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Generation of Short Pulses of Coherent Electromagnetic Radiation in a Free-Electron Laser Amplifier

F. Hartemann*, K. Xu and G. Bekefi

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Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 USA

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* Present address : THOMSON-CSF, 2 rue Latécoère, BP 23, 78140 Vélizy-Villacoublay, France and Laboratoire PMI, Ecole Polytechnique, 91128 Palaiseau Cedex, France
Abstract

Theoretical and experimental studies of the evolution of a frequency-chirped pulse under the influence of both phase and gain dispersive effects induced by the free-electron laser (FEL) interaction are presented. For the experimental parameters used (electron beam voltage $V \approx 150$ kV, wiggler periodicity $\ell_w = 3.5$ cm, gain $\sim 10$ dB, input pulse width $\Delta t \sim 200$ ns, frequency $\omega_0/2\pi \approx 10$ GHz and frequency chirp $\alpha/2\pi \sim 5$ MHz/ns), pulses of a few nanoseconds were generated after an interaction length of 2.30 m, in good agreement with theoretical expectations.
I. Introduction

The generation of short pulses of coherent electromagnetic radiation is a subject of considerable interest. Such pulses are needed in many diverse areas of physics, such as nonlinear spectroscopy \(^1\text{-}^2\), studies of transient phenomena, surface physics, accelerator physics \(^3\text{-}^4\) (RF linacs) and optical communication \(^5\text{-}^6\). In addition, the generation of short pulses in an active medium can lead to extremely high instantaneous peak powers.

In the microwave wavelength range, pulse compression can be achieved by propagating a frequency-chirped pulse in a passive medium with group velocity dispersion (this effect, which is the direct counterpart of the dispersive pulse broadening, will be referred to as "phase compression"). Colliding pulses in a microwave interferometer also results in the generation of short pulses of RF radiation. In the optical part of the electromagnetic spectrum, the generation of short pulses commenced with active mode-locking \(^7\text{-}^8\), was followed by passive mode-locking \(^9\text{-}^10\text{-}^11\) and culminated in the colliding pulses mode-locked systems \(^12\). Recently, soliton formation was observed in optical fibers \(^13\text{-}^14\text{-}^15\) and lead to another compression scheme, the soliton laser \(^16\text{-}^17\text{-}^18\text{-}^19\).

In this paper, we study both theoretically and experimentally a novel scheme of active pulse compression (referred to as "gain compression") in a free-electron laser (FEL) amplifier \(^20\text{-}^21\text{-}^22\). The experimental results represent what we believe to be the first measurement of frequency modulation effects in a FEL amplifier. The outstanding capabilities of the free-electron lasers include their inherent tunability, high radiation levels at high efficiencies and narrow bandwidth. At high power levels, nonlinear effects such as self-focusing (optical guiding) \(^23\text{-}^24\text{-}^25\), saturation \(^26\text{-}^27\) and sideband instabilities \(^28\) are also observed in FEL generators. These properties of FELs make them particularly attractive for the generation of short pulses of coherent electromagnetic radiation.

The pulse compression scheme studied in this paper is the following. A frequency-chirped pulse is injected into the FEL interaction region. Because of the high gain and narrow bandwidth of the FEL interaction, only the resonant frequency band of the pulse is actively amplified. This results in a high power, short pulse of coherent electromagnetic radiation at the output of the laser. For our experimental parameters (beam voltage \(V\) \(\approx\) 150 kV, wiggler periodicity \(\ell_w = 3.5\) cm, gain \(\sim 10\) dB), pulses of a few nanoseconds
at a frequency of 10 GHz (Δω/ω ~ 1%) were obtained after an interaction length of 2.30 m. For the same input pulses (width Δt > 100 ns, frequency chirp α/2π ~ 5 MHz/ns), such compression ratios would require hundreds of meters of conventional passive dispersive medium (waveguide, for example), with prohibitively high attenuation.

This paper is organized as follows. In section II, we present a simple theoretical model of the pulse dynamics during the FEL interaction, and computer calculations describing the evolution of the pulse envelope in the FEL. This model makes use of a Raman-type dispersion relation for the FEL instability, and allows us to predict the pulse behaviour as a function of various parameters. The experimental arrangement is outlined in section III, where we describe the measurements obtained with our Raman (collective) FEL and compare them with theoretical predictions. Finally, conclusions are drawn in section IV.

II. Theory

In this section, we describe the evolution of a frequency-chirped pulse in the FEL interaction region, under the influence of both phase and gain dispersive effects induced by the FEL instability. We make use of a Raman-type FEL dispersion relation and we also include the effects caused by the waveguide cutoff frequency associated with the TE01 mode of the electromagnetic wave in question. In the case of a Gaussian input pulse with a linear frequency chirp, we obtain a simple analytical expression for the output pulse shape after Taylor expanding the dispersion relation to second order.

We first briefly outline the general method used to describe the evolution of the pulse during the FEL interaction. Consider a pulse at the input position (z = 0)

\[ E(z = 0, t), \]

where \( E \) represents the electric field of the pulse. We can Fourier transform this pulse into its spectral frequency components

\[ \tilde{E}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} E(z = 0, t) \exp(-i\omega t) dt. \]

Under the assumption that the ω-Fourier spectrum remains unchanged during the interaction (linear interaction for the frequency spectrum), we can obtain the pulse shape at the
output of the FEL \((z = z_0)\) by Fourier transforming back again after taking into account the different phase shifts accumulated by the various frequency components of the pulse

\[
E(z = z_0, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{E}(\omega) \exp[i(\omega t - k(\omega)z_0)] d\omega,
\]

where \(\omega\) and \(k\) satisfy the complex dispersion relation \(D(\omega, k) = 0\) associated with the FEL interaction.

In the case of a purely phase dispersive medium, \(k(\omega)\) is real and only phase shaping effects will affect the pulse. On the other hand, for an active medium such as the FEL, \(k(\omega)\) is complex and we obtain both phase and gain dispersive effects that change the shape of the pulse.

We now take the special case of a Gaussian input pulse, with a linear frequency chirp

\[
E(z = 0, t) = E_0 \exp \left( -\frac{t^2}{\Delta t^2} \right) \exp[(\omega_0 + \alpha t)t].
\]

Here \(\Delta t\) is the initial pulse width, \(\omega_0\) its center frequency, and \(\alpha\) represents the frequency chirp. The corresponding input spectrum is determined by Fourier transforming Eq. (1)

\[
\tilde{E}(\omega) = \frac{E_0}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp \left( -\frac{i^2}{\Delta t^2} \right) \exp[((\omega_0 - \omega)t + \alpha t^2)] dt.
\]

This integral is readily calculated and we obtain the familiar result (see Fig. 1)

\[
\tilde{E}(\omega) = \frac{E_0}{i(1/\Delta t^4 - \alpha^2)^{1/4}} \exp \left[ -\frac{(\omega_0 - \omega)^2}{\Delta t^2(1/\Delta t^4 + \alpha^2)} \right] \exp \left[ i \left\{ \theta - \frac{\alpha(\omega_0 - \omega)^2}{4(1/\Delta t^4 + \alpha^2)} \right\} \right],
\]

where \(\theta = \frac{1}{2} \arctan(\alpha \Delta t^2)\) is a constant phase term.

At this point, we need to make use of the dispersion relation to evaluate the different phase shifts accumulated by the various frequency components in the interaction region. We use a Taylor expansion around \(\omega = \omega_0\)

\[
k(\omega) \approx k(\omega_0) + (\omega - \omega_0) \frac{\partial k}{\partial \omega} \bigg|_{\omega=\omega_0} - \frac{1}{2} (\omega - \omega_0)^2 \frac{\partial^2 k}{\partial \omega^2} \bigg|_{\omega=\omega_0}.
\]

Equation (4) can be written as follows

\[
k(\omega) \approx k_0 + i\chi + (\omega - \omega_0)^{-1} - \frac{1}{2} (\omega - \omega_0)^2 (\gamma + i\delta),
\]
where we recognize the following terms: $k_0$ is the real phase shift accumulated by the center frequency, $\chi$ represents the growth rate of the center frequency, $v_g$ is the (complex) group velocity at $\omega = \omega_0$, $\gamma$ is the group velocity dispersion (phase dispersive effects), and $\delta$ represents the gain dispersion (bandwidth), corresponding to gain dispersive effects. These parameters are directly determined from the dispersion relation $D(\omega, k) = 0$.

Making use of the Taylor expansion of the wavenumber [Eq. (5)], and the input spectrum [Eq. (3)], we can derive the output pulse shape by evaluating the following integral

$$E(z, t) = \frac{E_0}{2\sqrt{\pi(1/\Delta t^4 + \alpha^2)^{1/4}}} \int_{-\infty}^{+\infty} \exp \left[ \chi z - (\omega - \omega_0)^2 a(z) \right] \times \exp \left[ i \left( \theta - \omega t - k_0 z - (\omega - \omega_0)v_g^{-1} z + (\omega - \omega_0)^2 p(z) \right) \right] d\omega. \quad (6)$$

After some algebra, we obtain the following expression for the output pulse

$$E(z, t) = E_0 \frac{1}{f(z)} \exp(\chi z) \exp \left[ -\frac{\tau^2}{\Delta t^2(z)} \right] \exp \left[ i \left\{ \phi(z) + \omega_0 \tau + \alpha(z)\tau^2 \right\} \right], \quad (7)$$

where we have defined

$$a(z) = \frac{1}{\Delta t^2} \frac{1}{4(1/\Delta t^4 + \alpha^2)} - \frac{\delta}{2}, \quad (8)$$

$$p(z) = -\alpha \frac{1}{4(1/\Delta t^4 + \alpha^2)} - \frac{\gamma}{2}, \quad (9)$$

which are, in turn, used to introduce the following physical parameters. $\Delta t(z)$, which represents the evolution of the pulse width along the propagation axis, with

$$\Delta t(z) = 2\frac{\sqrt{a^2 + p^2}}{\sqrt{\alpha}}. \quad (10)$$

The chirp function $\alpha(z)$, defined as

$$\alpha(z) = \frac{-p}{4(a^2 + p^2)}. \quad (11)$$

The normalization factor $f(z)$, defined such that, in the absence of gain, the surface of the pulse remains constant:

$$f(z) = 2(1/\Delta t^4 + \alpha^2)(a^2 - p^2)^{1/2}. \quad (12)$$
The phase shift $\phi(z)$, defined by
\[ \phi(z) = \frac{1}{2} \arctan \left( \frac{\nu}{a} \right) + \theta - (k_0 + \omega_0 v_g^{-1}) z. \] (13)

Finally, $\tau = t - v_g^{-1} z$ reflects the propagation time of the center frequency of the pulse in the interaction region, and the term $\exp(\chi z)$ corresponds to the gain induced by the FEL interaction.

It can be easily verified that for $z \to 0$ we recover the input pulse. In the special case of the propagation of an unchirped pulse ($\alpha = 0$) in a passive dispersive medium ($\chi = 0$, $\delta = 0$), we find the well-known dispersive pulse broadening effect
\[ \Delta t(z) = \Delta t \sqrt{1 + \left( \frac{2\gamma z}{\Delta t^2} \right)^2}. \] (13)

We now study the evolution of the pulse width $\Delta t(z)$. We determine the minimum width by taking the derivative of Eq. 10 with respect to $z$ such that
\[ \partial_z \Delta t(z) = 0. \] (14)

Taking into account the fact that $\partial_z a(z) = -\delta/2$ and $\partial_z p(z) = -\gamma/2$ [see Eqs. (8) and (9)], we can reduce (14) to the following equation
\[ p^2(z) - 2\frac{\gamma}{\delta} a(z) p(z) - a^2(z) = 0. \] (15)

Solving (15) for $z = z^*$ yields the distance at which the maximum pulse compression is achieved. The corresponding minimum pulse width is then given by (10)
\[ \Delta t^* = \Delta t(z^*). \] (16)

For example, in the case of a passive dispersive medium ($\chi = \delta = 0$), we find that the maximum compression is obtained at
\[ z^* = -\frac{1}{\gamma} \frac{1}{2(1/\Delta t^4 + \alpha^2)}, \] (17)

where the pulse width is reduced to
\[ \Delta t^* = \frac{\Delta t}{\sqrt{1 + \alpha^2 \Delta t^4}}. \] (18)
Note that in this case of a passive dispersive medium, the minimum pulse width depends strongly on the width of the input pulse, and that compression is achieved only for $\alpha/\gamma < 0$. We shall henceforth refer to this phenomenon as “phase compression”.

We now consider the Raman FEL dispersion relation, from which we can derive the values of the parameters defined above. The two interacting modes, illustrated on Fig. 2, are the forward propagating waveguide mode

$$k = \frac{1}{c} \sqrt{\omega^2 - \omega_c^2},$$  \hspace{1cm} (19)

where $\omega_c$ is the cutoff frequency of the waveguide in the interaction region, and the beam mode

$$ k = \frac{\omega}{v} - k_w + \frac{\omega_p}{v\gamma_0^{3/2}}, \hspace{1cm} (20)$$

corresponding to the unstable slow space-charge wave. Here $v = c \beta_0$ is the electron beam velocity, $k_w$ is the wiggler wavenumber, $\omega_p$ is the beam plasma frequency and $\gamma_0 = (1 - \beta_0^2)^{-1/2}$ is the relativistic factor. In the limit of small $\omega_p$, and in the vicinity of the upshifted unstable (growing) root, the dispersion relation can be approximated as follows

$$ \left[ k - \frac{1}{c} \sqrt{\omega^2 - \omega_c^2} \right] \left[ k + k_w - \frac{\omega}{v} \right] = -\chi^2, \hspace{1cm} (21)$$

where $\chi$ is a small coupling term which determines the peak growth rate of the FEL instability (here, we neglect the implicit slow frequency dependence of $\chi$). Its magnitude $^{20,21,22,23}$ depends on the wiggler strength, electron beam current, frequency, electromagnetic wave filling factor etc. We can solve (21) for $k(\omega)$, with the result that

$$ k(\omega) = \frac{1}{2} \left\{ \frac{1}{c} \sqrt{\omega^2 - \omega_c^2} + \frac{\omega}{v} - k_w + \left[ \left( \frac{1}{c} \sqrt{\omega^2 - \omega_c^2} - \frac{\omega}{v} - k_w \right)^2 - 4\chi^2 \right]^{1/2} \right\}. \hspace{1cm} (22)$$

We introduce the FEL upshifted frequency $\omega_0 = k_w c \gamma_0^2 \beta_0 \left[ 1 + \beta_0 \sqrt{1 - (\omega_c/k_w c \beta_0 \gamma_0)^2} \right]$ and wavenumber $k_0 = \sqrt{\omega_0^2 - \omega_c^2}/c$, obtained by solving (19) and (20) simultaneously, and the small parameter $\delta \omega = \omega - \omega_0$. Expanding $k(\omega)$ to second order yields the following approximation

$$ k(\omega) \approx k_0 + i\chi + \delta \omega \frac{1}{2} \left( \frac{1}{v} - \frac{\omega_0}{k_0 c^2} \right) + \delta \omega^2 \frac{1}{4k_0 c^2} \left( 1 - \frac{\omega_0^2}{k_0^2 c^2} \right) - \delta \omega^2 \frac{i\chi}{\delta \omega_i} \frac{1}{2}. \hspace{1cm} (23)$$
This expansion is only valid for \( \chi \neq 0 \).

Here, we have made use of a parabolic approximation of the gain term (see Fig. 3)

\[
\sqrt{\delta \omega^2 - \delta \omega_i^2} \approx i \delta \omega_i \left[1 - \left(\frac{\delta \omega}{\delta \omega_i}\right)^2\right],
\]

with \( \delta \omega_i \) as the gain bandwidth of the FEL interaction

\[
\delta \omega_i = \frac{2\chi}{(\omega_0/k_0c^2) - (1/v)}.
\]

Comparing equations (5) and (23), we obtain the sought-after expressions for the expansion parameters. The group velocity \( v_g \) is given by

\[
v_g^{-1} = \frac{1}{2} \left(\frac{1}{v} + \frac{\omega_0}{k_0c^2}\right).
\]

The group velocity dispersion \( \gamma \), which vanishes for \( \omega_c = 0 \), is given by

\[
\gamma = \frac{1}{2k_0c^2} \left(1 - \frac{\omega_0^2}{k_0^2c^2}\right).
\]

Finally, the gain dispersive term of the FEL interaction is

\[
\delta = -\frac{2\chi}{\delta \omega_i^2}.
\]

At this point, it is possible to evaluate the distance of maximum “gain compression” induced by the FEL interaction. For the purpose of illustration, we first limit ourselves to the non-dispersive case (cutoff frequency \( \omega_c = 0, \gamma = 0 \)). We simplify matters further by considering a frequency-chirped input pulse with constant amplitude \( (\Delta t \to \infty) \). In this situation, the maximum pulse compression induced by the FEL interaction (“gain compression”) is obtained after a propagation distance

\[
z^* = \frac{\delta \omega_i^2}{\alpha \chi}.
\]

The corresponding minimum pulse width is readily calculated

\[
\Delta t^* = 2\sqrt{-\delta z^*} = \frac{\delta \omega_i}{\alpha} \sqrt{\frac{8}{\chi z^*}}.
\]
We obtain the expected $\xi \omega_i / \alpha$ scaling. We see that for a high gain and narrow bandwidth, very high compression ratios can be achieved within short interaction distances; we also note that the minimum pulse width obtained is basically independent of the initial pulse width, as opposed to the usual "phase compression" [see Eq. (18)].

We now take into account the full dispersive effects induced by the FEL interaction and consider the evolution of a frequency-chirped pulse with initial width $\Delta t = 100$ ns, frequency $\omega_0/2\pi = 10.0$ GHz and chirp $\alpha/2\pi = 3.5$ MHz/ns. We calculate the Taylor expansion parameters for the following FEL parameters: wiggler periodicity $\ell_w = 3.5$ cm, waveguide cutoff frequency $\omega_c/2\pi = 6.6$ GHz and peak growth rate $\chi = 2$ dB/m. The gain bandwidth is illustrated in Fig. 3. We can then make use of equation (7) to evaluate the pulse envelope after propagation through 3 m of FEL interaction region. The result is shown in Fig. 4, clearly indicating both amplification and compression of the pulse. The computer simulation (dots) is generated by using the input spectrum $\tilde{E}(\omega)$ [Eq. (3)] in conjunction with the full dispersion $k(\omega)$ [Eq. (22)] and Fourier transforming back at the output of the FEL. The agreement with analytical predictions is excellent. Higher order effects appear in the form of additional modulation in the wings of the pulse envelope.

Figure 5 illustrates the same situation for a quasi-monochromatic unchirped ($\alpha = 0$) input pulse. Again we see an excellent agreement between the analytical theory and the computer calculations. As expected, the unchirped pulse grows in amplitude, but does not compress.

Finally, the evolution of the pulse width as a function of the distance $z$ in the FEL interaction region can be predicted from Eq. (10). The result is shown in Fig. 6, for the same pulse and FEL parameters as those discussed above. The dashed curve, obtained for zero gain (passive medium) illustrates the dramatic difference between the gain and phase dispersive effects in the FEL.

Having developed a simple theoretical model for the evolution of a frequency-chirped pulse under the influence of both the phase and gain dispersive effects induced by the FEL interaction, we shall now compare our results with measurements made on our Raman free-electron laser $^{21,22,25,26,27}$.
III. Experiments

The overall experimental setup is illustrated in Fig. 7. The electron beam is generated by a thermionically emitting Pierce gun (250 kV, 250 A) from a SLAC klystron, energized by a Pulserad 615MR Marx type accelerator. Focusing coils compress the electron beam, and a pinhole emittance selector (0.25 cm in radius) removes all but the cold inner core of the beam. The ensuing 5.0 A, $\sim 150$ kV electron beam with an energy spread $< 0.5\%$ is confined radially by an axial magnetic field $B_1 \simeq 1.7$ kG. Beam excitation is provided by a 65 periods bifilar helical wiggler with a periodicity $\ell_w = 3.5$ cm and a wiggler amplitude that can be varied from zero to $\sim 1$ kG.

The FEL is operated as a single pass amplifier. A wave launcher injects the input pulse into the wiggler interaction region. The rectangular waveguide (2.25 cm × 1.00 cm cross section) has a lowest mode cutoff equal to 6.56 GHz. Only the fundamental $TE_{10}$ mode can propagate freely in the waveguide, all higher order modes being cutoff over the operating frequency range of the experiment (8-12 GHz).

The input signal is generated by a backward wave oscillator (BWO) operating between 8 and 12 GHz. Frequency chirping is obtained by applying a high voltage ramp signal to the helix of the BWO (typically, this signal rises from 0 to 150 V within 500 ns). This FM technique yields sweep velocities of a few MHz/ns. The frequency-chirped RF signal is subsequently gated by a microwave switch, then amplified to a power level of a few watts by a travelling wave tube (TWT) amplifier, and finally injected into the FEL interaction region. The input and output pulse shapes are measured by calibrated crystal detectors and displayed on a fast oscilloscope.

The frequency modulation of the microwave source (BWO) is calibrated by injecting the frequency-chirped pulse into a bandpass filter with narrow bandwidth. For our experiments, we used a filter with center frequency $f = 9.355$ GHz. and bandwidth $\Delta f = 70$ MHz. Because the instantaneous output frequency of the BWO is a function of the applied bias voltage, we can obtain a short pulse of microwaves at the output of the bandpass filter by sweeping the bias voltage. The pulse width will be approximately given by $\Delta t \simeq \Delta f (df/dV)(dV/dt) \simeq \Delta f (\alpha/2\pi)$. For example, a 20 ns output pulse indicates that $\alpha \simeq 2\pi \times 3.5$ MHz/ns. As the voltage is swept slower, the pulse broadens, in accordance
with expectations. For our experimental setup, we were able to obtain chirp parameters \( \alpha/2\pi \) as high as \( \pm 12 \) MHz/ns (for both increasing and decreasing sweeps of the frequency).

Experimental results are shown on Fig. 8. The FEL is fired by discharging the Marx accelerator. Because of an RC droop, the beam energy falls gradually as illustrated in Fig. 8a. The bias voltage applied to the helix of the BWO is shown on Fig. 8b. As a result, the frequency of the microwave signal injected into the FEL interaction region remains constant (@ 10.60 GHz) during the voltage pulse, except for a duration of 500 ns, during which the frequency is swept at a rate of \( \alpha/2\pi \approx 5 \) MHz/ns. The microwave power measured at the output of the FEL is shown on Fig. 8c. Amplification of the 10.60 GHz signal occurs at a beam energy for which the slow (negative energy) space charge wave on the beam is in near phase synchronism with the electromagnetic wave (see peak marked A in Fig. 8c). Later in time, at a lower beam energy, one observes a dip (marked B in Fig. 8c) corresponding to wave absorption. Here the wave energy is converted to electron kinetic energy, as is known to occur when the fast (positive energy) space charge wave is in synchronism with the electromagnetic wave. When the bias voltage is swept, however, we see an additional narrow peak (see arrow marked X on Fig. 8c), corresponding to the amplification of the instantaneous resonant frequency ("gain compression"). The width of this narrow peak is \( \sim 33 \) ns (to be compared with the 800 ns of the regular gain peak A), and agrees well with the expected value obtained by taking into account the effective bandwidth of the FEL interaction: \( \delta\omega_i/2\pi \sim 150 \) MHz, yielding \( \Delta t \sim \delta\omega_i/\alpha \sim 30 \) ns. The gain of the FEL interaction is \( \approx 10 \) dB; the narrow peak is observed at a lower energy than the 10.60 GHz gain peak A because the instantaneous resonant frequency corresponding to gain compression is somewhat lower than 10.60 GHz (see the voltage bias signal on Fig. 8b).

Additional evidence for the gain compression effect can be obtained by repeating the experiment for different parameters. We have changed the center frequency of the chirped pulse between 9 and 11 GHz and obtained similar results to those described above. We have also varied the width of the output pulse by changing the value of the chirp parameter, or by varying the gain of the FEL.

The growth rate of the FEL interaction is controlled by the strength of the wiggler
field and can be varied from 0 to 10 dB/m. Because of technical limitations, the data is collected for two different chirp ranges: high chirp, with $\alpha/2\pi$ ranging from 5.50 to 9.41 MHz/ns, and low chirp where $\alpha/2\pi$ varies between 2.63 and 3.37 MHz/ns. The results are presented in Fig. 9. The squares represent the low chirp data, while the circles correspond to data taken at high chirp. The shaded areas, bounded by the different values of $\alpha$, are from theory. We obtain reasonable agreement between the experimental data and the theory. In particular, since the effective bandwidth of the FEL interaction increases with the FEL growth rate, broader output pulses occur at higher gains (see Eq. 30). We also note that the output pulses are shorter for the high chirp. This is confirmed by Fig. 10. Here, we plot the pulse width measured at the output of the FEL for different values of the frequency chirp $\alpha$, keeping the input pulse width constant at 200 ns. The shaded area is limited by the theoretical curves obtained for the minimum and maximum values of the gain used in this experiment (1.88 and 9.66 dB/m, respectively). The agreement between theory and experiment is quite good, and the data clearly shows that the pulses compress more at higher chirp.

IV. Conclusion

We have studied theoretically and experimentally the behaviour of a frequency-chirped pulse propagating in a FEL under the influence of both the phase and gain dispersive effects induced by the FEL interaction. The analytical theory shows that the narrow bandwidth of the FEL instability allows only certain frequency components of the pulse to interact strongly with the bunched electron beam, thus resulting in the generation of a short pulse of coherent electromagnetic radiation at the output of the FEL. This effect can be described in terms of combined amplification and compression of the initial pulse, as it propagates into the FEL interaction region, and is somewhat analogous to the Q-switching process used for conventional lasers. Computer calculations, taking into account higher-order dispersive effects, confirm the theoretical analysis.

The experimental data generally agree well with theoretical expectations and confirm the principal effects expected: amplification and compression of the input pulse, broadening of the output pulses at higher FEL gains (larger effective FEL bandwidth), and generation of shorter pulses at higher frequency chirps.
Finally it is interesting to note the possibility of self-phase modulation in the FEL at high electromagnetic intensities, via non-linear effects, yielding chirped pulses that can subsequently compress as they propagate further into the interaction region and eventually reach a soliton-like equilibrium.

Acknowledgements

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Figure captions

Fig. 1 Typical Fourier transform of a Gaussian pulse with a linear frequency chirp. Here $\Delta t = 50$ ns, $\omega_0/2\pi = 10.0$ GHz and $\alpha/2\pi = 1.0$ MHz/ns.

Fig. 2 Schematic, showing the dispersion characteristics of the FEL experiment. The parameters are: $\ell_w = 3.5$ cm, $\omega_e/2\pi = 6.6$ GHz and $v/c = 0.62$. Only the slow (unstable) space-charge wave is represented here. All studies are carried out at the upshifted frequency.

Fig. 3 Imaginary part of the wavenumber as a function of frequency for $\chi = 2.0$ dB/m. The other FEL parameters are as in Fig. 2. The solid line is calculated from Eq. 22, while the dashed line represents the parabolic approximation of Eq. 24.

Fig. 4 Gain compression of a $\Delta t = 100$ ns, $\omega_0/2\pi = 10.0$ GHz and $\alpha/2\pi = 3.5$ MHz/ns chirped input pulse. The FEL parameters are as in Fig. 2 ($\chi = 2.0$ dB/m) and the interaction region is 3 m long. The dashed curve represents the input pulse, the solid line is from theory and the dots correspond to the computer simulation.

Fig. 5 Similar to Fig. 4, but without chirp ($\alpha = 0$). The pulse does not compress and shows only gain.

Fig. 6 Pulse width as a function of distance into the FEL interaction region. The parameters are as in Fig. 4. The dashed curve, shown for comparison, corresponds to zero gain ($\chi = 0$, passive dispersive medium).

Fig. 7 Overall experimental setup.

Fig. 8 (a) time history of the diode voltage (electron beam energy). (b) instantaneous frequency of the input pulse. (c) microwave intensity at the output of the FEL.

Fig. 9 Pulse width measured at the FEL output as a function of the growth rate. The squares represent the low chirp data, while the circles were obtained at high chirp; the shaded areas are from theory (see text) and correspond to the the range of $\alpha$ used in the
experiments.

Fig. 10 Pulse width measured at the FEL output as a function of the chirp. The shaded area is limited by the theoretical curves corresponding to the minimum and maximum growth rates used in the experiment (1.88 and 9.66 dB/m, respectively).
References


Fig. 1
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ELECTROMAGNETIC WAVEGUIDE MODE

\[ \omega^2 = k^2 c^2 + \omega_c^2 \]

UPSHIFTED FREQUENCY

SPACE-CHARGE WAVE

\[ \omega = (k + k_w)v - \omega_p / \gamma^{3/2} \]

Fig. 2
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Fig. 3
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Fig. 4
Hartemann, Xu, Bekefi
Fig. 5
Hartemann, Xu, Bekefi
Fig. 6
Hartemann, Xu, Bekefi
Fig. 8
Hartemann, Xu, Bekefi
Fig. 9
Hartemann, Xu, Bekefi
Fig. 10
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"X = 9.66 dB/m"

"X = 1.88 dB/m"