OBSERVATIONS OF FLUCTUATING ELECTROMAGNETIC
EMISSION AT THE PLASMA FREQUENCY IN ALCATOR
TOKAMAKS *

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Abstract

Measurements are presented of fluctuating millimeter-wave
radiation from the Alcator Tokamaks. Its characteristics are
1) rapid modulation, approaching 100% with rise time 3-4 µs
2) very narrow line width < 5 GHz at the plasma frequency
3) extremely large intensity, up to 200 time thermal. These
characteristics distinguish this radiation from the steady wpe
emission previously documented and are interpreted as indicating
nonlinear conversion of electrostatic oscillations as its origin.

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1. Introduction

The emission spectrum of Tokamak plasmas in the Far Infrared has been characterized\textsuperscript{1,2,3} into three distinct regimes. A primary macroscopic parameter determining the regime appears to be electron density. At lowest densities, typically $\omega_{po}/\omega_{co} \lesssim 3$ (where $\omega_{po}$ and $\omega_{co}$ are electron plasma and cyclotron frequencies at the plasma center) "broadband nonthermal" emission is observed extending in frequency from below $\omega_{co}$ to $\gtrsim 6\omega_{co}$ with intensity many times exceeding the thermal expectation. At somewhat higher density the spectrum is often characterized by "dominant $\omega_{pe}$" emission i.e. an intense superthermal emission band close to the plasma frequency, but with the emission level at the first few cyclotron harmonics close to thermal. As the density is raised still further ($\omega_{po}/\omega_{co} \gtrsim 5$) the dominance of the $\omega_{pe}$ emission diminishes, the feature eventually disappearing when $\omega_{po}$ exceeds the minimum of $\omega_{ce}$ within the plasma ($\omega_{cm}$). This is the "thermal" regime where the dominant emission is essentially thermal at the cyclotron harmonics and the "$\omega_{pe}$" emission, when visible, is small.

It has been shown\textsuperscript{4} that it is possible to understand the broadband nonthermal emission as arising from high perpendicular energy components of the electron distribution function. These almost certainly arise from pitch-angle scattering of runaway electrons by microscopic instabilities\textsuperscript{5} excited by the distortion of the distribution function.
The \( \omega_{pe} \) emission, whether dominant or not, is also attributed to phenomena related to runaways. Two different mechanisms have been proposed to explain the emission. The first\(^6,7\) presumes a highly elevated level of electrostatic (\( \omega_{pe} \)) waves to be excited by velocity space instability. These waves then scatter nonlinearly from low frequency fluctuations to produce the observed electromagnetic radiation. The second\(^8\) consists of direct Cerenkov radiation by relativistic electrons travelling faster than the local phase velocity of the extraordinary wave.

The Cerenkov mechanism predicts a rather broad emission feature extending from \( \omega_{po} \) to \( \omega_{cm} \). This is in accord with some experimental measurements\(^3,9\) of steady emission. In fact substantial agreement even in line shape has been demonstrated\(^10\) providing quantitative information on the runaway tail distribution. In particular, the total intensity observed is approximately proportional to the number of relativistic runaways in the plasma.

In contrast, the nonlinear emission is predicted to be in a very narrow line, close to \( \omega_{po} \), which should be unresolved by the experimental measurements. The expected power in the line depends upon the level of the two scattering waves.

A detailed theoretical analysis\(^10\) of the two mechanisms has shown that if the level of \( \omega_{pe} \) fluctuations is taken to be that necessary to maintain sufficient wave "friction" on runaways to balance the accelerating electric field\(^11\) then non-
linear scattering from thermal ion acoustic fluctuations gives rise to negligible radiation compared to the Cerenkov mechanism. However, enhancement of the \( \omega_{pe} \) or of the low frequency fluctuation levels could make the nonlinear mechanism significant or even dominant.

Our purpose here is to present new experimental results concerning the "dominant \( \omega_{pe} \)" emission regime which show that this emission is of a different character from that observed in steady, more modest emission regimes. The differences in the observed character of these two types of \( \omega_{pe} \) may be summarized as follows:

1) The dominant emission is rapidly fluctuating with modulation approaching 100%. The steady emission is almost totally quiescent by comparison.

2) The fluctuating emission is very narrow in spectral width, unresolved by standard techniques; whereas the steady emission has finite spectral width.

3) The total power in the fluctuating emission often exceeds that in the whole of the rest of the FIR spectrum and is in a very small bandwidth. Its intensity is very large.

In practice the characteristic by which we find it easiest to identify this form of emission is the first: its rapidly fluctuating nature.

The data here presented was observed on the Tokamak Alcator C (major radius \( R=64 \text{ cm} \), limiter radius \( a=17 \text{ cm} \)) primarily at
toroidal magnetic fields of 6T in hydrogen plasmas. However very similar observations have been made on Alcator A and no fundamental differences appear to exist between the two machines.

In section 2 we describe the salient features of the measurement techniques involved and present some 'averaged' spectra showing the fluctuating emission. In section 3 we demonstrate conclusively that it is the $\omega_{pe}$ radiation that is fluctuating and obtain an estimate of the maximum linewidth of the radiation. The temporal characteristics of the fluctuations and their spectrum is presented in section 4. An interpretation of the results in terms of the radiation mechanisms is presented in section 5.

2. FIR Spectrum of Plasma Frequency Emission

The observations are made with an InSb electron photoconductive detector. It has essentially flat spectral response from about 60 to 300 GHz, the range of interest. The video bandwidth of detector and preamplifier, confirmed with a modulated 138 GHz klystron, is 0.01 Hz to 200 kHz (-3dB). Spectral resolution is obtained using a rapid scan polarizing Michelson interferometer whose moving mirror vibrates at ~35 Hz. This determines the prima facie time resolution of the instrument as ~10 msec. However it is not that the system is insensitive to more rapid fluctuations. Rather, signal fluctuations lying in the range ~70 Hz - 10 kHz are falsely interpreted by the standard Fourier transform analysis of the interferogram as variation due to the changing path difference in the interferometer. They then give rise to spurious noise features on the spectrum obtained. Fluctuations
at higher frequencies than \( \sim 10 \text{ kHz} \), the maximum interferogram sampling rate, are filtered out prior to digitization to avoid aliasing problems. However the full bandwidth signal is simultaneously stored on an analog recording system having useful bandwidth \( > 100 \text{ kHz} \), thus allowing the high frequency fluctuations to be studied.

A brief summary of the action of the Michelson interferometer\(^{13}\) is appropriate here to lay the groundwork for understanding what the observations show. The output intensity at the detector of an ideal polarization Michelson interferometer is

\[
J(x) = \int_{0}^{\infty} I(\omega) \frac{1}{2} (1 - \cos \left[ \frac{\omega x}{c} \right]) d\omega
\]  

(1)

where \( I(\omega) \) is the spectral intensity at frequency \( \omega \) and \( x \) is the optical path difference (equal to twice the mirror displacement). The standard practice of Fourier spectroscopy is then to make measurements of \( J(x) \) at a number of values of \( x \) and to deduce the spectral intensity in the form

\[
I^*(\omega) = \frac{4}{\pi c} \int_{0}^{\infty} A(x) (J(x) - \overline{J}) \cos \left[ \frac{\omega x}{c} \right] dx
\]  

(2)

where \( \overline{J} \) is the average of \( J(x) \), the integral is actually performed as a finite sum (determining the maximum significant \( \omega \)) and \( A(x) \) is an apodizing function expressing the fact
that only a finite range of $x$ is available for the integration; $A(x) = 0$ for $x > \Delta$ where $\Delta$ is the maximum optical path difference.

By a fundamental theorem of Fourier transforms the spectrum determined is actually a convolution of the original spectrum with the instrumental line shape $B(\omega)$:

$$I'(\omega) = \int_\infty^{-\infty} I(\omega + \omega') B(\omega') \, d\omega'$$  \hspace{1cm} (3)$$

and $B(\omega)$ is just the Fourier transform of the apodisation function $A(x)$.

In Fig. 1 we show the detector output during a typical period of emission. It occurs close to the beginning of the plasma discharge and the line-averaged electron density $\bar{n}_e$ is rising owing to the pulsed gas input which is used to obtain the high densities, of interest later in the discharge.

We have found that the primary circumstance leading to the fluctuating emission is that a period of low density be allowed after the initial ionization of the background filling gas. If the pulsed gas is introduced so early as to prevent this, then the fluctuating emission and indeed all nonthermal features are suppressed. If a long delay of the pulsed gas is allowed then the 'broadband' non-thermal emission develops, and only later as $\bar{n}_e$ rises does the fluctuating $\omega_{pe}$ emission appear. With intermediate delays no broadband emission develops but rather a typically more prolonged episode of fluctuating $\omega_{pe}$ emission occurs, as is observed in Fig. 1,
frequently succeeded by steady $\omega_{pe}$ emission which eventually disappears.

We interpret this dependence as indicating that the runaways responsible for the non-thermal features are primarily generated at the time of density minimum and that their numbers, and thereby their effects, are determined by the magnitude and duration of this minimum.

In the plasma of Fig. 1 the fluctuating emission persists for about 40 ms as shown. The oscilloscope trace has been low-pass filtered above $\sim 10$ kHz for clarity. The sharp signal minima correspond to the zero path difference of the Michelson interferometer as the mirror moves. Fourier transform analysis of the signal during the periods lettered in Fig. 1 leads via Eq. (2) to a series of spectra as shown in Fig. 2. The apodizing function used for the spectra of Fig. 2 is essentially triangular, falling linearly to zero at $\Delta$. That defines the resolution indicated ($= 1.1 \times \text{FWHM of } B(\omega)$) which is $1/\Delta$ cm$^{-1}$.

During this period of the discharge experiments have shown\textsuperscript{14} that the density profile is rather flat. Thus we have indicated by arrows on Fig. 2 the mean value of $\omega_{pe}$ during the time of the interferogram, corresponding to $\bar{n}_e$ which is approximately equal to the peak density. Spectra 2(a), (b) and (c) are during the fluctuating emission episode. The effects of emission variation in the 70 Hz - 10 kHz band are evident in the noise level of the spectra. Nevertheless it is clear that the spectra are dominated by an unresolved peak at $\omega_{pe}$. The thermal emission is evident at $2\omega_{ce}$ above the
noise level and on 2(b) (and to a lesser extent 2(c)) emission between $\omega_{pe}$ and $\omega_{cm}$ (90 - 130 GHz) and between $\omega_c$ and $2\omega_c$ (200 - 250 GHz) is significant. Other features on (a) - (c) are to be attributed to noise.

Fig. 2(d) and (e) show non-fluctuating emission spectra. The spectrum noise level is greatly reduced and the steady $\omega_{pe}$ emission remains. Inspection of the interferogram indicates that the steady emission is resolved; i.e. the spectral shape plotted is reasonably accurate.

In experiments with a rapidly rotating polarizer we have attempted to observe any preferential polarization of the fluctuating emission. The result is essentially negative; i.e. no preferential polarization could be measured. The sporadic nature of the emission introduces significant uncertainties in such measurements. However, it is possible to state that the degree of (linear) polarization is at least a factor of 3 less than that of the thermal second cyclotron harmonic emission (which is $\sim$ 40% in these experiments).

3. Identification and Linewidth of the Fluctuating Component

That the fluctuations in the emission coincide with a dominant $\omega_{pe}$ feature on the spectrum is circumstantial evidence that it is this feature whose intensity is fluctuating and not others. However, thus far in our discussion we have not ruled out the possibility of fluctuations simultaneously occurring in some broadband background component not adequately visible in the spectra above the noise level. Indeed it is known that fluctuations do occur in the broadband non-thermal emission regime.
Figure 3 shows an expanded-scale oscillograph of a typical interferogram of the emission with effective bandwidth \( \sim 100 \) kHz. It is immediately apparent that the amplitude of the fluctuating component is modulated at a rather low frequency. The emission, which appears primarily as sharp bursts, evidences clear minima at approximately 2.3, 4.0, 5.6, and 7.5 ms after the zero path difference point, and maxima between these. It is precisely this modulation which corresponds to the interferometer's effect (Eq. (1)) on the narrow \( \omega_{pe} \) emission in the spectrum. Thus we are assured that this component is indeed the fluctuating one. The residual fluctuation level at the minima is so small as to be attributable still to the \( \omega_{pe} \) component. Thus no fluctuations in other parts of the spectra are discernable and our signal is dominated by \( \omega_{pe} \) fluctuations.

Further information may be deduced by direct inspection of interferograms such as Fig. 3. The Fourier analysis described by Eq. (2) treats the interferogram as if its modulation fell to zero at the maximum path difference, \( \Delta \). In that way spectra such as Fig. 2 are obtained. However, there is no sign that such an attenuation of the modulation is present on Fig. 3 and undoubtedly if we were able to continue to greater mirror displacements the modulation would continue, only finally falling to zero at some significantly larger displacement, \( \Delta' \). In just the same way as we obtain the resolution \( 1/\Delta \) cm\(^{-1}\) for an apodised interferogram, if we can determine \( \Delta' \), it will determine the bandwidth of the \( \omega_{pe} \) feature as \( \sim 1/\Delta' \) cm\(^{-1}\). Let us therefore suppose that the amplitude modulation of the
interferogram may be approximated as falling linearly. Then if at path difference \( A \) the modulation has fallen to a fraction \( f \) we can determine \( A' \) as

\[
A' = A \frac{1}{1 - f}. \tag{4}
\]

Examining interferograms such as Fig. 3 we find that the ratio of the maximum fluctuating signal to an adjacent minimum near maximum path difference is \( \frac{1}{4} \) (this ratio is equal to \( f/(1 - f) \)).

The mirror displacements we have used correspond to \( A = 1.78 \text{ cm, } 1/A = 0.56 \text{ cm}^{-1} = 17 \text{ GHz} \). Thus the estimate for the width of the \( \omega_{pe} \) feature from Eq. (4) is \( 1/5A' \) or \( \sim 3.5 \text{ GHz} \).

Now the assumption of linear fall off of modulation leads to a probably rather narrow estimate of linewidth; so, by comparison, suppose that the modulation falls as a Gaussian. This leads to a Gaussian line shape whose FWHM is

\[
\frac{2}{\pi} \left( \ln \ln 1/2 \right)^{1/2} \frac{1}{\Delta} \text{ cm}^{-1} \tag{5}
\]

giving in our case \( 0.25/\Delta \) or \( \sim 4.3 \text{ GHz} \).

In making these estimates we are making fuller use of our data than the apodised transform. Moreover we are in effect introducing further information such as the fact that negative intensity is unphysical. Such concepts can be embodied
systematically in a more sophisticated analysis of the interferogram. However the quality of our data does not warrant such a fuller treatment. Our conclusion is that conservatively the width of the $\omega_{pe}$ emission is less than $\approx 5$ GHz. The finite phase contrast of the interferometer prevents us from placing any lower bound on this width.

It should be noted that this means of estimating the linewidth is not affected by the line frequency rising continuously during the mirror scan. This is because we compare only the magnitudes at the points of constructive and destructive interference. The positions of these points are affected by the line frequency but their magnitudes only by linewidth.

Finally we note that the true specific intensity of the emission is enhanced over that shown in Fig. 2 by the same factor that its width is narrower than that shown (viz. $\approx 4$).

4. Temporal Characteristics

The qualitative nature of the fluctuation temporal characteristics is visible in Fig. 3. The emission consists primarily in fast bursts of radiation occurring sporadically in time. In order to study more qualitatively these characteristics, a spectrum analysis of the fluctuations has been performed. Because of the transient and irreproducible nature of the fluctuations a storage and analysis system was used. Fig. 4 shows schematically the elements employed. The output of detector and preamplifier for a complete shot is stored on an analog drum storage system. Subsequently any chosen five-millisecond time-slice is sampled with a transient digitizer which plays back repetitively the signal at real-time speed on an analog output. The playback is displayed on an oscilloscope.
and simultaneously input to a Tektronix 7L5 spectrum analyser. The analyser then sweeps at a rate conveniently slow to avoid transient effects. The overall frequency response of the system is determined by a test signal replacing the preamplifier.

A representative spectrum is shown in Fig. 5 together with the system's relative response, the latter plotted at approximately the system noise level. Above \( \sim 150 \) kHz the system noise contributes significantly to the spectrum. Below \( \sim 100 \) kHz the response is flat and system noise is negligible. The general spectrum shape is fairly reproducible: approximately an exponential roll off from 0 to \( > 100 \) kHz at about \(-0.12\) to \(-0.15\) dB per kHz. The fine structure is real but irreproducible from shot to shot or at different times in a shot.

The broad spectral width of this roll off is thus approximately 40 to 50 kHz at \(-6\) dB. This relates to the characteristic rise (and fall) time \((\tau)\) of the radiation bursts giving \(\tau \sim 3\) to 4 µs.

The time slice analysed in Fig. 5 shows seemingly nearly random occurrence of the emission. This is reflected in the lack of dominant features in the spectrum. A few milliseconds later a clearly periodic variation of the emission occurs as shown in Fig. 6. The spectrum obtained for a time-slice dominated by the emission of Fig. 6 is shown in Fig. 7. The relatively coherent periodicity is evident as a strong feature at \(\sim 13.5\) kHz with second harmonic at \(\sim 27\) kHz (third harmonic at \(\sim 40\) kHz is \(\sim 1\) dB below the baseline). The spectrum also reveals a second periodicity with frequency \(\sim 8.5\) kHz with its
harmonics 17 kHz, 25 kHz, 33 kHz also visible. In fact a feature at \( \sim 4 \) kHz may be a subharmonic of this series. These two periodicities appear to account for the bulk of the spectrum. Thus for this time-slice the fluctuations consist primarily of these two rather coherent components.

Even in less clearly structured spectra such as that of Fig. 5, examination of the fine structure reveals the presence of apparently quite coherent modes. However in such cases the number of significant modes is too large (greater than about five) to make any confident complete identification or enumeration of them. Those modes which can be identified generally have their fundamental at a frequency in the range 5-20 kHz.

We have attempted to correlate the emission fluctuations with simultaneous fluctuations in other diagnostics. In the case of the soft x-ray diodes and the external magnetic loops, which are sensitive to large-scale MHD oscillations, there is no significant correlation with the \( \omega_{pe} \) emission fluctuations. Inadequate count-rates are available for the limiter hard x-rays to enable a correlation to be made at the frequencies of interest.

5. Interpretation

The Cerenkov emission process appears to be able to explain, at least in some cases, the steady \( \omega_{pe} \) emission. Indeed it can be used to deduce the number of runaways in the discharge. If we apply the analysis of Swartz et al.\(^{10}\) to the spectrum of Fig. 2(d) we arrive at an estimate of approximately 10 kA of current carried by relativistic runaways. This estimate depends upon a number of factors including effective wall reflectivity etc. and so its uncertainty is at least a factor of 2.
On the other hand the Cerenkov process cannot account for
the fluctuating emission. The almost 100% modulation on
microsecond timescales appears impossible for this process. The
emission it predicts does not have the extremely narrow line-
width demonstrated for the fluctuating component, nor can it
provide adequate power, given the restrictions of the total plasma
current. We therefore conclude that the fluctuating emission
is a manifestation of nonlinear conversion of electrostatic
\( \omega_{pe} \) waves in the plasma.

The narrow linewidth observed is a natural consequence
of the nonlinear interaction of electron plasma waves with
low frequency ion (possibly acoustic) waves. If we assume
(what is not at all obvious or necessary) that the spectral
width of the electron plasma spectrum is negligible, then the
width of the observed radiation may be related directly to
the low frequency wave spectrum. If there is finite width to
the plasma spectrum this will only increase the linewidth observed.
Therefore we may deduce, from our measured upper bound on the
linewidth, that the low frequency waves have frequency
\( \omega_L/2\pi \lesssim 2.5 \text{ GHz} \). (Here we have assumed both scattering and
decay are possible so that the electromagnetic wave frequency
is \( \omega_t = \omega_{pe} + \omega_L \)). We note, for comparison, that the ion plasma
frequency under typical conditions is \( \omega_{pi}/2\pi \approx 2 \text{ GHz} \).

In order to identify more completely the nature of the
low frequency wave and the width of the electrostatic plasma spectrum,
higher resolution measurements of the radiation would be necessary.
In experiments performed on Uragan stellarator\textsuperscript{16} such high
resolution measurements have been made, under apparently related
conditions. They showed emission to occur at $\omega_{pe} \pm \omega_{pi}$, $\omega_{pe} + 2\omega_{pi}$. We are unfortunately unable to confirm any similar structure with the present techniques.

The extreme intensities, even averaged over several milliseconds, relate, in this interpretation, to the levels of plasma and low-frequency waves. As has been noted, if the low-frequency wave is supposed to be ion acoustic fluctuations at the thermal level then the average level of plasma oscillations required to produce the observed intensity greatly exceeds that necessary to balance the accelerating electric field on a runaway electron. In such a situation we might expect the runaway tail to be rapidly depleted and the emission to cease in a time short compared to a runaway acceleration time. This does not occur, the emission continuing for typically 40 ms.

On the other hand in these experiments the ratio of electron to ion temperature is typically $< 3$ which does not seem large enough to allow a highly elevated ion acoustic spectrum because of ion Landau damping. Perhaps this indicates that other types of low-frequency waves should be considered.

The temporal characteristics of the emission fluctuations are indicative of the timescales of the processes involved. The rise time of 3-4 $\mu$s for the bursts is presumably related to the growth rate (possibly non-linear) of the waves involved. The periodicity (typically 5-20 kHz) of the emission probably indicates the relaxation time of the system. This may be a relaxation oscillation in the electron tail distribution function or it may simply reflect oscillations in the wave intensities due for example to nonlinear decay instabilities.
It is obvious from this discussion that a great deal more work, both theoretical and experimental, is necessary to elucidate the many remaining uncertainties of interpretation.

Summary

We have reported a type of electromagnetic emission, at the plasma frequency, previously undocumented in Tokamaks. Its characteristics differ noticeably from the steady emission in that it is rapidly fluctuating, extremely narrowband and extremely intense. We believe that these characteristics are a clear indication that the emission arises from indirect nonlinear processes and not from the direct Cerenkov mechanism which appears able to explain the steady emission.

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5. P. Brossier, Nucl, Fusion 18, 1069 (1978) and references therein.


17. C.S. Liu and Y. Mok, Phys. Rev. Lett. 38, 169 (1977) have calculated time scales of 1-10 µs for wave growth, in the context of the broad-band nonthermal regime, which may apply also in the
Figure Captions

Fig. 1. A characteristic period of emission. The traces are:
- $V_L$, loop voltage, 6V/div; $I_p$, plasma current, 125 kA/div; far infrared emission, the output intensity of the Michelson interferometer;
- $n_e$, line average density, fractional phase shift of density interferometer, $0.55 \times 10^{14}$ cm$^{-3}$ per fringe.

Fig. 2. FIR spectra of extraordinary mode emission for the times indicated in Fig. 1. Intensity scale deduced from second cyclotron harmonic (assumed thermal) and known electron temperature. Arrows indicate the plasma frequency. The central cyclotron fundamental and 2nd harmonic are indicated and dashed lines show the extent of each harmonic due to magnetic field variation across the plasma.

Fig. 3. Interferogram of fluctuating emission. The zero path difference is marked by the distinct minimum at $\sim 2$ ms.

Fig. 4. Schematic of the system used for spectral analysis of the fluctuations.

Fig. 5. Fluctuation spectrum (solid line) at 10 dB/div, 3 kHz resolution, 20 kHz/div. Broken line indicates relative system response and also approximately the (absolute) system noise level.

Fig. 6. FIR emission oscillogram showing periodic bursts of $\omega_{pe}$ emission.
Fig. 7. Fluctuation spectrum of emission of Fig. 6, 2 dB/div, 1 kHz resolution, 5 kHz/div.
Fig 1

Fluctuating emission

V_t
I_p
FIR Emission
\bar{n}_e

10ms