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K. Hizanidis, [a] P.A. Zestanakis, [a] A.K. Ram, and Y. Kominis [a]

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Plasma Science and Fusion Center, Massachusetts Institute of Technology
Cambridge, MA 02139 U.S.A.

[a] National Technical University of Athens
Association EURATOM-Hellenic Republic
Zografou, Athens 15773. Greece

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Nonlinear heating of ions by electron cyclotron frequency waves

K. Hizanidis[1], P.A. Zestanakis[1], A.K. Ram[2], Y. Kominis[1]

[1] *NTUA, Association EURATOM-Hellenic Republic Zografou, Athens 15773. Greece*

[2] *Plasma Science and Fusion Center, MIT, Cambridge, MA 02139 U.S.A.*

Abstract

The conditions for nonlinear heating of ions by two high frequency electron cyclotron (EC) beams are determined analytically. It is found that the extraordinary X mode is best suited for imparting wave energy to ions and modifying their distribution function. The eventual aim is to determine the possibility of affecting the transport of fusion alpha particles in tokamak plasmas by EC waves.

Introduction

It has been previously shown that high frequency electrostatic lower hybrid waves can interact with ions and nonlinearly energize them [1, 2]. This occurs either for a spectrum with a bandwidth larger than the ion cyclotron frequency or for two plane waves with their frequencies separated by a low integer multiple of the ion cyclotron frequency. We have developed a more general theory which considers the nonlinear interaction of electromagnetic waves with ions in a magnetized plasma. Of particularl interest is the interaction with waves in the EC range of frequencies - the X mode and the O mode. The electromagnetic waves propagate as a Gaussian beam into the plasma and spatial region of interaction occurs where two such beams overlap in a plasma. The interaction with ions occurs when the wave frequency of each beam is modulated around the the ion cyclotron frequency. The conditions for the nonlinear interaction are studied using the Lie transform canonical perturbation theory for Hamiltonian systems [3].

Hamiltonian Perturbation Theory

The motion of a charged particle in the presence of electromagnetic fields is given by the Hamiltonian

$$h(\mathbf{q},\mathbf{p},t) = \left[1 + (\mathbf{p} - e\,\mathbf{A}(\mathbf{q},t))^2\right]^{1/2} + e\,\Phi(\mathbf{q},t). \tag{1}$$

where $\mathbf{A}$ and Φ are the vector and scalar potentials, respectively, for the fields, e is the charge of the particle, $\mathbf{q}$ is its position and $\mathbf{p}$ is its momentum. The mass m and the speed of light c have been normalized out. A particle's position and velocity are represented in phase space by the 6–dimensional vector $\mathbf{z} \equiv (\mathbf{q},\mathbf{p})$.

We assume that $\mathbf{A} = \mathbf{A_0} + \epsilon\mathbf{A_1}$ is a sum of two terms. The first term corresponding to the background steady state magnetic field while the second term is due to the EM field. Φ is the scalar potential corresponding to the electromagnetic field. We asume the amplitudes of $\mathbf{A_1}$ and Φ are small compared to the amplitude of $\mathbf{A_0}$ so that the EM waves are perturbations acting on the particle. The perturbations are indicated by the ordering parameter ϵ, so that Φ is of order ϵ. We order each term in the perturbation theory by using subscripts [3], e.g., for the Hamiltonian $h = h_0 + \epsilon h_1 + \epsilon^2 h_2 ....$ The particle velocity is given by

$$\mathbf{v} \equiv \frac{\partial h}{\partial \mathbf{p}} = \frac{\mathbf{p} - e\,\mathbf{A}}{\gamma} = \sum_i \frac{\partial h_i}{\partial \mathbf{p}} \equiv \sum_i \mathbf{v}_i, \tag{2}$$

$$\gamma = \left[1 + (\mathbf{p} - e\,\mathbf{A}(\mathbf{q}, t))^2\right]^{1/2} \tag{3}$$

The unperturbed Hamiltonian is

$$h_0 = \left[1 + (\mathbf{p} - e\,\mathbf{A_0}(\mathbf{q}, t))^2\right]^{1/2} \tag{4}$$

and the next two terms in the perturbation expansion lead to

$$h_1 = -e\,\mathbf{v}_0 \cdot \mathbf{A}_1 + e\,\Phi_1, \tag{5}$$

$$h_2 = -e\,\mathbf{v}_0 \cdot \mathbf{A}_2 + e\,\Phi_2 + \tfrac{1}{2} e^2 \gamma_0^{-1} \left[\mathbf{A}_1 \cdot \mathbf{A}_1 - (\mathbf{v}_0 \cdot \mathbf{A}_1)^2\right], \tag{6}$$

Consider a continuous canonical transformation operator $T(\mathbf{q}, \mathbf{p}, t; \epsilon)$ which maps the particle orbit $\mathbf{z}$ to $\mathbf{Z} \equiv T\mathbf{z}$. $T(\epsilon = 0) = I$ is the identity transformation. For every function of the phase space variables and time $g(\mathbf{z}, t)$, $g(T\ \mathbf{z}, t) = [T\ g](\mathbf{z}, t)$ The generator of this transformation w is related to T by

$$\frac{d\,T}{d\,\epsilon} = -T\ L_w, \tag{7}$$

where L_w is a Lie operator, whose action on an arbitrary function of the phase space variables $g(\mathbf{z})$ is given by $L_w g = \{w, g\}$. When the particle orbit $\mathbf{z}$ evolves under the Hamiltonian h, the transformed particle orbit $\mathbf{Z} \equiv T\mathbf{z}$ evolves under the Hamiltonian $K = T^{-1} h + T^{-1} \int_0^{\epsilon} T(\epsilon) \frac{\partial w(\epsilon)}{\partial t} d\epsilon$ [4]. Then to second order in ϵ

$$T_0 = I, \qquad T_1 = -L_1, \qquad T_2 = -\frac{1}{2} L_2 + \frac{1}{2} L_1^2,$$

$$K_0 = h_0, \qquad K_1 = h_1 + \frac{\partial\, w_1}{\partial t} + \{w_1, h_0\},$$

$$K_2 = h_2 + \frac{1}{2} L_1 \left[h_1 + K_1\right] + \frac{1}{2}\left[\frac{\partial w_2}{\partial t} + L_2 h_0\right], \qquad \text{where } L_i \equiv L_{w_i}.$$

Interaction of ions with a slowly varying envelope of EM fields

Assume that

$$\mathbf{A}_0 = q_x\, B_0\, \hat{\mathbf{y}} \quad \mathbf{A}_1 = \mathcal{A}_1(\mathbf{q}, t) \exp\left[i k_x q_x + i k_z q_z - i\omega t\right] \tag{8}$$

while the first order fields are given by the vector potential where $\omega \gg \Omega$, the ion cyclotron frequency, $\mathcal{A}_1(\mathbf{q},t)$ is the slowly varying envelope, k_x and k_z are the components of the wave vector perpendicular and parallel, respectivley, to the direction of the ambient magnetic field. The canonical transformation to the guiding centre variables (y_g, p_g), (ψ, μ) and (q_z, p_z) gives:

$$h_0 \ = \ \left[1 \ + \ 2\Omega\mu \ + \ p_z^2\right]^{1/2} \tag{9}$$

$$h_1 \ = \ -\frac{e}{\gamma_0}\sum_{n=-\infty}^{\infty} \mathbf{U}_n^* \cdot \mathcal{A}_1(\mathbf{r}_g + \boldsymbol{\xi}, t)\exp\left[ik_x x_g + in\psi + ik_z z - i\omega t\right] \ + \ cc, \tag{10}$$

where $\rho = [2\mu/\Omega]^{1/2}$, $\mathbf{U}_n = \frac{\upsilon}{\sqrt{2}}\mathcal{J}_{n+1}(k_x\rho)\,\hat{\mathbf{u}}_+ + \frac{\upsilon}{\sqrt{2}}\mathcal{J}_{n-1}(k_x\rho)\,\hat{\mathbf{u}}_- + p_z\hat{\mathbf{u}}_z$, $\hat{\mathbf{u}}_\pm = \dfrac{\hat{\mathbf{u}}_x \pm i\hat{\mathbf{u}}_y}{\sqrt{2}}$, $\upsilon = \rho\,\Omega$, $\mathbf{r}_g = (x_g, y_g, q_z)$, $x_g = p_g/\Omega$ and $\boldsymbol{\xi} = \hat{\mathbf{u}}_x\,\rho\sin\psi + \hat{\mathbf{u}}_y\,\rho\cos\psi$.

In our perturbation we choose w_1 so that $K_1 = 0$. Then

$$w_1 \ = \ -i\frac{e}{\gamma_0}\sum_{n=-\infty}^{\infty}\exp\left[ik_x x_g + in\psi + ik_z q_z - i\omega t\right]\frac{\mathbf{U}_n^* \cdot \mathcal{A}_1(\mathbf{r}_g + \boldsymbol{\xi}, t)}{n\frac{\Omega}{\gamma_0} \ + \ k_z\frac{p_z}{\gamma_0} \ - \ \omega} + cc, \tag{11}$$

provided that ω has an infinitesimally small positive imaginary part so that the fields decay exponentially as $t \to -\infty$. Also $\left|\left[\frac{\partial}{\partial t} \ + \ \frac{p_z}{\gamma_0}\frac{\partial}{\partial q_z} + \frac{\Omega}{\gamma_0}\frac{\partial}{\partial\psi}\right]\mathcal{A}_1\right| \ll \left|\left[n\frac{\Omega}{\gamma_0} \ + \ k_z\frac{p_z}{\gamma_0} \ - \ \omega\right]\mathcal{A}_1\right|$.

From eq. 11 we determine K_2, which is the second order Hamiltonian due to the nonlinear interaction of the particle with the EM fields. This can be done by using the $K - \chi$ theorem [3, 5]. We obtain,

$$K_2 = \frac{e^2}{\omega^2}\mathcal{E}_1^*(\mathbf{x},t)\left[\frac{1}{\gamma_0} + \sum_{n=-\infty}^{\infty}\left[\left[k_z\frac{\partial}{\partial p_z} + n\frac{\partial}{\partial\mu}\right]\frac{1}{\omega - n\frac{\Omega}{\gamma_0} - k_z\frac{p_z}{\gamma_0}} - \frac{1}{\gamma_0}\right]\frac{\mathbf{U}_n\mathbf{U}_n^*}{\gamma_0^2}\right]\cdot\mathcal{E}_1(\mathbf{x},t) \tag{12}$$

Interaction with a beat wave

The form of K_2, Eq. 12, reveals that there is a resonance between the unperturbed motion and the slowly varying envelope when the wave frequency of is very high compared to the characteristic frequencies of the cyclotron frequency. As an example, consider the consider two EM waves with frequencies $\omega_{1,2} = \omega \pm \Delta\omega/2$ and wave vectors $\mathbf{k}_{1,2} = \mathbf{k} \pm \tilde{\mathbf{k}}/2$ propagating at an angle θ with respective to the ambient magnetic field. The envelope takes the form:

$$\mathcal{E}_1(\mathbf{x},t) \ = \mathcal{E}_1^0\cos\left[\frac{1}{2}\left[\tilde{k}_x x + \tilde{k}_z z - \Delta\omega t\right]\right], \tag{13}$$

where $\mathcal{E}_1^0$ is, without loss of generality, real. The effect of this interaction can be approximated by the Hamiltonian:

$$\tilde{K} \ = \ \Omega\mu + k_z^2 I^2 - \Delta\omega I + \frac{1}{2}\left|\mathcal{E}_1^0\right|^2\sum_{m=-\infty}^{\infty}\mathcal{J}_m\left[k_x\rho\right]\cos\left[\phi + m\psi\right], \tag{14}$$

where $\phi = k_z z - \Delta\omega$ and $I = p_z/\tilde{k}_z$ is the conjugate momentum. We have assumed that the dynamics is non-relativistic and only the first term of K_2 in Eq. 12 is kept. Moreover, the envelope can be modulated in frequency so as to enhance the energization of ions [6]:

$$\mathcal{E}_1(\mathbf{x},t) \ = \mathcal{E}_1^0 \cos\left[\frac{1}{2}\left[\tilde{k}_x x + \tilde{k}_z z - \Delta\omega t - \frac{\omega_{\rm FM}}{\omega_{\rm mod}} \sin\left[\omega_{\rm mod} t\right]\right]\right], \tag{15}$$

where $\omega_{\rm FM}$ is the modulation bandwidth and $\omega_{\rm mod}$ is the modulation frequency. Then $\tilde{K}$ becomes

$$\tilde{K} \ = \ \Omega\mu + k_z^2 I^2 - \Delta\omega I + \frac{1}{2}\left|\mathcal{E}_1^0\right|^2 \sum_{n,m=-\infty}^{\infty} \mathcal{J}_n\left[\alpha_{\rm FM}\right]\mathcal{J}_m\left[k_x\rho\right]\cos\left[\phi - n\omega_{\rm mod} + m\psi\right], \tag{16}$$

where $\alpha_{\rm FM} = \omega_{\rm FM}/\omega_{\rm mod}$. It is evident that frequency modulation enables the interaction of the magnetized particles with a multitude of resonances.

The strength of the interaction can be estimated by applying the standard Chirikov criterion or by estimating the rms deviation of zero order invariants

$$\Delta\mu^{\rm rms} = \sqrt{\langle\frac{\partial\tilde{w}_1}{\partial\psi}\rangle}, \tag{17}$$

where $\tilde{w}_1$ is the first order Lie transform generator given by $\frac{\partial}{\partial t}\tilde{w}_1 + \{\tilde{w}_1, K_0\} = \langle K_1\rangle - K_1$.

Our calculations for a typical tokamak plasma show that X–mode interacts more effectively with ions that the O–mode, for $\omega_{\rm FM} \approx \omega_{\rm mod} \approx \Delta\omega \approx \Omega$ when the propagation of the wave is almost perpendicularly to the magnetic field and ω is slightly greater than the electron cyclotron frequency.

Conclusions

We have considered the dynamics of the interaction of magnetised particles with EM waves. In particular we have demonstrated that the modulation of the EM wave at a bandwidth larger than the cyclotron frequency of the particle can modify the topology of the phase space by introducing resonances between the particle motion and the envelope of the EM radiation. Aspects of such interactions in typical tokamak plasmas have been considered.

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