Plasma-Neutral Interaction in Tokamak Divertor for "Gas Box" Neutral Model

Sergei Krasheninnikov, T. K. Soboleva*

June 1995

MIT Plasma Fusion Center
Cambridge, Massachusetts 02139 USA

*Instituto de Ciencias Nucleares, UNAM, Mexico, D. F., Mexico

This work was supported by the US Department of Energy under contract DE-FG02-91ER-54109. Reproduction, translation, publication, use, and disposal, in whole or in part, by or for the US Government is permitted.

Submitted for publication in: Physics of Plasmas
Plasma-Neutral Interaction in Tokamak Divertor for "Gas Box" Neutral Model

S. I. Krasheninnikov a)
MIT, Plasma Fusion Center, Cambridge, MA 02139, USA

T. K. Soboleva a)
Instituto de Ciencias Nucleares, UNAM, Mexico D.F., Mexico

Abstract

We investigated plasma flow through the gas cloud in a tokamak divertor for "gas box" divertor geometry and Knudsen regime of neutral transport. We have shown that similar to the neutral models considered previously, the plasma parameters near the target is sensitive to the energy flux into the hydrogen recycling region and can change rapidly resulting in bifurcation like behavior, which might be interpreted as a transition to detached regime. Notice, that the critical values the energy flux below which the rapid change of plasma parameters occurs are very similar for all neutral transport models. However, for low plasma temperature near the target, the scalings of plasma parameters for "gas box" neutral model are different from those obtained for diffusive Knudsen neutral model considered previously. In particular, "gas box" neutral model is less efficient in the energy spreading onto the side walls than diffusive Knudsen neutral model.

PACS numbers: 52.30.-q, 52.25.Fi
I. Introduction

Experiments on most diverted tokamaks have demonstrated detached divertor regimes (see Refs. 1-4) which are characterized by: high energy radiation losses from the scrape off layer (SOL) region; low plasma temperature near the divertor plates; strong decrease of the plasma particle and energy fluxes onto the plates; and strong plasma pressure drop along magnetic field lines in the divertor volume. Due to a very low heat loads on the divertor plates observed in these regimes, they look attractive from the International Thermonuclear Experimental Reactor (ITER)\(^5\) divertor design point of view.

In Ref. 6 one dimensional physical model of the tokamak SOL was developed to investigate the main features of plasma - neutral interactions in the recycling region of a tokamak slot-like divertor for the two opposite extremes of fluid and diffusive Knudsen neutrals. Fluid approximation for neutral transport can be applied for relatively high plasma density in the hydrogen recycling region, when the neutral mean free path, \(\lambda_N\), is smaller than SOL plasma width, \(\Delta_p\). Diffusive Knudsen approximation describes the opposite extreme for which \(\Delta_p << \lambda_N << L\), where \(L\) is the length of the slot.

It was shown\(^6\) that in both cases of fluid and diffusive Knudsen neutrals the reduction of the heat flux into the hydrogen recycling region from upstream, \(q_{rc}\), due to impurity radiation, below some critical value leads to either thermal bifurcation or rapid change of the SOL plasma parameters. The resulting changes of plasma parameters in the recycling region are consistent with the detached divertor observations mentioned earlier. The physical mechanisms responsible for the decrease of the plasma flux onto the target and plasma pressure drop along magnetic field
lines in the divertor volume (depending on the SOL plasma parameters and divertor geometry) are the influence of the neutral gas pressure on plasma flow \(^7, 8\) for fluid neutrals, and friction between the plasma flowing toward the target and the neutral gas scattered by the sidewalls\(^9\) for diffusive Knudsen neutrals.

Since the balance of the SOL plasma sink and source must be sustained, the decrease of plasma flux onto the target (plasma sink) after detachment has to be accompanied by the reduction of the global neutral gas ionization (plasma source), while maximum neutral gas density increases. This effect only possible either in a slot like divertor or for the case of relatively dense divertor plasma, when \(\lambda_N\) is smaller than plasma scale length, which allow a significant reduction of neutral influx into hot plasma region in comparison with neutral free streaming flux. Note, that this very important feature of detached plasmas is usually missed in the simple models of plasma detachment\(^9,10\). For both neutral transport models considered in Ref. \(6\) the reduction of the global neutral gas ionization is related to the shift of high plasma temperature region, where ionization might occur, away from the target which result to the decrease of a local neutral gas density in the ionization region.

The main goal of this paper is to investigate with a simple model the influence of a tokamak divertor geometry on the physical picture of plasma-neutral interaction in the hydrogen recycling region drawn above. We consider Knudsen neutral gas flow in the divertor \((\lambda_N > \Delta_P)\) and assume that at a poloidal distance \(L_N\) from the plate there is the widening of the slot divertor (which we will call a "gas box") with the width, \(\Delta_{gb}\), is much bigger than the slot width far from the target, \(\Delta\) (see Fig. 1). For the case when the ratio \(\Delta_{gb}/L_N\) is not so small and \(\Delta/L_N << 1\) we can assume
uniform distribution of neutral gas within the "gas box" and neglect plasma-neutral interaction beyond the "gas box".

The equations and boundary conditions describing a plasma flow through the "gas box" are presented in Section II. Analytical and numerical solutions of these equations are discussed in Section III. The main conclusions are summarized in Section IV.

II. Equations and boundary conditions

We consider uniform distribution of neutral gas within the "gas box" and neglect plasma-neutral interaction beyond the "gas box". Following Ref. 6 we prescribe plasma pressure, $P_u$, and the energy flux, $q_{rc}$, at the entrance into the "gas box". Using "poloidal" coordinate $y$, directed from the target, plasma parallel momentum balance equation as well as plasma energy balance (describing both ion-neutral elastic and electron-neutral inelastic collisions) and continuity equations can be written in the form

\begin{align}
  b \frac{d}{dy} \left( MnV^2 + P \right) &= -MK_{iN}NnV, \\
  \frac{dq}{dy} &= -\varepsilon nNK_{iN} - E_i nNK_i, \\
  \frac{dj}{dy} &= nNK_i,
\end{align}

where $n$, $T$, and $V$ are plasma density, temperature (we assume electron and ion temperature equal), and velocity respectively; $P = 2nT$ is the
plasma pressure; $N$ is the neutral gas density; $M$ is the ion mass; $b = \sin \psi$ with $\psi$ the angle between the target and magnetic field line; $j = bnV$ is the poloidal plasma flux; $E_l$ is the neutral ionization "cost" describing plasma energy losses due to electron-neutral inelastic collisions

$$E_l(T) = I + E_R \left( K_R(T) / K_l(T) \right),$$

where $I = 13.6$ eV is the hydrogen ionization potential, $E_R$ the characteristic energy loss, and $K_R(T)$ and $K_l(T)$ are the electron impact excitation and ionization rate constants for hydrogen; $K_{IN}(T)$ is ion-neutral collision rate constant and $\varepsilon(T) = \varepsilon_0(T - T_0)$ is the characteristic energy loss due to elastic ion-neutral collisions, $\varepsilon_0 = \text{const.}$;

$$q = \left( MV^2 / 2 + 5T \right) j + q_e,$$

with $q$ and $q_e = -\kappa_e(T)b^2 dT/dy$ the total poloidal plasma heat flux and the parallel heat flux due to electron heat conduction along the field lines, $\kappa_e(T)$ is the electron heat conduction coefficient along the magnetic field. Following Ref. 6 we take $K_{IN} = 2.1 \sigma (T / M)^{1/2}$ and $\varepsilon_0 = 1.5$, where $\sigma$ is charge exchange cross section and $T_0$ is neutral gas temperature, which assume to be fixed ($T_0 \approx 1$ eV).

Assuming subsonic plasma flow from Eq. (1) one has

$$j = -\frac{b^2}{MNK_{IN}} \frac{dP}{dy}.$$
Then, plasma continuity and energy balance equations can be written as

\[ \frac{dj}{dy} = -\frac{d}{dy}\left\{ \frac{b^2}{MNK_iN} \frac{dP}{dy} \right\} = \frac{K_i(T)}{2T} PN, \]  

\[ -\frac{dq}{dy} = b^2 \frac{d}{dy}\left\{ \kappa_e(T) \frac{dT}{dy} + \frac{5T}{MNK_iN} \frac{dP}{dy} \right\} = \frac{(eK_iN + E_iK_i)}{2T} PN, \]

where we used the relation \( n = P/2T \).

The boundary conditions for Eqs. (7), (8) are

\[ q(y = L_N) = -q_{rc}, \]  

\[ j(y = L_N) = 0 \Rightarrow \frac{dP}{dy}(y = L_N) = 0, \]  

\[ P(y = L_N) = P_u, \]  

\[ q(y = 0) = \delta j(y = 0)T_d, \]

\[ j(y = 0) = -\frac{\alpha bP_d C_d}{2T_d}, \]

where Eqs. (9)-(11) represent boundary conditions for the plasma heat and particle fluxes and pressure at the entrance into "gas box", while Eqs. (12), (13) are the standard boundary conditions for the plasma heat and particle fluxes at the target; (...)\(_d\) is the (...) value near the target \( y=0 \),
\[ C_d = \left( \frac{T_d}{M} \right)^{1/2}, \alpha = 0.5, \delta \text{ is the plasma heat transmission coefficient.} \]

Recall that \( P_u \) and \( q_{rc} \) are prescribed upstream plasma pressure and the heat flux coming into the "gas box" region from upstream.

One can see that there are 5 boundary conditions for two secondary order differential equations (7) and (8) for plasma pressure and temperature. Therefore, they only can be satisfied for some specific neutral density \( N \), which has to be found from solution of Eqs. (7), (8) with the boundary conditions (9)-(13).

II. Analytic and numerical solutions

Before we proceed with numerical solution of Eqs. (7)-(13) it is useful to make some analytical estimates. Notice, that if we transform Eqs. (7)-(13) using

\[ \frac{P}{P_u} \rightarrow p, \quad \frac{y}{L_N} \rightarrow \xi, \quad \frac{q}{P_u} \rightarrow q', \quad \frac{j}{P_u} \rightarrow j', \quad (14) \]

we find that the solution of Eqs. (7)-(13) is only determined by the values of \( q_{rc} / P_u, L_N P_u \), and the factor \( b \); neutral density \( N \), which has to be found from the solution, should scale like \( 1/L_N \). The term \( L_N P_u \) appears only in the resulting set of equations and boundary conditions as a factor \( (L_N P_u)^{-1} \) in front of electron heat conduction coefficient and we will see below that it's influence on plasma parameters is rather weak. Therefore, it is convenient to characterize the solution of Eqs. (7)-(13) by it's temperature near divertor plate \( T_d \), which, omitting the weak impact of the parameter \( L_N P_u \), for the fixed \( b \) factor, depends only on the \( q_{rc} / P_u \).
ratio and might be written as an equation

\[ G(T_d) = \frac{q_{rc}}{P_u}, \tag{15} \]

where function \( G(T_d) \) has to be found from the solution of Eqs. (7)-(13). To characterize other plasma parameters, such as plasma pressure drop, plasma flux onto the target, and neutral density we introduce the functions

\[ p(T_d) = \frac{P_d}{P_u}, \quad J(T_d) = \frac{j_d}{P_u}, \quad H(T_d) = N L_N. \tag{16} \]

Let investigate the behavior of the functions \( G(T_d), p(T_d), J(T_d), \) and \( H(T_d) \) analytically for the extreme cases of high and low values of \( q_{rc} / P_u \) ratio.

For the high \( q_{rc} / P_u \) ratio, which corresponds to the high temperature \( T_d \), plasma parameters are practically uniform in the "gas box". Then, integrating the right hand sides of Eqs. (7), (8) and using the boundary conditions (9)-(13) after simple algebra we find

\[
G(T_d) = \frac{\alpha b C_d}{2} \left( \delta + \frac{E_i(T_d)}{T_d} + \frac{\varepsilon}{K_i(T_d)} \right), \quad p(T_d) = 1, \tag{17}
\]

\[ J(T_d) = \frac{\alpha b C_d}{2T_d}, \quad H(T_d) = \frac{\alpha b C_d}{K_i(T_d)}. \]

For the low \( q_{rc} / P_u \) ratio, which corresponds to the temperature \( T_d = T_0 \), plasma parameters are very non uniform in the "gas box" region. In this case plasma temperature is very low (\( \sim T_0 \)) and ionization rate
constant is negligible everywhere in the "gas box" except a small region near the entrance into the "gas box" where plasma temperature $T \sim T_1 \approx T(y - L_N) \gg T_0$ is sufficient for neutral ionization to be important.

Assuming that $P_d < P_u$ and the width of this neutral ionization region $y_1$ is much smaller than the length of the "gas box" $L_N$ from plasma continuity equation (7) we find

$$|j_\parallel| = \frac{b^2 P_u}{MK_{IN}(T_0)N L_N}, \quad |j_\parallel| = \frac{K_{IN}(T_1)P_u N y_1}{2T_1}. \quad (18)$$

where the first relation describes plasma diffusion through the neutral cloud in the "gas box", and the second one describes the origin of plasma flux due to neutral ionization.

Considering energy balance equation (8) in neutral ionization region we find the estimates

$$q_{rc} = \frac{b^2 \kappa_{IN}(T_1)T_1}{y_1}, \quad q_{rc} = \left(5T_1 + E_1(T_1)\right)|j_\parallel|. \quad (19)$$

where the first relation describes energy balance at the entrance into ionization region (where plasma flux is small and the energy is mainly transported by heat conduction), and the second one describes the energy balance at the exit from ionization region (where the energy is mainly transported by convection) and accounting energy loss due to neutral ionization ($E_1$ term).

Analyzing energy balance equation (8) in the main volume of the
"gas box" we have to request temperature rise from \( T_d \sim T_0 \) at the target to \( T \sim T_1 \) in the ionization region, which brings us to the following relation

\[
\Lambda_d = \frac{-\varepsilon K_{iN}(T_0)P_u N_L N}{4b^2 T_0} \left| j \right|
\]

where \( \Lambda_d = \ln\left(\frac{T_0}{(T_d - T_0)}\right) \gg 1 \).

From Eqs. (18)-(20), after some algebra, we find the asymptotics of the functions \( G(T_d), p(T_d), J(T_d), \) and \( H(T_d) \) for \( T_d \to T_0 \):

\[
G(T_d) \approx \frac{5T_1 + E_I(T_1)}{2T_0} \left( \frac{\varepsilon T_0}{\Lambda_d M} \right)^{1/2}, \quad p(T_d) \approx \frac{1}{\alpha b} \left( \frac{\varepsilon}{\Lambda_d} \right)^{1/2},
\]

\[
J(T_d) \approx \frac{1}{2T_0} \left( \frac{\varepsilon T_0}{\Lambda_d M} \right)^{1/2}, \quad H(T_d) \approx \frac{2b^2}{K_{iN}(T_0)} \left( \frac{\Lambda_d T_0}{\varepsilon M} \right)^{1/2},
\]

where temperature \( T_1 \) is determined by the relation

\[
\frac{P_u L_N}{b^2} \left( \frac{q_{rc}}{P_u} \right)^3 = \frac{\kappa_e(T_1) K_i(T_1) (5T_1 + E_I(T_1))^2}{2MK_{iN}(T_0)}.
\]

As one sees from Eq. (22), temperature \( T_1 \) has rather weak dependence on both \( q_{rc} / P_u \) and \( L_N P_u \) values as well as on the factor \( b \), since functions \( \kappa_e(T_1), K_i(T_1), \) and \( K_R(T_1) \) are very sharp for relatively low \( T_1 \sim \) few eV. Therefore, we can neglect weak dependence of \( T_1 \) on \( q_{rc} / P_u, L_N P_u, \) and \( b \), assuming that \( T_1 \sim 10 \) eV.

Thus, we find from the expressions (17) and (21) that for high
temperature $T_d$ (high value of $q_{rc}/P_u$ ratio) functions $G(T_d)$ and $J(T_d)$ behave just like the fluid and diffusive Knudsen neutral models from Ref. 6, being directly and inversely proportional to $T_d^{1/2}$ while $p(T_d)$ stays close to unity, and $H(T_d)$ increases for $T_d \to \infty$. For temperatures $T_d \to T_0$ all functions $G(T_d)$, $p(T_d)$, $J(T_d)$, and $H(T_d)$ exhibit a very sharp dependence on $T_d$ similar to diffusive Knudsen neutral model from Ref. 6. However, scaling dependencies of plasma parameters and neutral density are very much different. Indeed, from Eqs. (21) we find that $|\cos| \propto q_{rc}/P_u$, $N \propto N_{max}/P_u$, $Q_t \propto q_{rc}^2/P_u$, and the efficiency of energy spreading onto the sidewalls due to plasma-neutral interaction, $Q_t$, (defined as a ratio of the target heat loading, $q_t = (|\cos| + q_{sd})$, to $q_{rc}$) approaches some constant for $q_{rc} \to 0$. For diffusive Knudsen neutral model one has $|\cos| \propto q_{rc}^3/P_u$, $N \propto N_{max} \propto \sqrt{P_u}$, $Q_t \propto q_{rc}^2/P_u$, $L_{R} \propto P_u^{3/2}/q_{rc}^3$, which gives $N_{LR} \propto P_u^2/q_{rc}^3$. The reason for these differences is that diffusive Knudsen neutral model allow to keep very low neutral density in the ionization region for low $q_{rc}/P_u$ ratio, while for "gas box" model the neutral density is homogeneously distributed over whole "gas box" length.

The numerical solutions of Eqs. (7)-(13) are obtained by a shooting method for $P_u=10^{14} \text{ cm}^{-3} \times 100 \text{ eV}$, $L_N=100 \text{ cm}$, $\alpha=0.5$, $\delta=5$, $b=0.05$, $T_0=1 \text{ eV}$, and the exact rate constants from Ref. 11. Functions $G(T_d)$, $p(T_d)$, and $J(T_d)$ as well as $H(T_d)$, $T_l(T_d)$, and $Q_t(T_d)$ are shown in Fig. 1 and Fig. 2. Plasma density, temperature, particle and energy fluxes profiles in the "gas box" region for high and low $T_d$ values are shown in Figs. 3, 4.

One can see that numerical results confirm analytical analysis, in
particular, step like behavior of the functions \( G(T_d) \), \( p(T_d) \), \( J(T_d) \), and \( H(T_d) \) at \( T_d = T_0 \), and shrinking of neutral ionization region (where plasma flux varies) for low \( q_{rc} / P_u \). Notice, that like for diffusive Knudsen neutral model the temperature at the target is sensitive to \( q_{rc} \) and can change rapidly, but smoothly, resulting in behavior which may appear similar to bifurcation.

IV. Conclusions

We investigated plasma flow through the gas cloud in a tokamak divertor for "gas box" divertor geometry and Knudsen regime of neutral transport. We have shown that similar to the neutral models considered in Ref. 6, the plasma parameters near the target is sensitive to the energy flux into the hydrogen recycling region \( q_{rc} \) and change rapidly resulting in bifurcation like behavior, which and might be interpreted as a transition to detached regime. Notice, that the critical values the energy flux \( q_{rc} \) below which the rapid change of plasma parameters occurs are very similar for all neutral transport models.

However, for low target plasma temperature, the scalings of plasma parameters for "gas box" neutral model are different from those obtained for diffusive Knudsen neutral model. In particular, "gas box" neutral model is less efficient in the energy spreading onto the side walls than diffusive Knudsen neutral model. The reason for this difference is that diffusive Knudsen neutral model allow to keep very low neutral density in the ionization region resulting in a very low plasma flux onto the target, while for "gas box" model the neutral density is homogeneously distributed over whole "gas box" length enhancing neutral ionization and, as a
consequence, plasma flux onto the target.
Acknowledgments

This work was performed in part under Department of Energy grant DE-FG02-91-ER-54109. T.K.S. is supported by Instituto de Ciencias Nucleares, Universidad Nacional Autonoma de Mexico, Mexico D.F., Mexico. S.I.K. is pleased to acknowledge the hospitality and support of the Instituto de Ciencias Nucleares, UNAM at Mexico City, where this work was initiated.
References

a) Permanent address: Kurchatov Institute of Atomic Energy, Moscow, Russia.


10 Ph. Ghendrih, Physics of Plasmas 1, 1929 (1994).

Figure Captions

Fig. 1. Geometry of the problem.

Fig. 2. Functions $G(T_d) \times P_0$ [kW/cm$^2$], $J(T_d) \times P_0$ [10$^{20}$ cm$^{-2}$ s$^{-1}$], and $p(T_d)$ ($P_0$=10$^{14}$ cm$^{-3}$ x 100 eV).

Fig. 3. Functions $H(T_d)$ [10$^{15}$ cm$^2$], $T_1(T_d)$ [20 eV], and $Q_t(T_d)$.

Fig. 4. Plasma density ($n$, [10$^{14}$ cm$^{-3}$]), temperature ($T$, [eV]), and pressure ($p = P/P_u$) and plasma particle ($|j|$, [10$^{19}$ cm$^{-2}$ s$^{-1}$]) and energy ($|q|$, [kW/cm$^2$]) fluxes profiles in the "gas box" region for $T_d$=1.01 eV.

Fig. 5. Plasma density ($n$, [10$^{14}$ cm$^{-3}$]), temperature ($T$, [eV]), and pressure ($p = P/P_u$) and plasma particle ($|j|$, [10$^{19}$ cm$^{-2}$ s$^{-1}$]) and energy ($|q|$, [kW/cm$^2$]) fluxes profiles in the "gas box" region for $T_d$=20 eV.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.