Mechanisms and Control of Beam Halo Formation in Intense Microwave Sources and Accelerators

C. Chen and R. Pakter
February, 2000

Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge, MA 02139, USA

This work was supported in part by the Air Force Office of Scientific Research Grant No. F49620-97-1-0325, in part by the Department of Energy, Office of High Energy and Nuclear Physics Grant No. DE-FG02-95ER-40919, and in part by the Department of Energy through a subcontract with Princeton Plasma Physics Laboratory. Reproduction, translation, publication, use and disposal, in whole or part, by or for the United States government is permitted.

Mechanisms and Control of Beam Halo Formation in
Intense Microwave Sources and Accelerators

C. Chen and R. Pakter
Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

ABSTRACT

Halo formation and control in space-charge-dominated electron and ion beams are investigated in parameter regimes relevant to the development of high-power microwave (HPM) sources and high-intensity electron and ion linear accelerators. In particular, a mechanism for electron beam halo formation is identified in high-power periodic permanent magnet (PPM) focusing klystron amplifiers. It is found in self-consistent simulations that large-amplitude current oscillations induce mismatched beam envelope oscillations and electron beam halo formation. Qualitative agreement is found between simulations and the 50 MW 11.4 GHz PPM focusing klystron experiment at Stanford Linear Accelerator Center (SLAC) [D. Sprehn, G. Caryotakis, E. Jongewaard, and R. M. Phillips, “Periodic permanent magnetic development for linear collider X-band klystrons,” Proc. XIX International Linac Conf. (Argonne National Laboratory Report ANL-98/28, 1998), p. 689.] Moreover, a new class of cold-fluid corkscrewing elliptic beam equilibria is discovered for ultrahigh-brightness, space-charge-dominated electron or ion beam propagation through a linear focusing channel consisting of uniform solenoidal magnetic focusing fields, periodic solenoidal magnetic focusing fields, and/or alternating-gradient quadrupole magnetic focusing fields in an arbitrary arrangement including field tapering. As an important application of such new cold-fluid corkscrewing elliptic beam equilibria, a technique is developed and demonstrated for controlling of halo formation and beam hollowing in an rms-matched ultrahigh-brightness ion beam as it is injected from an axisymmetric Pierce diode into an alternating-gradient magnetic quadrupole focusing channel.

PACS numbers: 29.27, 52.75.V
I. INTRODUCTION

One of the most challenging tasks in the development of high-intensity microwave sources and high-intensity particle accelerators is to prevent intense electron or ion beams from beam losses [1,2]. In high-intensity microwave sources, such as those considered for directed energy applications and for powering the next linear collider (NLC), a small fractional loss of electrons into the radio-frequency (rf) structure will inevitably induce secondary emission of electrons which, in the presence of intense rf fields, may cause an avalanche of secondary electron emission and subsequent plasma formation and alteration in the frequency response or dispersion characteristics of the rf structure. It is likely that a sequence of such events ultimately leads to rf pulse shortening in high-power microwave (HPM) sources [1,3-7]. In high-intensity electron or ion accelerators, such as high-gradient electron linacs, rf proton linacs for spallation neutron source, and induction linacs for heavy ion fusion applications, losses of electrons or ions in the accelerating structure may also result in intolerable radioactivity in the structure [8], in addition to the secondary emission of electrons and/or ions.

While disruptive beam loss is caused by violent instabilities such as the beam-breakup (BBU) instability [9-11] in the beam, mild beam loss is often associated with the formation of a tenuous halo [12-21] around a dense core of a beam, making physical contact with the inner wall of a microwave tube or accelerator. From the point of view of beam transport, there are two main processes for halo formation in high-intensity particle (electron or ion) beams. One process is caused by a mismatch in the root-mean-square (rms) beam envelope [12-15], and the other is due to a mismatch in the particle phase-space distribution relative to an equilibrium distribution [16-21]. Both processes can occur when the beam intensity is sufficiently high, so that the particle beam becomes space-charge-dominated.

For a periodic focusing channel with periodicity length $S$ and vacuum phase advance $\sigma_0$, a *space-charge-dominated beam* satisfies the condition [20]

$$\frac{SK}{4\sigma_0\varepsilon} > 1,$$

whereas an *emittance-dominated beam* satisfies the condition

$$\frac{SK}{4\sigma_0\varepsilon} << 1.$$
Here, $K = 2q^2 N_b / \gamma_b^3 \beta_b^2 mc^2$ is the normalized self-field perveance, $\varepsilon$ is the unnormalized transverse rms emittance of the beam, $N_b$ is the number of particles per unit axial length, $q$ and $m$ are the particle charge and rest mass, respectively, $\beta_b c$ and $\gamma_b$ are the average axial velocity and relativistic mass factor of the particles in the beam, respectively, and $c$ is the speed of light in vacuo. The emittance, which is essentially the beam radius times a measure of randomness in the transverse particle motion, is often measured experimentally or calculated in terms of the normalized transverse rms emittance $\varepsilon_n = \gamma_b \beta_b \varepsilon$. For a uniform density beam with radius $a$ and temperature $T_b$, the normalized transverse rms emittance is given by

$$\varepsilon_n = \gamma_b \beta_b \varepsilon = \frac{a}{2} \left( \frac{\gamma_b \beta_b T_b}{mc^2} \right)^{1/2},$$

where $k_B$ is the Boltzmann constant. For an electron beam, the dimensionless parameter $SK / 4\sigma_0 \varepsilon$ can be expressed as

$$\frac{SK}{4\sigma_0 \varepsilon} = 2.9 \times 10^{-5} \frac{1}{\sigma_b} \left( \frac{S}{\varepsilon_n} \right) \frac{I_b}{\gamma_b^2 \beta_b^2},$$

where $I_b$ is the electron beam current in amperes, $\varepsilon_n$ is the normalized rms emittance in meter-radians, and $S$ is in meters. For an ion beam,

$$\frac{SK}{4\sigma_0 \varepsilon} = 1.6 \times 10^{-8} \frac{1}{\sigma_0 A} \left( \frac{q}{e} \right) \left( \frac{S}{\varepsilon_n} \right) \frac{I_b}{\gamma_b^2 \beta_b^2},$$

where $A$ and $q/e$ are the atomic mass and magnitude of the charge state of the ion, respectively, $I_b$ is the ion beam current in amperes, $\varepsilon_n = \gamma_b \beta_b \varepsilon$ is the normalized rms emittance in meter-radians, and $S$ is in meters.

In this paper, halo formation and control in space-charge-dominated electron and ion beams are investigated in parameter regimes relevant to the development of HPM sources and high-intensity electron and ion linacs. A mechanism for electron beam halo formation is identified in high-power periodic permanent magnet (PPM) focusing klystron amplifiers. A new class of cold-fluid corkscrewing elliptic beam equilibria is discovered for ultrahigh-brightness, space-charge-dominated electron or ion beam propagation through a linear focusing channel consisting of uniform solenoidal magnetic focusing fields, periodic solenoidal magnetic focusing fields, and/or alternating-gradient quadrupole magnetic...
focusing fields in an arbitrary arrangement including field tapering. As an important application of such new cold-fluid corkscrewing elliptic beam equilibria, a technique is developed and demonstrated for controlling of halo formation and beam hollowing in an rms-matched ultrahigh-brightness ion beam as it is injected from an axisymmetric Pierce diode into an alternating-gradient magnetic quadrupole focusing channel. In these studies, two-dimensional cold-fluid and self-consistent electrostatic and magnetostatic models are used whenever appropriate. The self-consistent model is based on a Green’s function technique rather than a particle-in-cell (PIC) technique.

In the study of electron beam halo formation in high-power PPM focusing klystron amplifiers, the two-dimensional self-consistent electrostatic and magnetostatic model [15] for the transverse beam dynamics is used to analyze equilibrium beam transport in a periodic magnetic focusing field in the absence of radio-frequency signal, and the behavior of a high-intensity electron beam under a current-oscillation-induced mismatch between the beam and the periodic magnetic focusing field during high-power operation of the device. Detailed simulation results are presented for choices of system parameters corresponding to the 50 MW, 11.4 GHz periodic permanent magnet (PPM) focusing klystron experiment [22] performed at the Stanford Linear Accelerator Center (SLAC). It is found that sizable halos appear once the beam envelope undergoes several oscillations.

In the analysis and applications of cold-fluid corkscrewing elliptic beam equilibria, the steady-state cold-fluid equations are solved with general magnetic focusing field profile. Generalized beam envelope equations for equilibrium flow are obtained. It is shown that limiting cases of cold-fluid elliptic beam equilibria include the familiar cold-fluid round rigid-rotor beam equilibrium in a uniform magnetic focusing field [23-25] and both the familiar round rigid-rotor Vlasov beam equilibrium [26-28] in periodic solenoidal focusing field and the familiar Kapchinskij-Vladimirskij beam equilibrium [29] in alternating-gradient quadrupole magnetic focusing field in the zero-emittance limit. As a simple example, a cold-fluid corkscrewing elliptic beam equilibrium in a uniform magnetic focusing field is discussed. As an application of the present equilibrium beam theory, a general technique is developed, and demonstrated with an example, for the controlling of beam halo formation and beam hollowing in ultrahigh-brightness beams. This technique is effective before any collective instability may develop to reach considerably large amplitudes.
The paper is organized as follows. In Sec. II, steady-state cold-fluid equations and two-dimensional self-consistent model are presented for transverse electrostatic and magnetostatic interactions in a high-intensity charged-particle beam propagating through a linear focusing channel with general magnetic focusing field profile. In Sec. III, both equilibrium beam transport and halo formation in high-power PPM focusing klystron amplifiers are studied. The equilibrium (well-matched) beam envelope is determined for intense electron beam propagation through a PPM focusing field, and self-consistent simulations of equilibrium beam transport are performed. The effects of large-amplitude charge-density and current oscillations on inducing mismatched beam envelope oscillations are discussed, and use is made of the self-consistent model to study the process of halo formation in a high-intensity electron beam during high-power operation of such a device. The results are compared with the SLAC PPM focusing klystron amplifier experiment [22]. In Sec. IV, a solution to the steady-state cold-fluid equations presented in Sec. II is obtained, and generalized beam envelope equations for equilibrium flow are derived. Examples of corkscrewing elliptic beam equilibria in a uniform magnetic field are presented. In Sec. V, a technique for controlling of beam halo and beam hallowing is developed and demonstrated as an important application of the cold-fluid equilibrium beam theory. Finally, conclusions are given in Sec. VI.
II. MODELS AND ASSUMPTIONS

We consider a thin, continuous, space-charge-dominated charged-particle beam propagating with axial velocity $\beta c \hat{e}_z$ through a linear focusing channel consisting of uniform solenoidal magnetic focusing fields, periodic solenoidal magnetic focusing fields, and/or alternating-gradient quadrupole magnetic focusing fields in an arbitrary arrangement. The fields can be tapered, and the quadrupole magnets are allowed to be at various angles in the transverse direction. In the thin-beam approximation, the focusing magnetic field is expressed approximately as

$$B_{\text{ext}}(x,y,s) = B_z(s) \hat{e}_z - \frac{1}{2} B_z'(s) \left( x \hat{e}_x + y \hat{e}_y \right) + \left( \frac{\partial B_y^2}{\partial y} \right)_0 \left( y \hat{e}_x + x \hat{e}_y \right),$$  \hspace{1cm} (1)

In Eq. (1), $s = z$ is the axial coordinate, $x_\perp = x \hat{e}_x + y \hat{e}_y$ is the transverse displacement from the $z$-axis in the laboratory frame, the prime denotes derivative with respect to $s$, $x_\perp = x \hat{e}_x + y \hat{e}_y$ is the transverse displacement from the $z$-axis in a frame of reference that is rotated transversely by an angle of $q_j$ with respect to the laboratory frame, and $(\partial B_x^0 / \partial y)_0 = (\partial B_y^0 / \partial x)_0$ with subscript ‘zero’ denoting $(x, y) = 0$.

In the present analysis, we consider the transverse electrostatic and magnetostatic interactions in the beam. We make the usual paraxial approximation, assuming that (a) the Budker parameter is small compared with $\gamma_b^2$, i.e., $q^2 N_b / \gamma_b n c^2 << 1$, (b) the beam is thin compared with the characteristic length scale over which the beam envelope varies, and (c) the kinetic energy associated with the transverse particle motion is small compared with that associated with the axial particle motion. In the following, we present steady-state cold-fluid equations describing equilibrium beam propagation in the magnetic focusing field defined in Eq. (1), and a two-dimensional self-consistent model describing the transverse dynamics of the beam.

A. Steady-State Cold-Fluid Equations

For an ultrahigh-brightness beam, such as a high-intensity heavy ion beam, kinetic (emittance) effects are negligibly small, and the beam can be adequately described by cold-fluid equations. In the paraxial approximation, the steady-state cold-fluid equations for time-stationary flow ($\partial / \partial t = 0$) in cgs units are
\[ \beta_b c \frac{\partial}{\partial s} n_b + \nabla \cdot (n_b \mathbf{V}_\perp) = 0, \quad (2) \]

\[ \nabla \cdot \mathbf{V}_\perp = \beta_b^{-1} \nabla \cdot A', = -4\pi q n_b, \quad (3) \]

\[ n_b \left( \beta_b c \frac{\partial}{\partial s} + \mathbf{V}_\perp \cdot \frac{\partial}{\partial \mathbf{x}_\perp} \right) \mathbf{V}_\perp = \frac{q n_b}{\gamma_b m} \left[ -\frac{1}{\gamma_b} \nabla \cdot \phi' + \beta_b \hat{\mathbf{e}}_z \times \mathbf{B}_{0z} + \frac{\mathbf{V}_\perp}{c} \times B_z(s) \hat{\mathbf{e}}_z \right], \quad (4) \]

where \( \gamma_b = (1 - \beta_b^2)^{-1/2} \), use has been made of \( \beta_z \equiv \beta_b = \text{const.} \), and the self-electric and self-magnetic fields \( \mathbf{E}' \) and \( \mathbf{B}' \) are determined from the scalar and vector potentials \( \phi' \) and \( A'_z \hat{\mathbf{e}}_z \), i.e., \( \mathbf{E}' = -\nabla \cdot \phi' \) and \( \mathbf{B}' = \nabla \times A'_z \hat{\mathbf{e}}_z \). It will be shown in Sec. IV that the steady-state cold-fluid equations (2)-(4) support a class of solutions that, in general, describe corkscrewing elliptic beam equilibria in the magnetic focusing field defined in Eq. (1).

**B. Two-Dimensional Self-Consistent Model**

For moderately high-brightness beams, such as electron beams in high-power PPM focusing klystron amplifiers, kinetic (emittance) effects play an important role in the beam dynamics, and the evolution of the phase space of such beams must be studied. In the paraxial approximation, the self-consistent electrostatic and magnetostatic interactions in such a charged-particle beam can be described by a two-dimensional model involving \( N_p \) macroparticles (i.e., charged rods). In the laboratory frame, the transverse dynamics of the macroparticles is governed by [15,30,31]

\[ \frac{d^2 x_i}{ds^2} + \kappa_q(s)(x_i \cos 2\varphi_q + y_i \sin 2\varphi_q) - 2\sqrt{\kappa_z(s)} \frac{dy_i}{ds} - y_i \frac{d}{ds} \sqrt{\kappa_z(s)} + \frac{q}{\gamma_b^3 B_b^2 mc^2} \frac{\partial \phi'}{\partial x_i} = 0, \quad (5) \]

\[ \frac{d^2 y_i}{ds^2} - \kappa_q(s)(-x_i \sin 2\varphi_q + y_i \cos 2\varphi_q) + 2\sqrt{\kappa_z(s)} \frac{dx_i}{ds} + x_i \frac{d}{ds} \sqrt{\kappa_z(s)} + \frac{q}{\gamma_b^3 B_b^2 mc^2} \frac{\partial \phi'}{\partial y_i} = 0, \quad (6) \]

where \( i = 1, 2, ..., N_p \), and the focusing parameters \( \kappa_z(s) \) and \( \kappa_q(s) \) and self-field potential \( \phi'(x_i,y_i,s) \) are defined by

\[ \sqrt{\kappa_z(s)} = \frac{q B_z(s)}{2\gamma_b B_b mc^2} = \Omega_z(s), \quad (7) \]
\[ \kappa_q(s) = \frac{q}{\gamma_0\beta s mc^2} \left( \frac{\partial B_T^q}{\partial y} \right)_0, \]  

\[ \phi^s(x_i, y_j, s) = \frac{qN_b}{N_p} \sum_{j=1}^{N} \ln \left( \frac{(x_i - x_j)^2 + (y_i - y_j)^2}{(x_i - x_j r_i^2 / r^2_j)^2 + (y_i - y_j r_i^2 / r^2_j)^2} \right) \] 

respectively. Here, \( \Omega_c(s) \) is the (local) relativistic cyclotron frequency associated with the axial magnetic field \( B_z(s) \), and \( r_i \equiv \left( x_i^2 + y_i^2 \right)^{1/2} \). The beam is assumed to propagate inside a perfectly conducting cylindrical tube of radius \( r_w \), such that the self-field potential satisfies the boundary condition \( \phi^s(r_i = r_w, s) = 0 \). Note that the parameter \( \sqrt{\kappa_c(s)} \) can be positive, negative or zero at any given axial position.

The two-dimensional self-consistent model described by Eqs. (5) and (6) will be used to simulate equilibrium beam transport in a PPM focusing field in the absence of rf signal and electron beam halo formation in the transverse direction induced by large-amplitude longitudinal current oscillations in a PPM focusing klystron amplifier (Section III). It will also be used to verify cold-fluid corkscrewing elliptic beam equilibria in a linear focusing channel consisting of uniform solenoidal magnetic focusing fields, periodic solenoidal magnetic focusing fields, and/or alternating-gradient quadrupole magnetic focusing fields in an arbitrary arrangement, and to demonstrate control of halo formation and beam hollowing in an rms-matched ultrahigh-brightness ion beam as it is injected from an axisymmetric Pierce diode into an alternating gradient focusing channel (Sec. V).
III. ELECTRON BEAM HALO FORMATION IN PPM FOCUSING KLYSTRONS

In this section, we study the dynamics of relativistic electron beams in high-power PPM focusing klystron amplifiers. Of particular interest are the properties of equilibrium beam transport in the absence of rf signal and the mechanism for electron beam halo formation during high-power operation of such a device. To make comparisons with experiment, the following analysis is carried out with system parameters corresponding to those in the SLAC 50 MW, 11.4 GHz PPM focusing klystron experiment [22].

A. Equilibrium Beam Transport

In the absence of rf signal, the relativistic electron beam propagates through the PPM focusing field in an equilibrium state. It has been shown previously [26-27] that one of the equilibrium states for the system described by Eqs. (5) and (6) is the rigid-rotor Vlasov equilibrium in which the beam density is uniform transverse to the direction of beam propagation. The outermost beam radius \( r_\text{b}(s) = r_\text{b}(s + S) \) obeys the envelope equation [26]

\[
\frac{d^2 r_\text{b}}{ds^2} + \kappa_z(s) r_\text{b} - \frac{K}{r_\text{b}^3} - \frac{\left\langle \hat{P}_\theta \right\rangle^2}{r_\text{b}^3} - \frac{(4\epsilon)^2}{r_\text{b}^3} = 0,
\]

where \( \gamma_\text{b} \beta_\text{b} m_c \left\langle \hat{P}_\theta \right\rangle = \text{constant} \) is the macroscopic canonical angular momentum of the beam at \( r = r_\text{b}(s) \), and \( \epsilon \) is the unnormalized transverse rms emittance associated with the random motion of the electrons. If there is no magnetic field at the cathode, then \( \left\langle \hat{P}_\theta \right\rangle = 0 \). Any residual magnetic field at the cathode will lead to \( \left\langle \hat{P}_\theta \right\rangle \neq 0 \).

We analyze the beam envelope for equilibrium beam transport in the SLAC 50 MW, 11.4 GHz PPM focusing klystron experiment [22]. The system parameters of the experiment are shown in Table 1. To examine the influence of small residual magnetic field on the beam transport, we analyze two different cases shown in Table 2. In Case I, we assume no residual magnetic field at the cathode, such that \( \left\langle \hat{P}_\theta \right\rangle = 0 \). In Case II, however, a residual field of 6.86 G is assumed, corresponding to a beam with a finite canonical angular momentum given by \( \gamma_\text{b} \beta_\text{b} m_c \left\langle \hat{P}_\theta \right\rangle = 4.5 \times 10^{-26} \text{ Kgm}^2/\text{s} \). The following
dimensionless parameters are derived from Table 2: 
\[ S^2 k\zeta(s) = \left[1.04 \times \sin \left(\frac{2\pi s}{S}\right)\right]^2 \]  
(with \( S = 2.1 \text{ cm} \), \( \sigma_0 = 42.3' = 0.738 \), \( SK/4\sigma_0\varepsilon = 10.1 \), and \( \langle \hat{P}_o \rangle/4\varepsilon = 0.0 \) in Case I and \( \langle \hat{P}_o \rangle/4\varepsilon = 6.93 \) in Case II.

Figure 1 shows plots of the axial magnetic field \( B_z(s) \) and outermost beam radius \( r_b(s) \) versus the propagation distance \( s \) for Cases I and II. In both cases, the amplitude of well-matched (equilibrium) envelope oscillations about the average beam radius is only about 0.005 mm, as seen in Figs. 1(b) and 1(c).

Self-consistent simulations based on the model described in Sec. II.B are performed to further investigate the equilibrium beam transport. In the simulations, 4096 macroparticles are used. The macroparticles are loaded according to the rigid-rotor Vlasov distribution [26] with an initial beam radius equal to the equilibrium (matched) beam radius at \( s = 0 \) [see Figs. 1(b) and 1(c) for Cases I and II, respectively].

Figure 2 shows, respectively, the initial and final phase-space distributions at \( s = 0.0 \) cm and \( s = 42.0 \) cm for Case I. The final distribution in the configuration space shown in Fig. 2(d) agrees very well with the initial distribution shown in Fig. 2(a), and the effective beam radius obtained from the simulation agrees with that obtained from Eq. (10) within 0.2%. In the simulation, no beam loss is detected. Comparison between the final phase-space plots in Figs. 2(e) and 2(f) and the initial phase-space plots in Figs. 2(b) and 2(c) shows a slight emittance growth. This is because of numerical noise in the simulation. Nevertheless, the emittance growth has little effect on the beam transport properties because the beam transport is dominated by space charge. Similar results are also obtained for Case II [32], showing preservation of the initial distribution and no beam loss. In both cases, we find that the equilibrium beam transport in the PPM focusing klystron is robust, and that there is no beam loss in the absence of rf signal. Within the experimental error, these results are in good agreement with the experimental observation [22] of 99.9% beam transmission in the absence of rf signal.

B. Halos Induced by Large-Amplitude Current Oscillations
Microwave generation in a klystron is due to the coupling of large-amplitude charge-density and current oscillations in the electron beam with the output rf cavity or structure. The charge-density and current oscillations result from the beating of the fast- and slow-space-charge waves on the electron beam, and are primarily longitudinal. From the point of view of beam transport, the charge-density and current oscillations perturb the equilibrium beam envelope. Although a quantitative understanding of the effects of such large-amplitude charge-density and current oscillations on the transverse dynamics of the electron beam requires three-dimensional modeling which is not available at present, a qualitative two-dimensional study of such effects is presented in the remainder of this section.

The amplitude of the envelope mismatch induced by longitudinal current oscillations can be estimated using the standard one-dimensional fluid model based on the continuity, Lorentz force and full Maxwell’s equations. It follows from the linearized continuity equation that the current perturbation $(\delta I_b)_{f,s}$ is related to the axial velocity perturbation $c(\delta \beta_b)_{f,s}$ by [33,34]

$$
\frac{(\delta I_b)_{f,s}}{I_b} \equiv -\frac{\omega}{\omega - \beta_b c k_{f,s}} \frac{(\delta \beta_b)_{f,s}}{\beta_b},
$$

where subscripts $f$ and $s$ denote the fast- and slow-space-charge waves, respectively, and $\omega$ and $k_{f,s}$ are the frequency and wave numbers of the perturbations, respectively. Making the long-wavelength approximation for a thin beam, it can be shown that the dispersion relations for the fast- and slow-space-charge waves can be expressed as [33]

$$
\omega - \beta_b c k_{f,s} = \pm \sqrt{\varepsilon_{sc}} \gamma_b \beta_b^2 \omega, \tag{12}
$$

where $k_f$ assumes plus sign, and $k_s$ assumes minus sign. In Eq. (12), $\varepsilon_{sc}$ is the longitudinal space-charge coupling parameter. The effective value of $\varepsilon_{sc}$ is estimated to be $\varepsilon_{sc} = 0.012$ for the SLAC PPM focusing klystron [22]. In the klystron, the total current oscillations are the sum of fast- and slow-space-charge waves with a phase difference of $\sim 180^\circ$. As a result, the total current oscillations and the total velocity oscillations are out of phase by $\sim 180^\circ$. Therefore, the amplitude of the total current oscillations is given by
\[
\left(\frac{\delta I_b}{I_b}\right)_{\text{total}} \equiv -\frac{2\gamma_b\beta_b^2}{\sqrt{\epsilon_{sc}}} \frac{\delta \beta_b}{\beta_b}.
\] (13)

This has the important consequence that the perveance of the electron beam varies dramatically along the beam during high-power operation. From the definition of the perveance, i.e.,

\[ K = \frac{2e^2 N_b}{\gamma_b^3 \beta_b^2 m_e c^2}, \]

it is readily shown that the amplitude of perveance variation is given by

\[
\frac{\delta K}{K} \equiv \left(1 + \frac{3\gamma_b \sqrt{\epsilon_{sc}}}{2\beta_b^2}\right) \left(\frac{\delta I_b}{I_b}\right)_{\text{total}}.
\] (14)

For the SLAC PPM focusing klystron, Eq. (14) yields \( \delta K / K = 1.45 \times \left(\frac{\delta I_b}{I_b}\right)_{\text{total}} / I_b \). At the rf output section, \( \delta K / K \) exceeds unity considerably because \( \delta I_b / I_b = 1 \). (Note that the current oscillations are highly nonlinear in the rf output section and the maximum current exceeds \( 2I_b \) during high-power operation.) From the beam envelope equation (10), the relative amplitude of beam envelope mismatch is estimated to be \( \delta r_b / r_b = 0.56 \), where \( r_b \) is the equilibrium beam radius and \( \delta I_b / I_b = 1 \) is assumed.

In the self-consistent simulations presented below, we use \( \delta r_b / r_b = 1.0 \) in order to take into account the fact that the instantaneous current exceeds \( 2I_b \) during high-power operation of the klystron.

The process of halo formation in intense electron beams is studied using the two-dimensional self-consistent model described in Sec. II.B. In the simulations, 4096 macroparticles are used, and the macroparticles are loaded according to the rigid-rotor Vlasov distribution [26] with an initial beam radius of \( 2r_b(0) \), where \( r_b(0) \) is the equilibrium beam radius at \( s = 0 \) [see Figs. 1(b) and 1(c) for Cases I and II, respectively]. The effect of current oscillation buildup in the PPM focusing klystron, which requires three-dimensional modeling, is not included in the present two-dimensional simulation. In the limited space of this paper, we discuss only the results of the self-consistent simulation for Case I, although the effect of small residual magnetic field at the cathode in the halo formation process is also studied for Case II and is reported elsewhere [32].

Figure 3 shows the phase-space distributions of the electrons at several axial distances during the fourth period of the beam core radius oscillation for Case I. In contrast to the equilibrium phase-space distribution (Fig. 2), significant halos appear at \( s = 34.7, 37.8, 42.0, 44.1, \) and \( 46.2 \) cm. In the configuration space plots shown in Figs. 3(a) to 3(e), we observe a large variation in the beam core
radius during the mismatched envelope oscillation period. The halo particles reach a maximum radius of 
\[ r_h = 6.4 \text{ mm} \] at \[ s = 42.0 \text{ cm} \], where the beam core radius is a minimum and the traveling-wave RF output section is located. Around 1.5\% of the electrons are found in the halo at that axial position. Because the maximum halo radius of \( r_h = 6.4 \text{ mm} \) is greater than the actual beam tunnel radius \( r_T = 4.7625 \text{ mm} \), these halo electrons are lost to the waveguide wall. Therefore, the simulation results show that there will be 1.5\% beam electron loss. In terms of beam power loss, 1.5\% beam electron loss in the simulation corresponds to 0.2\% beam power loss because the lost electrons have given up 88\% of their kinetic energies (or have slowed down by about a factor of 2 in their axial velocities). The simulation results agree qualitatively with 0.8\% beam power loss observed in the experiment [22]. The discrepancy between the simulation and experimental measurements may be caused by nonlinearities in the applied magnetic fields which are not included the present simulation.

As the beam propagates in the focusing field, its distribution rotates clockwise in the \( (x, dx/ds) \) phase space, as shown in Figs. 3(f) to 3(j). The particles are initially dragged into the halo at the edges of the phase space distribution, where a chaotic region is formed around an unstable periodic orbit that is located just outside the beam distribution [13]. The unstable periodic orbit is a result of a resonance between the mismatched core envelope oscillations and the particle dynamics. As the halo particles move away from the beam core, the influence of space charge forces decreases and these halo particles start rotating faster than the core particles, creating the S-shaped distributions observed in Figs. 3(f) to 3(j).

The halo formation is also observed in the \( (x, dy/ds) \) phase space distributions shown in Figs. 3(k) to 3(o). Although the macroscopic (average) canonical angular momentum \( \langle \hat{P}_\theta \rangle \) is constant in the simulation, the distributions presented in Figs. 3(k) to 3(o) indicate that the distribution of single particle canonical angular momenta induces spread in the \( (x, dy/ds) \) phase space.

Figure 4 shows the halo radius and effective beam core radius as a function of the propagation distance for Case I. The halo radius is the maximum radius achieved by all of the macroparticles in the self-consistent simulation. It is apparent in Fig. 4 that the halo formation process takes place essentially during the first 4 periods of the envelope oscillations. After reaching \( r_h = 6.4 \text{ mm} \) at \( s = 42.0 \text{ cm} \), the
halo radius saturates. It is interesting to note that once the halo is developed, the halo radius and core envelope radius oscillate in opposite phase, with the former being maximum when the latter is minimum [as seen in Fig. 3(c)] and vice versa.

To summarize briefly, we studied equilibrium beam transport in a periodic magnetic focusing field in the absence of RF signal and the behavior of a high-intensity electron beam under a current-oscillation-induced mismatch between the beam and the magnetic focusing field. Detailed simulation results were presented for choices of system parameters corresponding to the SLAC 50 MW, 11.4 GHz periodic permanent magnetic (PPM) focusing klystron experiment [22]. We found that in the absence of RF signal, the equilibrium beam transport is robust, and that there is no beam loss in agreement with experimental measurements. During the high-power operation of the klystron, however, we found that the current-oscillation-induced mismatch between the beam and the magnetic focusing field produces large-amplitude envelope oscillations whose amplitude is estimated using a one-dimensional cold-fluid model. From self-consistent simulations, we found that for a mismatch amplitude equal to the beam equilibrium radius, the halo reaches 0.64 cm in size and contains about 1.5% of total beam electrons at the RF output section for a beam generated with a zero magnetic field at the cathode. In terms of beam power loss, 1.5% beam electron loss in the simulation corresponds to 0.2% beam power loss because the lost electrons have given up 88% of their kinetic energies, which agrees qualitatively with 0.8% beam power loss observed in the experiment [22].
IV. CORKSCREWING ELLIPTIC BEAM EQUILIBRIA

In this section, we show that there exists a class of solutions to the steady-state cold-fluid equations (2)-(4) which, in general, describe corkscrewing elliptic beam equilibria [35] for ultrahigh brightness, space-charge-dominated beam propagation in the linear focusing channel defined in Eq. (1).

We seek solutions to Eqs. (2)-(4) of the form [35]

\[ n_b(x, s) = \frac{N_b}{\pi a(s) b(s)} \Theta \left[ 1 - \frac{\tilde{x}^2}{a^2(s)} - \frac{\tilde{y}^2}{b^2(s)} \right], \]  

(15)

\[ V_\perp(x, s) = [\mu_x(s) \tilde{x} - \alpha_x(s) \tilde{y}] b_s c \hat{e}_z + [\mu_y(s) \tilde{y} + \alpha_y(s) \tilde{x}] b_s c \hat{e}_\gamma. \]  

(16)

In Eqs. (15) and (16), \( x = \tilde{x} \hat{e}_x + \tilde{y} \hat{e}_y \) is a transverse displacement in a rotating frame illustrated in Fig. 5; \( \Theta(s) \) is the angle of rotation of the ellipse with respect to the laboratory frame; \( \Theta(x) = 1 \) if \( x > 0 \) and \( \Theta(x) = 0 \) if \( x < 0 \); and the functions \( a(s), b(s), \mu_x(s), \mu_y(s), \alpha_x(s), \alpha_y(s) \) and \( \Theta(s) \) are to be determined self consistently.

Substituting Eqs. (15) and (16) into Eq. (2) and expressing the result in terms of the tilde variables, we find

\[ \left( \mu_x + \mu_y - \frac{a' a}{b'} \right) \Theta \left[ 1 - \frac{\tilde{x}^2}{a^2} - \frac{\tilde{y}^2}{b^2} \right] + 2 \left( \frac{a'}{a} - \mu_x \right) \tilde{x}^2 \left( \frac{b'}{b} - \mu_y \right) \tilde{y}^2 \right) \delta \left[ 1 - \frac{\tilde{x}^2}{a^2} - \frac{\tilde{y}^2}{b^2} \right] = 0, \]  

(17)

where the ‘prime’ denotes derivative with respect to \( s \), \( \delta(x) \equiv d\Theta(x)/ds \), and use has been made of the identities \( \partial \tilde{x}/\partial s = \Theta' \tilde{y}, \partial \tilde{y}/\partial s = -\Theta' \tilde{x} \), and \( \nabla \cdot \mathbf{F} = \partial F_x/\partial \tilde{x} + \partial F_y/\partial \tilde{y} \) for any vector field \( \mathbf{F} \).

Since Eq. (17) must be satisfied for all \( \tilde{x} \) and \( \tilde{y} \), the coefficients of the terms proportional to \( \Theta, \tilde{x}^2 \delta, \tilde{y}^2 \delta, \) and \( \tilde{x} \tilde{y} \delta \) must vanish independently. This leads to the following equations

\[ \mu_x = \frac{1}{a} \frac{da}{ds}, \quad \mu_y = \frac{1}{b} \frac{db}{ds}, \]  

(18)

\[ \frac{d\Theta}{ds} = \frac{a^2 \alpha_x - b^2 \alpha_y}{a^2 - b^2}, \]  

(19)

where the functions \( a(s), b(s), \alpha_x(s), \) and \( \alpha_y(s) \) still remain to be determined.
Solving for the scalar and vector potentials from Eq. (3), we obtain

\[ \phi = \beta^{-1} \mathcal{A} = -\frac{2qN_b}{a+b} \left( \frac{\tilde{x}^2}{a} + \frac{\tilde{y}^2}{b} \right) \]  

(20)

in the beam interior with \( \tilde{x}^2/a^2 + \tilde{y}^2/b^2 < 1 \). In deriving Eq. (20), use has been made of \( \nabla_\perp = \partial^2 / \partial \tilde{x}^2 + \partial^2 / \partial \tilde{y}^2 \).

To solve the force equation (4) we substitute Eqs. (15), (16), (18)-(20) into Eq. (4), express the results in terms of the tilde variables, and use the relations \( \partial \tilde{x} / \partial s = \theta \tilde{e}_\tilde{y} \), \( \partial \tilde{y} / \partial s = -\theta \tilde{e}_\tilde{x} \), \( \partial \tilde{e}_\tilde{x} / \partial s = \theta \tilde{e}_\tilde{y} \) and \( \partial \tilde{e}_\tilde{y} / \partial s = -\theta \tilde{e}_\tilde{x} \). We obtain

\[ \{ f_x + \kappa \cos[2(\theta - \varphi)]\}\tilde{x} - \{ g_y + \kappa \sin[2(\theta - \varphi)]\}\tilde{y} = 0, \]  

(21a)

\[ \{ f_y - \kappa \sin[2(\theta - \varphi)]\}\tilde{x} + \{ g_x - \kappa \cos[2(\theta - \varphi)]\}\tilde{y} = 0 \]  

(21b)

in the \( \tilde{x} \) and \( \tilde{y} \) directions, respectively. In Eq. (21),

\[ f_x = \frac{1}{a} \left( \frac{d}{ds} \frac{b^2}{a} \left( \alpha_x^2 - 2\alpha_x \alpha_y + a^2 \alpha_y^2 \right) - 2\alpha_y \sqrt{\kappa} - \frac{2K}{a(a+b)} \right), \]  

(22a)

\[ f_y = \frac{1}{b} \left( \frac{d}{ds} \frac{a^2}{b} \left( \alpha_y^2 - 2\alpha_x \alpha_y + b^2 \alpha_x^2 \right) - 2\alpha_x \sqrt{\kappa} - \frac{2K}{b(a+b)} \right), \]  

(22b)

\[ g_y = \frac{1}{b^2} \left( \frac{d}{ds} \frac{b^2}{a^2} \left( \alpha_x + \sqrt{\kappa} \right) - \frac{a^2 b (\alpha_x - \alpha_y)}{a^2 + \sqrt{\kappa} \sqrt{a^2 + b^2}} \frac{d}{ds} \frac{b}{a} \right), \]  

(22c)

\[ g_x = \frac{1}{a^2} \left( \frac{d}{ds} \frac{a^2}{b^2} \left( \alpha_y + \sqrt{\kappa} \right) - \frac{a b^2 (\alpha_y - \alpha_x)}{a^2 + \sqrt{\kappa} \sqrt{a^2 + b^2}} \frac{d}{ds} \frac{a}{b} \right), \]  

(22d)

Since Eqs. (21a) and (21b) must be satisfied for all \( \tilde{x} \) and \( \tilde{y} \), we obtain the generalized beam envelope equations

\[ f_x + \kappa \cos[2(\theta - \varphi)] = 0, \]  

(23a)

\[ f_y - \kappa \sin[2(\theta - \varphi)] = 0, \]  

(23b)

\[ g_y + \kappa \sin[2(\theta - \varphi)] = 0, \]  

(23c)

\[ g_x - \kappa \sin[2(\theta - \varphi)] = 0. \]  

(23d)

Making use of Eq. (22), we can express the generalized beam envelope equations as [35]
\[
\frac{d^2 a}{ds^2} + \left\{ \kappa_q(s) \cos[2(\theta - \varphi_q)] - \frac{b^2(\alpha_x^2 - 2\alpha_x\alpha_y) + a^2\alpha_y^2}{a^2 - b^2} - 2\alpha_y\sqrt{\kappa_z} \right\} a - \frac{2K}{(a+b)} = 0, \tag{24a}
\]

\[
\frac{d^2 b}{ds^2} + \left\{ -\kappa_q(s) \cos[2(\theta - \varphi_q)] + \frac{a^2(\alpha_y^2 - 2\alpha_x\alpha_y) + b^2\alpha_x^2}{a^2 - b^2} - 2\alpha_x\sqrt{\kappa_z} \right\} b - \frac{2K}{(a+b)} = 0, \tag{24b}
\]

\[
\frac{d}{ds} \left[ b^2(\alpha_x + \sqrt{\kappa_z}) \right] - \frac{a^2b(\alpha_x - \alpha_y)}{a^2 - b^2} \frac{d}{ds} \left( \frac{b}{a} \right) + \kappa_q(s) b^2 \sin[2(\theta - \varphi_q)] = 0, \tag{24c}
\]

\[
\frac{d}{ds} \left[ a^2(\alpha_y + \sqrt{\kappa_z}) \right] - \frac{ab^2(\alpha_x - \alpha_y)}{a^2 - b^2} \frac{d}{ds} \left( \frac{a}{b} \right) - \kappa_q(s) a^2 \sin[2(\theta - \varphi_q)] = 0, \tag{24d}
\]

\[
\frac{d\theta}{ds} - \frac{a^2\alpha_y - b^2\alpha_x}{a^2 - b^2} = 0, \tag{24e}
\]

\[
\mu_x = \frac{1}{a} \frac{da}{ds}, \tag{24f}
\]

\[
\mu_y = \frac{1}{b} \frac{db}{ds}. \tag{24g}
\]

Equations (18) and (19) are added here as Eqs. (24e)-(24g) for completeness. Equations (24a)-(24g), together with the density and velocity profiles defined in Eqs. (15) and (16), describe cold-fluid equilibrium states for variably focused ultrahigh brightness beams.

A wide variety of cold-fluid beam equilibria can be constructed with Eqs. (15), (16) and (24) for proper choices of magnetic focusing field profiles. While cold-fluid beam equilibria are elliptic and corkscrewing in general, they do recover familiar beam equilibria in proper limits. In particular, such limiting cases of cold-fluid elliptic beam equilibria include: a) the familiar cold-fluid round rigid-rotor beam equilibrium [23-25] in a uniform magnetic focusing field with \( \kappa_z(s) = \text{const.} \neq 0, \kappa_q(s) = 0, \theta(s) = 0 \), \( a(s) = b(s) = \text{const.} \), and \( \alpha_x(s) = \alpha_y(s) = \text{const.} \) as discussed in more detail below, b) the familiar round rigid-rotor Vlasov beam equilibrium [26-28] in a periodic solenoidal focusing field in the zero-emittance limit with \( \kappa_z(s) = \kappa_z(s + S) \neq \text{const.}, \kappa_q(s) = 0, \theta(s) = 0 \), \( a(s) = a(s + S) = b(s) \neq \text{const.} \), and \( \alpha_x(s) = \alpha_y(s) \neq \text{const.} \), and c) the familiar Kapchinskji-Vladimirskij beam equilibrium [29] in alternating-gradient quadrupole magnetic focusing field in the zero-emittance limit with \( \kappa_z(s) = 0, \kappa_q(s) = \kappa_q(s + S) \neq \text{const.}, \theta(s) = 0 \), \( a(s) = a(s + S) \),
\( b(s) = b(s+S) \), and \( \alpha_x(s) = \alpha_y(s) = 0 \). Furthermore, for \( \theta(s) = 0 \) and \( \alpha_x(s) = \alpha_y(s) = 0 \), the present corkscrewing elliptic beam equilibria also recover geometrically non-rotating beam equilibria reported recently [36].

As a simple example, we consider corkscrewing elliptic beam equilibria in a uniform magnetic field with \( \mathbf{B}^{\text{ext}} = B_z \hat{z} \). Setting \( \sqrt{k_z(s)} = \sqrt{k_z(0)} = \frac{qB_{z0}}{2 \gamma_B^2 \beta_B mc^2} = \text{const.} \) and \( \kappa_q(s) = 0 \), it can be shown that Eq. (24) has the following two branches of physically acceptable special solutions:

\[
\begin{align*}
\alpha_x(s) &= \frac{\alpha_x}{\alpha_x + \alpha_y} s + \theta(0), \\
\alpha_y(s) &= \frac{\alpha_y}{\alpha_x + \alpha_y} s + \theta(0).
\end{align*}
\]

for branch A, and

\[
\begin{align*}
\alpha_x(s) &= \frac{\alpha_x + 2 \sqrt{k_z(0)}}{\alpha_x + 2 \sqrt{k_z(0)}} s + \theta(0), \\
\alpha_y(s) &= \frac{\alpha_y + 4 \sqrt{k_z(0)}}{\alpha_x + 4 \sqrt{k_z(0)}} s + \theta(0).
\end{align*}
\]

for branch B. In Eqs. (25) and (26), both \( \alpha_x \) and \( \alpha_y \) are constant.

For branch A, the conditions for the confinement of corkscrewing elliptic beam equilibria are

\[
\begin{align*}
\alpha_x < 0, & \quad \alpha_y < 0 \quad \text{and} \\
(\alpha_x + \sqrt{k_z(0)})(\alpha_y + \sqrt{k_z(0)}) < k_z(0),
\end{align*}
\]

for positively charged particle beams with \( \sqrt{k_z(0)} = \frac{qB_{z0}}{2 \gamma_B^2 \beta_B mc^2} > 0 \), and

\[
\begin{align*}
\alpha_x > 0, & \quad \alpha_y > 0 \quad \text{and} \\
(\alpha_x + \sqrt{k_z(0)})(\alpha_y + \sqrt{k_z(0)}) < k_z(0),
\end{align*}
\]

for negatively charged particle beams with \( \sqrt{k_z(0)} = \frac{-qB_{z0}}{2 \gamma_B^2 \beta_B mc^2} < 0 \).
for negatively charged particle beams with $\sqrt{\kappa_{z0}} = -|q|B_{z0} / 2\gamma_{z}\hat{B}_{z}mc^{2} < 0$. Because $\alpha_{x}$ and $\alpha_{y}$ have the same sign, the internal flow for branch A is always rotation-like.

For branch B, the conditions for the confinement of corkscrewing elliptic beam equilibria are

$$\alpha_{y} > -2\sqrt{\kappa_{z0}} , \alpha_{x} > -2\sqrt{\kappa_{z0}} \text{ and } (\alpha_{x} + \sqrt{\kappa_{z0}})(\alpha_{y} + \sqrt{\kappa_{z0}}) < \kappa_{z0}$$

(29)

for positively charged particle beams with $\sqrt{\kappa_{z0}} = qB_{z0} / 2\gamma_{z}\hat{B}_{z}mc^{2} > 0$, and

$$\alpha_{y} < -2\sqrt{\kappa_{z0}} , \alpha_{x} < -2\sqrt{\kappa_{z0}} \text{ and } (\alpha_{x} + \sqrt{\kappa_{z0}})(\alpha_{y} + \sqrt{\kappa_{z0}}) < \kappa_{z0}$$

(30)

for negatively charged particle beams with $\sqrt{\kappa_{z0}} = -|q|B_{z0} / 2\gamma_{z}\hat{B}_{z}mc^{2} < 0$. In contrast to the internal flow for branch A, the internal flow for branch B can be either rotation-like with $\alpha_{x}$ and $\alpha_{y}$ in the same sign, or quadrupole-flow-like with $\alpha_{x}$ and $\alpha_{y}$ in the opposite signs.

Figure 6 shows the regions in parameter space for the confinement of corkscrewing elliptic beam equilibria in a uniform magnetic field applicable for both positively and negatively charged particle beams. It is important to point out that the familiar cold-fluid round rigid-rotor beam equilibria [23-25] are recovered in the present analysis by setting $\alpha_{x} = \alpha_{y}$ in either Eq. (25) or Eq. (26), as indicated by the dark solid line shown in Fig. 6.
V. CONTROL OF HALO FORMATION AND BEAM HOLLOWING

As discussed in the Introduction, one of the key mechanisms for halo formation in high-intensity electron or ion beams is due to a mismatch in the particle phase-space distribution relative to an equilibrium distribution. In general, distribution mismatch can lead to rather complex evolution in a beam, including not only halo formation but also beam hollowing. This mechanism for halo formation and beam hollowing occurs for rms matched beams because rms beam matching does not necessarily guarantee the beam in an equilibrium state.

For example, both halo formation and beam hollowing were observed in the heavy ion beam injector experiment at Lawrence Berkeley National Laboratory (LBNL) [19], in which an ultrahigh-brightness, space-charge-dominated potassium ion beam was generated with an axisymmetric Pierce diode and then accelerated by a set of electrostatic quadrupoles. More recently, experimental evidence of beam hollowing was found in a high-brightness, space-charge-dominated electron beam experiment at University of Maryland [37,38].

As an important application of the equilibrium beam theory presented in Sec. IV, we develop and demonstrate a technique for controlling of beam halo formation and beam hollowing in ultrahigh-brightness beams. This technique is widely applicable in the design of ultrahigh-brightness beams, and is effective before any collective instability develops to reach considerably large amplitudes.

To demonstrate the efficacy of this technique, we consider here a specific example, namely, the matching of a round particle beam generated by an axisymmetric particle source into alternating-gradient magnetic quadrupole focusing channel. For comparison, we analyze two non-rotating rms matched beams with the same intensity; one beam will be in equilibrium and the other beam has an initial perturbation about the equilibrium transverse flow velocity. At the entrance of the alternating-gradient magnetic focusing channel \((s = 0)\), both beams have the same density profile defined in Eq. (15), but the transverse flow velocities of the beams are of the form [31]

\[
\frac{dx_{\perp}}{ds} = \frac{x_{\perp}}{a} \left( \frac{da}{ds} \right) \left[ 1 + \sqrt{1 - \frac{2x_{\perp} \cdot x_{\perp}}{a^2}} \right],
\]

(31)
where \( \nu \) is a parameter that measures the nonlinearity in the velocity profile. For example, an initial velocity profile with \( \nu > 0 \) in Eq. (31) may model the effects of the concave shape of a Pierce-type ion diode [19]. For equilibrium beam propagation, \( \nu = 0 \).

The rms matching is obtained by numerically solving the rms envelope equations [39]

\[
\frac{d^2\bar{a}}{ds^2} + \kappa_q(s)\bar{a} - \frac{K}{2(\bar{a} + \bar{b})} = 0, \tag{32a}
\]

\[
\frac{d^2\bar{b}}{ds^2} - \kappa_q(s)\bar{b} - \frac{K}{2(\bar{a} + \bar{b})} = 0, \tag{32b}
\]

where \( \bar{a} \equiv \langle x^2 \rangle^{1/2} \) and \( \bar{b} \equiv \langle y^2 \rangle^{1/2} \) are the rms envelopes, \( \langle \cdots \rangle \) denotes average over the particle distribution, and emittance terms are neglected. For given beam intensity \( K \) and focusing channel parameters \( C_3 \) and \( \eta \) shown in Fig. 7, we make use of Eq. (32) to determine the injection parameters for the axisymmetric beam, namely, \( \bar{a}(0), \bar{b}(0), \bar{a}'(0) \) and \( \bar{b}'(0) \), as well as the strengths of the two quadrupoles centered at \( s = S / 4 \) and \( s = 3S / 4 \) in the first lattice, \( C_1 \) and \( C_2 \), as shown in Fig. 7, assuming all quadrupoles having the same width \( \eta \) and equally spaced. Because Eq. (32) has a unique solution for an rms matched beam in the constant-parameter alternating-gradient focusing section with \( s / S > 1 \), integrating Eq. (32) from \( s = S \) to \( s = 0 \) yields four implicit functions \( \bar{a}(C_1, C_2), \bar{b}(C_1, C_2), \bar{a}'(C_1, C_2), \text{ and } \bar{b}'(C_1, C_2) \). The conditions for an initially converging round beam, i.e., \( \bar{a}(0) = \bar{b}(0) = a(0) / 2 = b(0) / 2 \) and \( \bar{a}'(0) = \bar{b}'(0) \), uniquely determine the parameters \( C_1 \) and \( C_2 \), which is done numerically with Newton’s method. The results are presented in Figs. 7 and 8.

Figure 7 shows the focusing field parameter \( S^2 \kappa_q \) as a function of \( s \), where \( \eta = 0.3, C_1 = 2.31, C_2 = 7.44 \) and \( C_3 = 10.0 \). In Fig. 8, the solid and dashed curves show, respectively, the rms matched envelopes \( \bar{a}(s) \) and \( \bar{b}(s) \) for the focusing channel with vacuum phase advance \( \sigma_o = 70.8^\circ \) and beam perveance \( SK / 4\epsilon(0) = 16.0 \) (corresponding to a space-charge-depressed phase advance of \( \sigma = 5.4^\circ \)), where a negligibly small unnormalized rms emittance of \( \epsilon(0) = 0.15 \times 10^{-6} \) m-rad has been assigned to the beam at \( s = 0 \).
Self-consistent simulations are performed with \( N_p = 3072 \) and free-space boundary conditions to study the phase space evolution for the two beams in the focusing channel shown in Fig. 7. In Fig. 8, the solid dots and open circles correspond to the rms envelopes \( \bar{a}(s) \) and \( \bar{b}(s) \) obtained from a self-consistent simulation for a beam initially with a nonlinear velocity profile with \( \nu = 0.25 \). It is evident in Fig. 8 that there is excellent agreement between the prediction of the rms envelope equations (32a) and (32b) and the results of the self-consistent simulation, despite that the transverse flow velocity is perturbed substantially.

We now examine the evolution of the particle distribution if the nonlinearity in the initial transverse flow velocity profile is introduced, and compare with equilibrium beam propagation. The results are summarized in Figs. 9 and 10. Figure 9 shows a comparison between particle distributions in the configuration space with and without nonlinearity in the initial transverse flow velocity at three axial positions: \( s/S = 0, 1.0 \) and 2.5. These axial positions are chosen such that \( \bar{a}(s) = \bar{b}(s) \). In Fig. 9, the plots shown on the left correspond to \( \nu = 0 \) and those on the right to \( \nu = 0.25 \). For \( \nu = 0.25 \), the initially round beam develops sharp edges after the first lattice, becoming partially hollow subsequently at \( s/S = 2.5 \). In Fig. 10(b), the radial distribution of 3072 macroparticles at \( s/S = 2.5 \) shows that the density at the edge is twice the density at the center of the beam, and that there is a small halo extending outward beyond the radius where the density reaches its maximum. The partially hollow density profile shown in Fig. 10(b) is similar to, but not as pronounced as, that observed in the heavy ion beam injector experiment at LBNL [19]. In contrast to the case with \( \nu = 0.25 \), the beam propagates in an equilibrium state for \( \nu = 0 \) without beam hollowing and without any significant beam halo formation, as shown in Fig. 10(a).
VI. CONCLUSIONS

Halo formation and control in space-charge-dominated electron and ion beams have been investigated analytically and computationally in parameter regimes relevant to the development of high-power microwave (HPM) tubes and high-intensity electron or ion linear accelerators. In particular, a mechanism for electron beam halo formation was identified in high-power periodic permanent magnetic focusing klystron amplifiers, and a new class of cold-fluid corkscrewing elliptic beam equilibria was discovered for ultrahigh-brightness beam propagation in linear focusing channel consisting of uniform and periodic solenoidal and alternating-gradient quadrupole magnetic fields in an arbitrary arrangement including field tapering.

In the exploration of electron beam halo formation in PPM focusing klystron amplifiers, equilibrium beam transport was analyzed in a periodic magnetic focusing field in the absence of RF signal, and the behavior of a high-intensity electron beam was studied under a current-oscillation-induced mismatch between the beam and the magnetic focusing field. Detailed simulation results were presented for choices of system parameters corresponding to the SLAC 50 MW, 11.4 GHz periodic permanent magnetic (PPM) focusing klystron experiment. It was found that in the absence of RF signal, the equilibrium beam transport is robust, and that there is no beam loss in agreement with experimental measurements. During high-power operation of the klystron, however, it was found that the current-oscillation-induced mismatch between the beam and the magnetic focusing field produces large-amplitude envelope oscillations whose amplitude was estimated using a one-dimensional cold-fluid model. Self-consistent simulations showed that for a mismatch amplitude equal to the beam equilibrium radius, the halo reaches 0.64 cm in size and contains about 1.5% of total beam electrons at the RF output section for a beam generated with a zero magnetic field at the cathode. Because the halo radius is greater than the actual beam tunnel radius, these halo electrons are lost to the waveguide wall, yielding 0.2% beam power loss. The simulation results agree qualitatively with 0.8% beam power loss observed in the experiment [22]. The discrepancy between the simulation and experimental measurements may be caused by nonlinearities in the applied magnetic fields which are not included the present simulation.

In the analysis and applications of cold-fluid corkscrewing elliptic beam equilibria, the steady-state cold-fluid equations were solved for an ultrahigh-brightness, space-charge-dominated beam in general
magnetic focusing field profile including periodic and uniform solenoidal fields and alternating-gradient quadrupole magnetic fields. Generalized beam envelope equations for equilibrium flow were obtained. It was shown that limiting cases of cold-fluid corkscrewing elliptic beam equilibria include the familiar cold-fluid round rigid-rotor beam equilibrium in a uniform magnetic focusing field and both the familiar round rigid-rotor Vlasov beam equilibrium in periodic solenoidal focusing field and the familiar Kapchinskij-Vladimirskij beam equilibrium in alternating-gradient quadrupole magnetic focusing field in the zero-emittance limit. As a simple example, a cold-fluid corkscrewing elliptic beam equilibrium in a uniform magnetic focusing field was discussed. As an application of the present equilibrium beam theory, a general technique was developed, and demonstrated with an example, for the controlling of beam halo formation and beam hollowing in ultrahigh-brightness beams. This technique is effective before any collective instability may develop to reach considerably large amplitudes.
ACKNOWLEDGMENTS

This work was supported in part by the Air Force Office of Scientific Research Grant No. F49620-97-1-0325, in part by the Department of Energy, Office of High Energy and Nuclear Physics Grant No. DE-FG02-95ER-40919, and in part by the Department of Energy through a subcontract with Princeton Plasma Physics Laboratory.
REFERENCES


30. See, for example, Chapter 10 of Ref. 25.


Table 1. SLAC 50 MW, 11.4 GHz, PPM Focusing Klystron Experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Current $I_b$</td>
<td>190 A</td>
</tr>
<tr>
<td>Beam Voltage</td>
<td>464 kV</td>
</tr>
<tr>
<td>Cathode Radius</td>
<td>2.86 cm</td>
</tr>
<tr>
<td>Cathode Temperature $T_b$</td>
<td>800°C †</td>
</tr>
<tr>
<td>Beam Radius</td>
<td>2.38 mm†</td>
</tr>
<tr>
<td>Pipe Radius</td>
<td>4.7625 mm</td>
</tr>
<tr>
<td>Total Tube Length</td>
<td>90.0 cm</td>
</tr>
<tr>
<td>Focusing Field Period Length</td>
<td>2.1 cm</td>
</tr>
<tr>
<td>PPM Focusing Section Length</td>
<td>42.0 cm</td>
</tr>
<tr>
<td>RMS Axial Magnetic Field</td>
<td>1.95 kG</td>
</tr>
</tbody>
</table>

† estimated

Table 2. System Parameters Used in the Simulation

<table>
<thead>
<tr>
<th>BASIC PARAMETER</th>
<th>CASE I</th>
<th>CASE II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Current $I_b$</td>
<td>190 A</td>
<td>190 A</td>
</tr>
<tr>
<td>Beam Voltage</td>
<td>464 kV</td>
<td>464 kV</td>
</tr>
<tr>
<td>Cathode Radius</td>
<td>2.86 cm</td>
<td>2.86 cm</td>
</tr>
<tr>
<td>Residual Magnetic Field at Cathode</td>
<td>0.0 G</td>
<td>6.86 G</td>
</tr>
<tr>
<td>Cathode Temperature $T_b$</td>
<td>800°C</td>
<td>800°C</td>
</tr>
<tr>
<td>Beam Radius</td>
<td>2.05 mm</td>
<td>2.38 mm</td>
</tr>
<tr>
<td>Pipe Radius</td>
<td>9.0 mm</td>
<td>9.0 mm</td>
</tr>
<tr>
<td>Total Tube Length</td>
<td>90.0 cm</td>
<td>90.0 cm</td>
</tr>
<tr>
<td>Focusing Field Period Length</td>
<td>2.1 cm</td>
<td>2.1 cm</td>
</tr>
<tr>
<td>PPM Focusing Section Length</td>
<td>42.0 cm</td>
<td>42.0 cm</td>
</tr>
<tr>
<td>RMS Axial Magnetic Field</td>
<td>1.95 kG</td>
<td>1.95 kG</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

Fig. 1 Plots of the axial magnetic field in (a) and outermost beam radius $r_b(s)$ versus the propagation distance $s$ for equilibrium beam propagation corresponding to Case I in (b) and Case II in (c). The dimensionless parameters are: $S^2 \kappa_z(s) = [1.04 \times \sin\left(2\pi s / S\right)]^2$, $\sigma_0 = 42.3^\circ = 0.738$, $SK/4\sigma_0\epsilon = 10.1$, and $\left\langle \tilde{P}_0 \right\rangle / 4\epsilon = 0.0$ in (b) and $\left\langle \tilde{P}_0 \right\rangle / 4\epsilon = 6.93$ in (c).

Fig. 2 Plots of the initial and final particle distributions at $s = 0.0$ and 42.0 cm for the equilibrium beam corresponding to the parameters in Case I.

Fig. 3 Plots of particle distributions in phase space at $s = 34.7, 37.8, 42.0, 44.1, \text{ and } 46.2$ cm for Case I.

Fig. 4 Plots of the halo radius (solid curve) and core radius (dashed curve) as a function of the propagation distance $s$ for Case I.

Fig. 5 Laboratory and rotating coordinate systems.

Fig. 6 Regions in the parameter space for the confinement of corkscrewing elliptic beam equilibria in a uniform magnetic field.

Fig. 7 Plot of the focusing parameter $S^2 \kappa_q$ as a function of the propagation distance $s$.

Fig. 8 Plots of rms beam envelopes versus propagation distances. Here, the solid and dashed curves are obtained from Eq. (32), whereas the solid dots and open circles are from the self-consistent simulation for a beam with $\nu = 0.25$.

Fig. 9 Particle distributions in the configuration space for $\nu = 0$ (left) and $\nu = 0.25$ right. Here, the coordinates $x$ and $y$ are normalized to $\sqrt{\epsilon(0)}S$.

Fig. 10 Radial distribution of the macroparticles at $s / S = 2.5$ for (a) $\nu = 0$ and (b) $\nu = 0.25$. 
Fig. 3
Fig. 4
Fig. 6
Fig. 8
Fig. 10