{\nu/2}} \left[ \left( \frac{1}{y_1 \sqrt{b_1}} \right)^n + \left( \frac{1}{y_2 \sqrt{b_2}} \right)^n + \ldots \right].$$  \hspace{1cm} (7.3)$$

Combining Eqs. 7.1, 7.2 and 7.3 gives

$$N_f = \frac{\Delta b}{c \pi^{\nu/2} (1 - R)^{mn} \sigma_{\text{max}}^n} \times \left[ \left( \frac{1}{y_1 \sqrt{b_1}} \right)^n + \left( \frac{1}{y_2 \sqrt{b_2}} \right)^n + \ldots \right]$$  \hspace{1cm} (7.4)$$

where the $Y$ factor is a function of crack depth $b$, and is obtained from Figs. 5.1, 5.3, 5.5, and 5.7.

For a 3D crack, the $Y$ factor varies along the periphery of the crack front line. It is then calculated at both the deepest and edge points. The crack depth and semi-crack length are obtained by integrating Eq. 4.30 at those two points.

7.4 Bending stress and strain

Assume that a conductor with a total thickness $2t$ is bent in a radius $r$. The maximum tensile strain, $\varepsilon_b$, is obtained by:

$$\varepsilon_b = \frac{t}{r} .$$  \hspace{1cm} (7.5)$$

For CS1, $t = 25.5$ mm, $r = 800$ mm, and the plastic bending strain is

$$\varepsilon_b = \frac{t}{r} = \frac{25.5}{800} = 0.03 .$$

By checking the tensile curve of Incoloy 908 at annealing condition, at plastic strain=3%, the maximum bending stress is about 760 MPa.
8. Appendix 1 - $K$ expressions for different types of crack[13.14]

$$K_i = \frac{\sigma \sqrt{\pi b}}{E(k)} F\left(\frac{b}{t}, \frac{b}{a}, \frac{R}{t}, \phi\right)$$

(1)

(a) $b/a < 1$

(b) $b/a > 1$

Fig. Coordinate system used to define parametric angle.

$$E(k) \equiv \left[1 + 1.464 \left(\frac{b}{a}\right)^{1.65}\right]^{\frac{1}{2}} \text{ for } \frac{b}{a} \leq 1 \quad (2a)$$

(Accuracy 0.13%)

$$E(k) \equiv \left[1 + 1.464 \left(\frac{a}{b}\right)^{1.65}\right]^{\frac{1}{2}} \text{ for } \frac{b}{a} > 1 \quad (2b)$$

Fig. Embedded crack

$$K_i = \frac{\sigma \sqrt{\pi b}}{E(k)} F_e\left(\frac{b}{a}, \frac{b}{t}, \frac{a}{W}, \phi\right) \quad (3)$$
Range of applicability: \( 0 \leq b/a \leq \infty , a/W < 0.5 \)
and \(-\pi \leq \phi \leq \pi \)

Necessary conditions:
\[
\begin{align*}
\frac{b}{t} < 1.25 \left( \frac{b}{a} + 0.6 \right) & \quad \text{for} \quad 0 \leq \frac{b}{a} \leq 0.2 \\
\frac{b}{t} < 1 & \quad \text{for} \quad 0.2 \leq \frac{b}{a} \leq \infty
\end{align*}
\]

\[ F_e = \left[ M_1 + M_2 \left( \frac{b}{t} \right)^2 + M_3 \left( \frac{b}{t} \right)^3 \right] g f_\phi f_w \]  

\[ M_1 = 1 \quad \text{for} \quad b/a \leq 1 \]  

\[ M_1 = \left( \frac{a}{b} \right)^{0.05} \quad \text{for} \quad b/a > 1 \]  

\[ M_2 = \frac{0.05}{0.11 + (b/a)^{3/2}} \]  

\[ M_3 = \frac{0.29}{0.23 + (b/a)^{3/2}} \]  

\[ g = 1 - \frac{\left( \frac{b}{t} \right)^n \cos \phi}{1 + 4 \left( \frac{b}{a} \right)} \]  

\[ f_\phi = \left[ \left( \frac{b}{a} \right)^2 \cos^2 \phi + \sin^2 \phi \right]^{1/4} \quad \text{for} \quad b/a \leq 1 \]  

\[ f_\phi = \left[ \left( \frac{a}{b} \right)^2 \sin^2 \phi + \cos^2 \phi \right]^{1/4} \quad \text{for} \quad b/a > 1 \]  

\[ f_w = \left[ \sec \left( \frac{\pi a}{2W \sqrt{t}} \right) \right]^{1/2} \]  

\[ K_I = \frac{\sigma \sqrt{\pi b}}{E(k)} F_s \left( \frac{b}{a}, \frac{b}{t}, \frac{a}{W}, \phi \right) \]  

Fig. Surface crack

Range of applicability: \( 0 \leq b/a \leq 2 , a/W < 0.5 \)
and \( 0 \leq \phi \leq \pi \)
Necessary conditions: The same as Eq.(4)

\[ F_s = \left[ M_1 + M_2 \left( \frac{b}{t} \right)^2 + M_3 \left( \frac{b}{t} \right)^4 \right] g f_{\phi} f_W \]  \hspace{1cm} (13)

\begin{align*}
M_1 &= 1.13 - 0.09 \left( \frac{b}{a} \right) \quad \text{for } b/a \leq 1 \\
M_1 &= \sqrt{\frac{a}{b}} \left( 1 + 0.04 \frac{b}{a} \right) \quad \text{for } b/a > 1 \\
M_2 &= -0.54 + \frac{0.89}{0.2 + \left( \frac{b}{a} \right)} \quad \text{for } b/a \leq 1 \\
M_2 &= 0.2 \left( \frac{a}{b} \right)^{4} \quad \text{for } b/a > 1 \\
M_3 &= 0.5 - \frac{1}{0.65 + \frac{b}{a}} + 14 \left( 1 - \frac{b}{a} \right)^{24} \quad \text{for } b/a \leq 1 \\
M_3 &= -0.11 \left( \frac{a}{b} \right)^4 \quad \text{for } b/a > 1 \\
g &= 1 + \left[ 0.1 + 0.35 \left( \frac{b}{t} \right)^2 \right] \left( 1 - \sin \phi \right)^2 \quad \text{for } b/a \leq 1 \\
g &= 1 + \left[ 0.1 + 0.35 \left( \frac{a}{b} \right) \left( \frac{b}{t} \right)^2 \right] \left( 1 - \sin \phi \right)^2 \quad \text{for } b/a > 1 \\
f_{\phi} &= \text{Eq.(10)} \\
f_W &= \text{Eq.(11)}
\end{align*}  \hspace{1cm} (14)-(17)

Fig. Corner crack

\[ K_I = \frac{\sigma \sqrt{\pi d}}{E(k)} F_c \left( \frac{b}{a}, \frac{b}{t}, \phi \right) \]  \hspace{1cm} (18)

Range of applicability: \( 0.2 \leq b/a \leq 2 \), \( a/t < 1 \)
and \( 0 \leq \phi \leq \pi/2 \)
for \( a/W < 0.2 \)

\[ F_c = \left[ M_1 + M_2 \left( \frac{b}{t} \right)^2 + M_3 \left( \frac{b}{t} \right)^4 \right] g_1 g_2 f_{\phi} \]  \hspace{1cm} (19)
\( M_1 = 1.08 - 0.03 \left( \frac{b}{a} \right) \) for \( b/a \leq 1 \)  
\( M_1 = \frac{1.08}{\frac{b}{b}} \left( 1.08 - 0.03 \frac{a}{b} \right) \) for \( b/a > 1 \)  
\( M_2 = -0.44 + \frac{1.06}{0.3 + \left( \frac{b}{a} \right)} \) for \( b/a \leq 1 \)  
\( M_2 = 0.375 \left( \frac{a}{b} \right)^2 \) for \( b/a > 1 \)  
\( M_3 = -0.5 + 0.25 \left( \frac{b}{a} \right) + 14.8 \left( 1 - \frac{b}{a} \right)^{1.5} \) for \( b/a \leq 1 \)  
\( M_3 = -0.25 \left( \frac{b}{a} \right)^2 \) for \( b/a > 1 \)  
\( g_1 = 1 + \left[ 0.08 + 0.4 \left( \frac{b}{a} \right)^2 \right] \left( 1 - \sin \phi \right)^3 \) for \( b/a \leq 1 \)  
\( g_1 = 1 + \left[ 0.08 + 0.4 \left( \frac{a}{b} \right)^2 \right] \left( 1 - \sin \phi \right)^3 \) for \( b/a > 1 \)  
\( g_2 = 1 + \left[ 0.08 + 0.15 \left( \frac{b}{a} \right)^2 \right] \left( 1 - \cos \phi \right)^3 \) for \( b/a \leq 1 \)  
\( g_2 = 1 + \left[ 0.08 + 0.15 \left( \frac{a}{b} \right)^2 \right] \left( 1 - \cos \phi \right)^3 \) for \( b/a > 1 \)  
\( f_\phi = \text{Eq. (10)} \)

Fig. Single edge notch crack

\[ K_i = \sigma \sqrt{\pi b} F \left( \frac{b}{t} \right) \]

\[ F \left( \frac{b}{t} \right) = \sqrt{\frac{2t \tan \frac{\pi b}{2t}}{\pi b}} \times \frac{0.752 + 2.02 \left( \frac{b}{t} \right) + 0.37 \left( 1 - \frac{\pi b}{t} \right)}{\cos \frac{\pi b}{2t}} \]
9. Appendix 2 - Computer code to estimate fatigue life
program fatlife

C terminology:
- da/dN = c * dk**rm
- fn = life
- b0 = initial crack depth
- rkc = fracture toughness
- st = applied maximum stress
- r = load ratio
- wm = Walker's coefficient for R ratio effect, dkeff = dk*(1-r)**wm
- dn = integration step of cycle
- t = specimen thickness
- w = half specimen width
- b_a = b/a
- b = crack depth
- a = half crack length
- ntype = 1, surface crack, cross section: 2a x b, 2w x t
  - crack area: pi * ab/2
- ntype = 2, single edge notch crack: thickness: b, 2w x t
  - crack area: b*2w
- ntype = 3, corner crack: a x b, w x t
  - crack area: pi * ab/4
- ntype = 4, embedded crack: 2a x 2b, 2w x 2t
  - crack area: pi * ab

C

common /input/ c, rm, rkc, r, wm, dn, st, t, w, b_a0, bO
common /result/ af, bf, b_a, l, fn, rka, rkb
read(*,*) ntype, c, rm, rkc, r, wm, dn, st, t, w, b_a0, bO
if (ntype.eq.-1) goto 13

C

life vs. stress at given initial crack length
210 read(*,*) b0, st0, stf, nst
if (b0.eq.-1) goto 211
write(8,*), i, b0, st, fn, af, bf, b_a, l
21 continue

211 continue
C

goto 11
13 close(8)
close(9)
stop
end

C

subroutine cracksc_an
common /input/ c, rm, rkc, r, wm, dn, st, t, w, b_a0, bO
common /result/ af, bf, b_a, l, fn, rka, rkb
pi = 3.14159
b = b0
a = b0 / b_a0

52
c go ahead if not fracture
1  if (a.lt.w) then
    atemp=a+da/2
    btemp=b+db/2
    if (btemp.gt.t) goto 101
    call surface_a(atemp,btemp,t,w,yatemp)
    call surface_b(atemp,btemp,t,w,ybtemp)
    rkatemp=yatemp*st*sqrt(pi*btemp)
    rkbtemp=ybtemp*st*sqrt(pi*btemp)
    dkatemp=rkatemp*(1.-r)**wm
    dkbttemp=rkbtemp*(1.-r)**wm
    dadn=c*dkatemp**rm
    dbdn=c*dkbttemp**rm
    da=dadn*dn
    db=dbdn*dn
    a=a+da
    b=b+db
    cn=cn+dn
    l=l+1
    if (b.gt.t) goto 101
    call surface_a(a,b,t,w,ya)
    call surface_b(a,b,t,w,yb)
    rka=ya*st*sqrt(pi*b)
    rkb=yb*st*sqrt(pi*b)
else
    btemp=b+db/2
    if (btemp.gt.t) goto 101
    call sen(btemp,t,ybtemp)
    rkbtemp=ybtemp*st*sqrt(pi*btemp)
    dkbttemp=rkbtemp*(1.-r)**wm
    dbdn=c*dkbttemp**rm
    db=dbdn*dn
    b=b+db
    cn=cn+dn
    l=l+1
    if (b.gt.t) goto 101
    call sen(b,t,yb)
    rkb=yb*st*sqrt(pi*b)
endif
   c check if reaching at fracture point
   if ((rkb.gt.rkc).or.(rka.gt.rkc)) then
     goto 101
   else
     goto 1
   endif
   c final fracture
101  fn=cn
    bf=b
    af=a
    b_a=bf/af
    return
end

subroutine cracksen_an
common /input/ c,rm,rkc,r,wm,dn,t,w,b_a0,b0
common /result/ af,bf,b_a,1,fn,rka,rkb
pi=3.14159
b=0
bn=0.
bn=0.
l=0

c go ahead if not fracture
1  if (btemp.ge.t) goto 101
  call sen(btemp,t,ybtemp)
  rkbtemp=ybtemp*st*sqrt(pi*btemp)
  dkbttemp=rkbtemp*(1.-r)**wm
  dbdn=c*dkbttemp**rm
\[ db = dbdn*dn \\
\[ b = b + db \\
\[ cn = cn + dn \\
\[ l = l + 1 \\
if (b \geq t) \text{ goto } 101 \\
call \text{ sen}(b, t, yb) \\
rkb = yb * st * \sqrt{\pi b} \\
c \]

check if reaching at fracture point 
if (rkb \lt rkc) \text{ goto } 1

c final fracture
101 
\[ fn = cn \\
f b = b \\
return 
end 

c subroutine crackcc_an 
c \]

common /input/ c, rm, rkc, r, wm, dn, st, t, w, b_a0, b0 
c \]

common /result/ af, bf, b_a, l, fn, rka, rkb 
pi = 3.14159 
b = b0 
a = b0 / b_a0 
cn = 0. 
da = 0. 
\[ db = 0. 
\]
l = 0 
c check if reaching at fracture point 
l if ((a \geq w) \& \& (b \geq t)) \text{ goto } 101 

c go ahead if not fracture 
if ((a \geq w) \& \& (b \lt t)) then 
\[ btemp = b + db/2 \\
if (btemp \geq t) \text{ goto } 101 \\
call \text{ sen}(btemp, t, ybtemp) \\
rkbtemp = ybtemp * st * \sqrt{\pi btemp} \\
dkbttemp = rkbtemp * (1. - r)^**wm \\
dbdn = c * dkbttemp**rm \\
\[ db = dbdn*dn \\
\[ b = b + db \\
cn = cn + dn \\
l = l + 1 
\]
if (b \geq t) \text{ goto } 101 \\
call \text{ sen}(b, t, yb) \\
rkb = yb * st * \sqrt{\pi b} \\
elseif ((a \lt w) \& \& (b \geq t)) then 
\[ atemp = a + da/2 \\
if (atemp \geq t) \text{ goto } 101 \\
call \text{ sen}(atemp, t, yatemp) \\
rkatemp = yatemp * st * \sqrt{\pi atemp} \\
dkatemp = rkatemp * (1. - r)^**wm \\
dadn = c * dkatemp**rm \\
da = da + da \\
cn = cn + dn \\
l = l + 1 
\]
if (a \geq t) \text{ goto } 101 \\
call \text{ sen}(a, t, ya) \\
rka = ya * st * \sqrt{\pi a} \\
else 
\[ atemp = a + da/2 \\
btemp = b + db/2 \\
call \text{ corner}_a(atemp, btemp, t, w, yatemp) \\
call \text{ corner}_b(atemp, btemp, t, w, ybtemp) \\
rkatemp = yatemp * st * \sqrt{\pi atemp} \\
rkbttemp = ybtemp * st * \sqrt{\pi btemp} \\
dkatemp = rkatemp * (1. - r)^**wm \\
dkbtemp = rkbtemp * (1. - r)^**wm \\
dadn = c * dkatemp**rm \\
dbdn = c * dkbtemp**rm \\
da = da + da \\
b = b + db \\
cn = cn + dn \\
54
l=l+1
call corner_a(a,b,t,w,ya)
call corner_b(a,b,t,w,yb)
rka=ya*st*sqrt(pi*b)
rkb=yb*st*sqrt(pi*b)
endif

c c check if reaching at fracture point
if ((rkb.gt.rkc).or.(rka.gt.rkc)) then
goto 101
else
goto 1
endif

c c final fracture
101
fn=cn
bf=b
af=a
b_a=bf/af
return
end

c subroutine crackec_an
common /input/ c,rm,rkc,r,wm,sn,t,t,w,b_a0,b0
common /result/ af,bf,b_a,l,fn,rka,rkb
pi=3.14159
b=b0
af=a0/cn=0.
da=0.
db=0.

l=0
c c go ahead if not fracture
1
atemp=a+da/2
btemp=b+db/2
if ((atemp.gt.w).or.(btemp.gt.t)) goto 101
call embed_a(atemp,btemp,t,w,yatemp)
call embed_b(atemp,btemp,t,w,ybtemp)
rkatemp=yatemp*st*sqrt(pi*btemp)
rkbtemp=ybtemp*st*sqrt(pi*btemp)
dkatemp=rkatemp*(1.-r)**wm
dkbtemp=rkbtemp*(1.-r)**wm
dadn=c*dkatemp**rm
dbdn=c*dkbtemp**rm
da=dadn*dn
db=dbdn*dn
a=a+da
b=b+db
cn=cn+dn
l=l+1
if ((a.ge.w).or.(b.ge.t)) goto 101
call embed_a(a,b,t,w,ya)
call embed_b(a,b,t,w,yb)
rka=ya*st*sqrt(pi*b)
rkb=yb*st*sqrt(pi*b)

c c check if reaching at fracture point
if ((rkb.lt.rkc).and.(rka.lt.rkc)) goto 1

c c final fracture
101
fn=cn
bf=b
af=a
b_a=bf/af
return
end

c subroutine surface-b(a,b,tt,w,yf)
pi=3.141593
if (b.le.a) then
rml=1.13-0.09*(b/a)
rm2=0.54+0.89/(0.2+b/a)
rm3=0.5-1/(0.65+b/a)+14*(1.-b/a)**24
g=1.
fphi=1.
fw=sqrt(1/(cos(pi*a*sqrt(b/tt))/(2*w)))
fs=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
ek=sqrt(1.+1.464*(b/a)**1.65)
yf=fs/ek

else
  rml=sqrt(a/b)*(1.+0.04*a/b)
  rm2=0.2*(a/b)**4
  rm3=-0.11*(a/b)**4
  g=1.
  fphi=sqrt(a/b)
  fw=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
  fs=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
  ek=sqrt(1.+1.464*(a/b)**1.65)
  yf=fs/ek
endif
return
end

subroutine surface_a(a,b,tt,w,yf)
cc
Y factor at edge points (10/13/94)
pi=3.141593
if (b.le.a) then
  rml=1.13-0.09*(b/a)
  rm2=-0.54+0.89/(0.2+b/a)
  rm3=0.5-1/(0.65+b/a)+14*(1.-b/a)**24
  g=1.+(0.1+0.35*(b/tt)**2)
  fphi=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
  fw=sqrt(1/(cos(pi*a*sqrt(b/tt))/(2*w)))
  fs=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
  ek=sqrt(1.+1.464*(b/a)**1.65)
  yf=fs/ek
else
  rml=sqrt(a/b)*(1.+0.04*a/b)
  rm2=0.2*(a/b)**4
  rm3=-0.11*(a/b)**4
  g=1.+(0.1+0.35*(a/b)*(b/tt)**2)
  fphi=1.
  fw=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
  fs=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
  ek=sqrt(1.+1.464*(a/b)**1.65)
  yf=fs/ek
endif
return
end

subroutine sen(b,tt,yf)
pi=3.141593
if (b.ge.tt) b=0.999999*tt
temp=0.5*pi*b/tt
yfl=sqrt (tan(temp) /temp)
yf2=0.752+2.02*b/tt+0.37*(1.-sin(temp))**3
yf3=cos(temp)
yf=yfl*yf2/yf3
return
end

subroutine corner_b(a,b,tt,w,yf)
pi=3.141593
if (b.le.a) then
  rml=1.08-0.03*(b/a)
  rm2=-0.44+1.06/(0.3+b/a)
  rm3=-0.5+0.25*b/a+14.8*(1.-b/a)**15
  gl=1.
  g2=1.+(0.08+0.15*(b/tt)**2)
  fphi=1.
  fc=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*gl*g2*fphi
  ek=sqrt(1.+1.464*(b/a)**1.65)
  yf=fc/ek
else
  rml=sqrt(a/b)*(1.08-0.03*a/b)
  rm2=0.375*(a/b)**2
  rm3=-0.25*(a/b)**2
  gl=1.
  g2=1.+(0.08+0.15*(a/tt)**2)
end
fphi=sqrt(a/b)
fce=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*g1*g2*fphi
ek=sqrt(1.+1.464*(a/b)**1.65)*g2*g1
yf=fc/ek
endif
101
return
e nd

subroutine corner_a(a,b,tt,w,yf)
pi=3.141593
if (b.le.a) then
  rml=1.08-0.03*(b/a)
  rm2=-0.44+1.06/(0.3+b/a)
  rm3=-0.5+0.25*b/a+14.8*(1.-b/a)**15
  gl=1.+(0.08+0.4*(b/tt)**2)
  g2=1.
  fphi=sqrt(b/a)
  fc=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*gl*g2*fphi
  ek=sqrt(1.+1.464*(a/b)**1.65)
  yf=fc/ek
else
  rml=sqrt(a/b)*(1.08-0.03*a/b)
  rm2=0.375*(a/b)**2
  rm3=-0.25*(a/b)**2
  gl=1.+(0.08+0.4*(a/tt)**2)
  g2=1.
  fphi=1.
  fc=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*g1*g2*fphi
  ek=sqrt(1.+1.464*(b/a)**1.65)
  yf=fc/ek
endif
101
return
e nd

subroutine embed_b(a,b,tt,w,yf)
pi=3.141593
if (b.le.a) then
  rml=1.
  fphi=1.
  ek=sqrt(1.+1.464*(b/a)**1.65)
else
  rml=sqrt(a/b)
  fphi=sqrt(a/b)
  ek=sqrt(1.+1.464*(a/b)**1.65)
endif
rm2=0.05/(0.11+(b/a)**1.5)
rm3=0.29/(0.23+(b/a)**1.5)
g=1.
fw=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
fs=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
yf=fs/ek
110
return
e nd

subroutine embed_a(a,b,tt,w,yf)
pi=3.141593
if (b.le.a) then
  rml=1.
  fphi=sqrt(b/a)
  ek=sqrt(1.+1.464*(b/a)**1.65)
else
  rml=sqrt(a/b)
  fphi=1.
  ek=sqrt(1.+1.464*(a/b)**1.65)
endif
rm2=0.05/(0.11+(b/a)**1.5)
rm3=0.29/(0.23+(b/a)**1.5)
g=1.0-(b/tt)**4/(1.0+4*b/a)
fw=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
fs=(rml+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
yf=fs/ek
110
return
e nd
## 10. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>semi-crack length</td>
</tr>
<tr>
<td>$a_c$</td>
<td>critical semi-crack length</td>
</tr>
<tr>
<td>$a_i$</td>
<td>initial crack size</td>
</tr>
<tr>
<td>$a_f$</td>
<td>final crack size</td>
</tr>
<tr>
<td>$b$</td>
<td>crack depth</td>
</tr>
<tr>
<td>$b_c$</td>
<td>critical crack depth</td>
</tr>
<tr>
<td>$b_0$</td>
<td>mean crack depth</td>
</tr>
<tr>
<td>$b_{all}$</td>
<td>maximum allowable crack depth at given odds</td>
</tr>
<tr>
<td>$b/a$</td>
<td>crack aspect ratio</td>
</tr>
<tr>
<td>$b_c/a_c$</td>
<td>critical crack aspect ratio</td>
</tr>
<tr>
<td>$\Delta b$</td>
<td>integration step of crack depth</td>
</tr>
<tr>
<td>$c$</td>
<td>coef. in Paris equation</td>
</tr>
<tr>
<td>$c_i$</td>
<td>linear coefs. (i=1,2...)</td>
</tr>
<tr>
<td>$CW$</td>
<td>cold work</td>
</tr>
<tr>
<td>$\frac{da}{dN}$</td>
<td>crack growth rate</td>
</tr>
<tr>
<td>$f(\theta)$</td>
<td>function of orientation at crack tip</td>
</tr>
<tr>
<td>$K$</td>
<td>stress intensity factor at crack tip</td>
</tr>
<tr>
<td>$K_1$</td>
<td>stress intensity factor in mode I</td>
</tr>
<tr>
<td>$K_{10}$</td>
<td>stress intensity factor of mode I for a crack in an infinite plate</td>
</tr>
<tr>
<td>$K_{1c}$</td>
<td>critical stress intensity factor of mode I</td>
</tr>
<tr>
<td>$K_{1e0}$</td>
<td>mean value of $K_{1c}$</td>
</tr>
<tr>
<td>$K_{\text{max}}$</td>
<td>maximum stress intensity factor during cyclic loading</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>stress intensity factor range during cyclic loading</td>
</tr>
<tr>
<td>$m$</td>
<td>Walk coef. for R ratio effect</td>
</tr>
<tr>
<td>$n$</td>
<td>exponent in Paris equation</td>
</tr>
<tr>
<td>$N_f$</td>
<td>fatigue life</td>
</tr>
<tr>
<td>$r$</td>
<td>polar coordinate at crack tip</td>
</tr>
<tr>
<td>$r$</td>
<td>bending radius</td>
</tr>
<tr>
<td>$R$</td>
<td>load ratio</td>
</tr>
<tr>
<td>$s_i$</td>
<td>standard deviation for variable i (i=1,2...)</td>
</tr>
<tr>
<td>$s_y$</td>
<td>standard deviation of $y$</td>
</tr>
<tr>
<td>$s_{\sigma}$</td>
<td>standard deviation of stress</td>
</tr>
<tr>
<td>$s_K$</td>
<td>standard deviation of stress intensity factor</td>
</tr>
<tr>
<td>$s_Y$</td>
<td>standard deviation of $Y$ factor</td>
</tr>
<tr>
<td>$t$</td>
<td>specimen thickness</td>
</tr>
<tr>
<td>$u_i$</td>
<td>uncertainty for variable i at given odds (i=1,2...)</td>
</tr>
<tr>
<td>$u_i^{\text{up}}$</td>
<td>upper limit of $u_i$</td>
</tr>
<tr>
<td>$u_i^{\text{low}}$</td>
<td>lower limit of $u_i$</td>
</tr>
<tr>
<td>$u_{\sigma}$</td>
<td>uncertainty of stress at given odds</td>
</tr>
<tr>
<td>$u_K$</td>
<td>uncertainty of $K$ at given odds</td>
</tr>
</tbody>
</table>
\( u_r \) uncertainty of Y factor at given odds
\( u_{rs} \) root square of sum of square of all variable uncertainties
\( x_{i}(i=1,2...) \) independent variable i in uncertainty analysis
\( x_{\bar{u}i} \) mean value of \( x_i \)
\( x_{\bar{u}i} \) limits of \( x_i \)
\( y \) dependent variable in uncertainty analysis
\( y_0 \) mean value of \( y \)
\( y_{\text{all}} \) maximum allowable value of \( y \)
\( y_{u}(i=1,2...) \) y limits due to variable i
\( \Delta y_{i}(i=1,2...) \) uncertainty of \( y \) due to variable i
\( \Delta y_{i}^{\text{max}} \) upper limit of \( \Delta y_i \)
\( \Delta y_{i}^{\text{min}} \) lower limit of \( \Delta y_i \)
\( Y \) Y factor of a crack
\( Y_0 \) mean value of \( Y \)
\( \sigma \) applied tensile stress
\( \sigma_0 \) mean value of \( \sigma \)
\( \sigma_b \) maximum bending stress
\( \Delta \sigma \) applied stress range
\( \sigma_{\text{max}} \) maximum applied stress
\( \varepsilon_b \) plastic bending strain
\( \xi \) integration factor for fatigue life
11. References


