Phase Stabilization of Laser Beams in a Cold Atom Accelerometer

by

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Bachelor of Science, Mechanical Engineering
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Submitted to the Department of Mechanical Engineering
in Partial Fulfillment of the Requirements for the Degree of

Master of Science in Mechanical Engineering
at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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ABSTRACT

A cold atom accelerometer measures the displacement of a proof mass of laser cooled atoms with respect to an instrument reference frame. The cold atom interferometer’s reference frame is defined by a pair of specially prepared, counter-propagating laser beams, that measure inertially induced atom displacements with nm scale resolution. This corresponds to acceleration sensitivities comparable to state of the art electro-mechanical accelerometers. In dynamic environments, sensitivity is limited by the stability of the relative laser phase of the two interrogation laser beams, which is adversely affected by vibrations and temperature fluctuations of the interrogation beam optics. Without an independent measurement, the cold atom interferometer cannot distinguish platform acceleration from laser phase fluctuations, which thus are a potentially serious source of error.

In this thesis, a Michelson optical interferometer and an optical feedback loop were used to stabilize the relative phase of the interrogation laser beams in a cold atom accelerometer. A digital controller stabilized the relative phase via an electro-optic phase modulator. This control loop’s bandwidth encompasses 98.8% of the noise power as determined from the power spectral density of the open loop 795nm Michelson signal. Increasing the controller bandwidth would gain the system marginal improvement in noise reduction. At an atom interferometer dwell time of 1 msec, active laser phase stabilization improved the atom interferometer sensitivity; at an atom interferometer dwell time of 8 msec, an improvement was no longer evident. Improvements to the laser phase stabilization system are proposed to increase atom interferometer stability at longer dwell times.

Thesis Supervisor: Dr. Richard Stoner
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Nicole Byrne
For Andrew and my family
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Figure A-1: Hyper fine structure of a Cesium atoms' \( F=4 \) ground state. This image is a typical saturation absorption scan of the \( F=4 \) transitions.

Figure A-2: Hyper fine structure of a Cesium atoms' \( F=3 \) ground state. This image is a typical saturation absorption scan of the \( F=3 \) transitions.
Table 1: Summary of existing Cold Atom interferometer sensor performance. Note that this is only a sample of all the existing experiments and not an extensive list.
Advancements in atom interferometry with laser cooled atoms since the 1990’s have shown that these cold atom sensors can surpass the current sensitivities of state of the art mechanical and optical sensors. Atom interferometry-based inertial sensing derives its precision and accuracy from its use of clouds of non-interacting atoms as inertial proof masses. Atoms are near-ideal proof masses because they have well-known and invariant physical properties. Cold atom inertial sensors use ensembles of laser-cooled atoms, along with specially prepared laser beam pairs as precision yardsticks to measure inertially induced atom displacements.

In spite of their use of near ideal proof masses, cold atom inertial sensors are not perfect. The stability of the cold atom inertial sensor is only as good as that of the relative laser phase between the pair of laser beams used for the displacement measurement. Relative phase stability of these lasers is affected by variations in the physical path along which the lasers must propagate to interact with the atoms. This propagation path is defined by a mechanical system, affected by its environment, which must be made stable for effective operation of a cold atom inertial sensor. This thesis describes a system for the electro-optic control of the relative phase of a laser beam pair used for acceleration sensing using atom interferometry.
1.1 Cold Atom Inertial Sensing

This thesis pertains to an emergent method for inertial sensing using cold atoms, a method usually placed into the category of atomic and laser physics. A brief description of Cold Atom inertial sensing will clarify how a mechanical engineering investigation fits into that context. The basic operating principle of a Cold Atom accelerometer is illustrated in Figure 1. A proof mass is released from a known location with zero initial velocity and allowed to freely fall in an instrument reference frame. If one measures the change in the particle position occurring during a specified time interval, one can deduce the average dynamics of the sensor reference frame during that time interval. For this approach to work in highly dynamic conditions, one can allow the particles to freely fall for only a few msec in a compact sensor. For acceleration sensitivity at the micro-g level, displacement measurements with nanometer scale precision is required. To achieve this level of precision, we choose to use free atoms as our proof mass. This choice enables the use of a highly precise modality called atom interferometry [2] to make displacement measurements.

![Diagram](image)

**Figure 1-1:** The basic idea. A proof mass is released initially at rest in a sensor reference frame. Its subsequent motion during a specified time interval is measured, permitting determination of the mean sensor frame dynamics during the measurement interval.

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However, for use as proof masses, atoms must be prepared in a state of very low temperature. Room temperature atoms, e.g., travel many cm in a millisecond, exceeding the physical extent of a compact sensor. Alkali atoms, which interact strongly with light, can be cooled using laser beams to very low temperatures [15]: the atoms used in this work had temperatures of 20 uK, and mean speeds of order cm/sec, four orders of magnitude smaller than room temperature atoms.

Thus, Cold Atom accelerometry comprises a simple "catch and release" scheme, in which: laser cooling confines and cools a cloud of atoms; the atoms are released into free fall; and atom interferometry is used to measure their motion during a specified time interval after release. The specific type of atom interferometry used in this work, Light Pulse Atom Interferometry (LPAI: also referred to as Raman pulse interferometry) [3], uses a specially prepared pair of oppositely directed laser beams referred to as Raman beams. LPAI measures motion along the common axis of that laser beam pair. Note that atoms must therefore remain in the confines of the laser beams during the measurement sequence.

It is this Raman beam pair that connects the somewhat otherworldly domain of atomic physics to the mechanical world—these beams are prepared by and travel through optical components. The alignment, position, and motion of these components bear directly on the stability and quality of the inertial measurement. This thesis comprises an effort to measure and stabilize the relative phase of the atom interferometer Raman beams, in real time, as a means of improving the stability of inertial measurements using those laser beams.
1.2 Light Pulse Atom Interferometry (LPAI)

This section provides an overview summary of how an LPAI-based accelerometer is physically realized, and briefly describes the sequence of steps in an acceleration measurement sequence. Figure 1-2 is a schematic representation of an LPAI accelerometer. LPAI is implemented using a pair of coaxial, counter-propagating laser beams (often referred to as a Raman laser beam pair) directed onto the cold atoms. The Raman beam diameter used in this work was 7 mm, a reasonably typical value. The accelerometer measures inertially induced displacement along the axis of the laser beam pair; to lowest order, transverse motion is not registered by the accelerometer. Atoms must remain within the confines of the Raman beam pair for the duration of the measurement sequence, under accelerations either parallel or normal to the Raman beam pair. Note that this constraint restricts the maximum measurement time for a given acceleration environment or, conversely, a maximum acceleration given a specified measurement time.

![Figure 1-2: Light Pulse Atom Interferometry (LPAI) accelerometer mechanization.](image)

**Figure 1-2:** Light Pulse Atom Interferometry (LPAI) accelerometer mechanization.
Each measurement cycle consists of four phases: atom cooling and trapping; atom state preparation; LPAI laser pulse sequence; and measurement of atoms' internal quantum state population balance. Laser cooling uses specially configured laser beams and magnetic fields to cool the atoms and confine them in a Magneto-optic Trap (MOT). Room temperature thermal atoms have velocities of \(-200\text{m/s}\) and the MOT slows the atoms down to velocities on the order of \(-1\text{cm/s}\) and to temperatures of \(-20\text{μK}\). This degree of cooling permits interrogating the atoms for many milliseconds at a time.

The atom state preparation consists of a series of laser and microwave pulses that place the atoms in a specific magnetically insensitive atomic ground state. The LPAI interrogation sequence is composed of a series of three laser pulses. This series of laser pulses separate the atoms into a superposition of two internal quantum ground states and then recombine them. Finally, the relative populations of the two states are measured using lasers to fluoresce the atoms. The ratio of these two state populations is a measure of the inertial affects that the atoms have undergone through the interrogation process. More detail about atom interferometry will follow in chapter 2.

### 1.3 A Brief History of Cold Atom Accelerometers

Acceleration measurement with LPAI has shown promising results in several laboratory and a few commercial sensors. Much work has been done using atom interferometry to develop precise accelerometers to measure gravity (gravimeters) and also gravity gradients (gradiometers)[4,5,6,7]. The performance of cold atom LPAI laboratory gravimeters rivals even
the most precise falling corner cube sensors [6,7]. At Stanford, the work of the groups of Steve Chu and Mark Kasevich exemplify LPAI's potential for highly precise and stable acceleration sensing. The atomic fountains used in the work at Stanford take advantage of long interrogation times to achieve high sensitivities by launching atoms vertically upward and letting them fall back down [4,5,6]. The long interrogation times improve the sensitivity of a three pulse cold atom interferometer, which scales with (interrogation time)$^2$, resulting in a more sensitive measurement [4,5,6]. Advancements have also been made to make these sensors mobile and into more practical form factors [6,7]. Biedermann used two simultaneous atomic fountains to create a compact mobile gradiometer which was later installed with a vibration isolation system into a truck to map out local gravity gradients [6,11]. Barrett et al. have made advancements in mobile gravimeters by using a unique retro-reflecting hollow pyramid mirror to reduce the required optical axes down to a single beam of which multiple frequencies would pass through [7]. Barrett et al.'s mobile hollow pyramid MOT gravimeter "Miniatom" spawned µQuanS company which produces a commercial version of the "Miniatom" with improvements to electro-optics to increase sensitivity and reduce the size of the physics package. All the previous experiments were conducted using a retro-reflected Raman beam configuration but McGuiness et al. showed that counter propagating the Raman frequencies, by physically routing the two frequencies to opposite positions with respect to the atoms, permitted them to achieve high data rates [10]. High data rates allow McGuiness et al. to still reach practical sensitivities without needing the long interrogation times of the atomic fountains [10]. A summary performance comparison of these sensors can be seen in Table 1.
Table 1: Summary of existing Cold Atom interferometer sensor performance. Note that this is only a sample of all the existing experiments and not an extensive list.

<table>
<thead>
<tr>
<th>Author</th>
<th>Experiment</th>
<th>Volume of Vacuum chamber [cc]</th>
<th>Measurement Repetition Rate [Hz]</th>
<th>Interrogation time Tdwell [msec]</th>
<th>Scale Factor [rad/(m/s²)]</th>
<th>Short term Sensitivity [μg/Hz¹/²]</th>
<th>Short term phase stability [mrad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. Barrett et al [7]</td>
<td>“Miniatom” Gravimeter (Rb)</td>
<td>2e3</td>
<td>1</td>
<td>40</td>
<td>2.58E+04</td>
<td>0.17</td>
<td>42</td>
</tr>
<tr>
<td>μQuanS [7]</td>
<td>Commercial Gravimeter (Rb)</td>
<td>2e3</td>
<td>-</td>
<td>~50</td>
<td>4.00E+04</td>
<td>0.03</td>
<td>-</td>
</tr>
<tr>
<td>B. Barrett et al [7]</td>
<td>ICE Gravimeter to test Weak Equivalence Principle (tested in dynamic micro-g environment) (Rb)</td>
<td>~4e3</td>
<td>-</td>
<td>3</td>
<td>1.45E+02</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>Stanford-M. Kasevich [4]</td>
<td>Atomic Fountain- (Na)</td>
<td>1e5</td>
<td>1</td>
<td>100</td>
<td>2.00E+05</td>
<td>0.07</td>
<td>14</td>
</tr>
<tr>
<td>Stanford-A. Peters [5]</td>
<td>Precise Measure of g; Atomic Fountain- (Cs)</td>
<td>~7e3</td>
<td>0.77</td>
<td>160</td>
<td>3.33E+05</td>
<td>0.02</td>
<td>74</td>
</tr>
<tr>
<td>Stanford-G. Biedermann [6]</td>
<td>Mobile Gradiometer (Cs)</td>
<td>&gt;200</td>
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<td>85</td>
<td>1.11E+05</td>
<td>0.0042</td>
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<tr>
<td>McGuinness et al [10]</td>
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<td>50</td>
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<td>0.57</td>
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</tr>
<tr>
<td>Draper-Byrne</td>
<td>Compact Accelerometer test bed oriented to measure gravity (Cs)</td>
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<td>15</td>
<td>8</td>
<td>1.00E+03</td>
<td>2</td>
<td>75</td>
</tr>
</tbody>
</table>

The short term sensitivity values in Table 1 are dependent on the scale factors of the experiments as well as the data rates. Scale factor relates the atomic phase shift $\phi$ registered by the interferometer measurement to the applied acceleration, and is defined as the following [1]:
\[ \text{Scale Factor} = 2 \cdot \frac{2\pi}{\lambda} \cdot T^2; \quad \phi = a \times (\text{Scale Factor}) \]

\( \lambda \) is the wavelength of the alkali resonance and \( T \) is the dwell time. The relationship between noise per interrogation on the phase measurement signal, \( \delta \phi \), and the resultant noise density of the acceleration measurement, \( \delta a \), is as follows [1]:

\[ \delta a = \frac{\delta \phi}{(\text{Scale Factor}) \cdot \sqrt{f_i}} \]

\( f_i \) is the rate of interferometer measurements per unit time. These relationships highlight that the laser phase stability requirements are greater for a sensor with short interrogation times than sensors with long interrogation times. Operation at short interrogation time is critical to increasing the bandwidth and dynamic range of a sensor. High bandwidths, and the ability to sense large accelerations, are often requirements in precise navigation systems which are of particular concern for the work at Draper Laboratory and of McGuiness et al.

### 1.4 Sources of error in LPAI Measurements

There are multiple sources of systematic and random error in LPAI measurements. We now describe three primary sources of error in LPAI measurements: AC Stark shifts, magnetic field fluctuations and drift, and laser phase fluctuations and drift. Both AC Stark shifts and magnetic field variations can be compensated for but it should be noted that they are not the central concern of this thesis: the control loop primarily handles the affect of laser phase fluctuations and drifts. Two of these error sources arise from the application of the Raman laser light (AC Stark shift and laser phase fluctuations); the resulting phase errors are not directly dependent on
interrogation time $T$. However, the phase error arising from magnetic field variation does depend on $T$ because unlike the interferometer laser light, the magnetic field is present at all times.

**AC Stark shifts and Magnetic Field Fluctuations and Drift**

We can organize the effects of AC Stark shifts, and magnetic field fluctuations and drift into two categories: per shot phase errors which manifest during the Raman pulse times and ongoing phase errors which are occurring during the dwell times when the Raman pulses are off.

Both AC Stark shift and Magnetic field fluctuations alter the energy splitting between the two Cesium ground states at each pulse. Raman transitions utilize the two photon hyperfine splitting frequency to coherently manipulate the atoms [3]. Figure 1-3 shows the energy levels in an atom with two ground states: the hyperfine splitting is the energy difference denoted by $\omega_{HF}$.

![Energy Level Diagram](image)

Figure 1-3: Energy level diagram for a two state atom
The hyperfine splitting frequency, $\omega_{\text{HF}}$, for a cesium atom in field-free space is 9.192631770 GHz [17]. An oscillating electric field such as laser light, or a time varying magnetic field, can shift the hyperfine splitting frequency. The atom interferometer is insensitive to a constant AC stark shift or constant magnetic field shifts but if either error source changes with time, then the interferometer will see a phase shift. The phase shift from per shot errors is shown in the following expression [1]

$$\Delta \varphi_{\text{res}} = \frac{\delta_3}{\Omega_{\text{eff}}} - \frac{\delta_1}{\Omega_{\text{eff}}}$$

where $\delta_3$ and $\delta_1$ (in units of $\text{sec}^{-1}$) are the differences between the difference in Raman laser frequencies and the hyperfine splitting (which can be shifted by AC Stark or magnetic field effects) extant during the first and third laser pulses, respectively. $\Omega_{\text{eff}}$ is a parameter (also having units $\text{sec}^{-1}$) proportional to the laser field strength. Note that the atom interferometer is sensitive to AC Stark and magnetic field variations that are linearly evolving in time, as opposed to laser phase variations for which the AI is only sensitive to non-linear phase fluctuations. The reason that only the first and third pulses contribute to the AC Stark shift is due to physics details beyond the scope of this thesis. In brief, the second pulse of the interferometer pulse sequence does not induce AC Stark phase error but it does impart error due to laser phase.

Ongoing phase errors which accrue in the times between the three Raman pulses can also disrupt the atom interferometer phase. Stray resonant light that is present in the times between pulses reduces the number of atoms which undergo the interferometer sequence. This stray resonant light can be absorbed by some of the atoms; absorption causes an atom to lose all the phase information which was imprinted during the sequence. This will cause a reduction in contrast
and thus the sensitivity of the interferometer measurement. Time varying magnetic fields can cause the atoms in the interferometer to accrue more phase than they would normally which results in an inaccurate measure of interferometer phase.

The AC Stark shift can be minimized by properly balancing the intensities of the two Raman beams. With proper control of the magnetic field (either through passive shielding or actively correcting the field with trim coils) a consistent magnetic field can be generated to minimize the negative phase impacts of a time varying magnetic field on the atom interferometer. Also stray light errors are avoided by properly attenuating any light sources during the dwell times of the interferometer.

*Laser Phase fluctuations*

As discussed earlier, an interferometer measurement consists of three pulses from the interferometer laser beam pair, applied at a fixed interval $T$. The atoms, in effect, record the phase difference of the two lasers at the locations of the atoms at the times of pulse application. Thus, the lasers act as a means of measuring the atom position at three points during its motion after release, which enables determination of the average acceleration over the time of the interferometer interrogation. However, spurious changes to the laser phase difference occurring during the interrogation may be falsely registered as atom motion. Specifically, the interferometer phase is altered according to

$$\{\text{interferometer}\} = \phi = \phi_1 - 2\phi_2 + \phi_3$$
where $\mathcal{Q}_k$ is the laser phase difference at the time of pulse $k = 1, 2, 3$. The laser phases $\mathcal{Q}_k$ are directly proportional to the atom's position at the respective pulse times. Thus, the interferometer phase shift is [ideally] proportional to the change in average velocity occurring during the interrogation sequence. However, if the relative phase difference between the sensing lasers is fluctuating, it is analogous to measuring the distance the atoms have traveled with a randomly moving ruler [1]. Interferometer phase is insensitive to laser phase errors that are evolving linearly in time (or constant velocity) but is sensitive to higher order time dependence in phase. Therefore if there is any non-linear time variation in the relative phase difference between the two sensing lasers, it would contribute to measurement error. As discussed earlier, any change in the path length of one interferometer laser that is not common to the other laser will show up as a phase shift between the two lasers. The laser phase change due to path length changes occur based on the following relationship

$$\delta \varphi_{\text{pathlength}} = K \Delta x$$

where $K$ is the wave number and $\Delta x$ is the relative change in propagation path lengths of the interferometer lasers.

1.5 Motivation for laser phase stabilization

1.5.1 LPAI Sensitivity to Laser Phase Fluctuations

Since the two lasers are counter-propagating, any changes in the relative path lengths that each beam travels along, will affect the phase of one laser with respect to the other [1]. Path length differences are affected by a variety of environmental disturbances, but two major contributors
are temperature fluctuations and vibrations. Temperature fluctuations can impact static optics like mirrors and fiber along the beam path and cause them to expand and contract resulting in a path length variation. E.g., a path length change of 10nm results in a 70mrad change in phase. Such a phase shift is easily resolved in a single LPAI measurement.

We now discuss the effect of oscillatory laser phase disturbances. The fixed time interval between laser interferometry pulses means that the interferometer will be insensitive to oscillatory laser phase disturbances at certain frequencies. For instance, if the phase difference between the two sensing beams is oscillating in a sinusoidal manner at a frequency of $1/T$, the atom interferometer would see the same relative phase at each pulse and therefore not detect the oscillatory behavior. Figure 1-4 illustrates this scenario.

![Figure 1-4: Relative laser phase fluctuating as $\sin(\alpha + \omega t)$ with $\omega$ equal to $1/T$](image)

However, if the relative laser phase of the two beams is oscillating at a period of $1/2T$, then the atom interferometer is maximally sensitive to the relative laser phase fluctuations. Figure 1-5 shows this worst-case relative phase oscillation in which the atom interferometer would see the largest variation of phase.
We now discuss in more detail how time dependent phase variations are registered by an atom interferometer. Cheinet et al. experimentally determined the sensitivity function of a three pulse Raman interferometer by perturbing the phase of one of the Raman frequencies and then mapping out the frequency response of the atom interferometer to phase disturbances [8].

Figure 1-6: Atom interferometer laser phase sensitivity $H(2\pi f)^2$ full scale of frequencies DC-1MHz. The predicted weighting function was developed by Cheinet et al. For this experiment the time between pulses $T = 8$ms and the pulse duration $\tau = 6$us
The weighting function introduced by Cheinet et al agrees with the qualitative discussion above: the atom interferometer is insensitive to disturbances at frequencies of $1/T$ and most sensitive to disturbances at frequencies of $1/(2T)$. The low frequency range of disturbances (DC to kHz) could stem from mechanical vibrations and thermal fluctuations which could impact the relative phase of the sensing beams. The atom interferometer acts in a sense as a band pass filter with a lower bound of $1/2T$ and an upper bound of $1/\tau$, where $\tau$ is the duration of the interferometer laser pulse.

1.6 Methods of Laser Phase Stabilization

A common implementation of the interferometer laser optical circuit is to ensure that the two Raman beams share the same propagation path to the greatest possible extent [4,5,6,7,9]. In this retro-reflected configuration, the retro mirror acts as an inertial reference for acceleration measurements [4]. Mirror vibration changes the phase of the reflected beam that interacts with the atoms, and is indistinguishable from atom motion. To ensure a stable laser phase, this mirror needs to be stabilized. One method of retro-mirror stabilization is passive isolation, as demonstrated in the Ph.D. thesis of M. Kasevich [4]. The mirror was suspended over the vacuum system on a platform hung with surgical tubing [4]. The tubing reduced the natural frequency of the platform to 0.5 Hz and damping was added using high vacuum grease which weakly coupled semi-rigid posts to the platform [4]. A seismometer was also mounted to the platform to track the mirrors motion [4]. Active stabilization systems have also been implemented with retro-reflected Raman beam configured experiments [9]. M. Schmidt implemented an active feedback system to stabilize the retro-mirror in his system by mounting the mirror to a commercial passive isolation stage which was retro-fitted with voice coil actuators [9].
The approach taken in this thesis, and the work of McGuiness et al., is to separate the Raman frequencies and counter-propagate them through the sense head. In this configuration a retro mirror is not used, therefore phase disturbances due to shake of optics or fiber is no longer common to both frequencies and will result in a change of the beams’ path length difference. One method of path length compensation is to adjust a free space optic along one of the optic paths. If the phase change between the two frequencies is measured with a Michelson optical interferometer, this error signal could be sent into a controller which then would actuate an optic to adjust for the path length change. This could be done by mounting a piezo actuator to a mirror along one of the beam paths and using feedback control from the Michelson error signal to drive the piezo. The drawbacks with this method are bandwidth limitations of the mechanical system and the difficulty in moving a mirror without steering a beam. The resonance of the mechanical system will limit the controller’s bandwidth. Also, due to the critical alignment of the beams into fibers and other electro-optic components like Acoustic-Optic Modulators (AOMs), adjusting a mirror along an optics path could not only change the path length but also affect coupling efficiencies. The method implemented in this thesis was to compensate for path length changes by directly changing the phase of one of the beam paths by using an electro-optic phase modulator (EOM). This method allows for controller bandwidths up to GHz frequencies without steering the beam. More detail will be given about this implementation in the Controller chapter.

1.7 Thesis Contributions

1. First, significant time was spent working on the design, construction and characterization of the compact accelerometer test bed developed for this thesis. The system was designed to be a universal mobile cold atom sensor platform for use in testing new atom beam splitting technologies as well as interferometer sequences developed in the Draper
Cold Atom Research and Development group. The compact size permits dynamic testing on index heads and rate tables to test various sensors' bias stabilities and scale factor characterizations. The initial characterization of the system set up as an accelerometer yielded a short term sensitivity of $2 \mu g/Hz^{1/2}$ with a $T_{\text{dwell}} = 8\text{msec}$, data rate of 15Hz and the sensor mounted to an inertially quiet test pier.

2. Second, the laser phase noise content in the system was characterized. The phase variation between the two frequencies was measured by setting up a Michelson optical interferometer in the beam paths. The free running Michelson signal indicated that the significant laser phase noise content occurred at low frequencies (from DC to kHz scale). This permitted the controller bandwidth to be designed to compensate disturbances within this band of frequencies.

3. Third, a feedback system was implemented with the separated frequency accelerometer system to compensate for laser phase fluctuations in the Raman beams. The initial characterization of the feedback loop indicates that the frequency response of the controller performs per its design specifications (compensation of disturbances at frequencies within the range of DC to 12.5kHz). Some improvement was seen in the atom interferometer performance with the controller on for short interrogation times ($T_{\text{dwell}} = 1\text{msec}$); however, when a longer dwell time of 8 msec was used, it became difficult to discern the impact of the controller on the atom interferometer's short term sensitivity.

1.8 Thesis Outline

This thesis focuses on laser phase stabilization in counter-propagating Raman beams for a cold atom LPAI accelerometer. Chapter 2 summarizes the theory of atom interferometry, how a Magneto-Optic Trap (MOT) works to create an ideal atom proof mass, as well as how a Michelson optical interferometer can be used to compare the phase of two laser beams. Next, chapter 3 explains the apparatus design of the compact accelerometer test bed used in this thesis. Chapter 4 gives a description of the controller used for laser phase stabilization. In chapter 5 the experimental results of the controller characterization are discussed. Finally, chapter 6 concludes the thesis with a summary of results and suggestions for future work.
CHAPTER 2

This chapter is to provide background information for the reader on the subjects of magneto-optical traps (MOTs), light pulse atom interferometry (LPAI), as well as a discussion of how a Michelson optical interferometer works.

2.1 Magneto-Optic Trap (MOT)

Figure 2-1: Optical scattering force. Photons (depicted in yellow) moving in a single direction are absorbed, then re-emitted ("scattered") in all directions by an atom (violet), resulting in a net force.

Magneto-optic trapping is the method by which a cold atom proof mass is created for use in atom interferometry. Room temperature atoms are slowed down to micro-Kelvin temperatures through the use of lasers and magnetic fields. Laser photons, traveling in one direction, are
absorbed by atoms, and then re-emitted by the atoms in all directions, resulting in a net force on the atoms in the direction of the laser beam (see Figure 2-1). Alkali atoms absorb photons at very high rates over very narrow ranges of optical wavelength (i.e., they have very narrow optical resonances). This narrow resonance property can be exploited to "trick" the atoms. Making the laser wavelength slightly longer than the resonant wavelength means that the Doppler shift will cause the atom to absorb more photons when the atoms move in a direction opposite to the laser beam direction. A pair of oppositely directed lasers, both with slightly longer than resonant wavelength, will then tend to retard atom motion along the common beam axis. This damping scheme can be extended to three dimensions using three orthogonally oriented beam pairs [13].

However, while this damping scheme (called "optical molasses", first reported by Chu in 1985 [12]) can cool atoms' motion, it cannot confine them. Addition of a quadrupole magnetic field permits the lasers to act also as a spring, by perturbing the atoms' internal energy levels (the magnetic field by itself exerts only a very small force on the atoms) [14]. Atoms concentrate into a small volume near the magnetic field zero [15]. The magnetic field is created using two identical magnetic coils oriented in an anti-Helmholtz configuration, in which the coils are spaced a radius apart with oppositely directed magnetic dipoles [1]. A magnetic field zero is created midway between the coils [15]. Figure 2-2 depicts the three laser beams axes along with the quadrupole coils.
2.2 Cold Atom Interferometry

Atom interferometry is the means by which we measure atom proof mass displacements caused by inertial input. Atom interferometry utilizes the particle-wave duality principle in quantum mechanics developed by Louis de Broglie, to manipulate atoms to produce an interference signal which can be used as a sensor much like an optical interferometer [2]. We have already described the method used in this thesis, Light Pulse Atom interferometry (LPAI), as a process using the application of three laser pulses evenly spaced in time. LPAI uses so-called stimulated Raman transitions to create the atom interferometer [3]. These pulses coherently split and recombine atoms to produce an interference signal, which gives information about the inertial input seen by the atoms [1]. This process can be used to measure an atom’s acceleration with
respect to the sensor housing since the wave packets will accrue different phases from the light pulses if they are undergoing acceleration [1]. Figure 2-3 depicts how phase is accrued on the atoms at each pulse. Each Raman beam can be thought of as a light wave with an associated phase. The phase of each light wave has three components: a spatially dependent phase component, a frequency dependent phase component and a phase offset term [1].

\[
\begin{align*}
\Phi_{jT} &= K_T Z + \omega_T t + \phi_{0T} \\
\Phi_{jB} &= K_B Z + \omega_B t + \phi_{0B}
\end{align*}
\]

**Figure 2-3:** Diagram representing manner in which laser phase is imprinted on the atoms at each pulse.

In the figure above, \(\Phi_{jT}\) and \(\Phi_{jB}\) are the phases of each light wave ("T" and "B" correspond to top and bottom beams), \(K_T\) and \(K_B\) are the wave numbers of each Raman beam (defined as \(\frac{2\pi}{\lambda}\) for both), \(Z\) is the position of the atom cloud, \(\omega_T\) and \(\omega_B\) are the frequencies of each Raman beam and the \(\phi_0\) terms are the phase offsets. The effective phase that is "imprinted" on the atoms at each pulse is the difference in the two phases of the Raman beams indicated as \(\Phi_j\). It should be noted that the resulting expression for \(K_{eff}\) is the sum of the absolute values of the two \(K\) values shown below because the two waves are propagating in opposite directions.
\[ K_{eff} = K_T - K_B = \frac{2\pi}{\lambda} - \frac{-2\pi}{\lambda} = \frac{4\pi}{\lambda} \]

Also, \( \omega_{eff} = 2\pi \times 9.192631770 \) GHz is the hyperfine splitting frequency discussed in section 1.4.

The sequence of pulses used for the acceleration measurement in this thesis is referred to as a Mach-Zehnder light pulse atom interferometer (depicted in Figure 2-4 [1]).

![Mach-Zehnder three pulse atom interferometer](image)

**Figure 2-4: Mach-Zehnder three pulse atom interferometer**

The Mach-Zehnder interferometer measures acceleration by comparing the relative velocities of the atoms over the course of two dwell times. Acceleration is measured by comparing the relative laser phase (from Figure 2-3) that is imprinted on the atoms, based on the following equations (previously presented in the Introduction)

\[
\begin{align*}
\text{interferometer phase shift} &= \phi = \varphi_1 - 2\varphi_2 + \varphi_3 \\
\text{Scale Factor} &= 2 \cdot \frac{2\pi}{\lambda} \cdot T^2, \quad \phi = a \times (\text{Scale Factor}) \\
\Rightarrow a &= \frac{\phi}{2 \cdot \frac{2\pi}{\lambda} \cdot T^2}
\end{align*}
\]
where \( a \) is acceleration, \( \Phi_c \) are the imprinted phase differences at each respective pulse, and \( T \) is the time in between each pulse [1]. Any changes in the relative path lengths of the independent beams will result in a shift in the phase difference between the two beams, which in turn is registered as a phase shift as per equation (1), and a false acceleration as per equation (3). The path length difference is affected by a variety of environmental disturbances: two major contributors are temperature fluctuations and vibrations.

Atom interferometer phase is physically realized by measuring the populations of atoms in the two ground states of Cs atoms at the conclusion of the LPAI pulses. This is done by using a series of read out beam pulses separated by a re-pump pulse as indicated in Figure 2-5.

![Figure 2-5: Readout pulse sequence following interferometer pulses](image)

The light used for the read out pulses is resonant with one of the ground states (\( F=4 \)) so the first pulse causes the atoms in the \( F=4 \) state at the end of the interferometer sequence to fluoresce and...
the signal is picked up by a photo-detector. The following re-pump pulse then transfers the atoms that were in the other ground state (F=3) at the conclusion of the interferometer, into the F=4 state. The final read out pulse measures the total atom fluorescence signal. By comparing the two readout signals, a ratio between the two state populations can be determined.

The ratio of the two states’ populations will oscillate in a sinusoidal manner as the phase of the atom interferometer is changed. By scanning the laser phase difference of the last pulse in the interferometer sequence, atom interference fringes can be mapped out. An example of an interference fringe is seen in Figure 2-6. The peak to peak amplitude of this sinusoidal scan is referred to as the contrast. In Figure 2-6 the contrast of the interferogram is 0.48

![Atom Interferometer Scan](image)

Figure 2-6: Interference fringes generated by scanning the phase difference in the last pulse of the LPAI sequence.

The expression which relates the ratio of the two atoms states to interferometer phase is as follows:

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Chapter 2. Theory
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\[ P = \frac{1}{2} + \frac{C}{2} \cos \Phi + B \]

where \( P \) is the ratio of the two states, \( C \) is the contrast in the fringes, \( \Phi \) is the phase of the atom interferometer and \( B \) is an offset constant for imperfections in the fringe. To measure the sensors sensitivity the third pulse is fixed such that the sensor is operating at the most sensitive part of the fringe or at \( P=0.5 \). A series of data is taken at this fixed position to determine fluctuations about the mid fringe point. We use the following expressions to determine the phase sensitivity at a given ratio of states.

\[
\Phi = \cos^{-1} \left( \frac{2(P - B - \frac{1}{2})}{C} \right)
\]

\[
\frac{d\Phi}{dP} = -\frac{1}{\sqrt{1 - \frac{4(P - B - \frac{1}{2})^2}{C^2}}} \times \frac{2}{C}; \quad \frac{d\Phi}{dP} \approx -\frac{2}{C}
\]

Assuming that \( P = 0.5 \) and \( B = 0 \), the expression simplifies to \( -\frac{2}{C} \). Therefore the data can be converted to phase using the approximate \( \frac{d\Phi}{dP} \) expression. This is the manner in which the experimental data is analyzed later on in this thesis.

It should be noted that in the presence of acceleration (like gravity), the interferometer will flip through many of these fringes due to the Doppler shift that the atoms experience as they accelerate with respect to the laser beams. A chirp on the Raman laser frequency difference counters the Doppler shift to maintain a fixed position on one fringe location over the measurement. The chirp rate is a measure of the acceleration and is 23 kHz/ms for acceleration due to gravity. In principle, a very short dwell time could be used in place of the chirp to remain on one fringe but this approach would degrade the sensitivity of the sensor.
2.3 Michelson interferometer

A Michelson optical interferometer is a tool which precisely measures changes in the path lengths over which two laser beams traverse with respect to each other [16]. A Michelson interferometer has been installed, using a laser that does not interact with the atoms, into the optical circuit of the Cold Atom accelerometer. This Michelson interferometer continuously measures changes in the optical path length difference between the two interferometer laser beams.

A typical Michelson interferometer works by taking a single laser source, splitting it into two separate optical paths and then recombining the two light paths to create an interference pattern [16]. Figure 2-7 shows an example of a Michelson interferometer in which the source light (purple) is split at the beam splitting cube. Half of the light (red) travels up to the top mirror and is reflected back through the cube to the detector, traveling a distance \( x_1 \). The other half of the light (blue) passes through the beam splitting cube and is first reflected off right side mirror and then reflected again off the cube traveling a distance of \( x_2 \). Since the two beams come from the same source their wave numbers and frequencies are equal.
Figure 2-7: Diagram of a typical Michelson optical interferometer which uses a single light source.

The phase difference between the two beams when they are recombined on the detector is as follows

\[
\{\text{Michelson Phase difference}\} = [K(x_2 - x_1)] = K\Delta x
\]

If the two beams have a uniform phase front when overlapped they will produce an interference fringe pattern that will have either a bright spot when the phase difference is 0 radians or a dark spot when the phase difference is \(\pi\) radians due to constructive and destructive interference [16]. Using a photo-detector one can measure the intensity of the interference pattern to determine how much the path lengths are changing with respect to each other [16]. The intensity of the
interference pattern created by the Michelson interferometer corresponds to the following expression.

\[ V = I \cos(k \Delta x) + A \]

\( I \) is the combined intensity of the two beams, \( k \) is the wave number of the light and \( A \) is a constant offset which accounts for imperfections in interference signal.

Michelson interferometers are often used to precisely measure distances because of this relationship between the path length differences in the two arms and the differential phase [16]. For a Michelson interferometer with two separate beam sources, a more general expression for the interference intensity is as follows

\[ V = I \cos(k \Delta x + \omega_{eff} t + \varnothing_{eff}^0) + A \]

where \( \omega_{eff} \) is the difference in the frequencies of both beams and \( \varnothing_{eff}^0 \) is the difference in the initial phase offsets of both beams.
3.1 The Cold Atom Compact Accelerometer Test bed (CTB)

The Cold Atom Compact Accelerometer Test Bed was designed to be relatively mobile and compact, as compared to the original test bed developed by Butts et al. [1,19]. In the Compact Accelerometer Test Bed (CTB) smaller vacuum components were used to reduce the size and weight of the assembly. Also the vacuum system, magnetic coils and output collimating optics all mounted onto an 18”x18”x0.5” optical bread board as shown in Figure 3-1. Fiber connections were used to deliver the light frequencies required to create the MOT and perform atom interferometry. The entire sense head was contained within an 18”x18”x15” volume and could be moved to different locations and mounted on a rate table and/or an index head for inertial testing to characterize bias and scale factor stability of the sensor. See Figure 3-2 in which the CTB was mounted on an index head on a stable pier.

Figure 3-1: Model of Cold Atom Compact Accelerometer Test Bed
Within the 18”x18”x15” volume the sense head contained a quartz vacuum cell where the MOT was formed and where the atom interferometry took place. Surrounding the cell were optics used to deliver the laser frequencies necessary to create the MOT, prepare the atom proof mass, and sense acceleration with atom interferometry. The sense head also contained a variety of magnetic coils that were used to null earth’s magnetic field, create the Magneto-optic trap and create a bias field along the sensing direction. More detail will be given about these coils and optics in the following sections.

![Cold Atom Compact Accelerometer Test Bed (CTB) mounted to an index head and on a stable pier and Minus K vibration isolation stage for inertial testing](image)

**Figure 3-2: Cold Atom Compact Accelerometer Test Bed (CTB) mounted to an index head and on a stable pier and Minus K vibration isolation stage for inertial testing**

### 3.2 Vacuum System

Figure 3-3 shows a model of the quartz vacuum cell. Its octagonal shape permitted multiple beams to enter the cell through its 1” diameter windows. The internal volume of the quartz cell in which acceleration measurements were taken was 76cc.
Figure 3-3: Octagonal Glass Vacuum Chamber used for atom interferometry

Figure 3-4 depicts a diagram of the vacuum hardware in the CTB. A vacuum of $10^{-9}$ torr was achieved in this cell by initially using a turbo-pump connected to the valve indicated in the diagram below to pump the cell down to $10^{-6}$ torr. The turbo-pump was then detached and the final pressure was achieved with a combination of getters and ion pumps. The getter provided high pumping speed for non-noble gases and the ion pump removed helium that permeated through the quartz. A Cesium alkali source was routed through the neck of the quartz cell to direct the Cs atoms at the trapping region. This Cs source was powered by a current source through the electrical feed through indicated in the diagram. All these components were mounted to a 90° bracket which mounted the vacuum system to the base 18”x18”x0.5” bread board shown in Figure 3-1.
3.3 Magnetic Coils and CTB beam paths

There were a variety of magnetic coils required in the CTB system. The first were the nulling coils, which were used to null out earth’s magnetic field at the center of the quartz cell where the MOT is formed. The model of the test bed depicted in Figure 3-1 indicates the 12”x12” square nulling coils. These were arranged to cancel earth’s field in three orthogonal directions.

Another type of magnetic coil used in the test bed was the MOT trapping coil. As previously stated in chapter 2, the MOT trapping coils were used in combination with the trapping beams to generate the MOT. Two magnetic coils were arranged in the anti-Helmholtz configuration in order to create a magnetic field zero where the MOT was formed. The three orthogonal retro-reflected trapping beams and MOT coils are indicated in Figure 3-5.

The final set of magnetic coils was the bias coils which were required for state preparation of the atoms and for atom interferometry. The bias coils were oriented along the Raman beam axis, as
indicated in Figure 3-5, so that the resulting uniform field defined the quantization axis of the atoms. The Raman beam axis was the axis along which acceleration was measured.

![Diagram of optical axes in CTB]

**Figure 3-5: Identification of optical axes in CTB**

### 3.4 Optical system

Three 852nm Toptica laser systems were used to generate the six required frequencies which consisted of trapping light, re-pump light, state prep light, probe light, and two Raman frequencies.

#### 3.4.1 Trapping and Probe Light

As previously stated in the theory section on magneto-optic traps, three pairs of orthogonal beams were used for laser cooling and trapping. For archival purposes, I document here the details of laser frequencies used in the system, in the context of the internal hyperfine structure of the Cs atom [17]. The cooling or trapping beams were detuned by approximately two line widths from the F=4 to F'=5 transition (~10-15 MHz for cesium). Probe light which was used to
read out acceleration from the sensor is also very close to the F=4 to F'=5 transition as indicated in Figure 3-6. These frequencies were generated from the same Toptica system, F=4 laser, but in different ways to allow independent control of the two lasers.

![Figure 3-6: Cesium atomic energy level diagram showing laser frequencies used in CTB system. Note that this is not drawn to scale.](image)

The F=4 laser system was locked by saturation absorption spectroscopy to the F=4 to F'=4/5 frequency indicated in Figure 3-6 by the F=4 laser line. An image of the saturation absorption scan for the F=4 transitions can be seen in the Appendix A. This light then passed through an Acousto-optic modulator (AOM), labeled the Constant AOM in Figure 3-7, which shifted half of the light -125 MHz and passes the other half through with no change.

The AOM is very useful at modulating the intensity and frequency of light and also works as a very fast shutter, switching off in ~10-100ns. In the optic boards for the Compact Accelerometer test bed, AOM’s were primarily used to modify the frequency of the light passing through it and also to shutter the various light frequencies required in the measurement sequence. The AOM was driven at the desired light shift frequency; the light frequency could be shifted up or down. The positive and negative first order sidebands are the predominant sidebands produced by the AOM but higher order sidebands can also be seen. The first order sidebands can be seen physically displaced from the carrier frequency at the output of the AOM, which is why the
AOM works so well as a shutter. Alignment of the input light permits the output to favor either the +1 or -1 order light. The Constant AOM was oriented to favor the -1 sideband light. Typically in this experiment, half of the input power could be measured in the desired shifted frequency beam. The efficiency of this process was dependent on alignment and beam shape through the crystal.

The un-modulated light of the Constant AOM then was passed through another AOM, labeled the Probe AOM in Figure 3-7, where it was modulated +125 MHz. The -125 MHz modulated light from the Constant AOM was then sent through the Detuning AOM where the frequency was modulated twice by +125 MHz or +250 MHz total by passing the light through the AOM twice. This light was then sent to a tapered amplifier to amplify the intensity before being sent to the F=3 optic board where it was overlapped with re-pump light and then delivered to the cell.
3.4.2 Re-pump and State Prep Light

The F=3 laser system was locked to the F=3 to F'=3/4 frequency again by saturation absorption spectroscopy. The output of the F=3 laser was coupled into a fiber in order to send the light to the location on the board where the light was modulated by the Re-pump AOM. The Re-pump AOM modulated the light +99 MHz so that the light was at the F=3 to F'=4 frequency which was used for re-pump. The re-pump light optically pumped atoms into the correct state for state prep. Also on the F=3 board was the output of the TA from the F=4 optics board. The TA amplified the output of the detuning AOM from 8mW of optical power to 100mW of optical power. Ten percent of this light was sent to the state prep AOM where it was double passed to shift the frequency -250 MHz to the F=4 to F'=4 frequency. This state prep light was used with the re-pump light for state preparation of the atoms. The remaining 90% of the light out of the TA, or MOT light, was overlapped with the re-pump light on to one collimator. This overlapped light was sent to the test bed.

![Figure 3-8: F=3 Laser optics board](image-url)
3.4.3 Raman Light

The two Raman frequencies were generated on the Raman Frequency Generation Board. In the CTB, each Raman frequency was sent along its own beam path and delivered to the cell as two separate counter-propagating beams. The two Raman frequencies were counter-propagating to enable inertial measurement. This separated frequency method of delivering the Raman light to the cell differs from the more commonly used method of co-propagating the two frequencies along one beam path and using a retro reflecting mirror and $\frac{\lambda}{4}$ wave plate after the cell to redirect the light back at the atoms and rotate the light’s polarization by 90°. In the retro reflected orientation there are two pairs of Raman beams that interrogate the atoms as seen in Figure 3-9.

![Diagram comparing Co-propagating reflected Raman frequency orientation to Separated Raman frequency orientation.](image)

Figure 3-9: Diagram comparing Co-propagating reflected Raman frequency orientation to Separated Raman frequency orientation.

The reflected configuration works well when the interferometer is sensitive along a fixed orientation parallel to gravity. Atoms accelerate along the Raman axis and experience a Doppler shift: this Doppler shift distinguishes between the two different wave vectors or $K_{eff}$ produced by the beam pairs. For vertical orientation, the atoms experience a Doppler shift of 23 kHz/msec. One beam pair’s resonance would be shifted up by the Doppler shift and the other would be shifted down based on the orientation of the carrier frequency in each beam pair. If the drop
time is long enough, the resonances of the two $K_{\text{eff}}$ will separate, making them clearly distinguishable from each other. In an instance where there is very little acceleration along the Raman beams, both of the beam pairs act on the atoms, because the separation is negligible. In the separated frequency configuration, there is only one beam pair and therefore only one $K_{\text{eff}}$ to consider.

To create the two interferometer laser frequencies, an 852nm laser source was detuned 800 MHz from the $F = 3$ to $F' = 3/4$ transition as indicated by the "Raman Master" line in Figure 3-6. This detuning was achieved with an Electro-optic modulator from EOSpace (PM-0K5-10-PFA-850). This EOM created positive and negative sidebands modulated at 800 MHz. The laser was locked to a feature in the -800 MHz sideband saturation absorption signal. This Raman master light was then passed through another EOM, this time a Photline 10 GHz phase modulator (NIR-MPX-800-LN-10). This second EOM modulated the Raman Master light by 9.192631770 GHz. This modulation created $\pm$ sidebands that were spaced from the Raman Master frequency by 9.192631770 GHz. This frequency difference between the two lasers was the two photon detuning or the Cs clock frequency.

![Figure 3-10: Spectrum of Raman seed light produced by the Photline 10 GHz EOM. The $\omega_{\text{carrier}}$ and $\omega_{\text{carrier}} - 9.2$ GHz (green) are the frequencies used for the Raman beams.](image)

The modulated Raman master light was used as the seed light into the Raman Frequency Generation Board. This seed light was used to injection lock lasers 1 and 2 on the board in Figure 3-11. One laser was locked to the carrier frequency and the other laser was locked to the -
9.2 GHz side band (indicated as the green spike to the left of the $\omega_{\text{carrier}}$ in Figure 3-10). Injection locking primarily amplified only one frequency in each locked laser. The current of the injection locked laser was adjusted to lock the laser to either carrier or the -9.2GHz sideband. To verify that the laser was locked to one frequency and not the other, some of the light out of the injection locked laser was picked off and sent to a Fabry-Perot cavity. The Fabry-Perot cavity was used as a diagnostic to view the spectrum of the injection locked light. When the injection locked laser was locked to a single frequency, a single spike would appear on the output signal of the cavity. The other sidebands were suppressed by ~20dB.

Figure 3-11 shows that the two injection locked beams were overlapped on a polarizing beam splitting cube (PBS). The $\lambda/2$ wave plates before the PBS were used to rotate the polarization of the light to adjust the intensity of each frequency that passed through the PBS. Setting the correct ratio of intensities between the two frequencies was important to cancel the AC Stark shift that occurred when the Cesium atoms were presented with off-resonant oscillating electric fields. The AC Stark effect shifted the separation frequency or clock frequency between the F=3 and F=4 ground states. The AC Stark produced by one of the beams could be compensated for by adjusting the intensity of the opposite counter propagating beam to put an equal but opposite shift on the separation of the ground states.
After the PBS both frequencies passed through the Raman AOM where they were shifted + 80 MHz. This AOM acted as a shutter for the Raman light. These frequencies were then reflected off a dichroic mirror which reflected 850nm light and passed 795nm light. The two frequencies were separated again and sent to two separate collimators which sent the frequencies to the test bed. A 795nm Toptica laser was used to generate the light for the Michelson interferometer. The 795nm light was injected into the two Raman frequency paths at the dichroic mirror. The 795nm light was divided at the same PBS that the Raman light was split and aligned into the Raman frequency collimators. Finally the Probe light that was generated on the F=4 laser board was overlapped downstream of the 795nm light. All three beams were passed through the same fibers and directed at the cell.

### 3.5 Michelson Interferometer

To obtain a phase error signal between the two Raman beams in the CTB system, a Michelson interferometer was used. To set up the Michelson interferometer, a 795nm laser source was
This wavelength was chosen because it is not resonant with the cesium atoms and could be left running continuously without disrupting atom interferometry. The 795nm light was split and combined along the two Raman beam paths on the optics table where the Raman frequencies were generated (refer to Figure 3-11). The 795nm was inserted as far upstream as possible so that it could overlap much of the same path that the Raman frequencies take and therefore observe the same disturbances. Figure 3-12 shows a diagram of how the interference signal was generated. A polarizing beam splitting cube (PBS) at the output of one of the Raman beam paths was utilized to recombine the two 795nm paths. As seen in the diagram, Path 1 was reflected up off the cube. Then Path 1 was reflected off a mirror and passed twice through a $\lambda/4$ wave plate which rotated the polarization $90^\circ$ so that it could pass through the cube toward the photo-detector. Path 2 was reflected off the PBS and directed toward the photo-detector where it was combined with Path 1. An interference pattern was created and this signal was measured by the photo-detector to create an error voltage which was fed into the control circuit.

**Michelson Optical Interferometer**

![Michelson interferometer diagram](image)

Figure 3-12: Michelson interferometer diagram
3.6 Magnetic Shielding

Magnetic shielding was designed to attenuate stray magnetic fields in the test bed that could cause systematic phase drifts. Disturbances to the bias field from external magnetic fields can disrupt atom interferometry and were a potential source of error in the acceleration measurement. A double layer nested cube design was constructed out of a nickel-iron alloy, which is highly magnetically permeable, to guide unwanted magnetic fields around the sense head contained inside. Each layer was constructed with a shallow base layer and a large cubic cover that fit within the base. Figure 3-13 shows an expanded view of the CTB with the Magnetic shielding on.

![Diagram of Magnetic Shielding Design for CTB](image)

**Figure 3-13: Magnetic shield design for CTB**

A spacer plate was used in between the inner and outer layer base shields to space the layers off each other and also to provide a surface for each base shield to mount to. Each layer’s cubic cover had ¼”-20 nuts welded along the bottom edge. These nuts, indicated in Figure 3-14, were used to bolt the covers to the base shields. When the test bed was mounted on to a rate table or index head, clearance holes in the base shields provided access to mount the CTB directly to the
rate table or index head so that the shields were not supporting the weight of the test bed. Putting this additional stress on the shields would degrade the magnetic shields.

Figure 3-14 Cross section view of mounted magnetic shields
CONTROLLER DESIGN

4.1 Purpose of Controller

We have already seen that in atom interferometry, optical phase difference variations over time are what cause acceleration measurement errors. Phase difference variation linear in time, however, has no effect on the acceleration registered by the interferometer: recall that

\[ a = \frac{\varphi_1 - 2\varphi_2 + \varphi_3}{2 \cdot \frac{2\pi}{\lambda} \cdot T^2} \]

If \(\varphi_1, \varphi_2\) and \(\varphi_3\) are changing linearly, there is no false acceleration signal: only laser phase difference variation of higher than linear order in time causes acceleration error. We will see in this chapter that the atom interferometer itself acted as a band-pass filter to laser phase variations, effectively filtering out DC and very high frequency laser phase variations. The controller then needed to effectively reject disturbances in the band of frequencies that the atom interferometer was most sensitive to.

4.1.1 Atom Interferometer Laser Phase Sensitivity

The three pulse Mach-Zehnder light pulse atom interferometer acted as a band pass filter to laser phase fluctuations based on the duration of the light pulses and the dwell time in between pulses.
The lower break point of the band pass filter is \( \frac{1}{2\pi T_{\text{dwell}}} \) where \( T_{\text{dwell}} \) the time in between pulses [8]. For laser phase noise with period longer than the dwell time, the atom interferometer sees a reduced phase affect over the three pulses. The upper break point of the band pass filter is set by \( \frac{1}{\tau} \) where \( \tau \) is the pulse duration. Laser phase noise which oscillates faster than \( \frac{1}{\tau} \) would begin to average over the duration of the pulse and have a reduced affect on the atom interferometer phase. An experiment was conducted with the Compact Accelerometer Test Bed where programmed phase disturbances were introduced into one Raman beam path via an Electro-optic modulator (EOM) driven by a signal generator. The signal generator was programmed to drive the EOM at fixed amplitude sinusoidal inputs over a variety of frequencies, which produced sinusoidal variation in the path length difference of the atom interferometer. The noise of the three pulse atom interferometer in radians was then measured and compared to the disturbance input in radians. Figure 4-1 and Figure 4-2 below feature the experimentally measured data as well as a fit function from the work of Cheinet et al. in their measurement of the sensitivity function of an atomic interferometer [8]. It should be noted that the experimental data was limited by a sensor per shot phase SNR of 40. The experimental data agreed with the predicted frequency response developed by Cheinet et al and indicated the lower and upper break points of the band pass filter for an atom interferometer with a \( \pi \) pulse duration of 6usec and dwell time in between the pulses of 8ms. The data also displayed the notches in the sensitivity function of the atom interferometer at frequencies of \( \frac{1}{(T_{\text{dwell}} + \tau)} \).
Figure 4-1: Atom interferometer laser phase sensitivity $H(2\pi f)^2$ for frequencies DC-10kHz. The predicted weighting function developed by Cheinet et al was used to fit the experimental data. For this experiment the time between pulses $T = 8\text{ms}$ and the pulse duration $\tau = 6\text{us}$. The Atom interferometer is insensitive to frequencies of $\frac{1}{(T_{\text{dwell}} + \tau)}$ as indicated by the notches in the frequency response.

Figure 4-2 Atom interferometer laser phase sensitivity $H(2\pi f)^2$ full scale of frequencies DC-1MHz. The predicted weighting function developed by Cheinet et al was used to fit the
experimental data. For this experiment the time between pulses $T = 8\text{ms}$ and the pulse duration $\tau = 6\text{us}$. The Atom interferometer is insensitive to frequencies of $\frac{1}{(T_{\text{dwell}} + \tau)}$ as indicated by the notches in the frequency response.

This data determines the effective bandwidth of frequencies over which the controller needed to effectively reject laser phase fluctuations in order to ensure an accurate acceleration measurement.

### 4.1.2 Observed Noise Content

In order to see what the laser phase noise content actually was for this system, a Fast Fourier Transform (FFT) was performed on the free running $795\text{nm}$ Michelson signal. This open loop data indicated that 98% of the noise content was observed below 2 kHz. Due to significant filtering in the RF electronic circuit, it is not believed that this noise content was due to electronic noise in the frequency generation of the laser frequencies. This measured noise content was predominantly due to inertial shake of the optics/fiber along the Raman beam paths and temperature fluctuations in the fiber. This permitted the controller bandwidth to be on the order of kHz as opposed to MHz as indicated by the Atom interferometer sensitivity data.
Figure 4-3: Laser Phase noise power spectral density in first floor lab at Draper Laboratory. An FFT of the free running 795nm Michelson signal was taken to determine the major noise components.

4.2 Block diagram of controller

The feedback system implemented to correct for laser phase fluctuations between the two Raman beams was, in effect, a simple proportional feedback controller. Phase error was fed into a digital proportional controller and then the phase was changed directly by an electro optic actuator. Since the proportional controller is digital, its bandwidth was limited by the sampling time and therefore is not uniform over all frequencies. In this configuration an Electro-optic modulator EOM, was used as the actuator and a Michelson interferometer acted as the sensor to measure fluctuations in the phase difference of the two Raman beams paths. The constant “B” indicated in the EOM block of Figure 4-4 was the Vpi constant or the voltage required to change the phase of the light passing through the EOM by π radians. The three major disturbances
indicated in this feedback loop were temperature fluctuations, mechanical vibration and RF electronics noise. These disturbances will be further examined in the next section.

![Diagram](image)

**Figure 4-4: Feedback loop diagram for Raman beam relative phase stabilization.**

The error signal fed into the controller was calculated by modulating the Michelson interference signal $\pm \frac{\pi}{2}$ radians and taking the difference of an averaged Michelson signal at $\frac{\pi}{2}$ from an averaged signal at $-\frac{\pi}{2}$. This modulation frequency was much less than the sampling time (25 kHz modulation frequency versus 1 MHz sampling frequency) so it set the true bandwidth of the controller. Figure 4-5 describes in further detail how the modulation generated the error for the control loop. The controller effort adjusted the DC offset of this modulation signal to maintain a zero error signal between the A and B pulses.
Figure 4-5: Graph 1 depicts the A-B modulation applied to the electro-optic phase modulator in the Raman optics path. This square wave with amplitude of Vpi/2 and frequency 25 kHz, modulates the phase of the Michelson interference signal by ± π/2 radians to generate an error signal. Error is calculated by comparing the average intensities of the interference signal at the A and B points shown in graph 2. With an added phase disturbance (Wk) the difference between the A and B points will be non-zero indicates by the red locations.

The controller's frequency response can be predicted in the following manner. The controller is working to stay on one fringe of the interference signal. The error signal generated by the modulation tells the controller how much it needs to adjust the DC offset of the modulation signal to maintain the mid fringe A and B positions. A phase disturbance will cause the difference between the A and B positions to deviate from zero as indicated in Figure 4-5. The control law can be written as shown below in equation 4. In this equation K is the loop gain, C_k is the control effort (in phase) and the Y_k terms are the system phase outputs averaged over half
the modulation period. The difference between $Y_k$ and $Y_{k-1}$ is the phase error signal generated between the A and B modulation points in Figure 4-5.

$$C_k = -K \ast (Y_k - Y_{k-1}) \quad (4)$$

The controller responds to disturbances in the system after it has taken the difference of the A and B modulation points (indicated in the difference of $Y_k - Y_{k-1}$). The control effort is then applied to the start of the next modulation period and thus the system response at $K+1$ is defined as the following.

$$Y_{k+1} = (-1)^k W_{K+1} + C_k + v_k \quad (5)$$

where $W_{k+1}$ is the input phase disturbance that the system has yet to act on (the $(-1)^k$ terms indicates the change in sign based on which modulation point in the fringe the measurement is on), $C_k$ is the control effort from the previous period and $v_k$ is residual noise in the system. If you substitute equation 5 into equation 4 with the correct corresponding coefficients and assume no noise, equation 4 simplifies to what written below

$$C_k = -K \ast \left( \frac{W_{k+1} + W_{k-1}}{2} \right) \quad (6)$$

The change in the system response over the modulation period then becomes equivalent to the average disturbance input. Since this controller is digital, equation 6 can be written in the Z transform domain as such.

$$C(z) = -K \ast W(z) \left( \frac{z+1}{2z} \right) \quad (7)$$

To determine the system response of the controller, we can re-write equation 7 as follows
In the Z domain the frequency response is determined by substituting \( z = i\omega T_s \), where \( T_s \) is the sampling time of the controller and \( \omega \) is frequency of the disturbance in \( \text{radians/sec} \), into the transfer function and calculating the magnitude of the transfer function at various \( \omega \) values. The bode plot of this transfer function with a sampling time of \( T_s = \frac{1}{25kHz} \) or 40e-6 sec is as follows. This sampling time produced a bandwidth of 3.9e4 rad/s or 6.2 kHz which is sufficient to respond to the observed noise content previously discussed.

**Figure 4-6: Predicted frequency response of the digital proportional controller. Loop gain equals 1**

Substituting the equation 6 into equation 5 and assuming no noise, we then get the following equation for the system response at time \( k+1 \) with respect to disturbance inputs.
\[ Y_{k+1} = W_{k+1} - K \times \left( \frac{W_k + W_{k-1}}{2} \right) \]  \hspace{1cm} (9)

The transfer function for the system response to input disturbances in the z domain is as follows

\[ \frac{Y(z)}{W(z)} = -K \frac{2z^2 - z - 1}{2z^2} \]  \hspace{1cm} (10)

The predicted system frequency response is as follows.

Figure 4-7: Predicted closed loop system frequency response. Loop gain equals 1

In chapter 5 experimental data will show how these predictions align with the physical response of the system.
4.2.1 Disturbance Sources

Temperature Fluctuations

Temperature fluctuations were one driver of laser phase fluctuations. In each Raman beam optical path there was over 7 meters of fiber. A change of 1 °C could cause the fiber's length to change by 10 parts per million (ppm). A micron scale change in either of the paths would result in radians of laser phase variation because the relative phase difference between the two beams in the Michelson interferometer was proportional to path length change multiplied by the wave number of the laser light \( \left( \frac{2\pi}{\lambda} \right) \) as explained in the theory chapter. Therefore it would not take a large change in temperature to greatly impact the laser phase of the light passing through the fibers.

Mechanical Vibrations

Mechanical vibrations in optical components also contribute to laser phase noise. If any of the optics that are not common to both Raman beams paths were to vibrate with respect to each other it would cause a change in the path length of the respective beam path that the optic is on. A change in one of the beams path's length would result in a change in the relative phase of the two beams. Figure 4-8 indicates the optics on the Raman board which would cause path length changes in one Raman beam path and not the other. It is also important to note that any path length difference occurring in the red highlighted region at the far right of the diagram in Figure 4-8 was not sensed by the Michelson interferometer. However, all of the optical paths in this region were free space (not fiber) and were reasonably stable in time.
Figure 4-8: Raman frequency generation board indicating which optics could cause a change in the relative laser phase of the two Raman beams. The optics in the highlighted red areas were not shared by both Raman beams, therefore any vibrations would cause correlated path length changes in one beam and not the other.

RF Electronic Noise

RF electronic noise on the signal applied to the EOM that generates the Raman sideband noise added directly to noise on the laser phase difference, and thus accelerometer noise.

4.2.2 Actuator- Electro-optic Phase Modulator

As stated earlier the actuator in this feedback loop was an electro-optic phase modulator. A Photline 10 Ghz Phase modulator (P/N: NIR-MPX-800-LN-10) was installed in one of the Raman optics paths (indicated by the yellow “EOM” in Figure 4-8). Figure 4-9 is an example of a similar phase modulator from ThorLabs. When a voltage was applied across the electrodes in the modulator’s lithium niobate crystal the path that the light travels through the crystal was lengthened resulting in a phase delay on light passing through it [18].
The voltage associated with a $\pi$ phase shift, also known as Vpi, for this modulator was 5.34V. This number was determined experimentally by applying a triangle waveform to the EOM and monitoring the output of the 795nm Michelson photo detector. The applied triangle wave and associated 795nm Michelson response is seen in graph 1 of Figure 4-10. The Vpi was calculated by comparing the output response phase to the applied voltage and determining the corresponding slope of phase to voltage. To get Vpi the inverse of this slope was multiplied by $\pi$ radians. The slope of $0.58\frac{\text{rad}}{\text{volt}}$ and Vpi of 5.34V is indicated in graph 2 of Figure 4-10.
Figure 4-10: Experimental calibration of voltage required for $\pi$ phase shift or $V_{pi}$ in Photline 10 Ghz Phase modulator (P/N: NIR-MPX-800-LN-10). Graph 1 shows the applied triangle wave signal to the EOM and the measured response of the Michelson interferometer converted to phase. Graph 2 looks at a linear segment of the data and compares the response phase to the applied voltage to obtain a slope of $0.58 \frac{\text{rad}}{\text{V}}$ and $V_{pi}$ of 5.34V.
4.2.3 Phase Sensor- Michelson Optical Interferometer

A Michelson interferometer was selected as the sensor for the feedback loop because it provided a way to measure the change in phase that occurs along the two Raman beams paths. As explained in the theory section, the interference signal at the output of the Michelson interferometer is a very accurate measure of the path length changes that occurs in the two beam paths that are being combined. Since phase is related to changes in path length by the wave number, the interference intensity measured on the photo-detector can be used as a direct measure of phase fluctuations. By using 795nm light, a continuous error signal could be measured. If the 852nm signal was used as the error signal, data would only be acquired over the microsecond pulse lengths used for atom interferometry. The 795nm light could be left running continuously because it is not resonant with the cesium atoms used to create the cold atom proof mass and thus would not disrupt the atom interferometer. As previously explained in the Apparatus chapter, the 795nm light is overlapped with the 852nm Raman beams as far upstream as possible so that the two wavelengths undergo approximately the same path length changes.
EXPERIMENTAL RESULTS

5.1 Closed loop vs Open loop 795nm Michelson Response

With the goal of assessing the phase stabilization control system, I carried out a comparative study of Michelson interferometer signals obtained in open and closed loop operation. As discussed earlier, the Michelson interference is modulated to create the error signal fed into the proportional controller. Figure 5-1 shows the open loop modulated 795nm Michelson signal (blue) and the closed loop locked Michelson signal (red). The locked signal sits at 0 radian phase difference, indicating that the controller has minimized the error signal between the A and B modulation values. The reason that the red locked Michelson signal is not perfectly flat and has spikes at the modulation steps is due to the finite slew rate of the voltage source driving the EOM: the interferometer phase must cross an intensity peak or valley during transitions between modulation phase states from \(+\frac{\pi}{2}\) to \(-\frac{\pi}{2}\). The transition is not instantaneous, so a spike in the Michelson intensity bleeds through. This could be remedied with faster electronics to reduce the interruptions in the locked phase data.
Figure 5-1: 795nm Michelson interference signal with and without feedback. The open loop data displays the 25 kHz phase modulation which is applied to the control EOM to generate the error signal.

5.2 852nm vs 795nm Michelson Signal Comparison

The success of the laser phase control loop at stabilizing the relative phase of the 852nm Raman beams is dependent on how well the 795nm light captures the phase disturbances which occur in the 852nm Raman beam paths. During actual accelerometer operation, the 852nm Raman light is only on for short micro-second pulse durations during the interferometer sequence, but for the diagnostic purpose of comparing its behavior to the 795nm light, it was left running continuously. Also for comparison to the 795nm light, both 852nm injection-locked lasers were locked to the same frequency so that interference between the two beam paths could be observed without needing to demodulate the 9.192631770 GHz modulation between the carrier frequency and the sideband nominally used for Raman transitions.

The two free running 795nm and 852nm Michelson signals would stay nominally correlated or anti-correlated for many seconds at a time but a slow drift of $\pi$ radians was observed between the
two signals which occurred over tens of seconds. Figure 5-2 shows how the two signals' free running oscillations would go from being anti-correlated to being correlated over a 40 second interval. These slow drifts could be attributed to drifts in the laser frequencies of either the 795nm and 852nm Toptica laser systems. A change on the order of 1 MHz on a minute time scale to either laser frequencies could result in this slow \( \pi \) radian shift because the paths of the two frequencies are not the same length. This type of instability in laser frequency has been observed at Draper in the Toptica laser systems and is well within the lock specifications required for atom interferometry.

![Graph 1: Open Loop Comparison of 795nm and 852nm Signals](image1)

![Graph 2: Correlation Coefficients](image2)

**Figure 5-2:** Open loop comparison of continuously running 795nm and 852nm Michelson signals. Plot 1 shows the photo-detector signals of the 795nm and 852nm open loop Michelson interference signals. The two open loop data streams were compared with each other to see how well the two signals were correlated. Plot 2 is a graph of the correlation coefficients of the two data streams. The two signals slowly drift from being anti-correlated to being correlated. This drift is attributed to MHz scale fluctuations in the output laser frequencies of each wavelengths laser source.
This slow drift that is occurring between the two Michelson signals is too slow to impact the LPAI measurement. The atom interferometer is insensitive to drifts that occur at sub-hertz levels (as indicated in Figure 4-1).

The slow drifts do impact how the 852nm Michelson signal appears when feedback is on. Since the error into the controller is the 795nm Michelson signal, the controller effort will work to minimize the error between the A and B modulation pulses that are applied to the 795nm light. If a drift has occurred such that the 795nm and 852nm signals are not correlated, when the modulation is applied to the 852nm light, it may not be modulating at the same points on the interferogram as it is for the 795nm light. This affect is indicated in Figure 5-3.

![Figure 5-3: Description of how 852nm Michelson signal can modulate at different phase points than the 795nm Michelson due to the slow phase drift that occurs between the two signals.](image)

The 852nm signal is locked to a slowly evolving phase when the feedback is on but when the 852nm and 795nm signals are out of phase with each other, the 25 kHz modulation will be seen in the locked 852nm signal. This modulation that bleeds through is not an issue for atom
interferometry as long as the Raman pulses are placed such that all three occur on either an A pulse or a B pulse over the course of a measurement as seen in Figure 5-4. If in the next repetition of the acceleration measurement all three Raman pulses occur on the B pulse instead of the A pulse (or vice versa), this would also not be an issue because what is important to the acceleration measurement is that the laser phase is stable over the course of the three pulses of a single interrogation, and not from one interrogation to the next.

![Diagram](image)

**Figure 5-4: Placement of Raman pulses. The red spikes indicate the Raman optical pulses and the blue square wave indicates the modulation applied to the Raman and 795nm light.**

Since the residual modulation in the locked 852nm signal has no affect on the stability of the atom interferometer readout, a two point average was taken in Figure 5-5, of the locked 852nm, 795nm and applied control voltage data to show the deviations in the locked signals over time. The two point average was converted to phase to show both the 795nm and 852nm locked Michelson signals nominally reside close to 0 radians. The control signal plot shows that over the course of the 20 second data the phase error changed by $\sim 7$ radians. It should be noted that the step changes that occur in both locked signals are a results of the flybacks that occur when the controller effort reaches the maximum voltage limit of the EOM. The controller does a $\pi$ radian jump to reset the EOM control voltage to a level in between its minimum and maximum voltage levels of $\pm 5$ volts. If the programmed Vpi values are slightly off from the actually Vpi values when a flyback occurs, a discontinuity or step can be seen in the controlled signal. Since the Vpi calibration was performed using the 795nm light, the steps in the 795nm signal are smaller than those of the 852nm locked signal. It’s preferred to have the programmed Vpi value calibrated to...
the 795nm light since the controller makes phase adjustments based on the 795nm signal and not the 852nm signal.

![Graphs of 795nm and 852nm Michelson responses and control signal with phase unwrapped.](image)

Figure 5-5: Comparison of the two point average of 795nm and 852nm Michelson interference signals with feedback turned on. The controller works to keep the phase difference between the two legs of both the 795nm and 852nm Michelson interferometers nominally at 0 rad. The applied phase correction (seen in bottom graph of Figure 5-5) has been unwrapped. In actuality the controller would do π radian flybacks when it reached the voltage limit of the electro-optic modulator. Step deviations occur in both the 795nm and 852nm locked signals at the π radian flybacks due to variations in the voltage required to shift the phase π radians for 795nm and 852nm wavelengths.
5.3 Frequency Response

To assess the bandwidth of the control system, an experiment was conducted to determine the system and controller’s frequency responses. An additional electro-optic modulator (EOM) was inserted into the opposite optical paths of the control EOM and phase disturbances were introduced into the system by driving the disturbance EOM with a signal generator set at fixed amplitude and varying the frequency. The amplitude of the disturbance was approximately constant at 350 mrad ± 25 mrad over the frequencies investigated, measured by turning off feedback and observing the amplitude of the disturbance on the open loop 795nm Michelson signal.

![Test Bed Diagram](image)

**Figure 5-6: Additional EOM added into Raman Frequency generation optics to insert disturbances for frequency response tests.**

As expected, the controller responded to the disturbance inputs out to a -3dB point of ~3e4 rad/s or 4.7 kHz. See below for the experimental results as well as the previously shown expected curve.
Figure 5-7: Magnitude of controller response $C(s)$ to input phase disturbances $W(s)$ at fixed amplitude of $\sim-350$ mrad at frequencies from 20 Hz to 80 kHz. Disturbances were introduced using an electro-optic modulator (EOM) driven with a sinusoidal signal generator to directly change the phase of the light in one of the Raman paths. The -3dB point is $-3e4$ rad/s or 4.7 kHz matches well with the expected frequency response shown by the dashed curve.

This controller bandwidth is appropriate to address the observed noise content. Figure 5-8 features a cumulative distribution function of the observed laser phase noise content. This function was produced by integrating the power spectral density of the open loop noise in Figure 4-3 from 0 – 250 kHz. Figure 5-8 indicates that the controller bandwidth encompasses 98.8% of the noise power as determined from the power spectral density of the open loop 795nm Michelson signal. Increasing the controller bandwidth will gain the system marginal improvement in noise reduction.
Figure 5-8: Cumulative Distribution function of observed noise content from open loop 795nm Michelson signal data. The dotted line indicates the controller bandwidth below which 98.8% of the noise power resides.

The system response to disturbance input indicates that at lower frequencies the closed loop system does a better job at rejecting disturbances. Any disturbances above $10^4$ rad/s are not rejected and show up in the system response without any attenuation from the controller. The system response curve is similar in shape to the expected system response curve in which the dynamics of the system are determined by the sampling period. The discrepancy in the measured data and the predicted curve at lower frequencies is due to the noise floor of the photo detector used to measure the Michelson interferometer output. See below the experimental system response data as well as the previously shown predicted fit.
Figure 5-9: Magnitude of response of 795nm interference signal with feedback control \( Y(s) \) to phase disturbance \( W(s) \) of fixed amplitude \( \approx 350 \) mrad at frequencies from 20Hz to 80 kHz. Disturbances were introduced using an electro-optic modulator (EOM) driven with a sinusoidal signal generator to directly change the phase of the light in one of the Raman paths. Note that the noise floor of the system with no disturbance is -34dB just due to electronic noise in the detection of the interference signal.

In this closed loop system the EOM directly changes the laser phase to correct for the error so there is essentially no lag between the control effort and output response as indicated in Figure 4-7.
5.4 Atom Interferometer response with feedback control

A series of inertial measurements were taken to assess the effect of phase control on accelerometer performance. First, a short dwell time of 1msec was chosen for the three pulse LPAI sequence. Five minutes of stability data were taken with no control effort and no disturbance input resulting in a short term phase sensitivity of 85mrad/Hz$^{1/2}$. Then the feedback control was turned on and very little change was seen in the short term sensitivity as indicated by the yellow curve in Figure 5-10. Next a 350mrad, 200 Hz disturbance was added into the one of the Raman beam paths with no feedback which degraded the short term sensitivity to 222mrad/Hz$^{1/2}$. A 200 Hz disturbance was chosen because the LPAI was sensitive to this disturbance frequency and the disturbance fell within the laser phase controller’s bandwidth. Finally both the disturbance and feedback were introduced to the system and the short term sensitivity improved from 222mrad/Hz$^{1/2}$ to 140mrad/Hz$^{1/2}$.

![Figure 5-10: Atom interferometer sensitivities with disturbance and feedback loop. The separation time between the Raman pulses for this measurement was set at T =1.03ms. The feedback loop works to reduce the affects of the added disturbance as evident by the](image)

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improvement in the short term sensitivity of the data with the 200 Hz disturbance when the feedback is turned on.

A second inertial measurement was taken at a longer dwell time of 8msec. In Figure 5-11, the short term sensitivities do not change by more than a factor of 2 with the added disturbance or feedback turned on. With such a small discrepancy in the sensitivities, it made it difficult to discriminate a clear effect that the disturbance or feedback had on the short term sensitivity of the LPAI. Time constraints prevented a thorough investigation of the performance of the LPAI at longer interrogation time.

Allan Deviation: DS three pulse $T = 8.005 \text{ msec mod on}$, $f_{\text{amp}} = 2 \text{ Hz}$.

Figure 5-11: Atom Interferometer Sensitivities with disturbance and feedback loop. The separation time between the Raman pulses for this measurement was set at $T = 8.005 \text{ ms}$. It was difficult to discern from this data what if any improvement the controller made on the noisy signal.

The poor results seen in Figure 5-11 could possibly be due to effects from the $\pi$ radian fly-back resets which occur when the control output reaches the maximum voltage limits on the EOM.
actuator. Figure 5-12 depicts the transient structure that was imposed on the 852nm Michelson phase leading up to a fly-back reset event.

Figure 5-12: Observation of added transient structure added to 852nm Michelson phase when flyback occurs

If a LPAI sequence were to fall within the non-linear phase structure prior to the fly-back, the LPAI would measure an acceleration error. With a dwell time of 8msec, the LPAI sampled 64X more of this non-linear curvature than it did with a dwell time of 1msec. Flyback events occurred fairly frequently, and would affect LPAI interrogations at least several times a minute or more. Further analysis and investigation will be needed to determine the actual impact of the fly-back affects.
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6.1 Summary

In this thesis a control system was implemented to stabilize the relative laser phase between the Raman beams in a cold atom accelerometer test bed. Laser phase noise in the Raman beams due to vibrations and temperature fluctuations along the two beam trajectories can corrupt the atom interferometers ability to sense inertial inputs. Over the course of this master’s thesis, a compact cold atom accelerometer test bed was designed and built, which will be used as a universal platform to test new concepts for cold atom sensors at Draper Laboratory. The two Raman beams used in atom interferometry were configured in a separated frequency orientation in which they were routed along separate paths to be oppositely directed at the cold atom proof mass. Initially this compact test bed (CTB) was established as an accelerometer measuring gravity and demonstrated a sensitivity of 2\(\mu\)g/Hz\(^{1/2}\) at a data rate of 15Hz. A Michelson interferometer was installed to sense path length variations along the two Raman beam paths, which provides sensitivity to path variations that would create noise in the inertial signal. The power spectral density of the observed laser phase noise content (refer to Figure 4-3) indicated that the controller’s demonstrated bandwidth of 4.7 kHz, encompassed 98% of the noise power. The attenuation of the controller was as follows: -20dB at 20Hz, -16dB at 150Hz, -9dB at 300Hz, -3dB at 1.6 kHz and ~0dB at 5kHz. Inertial measurements taken at a dwell time of 1msec with the control loop running indicated an improvement in the atom interferometer sensitivity. A similar
inertial experiment taken at a longer dwell time of 8msec, however, did not show improvement. Time constraints prevented further investigation into the reasons for the behavior at the longer dwell time but it was hypothesized that the controller’s π radian fly-backs resets could have caused the degradation of the LPAI sensitivity. The fly-back resets occur when the control effort approaches the maximum voltage of the EOM actuator and have shown to impart structure on the phase. Further investigation will be required to validate this claim.

6.2 Future Work

As stated above, a thorough examination of the controller’s effects on the atom interferometer was not conducted. Moving forward a more comprehensive set of measurements should be taken to characterize the effectiveness of the controller at different disturbance frequencies. For the sake of time one disturbance frequency, 200 Hz, was examined but in the future it would be helpful to ensure that the controller behaves as expected for a variety of frequencies which the LPAI is sensitive to and the controller can respond to. Also, in the future the use of an integral gain should be investigated to produce a more robust control loop. In the quiet environment of the first floor lab, a proportional controller was sufficient to stabilize the laser phase but given a larger disturbance, a proportional controller has a greater likelihood of driving the system to instability. In addition to the use of integral gain, a more thorough examination on the effects of the controller fly-backs resets on the LPAI phase should be conducted to see if improvements can be made to the laser phase stability. Modifying the speed of the control electronics may be one remedy to reducing the added non-linear phase structure the fly-back transitions impart on the laser phase.
REFERENCES


A.1 Hyperfine F=4 transition

Figure A-1: Hyperfine structure of a Cesium atoms’ F=4 ground state. This image is a typical saturation absorption scan of the F=4 transitions.
A.2 Hyperfine F=3 transitions

Figure A-2: Hyperfine structure of a Cesium atoms' F=3 ground state. This image is a typical saturation absorption scan of the F=3 transitions.