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ABSTRACT

In the electron cyclotron range of frequencies, the high-β NSTX plasmas are overdense to the traditional extraordinary X mode and/or the ordinary O mode. For low harmonics of the electron cyclotron frequency, the X and O modes are cutoff at the edge and for high harmonics the plasma is essentially optically thin to these modes. Thus, heating of such plasmas by electron cyclotron waves or diagnosing of such plasmas by electron cyclotron wave emission becomes problematic. However, such plasmas are optically thick to electron Bernstein waves (EBW) which are strongly absorbed or emitted by thermal electrons in the Doppler shifted vicinity of the fundamental or harmonics of the electron cyclotron resonance [1]. In order to propagate out into the vacuum region, EBWs have to mode convert to the X and O modes at the upper hybrid resonance. In this paper we discuss the approximate kinetic model that has been developed to study this mode conversion process. From this model we show that the emission coefficients for the X and O modes from EBWs are directly related to the excitation coefficients of EBWs from X and O modes.

INTRODUCTION

We have previously shown that the high-β NSTX-type plasmas can be heated by waves in the electron cyclotron range of frequencies by mode converting the X mode or the O mode to electron Bernstein waves (EBW) at the upper hybrid resonance (UHR) [1]. From ray tracing analysis, we have also shown that EBWs are locally and strongly absorbed at the Doppler shifted electron cyclotron resonance or its harmonics. For spherical tokamak plasmas, the location of the EBW energy deposition is significantly different for excitations on or away from the equatorial plane [1,2]. The strong and localized absorption implies that thermal emission of EBWs can occur for frequencies corresponding to the local electron cyclotron frequency. This emission then converts at the UHR to the X and O modes which are then observed in the vacuum region.

The excitation and emission of EBWs has been studied on the Wendelstein 7-AS [3]. More recently, experiments on CDX-U and NSTX [4] and MST [5] have observed emission of EBWs via the mode conversion process and studied its dependence on the edge properties of the plasma.

In this paper we formulate an approximate kinetic description of the mode conversion process that includes the propagating EBWs. From numerical solutions of this model we find that, for the same plasma parameters, the fraction of the EBW power that is mode converted to the X mode (emission coefficient) is the same as the fraction of the X mode power that is converted to the EBWs (excitation coefficient) when the X mode is launched into the plasma. A fraction of the emitted EBW power also converts to the O mode. This fraction is the same as the fraction of the O mode power that is converted to the EBWs if the O mode was to be launched from the outside. Thus, the X and O mode emission coefficients are the same as the X and O mode excitation coefficients. This symmetry between the emission coefficients and the excitation coefficients is useful for designing experiments to heat and drive plasma currents in NSTX-type plasmas by waves in the electron cyclotron range of frequencies. Since the X mode and O mode emission coefficients have to be less than 1, the parameter space for which the excitation coefficient from one of these modes is optimized is complimentary to the parameter space for the optimum excitation coefficient for the other mode [1]. While the relationship between the excitation and emission coefficients may seem intuitive, at this stage it is simply based on numerical results. An analytic derivation of these results is presently a challenge. The result has important consequences for experimentally achieving optimum plasma edge parameters for the excitation of EBWs; such optimized parameters can be ascertained by monitoring the emission from EBWs.

APPROXIMATE KINETIC DESCRIPTION OF MODE CONVERSION

Our previous studies have shown that efficient coupling between EBWs and the X or O modes occurs when the UHR is in the steep density gradient regions near the edge of the plasma. For this region we use the slab geometry model where the x axis is the direction of inhomogeneity and corresponds to the radial direction, the y axis is along the poloidal direction, and the z axis is along the toroidal direction. The equilibrium magnetic field is assumed to be sheared with the form

$$\vec{B}_0(x) \equiv B_y(x)\hat{y} + B_z(x)\hat{z} = B_0(x)\sin\Psi(x)\hat{y} + B_0(x)\cos\Psi(x)\hat{z} \tag{1}$$

where Ψ is the angle between $\vec{B}_0$ and the z-axis. The variation of the fields in the y and z directions is assumed to be of the form $\exp(ik_y y + ik_z z)$ where k_y and k_z are the appropriate components of the wave vectors. The time dependence is assumed to be of the form $\exp(i\omega t)$ where ω is the wave angular frequency. Assuming that the EBWs are electrostatic, we find that the approximate full wave description of the propagation of the X mode, O mode, and the EBWs is given by [1]

$$\frac{d\vec{F}}{d\xi} = i \overleftrightarrow{A}_K \cdot \vec{F} \tag{2}$$

where $\xi = \omega x/c$ is the normalized spatial variable,

$$\vec{F}^T = [E_x \; E_y \; E_z \; (i\tilde{\chi}E_x') \; cB_z \; (-cB_y)] \tag{3}$$

is the transpose of the field vector $\vec{F}$, $E_x' = (dE_x/d\xi)$,

$$\overleftrightarrow{A} = \begin{bmatrix} 0 & 0 & 0 & -\chi_1^{-1} & 0 & 0 \\ n_y & 0 & 0 & 0 & 1 & 0 \\ n_z & 0 & 0 & 0 & 0 & 1 \\ K_{xx} & \chi_{xy} & \chi_{xz} & 0 & n_y & n_z \\ -\chi_{xy} & K_{yy} - n_z^2 & \chi_{yz} + n_y n_z & 0 & 0 & 0 \\ -\chi_{xz} & \chi_{yz} + n_y n_z & K_{zz} - n_y^2 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$\overleftrightarrow{K} = \overleftrightarrow{I} + \overleftrightarrow{\chi} \tag{5}$$

$$\overleftrightarrow{\chi} = \frac{-\omega_p^2}{\omega^2 - \omega_c^2} \begin{pmatrix} 1 & -i\omega_{cz} & i\omega_{cy} \\ i\omega_{cz} & 1 - \omega_{cy}^2 & -\omega_{cy}\omega_{cz} \\ -i\omega_{cy} & -\omega_{cy}\omega_{cz} & 1 - \omega_{cz}^2 \end{pmatrix} \tag{6}$$

where $\omega_p(x)$ is the electron plasma angular frequency, $\omega_{cy}(x) = eB_y(x)/m_e$ $\omega_{cz}(x) = eB_z(x)/m_e$ are the electron cyclotron angular frequencies for the poloidal and toroidal fields, respectively, $\omega_c = \sqrt{\omega_{cy}^2 + \omega_{cz}^2}$,

$$\chi_1 = \left(\frac{v_T}{\omega_c}\right)^2 \left(\frac{\omega_p^2}{\omega^2 - \omega_c^2} - \frac{\omega^2}{\omega^2 - 4\omega_c}\right) \tag{7}$$

and $v_T = \sqrt{T(x)/m}$ is the electron thermal velocity for corresponding to the temperature $T(x)$.

EXCITATION AND EMISSION OF ELECTRON BERNSTEIN WAVES

For NSTX-type parameters discussed in [1], and for a wave frequency of 14 GHz, Figure 1a shows the mode conversion efficiency C_{XB} for the excitation of EBWs when an X mode is launched from outside the plasma. Figure 1b shows the mode conversion efficiency C_{OB} when, for the same parameters, an O mode is launched from the outside.

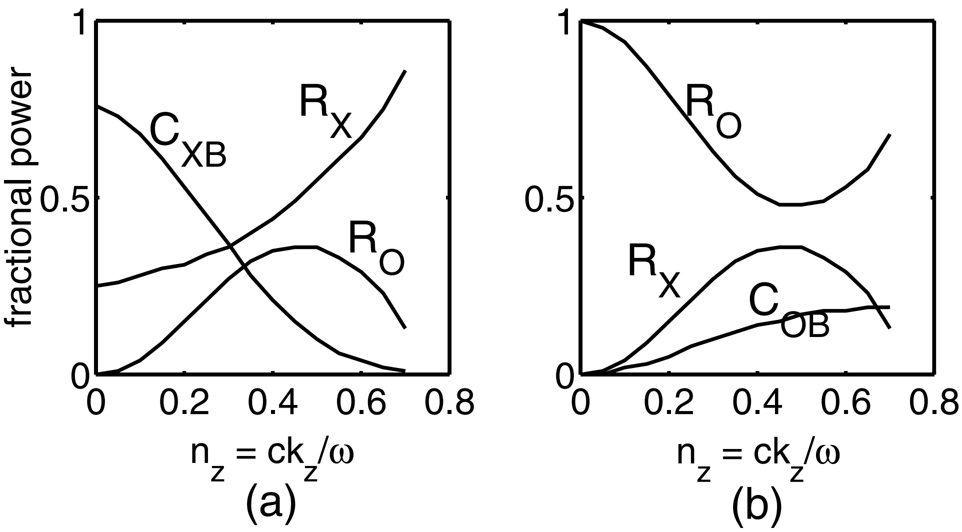

Figure 1: The fraction of the input power, as a function of k_z for $k_y = 0$, that is reflected out on the X mode R_X, on the O mode R_O, and mode converted to Bernstein waves: (a) for incoming X mode the conversion coefficient is C_{XB}; (b) for incoming O mode the conversion coefficient is C_{OB}.

When EBWs are emitted from inside the plasma, they propagate out to the edge where they can couple to the X and O modes near the UHR. Some of the EBW power reaching the UHR will get reflected back into the plasma. For the same parameters as discussed above, Figure 2 shows the fraction of the emitted EBW power, as a function of k_z, that is converted to the X mode E_X, to the O mode E_O, and reflected back into the plasma R_B. Upon comparing the results from Figures 1a and 1b with those given in Figure 2, we note the following relationships

$$E_X = C_{XB} \quad \text{and} \quad E_O = C_{OB} \tag{8}$$

Thus, the fraction of the EBW emitted power that is mode converted to the X mode and the O mode is the same as the power that is mode converted from the X mode to the

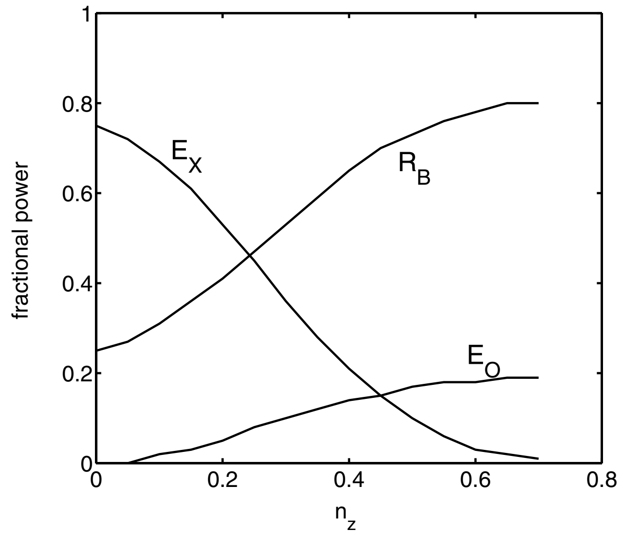

Figure 2: The fraction of the outgoing EBW power, as a function of k_z for $k_y = 0$, that is converted to the X mode E_X and on the O mode E_O. R_B is the fraction of the emitted EBW power that is reflected back into the plasma at the upper hybrid resonance.

EBW and, separately, from the O mode to EBW. Since $E_X + E_O \leq 1$, it follows that if the plasma conditions are such that the X-B conversion process is optimized, then the O-B conversion process is not effective, and vice-versa. We have noted this effect before when studying the mode conversion excitation of EBWs [1]. It was found that the X-B and the O-B conversion processes were optimized in, essentially, mutually exclusive regions of the parameter space spanned by $(\omega, k_{\|})$ (where $k_{\|}$ is the component of $\vec{k}$ parallel to $\vec{B}_0$). Consequently, from the relationship in (8), we can conclude that the experimental design for the excitation of EBWs can be completely based on the EBW emission characteristics of the plasma.

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