A Shaped-Foil Recoil Spectrometer for Neutron Time-of-Flight Measurements

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I. INTRODUCTION

With the progress towards higher yields and $\rho R$s in inertial confinement fusion experiments, it has become important to develop new designs for neutron spectrometers to enable measurements of parameters such as fuel and shell $\rho R^3$. In particular, recoil spectrometers are of interest because the elastic scattering cross-sections and the expected energy distributions are well-defined. This has the added advantage that the measured signal is absolutely calibrated, allowing accurate measurements of neutron yield.

A typical recoil spectrometer design requires that only recoils scattered into a small solid angle be observed so that there is a one-to-one relationship between the observed recoil energy and the neutron energy. A detector operated in current-mode can then determine the exact fluence of neutrons. The problem with this approach is that the detected signal is small.

Moran2 attempted to mitigate this low-signal problem by using a proton-recoil generator in the shape of an annulus, centered on the line of sight between the target and the time-of-flight detector. All parts of the annulus subtend the same angle at the detector and contribute recoils of the same energy. All these recoils travel the same path length from the target to the detector and thus the signal is still time-of-flight dispersed, i.e. neutrons of a given energy, interacting with different portions of the annulus, produce recoils which arrive at the detector at the same time. To increase the signal further, however, requires that the annulus be broadened to a shell-like structure, which necessarily allows recoils scattered at different angles (and thus of different energies) to reach the detector.

The purpose of this discussion is to show that there is a precise, mathematically-defined shape of the recoil foil which ensures that all recoils generated by neutrons of the same energy arrive at the detector at the same time, regardless of their angle and energy. As will be shown, this shape is defined only if the recoils are heavier than protons. An analysis of the expected signal, spectrum, and signal-to-noise of a deuteron-recoil spectrometer of this kind will be performed. In its present form, a suggested detector would be a silicon PN-diode operated in current-mode. Note that, in this discussion, it is the time-integrated neutron spectrum that is desired. Time-of-flight measurements are operating in the regime where the neutrons are completely Doppler separated, i.e. neutrons of the highest energies arrive first, the lowest arrive last.

II. SCATTERING FOIL SHAPE FOR CONSTANT TIME-OF-FLIGHT

Consider the scattering of neutrons from a foil as shown in Fig. 1. If the system is far enough from the source, neutrons of a given energy arrive together in a ‘wave front’. Depending on the shape of the foil, neutrons from this front scatter at different times from various parts of the foil. The basis of this concept is that recoils generated at any point on the foil (by this wave front) arrive at the detector at the same time.

Consider a planar wave front of neutrons arriving at a height $2h$ at time $t = 0$. Recoils produced at point $C$ arrive at the detector at time $t = 2h/v_r^{\text{max}}$, where $v_r^{\text{max}}$ is the maximum recoil velocity, as produced by forward scattering. During this period, a neutron from the same wave front can travel on to point $P$ in time $t_1$, produce a recoil at an angle $\theta$, which then travels on to the detector in a time $t_2$. If the foil is shaped such that $t = t_1 + t_2$ then

$$\frac{2h}{v_r^{\text{max}}} = \frac{2h - y}{v_r} + \frac{\sqrt{x^2 + y^2}}{v_r}$$

where $v_r$ is the recoil velocity.

Now a particle, with mass number $A$, elastically scattered by a neutron, acquires a velocity, $v_r$ given by

$$v_r = v_n \zeta \cos \theta$$

where $\zeta = \frac{2}{1 + A}$.
\[ \theta \] is the angle of scattering with respect to the initial neutron direction, and \( v_n \) is the neutron velocity. From this it can be seen that \( v_{r,\text{max}} = v_n \zeta \).

Substituting this into Eq. 1, and using the relationship \( \cos \theta = y / \sqrt{x^2 + y^2} \), gives:

\[ \frac{2h}{\zeta} = (2h - y) + \frac{x^2 + y^2}{y \zeta} \]  \hspace{1cm} (4)

which, through further manipulation, gives the equation:

\[ \frac{x^2}{(1 - \zeta)h^2} + \frac{(y - h)^2}{h^2} = 1 \]  \hspace{1cm} (5)

This is simply the equation of an ellipse, centered at the point \((0, h)\), with major axis (in the \(y\)-direction) of length \(h\), and minor axis (in the \(x\)-direction) of length \(h \sqrt{1 - \zeta}\). In three dimensions, this shape is, of course, an ellipsoid.

Now for protons, \( \zeta = 2/(1 + A) = 1 \), which makes the minor axis of the ellipse zero and Eq. 5 undefined. Thus it is impossible to construct a foil of this kind for use with proton recoils. The obvious alternative is to use deuterons, where \( \zeta = 2/3 \). The shape of the deuteron foil is shown in Fig. 2 for \( h = 0.5 \). It is worthwhile noting that for very large \( A \), where \( \zeta \to 0 \), the foil shape approaches a circle, or, in 3-D, a sphere.

\section*{III. CALCULATION OF SIGNAL FROM A DEUTERON-RECOIL ELLIPSOID}

The function of the shaped foil is thus to increase the surface area for producing recoils without compromising the time-of-flight dispersion originally present in the neutrons. Calculating the total signal produced requires integrating over all the angles over which recoils are generated, taking into account the differential scattering cross section in conjunction with the geometry of the foil.

Consider a small portion of the foil volume, \( dV \). From this volume, the detector subtends a solid angle \( d\Omega_{\text{det}} \). If the fluence of neutrons is \( I \), the number density of recoil particles is \( n_r \), and the differential scattering cross section in the laboratory frame is \( d\sigma / d\Omega_{\text{lab}} \), then the number of scattered recoils, \( dN \), produced within this differential volume that reach the detector is given by

\[ dN = n_r I dV \frac{d\sigma}{d\Omega_{\text{lab}}} d\Omega_{\text{det}} \]  \hspace{1cm} (6)

Now, if the detector has a flat surface of area \( A_D \), normal to the incident neutron direction, then

\[ d\Omega_{\text{det}} = \frac{A_D \cos \theta}{r^2} \]  \hspace{1cm} (7)

where \( r \) is the distance from the detector to \( dV \). The \( \cos \theta \) term takes into account the projection of the area \( A_D \) in the direction of the foil volume element.

The volume element \( dV \) is defined in the spherical coordinate system centered at the detector and is given by

\[ dV = r^2 \sin \theta d\theta d\phi dr \]  \hspace{1cm} (8)

Now, if the foil has a thickness, \( \delta \), which is very small compared to the scale of the entire foil, then, by using the shape of the ellipsoid, as given by Eq. 5, it can be shown that
\[ dr = \delta \left( 1 + \tan^2 \left( \frac{1 + \zeta \cos^2 \theta}{1 - \zeta \cos^2 \theta} \right)^{1/2} \right) = \delta S(\theta) \] (9)

where the shape factor \( S(\theta) \) carries the information about foil geometry.

The differential cross section in the laboratory frame is related to the differential cross section in the center of mass frame by:

\[
\frac{d\sigma}{d\Omega_{\text{lab}}} = 4 \cos \theta \frac{d\sigma}{d\Omega_{\text{CM}}} (10)
\]

where the scattering angle in the center of mass frame, \( \gamma \), is simply twice that in the laboratory frame, or, \( \gamma = 2\theta \).

Using Eqs. 7 - 10 in Eq. 6, and, integrating over \( 0 < \phi < 2\pi \) and \( \theta_{\text{min}} < \theta < \theta_{\text{max}} \), gives the total number of recoils reaching the detector:

\[
N_{\text{tot}} = n_\tau I_A \delta \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} 8\pi \cos^2 \theta \sin \theta S(\theta) \frac{d\sigma}{d\Omega_{\text{CM}}} d\theta (11)
\]

This allows the total number of recoils, for a given fluence of monoenergetic neutrons, to be determined, provided that \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \) are specified. The values of \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \) would be fixed by constructing only the portion of the foil ellipsoid between these angles.

To determine the practical values of \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \), a number of factors must be considered, including the following:

1. The recoil yield is raised by increasing the difference between \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \).
2. The spread in recoil energies is reduced by decreasing the difference between \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \).
3. For large values of \( \theta_{\text{max}} \), low energy recoils can reach the detector; however, such low energy ions will incur significant energy losses while escaping from the foil. This puts a practical upper limit on \( \theta_{\text{max}} \).
4. A neutron shielding plug should be installed to block the immediate line-of-sight between the detector and the target. This would place a lower limit on \( \theta_{\text{min}} \).

To investigate factors 1 and 2, the recoil yield distribution with angle, \( dN/d\theta \), and with energy, \( dN/dE \), will be calculated. The angular distribution is determined simply by differentiating Eq. 11 with respect to \( \theta \), giving:

\[
\frac{dN}{d\theta} \sim \cos^2 \theta \sin \theta S(\theta) \frac{d\sigma}{d\Omega_{\text{CM}}} (12)
\]

The energy distribution is given by \( dN/dE = (dN/d\theta)/(dE/d\theta) \). Now the energy of the recoils is determined from Eq. 2 and is given by

\[
E = \frac{4A}{(1 + A)^2} E_n \cos^2 \theta (13)
\]

\[FIG. 3. \text{Recoil deuteron angular and energy distributions after scattering by 14.1-MeV neutrons. (a) Differential angular yield of recoils for various angles as given by Eq. 12. (b) Energy distribution of recoils for scattering angles from 0 - 60°.}\]

Dividing Eq. 12 by the differential of Eq. 13 (with respect to \( \theta \)) gives:

\[
\frac{dN}{dE} \sim \cos \theta S(\theta) \frac{d\sigma}{d\Omega_{\text{CM}}} (14)
\]

Note that this is simply the well-known energy spectrum of knock-on deuterons (given by \( d\sigma/d\Omega_{\text{CM}} \)) modified by the geometric factors \( \cos \theta \) (the projection of the detector plane) and \( S(\theta) \) (the shape factor of the foil).

For 14.1-MeV neutrons, \( dN/d\theta \) and \( dN/dE \) are shown in Fig. 3.

From these distributions, it appears that a reasonable value for \( \theta_{\text{max}} \) is given naturally by the low energy dip in the cross-section at \( \sim 10 \text{ MeV} \), corresponding to an angle of \( \sim 25° \). This means, if only deuterons scattered between \( 0 < \theta < 25° \) reach the detector, they will have a relatively narrow energy spread between 10 - 12.5 MeV (neglecting energy losses incurred during escape from the foil).

The calculation of total recoil yields can now be performed. Using \( 0 < \theta < 25° \), the integral in Eq. 11 (which is the effective scattering cross section) is calculated to be 0.135 barns.

Now a polyethylene (CH₄) scattering foil with a density of 0.93 g/cm², has a number density of hydrogen nuclei of \( 8 \times 10^{22} \text{ cm}^{-3} \). In this calculation, it will be assumed that a deuterated foil (CD₂) has the equivalent
TABLE I. Sample calculation for the number of deuteron recoils from a D-T shot with a yield of \(10^{13}\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-T Yield</td>
<td>(10^{13})</td>
</tr>
<tr>
<td>Foil / Detector distance from target</td>
<td>160 cm</td>
</tr>
<tr>
<td>Number density of deuterons in foil (CD2)</td>
<td>(8 \times 10^{22}) cm(^{-3})</td>
</tr>
<tr>
<td>Foil Thickness</td>
<td>50 (\mu)m</td>
</tr>
<tr>
<td>Detector Area</td>
<td>1 cm(^2)</td>
</tr>
<tr>
<td>Accepted recoil scattering angles</td>
<td>0 - 25(^\circ)</td>
</tr>
<tr>
<td>Energy spread of recoils (no ranging)</td>
<td>10 - 12.5 MeV</td>
</tr>
<tr>
<td>TOTAL NUMBER OF RECOILS</td>
<td>1700</td>
</tr>
</tbody>
</table>

number density of deuterons. A foil thickness of \(d = 50\) \(\mu\)m will be used - a 10 MeV deuteron would lose approximately 0.5 MeV in traversing a 50 \(\mu\)m foil.

The flux of neutrons is given by \(I = Y/(4\pi R^2)\), where \(Y\) is the yield of neutrons and \(R\) is the distance of the foil from the target. A 14.1-MeV neutron yield of \(10^{13}\) and a foil distance of 100 cm (the OMEGA target chamber radius) will be assumed.

Using all these values, along with a detector area of \(A_D = 1\) cm\(^2\), in Eq. 11 gives a total recoil yield for a \(10^{13}\) shot of 1700 recoils. This yield of 1700 recoils should be sufficient to generate a spectrum using a current-mode TOF detector. These numbers are summarized in Table I. Although these numbers will not be sufficient to determine fuel \(\rho R\) on OMEGA using secondary neutrons, they are adequate to measure the primary D-T yield as well as the fuel ion temperature. Recoil spectrometers have the advantage over standard scintillators and activation counters in that they provide an absolute measure of neutron yield. The higher yields on the future National Ignition Facility (NIF) will enable the lower yield secondary particles to be detected.

On the same shot, the number of direct neutron noise events is given by \(N_n = I_{\text{neut}} A_D\sigma_{\text{det}}\Delta_{\text{det}}\), where \(n_{\text{det}}\), \(\sigma_{\text{det}}\), and \(D_{\text{det}}\) are the number density, neutron cross section, and sensitive depth of the detector respectively. Thus, the signal-to-noise ratio is given by

\[
\frac{N}{N_n} = \frac{n_{\text{det}}\sigma_{\text{eff}}\delta}{n_{\text{det}}\sigma_{\text{det}}\Delta_{\text{det}}}
\]

where \(\sigma_{\text{eff}}\) is the effective recoil scattering cross section calculated above - 0.135 barns for \(0 < \theta < 25^\circ\).

Now, if a silicon detector is used, \(\sigma_{\text{det}}\) would be approximated by the total neutron interaction cross section for silicon (\(\sim 1.5\) barns). If the sensitive depth is \(\sim 20\) \(\mu\)m and \(n_{\text{det}} = 5 \times 10^{22}\) cm\(^{-3}\) (the number density for silicon), the signal-to-noise ratio would be \(\sim 0.4\). A second detector, without the scattering foil, would need to be run concurrently in order to generate a background spectrum which can be subtracted.

Signal-to-noise will be increased by utilizing a narrow, neutron shielding 'plug' along the line-of-sight between the detector and the target, sacrificing a small number of forward scattered deuteron recoils. This plug would extend from the apex of the foil (at \(\theta = 0\)) out towards the target for several tens of cm. Note that the signal-to-noise can be improved further for higher energy neutrons since thicker foils (greater than the 50 \(\mu\)m used in this calculation) can be used - the higher energy deuterons being able to pass through more foil material with less energy loss. This is important if the spectrometer is to be used to measure tertiary neutrons on future, high-yield experiments on the NIF.

IV. CONCLUSION

The conceptual design for a deuteron recoil time-of-flight spectrometer has been presented. This design uses a scattering foil in the shape of an ellipsoid that increases the number of expected recoils above that of other proposed designs while still maintaining the time-of-flight dispersion originally present in the neutrons. Such a design works only for recoils heavier than protons. By utilizing a foil which allows deuteron recoils scattered by angles from 0 - 25\(^\circ\), the energy spread of deuterons is from 10 - 12.5 MeV. For a D-T yield of \(10^{13}\), approximately 1700 recoils, generated by the primary neutrons, would be expected, allowing an absolutely calibrated measurement of primary yield, as well as core ion temperature, on current experiments at OMEGA.

V. ACKNOWLEDGEMENTS

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