Fluid damping with elastic medium
in 3-D Printing Process

by

Jorge A. Broggio

Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of

Bachelor of Science

at the

Massachusetts Institute of Technology

May 1998

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ABSTRACT

The fluid recirculation system that feeds the inkjet printhead in 3-D Printing utilizes a
peristaltic pump. The peristaltic pump imparts periodic flow and the pulsations in flow rate and
pressure affect the inkjet printhead stability. The work herein contained addresses the design and
testing of a capacitance-resistance damper used to minimize the effect from these pulsations.

The capacitance-resistance damper is fabricated using rubber (typically latex) tubing for
capacitance elements and orifices as resistors. This system was found to be experimentally
effective in damping out the pulsations from the peristaltic pump. A series of experiments
circulating carrier fluid through three different types of tube materials, rubber, silicone, and PVC,
for different pressures and flows of the circulating carrier fluid, length and thickness of the pipes,
were performed to determine the damping behavior of the three different materials. The Young’s
modulus of each material was tested experimentally. Modeling the damper as a capacitance
device an inverse correlation between the damping capacity and the Young’s modulus was
observed. Observations also verified a direct correlation of damping capacity with tube length
and diameter. Observations of flow variations with strobe photography defined the criteria for
low and high capacity damping.

The effects of orifices were tested on an experimental damper. Orifices were intercalated
to disrupt laminar flow, increasing the Reynold’s number, the resistance, and the damping
capacity. The orifice diameter was observed to be inversely proportional to damping capacity.
The number of orifices placed in series was observed to be directly proportional to damping
capacity.

Finally, the integrated damping effect of the RC device was observed to be directly
proportional to resistance and capacitance. Significant steps were taken towards finding an
optimal solution for a given peristaltic pump. The length, and inner diameter in the capacitor, the
diameter and number of orifices in the resistor, and the Young’s modulus of the pipe material in
the capacitor were interrelated variables for damping success.

The beginnings of an analytical treatment of this resistance capacitance network were
made by deriving expressions for the capacitance of a section of tubing, the resistance of an
orifice, and the resistance of a section of tubing. Future work can use these analytical expressions
to design dampers better matched to specific requirements.

Thesis Supervisor: Emanuel Sachs
Title: Professor of Mechanical Engineering
ACKNOWLEDGEMENT

I am grateful to the many professors at MIT who have dedicated their time to the academic disciplines that they enjoy. I have learned much from sharing experiences with them.

I thank the many other people in elementary and secondary school who through education have attempted to share knowledge from their field of expertise with me.

Finally, I acknowledge the many other people who have helped me.

Jorge A. Broggio
Cambridge, May 18, 1998
BIOGRAPHY

Born in Buenos Aires, Argentina, on the twentieth of October of 1975, Jorge Antonio Broggio spoke Spanish from his infancy. His family left Buenos Aires when Jorge had two years of age and resided in Caracas and Valencia, Venezuela, for a span of one year. When he was four, Jorge’s family began to live in Zaragoza, Spain, and placed him in a preschool where he began to learn English.

Elementary school for Jorge began the next year at The American School of Zaragoza. He attended this school for four years. For his fourth grade year Jorge attended the elementary school at the U.S. Air Force base in Zaragoza.

By the fall of his fifth grade year Jorge’s parents were living in Mansfield, Ohio, U.S.A. Jorge attended the Lexington School Systems between the ages of nine and seventeen. He graduated from Lexington High School in June of 1993.

Jorge then attended MIT and will receive a Bachelor’s degree in Mechanical Engineering and Chemistry with a minor in Biology in June of 1998.
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INTRODUCTION

Manufacturing has developed over a broad range of time. Recently, the industrial revolutions have provided an environment that allowed manufacturing to grow rapidly. Many different types of processes have been created and the three-dimensional printing process is distinguished among these for the uniqueness and precision of its products.

The machine schematic shown in Figure 1 is used in the three-dimensional printing process. The machine has degrees of freedom along the three coordinate axes. An air bearing is used for the horizontal fast axis marked as x. A slow geared mechanism is used for the horizontal slow axis marked y. The vertical, z, axis is controlled by a piston that lowers the printed surface. The fluid is delivered to the printer head through a series of tubes called the fluid system.

![Diagram of 3-D Printing Machine Schematic](image1)

**Figure 1.** 3-D Printing Machine Schematic.
The fluid system is diagramed in Figure 2 for a non-re-circulating system. This system does not require a fluid damper since the pressure vessel and its pressure regulator isolate the fluid flowing through the discharge nozzle from the fluctuations in pressure and displacement produced by the pump. In this case the design requirement of interest will be to determine the best way to refill the pressure vessel.

Figure 2. Non-Re-circulating Fluid Delivery System for 3-D printing.
The fluid system shown in Figure 3 consists of a loop of re-circulating fluid kept at a constant pressure by a pressure vessel. The re-circulation becomes a design requirement when sedimentation of colloidal printing fluids disrupts the desired characteristics of the fluid at the nozzle. Furthermore, a new design requirement develops from having to re-circulate the fluid with a pump, which causes inconsistencies in flow rate and pressure, while the desired output from the nozzle must be at a constant volumetric flow rate and constant pressure. To attain this requirement an in-line fluid damper must be located between the pump and the nozzle.

![Figure 3. Re-circulating Fluid Delivery System for 3-D printing.](image)

The in-line fluid damper must reduce the magnitude of pressure and flow rate variations along the moving stream of fluid that will exit through the nozzle. The energy from these oscillations must be redirected to level spikes or depressions in pressure and flow rate. An ideal situation would incorporate a capacitor parallel to the flow that would...
absorb and compensate flow rate spikes and depressions respectively. This situation must also incorporate a constriction of flow that induces a counteracting pressure or dissipates the energy from any increases or decreases in pressure as the pump delivers them. The constriction used in this study dissipates energy instead of storing it. The in-line alternating arrangement of a series these capacitors and resistors ensures steady flow. The capacitor is a tube that transfers energy as its walls flex parallel to flow. These walls are typically made of an elastic material such as latex rubber. The resistor is a singularity inserted into the flow in the form of an orifice. This orifice is made by constricting the tube wall with a round clamp around the outside of the wall.

The three-dimensional printing process works by placing a layer of material on a plane and then adding a second substance on selected points of that plane determined by a graphics file specifying the shape of the part. Upon mixing, the two substances become hard, and one horizontal plane of the part is made. The machine pauses to verify that everything is working as expected and then proceeds to print another horizontal plane. The summation of all the horizontal planes in the z-direction results in a finished part. The flow of operations is diagrammed in Figure 4.

Figure 4. Process Flow Diagram.
The machine must meet certain design requirements to be able to print parts to specification. These design requirements are shown in Figure 5 along with the customer specifications that they are intended to fulfill.

<table>
<thead>
<tr>
<th>Pump type</th>
<th>Pump size</th>
<th>Damper type</th>
<th>Hoses</th>
<th>Filters</th>
<th>Drying Aid</th>
<th>Drop deflection</th>
<th>Bearings</th>
<th>Recirculating</th>
<th>Non-Recirculating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allows easy switch of material</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
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<td>Even deposition of material</td>
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<td>Fast polymerization</td>
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<td>Precision</td>
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<td>Print data files</td>
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**Figure 5.** Needs-Metrics Matrix for 3-D Printing.

As can be seen the flow from the nozzle must have flow and pressure characteristics that are as close to constant as possible. The quality of the finished product, part being printed, will be better when the output of the nozzle has the same magnitude for every point on the printed plane making every building block of the final product the same size. To achieve this goal a damper must be used to level the flow and pressure characteristics of the fluid system in the region before the nozzle. A grid showing the product, flow out of the damper, specifications, and the correlating design requirements of the damper is displayed in Figure 6.

<table>
<thead>
<tr>
<th>pump</th>
<th>hose type</th>
<th>hose diameter</th>
<th>restriction diameter</th>
<th>number of restrictions</th>
<th>hose length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant P</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant Q</td>
<td></td>
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</tbody>
</table>

**Figure 6.** Needs-Metrics Matrix for Damper.
Background:

The fluid damper made according to the specifications discussed above meets its requirements through the application of proved theories about fluid flow and material behavior. The Young's Modulus, Poiseuille flow, flow through an orifice, the deformation of solids, and filter theory lead to the design of a successful damper.

The Young's Modulus defined by Thomas Young defines the elastic deformation, strain, of a material to be a first order variable of stress or vice-versa from the moment when a stress or strain is applied until the yield stress or yield strain is reached for certain materials. The typical experimental setup for the determination of the Young's Modulus of a material is shown in Figure 7.

![Diagram showing determination of Young's Modulus](diagram)

**Figure 7.** Determination of Young's Modulus.

The Young's Modulus explains how a material will deform or strain when a force or stress is applied to the material. The way in which a material deforms depends upon the geometry of the material and the mode of application of the force. For the case of a cylindrical pressure vessel such as a pipe when the pressure on the inside is larger than
that on the outside, the pressure acts evenly throughout the inner surface area of the pipe and the strain occurs along the thickness and length of the pipe. A free-body diagram of the pipe along these two directions is shown in Figure 7.1

**Figure 7.1.** Free Body Diagram of the Longitudinal and Cross-Sectional Forces Acting on a Cylindrical Pipe.

This diagram can be used to determine the deformation that a pipe undergoes when it has a pressure applied along the inner surface. The force balances in each free body diagram from the above figure yield the stress in the longitudinal and the angular directions.

\[ \text{LPD} = \sigma_2 \cdot 2tL \]

\[ \frac{PD}{4} = \frac{\pi}{4} \cdot \sigma_1 \cdot t \cdot D \]

\[ \sigma_1 = \frac{PD}{4t} \]

\[ \sigma_2 = \frac{PD}{2t} \]
The stress in the radial direction is zero. In Figure 7.2 equations relating stress to strain in Cartesian and in cylindrical coordinates are shown. From these equations the strain can be determined in the direction of each of the two stresses.

\[
\varepsilon_x = \frac{\sigma_x}{E} - \left( \frac{v}{E} \right) \cdot (\sigma_y + \sigma_z)
\]

\[
\varepsilon_y = \frac{\sigma_y}{E} - \left( \frac{v}{E} \right) \cdot (\sigma_z + \sigma_x)
\]

\[
\varepsilon_z = \frac{\sigma_z}{E} - \left( \frac{v}{E} \right) \cdot (\sigma_x + \sigma_y)
\]

**Figure 7.2.a.** Strain equations.

\[
\sigma_x = \frac{E}{(1 + v) \cdot (1 - 2v)} \left[ (1 - v) \cdot \varepsilon_x + v \cdot (\varepsilon_y + \varepsilon_z) \right]
\]

\[
\sigma_y = \frac{E}{(1 + v) \cdot (1 - 2v)} \left[ (1 - v) \cdot \varepsilon_y + v \cdot (\varepsilon_z + \varepsilon_x) \right]
\]

\[
\sigma_z = \frac{E}{(1 + v) \cdot (1 - 2v)} \left[ (1 - v) \cdot \varepsilon_z + v \cdot (\varepsilon_x + \varepsilon_y) \right]
\]

**Figure 7.2.b.** Stress equations.
The change in volume for the applied pressure can now be determined when $v$ is one half.

$$
\varepsilon_2 = \frac{3}{8} \frac{PD}{Et}
$$

$$
\Delta A = \pi D \frac{\varepsilon_2 P}{2}
$$

$$
\Delta V = \Delta A \cdot L
$$

$$
\Delta V = \frac{3}{16} \pi L D^3 \frac{\Delta P}{Et}
$$

So the change in volume can be divided by the change in pressure to yield the capacitance.

$$
C = \frac{\Delta V}{\Delta P} = \left( \frac{3}{16} \right) \pi L D^3 \frac{1}{Et}
$$

The viscosity of a fluid as defined by Jean Leonard Marie Poiseuille is found by locating a fluid between two parallel planes of a solid and applying parallel force vectors in opposite directions on the plane and perpendicular to the normal of each plane. The experimental setup is shown in Figure 8. The area of the solid plane and the distance separating the two planes will determine at what speed the two planes move relative to each other. The viscosity is defined as a function of force, area, relative speed, and distance between planes. Permutating the number of the dependent variables(s) from one to four across the four variables while maintaining the other variable(s) in each of the
thirteen cases independent a set of data points that govern the interrelation of these four properties can be collected. This set of data points can be used to define the mathematical relation that couples each of these variables to each other. Viscosity is the product of the magnitude(s) of some of the variable(s) and the reciprocal of the magnitude(s) of some of the variable(s). Viscosity has been found to be the product of the force, the distance separating the plates, the reciprocal of the area, and the reciprocal of the relative speed.

$$\eta = \frac{FH}{V_1 A}$$

![Diagram of experimental setup](image)

**Figure 8.** Experimental Setup to Determine Viscosity.
The Navier-Stokes equations in cylindrical and Cartesian coordinates are shown in Figure 8.1 and 8.2. They equate the forces acting on a volume element.

\[
\rho \frac{Dv_x}{Dt} = \frac{-\delta P}{\delta x} + \eta \left( \frac{d^2 v_x}{dx^2} + \frac{d^2 v_x}{dy^2} + \frac{d^2 v_x}{dz^2} \right) + \frac{1}{3} \eta \frac{d}{dx} (\text{Grad} \cdot v) + \rho g_x
\]

\[
\rho \frac{Dv_y}{Dt} = \frac{-\delta P}{\delta y} + \eta \left( \frac{d^2 v_y}{dx^2} + \frac{d^2 v_y}{dy^2} + \frac{d^2 v_y}{dz^2} \right) + \frac{1}{3} \eta \frac{d}{dy} (\text{Grad} \cdot v) + \rho g_y
\]

\[
\rho \frac{Dv_z}{Dt} = \frac{-\delta P}{\delta z} + \eta \left( \frac{d^2 v_z}{dx^2} + \frac{d^2 v_z}{dy^2} + \frac{d^2 v_z}{dz^2} \right) + \frac{1}{3} \eta \frac{d}{dz} (\text{Grad} \cdot v) + \rho g_z
\]

**Figure 8.1.** Navier-Stokes in Cartesian Coordinates.

\[
\rho \left[ \frac{\delta v_r}{\delta t} + v_r \frac{\delta v_r}{\delta r} + \frac{v_\theta}{r} \frac{\delta v_r}{\delta \theta} + \left( \frac{v_\theta}{r} \right)^2 + \frac{\delta v_r}{\delta r} \right] =
\]

\[
\frac{-\delta P}{\delta r} + \eta \left[ \frac{\delta}{\delta r} \left( \frac{1}{r} \frac{\delta v_r}{\delta r} \right) + \left( \frac{\delta^2 v_r}{\delta r^2} \right) - \frac{2}{r^2} \left( \frac{\delta v_\theta}{\delta \theta} \right) + \frac{\delta^2 v_r}{\delta z^2} \right]
\]

**Figure 8.2.** Navier-Stokes in Cylindrical Coordinates

These equations can be solved for a fluid flowing through a pipe by considering the boundary conditions that act on the fluid. In cylindrical coordinates, the velocity in the z direction is a function of only radius, and the velocities in the theta and radial
directions equal zero for a well-developed flow. As a result the radial and angular components of the Navier-Stokes equations are non-critical, while the z component is critical for describing the forces acting on a volume element of the fluid as it flows through a cylindrical pipe.

\[
0 = -\frac{dP}{dz} + \left( \eta \frac{d}{r} \right) \left( r \frac{d}{dr} \frac{dz}{d\theta} \right)
\]

Pressure does not vary with time along the radial or angular coordinates, so the first term in the above equation does not vary along the radial or angular coordinates and only varies with the z coordinate. The velocity in the z direction for a well developed flow through a pipe does not vary along the z or the theta coordinates and only varies along the radial coordinate, so the second term in the above equation only varies with the radial coordinate. Since the terms can be rearranged to equal each other, they depend on different variables, and the radial coordinate boundary does not change because the diameter of the pipe remains constant, and the change of velocity in the z direction is related linearly to the change of pressure, then the two terms in the above equation are constant and equal to each other.

The above simplification of the Navier-Stokes equations can be multiplied by \( r \), integrated over the radial coordinate from zero to one half of the pipe diameter, and divided by \( r \).

\[
\frac{d}{dr} v_z = \left( \frac{1}{2\eta} \right) \left( \frac{\Delta P}{L} \right) r
\]
The integration constant that is introduced can be enumerated when a radial coordinate value of zero is introduced into the equation since the left side of the equation equals zero when the radial coordinate equals zero. This integration constant therefore equals zero. Now another integration over the radial coordinate from zero to one half of the pipe diameter yields the velocity in the $z$ direction.

$$v_z = \frac{1}{4 \eta} \left( \frac{\Delta P}{L} \right) r^2 + C_2$$

This second integration produces a constant that can be determined by introducing a value for the radial coordinate that enumerates the distance from the centerline of the flow to the outer edge of the flow, because the velocity in the $z$ direction equals zero at the boundary of the liquid against the wall of the pipe.

$$C_2 = \frac{-1}{4 \eta} \left( \frac{\Delta P}{L} \right) r^2$$

$$v_z = \frac{R^2}{4 \eta} \left( \frac{-\Delta P}{L} \right) \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

The average velocity in the $z$ direction can be obtained by integrating this equation over area and dividing by the total area.

$$v_{zav} = \frac{1}{\pi R^2} \int v_z \, dA = \frac{R^2}{8 \eta} \frac{-\Delta P}{L}$$
The volumetric flow rate of the fluid as it travels through a length of pipe from a high pressure to a low pressure is obtained by multiplying the above equation with the area of the cross-section through which the flow moves.

\[ Q = \left( \frac{\pi}{128} \right) \left( \frac{\Delta P}{\varphi} \right) \frac{D^4}{\eta} \]

The flow rate described above is the expected flow rate when the fluid moves through a pipe. The modification of the Navier-Stokes equations that is necessary to derive the above volumetric flow rate occurs because of the constraints that are imposed on the fluid by its surroundings. When the fluid flows between two planes the Cartesian Navier-Stokes equations can be simplified by the use of physical constraints on the fluid shown in the experimental setup of Figure 9.

![Figure 9. Schematic Showing Conditions That are Easily Approximated with the Navier-Stokes Equations in Cartesian Coordinates.](image)

The experimental setup displayed in Figure 9 resembles the setup used to define the viscosity of a fluid. The highly constrained fluid flow that is possible in this idealized
setup allows the determination of viscosity with a small set of data from four variables. Due to the setup these four variables are the important ones as long as the fluid is Newtonian and the viscosity is linear with respect to any one of these variables. The volumetric flow rate is equal to the speed of the fluid times the area of the cross-section over which the fluid travels. The cross-sectional area will depend on the area of the parallel plates and the distance separating them. The velocity component of the volumetric flow rate has the same value as the speed component in the experiment to determine viscosity. So, volumetric flow rate must be a function of at most four of the five variables or dimensions shown in Figure 9, and one of these variables can not be speed or bulk velocity of flow.

![Diagram](image)

**Figure 10.** Schematic Showing Conditions That are Easily Approximated with the Navier-Stokes Equations in Cylindrical Coordinates.

When the fluid flows inside a pipe the cylindrical Navier-Stokes equations can be simplified by use of the physical constraints on the fluid shown in the experimental setup of Figure 10. The setup is analogous to that in Figure 9 and consequently to the setup used to determine viscosity. The radial direction of Figure 10 corresponds to the x direction of Figures 9 and 8. Dimensionally, the diameter of the pipe shown in Figure 10
is analogous to the distance between the plates of Figures 9 and 8. The lengths of the experimental section along the z coordinate for the three setups of Figures 9, 10 and 8 are also analogous. Similarly, the angular coordinate of Figure 10 is analogous to the y coordinate of Figures 9 and 8 where traversing all the angles is the same as moving from one end of the parallel planes to the other in a direction perpendicular to the flow direction. Consequently, the velocity will be a function of the other four variables in Figure 10 and the volumetric flow rate may be a function of all four but it must be a function of force and length.

The Reynolds number is obtained by dividing the inertial forces on a fluid element by the viscous forces on that element. Figure 11 is a schematic for this gedanken experiment where the forces and relevant dimensions are labeled. The Reynolds number is also shown as it relates to the inertial and viscous forces.

\[ F_{\text{inertial}} = \frac{mv^2}{\Delta l} \]

\[ F_{\text{viscous}} = \frac{\eta v A}{\Delta l} \]

\[ Re = \frac{F_{\text{inertial}}}{F_{\text{viscous}}} \]
Figure 11. Reynolds Number in Relation to Viscous and Inertial Forces.

The inertial terms will dominate when the Reynolds number approaches infinity and fluid flow will exhibit high inductance and low resistance. The viscous terms will dominate when the Reynolds number approaches zero and the fluid flow will exhibit low inductance and high resistance. At intermediate values for the Reynolds number the fluid flow will have a lower force acting against the force that moves the fluid while at the very low Reynolds number frictional forces will act against the fluid's driving force and at very high values of the Reynolds number inertial forces act against the changes of force in the fluid's driving force.

These idealizations of the forces on a fluid element become important not only when the force on an infinitesimal volume element is considered such as it is in the Navier-Stokes equations, but the balance between viscous and inertial forces also becomes important when the energy entering and leaving a volume element is considered.
such as it is in the Bernoulli equation. Figure 12 shows a control volume used to determine the engineering Bernoulli equation by balancing the energy changes that act on the control volume. To determine the engineering Bernoulli equation the internal energy is presented in thermodynamic terms.

![Diagram](image)

**Figure 12.** Control Volume for the Bernoulli Equation.

\[
\begin{align*}
(\rho (K\text{E} + I\text{E} + P\text{E}) \, V_A_1 - \rho (K\text{E} + I\text{E} + P\text{E}) \, V_A_2 + \rho vA_1) - \rho vA_2 + \frac{d}{dt} \dot{Q}_H + \frac{d}{dt} \dot{W}_s \\
\end{align*}
\]

\[
\begin{align*}
\frac{d}{dt} \int_{z_1}^{z_2} \rho A (I\text{E} + K\text{E} + P\text{E}) \, dz
\end{align*}
\]
In the limiting cases when the Reynolds number approaches zero and infinity the engineering Bernoulli equation can be rewritten to yield the order of pressure required to generate the flow.

\[ \Pi_i = \rho v^2 \]

\[ \Pi_v = \frac{\eta v}{L} \]

The Navier-Stokes equations can be rewritten non-dimensionally in terms of the Reynolds number and the Strouhal number, and then the equation can be simplified for viscous dominated flow and inertial dominated flow.

\[ \text{Re} \left( \text{Sr}^{-1} \frac{\delta v}{\delta t} + v \cdot \text{Grad} v \right) = \left( -\frac{\Pi L}{\eta v} \right) \text{Grad} P + \text{Grad}^2 v \]

\[ \text{Re} \left( \text{Sr}^{-1} \frac{\delta v}{\delta t} + v \cdot \text{Grad} v + \text{Grad} P \right) = \text{Grad}^2 P \]

Inertially dominated

\[ \text{Re} \left( \text{Sr}^{-1} \frac{\delta v}{\delta t} + v \cdot \text{Grad} v \right) = -\text{Grad} P + v \]

Viscous dominated
From the Navier-Stokes equations for viscous dominated and inertial dominated flow, the functional relationship between pressure and volumetric flow rate can be determined for the cases in which the Reynolds number is both very high and very low. Figure 13 shows the physical conditions that must be implemented to create both a very high and a very low Reynolds number in a fluid that flows in a given direction.

![Diagram showing inertial and viscous effects](image)

**Figure 13.** Inertial vs. Viscous Effects Determined by Reynolds Number.

The degree of impedance against a driving force will be determined by the absolute and relative magnitudes of the two physical processes shown in Figure 13. The actual physical implementation of an impedance device may or may not be easily modeled as a pure resistance, Reynolds number approaches zero, or as a pure inductance, Reynolds number approaches infinity. In fact if the unidirectional flow approximation is not maintained the above idealizations will not hold true. The unidirectional flow constraint demands that the velocity of the fluid in the dimensions defining the cross-
section of the flow be zero at all times. Deviations from this constraint are counterproductive because the fluid no longer travels in the desired direction.

However, when resistance is the desired mode of impedance such deviations at the microscopic level will be welcome. Indeed at the microscopic level when the Reynolds number approaches zero, the molecular fluctuations of the fluid in question will cause the bulk velocity to no longer be unidirectional since bulk molecular fluctuations tend to cause an equal displacement in every direction over the time intervals of interest. So the resistance created in a fluid as the Reynolds number approaches zero originates from the violation at the microscopic level and ensuing macroscopic level of the unidirectional flow condition. The conditions necessary for a low Reynolds number to exist are detrimental to the goal of the three-dimensional printing process. Thus, a balance exists between primarily inductive impedance occurring when the Reynolds number is large and the flow is unidirectional and its resistive counterpart.

The difficulty in creating an inductance versus the ease with which a resistance can be imposed in a flowing fluid lead to the use of resistance as a source of impedance. A small orifice provides an impedance that can be modeled as a resistor when the inductance effects are ignored. Figure 14 shows such an orifice and the fluid flow characteristics downstream of it. The resistance is created by the zone of turbulent flow that forms in the fluid.
Figure 14. Resistance at the Orifice Due to Viscous Effects.

When the flow through the orifice is assumed to be impeded only by the resistance in the fluid the value of that resistance can be determined by balancing the Bernoulli equation for energy changes in a volume element.

\[
\frac{1}{2} (v_2)^2 = \left(\frac{1}{2}\right) (v_2)^2 + \frac{p_1 - p_2}{\rho} - l_v
\]

The pressure at which the resistance is created equals the pressure before the resistance for the related equations. The relation of areas to velocities,

\[A v_1 = A_o v_2\]

the expression for losses,

\[l_v = \frac{1}{2} (v_2)^2 K\]
and an estimate for the value of $K$ for small orifices (see The Chemical Engineering Handbook),

$$K = 1.6$$

can be placed into the engineering Bernoulli equation to obtain the velocity at the orifice.

$$v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 + K)}}$$

Considering this flow velocity over the cross-section of the orifice gives the volumetric flow rate at the orifice.

$$Q = A_o \cdot v_2 = A_o \cdot \sqrt{\frac{2P}{\rho(1 + K)}}$$

The pressure in the resistor varies with the square of the flow rate as is shown in Figure 14.1

$$P(Q) = \frac{2.6 Q^2 \rho}{2 (A_o)^2}$$
Figure 14.1. Pressure as a Function of Flow Rate for an Orifice Type Fluid Resistance.

To determine the fluid resistance, the rate of change of pressure with a change in flow rate must be determined for the specific flow rate seen in the orifice at steady-state. A graph of the resistance, or the rate of change of pressure for a value of flow rate for an orifice with a small cross-section relative to pipe diameter is shown in Figure 14.2.

\[
R(Q) = 5.2 \frac{Q \rho}{2 (A_o)^2}
\]
Figure 14.2. Resistance as a Function of Flow Rate for an Orifice Type Fluid Resistance.

The derivative operator determines this rate of change accurately and the pressure equation determines the pressure accurately only at points where the flow rate is close to that seen in the orifice. For this reason pressure at the orifice and resistance for it must be determined using the exact solution of flow rate at the orifice. So the value of flow rate found at the orifice at steady-state used along with the orifice cross-section and the density of the fluid are used to determine the pressure and the resistance at those conditions.

The resistance and capacitance expressions developed for the fluid system damper are joined to create a low-pass filter that performs at a cutoff frequency by removing signals with frequencies higher than this cutoff frequency. The values of the resistor and capacitor used determine the frequency at which the low-pass filter works. Furthermore if n resistor in series with capacitor in parallel systems are placed in series the cutoff will
be much quicker. The quicker cutoff results from adding poles to the system. Figure 15 shows the electrical diagram of a one-resistor one-capacitor low pass filter and its Bode phase and magnitude plots.

\[ \begin{align*}
&\text{Circuit for RC Low Pass Filter} \\
&\text{Bode Magnitude Plot for RC Low Pass Filter} \\
&\text{Bode Phase Plot for RC Low Pass Filter}
\end{align*} \]

Figure 15. Electrical Circuit and Bode Plots for a Low-Pass RC Filter.
When four resistor capacitor setups are used instead of one the circuit and Bode phase and magnitude plots will be as those shown in figure 16.

Figure 16. Electrical Circuits and Bode Plots for a cascaded RC Low-Pass Filter.
The output voltage or pressure for the one resistor one capacitor setup is a function of the input voltage or pressure and its frequency.

\[ P_o = P_i \frac{1}{(1 + j\omega RC)} \]

The cutoff frequency can be defined in radians per second or in Hertz.

\[ \omega = \frac{1}{RC} \]

\[ f = \frac{1}{2 \pi RC} \]

The transfer function, magnitude, and phase of the frequency response are shown in the following equations.

\[ H(j\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC} = \frac{e^{j\arctan(\omega CR)}}{\sqrt{1 + (\omega CR)^2}} \]

\[ |H(j\omega)| = \frac{1}{\sqrt{1 + (\omega CR)^2}} \]

\[ \Phi(j\omega) = -\arctan(\omega CR) \]
Joining the resistance and capacitance concepts yields a damper that acts as a low pass filter. A one resistance with one capacitance damper along with a cascaded four resistance with four capacitance filter is shown in Figure 17.

![Diagram of damper](image)

**Figure 17.a.** One Resistance and One Capacitance Damper.

![Diagram of cascaded filters](image)

**Figure 17.b.** Cascaded Four Resistances and Four Capacitance Dampers.

The functioning of this filter depends on certain properties of the filter, which are tested in the experimental section of this work. The relation between the length, diameter and Young’s modulus of the capacitor section will be qualitatively assessed and established by determining the success of a filter that depends on the capacitance generated by these dimensions for success. Also, the relation between the restriction size and the number of restrictions with the effectiveness of a filter that uses resistance to stop high frequency pressure and volume variations will be qualitatively established. The
effectiveness of the joint operation of the capacitor and resistor will also be qualitatively established with the final determination of a successful low pass filter design to damp variations in pressure and volume that originate at the peristaltic pump being used by the three dimensional printing machine.
EXPERIMENTAL PROCEDURE

Experimental Setup 1

The experimental setup used to test the damper when it was not on the machine is shown in Figure 18. This setup is similar to the setup that the fluid experiences in the printing machine. One major difference exists between the experimental setup and the machine. The experimental setup does not include the motion effects that the machine has as the printer head swings back and forth quickly along the fast axis.

![Figure 18. Laboratory Setup](image)

The camera in the setup displays the output from the nozzle after it has formed into drops. This output shows well-developed drops that eventually hit the printed surface although in the setup presented the drops enter a beaker. The drop pattern over time for a good damper is shown in Figure 19. In this figure the drop pattern does not deviate by a large amount as time proceeds.
The output when a poor damper or no damper at all is used is shown in Figure 20. In this figure the drop pattern deviates by a large amount as time proceeds.

Figure 20. Time Sequence of Drop Patterns for a Low Damping Capacity Damper.

Test of the Capacitance Behavior of the Damper:

The capacitance unit of the filter was tested by use of the setup in Figure 18. A capacitor section was introduced into the fluid system of this setup in the region between
the pump and the damper. The capacitor was varied according to the experiment being performed.

A test was performed to determine the effect that the Young's Modulus has on the performance of a capacitor. A section of tubing made from natural rubber, and one of silicone was used as a capacitor. The pump was started and once the flow and drop formation at the nozzle were well developed the variation in the pattern of the drops was used to determine the success of the capacitor.

To relate this finding to the Young's modulus of each of the materials the experimental setup shown in Figure 21 was used to determine the Young's modulus of the hoses.

![Diagram of experimental setup](image)

**Figure 21.** Laboratory Setup for Young's Modulus Calculations.

The capacitors shown in Figure 22 were used to determine how the length of a capacitor affects the performance of that capacitor as a fluid damper. The capacitors
from Figure 22 were introduced into the fluid system between the nozzle and in the section between markers A and B in Figure 18. After the pump was started and flow and drop formations at the nozzle were well developed the variation in the pattern of the drops was used to determine the success of the capacitor.

![Diagram of damper with different lengths](image)

**Figure 22.** Damper With Different Lengths.

The diameter of the capacitor section was varied as shown in Figure 23 and the same procedure as that described for the variation of length was used to determine the effectiveness of the different capacitors.
Test of the Resistance Behavior of the Damper:

The capacitor in Figure 24 was inserted into the setup of Figure 18 between points A and B. The experiment was then completed with the same procedure as the experiment to determine the effect of the diameter of the capacitor section.

The effect of varying the size of the restriction was found by placing restrictions of different diameters into the system and following the same procedure as in the
determination of resistance effects on damping ability. Dampers with varying restriction sizes are shown in Figure 25.

Figure 25. Damper With Different Diameters of Restrictions.

Figure 26 shows a damper with more than one restriction on it. This damper was placed in the experimental apparatus of Figure 18 and the same procedure as for the determination of restriction size effects on damping ability was used to determine the effect of varying numbers of restrictions on the damping ability.

Figure 26. Damper With Multiple Restrictions.
Test of the Combined Effect, Capacitance and Resistance of the Damper:

Finally, the effects of a damper with both resistance and capacitance elements connected to make a low pass filter were tested on the three-dimensional printing machine. The experimental setup shown in Figure 27 was used. This setup resembles the setup displayed in Figure 18, and the major difference between the two setups is that the setup of Figure 26 includes the effects that printing motion has on the flow from the nozzle.
Figure 27. Strobe Photography Setup in 3-D Printing Machine.
RESULTS

The effectiveness of the damper was found to correlate inversely to the value of the Young’s modulus of the material used as a capacitor. Figure 28 shows this relationship.

![Graph showing the relationship between Young's modulus and damping capacity.](image)

Figure 28. Damping Capacity in Relationship to Young’s Modulus.
Below are values and graphs found from experimental analysis of the testing performed on the different hoses. These were used to find the Young’s Modulus for each material.

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<th>Rubber</th>
<th>Silicone</th>
<th>Tygon PVC</th>
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<td>275</td>
<td>4 3/16</td>
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<td>4 9/16</td>
<td>569</td>
<td>4 5/16</td>
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<td>4 7/8</td>
<td>823</td>
<td>4 ½</td>
</tr>
<tr>
<td>5 5/16</td>
<td>1102</td>
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</tr>
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<td>5 12/16</td>
<td>1328</td>
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<tr>
<td>6 ½</td>
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<tr>
<td>t=.0610”</td>
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<td>t=.0580”</td>
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Figure 28.1. Experimentally Determined Values for Young’s Modulus Calculations.

Figure 28.2. Experimental Young’s Modulus for Rubber.
Figure 28.3. Experimental Young’s Modulus for Silicone.

Figure 28.4. Experimental Young’s Modulus for PVC.
In contrast the effectiveness of the damper was directly proportional to the length of the capacitor section included in the damper. This proportionality is shown in Figure 29.

![Figure 29. Damping Capacity as a Function of the Length of the Hose.](image)

As was found for the length, the diameter of the capacitor was determined to affect damping ability in direct proportion to its magnitude. Figure 30 shows relation of the capacitor’s diameter to the damper’s damping ability.

![Figure 30. Damping Capacity as a Function of the Diameter of the Hose.](image)
The effectiveness of the damper also varied when a resistance element was added to the damper to complete the low-pass filter characterization. The filter’s effectiveness increased when a resistance in the form of an orifice that disrupted flow was added. This effect is shown in Figure 31.

Figure 31. Damping Capacity of a Hose Without Restrictions and With Restrictions
Interestingly, the size of the restriction was inversely proportional to the effectiveness of the damper. Figure 32 demonstrates this result.

\[ d = \frac{1}{6}''\]

**Figure 32.** Damping Capacity as a Function of the Diameter of the Restriction.

Furthermore, the number of resistances was directly proportional to the effectiveness of the damper. This result is shown in Figure 33.

\[ n = 3\]

**Figure 33.** Damping Capacity as a Function of the Number of Restrictions.
The damper was assembled by arranging a capacitance and resistance into the form of a low-pass filter. The effectiveness of the low pass filter as a damper for the three dimensional printing machine was found to be directly proportional to the values of resistance and capacitance. Figure 34 shows this proportionality.

Figure 34. Damping Capacity as a Function of Capacitance and Resistance.
DISCUSSION

The fluid damper studied in this system was found to have certain attributes that enhanced its performance and some that worsened it. Modeling the fluid damper system as a low pass RC filter describes the function of the system well. In accordance with the experimental findings and with the low pass filter theory the increase of the value of capacitance in the damper results in a better performance for the damper. Likewise improved performance was observed experimentally when the value of the resistor in the low pass filter idealization of the damper was increased. This result also agrees with the theory on low pass filters.

The equations for capacitance and resistance for fluid in the damper vary with the value of the variable values in these equations. The capacitance is dependent on the Young’s modulus, the length of the deformable section, the diameter of the deformable section, the thickness of the deformable section, and the pressure gradient exerted upon the capacitor surface. The resistance varies with the flow rate, the density of the fluid, and the cross-section of the fluid. These parameters for the resistance were varied by varying the size of the orifice and the number of orifices present.

The capacitance proved to vary as was expected in the equation describing capacitance excepting that no conclusive tests could be done on the thickness effect on capacitance.

The decrease in size of the restriction followed in a higher resistance. This result substantiates the directly proportional nature of the flow rate versus resistance that is expected from the equation for resistance. The same result as the diameter of the
restriction is decreased also substantiates the inverse correlation that is found in the equation for resistance between the resistance and the cross sectional area. No experiments were conducted to determine the veracity of the direct correlation found in the theoretical equation for the dependence of resistance on density of fluid.

To obtain better parameters for the design of a fluid damper the effects of the fluid should be considered when determining the value of the capacitance for the filter. Also, a distinction should be made between inductance and resistance in the resistor and the calculation of the filter’s aptitude should proceed from that finding. Different orifices should be tried to determine what type of orifice provides the best resistance. Furthermore the low pass filter concept should be used to optimize the damper’s operational flexibility and convenience.
CONCLUSIONS & RECOMMENDATIONS

Certain parameters that can vary during design affect the fluid damper's effectiveness. Because the damper is a low pass RC filter, the values of importance to obtaining the proper cutoff frequency are the values of the resistor and capacitor. Furthermore, the number of cascaded RC units determines the effectiveness with which a disturbance is damped at the cutoff frequency.

The resistor's effectiveness varies with the size and number of orifices. The capacitor's effectiveness varies with the Young's modulus of the tube material, the length of the tube, and the diameter of the tube. These criteria for resistor and capacitor effectiveness determine those for damper effectiveness.

The above conclusions were drawn from qualitative results. The same conclusions should be arrived at through quantitative measurements. Other considerations should also be evaluated through quantitative measurements. The type of fluid flowing through the system should be varied to observe its effect on the damper. Differently shaped orifices should be tried to observe their effect on the damper. Inductive impedance should be tried as a replacement for the resistive impedance currently used. An optimal damper should be experimentally developed by adjusting the above criteria after quantitative characterizations of the relationships between them.
BIBLIOGRAPHY


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