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ABSTRACT

A novel two-stream relativistic klystron amplifier (RKA) is presented in which the amplification of stimulated beam modulation is achieved via the unstable two-stream interaction rather than the use of passive cavities. After a calculation of the limiting current, the amplification and saturation of the stimulated beam modulation are analyzed using a cold-fluid model and particle-in-cell simulation. Good agreement is found between theory and simulation in the linear regime. Almost fully modulated intense relativistic electron beams are obtained at saturation.

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The generation of intense coherent radiation based on the principle of the relativistic klystron amplifier (RKA) has been a subject of current experimental and theoretical research [1]-[3]. High-power RKAs have applications in wide areas, including the development of high-gradient, radio-frequency (RF) accelerators and high-power radar. The primary goals of the RKA research attempt to address the following critical issues: (1) the suppression of self-oscillations in high-gain, multi-cavity RKA operation, (2) the problem of RF breakdown, particularly at the output cavity, and (3) the output coupling of an intense electromagnetic wave with the space-charge waves on the electron beam.

In this Letter, we present a novel two-stream relativistic klystron amplifier in which the interaction of the slow space-charge wave on a fast electron beam with the fast space-charge wave on a slow electron beam leads to an amplification of stimulated beam modulation. (The modulation comes about as a results of the beating of fast and slow space-charge waves.) In this two-stream RKA concept, the input and output cavities are connected with a smooth drift tube without intermediate passive cavities, as shown schematically in Fig. 1. The removal of the passive cavities prevents the amplifier from self-oscillation, a problem often arising from a traveling-wave tube (TWT) type of interactions in the RKA with multiple passive cavities. The possibility of RF breakdown at the passive cavities is eliminated. Thus, the two-stream RKA has an advantage over the conventional single-stream RKA, particularly in high-gain, high-power operation.

Unlike the two-stream amplifier which was proposed by Pierce and Hebenstreit [4] in the late 1940's and which yielded poor efficiency ($< 5\%$) experimentally due to inefficient helix input and output coupling [5], we propose using resonant cavities in the two-stream RKA in the same way as in the conventional RKA. We also note that the basic system configuration of this two-stream RKA differs from the recently proposed double-stream cyclotron maser concept [6], which is more difficult to implement since it relies crucially on the cyclotron motion of the electrons in the applied axial magnetic field. In contrast to previous work [4],[5], we address the important problem of the coupling of an input electromagnetic wave with the relativistic electron beams. To substantiate our two-stream RKA concept, we report the results of analytical and numerical studies of the growth and saturation of stimulated beam modulation in a two-stream RKA.

We first consider two concentric annular relativistic electron beams co-propagating in the z direction through a perfectly conducting drift tube of radius b immersed in an applied uniform axial magnetic field $B_0\vec{e}_z$. The assumptions in the present analysis are (1) both beams are infinitely thin, (2) the strength of the applied magnetic field is infinite, and (3) there is no background plasma. Under these assumptions, the equilibrium motion of the electrons is one-dimensional; the two cold beams are described completely by the axial velocities $V_\alpha\vec{e}_z$, currents I_α, and radii a_α, where the index $\alpha = 1, 2$ designates the inner and outer beams, respectively. For given injection kinetic energies for the beams, $\gamma_{0\alpha} - 1$, as measured in units of the electron rest mass energy mc^2, the time-independent electrostatic potentials between the beams and the drift tube impose an upper bound [7] on the beam currents under which beam propagation is possible. Indeed, energy conservation requires that

$$\gamma_{01} = \gamma_1 + \frac{I_1}{\beta_1 I_{01}} + \frac{I_2}{\beta_2 I_{02}} \tag{1}$$

and

$$\gamma_{02} = \gamma_2 + \frac{I_1}{\beta_1 I_{02}} + \frac{I_2}{\beta_2 I_{02}} \ , \tag{2}$$

where $\beta_\alpha = V_\alpha / c > 0$ and $\gamma_\alpha = (1 - \beta_\alpha^2)^{-1/2}$ are the normalized axial velocity and relativistic mass factor of the αth beam in equilibrium, respectively, $I_{0\alpha} = I_A / [2 \ln(b/a_\alpha)]$, $I_A = mc^3/e \cong 17$ kA is the Alfvén current, c is the speed of light in vacuum, and $-e$ is the electron charge. In general, the upper bound on the beam current has to be determined numerically by seeking physical solutions for γ_1 and γ_2 in Eqs. (1) and (2).

We have derived with a cold-fluid model a dispersion relation describing the lowest azimuthally symmetric, small-amplitude space-charge waves on the two relativistic electron beams subject to transverse-magnetic (TM) like boundary conditions. The result is:

$$D(\omega, k_z) = 1 - \sum_{\alpha=1}^{2} \frac{\epsilon_\alpha R_\alpha (c^2 k_z^2 - \omega^2)}{(\omega - k_z V_\alpha)^2} + \frac{\ln(a_2/a_1)}{\ln(b/a_1)} \frac{\epsilon_1 \epsilon_2 R_{12} R_2 (c^2 k_z^2 - \omega^2)^2}{(\omega - k_z V_1)^2 (\omega - k_z V_2)^2} = 0 \ . \tag{3}$$

In Eq. (3), $\omega = 2\pi f$ and k_z are, respectively, the angular frequency and axial wave number of the wave,

$$\epsilon_\alpha = \frac{2}{\gamma_\alpha^3 \beta_\alpha} \left(\frac{I_\alpha}{I_A} \right) \ln \left(\frac{b}{a_\alpha} \right) , \tag{4}$$

is the dimensionless coupling constant for the αth beam. The quantities

$$R_\alpha = \frac{1}{\ln(b/a_\alpha)} \frac{I_0(pa_\alpha)}{I_0(pb)} [I_0(pb) K_0(pa_\alpha) - I_0(pa_\alpha) K_0(pb)] \tag{5}$$

and

$$R_{12} = \frac{1}{\ln(a_2/a_1)} \frac{I_0(pa_1)}{I_0(pa_2)} [I_0(pa_2) K_0(pa_1) - I_0(pa_1) K_0(pa_2)] \tag{6}$$

are wavelength-dependent geometric factors, and $I_0(x)$ and $K_0(x)$ are the first- and second-kind modified Bessel functions of the zeroth order, respectively, where $p^2 = k_z^2 - \omega^2/c^2$. Note that the single-beam dispersion relation in Ref. [8] can be recovered from Eq. (3) by setting $\epsilon_2 = 0$. In the long-wavelength limit with $k_z^2 b^2 \ll \gamma_1^2$ and $k_z^2 b^2 \ll \gamma_2^2$, dispersion relation (3) can be approximated by

$$1 - \sum_{\alpha=1}^{2} \frac{\epsilon_\alpha (c^2 k_z^2 - \omega^2)}{(\omega - k_z V_\alpha)^2} + \frac{\ln(a_2/a_1)}{\ln(b/a_1)} \frac{\epsilon_1 \epsilon_2 (c^2 k_z^2 - \omega^2)^2}{(\omega - k_z V_1)^2 (\omega - k_z V_2)^2} = 0 \ , \tag{7}$$

where use has been made of the approximation $R_1 \approx R_2 \approx R_{12} \approx 1$.

In order to find the condition for the two-stream instability and how the maximum growth rate scales with system parameters, such as γ_α, ϵ_α, a_α and b, we first consider, for simplicity, a special long-wavelength case in which one electron beam overlaps the other beam ($a_1 = a_2$), both beams are tenuous, and have the same coupling strength ($\epsilon_1 = \epsilon_2 = \epsilon \ll 1$). Making the approximation $c^2 k_z^2 - \omega^2 \approx \langle \gamma \rangle^{-2} c^2 k_z^2$, it is readily shown from Eq. (7) that the space-charge waves on the electron beams are unstable whenever

$(\Delta\beta)^2 < 8\langle\gamma\rangle^{-2}\epsilon$, and that for a given ϵ the maximum spatial growth rate of the field amplitude squared,

$$\Gamma = \frac{\sqrt{\epsilon}}{\langle\gamma\rangle\langle\beta\rangle^2}\left(\frac{\omega}{c}\right) , \tag{8}$$

occurs when $(\Delta\beta)^2 = 3\langle\gamma\rangle^{-2}\epsilon$. Here, we have introduced the notations $\Delta\beta \equiv |\beta_2 - \beta_1|$, $\langle\beta\rangle \equiv (\beta_1 + \beta_2)/2$, and $\langle\gamma\rangle \equiv (1 - \langle\beta\rangle^2)^{-1/2}$. Such a two-stream instability is convective for co-propagating beams. These results are analogous to those for the one-dimensional two-stream system if one makes the identification that the corresponding nonrelativistic plasma frequency is $\omega_p = \epsilon^{1/2}\omega$ [9]. It is this two-stream instability that is responsible for the amplification of stimulated beam modulation in the two-stream RKA. Furthermore, the spatial period of the envelope of the current modulation is found be to approximately $\lambda_r = 0.5\pi/\Gamma$. Therefore, for this special long-wavelength case ($a_1 = a_2$ and $\epsilon_1 = \epsilon_2 = \epsilon \ll 1$), the maximum gain in the beam current modulation amplitude squared is about 14 dB per spatial modulation period. Typically, as the beam separation $|a_2 - a_1|$ increases, the maximum gain decreases due to the third (stabilizing) term in Eq. (7).

It should be emphasized that based on Eq. (4), comparable coupling strengths for both beams can also be achieved using beams with quite unequal currents and voltages; that is, a high-power (high-current, more energetic) outer electron beam and a low-power (low-current, less energetic) inner electron beam. Hence, the density modulation of the high-power, primary driving electron beam can be amplified by employing a low-power, secondary electron beam of the two-stream RKA, as will be shown later in our simulation (see Figs. 2 and 3). This unique feature makes the two-stream RKA an attractive concept from a practical point of view, because the addition of the secondary electron beam will not result in any significant degradation in the overall efficiency.

We have conducted an extensive simulation study of the amplification and saturation of stimulated beam modulation in two-stream RKAs using MAGIC [10], a two-dimensional particle-in-cell code. Particle-in-cell simulations are needed in order to study the effect of the time-independent space-charge associated with the intense relativistic electron beams as well as the coupling between the beams and the RF field at the input. A configuration similar to the one shown in Fig. 1 is used in the simulations. The coupling of the input RF power to the electron beams is modeled with an input cavity, while the output cavity has not been included. The results of the simulations on the beam modulation are summarized in Fig. 2 and 3.

The amplitude of beam current modulation is plotted as a function of the interaction length z in Fig. 2. The center of input cavity gap is at $z = 0$. In the linear regime, the intensity growth rate is found to be 30 dB/m from the simulation, which is in good agreement with an intensity growth rate of 31 dB/m as calculated from the full dispersion relation in Eq. (3). By calculating the Poynting flux through the input cavity window, the input RF power is estimated to be $P = 0.9$ MW; the input cavity admittance, $Y = 0.02$ $(\text{Ohm})^{-1}$. The saturated current modulation amplitude at $z \approx 110$ cm is $\delta I_1 = 3.4$ kA (that is, a current modulation of 68%) for the outer beam, and $\delta I_2 = 0.9$ kA (or 90% current modulation) for the inner beam.

Figure 3 shows the longitudinal electron phase space of the two beams, $(z, \gamma v_z)$, as obtained from the simulation for the case corresponding to Fig. 2. Electrons are emitted axially from two annular cathodes located at $z = 43.5$ cm. Because of the time-independent electrostatic potentials between the drift tube and the beams, the electrons experience significant momentum depression shortly after being emitted. As they pass through the input cavity gap which is located 10 cm downstream from the cathodes (at $z = 53.5$ cm), there is additional momentum depression due to an increase of the electrostatic potential near the input cavity gap. Further downstream, the axial momenta of the electron beams are modulated spatially with a wavelength corresponding to an input frequency of $f = 3.375$ GHz. The phase difference between the inner and outer beam modulations is about 180°, as expected for two-stream interactions. The amplitude of the modulation grows till it reaches saturation at $z \approx 170$ cm, where the momentum modulation amplitude is approximately equal to the momentum difference between the fast and slow electron beams in equilibrium.

The production and transport of two relativistic electron beams and the conversion from the beam modulation to an electromagnetic wave at the output cavity are being investigated. With two split cathodes, two intense relativistic electron beams with voltage difference up to 100 kV have been demonstrated in the experiment by Fink, *et al.* [11]. Such a voltage differece, or larger, appears necessary to achieve spatial growth of the modulated beams. In fact, recent experiments and calculations [12] have indicated that two annular electron beams with the *same* injection energy from a single cathode appear to yield a stable two-stream interaction. Finally, the conversion efficiency at the output cavity in the two-stream RKA is expected to be comparable to that in the conventional single-stream RKA.

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FIGURE CAPTIONS

Fig. 1 Schematic of a two-stream relativistic klystron amplifier.

Fig. 2 Amplitude of the beam current modulation vs the interaction length obtained from the simulation. The triangles are for the inner beam; the circles for the outer beam. The parameters are: $a_1 = 2.08$ cm, $a_2 = 2.29$ cm, $b = 2.54$ cm, $I_1 = 1.0$ kA, $I_2 = 5.0$ kA, $\gamma_{01} = 1.43$ ($\gamma_1 = 1.32$), $\gamma_{02} = 1.78$ ($\gamma_2 = 1.69$), and $f = 3.375$ GHz. The beam thickness is 0.1 cm for both beams. Total power in the outer beam is 2.0 GW; total power in the inner beam is 0.22 GW.

Fig. 3 Longitudinal electron phase space of the two beams.

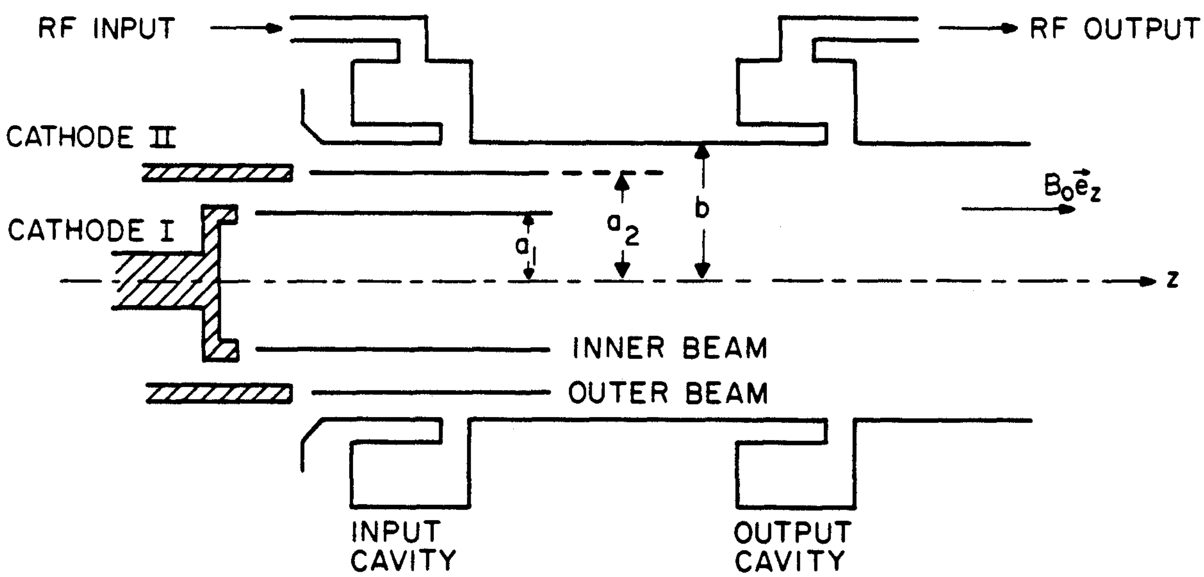
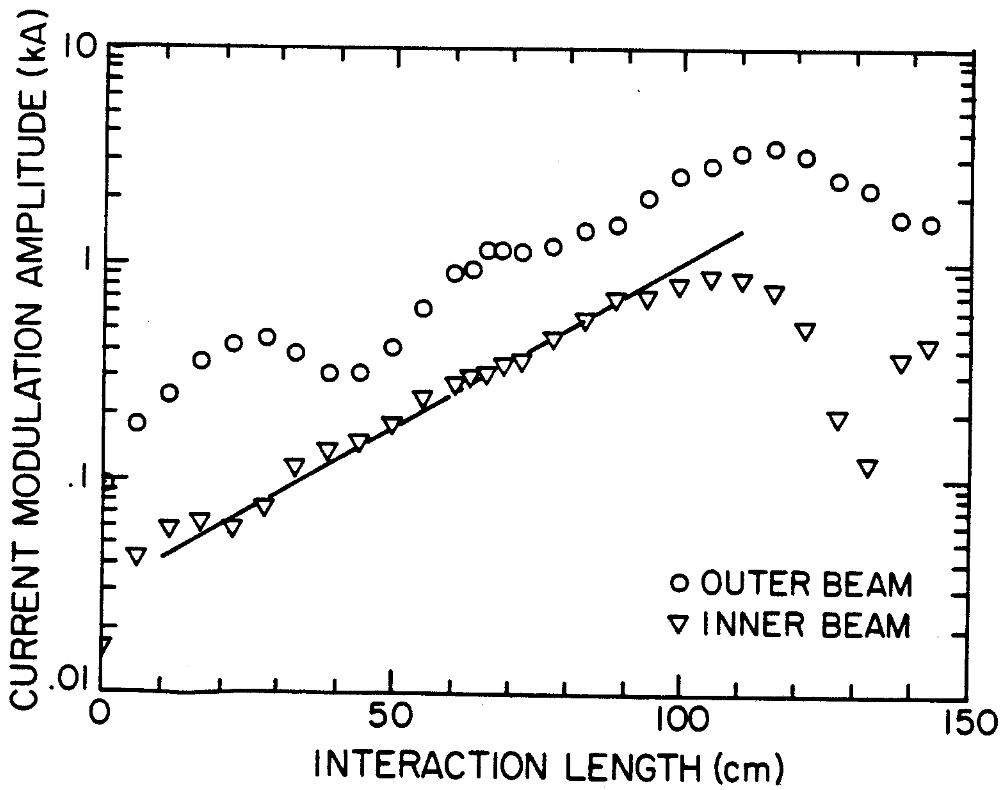

RF INPUT
RF OUTPUT
CATHODE II
CATHODE I
a_1
a_2
b
$B_0\vec{e}_z$
z
INNER BEAM
OUTER BEAM
INPUT CAVITY
OUTPUT CAVITY

Fig. 1



CURRENT MODULATION AMPLITUDE (kA)
10
1
.1
.01
0
50
100
150
INTERACTION LENGTH (cm)
○ OUTER BEAM
▽ INNER BEAM

Fig. 2

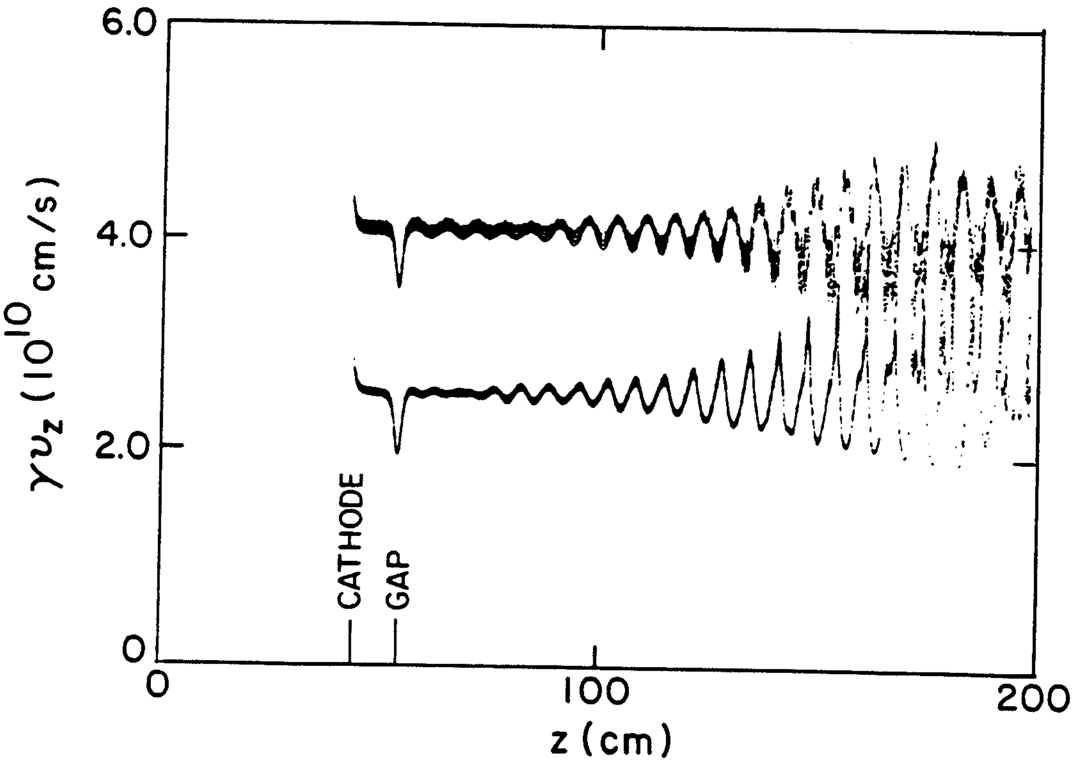


Fig. 3