Fluctuations and State Preparation in Quantum Degenerate Gases of Sodium and Lithium

by

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Abstract

Ultracold atoms enable the precise study of novel systems where the correlations between particles are strong. These systems can be simple to describe yet impossible to efficiently simulate on a classical computer; understanding their behavior addresses fundamental questions in condensed-matter physics.

The first part of this thesis describes measurements of spin and density fluctuations in degenerate Fermi gases. We begin by presenting a proof-of-principle experiment that demonstrates how information about atomic fluctuations can be extracted from experimental images and used to measure the temperature of a noninteracting system. We then describe a new technique for measuring spin fluctuations that employs an effect analogous to optical speckle, using it to characterize the pair correlations in a strongly attractive Fermi gas. Finally, we use the methods we have developed to characterize the magnetic correlations of a Fermi gas with strong repulsive interactions on the upper branch of a Feshbach resonance, and show that, contrary to earlier experimental and theoretical predictions, this system does not undergo a ferromagnetic phase transition.

The second part describes the development of an apparatus for performing experiments with sodium and lithium in optical lattices. We describe progress towards the implementation of synthetic magnetic fields in systems of lattice fermions, which would enable the study of new topological phases. This includes the development of general precursors such a Bose-Einstein condensate and a stable Mott insulator of bosons, as well as more specific studies of heating and dynamical instabilities in tilted and shaken lattices.

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Chapter 1

Introduction

At the heart of low-energy physics are the ways in which complex systems result from simple rules. Ultracold atoms are, in many ways, simplicity itself - at temperatures low enough that the details of the interatomic potential become irrelevant, their behavior can be described as that of ideal quantum particles. By localizing them to the sites of an optical lattice, one converts their continuous degrees of freedom into discrete ones and makes things simpler still. This confinement also strengthens the interactions between particles so that they can no longer be considered independently, vastly increasing the complexity of the resulting system.

In part because they are so simple, the Hamiltonians describing cold atoms in optical lattices often resemble those used by theorists as qualitative 'toy' models of condensed-matter systems. For example, applying a deep optical lattice to fermionic atoms implements the Fermi-Hubbard model, which is believed to result in $d$-wave pairing and qualitatively model the behavior of the high-$T_c$ cuprate superconductors. These experiments can therefore be understood as a sort of quantum simulation, where one overcomes the difficulty in calculating the time evolution of a quantum system with a classical computer by modeling it instead with a second, tunable, quantum system [1, 2].
1.1 Digression: sign problems and quantum simulation

Cold-atom implementations of systems that are already well-understood are important in validating methods and building technical foundations for further work. But since the ultimate value of quantum simulations is based on their ability to realize systems whose behavior cannot be predicted using computers, we digress briefly to consider what sort of systems those might be [3, 4]—this material is general knowledge to theorists, but often unfamiliar to experimentalists.

Suppose we want to use a computer to model a classical system in thermal equilibrium, consisting of $N$ particles each with $m$ degrees of freedom. Then the number $\Omega$ of possible states of the system is $m^N$, and if we want to calculate a thermal average of some quantity $A$, that is:

$$\langle A \rangle = \frac{\sum_{c \in \Omega} A(c) \exp\left(-\frac{E(c)}{k_BT}\right)}{\sum_{c \in \Omega} \exp\left(-\frac{E(c)}{k_BT}\right)}$$

(1.1)

then we must sum over each of the $m^N$ configurations and it seems that the computational effort is exponential in $N$.

If this were the end of the story then perhaps there would be a large interest in classical, as well as quantum, simulations; sums of this type are ubiquitous in areas ranging from Bayesian inference [5] to financial engineering. Fortunately, there are Monte Carlo methods for approximating them in time polynomial in $N$. By constructing a sequence of samples $c_1, c_2 \cdots c_M$ from $\Omega$ with probability $p(c) = \exp(-E(c)/k_BT)$, we can make the approximation

$$\langle A \rangle \approx \frac{1}{M} \sum_{i=1}^{M} A(c_i)$$

(1.2)

with errors that scale as $1/\sqrt{M}$. There exists a large class of methods for generating these sequences $\{c_i\}$, of which the Metropolis-Hastings algorithm is probably the most
well-known.

So why are quantum systems harder? We might imagine that the problem stems from the fact that a quantum state consists of, instead of a point \( c \) in configuration space, a real number associated with all of those points, \( |\psi\rangle = \sum_{c \in \Omega} \alpha(c)|c\rangle \). While this is true, it does not necessarily imply worse scaling of the computational effort, at least in the case of thermodynamic observables. These can be written as a trace and expanded as a discrete path integral in imaginary time, approximating the continuous one in the limit where the number \( K \) of timesteps is large:

\[
\langle A \rangle = \frac{Tr(A \exp(\frac{-H}{k_B T}))}{Tr(\exp(\frac{-H}{k_B T}))} \tag{1.3}
\]

\[
Z = \sum_{c_1, c_2, \ldots, c_K \in \Omega} \langle c_1|1 - \frac{\beta H}{K}|c_2\rangle \cdots \langle c_{K-1}|1 - \frac{\beta H}{K}|c_K\rangle \langle c_K|1 - \frac{\beta H}{K}|c_1\rangle \tag{1.4}
\]

The resulting sum is over \( K \) copies of configuration space: instead of integrating over all possible configurations, we integrate over a set of 'worldlines', one of which consists of a configuration at each of \( K \) points in imaginary time. In other words, our quantum system in \( d \) dimensions is represented as a classical system in \( (d + 1) \), and we can apply the same Monte Carlo techniques employed for ordinary integrals.

There is, however, a major complication: in the quantum setting the weight of the configuration is not always positive. For example, if fermion worldlines permute as in Fig. 1-1, then that configuration gives a negative contribution to the sum contributing to \( Z \). The average then becomes a sum over large oscillating terms and the relative error scales exponentially with \( N \) and \( \beta \).

The upshot is that there are at least two classes of systems where quantum simulation offers the possibility of categorical improvement over what is possible with classical computers, because the problems in question are NP-hard [4]. The first consists of fermions in more than one dimension, which suffer from the above sign problem. The second consists of non-equilibrium dynamics where the above approach also fails because the integral must be taken over real, rather than imaginary time.
Figure 1-1: Sketch of permuting fermion worldlines, representing hopping in a 2D system. The vertical axis represents imaginary time. The particles represented by blue and green worldlines change places, and since the particles are identical this term contributes to the trace. However, this exchange is an odd permutation and the term enters into the overall sum with a negative sign. In one dimension Pauli exclusion prevents the fermion worldlines from crossing and hence there is no sign problem.

1.2 Cold atoms as quantum simulators

In the quantum simulation paradigm we typically apply some known (possibly time-dependent) Hamiltonian to a system in thermal equilibrium, and then attempt to characterize the resulting state. Progress in this field therefore proceeds in two directions: implementing more interesting Hamiltonians (at sufficiently low temperatures so that their interesting features can be observed), and improving the ability to characterize the resulting states.

At present the main obstacle to preparing interesting states of fermions is that the temperatures in current experiments are too high. In a system with two spin states an entropy of $\ln(2)$ per particle is sufficient for full spin disorder, so observation of antiferromagnetic order in the Fermi-Hubbard model, for example, requires an entropy below that [6]. At the same time, work continues on realization of new Hamiltonians, with a major source of recent attention coming from progress in the realization of synthetic gauge fields [7] and the resulting topological band structures [8].
The development of state characterization involves building up a diverse array of tools. A few examples include novel RF spectroscopy techniques [9], Bragg scattering for detecting long range order [10, 11], and measurement of correlations by merging wells and measuring double occupancies [12]. A major recent development has been experiments capable of achieving single-site resolution [13, 14], which allow direct imaging of atoms and direct measurement of correlation functions.

1.3 Thesis outline

My time as a member of the BEC2 lab bridged two generations of the experimental apparatus. Work in the old and new machines correspond to an emphasis on state detection and preparation, respectively.

- Chapter 2 describes new techniques for state characterization: measurements of correlations and thermodynamic response functions of fermionic systems using spin and density fluctuations. These experiments were conducted in the old apparatus and include a proof-of-principle study of suppressed fluctuations in non-interacting systems and characterization of strongly attractive and repulsive systems.

- Chapter 3 discusses technical details of the development of the new machine after its initial construction, including the creation of its first Bose condensates and the installation of optical traps, imaging, and lattices.

- Chapter 4 introduces state preparation in optical lattices, and describes initial efforts to realize different lattice Hamiltonians with bosonic Na. It also discusses our studies of a dynamical instability present in tilted lattices and progress towards implementing synthetic magnetic fields.

- Chapter 5 presents an outlook for the future of our lab and apparatus.
Chapter 2

Fluctuations in Quantum Degenerate Fermi Gases

This chapter describes fluctuation experiments performed in the old apparatus on spin and density fluctuations in bulk Fermi systems [15-19].

Cold atom state characterization tends to be complementary to study of analogous systems in condensed matter, because the simplest measurements in each case tend to be very different. In cold atom systems it is simple to measure momentum and quasimomentum distributions by rapidly switching off the trap [20], but it is difficult to perform transport measurements and to characterize thermodynamic response functions without taking derivatives of noisy signals. The quantities that are hard to measure include some of the most important characteristics of the phases whose boundaries one would like to map out in quantum simulations: for example, in a gas of Cooper pairs or an antiferromagnetic phase the spin susceptibility vanishes, and for a ferromagnetic systems it diverges.

With some difficulty, one can measure the response functions and transport properties in direct ways. By precisely characterizing the trapping potential and taking advantage of cylindrical symmetry, it is possible to accurately take the derivative of the atom number with respect to the chemical potential [21]. Alternatively, one can measure transport directly by separating different spin components and allowing them to recombine [22].
Another option is to take advantage of the fact that the system is in thermal equilibrium and therefore intrinsically 'sampling' all of the possible states with an energy width $k_B T$. The fluctuations in observed quantities therefore characterize their response to thermodynamic potentials.

### 2.1 Fluctuation response theorem

This intuition is formalized by the fluctuation-response theorem. Given conjugate variables $x$ and $f$, that is, given a Hamiltonian of the form $H = H_0 + x f$, the fluctuation-response theorem relates the susceptibility of $x$ to its equilibrium variance [23]:

$$\frac{\partial \langle x \rangle}{\partial f} = \frac{\text{Var}(x)}{k_B T} \quad (2.1)$$

So, for example, the variance of the atom number $\text{Var}(N)$ is proportional to the product of compressibility $\partial N / \partial \mu$ and temperature. For a two-component gas, making the analogy between magnetization and spin imbalance such that $M = n_\uparrow - n_\downarrow$ and $H = \mu_\uparrow - \mu_\downarrow$, the spin susceptibility $\partial M / \partial H$ is proportional to the product of temperature and $\text{Var}(M)$. The size of the fluctuations determines the product of temperature and response; this means that measuring the response function requires measuring the temperature in addition to the fluctuations, or if the response is known we can use the fluctuations as a form of thermometry.

Our work on local fluctuations in Fermi systems builds off of earlier work on using fluctuations to characterize momentum-space correlations [24–26] and in bosons [27], and several of our experiments were performed nearly simultaneously with similar observations from another group in Zurich [28, 29].

### 2.2 Fluctuations in an ideal Fermi gas

At zero temperature, the fluctuations in an ideal Fermi gas of sufficiently large volume disappear. We can imagine the volume as an isolated system with its own chemical
potential and count up the occupation of states. Since the Fermi function approaches a step, every state with energy below the chemical potential is occupied and every state with energy above it is not, so there is no uncertainty in the measured number of particles. At the other extreme, the distribution spreads out at high temperature so that even the lowest-energy states have only a small chance of being occupied. Counting a large number of unlikely events results in Poisson statistics: when \( n \) independent events each occur with probability \( p \), the variance in the number of successes \( np(1 - p) \) approaches the expected number \( np \) as \( p \) approaches zero.

This full-scale suppression of density fluctuations due to Pauli exclusion is a dramatic effect, which makes the temperature-dependence of the number fluctuations in an ideal Fermi gas an excellent proof-of-principle experiment as well as an illustration of a fundamental physical concept. In principle all we need to do is to repeatedly measure the number of atoms in a particular volume to calculate the variance, and counting the local atom number is one of the most common measurements in atomic physics.

### 2.2.1 Fluctuation in absorption imaging

Typically one counts atoms using an absorption imaging procedure that involves taking three images: a probe with atoms (PWA), a probe without atoms (PWOA) to normalize the probe intensity, and a dark field (DF) to normalize the detector. We try to work far below saturation, where absorption is multiplicative and the fraction of transmitted light is exponential in the atom count.

\[
\frac{PWA - DF}{PWOA - DF} = \exp\left(-\frac{\sigma N}{A}\right)
\]

where \( \sigma \) is the cross section, \( 3\lambda^2/2\pi \) on resonance, \( N \) is the atom count in a particular pixel, and \( A \) is the pixel area. How are fluctuations in atom number reflected in absorption? The variance in optical density has contributions from photon
and atom shot noise, doing the error propagation gives the following formula:

\[
(\Delta(OD))^2 = \frac{1}{\langle N_1 \rangle} + \frac{1}{\langle N_2 \rangle} + \frac{\sigma}{A} (\Delta N_{atom})^2
\]  

(2.3)

\(N_1\) is the average number of photons measured in a given pixel position for the probe with atoms, \(N_2\) is the average number of photons measured in that pixel for the probe without atoms, and \((\Delta N_{atom})^2\) is the variance in atom number for that pixel. By calculating the first two photon shot noise terms (which are uninteresting, because our light source is classical) and subtracting them from the total variance, we isolate the atom number variance. When we analyze our experiments we also subtract contributions from detector read noise and the photon shot noise in the dark field, but these are fairly small contributions.

Naively, the fact that OD variance is proportional to atom number variance may suggest that it would be advantageous to use a large number of atoms per pixel. However, as the atom number increases, the relative photon shot noise contribution from the probe with atoms \(N_1\) increases as well. Since \(N_1 \propto e^{-OD}\), the ratio of atom shot noise \((\propto d)\) to photon shot noise \((\propto (1 + e^d))\) is maximized for a \(OD \approx 1.3\). Moreover, measurement errors from imaging fringes and other sources of technical noise are not constant in the optical density. They are, if anything, closer to constant in the transmission, and exponential in the optical density, favoring imaging at lower densities.

The result is that typical in-trap optical densities are too high to make this a realistic measurement. Thankfully, noninteracting Fermi gases have a convenient property allowing us to reduce the optical density to its optimum value.

### 2.2.2 Rescaling in time of flight

Optically thick clouds are typically studied by releasing them from the trap and allowing them to expand in time-of-flight. This technique is particularly useful in the case of noninteracting Fermi gases because an exact rescaling [30] allows one to regard the sample after ballistic expansion (and, in fact, most sequences of harmonic con-
finements) as identical to the equilibrium sample with spatially rescaled coordinates, even though the sample is far from thermal equilibrium. This is why, for example, the shape fit functions used to determine the temperature of a noninteracting Fermi gas do not depend on the time of flight, and the rescaling holds not just for the overall envelope but for all correlation functions as well.

Figure 2-1: Phase space diagram of ballistic expansion of a harmonically trapped Fermi gas. Ballistic expansion conserves phase space density and shears the initially occupied spherical area into an ellipse, but the occupation probability of phase space cells does not change. In the center of the cloud, the local Fermi momentum and the sharpness of the Fermi distribution are scaled by the same factor, keeping the ratio of local temperature to Fermi energy constant. The same is true for all points in the expanded cloud relative to their corresponding unscaled in-trap points.

This is intuitively illustrated in Fig. 2-1. In the trap the Hamiltonian is symmetric between position and momentum and the cloud occupies a circle in phase space. After release from the trap, momentum is ‘frozen’ in but the cloud shears in phase space as the original momentum is reflected in the position of each phase space cell. Importantly, the occupation of the phase space cells is unchanged, so the rescaling $x \rightarrow x\sqrt{1 + \omega^2 t^2}$ maps the old cloud onto the new.
2.2.3 Image analysis

Within- and between-images variances

Even under the favorable conditions of time-of-flight, the atom noise signal is small enough to be easily dwarfed by technical noise and errors in normalization. One of the most important choices in conducting the experiment is to select which 'identical' atom number measurements to group together to calculate the variance. Within a single image, the atom number varies slowly; it is possible to do a 'within-image' comparison of pixels that are near each other, or at the same radius from the cloud center. Unfortunately the envelope still varies quickly enough that its derivative makes a non-negligible contribution to the difference in atom count for adjacent pixels. This can be corrected for, but the method still remains sensitive to spatial variation in the probe intensity. Alternatively, we can take a series of images and compare the atom count in the same pixel in a 'between-image' comparison. This method has the problem that the atom number often changes on the order of $10\%$ from shot to shot, potentially overwhelming the shot noise contribution.

While trying to determine the optimal method of processing images, we conceived of a potentially elegant solution to the problem. Using an interline CCD camera it is possible to take two images, one of each spin state, separated by only $2\mu s$. As long as the cloud is well-balanced, these images should be truly identical apart from atom number fluctuations, without being susceptible to shot-to-shot fluctuations or a difficulty in identifying 'identical' locations within the image. Unfortunately, this approach was derailed by technical problems. Both types of interline cameras we used had technical noise issues (in one case, linear bands of noise, and in the other, a spatially-varying difference in gain between the two closely-timed images shown in Fig. 2-2) significantly in excess of the atomic shot noise signal.

After abandoning two-state imaging, we ultimately settled on a between-images approach that incorporated within-images information in an aggregate way. By computing a 2D Thomas-Fermi profile from all of the pixels in each image and subtracting it from the image before calculating the between-images atom number variance, we
Figure 2-2: Spatial gain variation of the Coolsnap HQ DIF from Princeton Instruments. Two identical images are taken after a variable delay and the normalized difference, which should be uniformly zero, is shown for two delay times. A problem with the CCD clocking causes the detector gain to depend on the exposure time and on spatial position. Because the two pictures taken in rapid succession cannot be effectively compared, this imaging configuration is poorly suited to atom noise experiments.

were able to minimize the effect of global shot-to-shot fluctuations in atom number.

**Cross-section determination**

The variances have components from atom noise, photon noise, and technical detector noise, shown in Fig 2-3. We subtract the latter two components, which requires calibrating the camera conversion gain between electrons (e\(^{-}\)) and detector digitizer units (ADU) of (1.070(5)ADU/e\(^{-}\)) and read noise (3.5(3)ADU) using pictures without atoms. To interpret the atom noise we need to calibrate the absorption cross section, which depends on the imaging intensity.

Atomic shot noise dominates over photon shot noise only if each atom absorbs several photons. As a result, the absorption images were taken using the cycling transition to the lowest lying branch of the \(^2P_{3/2}\) manifold. At the zero-interaction field of 520G the decay into other states is negligible and the dominant effect on the cross-section is from acceleration of the atoms by the photon absorption and recoil, which Doppler shifts the atoms out of resonance. To limit the need for nonlinear
Figure 2-3: Determination of profiles of the atom number variance. For each bin, the total photon count is determined, and its contribution (red) to the total variance of the optical density (blue) is subtracted. The atom number variance (green) is compared to the average atom number.

normalization procedures, we chose a probe laser intensity corresponding to an average of only 6 absorbed photons per atom during 4 μs of exposure time. At this intensity, about 12% of the ⁶Li saturation intensity, the measured optical density was found to be reduced by 20% from its low-intensity value, shown in Figure 2-4.

Even for low imaging intensities, the resonant absorption cross-section of \(2.14 \times 10^{-13} \text{ m}^2\) turns out to be an inaccurate estimate of the cross-section observed in experiments, presumably due to imperfections in polarization. Applying the measured 20% reduction for the experimental imaging intensities leads to a value of \(1.71 \times 10^{-13}\)
Figure 2-4: Absorption cross-section as a function of imaging intensity. The line is a quadratic fit to the data. The reduction of the cross section is mainly due to the Doppler effect caused by acceleration of the atoms by radiation pressure, and to a smaller extent, by partial saturation of the optical transition. At the probe light intensity chosen in this study (the shaded region on the x axis), the number of photons absorbed per atom is about 6. The decrease of the cross section is slightly larger than accounted by simple models.

$\text{m}^2$, substantially higher than either the estimate obtained from fits to the ballistic expansion ($1.48 \times 10^{-13} \text{ m}^2$) or that obtained from assuming that the atom shot noise in the wings of the hot cloud is fully Poissonian ($1.50 \pm 0.12 \times 10^{-13} \text{ m}^2$). Since the atom noise in the wings must be Poissonian, and since the value obtained from making this assumption is close to other estimates, we use the cross-section obtained from atom noise as the 'final' value for data analysis.
2.2.4 Probe volume considerations

Two factors make the size of the volume used for counting atoms important. First, since the cloud expands in time of flight, not all of the cloud will be in focus. So even though the diffraction-limited resolution of the imaging system is much smaller than the effective pixel size of 6.5 μm, atoms that are closer to or further away from the imaging plane may be spread out over several pixels, thus distorting the atom noise statistics. Instead of attempting to optically characterize the depth of field of our imaging system, we apply software binning to the raw images for a range of bin sizes and find the effective pixel size (26μm) where the atom noise stops increasing, as shown in Fig 2-5.

Second, for small enough probe volumes we can no longer make the local density approximation and quantum, as well as thermal fluctuations become important [31]. Pauli exclusion prevents there from being multiple particles within a range 1/kF of each other, but as the volume decreases below this value the measured atom number noise should approach Poissonian noise. If we write the variance of the number of fermions \( N = \int_V dr \psi_\dagger(r) \psi(r) \) in a volume \( V \) as:

\[
\text{Var}(N) = \langle N \rangle - \langle \int_V dr \int_V dr' \psi_\dagger(r) \psi_\dagger(r') \psi(r) \psi(r') \rangle \tag{2.4}
\]

Fourier transforming and shifting the integrals gives

\[
\text{Var}(N) = \langle N \rangle - \int \frac{d^3k}{2\pi^3} \int \frac{d^3k'}{2\pi^3} n_k n_{k'} F(k - k') \tag{2.5}
\]

where

\[
F(q) = | \int_V d^3re^{iq\cdot r} |^2 \tag{2.6}
\]

\( F \) is a purely geometric integral and for a sphere it has a width \( \approx 1/R \), so for large probe volumes relative to \( 1/k_F \) it can be treated effectively as a delta function. In that case Var(N) is simple enough to calculate numerically, with the slope near zero.
agreement with the first order term

$$\text{Var}(N) \approx \langle N \rangle \frac{3k_B T}{2E_F}$$

(2.7)

given by, for example, using the zero-temperature expression for the compressibility $3/(2nE_F)$. For small probe volumes, however, the probability of measuring a nonzero number of particles is small (and the probability of measuring more than one particle is vanishing) and so the noise is trivially Poissonian.
Because the variance in atom number is uncorrelated, it allows us to characterize the transfer function of our imaging system. Fig. 2-6 shows the average power spectrum of the optical density images as a function of spatial frequency. Because the Fourier transform of uncorrelated fluctuations is flat, the deviation from flatness of the density noise corresponds to blurring induced by the lens, barring the central peak corresponding to the shape of the cloud. We can see that the power spectrum decays to negligible amounts well before $q_f$ where quantum fluctuations become large. The finite background at large $|q|$, to which atom noise cannot contribute due to the resolution limit, is due to the photon shot noise.

### 2.2.5 Fluctuations and temperature

Fig. 2-7 shows an absorption image of an expanding cloud of fermionic atoms. The 'between-images' method of image analysis calculates the variance in atom number at one (binned) pixel over all of the images. Combining these variances into one image results in a 'variance image', which for a Poissonian sample (with no suppression of fluctuations), would be identical to a scaled mean image showing the number of atoms per probe volume. This is close to the situation for the hottest cloud (the temperature was limited by the trap depth), whereas the colder clouds show a distinct suppression of the atom number variance, especially in the center of the cloud where the local $T/\mu$ is smallest.

In Fig. 2-11, profile of the variance are compared to theoretical predictions, showing good agreement with temperatures computed from 2D Thomas-Fermi fits of the shape of the cloud. The predictions (Fig 2-8) are made by making the local density approximation and integrating $p(1 - p)$ over all momenta using the Fermi function, and then integrating once more over the line of sight:

$$
\langle N^2 \rangle = \int_{-\infty}^{\infty} dz \int_{0}^{\infty} \frac{V dp}{2\pi^2} p^2 F(\frac{p^2}{2m} - (\mu_0 - \omega^2 z^2), T)(1 - F(\frac{p^2}{2m} - (\mu_0 - \omega^2 z^2), T)) \ (2.8)
$$

Where $F(E - \mu, T)$ is the Fermi function.
Figure 2-6: (a) Radially averaged power spectra of average optical density images for hot (solid line) and cold (dashed line) samples (b) Power spectrum of cold sample (arbitrary units) (c) Power spectrum of hot sample (arbitrary units)

The temperatures are obtained from standard Thomas-Fermi shape fits, but as described in detail in [32], this process is not always as straightforward as it seems. Figures 2-9 and 2-10 show systematic changes in the fit temperature as a function of the time of flight and of the size of the region used for fitting. When the time of flight is too short, the effect of small intensity variations in the probe beam or saturation effects are magnified. We make sure to conduct thermometry in a parameter regime
Figure 2-7: Comparison of density images to variance images. For Poissonian fluctuations, the two images at a given temperature should be identical. The variance images were obtained by determining the local density fluctuations from a set of 85 images taken under identical conditions. (a) Two dimensional image of optical density of an ideal Fermi gas after 7 ms of ballistic expansion. The noise data were taken by limiting the field of view to the dashed region of interest, allowing for faster image acquisition. (b) For the heated sample, variance and density pictures are almost identical, implying only modest deviation from Poissonian statistics. (c) Fermi suppression of density fluctuations deep in the quantum degenerate regime manifests itself through the difference between density and variance picture. Especially in the center of the cloud, there is a large suppression of density fluctuations. The variance images were smoothed over 6×6 bins. The width of images (b) and (c) is 2 mm.

where the fit temperature is locally flat with respect to the time of flight and analysis region, and where shape fits from images taken from different directions agree on the temperature.

As shown in Fig.2-11, the measured fluctuations demonstrate excellent agreement with predictions, suggesting that fluctuations can be measured quantitatively and used as an alternative form of thermometry.

2.2.6 Beyond the second moment

In principle the higher-order moments of the density should reflect the higher-order correlations [34], and we know that at zero temperature the fourth moment of density $\langle (n - \langle n \rangle)^4 \rangle$, like all of the higher moments, must be zero. Because of technical noise
we do not, in fact, observe the expected suppression of higher moments. The shaded areas in Figure 2-12 are derived from theoretical values for the variance of the sample variance for a Poisson distribution. This is given by

\[
\text{Var(Var}(N)) = \frac{(m-1)^2}{m^3} \mu_4 - \frac{(m-1)(m-3)}{m^3} \mu_2
\]

(2.9)

where \( m \) is the number of observations in each sample. The moments \( \mu_2 \) and \( \mu_4 \) are the central moments of the population distribution. For a Poisson distribution,

\[
\text{Var(Var}(N)) = \frac{(m-1)^2}{m^3} \langle N \rangle (1 + 3\langle N \rangle) - \frac{(m-1)(m-3)}{m^3} (\langle N \rangle)^3
\]

(2.10)

where \( \langle N \rangle \) is the population mean. For \( m, \langle N \rangle \gg 1 \), this is simply \( 2\langle N \rangle^2/m \). So, surprisingly, the variance of the variance of the cold cloud matches the Poissonian predictions better than it does the expected suppression. Note that the above analysis
Figure 2-9: Temperature versus time of flight for two cold clouds at different temperatures. Clouds are in a 'tight' optical trap, a slightly different from that used in the fluctuation experiments. For short times of flight and high optical densities, the measured temperature is systematically too low.

corrects errors in our previous discussions of this subject.

2.3 Spin fluctuations in an interacting Fermi gas

After proving that the fluctuations in a noninteracting Fermi gas behave as expected we would like to characterize them in interacting systems. When a gas consists, around the Fermi surface, largely of molecules or generalized Cooper pairs, one expects the number fluctuations to consist of pairs and the fluctuations in relative number to be heavily suppressed. Unfortunately the exact rescaling that allows time-of-flight images of noninteracting Fermi ages to be treated, essentially, as rescaled in-situ
Figure 2-10: Temperature versus analysis region size for different values of the time of flight. Clouds are in a 'tight' optical trap, a slightly different from that used in the fluctuation experiments.

images does not hold for interacting ones. Nor is it straightforward to calculate, for a strongly interacting gas, what the interactions do to the correlation functions as the cloud expands. Off-resonant absorption imaging is one possibility, but then one has to contend with the lensing of the probe beam by the optically thick, dispersive cloud.

2.3.1 Disperive imaging

The dispersion that distorts off-resonant images of dense clouds can be used instead to characterize them. How does one image a phase object in focus? One can think of absorption imaging as a form of homodyne detection where the scattered light is mixed again with the incident light on the detector. By using a phase spot in the Fourier plane to phase shift the incident light, one can shift the phase of the homodyne
Figure 2-11: Comparison of observed variances (black dots) with a theoretical model (black line) and the observed atom number (gray), at three different temperatures (a, b, and c), showing 50, 40, and 15% suppression. Noise thermometry is implemented by fitting the observed fluctuations, resulting in temperatures $T/T_F$ of $0.23\pm0.01$, $0.33\pm0.02$, and $0.60\pm0.02$. This is in good agreement with temperatures $0.21\pm0.01$, $0.31\pm0.01$, and $0.6\pm0.1$ obtained by fitting the shape of the expanded cloud [33]. The quoted uncertainties correspond to one standard deviation and are purely statistical.

or, in the case of dark ground imaging, remove it completely.

In fact, this can be done more simply without employing a phase spot at all and using the Faraday effect [35, 36]. If we write the light polarizations in the circular basis where $\sigma^+ = \left(\begin{array}{c} 1 \\ 0 \end{array}\right)$ and $\sigma^- = \left(\begin{array}{c} 0 \\ 1 \end{array}\right)$, then we can apply linearly polarized light at $\left(\begin{array}{c} 1 \\ 1 \end{array}\right)$ and a weak dispersive object will convert that to $\left(\begin{array}{c} (1-\varepsilon)+\varepsilon i \\ 1 \end{array}\right)$. Then, by putting a crossed linear polarizer after the atoms at $\left(\begin{array}{c} 1 \\ -1 \end{array}\right)$, we can reproduce dark ground imaging, with a signal proportional to $\varepsilon^2$ with zero background, or by putting it at 45°, or $\left(\begin{array}{c} 1 \\ i \end{array}\right)$ in the circular basis, we can reproduce phase contrast imaging with a signal linear in $\varepsilon$. In fact, this polarization contrast imaging technique is cleaner than the Fourier technique because it does not distort the image by high-pass-filtering it via the phase-shifting spot in the Fourier plane.

Surprisingly, when we performed the experiments we observed similar intensity fluctuations with and without the polarizer in place, despite the fact that for our choice of detuning the phase effect should dominate. The fluctuations without the polarizer were a factor of ten larger than expected for full Poissoninan noise. Ulti-
Figure 2-12: Variance of the variance for hot (red) and cold (blue) clouds. The bars reflect the observed variance of variance, and the shaded areas correspond to theoretical predictions assuming Poissonian noise. We expect, and do not observe, strong suppression of the fourth moment of density for the cold cloud.

Ultimately we explained this surprising effect as being due to a speckle-like effect.

2.3.2 Speckle imaging

Light propagating through a phase grating acquires an intensity modulation after spatial propagation, as in the Talbot effect [37]. For a sinusoidal grating maximum contrast is achieved after a propagation distance $2a^2/\lambda$, where $a$ is the grating spacing, in other words, after a distance proportional to the Rayleigh length for a spot the size of the grating spacing.

Dispersive effects therefore manifest themselves in images whenever the defocus, or the cloud size, is substantially larger than this distance. This is similar in some ways
to defocus contrast imaging, with the important difference that the phase objects being imaged are small local fluctuations rather than the large cloud envelope, dramatically reducing the required defocus distance [38].

For a random phase object, the phenomena that results is similar to that of laser speckle, which one observes, for example, when one views the light scattered from a laser beam incident on an index card. The surface roughness of the card causes the scattered beam to experience random phase shifts, which spatial propagation converts to intensity noise [39]. This effect is simulated in Figure 2-13.

2.3.3 Susceptibility and compressibility

Sample variance images resulting from this speckle effect are shown in Figure 2-14. The speckle is a highly sensitive probe of the phase fluctuations in the cloud, and we can easily resolve fluctuations less than 10% of atom shot noise. Unfortunately, the fact that it appears as a consequence of spatial propagation makes it difficult to definitely isolate the contributions from different spatial regions, so in analyzing the speckle images we tend to aggregate the cloud as a whole.

By changing the probe detuning, the relative phase shifts of the two spin components can be either additive or subtractive, as shown in Figure 2-15. This allows us to measure both the compressibility and spin susceptibility of the cloud. To do so at a variety of fields requires scanning the imaging frequency over several GHz, so we stabilized an additional diode laser with a variable frequency offset. Because a narrow relative linewidth is not important, in our experiment, our offset lock was constructed using an off-the-shelf phase detector/loop filter (RF-BAY PDF100, FPS8-18).

Our goal was to determine at various interaction strengths the normalized susceptibility \( \tilde{\chi} = \chi/\chi_0 \) and compressibility \( \tilde{\kappa} = \kappa/\kappa_0 \), where \( \chi_0 = 3n/2E_F \) and \( \kappa_0 = 3/2nE_F \) are the susceptibility and compressibility of a non-interacting Fermi gas at zero temperature. We therefore calibrate our measurement by measuring the spin fluctuations in a noninteracting mixture. Fig. 2-14 shows raw profiles of the difference and sum variances of the measured optical densities \( \Delta^2_- = (c \Delta(N_1 - N_2))^2 \) and \( \Delta^2_+ = (c' \Delta(N_1/3 + N_2))^2 \). In these relations \( c \) and \( c' \) are the conversion factors be-
Figure 2-13: Simulation of propagation effects after light has passed through a Poissonian phase noise object. Shown are the variance measured in the amplitude or in-phase quadrature (black line) and the out-of-phase quadrature (gray line) as a function of defocus distance, for an imaging system with a numerical aperture of 0.14. Within a distance less than 5 percent of our cloud size, noise becomes equally distributed between the two quadratures and the variances in transmission and phase-contrast images become the same. (Top inset) For small phase fluctuations, an in-focus phase noise object gives no amplitude contrast, but when it is out of focus it does. (Bottom inset) Sample intensity patterns for a defocused phase object.

tween number fluctuations and fluctuations in optical density in the specific probe volume $V$. Without interactions, $N_1$ and $N_2$ are uncorrelated, and one predicts $\frac{(\Delta(N_1 - N_2))^2}{(\Delta(N_1/3 + N_2))^2} = 2/(1 + (1/3)^2) = 1.8$. The observed ratio of $\Delta^2 / \Delta_+^2 = 1.56(14)$ reflects excess noise contributing to $\Delta_+^2$ due to residual systematic dispersive effects and is accounted for by setting $c'/c = \sqrt{1.8/1.56}$. For high temperatures, the atomic noise of the non-interacting gas approaches shot noise; for
Figure 2-14: (Top) Example speckle noise image, with white box indicating analysis region. (Bottom) Noise data for noninteracting (left) and resonantly interacting (right) cold clouds, showing $\Delta_2^2$ (black dots) and $\Delta_4^2$ (grey dots). Solid lines are Gaussian fits to the data, and dotted lines are expected full Poissonian noise for the corresponding quantities based on density determined from off-resonant absorption.

lower temperatures we observe a reduction in noise due to Pauli blocking as in the previous section.

Recomposing the variances from the two experimentally accessible linear combinations these relations become $\Delta_2^2/Nc^2 = 3/2 (T/T_F) \tilde{\chi}$ and $9/4 \Delta_4^2/Nc^2 - 1/4 \Delta_2^2/Nc^2 = 3/2 (T/T_F) \tilde{\kappa}$ and allow us to determine $c$ and $c'$ by using the 527G noise measurements for which $\tilde{\chi} = \tilde{\kappa} = 1 + O \left( (T/T_F)^2 \right)$.

Fig. 2-16 shows the spin susceptibility, the compressibility, and the ratio between the two quantities for the interacting mixtures as the interaction strength is varied through the BEC-BCS crossover. The susceptibility and compressibility reproduce the expected qualitative behavior: for the sample at unitarity, where the transition temperature is sufficiently high that a sizable portion of the sample is superfluid, and for the sample on the BEC side, where even the normal-state atoms form molecules,
the spin susceptibility is strongly suppressed relative to the compressibility. This reflects the fact that the atoms form bound molecules or generalized Cooper pairs; the spin susceptibility should be exponentially small in the binding energy, while the increased compressibility reflects the bosonic character of the molecular condensate. At 915G and 1000G, where the sample is above the superfluid critical temperature, the susceptibility is larger but still below its value for the noninteracting gas, reflecting the persistence of pair correlations even in the normal phase of the gas.
Figure 2-16: (a) The ratio $\chi/\kappa$, (b) the normalized susceptibility $\chi/\chi_0$, and (c) the normalized compressibility $\kappa/\kappa_0$ in the BEC-BCS crossover. The variances derived from sequences of images are converted into thermodynamic variables using the measured temperatures and a calibration factor determined from the noninteracting gas. The vertical line indicates the onset region of superfluidity, as determined via condensate fraction measurements. The curves show theoretical zero temperature estimates based on 1st (dotted) and 2nd order (solid) perturbative formulas obtained from Landau’s Fermi-liquid theory integrated along the line of sight, and results from a Monte Carlo calculation (dashed) for the compressibility in a homogeneous system [? ].

2.4 Dynamics of spin fluctuations in a repulsive Fermi gas

In situ noise measurements are robust in the sense that they depend only on the local properties of the sample, without requiring that the cloud be overall in the ground state of a particular trapping potential. As a result they are well-suited to dynamical measurements where the cloud shape is changing or the potential is not exactly known.

This makes them ideal for characterizing systems on the upper branch of a Feshbach resonance where the sample is intrinsically unstable with respect to a molecular bound state. Strong, short-range repulsive interactions cannot be achieved with purely repulsive potentials: a hard core potential is characterized by a positive scat-
tering length $a$ identical to the radius of the hard core. Creating strong interactions with $k_Fa \geq 1$ therefore requires that the hard core radius to be comparable to the inverse of the interatomic spacing and therefore no longer short range. On the other hand arbitrarily large repulsive scattering lengths can be realized with short-range attractive potentials if the energy of the interacting atoms is nearly resonant with a bound state with binding energy $\hbar^2/(ma^2)$, but then the repulsive gas is then by necessity only metastable with respect to pair formation.

2.4.1 Stoner instability

Strong repulsive interactions create a competition between kinetic and interaction energy. When the system polarizes, there are more particles of one type, and Pauli exclusion requires that the additional particles occupy levels with higher kinetic energy. By doing so, however, it avoids the interaction energy, and one might expect that the system transitions to a ferromagnetic phase when interactions become large.

This is borne out by a simple mean field analysis. If we start with an initially balanced sample and shift particles within an energy width $\delta \varepsilon$ from the Fermi level from one spin component to the other, then

$$\Delta E_{\text{int}} = \Delta U n_\uparrow n_\downarrow = U(N/2 - D(\varepsilon)\delta \varepsilon)(N/2 + D(\varepsilon)\delta \varepsilon) - U(N/2)^2$$  \hspace{1cm} (2.11)

$$\Delta E_{\text{kin}} = (D(\varepsilon)\delta \varepsilon)\delta \varepsilon$$  \hspace{1cm} (2.12)

where $D(\varepsilon)$ is the density of states at the Fermi level, and so the system is unstable to domain formation when $UD(\varepsilon) > 1$. However, it is an open question whether this mean-field treatment is even qualitatively accurate – for example, it incorrectly predicts a ferromagnetic phase transition in one dimension where it is rigorously forbidden [40]. The Stoner model has been studied extensively by theorists, but there is no consensus as to the nature, order, or location of the ferromagnetic phase transition. [41–44].

An earlier experiment in our group observed indirect signatures they deemed to be indicative of the ferromagnetic phase transition: a minimum in the three-body
recombination rate, minimum in kinetic energy, and a maximum in the cloud volume [45]. However they were unable to observe domains and eddy currents from the steel vacuum chamber limited their time resolution to 4.5ms.

A direct measurement of the fluctuations would definitively confirm the presence or absence of domains because the spin fluctuations are dramatically enhanced by ferromagnetic correlations. The spin fluctuations are enhanced by a multiplicative factor equal to the average number of particles in a domain, and our previous measurement revealed more than enough sensitivity to measure domains of a few particles.

2.4.2 Spin fluctuations after a rapid quench

In addition to measuring the spin fluctuations directly, we dramatically improve the speed of measurements by combining the reduced eddy currents of a glass cell vacuum chamber with a fast current jump. By using an external capacitor to charge a capacitor to 370V, we can overcome the inductance of the coils to switch the magnetic fields in 350\(\mu\)s. The experimental timing is shown in the inset of Fig 2-18.

![Schematic for capacitor 'booster' box for fast current jumps.](image)

**Figure 2-17:** Schematic for capacitor 'booster' box for fast current jumps. The shaded box represents the rest of the high power circuitry used during normal operation. Component values are chosen to achieve critical damping.

Figure 2-19 shows the observed spin fluctuations enhancement compared to the
Figure 2-18: Diagram showing energy levels and timing of the experiment. The upper (repulsive) and lower (attractive) branches of the Feshbach resonance are connected by 3-body collisions. In our experiment, we quickly jump from a weakly interacting Fermi gas (A) to a strongly interacting one (B) with a rapid magnetic field change. The evolution of correlations and domains and the molecule formation (population of the lower branch) are studied as a function of hold time $t$.

non-interacting cloud at 527 G. The variance enhancement factor $\text{Var}(M)/\text{Var}(M)_0$ reaches its maximum value of 1.6 immediately after the quench, decreasing during the 2 ms afterward. The absence of a dramatic increase shoes that no domains form and that the sample remains in the paramagnetic phase throughout. Similar observations were made for a wide range of interaction strengths and wait times. Note that first-order perturbation theory [46] predicts an increase of the susceptibility by a factor of 1.5 at $k_Fa = 0.5$ and by a factor of 2 at $k_Fa = 0.8$ (i.e. no dramatic increase for $k_Fa < 1$). Therefore, our data shows no evidence for the Fermi gas approaching the Stoner instability.

### 2.4.3 Pair formation

What happens to the system instead? One possibility is that a significant fraction of the atoms are converted into molecular pairs on the lower branch of the Feshbach resonance in a process that competes with domain formation [47]. We characterize the pair formation by comparing the total number of atoms and molecules $N_a + 2N_{mol}$.
(determined by taking an absorption image after ballistic expansion at high magnetic field where molecules and atoms have the same absorption resonance) to the number of free atoms (determined by rapidly sweeping the magnetic field to 5G before releasing the atoms and imaging the cloud, converting pairs into deeply bound molecules that are completely shifted out of resonance) [48].

The time evolution of the molecule production (Fig. 2-20) shows two regimes of distinct behavior. For times less than 1 ms, we observe a considerable number of atoms converted into molecules, while the total number $N_{a} + 2N_{mol}$ remains constant. The initial drop in atom number becomes larger as we increase the final magnetic field, and saturates at around 50% near the Feshbach resonance.

For longer times on the scale of 100 ms we observe a steady increase of the molecule fraction to 90% for the longest hold time. We interpret this as a result of system reaching thermal equilibrium between atoms and molecules, where the system thermalizes after the rapid quench and begins to cool itself evaporatively. This cooling eventually shifts the atom-molecule equilibrium towards high molecule fractions. During the same time scale, a slow loss in both atom number and total number is observed, resulting from evaporation loss and inelastic atom-molecule and molecule-molecule collisions, which lead to a large energy release via the creation of deeply-bound molecules, and subsequent ejection of particles from the trap.

Does our experiment provide evidence that the rapid conversion into molecules is faster than the evolution of ferromagnetic domains for all parameter values? We believe that it does. First, for strong interactions with $k_{F}a$ around 1, one expects both instabilities (pair formation and Stoner instability) to have rates which scale with the Fermi energy $E_{F}$ and therefore with $n^{2/3}$. Therefore, one cannot change the competition between the instabilities by working at higher or lower densities. According to Ref. [47] the fastest unstable modes for domain formation have a wavevector $q \approx k_{F}/2$ and grow at a rate of up to $E_{F}/4\hbar$ when the cloud is quenched sufficiently far beyond the critical interaction strength. Unstable modes with such wavevectors will develop "domains" of half a wavelength or size $\xi = \pi/q = 2\pi/k_{F}$ containing 5 atoms per spin state in a volume $\xi^{3}$. This rate is comparable to the observed conver-
sion rates into pairs of $0.13E_F$. Therefore, at best, “domains” of a few particles could form, but before they can grow further and prevent the formation of pairs (in a fully polarized state), rapid pair formation takes over and populates the lower branch of the Feshbach resonance.

Still, theoretical analysis of the ferromagnetic instability has revealed that the phase transition depends sensitively on system parameters including dimensionality [49, 50], band structure [51, 52], and interactions [53]. By imposing a weak lattice or changing the interactions, it may be possible to observe the transition to a ferromagnetic phase. As an example of the latter, it may be possible to suppress pair formation by using the interactions near narrow Feshbach resonances, where the pairs should have a much smaller overlap matrix element with the free atoms, but where the interatomic potential can no longer be considered as short-range.

Figure 2-19: Spin fluctuations (a) after 350 $\mu$s as a function of magnetic field and (b) on resonance as a function of hold time scaled to the value measured at 527G. Even at strong repulsive interactions, the measured spin fluctuations are barely enhanced, indicating only short-range correlations and no domain formation. The spin fluctuations were determined for square bins of 2.6 $\mu$m, each containing on average 1000 atoms per spin state.
Figure 2-20: Characterization of molecule formation at short and long hold times, and at different values of the interaction strength. The closed symbols, circles (black) at 790G with $k_F a = 1.14$, squares (blue) at 810G with $k_F a = 2.27$ and diamonds (red) at 818G with $k_F a = 3.5$ represent the normalized number of free atoms, the open symbols the total number of atoms including those bound in Feshbach molecules (open circles at 790G with $k_F a = 1.14$). The crosses (green) show the molecule fraction. The characteristic time scale is set by the Fermi time $\hbar/E_F = 43\mu$s, calculated with a cloud averaged Fermi energy.
Chapter 3

Development of an Apparatus for Sodium and Lithium Optical Lattice Experiments

The fluctuation experiments in the previous chapter were performed in an early BEC apparatus built in 1998 [54] that had been upgraded to become one of the first machines to achieve quantum degeneracy in fermions [55]. After more than a decade of pioneering work, including the first observation of fermionic superfluidity in optical lattices [56], this apparatus had finally begun to show its age. Not only had key apparatus components such as gate valves and titanium filaments begun to fail, but the precision of cold atom experiments and consequently the demands in terms of stability and of optical quality had advanced far beyond what anyone could have envisioned more than a decade prior.

Consequently, while the noise measurements were being made, primarily by Christian Sanner and myself, a next-generation apparatus was already in the process of being designed and prepared by the other members of the lab. Its initial design and construction are described in detail in Aviv Keshet's thesis [19].

This chapter describes the process of developing that machine from the point of its first magneto-optical trap into its current status as a versatile tool for studying cold atoms in optical lattices. The design of our next-generation apparatus was focused on
versatility rather than on one particular strength or area of research such as single-site resolution [13] or electron microscopy [57]. This flexibility proved valuable in allowing for a number of 'pivots' in the scientific research agenda of our lab. During apparatus construction the goal we envisioned was to observe antiferromagnetic ordering or correlations in the Fermi-Hubbard model [11, 58]. But during that time another opportunity presented itself: a different lab in our group working with $^{87}$Rb was in the process of realizing a novel scheme for implementing synthetic gauge fields, but they observed anomalous heating that prevented them from reaching the ground state of the Harper Hamiltonian [59]. It appeared that the higher recoil frequency of Na, coupled with the ease of achieving large gradients in our apparatus, might allow the same scheme to be implemented with a lower heating rate. The suspicion that those experimental difficulties were related to collisional physics motivated an extended study of the relationship between interactions and Bloch oscillations described in the next chapter. And now with the rubidium team’s strong evidence that they have observed the ground state of the Harper Hamiltonian, we have largely turned our focus towards implementing synthetic gauge fields with fermions. The result of these rapid pivots, is unfortunately, that these next two chapters may resemble a technical sketch more than an exposition of new scientific results.

3.1 Na Bose condensate in a plugged quadrupole

In our dual-species machine, the starting point for lattice experiments with both sodium and lithium is a Bose condensate of sodium. When operated in the dual-species mode the thermalization of Na with Li prevents it from condensing, but a large Na condensate in the single-species configuration, on the order of several $10^6$ atoms, demonstrates that enough cooling power is present to create a large degenerate sample of lithium.
3.1.1 Motivation for a plugged trap configuration

In the simplest magnetic trap for neutral atoms, two coils in anti-Helmholtz configuration are used to generate a 3D quadrupole field, where atoms experience a constant restoring force. This setup is not sufficient to achieve quantum degeneracy because the magnetic field at the trap center is zero, and so as the atoms cool they spend increasing amounts of time in its vicinity, where they are likely to undergo Majorana spin flips to untrapped states and subsequent heating and ejection from the trap.

There are three popular approaches to solving this problem. First, in the Ioffe-Pritchard design one can use additional coils to make the magnetic field nonzero at its local minimum [60]. Second, one can create a time-orbiting potential (TOP) trap by moving the magnetic field zero in a periodic way so that the time-averaged field is nonzero [61]. Third, one can combine the magnetic potential with an attractive [62] or repulsive [63] optical potential so that the atoms are trapped away from the region of zero field.

For comparable coil sizes and currents, the last 'plugged-trap' approach offers the tightest confinement and hence the fastest evaporation and cycle time – as modern atomic physics experiments become more data-intensive and time is at a premium, it has experienced a resurgence in popularity [64, 65]. The coil geometry is also the simplest, especially considering that with by reversing the current direction in one coil of the pair, it is nearly ideal for generating the large homogeneous bias fields used for tuning interactions near a Feshbach resonance [66].

The main disadvantages of the plugged quadrupole are the requirement of an additional blue-detuned laser and imaging along the plug axis, and its sensitivity to small pointing drifts of the optical plug, which are compounded by environmental instability.

3.1.2 Experimental implementation

We reduce the workflow effects of plug drift by making the plug easy to realign. By adding motorized actuators (Newport TRA12CC) to the last mirror before the vac-
uum chamber, the plug can be moved in repeatable 2 \mu rad steps without the need to physically manipulate the actuator, dramatically increasing the speed and reproducibility of plug realignments.

The plug beam itself comes from a Spectra-Physics Millenia Xs (diode-pumped solid-state laser) that outputs 9.5 W at 532 nm. The plug waist is estimated to be around 80 \mu m. Because the beam is repulsive, the potential near the trap minimum tends to be subject to corrugations from imperfections in the plug beam. Often, as shown in Fig 3-2, the trap will have multiple local minima. Optimization of the sample requires maximizing the condensate number in one specific minimum; to ensure a strong asymmetry the plug is moved off of the magnetic trap center, typically by a distance approximately 40 \mu m.

Forced rf evaporation of sodium typically occurs on either the $|F, m_F\rangle = |1, -1\rangle \rightarrow |2, -2\rangle$ or the $|F, m_F\rangle = |1, -1\rangle \rightarrow |1, 0\rangle$ transition. The first transition is often easier to use because it occurs at rf frequencies, but the large amount of rf radiation can interfere with the normal operation of electronic devices ranging from power supplies and actuators to flow and temperature meters. Since evaporation on a microwave transition near 1.7 GHz is necessary anyway when trying to evaporate sodium without evaporating lithium, we choose that transition for our experiment. The final atom number seems quite sensitive to the Rabi frequency for forced evaporation. We use a 50W microwave amplifier (EMPOWER 1119) located < 50 cm from the experimental apparatus to minimize attenuation and a single wire loop as the microwave antenna, since owing to the the evaporation frequencies ranging from 1.4 to 1.8 GHz it's impractical to use a resonant circuit to increase the RF power.

We load about $8 \times 10^8$ atoms into a magnetic trap with a gradient of 50 G/cm from a dark-spot MOT [67]. The trap is then ramped up to 750 G/cm for evaporation. While we are able to achieve BEC in 8s without trap decompression, we obtain a larger condensate of about $5 \times 10^6$ atoms after performing 9s of evaporation, then reducing the gradient to 75G/cm in 3s and performing 3s of evaporation in the decompressing trap. The number and temperature during the evaporation ramp are shown in Figs. 3-3 and 3-4.
Figure 3-1: Condensate in a plugged trap after 15ms of time of flight. Because of corrugations near the edges of the repulsive potential provided by the plug, the atoms are trapped in multiple local minima. Small misalignment of the plug maximizes the condensed atom number in a single minimum.

3.2 Optical potentials

3.2.1 Optics layout

Just as the choice between different magnetic trapping geometries has a cascading effect on the rest of the apparatus design, so does the choice of vacuum chamber. Experiments typically take place within either a small (~5cm wide) glass cell, which offers a large solid angle but suffers from stray reflections, or within a larger steel chamber with many viewports, where the range of incident angles is smaller but where beams can enter normally incident to antireflection coatings and where the vacuum chamber itself is more mechanically robust.

The large solid angle available for a glass cell, combined with the typically lower optical quality, means that it is generally preferable to direct beams into the exper-
Figure 3-2: Condensate imaged in trap, illustrating the effect of the plug on the trapped atoms.

iment at slightly different angles. In a steel chamber this is less desirable and so inevitably different beams must share the same optics. We prioritize the different beam paths based on the following rules:

- The MOT is the lowest priority – so low, in fact we can move the MOT mirrors out of the way during the RF evaporation using pneumatic actuators, without letting the accumulated drift impact the experiment repeatability.

- The lattice is the highest priority, both in terms of stability and in terms of the quality of the beam profile. Consequently the lattice should not be transmitted through more optical elements than necessary, which means that other than the MOT it should enter the chamber last. Since we would like there to be imaging along the lattice directions, the final lattice mirrors and lat-
Figure 3-3: Temperature plotted versus evaporation time. The blue dots correspond to temperatures in a tight magnetic trap used for the initial stage of evaporation, whereas the red dots correspond to temperatures in the decompressed magnetic trap normally used at the very end of evaporation. The temperature was calculated by fitting a Gaussian profile to the cloud in time of flight and using the width to estimate the average velocity in trap.

- The situation is slightly different for the top axis, owing to mechanical considerations – because of gravity, it’s hard to mount large numbers of optics in a vertical column in an extremely stable fashion. Therefore, we choose to attach the final mirrors for the vertical lattice to the vacuum chamber itself with short lever arms, requiring the MOT beams as well as the imaging beam to pass through the lattice dichroics. Below the vacuum chamber we use a mir-
Figure 3-4: Atom number plotted versus evaporation time. Again the blue and red dots are compressed and decompressed samples, respectively. Atom numbers are estimated using the resonant cross section.

ror mounted on a rotation stage (Newport URS75BCC) to switch between the MOT and imaging configurations.

Applying these rules generates the optics layouts shown in Figs. 3-5 and 3-6.

3.2.2 High power IR considerations

Optical lattice experiments work best when as much power as possible is available, particularly in the lattice beams. Since increasing confinement is easier than reducing it with blue-detuned beams, and since it is often desirable to work at low densities near one particle per site, we want large lattice beams with low curvature and simultaneously enough intensity to reach deep into the Mott insulator regime.
Figure 3-5: Main experiment layout (not to scale). This diagram illustrates the layout of the optics in the horizontal plane of the atoms, including the plug and the two horizontal lattice/imaging axes.

For us, this means that we want up to 10W per beam, which is large enough that some care must be taken in the various parts of the optical system. We use photonic crystal fibers from NKT photonics with the maximum possible mode area, the LMA-25 photonic crystal fibers that are not polarization maintaining. They are end-sealed (we find fused silica end caps unnecessary) and connectorized to SMA-905 high power connectors by a French technology center, Alphanov.

Because of its superior thermal properties, it is important that optics experiencing high intensities for long times be made from fused silica rather than a borosilicate glass like BK7. Our fiber collimators are fused silica components from Micro Laser Systems (FC5-Y-FS) or large area collimators from Kirchoff and Schacter (60FC-SMA-T-4-A100-03).
As a precaution to prevent cavity formation and damaging back-reflections into the fiber laser, we insert an optical isolator immediately after the fiber output on the experiment table. One consequence of this is that when we optimize the mode overlap of the retroreflected beam by maximizing the back-transmission of the retroreflected beam through the fiber, we use lock-in detection where the lattice beam is amplitude-modulated at 100 kHz.

Two other design choices involve significant tradeoffs. First, we use variable beam expanders (Special Optics 53-26-2-10X-İz) for the lattice beams, sacrificing some stability for flexibility. It turns out that finding the correct balance between density and lattice depth is somewhat difficult so we choose to make changing the beam size less cumbersome.
Second, to avoid increasing the number of lattice optics with an additional mirror, the lattice retroreflector is a dichroic that transmits the imaging beam, and the lens that collimates the lattice beam before the retroreflector is also the imaging front lens. This means that the imaging front lenses along the lattice directions are singlets, significantly limiting the usable f-number.

The left/right lattice and plug imaging axes are therefore 'rough' imaging axes and are fitted with a compact, non-cooled camera with higher noise and lower quantum efficiency (pixelfy QE). High-quality imaging occurs on the side and top axes, where bucket windows allow smaller focal lengths and lower f-numbers. Currently we work far below the maximum numerical apertures of 0.14 (side) and .9 (bottom): while the side bucket window was originally designed to house a specific microscope objective (5x Mittutoyo Plan APO NIR), mechanical errors in the bucket fabrication meant that the magnetic trap center is out of the field of view when the original high resolution objective is used.

For blocking IR and plug light from imaging pathways, we find that the best-performing filter is the reflective StopLine notch filter from Semrock, though if maximizing transmission is not a concern then Schott glass is both cheap and effective at blocking 532nm light.

Originally we intended to use LCD waveplates so the same beams could be used as lattices and ODTs, reducing the mass transport required throughout the system when ramping up the lattice. Unfortunately after investigating the Mott insulator lifetime we realized that the nematic liquid crystal waveplates we used for this purpose have intrinsic intensity noise that heats the sample: they are typically controlled by amplitude-modulating a square wave, with the dynamic range dropping off sharply above a modulation frequency of 5 kHz. This square wave ends up creating residual intensity modulation at that frequency of several percent, which rapidly heats the sample in the lattice, forcing us to return to the traditional configuration of having separate lattice and ODT beams.
3.2.3 Alignment considerations

Aligning the vertical lattice so that it points along the direction of the field gradient is important for preventing excess heating in modulation schemes. This is difficult to do with high accuracy but rough alignment can be performed using Stern-Gerlach separation of the different spin components in a magnetic field. By applying an RF sweep and then a magnetic field gradient, the gas is split into two pieces, displaced along the direction of the gradient. Applying Kapitza-Dirac pulses to the two-component gas separates each component into $0, \pm 2\hbar k$ momentum modes along the beam direction for six components in total. The lattice is roughly aligned with the magnetic field if all six components lie on a line, as shown in Fig. 3-7. Imaging in two horizontal directions verifies that the beam is aligned along both degrees of freedom.

![Figure 3-7: Stern-Gerlach calibration of the vertical lattice angle to lie on the magnetic coil axis. An rf pulse is applied to place the condensate in a mixture of $|F, m_F\rangle = |1, -1\rangle$ and $|1, 0\rangle$ states, then a magnetic field gradient separates the components of the mixture. After separation, a Kapitza-Dirac pulse diffracts the cloud into $\pm 2\hbar k$ components along the lattice direction.](image)

3.3 Data management

Each 'shot' of an atomic physics experiment is associated with a large amount of information: the experimental sequence and parameters, associated image files and
derived quantities, and metadata about the nature and purpose of performing that iteration.

In the current state of the field, most of that information is discarded or rendered inaccessible over time. Even if sequence files are saved, unless information is explicitly recorded in a lab book the process of unearthing the records, extracting relevant information, and associating them with images is sufficiently tedious that it is rarely performed. Searching old logs in a systematic way is even less feasible – even though information about earlier experiments may be highly relevant.

Inspired by the success of Cicero Word Generator, an experimental control program written by Aviv Keshet for our lab and now widely used elsewhere [68], we developed the 'Clinamen' application (screenshot in Figure 3-8) is to store all of the data associated with an experiment, including information about raw and processed images, in a structured database so that it can be easily searched and retrieved. It is built using a standard web application stack: we store data from Cicero Word Generator and image files into a MySQL relational database, and provide a Django web application (with user interface built on JQuery) for searching and accessing the data, and performing image processing.

The table schema is shown in Fig 3-9. Runs of the experiment are associated with variables and their values, and also with images and various processed images and derived quantities extracted from the raw frames.

One core recordkeeping problem is the association of image files with data about the experimental sequence. Like many other groups, we use a number of different cameras with proprietary and mutually incompatible software. In principle these cameras tend to have APIs so one could write a single piece of custom software for operating the cameras that also performs recordkeeping and analysis. What we decide to do instead is to use the image files saved to disk, and then after the fact associate experimental sequence data with image data using the timestamps of the image files.

Data is inserted into the database after every iteration in Cicero Word Generator – the sequence file, length, etc. are recorded along with the names and values of relevant variables. However, at the time of insertion the relevant image files may not
yet exist on disk, so we monitor the directories in which image files are stored and associate incoming files with experimental sequences based on their timestamps.

To facilitate the processing of images we create user-defined image 'types' such that every image of a given type is processed in the same way. This is necessary because the features of the atomic sample are typically not obvious from raw image files, but the processing that converts raw frames into, for example, absorption or fluorescence images depends on the imaging configuration since different configurations may have different numbers of frames in a different order. Therefore, we allow users to define an image type in the sequence description, and to use the web application to associate each image type with one or more processing methods that can generate either new, processed images (such as a transmission image) or derived quantities (such as a count of the atom number).
3.4 Environmental stability

Regulating the ambient temperature and humidity is crucial for the success of atomic physics experiments, and often neglected before making changes to the HVAC system becomes prohibitively difficult. Stabilizing the environment was one of the major goals for our recent apparatus upgrade, with a focus on enclosures and individual temperature control for the experiment and laser tables.

While this dramatically improved environmental stability, we still noticed alignment drifts over the period of weeks, as well as some unexplained fluctuations in the sample size. We improved temperature and humidity monitoring by installing the
Weathergoose II, a monitor designed for IT applications (server farms) which made it straightforward to log dozens of temperature monitors on a long-term basis. By logging the temperature at eleven different locations as well as the air flow, humidity, and air conditioning control we were able to identify additional improvements:

- The most important improvement was to identify temperature gradients that affect experiment stability even when enough cooling capacity is present. For example, in Fig. 3-10, on April 9 there is a large temperature increase in some parts of the lab and a large decrease in others, suggesting that cooling capacity is incorrectly distributed in the lab rather than globally insufficient. While higher air velocity reduces temperature gradients due to faster air mixing, it creates different sources of instability due to vibration and air currents. Once gradients are identified, however, they can be addressed by redirecting the output of the air conditioners.

- We also identified large fluctuations in the humidity, which we addressed by installing a dehumidifier to supplement the existing dehumidification performed by the air conditioners themselves.

- While substantially better than before, the environmental stability can still be improved, especially on days with extreme temperatures, by adding additional cooling capacity.

### 3.5 Outlook

Progress in realizing novel Hamiltonians in our apparatus has been slower than expected, due in part to a number of major equipment failures including the implosion of one of our vacuum windows and at least three catastrophic failures of our yellow laser system. However, we have developed an apparatus that is fast, versatile, and
reliable, and we believe that we are well-positioned to soon realize topological phases of lattice fermions as discussed in the next chapter. I look forward to learning about the exciting discoveries in the years ahead.

Figure 3-10: Sample temperature log from the Weathergoose temperature monitor. The ambient temperature is monitored at twelve measurement points spread out across the lab. On Apr 09, some of the sensors report a large increase in temperature while others report a large decrease, indicating that the air temperature is not homogenized throughout the lab.
Chapter 4

Bose-Einstein Condensates of Sodium in Tilted and Shaken Lattices

4.1 Optical dipole potentials

Because cold atoms' behavior can be so effectively described as that of ideal quantum particles, studying them resembles starting from a blank slate. One builds up, piece by piece, the potentials and interactions where complexity will manifest itself. Optical confinement is essential for this process – both for trapping the overall atom cloud [69], and for localizing atoms in a lattice so that their continuous degrees of freedom become discrete [70].

Light that is red-detuned to an atomic transition generates an attractive potential. The interaction of an atom with an oscillating electric field can be written as:

\[ H = -e \mathbf{E} \cdot \mathbf{d} \cos(\omega t) \]  \hspace{1cm} (4.1)

For a given pair of states \(|g\rangle\) and \(|e\rangle\) coupled to the radiation field, the ground state energy is shifted by two virtual processes; the atom can go to the excited state by either absorbing or emitting a photon. If we define the Rabi frequency \(\hbar \Omega = -eE \langle g|z|e\rangle\), then this is also the coupling to each dressed state and the perturbation
is:

\[ H'_\pm = \frac{1}{2} \begin{pmatrix} 0 & \Omega \\ \Omega & \omega_{eg} \pm \omega_L \end{pmatrix}. \] (4.2)

As long as \( \delta \gg \Omega \) the energy shift is proportional to the light intensity:

\[ \Delta E \approx \frac{\hbar \Omega^2}{4} \left( \frac{1}{\omega_L + \omega_{eg}} - \frac{1}{\omega_L - \omega_{eg}} \right) \] (4.3)

The heating rate is proportional to \( \gamma/\delta^2 \) and so optical traps are typically detuned by hundreds of nanometers. This means that the contribution from the counter-rotating term is non-negligible on the order of 30%.

We can create periodic potentials by interfering beams at the same frequency pointing in different directions. If \( E_1 = E_0 \hat{e} e^{i(k_1 \cdot r - \omega t)} \) and \( E_2 = E_0 \hat{e} e^{i(k_2 \cdot r - \omega t)} \) then the time averaged intensity is

\[ |E|^2 = 2E_0^2 \cos((k_1 - k_2) \cdot r) + 1. \] (4.4)

The most common configuration is with a retroreflected beam and \( k_2 = -k_1 \), creating a lattice with periodicity \( \lambda/2 \).

## 4.2 Optical lattices

### 4.2.1 Bloch Waves

When the Hamiltonian is periodic – that is, invariant under finite translations by the lattice spacing – its eigenfunctions take on a specific form. Since the Hamiltonian is a comb of spatial frequencies with evenly spaced teeth, we might expect its eigenfunctions to take on such a form as well. This is the intuition formalized by Bloch’s theorem [23]:

Let \( T = e^{iP \cdot a} \) be a unitary translation operator along a lattice basis vector \( a \), such
that \( T\psi(x) = \psi(x + a) \). If \( T \) is a symmetry of the Hamiltonian, that is,

\[
H = THT^\dagger
\]  

(4.5)

Then eigenfunctions of \( H \) are also eigenfunctions of \( T \):

\[
T\phi(x) = e^{ik\cdot a}\phi(x)
\]  

(4.6)

Rewriting this expression using \( \phi(x) = e^{ik\cdot x}u(x) \),

\[
Tu(x) = e^{-ik\cdot(x+a)}\phi(x + a) = u(x)
\]  

(4.7)

we see that the eigenstates can be written such that they are products of two parts, with the first a plane wave and the second a function \( u(x) \) with the lattice periodicity.

For the case of an optical lattice, which generates a cosine potential:

\[
H = \frac{p^2}{2m} + V_0 \cos^2(kx)
\]  

(4.8)

the Hamiltonian takes on a simple form.

\[
|q\rangle = e^{iq\cdot x} \sum_n c_n(q) e^{in\cdot q\cdot L\cdot x}
\]  

(4.9)

For a given \( q \), \( H \) is tridiagonal in the \( c_n \)'s and straightforward to solve numerically. Representative solutions are shown in Fig 4-1.

\[
H_{n,n} = \frac{(q + nq_j)^2}{2m}
\]

\[
H_{n,n\pm 1} = -V_0/4
\]  

(4.10)

4.2.2 Kapitza-Dirac diffraction

When setting up an optical lattice, the power and beam waist, and therefore the local intensity, are known. Nonetheless there are often small errors in focusing or
beam overlap and the atoms themselves are the best probe of the lattice depth. The standard technique for performing this measurement is Kapitza-Dirac diffraction and involves applying the lattice potential as a brief pulse. The zero-momentum state is projected onto the zero-quasimomentum state in each of the different bands of the lattice, which evolve at different energies. After the pulse the atomic wavefunction will have additional momentum components at $\pm 2\hbar k, \pm 4\hbar k$, etc given by:

$$\langle \psi | p = 2n\hbar k \rangle = \sum_i \left( \langle p = 2n\hbar k | \phi_i \rangle e^{iE_i(0)t} \langle \phi_i | p = 0 \rangle \right).$$

Where the $\phi_i$s are the Bloch band wavefunctions. A sample image of Kapitza-Dirac diffraction is shown in Fig 4-2, as is a plot of the calculated time-evolution.

### 4.2.3 Wannier basis and tight binding

While the Bloch functions diagonalize the Hamiltonian, they are spread out over all of the lattice sites and it is often preferable to work with localized wavefunctions when considering interactions or tilted lattices. Since the Bloch functions are narrow
Figure 4-2: Kapitza-Dirac diffraction picture after time of flight (top) and calculated occupancies (bottom). The blue, green, and red curves show the populations of the zeroth, first, and second orders, respectively.
combs in momentum space, intuition suggests that a localized function should be a broad sum over Bloch functions in quasimomentum space

\[ w_n(x - x_i) = \sqrt{\frac{a}{2\pi}} \int dq u_q^n(x) e^{-iq\cdot x_i} \]  

(4.12)

These are known as Wannier functions, and because the Bloch functions are only defined up to a phase factor, there are multiple choices for the Wannier basis. The naive choice for a localized function around \( x = 0 \) is to set the phases so that the \( u(x) \)s are positive real there and add up constructively. This also turns out to be the correct choice (for even bands) after a more rigorous analysis [71].

Conveniently, for deep lattices the Hamiltonian is simple in the Wannier basis:

\[ H = \sum_k \varepsilon_k a_k^\dagger a_k \]

\[ = \sum_{ij} (\int dk \varepsilon(k) e^{-ik\cdot (R_i - R_j)} b_i^\dagger b_j) \]  

(4.13)

In other words, given two sites separated by a distance \( d = R_i - R_j \), the lattice Hamiltonian introduces a hopping amplitude between them proportional to the Fourier component of the band dispersion \( \varepsilon(k) \) corresponding to \( d \). For a deep lattice the dispersion approaches a sine function so hopping only takes place between nearest neighbors.

Interactions are also simple in the Wannier basis. The usual pseudopotential approximation for s-wave scattering \( V = (4\pi a/\hbar)\delta(r) \) gives the following integral, since off-site interactions disappear quickly as the lattice depth increases:

\[ U = \int d^3x |w(x)|^4 \]

(4.14)

The dependence of \( J \) and \( U \) on the lattice depth \( V_0 \) is shown in Fig 4-3.
Figure 4-3: Tunneling rate $J$ and interaction strength $U$ plotted logarithmically as a function of lattice depth in recoils.

### 4.3 Superfluid and Mott insulating phases

The combination of tunneling and on-site interactions gives the Bose-Hubbard Hamiltonian:

$$H = \sum_{<ij>} -Ja_i^+a_j + \sum_i Ua_i^+a_i(a_i^+a_i - 1) + \sum_i \mu(x)a_i^+a_i$$  \hspace{1cm} (4.15)

where $\mu$ is the local chemical potential in the local density approximation, and the sum $<ij>$ is taken over nearest neighbors. The tunneling and interaction terms favor states with radically different character. If the first, kinetic term dominates, then the ground state has all of the particles at zero momentum. The superfluid nature of this state is apparent by factoring it into a product of local coherent states
with the same phase:

\[ |\psi\rangle = \prod_i \alpha_i^{t_1}|0\rangle = \prod_i (e^{\sqrt{n_i}b_i^\dagger}|0\rangle) \]  

(4.16)

If the second, onsite interaction dominates, then the ground state is a Mott insulator, is a product of Fock states with the same number of particles on each site.

\[ |\psi\rangle = \prod_i b_i^{t_{nm}}|0\rangle \]  

(4.17)

4.3.1 Mean-field theory

In between these two extremes the Hamiltonian cannot be solved analytically. We can use as an ansatz Gutzwiller's variational wavefunction that has the pure superfluid and pure Mott insulator as special cases, where the many-body wavefunction again factorizes as a product of on-site wavefunctions [72]:

\[ |\psi\rangle = \prod_i (f_0 + f_1 b_i^\dagger + f_2 b_i^{12} \cdots + f_n b_i^{t_{nm}})|0\rangle \]  

(4.18)

The interaction and kinetic energies are given by:

\[ H_{int} = \sum_n |f_n|^2(\mu n + Un(n-1)/2) \]  

(4.19)

\[ H_{kin} = -zJ|f_0|^2 f_1 + \cdots + \sqrt{3}f_2^* f_3 + \cdots|^2 \]

We can identify the boundary of the Mott insulator phase by considering its stability with respect to a small perturbation. Let almost all of the amplitude be in \( n \) with the a small fraction \( \alpha \) in states \( (n + 1) \) and \( (n - 1) \), . Then we expand to second order in \( \alpha \). The Landau theory of phase transitions suggests that the critical point is where the second derivative changes sign:

\[ E(\alpha) = zJa^2(\sqrt{n} + \sqrt{n+1})^2 + U\alpha^2 \]  

(4.20)

so the phase transition occurs, in this approximation, at \( U/J = 4nz \), where \( z \) is
the number of nearest neighbors.

### 4.3.2 Mott insulator lifetime

In real experiments, of course, the chemical potential varies across the trap so the system will be in a combination of Mott and superfluid phases with different occupations. Still, by expanding the cloud in time of flight is possible to observe the decay of coherence across the sample as the lattice depth is increased past about $20E_R$, as shown in Fig 4-4.

![Images of interference patterns](5Er.png, 8Er.png, 10Er.png, 12Er.png, 14Er.png, 16Er.png, 17Er.png, 18Er.png, 20Er.png, 22Er.png)

Figure 4-4: Superfluid to Mott insulator phase transition. In 100ms the lattice is adiabatically ramped up to its final value, then released from the trap. The sharp interference peaks in the expanded cloud indicate superfluid coherence and a uniform phase across the sample.

To demonstrate that the loss of coherence is in fact due to the Mott insulator phase transition, rather than from heating of the sample caused by the deeper lattice, and to characterize the technical heating in our apparatus, we also perform an experimental sequence where we ramp from the superfluid to the Mott insulator, then hold the sample for a variable time in the Mott insulator, then ramp back to the superfluid. The results are shown in Fig 4-5, and coherence mostly disappears from the sample after 100 ms. This reflects a significant increase from our initial measurements where vibrations from the cooling fan used for the microwave amplifier were limiting the lifetime, but it still offers room for improvement. The physics of the superfluid-Mott insulator phase transition was first explored in 2002 [73], but it remains the starting point for studying many different kinds of lattice physics.
Figure 4-5: Mott insulator lifetime. The lattice depth is ramped from $8E_R$ to $22E_R$ in 100ms, then held for a variable time (the last number in the individual image captions), then ramped back down to $8E_R$ in 20ms. The destruction of coherence with increasing hold time in the Mott insulator occurs as a result of technical heating, typically due to vibrations that shift the phase of the retroreflected lattice beam.

4.3.3 Return of coherence after a rapid quench

Below are the results of a preliminary experiment on the restoration of coherence after a rapid quench, in the 1D analogue of [74]. We start with a system of 'pancakes' held in a deep 1D lattice of $44E_R$ for 100 ms, where mean-field effects induce dephasing of the pancake BECs [75]. In 400µs we rapidly switch the lattice to $13E_R$ and hold it there for a variable time before releasing the sample from the trap. Fig 4-6 shows the restoration of coherence as determined by the number of atoms in $±2\hbar k$ momentum components.

These results were initially surprising in two ways: First, the return and loss of coherence occurs much faster than the tunneling timescale at that lattice depth $J = (2\pi) \times 75$Hz; the 'light cone' spreading of correlations observed in [76] happens on that scale. We expect that this discrepancy can be explained by the large number of particles in each pancake and the consequent Bose enhancement of tunneling: the tunneling amplitude scales as $J\langle N \rangle$ where $\langle N \rangle$ is the average number of particles per pancake. Second, the coherence decays quickly again after reappearing. This is reminiscent of the revival observed in [77], and we hope that additional theoretical and experimental work will provide insight into this system.
Figure 4-6: Restoration of coherence after a rapid ramp back to a shallow 13 $E_R$ lattice. The fraction of atoms in the $\pm 2\hbar k$ momentum components, an indication of the superfluid coherence of the sample, is plotted as a function of the hold time in the shallow lattice. The coherence undergoes a rapid revival and decay on a timescale much faster than the tunneling time $J \approx 75\text{Hz}$, indicating the Bose enhancement of tunneling.

4.4 Tilted lattices and Bloch oscillations

The competition between tunneling and on-site interaction produces the Mott insulator phase transition, and to engineer more complex systems it is often useful to introduce another relevant energy scale in addition to $J$ and $U$. One way this can be done is to apply a tilt, a uniform gradient potential which offsets the lattice sites by an energy $\Delta$ along one dimension.

Thought of in terms of quasimomentum, atoms in tilted lattices have their quasimomentum increase linearly until they are 'reflected' by the periodic boundary conditions at the edge of the Brillouin zone. These correspond to Bloch oscillations of the center of mass in position space.
For small tilts the clearest treatment of Bloch oscillations is semiclassical [78].

\[ \dot{k} = \hbar F \]
\[ \dot{r} = \frac{\partial \varepsilon}{\partial k} \]  

For a constant external force in the tight-binding limit,

\[ x - x_0 = \frac{2Jd}{F} \cos[(k_0 + Ft)d] \]  

Alternatively, in the limit of small interactions and strong tilts the Hamiltonian is diagonalized by the Wannier-Stark functions [79],

\[ |\phi(m)\rangle = \sum_n J_{n-m}(\frac{2J}{\Delta})w(x - x_n) \]

where the \( J \)'s are Bessel functions.

The evolution of the momentum distribution is shown in Fig 4-7. In these measurements we turn on a uniform magnetic field using magnetic coils in Helmholtz configuration, then rapidly turn one of them off. The rationale for this is as follows: due to the harmonic confinement, the gradient cannot be applied gradually because that will simply create an adiabatic shift to the new trap minimum. However, the large inductance of the coils means that the rate of change of the gradient is proportional to the voltage across the coil. By switching the coil off, rather than on, it is possible to switch at the maximum safe voltage for the system without the need to apply a power supply / capacitor combination.

Figure 4-7: Images of Bloch oscillations in a 1D lattice. A tilt potential of \( \Delta = \hbar \times (2\text{kHz}) \) is applied for a variable time ranging from 0 to 2 ms.
4.4.1 Dynamical instability

In the presence of interactions Bloch oscillations are damped by a dynamical instability that occurs above a certain critical value of the quasimomentum [80, 81]. One might expect the sample to be collisionally unstable anywhere above $q = 0$, but collisions must conserve energy and quasimomentum, and the concavity of the dispersion relation means that for a condensate at low quasimomentum, the particle that gains energy does not gain enough to compensate for the particle that loses energy. This is shown in Fig 4-8. Above that value, two-body collisions are allowed and the sample undergoes rapid heating.

![Figure 4-8: Rough illustration of the dynamical instability. A sinusoidal dispersion relation for a deep lattice is shown in blue. The red curve shows the final energy and momentum from a particle at point (1), provided that a second particle at (1) scatters to a lower-energy state. Since the curve lies below the dispersion relation, there is no way for this collision process to conserve energy and momentum. The green curve shows the same energy for a particle at point (2). Since it lies above the blue curve, the collision is allowed provided that the excess energy goes into transverse excitations. The extension of the curve beyond the boundary reflect so-called 'unklapp' scattering processes reflecting the periodicity of the Brillouin zone.](image)

We first observe this effect using an alternate experimental configuration for observing Bloch oscillations, where we create the gradient in a moving frame through constant acceleration of the lattice. This is most easily done by using a non-retroreflected
Raman lattice, where the velocity of the lattice is the product of the wavelength and the difference in frequency, and applying a constant frequency ramp to one of the lattice arms. Because the imaging system and transverse confinement exist in the stationary lab frame, this method has the disadvantage that the cloud can move out of the trap or the field of view in a matter of milliseconds. However, it has the advantage that the tilt can be easily stopped, restarted, and reversed, to observe behavior at different quasimomenta. For example, in Fig 4-9 we ramp up the quasimomentum to a target value and then hold it there, rather than letting it continue to increase linearly as in the case of a constant gradient, to observe the onset of the dynamical instability.

Figure 4-9: Observation of dynamical instability in a $4E_R$ Raman lattice. We ramp up the frequency to a variable final value, then the sample is held there for 2 ms before release from the trap. Above a critical value of the quasimomentum, there is rapid heating and loss of the sample. The image captions are arbitrary units proportional to the value of the quasimomentum before the hold.
4.4.2 Relaxation to zero-momentum mode

Surprisingly, in further measurements we observe a decay of the cloud down to the zero-quasimomentum mode, even though two-body collisions are forbidden at low quasimomenta. The experiments shown in Fig. 4-11 were done in a stationary lattice, where we first turn off one of the bias coils to initiate Bloch oscillations, then turn off the second coil in the pair to 'freeze' the quasimomentum. The images below are taken with a cloud initially at the Brillouin zone edge that can decay by either increasing or reducing its quasimomentum, a similar decay occurs when the cloud acceleration ends well below the zone boundary and decay occurs only in one direction.

Figure 4-10: Relaxation of Bloch oscillations. A tilt of $\Delta = \hbar \times (5.5\text{Khz})$ is applied to increase the quasimomentum until it reaches the edge of the Brillouin zone, then the tilt is switched off. Images are taken after an increasing hold time on the horizontal axis. The decay to the zero-quasimomentum mode cannot be explained by simple two-body collisions.

As in the earlier quench, the decay of the quasimomentum occurs much faster than the tunneling time scale $J$, as shown in Fig. 4-11. What is unknown is the nature of the decay mechanism: since it continues past the point where two-body collisions should be forbidden, one suspects that it should be a many-body process.
Figure 4-11: Decay of occupation of higher-momentum states in lattices of 6 $E_R$ (triangles) and 16 $E_R$ (squares). Bloch oscillations are stopped when the condensate’s quasimomentum is at the Brillouin zone edge, then after a variable delay the fraction of atoms remaining at the zone edge is plotted as a function of the hold time.

4.5 Gauge fields

By combining the ingredients above we can manipulate the tunneling phases so that the atoms in the lattice experience the Hamiltonian of a charged particle in a strong magnetic field:

Suppose we have an electron moving in a lattice in the $x-y$ plane with a constant magnetic field in the $z$ direction. If we take the tight-binding Hamiltonian, which simply consists of translations by the lattice spacing in the four in-plane directions:

\[
H = 2J[\cos(k_xa) + \cos(k_ya)]
\]  

(4.24)

and account for the magnetic vector potential by making the usual Peierls substitution
$p \rightarrow p + \frac{e}{c} A$, then for the Coulomb gauge $A = (0, x, 0)$ and $\phi = A c / h c$ we have

$$H = 2J [a_{n,m}^{\dagger} a_{n+1,m} + a_{n+1,m}^{\dagger} a_{n,m} + e^{i\phi} a_{n,m}^{\dagger} a_{n,m+1} + e^{-i\phi} a_{n,m+1}^{\dagger} a_{n,m}]$$

which corresponds to a magnetic flux through the plaquette of

$$B = \frac{1}{a^2} \oint A \cdot dl = \frac{h \phi}{ea^2}$$

where the line integral is taken over the straight line connecting the two lattice sites.

So an effective magnetic flux can be introduced by altering the tunneling phases in the $y$ direction so that they depend on $x$ [82].

Just as a finite translation in position is equivalent to imprinting a momentum-dependent phase, a finite translation in momentum is equivalent to imprinting a position-dependent phase. So to achieve an effective magnetic field we need a momentum kick in the $y$ direction that is different for the two directions of $x$ hopping.

We can do this by tilting the lattice and then applying an additional Raman lattice at the frequency of the tilt [59]. This is similar to methods that use a Raman lattice imposed on lattice of double wells [8, 83–85], and builds upon earlier work that constructs synthetic magnetic fields using internal states [7] and driven lattices [86].

$$V_K = \frac{V_0^r}{2} \cos((k_1 - \mathbf{k}_2) \cdot \mathbf{r} - \omega t)$$

$$V_K = V_0^r (e^{i(k \cdot r - \omega t)} + e^{-i(k \cdot r - \omega t)})$$

Treating the Raman beams as a perturbation and applying the rotating wave approximation, the additional tunneling term is:

$$H' = \sum_{nm} (b_{n,m}^{\dagger} b_{n+1,m} (\int d^2 r w(r - R_{nm}) w(r - R_{nm} - d_x) e^{i k \cdot r} + h.c.)$$

$$H' = \sum_{nm} (b_{n,m}^{\dagger} b_{n+1,m} e^{i k R_{nm}} \int dx w(r) w(r - d_x) e^{i k \cdot r} + h.c.)$$

which is exactly the Harper Hamiltonian. It has the remarkable feature that its band structure has a non-trivial topology, that is, the integral over the Brillouin zone of
the Berry curvature

$$\nu = \frac{i}{2\pi} \int_{BZ} \left( \langle \partial_{k_z} u(k) | \partial_{k_y} u(k) \rangle - \langle \partial_{k_y} u(k) | \partial_{k_z} u(k) \rangle \right)$$

(4.29)

can nonzero. For example, for the Raman lattice at 45° with period $\lambda/\sqrt{2}$, which corresponds to $\phi = \pi/2$, $\nu = 1$. The integral $\nu$ is a topological invariant of the band structure known as the Chern number and is always an integer. It is connected to a variety of mathematical objects that are not obviously related (and largely beyond the understanding of the author). For example, in a similar fashion to how the Gauss-Bonnet theorem relates the integral of the curvature to the number of 'handles' of a Riemann surface, the integral of the Berry curvature is a member of the integer cohomology class $H^2(X, \mathbb{Z})$. Moreover, it has a physical manifestation as the integer Hall conductivity:

$$\sigma_{xy} = \frac{e^2}{h} \sum \nu$$

(4.30)

where the sum is over all occupied bands.

Before we can measure the Chern number and other interesting properties of the band structure, we must prevent the sample from rapidly heating in the Raman lattice – as of the writing this reflects the immediate challenge facing our experiment. This heating is extremely fast and broad – the effect of adding a Raman lattice with different Raman detunings is shown in Fig 4-12

4.6 Outlook

If the heating is caused by interaction effects then reducing it may require lowering the density to less than one particle per site. But applying the Raman lattice to a system of fermions rather than bosons offers the possibility of filling the band so that the Fermi level sits in the gap between the first and second bands, reducing the sensitivity of the system to excitations whose energies are small on the scale of the bandgap. Normally such a system would be uninteresting, but in a strong magnetic field the full filling of the band makes the measurement of the Hall conductivity
more straightforward. Some details have to be worked out about the measurement of conductivity in a pre-existing tilt, but it appears to be a promising next step.
Figure 4-12: Heating of the sample in a $1.6E_R$ Raman lattice applied for 10ms. The individual image captions are the Raman detunings, in KHz. Heating occurs at frequencies within the first band, and also at fractions of the bandgap. Coherence is rapidly destroyed on a timescale faster than the tunneling.
Chapter 5

Outlook

We conclude by speculating upon prospects for future work: the future of fluctuation measurements, of experiments in our new apparatus, and of quantum simulation at large. In the Introduction we argued that over the long term, quantum simulations will focus increasingly on dynamics and on fermions. Here we consider a few topics of current interest:

5.1 Dynamics

Our work in the old apparatus has demonstrated that spin and density fluctuations can be measured quantitatively and used to characterize density and pair correlations (and the analogous thermodynamic response functions) in a variety of Fermi systems. These measurements have the potential to become a general tool and part of a set of overlapping and complementary methods used to characterize the state of quantum simulations. Fluctuation measurements, Bragg scattering, and Bragg spectroscopy measure the structure factor (and dynamic susceptibility) $S(k, \omega)$ in different regimes [87, 88], which coincide with the pair and density correlations that are measured microscopically by measuring double occupancies and individual occupations. As the space of interesting Hamiltonians that can be experimentally simulated grows, refinement of these techniques will be essential to characterize the states that result.

Fluctuation techniques offer perhaps the most obvious benefits in dynamical mea-
surements: because the fluctuations measurements are optical and in situ they are essentially instantaneous, allowing for the ability to measure the evolution of correlations after a rapid quench. Quench experiments have become increasingly popular [74, 76] and one expects that, owing to fundamental limitations to simulating them with computers, their importance will only grow. In particular, many questions about ferromagnetism in weakly-bound fermions remain unanswered – while we have demonstrated that there is no ferromagnetic phase transition for free atoms on the upper branch of a broad Feshbach resonance, it remains an open question whether a phase transition analogous to the Stoner instability can be realized in similar but slightly modified systems.

5.2 Quantum magnetism

In recent years the atomic physics research agenda has been propelled in no small part by the vision of determining the phase diagram of the Fermi-Hubbard model. This system continues to be an important target: historically important, conceptually simple to realize, and gateway to to a host of interesting questions about quantum magnetism and criticality [89]. Unfortunately the achievement of long-range order at the superexchange energy scale $J^2/U$ remains a formidable engineering challenge that after more than a decade nobody quite knows how to solve.

There are a variety of candidate schemes for achieving the requisite entropies, including use of a bosonic [90], single-component [91] or low-density [11] reservoir, or doubling the period of a band insulator [92], but it is still unclear which if any of these will be successful. In this regard our next-generation apparatus represents a hedged bet, not committed to any particular cooling scheme but focused on general stability and versatility to be prepared to seize opportunities should they present themselves.
5.3 Topology

In contrast, breakthroughs in the realization of topological physics with ultracold atoms have been happening at a breakneck rate, including the achievement of important milestones in the observation of topological states in single-particle band structures. The group of Tillman Esslinger observed the topological phase transition in the single-particle band structure of the Haldane model [93], and the group of Immanuel Bloch has measured the (integer) quantized Hall conductivity in the Harper Hamiltonian, which corresponds to the integrated Berry curvature that constitutes a topological invariant [8].

New frontiers include the realization of topological spin effects [94], non-Abelian physics [95, 96], time-reversal symmetric topological insulators [97], and strongly-coupled topological states in the fractional quantum Hall regime [98, 99]. We hope that in our apparatus we will be able to implement a uniform magnetic flux in a system of lattice fermions, and make important contributions to this conversation.
Appendix A

Suppression of Density Fluctuations in a Quantum Degenerate Fermi Gas

This chapter contains a reprint of Ref. [15]: C Sanner, EJ Su, A Keshet, R Gommers, Y Shin, W Huang, W Ketterle. Physical review letters 105 (4), 040402
Suppression of Density Fluctuations in a Quantum Degenerate Fermi Gas

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We study density profiles of an ideal Fermi gas and observe Pauli suppression of density fluctuations (atom shot noise) for cold clouds deep in the quantum degenerate regime. Strong suppression is observed for probe volumes containing more than 10,000 atoms. Measuring the level of suppression provides sensitive thermometry at low temperatures. After this method of sensitive noise measurements has been validated with an ideal Fermi gas, it can now be applied to characterize phase transitions in strongly correlated many-body systems.

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Systems of fermions obey the Pauli exclusion principle. Processes that would require two fermions to occupy the same quantum state are suppressed. In recent years, several classic experiments have directly observed manifestations of Pauli suppression in Fermi gases. Antibunching and the suppression of noise correlations are a direct consequence of the forbidden double occupancy of a quantum state. Such experiments were carried out for electrons [1–3], neutral atoms [4,5], and neutrons [6]. In principle, such experiments can be done with fermions at any temperature, but in practice low temperatures increase the signal. A second class of (two-body) Pauli suppression effects, the suppression of collisions, requires a temperature low enough such that the de Broglie wavelength of the fermions becomes larger than the range of the interatomic potential and p-wave collisions freeze-out. Experiments observed the suppression of elastic collisions [7] and of clock shifts in radio frequency spectroscopy [8,9].

Here we report on the observation of Pauli suppression of density fluctuations. This is, like the suppression of collisions between different kinds of fermions [10], a many-body phenomenon which occurs only at even lower temperatures in the quantum degenerate regime, where the Fermi gas is cooled below the Fermi temperature and the low lying quantum states are occupied with probabilities close to 1. In contrast, an ideal Bose gas close to quantum degeneracy shows enhanced fluctuations [11].

The development of a technique to sensitively measure density fluctuations was motivated by the connection between density fluctuations and compressibility through the fluctuation-dissipation theorem. In this Letter, we validate our technique for determining the compressibility by applying it to the ideal Fermi gas. In future work, it could be extended to interesting many-body phases in optical lattices which are distinguished by their incompressibility [12]. These include the band insulator, Mott insulator, and also the antiferromagnet for which spin fluctuations, i.e., fluctuations of the difference in density between the two spin states are suppressed.

Until now, sub-Poissonian number fluctuations of ultracold atoms have been observed only for small clouds of bosons with typically a few hundred atoms [13–16] and directly [17, 18] or indirectly [19] for the bosonic Mott insulator in optical lattices. For fermions in optical lattices, the crossover to an incompressible Mott insulator phase was inferred from the fraction of double occupations [20] or the cloud size [21]. Here we report the observation of density fluctuations in a large cloud of fermions, showing sub-Poissonian statistics for atom numbers in excess of 10,000 per probe volume.

The basic concept of the experiment is to repeatedly produce cold gas clouds and then count the number of atoms in a small probe volume within the extended cloud. Many iterations allow us to determine the average atom number \( N \) in the probe volume and its variance \( \langle \Delta N \rangle^2 \). For independent particles, one expects Poisson statistics, i.e., \( \langle \Delta N \rangle^2 / \langle N \rangle = 1 \). This is directly obtained from the fluctuation-dissipation theorem \( \langle \Delta N \rangle^2 / \langle N \rangle = n k_B T \kappa_T \), where \( n \) is the density of the gas, and \( \kappa_T \) the isothermal compressibility. For an ideal classical gas \( \kappa_T = 1/(n k_B T) \).

FIG. 1. Phase space diagram of ballistic expansion of a harmonically trapped Fermi gas. Ballistic expansion conserves phase space density and shears the initially occupied spherical area into an ellipse. In the center of the cloud, the local Fermi momentum and the sharpness of the Fermi distribution are scaled by the same factor, keeping the ratio of local temperature to Fermi energy constant. The same is true for all points in the expanded cloud relative to their corresponding unscaled in-trap points.
and one retrieves Poissonian statistics. For an ideal Fermi gas close to zero temperature with Fermi energy $E_F$, $\kappa_T = 3/(2nE_F)$, and the variance $(\Delta N)^2$ is suppressed below Poissonian fluctuations by the Pauli suppression factor $3k_B T/(2E_F)$. All number fluctuations are thermal, as indicated by the proportionality of $(\Delta N)^2$ to the temperature in the fluctuation-dissipation theorem. Only for the ideal classical gas, where the compressibility diverges as $1/T$, one obtains Poissonian fluctuations even at zero temperature.

The counting of atoms in a probe volume can be done with trapped atoms, or after ballistic expansion. Ballistic expansion maintains the phase space density and therefore the occupation statistics. Consequently, density fluctuations are exactly rescaled in space by the ballistic expansion factors as shown in Fig. 1 [22,23]. Note that this rescaling is a unique property of the harmonic oscillator potential, so future work on density fluctuations in optical lattices must employ in-trap imaging. For the present work, we chose ballistic expansion. This choice increases the number of fully resolved bins due to optical resolution and depth of field, it allows adjusting the optimum optical density by choosing an appropriate expansion time, and it avoids image artifacts at high magnification.

We first present our main results, and then discuss important aspects of sample preparation, calibration of absorption cross section, data analysis and corrections for photon shot noise. Figure 2(a) shows an absorption image of an expanding cloud of fermionic atoms. The probe volume, in which the number of atoms is counted, is chosen to be 26 $\mu$m in the transverse directions, and extends through the entire cloud in the direction of the line of sight. The large transverse size avoids averaging of fluctuations due to finite optical resolution. From 85 such images, after careful normalization [24], the variance in the measured atom number is determined as a function of position. After subtracting the photon shot noise contribution, a 2D image of the atom number variance $(\Delta N)^2$ is obtained. For a Poissonian sample (with no suppression of fluctuations), this image would be identical to an absorption image showing the number of atoms per probe volume. This is close to the situation for the hottest cloud (the temperature was limited by the trap depth), whereas the colder clouds show a distinct suppression of the atom number variance, especially in the center of the cloud where the local $T/T_F$ is smallest.

In Fig. 3, profiles of the variance are compared to theoretical predictions [25,26]. Density fluctuations at wave vector $q$ are proportional to the structure factor $S(q, T)$. Since our probe volume (transverse size 26 $\mu$m) is much larger than the inverse Fermi wave vector of the expanded cloud ($1/q_F = 1.1$ $\mu$m), $S(q = 0, T)$ has been integrated along the line of sight for comparison with the experimental profiles. Within the local density approximation, $S(q = 0, T)$ at a given position in the trap is the binomial variance $n_F(1 - n_F)$ integrated over all momenta, where the occupation probability $n_F(k, \mu, T)$ is obtained from the Fermi-Dirac distribution with a local chemical potential $\mu$ determined by the shape of the trap. Figure 4 shows the dependence of the atom number variance on atom number for the hot and cold clouds. A statistical analysis of the data used in the figure is in [24].

The experiments were carried out with typically $2.5 \times 10^9$ $^6$Li atoms per spin state confined in a round crossed dipole trap with radial and axial trap frequencies $\omega_r = 2\pi \times 160$ s$^{-1}$ and $\omega_z = 2\pi \times 230$ s$^{-1}$ corresponding to an in-trap Fermi energy of $E_F = k_B \times 2.15$ $\mu$K. The sam-
ple was prepared by laser cooling followed by sympathetic cooling with $^{23}\text{Na}$ in a magnetic trap. $^6\text{Li}$ atoms in the highest hyperfine state were transferred into the optical trap, and an equal mixture of atoms in the lowest two hyperfine states was produced. The sample was then evaporatively cooled by lowering the optical trapping potential at a magnetic bias field $B = 320 \pm 5$ G where a scattering length of $-300$ Bohr radii ensured efficient evaporation. Finally, the magnetic field was increased to $B = 520 \pm 5$ G, near the zero crossing of the scattering length. Absorption images were taken after 7 ms of ballistic expansion.

We were careful to prepare all samples with similar cloud sizes and central optical densities to ensure that they were imaged with the same effective cross section and resolution. Hotter clouds were prepared by heating the colder cloud using parametric modulation of the trapping potential. For the hottest cloud this was done near 520 G to avoid excessive evaporation losses.

Atomic shot noise dominates over photon shot noise only if each atom absorbs several photons. As a result, the absorption images were taken using the cycling transition to the lowest lying branch of the $^2P_{3/2}$ manifold. However, the number of absorbed photons that could be tolerated was severely limited by the acceleration of the atoms by the photon recoil, which Doppler shifts the atoms out of resonance. Consequently, the effective absorption cross section depends on the probe laser intensity and duration. To remove the need for nonlinear normalization procedures, we chose a probe laser intensity corresponding to an average of only 6 absorbed photons per atom during a 4 $\mu$s exposure. At this intensity, about 12% of the $^6\text{Li}$ saturation intensity, the measured optical density was 20% lower than its low-intensity value [24]. For each bin, the atom number variance $(\Delta N)^2$ is obtained by subtracting the known photon shot noise from the variance in the optical density $(\Delta \text{OD})^2$ [24]:

$$\frac{\sigma^2}{A^2} (\Delta N)^2 = (\Delta \text{OD})^2 - \frac{1}{\langle N_1 \rangle} - \frac{1}{\langle N_2 \rangle} \tag{1}$$

Here, $\langle N_1 \rangle$ and $\langle N_2 \rangle$ are the average photon numbers per bin of area $A$ in the image with (without) atoms and $\sigma$ is the absorption cross section.

The absorption cross section is a crucial quantity in the conversion factor between the optical density and the number of detected atoms. For the cycling transition, the resonant absorption cross section is $2.14 \times 10^{-13}$ m$^2$. Applying the measured 20% reduction mentioned above leads to a value of $1.71 \times 10^{-13}$ m$^2$. This is an upper limit to the cross section due to imperfections in polarization and residual line broadening. An independent estimate of the effective cross section of $1.48 \times 10^{-13}$ m$^2$ was obtained by comparing the integrated optical density to the number of fermions necessary to fill up the trap to the chemical potential. The value of the chemical potential was obtained from fits to the ballistic expansion pictures that allowed independent determination of the absolute temperature and the fugacity of the gas. We could not precisely assess the accuracy of this value of the cross section, since we did not fully characterize the effect of a weak residual magnetic field curvature on trapping and on the ballistic expansion. The most accurate value for the effective cross section was determined from the observed atom shot noise itself. The atom shot noise in the wings of the hottest cloud is Poissonian, and this condition determines the absorption cross section. Requiring that the slope of variance of the atom number $(\Delta N)^2$ vs atom number $N$ is unity (see Fig. 4) results in a value of $(1.50 \pm 0.12) \times 10^{-13}$ m$^2$ for the effective cross section in good agreement with the two above estimates.

The spatial volume for the atom counting needs to be larger than the optical resolution. For smaller bin sizes (i.e., small counting volumes), the noise is reduced since the finite spatial resolution and depth of field blur the absorption signal. In our setup, the smallest bin size without blurring was determined by the depth of field, since the size of the expanded cloud was larger than the depth of field associated with the diffraction limit of our optical system. We determined the effective optical resolution by binning the absorption data over more and more pixels of the CCD camera, and determining the normalized central variance $(\Delta N)^2/N$ vs bin size [24]. The normalized variance increased and saturated for bin sizes larger than 26 $\mu$m (in the object plane), and this bin size was used in the data analysis. We observe the same suppression ratios for bin sizes as large as 40 $\mu$m, corresponding to more than 10,000 atoms per bin.

For a cold fermion cloud, the zero temperature structure factor $S(q)$ becomes unity for $q > 2q_F$. This reflects the
fact that momentum transfer above $2q_F$ to any particle will not be Pauli suppressed by occupation of the final state. In principle, this can be observed by using bin sizes smaller than the Fermi wavelength, or by Fourier transforming the spatial noise images. For large values of $q$, Pauli suppression of density fluctuations should disappear, and the noise should be Poissonian. However, our imaging system loses its contrast before $q = 2q_F$ [24].

Observation of density fluctuations, through the fluctuation-dissipation theorem, determines the product of temperature and compressibility. It provides an absolute thermometer, as demonstrated in Fig. 3 if the compressibility is known or is experimentally determined from the shape of the density profile of the trapped cloud [17,27]. Because variance is proportional to temperature for $T \ll T_F$, noise thermometry maintains its sensitivity at very low temperature, in contrast to the standard technique of fitting spatial profiles.

Density fluctuations lead to Rayleigh scattering of light. The differential cross section for scattering light of wave vector $k$ by an angle $\theta$ is proportional to the structure factor $S(q)$, where $q = 2k \sin(\theta/2)$ [26]. In this work, we have directly observed the Pauli suppression of density fluctuations and therefore $S(q) < 1$, implying suppression of light scattering at small angles (corresponding to values of $q$ inversely proportional to our bin size). How are the absorption images affected by this suppression? Since the photon recoil was larger than the Fermi momentum of the expanded cloud, large-angle light scattering is not suppressed. For the parameters of our experiment, we estimate that the absorption cross section at the center of a $T = 0$ Fermi cloud is reduced by only 0.3% due to Pauli blocking [28]. Although we have not directly observed Pauli suppression of light scattering, which has been discussed for over 20 years [28–30], by observing reduced density fluctuations we have seen the underlying mechanism for suppression of light scattering.

In conclusion, we have established a sensitive technique for determining atomic shot noise and observed the suppression of density fluctuations in a quantum degenerate ideal Fermi gas. This technique is promising for thermometry of strongly correlated many-body systems and for observing phase-transitions or cross-overs to incompressible quantum phases.

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Note added in proof.—Results similar to ours are reported in Ref. [31].

EXPERIMENTAL DETAILS

To accurately measure the atom number variance it is necessary to eliminate patterns in the absorption images whose fluctuations increase the observed noise. Weak reflections of the probe beam from the walls of the glass cell and from optical elements in the imaging system can interfere with the probe beam itself, leading to spatial fluctuations in its intensity profile. To reduce interference fringes and ensure uniform illumination, the central area of the probe beam is imaged onto the sample through a 2 mm aperture. Even though the residual fringes are small, there are two significant effects of inhomogeneous illumination which must be addressed.

First, if the time elapsed between the image with atoms and the reference image without atoms is too large, mechanical vibrations of the optics will cause the intensity profile of the probe to change between the two images, creating artifacts in the absorption image. To reduce this effect, we operate our CCD in fast kinetics mode, with a time interval \( \approx 500\mu s \) between exposures. Since there is no longer enough time for the atoms to exit the frame between images, before taking the reference image we optically pump the atoms from the \( \left| 1 \right\rangle \) state to the \( \left| 16 \right\rangle \) state, and from the \( \left| 12 \right\rangle \) state to the \( \left| 15 \right\rangle \) state (\( \left| 1 \right\rangle \) refers to the lowest hyperfine state, etc.), by exciting them to the \( m_J = 1/2 \) excited state manifold. At the magnetic fields used in the experiment, these levels are separated in frequency from the \( \left| 2 \right\rangle \) state used for imaging by \( \approx 2 \) GHz and contribute negligibly to resonant imaging.

Second, if the average probe intensity is too high, the atoms subjected to higher intensities will have a lower effective cross-section, and so any spatial fluctuations in the beam intensity will be ‘imprinted’ onto the absorption images. As a result, in our experiment we use a probe beam with maximum intensity of 0.12 of the saturation intensity \( I_{\text{sat}} = 2.54 \text{ mW/cm}^2 \), where these effects are relatively small. The variation of optical density with intensity is shown in Fig. 1.

Additionally, the exposure time must be kept very short to prevent the atoms from moving between pixels during the exposure. The expected motion of atoms during the 4 \( \mu s \) exposure is on the order of 1 \( \mu m \), much smaller than the effective pixel size.

NOISE DETERMINATION

In this experiment the local atom number variance is determined by comparing the measured number of atoms in the same bin across a series of images. To do this, we must first eliminate the effect of fluctuations in the total atom number between experimental cycles. Initially, we select the 85 images used in the analysis from a larger group of \( \approx 150 \) images, using an automated procedure to choose the images whose total atom numbers are closest to the center of the distribution. A very small number of images (<1%) are manually excluded because of obvious artifacts in the frame due to dust particles or other large perturbations. Then, we subtract a fitted profile from each OD image before computing the variance. Initially we subtracted a fitted 2D Thomas-Fermi profile, but we replaced this with a Gaussian fit which had an insignificant effect on the variances, while taking considerably less computation time.

We then compute the variance in optical density at each position. That variance has contributions from photon and atom shot noise, given by the following formula:

\[
(\Delta(OD))^2 = \frac{1}{\langle N_1 \rangle} + \frac{1}{\langle N_2 \rangle} + \frac{\sigma^2}{A^2} (\Delta N_{\text{atom}})^2
\]

This equation holds bin by bin: \( N_1 \) is the average number of photons measured in a given bin position for the image with atoms, and \( N_2 \) is the average number of photons measured in that bin for the reference image. \( (\Delta N_{\text{atom}})^2 \) is the variance in atom number for that bin, \( \sigma \) is the
FIG. 2: Atom number variance vs. atom number. (a) Data for all of the resolution elements is plotted. Red points are from the hot cloud at $T/T_F = 0.6$, blue points from the cold cloud at $T/T_F = 0.21$. There is significant scatter in the variance data, and there are many “cold” pixels which actually have higher variance than their corresponding “hot” pixel. (b) The red and blue shaded regions indicate the expected 2σ scatter in sample variance that is expected due to atom and photon counting statistics. The large circles are variance data averaged over pixels with similar atom number for hot (red) and cold (blue) cloud. The bars show the measured 2σ scatter of the data points. The measured scatter agrees very well with the expected scatter, indicating that the scatter of the data is fully accounted for by counting statistics. Negative values of the observed atom number variance result from the subtraction of photon shot noise.

FIG. 3: Determination of profiles of the atom number variance for a cold cloud. For each bin, the total photon count is determined, and its contribution (red) to the total variance of the optical density (blue) is subtracted. The obtained atom number variance (green) is compared to the average atom number (black). The displayed trace reveals 50% noise suppression in the center of the cloud. The apparently high suppression in the wings is a statistical fluctuation. Fig. 2 shows that the suppression is monotonic in atomic density.

The atom number variance is isolated by calculating the first two photon shot noise terms and subtracting them. The analysis used in the paper also subtracts contributions from detector read noise and photon shot noise in the dark field, but these are fairly small contributions.

The determination of $N_1$ and $N_2$ depends on the CCD gain, which is measured to be 1.18 (counts/electron) from $\approx 240$ pairs of images without atoms, employing the assumption that the detector statistics are Poissonian. After the subtraction of photon shot noise (and technical noise), the remaining variance in optical density is due to the atom number variance. Fig. 3 shows the contributions of photon and atom number variance to the overall noise in optical density.

The large scatter of the measured atom number variance, as depicted in Fig. 2, is not primarily due to technical noise, but instead a statistical property of the sampling distribution of the variance. The shaded areas are derived from theoretical values for the variance of the sample variance. This is given by

$$\text{Var(Var}(N)) = \frac{(m-1)^2}{m^3} \mu_4 - \frac{(m-1)(m-3)}{m^3} \mu_2$$

where $m$ is the number of observations in each sample. The moments $\mu_2$ and $\mu_4$ are the central moments of the population distribution. For a Poisson distribution, $\mu_2 = \langle N \rangle^3$ and $\mu_4 = \langle N \rangle(1 + 3\langle N \rangle)$, and for $m, \langle N \rangle \gg 1$, this expression reduces to $2\langle N \rangle^2/m$. Fig. 2b shows the
comparison between the expected and measured variance in the sample variance.

IMAGING SYSTEM CHARACTERIZATION

The blurring of adjacent pixels due to finite optical resolution effectively decreases the measured atom number variance. This effect is avoided by binning the data using a sufficiently large bin size (Fig. 5). In our experiment, this bin size is determined by the extension of the cloud along the optical axis, which is much larger than the depth of focus of the diffraction limit of the lens system.

Atom noise allows us to characterize the transfer function of our imaging system. Fig. 4 shows the average power spectrum (modulus squared of the spatial Fourier transform) of the optical density images. Because the Fourier transform of uncorrelated fluctuations is flat, the deviation from flatness of the density noise corresponds to blurring induced by the lens, barring the central peak corresponding to the shape of the cloud. For wavevectors $q$ much larger than the resolution limit of the detection scheme, the atom number fluctuations are no longer imaged, and the power spectrum is the photon shot noise. For our experiment this happens for $q < q_F$. Comparison of the power spectra for the cold and the hot cloud shows, at small values of $q$, a 50% suppression, consistent with the results obtained using spatial bins. If the imaging system still had contrast at $q > 2q_F$, we would expect the ratio of the power spectra to approach unity, since momentum transfer $q > 2q_F$ to a Fermi cloud has negligible Pauli suppression.
FIG. 4: (a) Radially averaged power spectra of optical density images for hot (solid line) and cold (dashed line) samples. (b) Power spectrum of cold sample (arbitrary units). (c) Power spectrum of hot sample (arbitrary units). A constant offset is added to the power spectrum for the hot sample to equalize the levels of photon shot noise.

FIG. 5: Observed atom number variance versus bin size for heated (dashed line) and cold (solid line) samples, normalized to 1 for Poissonian statistics. A plateau is reached when the blurring of the bins due to finite optical resolution is negligible.
Appendix B

Speckle imaging of spin fluctuations in a strongly interacting Fermi gas

This chapter contains a reprint of Ref. [16]: C Sanner, EJ Su, A Keshet, W Huang, J Gillen, R Gommers, W Ketterle, Physical review letters 106 (1), 010402
Speckle Imaging of Spin Fluctuations in a Strongly Interacting Fermi Gas

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Spin fluctuations and density fluctuations are studied for a two-component gas of strongly interacting fermions along the Bose-Einstein condensate-BCS crossover. This is done by in situ imaging of dispersive speckle patterns. Compressibility and magnetic susceptibility are determined from the measured fluctuations. This new sensitive method easily resolves a tenfold suppression of spin fluctuations below shot noise due to pairing, and can be applied to novel magnetic phases in optical lattices.

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One frontier in the field of ultracold atoms is the realization of quantum systems with strong interactions and strong correlations. Many properties of strongly correlated systems cannot be deduced from mean density distributions. This has drawn interest toward novel ways of probing cold atoms, e.g., via rf spectroscopy [1,2], Bragg and Raman scattering [3], interferometric methods [4,5], and by recording density correlations [6-8]. Further insight into quantum systems is obtained by looking not only at expectation values, but also at fluctuations. Several recent studies looked at density fluctuations, either of bosons around the Mott insulator transition [9-11], or of a gas of noninteracting fermions [12,13].

In this Letter, we extend the study of fluctuations of ultracold gases in several ways. First, we introduce the technique of speckle imaging as a simple and highly sensitive method to characterize fluctuations. Second, we apply it to a two-component Fermi gas across the Bose-Einstein condensate (BEC)-BCS crossover. Third, we directly measure fluctuations in the magnetization, i.e., the difference of the densities in the two different spin components, bypassing the need to measure the individual densities separately.

Our work is motivated by the prospect of realizing wide classes of spin Hamiltonians using a two-component gas of ultracold atoms in an optical lattice [14,15]. An important thermodynamic quantity to characterize two-component systems is the spin susceptibility, which provides a clear signature of phase transitions or crossovers involving the onset of pairing or magnetic order [16-19]. At a ferromagnetic phase transition the susceptibility diverges, whereas in a transition to a paired or antiferromagnetic phase the susceptibility becomes exponentially small in the ratio of the pair binding energy (or antiferromagnetic gap) to the temperature. The fluctuation-dissipation theorem relates response functions to fluctuations, consequently the spin susceptibility can be determined by measuring the fluctuations in the relative density of the two spin components.

In our experiment, we image the atom clouds using light detuned from resonance so that each atom’s real polarizability, which contributes to the refractive index, is much larger than its imaginary polarizability, which contributes to absorption. Since the detunings for the two spin states are different, spin fluctuations lead to fluctuations in the local refractive index, resulting in phase shifts of the imaging light that vary randomly in space. We measure these phase shifts by imaging the resulting speckle patterns.

These speckle patterns are created by propagation, which converts the spatially varying phase shifts of the imaging light into an intensity pattern on our detector without the use of a phase plate. Spin and density fluctuations occur on all spatial scales down to the interatomic separation; the smallest observable fluctuations have a wavelength equal to the imaging system’s maximum resolution. In our system that length has a Rayleigh range, and hence a depth of field, smaller than the cloud size, so the recorded image is necessarily modified by propagation effects. Propagation mixes up amplitude and phase signals [Fig. 1]. This can be easily seen in the case of a phase grating, which creates an interference pattern further downstream; after propagating for a distance equal to the Rayleigh range of the grating spacing, the imprinted phase is converted into an amplitude pattern. This feature of speckle makes our imaging technique both simple and robust. It is insensitive against defocusing, and allows us to image fluctuations of the real part of the refractive index (i.e., a phase signal) without a phase plate or other Fourier optics.

Similar physics is responsible for laser speckle when a rough surface scatters light with random phases [20], and occurs when a Bose-Einstein condensate with phase fluctuations develops density fluctuations during expansion [21], or when a phase-contrast signal is turned into an amplitude signal by deliberate defocusing [22].

The experiments were performed with typically $10^6 \ ^6 \text{Li}$ atoms in each of the two lowest hyperfine states $|1\rangle$ and $|2\rangle$ confined in an optical dipole trap oriented at 45° to the imaging axis with radial and axial trap frequencies $\omega_r = 2\pi \times 108.9(6) \text{ s}^{-1}$ and $\omega_z = 2\pi \times 7.75(3) \text{ s}^{-1}$. For the
samples imaged at 527 G, the sample preparation was similar to that described in [13], with a temperature of 0.14(1)TF. The samples imaged at other magnetic fields were prepared in a similar fashion, except that evaporation was performed at 1000 G to a final temperature of $T = 0.13(1)T_F$ before ramping the magnetic field over 1.5 s to its final value. The temperature at 1000 G was determined by fitting a noninteracting Thomas-Fermi distribution in time of flight. The temperatures at other points in the crossover were related to that value assuming an isentropic ramp, using calculations presented in [23]. Using this method we obtain temperatures of 0.13(1)TF at 915 G, 0.19(1)TF at 830 G, and 0.19(3)TF at 790 G where additional evaporation was performed to achieve a central optical density similar to that at the other magnetic fields. The extent of the cloud along the imaging direction was 135 μm, much larger than the Rayleigh range of 8 μm for our imaging system with a NA of 0.14.

The superfluid to normal phase boundary was determined by measuring condensate fraction [Fig. 2] using the standard magnetic field sweep technique [24,25]. For this, the magnetic field was rapidly switched to 570 G to transfer atom pairs to more deeply bound pairs (molecules) which survive ballistic expansion. For resonant imaging of the molecules, the field was ramped back to 790 G over 10 ms. The condensate fraction was determined by fitting the one-dimensional density profiles with a bimodal distribution.

As previously described, propagation converts spatial fluctuations in the refractive index into amplitude fluctuations on the detector. For different choices of the probe light frequency, the two atomic spin states will have different real polarizabilities and the local refractive index will be a different linear combination of the (line-of-sight integrated) column densities $n_1$ and $n_2$. To measure the susceptibility we choose a probe light frequency exactly between the resonances for states $|1\rangle$ and $|2\rangle$, so that the real polarizabilities are opposite and the refractive index is proportional to the magnetization $(n_1 - n_2)$. The intensity fluctuations on the detector after propagation are consequently proportional to the fluctuations in magnetization. Since a refractive index proportional to $(n_1 + n_2)$ occurs only in the limit of infinite detuning, we measure the fluctuations in the total density by exploiting the fact that the fluctuations in total density can be inferred from the fluctuations in two different linear combinations of $n_1$ and $n_2$. For convenience, we obtain the second linear combination using a detuning that has the same value, but opposite sign for state $|2\rangle$, and therefore three times the value for state $|1\rangle$. With this detuning, we record images of the fluctuations in $(n_1/3 + n_2)$.

In principle, this information can be obtained by taking separate absorption images on resonance for states $|1\rangle$ and $|2\rangle$. However, the images would have to be taken on a time scale much faster than that of atomic motion and there would be increased technical noise from the subtraction of large numbers. The use of dispersive imaging has the additional advantage over absorption in that the number of scattered photons in the forward direction is enhanced by superradiance. As a result, for the same amount of heating, a larger number of signal photons can be collected [26]. This is crucial for measuring atomic noise, which requires the collection of several signal photons per atom. The choice of detuning between the transitions of the two states has the important feature that the index of refraction for an equal mixture fluctuates around zero, avoiding any lensing and other distortions of the probe beam. This is not

FIG. 1. Simulation of propagation effects after light has passed through a Poissonian phase noise object. Shown are the variance measured in the amplitude or in-phase quadrature (black line) and the out-of-phase quadrature (gray line) as a function of defocus distance, for an imaging system with a numerical aperture of 0.14. Within a distance less than 5% of our cloud size, noise becomes equally distributed between the two quadratures and the variances in transmission and phase-contrast images become the same. (Top inset) For small phase fluctuations, an in-focus phase noise object gives no amplitude contrast, but when it is out of focus it does. (Bottom inset) Sample intensity patterns for a defocused phase object.

FIG. 2. Measured condensate fraction as a function of dimensionless interaction strength $1/(k_F a)$. Insets show typical images from which the condensate fraction was extracted by fitting a bimodal distribution. The dashed line is a sigmoidal fit to guide the eye.
the case for other choices of detuning, and indeed, we observe some excess noise in those images (see below). At the detunings chosen, 10% residual attenuation is observed, some due to off-resonant absorption, some due to dispersive scattering of light out of the imaging system by small scale density fluctuations. The contribution to the variance of the absorption signal relative to the dispersive signal scales as $(2\Gamma)^2/s^2 \approx 0.006$ and can be neglected in the interpretation of the data.

The noise analysis procedure was nearly identical to that performed in [13]. A high-pass filter with a cutoff wavelength of 13 $\mu$m was applied to each image of the cloud to minimize the effect of fluctuations in total atom number. Then, for each pixel position, the variance of the optical densities at that position in the different images was computed. After the subtraction of the contribution of photon shot noise, the resulting variance image reflects the noise contribution from the atoms.

The goal of our noise measurements is to determine at various interaction strengths the normalized susceptibility $\tilde{\chi} = \chi/\chi_0$ and compressibility $\tilde{\kappa} = \kappa/\kappa_0$, where $\chi_0 = 3n/2E_F$ and $\kappa_0 = 3/2nE_F$ are the susceptibility and compressibility of a zero-temperature noninteracting Fermi gas with the same total density $n$ and Fermi energy $E_F$. Before studying spin fluctuations through the BEC-BCS crossover, we therefore calibrate our measurement by measuring the spin fluctuations in a noninteracting mixture, realized at 527 G where the scattering length between the two states vanishes. Figure 3 shows raw profiles of the variances $\Delta^2$ and $\Delta^2_+$ measured at the two detunings. These fluctuations in the speckle pattern are proportional to number fluctuations in the specified probe volume $V$: $\Delta^2 = [c\Delta(N_i - N_2)]^2$ and $\Delta^2_+ = [c'\Delta(N_i/3 + N_2)]^2$. In these relations $c$ and $c'$ are factors which have to be calibrated. Without interactions, $N_1$ and $N_2$ are uncorrelated, and one predicts $[\Delta(N_i - N_2)]^2/[\Delta(N_i/3 + N_2)]^2 = 2/[1 + (1/3)^2] = 1.8$.

The observed ratio of $\Delta^2_{+}/\Delta^2_+ = 1.56(14)$ reflects excess noise contributing to $\Delta^2_+$ due to residual systematic dispersive effects and is accounted for by setting $c'/c = \sqrt{1.8}/1.56$. For high temperatures, the atomic noise of the noninteracting gas approaches shot noise; for lower temperatures we observe a reduction in noise due to Pauli blocking as in our previous work [13]. With our new method, we easily discern spin fluctuations with a variance of less than 10% of atom shot noise.

The fluctuation-dissipation theorem connects the variances $[\Delta(N_1 - N_2)]^2$ and $[\Delta(N_1 + N_2)]^2$ to the susceptibility $\tilde{\chi}$ and the compressibility $\tilde{\kappa}$ via $[\Delta(N_1 - N_2)]^2 = 3N/2(T/T_F)\tilde{\chi}$ and $[\Delta(N_1 + N_2)]^2 = 3N/2(T/T_F)\tilde{\kappa}$ with $N = N_1 + N_2$ and $T/T_F$ being the temperature measured in units of the Fermi temperature $T_F$. Recomposing the variances from the two experimentally accessible linear combinations these relations become $\Delta^2_{+}/Nc^2 = 3/(2(T/T_F)\tilde{\chi})$ and $9/4\Delta^2_+/Nc^2 - 1/4\Delta^2_{+}/Nc^2 = 3/(2(T/T_F)\tilde{\kappa})$. The constants $c$ and $c'$ are determined using the noise measurements at 527 G for a noninteracting Fermi gas for which $\tilde{\chi} = \tilde{\kappa} = 1 + O((T/T_F)^2)$. This analysis ignores line-of-sight integration corrections.

Figure 4 shows the spin susceptibility, the compressibility, and the ratio between the two quantities for the interacting mixtures as the interaction strength is varied through the BEC-BCS crossover. The susceptibility and compressibility reproduce the expected qualitative behavior: for the sample at unitarity, where the transition temperature is sufficiently high that a sizable portion of the sample is superfluid, and for the sample on the BEC side, the spin susceptibility is strongly suppressed relative to the compressibility. This reflects the fact that the atoms form bound

![FIG. 3](color online). (Top panel) Example speckle noise image, with white box indicating analysis region. (Bottom panels) Noise data for noninteracting (left panel) and resonantly interacting (right panel) cold clouds, showing $\Delta^2$ (black dots) and $\Delta^2_+$ (gray dots). Solid lines are Gaussian fits to the data, and dotted lines illustrate the expected full Poissonian noise for the corresponding quantities based on density determined from off-resonant absorption.

![FIG. 4](a) The ratio $\chi/\chi_0$, (b) the normalized susceptibility $\chi/\chi_0$, and (c) the normalized compressibility $\kappa/\kappa_0$ in the BEC-BCS crossover. The variances derived from sequences of images are converted into thermodynamic variables using the measured temperatures and a calibration factor determined from the noninteracting gas. The vertical line indicates the onset region of superfluidity, as determined via condensate fraction measurements. The curves show theoretical zero temperature estimates based on 1st (dotted) and 2nd order (solid) perturbative formulas obtained from Landau’s Fermi-liquid theory integrated along the line of sight, and results from a Monte Carlo calculation (dashed) for the compressibility in a homogeneous system [32].
molecules or generalized Cooper pairs; the spin susceptibility should be exponentially small in the binding energy, while the enhanced compressibility reflects the bosonic character of the molecular condensate. At 915 G and 1000 G, where the sample is above the superfluid critical temperature, the susceptibility is larger but still below its value for the noninteracting gas, reflecting the persistence of pair correlations even in the normal phase of the gas.

Above the Feshbach resonance, for attractive interactions, we compare our results to first and second order perturbation theory in the small parameter $k_F a$. This ignores the instability to the superfluid which are present even at zero temperature to reveal the presence of a possible pseudogap phase.

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Appendix C

Correlations and pair formation in a repulsively interacting Fermi gas

This chapter contains a reprint of Ref. [17]: C Sanner, EJ Su, W Huang, A Keshet, J Gillen, W Ketterle. Physical review letters 108 (24), 240404

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Correlations and Pair Formation in a Repulsively Interacting Fermi Gas

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A degenerate Fermi gas is rapidly quenched into the regime of strong effective repulsion near a Feshbach resonance. The spin fluctuations are monitored using speckle imaging and, contrary to several theoretical predictions, the samples remain in the paramagnetic phase for an arbitrarily large scattering length. Over a wide range of interaction strengths a rapid decay into bound pairs is observed over times on the order of $10\hbar/\varepsilon_F$, preventing the study of equilibrium phases of strongly repulsive fermions. Our work suggests that a Fermi gas with strong short-range repulsive interactions does not undergo a ferromagnetic phase transition.

Many-body systems can often be modeled using contact interactions, greatly simplifying the analysis while maintaining the essence of the phenomenon to be studied. Such models are almost exactly realized with ultracold gases due to the large ratio of the de Broglie wavelength to the range of the interatomic forces [1]. For itinerant fermions with strong short-range repulsion, textbook calculations predict a ferromagnetic phase transition—the so-called Stoner instability [2].

Here we investigate this system using an ultracold gas of fermionic lithium atoms, and observe that the ferromagnetic phase transition does not occur. A previous experimental study [3] employing a different apparatus found indirect evidence for a ferromagnetic phase, but did not observe the expected domain structure, possibly due to the lack of imaging resolution. Here we address this shortcoming by analyzing density and spin density fluctuations via speckle imaging [4]. When spin domains of $m$ atoms form, the spin density variance will increase by a factor of $m$ [5], even if individual domains are not resolved. One main result of this paper is the absence of such a significant increase which seems to exclude the possibility of a ferromagnetic state in the studied system.

The Stoner model assumes a two-component Fermi gas with a repulsive short-range interaction described by a single parameter, the scattering length. The predicted phase transition to a ferromagnetic state requires large repulsive scattering lengths on the order of the interatomic spacing. They can be realized only metastable with respect to decay into the bound state. Over a wide range of interaction strengths a rapid decay into bound pairs is observed over times on the order of $10\hbar/\varepsilon_F$, preventing the study of equilibrium phases of strongly repulsive fermions. Our work suggests that a Fermi gas with strong short-range repulsive interactions does not undergo a ferromagnetic phase transition [19,20]. Theoretical studies concluded that the pairing instability is somewhat faster than the ferromagnetic instability [21]. The second major result of this paper is to show that pair formation occurs indeed on a very short time scale. The measured time constant of $10\hbar/\varepsilon_F$ (where $\varepsilon_F$ is the Fermi energy) indicates that the metastable repulsive state will never reach equilibrium and that, even in a metastable sense, a Fermi gas with strong short-range repulsive interactions does not exist. The fast pair formation could not be observed previously due to limited time resolution [3]. Instead, a much slower second phase in the conversion of atoms to pairs was observed leading to the wrong conclusion that the unpaired atoms have a much longer lifetime.

FIG. 1. Diagram showing energy levels and timing of the experiment. The upper (repulsive) and lower (attractive) branch energies, near a Feshbach resonance, are connected by three-body collisions. In our experiment, we quickly jump from a weakly interacting Fermi gas (A) to a strongly interacting one (B) with a rapid magnetic field change. The evolution of correlations and domains and the molecule formation (population of the lower branch) are studied as a function of hold time $t$. Adapted from [42].
The experiments were carried out with typically $4.2 \times 10^5$ $^6$Li atoms in each of the two lower spin states $|1\rangle$ and $|2\rangle$ confined in an optical dipole trap with radial and axial trap frequencies $\omega_r = 2\pi \times 100(1)$ s$^{-1}$ and $\omega_z = 2\pi \times 9.06(25)$ s$^{-1}$. The sample was evaporatively cooled at a magnetic bias field $B = 320$ G, identical to the procedure described in [22]. Then the magnetic field was slowly ramped to 730 G ($k_Fa = 0.35$) in 500 ms. The fraction of atoms being converted to molecules during the ramp was measured (see below for method) to be below 5%. The temperature of the cloud was typically $0.23(3)T_F$ at 527 G with a Fermi energy of $E_F = k_BT_F = h \times 6.1$ kHz. After rapidly switching the magnetic field from 730 G to the final value in less than 350 $\mu$s, spin fluctuations were measured by speckle imaging. Optionally an appropriate rf pulse was applied directly before imaging to rotate the spin orientation along the measurement axis. Due to the use of 20 cm diameter coils outside the vacuum chamber, the inductance of the magnet coils was 330 $\mu$H and the fast switching was accomplished by rapidly discharging capacitors charged to 500 V.

Experimentally, spin fluctuations are measured using the technique of speckle imaging described in Ref. [4]. For an appropriate choice of detuning, an incident laser beam experiences a shift of the refractive index proportional to the difference between the local populations of the two spin states $N_1$ and $N_2$. Spin fluctuations create spatial fluctuations in the local refractive index and imprint a phase pattern into the incoming light, which is then converted into an amplitude pattern during propagation. The resulting spatial fluctuations in the probe laser intensity are used to determine the spin fluctuations in the sample.

In Ref. [4] we prepared samples on the lower branch of the Feshbach resonance, where positive values of $k_Fa$ correspond to a gas of weakly bound molecules. At $k_Fa = 1.2$, we observed a sixfold suppression of spin fluctuations and a fourfold enhancement of density fluctuations. Typical fluctuations in the speckle images of a non-interacting Fermi gas at $T = 0.23T_F$ amount to 5% of the average optical signal per pixel, corresponding to about 50% of Poissonian fluctuations. Those fluctuations are modified by factors between 0.2 and 1.6 due to pairing and interactions.

In this study, on the upper branch of the Feshbach resonance, the situation is reversed. For unbound atoms, as the interaction strength increases, the two spin components should develop stronger and stronger anticorrelations and enhanced spin fluctuations. Previous experimental work [3] and several theoretical studies [10,11,13–15,18,23] predicted a phase transition to a ferromagnetic state where the magnetic susceptibility and therefore the spin fluctuations diverge. Recent Monte Carlo simulations [19] predict such a divergence around $k_Fa = 0.83$. We therefore expected an increase of spin fluctuations by one or several orders of magnitude, related to the size of magnetic domains.

Figure 2 shows the observed spin fluctuations enhancement compared to the non-interacting cloud at 527 G. The variance enhancement factor reaches its maximum value of 1.6 immediately after the quench, decreasing during the 2 ms afterward. The absence of a dramatic increase shows that no domains form and that the sample remains in the paramagnetic phase throughout. Similar observations were made for a wide range of interaction strengths and wait times. Note that first-order perturbation theory [24] predicts an increase of the susceptibility by a factor of 1.5 at $k_Fa = 0.5$ and by a factor of 2 at $k_Fa = 0.8$ (i.e., no dramatic increase for $k_Fa < 1$). Therefore, our data show no evidence for the Fermi gas approaching the Stoner instability.

Before we can fully interpret these findings, we have to take into account the decay of the atomic sample on the upper branch of the Feshbach resonance into bound pairs. We characterize the pair formation by comparing the total number of atoms and molecules $N_a + 2N_{\text{mol}}$ (determined by taking an absorption image after ballistic expansion at high magnetic field where molecules and atoms have the same absorption resonance) to the number of free atoms (determined by rapidly sweeping the magnetic field to 5 G before releasing the atoms and imaging the cloud, converting pairs into deeply bound molecules that are completely shifted out of resonance) [25].

The time evolution of the molecule production (Fig. 3) shows two regimes of distinct behavior. For times less than 1 ms, we observe a considerable number of atoms converted into molecules, while the total number $N_a + 2N_{\text{mol}}$ remains constant. The initial drop in atom number becomes larger as we increase the final magnetic field, and saturates at around 50% near the Feshbach resonance.

We attribute this fast initial decay in atom number to recombination [26,27] into the weakly bound molecular

![FIG. 2. Spin fluctuations (a) after 350 $\mu$s as a function of magnetic field and (b) on resonance as a function of hold time scaled to the value measured at 527 G. Even at strong repulsive interactions, the measured spin fluctuations are barely enhanced, indicating only short-range correlations and no domain formation. The spin fluctuations were determined for square bins of 2.6 $\mu$m, each containing on average 1000 atoms per spin state.](240404-2)
For longer time scales (hundred milliseconds) we observe a steady increase of the molecule fraction to 90% for the longest hold time. This occurs due to continuous evaporation which cools down the system and shifts the atom-molecule equilibrium towards high molecule fractions. During the same time scale, a slow loss in both atom number and total number is observed caused by inelastic collisions (vibrational relaxation of molecules) and evaporation loss.

Is the rapid conversion into molecules necessarily faster than the evolution of ferromagnetic domains? Our answer is tentatively yes. First, for strong interactions with \( k_F a \) around 1, one expects both instabilities (pair formation and Stoner instability) to have rates which scale with the Fermi energy \( E_F \) and therefore with \( n^{2/3} \). Therefore, one cannot change the competition between the instabilities by working at higher or lower densities. According to Ref. [21] the fastest unstable modes for domain formation have a wave vector \( q \approx k_F / 2 \) and grow at a rate of up to \( E_F / 4h \) when the cloud is quenched sufficiently far beyond the critical interaction strength. Unstable modes with such wave vectors will develop “domains” of half a wavelength or size \( 2 \pi / q \) containing 5 atoms per spin state in a volume \( \xi^3 \). This rate is comparable to the observed conversion rates into pairs of 0.13\( E_F \). Therefore, at best, “domains” of a few particles could form, but before they can grow further and prevent the formation of pairs (in a fully polarized state), rapid pair formation takes over and populates the lower branch of the Feshbach resonance. Based on our observations and these arguments, it seems that it is not possible to realize ferromagnetism with strong short range interaction, and therefore the basic Stoner model cannot be realized in nature.

One possibility to suppress pair formation is provided by narrow Feshbach resonances. Here the pairs have dominantly closed channel character and therefore a much smaller overlap matrix element with the free atoms. However, narrow Feshbach resonances are characterized by a long effective range and do not realize the Stoner model which assumes short-range interactions. Other interesting topics for future research on ferromagnetism and pair formation include the effects of dimensionality [30,31], spin imbalance [32,33], mass imbalance [34], lattice and band structure [35,36].

We now discuss whether ferromagnetism is possible after atoms and molecules have rapidly established local equilibrium. In other words, starting at \( T = 0 \), one could heat up the fully paired and superfluid system and create a gas of atomic quasiparticles which are similar to free atoms with repulsive interactions. Density and temperature of the atoms are now coupled. It is likely that such a state is realized in our experiments after a few ms following the quench, until evaporative cooling converts the system into a molecular condensate over \( \approx 100 \) ms. The possibility that such a quasiparticle gas could become ferromagnetic...
Ferromagnetic domain formation is analogous to phase picture, we could regard the atomic quasiparticles as free atoms, and then apply the Stoner model to them. Ferromagnetic domain formation is analogous to phase separation between the two spin components [3]. Since dimers interact equally with the two spin components, one might expect that even a noticeable dimer fraction should not suppress the tendency of the atomic gas to form domains. Therefore, in a simple model, one may neglect dimer-atom interactions.

If the Stoner model applies to this quasiparticle gas, the next question is whether the temperature is low enough for the ferromagnetic phase transition. Available theoretical treatments do not predict an exact maximum transition temperature to the ferromagnetic state and obtain an unphysical divergence for large scattering lengths. Since the only energy scale is the Fermi temperature, one would expect a transition temperature which is a fraction of the Fermi temperature [37], higher or around the temperature scale probed in our experiments. However, even above the transition temperature, the susceptibility is enhanced. A simple Weiss mean field or Stoner model leads to the generic form of the susceptibility $\chi(T) = \chi_0(T)/(1 + A\chi_0(T))$, where $\chi_0(T)$ is the Pauli susceptibility of the non-interacting gas and $w$ the interaction parameter. This formula predicts a twofold increase in the susceptibility even 50% above the transition temperature, which is well within the sensitivity of our measurements.

Therefore, our experiment can rule out ferromagnetism for temperatures even slightly lower than the experimental temperatures. Temperatures are very difficult to measure in a transient way for a dynamic system which may not be in full equilibrium. For example, cloud thermometry requires full equilibration and lifetimes much longer than the longest trap period. We attempted to measure the temperature after the hold time near the Feshbach resonance by quickly switching the magnetic field to weak interactions at 527 G and then performing noise thermometry using speckle imaging [4]. We measure column-integrated fluctuations that are 0.61(8) of the Poisson value which implies an effective temperature well below $T_F$, around 0.33(7) $T_F$, not much higher than our initial temperature of 0.23 $T_F$. Although the cloud is not in full equilibrium, an effective local temperature can still be obtained from noise thermometry.

Alternatively, we can estimate the temperature increase from the heat released by pair formation. A simple model [38] accounting for the relevant energy contributions predicts for $k_Fa = 1$ that molecule fractions of higher than 20% result in a final temperature above 0.47$T_F$, an estimate which is higher than the measurement reported above. One may hope that closer to resonance many-body effects lower the released energy; however, as we show in the Supplemental Material (Fig. 1 of [38]) this is not necessarily the case due to the repulsive interaction energy.

Our experiment has not shown any evidence for a possible ferromagnetic phase in an atomic gas in “chemical” equilibrium with dimers. This implies one of the following possibilities. (i) This gas can be described by a simple Hamiltonian with strong short range repulsion. However, this Hamiltonian does not lead to ferromagnetism. This would be in conflict with the results of recent quantum Monte Carlo simulations [19,20] and second order perturbation theory [11], and in agreement with conclusions based on Tan relations [39]. (ii) The temperature of the gas was too high to observe ferromagnetism. This would then imply a critical temperature around or below $0.2T_T$, lower than generally assumed. (iii) The quasiparticles cannot be described by the simple model of an atomic gas with short-range repulsive interactions due to their interactions with the paired fraction.

A previous experiment [3] reported evidence for ferromagnetism by presenting non-monotonic behavior of atom loss rate, kinetic energy and cloud size when approaching the Feshbach resonance, in agreement with predictions based on the Stoner model. Our measurements confirm that the properties of the gas strongly change near $k_Fa = 1$. Similar to [3], we observe features in kinetic and release energy measurements near the resonance (see Supplemental Material [38]). However, the behavior is more complex than that captured by simple models. The atomic fraction decays non-exponentially (see Fig. 3), and therefore an extracted decay time will depend on the details of the measurement such as time resolution. Reference [3] found a maximum of the loss rate of 200 s$^{-1}$ for a Fermi energy of 28 kHz. Our lower bound of the decay rate of $3 \times 10^3$ s$^{-1}$ is 15 times faster at a five times smaller Fermi energy. Our more detailed study rules out that Ref. [3] has observed ferromagnetic behavior.

Our conclusion is that an ultracold gas with strong short range repulsive interactions near a Feshbach resonance remains in the paramagnetic phase. The fast formation of molecules and the accompanying heating makes it impossible to study such a gas in equilibrium, confirming predictions of a rapid conversion of the atomic gas to pairs [21,40]. The Stoner criterion for ferromagnetism obtains when the effective interaction strength times the density of states is larger than one. This is a at least an approximately valid criterion for multi-band lattice models [41]. We have shown here that this criterion cannot be applied to Fermi gases with short-range repulsive interactions (the basic Stoner model) since the neglected competition with pairing is crucial.

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This is illustrated by a simplified model assuming Poissonian fluctuations in a given probe volume within the atom sample. With an average N atoms in this volume, one would measure a standard deviation in the atom number of \( \sqrt{N} \). However, if the atoms formed clusters each made of \( m \) atoms, the standard deviation of the number of clusters would be \( \sqrt{N/m} \), leading to a variance in atom number of \( \langle N \rangle/m = m N \).

Potentials with a positive scattering length \( a \) have no bound state only if the effective range \( r_c \) is larger than \( a/2 \). Otherwise, the s-wave scattering amplitude \( f(k) = 1/(-1/a + r_c k^2/2 - i k) \) has a pole on the imaginary axis corresponding to a bound state.

[29] Note that the drop in molecule formation after 1 ms cannot be explained by the drop in atomic density by a factor of 2 due to conversion into molecules and to an increase in the size of the sample due to the increased repulsive interactions.
Correlations and Pair Formation in a Repulsively Interacting Fermi Gas: Supplementary Information

In this supplement we present measurements of the release energy of fermions with strong repulsive interactions, derive estimates for sample heating due to molecule formation, adapt predictions for ferromagnetic domain formation after a quench to our experimental parameters, and discuss our temporal and spatial resolution for spin fluctuations.

KINETIC ENERGY AND RELEASE ENERGY

The energy of a trapped interacting gas is the sum of three contributions: the kinetic energy, the interaction energy, and the potential energy in the trapping potential.

\[ E = T + U_{\text{int}} + U_{\text{trap}} \] (1)

By suddenly releasing the atoms from the trap and measuring the radius of the cloud, it is possible to measure either the release energy \((T + U_{\text{int}})\) or the kinetic energy \(T\), depending on whether the interactions are left on or are switched off, respectively, at the time of release, by leaving the external magnetic field constant or rapidly switching it to a value away from the Feshbach resonance.

The system is prepared with variable interaction strength by rapidly switching the magnetic field to a value near the Feshbach resonance. For the kinetic energy measurement, the field is again rapidly switched to 5G, after which the atoms are released from the trap. After 8 ms of free expansion, the size of the cloud reflects the width of the in-trap momentum distribution and the average in-trap kinetic energy. The observed increase in kinetic energy with increasing interaction strength reflects the onset of pair (anti-) correlations of opposite-spin atoms — those anti-correlations reduce the repulsive potential energy at the price of increased kinetic energy.

The observed increase in kinetic energy is consistent with the observations in Ref. [3]. We do not observe a minimum in kinetic energy as in [3], since the interactions are suddenly increased and the cloud cannot adiabatically expand during the ramp as was the case in the earlier work. For fully spin-polarized domains, the kinetic energy would increase by a factor of \(2^{2/3}\) which provides an upper bound (since the true ground state must have an energy lower than or equal to the fully phase-separated state). The smaller observed energy increase (factor of 1.3) implies that fully spin polarized domains have not formed. It should be noted that the kinetic energy increase is insensitive to the correlation length or size of domains and cannot clearly distinguish between ferromagnetic domains and strong anti-correlations [4, 5].

In-trap kinetic and interaction energies are measured when the magnetic field is left at its value near the Feshbach resonance while the trap is switched off. After 4 ms of the 8 ms free expansion the field is switched to 5G for imaging. The resulting cloud size is directly related to the release energy of the cloud, the sum of the in-trap kinetic and interaction energies. Around \(k_{Fa} = 0.5\), we observe a strong increase of the transverse release energy by a factor of about 2. This implies that the total energy has increased by a factor of 4/3 compared to the non-interacting case since the interactions modify the expansion from ballistic to hydrodynamic. The comparison of kinetic energy and release energy at 790G and 810G shows that the extra energy due to repulsive interactions is clearly dominated by repulsive potential energy and not by kinetic energy. The latter would occur for ferromagnetic domains. In case of fully spin-polarized domains, the repulsive interaction energy would vanish, and
kinetic energy and release energy should be the same. Therefore, the energy measurements also rule out a ferromagnetic phase.

The interaction-strength dependence of the release energy shows a weak maximum at 790G. The ground state energy has to vary monotonically with the strength of repulsive interactions. This follows from a simple energy function at strong interactions is used as a trial wavefunction at weaker interactions. Therefore, the observed maximum is most likely related to non-equilibrium excitations caused by the sudden jump in the scattering length.

HEATING DUE TO MOLECULE FORMATION

Molecule formation heats the sample by transferring the binding energy and excess kinetic energy of the molecules to the remaining atoms, and also by creating holes in the Fermi sea. For small values of \( k_F a \), the total energy release per molecule is \( \hbar^2/ma^2 + 2EF \). From the energy per particle for an ideal homogeneous degenerate Fermi gas \( E = 0.6EF[1 + (5\pi^2/12)(T/T_F)^2] \) we obtain

\[
\frac{T}{T_F} = \sqrt{\frac{4\eta}{\pi^2}} \left( 1 + \frac{1}{(k_F a)^2} \right)
\]

for the final temperature with a small molecule fraction \( \eta \) (assuming initially \( \eta=0 \) and \( T=0 \)). Evaluating this result at \( k_F a = 1 \), where the two-body binding energy is \( 2EF \), one finds that molecule fractions of higher than 20 % result in a final temperature above \( 0.4T_F \), an estimate which is somewhat higher than the measurement reported in the main article.

DOMAIN GROWTH PREDICTIONS AND EXPERIMENTAL RESOLUTION

As described in the main paper, we have observed two dynamic time scales - a sub-ms timescale for rapid pair formation, followed by a slower time scale of tens of milliseconds where further cooling led to a full conversion of the atomic into a molecular gas. We now want to discuss over what time scale we could have observed ferromagnetic domain formation if it had happened, and some experimental aspects regarding temporal and spatial resolution.

Pekker et al. [6] predicted in uniform systems fast growth rates for small domains. Unstable modes with wavevector \( q \approx k_F/2 \) grow at a rate of up to \( EF/4h \) when the cloud is quenched sufficiently far beyond the critical interaction strength. This corresponds to a growth time of around 100\( \mu \)s. For a wide range of interactions and wavevectors, the predicted growth time is faster than \( 10h/EF \) or 250\( \mu \)s. During this time one would expect the thermal fluctuations to increase by a factor of \( e \).

Wavevectors \( q \approx k_F/2 \) will develop “domains” of half a wavelength or size \( \xi = \pi/q = 2\pi/k_F \), which is 2.3 \( \mu \)m at the experimental density of \( 3 \times 10^{13} \) cm\(^{-3} \) corresponding to \( k_F = 2.7 \times 10^4 \) cm\(^{-1} \). For speckle imaging the smallest effective probe volume is given by the nominal optical resolution or the optical wavelength times the aperture of the lens system which is equal to the bin size of 2.6 \( \mu \)m in our experiment. For a bin size \( d \), the measured fluctuations are an integral over fluctuations at all wavevectors \( q \) with an effective cutoff around \( 1/d \). Due to a mode density factor \( q^2 \) the largest contribution comes from wavevectors around \( \pi/d \) which is equal to 0.4 \( k_F \) for our experimental parameters, fortuitously close to the wavevector of the fastest growing unstable modes. Therefore, for a quench across a ferromagnetic phase transition, we would have expected a sub-millisecond growth time for the spin fluctuations, which was not observed.

Another analysis focuses on domains. As discussed in the main paper, random domains containing \( m \) atoms will increase the variance of spin fluctuations by a factor of \( m \). The fastest growing unstable modes contain around 5 atoms per spin state in a volume \( \xi^3 \). At our temperatures, the thermal fluctuations correspond to Poisson fluctuations reduced by the Pauli suppression factor \( (3/2)T/T_F \), i.e. in a small volume with about ten particles, the spin fluctuations correspond to plus/minus one particle, and will approach saturation (i.e. fully spin polarized domains) after a few growth times (when, of course, the unstable mode analysis is no longer applicable). If these domains randomize through spin diffusion, it will increase the variance of the spin fluctuations at all length scales larger than \( \xi \) by a factor of \( m \), without requiring optical resolution of the domain size \( \xi \). Spin diffusion effectively “converts” initial fluctuations at wavevector \( q \) to fluctuations at all wavevectors smaller than \( q \).

Since we start with a balanced two-state mixture with zero spin density, formation of random (i.e. uncorrelated) domains requires spin diffusion. Full sensitivity to domains requires spin transport over a distance equal to the bin size. Assuming diffusive motion at resonant interactions with a spin diffusivity \( D_s \approx h/m \) as observed in [7] results in a corresponding minimum wait time of about 300\( \mu \)s. This means that after crossing a ferromagnetic phase transition, formation of domains of a few to tens of particles should have occurred on ms scales and should have resulted in an observable increase of spin fluctuations. Therefore, we conclude that we have not entered any ferromagnetic phase within our experimental parameters.
Bibliography


