July, 1981

LIDS-P-1110

MODELING THE INTERACTING DECISION MAKER WITH BOUNDED RATIONALITY*

by

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ABSTRACT

An analytic characterization of the process of executing a well-defined decision-making task by a human decision maker is presented. A basic two-stage model of this process is introduced in which external situations are first assessed and then responses are selected. An information theoretic framework is used in which total internal activity is described in terms of internal coordination and internal decision-making, as well as throughput and blockage. A constraint on the rate of internal processing is suggested as a model of bounded rationality. The model is extended to include basic interactions in an organizational context: Direct control is modeled as a restriction on internal decision-making by external commands while indirect control is incorporated through an auxiliary situation assessment input received from the organization.

*This work was supported by the Air Force Office of Scientific Research under grant AFOSR-80-0229.

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INTRODUCTION

The role of the human decision maker is central to the design and evaluation of alternative organizational structures. Each structure includes a number of interacting decision makers (DMs) who must make compatible decisions in overlapping areas of responsibility using different data. If the decisions are based on organization-wide objectives, then the determination of decision strategies is a team-decision theoretic problem. In previous work on such problems [1], [2], [3], it has been assumed tacitly that the DMs are perfectly rational, i.e., each DM is allowed a given set of alternatives, has some knowledge of the consequences of choosing a particular alternative, and can rank order the alternatives with respect to some index of performance [4]. Optimal decision strategies are then obtained.

An alternative hypothesis, however, is that due to limitations in information processing and problem solving ability, the decision maker is unable to construct and consider all alternatives in a given situation, and cannot evaluate precisely the alternatives that he does consider [5]. To the extent that this is the case, the rationality of the decision maker cannot be perfect no matter how "intendedly rational" he is [6], i.e., he exhibits bounded rationality. March and Simon suggest that the DM with bounded rationality seeks to find an alternative which is satisfactory with respect to a given criterion, i.e., an alternative which satisfices [7].

Input-output models of the decision maker with bounded rationality have already been presented [8], [9], [10]. The basic departure in this paper from previous work is the modeling of the internal processing in transforming the inputs to the decision maker into outputs. This characterization of the decision-making processing is achieved through a synthesis of qualitative notions of decision-making with the analytic framework of information theory in which an <u>internal decision strategy</u> determines the input-output mapping. The characterization is such that

(1) an analytic representation of the total activity required to accomplish the internal processing can be given as a function of the internal decision strategy;

(2) the bounded rationality of a decision maker appears naturally as a constraint on the rate of total activity; and

(3) indirect and direct control through interactions with other organization members is included readily.

March and Simon [7] have hypothesized that the decision-making process of the satisficing decision maker is a two-stage process of "discovery and selection." The first stage is that of determining the situation of the environment, while the second addresses the question of what action to take in a particular situation. Selection in the first stage takes the form of choosing the degree and type of the "discovery" which the decision maker wishes to make regarding his environment, while discovery in the second stage pertains to generating possible courses of action for consideration. Clearly, the stages are coupled in that the type of alternatives sought depend on the situation perceived. Together they constitute the "construction of the decision situation" from which a decision emerges, since if the decision-making process has been carried out adequately, a satisfactory alternative is generated. Recent work by Wise [11] has supported this viewpoint.

Wohl [12] has suggested a similar two-stage model of the decision process through an extension of the classical stimulus-response model in psychology. When a stimulus is received, the initial reaction of the decision maker is to hypothesize about its origin. This is followed by the generation and evaluation of options, among which one response is selected. Wohl applies this Stimulus - Hypothesis - Option - Response (SHOR) model in a military context to the tactical decision process.

The model of the DM developed in the following sections yields pure internal decision strategies when the decision maker is unconstrained, and mixed strategies when bounded rationality is introduced. Similarly, in the satisficing context, it is shown that it is possible for the solutions to be only mixed strategies. Also, the greater the uncertainty in the input or stimulus to the DM, the greater the total activity required in the internal decision-making process. This is consistent with aspects of organization theory [13] that relate in a qualitative manner the uncertainty in the task to be performed and the amount of information that must be processed within the organization during task execution in order to achieve a given level of performance.

The paper is organized as follows. In the next section, the model of the decision making process is developed. In the following sections, the decision strategies for normative and satisficing problems are obtained and analyzed. Finally, the effect of interactions with the rest of the organization and the concepts of direct and indirect control are explored.

MODEL OF THE DECISION MAKING PROCESS

Based on the above discussion, the following two-stage model is assumed, and is illustrated in Figure 1. The decision maker receives an input x from his environment and uses it in the situation assessment (SA) stage of processing to "hypothesize about its origin." This results

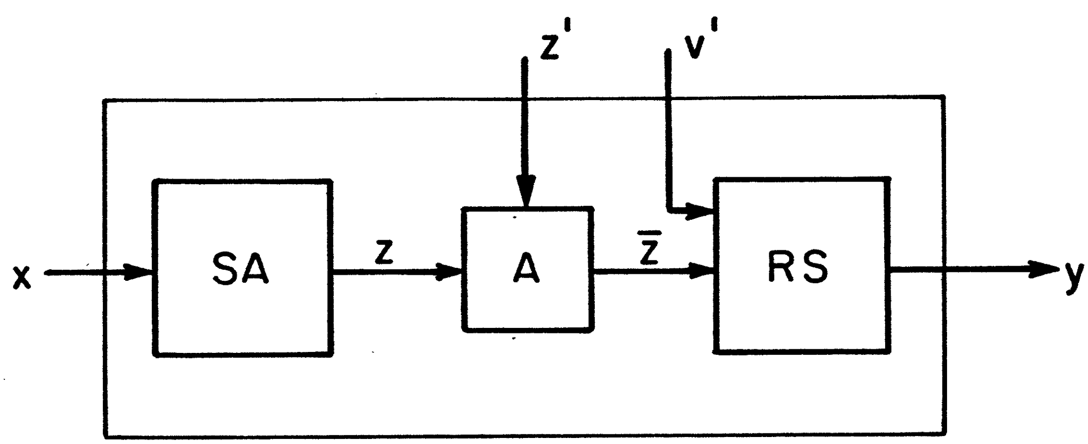

Fig. 1 The Basic Model of the Decision-Making Process

in the selection of a particular value of z, the variable that denotes the situation. Signals from the rest of the organization (RO) may modify the assessment of the situation and lead to the determination of a value for $\bar{z}$. Possible alternatives of action are then evaluated in the response selection (RS) stage. The outcome of this process is the selection of action or decision response y. A command input v' from the rest of the organization may affect the selection process.

Many classes of decisions can be represented by the process of Figure 1. Consideration in this paper will be restricted to decision-making tasks which are well-defined and which are performed in the <u>steady-state</u>, that is, the decision maker is assigned a particular task for which he is well trained and which he performs again and again for successively arriving inputs.

The first step, SA, can be considered as containing a set of well defined procedures or algorithms which map the input stimuli x to the assessed situation z. The algorithms differ in the amount of resources required to process the input and in the quality of the assessment they produce. However, no connection between these two attributes is assumed. The algorithms remain fixed as the process takes place; there is no adaptation or learning within each algorithm.

To be more precise, assume that the state of the decision maker's environment is given by x', an r-dimensional vector which takes values from a finite alphabet. However, the decision maker receives as input x, which is a noisy measurement of x'. The vector x is also r-dimensional and takes known values from a finite alphabet according to p(x).

The decision maker selects one of the U algorithms he possesses that map measurements x into assessed situations z, where z is an s-dimensional vector taking M values, with $s \leq r$. In the extreme case, the situation assessment would involve an estimation of the entire state x'. However, it is more likely that in order to choose an appropriate output or decision response, it is necessary to consider only some "sufficient statistic" determined from the measurement x. Thus, z represents a possible aggregation of input data. The input-output mappings of the algorithms used to determine z from x are denoted by $f_i(x)$ where i = 1, 2,..., U. For a given x, the situation assessment is obtained by the realization of the variable u. This variable is one of the internal choices in the decision-making process; indeed, according to the model defined above it represents the real decision made in accomplishing the assessment task. This process can be represented as shown in Figure 2, where q is the noise source in the measurement of x', and x = x' +q. The internal choice has been represented as a switch which takes positions according to the realization of u.

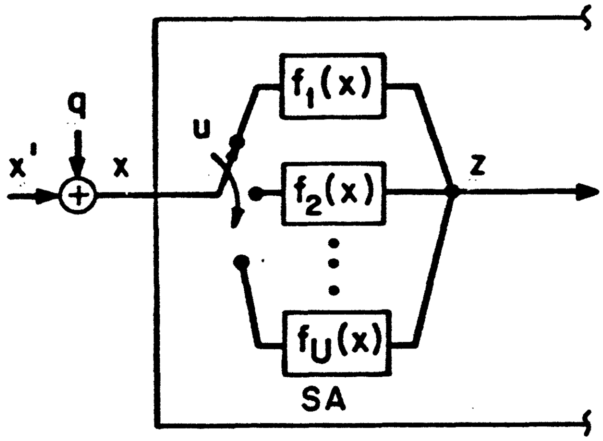

Fig. 2 Situation Assessment Stage

The inputs to the decision maker are considered to be symbols generated by a source according to p(x). A memoryless source is assumed, i.e., each symbol is generated independently. The quantity

$$H(x) = - \sum_{x} p(x) \log_2 p(x) \tag{1}$$

is defined to be the entropy of the source per symbol generated [14] measured in bits. If, in addition, the source is such that an input symbol is generated every τ seconds on the average, the entropy rate of the source is given by $H(x)/\tau$ which is measured in bits per second. The quantity τ is the mean symbol interarrival time and it is a description of the "tempo" of operations. The quantity H(x) can also be interpreted as the uncertainty regarding which value the random variable x will take.

To effect a mapping from x to z each algorithm consists of a series of steps, such as intermediate computations or comparisons. These steps specify the variables of the algorithm. Suppose algorithm i contains α_i variables denoted by

$$W^i = \{w_1^i, w_2^i, \ldots, w_{\alpha_i}^i\} \tag{2}$$

and let the algorithms have no variables in common, i.e.,

$$W^i \cap W^j = \emptyset \qquad i \neq j; \forall i,j \in \{1,2,\ldots,U\}. \tag{3}$$

Then, the model of the situation assessment stage consists of a system of variables, denoted S^I, where

$$S^I = \{u, W^1, W^2, \ldots, W^U, z\} \tag{4}$$

The interconnection of these variables is determined by the algorithmic interconnections within sets W^i, as well as the interconnection among algorithms determined by the variable u.

A key assumption in the following sections is that the mappings f_i are deterministic. Furthermore, because of the model structure, each algorithm is considered to be active or inactive, depending on the internal decision u. The probability distribution for each internal variable w_j^i therefore has two distinct modes. Consider the variable w_1^1 which is active when u = 1. Under this condition it takes values according to the values of the input x and also according to the characteristics of the algorithm. If $u \neq 1$, then w_1^1 is inactive, and in that case is assumed to take a fixed value which is not one of the values taken when active. The deterministic property of the algorithm implies that once the input is known and the algorithm choice is made, all other variables of the system are known.

Finally, because no learning takes place during the performance of a sequence of taks, the successive values taken by the variables of the model are uncorrelated, i.e., the model is memory-less. Hence, all information theoretic expressions written in the following development are on a per symbol basis (per symbol of the input); rates are determined by dividing the appropriate quantities by the mean symbol interarrival time τ.

G_n^I, the uncertainty in the system when the input is known, is by definition

$$G_n^I = H_x\ (u,\ W^1,\ W^2, \ldots\ W^U,\ z) \tag{5}$$

where $H_x(\ )$ is the conditional entropy (uncertainty) given by

$$H_x(z) = -\sum_x p(x) \sum_z p(z|x)\ \log_2 p(z|x). \tag{6}$$

In the present case, it reduces to

$$G_n^I = H(u). \tag{7}$$

The internal decision u is independent of the input x; x and u together determine the system S^I. The fundamental quantity in H(u) is the distribution p(u) which represents the inclination of the decision maker to select a particular algorithm, and is termed the internal decision strategy. For successively arriving inputs, the strategy reflects the relative frequency of a particular algorithm's use.

If an algorithm is used exclusively ($p(u = i) = 1$ for some i) then $H(u) = 0$, which indicates that no real decision is being made. On the other hand, when p(u) is uniform, i.e., each algorithm is equally likely to be chosen, then H(u) is at a maximum. G_n^I is therefore interpreted to be the amount of internal decision-making in the situation assessment stage.

The mutual information or transmission [14] between x and z written $T(x:z)$, describes the input-output relationship or throughput of the SA stage, which is denoted by G_t^I. Throughput is evaluated, by the definition, from

$$T(x:z) = H(z) - H_x(z). \tag{8}$$

Recall that the fundamental quantity in $H(z)$ is $p(z)$; similarly, $p(z|x)$ and $p(x)$ are needed to evaluate $H_x(z)$. A straightforward application of Bayes' rule, coupled with the knowledge of distribution $p(x)$ and the algorithm mappings $f_i(x)$, is sufficient to demonstrate that G_t^I is determined as an explicit function of the internal decision strategy $p(u)$.

A quantity complementary to the throughput is that part of the input information which was not transmitted by the system, i.e., the <u>blockage</u> of the system. It is denoted G_b^I and given by

$$G_b^I = H(x) - G_t^I. \tag{9}$$

The total coordination in the situation assessment (SA) stage is given by

$$G_c^I = T(w_1^1:w_2^1:\ldots:w_{\alpha_1}^1:w_1^2:\ldots:w_{\alpha_U}^U:u:z) \tag{10}$$

where T denotes the mutual information between all the variables. If the system S^I consists of U interconnected subsystems as shown in Figure 2, then the total coordination can be decomposed in terms of the internal coordination of each subsystem plus the coordination among subsystems [15], [16]; in this case the decomposition is given by

$$G_c^I = \sum_{c=1} [p_i g_c^i (p(x)) + \alpha_i \mathbf{H}(p_i)] + H(z) \tag{11}$$

where g_c^i denotes the internal coordination present in the i-th algorithm, p_i is the probability that the i-th algorithm has been selected, i.e., $p_i = p(u = i)$, and $\mathbf{H}(p)$ is the entropy of a random variable that can take one of two values with probability p [17]:

$$H(p) = p \log p + (1-p) \log (1-p). \qquad (12)$$

The function is shown in Figure 3. The expression for the total coordination, eq.(11), reflects the presence of switching within S^I. The weighting of the subsystem coordinations g_c^i is the relative frequency of each algorithm's use (the internal decision strategy). The value of each g_c^i depends on the internal variables of the algorithm and its implementation and on the characteristics of the input.

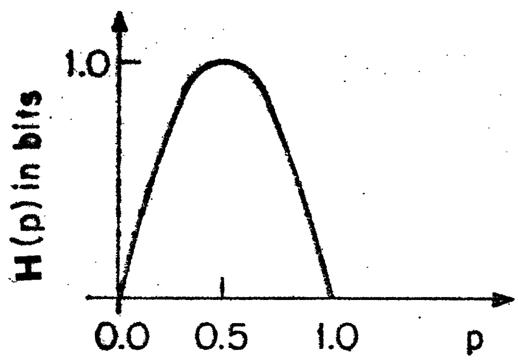

Fig. 3 Coordination Per Variable Required for Initialization

The second term of eq. (11) is interpreted to be the coordination required to switch among algorithms; it can also be regarded as the effort or resource use required to initialize the variables of an algorithm prior to its use. Examination of the mathematical expression for this coordination shows that it is dependent on p_i, the relative frequency of a particular algorithm's use, and furthermore, that each variable of the same algorithm makes an equal contribution to the total. The latter is not unreasonable, and the former is necessary because the coordination equation represents steady-state phenomena, i.e., the coordination required to initialize algorithms is very much related to the number of times on the average such initializations must take place. Reference to Figure 3 shows that the nature of this relationship is such that if a particular algorithm is always used (p = 1 in Figure 3), the initialization coordination is zero, as no initializations are taking place in the steady state. Similarly, if an algorithm is never used, it is never initialized ($\mathbf{H}(0) = 0$). In addition the symmetry of $\mathbf{H}(p)$ about $p = 0.5$ is significant because frequent use of an algorithm requires on the average the same number of initializations as equally infrequent use. This phenomenon arises because an often used algorithm is likely to be used for successive inputs, in which case no re-initialization would take place. Furthermore, since G_c^I measures the global coordination among all variables of S^I, and since z is the only variable within S^I which is related to all other variables, it is to be expected that G_c^I contains the term H(z).

Finally, the total uncertainty is defined as

$$G^I = \sum_{w \in S^I} H(w) + H(u) + H(z); \tag{13}$$

it is the sum of the entropy of each variable in S^I. The Partition Law of Information [15] yields the following identity:

$$G^I = G_n^I + G_t^I + G_b^I + G_c^I \tag{14}$$

Eq. (14) states that coordination, throughput, blockage, and internal decision making together describe the total activity in a system.

The four quantities on the right hand side of eq.(14) can be computed, if the probability distributions p(x) and p(u) are known and if the algorithms f_i and their specific realizations are given. A computer program that carries out the extensive calculations has been written.

The full realization of the basic model of Figure 1 is shown in Figure 4. The situation assessment stage is as described in the previous paragraphs. The variable z', the supplementary situation assessment received from the rest of the organization, combines with some subset of the elements of z to produce $\bar{z}$. The variables z and $\bar{z}$ are of the same dimension and take values from the same alphabet. The processing of z and $\bar{z}$ is accomplished by the subsystem S^A which contains the deterministic algorithm A; the latter defines a set of α_A variables, including $\bar{z}$, labelled W^A.

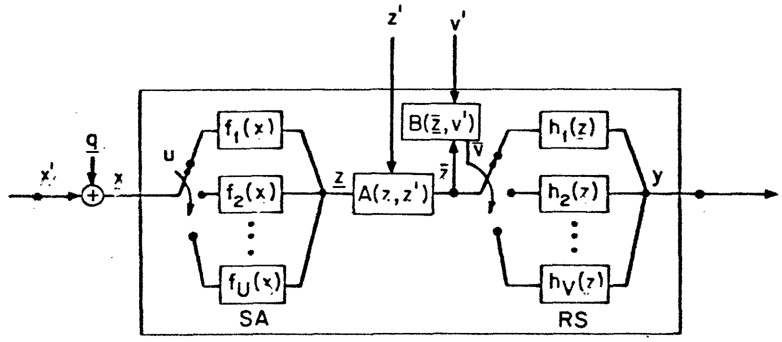

Fig. 4 Realization of the Basic Model

If there were no command input v', then $p(v|\bar{z})$ specifies the internal decision for selecting one of the V algorithms that map $\bar{z}$ into the output y, i.e.,

$$y = h_j(\bar{z}). \tag{15}$$

The development of the analytical description of the response selection stage (RS) is identical to that for the situation assessment but with $p(v|\bar{z})$ in place of p(u). Each algorithm h_j contains α'_j interconnected variables, denoted W^{U+j}, which specify the input-output mapping (15). The RS algorithm variables, together with the internal decision variable v and the output variable y, constitute the set of all variables of subsystem S^{II}.

The existence of a command input v' from the rest of the organization modifies the decision maker's choice v. A final choice $\bar{v}$ is obtained from an algorithm b

$$b: \bar{v} = b(v,v') \qquad \bar{v} = 1, 2,\ldots,V. \tag{16}$$

The specification of $b(v,v')$ defines a protocol according to which the command is used, i.e., the values of $\bar{v}$ determined by $b(v,v')$ reflect the degree of option restriction effected by the command.

The overall process of mapping the assessed situation $\bar{z}$ and the command input v' into the final choice $\bar{v}$ is represented by algorithm B in Figure 4, and the result of this process is a determinsitic modification of the strategy $p(v|\bar{z})$ into an effective strategy $p(\bar{v}|\bar{z}v')$. The processing of $\bar{z}$ and v' to yield $\bar{v}$ is done in subsystem S^B which contains the algorithm B; the latter has α_B variables.

If the model of the decision-making process, Figure 4, is viewed as a system S consisting of subsystems S^I, S^A, S^B, and S^{II} which inputs x, z', and v' and output y, then the Partition Law for Information can be expressed as follows:

- Throughput

$$G_t = T(x,z',v' : y) \tag{17}$$

- Blockage

$$G_b = H(x,z',v') - G_t \tag{18}$$

- Internal Decision-Making

$$G_n = H(u) + H_{\bar{z}}(v) \tag{19}$$

- Coordination

$$G_c = G_c^I + G_c^A + G_c^B + G_c^{II} + T(S^I : S^A : S^B : S^{II}) \tag{20}$$

where

$$G_c^A = g_c^A(p(z)) \tag{21}$$

$$G_c^B = g_c^B(p(\bar{z})) \tag{22}$$

$$G_c^{II} = \sum_{j=1}^{V} \left[p_j g_c^{U+j}(p(\bar{z}|\bar{v}=j)) + \alpha_j \mathbf{H}(p_j) \right] + H(y) \tag{23}$$

$$T(S^I:S^A:S^B:S^{II}) = H(z) + H(\bar{z}) + H(\bar{v},\bar{z}) + T_z(x':z') + T_z(x',z':v'). \tag{24}$$

The expression for the internal decision-making, eq. (19), shows that G_n depends on the two internal decision strategies $p(u)$ and $p(v|\bar{z})$, even though a command input v' may exist. This implies that the command input modifies his internal decision after $p(v|\bar{z})$ has been determined. Furthermore, it is possible for the command input to override totally the internal decision, i.e.,

$$\bar{v} = b(v,v') = v'.$$

The analysis and interpretation of the effects of z' and v' on the organization require further consideration of the types of interaction that can occur. These depend on the information structure of the organization. A general representation of the interaction between one DM and the rest of the organization is shown in Figure 5. The overall input to the organization from its environment is a vector X'. In this case, the information structure is defined by the two partioning matrices Π_1 and Π_2 with

$$x' = \Pi_1 X' \quad , \quad x_o' = \Pi_2 X' \tag{25}$$

where x' is the input vector to the DM and x_o' is the input vector to the rest of the organization (RO). In general, x_o' and x' can be disjoint, overlapping partially, or even identical. The only assumption that needs to

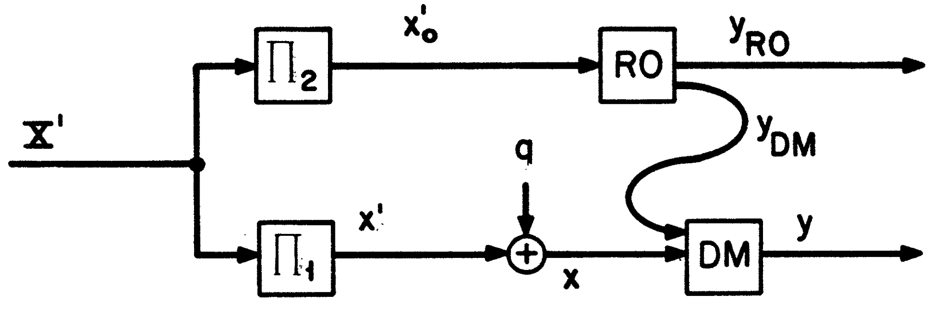

Fig. 5 Information Structure for Organization Member

be made at this time is that z', derived from x'_o, provides additional information to the DM. The assumption is necessary for deriving the detailed computable expressions for the throughput G_t. The expression for blockage reflects the fact that

$$G_t + G_b \equiv H(x,z',v')$$

i.e., all the uncertainty in the inputs to the DM either appears in the output (via the throughput) or is blocked internally.

The coordination for the system S, eq. (20), contains the internal coordination within each subsystem plus the coordination due to the interaction between subsystems. The first term in eq.(20) is identical to that of eq. (4) for the situation assessment stage. The second term, eq. (21), and the third term, eq. (22), depend on the probability distributions of the internal assessment z and the modified assessment $\bar{z}$, respectively. The internal coordination in the response selection stage, eq. (23), depends not only on the frequency with which each algorithm is selected, p_j, but also on the value of the assessed situation $\bar{z}$ and the value of the command input. The fourth term, eq. (24), includes the coordination due to the linkages betwen S^I and S^A, S^A and S^B, and (S^A, S^B) to S^{II}, respectively. The term $T_z(x':z')$ arises because of the relationship between the external input to the DM, x', and the supplementary situation assessment input, z'. Because of this relationship, it is possible to effect a greater coordination between S^I and S^A than that given by H(z), i.e., more information about the input x' can be forwarded to S^A than contained in z. For example, it is possible that the RO can resolve more finely a portion of the DM's input x' and a partial situation assessment which is more refined in some aspects than the DM's own assessment can therefore be made. In such an instance, an additional amount of the input is passed forward to S^A and the coordination between subsystems increases. This additional activity within the DM does not increase the total in general; rather, it can, at times, reduce significantly the activity required for subsequent processing, as will be illustrated later. A similar interpretation applies to the term $T_z(x',z':v')$.

BOUNDED RATIONALITY AND PERFORMANCE EVALUATION

The notion of bounded rationality refers to the limited ability of the human to process information. This qualitative notion translates readily into a restriction on the rate of total activity. Since steady state operation has been assumed, the rate of total activity can be expressed in terms of the tempo of operations (or mean symbol interarrival time) τ by

$$G/\tau \leq F \tag{27}$$

where F is the constraint expressing bounded rationality in bits per second. It will be assumed further that the task assigned to a decision maker and the choice of strategies must be such that constraint (27) is not violated.

In order to analyze the types of decision strategies used by a DM with bounded rationality, it is useful to introduce a mechanism for evaluating performance that is appropriate in both the normative and the descriptive context [5].

Let y' be the desired decision response to the input x' and let L(x') be a function or a table that associates a y' with each member of the input alphabet x'.

The actual response y and the desired one, y', are compared using a function d(y,y') which assigns a cost to each possible pair (y,y'). The expected value of this cost can be obtained by averaging over all possible inputs. This value can then serve as a performance index J for each pair of internal strategies p(u) and $p(v|\bar{z})$. For example, if

$$d(y,y') = \begin{cases} 0 & y = y' \\ 1 & y \neq y' \end{cases} \tag{28}$$

then

$$J(p(u),\ p(v|\bar{z})) = E\{d(y,y')\} = p(y \neq y') \tag{29}$$

which represents the probability of error in decision-making (Figure 6).

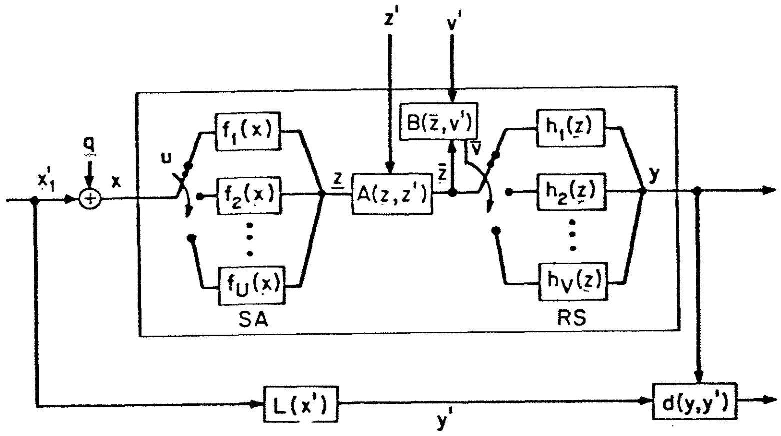

Fig. 6 Model of Decision-Making Process With Performance Evaluation Mechanism

The information obtained from performance evaluation can be used by the organization designer in defining and allocating tasks to the decision maker and in changing the number and contents of the situation assessment and response selection algorithms. This is achieved through training and learning; these processes, however, are outside the scope of this model, which is limited to decision-making in the steady-state.

The following problems can be posed:

Given the model of the decision-making process shown in Figure 6, where the internal processes are described by eqs. (14, (17)-(20), determine the internal strategies $p(u)$ and $p(v|\bar{z})$ such that either

(a) $J(p(u),\ p(v|\bar{z}))$ is minimized;

or (b) J is minimized subject to $G(p(u),\ p(v|\bar{z})) \leq F\,\tau$

or (c) $J \leq \bar{J}$

or (d) $J \leq \bar{J}$ subject to $G \leq F\tau$.

The first two are normative problems while the latter two are formulated so as to obtain satisficing strategies with respect to a performance threshold $\bar{J}$. The bounded rationality condition depends on τ; therefore, the internal strategies will also depend on the tempo of operations. The unconstrained cases (a) and (c) can be throught of as limiting cases when $\tau \to \infty$.

Decision strategies, i.e., situation assessment strategies p(u) and response selection strategies $p(v|\bar{z})$, can be described as pure or mixed. A pure strategy is one for which an algorithm f_i is selected with probability one, i.e.,

$$p(u = i') = 1 \qquad \text{for some } i'$$

and

$$p\left(v = j' \middle| \bar{z} = \bar{z}_m\right) \quad \text{for some } j' \text{ and for each } \bar{z}_m.$$

Since there are U pure situation assessment strategies and V·M response selection strategies, there are U·V·M possible pure decision strategies. All other strategies are said to be mixed and are represented by non-trivial distributions p(u) and $p(v|\bar{z})$. Every possible mixed strategy can be expressed as a convex combination of pure strategies.

A useful way of describing the properties of the solutions to the four problems (a)-(d) is by introducing the plane (J,G); each specific decision strategy is represented by a point, a pair (J,G), in that plane. Indeed, if a pure strategy is given, then, using eqs. (17)-(20) the components of G can be evaluated and, from eq. (14), G itself. Similarly, given the same decision strategy, the input-output pairs (x',y) can be determined and, consequently, J can be evaluated.

First, the convexity of G in the decision strategy is shown. The total activity G is defined as the sum of the marginal uncertainties H(w) of each system variable w [15]. If the possible distributions p(w) are elements of a convex distribution space, then H(w) is a convex function of p(w) [17]. Now, corresponding to each pure strategy D_k is a distribution $p_k(w)$ on w. Furthermore, any convex combination of pure strategies which defines a decision strategy determines a distribution p(w) of the arbitrary system variable w as a convex combination of the distributions of the same variable which correspond to the pure strategies, i.e., if

$$p(w) = (1-\delta)p_1(w)+\delta p_2(w)$$

then

$$H(w) \geq (1-\delta)H_1(w)+ \delta H_2(w) \qquad 0 \leq \delta \leq 1$$

and

$$\sum_w H(w) \geq (1-\delta) \sum_w H_1(w)+\delta\sum_w H_2(w) \qquad 0 \leq \delta \leq 1$$

The last expression is equivalent to

$$G \geq (1-\delta)G_1 + \delta G_2. \tag{30}$$

Consider the mixed strategy

$$D(\delta) = (1-\delta)D_1 + \delta D_2. \tag{31}$$

Then the objective function is given by

$$J(D) = (1-\delta)J_1 + \delta J_2. \tag{32}$$

Equations (30) and (32) are parametric in δ and can be used to describe the relationship of G and J as shown in Figure 7. The relative position of pairs (J_1,G_1) and (J_2,G_2) is arbitrary, i.e., it is not true in general that a smaller total activity G also realizes worse performance J.

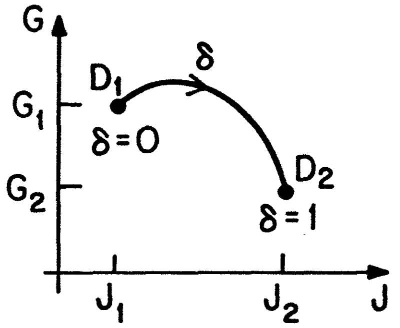

Fig. 7 (G,J) Locus for Binary Variation of Pure Strategies

Application of the above construction to all possible binary variations between pure strategies and then to successive binary combinations of mixed strategies leads to a region in the (J,G) plane that contains all possible strategies. Such a region for three pure strategies* is shown in Figure 8.

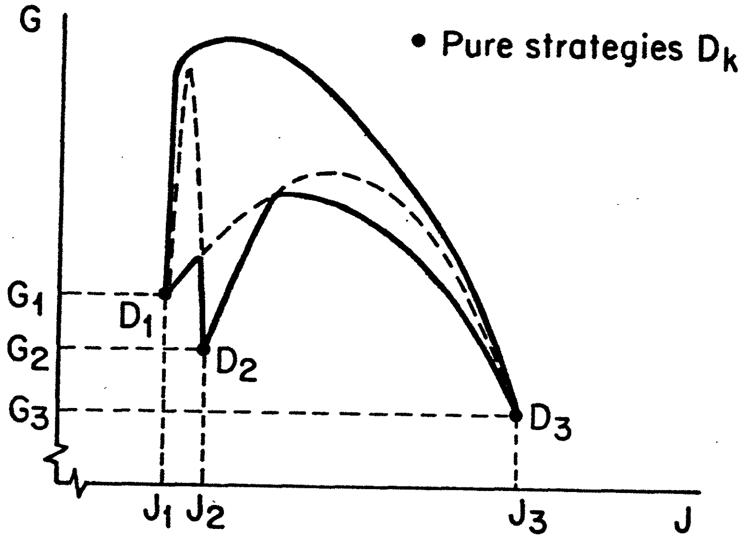

Fig. 8 Region of admissible (J,G) Pairs

The lower boundary of the region consists of the pure strategies and binary variations between pure strategies. It follows that the minimum J solution, D_1, is a pure strategy as is the minimum G one, D_3. The solution to the problems (a)-(c) can be analyzed using the (J,G) representation.

(a) The minimum error strategy will always be a pure strategy. This is evident from the construction of the (J,G) region.

(b) The bounded rationality condition is represented by a straight line parallel to the J axis. For a fixed value of the rate F, the bounded rationality threshold G_r is proportional to the tempo of operations, i.e.,

$$G_r = F \tau$$

*The (J,G) region shown represents the solution space for a specific numerical example [18] in which there are no inputs from the rest of the organization.

The decision strategy that minimizes the error can be pure or mixed, the latter a convex combination of two pure strategies. The specific solution depends on the intersection of the boundary of the (J,G) region with the G_r line.

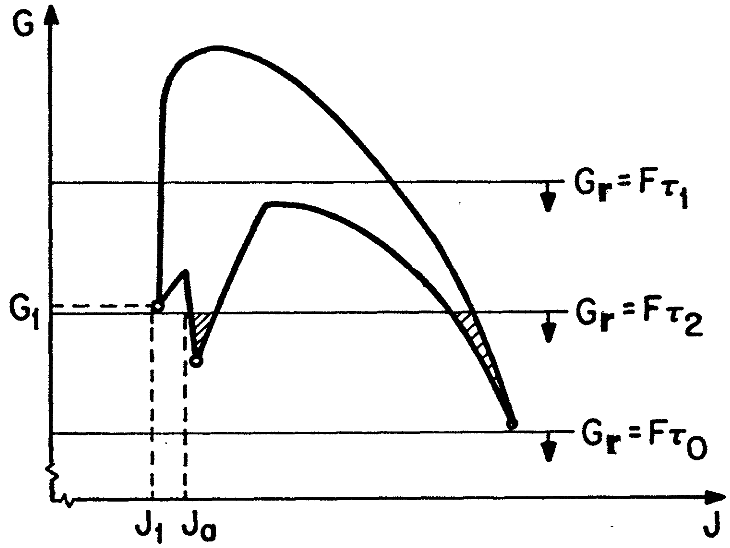

Fig. 9 Description of Solutions for Normative Problems (a), (b)

Two types of intersection are possible, as shown in Figure 9. For $\tau = \tau_1$, the minimum error is achieved by the (pure) strategy corresponding to the point (J_1, G_1), which is also the solution obtained to problem (a). However, as τ decreases, it may no longer be possible to use the optimal strategy, as illustrated in Figure 9 for $\tau = \tau_2$. In that case, the minimum error strategy is in general a <u>mixed</u> strategy, a binary variation between pure strategies.

As τ decreases, i.e., the tempo of operations increases, there exists in general some value $\tau = \tau_o$ below which the solution set is empty. This means that the rate of input arrivals is too fast for adequate processing. Such a condition represents on overload of the decision maker. Overload is represented by a line $G = G_0$ that is below the solution region in the (J,G) plane.

The solutions to the descriptive problems (c) and (d) can be characterized as the set of feasible solutions p_k to

$$\sum_{k=1}^{U.V.M} p_k J_k \leq \bar{J} \tag{33}$$

$$\sum_{k=1}^{U.V.M} p_k = 1 \quad ; \quad p_k \geq 0 \ \forall\, k \tag{34}$$

The condition (33) specifies a partition of the solution region by the vertical line $J = \bar{J}$ as shown in Figure 10.

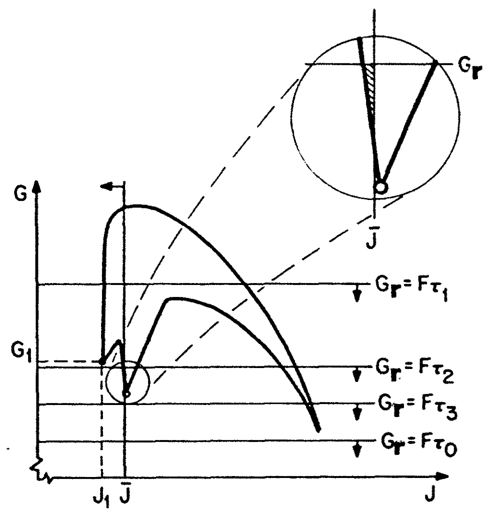

Fig. 10 Solutions to Descriptive Problems

Note that while, in general, an infinite number of decision strategies (shaded region) are satisficing, the difference in total activity between them can be quite large. Also, if the performance threshold $\bar{J}$ is less than the minimum J_k then no satisficing solution exists.

The solution to the descriptive problem with a bounded rationality constraint, (d), is obtained readily as the set of strategies, represented by values of p_k, which yield (J,G) pairs in the region defined by

$$R = \{(J,G) \mid G_r \leq F\tau\} \cap \{(J,G) \mid J \leq \bar{J}\} \tag{35}$$

Several types of intersection are possible depending on the values of τ and $\bar{J}$. For τ sufficiently large, the set of satisficing strategies includes the minimum error strategy ($\tau = \tau_1$ in Figure 10). As τ decreases, however, the solution set may contain only mixed strategies ($\tau = \tau_2$), i.e., strategies for which the amount of internal decision making G_n is non-zero. If τ is decreased sufficiently ($\tau = \tau_3$) then the satisficing solution set is empty. The decision task can be accomplished but the performance will not be good enough, i.e., the constraint $J \leq \bar{J}$ will be violated. Finally, there exist values of τ ($\tau \leq \tau_o$) for which the task cannot be accomplished.

The analysis using the (J,G) plane has shown that the minimum probability of error is realized by a pure strategy when no constraints are present. When the bounded rationality constraint is introduced, then it is possible for the optimal strategy to be a mixed one. In the satisficing context, it is also possible that, when the constraint of bounded rationality is imposed, all satisficing strategies be mixed ones.

Another confirmation of earlier results that used an information theoretic model of the decision maker [8], [9], [19] is that if the objective is to minimize the information processing activity G then pure strategies result. Indeed, the minimum G points in Figure 9 always correspond to pure strategies.

INTERACTIONS WITH THE ORGANIZATION

Before proceeding with the analysis of the DM's interactions with the rest of the organization through the supplementary situation assessment z' and the control input v', the magnitudes of the terms in the partition law, eq. (14), will be discussed.

First of all, the throughput and the blockage together are equal to the entropy of the inputs, eq. (26). The maximum value this entropy can take is

$$G_t + G_b = H(x,z',v') \leq \log_2 N + \log_2 M + \log_2 V \tag{36}$$

where N and M are the number of elements in the alphabets of x and z', respectively, and V is the number of response selection algorithms.

The internal decision-making, G_n, eq. (19) ranges from zero to a maximum value

$$0 \leq G_n \leq \log_2 U + \log_2 V \tag{37}$$

where U is the number of different situation assessment algorithms.

The coordination G_c, eq. (20), has five terms. The first four, eqs. (11), (21) - (24), include the internal coordination terms for each subsystem S^I, S^A, S^B, and S^{II} (namely, the g_c's) and terms proportional to the number of variables each algorithm contains. The fifth term, eq. (24), as well as the remaining terms in eqs. (11, (21)-(23) are of the order of $\log_2 N, \log_2 M, \log_2 U, \log_2 V$ or $\log_2 Y$, where the Y is the number of elements in the alphabet of y. An upper bound for G_c is:

$$G_c \leq \alpha^I(\log_2 N+2) + 2\alpha^A \log_2 M + 2\alpha^B \log_2 V + \alpha^{II}(\log_2 M+2)$$

$$+ 2\log_2 N + 6\log_2 M + \log_2 V \tag{38}$$

Since the number of variables in the algorithms is in general orders of magnitude larger than the algorithm of N,M,U,V, or Y, the internal coordination terms for the agorithms in the susbsytems dominate.

To illustrate the relative magnitudes or the various terms results from a simple example [18] are shown in Table 1. It is the same example from which Figure 8, 9, and 10 were extracted.

Note in Table 1 that the throughput and blockage add to 8.9 bits, the entropy of the input x:

$$G_t + G_b = H(x) = \log_2 486 = 8.9 \text{ bits}$$

Since only pure strategies are listed, it follows that G_n is identically zero. The internal coordination terms G_c clearly dominate.

Table 1. ILLUSTRATION OF THE PARTITION LAW

STRATEGY	G_t	G_b	G_n	G_c	G	J
D_1	1.4	7.5	0	114.5	125.0	0.23
D_2	1.2	7.7	0	109.9	118.8	0.25
D_3	0.3	8.6	0	102.5	111.4	0.38

The introduction of the interactions with the rest of the organization has a direct effect on the components of the partition law. While on the one hand, the subsystems S^A and S^B contribute to the coordination term, the information conveyed by z' and v' may reduce substantially the uncertainty and thus reduce the value of G.

Consider again Figure 5. The overall task of the organization is partitioned by the organization designer into subtasks by specifying the information structure [19], [20]. This, in turn, determines to a large extent the types of interactions possible between organization members. As discussed earlier, the relationship between z and z'depends on the relationship between x' and x'_o. The information structure will be assumed known in the following discussion.

Because the decision making model has been formulated as a process, it is possible to distinguish interactions occuring at different points in the process. Both interactions shown in Figure 4 are of the result sharing form of cooperating behavior [21]. The situation input z' represents the result of input processing in RO and which is passed to the DM. Similarly, the command input v' can be regarded as the result of another decision process which occurs within the RO.

In the context of organization theory, the functional characteristics of subsystems S^A and S^B that define the specific form of the interactions can be chosen so that lateral relationships (S^A) as well hierarchical ones (S^B) can be represented [13]. For example, the function B(v,v') may be such that the command input serves to coordinate the response selection activities of two DMs of the same rank (same echelon) or it can be that the command input may be a direct command from a superior.

Still another view of the interactions among DMs is through the concepts of influence and authority [6]. Influence is present when a DM's exercise of discretion is limited externally. Discretion is interpreted in this model as internal decision-making: $p(u)$ and $p(v|\bar{z})$. Simon writes [6] that the basic method for such limitation is the alteration of the premises on which the DM bases his decisions and the ability to do this is termed control. Both indirect and direct control are evident in the model. The former is possible through the suplementary situation input z' which can influence or modify the final value $\bar{z}$ of the assessed situation and, therefore, alter the premises upon which the response selection algorithm is chosen. Direct control corresponds to the direct modification, through v', of the decision strategy $p(\bar{v}|\bar{z})$.

To see more clearly the manner in which the interactions with the rest of the organization affect the performance of a decision maker, two special cases will be considered.

Indirect Control

Let the only input to the DM from the rest of the organization be z' ; in this case $\bar{v} = v$. Then the partition law expressions take the form;

$$G_n^{SI} = H(u) + H_{\bar{z}}(v) \tag{39}$$

$$\left.\begin{aligned} G_t^{SI} &= T(x,z':y) \\ G_b^{SI} &= H(x,z') - G_t \end{aligned}\right\} \tag{40}$$

$$\begin{aligned} G_c^{SI} = &\sum_{i=1}^{U} [p_i g_c^i + \alpha_i \mathbf{H}(p_i)] + H(z) + g_c^A (p(\bar{z})) \\ &+ \sum_{j=1}^{V} [p_j g_c^{U+j} (p(\bar{z}|v=j)) + \alpha_j' \mathbf{H}(p_j)] + H(y) \\ &+ H(z) + H(\bar{z}) + T_{\bar{z}}(x':z') \end{aligned} \tag{41}$$

One of the benefits of the situation input for the decision maker is that an improved and refined assessment of the situation is made, which contributes to better performance, all other things being equal,i.e., it does not produce sufficiently higher activity that a change in strategy is necessary to remain within rationality bounds. The ability of RO to alter performance through z' represents an indirect control on the decision maker. Such an influence need not be beneficial. If it is possible for RO to select z' based on x' such that performance is improved, it is also equally possible to construct a z',x' relationship which causes lower performance. In each case, once the strategy $p(v|\bar{z})$ has been selected, the DM is subject to control from the organization through z'.

While the control over performance that is possible by z' is easily seen, it is also true, though perhaps less clear, that the situation input can cause a significant change in either direction of the total activity G. Consider the following example, which is based on the model given in Figure 11 where z is assumed to take values z_1 and z_2 with equal probability.

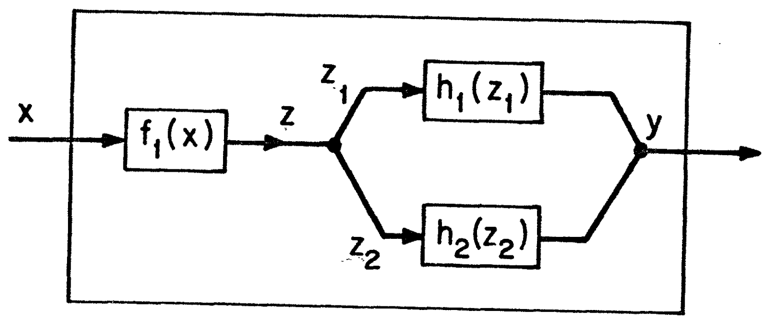

Fig. 11 Deterministic Switching

The coordination activity present in this model is then given by

$$G_c^{II} = g_c^1 + H(z) + (\alpha_1' + \alpha_2')\, \mathbf{H}(0.5) + H(y). \tag{42}$$

The internal coordination of algorithms h_1 and h_2 are zero because their respective inputs are deterministic from the point of view of the algorithm, i.e.,

$$p(z|v=1) = \begin{cases} 1 & z = z_1 \\ 0 & z = z_2 \end{cases}$$
$$p(z|v=2) = \begin{cases} 0 & z = z_1 \\ 1 & z = z_2 \end{cases} \tag{43}$$

Now consider the same model, but with a situation assessment subsystem, as shown in Figure 12.

Note that z still takes values z_1, z_2 with equal probability and that a correspondence $z_i \equiv z_i$, $i = 1,2$, is assumed. Suppose the relationship between x' and z' is such that z' is chosen so that

$$A(z,z') = \bar{z}_1 \tag{44}$$

is always the case. It is an extreme case, but possible within the framework of the model, and gives the result that algorithm $h_1(\bar{z}_1)$ is always used. It is easy to show that the difference in coordination activity between the models of Figures 11 and 12 is given by

$$G_c^{SI} - G_c^2 = [g_c^A(p(z)) + T_z^{SI}(x':z)] - [(\alpha_1'+\alpha_2') \mathbf{H}(0.5) + H^2(y)] \tag{45}$$

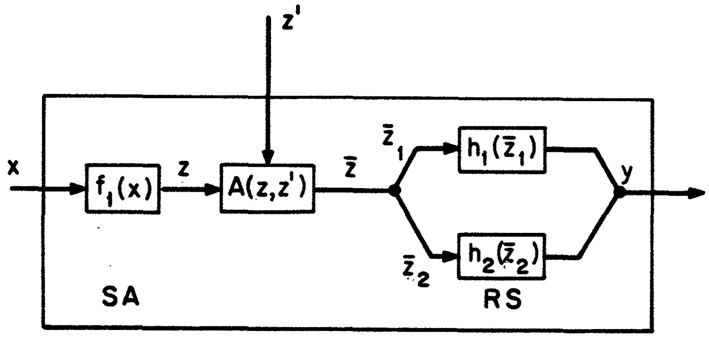

Fig. 12 Deterministic Switching With Situation Assessment

where the superscript 2 denotes the basic two-stage model. The first bracketed term in eq. (45) represents the extra coordination introduced into the model by the addition of the subsystem S^A. If the number of variables α_1' and α_2' in the response selection algorithms is large, the coordination required in switching algorithms in the model of Figure 11 may exceed the amount introduced by S^A. In that case, eq. (45) gives a negative result and the activity in the model of Figure 12 is less than that of the model of Figure 11 even though an additional stage of processing is present. This illustrates the significant effect that the situation input can have on the total activity of the model.

Direct Control

The possibility of direct control is present in the model through the command input v'. Consider the case where organization interactions are restricted to command inputs only. The expressions which characterize this model are obtained by appropriate reduction of eqs. (14), (17)-(20):

$$G_n^{CI} = H(u) + H_{\bar{z}}(v) \tag{46}$$

$$G_t^{CI} = T(x,v':y) \tag{47}$$

$$G_b^{CI} = H(x,v')-G_t \tag{48}$$

$$\begin{aligned} G_c^{CI} = & \sum_{i=1}^{U} [p_i g_c^i + \alpha_i \mathbf{H}(p_i)] + H(z) + g_c^B(p(z)) \\ & + \sum_{j=1}^{V} [p_j g_c^{U+j}(p(z|\bar{v}=j)) + \alpha_j' \mathbf{H}(p_j)] + H(y) \\ & + H(z) + H(z,v) + T_z(x':v') \end{aligned} \tag{49}$$

where CI denotes command input only.

The extreme case of direct control occurs when the values of v' selected as commands are such that the algorithm chosen is a determinsitic function of v'. Note that this implies that a protocol function b(v,v') is possessed by the decision maker which accomplishes the proper mapping.

The effective amount of decision-making in this case then becomes simply $H(u)$. Since the value of $\bar{v}$ is completely determined externally, an externally controlled switching is present, and considerable influence on the activity in the response selection stage is exerted. It is equally apparent that the performance will also be affected by v', and hence controlled directly, either beneficially or adversely.

CONCLUSION

Qualitative notions of decision-making have been combined with concepts from n-dimensional information theory into a working model which represents the decision-making process of a well-trained commander in the performance of a well-defined decision-making task. In particular, a basic model has been developed in the form of a two-stage process in which the situation is first assessed and then a response is selected based on the assessed situation. The model reflects explicitly internal choices made in the decision-making process. The stochastic version of the Partition Law of Information (PLI) has been used to characterize analytically the model as a function of the internal decision strategy.

The bounded rationality of the decision maker has been expressed in the form of a total activity rate constraint. It has been shown that the decision strategies which realize the optimal performance (normative) or satisficing performance (descriptive), subject to the boundedness of the decision maker, may only be <u>mixed</u> strategies, i.e., the decision maker alternates among options. It was also shown that alternating among options requires additional activity in the form of re-initialization of algorithm variables particular to each option. This activity and the coordination activity required to execute each option once it is chosen constitute a significant part of the total activity in the decision-making process. As such, they present a key consideration in the characterization of the decision maker with bounded rationality.

The extension of the basic model to include possible interactions in an organizational context was considered. In particular, two types of interaction, situation inputs and command inputs, are integral parts of the model. The notion of indirect control was shown to correspond to the former, while direct control was evident in the latter. It was seen that in terms of the model such control can be exercised to affect both the performance J and total activity G of the decision maker either beneficially or adversely.

The relative influence of each type of control is dependent on which stages dominate the overall performance and activity of the decision maker. For example, if the situation assessment stage accounts for a large fraction of the total activity and also dominates the overall performance, then it is possible that a partial assessment determined externally would reduce the total activity without compromising performance greatly. In such an instance the performance of a decision maker with bounded rationality would be more robust against increases in the tempo of operations. Similarly, variation in v' would have little impact on the total process, if the first stage were dominant. Therefore, there is a possibility that styles of command [12] may be explored through this model.

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