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ENHANCEMENT OF THE REACTION RATE BY ICRF HEATING*

by

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Abstract

ICRF heating at the second harmonic of deuterium in a D-T plasma produces an anisotropic tail of the deuterium ions in the direction perpendicular to the magnetic field. The non-Maxwellian distribution function leads to a significant enhancement of the reaction rate which corresponds to that of Maxwellian ions at much higher bulk temperature. The auxiliary power in the presence of ICRF is determined self-consistently from the power balance. We show that the enhanced reaction rate leads to savings in the peak power requirements.
1. Introduction

The supplementary heating, which is necessary to bring the plasma to ignition, will inevitably lead to some non-Maxwellian features in the ion distribution function ($f$). While considerations of ion transport may require that $f$ is very close to $f_{MAXW}$, one should examine also the gains in the enhanced reaction rate ($\bar{\sigma} \nu$) as a result of $f \neq f_{MAXW}$.

Previous studies [1,2] have considered a two-component plasma with ICRF heating at the fundamental harmonic of the minority deuterium ($D$). This reduces the effect due to the RF enhanced $\bar{\sigma} \nu$ since $D$ is less than 10% of the density of tritium and the power output is correspondingly lower. A more recent investigation [3] treats both the minority heating at the fundamental harmonic and the 50-50 second harmonic heating. Its main conclusions appear discouraging because they indicate that the deuterium $f$ is an isotropic Maxwellian. While this may be true under certain conditions, it is important to explore the whole range of parameters and to find realistic cases where the RF enhanced $\bar{\sigma} \nu$ is significant.

In this paper we examine the effect of ICRF heating at the second harmonic of deuterium in a D-T plasma. We develop a simple theoretical model based on a one-dimensional Fokker-Planck equation in the perpendicular direction with an RF diffusion term. The effects of trapped ions will not be considered. This picture allows us to examine the relevant parameters and to find analytic scaling laws. In this way one can find an ICRF heating scheme which will produce a sizable plateau on the tail of the ion distribution function. The energetic particles lead to the increase in $\bar{\sigma} \nu$, despite their small number. Experiments with second harmonic ICRF heating in a hydrogen plasma on PLT show anisotropic ion tails [4]. These results are explained by our theory. One can find also the enhanced $\bar{\sigma} \nu$ in a
deuterium plasma from the neutron flux and compare it with our results. If our predictions are correct, the ICRF should produce an anomalous neutron flux, i.e. one which corresponds to a higher temperature of a Maxwellian plasma than the one actually achieved. One practical result for a DT reactor is that the peak power requirement is reduced when $\sigma v$ is enhanced by the RF. This saving is not great, typically it is 20-30%, because the peak power occurs at lower temperatures when the enhancement is only moderate. One should note, however, that this is the minimum saving because it is based on the power balance. In reality one needs higher powers to heat the plasma and in that case the effect of the ICRF enhancement should be more pronounced. The power amplification factor, $Q = \text{fusion power/RF power}$ is typically of order 1. This makes a driven reactor not practical.

We shall not explore how the transport will be affected by the RF produced non-Maxwellian tails. This is an important problem to be studied in the future. At the same time one should not dismiss $f \neq f_{\text{MAXW}}$ as unacceptable and deleterious. One may point out as an example the lower hybrid current drive where the existence of a plateau on the tail of $f_e$ does not lead to significant changes in the electron transport. In any case the deviations from $f_{\text{MAXW}}$ will involve only the ions on the tail, whose number is small, but their effect on the reaction rate can be very considerable.

In Sec. 2 we derive the one-dimensional $F(v_\perp)$ for the fast wave ICRF heating at $\omega = 2\Omega_D$. The theory is compared with the anisotropic tail of $f$ [4] and is found to agree well with the experimental observations. In Sec. 3 we use $f \sim \exp\left(-\frac{v_\parallel^2}{2T}\right)F(v_\perp)$ to determine the reaction rate in the presence of RF. This RF enhanced reaction rate leads to fusion power which depends on $T$ and the RF power flow $P_x$. The power balance determines self-consistently the $P_x$ required and
we compare it with the usual auxiliary power. In Sec. 4 we give a summary and discuss some of the outstanding problems.

2. Ion Tails from Second Harmonic ICRF Heating

To find the deuterium distribution function we study the Fokker-Planck equation which includes the quasi-linear diffusion coefficient for the cyclotron absorption of the fast wave at the second harmonic [2]. This diffusion coefficient is averaged over a toroidal magnetic field surface. The result can be written as:

\[
\frac{\partial f}{\partial t} = \frac{1}{v} \frac{\partial}{\partial v} D v \frac{\partial f}{\partial v} + \left( \frac{\partial f}{\partial t} \right)_c
\]  

(1)

Here the velocities are normalized to the thermal velocity of deuterium \( v_t = \sqrt{\frac{T}{m_D}} \), the parallel wavenumber \( k_\parallel = 0 \) and the collisional term is taken in the Landau form.

\[
D = \frac{e^2 |E_+|^2 R J_0^2 \left( \frac{k \cdot v_\perp}{\Omega_D} \right)}{16 m_D \Omega_D r \sin \theta_0 \nu_t^2}
\]  

(2)

where \( E_+ = \frac{1}{2}(E_x + iE_y) \) is the l.c. polarized electric field, \( \Omega_D \) is the cyclotron frequency of deuterium, \( R \) is the major radius, and \( (r, \theta_0) \) give the position where the magnetic field surface crosses the resonance layer. Only after ray tracing analysis one can determine the geometrical effect. Here we shall use \( r \sin \theta_0 \approx a/4 \), where \( a \) is the minor radius. This gives a conservative estimate of \( D \) since in practice a deposition of the energy at smaller \( r \) is possible. The RF power flow \( P_x \) is related to the electric field:

\[
P_x = \frac{c}{8\pi} n_\perp |E_y|^2
\]  

(3)

where \( n_\perp \) is the perpendicular refractive index. The dispersion relation for the fast wave is [5]:

\[\text{dispersion relation}\]
\[ n_{\perp}^2 = K_{\perp} - \frac{K_x^2}{K_{\perp}} \]  

where \( K_\perp, K_x \) are the well known components of the dielectric tensor. For a DT plasma and \( \omega = 2\Omega_D \) we find:

\[ n_{\perp}^2 = 0.88 \frac{\omega^2_{D}}{\Omega_D^2} \]  

The ratio between the electric fields in the perpendicular direction is:

\[ \frac{E_x}{E_y} = \frac{i \cdot K_x}{K_{\perp}} = -1.64i \]

Now we can write \( D \) in the form:

\[ D = \frac{0.78 e^2 P_x J_i^2 (1.9 \frac{\omega_{pdf} v_D}{1}) R}{m_D^2 v_D^2 c\omega_{pdf}} \]

where all quantities are in CGS units. It is clear that the argument of the Bessel function becomes of order one for typical plasma parameters when \( V_i \) is in the range between 5–10.

The collisional term is:

\[ \left( \frac{\partial f}{\partial t} \right)_c = -\frac{2\pi \ln \Lambda e^4}{m_D} \sum \alpha \frac{\partial}{\partial v_i} \int d^3 v' (\frac{\partial^2}{\partial v_i \partial v_k} |\vec{v} - \vec{v}'|) (\frac{f_D}{m_D \partial v_k} - \frac{f_{\alpha}'}{m_D \partial v_k}) \]

where the index \( \alpha \) refers to D, T ions and electrons. The tritium and electron distribution functions are assumed to be Maxwellian at the same bulk temperature as the deuterium. The D-D collisional term can be "linearized" near a Maxwellian. This is true because \( f_D \neq f_{\text{Maxw}} \) only in the tail and the important contribution comes when the tail particles collide with the bulk. The tail-tail collisions are much less frequent since a small number of particles is involved. This is the meaning of the linearization and not that \( f_D \) is close to a Maxwellian everywhere. The next step is to simplify the equation by going to the limit \( v \gg 1 \). We are interested in the forming of a plateau on the tail, since the bulk of \( f \) will remain Maxwellian. The last
step is to assume that in the longitudinal direction the distribution function retains
its Maxwellian character and integrate over $v_{\parallel}$. It is known from studies of the lower
hybrid current drive $[7,8]$ that this is not entirely correct. The net effect usually
is broadening of the plateau in the direction perpendicular to the RF diffusion. If
this is the case here it will mean more particles in the tail and an even stronger
enhancement of the reactivity. So the assumption of $f = 1/\sqrt{2\pi} \exp(-v^2/2)F(v_{\perp})$
is a conservative estimate. The PLT experiment shows that an anisotropic tail is
present in agreement with the model here.

We shall derive the DD collisional term. The calculation for the DT term
follows the same steps and the final answer differs by a numerical factor.

$$C^{D/D} = -\frac{\ln \Lambda e^4 n_D}{(2\pi)^{1/2} m_D^2 v_D^3} \frac{\partial}{\partial v_i} I$$

(9)

where after some trivial integration by parts $I$ can be written in the form:

$$I = \int d^3v' \frac{(v - v')_i}{|v - v'|} \left( (v')^2 - 3 \right) f + v'_k \frac{\partial f}{\partial v_k} \exp \left(-\frac{(v')^2}{2}\right)$$

(10)

We have dropped the index of $f$ which from now on will refer to deuterium only.

By using the identity

$$\frac{1}{|v - v'|} = \frac{1}{2\pi^2} \int \frac{d^3k}{k^2} \exp[i\vec{k}(\vec{v} - \vec{v}')]$$

(11)

we can integrate easily over $d^3v'$ and the result is

$$I = \sqrt{2/\pi} \int d^3k \exp(-k^2/2 + i\vec{k} \cdot \vec{v}) \cdot \left[ \frac{k_i}{k^2} f \left( \frac{1}{k^2} (i k_k v_i + \delta_{ik} - k_k k_k) \frac{\partial f}{\partial v_k} \right) \right]$$

(12)

By choosing an appropriate co-ordinate system with $\vec{v}$ along the z-axis the $k$
integration can be performed and we find:

$$I = (2\pi)^{3/2} \left[ 2f \frac{\partial}{\partial v_i} \frac{1}{\sqrt{2}} \Phi(v/\sqrt{2}) - \frac{\partial f}{\partial v_k} \left( v_i \frac{\partial}{\partial v_k} + \delta_{ik} + \frac{\partial^2}{\partial v_i \partial v_k} \right) \frac{1}{\sqrt{2}} \Phi(v/\sqrt{2}) \right]$$

(13)
where $\Phi$ is the error function. Comparison with similar expressions in [2] shows that while the linearized Fokker-Planck equation in spherical co-ordinates, Eq. (22) is correct, the one in cylindrical co-ordinates, Eq. (35) is wrong. Therefore the one-dimensional Fokker-Planck equation derived there, Eq. (36) is also in error.

For $v \gg 1$, $\Phi(v/\sqrt{2}) \simeq 1$ with exponential accuracy. Now we integrate over $v_\parallel$ with a Maxwellian distribution in the parallel direction. The $v$ in the denomination is expanded to leading order in $(v_\parallel/v_\perp)^2 < 1$. This expansion is justified since we are interested in the tail in $v_\perp$ (for $v_\perp \lesssim 1$ the distribution function is Maxwellian) and $f$ falls off rapidly in $v_\parallel$. The result for the D-D collisional term from (9,13) is:

$$C^{D/D} = \frac{6\pi \ln \Lambda e^4 n_D}{m_D^2 v_i^2} \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \left( \frac{1}{v_\perp^2} \frac{\partial F}{\partial v_\perp} + \frac{1}{v_\perp} F \right)$$ (14)

Similarly for $C^{D/T}$ we get:

$$C^{D/T} = \frac{7n_T}{9n_D} C^{D/D}$$ (15)

The e-i collisional term can be written in the form:

$$C^{D/D} = -\frac{\ln \Lambda e^4 n_D}{(2\pi)^{1/2} m_D^2 v_i^2} \frac{\partial}{\partial v_i} I_e$$ (16)

Here we follow the same procedure, see Eqs. (9-13).

$$I_e = (2\pi)^{3/2} [2 \frac{T_e}{\delta^2 T_e} f \frac{\partial}{\partial v_i} v \Phi \left( \frac{\delta v}{\sqrt{2}} \right) - \frac{\partial f}{\partial v_k} \left( v_i \frac{\partial}{\partial v_k} + \delta_{ik} + \frac{\partial^2}{\partial v_i \partial v_k} \right) \frac{1}{v} \Phi \left( \frac{\delta v}{\sqrt{2}} \right)]$$ (17)

where $\delta = \left( \frac{m_i T_e}{m_D T_D} \right)^{1/2}$. Even when we consider the tail of the distribution function $\delta v < 1$, since $\delta \simeq 0.02$. For velocities which violate this inequality the ions will not be confined and they are of no interest in the present problem. Now we expand the error function for small arguments [9] and retain the leading terms. The result is:
\[ I_e = -\frac{8\pi}{3} \delta \left( v_i \frac{T_e}{T_e} f + \frac{\partial f}{\partial v_i} \right) \] (18)

After integrating over \( v_i \) we find the one-dimensional \( e-i \) collisional term.

\[ C^{D/e} = \frac{4(2\pi)^{1/2} \delta \ln \Lambda e^4 n_e}{3m_D v_{tD}^3} \frac{1}{v_i} \frac{\partial}{\partial v_i} \left( v_i^2 \frac{T}{T_e} F + v_i^2 \frac{\partial F}{\partial v_i} \right) \] (19)

Now we set \( T = T_e, n_D = n_T = n_e/2 \) and \( n = n_e \). After combining the collisional terms from (14, 15, 19) the 1D Fokker-Planck equation in dimensionless variables becomes:

\[ \frac{\partial F}{\partial \tau} = \frac{1}{v_i} \frac{\partial}{\partial v_i} \left[ 3 \left( \frac{1}{v_i^3} + \gamma v_i \right) \frac{\partial F}{\partial v_i} + 3 \left( \frac{1}{v_i^2} + \gamma v_i \right) F + D_v v_i \frac{\partial F}{\partial v_i} \right] \] (20)

Time is normalized to the collisional frequency, \( \gamma = \frac{\delta}{2\sqrt{2\pi}} = 3.3 \times 10^{-3} \) and \( D_v \) is the normalized diffusion coefficient.

\[ D_v = \frac{9 \frac{D m_D^2 v_{tD}^3}{16 \pi \ln \Lambda e^4 n}} {16 \pi \ln \Lambda e^4 n} \] (21)

Note that for \( v_i < \gamma^{-1/3} = 6.7 \) the ion-ion collisions are dominant, while above that value the electron-ion collisional term begins to dominate. In practical units the diffusion coefficient is \( \ln \Lambda \approx 15 \):

\[ D_v(v_i) = 4 \frac{T^{1/2}(\text{keV}) P_z(\text{cm}^2) R}{n^{3/2}(10^{14}) a} \left( \frac{2 n^{1/2}(10^{14}) T^{1/2}(\text{keV}) v_i}{B(T)} \right) \] (22)

Now we can find the steady state distribution function. In (20) we let \( \partial F/\partial \tau = 0 \) and consider that there is no perpendicular flux. The result is:

\[ F = N \exp \left( -\int_0^{v_i} \frac{x dx (1 + \gamma x^3)}{1 + \gamma x^3 + D_v(x) x^3/3} \right) \] (23)

where \( N \) is a normalization factor. Note that Eq. (20) was derived under the assumption \( v_i > 1 \), while the solution (23) extends to small \( v_i \). However, for \( v_i \) such that \( D_v v_i^3/3 \ll 1 \) we obtain a Maxwellian distribution, which is the solution
for the bulk of the original Fokker-Planck equation. The distribution changes from
the Maxwellian one at \( v_\perp \gg v_t \) for all cases of interest. The region in velocity space
where this change occurs depends on the plasma parameters and the intensity of
the RF field. The condition is:

\[
\frac{D_\nu(v_\perp)v_\perp^3}{3} \gg 1, \gamma v_\perp^3
\]  

(24)

Given the plasma parameters, (24) leads to a threshold for the power density.
The distribution function will form a significant tail only when the RF diffusion
dominates over the collisional diffusion for a wide range in velocity space. At very
high velocities the e-i collisions will reduce the tail and will eventually lead to a
cutoff. The energy at which this happens may be comparable to the maximum
energy of the confined ions, as determined by their banana orbits.

In any realistic case the number of particles in the plateau is a fraction of the
total number of particles. The normalization factor \( N \) can be approximated by
the usual value \( N = 1 \). The effect of \( F \) from (23) on \( \bar{\sigma} \bar{v} \) will be examined in the
next section. Here we would like to compare our results with the anisotropic tail
observed in the PLT experiment where ICRF heating at \( \omega = 2\Omega_H \) was applied in
a hydrogen plasma [4]. In that case \( D_H \) is different from \( D_\nu \) in (22) by a numerical
factor. From the corresponding dispersion relation we get:

\[
n_\perp = \frac{\omega_p H}{\Omega_H}, \quad \frac{E_z}{E_y} = -2i
\]  

(25)

\( D_H \) can be easily evaluated and in practical units is:

\[
D_H = 5.7 \times 10^{-3}\frac{T^{3/2}(\text{keV})P_z(\text{kW})R^2}{B^2(T)n_1/2(10^{14})a^2\gamma v_\perp^2}
\]  

(26)

where we have expanded the Bessel function for small arguments. The parameters
for PLT are \( B = 1.3 T, n = 3 \times 10^{13} \text{ cm}^{-3}, R/a = 3, a = 45 \text{ cm}. \) From Ref. 4,
We find the ion temperature which is $T = 2.4$ keV. At 800 kW of RF power from Ref. 4, fig. 3b we find 400 kW of radiated power and $P_x = .06$ kW/cm$^2$. From (26) we get $D_{II} = 4.1 \times 10^{-3} v_{\perp}^2$. Note that $\gamma_{II} = 4.7 \times 10^{-3}$ (it differs by $\sqrt{2}$ from $\gamma$ in the deuterium case). We write the distribution function from (23) as a function of energy.

$$F = F_0 \exp \left[ -\frac{1}{T} \int_0^E dE \frac{1 + 4.7 \times 10^{-3} (2E/T)^{3/2}}{1 + 4.7 \times 10^{-3} (2E/T)^{3/2} + 1.4 \times 10^{-3} (2E/T)^{5/2}} \right]$$  \hspace{1cm} (27)$$

The constant $F_0$ is found by normalizing to the plot in [4], fig. 2b at $E = 10$ keV. The results from (27) are shown in fig. 1 and are compared with the data from [4], fig. 2b. The agreement is excellent over the whole domain of the experimental data.

3. RF Enhanced Reaction Rate and Power Balance

For the thermonuclear reaction cross-section of D-T we use [10]:

$$\sigma = \frac{2.19 \times 10^{-20}}{E} \exp \left( -\frac{44.24}{E^{1/2}} \right)$$ \hspace{1cm} (28)$$

where $E$ is the energy of the deuteron in keV and $\sigma$ is in cm$^2$. To find the reaction rate we take the distribution function to be Maxwellian in the parallel direction and is given by (23) in the perpendicular direction. The result is:

$$\bar{\sigma}v_{RF} = 6.98 \times 10^{-13} T^{-1/2} \int_0^{v_{\perp max}} v_{\perp} dv_{\perp} \exp \left( -\int_0^{v_{\perp}} \frac{xdx(1 + \gamma x^2)}{1 + \gamma x^2 + P_x \frac{K_0}{3} T^{1/2} J_3^2 \epsilon T^{1/2} x^3} \right)$$

$$\int_0^{\infty} \frac{dv_{\parallel}}{v_{\parallel}^2 + v_{\perp}^2} \exp \left[ -\frac{v_{\parallel}^2}{2} - \frac{48.46}{\sqrt{T(v_{\parallel}^2 + v_{\perp}^2)}} \right]$$ \hspace{1cm} (29)$$

where $\bar{\sigma}v$ is in cm$^3$sec$^{-1}$, $T$ is in keV, $\gamma = 3.3 \times 10^{-3}$ and $K_0, \epsilon$ are defined from (22, 23): $K_0 = .4 \frac{R}{n^{3/2}(10^{14})}$, $\epsilon = .2 \frac{n^{1/2}(10^{14})}{B(T)}$. $v_{\perp max}$ is the maximum velocity.
of the confined deuterium ions. In the limit, \(P_x \to 0\) (no ICRF produced tail) we recover the usual reaction rate:

\[
\bar{\sigma} = 3.68 \times 10^{-12} T^{-2/3} \exp \left( -\frac{19.94}{T^{1/3}} \right) \tag{30}
\]

The integral over \(v_\perp\) in (29) should extend only to the maximum energy of the confined ions. For \(P_x \to 0\) this is not necessary because the Maxwellian provides a natural cutoff. However the ICRF diffusion leads to a plateau and then the place of the cutoff is of importance to establish the correct RF enhanced \(\bar{\sigma} v\). With a collision frequency

\[
\nu_i = 1.6 \times 10^3 \frac{n(10^{14})}{T^{3/2}(\text{keV})} \text{sec}^{-1} \tag{31}
\]

and an ion bounce frequency

\[
\omega_{Bi} = 2.2 \times 10^5 \frac{1}{qR(m)} \sqrt{\frac{\Delta BT(\text{keV})}{B}} \text{sec}^{-1} \tag{32}
\]

it is clear that we are in the banana regime, \(\nu_i \ll \omega_{Bi}\). The condition for \(v_{\perp \max}\) is roughly given by the size of the banana orbit compared with the minor radius \((\rho_B = a)\). For \(v_{\perp \max}\), normalized to \(v_{tD}\) this leads to:

\[
v_{\perp \max} = 22 \sqrt{\frac{a(m)R(m)}{T(\text{keV})}} B_p(\text{kG}) \tag{33}
\]

where \(B_p\) is the poloidal magnetic field in kG. While magnetic field ripples can reduce the value for \(v_{\perp \max}\) from (33) it is clear that in a reactor the maximum energy of confined ions should be in the Mev range.

As an example we shall use the parameters for a recently proposed fusion reactor D-LITE [11]: \(R = 2.5\) m, \(a = 1.1\) m, \(B = 4.3\) T, \(B_p = 7.9\) kG, \(n = 1.5 \times 10^{14}\) cm\(^{-3}\). We find \(K_0 = 0.5, \epsilon = 0.057, v_{\perp \max} = 288 T^{-1/2}\) in the units described above. Considering the uncertainty in the estimate (33) we reduce
\( v_{\perp \text{max}} \) to \( v_{\perp \text{max}} = 100 T^{-1/2} \). We use this value to find the RF enhanced \( \bar{\sigma} \bar{v} \). This corresponds to a maximum energy for the confined deuterium ions of 5 MeV. For \( P_z \) we use \( P_z = 1 \) kW/cm\(^2\), which gives approximately 40 MW of RF power delivered inside the plasma. The RF enhanced reaction rate is shown on Fig. 2 and is compared with the reaction rate of Maxwellian ions. The increase in \( \bar{\sigma} \bar{v}_{RF} \) is very substantial for temperatures above 6 keV and differs from \( \bar{\sigma} \bar{v}_M \) by an order of magnitude for \( T > 10 \) keV. This gain in the reaction rate will be used to find the power balance in the presence of RF.

It is clear that large powers are required to maintain a significant non-Maxwellian ion tail. This will reduce the power amplification factor \( Q = \) fusion power/RF power to a unacceptably low number, typically of the order of 1. The application of the ICRF enhancement is not to build a subignition (driven) reactor, but to achieve ignition with less power required. The reason for this is simple: the \( \alpha \)-particle heating is much stronger when the reaction rate is increased by ICRF.

The power balance is:

\[
P_{\text{aux}} = P_{\text{Br}} + P_{\text{diff}} - P_{\alpha}
\]

(34)

where \( P_{\text{aux}} \) is the supplementary power, \( P_{\text{Br}} \) is the power loss of the bremsstrahlung radiation, \( P_{\text{diff}} \) is the power loss of the thermal diffusion and \( P_{\alpha} \) is the \( \alpha \)-particle fusion power. For \( Z = 1 \) \( P_{\text{Br}} \) is:

\[
P_{\text{Br}} = 5.4 \times 10^{-3} n^2 (10^{14}) T^{1/2} \text{ (keV) W/cm}^3
\]

(35)

The thermal diffusion loss is:

\[
P_{\text{diff}} = 4.8 \times 10^{-2} n (10^{14}) T (\text{keV}) \left( \frac{3}{4} + \frac{1}{4} \frac{T_{\text{eff}}}{T} \right) \text{ W/cm}^3
\]

(36)

where we have taken into account the non-Maxwellian deuterium distribution function. The presence of the tail leads to an effective temperature \( T_{\text{eff}} > T \) where:
\[
T_{eff}/T = \frac{1}{2} \int_{0}^{v_{\perp \text{max}}} v_{\perp}^3 F(v_{\perp}) \, dv_{\perp}
\]
(37)

with \(F\) given by (23). The D ions contribute with a coefficient of 1/4 to the overall thermal pressure because 1/2 comes from the electrons and 1/4 from the \(T\) ions. The fusion power is given by [9]:

\[
P_{\alpha} = 1.4 \times 10^{15} \bar{\sigma} \bar{v} n^2 (10^{14}) \text{ W/cm}^3
\]
(38)

where \(\bar{\sigma} \bar{v}_M\) from (30) is used in the traditional power balance and \(\bar{\sigma} \bar{v}_{RF}\) from (29) should be used in the presence of second harmonic ICRF heating. At the ignition point \(P_{aux} = 0\) and the \(\alpha\)-particle power balance the losses at a certain temperature \(T_{ign}\) for \(n\tau_E\) given by the reactor parameters. To reach ignition \(P_{aux}(T)\) exhibits a maximum as a function of temperature for any given \(n\tau_E\). We shall show that this supplementary power requirement can be substantially reduced. Given the power flow from the antenna \(P_z\) the supplementary power per unit volume is \(P_{aux} = P_z/2\pi R\). The \(\alpha\)-particle fusion power \(P_{\alpha}\) is also a function of \(P_z\). The power balance equation (34) has to be solved for \(P_z\) for any given \(T\).

\[
\frac{10^3 P_z}{2\pi R} = P_{Br}(T) + P_{diff}(T, P_z) - P_{\alpha}(T, P_z)
\]
(39)

Here \(P_z\) is in kW/cm². The result \(P_z(T)\) is plotted on Fig. 3. It shows a maximum, which is compared with the maximum of \(P = 2\pi R P_{aux} 10^{-3}\) when the Maxwellian reaction rate is used. Obviously the results depend on the \(n\tau_E\) chosen. For smaller values of \(n\tau_E\) the savings in peak power are bigger. In our example we have taken for the D-LITE \(n\tau_E = 2 \times 10^{14}\) cm³ sec. The Alcator scaling [12] \(n\tau_E = 0.5 \times 10^{14} n^2 (10^{14}) a^2(m)\) gives \(n\tau_E = 1.4 \times 10^{14}\). The Mirnov scaling [12] \(n\tau_E = 3.5 \times 10^{14} n^{3/2} (10^{14}) a^2(m) B_p (\text{kG})\) gives \(n\tau_E = 6 \times 10^{14}\). We have chosen a value which is a conservative estimate. From Fig. 3 it is clear that one can save 20% on the peak power for a D-LITE reactor. While this gain is not spectacular, it is large enough to have practical applications.
4. Summary and Discussion

ICRF heating at $\omega = 2\Omega_p$ in a D-T plasma can produce significant anisotropic tails in the deuterium distribution function. This will lead to an enhanced reaction rate and an increase in the $\alpha$-particle fusion power, which will help to bring the reactor to ignition. There can be substantial savings in the peak power requirement, especially if the peak power is greater than determined by the power balance for the ideal auxiliary power. It is important to stress that ICRF heating should be optimized to produce these ion tails if there are no deleterious effects on the transport. In this scheme a driven reactor does not appear to be a viable option.

All conclusions about the RF enhanced reaction rate depend critically on the mechanism of power deposition. The question of mode conversion and the changes in $k_\perp$ and $E_+$ affect the diffusion coefficient, which determines the height of the plateau of the distribution function. It is expected that the diffusion coefficient will be larger than estimated by cold plasma dispersion relations. This in turn will give a greater enhancement in $\sigma v$ per unit of RF power. The spatial dependence and the ray focussing must be studied. The increase in the number of trapped ions may affect the transport and this important problem should be addressed also.

The above mentioned effects may change the simple picture presented here. We believe, however, that the basic physics is contained in the one-dimensional Fokker-Planck treatment and that the agreement with the PLT data [4] is not a coincidence. We hope that future ICRF heating experiments will focus attention on the tail production and help in establishing the scaling laws.

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References

Figure Captions

Figure 1 Comparison of theory and experiments on second harmonic ICRF produced anisotropic tails of ions in a hydrogen plasma on PLT. The solid line is a plot of $F$ from Eq. (27) with $T = 2.4$ keV.

Figure 2 Reaction rate for a D-T plasma. The dashed line corresponds to Maxwellian deuterium ions, the solid line is calculated on the basis of RF enhanced tails of the deuterium distribution function, see Eq. (29). $K_0 = 0.5$, $\epsilon = 0.057$, $\gamma = 3.3 \times 10^{-3}$, $v_{\perp \text{max}} = 100 T^{-1/2}$, $P_z = 1$ kW/cm$^2$.

Figure 3 Auxiliary power flow for a Maxwellian plasma (dashed line) and ICRF enhanced deuterium ion tail (solid line). $n_rE = 2 \times 10^{14}$ cm$^{-3}$ sec.
Figure 2