LHCD Accessibility Study with Reconstructed Magnetic Equilibria in PBX-M


February 1994

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(a)Supported by U. S. Department of Energy Contract DE-FG02-91-ER-54109.

This is a revised manuscript of PFC/JA-93-22. Accepted for publication in Nuclear Fusion.
Abstract: Ray tracing simulations based on experimental PBX-M equilibria show a limited range of the parallel wave number ($n_{\|}=k_{\|}/\omega$) along the ray trajectory. The range of $n_{\|}$ accessible to the excited wave is shown to have both a lower and an upper bound. The ray's phase volume is projected into ($n_{\|}, R, Z$) space to define a domain of wave accessibility for each toroidal mode number excited by the launcher. A comparison of circular and bean shaped plasmas with high aspect ratio ($A \sim 5.5$) shows that the rays fill a substantially larger portion of the accessible domain in bean shaped plasmas. Furthermore, in the bean shaped case the accessible domain tends to extend to higher $n_{\|}$. 
1. INTRODUCTION

Lower Hybrid Current Drive (LHCD) can modify the current density therefore influencing the stability and confinement properties of a tokamak. PBX-M is a high aspect ratio tokamak (A~5.5 with R₀ = 165 cm) with the capability of varying the poloidal cross-section from circular to bean shaped. Experimental observations showing the plasma response to the application of LHCD in different PBX-M configurations have been reported in Ref. [1] [2].

The examples we consider are for relatively low temperature ohmic plasmas (Tₑ₀ ~ 1 keV) or mildly heated plasmas with neutral beam injection (NBI). In this parameter range rays with an initial value of the refraction index, n₀∥ (n∥ = k∥c/ω) in the interval 1.0 < n∥₀ < 4.4 are weakly damped, typically with many internal reflections before depositing their energy [3] [4] [5] [6] [7]. We utilize reconstructed MHD equilibria [8] [9] in lower hybrid ray tracing codes [6] to obtain a consistent analysis of the wave behavior. Ray tracing results have indicated that the possible values of n∥ that a wave can reach along its path are bounded [10]. A simple analytic expression showing the existence of an upper bound to the n∥ upshift has been derived [11] [12]. Furthermore, we note a different behavior in the n∥ evolution as a function of the plasma shape. Analytical studies of LH ray tracing equations in the case of non circular plasma cross sections have been presented recently [13] [14]. In the present study, aimed at understanding the observed behavior, a domain where the wave can exist is compared for various experimental discharges.
2. **KINEMATIC DOMAINS AND WAVE ACCESSIBILITY**

We define a coordinate system $(\psi, \chi, \Phi)$ where, $\psi$ is the poloidal flux variable, $\chi$ the poloidal generalized angle parallel to the flux surfaces and $\Phi$ the toroidal angle. The components of the wave vector $k = (k_\psi, k_\chi, k_\phi)$ respectively parallel and perpendicular to the total magnetic field $B = (0, B_z, B_\phi)$ are defined as: $k_\parallel = k \cdot B / B$ and $k_\perp = |k \times B| / B$. These two expressions can be written as a function of the dimensionless parameter $\gamma(\psi, \chi) = B_z / B$ as follows:

\[
\begin{align*}
  k_\parallel &= k_\phi \sqrt{1 - \gamma^2} + k_z \gamma \\ 
  k_\perp^2 &= (k_z \sqrt{1 - \gamma^2} - k_\phi \gamma)^2 + k_\psi^2
\end{align*}
\]

With the assumption of toroidal symmetry $\Phi$ becomes a cyclic variable and, consequently the toroidal mode number ($n = Rk_\phi$) is a constant of motion. In the ray tracing simulations, individual rays are characterized by the invariant $n$ and the initial value of $k_z$ which define the parallel index of refraction $(n_{i0} = k_\parallel c / \omega)$ at the launched point.

Substituting $k_z$ from eq. (2.1) into eq. (2.2) we obtain an equation for $k_\parallel$:

\[
[k_\parallel \sqrt{1 - \gamma^2} - k_\phi \gamma]^2 = \gamma^2 [k_\perp^2(k_\parallel) - k_\psi^2]
\]

where the perpendicular wave vector $k_\perp$ is a function of $k_\parallel$ through the local dispersion relation [15]. From eq. (2.3), noting that $k_\psi^2 \geq 0$, we get the expression:

\[
[k_\parallel \sqrt{1 - \gamma^2} - k_\phi \gamma]^2 \leq \gamma^2 k_\perp^2(k_\parallel)
\]

3
In the electrostatic limit \( k_{\parallel}^2 = -(P/S)k_{\perp}^2 \), neglecting terms \( o(\gamma^2) \) with respect to terms \( o(1) \), eq. (2.4) breaks into the following two inequalities [11] [12]:

\[
k_{\parallel} \leq \frac{n/R}{\sqrt{1 - \gamma^2 - \sqrt{P/S}}} \gamma \equiv \frac{n/R}{1 - \gamma \omega_{pe}/\omega} \tag{2.5}
\]

\[
k_{\parallel} \geq \frac{n/R}{\sqrt{1 - \gamma^2 + \sqrt{P/S}}} \gamma \equiv \frac{n/R}{1 + \gamma \omega_{pe}/\omega} \tag{2.6}
\]

where we use Stix's notations [15]: \( S = 1 - \frac{\omega_{pe}^2}{\omega^2} + \frac{\omega_{pe}^2}{\Omega_{ce}^2} \equiv 1 \), \( P = 1 - \frac{\omega_{pe}^2}{\omega^2} \equiv -\frac{\omega_{pe}^2}{\omega^2} \). The left hand side of eqs. (2.5) and (2.6) are respectively the solutions of eq. (2.3) in the electrostatic approximation evaluated for \( k_{\parallel} = 0 \) (\( k_{\parallel}^+ \) and \( k_{\parallel}^- \)):

\[
k_{\parallel}^\pm \equiv \frac{k_o \pm k_o \gamma \frac{\omega_{pe}}{\omega}}{(1 + \gamma \omega_{pe}/\omega)(1 - \gamma \omega_{pe}/\omega)} \sqrt{1 - \left(1 - \frac{(1 + \gamma \omega_{pe}/\omega)(1 - \gamma \omega_{pe}/\omega)}{(\omega_{pe}/\omega)^2} \right) \frac{k_{\parallel}^2}{k_o^2}} \tag{2.7}
\]

Adopting the sign convention \( n > 0 \), eq. (2.5) represents an upper bound on \( k_{\parallel} > 0 \) as long as \( \gamma \omega_{pe}/\omega < 1 \). If this condition is not satisfied then the upper bound on \( k_{\parallel} \) extends to infinity in regions where \( \gamma \omega_{pe}/\omega > 1 \). In these regions there is a second solution of the electrostatic dispersion relation for which \( k_{\parallel} < 0 \), while \( n > 0 \). In fact, when \( \gamma \omega_{pe}/\omega > 1 \) only the solution \( k_{\parallel}^- \), in eq. (2.7), can be accessed for rays launched with \( k_{\parallel} > 0 \) while \( k_{\parallel}^+ \) is always negative. In regions where \( \gamma \omega_{pe}/\omega < 1 \) we have that also \( k_{\parallel}^+ > 0 \). Coupling to this branch can lead to rapid increases of the parallel wave number when \( 0 < (1 - \gamma \omega_{pe}/\omega) < 1 \) due to the dependence of the denominator in eq. (2.7).
It is particularly important to mention, at this point, that the upper bound on $n_\parallel$ discussed in Ref. [10] is not a kinematic boundary of the type obtained in eq. (2.5), but rather a KAM surface [16] [17] which prevents stochastic ray orbits from filling the entire accessible space. In fact, for the parameters considered in Ref. [10], the upper kinematic boundary on $n_\parallel$ is not closed, because $\gamma \omega_p/\omega < 1$ is not satisfied over the entire plasma cross section [16]. In these cases, the existence of a KAM surface prevents $n_\parallel$ from growing indefinitely.

From eq. (2.5) we observe that the upper boundary will rise as density increases. Furthermore, since $B_z/B \approx (1/qA)(1 + \kappa^2)/2$ with $\kappa$ the ellipticity, the upper boundary will also rise with decreasing aspect ratio and increasing elongation. When the electrostatic dispersion relation is valid eq. (2.6) represent a lower bound on $k_\parallel$. However, the electrostatic approximation breaks down in the region where slow-fast mode conversion occurs, in which case the lower bound on $k_\parallel$ is given by the well known accessibility condition [15]:

$$n_\parallel = \frac{k_\parallel c}{\omega} \geq \sqrt{S} + (\omega_p/\Omega_{ce})$$

(2.8)

Eqs. (2.4) and (2.8) have been studied for realistic PBX-M deuterium plasmas based on the reconstructed equilibria obtained with the FQ code [18] [19] [8] [9]. The function $\gamma(\psi,\chi)$ is numerically computed in each point of a grid covering the poloidal cross-section based on the magnetic structure of reconstructed equilibria which utilize Motional Stark Effect (MSE) magnetic pitch angle measurements [20] [21]. Since the electrostatic dispersion relation breaks down in the vicinity of the slow-fast mode conversion layer, we have utilized the electromagnetic cold dispersion relation in the study shown below.
3. A PARAMETRIC STUDY OF THE WAVE DOMAINS: IMPLICATIONS FOR PBX-M

In this section we calculate the parametric dependence of the wave domain, based on the experimental condition for a circular and bean shaped cross section of the PBX-M plasma. The main equilibrium parameters, for two characteristic discharges are summarized in Table I.

<table>
<thead>
<tr>
<th></th>
<th>circular shape</th>
<th>bean shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_p$ (kA)</td>
<td>117 kA</td>
<td>183 kA</td>
</tr>
<tr>
<td>$B_T$ (T)</td>
<td>1.53</td>
<td>1.53</td>
</tr>
<tr>
<td>$n_{e0}$ (cm$^{-3}$)</td>
<td>1.9 $10^{13}$</td>
<td>1.8 $10^{13}$</td>
</tr>
<tr>
<td>$T_{e0}$ (keV)</td>
<td>0.7</td>
<td>1.2</td>
</tr>
<tr>
<td>plasma area (cm$^2$)</td>
<td>3200</td>
<td>4500</td>
</tr>
<tr>
<td>magnetic axis (cm)</td>
<td>170</td>
<td>167</td>
</tr>
<tr>
<td>midplane half-width (cm)</td>
<td>31.5</td>
<td>30.1</td>
</tr>
<tr>
<td>$q_0$</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>$\beta_p$</td>
<td>0.43</td>
<td>0.65</td>
</tr>
<tr>
<td>ellipticity, $\kappa$</td>
<td>1.05</td>
<td>1.39</td>
</tr>
<tr>
<td>triangularity, $\delta$</td>
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<td>0.43</td>
</tr>
<tr>
<td>indentation, $i$ (cm)</td>
<td>0</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Table I
In general, the wave domain in the $R, Z$ cross section of the plasma (Z is the vertical axis in cylindrical coordinates), resembles a "volcano". Fig. 1 shows a 3-D plot of the upper $n_{//}$ boundary of the kinematic domain for the bean shaped plasma of table I.

Fig. 2 shows a cross section of the accessible wave domain in $(n_{//}, R)$ evaluated at $Z=0$. The boundaries of $n_{//}$ are for the launched value of $n_{//0} = 2.4$, in the circular cross section discharge of Table I. The wave cannot propagate outside the shaded regions. In region (A) only the slow mode can propagate whereas in region (B) both modes can exist. The two modes couple and convert into each other along the curve (C), which is the accessibility condition given by eq. (2.8). It is important to note that the domains are a result of the equilibrium. The rays are constrained to remain within these boundaries and do not necessarily fill the entire accessible $n_{//}$ regions.

The dependence of the domain on the initial $n_{//}$ is shown in Fig. 3. An increase of the launched $n_{//0}$ from 1.8 to 3.0 leads to a 75 % increase of the maximum permitted $n_{//}$.

The scaling with total plasma current is shown in Fig. 4 where we make the comparison between the cases with $I_p = 117$ kA and $I_p = 183$ kA based on the equilibrium reconstruction for a circular plasma. Increasing $I_p$ with a constant $q_0$ causes $\gamma$ to increase and results in a small increase of the upper $n_{//}$ boundary.

Due to the gamma dependence the $n_{//}$ boundary is sensitive to poloidal beta. Fig. 5 shows the $n_{//}$ domain for a simulated PBX-M high $\beta_p$ plasma ($n_{e0} = 6.8 \times 10^{13}$ cm$^{-3}$, $T_{e0} = 3.7$ keV, $\beta_p = 5$). Fig. 6 shows a comparison of the wave domains in circular and bean shaped configurations for discharges with similar average current density (see Table I).

The evolution of $n_{//}$, obtained by ray tracing calculations using the LSC code [6], is quite different for circular and bean shaped plasmas. Fig. 7a,b show
a projection of the ray trajectory points from the 3-D space \((n/, R, Z)\) on the plane \((n/, R)\) for the cases shown in Table I. The dashed curve shows a projection of the \(n/\) domain boundary on the same plane. It is important to notice the difference between the mid-plane cut \((Z=0)\) of the domain shown in Fig. 2 and this projection which includes all the \(Z\) values and enables one to visualize the ray trajectory points. Here, the evolution of \(n/\) is followed for an arbitrary number of passes (typically \(\sim 2000\) flux zone crossing) while the electron Landau damping is turned off. In the circular cross section case the waves only partially fill the accessible region (Fig. 7a) while in the bean shaped case (Fig. 7b) the ray trajectory points fill the accessible region more completely. In particular they are found in the regions close the maximum upper limit. When the electron Landau damping is included, the accessible region in Fig. 7a (circular case) does not allow for strong wave damping and quasi linear tail formation since the wave cannot find enough resonant electrons in a 0.7 keV plasma [6] [7]. However, non inductive current drive is experimentally observed [9]. This contradictory situation persists even after considerable study by modeling efforts on PBX-M [6] [7]. On the other hand, in the bean shaped case (Fig. 7b), due to the higher temperature \((T_e \sim 1.2\) keV) and to the taller \(n/\) upper bound the wave is damped off-axis in the region where the maximum \(n/\) up-shift takes place [6] [7].

In the bean shaped cases we observe a rapid increase of \(n/\) in the region of the plasma cross section between the magnetic axis and the pusher coil. In this region of strong poloidal asymmetry a rapid increase of the poloidal wave number can occur.
4. CONCLUSIONS

Results from ray tracing calculations in high aspect ratio tori suggest that the geometry of the poloidal plasma cross section plays an important role in lower hybrid current drive. The kinematics of the ray in these plasmas defines an upper and a lower bounds for \( n_\parallel \) in addition to the usual accessibility limitation due to the slow-fast mode conversion. We find a dependence of the wave domains on several parameters and most notably on \( \gamma \) (ratio between poloidal and total magnetic field) and plasma electron density. Therefore the evolution of \( n_\parallel \) is dependent on the plasma equilibrium. Ray tracing simulations on PBX-M plasmas show a larger toroidal upshift for bean shaped than for circular configuration. Plasma shaping leads to a stronger poloidal asymmetry and we observe that a wider portion of the accessible domain is reached by the wave in these cases compared with similar circular discharges.
Acknowledgments

We thank A. Cardinali, S.E. Jones, M. Ono, M. Okabayashi, H. Takahashi and S. von Goeler for useful discussions and comments on this work. We also want to thank C.E. Kessel for the computational work on the FQ code.

Supported by USDOE Grants: MIT: DE-FG02-91ER40666, PPPL: DE-AC02-76-CH03073
REFERENCES


FIGURE CAPTIONS

Fig. 1 3-D plot of the $n_{\parallel}$ upper kinematic boundary for a PBX-M bean shaped discharge.

Fig. 2 Cross section of the wave domain in the plain ($n_{\parallel}$, R) evaluated at $Z=0$ for a PBX-M circular plasma. The main parameters are: $\nu_{\text{LH}} = 4.6$ GHz, $n_{\parallel0} = 2.4$, $n_{e0} = 1.9 \times 10^{13}$ cm$^{-3}$, $I_p = 117$ kA $B_T = 1.53$ T. Region (A): area of existence of the slow mode, region (B): area of existence of the fast mode, (C): classical accessibility curve.

Fig. 3 Scaling of the $n_{\parallel}$ domain with the launched $n_{\parallel}$ for the circular plasma of Fig. 2. Region (A): $n_{\parallel0} = 1.8$, region (B): $n_{\parallel0} = 3.0$

Fig. 4 Dependence of the $n_{\parallel}$ domain with the total plasma current for two circular discharges at fixed $q_0 = 0.8$. Region (A): $I_p = 117$ kA, region (B): $I_p = 200$ kA, the other parameters are the same as in Fig. 2.

Fig. 5 $n_{\parallel}$ boundaries for a PBX-M high $\beta_p$ discharge: $n_{e0} = 7.0 \times 10^{13}$ cm$^{-3}$, $T_{e0} = 3.7$ keV, $\beta_p = 5$

Fig. 6 Comparison between the wave domains in a circular and a bean shaped PBX-M plasmas for a launched wave with $n_{\parallel0} = 2.4$. Region (A): circular plasma, same as in Fig. 2, region (B): bean shaped plasma: $n_{e0} = 1.8 \times 10^{13}$ cm$^{-3}$, $I_p = 183$ kA, $\kappa = 1.39$, $d = 0.43$, $i = 4.7$ cm.
Fig. 7 projection of the ray trajectory points from the 3-D phase space \((n_\parallel, R, Z)\) to the plane \((n_\parallel, R)\). Included in the dashed curve is the region covered by a projection of the \(n_\parallel\) kinematic domain on the plane \((n_\parallel, R)\).

section (A): circular plasma as in Fig. 2. Section (B): bean shaped plasma as in Fig. 6B.
Fig. 2
Fig. 3

\[ n_{\|0} = 3.0 \]

\[ n_{\|0} = 1.8 \]
Fig. 4
Fig. 5
Fig. 6
Fig. 7