MATERIALS CHARACTERIZATION AND STRUCTURAL DESIGN OF CERAMIC MICRO TURBOMACHINERY

by

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Abstract

Since 1995, MIT has been developing the technology for micro-gas turbine-generators capable of producing 50 W of electrical power in a package less than one cubic centimeter in volume. The initial goal of this project is to produce a turbine generator and an all-silicon cooled engine by micromachining single crystal silicon. The goal of this thesis is to explore the structural design issues associated with the micro silicon turbogenerator and the all-silicon cooled engine with the aim of establishing a methodology for the design and fabrication of highly stressed microfabricated structures. The major research contributions includes the effect of length scale on materials and structural response, material characterization, structural analysis and failure probability, and heat transfer analysis.

Due to the differences in length scales between the microengine and conventional macroscale turbomachinery, a detailed investigation of the effect of length scale has been conducted. This shows that a direct comparison of a microengine structural design with that embodied in macroscale engines is largely irrelevant, especially with regard to the material fracture strength. Strength test specimens must have similar length scales and be fabricated by the same processing route as the real structures.

The data for fracture strength, fracture toughness, yield strength, and creep resistance of single crystal silicon at various temperatures have been obtained from the literature and experimental investigation. The room temperature fracture strength has been shown to be extremely sensitive to the surface processing route and strength recovery methods have been proposed and tested. The fracture toughness has been found to be temperature independent below the brittle to ductile transition temperature (BDTT). The yield strength has been found to be extremely dependent on the temperature and strain rate above BDTT. High temperatures and low strain rates reduce the available yield strength.

The turbogenerator rotor has been analyzed using the finite element method. It was found that the stress critical location is located at the root of the turbine blade trailing edge. In order to reduce both stress and deflection levels, various structural redesign approaches were proposed. The finite element stress analysis and the fracture strength test data were combined using a probabilistic structural analysis to evaluate the reliability of the turbogenerator.

The all-silicon cooled engines will potentially run in the creep regime of silicon. As a result, thermo-structural analysis is required to assess the trade-off between structural reliability and engine performance. Since the heat transfer, thermal stress, fluid dynamics, and engine cycle performance analyses are coupled, an iterative analysis is required between structural, fluid, and performance design.

Finally, the results for material characterization and structural analysis are integrated with the overall designs of the turbogenerator and the all-silicon cooled engine. The analysis and design methodologies presented in this thesis can be applied to other highly stressed power microelectromechanical systems (MEMS), turbomachinery, and the structural design of ceramic components.

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CHAPTER 1
INTRODUCTION

1.1 Project Overview

In 1995, the Gas Turbine Laboratory (GTL) and the Microsystems Technology Laboratory (MTL) at MIT initiated a project to build micro gas turbine engines by micro fabrication technology [Grosheny, 1995] [Epstein et al., 1995]. Figure 1.1 shows a conceptual design of a microengine. Ultimately, as efficiencies are improved, these devices will be capable of producing more than 50 W of electrical power in a package less than one cubic centimeter in volume, while consuming 7-8 grams of hydrocarbon per hour. Such devices would represent a quantum leap in compact electric power sources, with the potential to achieve more than ten times the power density of current batteries at competitive costs [Epstein, 1995], [Epstein and Senturia, 1997], [Epstein, Senturia et al., 1997].

To achieve the design goal, the microengine will be operated at high temperatures and high stress levels [Epstein et al., 1997]. These structural requirements imply the use of refractory structural ceramics such as silicon nitride and silicon carbide. However, suitable microfabrication processing routes for refractory ceramics are not yet available. For this reason, the project has been divided into two phases. The first phase of the microengine program is to design and fabricate a turbogenerator from silicon [Epstein, 1995 ARO], with the possibility of extending it to function as a cooled silicon microengine [Epstein, 1997 ARO], for which fabrication processes have already been proven. In parallel, development is underway of processes suitable for more refractory materials. In phase two, based on the analysis and design experience obtained from the silicon article, as well as the process development in phase one, the final high temperature microengines will be built. In order to achieve the goal, the entire research team has been divided into several design group. Each design group has specific research goal in their own field. The analysis and experimental results from each group were then integrated by the system design group.
The focus of this thesis is to explore the structural design issues associated with the phase I device, i.e., a silicon turbogenerator and also to investigate the concept of a cooled silicon microengine. In particular, the following three issues will be addressed, with the aim of establishing a methodology for the design and fabrication of highly stressed microfabricated structures.

1. The effect of length scale on structural design with highly stressed brittle materials.

2. The effect of processing and service conditions on the mechanical behavior of silicon at the structural scales of interest.

3. The development of a methodology for the analysis and design of highly stressed microfabricated structures.

Figure 1.1 Schematic cross section of a microengine, note the device is axisymmetric about its center line ([Epstein, 1995])
Figure 1.2  SEM micrograph of a microengine turbine disk (Courtesy of Mr. Chung-Chia Lin).
1.2 Key Issues in Microengine Operation

Similar to large gas turbine engine, the design of the microengine requires careful trading between many requirements and considerations. Although the overall design, which is described in detail elsewhere [Epstein, AIAA, i997], is not the subject of the present work, it is important to highlight the key issues in the fabrication and operation of the microengine as they have strong interactions with the structural design process.

Fabrication:

Microfabrication processes suitable for low cost, large volume production, basically produce two-dimensional extruded geometries. Their capability to produce arbitrary three-dimensional geometries is very limited [Ayon et. al, 1998]. This imposes severe constraints on the designer's ability to tailor the geometry to optimize the design.

Turbomachinery and Fluid Mechanics:

In order to increase efficiency, the compressor pressure ratio should be relatively high. This implies a high rotating speed. A 500 m/s rotor tip speed was chosen in order to provide enough pressure ratio. Such high rotational speed would result in high centrifugal stress. In the microengine, the flow regime is unusual because it can reach supersonic velocities (maximum Mach number=1.4) but still remain in laminar flow (maximum Reynolds number = 20,000) [Mehra, 1997].

Bearing and Rotordynamics:

Low friction bearings are required to support the rotor against fluid and electrical forces, rotor dynamics and externally applied accelerations while operating at speeds of over two million rpm. Gas bearings were selected based on their superior load bearing capability at small scale and their relative ease of fabrication. The physical regime that the micro gas bearings operate in is unusual and well outside existing theory and empirical design practice [Piekos, et. al, 1997].

Electrical Machinery:

The base line design is a planar electric induction machine mounted on the compressor rotor. The major challenges of the electrical machinery design is the rotor material conductance at temperatures and the charge relaxation time. In addition, the
heat generation due to viscous drag and the electrostatic force in the rotor-stator air gaps will add to the complexity of the engine design [Mur Miranda, 1997].

**Combustion:**

Airbreathing combustion requires fuel injection, fuel-air mixing, and chemical reaction of the mixed reactants. The time required for these processes (combustor residence time) sets the combustor volume. It is almost independent of length scale. As a result, the combustor occupies most of the volume of the microengine. Initial efforts have focused on gaseous hydrogen as the fuel, as this offers an acceptable flame temperature (> 1600 K) while minimize the combustor volume. Subsequent work will attempt to introduce more operationally realistic hydrocarbon fuels [Tzeng, 1997], [Waitz, Gauba, and Tzeng, 1998],[Mehra and Waitz, 1998].

**Structures and Materials:**

The power generated by a turbine is proportional to the square of the tip speed [Kerrebrock, 1992]. However, the centrifugal stress due to rotation is also proportional to the square of the tip speed. As a result, a high power device is also a highly stressed device. Thus, to a large degree, the performance of the device is limited by the material strength. The microengine is configured to operate according to a Brayton cycle. The engine efficiency is determined by the pressure ratio and the maximum turbine inlet temperature, $T_{\text{inlet}}$ [von Wylen, 1985]. The higher the $T_{\text{inlet}}$ is, the higher the engine power will be. Usually the capabilities of the structural materials to withstand high temperatures and stresses limits $T_{\text{inlet}}$. The most widely utilized material in microfabricated devices, silicon, has relatively poor high temperature strength and creep resistance compared to the metallic alloys and ceramic coatings used in conventional engines. If silicon is used to construct the microengine, the turbine inlet temperature must be kept to an unusually low value or the engine must be aggressively cooled. In either case, the device efficiency will be very low. However, even at low efficiency, it may still be able to exceed the energy densities available from other compact power sources such as batteries. In order to increase the operating temperature, refractory ceramics will be required. However, currently there are no well defined micro fabrication routes for such materials.
1.3 Thesis Outline

There are several key issues in microengine structural design. These challenges define the present work. They are as follows:

1. The effect of length scale:

The small size of the microengine has a strong influence on its design. The scaling of material properties, mechanics, heat transfer, and fabrication constraints combine to produce very different constraints on structural design than are encountered in large engines.

2. Material characterization:

Accurate measurement of material properties is essential for structural design. Material properties, such as the elastic constants, strength, thermal expansion coefficients, thermal conductivity, and fracture toughness, need to be determined. For single crystal silicon, many properties are well defined and can be found in the literature. However, others, such as the fracture strength of silicon and the creep resistance, have been less thoroughly investigated and need to be determined by experiment [Spearing and Chen, 1997].

3. Stress / deformation analysis:

Analysis of stress and deformation are required to design a reliable structure. In the microengine, the most critical component is the rotor, especially the turbine disk. It experiences high centrifugal stresses and elevated temperatures, and potentially high impact loads. For the static structures, the major concern is the thermal stress resulting from the constrained thermal expansion. Therefore, these effects are to be investigated. Due to geometric complexity, the finite element method will be used as the primary analytical tool.

4. Probabilistic structural analysis:

The strength of brittle materials, such as silicon, is not deterministic. It is controlled by the processing route and material volume. As a result, strength is a stochastic variable. As a consequence, the structures do not necessarily fail at the most highly stressed location, and strength data can not be directly used to generate design
allowables. Probabilistic structural analysis methods utilizing Weibull statistics are applied in conjunction with the finite element analysis to predict the failure probability of structures.

5. Thermo-structural analysis:

The operation of the microengine is dependent on not exceeding the stress and temperature limits of the materials. Accurate thermo-structural analysis is therefore essential. Furthermore, this analysis provides performance goals for the cooling systems, and therefore is strongly coupled to the turbomachinery system design.

The preceding paragraphs define the major research tasks of this thesis. Their interactions are summarized in Figure 1.3. These tasks provide the subject matter for the following chapters. A more detailed introduction to microengine structural design is presented in chapter 2. The effect of length scale is discussed in chapter 3. The room temperature material characterization is presented in chapter 4. Chapter 5 contains microengine stress and deformation analysis. Probabilistic structural analysis is addressed in chapter 6. The high temperature material testing is presented in chapter 7. Chapter 8 presents the results of thermo-structural analysis. In chapter 9, the work of previous chapters is integrated to provide an overall structural design. Finally, chapter 10 summarizes the thesis work and provides conclusions, and suggestions for future work.
Figure 1.3  Flow chart to demonstrate the connectivity of the materials and structures tasks on the microengine project
1.4 Thesis Contribution

The contributions of this thesis can be subdivided into two areas; those more specific to the microengine project, and those with more general contributions to the design, testing, and analysis of microfabricated materials and structures. The microengine specific contributions are significant due to the potential overall impact of the microengine itself. As previously discussed, its performance is partially defined and constrained by the capabilities of the materials and the structural geometries achievable by microfabrication. Therefore, achieving an overall design which is consistent with these constraints is a key to the overall success of the project.

The following are the contributions to the general area of MEMS structures and materials:

1. Establishment of design methodologies for highly stressed MEMS devices: This includes understanding the effect of length scale on the analysis and design procedures for the structural design with brittle materials for room temperature devices, and also the ductile design methodology for inherently brittle structures.


3. Introduction of Weibull statistics into the MEMS area: The analytical results and procedure will be directly applicable to the structural reliability analysis of other MEMS devices.

4. High temperature silicon strength characterization and its introduction into MEMS structural design.
1.5 Reference


CHAPTER 2. INTRODUCTION TO MICROENGINE STRUCTURAL DESIGN

The objective of this thesis is to investigate structural design methodologies for gas turbine engines microfabricated from brittle materials. Therefore, as shown in Figure 2.1, it draws on three major areas of prior investigation: turbomachinery structures and materials, ceramic structural design, and MEMS design. The goal of this chapter is to provide a general background in each of these areas, highlighting the typical design considerations and the major constraints. With this background, the guiding principles of microengine structural design are then introduced. This chapter is organized as follows: Section 2.1 introduces the general considerations in the design of conventional turbomachinery structures. In section 2.2, the characteristics of ceramic materials and their structural applications will be presented. In section 2.3, MEMS and their structural design are addressed. The characteristics of microengine structures are discussed on section 2.4. Finally, in section 2.5, the principles to be applied to microengine structural design are briefly presented.

Figure 2.1 The required background for microengine structural design
2.1 Turbomachinery Structures and Materials

Gas turbine engines have been widely applied in modern aircraft propulsion and power generation. They are typically composed of a fan, a multi-stage compressor, a combustion chamber, and a multi-stage turbine. The combustor converts chemical energy from the fuel to kinetic energy of the turbine. The requirement for low weight in combination with operation at high tangential speed and high temperature (in the turbine), impose severe constraints on the design of turbomachinery, which are in turn reflected in structural characteristics unique to such machines [Kerrebrock, 1992]. Engine structures are composed of closely interconnected components of rather complex shape. Most of the unique structural features stem from the requirements of high speed and high temperature turbomachinery. The structural design concerns of turbomachinery can be classified broadly as: material properties, mechanical stress, thermal stress and heat transfer, rotordynamics and vibrations, aeroelasticity, bearings, and overall engine arrangement and static structures. They are addressed briefly in the following paragraphs:

**Materials:** Materials to construct turbomachinery must have high specific stiffness and specific strength to overcome the high centrifugal loads. For a high temperature turbine, the material strength, fatigue resistance, and creep resistance are critical in determining engine performance, durability, and reliability [Cybulsky and Bryant, 1993]. Some components also require significant impact and erosion resistance to reduce the damage due to impact with external or internal objects or particles in the flow [Vasco, 1998].

**Mechanical stresses:** The major source of stress is the centrifugal load in the rotating structure. A typical rotor stage usually consists of a disc, an outer rim, blades, retainers, damper, and seals. The disc experiences centrifugal loads from the mass of rim, blades, and the disc itself. In order to reduce the centrifugal stress, the cross-section of discs are usually designed to have a hyperbolic shape. Fillet radii are added to corners to reduce stress concentrations. An important criterion for the design of a gas turbine disc is to operate the disc below the "burst speed," i.e., the speed at which the disc will separate into pieces, due to excedance of the material strength.
For blades, in addition to the centrifugal loading, the gas dynamics result in bending forces, creating a further consideration. In general, this bending load becomes more important as the aspect ratio of blades increases [Kerrebrock, 1992].

**Heat transfer and thermal stresses:** The temperature at the turbine inlet of modern gas turbines is well above the melting temperature of the turbine materials. Cooling systems are essential to assure structural integrity.

Thermal stresses result from the lack of free thermal expansion and from non-uniform temperature distributions. Since the turbine materials (typically, Ni-based super alloys) have high elastic moduli (~ 200 GPa) and thermal expansion coefficients (~ $10^{-5}$) [Erickson, 1993], this implies that a small temperature difference will result in a considerable thermal stress.

**Rotordynamics and vibrations:** A gas turbine can be idealized as a flexible rotor consisting of a disk mounted on a flexible shaft. This is known as the Jeffcott rotor model [Ehrich, 1992]. There are several structural dynamics concerns associated with such a structure. Large amplitude vibrations will be generated as the gas turbine operates near the shaft critical speed [Ehrich, 1992]. Imbalance of rotating structures will also result in whirling [Ehrich, 1992]. The vibration of the blades and disk will also have characteristic natural frequencies, generally higher than those of the rotor. Due to the effect of centrifugal stiffening, the vibration frequencies of blades vary with rotating speed. The Campbell diagram is widely used to present the dynamic characteristics of engine spools. [Kerrebrock, 1992].

**Aeroelasticity:** The fluid forces acting on the blades can resulting in a forced vibration of the structures. Flutter is the term used for vibration which result from instabilities when the fluid does work on a vibrating blade to amplify or maintain the vibration. This can be a very complex phenomenon and will not be further addressed here. However, in general, increasing blade stiffness will reduce the flutter [Bisplinghoff and Ashley, 1962].

**Bearings:** Bearings are essential to any rotating or linear motion machinery. Bearings for conventional aircraft engines must be capable of sustaining high speed rotation, while supporting large thrust loads while minimizing weight. There are
numerous types of bearings, including rolling element bearings (ball or roller) and fluid-film (hydrodynamic or hydrostatic) bearings. In addition, many structural designs are directly with bearing systems such as structures for the secondary flow system.

**Overall engine arrangement and static structures:** The design concerns noted above reflect the demands of particular fields and components. It is also very important that an effective overall integration of the individual components is achieved, given the constraints on overall size and weight of such engines.

Due to the absence of centrifugal stresses, static structures have lower stress levels. However, they are generally exposed to higher temperatures, and can experience considerable thermal stress and pressure loading.
2.2 Ceramic Structural Design

Ceramic materials are non-metallic and inorganic solids, including glasses, vitreous ceramics, cements, and high performance structural ceramics. Except for the structural ceramics, these have been extensively used in civil engineering and construction.

Ceramics, can be divide into ionic and covalent ceramics according to the principal chemical bond between atoms. The ionic ceramics, such as zirconia and alumina, are basically compounds of metallic and non-metallic elements. The ionic bonding holds the atoms together to form crystalline structures. The covalent ceramics are usually compounds of two non-metals (e.g., silica), or pure elements (e.g., diamond or silicon). The covalent bonds hold the atoms together to form crystalline structures.

Both ionic and particularly covalent bonds are very stiff and strong. As a result, ceramics usually have much higher elastic moduli than metallic materials. These bonds also present a high lattice resistance to the motion of dislocations. This results in a very high yield strength and low fracture toughness. As a result, fast fracture becomes a major concern in ceramic structural design. With advances in material processing technology, structural ceramics such as silicon carbide and silicon nitride have been considered for turbomachinery components. These materials retained the general advantage of ceramics but have higher fracture toughness [Shaffer, 1991].

Compared to metals, structural ceramics have a higher inert strength, lower density, lower thermal expansion coefficient, and very high melting temperatures. High strength and low density implies a high specific strength. The higher melting temperature implies a potential for a higher maximum operating temperature. A lower thermal expansion coefficient implies better dimensional stability during operation. All of these attributes are desirable for a high performance turbomachinery design. Based on the above arguments, numerous research and development projects have been conducted to investigate the possibility of using ceramic materials in turbomachinery structures [ATTAP, 1994],[ATTAP, 1996],[Boyd, 1989],[Easley et.al, 1995],[Honjo et. al, 1993],[Kawase et. al, 1993],[Levine, 1992],[Takama et. al, 1993],[Takatori et. al, 1993],[Watanabe et. al, 1993].

Although structural ceramics have better performance than traditional ceramic materials, their toughness is still much lower than that of metallic materials. The
material fracture strength is still very sensitive to the surface or volume flaw distributions. As a result, the material strength is a stochastic variable and is volume dependent. This requires a totally different structural design methodology from that used for metallic structural design. [Nakakado, 1995] The approach commonly used is to design the structure such that the failure probability under operational loading is less than a threshold value. This requires structural analysis, material testing, and SEM fractography characterization.

Fracture in thermal shock occurs due to the local thermal tensile stress that arise during the thermal transient process. This restricts the service condition of ceramic structures and perhaps an important role in the choice of materials [Hasselmann, 1970].

Due to the low toughness and high melting temperature, ceramic structures can not be joined by conventional jointing processes used in metallic structures such as mechanical fasteners and welding. As a result, joining and joint reliability is an important issue in the design of ceramic structures [ATTAP, 1996].
2.3 Microelectromechanical Systems (MEMS) Structural Design

Semiconductor-based transducers have drawn remarkable attention in recent years and have come to be known generally as microelectromechanical systems (MEMS). MEMS encompass sensors, actuators, and electronics that interface with the physical world by converting physical stimuli from the mechanical, thermal, chemical, and optical domains to the electrical domain. MEMS fabrication utilizes the microelectronics technological base to fabricate sensors and actuators, thereby taking advantage of the sophisticated batch fabrication and production methods used in a modern integrated circuit foundry [Elliott, 1982]. Examples of MEMS devices include; accelerometers [Hsu, 1997], pressure sensors [Parameswaran, 1997], micromotors [Mehregany et. al, 1992], and flow nozzles.

The material used for MEMS devices are mainly semiconductor based ceramics, such as silicon and silicon oxide. These have excellent electrical and mechanical properties for MEMS device applications. Up to now, the structural analysis and design of MEMS devices has been focused on residual stresses, structural dynamics, and electro-mechanical coupling analysis.

**Residual stresses:** Many microfabrication processes include high temperature oxidation and deposition of materials different from the substrate. Due to the difference between coefficients of thermal expansion and other sources, considerable residual stresses can be generated when these devices are cooled down to room temperature. The residual stresses result in cracks, buckling, and distortion of MEMS devices. In addition, the nonuniform residual stress also generate stress gradients, which result in a distorted structure even in nominally residual stress free states [Hu, 1991],[Zhang et. al, 1998].

**Structural dynamics:** Most of the structural design in MEMS in the past has focused on the structural dynamics. Traditional MEMS can be represented by the combination of a mechanical system with electrostatic actuators, the structural dynamics of the mechanical system is important to the final device performance. Many material characterization schemes also rely on the structural dynamics behavior of materials [Attia et. al, 1998]. Natural frequencies and mode shapes of MEMS devices such as
accelerometers, need to be understood in order to define the operating regime and the
design of the control scheme.

**Electromechanical coupling:** The driving force in MEMS is usually electrostatics.
This is a highly nonlinear force with respect to driving voltages and the air gap. As a
result, the coupling between the driving force and the structural deformation can result
in a highly nonlinear dynamic system. Typical nonlinear system behavior such as limit
cycle and pull in can occur. This is a major research area in MEMS analysis [Mehregany
et. al, 1992], [Senturia, 1995],[Beerschwinger et. al, 1994],[Deqing et. al, 1994],[Temesvary
et al, 1995].

In recent years, with the rapid development in MEMS devices, other structural
concerns have been raised. The fatigue properties and life prediction of MEMS devices
have a potential importance in commercial MEMS transducers subject to dynamic
loading [Brown et. al, 1998]. The creep characteristics of silicon or silicide membranes
are potentially important for pressure transducers at elevated temperature [Maseeh and
Senturia, 1990]

In order to illustrate the structural design procedure for a typical MEMS, a
microfabricated accelerometer is used as a design example. The general procedure is
described at the following text. Figure 2.2 shows the schematic design flow.

Accelerometers are devices used to measure the acceleration experienced at their
location. The basic components for an accelerometer can be divided into the structures
and the instruments. The basic structural components are a proof mass, springs, and
dampers.

The first step in the structural design process is to design a prototype that is feasible
in microfabrication and potentially satisfies the basic specifications such as bandwidth
and sensitivity. This serves as the baseline design. In this step, the structural dynamics
modeling is performed by elementary beam or plate theory. The next step is to perform
a more detailed analysis, including behaviors such as deformation, vibration, damping
forces, and electrostatic analyses, on the baseline design. In this step, numerical
methods such as finite element, CFD, and boundary element techniques are usually
used.
After performing the detailed numerical modeling, the baseline design undergoes optimization and compromization due to the requirements of other disciplines and other structural design concerns. For example, the requirement to increase sensitivity of the sensing electrode will change the geometry of the proof mass as well as the structural dynamics.

After the optimization, the final design is selected. Detailed 3-D finite element analysis, CFD, and electrostatic analysis are required to predict the final performance of the device. If the prediction agrees with the design specification, the design analysis is finished. Otherwise, certain design alternations or specification goal adjustment is required to close the design.

Silicon is a very brittle material. Its strength is controlled by the surface flaws induced by the fabrication process. Under design loads, fast fracture of the structure may happen. Extensive research was conducted to determine the silicon fracture strength under particular treatments [Wilson and Beck, 1996], [Ericson and Schweitz, 1990], [Spearing and Chen, 1997]. However, the overall MEMS structural reliability has been less carefully studied. Therefore, the reliability or failure probability of MEMS devices is potentially an important issue to be addressed.

![Diagram of design flow for micro accelerometer structural design]

Figure 2.2 Schematic flow for micro accelerometer structural design
2.4 Characteristics of Microengine Structures

The materials used in microengines are microfabricable ceramics such as silicon, silicon carbide and silicon nitride. Most of these have the same advantages as for ceramic turbomachinery as described in section 2.2. They also have the disadvantages associated with ceramic engines. However, the reduced scale reduces their influence. Since the micro crack distribution in ceramic materials is volume or surface area dependent [Weibull, 1939], decreasing the overall dimensions of the engine will reduce the maximum flaw size in the ceramics, thus increasing the achievable strength. In addition, microfabrication processes, such as wet and dry etching, are much milder than traditional ceramic processing (for example, sintering and isostatic pressed). As a result, the process-induced flaw population is therefore much smaller in microfabricated structures. Previous experimental results show that the strength of semiconductor grade silicon can be as high as 3-5 GPa. This strength is much better than that usually found in macroscale structural ceramics (typically ~600 MPa [Takatori et. al, 1993] and metallic alloys (usually ~1 GPa) [Boyer, 1994].

The reduced need for attachments and interfaces between components is another advantage for microengines over macro scale ceramic turbomachinery. In the microengine spool design, there are no attachment concerns. Blades and disks are made from the same piece of material. This eliminates many of the concerns due to stress concentrations and thermal stresses induced by joining or fastening processes. Although the entire microengine is assembled from several wafers, diffusion bonding can be applied to join the wafers at low stress locations to maximize the reliability.

There are also some disadvantages associated with the microengine. Microfabrication basically produces a two-dimensional extruded geometry. Its capability to produce a 3-D shape is limited. As a result, the microengine design is constrained by manufacturing capability. Such designs also generate stress concentrations and disk deformation problems in structural design. With very high tip speed, as a result, the microengine has the similar primary stress levels and a higher stress concentrations than the traditional macroscale metallic turbomachinery. On the other hand, the key for success in microengine design is to use the higher available material strength to increase the performance in an unoptimized design.
Table 2.1 summarizes the material performance in terms of merit indices relevant to turbomachinery performance. The burst speed of rotating disks is proportional to the parameter $\sqrt{\sigma/\rho}$, where $\sigma$ is the fracture or yield strength and $\rho$ is the density. Microfabricated silicon, due to its low density and potentially high fracture strength, can have a very high burst speed. The non-dimensional thermal stress per unit degree, $\alpha E/\sigma$, is the merit index for thermal stress, where $E$ is the Young's modulus and $\alpha$ is the thermal expansion coefficient. It is desirable for this to be as small as possible. Again, silicon has the smallest $\alpha E/\sigma$, due to its low expansion coefficient and high strength. The specific stiffness, $E/\rho$, has great importance in the deformation of rotating structures. This should be as high as possible. The structural ceramics, such as silicon carbide, due to their high Young's modulus, results in a very high specific stiffness as does silicon. Metallic materials, on the other hand, have a relatively low specific stiffness due to their high density. The maximum temperature at which the structure can be operated is limited by several factors. For super alloys and silicon, the structural temperature is limited by creep. For titanium alloys, the tensile strength drops quickly as the temperature increases. For structural ceramics, strength is probably not the limiting factor for operation, since active oxidation is likely to be the controlling factor.

<table>
<thead>
<tr>
<th></th>
<th>Ni-based Super Alloys</th>
<th>Titanium Alloys</th>
<th>Macro Ceramics</th>
<th>Micro Silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\sigma/\rho}$ (m/s)</td>
<td>330</td>
<td>420</td>
<td>420</td>
<td>1000</td>
</tr>
<tr>
<td>$\alpha E/\sigma_{fly}$</td>
<td>$2.7 \times 10^{-3}$</td>
<td>$1.2 \times 10^{-3}$</td>
<td>$2.0 \times 10^{-3}$</td>
<td>$9.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>$E/\rho$ (MPa/Kg m$^{-3}$)</td>
<td>~26</td>
<td>~25</td>
<td>~95</td>
<td>~70</td>
</tr>
<tr>
<td>$T_{max}$ (°C) control factor</td>
<td>~1000 (creep)</td>
<td>~300 (strength)</td>
<td>~1500 (oxidation)</td>
<td>~600 (creep)</td>
</tr>
</tbody>
</table>
2.5 Microengine Structural Design Methodologies

For the silicon turbo-generator, the device will operate close to room temperature, and the silicon can be assumed to behave as a brittle material. However, for an all-silicon cooled engine, temperatures in parts of the engine structure may be in the range 800 - 1000 K. In this case silicon may exhibit significant ductility and creep deformation. The structural design methodology is summarized in Figure 2.3.

Design in the ductile and brittle regimes of material behavior requires different design approaches. For ductile structures, the material strength is essentially deterministic and a function of temperature. For structures that are not operated at high temperature and cyclic loading (static loading), the design methodology, therefore, is to identify the critical points and to design the structure with an appropriate safety factor.

For structures subjected to cyclic loading, the design methodology is to use the appropriate fatigue life prediction rules such as Coffin-Manson rule or Paris's law to predict the life of structures under low or high cycle fatigue.

For structures operating at elevated temperatures, the design methodology is to identify either the maximum allowable structural deformation or the maximum allowable strain to cause rupture, and use an appropriate creep model to calculate the life of the device. In both fatigue and creep loading situations, initially, structures have no static failure concern. However, the dynamic effects finally drive the structures to fail.

The structural design for brittle materials is quite different from the above approach. In brittle design, since the material strength is not deterministic, structural failure can potentially occur at any location. For static loading, the design method is to design the structure such that the probability of failure is less than an appropriate overall failure probability [ATTAP, 1996].

For ductile materials, during the tensile loading, crack tips tend to open due to plastic deformation. Fatigue crack growth is mainly caused by the closure of such cracks under compressive loading, which results in a net extension in crack size. Since brittle materials lack plastic deformation, as a result, fatigue effects are less of a concern. The major consideration is design against fast fracture. It is reasonable to assume, in the
absence of environmental degradation, that if the structure survives its initial loading, it will be able to survive indefinitely. This also suggests that proof testing might be a valid approach to ensuring reliability.

Silicon has a brittle-ductile transition temperature (BDTT) of approximately 550°C [Hirsch et. al, 1988]. Therefore, for a silicon turbogenerator, a brittle design methodology is appropriate. However, for a cooled silicon microengine, the design approach will vary by component. The temperature of the micro compressor is likely to be always less than the BDTT of silicon. Therefore, a brittle design approach is used. In contrast, the micro turbine and micro combustor structure will operate at higher temperatures, and a ductile design approach is required. For a future all silicon carbide engine, although the structural temperature may be potentially much higher than the all-silicon cooled engine, due to the high BDTT of silicon carbide (~ 1750 °C [Campbell et. al 1989]), it is not necessary to consider ductile design.

Figure 2.3  Microengine structural design methodology
2.6 Reference


CHAPTER 3. THE EFFECT OF LENGTH SCALE ON TURBOMACHINERY STRUCTURAL DESIGN

3.1 Introduction

As previously noted, the small size of the microengine has a strong influence on its design and performance. Consequently, it is important to quantify the effect of scale, so as to be able to assess possible design trade-offs and to avoid blindly applying design rules that have been derived for the macroscale. There are two sources of scale dependence in a system such as the microengine; those that arise directly from physical laws, and those that result indirectly from the scaling of the overall system. To elaborate on this second point; the design and performance of virtually all components of the microengine have some dependence on length scale. Since the structural design must be consistent with the overall system design, this imposes unusual constraints on the structural design. Furthermore, the reduction in scale permits the use of microfabrication processes that introduce a very different set of constraints on geometry than those commonly found at the macroscale.

If the system characteristic lengths are in a range where continuum theories can be applied, then the physical scaling can be applied directly to the design variables to compare macro and micro scale effects. On the other hand, if the physical scale is close to the scale defining the mechanisms governing the material behavior, e.g., atomic or molecular dimensions, continuum theory can no longer be applied, and other model must be invoked [Drexler, 1992]. In structural design, many design variables show scale dependences. They will be addressed in detail in this chapter.

Figure 3.1 shows schematically the influence of these factors on the structural design. These issues can potentially result in considerable differences between macro and micro scale structural design.

This chapter discusses the effect of length scale on turbomachinery structural design. The major turbomachinery structural design considerations will be addressed.
Macroscale metallic and ceramic turbomachinery structural design solutions will be used, where appropriate, as comparisons for the ceramic microengine structural design. The influence of length scale on the following factors will be described: materials, mechanics, heat transfer, and surface reactions. Each issue will be discussed in detail in the following sections.

![Diagram showing the relationship between Physical Scaling, Fabrication Technology, Other Fields (Fluid, electrical, combustion, etc), System Design, Structural Design, Materials, Mechanics, Heat Transfer, and Surface Reaction (e.g. Oxidation).]

Figure 3.1 The length scale effect in structural design
3.2 Material Considerations

The first major concern is the effect of length scale on the material properties. The key properties for turbomachinery design include the thermoelastic constants, strength, and fracture and creep resistance. These are determined by intrinsic and extrinsic factors including microstructure, surface condition, and operating temperature.

The first observation to make is that the structural scales of the microengine are significantly larger than the length scales that determine many aspects of material behavior, namely the atomic lattice spacing and the dislocation core size. Furthermore, since single crystal silicon is the principal structural material, no further scale effects are introduced by the presence of grain boundaries. These considerations imply that continuum mechanics will apply for the elastic, plastic, and heat conduction behaviors. However, as will be discussed later, the fracture strength, creep, and oxidation resistance may be affected.

3.2.1 Thermoelastic Constants:

Elastic stress and strain behavior can generally be described by the constitutive law,

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl},$$

(3.1)

where $\sigma$ and $\epsilon$ are the stress and strain tensors respectively. $C$ is the material stiffness tensor with potentially 21 independent coefficients. For a cubic symmetric material, such as silicon, this reduces to three constants, Young’s modulus $E$, Poisson’s ratio $\nu$, and shear modulus $\mu$. For isotropic materials, only two of these are independent. Since these constants are inherent to the continuum theory of elasticity, it follows that they are scale independent.

The assumption of scale independent elastic properties holds even if polycrystalline materials are employed providing that the grain size is much smaller than the structural dimensions. Since the most likely method of microfabricating polycrystalline materials is chemical vapor deposition, which yields submicron size grains, and the smallest structural dimensions in the microengine are greater than 10 $\mu$m, this is likely to hold true. If microstructures were generated with grain sizes approaching the structural
dimensions, it might become necessary to determine the local orientation of the grains and to introduce the corresponding local mechanical properties into the structural calculations. It should be noted that grains with these dimensions would be undesirable due to strength considerations.

### 3.2.2 Material Fracture Strength

The role of length scale in determining the strength of brittle materials has long been recognized [Griffith, 1921]. Experimental studies have shown that small specimens, on average, exhibit higher strengths than larger ones. In the particular case of microturbomachinery, there are two reasons for this; first, the processing route and second, the statistics of the strength controlling flaw population.

Ceramics usually have very high intrinsic strength. The ionic or covalent bonds possess an enormous lattice resistance to the motion of dislocations. As a result, they typically exhibit a very high (1-20 GPa) yield strength at room temperature. However, the penalty associated with the large lattice resistance is a low fracture toughness, i.e., brittle behavior. The presence of flaws inside the material volume or at the surface will control the material strength and reduce it dramatically below the intrinsic strength.

The design strength of a ceramic, then, is determined by its fracture toughness, \( K_{\text{IC}} \), and by the lengths of the largest microcracks, \( a_m \), it contains. The fracture strength is expressed as

\[
\sigma_f = \frac{K_{\text{IC}}}{Y\sqrt{\pi a_m}},
\]

where \( Y \) is a shape factor close to unity.

The fracture toughness is determined by the atomic bond strength (or energy) and lattice resistance to dislocation motion. Both of these parameters are controlled by length scale far below the scales of the microengine and the flaws which control its strength.

The use of microfabrication processes on single crystal materials results in very high quality surfaces on the finished structure. The reduction in the mean size of surface
flaws by utilizing appropriate microfabrication processes can result in very high material strengths [Hu 82],[Chen, Ayon, and Spearing 1997, 1998]. High strengths can also be achieved by careful mechanical grinding and polishing of polycrystalline ceramics at the macroscale [Rice, 1981]. However, these processes are not readily applied to surfaces with three dimensional curvatures such as airfoil sections and fillet radii, thus the use of microfabrication processes results in high mechanical strengths relative to the macroscale.

The second source of strength enhancement is the statistics of the flaw population. As shown in Figure 3.2, the strength of a structure made from a brittle material is not deterministic. Processing-induced flaw sizes and locations are random variables that combine with a deterministic value of the fracture toughness to make strength a stochastic quantity. Use of the empirically-determined Weibull probability density function [Weibull, 1939],[Ashby and Jones, 1987 ] allows a comparison of the ratio of the characteristic stresses ($\sigma_1/\sigma_2$) to give equal probability of failure for two volumes ($V_1,V_2$) with geometrically similar stress distributions,

$$\left(\frac{\sigma_1}{\sigma_2}\right) = \left[\frac{V_2}{V_1}\right]^\frac{1}{m},$$

(3.3)

where $m$ is the Weibull modulus. Thus, when comparing micro- and macro scale devices which have characteristic dimensions on the order of 1mm and 0.1m respectively, the volume ratio is 1x10^4. A value of $m$ of 12 is typical for sintered ceramics [Nakakado et al, 1995] and this gives an expected strength ratio of about 3 between the micro and macroscale. A caveat needs to be placed on this argument since extrapolation to this degree with a single probability density function generally cannot be justified. Using a Weibull distribution with the same fitting parameters (reference stress $\sigma_0$ and modulus $m$) assumes that the flaw population is the same between the two cases. Clearly this is unlikely to be the case since the processing routes are different at the micro and macro scales. With careful surface machining Weibull moduli of the order of 40 have been achieved at the macro scale, which would also produce a smaller increase in strength based on purely volumetric considerations. Nevertheless, the
comparison supports the argument for the potential of improved mechanical performance at the microscale.

![Diagram](image)

Figure 3.2 A flawed material subject to uniform tensile loading. Strength is controlled by the maximum flaw inside the material volume.

The statistical nature of the strength of brittle materials has another consequence in microturbomachinery design. Typically, macro scale ceramic turbomachinery components are designed with very low probabilities of failure (\(<10^{-4}\)). This reflects the total cost of the devices and is a function of the economic consequences of failure. Since micro-gas turbines are intended to be relatively inexpensive and failure of an individual device is likely to be of little consequence, a reduced safety margin can be tolerated. This further increases the allowable stress levels in the material in micro scale devices compared to those at the macro scale. The significance of the increased strength achievable at the micro scale is that it permits higher operating speeds, and hence improved specific performance of the engine.

An important consideration for the structural design approach is the size of the strength controlling flaws relative to the structural dimensions. The strengths achieved for Si indicate a largest flaw size in the range of 0.1 - 0.3 \(\mu m\), which is two orders of magnitude smaller than the smallest structural dimension. Thus, it is appropriate to use a stress-based design approach. If the flaw and structural dimensions were of a similar magnitude, it would be necessary to employ a fracture mechanics-based approach in which the effect of discrete flaws at critical locations would be analyzed.
3.2.3 Yield and creep strength

Yield is plastic deformation due to the motion of dislocations, and is usually the strength limiting process in ductile metal structures. It is not dependent on process induced flaws. As a result, the material yield strength is reasonably deterministic. Material plastic behavior is governed by dislocation motion. The dislocation core size is in the order of 0.1 - 1 nm (as determined by the Burger's vector, b, which is approximately equal to the lattice spacing). Significant increases in yield stress can be achieved by "pinning" dislocations with precipitate particles in alloys or with free surfaces and interfaces in whiskers or thin films. The microstructural dimensions at which such effects because appreciable are given by $\sim Gb/\tau_s$, where G is the shear modulus and $\tau_s$ is the shear yield strength. Since the smallest structural dimensions of the microengine (fillet) are at least four orders of magnitude larger than the dislocation core size and two orders of magnitudes larger than the particle spacing for hardening, the yield strength is unlikely to be affected at the microengine scale. As previously noted the room temperature behavior of silicon is determined by brittle fracture rather than yield. However, at elevated temperature, the structural limits may be determined by the yield strength.

For creep, the controlling mechanisms are dislocation climb, grain boundary diffusion, and surface diffusion. A generalized constitutive equation for deformation by creep processes can be expressed as, [Hahn and Averbac[, 1992].

$$\frac{d\varepsilon}{dt} = A(1/G)^n \sigma^q \exp(-Q/kT),$$

(3.4)

where $d\varepsilon/dt$ is the strain rate, G the grain size, $\sigma$ the applied stress, Q the activation enthalpy of the creep process and A, n, and q are constants. In the particular case of diffusional creep, in which dislocation climb is unimportant and creep is dominated by the flow of atoms from grain boundaries subject to a compressive stress to those under tensile stress (Coble creep), the constitutive equation can be expressed as

$$\frac{d\varepsilon}{dt} = D_{eff} \Omega \sigma / (kT^2),$$

(3.5)

with the effective diffusion coefficient defined by
\[ D_{ct} = B_1 D_v + B_2 \delta D_b / G, \]  

(3.6)

Here, \( \Omega \) is the atomic volume, \( D_v \) and \( D_b \) are the volume and grain boundary diffusion coefficients, respectively, \( \delta \) is the grain boundary thickness and \( B_1 \) and \( B_2 \) are constants. From Eq.(3.5) and Eq.(3.6), both the diffusion coefficient and the creep rate increase as the grain size decreases. For example, in ceramic sintering, the surface diffusion rate of 10 nm diameter powders is \( 10^9 \) times that of the same material particles with 10 \( \mu \)m diameter particles at the same temperature if Coble creep dominates. As a result, for fine size ceramics, the sintering temperature can be greatly reduced. [Hahn and Averback, 1992]. This certainly increases the relative rate of surface diffusion.

Since single crystal materials are not susceptible to grain boundary diffusion, this will result in a higher creep resistance. As previously noted, the structural scales of the microengine are sufficiently larger than the dislocation scale therefore a scale dependence is not introduced. It is possible that a length scale dependence in the creep response might be introduced by the increasing surface area to volume ratio at the microscale [Snedden and Sinclair, 1992]. However, the microengine scales are at least four orders of magnitude larger than the dislocations and grain boundary thickness where creep rates, due to surface diffusion, approach those of dislocation glide and climb.

An important consequence of the scale independence of the creep and yield behavior of silicon at the microengine scale is that it is appropriate to obtain material property data for these behavior from macroscopic standard test specimens. That is, the yield and creep properties can be tested by conventional tensile and bending test fixtures that can be used in conjunction with conventional load cells and loading forces.
3.3 Mechanics

3.3.1 Rotating structures

For rotating structures in turbomachinery, the major loading is the centrifugal stress. For a flat disk, the maximum stress appears at the center of the disk, and is given by:

\[ \sigma_{\text{max}} = \frac{3 + \frac{V}{\rho}}{8} V^2. \]  

(3.7)

If the tip speed \( V \) is fixed, the maximum stress is the same regardless of the size of the disk. Therefore, for a pure geometric scaling, the centrifugal stress is independent of scale. However, different materials, due to different densities, \( \rho \), will show different stress levels. Nickel-based super alloys and titanium alloys are widely used for turbine and compressor disks in conventional macroscale turbomachinery. For macro scale ceramic turbine applications, silicon nitride and silicon carbide are usually selected. In general, the metallic materials are denser and the stress levels are higher under the same operating conditions and for the same geometries.

As described in the previous section, the ability to fabricate brittle materials with higher strength than at the macro scale, combined with the lower density of ceramics and silicon than conventional alloys results in a fundamental scaling of the important performance metric, the specific strength \( \sigma_r/\rho_r \).

Table 2.1 shows a comparison between the nickel-based super alloys, titanium alloys, silicon carbide ceramic disk, and the mesoscale silicon disk. The table shows that a silicon micro turbine disk is potentially superior to any large scale engine in terms of its burst speed. However, at the macro scale, the manufacturing technologies permit optimization of the structural design. The disk cross-section can be made to have a hyperbolic shape to reduce the stress levels [Cook and Young, 1985]. Similarly, blades can be created with a 3-D shape to maximize the aerodynamic performance and reduce the stress. For the microengine, due to the limitations of the microfabrication process, this optimization can not be performed. In addition, axial flow type machines are difficult to fabricate as they require 3-D curvatures. The fabrication constraints thus drive the overall engine design to a centrifugal machine. Without the capability to create a three-dimensional geometry, the stress concentration at the blade roots and
other transitions between horizontal and vertical surfaces will be high. This will potentially outweigh the structural benefits of the small scale.

However, the use of microfabrication also largely eliminates the concern of assembling and joining components and subcomponents such as disks and blades. This is a challenging issue for the large scale ceramic turbines. The interfaces between components are often responsible for reduced reliability due to stress concentrations on bearing surfaces and fretting fatigue [ATTAP, 1997].

Continuum elasticity theory implies that geometrically similar structures, under identical stress states will experience identical strain states. Thus there is no inherent geometrical scaling in the deformation behavior from the macro to the microscale. However, due to the limitation imposed by the fabrication length scale, geometric similarities cannot be maintained and scaling issues do arise. The design is driven by two competing considerations; the power requirements scale the rotor diameter and blade height, and the fabrication requirements limit the etch depth, and hence the blade height. The overall effect is that the design is driven towards a relatively thin rotor disc with blades cantilevered off one side. This configuration is relatively compliant, and particularly susceptible to out-of-plane deformation due to the asymmetry of the blades, as well as other sources. These effects could be countered by tailoring the profile of the reverse face of the disk so as to balance the asymmetric loading. Despite the associated fabrication difficulties, it is important that such measures are taken as the axial and radial clearances are critical to maintain the performance of the bearings, the electric generator and to control leakage flows.

3.3.2 Static structures
The two principal loads on the static structure are pressure loadings and thermal stress. The fluid pressure inside the engine is always greater than the ambient pressure. A simple model for the stationary structure is an internally pressurized cylinder. The
hoop stress, $\sigma_{\text{hoop}}$, wall thickness, $t$, cylinder diameter, $D$, and the internal gage pressure, $p$, have the following relationship

$$\sigma_{\text{hoop}} = \frac{pD}{t}. \quad (3.8)$$

If the ratio $D/t$ is fixed, the hoop stress of the stationary structure is independent of scale. In the microengine design, the $D/t$ ratio is actually smaller than in a typical macroscale engine (typical $D$ is $\sim 0(10 \text{ inch or 250 mm})$ and $t \sim 0(0.1 \text{ inch or 2.5 mm})$ for PW 4000 high pressure compressor casing) due to the fabrication and heat transfer concerns. Furthermore the pressure ratio of the microengine is very low ($\sim 2-4$) and the sensitivity to overall weight is far less than in a large engine.

There are two major sources of thermal stresses; constrained thermal expansion or contraction and the non-uniform temperature distribution.

Consider a fixed-fixed rod with length $L$. It is uniformly heated to a temperature $T_f$. The thermal stress of this beam will be

$$\sigma_r = \alpha E (T_0 - T_f). \quad (3.9)$$

Where $\alpha$ is the coefficient of thermal expansion and $E$ is the Young's modulus of the material. $T_0$ is the initial structural temperature of the beam. There are no physical dimensions in Eq. (3.9). As a result, the thermal stress due to constrained thermal expansion is independent of length scale.

The expression of thermal stress due to non-uniform temperature distribution is more complicated and is geometry dependent although not length scale dependent. It is worth examining the case of a circular disc with a radial temperature distribution as an example for illustration purposes.

Consider a solid disk with outer radius $b$, which is subjected to a radial temperature distribution $T(r)$. The non-uniform temperature distribution results in the following thermal stress components [Jeager, 1941],

$$\sigma_n(r) = \alpha E \left( \frac{1}{b} \int_0^b T(r) r \, dr - \frac{1}{r^2} \int_0^r T(r) r \, dr \right), \quad (3.10)$$

$$\sigma_{\theta\theta}(r) = \alpha E \left( \frac{1}{b} \int_0^b T(r) r \, dr + \frac{1}{r^2} \int_0^r T(r) r \, dr - T(r) \right). \quad (3.11)$$
In order to understand the effect of length scale, let \( r^* = r/b \). Eq. (3.10) and (3.11) can be re-written as,

\[
\sigma_n(r^*) = \alpha E \left( \int_0^{r^*} T(r^*) r^* dr^* - \frac{1}{r^*} \int_0^{r^*} T(r^*) r^* dr^* \right),
\]

(3.12)

\[
\sigma_{00}(r^*) = \alpha E \left( \int_0^{r^*} T(r^*) r^* dr^* + \frac{1}{r^*} \int_0^{r^*} T(r^*) r^* dr^* - T(r^*) \right).
\]

(3.13)

If \( T(r^*) \) is identical in these disks, according to Eq.(3.12) and (3.13), their thermal stress distributions will be identical. Therefore, there are no length scale effect on non-uniform temperature induced thermal stress. However, the structural temperature distribution will have a strong dependence on length scale effect. This will be addressed in section 3.4.

3.3.3 Structural dynamics and vibration

A simple model for the dynamics of a turbine blade is to consider it as a cantilevered beam. The relevant variables are shown in Figure 3.3. The frequency of the first bending mode of a cantilever beam can be expressed as [Blevien, 1984]:

\[
f_1 \propto \sqrt{\frac{E}{\rho h^2 (\frac{h}{L})^4}},
\]

(3.14)

where \( h \) and \( L \) are the thickness and width of the beam respectively. For an exact geometric scaling, the aspect ratio \( h/L \) is fixed. Therefore, \( f_1 \) is proportional to \( 1/h \).

However, for a fixed tip speed, the rotating speed, \( \omega \), also inversely scales with the size of the disk. As a result, the important design variable, \( f_1/\omega \), the nominal frequency, is actually scale independent from purely geometric considerations.

However, the overall system design and material selection alter this conclusion. Eq.(3.14) also shows that \( f_1 \) depends on the square root of the specific stiffness, \( E/\rho \). Referring to Table 2.1, ceramic materials generally have higher specific stiffness than metallic materials. Therefore, both mesoscale and macroscale ceramic blades will have a higher \( f_1/\omega \) ratio for a given geometry.
Since the microengine is constructed to be a centrifugal (radial flow) machine, this further restricts the geometry. Radial flow blades inherently have a lower aspect ratio than axial flow blades, which means that radial blades are likely to be stiffer. However, since axial blades are long and extended parallel to the centrifugal force direction, they may benefit from significant centrifugal stiffening, which increases their resonant frequencies [Hibbit, 1979]. Radial type blades, on the other hand, benefit far less from this effect. In addition, the centrifugal force causes blade bending. As a result, blade height is restricted in order to avoid excessive deflection and stress.

![Figure 3.3 Schematic of a cantilevered beam](image)

### 3.3.4 Impact of Structures

Impact of projectiles on structures or between structures is another important structural design issue. For example, aircraft engines must pass the bird strike test before being certified by the FAA. The length scale dependence of the impact stress can be addressed by a simple dimensional analysis. As with all behaviors governed by elastic continuum mechanics, the elastic contact stress is found to be scale independent. This can be interpreted in the following way:

\[
\text{Stress} = \frac{P}{A} = m \frac{\delta V}{\delta t} / A,
\]

where \( P \) is the load and \( A \) is the interaction area and \( m \frac{\delta V}{\delta t} \) is the rate of the change in momentum. For a purely elastic impact, for the same impact velocity, \( V \), \( \delta V \) is essentially the same. Dimensional analysis shows that

\[
L(\text{stress}) \sim L(m)/L(A)L(\delta t), \text{ or, } L(\text{stress}) \sim L/L(\delta t).
\]
The length scale dependency of the impact duration, δt, can be found from the structural resonant frequency, and is proportional to the length scale, i.e., \( L(\delta t) = L \). As a result, \( L(\text{stress}) \) is length scale independent.

In order to obtain an analytical insight as to the effect of length scale on structural impact, it is useful to consider the Hertz contact problem. As shown in Figure 3.4, a spherical object collides with a solid space with an initial velocity \( V \). Both materials are assumed linear elastic and failure is not considered.

![Figure 3.4  Blunt impact](image)

A blunt projectile with a velocity \( V \) and mass \( m_1 \) hits a flat structure with mass \( m_2 \). During impact, the impact force, \( P \), can be expressed as [Zukas, 1984], [Hertz, 1881]

\[
P = n^{2/5} \left( \frac{5V^2}{4M} \right)^{3/5},
\]

where

\[
n = \frac{4\sqrt{R}}{3\pi(k_1 + k_2)},
\]

\[
M = \frac{1}{m_1} + \frac{1}{m_2} \quad \text{for } m_2 \gg m_1, \quad M = 1/m_1,
\]
\[ k_1 = \frac{1 - V_1^2}{\pi E_1} \]  

(3.18)

\[ k_2 = \frac{1 - V_2^2}{\pi E_2} \]  

(3.19)

As a result, for a fixed velocity \( V \), the dependency of \( P \) to length \( L \) is

\[ P \sim L^2. \]  

(3.20)

The contact radius, \( a \), can be expressed as [Hertz, 1881]

\[ a = \left( \frac{3nP}{4} (k_1 + k_2)R \right)^{\frac{1}{3}}. \]  

(3.21)

The dependency of \( a \) to length \( L \) is

\[ a \sim L. \]

Therefore, the average local stress \( \sim \frac{P}{\pi a^2} \), is independent of length scale.

The impact between the projectile and the flat structure results in a compressive stress on the flat structure during the impact period. Figure 3.4 shows a schematic plot of the stress distribution. The peak compressive \( q_0 \) can be expressed as [Zukas, 1984]

\[ q_0 = \left( \frac{3n}{2\pi R} \right) \left( \frac{5V^2}{4nM} \right)^{\frac{1}{4}}. \]  

(3.22)

In order to verify the lengthscale dependency of the impact duration, \( \delta t \), a finite element impact simulation was performed. As shown in Figure 3.5, a silicon ball collides with a thick steel flat plate with \( V = 10 \) m/s. Three length scales were investigated with ball diameter 4mm, 40cm, and 4m. The deformation history of point A of the flat structure is shown in Figure 3.6. The intervals for compressing, releasing, and rebounding are shown. From Figure 3.6, the duration of the compression phase is exactly proportional to the length scale of the structure and the projectile. Note that due to plastic deformation of the steel, the flat plate is permanently deformed. The maximum plastic deformation depth, \( \delta \), is also proportional to the length scale. The amount of \( \delta t \) and \( \delta \) are shown in Figure 3.6 for various length scales.
Although the elastic impact stress is independent of length scale, as discussed in section 3.1, brittle materials are stronger at small scales. As a result, the smaller scale structure would be expected to have better impact resistance.

The above analysis assumes that the colliding objects are elastic or elasto-plastic. For brittle materials, impact between structures is likely to result in fracture, which introduces an additional scale dependence. The impact of a projectile on a brittle material may result in the formation of one or more cracks [Zukas, 1984]. If these cracks are sufficiently long and exceed the critical length, for the applied stress level, the structure will fracture. If not, the residual strength is reduced due to the existence of larger flaws. This impact process is somewhat similar to the quasi-static indentation fracture which has been widely studied [Lawn, 1993] and this forms a useful starting point for the analysis of the effect of scale on fracture under impact loading.
Figure 3.6  Impact analysis result

Figure 3.7 shows a schematic of the indentation fracture testing process. A blunt indenter contacts the upper surface of a brittle specimen with exerted force \( P \). After indentation, the specimen is tested using the standard 4-pt bending test to measure the residual strength. The indentation process results in surface damage to the specimen. A larger indentation force induces larger damage. Therefore, the inert strength reduces as load increases. Figure 3.8 shows typical test results on soda-lime glass specimens.

As shown in Figure 3.8, Lawn [Lawn and Marshall, 1979] shows that the relationship between the post-indentation strength, \( \sigma_i \), and the indentation load, \( P \), can be expressed as

\[
\sigma_i = \left( \frac{K_{ic}}{\psi^3 \chi P} \right)^{\frac{1}{3}},
\]

where \( K_{ic} \) is the fracture toughness measured in a vacuum. \( \psi \) and \( \chi \) are the crack-geometry factor and the indentation residual-contact coefficient respectively. As a result, the post-indentation strength \( \sigma_i \sim P^{1/3} \).
To relate the impact and quasi-static blunt indentation process, the indentation force, $P_{im}$, is assumed to be equivalent to the impact force, $P_{im}$. $P_{im}$ can be expressed in the form of Eq.(3.15) and its scale dependency $L(P_{im}) = L^{2/3}$. As a result, the length scale dependence of the post-indentation strength can be expressed as

$$L(\sigma_I) = (1/L(P_{im}))^{2/3} = L^{-2/3}.$$  \hfill (3.24)

This shows that as the scale decreases, for a given geometry and velocity, the post-indentation strength increases significantly. For a factor of 100 reduction in length dimensions, the post-indentation strength would be expected to increase by a factor of 21.

Although pure geometric scaling on impact resistance favors the small scale, the material properties play an equally important role. Eq.(3.23) shows that the post-indentation strength is also very sensitive to fracture toughness. Single crystal silicon has an extremely low fracture toughness (0.8 -1.0 MPa√m), compared to polycrystalline ceramics (3 -6 MPa√m), composites (10 -30 MPa√m), and metals (30 -100 MPa√m). Thus some of the advantage of the small scale is negated by the constraints imposed by the material selection. Nevertheless, combining Eq.(3.23) and Eq.(3.24) suggests that the ratio of post impact strength for a macroscale ceramic engine structure ($L=0.1m$, $K_{IC} = 4$ MPa√m) to micro scale silicon ($L=1mm$, $K_{IC} = 0.9$ MPa√m) is approximately a factor of 3.

![Figure 3.7 Brittle material indentation fracture test](image)
Figure 3.8  Residual strength of material after indentation [Lawn, Wiederhorn, and Johnson, 1975]
3.4 Heat Transfer

3.4.1 Steady state temperature uniformity

The heat transfer for a body can be represented by the three heat transfer equations; conductive heat transfer per unit volume,

\[ \nabla \cdot (k \nabla T) + \dot{q}_v = \rho c \frac{\partial T}{\partial t}, \tag{3.25} \]

where \( k \) is the thermal conductivity of materials, \( \dot{q}_v \) is the thermal power generated inside the body, and \( c \) is the heat capacity of the material,

convective heat transfer:

\[ \dot{q} = h(T)A(T_s - T_\infty), \tag{3.26} \]

where \( h \) is the heat transfer coefficient, \( T_s \) is the surface temperature of a structure, and \( T_\infty \) is the temperature of the free stream, and

radiative heat transfer:

\[ \dot{q} = \varepsilon \sigma A(T_r^4 - T_\infty^4), \tag{3.27} \]

where \( \varepsilon \) is the emissivity, which is between 0 and 1. \( \sigma \) is the Stefan-Boltzmann constant and equal to \( 5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4 \), and \( T_\infty \) is the ambient temperature.

In most of structural applications, the radiative heat transfer is not important. In this section, discussion is restricted to conductive and convective heat transfer. From a structural point of view, the mechanical properties, especially strength at high temperature and the thermal transient response, which are related to thermal stress generation, are the most relevant. The effect of length scale on these issues is addressed.

Engine structures can be treated as a conductive-convective heat transfer system. A first order approach to calculate the structural temperature is the lumped-heat-capacity method [Chapman, 1984]. Assuming the temperature of the structure is uniform, the Laplacian term in Eq.(3.25) vanishes. For a structure with volume \( V \), Eq.(3.25) becomes

\[ \dot{q} = V \dot{q}_v = \rho c V \frac{\partial T}{\partial t}. \tag{3.28} \]
Putting (3.26) into (3.28), the equation becomes

$$hA(T - T_c) = \rho cV \frac{\partial T}{\partial t}, \quad (3.29)$$

with solutions of the form

$$\frac{T - T_c}{T_0 - T_c} = e^{-(hA/pcV)t).} \quad (3.30)$$

The time constant for the thermal transient is:

$$\tau = \frac{cpV}{hA}. \quad (3.31)$$

When the conductive thermal resistance is much smaller than the convective thermal resistance, the lumped-heat-capacity method is a reasonable estimation. That is,

$$\frac{L}{kA_c} \ll \frac{1}{hA_h} \quad (3.32)$$

or

$$\frac{hLA_h}{kA_c} = \frac{hs}{k} \ll 1 \quad (3.33)$$

where s is a characteristic dimension of the system. The dimensionless variable, $hs/k$, called the Biot number, is the ratio between conductive and convective thermal resistance. It represents the temperature uniformity of the solid structure. Biot numbers much smaller than 1 mean that the structural temperature is nearly uniform.

Both the thermal time constant and the Biot number are length scale dependent. For the same material subject to the same heat transfer condition, $\tau$ is proportional to $(V/A)$. That is, $\tau$ is proportional to the characteristic dimension. The larger the structure is, the longer the time to reach the steady state temperature. From Eq.(3.33), the Biot number is also proportional to the length scale. Larger Biot numbers imply a decrease in the uniformity of the structure's temperature.

A finite element case study is used to verify the above argument in a turbine structure. As shown in Figure 3.9, a radial turbine disk with blades is surrounded by fluid. The heat transfer boundary conditions are also shown in Figure 3.9. Three scales
were studied. The disc diameters are 4mm, 4cm, and 4 m. The aspect ratio is fixed during the scaling. Material and thermal boundary conditions are the same for the different length scales. The maximum temperature occurs at the blade tip and the minimum temperature is on the disk center of the bottom surface. The transient temperature responses of these turbines is shown in Figure 3.10. By Eq.(3.31), the larger the length scale, the larger the thermal time constant.

Figure 3.11 shows the variation of maximum and minimum temperature with the length scale. At a 4mm disk diameter, the difference in maximum and minimum structural temperature is about 30K. The non-uniformity increases as scale increases and the maximum temperature of the structure approaches the free stream fluid temperature.

Pure geometrical scaling shows that the likelihood of forming hot spots is less at small scales. The conductivities of structural ceramics and single crystal silicon are higher than titanium, and nickel and cobalt super alloys. These material considerations imply even lower Biot numbers. However, for macroscale gas turbine engines, various local cooling schemes have been developed for reducing temperatures at critical locations. This is achieved at a cost in efficiency to the cycle, but allows use of metallic materials to their temperature limits. It is worth noting that the Biot numbers associated with blade cooling are similar to those found in the microengine.

![Figure 3.9 Turbine FE model for length scale study](image_url)
3.4.2 Heat loss
The surface area to volume ratio increases as the scale decreases. For a gas turbine engine, the entire engine can be treated as a heat source. Part of the thermal power becomes mechanical power and the rest of the heat is transferred into the surrounding air via convection from free surfaces [Kerrebrock, 1992]. As shown in section 3.4.1, the surface heat transfer becomes more important at small scales.

As a result, the heat loss from the micro combustor is much greater than from a conventional combustor. A 50% heat loss has been measured in experiments [Mehra and Waitz, 1998]. This larger heat loss has two major implications; first, the overall efficiency decreases, although this can be partially recovered if a regenerative scheme is used, second, the high heat loss implies that the structural temperature is cooler at the microscale. It is probable that no additional cooling schemes will be necessary to reduce the wall temperature to below the material limits.

3.4.3 Thermal shock

In many applications such as gas turbine engines and thermal barrier coatings, ceramics are subjected to severe transient thermal conditions. Although ceramics can generally withstand steady high temperatures in service, their low toughness, high stiffness, and in some cases, relatively low thermal conductivity renders them highly susceptible to failure under rapid heating or cooling. This is often termed "Thermal Shock".

Poor thermal shock resistance has been a barrier for introducing ceramic materials in macroscopic gas turbine engine. Differential thermal expansion between the interior and exterior of a component, due to a transient thermal gradient, results in local stresses, which are sufficient to cause fast fracture. Thermal shock is controlled by the heat transfer at the surface and the heat conduction through the component as well as the thermal expansion coefficient and material strength/toughness.

The most basic calculation assumes that when the thermal stress is equal to the material tensile strength, the material will fracture. Therefore, the maximum allowable temperature drop $\Delta T_{\text{max}}$, called thermal shock resistance, $R$, can be expressed as [Hasselman, 1970]
\[ \Delta T_{\text{max}} = \frac{\sigma_s (1 - \nu)}{\alpha E}. \] (3.34)

Eq. (3.34) is a lower bound on the thermal shock resistance corresponding to a fully constrained ceramic layer or near infinite surface heat transfer. In most applications, structures are not fully constrained and are subject to finite surface heat transfer. If the variation of temperature is very slow, there will be no thermal stress. The dependency of the thermal shock resistance on heat transfer, therefore, needs to be included. As previously discussed, this is controlled by the Biot number, \( \text{Bi} \).

Thus Eq. (3.34) can be modified to become [Wang and Singh, 1995]

\[ \Delta T_{\text{max}} = C_1 \frac{\sigma_s (1 - \nu)}{\alpha E} f(\text{Bi}), \] (3.35)

where \( C_1 \) is a shape factor in the range of 1 to 5 (e.g., for a sphere, \( C_1 = 2.5 \)).

\( f(\text{Bi}) \) [Wang and Singh, 1995] can be expressed as \( C_x + C_y / \text{Bi} \). For a spherical object, \( C_x = 1, C_y = 2 \) [Wang and Singh, 1995]. Becher and performed quench tests using bar specimens with a square cross-section Warwick [Becher and Warwick, 1993]. The schematic geometry is shown in Figure 3.12. They fitted numerical results for \( f(\text{Bi}) \) as

\[ f(\text{Bi}) = 1.5 + 4.67 / \text{Bi} - 0.5 \exp(-51 / \text{Bi}) \] (3.36)

with \( C_1 = 1 \).

![Figure 3.12 Ceramic specimen for quenching experiment [Becher and Warwick, 1993]](image)

Figure 3.12 shows the critical temperature change to cause fracture of a square bar as a function of bar thickness. Predicted curves are shown for SiC, Al\(_2\)O\(_3\), and Si\(_3\)N\(_4\) with a constant surface heat transfer coefficient of 10\(^4\) W/m\(^2\)K and a strength of 550 MPa. Comparison of the predicted critical temperature drop for a macroscale component,
with a characteristic dimension of ~0.1m, with that for a mesoscale component, with an equivalent thickness of ~1mm, reveals nearly a factor of two increase in the critical temperature drop. These curves have been constructed based on materials with the same strength at the meso- and macroscale. When the increase in strength with reduction in scale is considered, the thermal shock concern is essentially eliminated for relatively high thermal conductivity materials such as Si and SiC. In addition, materials such as alumina, which cannot be considered at the macroscale are potentially viable at the mesoscale.

![Graph showing critical temperature change versus characteristic length for different materials](image)

**Figure 3.13** Dependence of critical temperature change to cause fracture via thermal shock for SiC, Al₂O₃, and Si₃N₄

In order to understand the effect of length scale on thermal stress history during the quenching process, a finite element simulation has been performed. A circular plate of silicon is heated up to 1300 K with 300 K water quenching (h = 5000 W/m²K) on the upper surface. The critical point is the center of the upper surface. Four case studies have been performed by changing the plate thickness while keeping the aspect ratio the same.
Figure 3.14 shows the time-dependent maximum stress at the critical point. For a thickness of 0.5 mm, which is close to the microengine dimension, the peak thermal stress is very small, only about 7 MPa. As the dimension is increased, the peak thermal stress also increases. For 5 cm thick bulk silicon, the peak thermal stress can be as high as 170 MPa. Note that the time when the peak stress occurred also increased with increasing length scale due to the increasing thermal time constant.

![Figure 3.14: Finite element quenching simulation result](image)
3.5 Oxidation

Key factors which affect the durability of engine structures in service are oxidation and corrosion. Oxidation is a process in which materials react with oxygen to form oxide. From a microscopic point of view, oxidation is controlled by two processes; chemical reaction and diffusion. For example, during the oxidation of silicon, the fresh reactant must pass through a silica layer before any reaction can occur. Two types of behavior are usually observed at high temperature; linear oxidation, with

\[ \Delta m = k_L t, \]

and parabolic oxidation, with

\[ \Delta m^2 = k_p t, \]

where \( \Delta m \) is the weight gain during the oxidation and \( t \) is time. \( K_L \) and \( K_p \) are linear and parabolic kinetic rate constants, respectively. Linear oxidation generally applies for short term behavior. For long term oxidation, the behavior is approximated as parabolic oxidation.

Oxidation rates follow Arrhenius Law, that is, the kinetic rate constant increase exponentially with temperature \( T \),

\[ K_{L,P} \propto e^{-Q_{L,P}/RT}, \]

where \( R \) is the universal gas constant.

The other major concern in material oxidation is the stability of oxidation. For stable oxidation, the products of the chemical reaction form a stable layer which prevents (or reduces) further reaction. In this case oxidation eventually stops. On the other hand, if the reaction products evaporate and can not prevent further oxidation, the oxidation rate can not be reduced. This is called active oxidation.

The structural materials for the microengine are initially silicon and subsequently silicon carbide. Therefore, it is essential to describe the oxidation mechanism for these materials. For silicon, the reactions governing the formation of silica (SiO\(_2\)) are given as,

\[ \text{Si}_{(s)} + \text{O}_{2(g)} \rightarrow \text{SiO}_{2(s)} \quad \text{Dry Oxidation}, \]

\[ \text{Si}_{(s)} + \text{H}_2\text{O}_{(g)} \rightarrow \text{SiO}_{2(s)} + 2\text{H}_{2(g)} \quad \text{Wet Oxidation}. \]
A simplified model based on the diffusion equation by Deal and Grove [Deal and Grove, 1965] is the most popular model to deal with thermal oxidation in silicon. It is called the "Linear-Parabolic Model". In this model, the relationship between the oxide thickness $x_o$ and oxidation time $t$ can be expressed as

$$x_o^2 + A x_o = B t,$$

where $A$ and $B$ are the linear and parabolic rate constants. They are functions of temperature, pressure, and oxidation type. Also, from chemistry, silicon with unit thickness after being fully oxidized will be converted to silica with 1.44 unit thickness, i.e., a 44% increase in thickness. Figure 3.15 shows the oxide growth under various conditions by the Deal and Grove model.

![Graph showing oxide thickness vs. oxidation time for different conditions](image)

**Figure 3.15 Oxidation of (100) silicon**

For silicon carbide, depending on reaction temperature and species, several possible oxidation reactions may occur. In most situations, a solid layer of silica will grow on the silicon carbide surface to prevent further oxidation. However, in the presence of
hydrogen and water vapor, such as in the microengine combustor, the final products of oxidation are not silica but a gaseous phase oxide or hydroxide. The volatilization of these materials leaves a fresh silicon carbide surface for further oxidation. [Kim and Reedy, 1987], [Jacobson 1993], [Opila and Hann,Jr., 1996].

Passive oxidation:

\[ \text{SiC}_{(s)} + \frac{3}{2} \text{O}_2(g) \rightarrow \text{SiO}_2(s) + \text{CO}_2(g) \]

\[ \text{SiC}_{(s)} + 3\text{H}_2\text{O}(g) \rightarrow \text{SiO}_2(s) + 3\text{H}_2(g) + \text{CO}_2(g) \]

Active oxidation:

\[ \text{SiC}_{(s)} + 2\text{H}_2\text{O}(g) \rightarrow \text{SiO}(g) + \text{CO}_2(g) + 2\text{H}_2(g) \]

\[ \text{SiO}_2(s) + \text{H}_2\text{O}(g) \rightarrow \text{SiO(OH)}_2(g) \]

The oxidation type is independent of structural scale but dependent on the oxidation environment. If the chemical species distribution and working temperature of the micro combustor are similar to the conventional macro scale combustor, there will be no difference in oxidation type. For passive oxidation of silicon carbide, the concern is the same as that of silicon, i.e., the oxide thickness. In active oxidation, from the chemical reaction, one can see a gaseous type of silicon oxide and/or silicon hydroxide and silicon oxyhydroxide. This represents a net loss of mass. The structures become weaker. Note that the oxidation rate is independent of scale but the surface area per unit volume increases when scale goes down. This means the mass reduction may be a serious concern at the micro scale. However, it should also be noted that the low Biot number referred as in Section 3.4.3 implies that for equal free stream fluid temperatures in the micro and macro scale, the surface temperature of structures will be less in the microscale. This reflects the proportionally increased contribution of surface heat transfer to the overall thermal resistance. This factor, combined with the Arrhenius dependence of oxidation rates, may lessen the adverse scaling of oxidation on the overall system performance.

Material oxides are usually highly brittle, which is very undesirable from a structural perspective. In addition, the mechanical properties of oxide are usually different from
the original material. Interfacial mechanical problems such as cracking may occur between the material and oxide interface.

The impact of oxidation on the scaled structures comes from two facts. First, the total amount of reaction is proportional to the surface area. For micro scale structures, the total surface area/volume is increased. This means the amount of oxide formation per unit volume increases. Secondly, the reaction rate is independent of the size of the materials, i.e., under same controlling parameters, the oxide thickness will be the same whatever the micro or macro structures. Since in Eq.(3.37), $x_0 \propto \sqrt{t}$ when $t \to \infty$, a thermal oxidation process carried out for several hours under 1200 K can usually achieve less than 5 μm oxide thickness. This amount is negligible for macro devices but it is a serious concern for critical dimensions of the microengine such as blade trailing edge width and bearing gap.
3.6 Summary and Conclusion

This chapter has discussed the effects of material and structural length scales on ceramic turbomachinery design. There are several direct effects of scale, notably on the strength of the brittle structural materials, heat transfer behaviors, and oxidation, arise from the overall scaling of the system, and in particular, the constraints imposed by the fabrication routes. This suggests that a direct comparison of a microengine structural design with that embodied in microscale engines is largely irrelevant. However, it is important to understand the effect of scale, particular in as far as they affect the applications of material test data and structural design methodologies. This understanding will underpin the detailed analyses presented in subsequent chapters.
3.7 Reference


Cook, R. D. and W. C. Young [1985]: Advanced Mechanics of Materials,


CHAPTER 4. ROOM TEMPERATURE MATERIAL CHARACTERIZATION

4.1 Introduction

The concept of the micro turbogenerator, as well as that of other mesoscopic machines such as the MIT motor compressor, is predicated on the ability to create very high speed moving parts. The high operating speeds generate high tensile stresses. In addition, high stresses can result from the thermal gradients and constrained thermal expansion that is inevitable in mesoscopic heat engines. These requirements demand that the materials for mesoscopic machines have a high strength and, furthermore, for rotating elements, a high strength to density ratio. The materials available for consideration are predominantly semiconductor materials (principally silicon), although refractory ceramics such as silicon carbide and silicon nitride may also become available. Since these are brittle materials, their fracture strength critically depends on the processing induced flaw population [Rice, 1974, 1981]. Furthermore, in common with all brittle materials, the statistics of the flaw population imply that the strength of the material is dependent on the volume or surface area under stress [Weibull 1939]. These considerations require that strength data be obtained from a statistically significant number of specimens produced by the same fabrication route and with a similar volume or area under stress as would be encountered in the mesoscopic machine. In this chapter we describe the use of microfabricated biaxial flexure specimens for this purpose.

Observations of deep reactive ion etched silicon parts show that the surface quality varies significantly with the process conditions used to etch the samples. Figure 4.1 shows a magnified view of the root of a turbine hub on the turbine rotor shown in Figure 1.2. The horizontal surface of the rotor disk is relatively smooth, the curved surface of the hub is covered by vertical striations and the transition region at the root is more rough than the other two surfaces. Similar features exist at the roots of turbine blades. The hub and blade roots have stress concentrations; therefore the reduced local
surface quality and associated reduction in strength have potentially serious
consequences for the design and operation of the device. In order to achieve a
successful design it is necessary to understand the factors affecting the local strength
and also to develop techniques to improve the strength by controlling the etched
surfaces at critical locations. To characterize the local variation in strength and to
explore methods for strength control we have developed the radiused hub flexure
specimen, which is described in section 4.5.

In addition to the requirements for high strength at room temperature, the feasibility
of an all silicon engine is dependent on retaining sufficient strength up to the desired
operating temperatures. A typical operating temperature of the rotor is 900 K and 1100
K in the static structure. At the higher temperatures above the brittle to ductile
transition temperature (~ 850 K) the strength will be limited by plasticity and creep
considerations. However, at intermediate temperatures, the behavior will be elastic-
brittle and thus it is important to find the effect of temperatures in the range of 300 - 800
K on the fracture toughness of silicon. The plastic and creep deformation of silicon at
temperatures above 850 K will be discussed in Chapter 7.

This chapter is organized so as to address these issues. Section 4.2 presents the
mechanical properties of silicon. Section 4.3 reviews previous research in silicon
fracture strength testing. Section 4.4 describes room temperature biaxial flexure
strength testing. Section 4.5 presents the radius hub flexure specimen testing and
results for strength recovery via isotropic etching. Section 4.6 discusses the procedure
and results for the fracture strength testing at intermediate temperatures. Section 4.7
summarized these issues.
Figure 4.1 SEM micrograph of the root of a hub of a turbine rotor.
4.2 Properties of Silicon

The basic structural material for MEMS and microturbomachinery is semiconductor grade silicon. The silicon turbogenerator is principally fabricated by Deep Reactive Ion Etching (DRIE) using a Surface Technology Systems™ (STS) inductively coupled plasma etcher. The equipment is characterized by a high density plasma and achieves an etch rate of ~ 3 μm/min. Depths of hundreds of microns can be obtained in one or two hours with highly anisotropic behavior [Ayon, Lin, and Schmidt, 1997], [Ayon et. al, 1998]. It is an enabling fabrication process for the micro gas turbine engine.

Silicon has a cubic (diamond) crystal structure and can be fabricated with extremely low lattice defect densities, and with essentially no mechanically significant defects in the bulk. Semiconductor grade silicon is fabricated by the Czochralski (CZ) growth or the floating zone (FZ) growth method with light p+ or n- doping wafers with <100> and <111> crystal orientations [Wolf and Tauber, 1986]. In this work only n- doped <100> wafers were investigated.

The elastic properties of <100> silicon have been calculated [Brantley, 1973]. The three components of the elastic stiffness tensor necessary to characterize cubic silicon are $C_{11} = 165.7$ GPa, $C_{12} = 63.9$ GPa, and $C_{13} = 79.6$ GPa. In terms of the engineering elastic constants the Young’s modulus $E$ varies from 169 GPa in the <110> direction to 131 GPa in the <100> direction. The Poisson’s ratio, $\nu$, also varies with orientation. However, the bi-axial modulus $E/(1-\nu)$ is an invariant. Table 4.1 shows the elastic properties of cubic silicon.

At temperatures below about 700 K silicon is a brittle, elastic material with extremely high inert strength [Kelly, 1986]. Chen and Leipold [Chen and Leipold, 1980] determined the fracture toughness, $K_{icr}$, of single crystal silicon for the principal cleavage planes. The fracture toughness $K_{ic}$ varies with orientation, but the difference is small and in all cases, $K_{ic}$ for silicon is between 0.8 and 1 MPa$\sqrt{m}$. The low toughness implies a strong sensitivity of the strength to processing and service induced flaws or cracks [Griffith, 1921], nonetheless, several studies listed in the following sections have shown that it is possible to obtain very high strengths [Hu, 1982] [Wilson and Beck,

Linear elastic fracture mechanics implies that to achieve a strength of 1 GPa, the maximum flaw size must be of the order of 0.3μm based on Eq.(3.2). The high strengths reported in the literature reflect that microfabrication processes are capable of achieving very good surface finishes, although there is considerable variation between processes. This observation reinforces the need to have adequate process control to achieve the desired surface quality, and furthermore to characterize the mechanical strength as a function of processing route.

Hirsch performed pre-cracked four-point bending test on FZ silicon [Hirsch et. al, 1989]. As shown in Figure 4.2, below 830 K, the failure stress is small and nearly constant. This indicates that failure is by brittle fracture. As the temperature increases, the failure stress increases suddenly and then at still higher temperature decreases smoothly. The sudden increase in failure stress indicates the failure mode has changed from brittle fracture to one of ductile rupture. The decrease in failure stress indicates the yield strength decreases with temperature.
Table 4.1  The room temperature elastic properties of cubic silicon

<table>
<thead>
<tr>
<th>$C_{11}$</th>
<th>$C_{12}$</th>
<th>$C_{44}$</th>
<th>$S_{11}$</th>
<th>$S_{12}$</th>
<th>$S_{44}$</th>
<th>$S^{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>165.7</td>
<td>63.9</td>
<td>79.56</td>
<td>0.00768</td>
<td>-0.00214</td>
<td>0.0126</td>
<td>0.00352</td>
</tr>
</tbody>
</table>

Note: 1. Unit of stiffness $C_{ii} = \text{GPa}$. Unit of compliance $S_{ij} = \text{GPa}^{-1}$

2. $S^c = S_{11} - S_{12} - 0.5S_{44}$

3. $S_{ij}$s are used to calculate equivalent $E$ and $\nu$

Minimum and Maximum Young’s Modulus in selected crystal directions

<table>
<thead>
<tr>
<th></th>
<th>[111]</th>
<th>[100]</th>
<th>[110]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{max}}$ (GPa)</td>
<td>168.9 (invariant)</td>
<td>168.9 &lt;011&gt;</td>
<td>187.5 &lt;111&gt;</td>
</tr>
<tr>
<td>$E_{\text{min}}$ (Gpa)</td>
<td>168.9 (invariant)</td>
<td>130.2 &lt;001&gt;</td>
<td>130.2 &lt;001&gt;</td>
</tr>
</tbody>
</table>

Figure 4.2  BDTT testing result [Hirsch et. al, 1989]
4.3 Previous Strength Testing

Without lattice defects and surface flaws, silicon is a strong covalent bonded solid with a theoretical strength in excess of 10 Gpa. Table 4.2 shows the theoretical strength for three crystal orientations [Kelly, 1986]. However, in reality, the presence of lattice defects generated during wafer manufacture and surface flaws generated by surface machining processes reduce the strength to relatively low values. For example, the strength of a die-sawed silicon beam can be lower than 100 MPa [Hu, 1982]. As shown in Figure 4.3, a silicon wafer after indentation with a sharp indenter at 1 N load have a strength of less 60 MPa [Chen, 1996]. For other milder surface micromachining processes, Hu examined the effect of ion-implantation damage in silicon and obtained an average flexural strength of 2.8 GPa for <100> silicon wafers [Hu, 1982]. Wilson and Beck examined the strength of KOH etched silicon cantilever beams [Wilson and Beck, 1996]. They found considerable variation in strength, with a significant dependence on orientation. Specimens orientated such that fracture occurred on <110> planes had an average strength of 3 GPa, while those which fractured on <111> planes had strengths of less than 1 GPa.

Table 4.2 Theoretical strength of silicon based on bond strength and densities [Kelly, 1986]

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Young's Modulus E</th>
<th>Theoretical Strength $\sigma_t$</th>
<th>$\sigma_t / E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;106&gt;</td>
<td>130 GPa</td>
<td>22.4 GPa</td>
<td>0.172</td>
</tr>
<tr>
<td>&lt;110&gt;</td>
<td>170 GPa</td>
<td>15.2 GPa</td>
<td>0.089</td>
</tr>
<tr>
<td>&lt;111&gt;</td>
<td>187 GPa</td>
<td>18.8 GPa</td>
<td>0.101</td>
</tr>
</tbody>
</table>
Figure 4.3 Residual strength of silicon [Chen, 1996]
4.4 Bi-axial Fracture Strength Testing

4.4.1 Biaxial Flexure Specimen

Biaxial flexure tests were used to obtain strength data for silicon as a function of fabrication route, and particularly the final surface finish. The specimen consists of a square plate which is simply supported over a circular hole. The specimen is loaded centrally to generate an axisymmetric biaxial stress distribution. The surface of interest is placed facing away from the loading point so as subject it to an in-plane tensile bending stress. The advantage of this test specimen is that incidental edge damage does not contribute to the fracture strength. The specimen is similar to that used for testing of ceramics [ASTM F394-78, F417-78] at the macroscale as well as that used by Hu in earlier work for testing silicon specimens [Hu, 1982].

To fabricate the biaxial specimens, semiconductor grade <100> silicon wafers were prepared with a range of five surface conditions. They are summarized in Table 4.3. The mechanically polished and the chemically polished wafers were as-received from the supplier. The KOH etched and deep reactive ion etched specimens were prepared using standard procedures (Please refer to Appendix A.2 for detail.) Surface roughness was measured to provide a comparison between surface finishes. This was performed using a DEKTAK 3 profilometer with a 5 μm radius stylus. Figure 4.4 shows the relative surface roughness of these specimens. Wafers were reduced in thickness to 230 - 280 μm except for the mechanically ground type A specimens, which are 500 μm thick. Wafers were cut into 10 mm x 10 mm square specimens using a diamond dicing saw.
Figure 4.4 Surface roughness of the biaxial specimens fabricated using five surface treatments.

4.4.2 Test method

Figure 4.5 shows a schematic of the apparatus used for strength testing both biaxial flexure and radiused hub flexure specimens. Figure 4.6 indicates the test procedures. Specimens were placed over a circular hole (7 mm in diameter) in a machined aluminum block. The block sat on a 150 N capacity load cell. The fixture and load cell were placed under a modified microhardness indenter. The microscope and X-Y stage of the indenter permitted accurate positioning and centering of the specimen. The indenter head was a steel ball bearing, 1.5 mm in diameter. Tests were performed in displacement control so as to cause fracture in approximately 30 s. The peak load was recorded via a digital data acquisition system running LABVIEW software. (Refer to Appendix A.3). In all cases the material behavior was observed to be elastic-brittle. After failure the overall fracture morphology was noted, and in some cases fractured specimens were examined using scanning electron microscopy (SEM).
Figure 4.5  Schematic of the strength test fixture

Figure 4.6  Schematic of the strength testing procedure
4.4.3 Specimen Analysis

Stress analysis of the specimens was carried out to ensure accurate data reduction and to assess possible sources of error inherent to testing at small scale. The general purpose finite element code, ABAQUS/Standard [ABAQUS,1996] was used to model the specimens. A two-dimensional axisymmetric analysis was used primarily, with some three dimensional analysis for substantiation of the 2-D results. Typical models consisted of 1000 and 6000 second order isoparametric elements for the 2-D and 3-D cases respectively. Calculations were performed on a dedicated DEC Alpha machine. The CPU time to generate a data point for an elastically deforming biaxial flexure specimen was 5-10 minutes for the 2-D case and 7 hours for the 3-D case.

Figure 4.7 shows the finite element mesh used to model the biaxial flexure specimen. The indenter was modeled as a hemisphere applied to the center of the specimen. The edge of the circular hole was modeled as a simply supported boundary condition. Hertzian contact between the punch and the specimen was assumed, with provisions for contact friction and plastic deformation of the indenter. For the axisymmetric analysis, the specimen material was modeled as isotropic with an equivalent Young’s modulus of 170 GPa and a Poisson’s ratio of 0.1, which are the approximated isotropic material constant for polycrystalline silicon. For the 3-D analysis the cubic elastic constants of silicon were used. The 3-D FE input file is shown in Appendix A.1.

Comparison of the elastically isotropic 2-D and cubic 3-D results show that the difference in the peak stress for a given applied load is less than 3%. Furthermore, the variation of stress at any given location is also within 3% for the two cases. This result justifies the use of axisymmetric, isotropic behavior for most of the subsequent analyses.

The corner region of the square specimen, which the axisymmetric model was not included, was shown to have a negligible effect on the magnitude and shape of the highest contours of the stress distribution. Thus, the axisymmetric FE model results are accurate when used to describe the behavior of the square silicon specimens used in the experiments. This simplification is reasonable since material at this region is essentially unloaded and only experiences rigid body displacements. Use of square specimens significantly simplifies the specimen manufacture.
The most significant potential source of error is the thickness variation present in the source wafers as a result of the etching process. Specimen thickness was measured by micrometer to be within an accuracy of ± 5μm. For the nominal thickness of 270 μm, this measurement uncertainty leads to a 4% error in stress level.

The effect of inelastic deformation and friction at the contact between the indenter and the test specimen was investigated. The indenter was modeled as an elastic-perfect plastic material. The uniaxial yield stress of the indenter was set at 0.8 GPa or 2 GPa, which are reasonable upper and lower bounds for ball bearing steel. Deformation was assumed to occur according to the von Mises yield criterion and its associated flow rule. The coefficient of friction between the ball and the Si specimens was set at 0.3. Figure 4.8 shows the FE results for the relationship between the applied load and maximum tensile stress in the specimen, for the elastic case and elastic-plastic contact with friction (μ=0.3). In addition, the approximate analytical solution obtained by Hu [Hu, 1982] is shown for comparison. There is a 10% decrease in the maximum stress due to the effect of plasticity. However, the specimen behavior has little dependence on the indenter yield strength varying between 0.8 and 2 GPa. A yield strength of 1.5 GPa was assumed for subsequent calculations. In addition, friction has little or no effect on the load-stress relationship. The stresses are in all cases somewhat lower than those predicted by Hu, who assumed, following Timoshenko, a parabolic stress distribution in the contact region [Hu, 1982]. Figure 4.8 serves as a calibration curve for reducing the experimental load data to flexural strengths.

One of the increased challenges to conducting mechanical testing at these scales is to minimize the effect of experimental positioning error. A microscope was used to ensure accurate placement of the indenter load. Finite element analysis was used to evaluate the effect of indenter misalignment on the error in the calculated maximum stress. Figure 4.9 shows the results of this analysis for an elastic contact. A positioning error of 50 μm, result in an error in the maximum stress of ~ 2%.
Figure 4.7  FE mesh of a biaxial flexure specimen (a), 2-D principal stress contours (b), and 3-D principal stress contours (c).
Figure 4.8  The relationship between applied load and specimen strength.

Figure 4.9  Effect of indenter positioning error on the calculated strength. Typical errors are estimated to be less than 50 μm, providing 98% accuracy.
4.4.4 Experimental result

As is conventional for brittle materials, the strength data is presented in the form of Weibull plots [Nemeth et. al, 1990], in which a distribution of specimen strengths obeying the Weibull probability density function will fall on a straight line. Figure 4.10 shows data for the two types of mechanically polished specimens, the KOH etched specimens, and the STS etched specimens. Table 4.3 summarizes these test results. Different data included results in different Weibull modulus. In Figure 4.10, if the circled data are included, the Weibull modulus in Table 4.3 will have the minimum value. If the circled data are treated as the result of external damage rather than the DRIE process, it should be excluded. The overall data without circled data will result in maximum Weibull modulus. The Weibull reference strength, $\sigma_r$, correlates to the measured surface roughness, ranging from 1.2 GPa for the mechanically polished material with a surface roughness of 3 µm to 4.6 GPa for the DRIE specimens with a surface roughness of 0.3 µm. The chemically polished silicon specimens had an even lower surface roughness (~0.1µm) than the other four surface conditions listed in Table 1, and exhibited a strength outside the measurement range at the thickness tested. The Weibull modulus also varies with the processing route, with values in the range 3 - 12 which are lower than those commonly found for polycrystalline ceramics produced by conventional processing routes.

The obvious deviation of the experimental data in Figure 4.10 from a straight line implies that a Weibull distribution is not a particularly good description of the data. In addition it is apparent that in several of the groups of specimens tested there were one or two specimens with anomalously low strengths. Data for polycrystalline ceramics is typically better represented by a Weibull distribution. It is likely that the relatively poor fit to a Weibull distribution results from there being more than one source of strength controlling flaws in the specimens. For instance flaws may be introduced by etching and also by handling and insertion in the testing apparatus. The lowest strength specimens reflect the extreme sensitivity to such flaws.
(a) $\text{Estimated } m = 2.7$
$\sigma_0 = 1.2 \text{ GPa}$

(b) $\text{Estimated } m = 3.4 - 4.2$
$\sigma_0 = 2.3 \text{ GPa}$
Figure 4.10  Weibull plots of the biaxial flexure data: (a) mechanically ground A), (b) mechanically ground (B), (c) KOH etched, and (d) DRIE silicon.
### Table 4.3: Strength characteristics of silicon with different surface conditions

<table>
<thead>
<tr>
<th></th>
<th>Mechanically ground (a)</th>
<th>Mechanically ground (b)</th>
<th>KOH Etched Silicon (c)</th>
<th>STS DRIE Silicon (d)</th>
<th>Chemically Polished</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>19</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Specimen thickness</td>
<td>500 μm</td>
<td>280 μm</td>
<td>280 μm</td>
<td>230 μm</td>
<td>280 μm</td>
</tr>
<tr>
<td>P-P Surface roughness</td>
<td>~ 3 μm</td>
<td>~ 1 μm</td>
<td>~ 0.3 μm</td>
<td>~ 0.3 μm</td>
<td>~ 0.1 μm</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>1.2 GPa</td>
<td>2.2 GPa</td>
<td>3.5 GPa</td>
<td>4.6 GPa</td>
<td>&gt; 4 GPa</td>
</tr>
<tr>
<td>Weibull modulus m</td>
<td>2.7</td>
<td>3.4-4.2</td>
<td>7.2-12</td>
<td>3.3</td>
<td>?</td>
</tr>
</tbody>
</table>
4.5 RHFS Fracture Strength Testing and Strength Recovery

4.5.1 Radiused Hub Flexure Specimen

In order to assess the effect of the degraded surface quality at horizontal-vertical transitions, a novel test specimen was introduced: the radiused hub flexure specimen (RHFS). As shown in Figure 4.11, the specimen is etched with a central raised hub. The load is applied centrally to the flat surface of the specimen, in the same apparatus as the biaxial flexure specimen, so as to produce an axisymmetric triaxial stress distribution, with a stress concentration at the radiused transition between the vertical surface of the hub wall and the horizontal surface of the specimen. The specimen dimensions are chosen so as to replicate the fabrication conditions likely to be encountered in the target application. It is also convenient to choose dimensions which produce fracture in the load range which can be accurately measured using the available apparatus. Referring to Figure 4.11, the following values were used: \( R = 2.5 \text{ mm}, \ r = 1 \text{ mm}, \ h_1 = 0.5 \text{ mm}, \ h_2 = 0.15 \text{ mm}, \ L = 10 \text{ mm}. \)

Radiused hub flexure specimens were produced from double polished silicon (<100>) wafers in the following manner: both surfaces were coated with photoresist AZ4620 spun at a speed of 4000 rpm to achieve a final thickness of 5.3 \( \mu \text{m} \). The topography was then transferred by exposing one surface in a Karl Suss aligner. The coating on the remaining surface was required to preserve the integrity of the wafer surface. After developing, the photoresist was used as a masking material during etching of the silicon substrate. The same etch conditions as those used during the microfabrication of the micro gas turbine generators was applied. The etch rate was measured at 3.4 \( \mu \text{m/min} \) with highly anisotropic profiles and selectivities of 80:1 (silicon to photoresist). After etching, the wafers were cut into 10mm \( \times \) 10mm square specimens using a diamond saw. In addition, some specimens were treated with wet or dry isotropic etches after the initial deep etch step to attempt to recover strength by reducing the surface roughness at the fillet radius. The wet etchant was composed of 5% HF, 55% HNO\(_3\), and 40% DI water. The etch rate was 1.8 \( \mu \text{m/min at room} \)
temperature. The dry isotropic etching was performed in an SF₆ plasma for 20 s with a measured etch rate of 8.3 μm/min. Both etches removed approximately 2 μm of material from the surface of the specimens.

![Figure 4.11: Geometry of the radiused hub flexure specimens (not to scale).](image)

4.5.2 Specimen analysis

Axisymmetric finite element analysis was used to calibrate the specimen. Figure 4.12 shows the FE mesh and the stress contours of this specimen. SEM observations revealed that deep reactive ion etching consistently produced a 10-12 μm fillet radius at the horizontal-vertical transition at the base of the specimen hub as well as the blade roots. A value of 11 μm was chosen for the FE modeling. The maximum stress at the stress concentration represents the local material strength. In a similar manner to that used for the biaxial flexure specimen, FE analysis was conducted to obtain the relationship between applied load and fracture strength.
Figure 4.12  FE mesh and stress contour of the radiused hub flexure specimen.
4.5.3 Experimental result

Figure 4.13 is a Weibull plot of the data from DRIE radiused hub flexure specimens, with strengths calculated assuming an 11 μm fillet radius. The Weibull reference strength, $\sigma_0$, of 1.50 GPa is 33% of that measured for DRIE biaxial flexure specimens. The dramatic reduction in strength reflects the poor etched surface in the region of the fillet radius.

Figure 4.14 shows a low magnification view of a fractured specimen. It is apparent that cracks propagate preferentially on the [111] planes, which are the lowest fracture energy cleavage planes of silicon. Figure 4.15 is a higher magnification view of the transition region in a fractured specimen. The increased surface roughness is clearly visible, and is comparable to that shown in Figure 4.1. The crack path follows a tangent to the hub root. This was evident at one or more points in all specimens tested, and it is reasonable to assume that this is the point at which fracture initiated.

The specimens in which an isotropic etch was used after the initial DRIE step showed the desired increase in strength. Figure 4.16 shows the Weibull plots for the isotropic etched specimens. The reference strengths were increased to 3.1 GPa and 4.0 GPa for wet and dry isotropic etching respectively. The increase in specimen strength further illustrates the importance of the surface quality in determining strength, and provides a useful tool for ensuring the required structural performance. Table 4.4 summarizes the radiused hub flexure specimen data.
Figure 4.13  Weibull plot of DRIE radiused hub flexure data.
Figure 4.14  SEM micrograph of a fractured radiused hub flexure specimen
Figure 4.15  SEM micrograph of a fractured radiused hub flexure specimen.
Figure 4.16  Weibull plot of the radiused hub flexure specimen data after a strength recovering isotropic etch step: (a) wet etch, and (b) dry SF₆ etch. Highest Weibull modulus in Table 4.4 will be reached if circled data are excluded.
<table>
<thead>
<tr>
<th></th>
<th>STS RHFS</th>
<th>STS RHFS + wet etch</th>
<th>STS RHFS + SF₆ dry etch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen size</td>
<td>35</td>
<td>31</td>
<td>28</td>
</tr>
<tr>
<td>Polishing thickness</td>
<td>NA</td>
<td>1.8 μm</td>
<td>2.7μm</td>
</tr>
<tr>
<td>σ₀</td>
<td>1.5 GPa</td>
<td>2.9 GPa</td>
<td>4 GPa</td>
</tr>
<tr>
<td>m</td>
<td>9.04</td>
<td>4.3 (including the circled data in Fig. 4.16a)</td>
<td>3.3 (including the circled data in Fig. 4.16b)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.3 (excluding the circled data in Fig. 4.16a)</td>
<td>8.8 (excluding the circled data in Fig. 4.16b)</td>
</tr>
</tbody>
</table>
4.6 Applicability of the Room Temperature Test Data

Although the fracture strength tests are only conducted at room temperature, it is possible to make estimates for the behavior at elevated temperatures. Since it is known that silicon is a brittle-elastic material at temperature below BDTT, and the flaw size is unlikely to be altered by increasing temperature, it follows that the only factor which may result in a temperature dependence of strength is the fracture toughness changes.

The fracture toughness of single crystal silicon has been tested at different temperatures by Rybicki [Rybicki and Priouz, 1988] using an indentation strength test. The results are shown in Figure 4.17. The fracture toughness, inferred from the post indentation strength, is approximately a constant for temperatures below 870 K. Above this temperature, the fracture toughness increases rapidly as dislocation motion permits deformation of the silicon at the crack tip.

From Figure 4.17, the conclusion can be drawn that the room temperature fracture test data can be applied for all temperature below 500 °C (~800 K). This is important, because for the all-silicon cooled engine, although the turbine and combustor structures may be operated above at higher temperatures, the compressor operates well below this limit. The compressor, therefore needs to be designed assuming brittle behavior whereas the combustor and turbine will be creep or yield limited (for more detail discussion, see Chapter 7 and 8.).
Figure 4.17  Silicon fracture toughness vs. temperature (Rybicki and Priouz, 1988)
4.7 Summary and Conclusion

The development of the microengine requires highly stressed structures to achieve high power densities. Material strength is, therefore, a critical issue for the design of such devices. Due to the stochastic nature of the strength of brittle materials, the length scales of the test specimens should be close to those of the structure in order to avoid excessive extrapolation of the test data. In this chapter, strength characterization and the supporting analysis of approximately sized (dimension 0.01 mm - 2mm) bi-axial flexure and radiused hub flexure single crystal silicon specimens are presented. The Weibull reference strength of planar biaxial flexure specimens was found to lie in the range of 1.2 to 4.6 GPa with a strong dependence on the surface quality. The Weibull exponents lay in the range of 3 - 12. The local strength at stress concentrations was obtained by testing radiused hub flexure specimens. For the case of deep reactive ion etched (DRIE) specimens, the strength at the fillet radii was found to be significantly lower than that measured from the planar bi-axial flexure specimens due to the inferior surface quality in such regions. It was found that the strength could be increased significantly by the introduction of an additional isotropic etch after the DRIE step. The test results reported herein have important implications for the development of highly stressed microfabricated structures. The sensitivity of the mechanical strength to surface processing and etching techniques must be accounted for in the design cycle, particularly with regard to the selection of the appropriate fabrication route. Furthermore the structural design of such devices should have a probabilistic component in order to properly account for the stochastic strength data. The results presented in this chapter therefore, served as the basis of the structural design for components subject to temperatures below 500 °C ( ~ 800 K).
4.8 Reference


CHAPTER 5. STRUCTURAL ANALYSIS OF BRITTLE TURBOMACHINERY STRUCTURAL COMPONENTS

5.1 Introduction

This chapter presents the structural analysis for the silicon turbogenerator and low temperature structural components of the all-silicon cooled engine. It includes stress, deformation, vibration frequencies/modes, and thermal stress/deformation. For the high temperature structural components of the all-silicon engine, the analysis is strongly dependent on the structural temperature. Both the heat transfer and thermal stress analysis for these components will be addressed in Chapter 8.

Microengine structural analysis can be divided into two major categories: rotating and static structures. The rotating structures include the turbines and compressors. The static structures include the combustor, guide vanes, diffuser, inlet and exhaust supporting structures. The rotating structure is potentially subjected to centrifugal forces, fluid loads, electrostatic forces, and thermal loads. For the static structures, internal pressure induced tensile stresses and the thermal stresses due to constrained expansion and temperature gradient are the potential important sources. In general, the rotating structure is more highly stressed and has more functional requirements imposed by the system design. There are less functional requirement for the static structure particularly at lower temperature. As a result, the rotating structure is more critical and be the sole focus of this chapter.

At temperature below the ductile to brittle transition temperature the material behavior is linear-elastic up until fracture. This considerably simplifies the structural analysis tasks. Nevertheless the rotating parts of the engine represent a complex three dimensional structure. The key to the efficient analysis of such a structure is to start with the simplest possible analyses, and refine them only where greater precision is required.

The sequence of structural analysis reflects this philosophy:
1. First, rotating elements are modeled as simple structural members with closed form solutions available from the theory of elasticity. These solutions provide order of magnitude estimates of the vibration frequencies, deflections, and stress levels. Although these models are relatively inaccurate, they provide the basic information to qualify the baseline design and to determine relative magnitudes of the stress levels and deflection due to different loading sources. More detailed analysis requires numerical solutions, such as those from finite element analysis.

2. Axisymmetric or two-dimensional finite element analysis. Although the actual geometries of the bladed turbine and compressor disks are not axisymmetric, this idealization provides a computationally efficient way to conduct parametric studies of disk thickness and hub geometry. Through these studies the design can be refined before restarting to a fully 3-D analysis.

3. Three-dimensional finite element analysis. Ultimately, the final design must be checked and fine meshed using a full 3-D FE analysis. In addition to accounting for the three dimensional nature of features such as blades, this analysis also permits consideration of the cubic symmetry of the material. Due to the computational expense of conducting 3-D analysis, this is only used where absolutely necessary.

Another important application of the structural analysis is that the finite element stress/deformation analysis model and its results are required for the structural failure probability calculation and reliability analysis. These are modeled to account for the stochastic nature of the material strength. This is presented in Chapter 6.

The rest of this chapter is organized as follows: Section 5.2 reviews the theory of elasticity as it pertains to the model of the turbine as a rotating disk. Section 5.3 presents the axisymmetric finite element stress/deformation analysis. Section 5.4 discusses the 3-D finite element stress/deformation analysis result. Section 5.5 addresses the issue of structural dynamics and vibration. The thermal stress/deformation and other non-centrifugal stresses are discussed in section 5.6. The possible application of stress analysis is presented in section 5.7. Section 5.8 is the overall summary and conclusion.
Figure 5.1  Schematic of microengine low temperature components structural analysis
5.2 Review of the Theory of Elasticity

Analytical models derived from elementary solid mechanics and the theory of elasticity, although extreme simplifications of the real turbomachinery, can provide a general insight and serve as the first step for structural analysis. For the rotating structures, a flat rotating disk is the most fundamental model for analysis.

As shown in Figure 5.2, consider a flat solid disk made of isotropic and homogeneous material with density \( \rho \). The radial and hoop stress can be expressed by [Timoshenko and Goodier, 1970]

![Figure 5.2 The flat disk model](image)

\[
\sigma_r = \frac{3 + \nu}{8} \rho \omega^2 (b^2 - r^2), \tag{5.1}
\]

and

\[
\sigma_\theta = \frac{3 + \nu}{8} \rho \omega^2 b^2 - \frac{1 + 3\nu}{8} \rho \omega^2 r^2, \tag{5.2}
\]

where \( \omega \) and \( b \) are the angular speed and the radius of the flat disk, respectively. \( \rho \) is the density of the material.

The maximum radial and hoop stresses are both at the disk center. Putting \( r = 0 \) into (5.1) and (5.2), the maximum stresses can be expressed as

\[
\sigma_{r, \text{max}} = \sigma_{\theta, \text{max}} = \frac{3 + \nu}{8} \rho \omega^2 b^2. \tag{5.3}
\]
For silicon, $\rho$ is 2330 Kg/m$^3$. Putting the baseline design geometry ($b=2$ mm) and speed ($\omega=250,000$ rad/s), the maximum stress is found to be 240 MPa.

The centrifugal force also results in radial expansion. The radial expansion of the disk can be expressed as

$$u = \frac{(1-\nu)}{8E}[(3+\nu)\rho\omega^2b^2r - (1+\nu)\rho\omega^2r^3], \quad (5.4)$$

where $E$ and $\nu$ are the Young's modulus and Poisson's ratio, respectively. The combined modulus, $E/(1-\nu)$, called the biaxial modulus, controls the deformation characteristics of the disk. Using representative engineering constants for silicon of $E=170$ GPa, $\nu=0.1$, the maximum radial growth due to centrifugal loading is 1.54\mu m. This is an important consideration for the journal bearing design as the performance of the bearings is strongly dependent on the gap between the rotor and the stator.

For flat disks, there is no disk bending deflection. However, for the microengine turbine disk, the mass of the blades in a single-sided bladed disk will contribute a bending moment. Such a moment will cause deflection of the disk. However, the existing circular plate bending model can be used for the first order estimation. Figure 5.3 shows a circular plate subject to a uniform line moment $M_0$. The analytical solution of this problem can be found as [Roda$k$ and Young, 1985]

$$\delta = \frac{6M_0r_0^2}{Eh^3}\left(\frac{1}{1+\nu} + \frac{\ln b}{r_0}\right), \quad (5.5)$$

where $\delta$ is the deflection of the disk edge. If we assume the line moment acts on the edge of the plate, Eq.(5.6) can be further reduced to

$$\delta = \frac{6M_0b^2}{Eh^3}. \quad (5.6)$$

In order to relate the mass of the blades to the line moment $M_0$ in Eq.(5.5), assume the blade is lumped as a mass $m$ at radius $r_0$, with blade height $t$. The centrifugal force
acting on the lumped blade is \( m r_0 \omega^2 \). The averaged moment arm is \( t/2 \). Therefore, the total moment acting on the disk is

\[
M = n t m r_0 \omega^2 / 2. \quad \text{And} \quad M_0 = M / 2 \pi r_0 = n t m \omega^2 / 4\pi,
\]

(5.7)

where \( n \) is the total number of blades. For the baseline turbogenerator design, \( \omega = 250,000 \text{ rad/s} \), \( n = 20 \), \( t = 200 \mu \text{m} \). The individual mass is calculated by I-DEAS™ as \( 4.8 \times 10^8 \text{ Kg} \). As a result, \( M_0 = 0.955 \text{ N-m/m} \). Put \( M_0 \) into Eq.(5.6) as \( r_0 \sim 1.5 \text{ mm} \), we got deflection as 3.2 \( \mu \text{m} \).

Turbine blades can be modeled as cantilever beams in order to estimate the order of magnitude of the vibration frequency. The vibration of a cantilevered beam can be expressed [Blevins, 1984] as

\[
f_1 = \frac{1.8751^2}{2\pi L^2} \sqrt{\frac{E I}{m}}.
\]

(5.8)

For a \( L = 200 \mu \text{m}, b = 400 \mu \text{m}, h = 50 \mu \text{m} \) cantilever beam. The blade inertia, \( I \), was estimated as \( 7.1 \times 10^{-7} \text{ Kg-m}^2 \) and \( m \) is estimated as \( 4.2 \times 10^{-5} \text{ Kg/m} \). As a result, the first mode of the turbine blade is estimated at 1.8 MHz.

These analytical models are used to decide the baseline design geometry and operating speed. As mentioned before, the baseline turbine disk radius was determined to be 2 mm with tip speed of 500 m/s based on engine performance requirement and the analytical estimation.

![Figure 5.3](image)

Figure 5.3  Deflection of a circular plate subjected to uniform line moment
5.3 **Axisymmetric FE Stress/Deformation Analysis**

The analytical model shown in section 5.2 can only provide a general insight into the problem. In order to obtain more accurate results, a more refined analysis is necessary. There are no analytical solution beyond the simple geometry. Numerical methods such as the finite element method (FEM) are required to perform such analysis.

As shown in Figure 1.2, the baseline turbine disk design has 20 blades. In order to establish a basis for design trades without incurring the cost of a full 3-D FE analysis, a 2-D axisymmetric model was used as an intermediate step. The commercial finite element package, ABAQUS STANDARD [ABAQUS, 1996], was used for this purpose.

Figure 5.4 shows the axisymmetric lumped mass model of the turbine disk. A raised hub was designed for carrying the axial thrust. The discrete turbine blades are lumped into a ring mass. This model can be used for calculating the stress near the disk center, the stress concentration near the hub and disk interface, the radial growth and out of plane deflection of the disk. However, it lacks the ability to describe the local stress and deformation near the blades.

The associated two-dimensional finite element model was created. Figure 5.5 shows the FE mesh. A typical FE model contains several hundreds of CAX8 second order eight node axisymmetric elements. The material is assumed isotropic with Young’s modulus equal to 170 GPa and Poisson’s ratio 0.1.

There are potentially three stress critical locations on the rotor: the disk center, the junction between the hub and the disk, and the junctions between the disk and the turbine blades. The asymmetric location of the blades on one side of the disk implies that not only the radial expansion, but also the out of plane deflection must be considered. To account for this, the blades were represented by an axisymmetric “lumped” mass. The lumped mass is obtained by matching the total mass and rotational inertia of the turbine blades. This model was verified subsequently by comparison to a full 3-D model described in Section 5.4.
Disk diameter = D  
Hub diameter = Dh  
Hub height = L  
Blade height = h  
Disk thickness = t

Figure 5.4  The axisymmetric lumped mass turbine disk model

Figure 5.5  The axisymmetric FE mesh

Figure 5.6 shows the rotor deformation. As shown in Figure 5.8, it is a function of blade height and disk thickness. For the baseline 200 µm tall blades, a 2.4 µm vertical deflection at the disk tip was calculated. The radial expansion is 1.5 µm which is essentially the same as that calculated for an unbladed disk. The stress level at the disk center is also close to the flat disk case. However, the stress distribution is quite different. As shown in Figure 5.7, the top surface of the disk, subject to centrifugal and tensile bending stresses, shows a higher stress level than the bottom surface, which is subjected to centrifugal and compressive bending stresses. The raised portion of the hub shows much lower stress level because the rim speed of the hub is much smaller. Different rim speeds result in different radial expansions. This deformation mismatch results in a stress concentration at the disk/hub interface. The magnitude of the stress concentration is a function of geometry and local stress level.
Figure 5.6  Deformation of the lumped mass turbine due to centrifugal load (magnification ratio = 100)

Figure 5.7  Stress contour of the axisymmetric FE model near the hub/disk fillet

Two parametric studies have been performed using the axisymmetric model. These explored, first, the relationship between disk thickness, blade height, and tip deflection, and secondly, the relationship between fillet radius, hub aspect ratio, and the stress concentration at the hub/disk fillet.
Figure 5.8a and 5.8b show the relationships between disk thickness, blade height, and out of plane deflection. This is an important design trade because the out of plane deflection must be minimized in order to achieve the required thrust bearing performance and design the seals used to maintain a differential pressurization for the journal bearing. Figure 5.8 shows that this can be achieved by reducing the blade height and increasing disk thickness. This conflicts with the demands of turbomachinery performance which drive the design toward taller blades. It also partly conflicts with the fabrication considerations which restricts the design to thinner disks and shorter blades in order to minimize etching cycle times.

The sensitivity of the stress concentration to the design variable is shown in Figure 5.9. Figure 5.9a shows the effect of placing the blades asymmetrically on one side of the disk as opposed to having a balancing mass on the reverse side of the disk. The stress level in the asymmetric case is larger due to the contribution of the bending moment.

Figure 5.9b shows that the stress concentration at the hub/disk transition is a strong function of the local fillet radius. The stress is reduced 40% if the fillet radius is increased from 10 μm to 30 μm. A larger fillet radius is clearly desirable; however, this significantly increases the fabrication complexity. The analysis also shows that the stress concentration is a weak function of the hub height, L. The stress concentration increases when L increases but quickly saturates as L becomes larger (∼ > 0.2 D).
Figure 5.8  
(a) The relationship between rotor deflection and rotor thickness 
(b) Rotor deflection vs. blade height
Figure 5.9 (a) The effect of out of plane asymmetry on the hub root stress concentration and (b) The relationship between stress concentration, fillet radius, and aspect ratio (L/D).
5.4 3-D FE Stress/Deformation Analysis

The two dimensional analysis provides useful input to the design process; however, the turbine geometry is not axisymmetric and furthermore, silicon has cubic material symmetry. Consequently, a three-dimensional FE analysis was used for the detailed design. In particular, the following issues were addressed:

(i) The effect of material anisotropy.

(ii) The state of stress around the turbine blades.

(iii) The vibration of blades and disk.

The effect of anisotropy was examined by applying the cubic material properties to a FE model of a flat rotating disk. As shown in Figure 5.10, the calculation shows that the anisotropic model has a 3% increase in maximum stress over the isotropic case in the highest stiffness direction. The <110> direction has the highest stress for a given radius. The radial expansion, however, is independent of direction, which follows from Eq.(5.5). This shows that the expansion depends on the invariant property bi-axial modulus E/1-v. The result indicates that use of isotropic material properties is a good approximation even for detailed analysis.

A full 3-D anisotropic turbine disk model has been constructed to examine the deformation and primary stress distribution. Figure 5.11 shows the mesh for the 3-D FE model. It consists of nearly 1000 C3D20 twenty-node brick isoparametric elements. The stress contour is shown in Figure 5.12. The associated material orientation is marked in the figure. Due to the cubic material symmetry, the stress contour is somewhat rectilinear rather than axisymmetric. Note that the stress concentration at sharp transitions are mesh dependent and thus can not be well represented by this model. Nevertheless, the mesh is fine enough to quantify the primary stress distribution. The deformation of the micro rotor at full speed is shown in Figure 5.13. Note that the bending of the blade at the trailing edge will cause high stress at the blade root. Figure 5.14 shows the relationship between deformation and rotating speed in both axisymmetric and full 3-D models. The axisymmetric model underpredicts the deformation by about 25%. This is because the continuous rim used to represent the blades provides greater stiffness than the discretized blades. If required for further
parametric studies, the stiffness of the blades could be artificially reduced to better match the 3-D results.

Figure 5.10 3-D isotropic and cubic anisotropy disk FE model
Figure 5.11  FE mesh for the whole 3-D micro rotor model

Figure 5.12  The primary stress contour of the micro rotor
To determine the blade root stress, a more detailed model was constructed. The critical point is at the root of the trailing edge of the blades. Figure 5.15 shows the corresponding FE mesh and calculated stress contours.
A parametric study was conducted using this FE model to examine the relationship between blade stress, blade height, and blade root fillet radius. Figure 5.16 presents the results of this parametric study. The maximum stress at the critical point triples as the blade height is doubled. This is due to the centrifugal bending of the blade. The fillet radius must be increased in order to reduce the stress level if the blade height is to be increased. As for the axisymmetric analysis, these results help guide the design trade-off between turbomachinery performance, fabrication process, and structural integrity.

It is worth pointing out that the stress of the blade root comes from the bending of the blade. A taller blade means that the beam or plate length and the centrifugal force and moment arm increases. Therefore, there are three effective ways to reduce the blade root stress. First, to reduce the mass of the blade, second, to increase the local moment of inertia of the blade trailing edge, and third, to constrain the bending of the blades.

![MAX. STRESS (MPa)](image)

Figure 5.15 FE model to calculate the blade root stress concentration.

The mass reduction can be achieved by creating hollow blades. Blades can be etched in a honeycomb shape to remove mass and still maintain the stiffness of the structure.
There are several concerns for this approach. First, the creation of interior fillet radii creates another region of high stress which must be controlled by using an appropriate etch process. Second, the heat transfer of high aspect ratio structures. Honeycomb structures have a high aspect ratio. This means that the thermal resistance will be increased. For the turbine blades, this will result in a hotter blade tip, which is a serious concern for the reliability of an all-silicon cooled turbine. However, it is not a concern for the turbogenerator design.

The other method to reduce the stress level at the critical point is to stiffen the blade trailing edge. For instance, design stiffer blades by changing the dimensions, especially increasing the thickness of blade trailing edge. From beam or plate theory, the stress level is expected to be inversely proportional to the square of thickness. Therefore, a small increase in thickness will greatly decrease the stress level. However, the associated constraint is the turbomachinery performance. Changing the blade shape will change the flow field; in general, slender blades are preferred. The performance may change greatly.

The hollowing blade approach also reduces the deflection of both blades and disk, which is a desired result. On the other hand, the blade stiffening approach will reduce the blade deflection but not the deflection of the disk. The third approach is to constrain the blade bending by adding a shroud on top of the blades. Since the bending of the blades is fully constrained, the stress level is significantly reduced. In addition to the reduction of the stress level, the disk deflection is also greatly reduced since the disk stiffness is also increased by the presence of the continuous shroud. Figure 5.17 shows the FE deformation plots of a micro compressor under steady state operation. The diameter of this micro compressor is 4 mm. The disk thickness and blade height are 260 \( \mu \)m. It is operated at a tangential speed of 500m/s. The tip deflection of the unshrouded compressor is 13.35 \( \mu \)m. If a 80 \( \mu \)m thick shroud is added to the rotor, the tip deflection of the compressor is reduced to 2 \( \mu \)m, i.e., only 1/6 of the unshrouded case. As shown in Figure 5.18, the maximum stress level is also reduced from above 1 GPa at the blade root to under 600 MPa at the shroud inner free surface. The presence of a shroud is currently the preferred structural design recommendation. However, it
also brings a significant challenge in the fabrication process by introducing additional DRIE and wafer bonding processes.

Figure 5.16 The relationship between blade root stress concentration and design parameters. (a) stress vs. blade height, (b) stress vs. fillet radius.
Figure 5.17  The deformation of micro compressor without and with shroud. The tip deflection of the unshrouded and shrouded compressor is 13.35 μm and 2 μm, respectively.

Figure 5.18  The stress contour of shrouded compressor. The critical point is now moved toward the shroud inner free surface.
5.5 Structural Dynamics and Vibration

As noted in Section 3.3, the resonant frequencies scale inversely with length [Meirovich, 1986][Blevins, 1984]. Moreover, in comparison with large scale turbomachinery, the micro turbine blades have relatively low aspect ratios. These two factors imply that the micro turbine blades will have extremely high resonant frequencies. Although the rotating speed also increases, the extremely high resonant frequencies and small number of blades may result in an unusually high operating margin between the turbomachinery frequency and the first structural mode. At 500 m/s tip speed, and a 4 mm diameter rotor with 20 blades, the turbomachinery frequency is nearly 0.8 MHz, while the first structural natural frequency is 2.5 MHz. It is interesting to note that this advantage occurs even though centrifugal stiffening is relatively unimportant in micro turbomachinery. Figure 5.19 shows the first, the second, and the third modes of vibration of the turbine blade. The effect of blade height on the first vibration frequency is shown on Figure 5.20. Again, this represents a trade off between engine performance and the structural reliability. Increasing the blade height reduces the vibration frequency. For the baseline design turbine blade, the first resonant frequency of a 400 µm height blade is almost identical to the turbomachinery frequency.

The vibration mode of turbine disk can be approximated by plate vibration formulae. Consider a circular plate (radius = a, thickness =h) made by a homogeneous material (Young’s modulus E, Poisson’s ratio ν, and density ρ) with free boundary condition. The vibration frequencies can be expressed as [Blevins, 1984]

\[
f_{ij} = \frac{\lambda_{ij}^2}{2a^2} \frac{Eh^2}{12\rho(1-\nu^2)}^{1/2},
\]

(5.9)

where \(\lambda_{00}^2 = 0\), \(\lambda_{10}^2 = 0\), \(\lambda_{20}^2 = 5.253\), and \(\lambda_{01}^2 = 9.084\). Putting this \(\lambda_{ij}^2\) into Eq. (5.9), with a = 2mm and h = 300 µm, gives natural frequencies of:

\[f_{00} = 0, f_{10} = 0, f_{20} = 0.156 \text{ MHz}, \text{ and } f_{01} = 0.269 \text{ MHz}.
\]
The corresponding mode shape is shown in Figure 5.21. There are two rigid body modes. For a free rotating disk, the frequency of these rigid body modes will be zero. However, for the microengine, the hydrostatic/dynamic journal bearing will provide stiffness to the rigid translation mode (i.e., $f_{10}$) and the axial thrust bearing will contribute stiffness to the rotating mode (i.e., $f_{10}$). As a result, the resonant frequencies of both modes will be increased. The detail work for the air bearing is described elsewhere [Piekos et al, 1997]. In addition, the effect of centrifugal stiffening will also change the natural frequencies. In order to address this issue, a 2-D FE model for a rotating disk was created using shell element. The journal and thrust bearings were modeled as springs in X, Y, and Z directions. Figure 5.22 shows the two fundamental modes. The potato chip mode has a natural frequency $f_{20} = 0.181 \text{ MHz}$. It is higher than the corresponding mode calculated at zero rotating speed by Eq.(5.8). On the other hand, the umbrella mode has a natural frequency $f_{01} = 0.254 \text{ MHz}$. It is slightly less than the corresponding value at stationary. Both of them come from the effect of centrifugal stiffening. [Hibbit, 1979].

The structural dynamic behavior of the bladed disk at full rim speed is described in Figure 5.23. Since the natural frequency of the turbine disk is much less than that of the turbine blades, the structural dynamic behavior is dominated by the disk. The fundamental mode of a bladed disk is the same as for a flat disk. Due to the presence of blades, the increased mass reduced the natural frequencies to 0.161 and 0.215 MHz.

Note that the natural frequencies of disk vibration are proportional to the thickness and inversely proportional to the square of the disk size. The future disk natural frequencies can be obtained by the baseline analysis result with simple geometry scaling.

Dynamic effects do limit the rotor design in other respects as the stiffness of the micro gas bearing is much less than that of the turbine disk. As a result, the first several modes of the turbine disk are essentially rigid body motions with frequencies much lower than the operating speed of the engine. The issues arising from this are described elsewhere [Piekos et al, 1997].
Figure 5.19  The first three vibration modes of a turbine blade.

Figure 5.20  The effect of turbine blade height to the first blade vibration mode
Figure 5.21 The fundamental modes of a rotating disk

Figure 5.22 FE vibration analysis for a rotating disk at rim speed = 500m/s:
(a) $f_{20} = 0.181$ MHz, (b) $f_{01} = 0.254$ MHz

Figure 5.23 FE vibration analysis for the turbogenerator at rim speed= 500 m/s:
(a) $f_{20} = 0.161$ MHz, (b) $f_{01} = 0.215$ MHz
5.6 Stress and Deformation Due to Other Loading

This section provides analyses to estimate the stress level caused by the non-centrifugal sources of loading, namely fluid loading and thermal loading.

5.6.1 Fluid loading

The fluid loading acting on a turbine blade is calculated by integrating the pressure on the surfaces to find the equivalent total drag and lift forces. The resultant lift and drag are 50 N/m (blade height) and 140 N/m respectively. For a 200 µm height blade, the total lift and drag are 0.01 N and 0.028 N. To find the influence of the fluid loading on the blade stress, the force is assumed to act on the blade aerodynamic center.

These forces were input to a single blade 3-D FE model to calculate the induced stress. The maximum stress is 50 MPa acting on the blade leading edge. The stress at the trailing edge due to aerodynamic forces is negligible. Since the stress level due to aerodynamic forces is much smaller than the centrifugal stress and the maximum stress does not happen at the same location as for centrifugal loading, the stress induced by aerodynamic force is not important in steady state operation.

The exciting frequency of the fluid load and the first mode of the blade, as shown in section 5.5, are 0.8 and 2.5 MHz respectively. The nominal operation frequency is therefore, 0.3. At this frequency, the dynamic loading factor, although dependent on the material damping, is essentially 1. The dynamic loading effect can be neglected. For the transient case, due to the lower rotating speed, the nominal operating frequency is even smaller.

5.6.2 Thermal stress and deformation

A rise in structural temperature expands the structure. The amount of thermal deformation influences the performance and design in several important engine components such as bearing, generator, and engine structure itself.

The average of coefficient of thermal expansion (CTE) of silicon from room temperature to 1100 K is around $3 \times 10^{-6}$ m/mK [Purdue, 1970]. Consider a one-dimensional structure with length $L$ and CTE $\alpha$. The thermal strain $\epsilon$ and deformation $u$ are
\[ \varepsilon = \alpha \Delta T \] \hspace{1cm} (5.10)

\[ u = L \alpha \Delta T \] \hspace{1cm} (5.11)

For a silicon structure with a length of 2 mm, a 500 K difference can result in a variation of length in 3 \( \mu \)m. This amount is larger than the centrifugal expansion.

Due to the possible non-uniform temperature distribution over the entire structure, the width of bearing gaps may vary. This is important for the operation of the bearing and generator. If the amounts of expansion are different between the rotor and stator, the width of the bearing air gap will be changed. If there is any significant temperature difference between the top and bottom surface of the rotor, the different thermal growth will result in deflection. The relative significance of both variations are strongly dependent on the structural design and thermal analysis.

If there are any constraints to prevent free expansion or the existence of non-uniform temperature distribution, thermal stress will appear [Boley and Weiner, 1985]. For a totally constrained one-dimensional silicon structure, the thermal stress can be expressed as

\[ \sigma_T = E \alpha \Delta T \] \hspace{1cm} (5.12)

For the turbogenerator, the maximum temperature difference is less likely to exceed 100 K. As a result, by Eq. (5.12) the maximum thermal stress for the static structure (assuming it is fully constrained) is less than 55 MPa. The stress level is small and not a major design concern. For the rotating structure, since it is free of any constraint, instead of using Eq. (5.12), Eq. (3.10), and Eq. (3.11) should be used for the analytical estimation. In the worst scenario, we assume there is a radial temperature gradient of 50 K/mm, which corresponds to a maximum temperature difference of 100 K. The maximum radial or hoop thermal stress obtained is 33 MPa. Again, this is much less than the centrifugal stress.

5.6.3 Stress due to internal pressure load

Turbogenerators are driven by injecting high pressure air into the device. The walls of the static structure surrounding the rotor must be sized to withstand this pressure.
The geometry is complicated and the pressure distribution is non-uniform. Before performing a complicated calculation, it is better to perform calculations based on a simple model to access the importance of this force. Modeling the static structure as a thin walled pressure vessel provides a conservative estimation for the stresses. Consider a thin wall cylinder with mean radius \( a \) and thickness \( t \). When it is subject to an internal gage pressure \( p \), the induced hoop stress is

\[
\sigma_h = \frac{ap}{t}
\]

The gage pressure is in the range of 1 to 3 atm. Typical dimensions for \( a \) and \( t \) are 2 mm and 100 \( \mu \)m, respectively. Assuming \( p = 3 \) atm, the hoop stress is then calculated as 6 MPa. This is a small number and can be neglected for the turbogenerator.
5.7 Application of the Room Temperature Structural Analysis

The stress, deformation, structural dynamics, and thermal stress analysis described in the previous sections can be used for several purposes.

1. Determination of important structural component's dimension: The parametric studies of section 5.3 and 5.4 provide the guidelines to determine key structural dimension such as disk thickness, fillet radii, and blade height.

2. Operating condition determination: The allowable stress and deflection restrict the rotating speed. For the operating regime, the relationship between blade resonant frequencies, blade geometries, and rotating speed determines whether the turbogenerator operates in the subcritical or supercritical regime.

3. Basis of structural re-design: Although the baseline design is far from the final design, the result of the baseline structural analysis is very important. It can indicate the direction for improvement. For example, both the stress and vibration analysis indicate that the stress level can be reduced significantly and the first mode can be increased considerably by a slight increase in blade trailing edge thickness.

4. Input for probabilistic structural analysis: It is important because that the strength of brittle materials is a stochastic variable. This implies that failure does not necessarily occur at the highest stress locations or at a deterministic way. As a result, volume of stress structure failure probability must be evaluated. Such evaluation requires both material and stress information. An appropriate mathematical model such as Weibull weakest link theory is used to calculate the overall structural survival probability. The stress analysis models presented in this chapter also provide the necessary information to calculate the structure reliability, as will be described in Chapter 6.

5. Stress and deformation for the all-silicon cooled engine: Although cooled engines will be operated at elevated temperatures, the structural temperature of some components such as the compressor is still below the BDTT. The structural analysis method of brittle structures at high temperature (below BDTT) is essentially the same as at room temperature. Therefore, the analysis of this chapter can be applied to the structural design of these components in cooled engines.
5.8 Summary and Conclusion

This chapter describes the stress and deformation analysis of the cold turbogenerator. For room temperature devices, the most critical component is the turbine. It is subjected to high centrifugal loading due to the high rotating speed. The analytical rotating disk model has been used for first order stress and radial growth calculations. For more complicated structures, 2-D and 3-D finite element models were constructed. Parametric studies have been performed to address the design trends on the hub/disk fillet radius, blade/disk fillet radius, and blade height. In addition, the vibration modes of turbine blades are calculated. At the baseline operating condition, it is subcritical. There are no significant loads acting on the stationary structure at room temperature. The order of magnitude analysis shows that the centrifugal force is the dominating load in the room temperature devices. Therefore, the effect of these non-centrifugal loads can be neglected or covered by a suitable design safety factor.

The finite element models created in this chapter are also served as the input for structural failure probability evaluation of brittle structures. It will be addressed in Chapter 6.

The structural analysis shown in this chapter is consistent with the overall engine development. The tendency of engine design is toward faster rotating speed and thinner and taller blades. The function of the structural analysis is to set constraints for these design spaces by either structural mechanics or structural dynamics analysis. Under these constraints imposed by structural analysis, the engine design can perform sub-optimization in these geometries and operating speed.
5.9 Reference


CHAPTER 6.
PROBABILISTIC STRUCTURAL DESIGN

6.1 Introduction

The state of stress in the rotating elements of the turbogenerator has been described in Chapter 5. The critical locations and the magnitude of stress have been identified and studied parametrically. These parametric studies can be used to determine allowable limits for dimensions such as blade height and fillet radius to refine the design in order to keep the stress level within the material capabilities. However, such an approach is not sufficient to predict the structural reliability structures made of brittle materials.

As described in Chapter 4, brittle materials usually have non-deterministic strength governed by the distribution of flaw sizes and locations. This has several implications for design against failure. First, it must be accepted that designing to a particular stress level implies accepting a non-zero probability of failure. Second, failure will not necessarily occur at the maximum stress location, and there may be a finite probability of failure at any location on the structure. Finally, it is important to reiterate that the strength of a given structure or specimen depends on its volume or surface area [Freudenthal, 1968]. Therefore material test data must be appropriately scaled before it can be applied to a structural design.

To deal with the stochastic nature of ceramic structural design, it is appropriate to use probabilistic (non-deterministic) design approach. This methodology has been used in ceramic structural design [Duffy et al., 1993], [Khandelwal et al., 1995], [Saith, Norton, and Parthasarathy, 1995], [She et al., 1991], [Sturmer et al., 1993]. Simply stated, in such an approach, the structural design goal is to keep the failure probability lower than a predetermined threshold. Figure 6.1 summarizes the probabilistic structural analysis procedure. Basically, three major tasks are included: the material testing, structural analysis, and failure probability prediction. Material testing and structural
analysis have already been presented in chapters 4 and 5 respectively. The failure probability prediction is described in this chapter.

This chapter is organized as the following: Section 6.2 reviews the use of Weibull statistics in structural design with brittle material. Section 6.3 presents the probabilistic structural design procedures. The material test data reduction and scaling is addressed in Section 6.4. The primary failure probability calculation based on the test data and the baseline design for the micro turbogenerator is described in Section 6.5. Finally, the summary and conclusion is presented in Section 6.6.

![Flow chart of the probabilistic structural design](image-url)

**Figure 6.1** Flow chart of the probabilistic structural design
6.2 Weibull Statistics and the Principle of Independent Action

In brittle materials, the critical flaw sizes are usually undetectable given the capabilities of non destructive evaluation (NDE) techniques and are generally accounted for by using a probabilistic methodology.

The most widely used approach to account for the stochastic nature of the strength of brittle structures is the weakest link model [Weibull, 1939]. In this model, the failure probability of a volume of material, \( V \), under a uniform stress, \( \sigma \), is assumed to be given by

\[
P_f(\sigma, V) = 1 - \exp\left(-\frac{V}{V_0}\left(\frac{\sigma - \lambda}{\sigma_0}\right)^m\right),
\]

(6.1)

where \( V_0 \) and \( \sigma_0 \) are the reference volume and strength respectively. \( m \) is the Weibull modulus that describes the scatter of material strength. \( \lambda \) is a threshold parameter which corresponds to the minimum material strength. For ceramics, \( \lambda \) is often taken to be zero. This leads to the two-parameter Weibull distribution.

\[
P_f(\sigma, V) = 1 - \exp\left(-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right).
\]

(6.2)

The two-parameter Weibull distribution has been shown as more conservative than the three-parameter Weibull distribution in structural design [Shih, 1980]. Depending on the nature of the strength controlling flaw population, the failure probability can also be treated as surface area dependent. In which case, an area term \( A/A_o \) is substituted for \( V/V_o \). The failure type (volume or surface flaw) can be determined by a combination of fractography, finite element analysis, process characterization, and material properties. Similarly, the choice between a two and three parameter Weibull model depends on the nature of the strength distribution and the statistical level of confidence required.

Equation (6.1) and Eq.(6.2) are suitable for states of uniform uniaxial tensile stress. For uniform multiaxial stress states, the method of application of Weibull statistics must be modified. As shown in Figure 6.2, the principle of independent action (PIA) model [Barnett, 1967] states that the Weibull survival probability of a uniformly stressed material element experiencing multiaxial loading is equal to the product of the survival
probabilities for each of the tensile principal stresses applied individually, i.e., Eq.(6.2) can be rewritten as

$$P_f(\sigma, V) = 1 - \exp\left(-\frac{V}{V_0}\left[\frac{(\sigma_1)}{\sigma_0}^m + \frac{(\sigma_2)}{\sigma_0}^m + \frac{(\sigma_3)}{\sigma_0}^m\right]\right), \quad (6.3)$$

or for a surface flaw population,

$$P_f(\sigma, A) = 1 - \exp\left(-\frac{A}{A_0}\left[\frac{(\sigma_1)}{\sigma_0}^m + \frac{(\sigma_2)}{\sigma_0}^m + \frac{(\sigma_3)}{\sigma_0}^m\right]\right). \quad (6.4)$$

In general, the principal stresses vary with position, thus Eq.(6.3) and Eq.(6.4) must be integrated over the volume or surface area of the structure, in order to obtain the overall failure probability. The finite element method is a convenient means of achieving this integration.

The failure probability calculation procedure is shown in Figure 6.3. Consider a finite element model of a structure. The structure is discretized into N finite elements. Each finite element has M Gaussian integration points with M associated subvolumes, each with a unique set of principal stresses.

The probability of failure of each Gaussian subvolume \(P_{\phi}\) can be directly calculated via Eq.(6.4). The overall structural failure probability is obtained by the following product:

$$P_f = 1 - \sum_{i=1}^{N} \sum_{j=1}^{M} (1 - P_{\phi_i^j}). \quad (6.5)$$

![Figure 6.2](image.png)

Figure 6.2 A cube with three principal stresses acting on the faces
Figure 6.3 The schematic process flow of structural failure probability calculation
6.3 Probabilistic Structural Design Procedures

Figure 6.1 illustrates the probabilistic structural design procedure. As shown in section 6.2, in order to calculate the failure probability of a structure, the material characteristics and stress distribution need to be obtained first.

The material characteristics are expressed by the two or three-parameter Weibull variables, i.e., \( \sigma_0 \), \( m \), and \( \lambda \) (if necessary). Chapter 4 has discussed the material characterization work required for the DRIE silicon microengine structures.

In order to calculate the entire failure probability by Eq. (6.5), the stress and sub-volume or sub-area information must be obtained using a finite element model. The finite element model constructed in Chapter 5 can be used to provide this information.

As mentioned previously it is important to distinguish between the effects of surface flaws and volumetric flaws. Surface flaws, such as scratches, are mainly caused by surface machining process or subsequent handling operations [Rice et al., 1974], [Rice et al., 1981]. Volumetric flaws, such as voids, lattice defects, and grain boundaries, are mainly induced by the initial ceramic forming process. Different flaw types will result in different failure probabilities. Given the absence of bulk defects in semiconductor grade single crystal silicon, it is assumed that all fractures initiate from surface flaws. However, for non-single crystal materials, such as CVD silicon carbide, depending on the processing route, structural failure may be caused by either type of flaw.

The state of stress of the mechanical test specimens is quite different from that experienced by the turbogenerator rotor under its operating condition. As a result, the stress-volume integrals are different. This implies that, the tested strength can not represent the real strength of the structure at the desired operating condition. The test data must be scaled appropriately before they can be used to estimate the failure probability of the structure. The data reduction and scaling will be addressed in section 6.4.

After obtaining appropriate \( m \) and \( \sigma_0 \) values, stress field, and volume information, the failure probability is calculated using Eq.(6.4). For structural reliability, a low stress \( \sigma \), high reference strength \( \sigma_0 \), and high Weibull modulus, \( m \), are desired. The calculated
failure probability then determines if the structure must be redesigned to reduce the stress level or the process needs to be improved to enhance $m$ and $\sigma_0$.

Table 6.1 illustrates the influence of the Weibull modulus. It is clear that in order to take advantage of the high strength of ceramic materials, the Weibull modulus of the material plays as important as the strength of materials.

<table>
<thead>
<tr>
<th>Weibull modulus $m$</th>
<th>Max. Allowable Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>635</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
</tr>
<tr>
<td>15</td>
<td>1600</td>
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<tr>
<td>30</td>
<td>2550</td>
</tr>
<tr>
<td>100</td>
<td>3480</td>
</tr>
<tr>
<td>200</td>
<td>3740</td>
</tr>
</tbody>
</table>
6.4 Test Data Reduction

From Eq.(6.4), it can be seen that the failure probability is controlled by the dimensionless variables \( m, (\sigma / \sigma_0), \) and \((V/V_0),\) or \((A/A_0).\) Specimens with different geometries will have different failure probabilities even if the applied loading and maximum stress levels are the same. Figure 6.4 illustrates this point. Figure 6.4 shows three specimens with the same maximum tensile stress level. The uniaxial specimen (Figure 6.4a) is subjected to this maximum stress everywhere. The rotating disk (Figure 6.4b) is subjected to the maximum tensile stress only at the disk center, with a quadratic decay of stress with radial position but not through thickness. Finally, the bending plate (Figure 6.4c) subject to bi-axial loading experiences the maximum tensile stress only at the bottom edge of plate center, with the stress decaying with radial and through-thickness location. The maximum stress region in these specimens are a volume, a line, and a point. As a result, if these specimens have the same total volume (or surface area), the failure probability under uniaxial loading will be much higher than that due to rotation and both will have a higher failure probability than that due to biaxial bending. Or, to have equal failure probabilities, the circular bending plate specimen must be considerably larger and the stress level must be higher than the rotating disk, etc. Such volume and stress scaling is a function of the Weibull modulus, \( m.\) The important consequence of this for the present work, and other highly stressed MEMS devices is that the volume and geometry of the test specimens are different than those of the microturbogenerator, therefore, the reference strength obtained in section 4 must be scaled before it can be applied to the structural design.

The data reduction procedures is shown in Figure 6.5. The first step is to construct a "virtual structure" with the same loading and geometry as the structure to be analyzed. For example, a flat rotating disk is a simple example of relevance to the turbogenerator. The next step is to calculate the failure probability of this rotating disk based on the bi-axial test data. The Weibull modulus is assumed equal in the two cases. The volume and area of both the bi-axial test specimen and the rotating disk, and the tested \( m \) and \( \sigma_0 \) are used to calculate the failure probability of both cases. The applied stress (rotational speed) in the virtual structure is adjusted until the failure probabilities are matched. This means that the stress at which the failure probability equal 63% defines a modified
reference strength for the real structure. The last step is to use this scaled strength as the strength of the real structure. Together with the value of \( m \), obtained from the tests, serves as the material data for failure probability calculation.

\[
V_E = 1 \\
\text{Uniform tensile stress } \sigma_{\text{max}}
\]

\[
P_f(\sigma_{\text{max}}) > P_f(\sigma_{\text{max}}) > P_f(\sigma_{\text{max}})
\]

Figure 6.4 For same material volume or surface area, the failure probability is different for different loading conditions

Figure 6.5 The data reduction procedure
In order to illustrate the scaling of strength it is convenient to introduce a stress ratio, $S$, which is arbitrarily defined as the ratio of the maximum stress in the mechanical test specimen to the maximum stress in the structure which gives the same probability of failure for the two cases [Chen, Ayon, and Spearing, 1997].

$$S \equiv \frac{(\sigma_{\text{max}})_{\text{specimen}}}{(\sigma_{\text{max}})_{\text{structure}}}.$$  \hfill (6.6)

This stress ratio, $S$, is a function of the form of the stress distributions and the dimensionless variables $A/A_v$, $\sigma_0$, and $m$. Figure 6.6 plots the stress ratio of the test specimen and a rotating disk with the same diameter as the micro turbine rotor. For comparison, results are shown for both volumetric and surface flaw distributions.

To obtain the stress ratio between a rotating disk and the biaxial flexure testing specimen, consider a one-dimensional approximation, the overall failure probability of a structure can be expressed by the following equations:

$$P_v = \int \frac{1}{V_0} \left( \frac{\sigma(r)}{\sigma_0} \right)^m dV,$$

and

$$P_s = \int \frac{1}{A_0} \left( \frac{\sigma(r)}{\sigma_0} \right)^m dA,$$

where $P_v$ and $P_s$ are the overall volumetric flaw and surface flaw based failure probabilities, $r$ is the radial position, $V_0$ and $A_0$ are reference volume and area respectively. For a rotating disk, the maximum principal stress distribution $\sigma(r)$ can be expressed as

$$\sigma_{\text{rotating disk}}(r) = \frac{3+v}{8} \rho R^2 \omega^2 - \frac{1+3v}{8} \rho r^2 \omega^2.$$  \hfill (6.9)

For a biaxial flexure specimen, the maximum principle stress distribution can be approximated by the ideal case shown in [Roda, 1965]. The stress distribution is

$$\sigma_{\text{spec}}(r, y) = \frac{12vy}{t^3} \frac{W}{16\pi} \left[ 4(1+v) \ln \frac{a}{r} + (1-v)(4 - \left( \frac{a}{r} \right)^2) \right],$$  \hfill (6.10)
where \( W \) is the applied load, \( t \) and \( a \) are the thickness and radius of the plate. Substituting Eq.(6.9) and (6.10) into (6.7) and (6.8) yields the failure probabilities for the two cases. \( W \) is adjusted such that the two individual failure probabilities are equal. The stress ratio \( S \) is then defined as

\[
S(m) \equiv \frac{(\sigma_{\text{spec}})_{\text{max}}}{(\sigma_{\text{rotating}})_{\text{max}}}.
\] (6.11)

![Graph showing Stress Ratio vs Weibull Modulus m with two curves labeled Volume Flaw and Surface Flaw, and a note that the curves intersect at equal failure probability at m=10.](image)

**Figure 6.6** The stress ratio

The key implication of Figure 6.6 is that for these particular test specimen and structural geometries, the allowable stresses in the structure are significantly lower than those measured in the mechanical tests. I.e., this implies that for the particular geometry, the stress level must be scaled by a factor of 0.67 (1/1.5) to achieve an equal probability of survival in the rotating structure for \( m=10 \). For more complicated geometries, where analytical solutions are not possible, the stress ratio can obtained by matching the experimentally determined failure probability of the test specimens and that calculated
for the structure. The failure probability of test specimens at a wide range of applied stress can be found in Figure 6.7. For the detailed structural design of the turbogenerator, the CARES/LIFE code developed by NASA [Nemeth, 1990],[Nemeth et. al, 1994], [Noor et. al, 1993] has been used in conjunction with ABAQUS for finite element analysis [ABAQUS, 1996]. A description of the CARES/LIFE code is included in the Appendix A.4.

Figure 6.7  Surface flaw based failure probability of the bi-axial test specimens (reference strength = 3.5 GPa)
6.5 Parametric Studies

Based on the information of section 6.3 and 6.4, the structure failure probability can be calculated. For this purpose a reference strength of 3.5 GPa and a Weibull modulus of 10 were used. These correspond to the test results for KOH etched bi-axial specimens and hence represent a realistic estimate for what can be achieved with a deep reactive ion etching followed by a strength recovering isotropic etch.

Several parametric studies have been conducted to calculate the overall failure probability of the turbogenerator rotor. In these studies, a 3.5 GPa reference strength and a Weibull modulus of 10 were used. Since the centrifugal stress varies with the square of the tip speed, as the rotating speed increases, the failure probability also increases. Figure 6.8 shows the calculated dependence of failure probability on rotating speed. The geometries investigated in this study were initially restricted to a flat disk and a flat disk with a center hub, in each case with the same dimensions as the baseline design. Both volumetric and surface flaw dominated cases were calculated for comparison although it is likely that the structural behavior is dominated by surface flaws. As shown in Figure 6.8, for a rotating disk, the failure probability of a surface-flaw dominated situation is less than the corresponding probability in a volumetric-flaw dominated situation. However, for the case of a disk with a central hub with a 15 μm fillet radius, the failure probability increases. Conversely, the corresponding volumetric-flaw dominated failure probability does not change significantly. The 15 μm fillet radius at the hub-disk transition represents a significant stress concentration. The ratio of this stress concentrated area to the overall surface area is much larger than the ratio between the stress concentrated volume to the overall material volume. As a result, this stress concentration is significant in the surface flaw dominated case but less important in the volumetric flaw dominated case.

It is more difficult to directly perform a parametric study of the full bladed disk because the problem (mesh) size would be too large to handle if the necessary refinement were implemented at the root of each blade. However, a good approximation can be obtained by combining the calculation for the failure probability of a single blade with that for the failure probability of a disk with a central hub. Since the effect of anisotropy is small and each blade is geometrically identical, the failure
probability for each blade is essentially the same. In addition, from the stress analysis, except at the blade root, the blade has very low stress. Letting the failure probability of a single blade and its root be \( P_{fb} \) and \( P_{fr} \) respectively, then

\[
P_{fr} = P_{fb}.
\] (6.12)

By weakest link statistics, the total failure probability \( P_{\text{total}} \) of the blade set is therefore,

\[
P_{\text{total}} = 1 - (1 - P_{fr})^N,
\] (6.13)

where \( N \) is the number of blades.

As \( P_{fr} \) is small \((< 10^2)\), Eq.(11) can be approximated by the linear term of the bilinear expansion, i.e.,

\[
P_{fb,\text{total}} = NP_{fr} - \frac{N(N-1)}{2} P_{fr}^2 + \ldots \approx NP_{fr}.
\] (6.14)

By the same approach, the overall failure probability of the entire turbine disk can be approximated by

\[
P_{f,\text{turbine, total}} \approx NP_{fb} + P_{f,\text{hub, disk}}.
\] (6.15)

Note that Eq.(12) and Eq.(13) hold only if the components and overall failure probability are small \((<10^2)\). For higher failure probabilities of an individual component, the quadratic or higher order terms in Eq.(11) must be included.

Figure 6.9 shows the failure probability of a single blade with a 26 \( \mu \)m root fillet radius as a function of blade height. Applying Eq.(14) gives the overall failure probability of the micro turbogenerator.

Consider a turbine disk with twenty 200 \( \mu \)m tall blades. The fillet radii at the hub and the blade roots are 15 \( \mu \)m and 26\( \mu \)m respectively. This is close to what initial fabrication experiments have yielded. The overall structural failure probability is obtained from Figure 6.8, Figure 6.9, and Eq. (13) as

\[
P_{f,\text{turbine, total}} = 5 \times 10^{-7} + 20 \times 2 \times 10^{-9} = 5.4 \times 10^{-7}.
\]
This predicted failure probability is low, although it must be recognized that structural failure is only one aspect of the device reliability. However, as the blade height is increased to 360µm, the failure probability for a single blade increases to $10^5$ and the overall probability of failure is raised to $10^4$. Although at this stage no target reliability has been set, a useful guideline is the $6\sigma$ ($\sim 3$ failures in $10^6$ parts) quality metric commonly used in microelectronics.

Note that the above study is based on KOH etched specimens. For STS DRIE test specimens, although they have a higher reference strength (4.6 GPa), their Weibull modulus (3.3) is much lower. If the material model is based on the current STS DRIE test results, the failure probability of each corresponding parametric study will be increased by a factor of $\sigma(10^6)$. It is not desirable to design a microengine structure with failure probability less than $10^5$. In order to reduce the failure probability, the STS DRIE process needs to be improved or a proper proof test needs to be included to reduce the scatter in material strength.

Figure 6.8 The effect of rotating speed
Figure 6.9  The effect of blade height
6.6 Conclusions

This chapter has presented the methodology for probabilistic structural design as applied to the silicon turbogenerator. The approach requires combination of the appropriate stochastic analysis and mechanics. Parametric studies were also performed to assess the overall failure probability for the baseline turbogenerator design.

The allowable failure probability has yet to be defined. This is an important decision that must be taken in combination with an assessment of the required reliability for the overall system. If the allowable failure probability is too high, the structural reliability is low. If the allowable failure probability is too low, the structural design is overly conservative. As a result, the overall performance of the device will be degraded. Therefore, it is very important to set an appropriate failure probability.

The probabilistic structural analysis not only provides the probability of structural failure, but also indicates the proper direction for improvement. A high Weibull modulus is the most effective way to reduce the failure probability. To achieve this, either a careful characterization in fabrication or proper proof testing is required to reduce the scattering.

As shown in Figure 6.8, the sensitivity of surface flaw dominated structures to sharp fillet radii indicates that a proper fillet radius is very important for structural reliability of structures fabricated from such materials. The strength of single crystal silicon is controlled by surface flaws. Therefore, better surface quality and large fillet radii are the most effective ways to increase structural reliability. On the other hand, the volume flaw dominated case shows a much lower sensitivity to the presence of sharp fillets. In this case, the strength is controlled by the microstructural features inside the volume such as grain boundaries, lattice defects, and inclusions. The best approach to enhance the material performance in this case is to improve the manufacturing process in order to produce fine grains and eliminate inclusions. It is less effective to increase the structural reliability by improving the surface quality and increasing the fillet radii.

Finally, it is important to point out that the probabilistic structural design flow shown in Figure 6.1 is also the brittle structural design methodology. Such a design methodology has been previously applied to other products and structures such as whiteware and ceramic turbines. It is also applicable for the brittle components of the
cooled engine structural design and is potentially important for the structural reliability evaluation of the microelectromechanical systems.
6.7 Reference


7.1 Introduction

In chapters 4, 5, and 6, the structural design methodology for the room temperature micro turbogenerator has been presented. This can be applied to some components of the all silicon cooled engine. As mentioned in Chapter 4, the maximum temperature of the turbine is above the brittle to ductile transition temperature (BDTT). In this regime, the material strength strongly depends on the structural temperature. In order to perform the design of the cooled engine, two extra tasks are required; First, material high temperature strength must be obtained by performing experiments or referring to the literature. Second, thermo-structural analysis is required in order to understand the temperature and stress of engine structure. Figure 7.1 indicates the structural design flow for the cooled engine. In this chapter, the high temperature material testing is addressed, including determination of the Young’s modulus, yield strength, and creep resistance. The thermo-structural analysis is discussed in Chapter 8.

Figure 7.1 Schematic of the structural design flow for the all-silicon cooled engine
7.2 High Temperature Behavior of Silicon

As shown in Chapter 4, silicon shows ductile behavior at temperatures above BDTT [Hirsch et. al, 1988], [Rybricki and Pirouz, 1988]. Mura et. al [Mura, et. al, 1996] obtained the stress-strain curves for single crystal silicon over a range of temperatures. As shown in Figure 7.2, the stress-strain curve for silicon is essentially invariant at temperatures less than 600 °C. However, at higher temperatures, the behavior becomes increasingly nonlinear elastic and then ductile. The yield strength is reduced to less than 100 MPa as the temperature approaches 1000 °C. A typical silicon high temperature stress/strain curve is shown in Figure 7.3. At small strains, the material response is linear elastic. As the strain increases, the stress reaches a peak value corresponding to the yield stress. After passing the peak, the stress decreases with increasing strain until it reaches a lower plateau. Corresponding to the flow stress.

The yield shear stress, \( \tau_y \), is also strain rate dependent [Alexander, 1986], [Alexander and Haasen, 1968]. This can be expressed by the following equation,

\[
\tau_y = c \left( \frac{\text{d}e}{\text{d}t} \right)^m \exp \left( \frac{U}{kT} \right),
\]

(7.1)

where \( U \) is the binding energy between atoms and dislocations, \( k \) is the Boltzmann constant, \( T \) is the absolute temperature. The coefficient \( c \) and the exponent \( m \) are material dependent constants. Eq.(7.1) shows that the higher the strain rate, the higher the material yield strength and as a consequence the material behavior is expected to be more brittle.

Pearson et. al [Pearson et. ai, 1957] performed creep testing of silicon whiskers at different temperatures. They found that the silicon whiskers yielded as the temperature exceeded 600 °C. Patel and Chander [Patel and Chander, 1963] examined the strength of CZochralski grown silicon at 815 °C and reported a value of 120 MPa. The yield strength in the range 400 - 600 °C has also been investigated by Castaing [Castaing, 1981] using hydrostatic compression. The results are between 2 and 1.1 GPa. Recently, with the development of MEMS technologies, microfabricated structures have also been used for material testing. Huff [Huff et. al, 1993] used cavity sealed membrane specimens to characterize the strength of silicon at various temperatures. Figure 7.4
summarizes these yield strength testing results. Note that Figure 7.4 shows the yield strength, not the flow strength of silicon, which will be substantially lower.

The creep of silicon was tested by several researchers. Reppich et. al [Reppich, Haasen, and Ilschner, 1964] tested single crystal silicon at temperatures between 800 °C - 940 °C with shear stress levels between 2 MPa to 7 MPa. Silicon creep behavior at 865 °C was investigated by Taylor and Barrett [Taylor and Barrett, 1972], [Gittus, 1975]. The test data showed logarithmic creep behavior in this regime. At temperatures of near 850 °C and of shear stress level 7 Mpa, the time elapsed to reach 5% strain is of the order of 5 minutes.

Myshlyeav et. al [Myshyaev, Nikitenko, Nesterenko, 1969], [Frost and Ashby, 1982] reported a higher creep resistance for silicon in the region. The fitted creep law

\[ \dot{\varepsilon} = 10^{11} \exp\left(-\frac{5.6e.V - 2.7 \times 10^{-21} \text{cm}^3\sigma}{kT}\right) \text{ (s}^{-1}\text{)}, \]  

(7.2)

shows that the strain rate at 865 °C and 100 MPa shear stress is $5.4 \times 10^7$ s$^{-1}$. This corresponds to a time to reach 5% strain of one hour.

The allowable operating temperature of the cooled silicon engine spool is determined by the silicon yield strength at temperature. Based on the data described above, it is unlikely that the maximum temperature of the spool should be allowable to exceed 1000K (727 °C). However, there is no silicon creep data reported for the range between BDTT and this temperature.

In steady state, the spool is subjected to centrifugal stress with an order of magnitude of several hundred MPa. Even though this stress level is less than the yield strength at temperature, the creep behavior of silicon may still result in failure of the structure in an unacceptably short time. There are two major reasons why creep is an important design concern. First, the stability of the micro bearing strongly depends on the air gap width. Creep will gradually change the air gap between the spool and the static structure and destroy the bearing performance. Second, if the strain is continuously increased, from the stress - strain curve of Figure 7.2, once the strain exceeds the yield strain, the
strength of silicon will drop from the yield strength to the flow strength, causing catastrophic failure.

![Stress-strain graph](image)

**Figure 7.2** Silicon stress-strain curve [Mura et. al, 96]

![Idealized stress-strain curve](image)

**Figure 7.3** Idealized silicon high temperature stress-strain curve
Figure 7.4 Yield strength of silicon as a function of temperature
7.3 Experimental Procedure

In order to find the mechanical behavior of silicon in the temperature range of interest, a test facility has been assembled. This facility consists of an INSTRON™ air furnace of split shell type and its associated controller, a MTS™ 810 material servo hydraulic test machine, an INSTRON™ 8500 controller, an INSTRON™ SiC four-point bend fixture, and a power PC for data acquisition. The entire system can be divided into four sub-systems.

1. The furnace sub-system: This consists of a MDS791 split furnace with maximum capacity of 1500 °C and its associated CU666D temperature controller, Molybdenum di-silicide (MoSi₂) heating elements, thermo couples, and cooling systems. Figure 7.5 shows the photograph of these components. Refer to Appendix A.5 for detailed specifications.

2. The sensing sub-system: This includes a 10 KN INSTRON™ load cell and an INSTRON™ LVDT contacting the specimen mid part with 2 mm sensing range and 1 μm resolution. Figure 7.6 shows the photos of these components.

3. The loading sub-system: This includes a 1 KN capable four-point bend fixture, a MTS 810 material testing system, an INSTRON™ 8500 controller, and alumina push rods. As shown in Figure 7.7, the four-point bend fixture was made of reaction bonded (RB) silicon carbide. The major and minor spans are 40 and 20 mm respectively. The MTS™ 810 machine and the INSTRON™ 8500 controller provide the controlled load for testing.

4. The data acquisition sub-system: The major data acquisition components include a power PC and associated A/D interfaces between computer and INSTRON™ 8500. LabView™ software is used for the purpose.

Figure 7.8 shows the overall high temperature mechanical testing system. The furnace and sensors are mounted on the MTS material testing machine.

---

1 The experimental setup was assisted by Mr. John Kane and Mr. Kevin Lohner.
Figure 7.5 The furnace system
Figure 7.6  The sensing system

Figure 7.7  The four-point bending fixture with a dummy specimen
Figure 7.8  Overall high temperature mechanical testing system
7.3.1 Determination of yield strength

The information obtained directly from the mechanical tests are load and deflection as a function of time. The mechanical properties of interest, such as the Young's modulus, yield strength, and creep parameters must be derived from this information.

A load-deflection curve can be easily constructed based on the raw data. The slope of the load-deflection curve is proportional to the Young's modulus. This fact can be used to obtain the Young's modulus as a function of temperature, T.

A room temperature load-deflection curve can be obtained from the room temperature test. Assume the slope is $S_{\text{RT}}$. At temperature T, the corresponding load-deflection curve can also be constructed. Assume the slope is $S_{\text{HT}}$. Given that the Young's modulus of single silicon at room temperature, $E_{\text{RT}}$, is well defined, the Young's modulus at temperature T can be obtained from

$$E(T) = \frac{S_{\text{HT}}}{S_{\text{RT}}} E_{\text{RT}}. \tag{7.3}$$

The value of $E_{\text{RT}}$ depends on the crystal orientation. For a (100) silicon wafer, the Young's modulus in the <110> direction and <100> direction are 169 and 132 GPa [Brantly, 1974] respectively.

For yield stress and yield strain, the load-deflection curve must be converted to stress-strain curves. In general, this is not straightforward because the stresses and strains are not uniform across an arbitrary section. The top and bottom surface of the specimen have the maximum tensile and compress stress and consequently they yield first. However, even when they yield, the interior of the specimen is still subject to elastic deformation. However, if we only have interest in the elastic regime and the yield stress, the conversion is straightforward.

For a four-point bending fixture, as shown in Figure 7.9, its major and minor span lengths are 2L and 2a respectively. The applied load is P. The maximum stress of the specimen can be expressed as

$$\sigma_{\text{max}} = \frac{My}{I} = \frac{3P(L-a)}{bh^2} \tag{7.4} ,$$
where \( M \) is the bending moment, \( h \) and \( b \) are the specimen thickness and width. Eq.(7.4) converts the applied loading into the maximum stress.

In the linear elastic regime, the stress and strain are related by the simple constitutive law

\[
\sigma_{\text{max}} = E(T)\varepsilon_{\text{max}}.
\]

\( E_{\text{tt}} \) can be obtained by Eq.(7.3).

### 7.3.2 Analysis for creep resistance

For characterization of creep resistance, an appropriate mathematical creep model must be assumed. In this work, power law creep is assumed. For a polycrystalline solid, power law is a frequently used creep model. It is not clear if it is suitable for single crystals. However, the objective of this creep characterization is not to identify the physics of creep but to fit a model with which to predict the life of the microengine structure. Therefore, a fitted power law creep model is convenient for design purposes.

Power law creep assumes that the strain rate \( \dot{\varepsilon} \), shear stress \( \tau \), and absolute temperature \( T \) have the following relationship,

\[
\dot{\varepsilon} = A\left(\frac{\tau}{\mu}\right)^n \exp\left(-\frac{Q}{RT}\right).
\]

where \( n \) is the creep exponent, \( Q \) is the activation energy, \( R \) is the universal gas constant, \( \mu \) is the shear modulus and \( A \) is a proportionality constant.

Creep experiments on ceramic materials are often performed in bending to avoid the problems of gripping and buckling associated with tensile and compressive tests, respectively. Although experimentally easy, bending tests do not always lend themselves to exact calculations of strains from measured beam deflections and stresses from applied loads.

In four-point bending, the ideal experimental procedure is one in which the deflection is measured at the center of the beam relative to the two inner load points.
Based on a geometric relationship, the radius of curvature can be extracted. However, this means using three probes to measure displacement. In our current testing system, there is only one probe to measure the center deflection. Certain mathematical modeling and assumptions must be applied in order to convert the center deflection to strain.

Hollenberg et al. [Hollenberg, Terwilliger, and Gorden 1970] described the method to calculate the stress and strain in a four-point bending creep test. However, they assumed the deflection was measured at one of the load points. Sato et al. [Sato, Chu, Kobayashi, Ando, 1996] modified their result so that the stress and strain could be obtained from the center deflection. Both of the above results are based on the assumption of an isotropically creeping material (i.e., creep behavior is the same under tensile or compressive loading.), power law creep, and small strain (maximum strain is less than 1%).

For a power law creeping material at a fixed temperature, the strain rate is a function of stress, given by

\[ \dot{\varepsilon} \propto \sigma^n. \]  \hspace{1cm} (7.7)

where \( n \) is the creep exponent.

The maximum stress \( \sigma_{\text{max}} \) for the 4-Pt specimen can be expressed as

\[ \sigma_{\text{max}} = J_1 P = \frac{P(L-a)}{bh^2} \cdot \frac{2n+1}{n} \]  \hspace{1cm} (7.8)

and

\[ \varepsilon_{\text{max}} = J_2 y_c = y_c \cdot \frac{4h(n+2)}{2L^2 + a \cdot n(2L-a)}. \]  \hspace{1cm} (7.9)

where \( J_1 \) and \( J_2 \) are the creep compliance of the structure. This allows the conversion of load and deflection to stress and strain.

Figure 7.10 plots the results of Eq.(7.8) and Eq.(7.9). It is interesting to note that the stress and strain can not be determined without knowing the creep exponent \( n \). However, in order to obtain \( n \), the stress and strain must be determined. As a result, an iteration process is required.
The following is the general procedure for creep analysis. It is also summarized in Figure 7.11.

1. Determine the test stresses and temperatures.

2. By Eq.(7.8) and (7.9), roughly estimate the required load.

3. Convert the center deflection and load to strain and stress for several assumed value of n.

4. Construct strain rate vs. time curves for several assumed n.

5. Plot log(strain rate) vs. stress for several n.

6. Measure the slopes of these curves. The slope is n'. If n' matches the assumed n, it means that the creep exponent has been found.

7. Plot log(strain rate) vs. 1/T for the corrected n. The slope is -(Q/R).

8. Insert the calculated Q and n into Eq.(7.6) to obtain the proportional constant A.

Figure 7.9 Schematic of a 4-point bending system
Figure 7.10  The creep compliance $J_1$ and $J_2$ as functions of creep exponent
Figure 7.11 Material creep parameter extraction procedure
7.4 Result for Silicon Yield Strength

The yield strength test specimens were die-sawed from 1000 µm thick 4" wafers. The specimen dimensions were 1000 µm × 9 mm × L (L > 45 mm). Refer to Appendix A.6 for detail. After die-sawing, the edge of the specimens were polished using SiC paper and slurry to minimize the surface damage. Figure 7.12 shows the die-sawed surface after several surface treatments. By comparison with the etched surface, it is rougher and contains more surface flaws with a larger characteristic flaw size. As a result, the four-point bending test is very difficult to performed at temperatures below or near BDTT since the surface flaws dominate the mechanical behavior of specimens, in which case brittle fracture results.

The specimen was then put into the test system. The system was heated up with a ramp rate of 12 °C/min until it reached the test temperature. An additional 15 - 30 minutes were required to stabilize the system. After the system reached an equilibrium temperature, the specimen was loaded in displacement control (1-10µm/s) until the specimen failed. The applied force and center deflection were monitored by the load cell and LVDT respectively. Those data were collected on a Macintosh G3 power PC via Lab View data acquisition software. Refer to Appendix A.8 for operating procedure.

Yield strength tests were performed at 600, 650, 700, 750, 800, 850, and 900 °C with displacement rates of 1, 2, 5, 7, and 10 µm/s. From Eq. (7.9) with n=1 (linear elastic case), the relationship between maximum strain rate and the center deflection rate, \( \dot{\epsilon}_{\text{max}} \), can be found as

\[
\dot{\epsilon}_{\text{max}} = 10.9 \dot{y}_c.
\]  

(7.10)

As a result, the maximum strain rate is in the range of 10⁻⁴ to 10⁻³/s.

Figure 7.13 shows a typical load deflection curve at 800 °C with a displacement rate of 2 µm/s. The relationship between yield strength and yield load can be obtained from Eq.(7.8) with n=1. This results in a conversion ratio of 3.3 MPa/N. At lower test temperatures (600 - 650 °C) with the high strain rates specimens usually failed by brittle

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2 The silicon yield strength test was assisted by Kevin Lohner, Mark Kempets, and Terran Mcronian
fracture due to the existence of surface cracks. The detail load-deflection plots of individual test included in Appendix A.9 for reference.

The results of the yield strength tests are summarized in Figure 7.14. They shows that the higher the temperature, the lower the yield strength. The yield strength is also shown to be strain rate dependent. The higher the strain rate, the higher the yield strength. It is also possible to find the exponent $m$ in Eq. (7.1) from Figure 7.14. Table 7.1 summarizes the range of exponents reported in the literature and that obtained from the current test data. It shows that the $m$ of test data is higher than the previous literature results. The temperature variation in $\langle 110 \rangle$ Young’s modulus is shown in Figure 7.15. Due to the noise of the load cell and the test machine, the curve fitted Young’s modulus shows considerable scatter. Nevertheless, it clearly indicates that the Young’s modulus decreases as temperature increases. The theoretical temperature sensitivity, $-70$ ppm/K, is within the scatter bounds.

The uncertainty of the yield strength testing comes from several aspects. Firstly, there is a $\pm 5 \, \mu m$ uncertainty (over 1000 $\mu m$) in specimen thickness due to the accuracy of the micrometer. As a result, it will result a $\pm 1\%$ difference in converting the load to stress. Secondly, the alignment error in the four-point bending test will result the reaction force redistribution on the supporting rollers. As a result, the shear force along the test specimen is changed. However, the bending moment at the specimen center is still remain the same. As a result, the misalignment has no contribution to the uncertainty. Thirdly, the noise level of the LVDT is less than 1 $\mu m$, or 0.1% of the total measurement range. Since the displacement information is not required for determining the yield strength, as a result, the yield strength is indendent to the LVDT accuracy. Finally, due to the response of the load cell, there is a 4 - 5 N noise level on the loading. As a result, it brings an uncertainty of about 15 MPa in yield strength. This is the dominant error in the test data. For yield strength testing at temperature at 600 - 700 $^\circ$C, in which the yield strength is between 200 - 500 MPa. Such a uncertainty represents a 7% to 3% error,, which is acceptable. However, at temperature range of 800 to 900 $^\circ$C. The yield strength is only in order of 150 to 80 MPa. Such a uncertainty will result 10 to 20% error. As a result, in order to have more accurate yield strength above 800 $^\circ$C, a better load cell is required. The detailed error analysis is in Appendix A.7.
Table 7.1  Strain rate exponent in silicon yield strength

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<th>m</th>
<th>Reference</th>
<th>Note</th>
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<td>Patel and Chaudhuri, 1963</td>
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<td>2.9 - 3.2</td>
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Figure 7.12  SEM micrographs of the specimen edge under different surface treatment. (a): after 500 grit sand paper and (b): after 1 μm alumina polishing
Figure 7.13  Silicon load vs. deflection curve at 800 °C.

Figure 7.14  Summary of silicon yield strength at different strain rate.
Figure 7.15 Young's modulus of <110> silicon
7.5 Creep of Silicon

Since it would be time-consuming to obtain the power law creep parameters due to extensive amount of required creep testing, the initial goal is to define a temperature limit based on the silicon creep resistance. As a result, the creep testing was focused on the temperature of about 1000 K and stress levels of about 100 MPa. The reason to choose such a temperature and stress level is that according to the thermo-structural analysis, 1000 K is a conservative upper bound on the turbine temperature and the turbine disk primary stress is of the order of 100 MPa after structural redesign. The major concern is that the radial creep of the turbine disk acts to close the bearing air gap.

Creep tests of silicon were initially carried out by NASA Lewis Research Center (LeRC) using a four-point bending configuration in a servohydraulic testing machine. The specimen dimension is $45 \times 9 \times 1$ mm. At 1000 K and 150 MPa, LeRC found that the specimen fractured almost instantaneously. At 1000K and 100 MPa maximum stress level after 12 hours, LeRC claimed that there was no observable creep deformation [Bansal, 1998].

At MIT, the creep of silicon at similar stress level and temperature was characterized using both dead weight and under load control in a servohydraulic test machine. For deadweight tests, in order to increase the sensitivity, the specimen thickness was reduced to 500 µm while maintaining the length (45 mm) and width (9 mm) as for. A deadweight of 2.1 lb. (~ 950 grams) was placed on the four-point bending test rig. This load results in 120 MPa maximum stress on the bottom surface of the specimen. Figure 7.16 shows the center deflection vs. time. Data were collected starting two hours after reaching the testing temperature (997 K). From Figure 7.16, the average center deflection rate is estimated as $30 \mu\text{m}/10^4\text{s}$ or $3\times10^{-7}$ m/s. From Eq.(7.10) the average strain rate on the bottom surface of the specimen is around $3\times10^8\text{s}^{-1}$. However, the alignment control of the deadweight over the test fixture is poor. This may introduce a significant error on the result.

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3 The silicon creep test at NASA was conducted by Mr. Narottam Bansal.
Figure 7.16  Silicon creep at 997 K subject to 2.1 lb. dead weight. (Specimen dimension = 45 x 9 x 0.5 mm, the maximum stress is around 120 MPa)

Load controlled creep tests at several temperatures and stresses level were also performed. Figure 7.17 shows the specimen center deflection vs. time at 600 °C (873 K) and 155 MPa. The creep behavior of silicon at 650 °C (923 K) and 155 MPa is shown in Figure 7.18. Both Figure 7.17 and 7.18 clearly shows two regime of creep. Those probably correspond to the secondary and tertiary creep stages. The average creep strain rates for the secondary creep are $1.25 \times 10^{-7} \text{s}^{-1}$ for 600 °C and $1.25 \times 10^{4} \text{s}^{-1}$ for 650 °C respectively. The result shows a strong temperature dependent effect in creep resistance. The creep resistance is reduced rapidly as temperature increases.

Figure 7.19 shows the silicon creep behavior at 700 °C (973 K) and 125 MPa. The average creep strain rate was estimated as $3 \times 10^{-5} \text{s}^{-1}$. Note that this is not consistent with the result performed by LeRC and the previous deadweight test. A more detailed investigation is required in the future.

Figure 7.20 shows the specimen center

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*The load controlled creep testing was mainly conducted by Mr. Douglas Walters*
deflection vs. time at 800 °C (1073 K) and 100 MPa. Since this stress level is equal to the silicon yield strength at 800 °C under low applied strain rates. The specimen experienced excessive deformations in less than 2 minutes.

The short term creep test goal is to perform the above test to define the temperature and stress limit for the all-silicon cooled engine based on the material creep performance. It is also valuable to model the silicon creep behavior at the operating temperatures and stress regimes of the engine by the method described in Section 7.3.2. In order to achieve this, more experimental work is required. This will be important future work.

Figure 7.17  Silicon creep at 600 °C under 155 MPa maximum stress (Specimen dimension = 45×9×1 mm)
Figure 7.18  Silicon creep at 650 °C under 155 MPa maximum stress (Specimen dimension = 45×9×1 mm)

Figure 7.19  Silicon creep at 700 °C under 125 MPa maximum stress (Specimen dimension = 45×9×1 mm)
Figure 7.20  Silicon creep at 800 °C under 100 MPa maximum stress (Specimen dimension = 45×9×1 mm)
7.6 Summary and Conclusion

This chapter presents the high temperature material characterization of silicon via the four-point bend test. The yield strength is found to be strain-rate dependent. The higher the strain rate, the higher the yield strength. The test yield strength at temperature between 600-750 °C is slightly less than those reported in the literature. Possible reason is because of the lower strain rate in testing. At temperature of 800-900 °C, the measured yield strength is higher than in the literature data but with 10 to 20% uncertainty.

The creep resistance reduces as temperature increases. However, due to the inconsistency between the deadweight and load controlled test, the limit of structural temperature based on creep performance is not well defined. Nevertheless, based on the worst case (the load controlled creep test), a temperature between 600 - 650 °C (873 - 923 K) should be able to satisfy the requirements for an all-silicon engine (~30 minutes duration of operation).
7.7 Reference


Bansal, N. [1998], personal communications on silicon creep testing.


CHAPTER 8. THERMO-STRUCTURAL ANALYSIS

8.1 Introduction

The structural analysis described in the previous chapters is based on the assumption that the structure is at a temperature below the brittle to ductile transition temperature (BDTT) of silicon. This is the certainly case for the silicon turbogenerator which is to be driven by a pressurized air jet, but the structural temperature in the all-silicon cooled jet engine is well in excess of BDTT. For this device, the maximum fluid temperature can potentially be as high as 1600 K. In this temperature regime it is necessary to account for plastic deformation and creep processes, in addition to brittle fracture. The structural temperature distribution strongly depends on the fluid field behavior, as characterized by parameters such as the free stream temperatures, velocities, and heat transfer coefficients. Furthermore, the material properties, especially the thermal conductivity, are also temperature dependent. Since heat transfer occurs between the structure and the fluid, there is a coupling between the fluid mechanics and the structural mechanics which is governed by the interfacial heat transfer. This coupling must be accounted for in the analytical modeling. It is also important to remember that the demands of the thermodynamic cycle drive the design towards the highest possible operating temperature, whereas the concerns for structural reliability and durability require the lowest allowable operating temperature.

This chapter is organized as the follows: Section 8.2 introduces the fundamental principles of heat transfer. Section 8.3 describes the characteristics of the micro jet engine and the relative importance of the three heat transfer mechanisms at its characteristic length scale. In section 8.4, the thermo-structural analysis of the rotating structure is presented in detail. The thermo-structural analysis of the static structures is briefly discussed in Section 8.5. In section 8.6, some requirements for packaging are addressed. Section 8.7 concludes this chapter.
8.2 Heat Transfer Fundamental

As shown in section 3.4, the thermo-structural analysis of the microengine is a conductive-convective heat transfer problem. For the stationary structure near the combustor, thermal radiation is also an important effect.

The governing equation of this problem is the same as Eq. (3.25)

\[ \nabla \cdot (k \nabla T) + q_v = \rho c \frac{\partial T}{\partial t}, \tag{8.1} \]

\[ q = h(T)A(T_s - T_w). \tag{8.2} \]

Where A is the surface area and \( \rho \) is the density.

In order to perform the heat transfer analysis, material thermal properties and the convective heat transfer parameters must be found. That is, thermal conductivity \( k \), heat capacity \( c \), surface heat transfer coefficient \( h \), and the free stream temperature \( T_w \).

Silicon has a temperature dependent thermal conductivity. As shown in Figure 8.1, its conductivity decreases as temperature increases. From 150 W/m K at room temperature to 32 W/m K at around 1000 K. This represents a five fold increase in thermal resistance. This effect can not be neglected even at this small scale because the material strength is very sensitive to the temperature within this range.

The specific heat capacity of silicon is also temperature dependent, as shown in Figure 8.2, it increases monotonically as temperature increases. At room temperature, a value of 700 J/Kg.K is reported [Purdue, 1970]. The variation of specific heat capacity is usually less of a concern because it only affects the transient thermal behavior of the structure.

Heat transfer coefficients at this scale are not easy to obtain by calculation or experiment. However they can be estimated by calculating the wall shear of the boundary layer flow. For flow at a free stream velocity \( U_w \), over a flat plate, a friction coefficient \( C_f \), is defined such that [Chapman, 1985]

\[ \tau_w = C_f \frac{\rho u^2}{2}. \tag{8.3} \]

Where the shear stress, \( \tau_{w'} \), is actually calculated from the following equation
\[ \tau_w = \mu \frac{\partial u}{\partial y} \mid_{w}. \] (8.4)

Where \( \mu \) is the dynamic viscosity and \( \frac{\partial u}{\partial y} \mid_{w} \) the velocity gradient at the wall.

For a flat plate,

\[ \frac{u}{u_w} = \frac{3y}{2\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3, \] (8.5)

where \( \delta \) is the boundary layer thickness. Making use of the relation for the boundary-layer thickness, gives

\[ \tau_w = \frac{3}{2} \frac{\mu u_w}{4.64} \left( \frac{u_w}{v_x} \right)^{1/2}. \] (8.6)

Combining Eq.(8.3) and Eq.(8.6), this leads to

\[ \frac{C_{f\mu}}{2} = \frac{3}{2} \frac{\mu u_w}{4.64} \left( \frac{u_w}{v_x} \right)^{1/2} \frac{1}{\rho u_w^2} = 0.323 \text{Re}_x^{-1/2}. \] (8.7)

Re-writing Eq.(8.7) as

\[ St_x = \frac{Nu_x}{Re_x \text{Pr}} = \frac{h_x}{\rho C_p u_w} = 0.332 \text{Pr}^{-2/3} \text{Re}_x^{-1/2}, \] (8.8)

where \( St_x, \text{Re}_x, \text{and Pr} \) are the local Stanton number, local Reynolds number, and Prandtl number respectively. \( \text{Re}_x \) can be thought of as a flow field index and \( \text{Pr} \) as a fluid property. Therefore, if the flow field can be solved, the local heat transfer coefficient can be calculated by Eq.(8.8).

The free stream fluid temperature is also not very clearly defined. Due to the high fluid speed, aerodynamic heating is expected. As a result, the adiabatic recovery temperature is used as \( T_- \) for the non-rotating surface. For rotating surfaces, the turbomachinery relative temperature is used.

To calculate the adiabatic recovery temperature, \( T_{awr} \), the recovery factor, \( r \), is defined by [Holman, 1981]

\[ r = \frac{T_{awr} - T_w}{T_0 - T_w}, \] (8.9)
where $T_e$ is the stagnation temperature of the fluid.

$$\frac{T_n}{T_e} = 1 + \frac{\gamma - 1}{2} M^2.$$  (8.10)

For laminar flow,

$$r = Pr^{1/2}.$$  (8.11)

Substituting Eq.(8.11) into Eq.(8.9) and Eq.(8.10), the adiabatic recovery temperature can be calculated.

Since most flow paths in the microengine are relative small, the fluid temperatures are particularly sensitive to the heat transfer. As a result, $T_m$ varies. The heat transfer dependent $h$ and $T_m$ define the coupling between the structures and fluid. The thermal state of both media must be computed simultaneously. Thermo-structural analysis must therefore be an iterative scheme. The analysis flow is shown in Figure 8.3. A thermal finite element heat transfer model of the structure is constructed. The surface heat transfer coefficients are obtained from the boundary layer correlation or CFD analysis. The initial free stream flow temperature is obtained from the ideal power cycle. Both the heat transfer coefficients and fluid temperature are then applied as the thermal boundary conditions for the structural heat transfer model. The output from the heat transfer model, (i.e., the temperature of the structure and the surface heat transfer rates) are then fed back into the fluid and cyclic models to modify the boundary conditions. This iteration between the structural model, fluid model, and cycle are repeated until a converged solution is obtained.
Figure 8.1  Temperature dependence of the thermal conductivity of microfabricable materials [Purdue, 1970].

Figure 8.2  Temperature dependence of the heat capacity of silicon
Figure 8.3  Schematic analysis and design flow for the thermo-structural analysis
8.3 Characteristic of the Micro Turbo Jet Engine

Unlike the initial hot micro turbogenerator, which is designed to generate electricity, the micro jet engine is designed to produce enough thrust for a micro unmanned air vehicle (mAV). There are some fundamental differences in the design. The major difference is that for the micro turbogenerator, the work extracted from the turbine is much larger than for the micro turbo jet engine because it represents the sum of the electrical power output plus the power required to drive the compressor and that required to overcome losses. On the other hand, in order to produce thrust, the exhaust gas from the turbojet must still have sufficient pressure and enthalpy. The power output of the micro turbo jet engine is just used to drive the compressor and overcome losses. As a result, the exhaust temperature of the micro turbo jet engine can be 200 K higher than the hot micro turbogenerator. This increases the structural reliability concerns. If the structure is not carefully designed, the high temperature material strength may not be sufficient to support the mechanical stress at high temperature.

For a turbojet engine design, the following design principle has been applied: The combustor and turbine inlet gas temperature should be as high as possible while the combustor structure and turbine must be restricted to a reasonable temperature for structural reliability; the compressor, on the other hand, should be kept as cool as possible to maximize its efficiency. The remainder of the stationary structure, should be as cool as possible to minimize heat losses to the surrounding environment. It is very hard to satisfy all of these requirement at the small scales. Thermal isolation is especially difficult. Conventional cooling schemes are also not suitable for application at this scale. As a result, performance must be sacrificed. However, the micro airplanes are designed to be non-reusable. The expected operational lifetime will be less than one hour. As a result, the structural design can be pushed to its limit.

The thermo-structural analysis is complicated by the coupling between structural, fluid, and cycle analysis. In particular, it is very difficult to accurately estimate heat transfer coefficients. An order of magnitude analysis between conductive, convective, and radiative heat transfer is essential. Based on this analysis, the unimportant terms can be ignored in order to simplify the overall analysis.
The micro turbojet engines are composed of a rotor and a stator structure. Heat transfer between solid structures and working or cooling fluids occurs on the surfaces. These surface can be grouped into the following categories.

1. External stator wall

2. Surfaces confining the mean flow path, such as disk, combustor, turbine, and compressor surfaces.

3. Walls of secondary flow passage such as blade tips and air bearings gaps.

Considering each of these classes of surface listed; for the external stator wall, depending on the outside air speed, natural or forced convective heat transfer is likely to apply. The radiative and conductive transfers (through air) are much less important.

For the mean flow path, the heat transfer coefficient estimates have ranged from 1000 to 5000 W/m²K. Although not precisely defined, these values provide an index to compare the relative importance of different heat transfer mechanisms.

The thermal conductance achieved by the three heat transfer laws can be expressed as the following:

\[ C_{Conductive} = \frac{KA}{L} \quad (8.12) \]

\[ C_{Conductive} = hA \quad (8.13) \]

\[ C_{Radiative} = \varepsilon\sigma(T_3^3 + T_i^2T_w^2 + T_w^3) \quad (8.14) \]

It is useful to introduce an "equivalent heat transfer coefficient", defined as

\[ h_{equivalent} = \frac{C}{A} \quad (8.15) \]

The equivalent heat transfer coefficient achieved through conduction is therefore,

\[ h_{Conductive} = \frac{K}{L} \quad (8.16) \]

For radiative heat transfer,

\[ h_{Radiative} = \varepsilon\sigma(T_3^3 + T_i^2T_w^2 + T_w^3) \quad (8.17) \]
where $\varepsilon$ is the emissivity, $\sigma$ is the Stefan-Boltzmann constant. From Eq.(8.16), there is a length scale effect in the equivalent conductive heat transfer coefficient. This implies that even though the thermal conductivity of stationary air is extremely low (~ 0.065 W/m K), if the air gap is sufficiently small, the magnitude of the equivalent heat transfer coefficient can be comparable to the typical microengine turbomachinery surface heat transfer coefficients. This is shown in Figure 8.4. For example, for the micro bearing with a 10 to 20 $\upmu$m air gap, the effect of heat conduction is of the same order of magnitude as the typical convective heat transfer into the turbine. For the compressor and turbine tip clearance, air gaps are of a few microns in width. As a result, conduction dominates the heat transfer in these regions. For the mean flow, since the cross-sectional diameter of the passages are of the order of 100 $\upmu$m or higher, the effect of conductive heat transfer is less important.

The above analysis has two important conclusions. First, it shows that conductive heat transfer is very important in the small air gaps, which are characteristic of the microengine. Since the flow speed is also reduced inside these air gaps due to viscous effects, the convective heat transfer coefficients are also reduced. Second, it provides a helpful guideline for engine design. The effect of conductive heat transfer must be incorporated in deciding the dimensions of the mean flow paths.

For radiation heat transfer, there is no length scale effect. It is also important to point out that the radiation heat transfer between the microengine static and rotating structures are similar to cavity radiation heat transfer [Chapman, 1984]. As a result, the emissivity is not important and can be treated as a black body radiation problem. The equivalent heat transfer coefficient depends on the hotter structure temperature ($T_{s1}$) and cooler surface temperature ($T_{s2}$). As shown in Figure 8.5, the equivalent heat transfer coefficients over the possible operating regime (surface temperatures 800 - 1100K) are 100 - 300 W/m$^2$K with an emissivity of 1. These are much smaller than the convective heat transfer coefficients. As a result, the radiative heat transfer is not an important factor. At most it only needs to be incorporated during the final detailed analysis.
Figure 8.4  The equivalent conductive heat transfer coefficient as a function of air gap.

Figure 8.5  The radiative heat transfer coefficient as a function of structure surface temperature. (The emissivity is assumed to be 1)
8.4 Thermo-Structural Design of the Rotating Structures

The spool is considered to be the most critical structure of the entire engine. For the turbogenerator, surface flaw induced brittle fracture under centrifugal stress is the major design concern. For the spool of the micro jet engine, the situation is complicated by both the stress and temperature fields. If the structural temperature is less than the BDTT, brittle fracture is still the primary concern. For structural temperatures higher than BDTT, fracture is less of a concern. However, the available yield strength and creep resistance are reduced and are very sensitive to temperature variation. Unlike the fracture strength, which can be enhanced by improving the surface quality, the yield strength and creep resistance can not be improved without changing the material composition and microstructure. Therefore, an upper limit for the allowable structural temperature must be pre-defined. The temperature at the highest stress location can not exceed this value. This limit can be obtained by a simplified first order analytical heat transfer model or simple FE calculation. Once the calculation indicates that the structure will not violate the constraint, the optimization between structural and cycle design must be performed to refine the design. For the turbomachinery design, it is desirable to thermally isolate the turbine and the compressor in order to maintain the compressor efficiency. Therefore, a turbine/compressor connection with very high thermal resistance is desired provided that the turbine temperature is within the material capability. In the ideal case, when the resistance is infinite, an isolated turbine disk is sufficient to describe the heat transfer characteristic of the spool system.
8.4.1 Flat rotating disk model

The simplest heat transfer model for the turbine and the compressor is a flat disk subject to convective boundary conditions. Although the geometry of the flat disk is far from the reality, it yields much useful information for guiding design iterations. Consider a disk with thickness \(d\). As shown in Figure 8.6, both sides of the disk surfaces are subject to heat transfer with fluid temperatures \(T_h\) and \(T_b\) and surface heat transfer coefficients \(h_r\) and \(h_b\) respectively.

The Biot number can be defined as \(Bi = \frac{h_r d}{k}\). Inserting the order of magnitude of these variables into the Biot number definition, \(o(k) = 100\), \(o(d) = 0.0001m\). As a result, \(o(Bi) = o(h_r \times 10^8)\). The order of the heat transfer coefficient is in \(o(10^3)\). Therefore, the order of the Biot number in the baseline disk is \(o(0.001)\). Such a small Biot number implies that the temperature of the disk will be nearly isothermal.

For sufficiently small Biot numbers, the approximate average structural temperature can be calculated as

\[
T_s = \frac{T_h h_r + T_b h_b}{h_r + h_b}.
\]  

(8.18)

Equation (8.18) shows that the average structural temperature is entirely determined by the thermal boundary condition. Another useful insight obtained from this simple model is that the thermal barrier coatings are not useful at such a small scale. For a 10 - 20 \(\mu m\) refractory ceramic coating, its associate Biot number can be calculated by the above expression. For example, fused silica has a thermal conductivity of the order of 1 \(W/mK\). As a result, the Biot number is still \(o(10^3)\). The temperature of the coating is uniform and provides no insulating effect. The same reasoning also argues against introducing a thermal isolation shaft between the turbine and the compressor.

This one dimensional model can also be used to indicate the possibility of thermally isolating the turbine. Table 8.1 shows the parametric study results obtained using Eq(8.18) and the associated FE analysis. \(T_h\) is assumed to be 1300 K. For the turbine, \(T_h\) and \(h_r\) represent heat input from the high pressure, high temperature gas. \(T_b\) and \(h_b\) represent the cooling system of the turbine. A value of \(T_h\) in the range 600 - 800 K is
assumed. The heat transfer coefficient is velocity dependent. As a result, \( h_r \) is greater than \( h_a \). Therefore, \( h_r = h_a \) represents the limiting situation. From Table 8.1, it can be found that the corresponding structural temperature is at least 1000 K, depending on the temperature on the reverse side of the disk. This is too high to carry the centrifugal stress. This implies that a thermally isolated turbine disk is not a viable design for an all silicon engine.

<table>
<thead>
<tr>
<th>( T_s ) (K)</th>
<th>( h_r ) (W/m(^2)K)</th>
<th>( T_s ) (K) (FE Solutions)</th>
<th>( T_s ) (K) (Eq. 7.4.1.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>1000</td>
<td>1045</td>
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<td>664</td>
</tr>
<tr>
<td></td>
<td>100</td>
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</tbody>
</table>

Figure 8.6 The 1-D heat transfer model
8.4.2 Baseline spool analysis

The baseline microengine design, as shown in Figure 1.1, uses a shaft to connect the micro turbine and the micro compressor. The shaft is not only a mechanical connection but also provides an additional heat transfer path to reduce the turbine temperature. The length and cross-sectional area control the thermal resistance of the connection. A shorter and wider shaft means a lower thermal resistance, and a more uniform temperature.

An axisymmetric (2-D) FE model was constructed to perform a parametric study for this configuration. The baseline design shown on Figure 1.1 serves as the spool geometry for this FE study. As shown in Figure 8.7, the turbine and compressor blades are modeled as hoops of material. The surrounding fluid temperatures are obtained from the engine cycle analysis. The averaged heat transfer coefficients on the turbine and compressor bladed surfaces are obtained from a boundary layer correlation analysis. They are 1600 W/mK and 2500 W/mK for compressor and turbine bladed surfaces respectively. The adiabatic wall temperatures of turbine are assumed to vary from 1300 K to 1000 K. This is equivalent to a fluid total temperature ranging from 1450 K to 1250 K. The thermal boundary condition of the remaining surfaces are not well defined. Reasonable estimates were applied and parametric studies were performed to determine the trend. In this case, fluid temperatures were assumed on the turbine and compressor backside surfaces of 600 K and 500 K, respectively. The heat transfer coefficients are 20% of the corresponding front side values.

Figure 8.8 shows the temperature contour of the entire spool. The highest temperature is located at the turbine blade. The coolest temperature appears at the compressor blade. Most of the temperature drop is across the shaft. The heat flow is indicated in Figure 8.9. The overall heat transfer from the turbine into the compressor is roughly equal to the heat flux times the shaft cross sectional area. For this geometry and the boundary condition described above, the total heat transfer is approximately 4 Watts. One method to control the turbine temperature is to control the conductance of shaft. That is, varying the length and the cross-sectional area of shaft. Figure 8.10 shows the effect of shaft conductance to the temperature distribution of the entire spool.

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1 The heat transfer coefficients was calculated by Dr. Stuart Jacobson using correlation method.
Higher shaft conductance implies a more uniform spool temperature and higher heat transfer from the turbine into the compressor. At steady state, this heat must be transferred from the compressor surface into the working fluids. This results in more power required for compressing gas and the efficiency of compressor would be reduced.

![Image of a 2-D axisymmetric heat transfer model](image)

**Figure 8.7** The 2-D axisymmetric heat transfer model

The small stage or polytropic efficiency of a compressor, $\eta_{m,r}$, is defined as the isentropic efficiency of an elemental stage in the process such that it is constant throughout the whole process. For a compression process between inlet 1 and outlet 2, this can be expressed as [Cohen et. al, 1972]

$$\eta_{m,r} = \frac{\ln\left(\frac{p_2}{p_1}\right)^{\gamma^{-1}/\gamma}}{\ln(\frac{T_2}{T_1})}.$$  \hspace{1cm} (8.19)

Since the heat transfer from the the compressor to the fluid will increase $T_{\nu}$ as a result, the small stage efficiency is decreased. The overall process can be treated as a combination of many small stages. Therefore, the overall compression efficiency is
reduced if heat transfer occurs from the compressor into the fluid. The source of this heat input comes from the turbine to compressor heat transfer.

The axisymmetric model cannot describe the temperature of the individual turbine blades. Since the blades have been modeled as a hoop of material, the aspect ratio is much lower than the reality. This results in a lower thermal resistance at the blade region. For real blades, due to the higher aspect ratio, the thermal resistance in the longitudinal direction is higher. The higher thermal resistance may result in a higher blade temperature, or, in the worst case, hot spots at the blade tip. A three-dimensional heat transfer model is required to obtain the necessary information. In addition, the 2-D model must also be calibration using the 3-D model.

![Figure 8.8 The 2-D temperature contour](image)

In order to reduce the computational cost so that the 3-D heat transfer model can be used for parametric studies, further simplifications and assumptions must be made. Since the major structural heat transfer analysis concern is the turbine, this component must be modeled in detail. On the other hand, the structural temperature of the

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1 The overall engine efficiency was calculated by Jonathan Protz
compressor is always less than turbine. As a result, if the turbine structure temperature can be kept below the limit, compressor structure will be too. Therefore, from the structural point of view, it is not necessary to include directly the compressor in the model, although the thermal conditions it imposes must be incorporated. In addition, at steady state, the amount of heat input from the hot working fluid into the turbine is equal to the heat transfer from the turbine into the compressor. This also equals the heat transfer from the compressor structure into the working fluid. Therefore, if the amount of heat transfer from the turbine into the compressor is known, the efficiency reduction can be obtained. As a result, the compressor has not been explicitly modeled in the 3-D heat transfer model. However, the thermal conditions imposed by the compressor on the turbine are obtained from the axisymmetric model and applied to the 3-D turbine model. This significantly reduces the size of the 3-D model.

![Compressor and Turbine Diagram](image)

**Figure 8.9** The 2-D heat flux contour

Figure 8.11 is the 3-D heat transfer model. Only the turbine appears in the model. All heat transfer boundary conditions are the same as in the 2-D model. The influence of the compressor and the shaft is replaced. This heat flux is obtained from the 2-D model. For the parametric study, the amount of heat flux can be used as a variable parameter so that the effect of heat extraction on the turbine temperature can be studied.
Figure 8.12 shows the temperature contours of the micro turbine. The maximum temperature is 988 K and is located at the blade tip. This is 30 K higher than the 2-D model. This result can be associated with the increase in total surface area in the 3-D model as well as the effect of the blade aspect ratio. However, there no "hot spots" are obtained. The reason for this is that due to the small length scale, although the aspect ratio is increased, the overall local Biot number is still low. On the other hand, the minimum temperature in the 3-D model, 802K, is similar to that found in the 2-D model at the corresponding location. Overall, the 2-D and 3-D models agree with each other sufficiently well to validate results obtained from the 2-D model. These models can be used for parametric analysis to understand the relative importance of each design parameter and to perform sensitivity studies.

![Figure 8.10](image_url)  
**Effect of thermal resistance**
Table 8.2  Values of the baseline thermo-structural analysis parameters

<table>
<thead>
<tr>
<th>Frontside fluid $T_1$</th>
<th>Frontside $T_{ad} \text{ubatic}$</th>
<th>Frontside $h$</th>
<th>Backside $h$</th>
<th>Backside fluid $T_1$</th>
<th>Shaft heat transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1450 - 1150 K</td>
<td>1300 - 1000 K</td>
<td>2500 W/m$^2$K</td>
<td>500 W/m$^2$K</td>
<td>600 K</td>
<td>6.6 W</td>
</tr>
</tbody>
</table>

A set of parameters and their ranges must be determined in order to perform the parametric study. Table 8.2 shows the baseline design parameters. The shaft heat transfer, 6.6 W, comes from the 2-D heat transfer analysis using constant room temperature thermal conductivity of silicon. The importance of each design factor has been studied parametrically by varying its value systematically while keep the other parameters at the baseline value.

![Schematic of the 3-D spool heat transfer model](image)

Figure 8.11  Schematic of the 3-D spool heat transfer model. Only the turbine is included

The effect of heat extraction on the turbine temperature reduction is shown in Figure 8.13. There are three curves in the figure. Corresponding to the maximum and minimum turbine temperature, and the blade root temperature. Although the blade tips experience the maximum temperature, their stress level is much lower than the root. Figure 8.14 shows that the higher the heat extraction, the lower the structural temperature. For this particular geometry and boundary condition, 10 watts of heat...
extraction is required to reduce the blade root temperature to 900 K. Note that the baseline spool design can only provide 4 W heat transfer, therefore, the length and cross section of the shaft must be changed to increase the heat transfer.

Figure 8.12 A typical 3-D turbine temperature contour. The highest temperature is at the blade tip. The lowest temperature is at the junction between shaft and disk.
The effect of turbine fluid inlet temperature has also been investigated. Figure 8.14 shows the result. The maximum fluid inlet adiabatic wall temperature varied from 1300 K to 1600 K and the turbine blade exit temperature was kept unchanged. As a result, the fluid temperature drop is not the same for each inlet temperature. This means that the pressure ratio of the turbine is different for each case. As a result, it does not exactly present the real situation. However, given such assumptions, the blade root temperature only increases 30 K for a 300 K increases in inlet temperature. For a constant pressure ratio turbine, if the inlet temperature is increased by $\Delta T$. The entire turbine surface fluid temperature will be raised by roughly the same amount. As a result, the structural temperature will be more sensitive to the fluid temperature than in this study.

The effect of the turbine front surface heat transfer coefficient on the turbine temperature is shown in Figure 8.15. This shows that the structural temperature increases as the heat transfer coefficient increases. A 20% increase in heat transfer coefficient above the baseline value will increase the blade root temperature by 20 K. This study shows the sensitivity of the structural temperature to the variation of heat transfer coefficients. This is important because it is very difficult to obtain an accurate measurement or to calculate the heat transfer coefficients.

The sensitivity of the structural temperature on the fluid conditions in the backside of the turbine is another important issue since the thermal boundary conditions of the disk backside come entirely from estimates and need to be verified by experiments. In Figure 8.16, the backside fluid temperature has been varied from 400 K to 900 K. The corresponding blade root temperature is changed from 925 K to 965 K. This indicates that a 4.3% temperature increase results from a 225% increment in backside fluid temperature. The effect of the backside heat transfer coefficient on the turbine temperature is shown in Figure 8.17. The heat transfer coefficient has been varied from 200 W/m²K (7% of the front side value) to 1200 W/m²K (46% of the front side value). The blade root temperature changes from 960 K to 900 K, i.e., a 6.7% blade temperature variation for a 600% change in backside heat transfer coefficient. Both Figure 8.16 and Figure 8.17 indicate that cooling due to backside convective heat transfer is not important. The major cooling path is the shaft heat transfer into the compressor.
Figure 8.13  The effect of heat extraction on turbine temperature

Figure 8.14  The effect of maximum turbine inlet temperature
Figure 8.15  The effect of heat transfer coefficient variation

Figure 8.16  The effect of backside fluid temperature variation
Figure 8.17 The effect of backside heat transfer coefficient variation
8.4.3 Micro turbojet engine spool analysis

The turbine inlet total temperature for the micro turbojet engine is 1600 K. This corresponds to a blade relative temperature of 1400 K. The turbine exit temperature is approximately 1300 K, which is higher than that used in the baseline parametric study since it is designed to store more energy in the fluid in order to obtain higher thrust. As a result, the temperature of the turbine will be increased to above 1100 K. This is beyond the material capability. The baseline geometry must be changed.

There are three major changes implemented in the redesign. Firstly, the shaft becomes solid and its diameter is increased and the length is reduced. When the shaft diameter is close to the turbine disk diameter, it essentially becomes a shaftless design. Secondly, the diameter of turbine is reduced while the compressor disk is fixed. And thirdly, in order to generate more thrust, the engine size is scaled up by a factor of 2. That is, the diameter of the compressor disk becomes 8 mm.

The reason to reduce the turbine diameter is to reduce the heat transfer. There are three major control mechanisms for reducing heat input to the structure. I.e., reducing fluid temperature, reducing heat transfer coefficient, and reducing the surface area. If the fluid temperature is reduced, the overall performance will degrade and the cycle may be broken. The heat transfer coefficient is proportional to the square root of the flow speed. Therefore, even if the flow speed is cut by a factor of two, the heat transfer coefficient can only be reduced by 30%. This is not, therefore, a practical solution. In comparison with the two previous choices, reducing heat transfer by reducing surface area is feasible. A smaller turbine has a smaller power level capacity. However, the only functional requirement for the turbine is to drive the compressor. If the extracted power is sufficient to drive the compressor, this is a feasible option. The major penalty is that the turbine blades needs to be redesigned in order to extract enough power in a smaller turbine.

The heat transfer coefficients have been re-calculated based on the laminar flow integral correlation for variable velocity and density flow [Jacobson, 1998]. Their values are shown in Table 8.3. The thermal boundary condition for the bearing surface is not well defined. This requires a convective-diffusive heat transfer model to solve. However, two worst-case scenarios can be identified. First, if the fluid temperature of
the bearing air gap is less than the structure's, the bearing fluid will cool the structure. Therefore, an adiabatic boundary condition at the bearing surface will represent the worst case. Second, if the bearing fluid temperature is higher than the structure temperature, heat will transfer into the structure. The bearing heat generation due to the viscous drag is calculated to be 13 W [Protz, 1998]. Therefore, the second worst case occurs if these 13 watts are transferred into the spool. Figure 8.18 shows the heat transfer model and the corresponding thermal boundary conditions for the turbojet spool.

![Figure 8.18](image1)

**Figure 8.18** The micro jet engine spool heat transfer model

![Figure 8.19](image2)

**Figure 8.19** Microjet engine spool temperature contour
Figure 8.19 shows the temperature contour for the 2-D heat transfer model. The compressor and turbine disk thickness are 200 μm and 600 μm. The compressor and turbine blade heights are increased to 400 μm and 330 μm respectively in order to extract the required power. The maximum and minimum structural temperature appear at the turbine and compressor blade tips as before. For the adiabatic bearing wall case, the spool temperature is between 773 K and 689 K. For the case with all the bearing generated heat transferred into the spool, the spool temperature is between 815 K and 723 K. These values correspond to a sufficient material strength (see Figure 7.14) to guarantee the structural integrity of a silicon structure. In the worst scenario, there is a 100 K margin, however, since the fluid is heated on the compressor side, the spool temperature also becomes higher. Therefore, the results for the initial structural heat transfer analysis must have some margin to account for these additional temperature increases.

The total heat transferred from the turbine to the compressor is obtained by integrating the heat flux through each heat transfer element on the turbine surface. At steady state, this is equal to the heat transfer from the turbine to the compressor. The total heat transfer from the turbine to compressor is 35 W. This does not include the bearing heat transfer. Table 8.4 indicates the amount of heat transfer by component.

The next step is to calculate the heat transfer into the fluid. This changes the fluid temperature and the heat transfer coefficients. The new fluid temperature and heat transfer coefficients are then used as the new boundary condition for re-calculating the temperature of the structure. This solid/fluid iteration is repeated until the solution converges.

The influence of the compressor and turbine disk thickness are also investigated parametrically in order to optimize the design. Figure 8.20 shows the effect of disk thickness on the initial maximum and minimum structural temperature. Finally, the turbine disk and the compressor disk thickness were set at 600 μm and 200 μm respectively, which also accounts for the constraints imposed by microfabrication.

Heat transfer parametric studies based on the new spool geometry were conducted to address the uncertainties and the sensitivity of parameters. The effect of compressor
exit temperature, turbine inlet temperature, and the convective heat transfer conductance, hA, were studied.

![Diagram](image)

Figure 8.20 Geometry parametric study

The heat transfer from the compressor structure into the fluid raises the fluid temperature. The temperature rise was estimated to be within 200 K. The heat transfer boundary conditions used to find the effect of the compressor exit fluid temperature are the same as those shown in Table 8.3 except those quantities with the symbol "①", which were varied from 400 to 700 K. Figure 8.21 shows the results. Higher compressor exit temperature result in higher structural temperatures. Significantly, at 700 K, the structure temperature is still below the material allowable limit. On the other hand, the total heat transfer between the structure and fluid is reduced as the fluid temperature increased. However, there is only a 20% change in heat transfer with 300 K variation in compressor exit temperature.

The effect of turbine inlet temperature is shown in Figure 8.22. The temperature drop is fixed at 150K, which means the turbine temperature is the same in each case. The
thermal boundary conditions are the same as shown in Table 8.3 except those quantities with the symbol "@", which have ±200 K variation. At a turbine inlet relative temperature of 1600 K, (which means the turbine inlet total temperature is 1800 K.), the structure will be at or above the temperature limit. The heat transfer increases with increasing inlet temperature. As a rule of thumb, about 3-5 watts heat transfer arises per 100 K increment in turbine inlet temperature.

In order to account for the surface area mismatch between the real geometry and the 2-D model, a third parametric study was conducted. The effect of the convective heat transfer conductance, hA, on the turbine blade region is shown in Figure 8.23. A ±50% variation in hA is investigated. The structural temperature increases as hA is increased and approaches the limit as the conductance is increased to 150% of its nominal nominal value. This also shows that there are 3 watts of heat transfer per 10% conductance variation.

Table 8.3 The initial boundary condition for the spool heat transfer analysis

<table>
<thead>
<tr>
<th>Component Surface</th>
<th>Heat transfer Coefficient (W/m²K)</th>
<th>Fluid Temperature K (total or relative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor Hub (top)</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>Compressor Hub (side)</td>
<td>70</td>
<td>300</td>
</tr>
<tr>
<td>Compressor Disk (front)</td>
<td>1000 - 2000</td>
<td>300</td>
</tr>
<tr>
<td>Compressor Blade (inlet)</td>
<td>2900</td>
<td>300</td>
</tr>
<tr>
<td>Compressor Passage</td>
<td>2900 - 960</td>
<td>300 - 500</td>
</tr>
<tr>
<td>Compressor Blade (exit)</td>
<td>960</td>
<td>150</td>
</tr>
<tr>
<td>Compressor Disk (back)</td>
<td>1000</td>
<td>150</td>
</tr>
<tr>
<td>Bearing Surface</td>
<td>assume heat flux BC</td>
<td></td>
</tr>
<tr>
<td>Turbine Blade (inlet)</td>
<td>3300</td>
<td>1450</td>
</tr>
<tr>
<td>Turbine Passage</td>
<td>3300 - 1100</td>
<td>1450 - 1300</td>
</tr>
<tr>
<td>Turbine Blade (exit)</td>
<td>1100</td>
<td>1300</td>
</tr>
<tr>
<td>Turbine Disk</td>
<td>1200 - 1300</td>
<td>1300</td>
</tr>
<tr>
<td>Turbine Hub (wall)</td>
<td>230</td>
<td>1300</td>
</tr>
<tr>
<td>Turbine Hub (top)</td>
<td>100</td>
<td>800</td>
</tr>
</tbody>
</table>
Table 8.4  The heat transfer from fluid to spool

<table>
<thead>
<tr>
<th>Component</th>
<th>Hub region</th>
<th>Flat disk region</th>
<th>Blade region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat transfer (W)</td>
<td>0.1</td>
<td>10.8</td>
<td>24.2</td>
</tr>
</tbody>
</table>

Figure 8.21  The effect of compressor exit temperature (a) on structural temperature and (b) heat transfer into turbine
Figure 8.22  The effect of turbine inlet temperature (a) on structural temperature and (b) heat transfer into turbine. Adiabatic recovery temperature is used (a 1600 K turbine inlet temperature ~ 1450 K of the adiabatic recovery temperature in current design.)
Figure 8.23  The effect of nominal convective conductance on structural temperature and (b) heat transfer into turbine
8.4.4 Summary of the parametric study

From the previous parametric study, the temperature of the compressor is below BDTT. As a result, the brittle design methodology is always applicable for the compressor. This is very important since as the turbine disk is shrunk, the centrifugal stress in the turbine is reduced and the compressor blade roots have the highest stress in the system. It is always possible to obtain a high strength in silicon by modifying the surface if the temperature is below BDTT.

This study also shows that the structural temperature increases as the fluid in the compressor and the turbine inlet temperature increases. However, the structural temperatures are considerably low even when fluid at these locations are at their possible maximum temperatures. As a result, if the current heat transfer coefficients are accurate, there is some margin available to accommodate the uncertainty in fluid temperature distribution. Increasing in turbine inlet temperature is also possible.

Unlike the fluid temperature, which can be predicted with a reasonable accuracy, the heat transfer coefficient prediction is probably only accurate to within 50%. A more detailed heat transfer analysis is required to obtain better estimates of the key parameters. However, basic conclusions can be drawn from the current analyses and estimates. In particular, it is concluded in order to reduce hA, it is helpful to design a hot turbine with a smaller surface area and with a smaller number of blades.
8.5 Thermo-Structural Design of the Static Structures

Figure 8.24 shows a schematic plot for the entire all-silicon turbojet engine. Except the rotor, which is composed by turbine and compressor, the rest of the engine structure can be treated as the stator.

Static structures occupy most volume of microengine. Its major components include combustor, casing, and structures for guiding fluid flow such as thrust and journal bearings. As the rotating structures, the behavior of static structures is strongly depended on the structural temperature. It also shows a strong trade off between the structural reliability and the efficiency of the microengine.

The thermo-structural analysis of the stationary structures were mainly analyzed using MSC/Nastran at MIT Lincoln Lab [Huang, 1998]. Since there are no rotating structures, the stress level is much lower than the spool. As shown in Chapter 5, the magnitude of the hoop stress induced by internal pressure force is only around 10 MPa. Therefore, the mechanical stress is not a concern if the structure temperature is not very high to maintain sufficient material strength. As a result, the initial focus of this analysis is not on the structural reliability but on the engine performance. At small scales, the combustor heat loss due to surface heat transfer becomes more important. The goal of the stationary structural analysis, is therefore, to provide information to guide redesign to reduce heat transfer while maintaining structural integrity.

Due to the presence of the combustor, the maximum temperature of the stationary structure is potentially even higher than that of the spool. In order to reduce the heat transfer to the environment, the surface temperature must be minimized (in order to reduce convective heat transfer.). From the principle of the conservation of energy, reducing the heat transfer to the surroundings means that the internal energy is increased. As a result, the basic design goal leads to three concerns in structural design.

1. The increase in internal energy raises the stationary structure temperature (except for the structure near the outside surface). Such a temperature increase will eventually affect the temperature of spool and reduce the reliability of the spool.

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1 The thermo-structural analysis of static structure was performed by Dr. Eugene Huang at MIT Lincoln Laboratory.
2. The increase in static structure temperature results in a reduction in available material strength. For example, if the temperature is above 1200 K, the material yield strength is less than 50 MPa.

3. High internal structure temperatures and low surface structural temperatures implies a steep temperature gradient, which can result in thermal induced stresses [Jaeger, 1945],[Boley and Weiner, 1985]. The order of magnitude of such thermal stresses can be as high as the available material yield strength if both the structural temperature and temperature gradient are sufficient high.

For item (1), since the spool design is robust, therefore, it is still stable even if the thermal state of the static structure is varied significantly. Items (2) and (3) are more serious concerns. Since the cycle design pushes toward a higher internal structure temperature and higher temperature gradient, the reduction in material strength and increase in thermal stress may lead to a conflict with the structural design and eventually, may result in a trade-off between engine performance and structural integrity.

Figure 8.25 shows the temperature and thermal stress contours for a particular thermal boundary condition. The maximum thermal stress at combustor is around 50 MPa at 1180 K. Although this is a small stress for lower temperature, at the current level of temperature (1180 K), it is comparable to the available strength (~ 80 MPa). This analysis indicates that for stationary thermo-structural analysis, not only the temperature level, but also the temperature gradient is also an important design consideration.

Parametric studies have been conducted by 2-D finite element models. The detailed work is described elsewhere [Huang, 1998]. These parametric studies provides the basic information for the static structure integrity and the heat loss with respect to different static structural design. Detail 3-D analysis have not been performed yet. This is an important future work.
Figure 8.24 The schematic plot of the all-silicon turbojet engine [Protz, 1998]
Figure 8.25  Temperature and thermal stress contours of the static structure [Huang, 1998]
8.6 Packaging

The thermo-structural analysis of the stationary structures is directly related to the device packaging. Unlike microelectronics packaging, which has as its principal goal to maximize the heat dissipation in order to maintain the device reliability, the major consideration of the micro jet engine packaging is to prevent the loss of thermal power to the environment. In addition to power loss prevention, important packaging issues include the formation of interconnects from the jet engine to the other modules such as the starter, fuel pipes, power electronics etc. In addition, the overall weight of device is also critical in order to maximize the thrust to weight ratio. In this section, only the heat loss prevention is discussed.

Thermal isolation can be achieve by two different approaches. Either by reducing surface heat transfer coefficients or by controlling the surface temperature. From the perspective of device operation, it is difficult to control the surface heat transfer coefficients since these directly relate to the flow fields. Control of the surface temperature, is more feasible. In order to reduce the surface temperature, high thermal resistance materials, must be used. The thermal conductivity of amorphous silica is only 1 - 3 W/mK. A several hundred micron thick layer of silica will be an effective thermal barrier. The major concern of using silica is its high temperature strength. The glass temperature of silica, depending on its composition, varies from 600 °C to 1000 °C. Silica loses its mechanical strength at temperatures near or above the glass temperature. The higher the percentage of sodium in the glass, the lower the glass transition temperature. Therefore, it is very important to select the correct glass for the desired purpose. An even higher degree of thermal insulation can be achieved using an air recirculating jacket. The conductivity of stationary air is only 0.01% of silicon. If there is an annular air jacket surrounding the stationary structure, the superior thermal resistance of the stationary air will greatly reduce the outside surface temperature. However, the air jacket requires support structures. Such support structures also represent a thermal flow path. Without careful design, at small scale, such thermal flow paths may dominate the heat transfer and prevent the air jacket from fulfilling its function.
It is important to point out that the introduction of any thermal barrier will result in interfacial thermal stress due to coefficient of thermal expansion mismatch. In addition to the interfacial thermal stress, these barriers will also alter the temperature distribution. In general, the temperature gradient will be increased. This implies the thermal stress due to non-uniform temperature distribution will be increased. A challenge for the design is to be able to accommodate the thermal stress levels at high temperatures. The worst scenario is that the thermal stress critical locations and the maximum temperature locations coincide. Careful analysis and design iteration must be made in order to keep these two locations separated.
8.7 Summary and Conclusion

This chapter presents the heat transfer and thermal stress analysis for the micro turbojet engine. The heat transfer of the micro turbojet engine is a complicated problem. The analysis requires the integration of structural modeling, fluid dynamics analysis, and power cycle analysis. The analysis is started using the 1-D free rotating disk model and is refined to 2-D axisymmetric and 3-D models. For the micro turbo jet rotating structures, structural reliability concerns drive the spool design to a shaftless structure. For the stationary structures, the demand to reduce the combustion heat loss drive the design to a highly isolated package. The thermal stress in the rotating structures is shown to be far less than the centrifugal stress. However, in the stationary structure, due to the higher temperature (and lower strength), the thermal stress may be a significant design restriction. More thermo-mechanical analysis on static structures and packaging are required to provide information on the overall engine redesign.
8.8 Reference

Huang, E. [1998]: Unpublished data on preliminary result in heat transfer and thermal stress analysis.
Jacobson, S. [1997]: Personal communication with Stuart Jacobson.
Protz, J. [1998]: Personal communication with Jonathan Protz.
CHAPTER 9. STRUCTURAL DESIGN INTEGRATION

9.1 Introduction

The detailed design and analysis of room temperature and high temperature structures has been discussed in Chapters 3 through Chapter 8. In this chapter, these previous analyses will be reviewed in order to provide recommendations for the redesign of the baseline engine and for the design of an all-silicon cooled turbojet.

As previously stated, the three major concerns in structural design are the stress levels, deformations, and structural dynamics. The stress level must be lower than the available material strength at the prevailing temperature or the structure will fail. Deformation of the structures must be within the limits imposed by consideration of the device operation. Allowable vibration modes and resonant frequencies are also directly related to the operation of the complete device.

For stress level considerations, the strength of the structural materials must be obtained over the range of temperatures of interest. Stress and heat transfer analyses are also required to assess the structural integrity of the device. To design against excessive deformation, the sensitivity of structural deformations such as disk growth and deflection to the geometric parameters need to be understood in order to optimize the final design. For design involving structural dynamics, the structural natural frequencies must be compared with the operating frequencies and their harmonics. These analyses have been presented in previous chapters. In this chapter, key analytical results and recommendations are presented. They will serve as the design guidelines for the structural re-design of the microengine as well as being guiding structural design principles for other MEMS heat engines.
9.2 Review of Baseline Microengine Structural Analysis

Although the baseline microengine is not the final design, the conclusions of the structural analysis presented in previous chapters can be used to guide the structural redesign of the turbojet engines and serve as the basis for design optimization. The conclusions from each analytical and experimental task are listed below:

9.2.1 Room Temperature Fracture Strength Testing:

1. The fracture strength strongly depends on surface processing route. The reference strength of STS DRIE silicon can as high as 4.6 GPa. Such a high reference strength makes the high speeds which are inherent to the baseline operation conditions possible.

2. The strength of STS DRIE silicon at blade and hub roots is significantly lower than that at other locations due to local variations in the plasma flow field. However, the loss of strength can be recovered by a secondary polishing etch to “smooth” the surface. Such a polishing step must be included in the design and fabrication cycle.

3. The Weibull modulus of both bi-axial STS DRIE (m=3.3) and RHFS DRIE (m=5 - 9) test results were not sufficient. Under such circumstances, the failure probability will be relatively higher (∼ 10³) under the operating conditions. The parametric studies show that a Weibull modulus of 10 is required to achieve a failure probability of less than 10⁴. Such a Weibull modulus was achieved on KOH etched silicon biaxial test specimens, and it is therefore, achievable. A detailed characterization of process flow to improve the uniformity in strength or a proof test to retire low strength structural components is suggested for increasing the uniformity in strength.

9.2.2 Room Temperature Structural Analysis:

1. The major source of stress in the micro turbogenerator is the centrifugal stress. Other loading such as fluid loading, thermal stress, and internal pressure are negligible. The centrifugal stress is proportional to the material density and square of the tip speed.

2. The stress critical point is the blade trailing edge root. Attaining sufficient fillet radii at hub and blade roots is extremely important. Parametric studies show that a fillet radius of greater than 25 μm over 200 μm blade height is desirable. At such a fillet
radius with an appropriate Weibull modulus, the overall structural failure probability will be lower than 10⁻⁶.

3. The power output is proportional to the blade height. However, the blade root stress, blade tip deformation, and disk deflection are also increased with the increase in blade height. From the parametric study, it is suggested that the blade height should be limited to 300 μm or less with a blade root fillet radius 25 μm. If the blade height needs to be increased, the design must be modified in one of the following ways: Firstly, by redesign of the blade geometry such that the local thickness at the blade trailing edge is increased. Secondly, by reducing the blade inertia by using hollow blades. Thirdly, by increasing the stiffness of the blades and disk by introducing a shroud.

4. The presence of a shroud is extremely helpful to the overall design. It stiffens the turbogenerator and reduces both the disk and blade deflections. As a result, the blade root stress is greatly reduced. It is the most effective way to reduce deformation and stress levels simultaneously. Although it may introduce additional difficulties from the fabrication perspective, a turbogenerator with a shroud is likely to be the best structural design for the turbogenerator.

5. The resonant frequencies of the baseline turbogenerator design is dominated by the bearing stiffness. The resonant frequencies of the rigid body modes are very low. For the structural vibration modes, their resonant frequencies are much higher than the rotating frequency and the blade passing frequency. The resonant frequencies of blades are higher than that of the disk. The structural dynamics are dominated by the disk vibration due to its relatively lower natural frequencies.

9.2.3 High Temperature Material Testing:

1. Above BDTT, the yield strength is very sensitive to temperature and strain rate. High temperature and low strain rate tend to reduce the yield strength. The yield strength of silicon drops rapidly at temperatures above 650 °C (920 K). For structural reliability considerations, the maximum temperature of the spool must be kept below 970 K to achieve the required material strength in excess of 600 MPa.

2. The one creep test at 1000 K and 120 MPa indicates that the creep strain rate is low enough for the all-silicon cooled engine structural design. However, at higher
temperatures, both the yield and creep strength reduce rapidly. At 1200 K and 10 MPa shear stress level, the strain rate reaches $10^5$. This strain rate is too rapid to be tolerated in the structural design.

9.2.4 Thermo-Structural Analysis:

1. For the all-silicon cooled engine, there is a trade-off between the engine performance and the structural integrity. In order to maximize the engine performance, a high working temperature and low heat transfer are required. However, from the structural integrity point of view, a lower turbine temperature is required to ensure structural reliability. In order to reduce the turbine temperature, an aggressive cooling scheme is required.

2. The parametric studies shows that the temperature of the spool structure is not very sensitive to variation in any of the parameters which can be readily attend in the design. Specifically, the peak structural temperature changes by less than 50 °C given a 100% change in any of the following parameters: fluid temperature, surface heat transfer coefficients, and surface area. Of those only the surface area can be easily adjusted, but a modest improvement in reliability and reduction in heat loss into the spool can be achieved by reducing the hot area, i.e., minimize the turbine disk size and the number of turbine blades.

4. The Biot number is relatively small at the microengine scale. The temperature gradient-induced thermal stress is less than 5% of the corresponding centrifugal stress in the spool. As a result, thermal stress is not important for the spool. However, for the static structure, due to the presence of the combustor, the structural temperature is higher. The design approach to reduce losses due to heat transfer will result in higher temperature gradients. The non-uniform temperature distribution induces significant thermal stresses, which are compatible with the yield strength of silicon at high temperatures. These stresses may relax with time, however, this can result in unacceptable deformation levels and sagging under gravity and pressure loading.

5. The small dimensions also preclude the use of thermal barrier coatings. The low Biot number prevents solid state insulators achieving sufficient thermal gradient to be useful.
9.2.5 Possible Failure Mode

The turbogenerator: Since the micro turbogenerator is operated at or near room temperature, the material behavior is essentially brittle. Fast fracture is the failure mode. Although the failure does not necessarily occur at the stress critical location, based on the RHFS testing, the stress critical location, i.e., the root of blade, also has inferior strength. As a result, blade trailing edge root is the most likely location of failure.

If the bearing is not well functioning, impact between the rotor and the stator may dominate the failure behavior. The most serious situation is the impact between turbine blade and the stator. The momentum of the entire high speed rotor acts on the hitted blade and results in high stress and fracture the blade. Shock wave may be generated and cracks may propagate through the entire rotor.

The all-silicon turbojet engine: The failure criteria for the all-silicon engine is complicated due to functional requirement of the structural component, loading rate, and the possible structural temperature.

For the micro compressor, since the structural temperature is likely always below BDTT, brittle fracture is the dominant failure mode and it is most likely initiated at the compressor blade root.

For the micro turbine, it is quite complex. The current prediction of turbine blade temperature is above BDTT, as a result, the local yield concern increased. Material yield at the blade root is the most likely failure criteria. During speed up period, the strain rate is much greater than that at steady state and the temperature is less. As a result, during the transient period, the turbine material behavior will be brittle. However, the maximum stress at compressor blade root is about twice than that at turbine root according the current design. As a result, the failure probability of compressor is much higher than that of turbine. The above discussion indicate that it is not a severe concern for turbine brittle failure. The major concern for turbine failure is still the high temperature plastic deformation.

Due to the nature of bearing operation, the variation of bearing air gap becomes another failure criteria. The bearing air gap is controlled by the deformation of both rotor and the stator. Deformation comes from centrifugal loading, thermal expansion,
and creep of structures. The centrifugal and thermal effects can be accounted during the
design by calculating the initial air gap. Therefore, the bearing life is controlled by the
creep deformation of structures. There are two possible results, i.e., the gap is closed or
becomes wider. In both case, the micro bearings lost their function. Note that unlike
the failures of turbine and compressor, which are typically local behaviors, the creep
induced air gap change is a global structural behavior. These creep deformation come
from the contribution of the creep strain through the entire structure.

As a summary, for the all-silicon turbojet engine, the compressor is subject to brittle
failure. Turbine may suffer from brittle or ductile failure. However, the ductile failure
is the major concern. The life of the device is controlled by the material creep behavior.
9.3 Structural Re-design of the Micro Turbojet Engine

The conclusions drawn from the baseline structural design and analysis are now applied to the new turbojet engine design. Figure 9.1 is a schematic of the turbojet engine. For detail specification, please refer to Appendix A.10 and A.11. The size of the rotor is doubled that of the baseline device in order to extract more power to overcome the loss due to heat transfer, viscous drag, and secondary flows. However, the tip speed of compressor is still kept as 500 m/s to keep the major stress levels the same as in the baseline design. The blade heights is also scaled up to 400 μm, which results in the same aspect ratio as the baseline blade.

For the compressor, since the operating temperature is always less than BDTT, as a result, it is always possible to control the material strength by surface processing. The strength of the material is, therefore, less of a concern. However, in order to reduce the stress levels, suitable etching processes should be used to maximize the allowable fillet radii.

In order to reduce the heat transfer to the spool, the turbine disk diameter is reduced. Since the disk radius is reduced, the centrifugal stress is also reduced. This is important since the temperature at stress critical locations in the turbine is potentially above BDTT. Such a reduction in stress level is required since in the baseline study, the stress at the blade roots was over 1 GPa, which is too high for a hot silicon turbine.

The shaft diameter is increased to become the same as that of the turbine disk. Therefore, this is essentially a shaftless design. The stability of the gas journal bearing and the conductance between turbine and compressor are maximized. Such a design will minimize the turbine temperature. The penalties associated with this design are the increased of losses due to viscous drag and the decrease in compressor efficiency due to heat transfer from the turbine.

In order to increase the thermal resistance between the top portion (near the combustor) and the bottom portion (near the turbine) of the static structure, a layer of glass was considered. As shown in Figure 8.1, since the conductivity of glass is on the order of 1 - 3 W/m K. Such a low thermal conductivity with the long thermal path will provide an effective way to isolate the top structure in order to reduce the heat transfer into the compressor air. However, with the presence of the glass, various problems
were introduced. The major problems are the low strength of glass at high temperature, the thermal stress in the static structure, and fabrication issues. For simplicity, the current design does not include the glass wafer. However, this serves as an optional approach for future improvement.

An air jacket was also considered for reducing the surface temperature of the static structure in order to reduce the surface heat loss. Basically, there are two design approaches. Firstly, the air jacket is included into the mask design, and is microfabricated along with the rest of the microengine. Secondly, equivalent thermal isolation can be achieved by external packaging. Currently, to provide flexibility and reduce mask complexity, the air jacket will be achieved by external packaging. However, in order to reduce the overall mass of the device and for efficient fabrication, once the isolation scheme is well established, the external packaging should be included into the device design as a component of the device.

Figure 9.1  The schematic drawing of the micro turbojet engine (Courtesy of Mr. Jonathan M. Protz)
9.4 Conclusions

This chapter summarizes the experimental investigation and analysis results in the baseline microengine structural design. This summary is important since it provides the recommendations for structural redesign of the all-silicon cooled engine.

Figure 9.2 is a schematic flow indicating the relationship between the experimental and analytical effort for the baseline structural design and the engine structural redesign. The structural design methodology and the studies of the effect of the structural length scale provide general guidelines. The information obtained through the room temperature material testing, structural analysis, and probabilistic structural design can be used in brittle component redesign. The results of the high temperature material characterization and the thermo-structural analysis will potentially guide the structural redesign for hot (ductile) components such as the turbine and combustor.

Finally, these structural analysis results are integrated with the overall system design to perform the redesign of the all-silicon cooled engine.

Figure 9.2 The relationship between the baseline study and the engine structural redesign
CHAPTER 10. SUMMARY AND CONCLUSION

10.1 Overall summary

The material characterization and structural analysis for a microfabricated turbogenerator and an all-silicon cooled micro engine have been presented in this thesis. The effect of structural length scale and the design methodologies for microfabricated highly stressed MEMS structures have been illustrated and developed.

The material characterization has been summarized in Table 10.1. Data for the fracture strength, fracture toughness, yield strength, and creep resistance of single crystal silicon at various temperatures has been obtained by either literature survey or experimental investigation. The room temperature fracture strength has been shown to be extremely sensitive to the surface processing route and strength recovery methods have been proposed and tested. The fracture toughness has been found that to be essentially temperature independent at temperatures below BDTT. The yield strength has been found that to extremely depend out on temperature and strain rate. High temperatures and low strain rates will reduce the available yield strength. However, there is insufficient creep data at 850 - 1000 K, this requires a more detailed experimental investigation in the future.

For the structural analysis, the turbogenerator rotor has been analyzed using the finite element method. It was found that the stress critical location is at the root of the turbine blade trailing edge. In order to reduce both stress and deflection levels, various structural redesign approaches were proposed. The finite element stress analysis and the fracture strength test data were combined using a probabilistic structural analysis to evaluate the reliability of the turbogenerator.

The all-silicon cooled engines will operate in the creep regime of silicon. As a result, thermo-structural analysis is required to assess the trade-off between structural reliability and engine performance. Since the heat transfer, thermal stress, fluid
dynamics, and engine cycle performance analyses are coupled, this requires an iterative analysis between structure, fluid, and overall performance.

Finally, the results for material characterization and structural analysis are integrated for the redesign of the turbogenerator and the preliminary design of the all-silicon cooled engine.

Table 10.1  Summary of material characterization

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<tr>
<th></th>
<th>RT</th>
<th>400 - 600 K</th>
<th>600 - 800 K</th>
<th>Near BDTT (830 K)</th>
<th>850 - 1000 K</th>
<th>1000 - 1300 K</th>
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<tbody>
<tr>
<td>Fracture Strength</td>
<td>√</td>
<td></td>
<td></td>
<td>NR</td>
<td>NR</td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>NR</td>
<td>NR</td>
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<tr>
<td>Yield Strength</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Creep Strength</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>?</td>
<td>√</td>
</tr>
</tbody>
</table>

√: Data available (by test or literature search)
NR: Not required
?: No reliable data. Focus of the future work
10.2 Overall conclusion

This thesis addresses the structural design and material characterization for the phase one research on the MIT microengine project. The effect of structural length scale has been investigated and the microengine structural design methodologies have been developed. The test methods for material characterization and the approach to structural analysis, structural reliability evaluation, and thermo-structural analysis have been established. These have been applied to the turbogenerator and the all-silicon cooled engine design. The example process provides an engine design and serve as the basis for the engine redesign.

Based on the experimental and analysis work presented in this thesis, the silicon turbogenerator is structurally feasible due to the extremely high material fracture strength at small scale. For the all-silicon turbojet engine, due to the heat transfer characteristic at small scale, it is also structurally feasible. However, such feasibility is accompanied by the penalty of sacrificing the device performance.

The experimental and analytical results shown in this thesis are applicable for other highly stressed / power MEMS devices. The methodology and analysis flow are also applicable to other related area such as turbomachinery structural design, MEMS material characterization and structural design, and the reliability evaluation of ceramic structures.
10.3 Suggested Future Work

This thesis represents a first iteration on the structural design of the microengine. In order to progress toward a final design, recommendations for future work are listed below:

A parameterization of silicon fracture strength vs. etch parameters:

There are two major issues that remain unresolved in the room temperature characterization of silicon. Firstly, the characterization shown in the Chapter 4 is for a particular etch recipe. The behavior of silicon after DRIE under different etching conditions may vary. Secondly, the low Weibull modulus indicates high degree of scatter. It is important that the Weibull modulus be improved in order to improve the structural reliability.

To address the first issue, an extensive mechanical testing program is underway. RHFS specimens produced by 50 different etching process parameters are being used to characterize the fillet radii and the fracture strength. The final goal is to optimize the process to achieve the required mechanical performance.

To address the second issue, a proof testing approach is suggested. A proof test can retire low strength products before packaging. As a result, the reference strength and the Weibull modulus will be enhanced and the structure reliability will be improved. The difficulty in proof testing is to apply the appropriate load. The strength of "low strength" product is in the order of 1 GPa. To retire these products, the proof tester must be able to exert a stress higher than that value. However, such stress level is very high for the most structures.

Crack healing:

Since the fracture strength is controlled by the surface flaws, if the tips of these surface micro cracks can be blunted, the stress intensity factor should be decreased. Previous researchers oxidized the surfaces to achieve the desired blunting [Yasutake et. al, 1986]. They reported that a considerable strength improvement can be achieved.
Such crack healing methodology may be applicable to increase the strength of microengine structures at the sharp transition regions.

**High temperature material testing:**

The high temperature material characterization facilities can be used for other materials. Since the major constraint for the current engine design is the high temperature mechanical behavior of silicon. Any material with higher yield strength and creep resistance at elevated temperatures has the potential to enhance the device performance. For example, the SiC/silicon hybrid structures could have advantages in both creep resistance and ease of fabrication. However, its mechanical behavior should be tested to verify its applicability, and a number of fabrication steps need to be developed and refined.

**Structural Analysis:**

The thermo-structural analysis reported in this thesis has mainly been performed using axisymmetric finite element models. The data obtained from these models provides information for reasonably detailed engine design. As the design converges, three-dimensional thermo-structural analysis will be required for the final detailed design.

**Running gaps**

The bearing air gap, tip clearance, and clearance for seals are very important for the overall engine performance. However, the deformation of the engine structures resulting from centrifugal and pressure loading, and the thermal expansion will change the air gaps during service. Since the running gap is an important parameter for effective bearing operation, an important task is to calculate the total deformation in order to design the initial air gap such that the running gap is consistent with the bearing design.
New materials and process development:

Since the material capability dominates the engine design and performance, the most fundamental approach to relief of such design constraints is to replace silicon with other refractory ceramics such as silicon carbide [Carter et. al, 1984], [Campbell et. al, 1989]. There are two major issues to be solved. First, the mechanical properties of the new materials must satisfy the requirements of the engine operation. Their properties must be fully understood for structural design purpose. It means a detailed characterization is required. Second, these new materials must be compatible with micro machining processes, which may require the development of new fabrication processes.

10.4 Reference


APPENDIX:

A.1 THE 3-D FINITE ELEMENT INPUT FILE FOR CONVERTING LOAD TO STRESS IN MATERIAL TESTING

The finite element meshes were created by I-DEAS™. After finishing the meshing, the graphic mesh were translated to the input file for ABAQUS /STANDARD. Such a file contains coordinates of each node and the connection for each element. Finally, the solid section, material properties, boundary conditions, initial conditions, and the loading were defined by author. In a finite element input file, 95% file size are for node and element definition. The following FE omits the detail in node and element definition and shows the important portion of modeling.

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XBC34,XBC35,XBC36
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245
A.2 ETCH RECIPES FOR MECHANICAL TEST SPECIMENS

1. KOH etch recipe

The fabrication of KOH etched specimens involved the utilization of double polished <100> silicon wafers which were coated with 1000 Å of silicon nitride deposited at a temperature of 800 °C. One surface was subsequently coated with photoresist (OCG 825, 20 cs) spun at 3000 rpm, exposed and developed in OCG 934 1:1. The exposed silicon nitride was etched in a LAM Research plasma etcher using a fluorinated chemistry and upon removal of the photoresist layer, the samples were immersed in a hot KOH solution (30% KOH by weight). At 85 °C, the measured etch rate was 2 μm/min. KOH etches preferentially along the <100> direction. After the etch, the wafers were cut to 10mm × 10mm dices and tested.

2. Radiused Hub Flexure Specimens

Radiused hub flexure specimens were produced from double polished silicon (<100>) wafers in the following manner: both surfaces were coated with photoresist AZ4620 spun at a speed of 4000 rpm to achieve a final thickness of 5.3 μm. The topography was then transferred by exposing one surface in a Karl Suss aligner. The coating on the remaining surface was required to preserve the integrity of the wafer surface. After developing, the photoresist was used as a masking material during etching of the silicon substrate. The same etch conditions as those used during the microfabrication of the micro gas turbine generators was applied. The etch rate was measured as 3.4 μm/min with highly anisotropic profiles and selectivities of 80:1 (silicon to photoresist). After etching, the wafers were cut into 10mm × 10mm square specimens using a diamond saw. In addition, some specimens were treated with wet or dry isotropic etches after the initial deep etch step to attempt to recover strength by reducing the surface roughness at the fillet radius. The wet etchant was composed of 5% HF, 55%HNO₃, and 40% DI water. The etch rate was 1.8 μm/min at room
temperature. The dry isotropic etching was performed in an SF, plasma for 20 s with a measured etch rate of 8.3 μm/min. Both etches removed approximately 2 μm of material from the surface of the specimens.

The masks to fabricate the biaxial and RHFS specimens are shown below.
Figure A3.1 The panel of the LabView™ data acquisition program
A.4 INTRODUCTION TO NASA CARES/LIFE

CARES/LIFE has been developed by the structural reliability group at NASA Lewis Research Center (LERC). It utilizes Weibull statistics for the structural failure probability evaluation. For more detail information, please refer to NASA Technical Report No. 2916, or contact Mr. Noel N. Nemeth at the NASA Lewis Research Center. The basic analysis flow is shown in the flow chart below.

There are three important core programs in the CARES/LIFE code. That is, the CPEST, Translator, and C5LIFE. All of them were written in FORTRAN format.

CPEST: It is the utility to estimate the material's Weibull information from the material test data. The Weibull modulus and reference strength, as well as the goodness of fit (i.e., the Kolmogorov-Smirnov (K-S) goodness-of-fit and the confidence level.)

Translator: CARES/LIFE provides three finite element translator, i.e., ABAQUS to CARES, ANSYS to CARES, and NASTRAN to CARES translator. In these finite element program, the desired stress and volume information are written into a binary output file. These translator then translates this output file to a CARES/LIFE readable neutral file in order to evaluate the structural failure probability.

C5LIFE: The parameter extracted by CPEST need to be written into a input file. This input file contains the following information: material Weibull parameters, flaw type, static fracture/dynamic fatigue crack propagation, and model to be used. In conjunction with the neutral file created by the translator. These two files are input into the C5LIFE. C5LIFE then use sub-element level integration to calculate the overall structural failure probability by either volumetric or surface flaws.

In addition to these core programs, Freelance™ and I-DEAS™ are also used in conjunction with the analysis result. Freelance™ is used to plot the Weibull statistic for material testing. On the other hand, I-DEAS™ is used to illustrate the failure probability of any location of the solid structure.

One important issue to use CARES/LIFE is that the for surface flaw dominated failure probability calculation, surface element is required. That is, for axisymmetric structural model, in addition to the axisymmetric elements for representing the
structure, extra axisymmetric membranes or axisymmetric shell elements are required. For 3-D finite element model, both brick and shell element are required. These surface elements are not to support the structure but to catch the stress information on the surface. For volume flow dominated case, these surface elements are not required.
A.5 SPECIFICATION OF FURNACE SYSTEM

1. **Dimensions Furnace**
   - Furnace Type: Single zone split cylindrical
   - Overall Height: 510 mm
   - Overall Diameter: 356 mm
   - Internal Bore: 100 mm
   - Hot Zone Height: 280 mm
   - Top Port Diameter: 48 mm
   - Bottom Port Diameter: 88 mm
   - Heating Elements: Six Kanthal Super 33 Molybdenum Disilicide
   - Weight: 36 Kg

2. **Operating Performance**
   - Maximum Operating Temperature: 1600 °C
   - Maximum Element Temperature: 1700 °C
   - Recommended Heating Rate: 10 °C/minute

3. **Services**
   - Rated Voltage: 230 Volts, 50 Hz, 1 Phase
   - Rated Current: 16 Amps
   - Rated Power: 3.6 kW
   - Cooling Water Flow: 1 ltr/minute

4. **Control Systems**
   - Control Instrument: Eurotherm Instruments type 902P
   - Alarm Instrument: Eurotherm Instruments type 92
   - Control Thermocouple: Type B
   - Alarm Thermocouple: Type B
   - Maximum Element Voltage: 80 V
   - Dimensions: Height 625 mm Width 550 mm Depth 500 mm
   - Circuit Diagram: 15655A2 Issue A
   - Weight: 85 Kg
A.6 HIGH TEMPERATURE MATERIAL TESTING SPECIMEN

The high temperature material testing specimens were cutted from \textit{<110>} n type single crystalline wafer. There were two thickness. The 1 mm (1000 \(\mu\)m) sample was used for yield and Young's modulus testing. The 0.5 mm (500 \(\mu\)m) sample was used for creep strength testing. The length and width of specimens are mainly constrained by the 20/40 four-point bending test fixture. That is, the length must be greater than 40 mm and the width must be less than 10 mm. The testing axis for these specimen are in \textit{<110>} crystalline direction, in which the maximum stiffness was reported.
A.7 ERROR ANALYSIS FOR SILICON YIELD STRENGTH TESTING

1. The effect of specimen thickness variation

From Eq. (7.4), for a given applied load $P$, the stress of specimen is inversely proportional to the square of the specimen thickness. The nominal thickness of the test specimen is 1000 $\mu$m with possible thickness variation $\pm 25\mu$m. The specimen thickness can be measured by micrometers. The accuracy of micrometers are $\pm 5\mu$m. As a result, there is a 0.5% uncertainty in thickness. Therefore, the uncertainty in strength due to the thickness variation will be $(1.005)^2 - 1 = 1\%$. The relationship between load and strength will be $100 \text{ N} = 330 \text{ MPa} \pm 3.3 \text{ MPa}$. For yield strength testing at 600 °C, this will result an uncertainty in strength of 5 MPa.

2. The effect of misalignment

![Diagram of a 4-pt bending system]

Figure A7.1  The schematic of a 4-pt bending system

The schematic of a 4-pt bending system is shown in Figure A8.1. The total exert force is $P$. If no mis-alignment, the reaction force on all SiC rollers will be $R_1 = R_2 = N_1 = N_2 = P/2$. A possible error is a misalignment of the top fixture. It result a net shift of the top two
pins with an distance $\Delta x$. It will be interest to examine the effect of such a misalignment.

Since there is no misalignment to locate the top SiC fixture itself. From the force and moment balance of the top fixture, it can be find that $N1 = N2 = P/2$. From the force balance of the specimen, the reaction force $R1$ and $R2$ can be solved as

$$R_1 = \frac{P}{2L}(L + \Delta x) \quad \text{and} \quad R_2 = \frac{P}{2L}(L - \Delta x)$$

The shear force diagram is shown in Figure A8.2. The bending moment at the center of the specimen, $M_{center}$, is the total area under the shear force diagram from the left pin to the center. That is, for a perfect aligned system,

$$M_{center} = \frac{P}{2}(L - a). \quad \text{And for a misaligned system,}$$

$$M_{center} = R_1(L - a + \Delta x) + \left(R_1 - \frac{P}{2}\right)(a + \Delta x)$$

$$= R_1L - \frac{P}{2}(a - \Delta x) = \frac{P}{2}(L - a)$$

![Figure A7.2](image) The shear force diagram

That is, the bending moment at the specimen center does not change for a misaligned system. As a result, the yield strength does not vary.

3. The effect of LVDT accuracy

The accuracy of LVDT only affect the displacement. Since there are no displacement information required for determining the yield strength, as a result, the LVDT accuracy
does not affect the yield strength result. However, it will affect the reported Young's modulus.

4. The effect of sensor noise

From the load-deflection curve obtained from the testing, there are a 4 to 5 N noise level in load. As a result, a 15 MPa uncertainty in yield strength exists due to the sensor noise. This uncertainty is independent to the testing temperature. As a result, at lower temperature (600 - 700°C), this uncertainty is only 3 to 5% of the yield strength. However, at higher temperature (800 - 900°C), due to lower yield strength, this uncertainty becomes more important. It can reach 10 - 20% of the yield strength.

5. Uncertainty in test temperature

There are two thermocouples inside the furnace. For a command test temperature, the main thermocouple shows the system can reach an accuracy level of ±3 °C. However, the minor temperature measurement of the other thermocouple always shows a temperature of 3 °C lower than the main thermocouple. Therefore, it is reasonable to assume the uncertainty in test temperature is ±7 °C.
A.8 THE FOUR-POINT BENDING TEST PROCEDURE

I. SPECIMEN PREPARATION

A. Obtain appropriate test material

1. Silicon: It is preferred that 1000um thick wafer is used over 500um. This allows a higher stress to be applied to the material and also increases the max allowable deflection before the fixture crashes.

2. Silicon carbide: Tests will be performed on varying thicknesses of SiC deposited onto Si substrates. Also, tests can be performed on monolithic SiC (freed from Si substrate) at large film thicknesses (>200um)

B. Cutting the specimens using MTL Diesaw (2nd floor)

1. Use program "C" for 1000um thick wafers (makes two passes). Use program "A" for 500um wafers.

2. Make sure the proper blade is being used (ask Dan Adams!)

3. Follow supplied instructions for Diesaw machine (don’t forget to reserve hours and log your hours when done!)

4. Make cuts parallel to Major Flat (110) plane

5. Cut in strips of 9mm

6. Should be able to get 13 specimens per wafer (cutting central 4 strips in half). Specimens must be at least 45mm long

C. Polishing specimen edges

1. Use grinding/polishing wheels on the 5th floor bldg 13 (CMSE) set to 150 RPM’s.
   a) Start on 1200 grit for 10-15 seconds per specimen edge. Use moderate water flow. Make sure you remove the sharp and chipped edges left behind by diesaw while not removing too much material so as to alter the specimen geometry significantly.
   
   b) Rinse specimens and move to 4000 grit for 15-20 seconds per edge with moderate water flow, then rinse. Stay only on each wheel just long enough to remove scathes left behind from the previous grit.

   c) Finish polishing the edges with 0.3 um wet polishing wheel, going for 30 seconds to one minute on each side. Refresh the slurry frequently. Rinse specimens again when done.

II. PREPARING THE WORKPLACE FOR TESTING

A. clean all SiC and Alumina ceramic surfaces with methanol to remove oils, residues, and particles

B. make sure the transducer tube-rod-probe assembly is seated properly on the LVDT pushpin.

C. make sure the LVDT and Load Cell are reading properly and are calibrated (flashing "SET-UP" lights indicate they need to be calibrated). Follow “Auto Calibration” procedure for Load cell calibration.

1. For LVDT Calibration, use the following “Manual Procedure”:
   a) Press “set-up”. Verify the LVDT is depressed so that it is at an appropriate ZERO point (use hydraulics).
b) Select "Calibrate", "Manual", "Coarse Balance", "Go".

c) Relay on. It will span the range. When it is settled and is reading 1mm. Press "Go" again.

d) Relay off. Fine balance --> Zero point --> "Go". Will stop blinking when finished.

D. Check that the channels are properly assigned on the Instron control panel. "A" for displacement. "X" for Load. "B" for position. "A" and "B" should be set to output "TRACK" values, while Load should output "FILTERED" values.

e) Check that the cables are connected from mechancal controller to circuit board properly. "A" goes to left receptacle, "B" to center, and "X" to right port.

f) Check that the AD converter is grounded (thin wire connected to wall outlet screw).

G. Load up "Kuo-Shen Data Acquisition" found in "41-219/KuoShen/labview". Ensure that the units and gain values are correct: Load (N) gain=50, Position(mm) gain=7.62, LVDT(mm) gain=0.1, Freq=5Hz. Run data collection and ensure that it is recording accurate values.

H. Plug in the power cable for the Temp controller

I. Make sure BOTH water valves are turned on and are flowing. The master water valve is on the wall by the Hydraulic pump, and the unit valve is on the floor between the MTS810 and the Instron 1332 Mech test units.

III. SETTING THE TEMPERATURE CONTROLLER

A. Make sure the master power toggle switch on back of controller is ON.

B. Hit the left green button to power ON the controller. Red lights will light up. Ignore these until you are ready to test. Do not press the right green button at this time.

C. Program in the desired temperature program. Follow directions supplied. Put the program on hold.

D. DO NOT HEAT /COOL THE FURNACE AT A RATE FASTER THAN 12C PER MINUTE!

IV. PROGRAMMING THE HYDRAULIC ACTUATOR CONTROLLER

A. MAKE SURE LOAD PROTECTS ARE SET and ACTION=UNLOAD!! Max load capacity of ceramic fixture is 1kN. Recommended load protect is +/- 0.4kN.

B. Program in desired control for actuator.

1. For position control. Select waveform and program piston speed (0.002mm/sec - 0.010mm/sec) and range (-5.0 mm).

V. LOADING THE 4-POINT BEND FIXTURE

A. BE VERY CAREFUL AND GENTLE WHEN HANDLING AND ASSEMBLING THE SiC FIXTURE!! IT IS VERY FRAGILE AND COSTS SEVERAL THOUSANDS OF DOLLARS $$!!! TO REPLACE!!!

B. Do not touch the SiC with metal components.

C. Load the specimen in the fixture and complete assembly. Follow the instructions in section 3.3.1 of INSTRON Ceramic Testing System Operator Manual

D. Setting fixture height close to upper pushrod

1. Leave an appropriate gap to allow for Thermal Expansion of pushrods (~3mm at 800C, 2mm at 650C, 0.5mm at RT) .
E. Carefully align the fixture so that the specimen is centered with respect to 1cm wide groove (so it can bend unrestricted). Make sure the 4 points of contact are centered relative to each other.

F. Raise the LVDT to be in contact with the underside of the specimen, and bring it to read ~0.95mm.

G. Slowly close the furnace door-halves and make sure it is centered with respect to the upper pushrod. Latch the two latches in the front and the two in the back.

VI. BEGINNING THE TEST SEQUENCE (CHECKLIST)
A. Press the right green button on the temp controller. The red lights should go out and your temp program should begin running.

B. Wait for at least 15 minutes once at temperature before beginning test (so transducer expansion catches up to lower pushrod) --system is at steady state.

C.

VII. PERFORMING MECHANICAL TEST / DATA COLLECTION
A. Once system is at steady state (i.e. LVDT readings have leveled off), turn on the hydraulics to the actuator.

B. Begin collecting data on Data Acquisition interface.

C. Press the START button on Instron controller to enact position waveform program.

D. Note the position that the upper pushrod comes in contact with the 4pt bend fixture. You will the the displacement value start decreasing steadily and the Load start increasing steadily at this point.

E. Figure out the position that the SiC 4pt bend fixture will crash. If you are using a 1000um thick specimen, you have ~1.8mm of position travel before this occurs. If you are using a 500um specimen, you have ~1.3mm of safe travel.

F. Keep running the piston until either the specimen fails or you have reached the Max allowable position travel (as outlined immediately above). To stop piston movement press the “hold” button. To return piston to the original position before you started travel, press “reset”.

G. Turn off the hydraulics.

H. Press “Stop and Record” on DAQ and save file to detailed filename:

1. i.e. “Si1000x9-800C-Y-2-A” stands for silicon specimen 1000um thick by 9mm wide tested at 800C for YIELD with piston speed =2um/sec, 1st (A) test of this kind.

VIII. CONCLUDING THE TEST SEQUENCE
A. Wait until the temperature is under 400C before opening the furnace to avoid thermally shocking the SiC fixture and the heating elements!

B. You may use indirect fan flow to expedite cooling.

C. Once you have opened the furnace, turn hydraulics to actuator back on. Slowly lower the piston down to lowest position so you have clearance to safely remove SiC fixture. Turn off hydraulics. Rotate the LVDT thumbwheel all the way right to retract the transducer probe safely away.

D. Once furnace has completely and safely cooled, carefully remove the SiC fixture from the lower pushrod wearing insulated gloves.
E. Remove tested specimen and appropriately serialize and store.

IX. CLEANING UP THE WORKPLACE
A. Shut off temp controller power and unplug.
B. Turn off unit water valve.
C. Gently wipe down ceramic fixtures with methanol and paper towel.

X. DATA REDUCTION AND ANALYSIS
A. Load tab delimited text file into Excel (select “Finish” --it does the rest)
B. To normalize data, create two new columns. In the first, put the formula “=average(d$1:d$100)-d1” and copy it down the column. This averages the first 100 data points and then subtracts each individual data point from it, correcting the sign and nonzero starting point of the LVDT data. In the next column, put the formula “=average(b$1:b$100)-b1”; this does the same thing for the load data. If there are fewer than 100 data points before the specimen started to bend, reduce the range accordingly.
C. To create a chart, select the two new columns you have just created, and pick “Chart -> As New Sheet” from the Insert menu. Use the default settings. Once the chart has appeared, change the temperature and rate so they are correct for the test which was done.
D. To create a plot of the linear regime, copy the sheet that has the chart on it using the “Move or Copy Sheet” option from the edit menu. Double click on the data series and adjust the range in Name and Values and then in X-Values so that only the linear region is displayed. With the data series still selected, pick “Trendline” from the Insert menu. For Type, select “linear”, and in Options, select “Display equation on chart.”
E. Save as an Excel File (denote with “x” to file end).
A.9 HIGH TEMPERATURE TEST DATA
Load vs Deflection for Silicon at 700°C with piston speed = 2μm/sec

Load vs Deflection Silicon at 700°C piston speed 1 μm/s
Load vs Deflection Silicon at 700°C
piston speed 7 um/s

Test Type: 4-Point Bend (40/20mm)
Specimen: Si 1000um x 9mm x 45 mm

Load vs Deflection for Silicon at 700°C
piston speed = 5um/sec, specimen = 1000um x 9mm
Load vs Deflection Silicon at 750°C
Piston speed 10 um/s

Load (N) vs Deflection (mm)

Load (N) vs Deflection (mm)

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Load vs Deflection for Silicon at 800°C

Test Type = 4-Point Bend (40/20mm)
Piston speed = 5μm/sec
Specimen = Si 1000μm x 9mm x 45mm

Load (N) vs Deflection (mm)
Load vs Deflection for Silicon at 850°C

Test Type = 4-Point Bend (40/20mm)
Piston speed = 10μm/sec
Specimen = Si 1000μm x 9mm x 45mm

Load vs Deflection Silicon at 850°C
piston speed 7 μm/s

Test Type: 4-Point Bend (40/20mm)
Specimen: Si 1000μm x 9mm x 45 mm
A.10  THE MIT MICRO BEARING TEST RIG

(Courtesy of Mr. Chuang-Chia Lin)
A.11 THE MIT/DARPA MICRO TURBO JET ENGINE

(Courtesy of Mr. Jonathan Protz)

Figure A11-1 The schematic plot of the micro jet engine

Figure A11-2 The three dimensional schematic of the micro jet engine
Figure A.11-3 The exploded engine stack