On The Impact Of Arbitrary Two-Dimensional Sections

by

Xiaoming Mei

B.S., University Of Science & Technology Of China (1995)

Submitted to the Department of Ocean Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Ocean Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1998

© Massachusetts Institute of Technology 1998. All rights reserved.

Author .................................................................

Department of Ocean Engineering

February 15, 1998

Certified by ............................................................

Dick K. P. Yue

Professor Of Hydrodynamics And Ocean Engineering

Thesis Supervisor

Accepted by ............................................................

J. Kim Vandiver

Chairman, Department Committee on Graduate Students
On The Impact Of Arbitrary Two-Dimensional Sections

by

Xiaoming Mei

Submitted to the Department of Ocean Engineering
on February 15, 1998, in partial fulfillment of the
requirements for the degree of
Master of Science in Ocean Engineering

Abstract

When an object drops into the water a slamming load with high pressure on the
body happens. Such an impact force often causes serious structure damage to the
ships and offshore structures. The study of the water-entry impact problem is thus
of fundamental interest and practical significance in naval architecture and marine
engineering.

We analytically study the water-entry problem of an arbitrary two-dimensional
object. The linearized formulation of Wagner (1932) for wedges of small deadrise
angles is adopted and extended to general body geometries with the boundary condition
on the body satisfied exactly. For wedges and circular cylinders, we derive closed-
form solutions by using the conformal mapping techniques for the exact solution of
the boundary-value problem at any instant. The analytical solutions are confirmed
by comparisons to the physical experiments and the fully-nonlinear simulation results
for wedges and by comparisons to the existing experiments for circular cylinders. For
arbitrary ship-like bodies, we also develop a general solution scheme based on the
use of Lewis-form representation of the body geometry. It is applied to wedge and
circular cylinder and agrees with the exact solutions very well. For illustration, the
solutions for the case of parabolic section and a bow flare section are also presented.

Thesis Supervisor: Dick K. P. Yue
Title: Professor Of Hydrodynamics And Ocean Engineering
Acknowledgments

I would like to express my deepest appreciation to Professor Dick K. P. Yue for his guidance and support during this research. During the past years his approach to the engineering science influenced me a lot. The present work could never have been completed without his tireless help.

Special thanks to Dr. Yuming Liu. Throughout my whole life at MIT he has given me invaluable helps in many aspects. I won’t never forget his continuous guidance and care.

I want to give my thanks to many members in our Vortical Flow Research Lab, especially Mr. Qiang Zhu and Mr. Lian Shen. They helped me in different ways.

I also want to take this opportunity to thank my parents. This thesis is dedicated to them.
Contents

1 Introduction .................................................. 12

2 Problem formulation ........................................... 15

3 Analytical solution scheme ................................... 20
   3.1 Boundary value problem solution ......................... 20
   3.2 Evolution of the intersection ............................. 22
   3.3 Pressure distribution on the wetted body surface ....... 25
   3.4 Impact force on the body ................................ 26

4 Water entry of wedges ....................................... 27
   4.1 Analytical solution ....................................... 27
      4.1.1 Splash-up of the water ............................... 27
      4.1.2 Pressure distribution on the body and Free surface shape ... 32
   4.2 Comparison to similarity solution ......................... 33
   4.3 Comparison to experiments ................................ 41
   4.4 Comparison to fully-nonlinear numerical simulations .... 43
      4.4.1 General numerical scheme ............................ 43
      4.4.2 Matching of the jet with the outer flow ............... 46
      4.4.3 Numerical Implementation ............................ 48
      4.4.4 Results ............................................... 49

5 Impact of circular cylinder .................................. 64
   5.1 Analytical solution ....................................... 64
5.2 Comparison to experiments ........................................ 70

6 Impact of arbitrary ship-like sections .......................... 73
   6.1 Lewis-form approximation ................................... 73
   6.2 Applications to wedge and circular cylinder ............... 75
   6.3 Application to parabolic section ........................... 76
   6.4 Application to bow flare section and comparison to experiments ... 76

7 Conclusions and future work .................................. 83
   7.1 Conclusions .................................................... 83
   7.2 Future work .................................................... 84
List of Figures

2-1 Definition sketch .......................................................... 16

3-1 Galilean transformation between \((y, z)\) and \((y', z')\) coordinate systems. 21

4-1 Conformal mapping for wedge entry ........................................ 28

4-2 Dimensionless Coefficient \(\gamma\) measuring the splash-up of the water ... 30

4-3 The location of the intersection between the wedge and the free surface as a function of the deadrise angle. Results plotted are: Pierson's hypothesis (Payne 1994) (— · —); the solution using the Lewis-form representation of the body (· · ·); the exact solution (——). ........... 31

4-4 The pressure distributions on the wedge and the associated free-surface profiles at different deadrise angles \(\alpha = 1, 4, 10, 30\) degrees. .............. 34

4-5 The pressure distributions on the wedge and the associated free-surface profiles at different deadrise angles \(\alpha = 45, 60, 81, 89\) degrees. .............. 35

4-6 Comparisons of pressure distribution on the dropping wedge with deadrise angle \(\alpha = 10^\circ\) between the present theory (——) and the similarity solution (— · —). ................................. 36

4-7 Comparisons of pressure distribution on the dropping wedge with deadrise angle \(\alpha = 30^\circ\) between the present theory (——) and the similarity solution (· · ·). ................................. 37

4-8 Comparisons of pressure distribution on the dropping wedge with deadrise angle \(\alpha = 45^\circ\) between the present theory (——) and the similarity solution (· · ·). ................................. 38
4-9 Comparisons of pressure distribution on the dropping wedge with dead-rise angle $\alpha = 60^\circ$ between the present theory (——) and the similarity solution ( - - - ). .................................................. 39

4-10 Comparisons of pressure distribution on the dropping wedge with dead-rise angle $\alpha = 81^\circ$ between the present theory (——) and the similarity solution ( - - - ). .................................................. 40

4-11 Comparisons of force coefficient on the dropping wedge among the present theory by direct pressure integration method (——), present theory by added mass method (- - -), asymptotic theories of Wagner (1932) (-----), von Karman (1929) (-----) and Von Karman momentum (· · ·), and the similarity solution ($\Delta$). ................................. 41

4-12 Comparisons of the instantaneous impact force on the dropping wedge among the measurement of Zhao & Faltisen (1996) (——), asymptotic theories of Von Karman (1929) (-----) and Wagner (1932) (-----), and the present theory (- - -). ................................................. 44

4-13 Jet flow near the intersection ......................................................... 46

4-14 The initial collocation for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$ with a constant dropping velocity. ............................................ 50

4-15 The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$ with a constant dropping velocity. $t = 0, 0.001, 0.01, 0.02, 0.03, 0.04, 0.05$ ......................... 51

4-16 The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$ with a constant dropping velocity. $t = 0, 0.001, 0.06, 0.07, 0.08, 0.09, 0.10$ ......................... 52

4-17 Collocation points near the intersection for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$ with a constant dropping velocity. $t = 0.06, 0.08, 0.10$ .................................................. 53

4-18 Comparison of the theory (——) and the fully nonlinear simulation ($\Delta$) for the pressure distribution on the wetted body surface during the water entry of a wedge with the deadrise angle $\alpha=30^\circ$. .................. 54
4-19 Comparisons of the theory (—- —-) and the fully-nonlinear numerical simulation (-----) for the free surface elevation near the intersection and the impact pressure distribution on the body for the water entry of a wedge with the deadrise angle \( \alpha = 30^\circ \). .................................................. 55

4-20 Initial free surface profile and free surface for the moment of starting matching of a wedge with the deadrise angle \( \alpha = 15^\circ \) with a constant dropping velocity. \( t = 0, 0.0002 \) .................................................. 56

4-21 The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle \( \alpha = 15^\circ \) with a constant dropping velocity. \( t = 0, 0.0002, 0.001, 0.002, 0.003, 0.004, 0.005 \) .... 57

4-22 Initial collocation of the free surface for the wedge entry with deadrise angle \( \alpha = 45^\circ \), \( t = 0 \) .................................................. 58

4-23 Initial free surface profile and free surface for the moment of starting matching of a wedge with the deadrise angle \( \alpha = 45^\circ \) with a constant dropping velocity. \( t = 0, 0.004 \) .................................................. 59

4-24 The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle \( \alpha = 45^\circ \) with a constant dropping velocity. \( t = 0, 0.004, 0.02, 0.04, 0.06, 0.08, 0.10 \) ............. 60

4-25 The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle \( \alpha = 45^\circ \) with a constant dropping velocity. \( t = 0, 0.004, 0.12, 0.14, 0.16, 0.18, 0.20 \) ............. 61

4-26 Dimensionless free surface profile for the water entry of a wedge with the deadrise angle \( \alpha = 45^\circ \) with a constant dropping velocity. \( t = 0, 0.004, 0.10 \) 
0.20 .................................................. 62

4-27 Dimensionless free surface profile near the intersection for the water entry of a wedge with the deadrise angle \( \alpha = 45^\circ \) with a constant dropping velocity. \( t = 0, 0.004, 0.10, 0.20 \) .................................................. 63

5-1 Circular cylinder impact .................................................. 65

5-2 Conformal mapping for circular cylinder impact .................................................. 65
5-3 Solution of the integral equation for water entry of a circular cylinder at constant dropping velocity. .................. 68

5-4 Horizontal motion of the intersection point during the water entry of a circular cylinder at constant dropping velocity. The results plotted are: the present exact solution (-----), Wagner's theory (1932) (--.--) and the Lewis-form approximate solution (- - -). .................. 69

5-5 Pressure distribution on the wetted body surface of a circular cylinder at constant dropping velocity. Vt/R=0.0025, 0.01, 0.02, 0.03, 0.04, 0.05, 0.075, 0.1 .................. 70

5-6 The impact force coefficient for the water entry of a circular cylinder with constant dropping velocity. The results plotted are: experiment of Campbell & Weynberg (1980) (- - -); experiments from Armand & Cointe (1986) with the falling velocity V=7.38m/s (-----) and V=2.33m/s (--.--) ; the classical theories of Von Karman (1929) and Wagner (1932) (-----); present theory with direct pressure integration (-----); and the present theory with added mass method (- - -). .... 72

6-1 Conformal mapping by using Lewis-form approximation ........ 74

6-2 Solution of the integral equation for the water entry of a parabolic section at constant dropping velocity. The results plotted are: (-----) the complete solution obtained with the use of Lewis approximation for the body surface; and (- - -) Wagner's (1932) theory. .... 77

6-3 Evolution of the intersection point between the free surface and the body in the water entry of a parabolic section at constant dropping velocity. The results plotted are: (-----) the complete solution obtained with the use of Lewis approximation for the body surface; and (-----) Wagner's (1932) theory. .................. 78

6-4 The geometry of the bow flare section. .................. 79

6-5 Solution of the integral equation for the impact of the bow flare section. 80
6-6 Evolution of the intersection point between the free surface and the body in the impact of bow flare section at constant dropping velocity.

6-7 Impact force on the bow flare section. The results plotted are: analytical solution (——) and experiment result ( - - -).
List of Tables

4.1 Water splash-up coefficients $\gamma$ for wedges with different deadrise angles 30

5.1 Convergence of the Chebshev polynomial expansion for the velocity of the intersection point between the free surface and the body during the water entry of the circular cylinder with increasing the order of the Chebyshev polynomial. $Y_{max} = 1$ ................................. 67

5.2 Convergence of the Chebshev polynomial expansion for the velocity of the intersection point between the free surface and the body during the water entry of the circular cylinder with increasing the order of the Chebyshev polynomial. $Y_{max} = 0.96$ ................................. 67
Chapter 1

Introduction

Water impact of bluff bodies striking a free surface is an universal phenomenon in the ocean engineering. One of the most important examples is forward bottom impact of ship. At certain ship speeds and in rough seas, the forward bottom of a ship emerges from the water as a result of large pitch and heave motions and violently impacts the water surface as it reenters. When this occurs the ship’s forward bottom experiences a heavy impulsive pressure from the water. Also flare impact of ship happens during extreme ship motions. An impact force is generated on the bow flare as the ship enters an oncoming wave. Recently slamming on the ocean platform is also becoming critically important. High slamming pressures can occur when breaking waves hit the column of a platform. Other applications include seaplane and spacecraft landing, torpedo and missile entry, etc.

Impact can cause a large load with high local pressure on bodies. The most immediate and evident consequence of slamming is the structural damage on the impact area. Slamming damage sustained by the ship hulls and marine facilities has been a substantial contributor to the repair cost. A study during a four year period in the early 1960s showed that of the 390 U.S.Flag, general vessels 199 sustained 229 impact damages. The total cost of repairs was $6,557,800 and the average was $28,700. In the early 1970s the average increased to $40,000. This problem turns to be more and more severe since the speed of the ship is increasing. Thus the study of the impact problem is of practical significance and has been the subject of a large
number of investigations in marine engineering and naval architecture.

An important pioneering work was due to Von Karman (1929) who developed an asymptotic theory for the flat or near-flat impact using the linearized free-surface and body boundary conditions. Later, Wagner (1932) modified the solution by accounting for the effect of water splash on the body. This work was further developed by Cointe & Armand (1987) and Wilson (1989) who considered nonlinearity of the local jet flow in the intersection between the body and free surface using the technique of asymptotic matching. Nevertheless, this theory is applicable and useful to the impact of bodies with small deadrise angles only.

For the special case of a wedge entering into the water at constant velocity, Dobroboskaya (1969) derived a similarity solution by making use of the geometric speciality of the body. The solution is valid for any deadrise angles, but it is given in terms of integral equations which must be resolved through numerical procedures. Moreover, such a similarity solution does not exist for arbitrary bodies.

Recently, Zhao and Faltinsen (1996) generalized the formulation of Wagner (1932) for the flat impact problem to arbitrary bodies with the body boundary condition satisfied exactly. They solved the linearized problem through a numerical procedure based on the boundary integral equation method. For wedges and ship bow-flare sections, the results were found to agree well with physical experiments and fully-nonlinear simulation predictions (Zhao & Faltinsen 1992) for both the impact pressure distribution and the slamming force on the body. This finding implies that the local thin-jet flow on the body plays a minor dynamic role in the general impact process.

In the present study, we are mainly motivated by the work of Zhao & Faltinsen (1996) but we solve the general two-dimensional impact problem of arbitrary bodies analytically. Following Zhao & Faltisfen (1996), we formulate the problem as an initial-boundary-value problem for the velocity potential by extending the linearization of Wagner (1932) to arbitrary bodies. The body boundary condition is imposed on the instantaneous position of the body and the effect of the local thin-jet flow near the intersection of the body with the free surface is ignored. An analytic procedure based on the use of conformal-mapping techniques is developed to derive the closed-form
solution to the general impact problem. For the cases of wedges and circular cylinders, the solutions are well confirmed by the existing experimental tests and full-nonlinear simulation predictions.

In our analysis, the boundary-value solution at each time is derived in a closed form using the conformal-mapping techniques. For wedges and circular cylinders, the mapping is exact and the mapping relations are expressed in explicit forms. For arbitrary bodies, the mapping is derived by employing the Lewis-approximation for the representation of the body surface. The evolution of the intersection point between the body and free surface in the impact is followed using the approach of Wagner (1932). For wedges and circular cylinders, simple closed-form solutions are obtained, which are compared well with the existing physical experiments and fully-nonlinear simulation results. For arbitrary geometries, an analytic procedure based on the use of Lewis-form approximation for the ship-like body representation is provided. For the water entry of a bow flare section the result compares well with the experiment of Zhao and Faltinsen (1996).

To further verify the analytic solution, fully nonlinear numerical simulations of the impact problem using the Mixed Eulerian Lagrangian Boundary Element Integral Method are performed. Since the velocity of the flow near the intersection between the body and the free surface is much larger than that of the major flows, it is well known that the simulations break down before satisfactory solutions are obtained. To overcome such a numerical difficulty we develop a new matching approach in tracking the motion of the jet near the intersection. With such an approach, the jet flow is described by an analytical solution and is matched to the major flow at any instant. The major flow is solved numerically. Such an approach is shown to be effective for two dimensional bodies and is also valid for three dimensional impact problems. For the case of wedges fully nonlinear simulation results confirm the analytical solution.
Chapter 2

Problem formulation

We consider the two-dimensional water-entry problem involving a slender (rigid) ship-like object which is forced to drop vertically at a constant velocity $V$ through the initially calm ocean surface. For simplicity, the body geometry is assumed to be symmetric about its vertical center-plane. For convenience, let the body surface be represented by the relation $h = H(\ell)$ with $h$ and $\ell$ respectively denoting the vertical and horizontal distances of a point on the body surface from the (low) apex of the body.

We introduce a two-dimensional Cartesian coordinate system $(y, z)$, as shown in figure 2-1, with the $y$-axis on the undisturbed water plane and the $z$-axis positive upwards and coinciding with the symmetry line of the body. Assume the fluid to be incompressible and inviscid and the fluid motion to be irrotational. The flow at any time $t$ can then be described by a velocity potential $\Phi(y, z, t)$ which satisfies the Laplace equation in the fluid domain.

$$\nabla^2 \Phi = 0$$  \hspace{1cm} (2.1)

On the free surface $z = \eta(y, t)$, the kinematic boundary condition takes the form:

$$\frac{D\eta}{Dt} = \frac{\partial \Phi}{\partial z} \quad \text{on} \quad z = \eta(y, t)$$  \hspace{1cm} (2.2)

where $D/Dt$ denotes the substantial derivative. Physically, the condition (2.2) implies
that once a fluid particle is on the free surface, it remains on the surface all the time. Upon assuming no air pocket involved in the impact and setting the atmospheric pressure to be zero, the dynamic free-surface boundary condition can be expressed as:

\[
\frac{D\Phi}{Dt} - \frac{1}{2}(\Phi_y^2 + \Phi_z^2) + g\eta = 0 \quad \text{on} \quad z = \eta(y,t) \tag{2.3}
\]

where \( g \) is the gravitational acceleration. On the body, no flux condition is imposed and it can be written in the form:

\[
\frac{\partial\Phi}{\partial n} = -Vn_z \quad \text{on} \quad S_B(t) \tag{2.4}
\]

where \( \mathbf{n}=(n_y, n_z) \) is the unit normal out of the body. In (2.4), \( S_B(t) \) represents the instantaneous wetted body surface which is determined by \( \ell \leq |Y(t)| \) with \( Y(t) \) being the horizontal coordinate of the intersection point between the body and free surface. For initial conditions, the body is considered to just touch the mean free surface (\( z=0 \))
at $t=0$ and the associated velocity potential and free-surface elevation equal to zero:

$$\Phi(y, z, 0) = \eta(y, 0) = 0 \quad \text{on} \quad z = 0.$$  \hfill (2.5)

The initial-boundary-value problem for $\Phi$ is complete with the imposition of the far-field radiation condition:

$$|\nabla \Phi| \to 0 \quad \text{as} \quad (y^2 + z^2)^{1/2} \to \infty.$$  \hfill (2.6)

In terms of the velocity potential $\Phi$, the pressure on the body is determined according to Bernoulli’s equation and the impact force on the body can be obtained by direct integration of the pressure over the wetted body surface.

Note that the stated problem is nonlinear owing to the free-surface boundary conditions (2.2) and (2.3). The numerical schemes must be used to obtain the exact solution, in general. To understand the mechanics of the impact process and the key behaviors of the solution, it is useful to derive the analytic solution of the linearized problem.

For the linearization, we need to normalize the stated problem. To do that, we choose the duration of the impact, $T$, as the characteristic time, the size of the body geometry, $L$, as the characteristic free-surface wavelength, and the water-entry depth of the body, $VT$, as the characteristic free-surface amplitude. From (2.2), the scale $VL$ obtains for the velocity potential. In terms of these basic characteristic quantities, the dynamic free-surface boundary condition (2.3) can be normalized to give:

$$\frac{D\tilde{\Phi}}{Dt} - \frac{\epsilon^2}{2}[(\tilde{\Phi}_y)^2 + (\tilde{\Phi}_z)^2] + \frac{\epsilon^2}{F_r^2} \tilde{\eta} = 0 \quad \text{on} \quad \tilde{z} = \epsilon \tilde{\eta}\quad \hfill (2.7)$$

where $F_r = V/(gL)^{1/2}$ is Froude number, $\epsilon = VT/L$ measures the free-surface steepness, and the variables with tildes denote quantities normalized by their scales.

Consider that the free-surface wave generated due to the water-body impact has a small amplitude, i.e. $\epsilon \ll 1$. Thus the nonlinear terms in (2.7) can be neglected. Also assume that the dropping velocity of the body ($V$) is so large (with Froude
number \( F_r > \epsilon^{1/2} \) that the gravity effect can be ignored. As a result, (2.7) reduces to \( D\tilde{\Phi}/Dt=0 \). Upon using the initial condition (2.5), we obtain the simplified dynamic free-surface boundary condition: \( \tilde{\Phi}(\tilde{y}, \tilde{z}, \tilde{t})=0 \) which is applied on \( \tilde{z}=\epsilon\tilde{\eta} \) at any time.

In the classical theory of Von Karman (1929), the free-surface boundary conditions are applied on the mean free surface (i.e. \( \tilde{z}=0 \)). For the impact problem involving bodies with surfaces of small curvatures, however, Wagner (1932) pointed out that this treatment misses a significant part of the contribution associated with the wetted body surface above the mean free surface whose area is comparable to the part of the body surface below the mean free surface. To include the effect of water splash-up on the body, Wagner proposed to impose the free-surface boundary conditions on the horizontal plane through the intersection between the body and free surface. Since we deal with the general impact problem of arbitrary bodies, like Zhao \\& Faltinsen (1996), we shall follow Wagner (1932) in the present study to apply the dynamic free-surface boundary condition \( \tilde{\Phi}=0 \) on the horizontal plane \( \tilde{z}=\epsilon\tilde{\eta}(\tilde{Y}, \tilde{t}) \).

Returning to physical variables, the linearized initial-boundary-value problem for \( \Phi \) can be summarized to contain the Laplace equation (\( \nabla^2 \Phi=0 \)), the initial condition (2.5), the radiation condition (2.6), the body boundary condition (2.4), and the free-surface dynamic boundary condition:

\[
\Phi(y, z, t) = 0 \quad \text{on} \quad z = \eta(Y, t) . \tag{2.8}
\]

Here the position of the intersection \( Y(t) \) between the free surface and the body in (2.8) is a priori unknown which must be solved together with the initial-boundary-value problem.

We remark that the linearized formulation is apparently invalid for the flow near the intersection between the body and free surface, where the free surface changes its shape sharply and the nonlinearity must be considered. With the method of asymptotic matching, such a local free-surface nonlinearity can be accounted for in theory (e.g. Wagner 1932; Cointe \\& Armand 1987). In this approach, the above linear formulation is used for the far-field problem, while a jet-like solution is found to exist.
in the near field. Recently, Zhao & Faltinsen (1996) showed that the local jet flow
gives a negligible contribution to the impact pressure as well as the slamming force on
the body. Thus, in practice, we can ignore the thin-jet flow and consider the global
problem only.
Chapter 3

Analytical solution scheme

In this section, we outline an analytic scheme to derive the general solution of the linearized impact problem. Unlike the work of Wagner (1932), where the body boundary condition is applied on the mean free surface, the present scheme imposes the body boundary condition at the instantaneous position of the body. Thus the solution derived is valid for any body geometry at any entry angle into the water.

The analytic solution to the linearized initial-boundary-value problem stated in chapter 2 can be derived through two steps of analyses: (i) given the intersection position $Y(t)$, solving the boundary-value problem for the vertical velocity of the free surface; and (ii) integrating equation (2.2) in time for the evolution of the intersection point.

3.1 Boundary value problem solution

As presented in chapter 2, at any instant, the boundary-value problem for $\Phi$ consists of the Laplace equation ($\nabla^2 \Phi = 0$), the free-surface boundary condition (2.8), the kinematic body boundary condition (2.4), and the radiation condition (2.6). The free-surface condition $\Phi = 0$, applied on the horizontal plane $z = \eta(Y) \equiv \mathcal{H}(Y) - Vt$, implies that $\Phi$ is symmetric about the plane $z = \eta(Y)$. Therefore, $\Phi$ is identical to the solution of the problem of a closed body moving in the negative $z$ direction at constant velocity $V$ in an infinite fluid. This fictitious body is made of the immersed
Figure 3-1: Galilean transformation between \((y, z)\) and \((y', z')\) coordinate systems.

part of the real body (bounded by \(S_B(t)\)) and its image about the horizontal plane \(z=\eta(Y)\).

Upon using the Galilean transformation, as shown in figure 3-1, we have

\[
\Phi(y, z, t) = \phi(y', z', t) - Vz'
\]  

(3.1)

where \(\phi\) represents the velocity potential for the problem of a uniform flow with constant velocity \(V\) past the stationary closed body. Here the coordinate system \((y', z')\) is chosen to be fixed on the closed body with the \(y'\)- and \(z'\)-axes respectively coinciding with the horizontal and vertical symmetry lines of the closed body. The \((y', z')\) coordinate system is related to the \((y, z)\) coordinate system by the relations:

\[
y' = y, \quad z' = z - \eta(Y)
\]  

(3.2)

To solve for \(\phi\), we employ the conformal mapping technique, with which the boundary condition on the body can be satisfied exactly. In general, the mapping relation depends on the body geometry and so does the solution of \(\phi\). Since the geometry of the closed body in Figure 3-1 is given in terms of \(Y(t)\), the velocity potential \(\phi\) is ultimately a function of the intersection position \(Y(t)\).
3.2 Evolution of the intersection

The determination of the intersection between the free surface and the body during the impact is essential for solving the boundary-value problem and finding the impact force on the body.

Following Wagner (1932), we define the intersection point to be the location where the fluid particle on the free surface first meets the body surface. According to this definition and applying the kinematic free-surface boundary condition (2.2), we obtain the governing equation for the motion of the intersection point \( y = \ell(t) \) at any time:

\[
\mathcal{H}(\ell) - Vt = \int_0^t v(\ell, \tau) \, d\tau
\]  \hspace{1cm} (3.3)

where \( \tau \) is the dummy time variable and \( v(\ell, \tau) \) is the vertical velocity of the free surface at \( y = \ell \). From the boundary-value solution, \( v(\ell, \tau) \) is determined to be given by

\[
v(\ell, \tau) \equiv \frac{\partial}{\partial z} \Phi(\ell, \eta(Y), \tau) = \frac{\partial}{\partial z'} \phi(\ell, 0, \tau) - V
\]  \hspace{1cm} (3.4)

and is ultimately a function of the intersection position \( Y(\tau) \). From (3.3), it is clear that the intersection position \( y = \ell \) at time \( t \) depends on the history of the intersection motion \( y = Y(\tau) \), \( \tau < t \), which is a priori unknown. Thus (3.3) presents the implicit time dependence for the intersection position \( y = \ell(t) \) only.

To derive the direct time dependence, we substitute (3.4) into (3.3), change the integral (with respect to \( \tau \)) into the one with respect to \( Y \), and obtain

\[
\mathcal{H}(\ell) = \int_0^t v_0(\ell, Y) \mu(Y) \, dY
\]  \hspace{1cm} (3.5)

where \( \mu(Y) = V(\, d\tau / dY) \) and \( v_0 = V^{-1} \partial \phi(\ell, 0, \tau) / \partial z' \). Equation (3.5) is an integral equation for the unknown \( \mu(Y) \) and in principle it can be solved provided that the dependence of the kernel \( v_0(\ell, Y) \) on \( Y \) is known.

For arbitrary bodies, the dependence of \( v_0 \) on \( Y \) is complex. In general, it is thus difficult to derive the closed-form solution to the integral equation (3.5) for \( \mu(Y) \). For bodies with smooth surfaces, the intersection point moves continuously and the
function \( \mu(Y) \) varies smoothly with \( Y \) during the impact. Thus, we can choose to make a Chebyshev-polynomial expansion for \( \mu(Y) \):

\[
\mu(Y) = \sum_{n=0}^{\infty} a_n T_n(Y) \quad \text{for} \quad Y \in [0, Y_{\text{max}}] \tag{3.6}
\]

where \( T_n \) denotes the \( n \)-th order Chebyshev polynomial of the first kind, \( a_n \) the unknown coefficient, and \( Y_{\text{max}} \) the maximum horizontal coordinate of the intersection point in the impact. \( T_n(Y) \) can be given as:

\[
T_n(Y) = \cos(n \cdot \arccos(2Y/Y_{\text{max}} - 1)) \quad \text{for} \quad Y \in [0, Y_{\text{max}}] \tag{3.7}
\]

which gives a set of orthogonal polynomials in \([0, Y_{\text{max}}]\). Note that for a sufficiently smooth function \( \mu(Y) \) for \( Y \in [0, Y_{\text{max}}] \), in principle, the series in (3.6) converges exponentially. In practice, we can truncate the series by keeping the first \( N \) terms only with an assurance of the requisite accuracy. To solve for \( Y \) from (3.6), it is more convenient to further expand the Chebyshev series into a normal polynomial:

\[
\mu(Y) = \sum_{n=0}^{N-1} a_n T_n(Y) = \sum_{n=0}^{N-1} b_n Y^n \quad \text{for} \quad y \in [0, Y_{\text{max}}] \tag{3.8}
\]

where the coefficient \( b_n \) is given in terms of \( a_n \) by the relation:

\[
b_n = \sum_{k=0}^{n} a_k c_{kn} \tag{3.9}
\]

with \( c_{kn}, k=0,1,\ldots,n \), being the known coefficients of the Chebyshev polynomial \( T_n(Y) \). From (3.8), the time dependence of the intersection position \( Y(t) \) can be obtained to be given by the relation:

\[
Vt = \sum_{n=0}^{N-1} \frac{b_n}{n+1} Y^{n+1}. \tag{3.10}
\]

To determine the unknown coefficients \( a_n, n=0,1,\ldots,N-1 \), we substitute (3.8)
into (3.5) to obtain
\[ \mathcal{H}(\ell) = \sum_{n=0}^{N-1} a_n \beta_n(\ell) \quad \text{for} \quad \ell > 0 \quad (3.11) \]
where the influence coefficient \( \beta_n \), \( n = 0, 1, \cdots, N - 1 \), is given by
\[ \beta_n(\ell) = \int_0^\ell v_0(\ell, Y) T_n(Y) \, dY, \quad n = 0, 1, \cdots, N - 1 \quad (3.12) \]

For simple geometries like wedges the dependences of \( \mathcal{H} \) and \( \beta_n \) on \( \ell \) are simple, so equation (3.11) can be solved analytically (see §4.1). For arbitrary bodies, in general, equation (3.11) must be solved numerically. To do that, in practice, we apply (3.11) at \( N \) discrete points, \( \ell_m, m = 1, 2, \cdots, N \) on the body surface which is possibly immersed during the impact. For exponential convergence, these points should be collocated at the maximas of the Chebyshev function \( T_N(Y) \) for \( Y \in [0, Y_{\text{max}}] \). The solution for the unknowns \( a_n \), \( n = 0, 1, \cdots, N - 1 \) is obtained by solving the resulting linear system of \( N \) algebraic equations.

\[ \sum_{n=0}^{N-1} \beta_n(\ell_m) a_n = \mathcal{H}(\ell_m) \quad \text{for} \quad m = 1, 2, \cdots, N \quad (3.13) \]

We must note that the intersection point defined above purely based on the solution of the linearized problem is different from the physical intersection point, which is located at the tip of the local thin jet flow. Owing to the fact that the dynamic pressure on the body associated with the thin jet flow is negligible small, however, the determination of the real physical intersection point between the free surface and the body is of no practical importance. On the other hand, the intersection point obtained from the linear solution, though not physical, corresponds to the location of the maximum impact pressure which is of significant concern in practical applications.
3.3 Pressure distribution on the wetted body surface

After obtaining the boundary-value solution and the motion of the intersection point, the impact pressure on the body can be evaluated according to Bernoulli's equation. In term of the velocity potential $\Phi$, the pressure at a point on the wetted body surface (say, $h = H(\ell)$) takes the form:

$$\frac{P(\ell,t)}{\rho} = -\frac{\partial \Phi}{\partial t} - \frac{1}{2}(\Phi_y^2 + \Phi_z^2)$$
$$\quad = -\frac{D\Phi}{Dt} - V\Phi_z - \frac{1}{2}(\Phi_y^2 + \Phi_z^2) \quad (3.14)$$

where $\rho$ is the fluid density, the pressure at infinity is set to be zero, and the gravity effect is ignored. Upon using (3.1) and (3.2),

$$\Phi_z = \phi_z' - V \quad (3.15)$$

$$\frac{D\Phi}{Dt} = \frac{D\phi}{Dt} - V\frac{Dz'}{Dt}$$
$$\quad = \frac{D\phi}{Dt} - V(\frac{Dz}{Dt} - \frac{D\eta}{Dt})$$
$$\quad = \frac{D\phi}{Dt} - V[-V - \frac{D}{Dt}(H - Vt)]$$
$$\quad = \frac{D\phi}{Dt} + V\frac{D}{Dt}H(Y) \quad (3.16)$$

After substituting (3.15) and (3.16) into (3.14) the pressure can be rewritten in terms of $\phi$ as:

$$\frac{P(\ell,t)}{\rho} = -\frac{D\phi}{Dt} - \frac{1}{2}(\phi_y^2 + \phi_z^2) - V\frac{D}{Dt}H(Y) + \frac{1}{2}V^2. \quad (3.17)$$

Note that the nonlinear terms in (3.14) and (3.17) are included for the purpose of removing the singularity of the linear solution at the intersection point (Wagner 1932; Zhao and Faltinsen 1993). Although not consistent with the linearization of the problem, this solution behaves properly near the intersection point and compares
well with physical experiments and fully-nonlinear numerical simulations (§4).

3.4 Impact force on the body

The impact force on the body can be determined by direct integration of the pressure over the wetted body surface.

Besides, it can also be obtained by using the so-called added mass theory. With the added mass theory, the impact force \( F \) is evaluated by the formula:

\[
F(t) = \frac{d(M_a V)}{dt}
\]

(3.18)

where \( M_a \) is the infinite-frequency heave added mass of the portion of the body immersed in the water in the impact. The added-mass coefficient depends on the body geometry only. From a computational standpoint, the added-mass method is much simpler than the direct pressure integration.

Note that the results for the force on the body obtained by these two methods may differ (cf. §4.3) since the nonlinear terms in Bernoulli's equation (3.14) are not considered in the classical added-mass theory.
Chapter 4

Water entry of wedges

4.1 Analytical solution

In this section, we present the analytic solution for the case of a two-dimensional wedge, whose surface is described by \( h = \ell \tan \alpha \) with \( \alpha \) denoting the deadrise angle. The solution is verified by making comparisons to the existing physical experiments and the fully-nonlinear simulation results. Unlike other works (e.g. Wagner 1932), the present theory is valid for any deadrise angle.

4.1.1 Splash-up of the water

For the wedge, the velocity potential \( \phi \) describes the problem of a uniform vertical flow past a rhombus with maximum width \( Y(t) \) and entry angle \( \pi/2 - \alpha \). The solution to this problem can be obtained in a closed form by mapping the flow in the physical plane \( (Z = y' + iz') \) into the uniform vertical flow in the mapped plane \( (W = p + iq) \) by making use of the Schwartz-Christoffle transformation. We write the solution in the form:

\[
\phi(y', z', t) \equiv \phi(Z, t) = \text{Re}\{-iW\} VY(t)/A
\]  

(4.1)

where \( Z \) and \( W \) are related by the one-on-one mapping relation:

\[
Z/Y(t) = A^{-1} \int_0^W \left( \frac{w^2}{w^2 + 1} \right)^\theta \, dw + 1
\]  

(4.2)
which maps the rhombus in the $Z$ plane into a vertical line segment from $(0,-1)$ to $(0,1)$ in the $W$ plane. Here the coefficient $A$ is a function of $\alpha$ and is given by:

$$A(\alpha) = \cos \alpha \int_0^1 \left( \frac{w^2}{1 - w^2} \right)^\theta \, dw \quad \text{with} \quad \theta = (\pi - 2\alpha)/2\pi. \quad (4.3)$$

The complex velocity of the flow is obtained from (4.1) to be:

$$u - iv = -iV Y(t) A^{-1} \frac{dW}{dZ} = -iV \left( \frac{W^2 + 1}{W^2} \right)^\theta. \quad (4.4)$$

Knowing $\phi$ at any instant, the motion of the intersection point $Y(t)$ can be determined according to (3.10) with the coefficients $b_n$, $n=0,1,\ldots,N-1$, given by (3.9) and (3.11). To evaluate the influence coefficient $\beta_n$, $n=0,1,\ldots,N-1$, the vertical velocity at $Z=\ell$ is first found from (4.4) to be:

$$v_0(\ell,Y) = \left( \frac{p^2 + 1}{p^2} \right)^\theta \quad (4.5)$$

where $W=p$ is the image of the physical point $Z=\ell$ and they are related by the mapping (4.2).
\[ \ell = A^{-1} \int_0^p \left( \frac{w^2}{w^2 + 1} \right)^\theta \, dw + 1 \quad (4.6) \]

Upon substituting (4.5) into (3.12) and using (4.6), we can easily show that

\[ \beta_n = \sum_{k=0}^n c_{kn} f_k(\alpha) \ell^{k+1} \quad n = 0, 1, \ldots, N - 1 \quad (4.7) \]

where the constant \( f_k(\alpha) \) is given by

\[ f_k(\alpha) = \int_0^1 \left( \frac{p^2 + 1}{p^2} \right)^\theta \lambda^k \, d\lambda \quad k = 0, 1, \ldots, N - 1 \quad (4.8) \]

Here \( \lambda \equiv Y/\ell \) and \( p \) is a function of \( \lambda \) given by the relation:

\[ (\lambda^{-1} - 1) \lambda = \int_0^p \left( \frac{w^2}{w^2 + 1} \right)^\theta \, dw \quad (4.9) \]

Since \( H(\ell) = \ell \tan \alpha \) for the wedge, it follows from (3.11) that \( a_0 = \tan \alpha / f_0 \) and \( a_n = 0 \) for \( n = 1, \ldots, N - 1 \). Thus, we have \( b_0 = a_0 \) from (3.9) and \( Y(t) = Vtf_0(\alpha) / \tan \alpha \) from (3.10).

The movement of the intersection between the body and free surface during the impact can also be described in terms of the vertical coordinate \( h(t) = Y(t) \tan \alpha \). Clearly we have \( h(t) = Vtf_0(\alpha) = Vt \gamma(\alpha) \) where the dimensionless coefficient \( \gamma \) measures the splash-up of water on the body (see figure 4-2) and can be determined from (4.8) and (4.9) to take the form:

\[ \gamma(\alpha) \equiv f_0(\alpha) = A \int_0^\infty \left[ \int_0^p \left( \frac{w^2}{w^2 + 1} \right)^\theta \, dw + A \right]^{-2} \, dp \quad (4.10) \]

Table 4.1 presents the numerical values of \( A \) and \( \gamma \) for a number of deadrise angles. Figure 4-3 plots the solution of \( \gamma \) as a function of \( \alpha \). For \( \alpha = 0 \), it shows \( \gamma = \pi / 2 \) which agrees with the asymptotic solution of Wagner (1932) for \( \alpha \ll 1 \). For \( \alpha = \pi / 2 \), \( \gamma = 1 \) which is identical to the solution of Von Kamman (1929). This result agrees with the general expectation that there exists no water splash-up on a vertical thin
Figure 4-2: Dimensionless Coefficient $\gamma$ measuring the splash-up of the water

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$A(\alpha)$</th>
<th>$\gamma(\alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99229044</td>
<td>1.56929479</td>
</tr>
<tr>
<td>4</td>
<td>0.96886070</td>
<td>1.56495351</td>
</tr>
<tr>
<td>10</td>
<td>0.92077805</td>
<td>1.55553150</td>
</tr>
<tr>
<td>30</td>
<td>0.74684426</td>
<td>1.51473011</td>
</tr>
<tr>
<td>45</td>
<td>0.59907209</td>
<td>1.46956195</td>
</tr>
<tr>
<td>60</td>
<td>0.43117780</td>
<td>1.40166402</td>
</tr>
<tr>
<td>81</td>
<td>0.14711515</td>
<td>1.21486118</td>
</tr>
<tr>
<td>89</td>
<td>0.01731911</td>
<td>1.04647318</td>
</tr>
</tbody>
</table>

Table 4.1: Water splash-up coefficients $\gamma$ for wedges with different deadrise angles

plate dropping into the water. Figure 4-3 also shows the solution obtained using Pierson's hypothesis (Payne 1994) the $\gamma$ is linearly dependent upon the deadrise angle $\alpha$. Overall, Pierson’s hypothesis is seen to considerably underestimate the value of $\gamma$. It seems to be useful only for small deadrise angles ($\alpha \ll 1$) while its prediction of $\gamma$ is off more than 100% from the exact solution for large deadrise angles ($\alpha > \pi/3$).

After obtaining the motion of the intersection, the velocity potential $\phi$ can be evaluated from (4.1) and (4.2) at any instant $t$ and thus the problem is solved completely.
Figure 4-3: The location of the intersection between the wedge and the free surface as a function of the deadrise angle. Results plotted are: Pierson’s hypothesis (Payne 1994) (--- · --); the solution using the Lewis-form representation of the body (- - -); the exact solution (——).
4.1.2 Pressure distribution on the body and Free surface shape

The pressure on the body can be obtained from (3.17) and the pressure coefficient \( C_p(\ell, t) = \frac{P(\ell, t)}{\rho V^2 / 2} \) is given by

\[
C_p = \frac{2\gamma}{A \tan \alpha} \left[ \left( \frac{1 - q^2}{q^2} \right)^\theta \int_{|q|}^1 \left( \frac{w^2}{1 - w^2} \right)^\theta dw - q \right] - \left( \frac{1 - q^2}{q^2} \right)^{2\theta} - 2\gamma + 1 \tag{4.11}
\]

where the real variable \( q \) is related to the position on the body by

\[
\frac{h(\ell)}{Vt} = \gamma A^{-1} \cos \alpha \int_0^q \left( \frac{w^2}{1 - w^2} \right)^\theta dw + \gamma \quad \text{for} \quad 0 \leq \frac{h(\ell)}{Vt} \leq \gamma \, . \tag{4.12}
\]

Near the intersection \((Y(t) - \ell \ll 1)\), the solution (4.11) behaves asymptotically as:

\[
C_p = -|q|^{-4\theta} + \frac{2\gamma}{\sin \alpha} |q|^{-2\theta} + O(1) \tag{4.13}
\]

for

\[
|q| = [A(2\theta + 1)(1 - \ell/Y) / \cos \alpha]^{\frac{1}{2\theta+1}} \ll 1 \, . \tag{4.14}
\]

Clearly, the pressure is singular at the intersection \((\ell=Y)\), but the resulting impact force is finite since the singularity is integrable.

The instantaneous free-surface shape can be obtained by integrating the kinematic free-surface boundary condition (2.2) with time. The result can be expressed in terms of a similarity variable \( \chi = y/Vt \) as:

\[
\frac{\eta(y, t)}{Vt} = \frac{\eta(\chi)}{Vt} = \frac{\chi \tan \alpha}{A\gamma} \int_{p_0}^\infty \left[ A^{-1} \int_0^p \left( \frac{w^2}{1 + w^2} \right)^\theta dw + 1 \right]^{-2} dp - 1 \tag{4.15}
\]

where the low limit of the integral \( p_0 \) depends on \( \chi \) and is determined from:

\[
\chi \tan \alpha = \gamma A^{-1} \int_0^{p_0} \left( \frac{w^2}{1 + w^2} \right)^\theta dw + \gamma \, . \tag{4.16}
\]

Figures (4-4) and (4-5) display the pressure distributions on the wetted body.
surface as well as the associated free-surface profiles for a range of deadrise angle \( \alpha \). The negative pressure due to the singularity at the intersection in the present solution is not shown in Figures (4-4) and (4-5) since it is not physical and will not be present if the free-surface nonlinearity is accounted for (Zhao & Faltinsen 1993). Also note that since the present solution agrees with the numerical result of Zhao & Faltinsen (1996) within the graphical error, their result is omitted in Figures (4-4) and (4-5). From the results in Figures (4-4) and (4-5), it is observed that for small \( \alpha \), the maximum pressure obtains near the intersection between the body and the free surface while for large \( \alpha \), it locates at the keel point.

### 4.2 Comparison to similarity solution

Figures (4-6)–(4-10) compare the pressure distribution on the dropping wedge with different deadrise angles, \( \alpha = 10^\circ, 30^\circ, 45^\circ, 60^\circ, 81^\circ \), between the present theory and the similarity solution. The results show good agreement, especially for small deadrise angles. For larger deadrise angles the result near the intersection point doesn’t compare very well. However, for the larger deadrise angles the maximum pressure locates at the keel of the wedge instead of near the intersection point. From a design point of view this disagreement has no much effect for the slamming force and maximum pressure.

Figure 4-11 shows the result of the impact force coefficient as a function of the deadrise angle (\( \alpha \)) of the wedge. For comparisons, the similarity solution from Zhao & Faltinsen (1996), theories of Wagner and Von Karman are shown in Figure 4-11. For the present theory, in addition to the solution by the direct pressure integration, the result obtained using the added mass theory is also shown. With the added mass theory, the impact force is calculated by the formula: \( F = d(M_a V)/dt \), where \( M_a \) is the infinite-frequency heave added mass of the portion of the body immersed in the water during the impact. (details about \( M_a \) are shown in the next section). It is seen that the result from the added mass theory is always larger than that by the direct pressure integration. This is caused by the fact that the force according to
Figure 4-4: The pressure distributions on the wedge and the associated free-surface profiles at different deadrise angles $\alpha = 1^\circ, 4^\circ, 10^\circ, 30^\circ$. 
Figure 4-5: The pressure distributions on the wedge and the associated free-surface profiles at different deadrise angles $\alpha = 45^\circ, 60^\circ, 81^\circ, 89^\circ$. 
Figure 4-6: Comparison of pressure distribution on the dropping wedge with deadrise angle $\alpha = 10^\circ$ between the present theory (----) and the similarity solution (- - -).
Figure 4.7: Comparison of pressure distribution on the dropping wedge with deadrise angle $\alpha = 30^\circ$ between the present theory (——) and the similarity solution (- - -).
Figure 4-8: Comparison of pressure distribution on the dropping wedge with deadrise angle $\alpha = 45^\circ$ between the present theory (---) and the similarity solution (- - -).
Figure 4-9: Comparison of pressure distribution on the dropping wedge with deadrise angle $\alpha = 60^\circ$ between the present theory (-----) and the similarity solution (- - -).
Figure 4-10: Comparison of pressure distribution on the dropping wedge with deadrise angle $\alpha = 81^\circ$ between the present theory (——) and the similarity solution (- - -).
Figure 4-11: Comparisons of force coefficient on the dropping wedge among the present theory by direct pressure integration method (——), present theory by added mass method (· - ·), asymptotic theories of Wagner (1932) (— ⋅ ⋅ —), von karman (1929) (— ⋅ —) and Von Karman monomentum (⋅ ⋅ ⋅), and the similarity solution (△).

added mass corresponds to the use of the linear pressure only. The nonlinear terms in Bernoulli’s equation gives higher-order singularity which gives a negative contritubition to the impact force which is not included in the added mass theory. Solution from the direct pressure integration agrees with similarity solution very well. Our analytical soultion is exactly same as numerical solution of Zhao & Faltinsen (1996).

4.3 Comparison to experiments

To verify the analytic solution, we make a comparison to the experiment of Zhao & Faltisen (1996) who examined the problem of free falling of a wedge into the calm water. The model has a deadrise angle of 30°. Owing to the fact that the velocity
of the body varies during the falling in the experiment, the above analytic results for constant falling velocity must be modified for direct comparisons.

Upon applying the momentum theorem and ignoring the gravity effect, the instantaneous velocity of the wedge during the impact is obtained to be given by

\[ V(t) = \frac{V_0}{1 + M_a/M} \]  \hspace{1cm} (4.17)

where \( V_0 \) the initial falling velocity of the body, \( M \) the mass of the body, \( M_a \) the infinite-frequency heave added mass of the submerged portion of the body. Note that the added mass here depends on the position of the intersection \( Y(t) \) between the free surface and the body. For the wedge, it can be written in the form:

\[ M_a = C_a \rho Y^2 \]  \hspace{1cm} (4.18)

where the added mass coefficient

\[ C_a = \frac{\lambda (1 - \alpha/2\pi)^2 \pi}{2} \]  \hspace{1cm} (4.19)

with \( \lambda \) denoting the three-dimensional effect associated with non-infinitely-long bodies.

From Newton's second law, the impact force acting on the body is determined to be:

\[ F(t) = -M \frac{dV}{dt} = \frac{2C_a \rho Y V_0}{(1 + M_a/M)^2} \left( \frac{dY}{dt} \right) \]  \hspace{1cm} (4.20)

Upon using the relation \( \frac{dY}{dt} = V \frac{\gamma}{\tan \alpha} \) for the wedge, the above equation becomes:

\[ F(t) = 2C_a \frac{\gamma}{\tan \alpha} \frac{\rho V_0^2 Y}{(1 + M_a/M)^3} \]  \hspace{1cm} (4.21)

Using (4.17) and the solution of \( \frac{dY}{dt} \) for the wedge, the motion of the intersection position in the free-falling of the wedge can be determined by the relation:

\[ \frac{\rho C_a}{3M} Y^3 + Y = \frac{\gamma V_0 t}{\tan \alpha} \]  \hspace{1cm} (4.22)
In the experiment, the mass of the wedge is $M = 241 \text{ kg/m}$. Since the aspect ratio of the model is about 5, we follow Zhao & Faltisen (1996) to use $\lambda = 0.8$ for the three-dimensional effect in the experiment. This may underestimate the force in the initial stage since the 3D effect is not so large in the beginning, however, the added mass method overestimates the force. For $\alpha = 30^\circ$, our analytic solution gives $\gamma = 1.515$. The comparison between the present theoretical prediction and the measurement of Zhao & Faltisen (1996) for the impact force on the body is shown in Figure 4-12. The predictions by the classical theories of Von Kamman (1929) ($\gamma = 1$) and Wagner (1932) ($\gamma = \pi / 2$) are also shown in Figure 4-12. Clearly, the present theory is seen to agree well with the experiment up to the instant when flow separation is observed to occur on the body in the test. On the other hand, Von Kamman's solution significantly underestimates while Wagner's solution over-predicts the impact force on the model.

4.4 Comparison to fully-nonlinear numerical simulations

4.4.1 General numerical scheme

To further verify the validity of the analytic solution we also developed a fully-nonlinear numerical time-simulation method. The general numerical scheme used is that of Vinje & Brevig (1981).

The potential flow theory is used. The fluid motion can be described by the velocity potential $\phi$ and the stream function $\psi$. We use the complex potential given as $\beta(Z, t) = \phi + i\psi$, where $Z = y + iz$. In the fluid domain Laplace's equation is satisfied for both the velocity potential $\phi$ and the stream function $\psi$, therefore $\beta(Z, t)$ is an analytic function of $Z$. Then Cauchy's Integral Theorem holds:

$$\oint_C \frac{\beta(Z, t)}{Z - Z_0} dZ = 0$$  \hspace{1cm} (4.23)
Figure 4-12: Comparisons of the instantaneous impact force on the dropping wedge among the measurement of Zhao & Faltisen (1996) (-----), asymptotic theories of Von Karman (1929) (---·---) and Wagner (1932) (----·----), and the present theory (- - - -).
with $Z_0$ outside the contour of integration $C$, which is taken to include the free surface, body surface and infinity boundary. $C$ is divided into two parts, $C_\phi$ on which $\phi$ is known and $C_\psi$ on which $\psi$ is known. Letting $Z_0$ move onto the contour of integration $C$ and taking real or imaginary parts of (4.23) yield Fredholm integral equations of the second kind which have good properites for numerical solution. Thus,

\[ \theta_0 \psi(y, z, t) + \text{Re} \left( \int_C \frac{\phi + i\psi}{Z - Z_0} dZ \right) = 0, \quad \text{for} \quad Z_0 \in C_\phi \quad (4.24) \]

\[ \theta_0 \phi(y, z, t) + \text{Re}(i \int_C \frac{\phi + i\psi}{Z - Z_0} dZ) = 0 \quad \text{for} \quad Z_0 \in C_\psi \quad (4.25) \]

Here $\theta_0$ is the actual angle between two elements adjacent to $Z_0$ on $C$. For a smooth boundary, $\theta_0 = \pi$. The contour of integration $C$ consists of $S_B, S_F$ and $S_\infty$, where $S_B$ is the wetted body surface. $S_\infty$ is chosen so far away that its contribution in integration (4.23) is zero. The contribution from the free surface integral can be rewritten. For $|y| \geq b(t)$, where $b(t)$ is a large number dependent on time $t$, the flow can be represented by an vertical dipole in infinite fluid which locates at origin.

\[ \beta(Z, t) = \frac{a(t)}{Z} i \quad (4.26) \]

Then the integral from $-\infty$ to $-b$ and from $b$ to $\infty$ of (4.23) can be written as

\[ \pm \frac{a(t)}{Z_0} \ln(1 \pm \frac{Z_0}{b}) i \quad (4.27) \]

where the plus sign is valid for the integral from $-\infty$ to $-b$ and the minus sign for the integral from $b$ to $\infty$. The coefficient $a(t)$ is solved as an unknown with the requirement of continuity at $|y| = b$.

Numerically (4.24) and (4.25) are discretised at collocation points along the body boundary and the free surface boundary inside $b(t)$. This results in a linear equation system $AX = B$ where $X$ is a vector of unknowns corresponding to the unknown part of $\beta(Z)$, either $\phi$ or $\psi$. By solving this matrix equation at each time instant the solution of the boundary value problem is given. The whole problem is now solved as

45
an initial boundary value problem given the position and velocity of the body as well as the velocity potential and the elevation along the free surface. The new positions of the fluid particles on the free surface are integrated in time from the kinematic boundary condition stating that a fluid particle at the free surface will remain at the free surface, i.e.

\[
\frac{DZ}{Dt} = W^* \tag{4.28}
\]

\(W\) is the complex velocity and \(W^*\) denotes the complex conjugation of the complex velocity. The new velocity potential \(\phi\) on the free surface are integrated in time from the dynamic free surface boundary condition, i.e.

\[
\frac{D\phi}{Dt} = \frac{1}{2} WW^* \tag{4.29}
\]

where gravity is neglected.

### 4.4.2 Matching of the jet with the outer flow

In the water impact of the body, a jet flow with large velocity and small thickness is generally developed in the region near the intersection between the free surface and the body. The treatment of the intersection points and the jet flow is very critical for the numerical simulation. The problem arises because of the need to satisfy two boundary conditions at this point, namely the body boundary condition and free
surface boundary condition. It is well known that the confluence of the boundary conditions can result in a singularity in the complex potential $\beta(Z)$. Peregrine(1972) has analysed the impulsive start-up to velocity $U$ of a wavemaker in water of depth $h$. His calculations predict a logarithmically singular free surface profile at the intersection point. A related concern of considerable importance in the numerical implementation of (4.24) and (4.25) is how well one should try to resolve the behaviour of the free surface near the possibly singular intersection points. For the wedge entry problem, good resolution around this point leads to a very high velocity jet which almost immediately leads to numerical breakdown. On the other hand, poor resolution here leads to a violation of the conservation of fluid mass as experienced by Yim(1985). Between these two extremes there is a range of resolutions where practical calculations can take place for large deadrise angles ($\alpha > 60^\circ$). For smaller values of $\alpha$ the numerical simulations encounter very severe difficulties because of the fast moving jet (Greenhow,1987). The jet causes the numerical procedure to breakdown before satisfied solution is attained. It can be seen that how to treat the jet is the key to proceed numerical simulation for moderate and small deadrise angles. The jet flow is thin, long and moves very fast which is the reason of numerical breakdown. However, the pressure inside the jet is very close to the atmospheric pressure and practically not important, so we can approximate the flow solution inside the jet under the condition of not affecting the pressure distribution and free surface profile in the other area.

Zhao & Faltisen (1993) proposed a simple approach to overcome the difficulty by chopping off the jet from the global flow and completely ignoring the effect of the jet in the simulation. Here, we propose an alternative scheme to track the jet flow and include its influence upon the solution in the simulation.

The present scheme employs the matching idea to couple the local jet flow with the global flow. The jet flow is described by an analytic solution while the global flow is solved numerically. For the thin jet, we expand complex potential $\beta(Z, t)$ in a Taylor series around the intersection point ($Z=Z_i$):

$$\beta(Z, t) = \beta(Z_i, t) + \sum_{m=1}^{M} A_m(t)(Z - Z_i)^m$$  \hspace{1cm} (4.30)
where unknown coefficients $A_m, m=1, \cdots, M$, are functions of time. The potential at the intersection $\beta(Z_i, t)$ at time $t$ is determined by integrating the body and free-surface boundary conditions with time. Upon incorporating the jet to the global problem, the unknown coefficients $A_m, m=1, \cdots, M$, can then be determined. The velocity of the fluid particle inside the jet can be determined by differentiating (4.30) with respect to $Z$.

### 4.4.3 Numerical Implementation

For the simulation of water entry of wedges, we choose to use $M=2$ in (4.30) for the jet flow. We expand complex potential $\beta(Z)$ around the intersection point $Z_i$.

\[
\beta(Z) = \beta(Z_i) + A(Z - Z_i) + B(Z - Z_i)^2
\]

(4.31)

Where $A$ and $B$ are unknown complex coefficients. They can be solved by enforcing the body boundary condition and matching the jet flow solution with the rest numerical solution. Inside the jet, we can use the approximate analytical expression and hence avoid the numerical discretization there which is the reason of numerical breakdown.

In implementation, the matching scheme is introduced after the jet near the intersection emerges. We don’t know exactly a criterion for when the jet occurs. Practically we can start to match the jet just before the general numerical simulation breaks down. In the beginning, the jet is considered as the flow between the body surface and the first free-surface segment near the intersection. When the second free-surface segment becomes tangential to the jet free surface, the jet flow is extended to include the region between the body surface and the second free-surface segment. This process continues as the jet flow develops during the impact.

Numerical simulation starts with an initial submergence $h_0$. The free surface shape at initial time instant is given by analytic solution in (§4.1). In order to catch the high curvature near the jet uneven collocation is used with more collocation points at the root area. For the time stepping a fourth order Runge-Kutta procedure with
a second order Runge-Kutta starting is used.

The convergence of the numerical results is checked with the time step and the discretizations of the free surface and body surface. In addition, the global behaviors of the problem such as the conservation of mass, the conservation of momentum, and the conservation of energy are also examined.

### 4.4.4 Results

The fully nonlinear numerical results for wedges with deadrise angles $\alpha = 15^\circ, 30^\circ, 45^\circ$ are shown here. Initial submergence is chosen as $h_0 = 0.05$ and the constant dropping velocity is chosen as $V = 1$.

The results for wedge with deadrise angle $\alpha = 30^\circ$ are presented at first. Figure 4-14 shows the initial collocation according to analytical solution in (§4.1). More points are used in the high curvature root area. Then the general numerical procedure without matching jet flow is used until $t = 0.001$. At this moment the jet already emerged. Due to it the general numerical method will break down immediately, so we start the numerical method with the jet approximation. Figures (4-15) and (4-16) display the simulation results for the evolution of the free-surface profile. The results demonstrate that the present treatment of the local thin-jet flow near the intersection between the free surface and the body is quite effective. Figure 4-17 shows the collocation points near the jet area which describe the free surface very well. In figure 4-18, the analytic solution for the pressure distribution on the body during the impact of the wedge is compared with the fully-nonlinear simulation result. It is seen that for both the location and the magnitude of the maximum impact pressure, the agreement between the present theory and the fully-nonlinear simulation result is excellent. The comparison between the analytic prediction and the fully-nonlinear simulation result for the free surface profile near the body during the impact is shown in figure 4-19. For illustration, the comparison for the pressure distribution on the body is also shown there. Except the shape of the jet near the intersection which is not predicted by the analytic solution, the comparison between the analytic prediction and the fully-nonlinear simulation result is again excellent. As expected, both the
Figure 4-14: The initial collocation for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$ with a constant dropping velocity.

Theory and numerical simulations show that the maximum impact pressure occurs around the root of the jet flow, where the free surface retains the maximum curvature. The simulation results also indicate that the dynamic pressure due to the jet flow is actually negligible.

Figures (4-20)–(4-21) show the results for $\alpha = 15^\circ$ and figures (4-22)–(4-27) for $\alpha = 45^\circ$. It can be seen that for the wedges the similarity solutions are obtained. Also it is noted that the jet flow develops faster for smaller deadrise angles.
Figure 4-15: The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$ with a constant dropping velocity. $t = 0, 0.001, 0.01, 0.02, 0.03, 0.04, 0.05$
Figure 4-16: The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$ with a constant dropping velocity. $t = 0, 0.001, 0.06, 0.07, 0.08, 0.09, 0.10$
Figure 4-17: Collocation points near the intersection for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$ with a constant dropping velocity. $t = 0.06, 0.08, 0.10$
Figure 4-18: Comparison of the theory (——) and the fully nonlinear simulation (Δ) for the pressure distribution on the wetted body surface during the water entry of a wedge with the deadrise angle $\alpha=30^\circ$. 
Figure 4-19: Comparisons of the theory (—·—) and the fully-nonlinear numerical simulation (——) for the free surface elevation near the intersection and the impact pressure distribution on the body for the water entry of a wedge with the deadrise angle $\alpha=30^\circ$. 
Figure 4-20: Initial free surface profile and free surface at the moment of starting matching of a wedge with the deadrise angle $\alpha=15^\circ$ with a constant dropping velocity. $t = 0, 0.0002$
Figure 4-21: The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle $\alpha=15^\circ$ with a constant dropping velocity. $t = 0, 0.0002, 0.001, 0.002, 0.003, 0.004, 0.005$
Figure 4-22: Initial collocation of the free surface for the wedge entry with deadrise angle $\alpha = 45^\circ$, $t = 0$
Figure 4-23: Initial free surface profile and free surface at the moment of starting matching of a wedge with the deadrise angle $\alpha=45^\circ$ with a constant dropping velocity. $t = 0, 0.004$
Figure 4-24: The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle $\alpha=45^\circ$ with a constant dropping velocity. $t = 0, 0.004, 0.02, 0.04, 0.06, 0.08, 0.10$
Figure 4-25: The evolution of the free surface profile near the intersection for the water entry of a wedge with the deadrise angle $\alpha=45^\circ$ with a constant dropping velocity. $t = 0, 0.004, 0.12, 0.14, 0.16, 0.18, 0.20$
Figure 4-26: Dimensionless free surface profile for the water entry of a wedge with the deadrise angle $\alpha=45^\circ$ with a constant dropping velocity. $t=0,0.004,0.10,0.20$.
Figure 4-27: Dimensionless free surface profile near the intersection for the water entry of a wedge with the deadrise angle $\alpha=45^\circ$ with a constant dropping velocity. $t = 0, 0.004, 0.10, 0.20$
Chapter 5

Impact of circular cylinder

Next we consider the water entry of a horizontal circular cylinder of radius $R$ (see figure 5-1). In this case, all lengths can be normalized by the radius of the cylinder. Thus, we can set $R=1$ in the following analysis. The cylinder surface below its center can be represented by $h(\ell) = 1 - (1 - \ell^2)^{1/2}$.

5.1 Analytical solution

For this case, the velocity potential $\phi(y', z', t)$ describes the vertical uniform flow past an oblong with maximum width $2Y(t)$ and maximum height $2h(Y)$. This oblong is composed of the sector of the circular cylinder, which is immersed in the water at time $t$, and its image about the plane $z'=0$. Like wedges, the closed-form solution for $\phi$ can also be derived with the use of conformal mapping.

To solve for $\phi$ at any instant, the physical flow in the $Z$-plane is mapped into a vertical uniform flow in the $W$-plane through a double conformal mapping (see figure 5-2).

\[
Z = -\frac{iY}{\tan Q} \quad \text{and} \quad W = -\frac{iY\nu}{\sin(\nu Q)}
\]  
(5.1)

where $Q = \zeta + i\zeta$ is an intermediate complex variable and the dimensionless coefficient $\nu$ is defined by

\[
\nu = \frac{\pi/2}{\arctan(Y/[1 - (1 - Y^2)^{1/2}]})
\]  
(5.2)
Figure 5-1: Circular cylinder impact

Figure 5-2: Conformal mapping for circular cylinder impact
In terms of mapping variables $W$ and $Q$, the velocity potential $\phi$ can be written in the form:

$$\phi(y', z', t) = \text{Re}\{-iVW\} \quad (5.3)$$

and the velocity components are obtained to be given by

$$u - iv = -iV \frac{\nu^2 \sin^2 Q}{\sin(\nu Q) \tan(\nu Q)} . \quad (5.4)$$

In particular, the vertical velocity component on the free surface ($z'=0$) at $y'=\ell$ is determined from (5.4) to be:

$$v_0(\ell, Y) = \frac{\nu^2 \sinh^2 \zeta}{\sinh(\nu \zeta) \tanh(\nu \zeta)} \quad (5.5)$$

where the real variable $\zeta$ is related to $\ell$ by:

$$\zeta = \frac{1}{2} \ln \left(\frac{\ell/Y - 1}{\ell/Y + 1}\right) . \quad (5.6)$$

After obtaining the boundary-value solution in terms of the intersection position $Y(t)$, the motion of the intersection point between the free-surface and the cylinder can be solved from (3.10). To determine the unknown coefficients $a_n$ from equation (3.11), the influence coefficients $\beta_n$, $n = 0, 1, \cdots, N - 1$, are evaluated by substituting (5.5) into (3.12) to obtain:

$$\beta_n(\ell) = \int_0^\ell \frac{\nu^2 \sinh^2 \zeta T_n(Y)}{\sinh(\nu \zeta) \tanh(\nu \zeta)} \, dY \quad \text{for} \quad n = 0, 1, \cdots, N - 1 . \quad (5.7)$$

Since the dependences of $\nu$ and $\zeta$ upon $Y$ are complex (cf. (5.2) and (5.6)), the integral in (5.7) cannot be evaluated analytically. Thus equation (3.11) must be solved numerically. For a number of $N$‘s, the values of $a_n$‘s are displayed in table ???. As expected, the coefficients $a_n$‘s are seen to vanish with increasing the value of $n$. For $N \geq 10$, it is seen that the solution of $a_n$‘s converges to the fourth decimal place. $\mu(Y)$ has some kind of singularity at $Y = 1$ for circular cylinder impact, which affects the convergence of the Chebyshev expansion. If we choose $Y_{\text{max}}$ a little bit
Table 5.1: Convergence of the Chebyshev polynomial expansion for the velocity of the intersection point between the free surface and the body during the water entry of the circular cylinder with increasing the order of the Chebyshev polynomial. $Y_{\text{max}} = 1$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$a_7$</th>
<th>$a_8$</th>
<th>$a_9$</th>
<th>$a_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.717311</td>
<td>1.052850</td>
<td>0.506851</td>
<td>0.352121</td>
<td>0.081440</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.745766</td>
<td>1.079061</td>
<td>0.583992</td>
<td>0.327875</td>
<td>0.155867</td>
<td>0.050481</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.768474</td>
<td>1.100174</td>
<td>0.633563</td>
<td>0.350079</td>
<td>0.216011</td>
<td>0.099240</td>
<td>0.031859</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.786767</td>
<td>1.201096</td>
<td>0.688380</td>
<td>0.446078</td>
<td>0.272286</td>
<td>0.146436</td>
<td>0.063983</td>
<td>0.020378</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.806371</td>
<td>1.240116</td>
<td>0.725113</td>
<td>0.493316</td>
<td>0.318191</td>
<td>0.190994</td>
<td>0.100130</td>
<td>0.041773</td>
<td>0.013422</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.823602</td>
<td>1.272391</td>
<td>0.765664</td>
<td>0.531383</td>
<td>0.360154</td>
<td>0.229833</td>
<td>0.134027</td>
<td>0.068333</td>
<td>0.027318</td>
<td>0.008549</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.837369</td>
<td>1.303755</td>
<td>0.797741</td>
<td>0.567838</td>
<td>0.396661</td>
<td>0.267153</td>
<td>0.167355</td>
<td>0.094901</td>
<td>0.047387</td>
<td>0.019215</td>
<td>0.005798</td>
</tr>
</tbody>
</table>

Table 5.2: Convergence of the Chebyshev polynomial expansion for the velocity of the intersection point between the free surface and the body during the water entry of the circular cylinder with increasing the order of the Chebyshev polynomial. $Y_{\text{max}} = 0.96$

smaller than 1, for example, $Y_{\text{max}} = 0.96$, the Chebyshev expansion converges faster than $Y_{\text{max}} = 1$. The results of $a_n$ for $Y_{\text{max}} = 0.96$ are shown in table ??, but we still choose $Y_{\text{max}} = 1$ to give results since it also converges and gives wider solution.

For the initial stage of the impact, a simple asymptotic solution can be derived. For $Vt \ll 1$, we have $Y \ll 1$. In this case, the cylinder surface can be represented by $h(\ell) = \ell^2/2 + O(\ell^4)$ for $\ell \ll 1$. From (5.2), we have $v = 1 + O(Y)$ which leads to $v_0(\ell, Y) = \ell/(\ell^2 - Y^2)^{1/2} + O(Y)$ from (5.5). From (3.12), it obtains that

$$
\beta_n = \sum_{k=0}^{n} \ell^{k+1} c_{kn} \int_0^1 \lambda^k / (1 - \lambda^2)^{1/2} \, d\lambda [1 + O(\ell)].
$$

Solving (3.11), we have $a_0 = a_1 = 1/4$ and $a_n = 0$ for $n \neq 0, 1$. Thus, it follows that $b_1 = 2a_1 = 1/2$ from (3.9) and $Y = 2(Vt)^{1/2} + O(Vt)$ for $Vt \ll 1$ from (3.10). We remark that this solution is identical to that obtained with the use of Wagner’s theory (Faltinsen 1990).

The solution of the integral equation for the circular cylinder impact is shown in...
Figure 5-3: Solution of the integral equation for water entry of a circular cylinder at constant dropping velocity.

Figure 5-3. Figure 5-4 shows the result for the time variation of the intersection point between the free surface and the body in the water entry of the circular cylinder with constant dropping velocity. The exact solution shown is obtained with $N=6$. For comparison, the Wagner's solution is also shown in Figure 5-4. Clearly, as expected, these two solutions agree well for small time only and differ significantly beyond the initial stage of the impact process. In particular, the Wagner's solution considerably overestimates the speed of the motion of the intersection point.

According to (3.17), (5.1), (5.2), (5.3) and (5.4), the pressure on the wetted body surface can be derived analytically. Since its expression is quite complicated the results are given numerically. Figure (5-5) shows the pressure distribution at the
Figure 5-4: Horizontal motion of the intersection point during the water entry of a circular cylinder at constant dropping velocity. The results plotted are: the present exact solution (---), Wagner’s theory (1932) (---·---), and the Lewis-form approximate solution (- - -).
Figure 5-5: Pressure distribution on the wetted body surface of a circular cylinder at constant dropping velocity. \( \text{Vt}/R = 0.0025, 0.01, 0.02, 0.03, 0.04, 0.05, 0.075, 0.1 \)

different time instants during the impact. We can see that the maximum pressure locates near the intersection point initially and eventually moves to the keel point.

### 5.2 Comparison to experiments

For confirmation, we make comparisons for the impact force on the cylinder to the experiments (Armand & Cointe, 1986 and Campbell & Weynberg, 1980) in which the cylinder is forced to drop into the water with constant velocities.

Two method can be used to calculate the impact force as discussed in (§3.4). At
first direct pressure integration method is used. For simplicity we can also employ the added mass theory, i.e. \( F(t) = \frac{d(M_a V)}{dt} \). The heave added mass of the sector of the circular cylinder immersed in the water during the entry is determined using the formula \( M_a = C_a \rho Y^2 \). For the present solution, we evaluate the added mass coefficient by using the formula from Taylor (1930):

\[
C_a = \pi (\nu^2 + 2) / 3 \left[ \pi (1 - 1/\nu) \csc^2 (\pi / \nu) + \tan^{-1} (\pi / \nu) \right]. \tag{5.9}
\]

The comparisons are shown in Figure 5-6, where the results from the classical theories of Von Karman (1929) and Wagner (1932) are also plotted. From Figure 5-6, it is clear that Wagner's theory significantly overestimates while Von Karman's theory under-predicts the impact force coefficient. The present theory with direct pressure integration is seen to agree very well with the experiments. The present theory with added mass method overestimates the force, but it still compares with experiments reasonably well.
Figure 5-6: The impact force coefficient for the water entry of a circular cylinder with constant dropping velocity. The results plotted are: experiment of Campbell & Weynberg (1980) (---); experiments from Armand & Cointe (1986) with the falling velocity $V=7.38 \text{ m/s}$ (---) and $V=2.33 \text{ m/s}$ (---); the classical theories of Von Karman (1929) and Wagner (1932) (· · ·); present theory with direct pressure integration (---); and the present theory with added mass method (---).
Chapter 6

Impact of arbitrary ship-like sections

6.1 Lewis-form approximation

For arbitrary body geometries, it is difficult to solve the boundary-value problem for $\phi$ analytically. For ship-like sections, however, a semi-analytical closed-form solution can be obtained by making use of the Lewis approximation for the body surface.

To solve for $\phi$ at any instant, we first adopt the Lewis form to approximate the wetted body surface:

$$Z_0(t) = \ell + ih = H(t) \sum_{n=1}^{3} A_n e^{i(3-2n)\theta} \quad \text{for} \quad \pi \leq \theta \leq 2\pi \quad (6.1)$$

where $H(t) = \mathcal{H}(Y)$ is the maximum immersed draft of the body and the dimensionless coefficients, $A_1$, $A_2$ and $A_3$, are defined by

$$A_1 = \frac{1}{2}(\lambda + 1) - A_3, \quad A_2 = \frac{1}{2}(\lambda - 1) \quad (6.2)$$

and

$$A_3 = -\frac{1}{4}(\lambda + 1) + \frac{1}{4} \left[ (\lambda + 1)^2 + 8\lambda \left(1 - \frac{4\sigma}{\pi}\right) \right]^{1/2} \quad (6.3)$$

where $\lambda = Y/H$ and $\sigma = S/2YH$ with $S$ the area of the wetted body section. Since the
intersection position $Y$ varies with time, in general, these coefficients are functions of time $t$. With this approximation for the body surface, the physical flow round the body in the $Z$-plane can be mapped into a vertical uniform flow past a unit circle in the $W$-plane based on the mapping relation:

$$Z/H(t) = \sum_{n=1}^{3} A_n W^{3-2n}.$$  \hspace{1cm} (6.4)

The velocity potential $\phi$ can then be written in the form:

$$\phi(y', z', t) = \text{Re} \left\{ -iA_1 VH(W - W^{-1}) \right\}$$  \hspace{1cm} (6.5)

and the velocity components are obtained to be given by

$$u - iv = -iA_1 V (1 + W^{-2}) \left[ \sum_{n=1}^{3} (3 - 2n) A_n W^{2-2n} \right]^{-1}.$$  \hspace{1cm} (6.6)

After obtaining the boundary-value solution for a given intersection $Y(t)$, the evolution of the intersection point is determined by following the procedure in §3.2
and the impact pressure on the body surface is obtained from (3.17).

6.2 Applications to wedge and circular cylinder

To examine the validity and accuracy of the proposed solution scheme, let us reexamine the impact problem of a wedge for which the exact solution is derived in §4.1. For the wedge,

$$\lambda = \cot \alpha, \quad \sigma = 1/2$$

(6.7)

and the coefficients $A_1$, $A_2$ and $A_3$ are given by

$$A_1 = \frac{1}{2} (\cot \alpha + 1) - A_3 , \quad A_2 = \frac{1}{2} (\cot \alpha - 1)$$

(6.8)

and

$$A_3 = -\frac{1}{4} (\cot \alpha + 1) + \frac{1}{4} \left[ (\cot \alpha + 1)^2 + 8 \cot \alpha \left( 1 - \frac{2}{\pi} \right) \right]^{1/2}$$

(6.9)

They are functions of $\alpha$, but independent of $t$ according to (6.8) and (6.9). From (6.6), the velocity of the free surface $v_0(\ell, y)$ is obtained, which is then used to determine the influence coefficient $\beta_n$ in (3.12). From (3.11), the unknown coefficients $a_n$'s are solved. The final solution for the motion of the intersection point can be expressed as: $h(Y) = V t \gamma(\alpha)$ with $\gamma(\alpha)$ now given by

$$\gamma(\alpha) = \int_{1}^{\infty} \frac{A_1 (1 + p^{-2}) \cot \alpha}{(A_1 p + A_2 p^{-1} + A_3 p^{-3})^2} \, dp .$$

(6.10)

This solution derived by using the Lewis-approximation for the body surface is shown in Figure 4-3 and compares very well with the exact solution (§4.1).

For the case of a circular cylinder, we have

$$\lambda = Y / [1 - (1 - Y^2)^{1/2}]$$

(6.11)

$$\sigma = [\arcsin Y - Y (1 - Y^2)^{1/2}] / [Y - Y (1 - Y^2)^{1/2}]$$

(6.12)

With (6.6), the influence coefficients $\beta_n$'s can be evaluated from (3.12) and then the
coefficients $a_n$'s are solved from (3.11). The result obtained with $N=6$ is shown in Figure 5-4. It is observed that the comparison with the exact solution is excellent.

According to the results of the wedge and circular cylinder we can conclude that this approximation by lewis Form can be used for arbitrary ship-like section. The general procedure has been given in (§6.1). For given section we only need to consider their differences in $A_n$ which denote the geometry of the body. Two more applications for parabolic section and bow flare section are shown.

6.3 Application to parabolic section

For the case of arbitrary bodies, as an illustration, we consider a parabolic sector whose surface is given by $h=\ell^2$. For this body geometry, we have

$$\lambda = 1/Y, \quad \sigma = 2/3. \quad (6.13)$$

Using these parameters and (6.6), the unknown coefficients in (3.10) can be resolved from (3.11) and (3.12). For $Y \in [0, 2]$, the solution of $Y(t)$ obtained with $N=6$ is plotted in figures (6-2) and (6-3). In this case, the result by Wagner's theory is obtained to be $Y(t)=2(Vt)^{1/2}$. This result is also shown in figures (6-2) and (6-3). As expected, it agrees with the complete solution only at the initial stage of the impact.

6.4 Application to bow flare section and comparison to experiment

In this section the method is applied to the bow flare which is used for dropping test by Zhao & Faltinsen (1996). The solution before the flow separation at the knuckle is considered here. The data of the test section in their experiment is: half breadth of section is $B = 0.155m$, vertical distance from keel to knuckles is $H = 0.203m$, length of measuring section is $L = 0.10m$. The body geometry is shown in the figure 6-4. We use half breadth of section $B$ as dimensionless scale of the length. The
Figure 6-2: Solution of the integral equation for the water entry of a parabolic section at constant dropping velocity. The results plotted are: (——) the complete solution obtained with the use of Lewis approximation for the body surface; and ( - - -) Wagner’s (1932) theory.
Figure 6-3: Evolution of the intersection point between the free surface and the body in the water entry of a parabolic section at constant dropping velocity. The results plotted are: (——) the complete solution obtained with the use of Lewis approximation for the body surface; and (---) Wagner's (1932) theory.
body surface is approximated by a smooth function in the dimensionless form. The dropping velocity changed slightly during the experiment. For simplicity we assume it's a constant $V = 2.6m/s$ and use this value as the dimensionless scale of the velocity.

After applying the general method in (§6.1) the problem is solved completely. Figure 6-5 shows the solution of the integral equation, while the evolution of the intersection point is shown in figure 6-6. The pressure distribution can be calculated from (3.17). Then the impact force on the bow flare is given by direct pressure integration along the wetted body surface. Good agreement between the theory and the experiment is obtained which is shown in figure 6-7.
Figure 6-5: Solution of the integral equation for the impact of the bow flare section.
Figure 6-6: Evolution of the intersection point between the free surface and the body in the impact of bow flare section at constant dropping velocity.
Figure 6-7: Impact force on the bow flare section. The results plotted are: analytical solution (——) and experiment result (---).
Chapter 7

Conclusions and future work

7.1 Conclusions

We derive an analytic solution to the general impact problem of arbitrary two-dimensional bodies by using the conformal-mapping techniques. The linearized formulation of Wagner (1932) for the flat impact problem is extended to arbitrary bodies. The boundary condition on the body is satisfied exactly and the effect of local thin-jet flow near the intersection of the body with the free surface is ignored. For wedges and circular cylinder, the exact closed-form solutions are obtained and confirmed through comparisons to the existing experiments and the full-nonlinear numerical simulations results. For arbitrary bodies, we develop an analytic-solution scheme based upon the use of the Lewis-form representation of the body surface. The solutions derived in this study are effective and ready for practical applications.

The Mixed Euler Lagragian Boundary Element Integral Method is used for numerical simulation. The exact nonlinear free surface boundary conditions without gravity are satisfied. Since the jet flow near the intersection point between the body and the free surface brings the difficulty for numerical simulation a special treatment is used in this area. At each time instant a reasonable analytical expression of the jet flow is given with some unknown coefficients. By matching the jet flow and the outer flow during the numerical simulation the whole flow field can be solved and the simulation can go forward smoothly. For the wedge entry with different deadrise
angles satisfactory results are obtained.

7.2 Future work

In the near future, we can consider the extension of the present analytical method for the three-dimensional impact problem, such as sphere entry, cone impact, etc. For the fully-nonlinear numerical simulation, it may be extended to arbitrary ship-like sections and also to three-dimensional bodies.
Bibliography


