An Experimental Investigation of a Low Reynolds Number, High Mach Number Centrifugal Compressor

by

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B.S.E., Mechanical and Aerospace Engineering
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ABSTRACT

Experiments were performed on a super-scale test facility to study the performance of very small centrifugal compressors suitable for microfabrication. The test facility was 75 times larger than the full-scale device and operated under 1/75 atmospheric pressure conditions to correctly match the Reynolds number, which is about 20,000. The impeller tip speed of the test facility matched those in the design full-scale device to capture the effects of compressibility.

Two compressor geometries were tested, one with and one without diffuser vanes; the same impeller was used in both cases. For each of the diffuser designs, speedlines were determined at impeller tip speeds of 400 m/s, corresponding to 100% of the design value, and 170 m/s, corresponding to 42% of design. Detailed measurements were made at a single operating point on each of the speedlines.

The test results show that the total-to-static pressure ratios developed by the facility are in agreement with CFD prediction. The measured mass flow rates are between the values predicted by 2-D and 3-D CFD. The performance of the vaned diffuser (Cp=0.48) is superior to that of the vaneless diffuser (Cp=0.24) in recovering the dynamic pressure of air exiting the impeller. The impeller isentropic efficiencies are about 0.48 and 0.27 for 100% and 42% design impeller speeds, respectively. These values are substantially lower than the predictions of CFD. The difference is thought to be due in part to the interaction of the flow in the impeller with the impeller casing, and to the effects of inlet separation.

This study confirms computational predictions for pressure rise, and is consistent with computational predictions of mass flow rate. However, the measured efficiency is lower than CFD predictions. Additional testing is required to determine the sources of loss in the impeller.

Thesis Supervisor: Alan H. Epstein
Title: R.C. Maclaurin  Professor of Aeronautics and Astronautics
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Contents

1 Introduction 15
   1.1 Background .............................................. 15
   1.2 Motivation and Technical Objectives ....................... 17
   1.3 Review of Previous Research ................................ 18
   1.4 Overview of the Thesis .................................... 19

2 Design of the Macro Compressor 21
   2.1 Design Motivations ......................................... 21
   2.2 Scaling Issues ............................................. 22
   2.3 Technical Challenges Faced in the Design and
       Operation of the Facility ................................ 24
       2.3.1 Material Stresses in the Impeller ..................... 24
       2.3.2 Rotordynamics ........................................ 24
       2.3.3 Low Pressure Operation ............................... 25
       2.3.4 Bearing Failures .................................... 26
   2.4 Overview of the Design ..................................... 27

3 Instrumentation and Data Acquisition 31
   3.1 Pressure Transducers ....................................... 32
   3.2 Arrangement of the Pressure Probes ......................... 33
       3.2.1 Inlet Duct ........................................... 33
       3.2.2 Impeller Exit ........................................ 33
       3.2.3 Diffuser Exit ....................................... 34
       3.2.4 Plenum Section ...................................... 34
   3.3 Pressure Line System ...................................... 35
       3.3.1 Time Response ...................................... 35
   3.4 Errors in the Measurement of Total and Static Pressures .... 38
       3.4.1 Effects of Rarefaction ............................... 38
       3.4.2 Effects of Viscosity ................................. 39
       3.4.3 Effects of Leaks and Outgassing ..................... 40
   3.5 Initial Arrangement of the Thermocouples .................. 43
3.6 Analysis and Redesign of the Initially Installed Thermocouples. 45
   3.6.1 Inlet Thermocouple. 45
   3.6.2 Diffuser Exit Thermocouple. 50
   3.6.3 Diffuser Exit Thermocouple Redesign. 52
   3.6.4 Rig Structure Temperature Acquisition. 60

4 Performance Parameters and Error Analysis 71
   4.1 The Measurement of Mass Flow. 71
      4.1.1 Mass Flow Relations. 71
      4.1.2 Errors In The Measurement of Mass Flow 73
   4.2 Measurement of Compressor Efficiency. 79
      4.2.1 Efficiency Measure 1: Rake Method. 79
         4.2.1.1 MISES Heat Transfer Calculation 80
      4.2.2 Efficiency Measure 2: Euler Method. 82
      4.2.3 Efficiency Measure 3: Shaft Efficiency. 85
         4.2.3.1 Measurement of the Impeller Speed. 86
         4.2.3.2 Mechanical Dissipation. 88
         4.2.3.3 Viscous Work on Sides and Back of Impeller. 90
         4.2.3.4 Errors in the Measurement of Efficiency using the
            Spindown Method. 91

5 Experimental Results 97
   5.1 Speed Lines. 97
   5.2 Impeller Performance. 99
   5.3 Diffuser Performance. 102
   5.4 Performance at Off-Design Reynolds Numbers. 105

6 Conclusions 111
   6.1 Summary of the Research. 111
   6.2 Recommendations for Future Work. 112

A Calculation of the Impeller System’s Rotational Inertial 113
   A.1 Impeller Disk. 113
   A.2 Impeller Shaft. 114
   A.2 Motor. 114

B Speed Line Data Points 117
# List of Figures

1.1 Micro-motor compressor baseline design ........................................... 20

2.1 Impeller rotordynamics prediction for rigid and soft mounted bearings ........ 29
2.2 Schematic of the final macro-compressor bearing configuration .......... 29
2.3 Side view of the macro-rig .............................................................. 30
2.4 Cross section showing detail of the compressor assembly .................... 30

3.1 Schematic of instrumentation locations ............................................. 61
3.2 Pressure line system ................................................................. 62
3.3a Viscous effects on pitot probes .................................................. 62
3.3b Viscous effects on pitot, experimental results by Sherman [13] ............. 63
3.4 Pitot probe designed for use in inlet ............................................... 63
3.5 Error in inlet pressure head due to the effects of tubing outgassing ...... 64
3.6 Experimental setup to investigate the importance of outgassing .......... 64
3.7 Results of outgassing experiment for 0.16 cm dia. Teflon .................. 65
3.8 Outgassing rates from experiment and literature compared ................. 65
3.9 Schematic of the inlet thermocouple in cross flow ............................. 66
3.10 Initial design of the diffuser exit total temperature probe ................. 66
3.11 Inlet thermocouple reading as a function of experiment time 
during the vaned diffuser run ......................................................... 67
3.12 Inlet thermocouple excess temperature distribution, during 
100% impeller speed operation ...................................................... 67
3.13 Thermocouple installed at the exit of the heat exchanger ................. 68
3.14 Measurements of the inlet thermocouples during the 100% 
speed vanless diffuser run ............................................................. 68
3.15 Diffuser thermocouple measurement during the 100% speed 
vaned diffuser experiment ............................................................. 69
3.16 New design thermocouple for impeller exit discharge 
temperature measurement 'Long Probe' .......................................... 69
3.17 'Long Probe' impeller exit thermocouple in its installed 
configuration ................................................................. 70
3.18 Measurements taken with the long (L) and short (S) impeller exit thermocouples .................................................. 70

4.1 Normalized inlet duct velocity profiles ................................................. 94
4.2 Flow factor vs. inlet duct centerline Reynolds number ......................... 94
4.3 Uncertainty in the corrected mass flow vs corrected mass flow for 42% and 100% impeller speed ......................................................... 95
4.4 Impeller control volume .................................................................. 95
4.5 Data from 100% speed spindown tests, compressor in vaned configuration ................................................................. 96

5.1 Speedlines at 100% impeller speed for the vaned and vaneless configuration ........................................................................ 107
5.2 Speedlines at 42% impeller speed for the vaned and vaneless configuration ........................................................................ 107
5.3 Diffuser wall and impeller exit gas temperature vs. time during the 100% speed, vaneless diffuser experiment. ......................... 108
5.4 Impeller exit angle distribution, 42% and 100% design speed and vaneless diffuser ............................................................................. 108
5.5 Impeller exit radial velocity distribution, 42% and 100% design speed and vaneless diffuser ................................................................. 109
5.6 Impeller exit circumferential velocity distribution, 42% and 100% design speed ................................................................. 109
5.7 The dependence of total-to-static pressure ratio on inlet Reynolds number .................................................................................. 110
5.8 The dependence of 100% speed total-to-static efficiency measurements on inlet Re. ................................................................. 110

A.1 Impeller disk cross section ................................................................. 115
A.2 Impeller shaft cross section ................................................................. 115
## List of Tables

1.1 Motor compressor performance predictions based on CFD .......................... 18  
2.1 Micro motor-compressor geometry and operating parameters .................. 21  
2.2 Operating parameter comparison of micro and macro compressors ............ 23  
3.1 Experimental measurements taken, and information provided .................. 31  
3.2 Minimum Knudsen numbers for pressure probes ................................... 39  
3.3 Reynolds numbers of the total pressure probes ................................... 40  
3.4 Steady state pressure errors (in torr) due to line outgassing as a function of pressure 43  
3.5 Observed variations and predicted inlet thermocouple error due to radiation 49  
3.6 Geometry and design flow conditions for the diffuser exit thermocouple .... 51  
3.7 Location and flow geometry of the long probe's shroud .......................... 55  
3.8 Evaluation of the primary junction wire heat transfer coefficient ............. 57  
4.1 Measurement sensitivities in the calculation of mass flow ...................... 74  
4.2 Results of 100% speed spindown test with rig in vaned configuration ....... 87  
4.3 Sensitivities in the calculation of spindown efficiency .......................... 91  
4.4 Uncertainties in the primary performance parameters ........................... 93  
5.1 Impeller performance parameters of the macro-compressor compared to CFD ................................................. 100  
5.2 Impeller isentropic efficiency measurement comparison .......................... 101  
5.3 Internal pressure ratios for the macro-compressor, all measurements ±0.2% 102  

B.1 Speed line data points for 42% speed vaned run ................................ 117  
B.2 Speed line data points for 100% speed vaned run ................................ 117  
B.3 Speed line data points for 42% speed vaneless run .............................. 118  
B.4 Speed line data points for 100% speed vaneless run ............................ 118
Nomenclature

A  Area
C_D  Flow coefficient
D, d  Diameter
C_p  Constant pressure heat capacity of air
E  Energy
F  Force
FF  Flow factor
h  Convective heat transfer coefficient
I  Rotational inertial
k  Thermal conductivity
Kn  Knudsen number
K_s  Sutherland constant
L  Length
M  Mach number
m  Mass
Nu  Nusselt number
P  Power
p  Pressure
Pr  Prandtl number
Q  Flow rate, heat energy
Re  Reynolds number
St  Stanton number
T  Temperature
t  Time
U  Gas velocity

Greek

Δ  Change
δ  Error
ε  Emmissitiity
\gamma \quad \text{Ratio of air specific heats}
\eta \quad \text{Adiabatic efficiency}
\lambda \quad \text{Air mean free path}
\mu \quad \text{Dynamic viscosity}
\theta \quad \text{Air flow angle}
\rho \quad \text{Density}
\sigma \quad \text{Stefan-Boltzmann constant}
\sigma_c \quad \text{Molecular size parameter}
\tau \quad \text{Torque, wall shear}
\omega \quad \text{Angular velocity}

\textbf{Subscripts}

act \quad \text{Actual}
atm \quad \text{Atmospheric condition}
c \quad \text{Compressor}
cent \quad \text{Centerline value}
crit \quad \text{Critical}
cor \quad \text{Corrected}
des \quad \text{Design}
e \quad \text{Edge}
ex \quad \text{Excess}
i \quad \text{Impeller in}
imp \quad \text{Impeller}
j \quad \text{Thermocouple junction}
L \quad \text{Long thermocouple probe}
o \quad \text{Impeller out}
p \quad \text{Pressure surface}
rad \quad \text{Radiation}
s \quad \text{Suction surface}
shroud \quad \text{Thermocouple shroud}
t, \text{tot} \quad \text{Total condition}
wall \quad \text{Surrounding wall}
Chapter 1

Introduction

1.1 Background

In proportion to their weight, gas turbine engines are among the most powerful and versatile energy sources available. In addition to their familiar role in commercial and military aircraft propulsion, they find applications in electricity generation as well as in tank and ship propulsion. Modern jet engines are true marvels of technical innovation, machines pushing the very edge of achievement in the areas of materials, mechanical design, control systems and fluid dynamics. As such, the design and development of a new engine demands a tremendous investment in human effort, and a project may span several years with a budget of several billion dollars.

There is an ongoing effort at MIT to develop a complete gas turbine engine with a maximum length scale on the order of only one centimeter [2]. This device will have a mass flow on the order of 0.1 g/sec and a power output of 10-100 Watts, which is about one millionth that of a conventional scale engine. It will be fabricated using micro-fabrication techniques and will contain clearances and features on the order of microns. To reflect the importance of the small scale of these devices, they have been dubbed 'microengines'. Such machines would bring the advantages of hydrocarbon fueled gas turbines, including their large power density and specific energy, down to a scale small enough to be easily transportable by a single person.

The range of power output delivered by a micro-engine would make them natural competitors to the conventional chemical batteries widely used to power radios and other electronic equipment of soldiers. The most common type of portable battery used by soldiers in the field during wartime is the BA5590. These LiSO batteries pose a major dilemma for the Army because of their cost, toxicity and relatively low specific energy. These batteries have a specific energy of 170 W-hr/kg, which is high for current technology (the best Li-ion batteries have a specific energy of 300 W-hr/kg, but are very dangerous). In contrast, a hydrocarbon such as butane has a theoretical specific energy of about 13,000 W-hr/kg. With a 20% efficient conversion cycle, the available specific energy is 2600 W-hr/kg, roughly 15 times larger than that of the BA5590. In addition, the fuels that will be burned in the micro-engine, which may also include hydrogen, JP-8 and methanol are cheap, readily available and comparatively clean.

An increasingly important device for converting fuel to electrical energy is the fuel cell. While the fuel cell may achieve greater efficiency than any thermodynamic cycle driven
device, they are currently very expensive and relatively bulky. One idea currently under development is to use an electrically driven micro-compressor to pressurize the air entering fuel cells, thereby increasing their power density.

Another recent application in development at MIT is a micro-engine for use in micro UAVs. These devices would need to carry out a single mission of about an hour, and be propelled by one or more micro-engines either directly through their jet exhaust, or via electricity that is generated by the engine and used to power a motor-driven propeller. A host of other applications have been envisioned for micro-turbomachinery. These include fluid pumps for use in the medical field, miniature refrigeration systems and compact power sources for use in hobbyist applications.

There are several scientific motivations for drastically reducing the size of turbomachinery. The most important of these comes from the cube-square law. While the mass and volume of any engine scales like the cube of its length scaling parameter, the power output, which is proportional to the frontal area and mass flow, depends only on the square of the length scale. This means that, all else being equal, the power density of an engine will vary linearly with the inverse of its size. In addition, micro-engines will be fabricated from single crystal silicon (Si) wafers with near perfect internal structure and a much higher strength/density ratio than the strongest conventionally formed metals. Another potential advantage of the fabrication method, which is very similar to that used to produce silicon chips, is that the engine components may be mass produced and assembled with relatively few manufacturing steps.

As might be expected, there are also inherent difficulties associated with such a drastic reduction in size. A central limitation on the design of these devices is that the silicon fabrication technique used can only produce 2-D extruded shapes. This makes it difficult to produce stress-relieving fillets at the base of blades, and near impossible at the current state of the art to incorporate blades with 3-D variations in shape. Another direct consequence of the chosen fabrication technique is that, unlike conventional radial turbomachinery that rely on variation in blade height to control the radial distribution of flow path area, 2-D micro-turbomachinery must achieve their flow area control from the shape of the blades themselves, thus compromising their design. A direct consequence of the small size of these machines is that as the Reynolds number decreases in proportion to their size, the boundary layers become relatively large compared to the scale of the flow path, producing blockage and a significant reduction in mass flow. Another result of small Reynolds numbers is that the flow is near laminar, which makes it more susceptible to separation. Both of these effects are especially detrimental to a compressor, which must sustain an adverse pressure gradient.
Practically all of the computational fluid dynamics (CFD) analysis tools available were designed and calibrated for conventional scale turbomachinery, and are unproven in the low Reynolds number regime of interest to this project. Uncertainty with the accuracy of CFD results led us to construct an experimental test facility capable of simulating these devices. A super-scale test rig large enough to accommodate conventional metal working fabrication facilities, along with standard sized instrumentation was constructed. This avoided the difficulty inherent in the instrumentation of micro-scale devices.

Figure 1.1 shows the micro-compressor that is the subject of the present study. The blading was designed by Jacobson [1] using the 2-D Euler code MISES [16] to provide a 2:1 compression ratio at a tip speed of 400 m/s. It is driven by an integral electric motor and will henceforth be called the “motor-compressor”. The super-scale test rig, also designed by Jacobson, was built to simulate the motor-compressor and has been dubbed the “macro-compressor”.

1.2 Motivation and Technical Objectives

The ultimate goal of the micro-engine project is the design and fabrication of high performance micro-turbomachinery for use in a new class of power plants. The macro-compressor project plays a critical role in the design process in that it provides answers to questions that before could only be answered using computational methods that are unproven in this physical regime. However, because of the inherent limitations of the experiment, we are only able to answer basic questions, such as pressure ratios and efficiencies, and not detailed issues such as the exact internal flow structure in the device.

In spite of the great efforts made to design a rig that would simulate the micro-compressor, there are some inconsistencies between the two devices. Perhaps the most important of these is wall temperature. Recent work on the micro-motor that will be used to drive the full scale compressor has shown that its associated gap produces a relatively large amount of heat, most of which is dissipated in the flow path of the compressor itself. This is expected to have a significant effect on the fluid dynamics of the compressor, but will not be captured by the present study. In this instance, we planned to correct the aerodynamic measurements for heat transfer effects with analysis.

An analysis of the electric motor designed to drive the micro-compressor has indicated that the initial device will be power-limited to a rim speed of about 200 m/s, which is 50% of the nominal value. It was therefore desired to perform experiments on the macro-compressor at 50% impeller speed. However, the presence of a resonance made
operation at this speed unfeasible, so a value of 42% of nominal (which permits smooth operation) was selected for study.

The achievements of this project are as follows:
1. Demonstrated the feasibility of low Reynolds number, high Mach number turbo-machinery.
2. Measured the total-to-static pressure ratio, flow angle and efficiency as a function of flow rate and rotor speed.
3. Compared the performance with a vaned and a vaneless diffuser.
4. Compared the measurements with those predicted by CFD.
5. Used the results as a guide for the redesign of components for future designs.

1.3 Review of Previous Research

Before the macro-compressor experiment, all of our knowledge about micro-turbomachinery came from CFD analysis. The initial blading for the motor compressor was designed by Jacobson [1] using a 2-D design tool (MISES). The plan view and predicted performance for the design is shown in Figure 1.1 and Table 1.1 respectively. Using 3-D computations, Mehra [14] showed that the presence of endwalls introduces considerable 3-D effects on the flow. He used a 3-D code developed by Dawes [17] to determine the impact of a 3-D flow field on a 2-D design. One of Mehra’s findings indicates that the growth of boundary layers on the endwalls of the diffuser causes the flow area of the passages to remain almost constant. This restricts the diffusion to only the small vaneless space at the entrance of the diffuser. A comparison of MISES 2-D, Dawes 2-D and Dawes 3-D results are shown in Table 1.1. A vaneless diffuser was constructed and tested in addition to the nominal vaned configuration.

<table>
<thead>
<tr>
<th></th>
<th>MISES 2-D</th>
<th>Dawes 2-D</th>
<th>Dawes 3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Ratio (t-t), Impeller</td>
<td>2.2</td>
<td>2.2</td>
<td>2.4</td>
</tr>
<tr>
<td>Pressure Ratio (t-t), Full Compressor</td>
<td>2.1</td>
<td>2.1</td>
<td>2.0</td>
</tr>
<tr>
<td>Isentropic Efficiency (t-t) Impeller</td>
<td>0.95</td>
<td>0.94</td>
<td>0.74</td>
</tr>
<tr>
<td>Isentropic Efficiency (t-s) Full compressor</td>
<td>0.89</td>
<td>0.85</td>
<td>0.56</td>
</tr>
<tr>
<td>Mass flow rate (g/sec)</td>
<td>0.11</td>
<td>0.11</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 1.1: Motor compressor performance predictions based on CFD.
1.4 Overview of the Thesis

This chapter introduces the concept of micro-turbomachinery and describes some of the challenges presented by their design and fabrication. The technical objectives of this experimental research are then presented along with its inherent limitations.

Chapter 2 describes the motivations for the construction of the super-scale compressor test facility, and the technical challenges that it introduced. It then summarizes the most important features of the design and operation of the macro-rig.

Chapter 3 describes the instrumentation and data acquisition hardware used in the experiments. The performance of the pressure and temperature instrumentation is then discussed and sources of experimental error associated with them are evaluated.

Chapter 4 presents the methods used to reduce the experimental data to compressor performance parameters, including mass flow, pressure ratio and isentropic efficiency. An error analysis is performed to determine the level of uncertainty for each parameter.

Chapter 5 presents the experimental results of this research and discusses them in the context of CFD predictions.

Chapter 6 summarized the findings of this research and makes suggestions for future work.
Figure 1.1: Micro-motor compressor baseline design
Chapter 2

Design of the Macro Compressor

2.1 Design Motivations*

The key geometries and flow parameters for the full-scale micro motor-compressor are shown in Table 2.1. The small size and high rotational speeds of this device make it difficult to experimentally test the turbomachinery. Instrumentation of the size required for the full scale compressor is generally unavailable. Although pressures could be measured with external pressure transducers, the probes needed to acquire total and static pressures would require technology development. The measurement of flow temperatures would also be very difficult due to the relatively large size of even the smallest thermoelectric instrumentation.

<table>
<thead>
<tr>
<th>Inlet $p_i$</th>
<th>1 atm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Temperature</td>
<td>300 K</td>
</tr>
<tr>
<td>Design Total Pressure Ratio</td>
<td>2.1</td>
</tr>
<tr>
<td>Impeller Diameter</td>
<td>4 mm</td>
</tr>
<tr>
<td>Blade to Shroud Clearance</td>
<td>5 $\mu$</td>
</tr>
<tr>
<td>Blade Height</td>
<td>200 $\mu$</td>
</tr>
<tr>
<td>Impeller Speed</td>
<td>1,909,860 RPM</td>
</tr>
<tr>
<td>Tip Speed (Mach Number)</td>
<td>400 m/sec (0.96)</td>
</tr>
<tr>
<td>Number of Impeller Blades</td>
<td>6</td>
</tr>
<tr>
<td>Number of Diffuser Blades</td>
<td>11</td>
</tr>
<tr>
<td>Design Mass Flow (MISES)</td>
<td>0.11 g/sec</td>
</tr>
</tbody>
</table>

* Table 2.1: Micro motor-compressor geometry and operating parameters

One solution to the problems described above is to build a geometrically scaled-up model of the motor compressor. As the size increases, the use of conventional instrumentation becomes feasible. Ball bearings can be used to support the rotor and

* The Macro facility was designed by Dr. Stuart Jacobson [1]
conventional electric motors or air turbines can be used to drive the compressor. However, cost increases with size, so it is desirable to keep the facility as small as possible.

A scale factor of 75 was chosen for two reasons. First, this increases the height of the flow passage to above 1.3 cm, the minimum required by most conventional instrumentation. Second, the rotational speed is consistent with available drive technology. The impeller rotation rate required to accurately simulate the motor-compressor tip speed scales linearly with the inverse of its size. Several drive options were available. An air turbine would allow operation at speeds in excess of 100,000 RPM, but would be difficult to control and very expensive. A gear system driven by an electric motor is easier to control, but is still expensive. The most straightforward method of powering the impeller is to use an electric motor directly driving the shaft. This option allows precise control of rotation rates and is relatively simple to install. With a test facility scaling factor of 75, the rotation rates required to match the tip speed are about 26,000 RPM, within the range of operation of conventional electric motors. For use in this project, a 15 Hp, 39,000 RPM AC induction motor manufactured by Reuland Electric Co. was selected.

2.2 Scaling Issues

The motor compressor flow is greatly influenced by compressibility and viscosity effects. These effects are described using two non-dimensional parameters: the Reynolds number and the Mach number:

\[ \text{Re} = \frac{\rho UL}{\mu}, \quad \text{M} = \frac{U}{\sqrt{\gamma RT}}, \]

where

\begin{align*}
\rho &= \text{fluid density (kg/m}^3), \\
U &= \text{flow velocity (m/sec)}, \\
L &= \text{length scale (m)}, \\
\mu &= \text{Absolute viscosity (N} \cdot \text{sec/m}^2), \\
\gamma &= \text{Specific heat ratio}, \\
R &= \text{Gas constant (J/kg K)}, \\
T &= \text{Absolute Temperature (K)}. \\
\end{align*}

The Reynolds number expresses the relative importance of inertial with respect to viscous effects in a flow. The micro-compressor operates with Reynolds numbers of about
20,000 (based on impeller blade chord, tip speed, and inlet static properties); two orders of magnitude lower than in conventional systems, indicating the increased importance of viscosity.

The Mach number, which expresses velocities as a fraction of the speed of sound, describes the relative importance of compressibility effects within the flow. Like conventional turbomachinery, the motor compressor needs a near sonic tip Mach number, about 0.96, to achieve the design pressure ratio. Thus, compressibility effects are also expected to play an important role in the compressor behavior and requires that Mach number be correctly matched in the scaled up device to produce similar results.

To maintain similarity of these parameters, one or more of the other quantities involved must be changed. To simplify the design, room temperature air was chosen as the working fluid, as it is in the actual device. However, because viscosity is almost entirely a function of temperature and composition, it is eliminated from our control. Similarly, \( \gamma \) and \( R \) are fixed. Therefore the velocity, \( U \), must be maintained at its full-scale value to correctly match \( M \). The only remaining parameter is the density, \( \rho \). To match the Reynolds number given unchanged \( U \) and \( \mu \), it is necessary to reduce the density in the same proportion that the length scale is increased. Because \( \rho \) is proportional to \( p \), this reduction in density may be achieved by operating the rig (macro-compressor) at a decreased pressure. The important scaling parameters are quantified in Table 2.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Micro-compressor</th>
<th>Macro-compressor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impeller radius (mm)</td>
<td>4</td>
<td>300</td>
</tr>
<tr>
<td>Pressure Ratio (design)</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>Impeller tip speed (m/s)</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Impeller rotation rate (RPM)</td>
<td>1,910,300</td>
<td>25,500</td>
</tr>
<tr>
<td>Inlet p (Atm.)</td>
<td>1</td>
<td>1/75=10.1 mm Hg (torr)</td>
</tr>
<tr>
<td>Inlet Temperature (K)</td>
<td>300</td>
<td>300 (approx.)</td>
</tr>
<tr>
<td>Inlet Density ((Kg/m^3))</td>
<td>1.176</td>
<td>0.0157</td>
</tr>
<tr>
<td>Mass flow (design) ((g/sec))</td>
<td>0.11</td>
<td>8.25</td>
</tr>
</tbody>
</table>

Table 2.2: Operating parameter comparison of micro and macro compressors.
2.3 Technical Challenges Faced in the Design and Operation of the Facility.

The design and eventual operation of the macro-facility posed a series of technical challenges. A discussion of these challenges follows.

2.3.1 Material Stresses in the Impeller

Rotor stress in the micro-compressor is not an issue at 400 m/s due to the very high strength/density of single crystal silicon. In contrast, the macro-facility must make use of metals, which have much lower values of strength/density. At this tip speed, the centrifugal forces produced maximum stress levels that are beyond the range of conventional ferrous-based steels [3]. The only suitable materials are alloys of Titanium, and AMS4928 was chosen.

Despite efforts to maximize material strength/density, the impeller is still a highly stressed structure. A finite element analysis of the centrifugal stresses produced in the impeller was performed by Chen [3]. This analysis showed that there are stress concentrations at the roots of the blade leading and trailing edges. To alleviate the stresses at the leading and trailing edges, which were anticipated to be exacerbated by the increased temperatures there, the fillet radius at the base of the blade in that region was increased over the correct scaled up value. Although this geometric non-similarity will effect the flow, these effects are expected to be minor. Another important design feature of the macro-impeller, intended to help counteract the out of plane bending stresses produced by the centrifugal force of the blades, is a thickening of the disk and the addition of a counterbalance mass at the back of the disk. At design speed, the impeller has a safety factor of 1.3 for stress, which is 1.14 on operating speed.

2.3.2 Rotordynamics

Rotordynamics is always an important issue in the design of high speed rotating machinery. Both the micro-compressor and the macro-facility are no different in this respect. However, whereas the micro-device will make use of air bearings (see Figure
1.1), the macro-rig's impeller is supported by a short shaft resting on two pairs of high speed ball bearings (see Figure 2.4). One feature of the macro-rig is that in the interest of geometric similarity to the micro-device, the impeller had to be supported from only one side. This overhung configuration is much less stiff than if it had been supported on both sides as is often done for high-speed spools. A rotodynamic study performed by Jacobson [1] predicted that when both pairs of bearings are hard mounted to the structure of the rig, the first critical frequency of the system, shaft bending, would lie within the desired operating speed range. Ball-bearings alone provide insufficient damping to prevent an unstable shaft resonance from occurring, so an attempt was made to stiffen the system by increasing the shaft thickness¹ thereby increasing the resonant frequency. However, the first critical could not be moved above the desired speed range. Instead, the first critical was moved below the desired range. The pair of bearings nearest the impeller was soft mounted on four rubber O-rings. In addition to providing some damping, the soft mounted bearings reduced the first critical frequency below the desired operating speed, allowing supercritical operation. A comparison of the computed frequency response for both the rigid and soft bearing mount systems is shown in Figure 2.1. In practice, the rig has not exhibited any well defined resonances (as expected for well balanced rotors). Instead, during acceleration and deceleration of the rotor, relatively long episodes of audible vibrations are observed at about 5,400 RPM, 7,800 RPM, 13,800 RPM and 21,000 RPM. These events extend over 600-900 RPM intervals. In the early stages of the experiment, these loud humming noises were a major source of concern as they appeared to be correlated with several bearing failures. Although the sounds have continued through each experiment, they are no longer a source of major concern as the bearings have remained intact.

2.3.3 Low Pressure Operation

To maintain the low air pressures required to correctly set the non-dimensional flow parameters, the entire compressor facility must be made airtight. This was achieved by installing O-rings onto all faces of adjoining parts. Protruding instrumentation such as thermocouples and Kiel probes were sealed using a combination of Swagelok and Cajon Ultratorr fittings. The Cajon fittings with their O-ring sealing mechanisms allowed the probes to be traversed while maintaining their seal. To eliminate the need for a high volume vacuum source, a closed loop system is employed in which the compressor exhaust air is

¹ The shaft thickness is limited by ball bearing speed, which must be kept within an allowable range to preserve their lifetime.
expanded through a throttle, cooled to the required inlet temperature using a heat exchanger, straightened by passing it through a fine wire mesh and honeycomb, and then returned to the compressor inlet. Somewhat more difficult is the seal between the impeller cavity, which is at low pressure, and the shaft, which is at atmospheric pressure. This is achieved with the use of a carbon rotating face seal, which is mounted flush to a surface of the shaft adjacent to the impeller/ shaft interface. This seal has performed well, withstanding even the shock produced by bearing failures, and maintaining sufficiently low leak rates after hours of operation.

2.3.4 Bearing Failures

One difficulty encountered in the initial stages of rig operation was four bearing failure events. These events usually occurred after several minutes of operation with impeller speeds exceeding 12,000 RPM. At the onset of each failure, a loud grinding sound was heard as the motor cut out, and the impeller decelerated. Upon inspection of the disassembled bearing housing, the second bearing in line after the impeller (number 2 in Figure 2.2) was observed to have been destroyed. In one case, bearing number 1, first in line after the impeller disk, was also destroyed.

In their initial configuration, both the front and rear pairs of bearings were oriented with their thrust faces in contact to provide axial support in both the forward and aft directions. The inner race of each bearing was pre-ground by the manufacturer so that pressing a matched pair of bearings together would supply a preload to each. This preload is required for the correct bearing operation by preventing ball slippage and associated friction and wear. The bearings were installed such that the aft pair (3&4) was allowed to 'float' in the axial direction but were rigid in the radial direction. Bearings 1 and 2 were supported on rubber O-rings in both the axial and radial directions. Relative to the back pair, bearings 1 and 2 were rigid axially but soft radially. It is theorized that during operation, nearly all of the axial thrust in the shaft (which was directed towards the impeller due to the pressure difference across the rotating seal) was sustained by bearing number 2. In addition, because of their proximity to the spinning mass of the impeller, bearings 1 and 2 are expected to bear most of the radial loads as well. This was verified by accelerometer readings taken at both bearing pairs. It is thought that because the front pair of bearings was forced to carry both the radial and axial loads, while the back pair operated under very light loading, the front pair may have become overloaded, leading to the observed failures.

In an attempt to more evenly distribute the axial and radial loads, the bearings were reconfigured as shown schematically in Figure 2.2. In this arrangement, bearings 3 and 4 are both oriented to sustain a forward axial thrust, while bearings 1 and 2 are both oriented
to sustain an aft thrust on their inner races. In this new arrangement, bearings 3 and 4 are now rigidly fixed in both the axial and radial directions. To provide an axial preload to both pairs of bearings, a spring producing about 580 N (130 lbs.) of force was installed as shown in Fig. 2.2. With this configuration, the rear pair of bearings will accept a larger fraction of the axial loads, leaving the front pair to carry mostly radial loads. This arrangement has proven successful, as no further bearing failures have occurred.

2.4 Over-view of the Rig Design

Figure 2.3 illustrates the flow conditioning and delivery system. Beginning at the compressor inlet, air enters axially, and is compressed as it passes through the impeller radially. The air exits the compressor through a metal duct and passes through a water-chilled heat exchanger that removes the heat produced during the compression process. After exiting the heat exchanger, the air passes through a fine mesh screen, which produces a pressure drop, thereby removing flow irregularities and then passes through a honeycomb flow straightener before re-entering the compressor inlet.

Figure 2.4 provides a more detailed view of the compressor assembly. The heavy black line shows the flow path of air through the machine. Beginning at the inlet, air enters axially, turns 90° and passes radially through the rotating impeller blades. After exiting the impeller, air passes through the diffuser section, which contains vanes mounted on a turntable, allowing their rotation. These vanes are removed in the vaneless configuration. After being decelerated in the diffuser, air continues radially outward until reaching a ring of holes designed to simulate the combustor inlet section of the micro-engine. Air passes axially through these holes and into a plenum. The plenum is designed to simulate the combustion chamber of the micro-engine. Its volume is adjustable by means of a movable plate from almost zero to nearly double the scaled volume in the micro-engine to allow the simulation of compression instabilities, such as surge. Air then passes through a throttle, which is adjustable by means of an external handle. After being throttled to a lower pressure, air exits the compressor and is passed through the conditioning system described above before being returned to the inlet.

The rig is evacuated using a vacuum pump, which is attached to the inlet pipe just upstream of the wire screen. This vacuum pump can take the rig from atmospheric pressure to 10 torr in about two minutes. Once the correct pressure has been attained, the vacuum pump is then valved off. To compensate for air leakage into the rig during operation, a
small adjustable bypass line running from the rig to the vacuum pump has been installed, allowing operation at constant pressure by controlling the removal of air.

The entire compressor assembly is supported by a steel frame mounted on wheels, which are guided by rails. This allows the rig to be pulled away from the heat exchanger and ducts, facilitating assembly and disassembly. The back plate of the rig is supported on bearings, which with the help of a hand operated gear crank, allows the compressor assembly to be rotated through 90° so that the inlet faces upwards. With the compressor rolled back and rotated, the impeller assembly may be installed and removed vertically using a crane.
**Figure 2.1:** Impeller rotordynamics prediction for rigid and soft mounted bearings.

**Figure 2.2.** Schematic of the final macro-compressor bearing configuration.
Figure 2.3: Side view of the macro-rig.

Figure 2.4: Cross section showing detail of the compressor assembly.
Chapter 3

Instrumentation and Data Acquisition

The measurements taken in a typical experiment are summarized in Table 3.1. The locations of these measurements are shown schematically in Figure 3.1. Pressures are measured using two absolute and one differential pressure transducer described below. Measurements of temperature are acquired via K-type thermocouples with a Fluke Helios I Computer Front End. The shaft rotation rate is monitored using a Hall effect remote pick-up installed in the motor housing, and a Hewlett Packard 5316A Universal Counter.

<table>
<thead>
<tr>
<th>Location</th>
<th>Measurement</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Duct</td>
<td>Total Pressure (Radial Traverse)</td>
<td>Compressor Mass Flow</td>
</tr>
<tr>
<td></td>
<td>Static Pressure (Wall)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Temperature (Centerline)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wall Temperature</td>
<td></td>
</tr>
<tr>
<td>Impeller Exit</td>
<td>Flow Angle (Spanwise Traverse)</td>
<td>Total-to-Total Pressure Ratio</td>
</tr>
<tr>
<td></td>
<td>Total Pressure (Spanwise Traverse)</td>
<td>Euler Efficiency</td>
</tr>
<tr>
<td></td>
<td>Static Pressure (Wall)</td>
<td>Temperature Efficiency</td>
</tr>
<tr>
<td></td>
<td>Total Temperature (centerline)</td>
<td>Mass Flow</td>
</tr>
<tr>
<td></td>
<td>Wall temperature</td>
<td></td>
</tr>
<tr>
<td>Diffuser Exit</td>
<td>Total Pressure (Spanwise Traverse)</td>
<td>Total-to-Total Pressure Ratio</td>
</tr>
<tr>
<td></td>
<td>Static Pressure (Wall)</td>
<td>Diffuser Pressure Ratio</td>
</tr>
<tr>
<td></td>
<td>Total Temperature (Centerline)</td>
<td>Temperature Efficiency</td>
</tr>
<tr>
<td></td>
<td>Wall Temperature</td>
<td></td>
</tr>
<tr>
<td>Plenum</td>
<td>Static Pressure (Wall)</td>
<td>Total-to-Static Pressure Ratio</td>
</tr>
<tr>
<td>Exchanger Exit</td>
<td>Total Temperature</td>
<td>Compressor Mass Flow</td>
</tr>
<tr>
<td>Motor Mount</td>
<td>Shaft Rotation Rate</td>
<td>Impeller Speed, Efficiency</td>
</tr>
</tbody>
</table>

*Table 3.1:* Experimental measurements taken, and information provided.
3.1 Pressure Transducers

Measurements of absolute pressure are made using two MKS Baratron type 220C capacitive pressure transducers housed in 45°C controlled enclosures. Each unit is specified to have a pressure range of 100 torr, an accuracy of 0.15% of the reading, and a resolution of 0.01 torr. These transducers have been designated ‘P1’ and ‘P2’. To measure fine differences in pressure, a type 220C differential transducer of similar configuration is used. This unit has a full scale of 1 torr, an accuracy of within 0.15% of readings, and a resolution of 0.0001 torr. It has been designated as ‘DP3’ in the pressure line system.

The analog signals produced by P1 and P2 are digitized by a Mark V Scanivalve Digital Interface Unit (SDIU) fitted with an internal 16 bit A/D converter. The signal produced by DP3 is digitized by another similar SDIU. These digital signals are relayed via GPIB cables to a 100 MHz Pentium PC running a data acquisition program written with Lab Windows². To ensure the accuracy of the digital output of the SDIUs, each was calibrated using a Keithley 191 Digital Multimeter with an accuracy of 0.01%. Each SDIU had approximately 1% error in slope, which was corrected for in software.

Although the absolute pressure transducers have the same specification, they differ markedly in performance. In particular, it was observed that measurements from P2, unlike P1, drifted down continuously. This effect occurred at the rate of about 0.5 torr per month and is suspected to be due to a slowly leaking built-in vacuum reference. In addition, the linearity of the instrument has been observed to change with time. During one of the experiments conducted in this study, P2 seriously malfunctioned, requiring the unit to be recalibrated by the manufacturer. In contrast, pressure transducer P1 held its zero point for many months. To overcome the day-to-day non-repeatability of P2, the readings of P1 were used to calibrate P2. This was done by fitting a quadratic least squares curve to the readings of both transducers over a range of pressures from 5 to 80 torr in 5 torr increments. During this calibration operation, which was performed before and after an experiment, the transducers were exposed to the same pressure by opening a zeroing valve between them. The drift in the measurement of P2 was negligible over the time period of an experiment.

A zeroing operation was also applied to the differential transducer, DP3, before and after each experiment. The two ports of DP3 were connected so that the pressure

² Lab Windows is a C code compiler from National Instruments containing libraries useful for data acquisition and analysis.
difference was zero, and an offset measured. The offset was due to the effects of tubing outgassing (Section 3.4.3) and was of order 0.0003 torr at a design pressure of 10 torr.

3.2 Arrangement of the Pressure Probes

Pressure readings at locations of interest throughout the rig are acquired by pressure instrumentation of two types. Total pressure probes are designed to stagnate the flow at their impact holes and are either set at the centerline position of the flow path or mounted on a micrometer traverse apparatus. Static pressure ports consist of flush holes in the walls of locations of interest and are designed to convey the static pressure at that position. It has been assumed that because flow is at right angles to the locations of static pressure measurements, the gradients in static pressures in this direction are small. An exception to this case occurs at the inlet, and will be discussed later.

The location and function of pressure probes installed in the test rig are described below.

3.2.1 Inlet Duct

The inlet duct contains a pitot tube and a static pressure port as shown in Fig. 3.1. The pitot tube is located at the top wall of the duct leading into the entrance of the impeller. It is mounted to a micrometer traverse that allows the head of the probe to be moved from the centerline of the duct to the duct wall, a distance of 8.1 cm. The upper static port is located about five stem diameters to the side of the pitot tube. During an experiment, pressure readings are acquired simultaneously from the stationary static port and the traversible total probe. This data is used to determine velocity profiles across the inlet duct from which estimates of compressor mass flows are made.

Part way through the series of tests performed in this study, a concern was raised that the upper static pressure port in the inlet duct was located too close to the base of the pitot tube stem, possibly leading to erroneous measurements of dynamic pressure. To see if this was the case, an additional static tap was installed at the opposite wall from the first, leaving the tap free from obstructions.

3.2.2 Impeller Exit

A wall static port, a cobra probe, and a Kiel head pitot tube are installed at the exit of the impeller, at a distance of 0.5 cm from the disk rim. A cobra probe has three impact holes aligned in the plane of the impeller. The two side holes are located at an angle of
about 15° from the centered third hole. The cobra probe is useful for determining the
direction of fluid flow out of the impeller. This is done by rotating the probe's head until
the pressures measured at the two side holes are approximately equal. Once the cobra probe
has been aligned with the flow in this way, the angle is read from an external dial fitted
with a vernier with a precision of 0.1°. With the cobra probe aligned in the direction of the
flow, the total pressure was determined by measuring the pressure at the center hole.

The zero incidence angle point of the cobra probe was originally found by mounting
it in a 1' x 1' wind tunnel so that its head was in the flow centerline, and then recording the
angle at which the pressure difference across its outer impact holes was minimized. The
accuracy of this technique is expected to be about 1°.

The cobra probe is mounted on a micrometer traverse that allows it to be moved
through the full air passage height of 1.52 cm. The cobra probe is used in conjunction with
the impeller exit static port to determine the pressure ratio of the impeller and to provide an
estimate of the efficiency. It should be noted that in the compressor flow path,
corresponding static and total pressure probes are mounted at the same radius but with a
90° offset in their circumferential position. It is thus assumed that the flow in the
compressor flow path is radially symmetric.

Also mounted at the impeller exit is a Kiel head probe. The Kiel probe is a pitot
probe fitted with a shroud that is intended to make the instrument less sensitive to flow
angle in its indication of total pressure. It was discovered, however, that at the low
Reynolds number of the flow, the Kiel is quite sensitive to angle, and was seldom used.

3.2.3 Diffuser Exit

A wall static port and a Kiel pitot probe mounted in the centerline of the flow
passage are located at the exit of the diffuser. The Kiel probe may be rotated to face directly
into the flow, at which point its indication of total pressure is maximized. The pressures
obtained from these two sources are used to give information about the performance of the
diffuser.

3.2.4 Plenum Section

The plenum is fitted with a wall static pressure port located in the region just before
air passes through the throttle. The pressure ratio developed between this location and the
inlet corresponds to the available pressure ratio of the full scale device, and is of particular
importance to this study. This parameter will be used frequently in the following chapters
when the compressor's performance is discussed.
3.3 Pressure Line System

Pressure signals from various locations on the rig are transmitted to the pressure transducers through a system of plastic tubing and valves, shown schematically in Figure 3.2. The system is divided into a 'high side', and a 'low side' because most of the pressure related parameters are found by comparing pairs of pressures acquired at the same time. The high side, which is controlled by valves 1-5, incorporates pressure sources that are expected to be high relative to their counterpart pressure. Valves 6-10 control the low halves of each pressure pair. Valves 1 and 6 are connected to either side port of the cobra probe. Valves 2 and 7 are connected to the probes monitoring the impeller exit total pressure (as measured by the center port of the aligned cobra probe), and the impeller exit wall static pressure respectively. Valves 3 and 8 are connected to the diffuser exit total and static pressure ports respectively. Valves 5 and 9 control the plenum static and inlet static pressures respectively. Valve 4, which is also used in combination with valve 9, controls the inlet total pressure. The static pressure tap controlled by valve 10 is used to monitor the pressure drop across the inlet mesh in order to quantify its effectiveness in removing disturbances from the inlet flow.

During an experiment, at most two valves, 1 through 10, are opened at the same time. The high side is connected to absolute transducer P1 and the low side to P2. The differential pressure transducer DP3 is positioned so that if the difference between P1 and P2 is less than +1 torr, a higher resolution measurement of the difference is taken. For use in checking the transducer calibrations, a zeroing valve between P1 and P2 was included.

3.3.1 Time Response

Concerns over time response of the pressure line system under reduced air pressure arise because of the increased importance of viscous forces at low Reynolds numbers.

The settling time of a pressure signal decreases with the flow rate of air through the pressure lines and increases with the total internal volume that must adjust to a pressure change. This volume consists of the sum of the tubing volume, internal volume of the valves and the pressure transducers themselves. The flow rate is adversely affected by the pressure loss from the source to the transducer, which is due to viscous loss in the circular plastic tubing as well as to flow restrictions in the pressure probes. The Cobra probe, which is used to determine flow angles, has the most severe restriction in the system.
because of its relatively small impact holes and tube diameters. Before this effect was understood and provisions made, any adjustment of the Cobra probe required many minutes of settling time before a measurement could be made.

To understand the trends in settling time required to change the pressure internal to the transducer, the flow regime inside the tubing must be determined. The Reynolds number of the flow inside the tubing is given by

$$\text{Re} = \frac{\rho \bar{U} D}{\mu}.$$  

The flow will be completely laminar for $\text{Re}<2000$. The largest tubing size under consideration has an internal diameter of 0.43 cm and the highest pressure air that is passed through the tubing is about 25 torr. With these parameters and room temperature air, the critical velocity for turbulent flow is given by:

$$\bar{U}_{\text{crit}} = \frac{\text{Re}_{\text{crit}} \mu_{300K}}{\rho_{25} D} = \frac{2000 \times 1.8 \times 10^{-3} \text{ N} \cdot \text{s} \cdot \text{m}^{-2}}{0.039 \text{ kg} \cdot \text{m}^{-3} \times 4.32 \times 10^{-3} \text{ m}} = 214 \text{ m/s}$$

For tubing with an ID of 0.43 cm, the incompressible viscous pressure gradient required to achieve this velocity, assuming room temperature conditions is given by Fox [4]:

$$\frac{\partial p}{\partial x} = \frac{8 \bar{U} \mu}{R^2}.$$  

Substituting in $\bar{U}_{\text{crit}}$ and $\mu = 1.8 \times 10^{-5} \text{ N sec/m}^2$:

$$\frac{\partial p}{\partial x} = 50 \text{ torr/m}.$$  

Because the maximum pressure gradients are expected to be much lower than this, it is concluded that flow within the pressure lines will always be laminar. For laminar flow, the volume flow rate as a function of the tube length, $L$, the tube diameter, $D$, and the pressure drop across the tube $\Delta p$, is given by Fox [4]:

$$Q = \frac{\pi \Delta p D^4}{128 \mu L}.$$  

36
From this equation, it is seen that for a given pressure gradient, the flow rate increases much faster than the internal volume of the tubing, which scales with the square of the diameter. This seems to indicate that tubing with as large a diameter as possible should produce the shortest time response. A complication arises when the tubing system must accommodate instruments with severe flow restriction such as the Cobra probe, and to a lesser extent the Kiel head probes. Because both of these devices have small impact holes (about 0.5 mm), and a small stem ID in the case of the cobra probe, it is theorized that the pressure drop between the entrance of the probes and the intersection of the plastic tubing is much larger than the pressure drop across the tubing. This means that it is more important to minimize the total volume to be brought to equilibrium (by decreasing its diameter) rather than to further decrease the pressure drop across the tubing by increasing its diameter.

This theory was substantiated by experiment. The time response of the Cobra probe was experimentally measured by venting the pressure transducers from atmospheric pressure to the rig pressure (15 torr) through the Cobra probe and lengths of 0.64 cm, 0.32 cm and 0.16 cm ID Teflon tubing. This test found that the 0.16 cm tubing produced the fastest response. This same test was performed on a static pressure tap, which had a much larger internal diameter than the cobra, it was found that the fastest time response was associated with the 0.64 cm tubing, followed closely by the 0.32 cm. The 0.64 cm tubing was found to produce a much longer lag than the other two sizes.

To reduce the overall time response of the pressure acquisition system, the lengths of tubing running from the instruments to the valves are of two different diameters. The lengths connected to the Cobra probe are each 0.16 cm, while all of the others are 0.32 cm. The 0.64 cm tubing, although producing a slightly faster response than the 0.32 cm tubing was not used because of the inconvenience of its stiffness and bulk. To further accommodate the Cobra probe, the volume after the valves was minimized by using only 0.16 cm tubing. The internal volume in the transducers themselves was reduced by installing internally fitting brass plug pieces. In laying all of the lines, the total length of tubing used was kept to a minimum by placing the transducers close to the pressure taps, and placing the ball valves adjacent to the transducers. With this configuration, the time required to allow for stabilization is about 30 seconds for pressures taken at static ports and the inlet pitot probe, and about 90 seconds for pressures taken using the cobra probe.
3.4 Errors in the Measurement of Total and Static Pressures

In addition to the transducer error, experimental errors in the measurement of static and total pressures can arise from three effects, 1) rarefaction phenomenon, 2) viscous effects at low Reynolds numbers, 3) leaks and outgassing in the pressure line system.

3.4.1 Effects of Rarefaction

Rarefaction concerns can arise when air no longer behaves as a continuum at the length scale of a pressure probe. The importance of rarefaction on the performance of the pressure probes may be evaluated by calculating the Knudsen number.

\[ Kn = \frac{\lambda}{L}, \]  \hspace{1cm} (3.1)

where

\[ \lambda = \text{local mean free path of air molecules}, \]
\[ L = \text{length scale of probe=internal radius}. \]

Flow may be considered to behave as a continuum for Kn<0.01. In this regime, pressure instrumentation will function as expected. To evaluate the Knudsen number for the pressure instrumentation installed in the macro-compressor, the internal radii of these devices is compared to the largest mean free path expected during normal experimental operation, when the rig is at a pressure of 10.1 torr and 300K. The average mean free path of air is given by Fiszdon [5]:

\[ \lambda = \frac{1}{\sqrt{2\pi}} \frac{m}{\sigma_e^2} \frac{1}{\rho(1 + K_s/T)}, \]  \hspace{1cm} (3.2)

where

\[ m = \text{mass of a molecule}=4.81 \times 10^{-26} \text{ kg}, \]
\[ \rho = \text{density}=0.017 \text{ kg/m}^3, \]
\[ T = \text{air temperature}=300 \text{ K}, \]
\[ K_s = \text{Sutherland constant} = 110K, \]
\[ \sigma_e^2 = \text{molecular size parameter for air}=10^{-19} \text{ m}^2. \]
\[ p = \text{pressure} = 10.1 \text{ torr} = 1350 \text{ Pa}, \]

so that

\[ \lambda = 4 \times 10^{-6} \text{ m}. \]

Table 3.2 shows the computed Knudsen numbers for probes used in this experiment. These results indicate that the minimum Knudsen numbers for the cobra and diffuser exit Kiel probes are on the threshold of continuum state. It should be noted, however, that during design operation, the pressures at the locations of these probes is higher than that used in the calculation and will tend to decrease the mean free path and Knudsen number below that shown in the table. It is acknowledged, however, that rarefaction effects may introduce errors into measurements taken from these probes. One recommendation for future work, is that replacement instruments with larger impact holes be utilized.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Probe internal radius, m</th>
<th>Knudsen number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall static ports</td>
<td>(1.6 \times 10^{-3})</td>
<td>0.0025</td>
</tr>
<tr>
<td>Inlet duct total</td>
<td>(2.4 \times 10^{-3})</td>
<td>0.0017</td>
</tr>
<tr>
<td>Impeller exit cobra</td>
<td>(2.5 \times 10^{-4})</td>
<td>0.016</td>
</tr>
<tr>
<td>Diffuser exit total Kiel</td>
<td>(2.5 \times 10^{-4})</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Table 3.2: Minimum Knudsen numbers for pressure probes

3.4.2 Effects of Viscosity

The measurement of total pressures under low Reynolds number conditions may result in erroneous measurements due to the effects of viscosity. This error, which manifests itself as pressure measurements that are higher than actual values, results because the flow does viscous work on the stagnation streamline entering the head of the pitot tube. This effect has been established by Sherman [13] and MacMillan [18] as a function of \(Re\) for a number of pitot tube designs, and is summarized in Figure 3-3(a) [6]. Here, a pressure coefficient, defined as the ratio of measured to actual dynamic head is plotted against the Reynolds number based on external pitot tube radius for a number of tube designs. This plot shows that viscous effects begin to become a concern when Reynolds numbers drop below about 50, at which point a 1% error in velocity results. To determine the importance of viscosity on the measurement of total pressures in the current experiment,
the approximate external radius based, design point Reynolds numbers for the total probes are given in Table 3.3.

<table>
<thead>
<tr>
<th>Location</th>
<th>Probe rad, external, m</th>
<th>U m/s</th>
<th>Static p, torr</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet tot</td>
<td>$2.4 \times 10^{-3}$</td>
<td>24</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Imp Exit tot</td>
<td>$0.6 \times 10^{-3}$</td>
<td>200</td>
<td>20</td>
<td>115</td>
</tr>
<tr>
<td>Diff Exit tot</td>
<td>$0.6 \times 10^{-3}$</td>
<td>50</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

*Table 3.3: Reynolds numbers of the total pressure probes*

The region of the rig where it is most difficult to achieve high Reynolds numbers is at the inlet section, where velocities and static pressures are lowest. In order to avoid large errors in the measurement of mass flow, a new pitot probe was designed. This tube, shown in Figure 3.4 incorporates a relatively large impact hole and a sharp inlet lip to lessen bluff body effects. In addition, the inlet of the probe is located five tube diameters away from the stem to lessen the effect of the latter's flow distortion. These features will reduce the design-point errors on mass flow below 1% (by producing Re>50). It is expected, however, that off-design measurements, such as operation of the rig at 42% speed, result in lower Re and additional viscous errors which will be corrected for using curve ‘C’ of Figure 3.3a. The data points that were used to draw curve ‘C’ in Fig. 3.3a are shown in Figure 3.3b (Type C probe). The level of scatter in this plot is used to indicate the uncertainty in the correction factor. From Table 3.3, the Kiel probe downstream of the diffuser may also suffer the effects of low Reynolds number under some experimental conditions and measurements there are corrected according to curve ‘E’ of Figure 3.3a.

### 3.4.3 Effects of Leaks and Outgassing

The measurements of total and static pressures under low pressure conditions are complicated by the phenomenon of outgassing from the walls of the pressure system. This is manifested as pressure measurements that are higher than the actual values. Outgassing is defined as a combination of the steady evolution of gasses from a surface exposed to low pressures, and the diffusion of air through the material. Outgassing of elastomers are generally hundreds of times larger than for metals [7], so it is reasonable to assume that all of the gas loads arise from the plastic tubing used in the pressure line system (assuming no leaks). These effects are especially important in the measurement of mass flows at the inlet, where the dynamic pressure head is only on the order of 0.05 torr, corresponding to
velocities of about 25m/s under 10 torr conditions. The effect is also important in the measurement of total pressures because of the relatively small size of the impact hole when compared to static pressure ports, which lead to increased blockage.

To assess the importance of outgassing on the measurement of mass flow as a function of rig pressure, the steady state pressure difference across the inlet total and static ports, with no flow in the rig, was measured with the rig at 5, 10, 15, 20, 25 and 30 torr. The results are shown in Figure 3.5. This plot indicates that as the rig pressure decreased, the error in mass flow due to outgassing effects increased rapidly, for two reasons. First, as Reynolds numbers decreased and the effects of viscosity became more important, larger pressure gradients in the tubing were required to pass the gas loads due to outgassing. Secondly, as the rig pressure decreased, the magnitude of the dynamic pressure also decreased proportionately. This point may be explained by using the data in Fig. 3.5 as an example. If a 100% design Reynolds number (10.1 torr inlet pressure) experiment was run, and the pressure head in the inlet was around 0.05 torr, then the dynamic pressure error due to outgassing would be 0.002 torr. This 4% error in the pressure head would lead to a 2% error in mass flow. On the other hand, if the rig was run at only 5 torr, corresponding to 50% Reynolds number, the pressure head would be only around 0.025 torr (assuming similar average flow velocity), while the head error due to outgassing would increase to 0.0045 torr. This 18% error in head would lead to almost 10% error in the mass flow calculated.

It should be noted that the only other measurement to suffer appreciably from the effects of outgassing was the velocity downstream of the impeller. Here outgassing became important because total pressure measurements were taken with the center impact hole of the cobra probe, a device with a very severe flow restriction. In this case, the total pressure head was on the order of 3.25 torr during an experiment with inlet pressure set to 10 torr, while outgassing offsets amount to 0.09 torr. This 3% error in pressure head result in a 1.5% error in the velocity measured downstream of the impeller.

The effect of outgassing on measurements taken from static pressure taps was negligible because of the low flow restriction associated with these devices. Similarly, outgassing was not an important source of error on angle measurements made with the cobra probe, since the level of outgassing in either arm of the device was balanced.

It is obvious from the above example that outgassing poses an important problem in the measurement of mass flow at the inlet and velocity downstream of the impeller. To try to better understand this effect, a tubing outgassing experiment was conducted. The experimental setup is shown in Figure 3.6. A length of tubing was connected between an absolute pressure transducer and a needle valve that was in turn connected to a vacuum

41
pump. At the start of a test, the valve was opened and the tubing and transducer were exposed to pressures <0.1 torr for three hours to remove any volatile substances such as water. The valve was then closed and the time taken for the pressure in the transducer to increase from 1.0 to 1.5 torr was recorded. This test was performed for several lengths of 0.16 cm ID Teflon tubing as well as several other lengths and thicknesses of polyethylene tubing. The experimental results of this test for lengths of Teflon are shown in Figure 3.7. Figure 3.8 shows these results in units of torr.L/sec.cm², a common unit for outgassing comparisons. Figure 3.8 also includes results from published literature [8, 9]. The outgassing rates of various lengths of 0.16 cm Teflon tubing all fall on a straight line lying between values of “Teflon as received” and “Teflon degassed” as reported in the literature. The literature also indicates that polyethylene should produce outgassing rates nearly an order of magnitude lower than that of Teflon. This result was matched or slightly improved upon by a test of 0.64 cm ID polyethylene tubing.

To test the benefits of polyethylene over Teflon, the Teflon tubing in the pressure system was replaced by a combination of 0.16 cm and 0.32 cm polyethylene. Although the performance seemed at first very promising, it was later found that the 0.16 cm polyethylene tubes developed cracks near the connection tubulations after being exposed to vacuum for a few hours. The reason for this behavior is not know, however the problem was intolerable because around 50% of the connection were found to have developed the cracks over a 12 hour period. As a compromise, the 0.16 cm tubing after the valves was left as Teflon, while some of the longer sections of 0.32 cm tubing running from the probes to the valves was switched to a thicker walled polyethylene line.

The only up-side of these outgassing errors is that they are reproducible, and may therefore be tabulated as a function of pressure and used to correct the experimental results. This calibration was performed by opening pairs of valves of interest with the stationary rig held at several pressures. When a static and total probe were valved open to the transducer, there was a steady state pressure differential. The differential pressures for pairs of interest were recorded before and after an experiment. An example of such a calibration table is shown in Table 3.4. This data is then used by subtracting the differentials from the corresponding experimentally measured total pressure.
<table>
<thead>
<tr>
<th>Valves</th>
<th>Rig p.</th>
<th>torr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>[1, 6]</td>
<td>0.0169</td>
<td>0.0101</td>
</tr>
<tr>
<td>[2, 7]</td>
<td>0.1547</td>
<td>0.0830</td>
</tr>
<tr>
<td>[4.5, 7]</td>
<td>0.0638</td>
<td>0.0338</td>
</tr>
<tr>
<td>[4, 9]</td>
<td>0.0004</td>
<td>0.0001</td>
</tr>
<tr>
<td>[5, 9]</td>
<td>0.0001</td>
<td>-0.0003</td>
</tr>
</tbody>
</table>

Table 3.4: Stationary rig pressure errors (in torr) due to line outgassing as a function of pressure.

From the table it is seen that the inlet static vs. plenum static measurement, [5, 9] was virtually unaffected by outgassing. This is because both arms were attached to static ports with relatively large holes. On the other hand, the measurement of impeller exit total vs. impeller exit static pressure [2, 7], exhibited a large pressure error due to outgassing. Again, this is due to a large difference in size of the impact holes used on either arm, which produced a pressure imbalance. It should be noted that the data listed in the table is an example only, with the values showing a variation of about 20% from experiment to experiment.

The importance of outgassing on the inlet pressure measurements that are indicated by Table 3.4 is significantly less than that shown in Figure 3.5. This improvement was due to two effects. The first was due to the reduction of outgassing by the replacement of 0.32 cm Teflon with polyethylene tubing. The second benefit came as a result of changing the original inlet total probe for a newly designed one that made use of a much larger impact hole, thereby reducing the flow restriction for the device. The net effect is that the importance of outgassing on measurements of total and static pressures at the inlet [4, 9], once a major source of concern, is now of the same order as the resolution of the differential pressure transducer (0.0001 torr).

3.5 Initial Arrangement of the Thermocouples

There were two series of experiments performed over the course of this project. The first series was carried out with the nominal vaned diffuser installed, while the second
was done with a simple vaneless space after the impeller. During the first series of experiments, there were four K-type thermocouples located throughout the facility. These were at the inlet duct, at the exit of the diffuser, and two downstream of the diffuser. Data from each of these thermocouples was acquired continuously throughout the experiment at the rate of about once every two seconds and saved to disk. The readings were acquired by a 20 channel Fluke Helios I Computer Front End.

The inlet thermocouple, shown schematically in Figure 3.9, was used to measure the total temperature of the flow entering the compressor. It was located in the inlet duct about 1 cm downstream of the location of the inlet pitot and static pressure probes. It consisted of a 0.305 cm dia. hollow stainless steel stem with a thermocouple junction located at its endpoint. Although the chromel and alumel internal wires of the thermocouple were insulated from the wall of the stem, the bead was in thermal contact with the end of the stem through a drop of epoxy, used to seal the device. The thermocouple was installed with its axis at right angles to the axis of the duct, and protruded into the duct so that the thermocouple junction was located at the centerline of the pipe. In this way, the stem of the inlet thermocouple was exposed to one duct radius (8.1 cm) of flow. This thermocouple was later found to be subject to large errors (discussed in section 3.6.1) and was replaced by another design for the second series of tests with the vaneless diffuser.

The thermocouple located at the exit of the vaned diffuser, at a radius of 0.425 m from the axis of the impeller, was intended to measure the air total temperature at this location. This thermocouple, which was designed and built specifically for this application, is pictured in Figure 3.10. It was installed with its impact hole centered in the flow path. The instrument was initially aligned with the flow by rotating the probe until its temperature reading was maximized, while the compressor was in operation. The performance of this thermocouple will be considered in Section 3.6.2.

There was an additional thermocouple installed at the exit of the diffuser. The stem of this thermocouple extended across the flow path and its junction was in contact with the metal surface of the diffuser. This device was intended to monitor the temperature of the diffuser throughout an experiment. It was found, however, that convective heat transfer through its stem resulted in erroneous readings. An improved configuration for measuring the temperature of the rig internal structure was devised and implemented in the second series of experiments.

The final thermocouple was installed further downstream of the diffuser, at a radius of 0.705 m. It was of similar construction to the one used in the inlet and its junction was centered in the compressor flow path. It was intended to measure the air total temperature at this location, but due to the combined effects of heat loss into the rig and probe conduction,
the information provided by this probe was of little value, and it was removed for the second series of experiments.

3.6 Analysis and Redesign of the Initially Installed Thermocouples

This section will discuss the performance of the thermocouples that were used during the first series of experimental tests with the vaned diffuser installed. After evaluating the causes of their shortcomings, the improved designs that were used in the second series of experiments are presented and analyzed.

3.6.1 Inlet Thermocouple

Figure 3.11 shows the evolution of the inlet thermocouple reading during the vaned diffuser run. This graph has been annotated to show the correspondence of the features in the plot with events in the experiment. Some general observations may be made about the results shown in Figure 3.11.

1) The inlet thermocouple measurement, \( T_j \) varied considerable (20° C) over the course of the experiment.

2) \( T_j \) decreased as the Reynolds number increased, where Re depended on the inlet pressure and the flow velocity (which increased with impeller speed and throttle setting).

3) The highest temperatures were observed late in the experiment with the rig at vacuum. Under these conditions, the convective heat transfer into the thermocouple was very low.

4) The peak temperatures reached by the inlet thermocouple were above the maximum measured wall temperature of 29° C at the base of the thermocouple.

5) The lowest \( T_j \) occurred during the termination of the experiment, when the rig was vented to atmospheric pressure, and the convective heat transfer into the thermocouple was the highest.

The air temperature entering the compressor is an important parameter in the determination of its performance. As such, it is important to determine if the observed thermocouple readings were indicating the true air temperature. In fact, the variations in temperature result from a poor thermocouple design, as described in the following sections.
Heat Exchanger Effectiveness

As was described in section (2.3), the purpose of the heat exchanger was to cool the compressor exhaust before it re-entered the inlet. The following qualitative analysis will attempt to determine if a variation of heat exchanger effectiveness may have been the cause of the observed variations in inlet thermocouple measurements.

A possible cause of variation in heat exchanger effectiveness is that the convective heat transfer from the air to the water-cooled passage walls in the device decreased as Re decreased with density and velocity. This would be in agreement with the observed dependence of temperature with Re. However, this effect would be offset by a lower mass flow, which, for a given hydraulic diameter and viscosity, is directly proportional to the Reynolds number. The relative importance of these two competing effects is evaluated by considering the variation of Nusselt number with Reynolds number for internal flow through tubes and ducts. It is shown by Kreith [11] that

$$Nu = \frac{hD}{k} \alpha Re^{0.3}, \quad \text{(laminar flow)}$$

$$Nu = \frac{hD}{k} \alpha Re^{0.8}, \quad \text{(turbulent flow)}$$

where

- $h$=convective heat transfer coefficient,
- $D$=hydraulic diameter,
- $k$=thermal conductivity of air.

Either of these equations indicates that the Reynolds number (which is proportional to mass flow rate) should increase more quickly than the heat transfer coefficient. This indicates that the heat exchanger effectiveness in cooling the air should be improved as the Reynolds number is decreased. This result is contrary to the experimental observations, indicating that the heat exchanger could not be the cause of the observed trends in thermocouple reading with inlet Reynolds number.

Conduction Errors

One potential source of error in the measurement of inlet temperature is steady-state conduction through the stem, leading to a temperature difference between the thermocouple junction and the surrounding air. The importance of this effect will be evaluated in the following analysis.
A schematic of the inlet thermocouple is shown in Figure 3.9. The average heat transfer coefficient for an infinite cylinder in a cross flow is given empirically by Lienhard [10] (Valid for Re×Pr>0.2):

\[ h = \frac{k}{D} \left( 0.3 + \frac{0.62 \text{Re}^{1/2} \text{Pr}^{1/3}}{1 + (0.4 / \text{Pr})^{2/3}} \left[ 1 + \left( \frac{\text{Re}}{282000} \right)^{5/8} \right]^{4/5} \right) \quad (3.3) \]

where, for our conditions

\[ \text{Re} = \frac{D \rho V}{\mu} = \frac{3.05 \times 10^{-3} \times 0.0155 \times 20}{1.85 \times 10^{-5}} = 51, \]

\[ k = 26 \times 10^{-3} \text{W} / \text{m} \cdot \text{K}, \]

\[ \text{Pr} = 0.7, \]

resulting in

\[ h = 32.1 \ \text{W/m}^2\cdot\text{K}. \]

The temperature distribution along the thermocouple stem is controlled by the 1D convection, conduction equation [10]

\[ \frac{d^2 T}{dx^2} - \frac{hP}{kA} (T - T_w) = 0, \quad (3.4) \]

where

- \( P \)=perimeter of thermocouple stem,
- \( A \)=metal cross section area of thermocouple stem,
- \( k \)=thermal conductivity of thermocouple stem,

The solution to (3.4) may be expressed in the form of a dimensionless excess temperature [10]

\[ \Delta T_{ex}(x) = \frac{T_x - T_w}{T_{wall} - T_w} = \frac{\cosh \left[ mL \left( 1 - \frac{x}{L} \right) \right]}{\cosh[mL]}, \quad (3.5) \]

where

\[ m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{32.1 \times \pi \times 3.05 \times 10^{-3}}{14 \times (\pi \times (3.05 \times 10^{-3} - 2.29 \times 10^{-3})/4)}}, \]

47
L=total length of thermocouple stem,
\( T_a \)=free stream static temperature,
\( T_{wall} \)= temperature at the base of the thermocouple stem,
\( T_x \)=temperature of thermocouple stem at a distance of \( x \) from the base.

Equation (3.5) is plotted for \( x \) varying from 0 to 0.081\( \text{m} \) (which is the location of the junction) in Figure 3.12. This figure shows that the excess temperature at the thermocouple junction is 0.0037. Using this value, and supposing a 20\(^\circ\) C temperature difference between the wall and the air, the error due to conduction will be 0.0037\( \times \)20=0.07\(^\circ\) C. This indicates that measurements of air inlet temperature were not strongly affected by conduction through the stem.

**Radiation Effects**

Errors due to radiation may result if the temperature of the thermocouple differs significantly from its solid surroundings. The temperature difference will cause a net transfer of radiant energy between the thermocouple and its surrounding walls. The magnitude of the temperature measurement error due to radiation depends on the relative importance of convective and radiative heat transfer to the thermocouple. For a situation in which a thermocouple is completely surrounding by an emissive wall (in which case the walls will behave as a black body source), the following energy balance by Kreith [11] will apply:

\[
h(T_a - T_j) = \varepsilon\sigma(T_j^4 - T_{wall}^4),
\]

where

\( h \)=convective heat transfer coefficient between the thermocouple and the air,
\( T_a \)=true gas temperature,
\( T_j \)=thermocouple junction temperature,
\( T_{wall} \)=average temperature of the surrounding walls,
\( \varepsilon \)=emissivity of the thermocouple material,
\( \sigma \)=Stefan-Boltzmann constant= \( 5.669 \times 10^{-8} \text{W/m}^2\cdot\text{K}^4 \).

To determine the importance of radiation on the temperature measured at the inlet, equation (3.6) has been evaluated and compared to experimental observations in Table 3.5. For the purpose of this calculation the following values have been assumed:
h = value predicted by Eq. (3.3),
ε = emissivity of weathered stainless steel = 0.85 [11],
T_w = 320 K = measured front plate temperature
T_j = values taken from experimental observations

The observed temperature difference will be taken to be the difference between T_j and the lowest temperature measured over the course of the experiment (16°C), which occurred during venting of the rig at the end of the experiment. During this venting operation, radiation error was relatively small due to higher values of h associated with higher air pressure. It is assumed that the true air temperature was independent of inlet pressure.

<table>
<thead>
<tr>
<th>Inlet pressure, torr</th>
<th>h (W/m².K)</th>
<th>ΔT (observed), °C</th>
<th>ΔT (predicted), °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>33.0</td>
<td>5.0</td>
<td>4.9</td>
</tr>
<tr>
<td>20.1</td>
<td>45.7</td>
<td>2.8</td>
<td>3.2</td>
</tr>
<tr>
<td>30.0</td>
<td>55.5</td>
<td>2.2</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table 3.5: Observed variations and predicted inlet thermocouple error due to radiation

Table 3.5 shows that the relatively small values of h cause appreciable errors due to radiation at the inlet. In addition, the assumption of T_w = 47°C (320 K) results in predicted errors that are consistent with the observed variations. The radiation hypothesis can also explain the observation that during rig evacuation near the end of the experiment, the inlet thermocouple read hotter than the local wall temperature. With the inlet thermocouple installed, its length near the junction was exposed to thermal radiation emitted by the center portion of the impeller. Although the temperature attained by the impeller is not known, it is expected to be significantly higher than the temperature at the front plate, which was measured to be 47°C (because the front plate is convectively cooled by the outside air). The influence of the impeller is interesting, but unfortunate because it makes correcting for the effects of radiation difficult.

Inlet Thermocouple redesign

In an attempt to lessen the errors in the inlet air temperature observed in the first series of experiments, two new thermocouples were designed and installed. A vented Kiel head design identical to the "short probe" described in section 3.6.3 replaced the simple cylindrical unshielded thermocouple.
An additional thermocouple was also installed at the exit of the heat exchanger where the influence of the hot impeller is not as great. This new thermocouple, which is pictured in Figure 3.13, consists of a two inch length of uninsulated thermocouple wire pair with a circular piece of copper mesh soldered to the junction. The mesh is intended to maximize the convective heat transfer between the air and the junction.

The measurements that were acquired by these two thermocouple designs during the 100% impeller speed, vaneless diffuser run are shown in Figure 3.14. The Kiel head thermocouple at the inlet consists of three separate thermocouples: a primary junction to measure the fluid temperature, a junction embedded in the head of the device to measure the effective shroud temperature, and a junction embedded in the stem about 0.8 cm below the shroud to measure the effective wall temperature. The plot of temperature vs. time shows the same trends encountered with the previous thermocouple design. For the case of the Kiel head design, the reading of all three thermocouples coincide closely with each other, indicating that the shroud provided very little shielding benefit for the primary junction. This was due to the low Reynolds number flow, which may have caused the primary thermocouple junction to be within the thermal boundary layer of the shroud. Low Reynolds numbers also resulted in low convective heat transfer into the primary thermocouple leads, which led to large conduction errors. These issues will be considered in depth in section 3.6.2.

In the case of the thermocouple installed at the exit of the heat exchanger, the variations in temperature reading with Reynolds number were much less pronounced than that encountered at the impeller inlet. This occurred because the internal wall temperatures were closer to air temperatures than was the case at the inlet, leading to smaller radiation errors. For use in the calculation of compressor performance parameters, the inlet air temperature will be taken to be the minimum indicated by the heat exchanger exit thermocouple, plus an uncertainty to account for the variability of the reading.

3.6.2 Diffuser Exit Thermocouple

The thermocouple used to measure the total temperature at the exit of the diffuser during the first series of experiments is shown in Figure 3.9. The temperature readings of this instrument over the course of the vaned diffuser run that was discussed in section 3.6.1, is plotted in Figure 3.15. This plot shows features similar to those described for the inlet thermocouple in section 3.6.1. In this case, however, the temperature errors were of opposite sign because the surroundings were cooler than the airflow being measured. In particular, it was observed that higher flow Reynolds numbers, which are associated with higher heat transfer rates, led to significant increases in the temperature measured by the
thermocouple. The specific sources of error affecting the diffuser exit thermocouple are examined below.

**Conduction through the thermocouple stem**

An analysis similar to that performed for the inlet thermocouple in section 3.6.1 may be applied for the case of the diffuser exit thermocouple. The geometry and typical flow conditions for this device under 100% operating conditions are listed Table 3.6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of stem exposed to flow</td>
<td>0.76 cm</td>
</tr>
<tr>
<td>External diameter of shroud</td>
<td>0.30 cm</td>
</tr>
<tr>
<td>Internal diameter of shroud</td>
<td>0.23 cm</td>
</tr>
<tr>
<td>Thermal conductivity of stainless steel</td>
<td>14 W-m/k</td>
</tr>
<tr>
<td>Flow velocity external to thermocouple</td>
<td>56 m/s</td>
</tr>
<tr>
<td>Heat transfer coeff into stem (Eq 3.2)</td>
<td>62 W/(m²-K)</td>
</tr>
</tbody>
</table>

**Table 3.6**: Geometry and design flow conditions for the diffuser exit thermocouple.

These factors may be applied to Eq. 3.5 to indicate the effectiveness of the stem in thermally isolating the shroud from the wall:

\[
\frac{T_{\text{shroud}} - T_\infty}{T_{\text{wall}} - T_\infty} = 0.71,
\]

where

- \(T_{\text{shroud}}\) = temperature of the shroud enclosing the thermocouple bead.
- \(T_{\text{wall}}\) = temperature at the wall (assumed close to the base temperature of the thermocouple).

This calculation shows that for the initial design of the diffuser exit thermocouple, the stem was relatively ineffective in isolating the shroud of the probe from the cooler rig structure.

**Entrance length of the thermocouple shroud**

A potential source of error in the measurement of the diffuser exit total temperature was heat transfer from the air entering the shroud to the cooler structure of the shroud. To evaluate the importance of this effect, the thermal entry length of the shroud may be computed from the equation by Fox [4] (assuming Pr=1)
\[
\frac{L}{D} = 0.06 \frac{\rho U_1 D}{\mu},
\]  

(3.7)

where

\(D\) = internal diameter of shroud,

\(U_1\) = shroud internal flow velocity.

The ideal flow rate through the vent holes of the shroud assuming the conservation of total pressure\(^3\) may be related to the free stream conditions using the conservation of mass:

\[
m = \rho_1 U_1 A_{\text{inlet}} = \rho_{\text{vent}} U_{\text{vent}} A_{\text{vent}} = \rho_{\infty} U_{\infty} A_{\text{vent}},
\]

(3.8)

where subscript 1 refers to conditions inside of the shroud. Therefore the velocity within the probe is

\[
U_1 = C_D U_{\infty} \left(\frac{A_{\text{vent}}}{A_{\text{inlet}}}\right),
\]

(3.9)

where

\(A_{\text{vent}}\) = area of vent holes in shroud,

\(A_{\text{inlet}}\) = inlet area of the shroud,

\(C_D\) = flow coefficient through the shroud = 0.6 [15].

Although each of the two vent holes were designed to have a diameter of 0.061 cm, it was later discovered that they were about 2/3 blocked with ceramic when they were constructed. This means that the flow velocity within the shroud was only about 1/3 of that intended in the design. Evaluating (3.9) using a measured free stream velocity of 50m/s gives \(U_1 = 2.2\) m/s. Now evaluating (3.7) with an assumed gas temperature of 400K, it is found that flow within the shroud becomes thermally fully developed when

\[
\frac{L}{D} = 0.3.
\]

This is before the thermocouple bead location of 0.5D, indicating that the junction was well within the thermal boundary layer of shroud and may have been seriously underestimating the gas temperature.

3.6.3 Diffuser Exit Thermocouple Redesign

The lessons learned from the analysis and experimental results described above indicate some measures that should help to lessen the magnitude of errors in the

\(^3\) Meaning that the velocity and density exiting the vent holes is the same as their free stream values

52
measurement of impeller discharge temperature during subsequent experiments. These include

1) Thermally isolating the shroud from the wall of the flow path, this will
   - provide radiation shielding to the junction,
   - lessen the concerns of thermal boundary layers inside the shroud,
   - lessen conduction through the exposed thermocouple leads,

2) Increasing the flow velocity through the shroud, this will
   - increase the heat transfer to the bead and wires,
   - increase the thermal entry length into the shroud.

A replacement probe was designed with these goals in mind, and is pictured in three views in Figure 3.16, and in its installed orientation in Figure 3.17. The horizontal stem, which extends into the flow path, was intended to thermally isolate the shroud from the wall. It does this by increasing the thermal resistance by virtue of its length, and by acting as a heat transfer fin, thereby allowing the shroud to attain a temperature close to that of the free stream. This instrument will be referred to as the ‘long probe’.

To help correct for the large errors observed with the original design, two additional thermocouples were incorporated into the long probe. In addition to the primary junction, which is housed in the shroud, there is a junction embedded in the ceramic base plug of the shroud, which indicates the temperatures of the shroud and the base of the lead wires. There is also a junction embedded in the thermocouple stem, in a location corresponding to the position of the flow path wall. This thermocouple is intended to indicate the effective stem wall temperature.

The geometry of this newly designed thermocouple probe precluded its installation and removal without disassembling the rig. In the chance that a failure of the probe might occur, a second removable probe was designed and installed. This other probe is similar to the first, but lacks the horizontal stem. It is geometrically identical to the original probe used during the first series of experiments, with the exception that the newer design contains larger vent holes (which are completely unblocked). This alternative probe, although lacking the wall-to-shroud temperature isolation provided by the horizontal stem, provides a useful back-to-back comparison of the two designs. This design also contains the two auxiliary thermocouples described above. It will be referred to as the ‘short probe’.

Conduction through the stem

Repeating the calculations performed above for the original design using a measured stem base velocity of 113m/s, the effectiveness of the horizontal stem in isolating
the long probe’s shroud from conductive heat transfer to the wall during 100% design conditions is found to be

\[
\frac{T_{\text{shroud}} - T_w}{T_{\text{wall}} - T_w} = 0.001 \quad (100\% \text{ speed}).
\]

This indicates that the end of the horizontal stem has achieved an excess temperature ratio which is only 0.1% of that existing between the stem base and the air (neglecting radiation). At 42% impeller speed, the velocity at the base of the stem was measured to be about 33 m/s, and the excess temperature is calculated to be 1.6%. These values are a significant improvement over the original design, which was calculated to have an excess temperature ratio of 71%.

**Free stream centerline velocity at shroud location**

An additional benefit of the long probe is that the horizontal arm allows the head of the probe to be positioned closer in to the impeller, reducing the amount of heat loss from the air into the cooler structure of the rig prior to the probe. Reducing the distance from the impeller exit also increased the velocity of flow incident upon the head. This leads to improved heat transfer between the air and the thermocouple junction inside of the shroud.

The free stream velocity at the location of the long probe’s shroud may be estimated from the conservation of mass and angular momentum at the impeller exit. With the assumption that the static temperature remained constant between the impeller exit and the thermocouple shroud location, conservation of mass gives

\[
R_i U_i \cos \theta_i = R_s U_s \cos \theta_s, \tag{3.10}
\]

where

- \(R_i\) = radius at which impeller exit velocity is measured,
- \(U_i\) = impeller exit centerline velocity,
- \(\theta_i\) = impeller exit centerline flow angle.

Subscript \(S\) represents the corresponding values at the thermocouple shroud location. The conservation of angular momentum gives

\[
R_i U_i \sin \theta_i = R_s U_s \sin \theta_s. \tag{3.11}
\]

The measured numerical values of terms in these equations are given in Table 3.7 for both 42% and 100% impeller speed.
<table>
<thead>
<tr>
<th></th>
<th>42% speed</th>
<th>100% speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$, m</td>
<td>0.155</td>
<td>0.155</td>
</tr>
<tr>
<td>$R_s$, in</td>
<td>0.171</td>
<td>0.171</td>
</tr>
<tr>
<td>$U_1$, m/s</td>
<td>100</td>
<td>231</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>86°</td>
<td>84°</td>
</tr>
</tbody>
</table>

Table 3.7: Location and flow geometry of the long probe's shroud

Solving the conservation of mass and angular momentum equations for the velocity at the location of the thermocouple shroud gives

\[
\begin{align*}
U_{long, 42\%} &= 90.5 \text{ m/s}, \quad \theta_5 = 86^\circ \quad 42\% \text{ speed}, \quad \text{long probe}, \\
U_{long, 100\%} &= 209 \text{ m/s}, \quad \theta_5 = 84^\circ \quad 100\% \text{ speed}, \quad \text{long probe}.
\end{align*}
\]

It should be noted however, that (3.11) will not hold exactly due to the viscous shear work done at the walls of the flow path, which would lead to a reduction in the flow angle, $\theta_5$. It is also expected that the velocity at $R_s$ would have been very sensitive to this effect because of the large flow angles involved (because $\frac{d}{d\theta}\cos\theta$ is large for $\theta = 90^\circ$ in the conservation of mass equation).

A more conservative way of calculating the free stream velocity at the location of the thermocouple shroud is to apply conservation of mass between the impeller exit and the base of the thermocouple stem to calculate the flow angle at the latter. The flow angle at the shroud is then found by linearly interpolating between the two known flow angles. Substituting this angle into Eq. 3.10 gives

\[
\begin{align*}
U_{long, 42\%} &= 75 \text{ m/s}, \quad \theta_5 = 85.2^\circ \quad 42\% \text{ speed}, \quad \text{vaneless diffuser}, \\
U_{long, 100\%} &= 184 \text{ m/s}, \quad \theta_5 = 83.2^\circ 100\% \text{ speed}, \quad \text{vaneless diffuser}.
\end{align*}
\]

The extreme sensitivity of flow velocity to the assumed flow angle demonstrates the difficulty in evaluating the performance of the impeller exit thermocouple. For this reason, the calculations performed below for the long probe can only be expected to give approximate results.

At the short probe location, the free stream velocity was measured directly with a Kiel head probe and a static pressure tap:
$U_{\text{short,42\%}} = 33 \text{ m/s}$  
$U_{\text{short,100\%}} = 113 \text{ m/s}$

These results are used in the following section.

**Entrance length of the thermocouple shroud**

The new probe shroud has vent holes that are 2.25 times larger than that of the original design. However, an error during its fabrication resulted in the vents being about 1/2 blocked. Repeating the calculation that was performed above for the original probe, the entrance lengths for the long probe under 42% and 100% speed conditions are approximately

$U_i = 10 \text{ m/s}$  
$U_i = 30 \text{ m/s}$

(42% speed),

(100% speed).

The corresponding entrance lengths are calculated to be

$(L/D) = 1$  
$(L/D) = 4.3$  

(42% speed),

(100% speed).

For a junction position of $(L/D) = 0.5$, it is expected that temperature measurements are minimally affected by boundary layer growth on the inside surface of the shroud, in the case of 100% impeller speed operation. For operation at 42% speed, some boundary layer interference with the junction may be expected.

In the case of the short probe, the vent holes were observed to be completely free of obstruction and its internal shroud velocity for 100% and 42% impeller speeds are calculated to be

$U_i = 10 \text{ m/s}$  
$U_i = 35 \text{ m/s}$

(42% speed),

(100% speed).

These values indicate that the short probe has approximately the same entrance lengths, for each impeller speed, as for the long probe. The junction position for the short probe is also at $L/D = 0.5$.

**Calculation of the heat transfer coefficient into the exposed lead wires**

The calculation of the heat transfer coefficient for the primary junction wires is complicated by rarefaction effects which arise because the mean free path of the air under operating conditions is an appreciable fraction of the wire diameter (0.0127 mm). At 100%
impeller speed and design pressure ratio, corresponding to $p=20$ torr and $T=400K$ at the exit of the impeller, the Knudsen number is calculated from (3.1) and (3.2) to be $Kn=0.3$, indicating that flow was not a continuum. The effects of large Knudsen numbers on the convective heat transfer for fine thermocouple wires was determined experimentally by Kuz'menko and Merkulov [12]. It was found that for $0.005<Re<0.5$ and $Kn>0.006$, results are described by the following empirical equations:

$$Nu = \frac{Nu_0}{1 + 0.017(\rho_0/\rho)(d_0/d)^{2.5}}, \quad (3.10)$$

$$Nu_0 = 1.48Re^{0.33} Pr,$$

where

$$Re = \frac{Upd}{\mu},$$

$$\rho_0 = 0.16kg/m^3,$$

$$d_0 = 50\mu m.$$

The numerator of (3.10) is the Nusselt number that applies when the continuum assumption holds ($Kn<0.006$). The denominator is a correction factor that takes into account the effects of larger $Kn$. For typical 100% speed experimental conditions, the correction term reduces the continuum based heat transfer coefficient by a factor of about five. Equation (3.10) is evaluated in Table 3.8 for the conditions at both 100% and 42% impeller speeds:

<table>
<thead>
<tr>
<th></th>
<th>42% impeller speed</th>
<th>100% impeller speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$ (m/s)</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>0.0218</td>
<td>0.0470</td>
</tr>
<tr>
<td>$d$ (m)</td>
<td>12.7 $\mu m$</td>
<td>12.7 $\mu m$</td>
</tr>
<tr>
<td>$T$ (K) (approx.)</td>
<td>325</td>
<td>420</td>
</tr>
<tr>
<td>$h$ (W/m$^2$K)</td>
<td>140</td>
<td>326</td>
</tr>
</tbody>
</table>

Table 3.8: Evaluation of the primary junction wire heat transfer coefficient

These results will be used in the following sections.
Temperature deficit of primary junction due to radiation

From (3.4), the temperature error due to radiation is given by

\[ \Delta T_{\text{rad}} = \frac{\sigma \varepsilon (T_j^4 - T_{\text{shroud}}^4)}{h}, \]

where

\[ \varepsilon = 0.3 \text{ for Alumel/chromel junction [15].} \]

Typical values of \( T_j \) and \( T_{\text{shroud}} \) are taken from measurements to be 405K and 400K respectively, under 100% impeller speed conditions. They result in a calculated radiation error of about 0.07K. Under 42% impeller speed conditions, typical \( T_j \) and \( T_{\text{shroud}} \) were measured to be 323K and 321.5K respectively, corresponding to a calculated radiation error of 0.02K. These predicted radiation errors are negligible compared to the observed variation and it is concluded that the effects of radiation are not important for either the long or short probe.

Temperature error due to conduction through thermocouple leads

The non-dimensional excess temperature due to steady state conduction through the exposed junction lead may be calculated from (3.3)

\[ \Delta T_{\text{ex}} = \frac{T_m - T_j}{(T_m - T_{\text{wall}})} = \cosh\left[ \frac{mL(1 - \frac{l}{L})}{\cosh[mL]} \right], \]

where \( m \) was defined previously and \( l = 0.140 \text{ cm} \) is the length of thermocouple wire from the junction to the ceramic post. Using \( k = 29.7 \text{ W/m-K} \) for Alumel wire [15], and the values of \( h \) computed above, the non-dimensional excess temperatures at the junction are calculated to be

\[ \Delta T_{\text{ex}} = 0.35 \quad (42\% \text{ speed}), \]
\[ \Delta T_{\text{ex}} = 0.15 \quad (100\% \text{ speed}). \]

These results demonstrate that the wire leads performed poorly at thermally isolating the primary junction from the shroud. This suggests that it is important to apply a correction factor to take into account the heat loss through the lead wires. The problem with using the
above excess temperature results directly is that the values of \( h \) used in their calculation apply only to the ideal situation of cross flow over infinite cylinders. In the true situation, the wires are angled with respect to the flow, and the flow itself is complicated by the presence of boundary layers on the shroud wall and cross flow due to presence of the vent holes.

One method of predicting a correction factor that avoids having to compute an accurate value of \( h \) is to compare the measurements produced by the long and short probes. If it is assumed that both probes have the same wire geometry, are exposed to the same air temperature and pressure, and as determined above, have roughly the same flow velocity over their primary junctions, then it is expected that \( h \) is nearly the same for both instruments. If it is also assumed that the junctions are not within the boundary layer of the shroud walls (true under 100% impeller speed conditions), then the excess temperature ratio should be the same for both devices, and the following equation will apply,

\[
\frac{T_w - T_{j,\text{long}}}{T_w - T_{\text{shroud, long}}} = \frac{T_w - T_{j,\text{short}}}{T_w - T_{\text{shroud, short}}}.
\]

Solving this equation for the true gas temperature:

\[
T_w = \frac{T_{j,\text{long}}T_{\text{shroud, short}} - T_{j,\text{short}}T_{\text{shroud, long}}}{T_{j,\text{long}} - T_{j,\text{short}} - T_{\text{shroud, long}} + T_{\text{shroud, short}}}. \tag{3.11}
\]

It should be noted that (3.11) is applicable only for 100% impeller speed conditions, with the vaneless diffuser installed.

The measurements of the long and short probe during the 100% speed, vaneless run are compared in Figure 3.18. To apply the correction developed above, the bead (primary junction) and shroud temperatures were taken from the data at a time corresponding to steady 100% design speed conditions, which occurred at about 225 minutes into the experiment:

\[
\begin{align*}
T_{j,\text{long}} &= 405 \text{ K}, \\
T_{j,\text{short}} &= 380 \text{ K}, \\
T_{\text{shroud, long}} &= 399 \text{ K}, \\
T_{\text{shroud, short}} &= 369 \text{ K}.
\end{align*}
\]

Substituting these values into (3.11), the conduction corrected air temperature is predicted to be
\[ T_m = 435.8 \text{ K (162.8° C)} \]

It should be noted that the value predicted above may be an overestimate because the correction method does not take into account the drop in air temperature between the locations of the long and short probe, due to heat loss into the rig walls.

### 3.6.4 Rig Structure Temperature Acquisition

To aid in the understanding of the temperature variations observed during the first series of experiments, several K-type thermocouples were embedded in the structure throughout the rig.

The first of these thermocouples consists of a copper washer to which a thermocouple junction is soldered. It is fastened to the external portion of the inlet duct, situated to indicate the temperature at the base of the shrouded inlet thermocouple.

To monitor the temperature of the impeller shroud plate, which is the 0.3 m diameter, 1.3 cm thick stainless steel plate covering the impeller, a thermocouple junction is incorporated into a bolt used to fasten the shroud in place. To indicate the temperature gradient between the shroud and the outside of the rig, a washer-type thermocouple is fastened to the front plate in the same radial location as the bolt. A similar combination of thermocouple bolt and washer is used to monitor the temperature at the center of the shroud plate opposite the vaneless diffuser, and at the corresponding location on the front plate.

Two setscrews fitted with thermocouple junctions are embedded in the walls at either side of the flow path at a distance of 2.5 cm radially out from the exit of the impeller rim. These thermocouples are intended to monitor the wall temperature in the vicinity of the head of the impeller discharge thermocouple, and to provided information about the magnitude of radiation errors there.
Figure 3.1: Schematic of instrumentation locations
Figure 3.2: Pressure line system

Figure 3.3a: Viscous effects on pitot probes [6]
Figure 3.3b: Viscous effects on pitot, experimental results by Sherman [13].

Figure 3.4: Pitot Probe designed for use in inlet.
Figure 3.5: Error in inlet pressure head due to the effects of tubing outgassing

Figure 3.6: Experimental setup to investigate the importance of outgassing
**Figure 3.7**: Results of outgassing experiment for 0.16 cm dia. Teflon.

**Figure 3.8**: Outgassing rates from experiment and literature compared

A), B) ‘Hand Book of High Vacuum Engineering’ [8]

C), D) ‘Vacuum Engineering’ [9]
Figure 3.9: Schematic of the inlet thermocouple in cross flow.

Figure 3.10: Initial design of the diffuser exit total temperature probe (used with vaned diffuser tests)
Figure 3.11: Inlet thermocouple reading as a function of experiment time during the vaned diffuser run.

Figure 3.12: Inlet thermocouple excess temperature distribution, during 100% impeller speed operation.
Figure 3.13: Thermocouple installed at the exit of the heat exchanger

Figure 3.14: Measurements of the inlet thermocouples during the 100% speed vaneless diffuser run. ‘Inlet Kiel’ consists of three separate measurements.
Figure 3.15: Diffuser exit thermocouple measurement during the 100% speed vaned diffuser experiment

Figure 3.16: New design thermocouple for impeller exit discharge temperature measurement 'Long Probe'
Figure 3.17: ‘Long Probe’ impeller exit thermocouple in its installed configuration

Figure 3.18: Measurements taken with the long (L) and short (S) impeller exit thermocouples
Chapter 4

Performance Parameters and Error Analysis

This chapter describes the individual measurements made during an experiment to determine the compressor performance parameters; mass flow, pressure ratio and efficiency. In addition, an error analysis on the parameters is performed.

4.1 The Measurement of Mass Flow

The compressor mass flow rate was measured at the inlet duct using dynamic pressure measurements. The following section describes relations used in the mass flow calculation, and estimates the expected experimental error.

4.1.1 Mass Flow Relations

The local Mach number is related to the isentropic ratio of total to static pressures by

\[
\frac{p_t}{p} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}},
\]

where \( p_t \) and \( p \) are the total and static pressures respectively, and the Mach number is related to the flow velocity through the speed of sound:

\[
M = \frac{U}{\sqrt{\gamma RT}},
\]

Combining 4.1 and 4.2 gives the flow velocity

\[
U = \sqrt{\left(\frac{\Delta p + p}{p}\right)^{\frac{\gamma - 1}{2}} - 1} \frac{2\sqrt{\gamma RT}}{\gamma - 1},
\]

where

\( \Delta p = p_t - p \) = dynamic pressure.
Static pressures are measured at the wall of the inlet duct and total pressures are surveyed radially along a single axis in the duct. A continuous velocity profile is generated across the inlet using a cubic spline fit, which allows a smooth interpolation of the data. The actual mass flow is estimated by integrating this profile over the area of the duct using

$$
\dot{m}_{act} = \rho \int_0^R 2\pi r U(r) dr, \quad (4.4)
$$

where $R=0.081m$ is the measured radius of the inlet duct, and the density is calculated using the perfect gas law:

$$
\rho = \frac{p}{RT}.
$$

It is assumed that there is no cross flow so that $p$ is independent of $r$. It is also assumed that $T$ is constant across the inlet. Thus $\rho$ is independent of $r$.

**Flow Factor**

The mass flow is dependent upon the shape of the velocity profile in the inlet, which is a function of the centerline Reynolds number

$$
Re_{cent} = \frac{U_{cent} D \rho}{\mu},
$$

where the viscosity of air is related to the static temperature using the expression by Fox [4],

$$
\mu = \frac{1.458 \times 10^{-6} T^{3/2}}{110.4 + T}. \quad (4.5)
$$

The dependence may be expressed using a calibration term called the flow factor (FF), so that

$$
\dot{m}_{act} = FF \cdot \rho \cdot A_{duct} \cdot U_{cent}, \quad (4.6)
$$

where

- $A_{duct}$ = duct cross section area,
- $U_{cent}$ = duct centerline velocity.

The flow factor dependence on centerline Reynolds number is determined by performing a radial dynamic pressure survey at several $Re$ values and then fitting a quadratic curve to the data points. This avoids having to perform a time consuming inlet traverse operation at each throttle setting during an experiment.
In presenting the flow factor relationship, it is useful to normalize the centerline Re by a typical value at 100% impeller speed. At the throttle setting that corresponds to a total-to-static pressure ratio of 2.05 (with the vaned diffuser installed), it was found that

\[ U_{cen} = 21.5 \text{ m/s}, \quad \rho = 0.0167 \text{ kg/m}^3, \quad \mu = 1.8 \times 10^{-5} \text{ N \cdot sec/m}^2 \text{ (Eq 4.5)}, \quad D = 0.162 \text{ m}, \]

so that

\[ \text{Re}_{cen}^{(\text{nominal})} = 3230. \]

Figure 4.1 shows the normalized inlet velocity profiles at 30%, 70%, 100%, and 200% of nominal inlet centerline Re. Figure 4.2 shows the least squares quadratic fit of flow factor vs. normalized Re for several data points. Curves showing the upper and lower bounds on the data (which give a rough indication of the uncertainty) have also been included.

**Corrected Mass Flow**

In order to compare mass flow measurements made at different inlet pressures, temperatures and geometric scales, it is necessary to correct the mass flows to some standard inlet condition and geometry, chosen in this case to be 1/75th of sea level pressure (10.1 torr), 300K and full scale (4mm diameter impeller):

\[
m_{cor} = \left( \frac{1}{75} \right)^2 \frac{m_{act} \sqrt{T_{act}/T_{des}}}{p_{act}/p_{des}}. \quad (4.7)
\]

**4.1.2 Errors In The Measurement of Mass Flow**

This section will present an error analysis for both the actual and corrected mass flow rates. To determine the sensitivity of the experimentally determined mass flow to the individual measurements of temperature and pressure, equations (4.6) and (4.7) are differentiated with respect to \( p, \Delta p, T, D \) and FF. These sensitivities, calculated at design point conditions are shown in Table 4.1.
<table>
<thead>
<tr>
<th>Measured Quantity</th>
<th>$\dot{m}$ (act) change due to +1% change in meas.</th>
<th>$\dot{m}$ (corr) change due to +1% change in meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static pressure, $p$</td>
<td>+0.5%</td>
<td>-0.5%</td>
</tr>
<tr>
<td>Dynamic press., $\Delta p$</td>
<td>+0.5</td>
<td>+0.5</td>
</tr>
<tr>
<td>Total Temperature, $T_i$</td>
<td>-0.6</td>
<td>-0.1</td>
</tr>
<tr>
<td>Inlet Diameter, $D$</td>
<td>+2.0</td>
<td>+2.0</td>
</tr>
<tr>
<td>Flow Factor, $FF$</td>
<td>+1.0</td>
<td>+1.0</td>
</tr>
</tbody>
</table>

Table 4.1: Measurement sensitivities in the calculation of mass flow

These sensitivities are used to compute the uncertainty in mass flow from the estimated measurement errors. Impeller speeds of 100% and 42% of design, and an inlet pressure of 100% of design are considered.

Transducers

Static pressures are measured using absolute transducer P2, which is calibrated using the measurements of P1 to within the resolution of the device (0.01 torr), (see Section 3.1). P1, which is believed to be correctly calibrated within its resolution, is rated to an accuracy within 0.15% of the reading.

At both 100% and 42% impeller speed, the accuracy of the static pressure measurement is limited by the accuracy of P1:

$$\delta p = \pm 0.15\%.$$  

The dynamic pressure $\Delta p$ is measured using the differential transducer DP3, which is rated to an accuracy within 0.15% of the reading. However, the resolution of the instrument is 0.0001 torr, so it is expected that the accuracy of measurements of dynamic pressure start to become limited by resolution to within $\pm 0.00005$ torr when

$$\frac{0.00005}{\Delta p} > 0.15\%.$$ 

This occurs for $\Delta p < 0.033$ torr. With the impeller set at 100% speed, it was found experimentally that centerline dynamic pressures ranged from about 0.02 to 0.05 torr as the throttle setting (which determined the mass flow and inlet velocity) was varied. With the impeller at 42% speed, it was found that dynamic pressures ranged from 0.00067 to 0.007 torr. It is therefore concluded that dynamic pressure measurements taken at 100% speed are mostly accuracy limited, while those taken at 42% speed are resolution limited. In this case the uncertainty in dynamic pressure is given by
\[ \delta \Delta p = \pm \frac{0.00005}{\Delta p} \times 100\% . \]

Using the sensitivities in Table 4.1, the uncertainties in the measurements of mass flow due to the pressure transducers are

\[ \delta \dot{m}_{\text{act}}, \delta \dot{m}_{\text{cor}} = \pm 0.075\% \quad \text{(static pressures)}, \]

\[ \delta \dot{m}_{\text{act}}, \delta \dot{m}_{\text{cor}} = \pm 0.075\% \quad \text{(dynamic pressures, 100\% speed)}, \]

\[ \delta \dot{m}_{\text{act}}, \delta \dot{m}_{\text{cor}} = \pm \frac{1}{2} \left( \frac{0.005}{\Delta p} \right) \% \quad \text{(dynamic pressures, 42\%).} \]

**Flow Factor**

The uncertainty in the flow factor is indicated by the level of scatter in Figure 4.2, showing measurements of flow factor vs. Re. Since Re depends on the measured temperature, velocity and pressure, the uncertainty in FF will also reflect the uncertainty in these quantities. The upper and lower bounds on the data differ from the best-fit line by about 1% of the value of the flow factor. This uncertainty introduces an error in the measurement of mass flow given by

\[ \delta \dot{m}_{\text{act}}, \delta \dot{m}_{\text{cor}} = \pm 1\%. \]

**Inlet Temperature**

The difficulties in the measurement of inlet temperature due to the effects of radiation were discussed in Section 3.6.1. The uncertainty in the inlet temperature will be taken to be the amount of variation observed in the readings of the thermocouple located at the heat exchanger exit, which was about \( \pm 3^\circ\text{C} \), or \( \pm 1\% \) on the absolute scale. This uncertainty in inlet temperature introduces an error in the measurement of mass flow (including its influence on the flow factor) given by

\[ \delta \dot{m}_{\text{act}} = \pm 0.6\%, \]

\[ \delta \dot{m}_{\text{cor}} = \pm 0.1\%. \]

**Inlet Duct Diameter**

The inlet duct inner diameter is bounded by the dimension and tolerance information that was supplied to the manufacturer of the part:

\[ D = 0.162 \pm 0.00025\text{m}. \]

This \( \pm 0.15\% \) uncertainty in the inlet diameter will introduce an uncertainty in the mass flow given by
\[ \delta \dot{m}_{\text{act}} = \pm 0.3\%, \]
\[ \delta \dot{m}_{\text{cor}} = \pm 0.3\%. \]

**Viscous Effects**

The importance of viscous effects in the measurement of total pressures in the inlet was discussed in section 3.4.2. The pressure coefficients determined by Sherman [13] for subsonic pressure measurements using pitot tubes of design ‘C’ (shown in Figure 3.3b) are used to correct the experimental measurements. The scatter in the data points from the study by Sherman, taken directly from Figure 3.3b, is used to give a rough indication of the uncertainty in the correction factors.

At 100% impeller speed, the inlet pitot probe Reynolds number based on external diameter is about

\[ \text{Re}_{100\%} = \frac{UD\rho}{\mu} = \frac{22 \times 4.8 \times 10^{-3} \times 0.0163}{1.8 \times 10^{-5}} = 96. \]

At 42% impeller speed, the inlet pitot probe Reynolds number is about

\[ \text{Re}_{42\%} = \frac{6 \times 4.8 \times 10^{-3} \times 0.0163}{1.8 \times 10^{-5}} = 26. \]

From Fig. 3.3b, the corrections in centerline dynamic pressures at these Reynolds numbers for probes of design ‘C’ are given by

\[ \delta \Delta p = -1 \pm 0.5\% \quad (\text{100}\% \text{ speed}), \]
\[ \delta \Delta p = -1.5 \pm 5\% \quad (\text{42}\% \text{ speed}). \]

Using the sensitivities in Table 4.1, the corrections and uncertainties in mass flow due to the effects of viscosity are

\[ \delta \dot{m}_{\text{act}}, \delta \dot{m}_{\text{cor}} = -0.5 \pm 0.25\% \quad (\text{100}\% \text{ speed}), \]
\[ \delta \dot{m}_{\text{act}}, \delta \dot{m}_{\text{cor}} = -7 \pm 2.5\% \quad (\text{42}\% \text{ speed}). \]

**Outgassing**

The effects of outgassing on the measurement of dynamic pressure at the inlet were discussed in Section 3.4.3. From Table 3.2, the uncertainty in the measured dynamic pressure due to outgassing at a rig pressure of 10 torr is \( \pm 0.0001 \) torr. As a percentage of the dynamic pressure (in torr), the error is

\[ \delta \Delta p = \pm \frac{0.0001}{\Delta p} \times 100\%. \]

76
This uncertainty introduces an error in the measurement of mass flow given by

\[ \delta \dot{m}_{\text{act}}, \delta \dot{m}_{\text{cor}} = \pm \frac{1}{2} \left( \frac{0.01}{\Delta \rho} \right) \% . \]

**Turbulence**

The dynamic pressure distribution across the inlet duct is found by measuring the difference between the pressure developed by the traversible inlet pitot probe and the wall static port. However, the presence of turbulent velocity components in an internal flow will cause the distribution of static pressure across the flow path to differ from that measured at the wall leading to an error in the measured dynamic pressure. This effect has been established for fully developed flow in pipes of circular cross section as a function of Reynolds number. The duct centerline Reynolds number at design conditions is 3230. The duct length required for flow in the inlet to become fully developed is shown by Fox [4] to be

\[ \frac{L}{D} \approx 0.06 \text{Re}_{\text{bulk}}, \]

where

\[ \text{Re}_{\text{bulk}} = \frac{U_{\text{avg}} D \rho}{\mu} = \text{Re}_{\text{cent}} \text{FF} = 3230 \times 0.885 = 2860, \]

so that

\[ \left( \frac{L}{D} \right)_{\text{developed}} = 170. \]

The actual duct entrance length is only about L/D=1.5, indicating that the flow is far from fully developed.

Owen [6] states that for a Reynolds number of 3000 under fully developed conditions, the percentage correction in mass flow that must be applied is –1.0%. Although the flow in the macro-compressor’s inlet is not fully developed, a correction of -2% to 0% is assumed to bound the true correction:

\[ \delta \dot{m}_{\text{act}}, \delta \dot{m}_{\text{cor}} = -1 \pm 1\% \quad (42\%, \ 100\% \ speed). \]

**Stem Blockage**

Stem blockage refers to the interference effect of a pitot probe stem on the surrounding flow. The flow area reduction caused by the presence of the stem and head of
a pitot probe causes a local acceleration of flow, leading to a reduction in the local static pressure, and an overestimate of the flow velocity. The importance of the effect at the inlet may be evaluated using the semi-empirical equation by Owen [6]:

$$\delta Dp = \frac{0.7kS}{A},$$

where

- $\delta Dp$ = fraction by which total pressure head must be reduced,
- $k$ = stem blockage factor = 1 (stem and static port on the same axis),
- $S$ = stem area = 2.8 cm$^2$,
- $A$ = pipe area = 206 cm$^2$.

Using these values for the inlet, it is predicted that dynamic pressures should be overpredicted by about 1%, resulting in a 0.5% over-prediction of mass flow. This correction will be applied to the measurements and an uncertainty of $\pm 0.5\%$ in mass flow will be assumed:

$$\delta m_{act}, \delta m_{cor} = -0.5 \pm 0.5\% \quad (42\%, \ 100\% \ speed).$$

**Propagation of Experimental Errors**

Since an attempt has been made to remove all systematic errors from the data by applying appropriate corrections factors, the remaining random errors are combined by computing their RMS value:

$$\delta m_{tot} = \left(\delta m_1^2 + \delta m_2^2 + \cdots + \delta m_n^2\right)^{1/2}. \quad (4.8)$$

Figure 4.3 shows the estimated uncertainty in corrected mass flow for both 42% and 100% impeller speeds, plotted against the corrected mass flow, over the range investigated in this study. Note that the uncertainty at 100% speed is nearly independent of mass flow, while the uncertainty at 42% speed increases rapidly as mass flow decreases. This occurs because the inlet duct’s centerline dynamic pressure approaches the resolution of the differential pressure transducer as mass flow decreases for the 42% speed case.
4.2 Measurement of Compressor Efficiency

The isentropic efficiency of the macro-compressor is defined as the ratio of the actual work to the ideal isentropic work required to achieve a particular pressure ratio. Although easily defined, the experimental measurement of the efficiency has proven to be one of the most challenging aspects of this research. To arrive at an accurate estimate, three independent measurement techniques are explored. The first makes use of air total temperature measurements upstream and downstream of the impeller (so called ‘rake’ efficiency). The second method attempts to determine the torque forces on the impeller by measuring the rate of change of the angular momentum across the impeller (Euler efficiency). The third method attempts to find the power delivered by the impeller by measuring the deceleration rate when it is allowed to coast (shaft efficiency).

4.2.1 Efficiency Measure 1: Rake Method

The definition of the isentropic compressor efficiency, assuming constant specific heats and adiabatic flow is

\[
\eta_c = \left( \frac{p_{2t}}{p_{1t}} \right)^{\frac{(\gamma - 1)}{\gamma}} - 1
\]

\[
\left( \frac{T_{2t}}{T_{1t}} \right) - 1
\]

(4.9)

In this study, the ratio of static pressures at the inlet and plenum is used instead of total values to allow the two pressures to be acquired simultaneously using the existing pressure line system. Since the Mach numbers at the inlet and plenum sections are below 0.07, the difference between the total and static pressures are less than 0.3% which is of the same order as the accuracy of the pressure transducers, and so will be neglected. To obtain the temperature ratio, the total temperature at the inlet duct and at the impeller exit are measured. A complication arises, however, due to the large mass of the rig, on the order of 1,500 kg, when compared to a typical air mass flow rate of 6 g/s. The rig takes much longer to reach equilibrium than the time period of a typical experiment, about five hours. Furthermore, the steady loss of heat from the rig to the cooler surroundings means that adiabatic conditions are never attained.
An attempt is made in the following section to calculate the magnitude of this effect using results from the 2-D CFD solver MISES [1, 16]. Although it is shown in this thesis that there are large differences in the performance as predicted by MISES compared to that observed during the experiment, the general trends reached by the following calculation should still be valid.

4.2.1.1 MISES Heat Transfer Calculation

Calculation Methodology

In this section, the CFD code MISES [16] will be used to estimate the heat transfer into the airfoils of the impeller and diffuser to ascertain the importance of heat loss on the measurement of rake efficiency. MISES is an adiabatic code, hence it gives no direct information about heat transfer. To determine a rough estimate of the heat transfer, it is noted that when boundary layers are thin, and for a Prandtl number close to unity, the thermal and momentum boundary layer equations have the same form and Reynolds Analogy may be invoked:

\[
\frac{C_f}{2} = St \text{Pr}^\frac{2}{3},
\]

(4.10)

where

\[
C_f = \frac{\tau}{0.5 \rho U_e^2},
\]

\[
St = \frac{\tau}{\rho U_e^2}.
\]

The subscript e denotes conditions at the edge of the boundary layer. By substituting in the definitions of Cf, St, and using Pr=0.7 for air, the following expression for the heat transfer coefficient results:

\[
h = \frac{\tau C_p}{0.788 U_e},
\]

(4.11)

where

- \(C_p\)=constant pressure heat capacity of air,
- \(\tau\)=Wall shear (predicted by MISES),
- \(U_e\)=velocity at edge of boundary layer (predicted by MISES).
Since MISES is a 2-D code, the endwalls are not included in the calculation results, and values must be extrapolated from the bladewall values.

The impeller is modeled in MISES with 30 points on both the pressure and suction surface contours of each blade. The Diffuser is modeled with 26 points on the pressure surface and 20 points on the suction surface. To facilitate the calculation of the heat transfer rate, an area radially centered on each point containing both bladewall and endwall surfaces is constructed. In following this procedure, it is therefore assumed that the endwalls have similar heat transfer coefficients to the blade walls at a given radial position. The heat transfer at each element \( i \) is given by

\[
Q_i = h_i A_i (T_i - T_{\text{wall}}),
\]  

(4.12)

where

- \( h_i \) = heat transfer coeff at point \( i \),
- \( A_i \) = area associated with point \( i \),
- \( T_i \) = gas temperature at point \( i \),
- \( T_{\text{wall}} \) = metal temperature at point \( i \).

To simplify the calculation, adiabatic conditions were assumed to exist within the impeller and diffuser sections, and air temperature corrections were only made at the exits of each section. The calculation was performed assuming a metal temperature of 300 K. The temperature drop across each section is given by

\[
\Delta T_{\text{gas}} = \frac{\sum Q_i}{m C_p}.
\]

Where \( m \) is the mass flow through the compressor, predicted by MISES.

When the above analysis is performed using the 100% MISES design case, (which predicts a pressure ratio of 2.16, a mass flow of 100% design and an efficiency of 94.5%) the temperature drops (below adiabatic conditions) across the impeller and diffuser are found to be

- temp drop across impeller = 7.3 K,
- temp drop across diffuser = 12.9 K,
- temp drop across both sections = 20.2 K.
The importance of this temperature drop on the measured efficiency is evaluated by computing the change in the efficiency caused by subtracting 20.2 K from the adiabatic exhaust temperature predicted by MISES. Under adiabatic conditions, MISES predicts that

impeller exhaust total temperature = 377.1 K,
total-to-static pressure ratio = 2.14,
efficiency = 0.945.

Using the temperature drop due to non-adiabatic conditions predicted above, these values become

impeller exhaust total temperature = 377.1-20.2 = 357K,
total-to-static pressure ratio = 2.14 (no change),
efficiency = 1.28.

This calculation indicates that during the start of the experiment, the heat transfer from the air into the rig will result in an overestimate of the efficiency. It expected that as the components in the rig heat up during an experiment, the amount of heat transferred to the walls, and the efficiency error, will decrease.

It is concluded that the difficulty in computing suitable correction factors means that direct air temperature measurements will not be useful in obtaining accurate efficiency estimates, but instead will provide an upper bound for the efficiency.

4.2.2 Efficiency Measure 2: Euler Method

This section will examine an efficiency measurement technique called the Euler method that is commonly used for conventional axial flow turbomachinery. The feasibility of this technique for use in this project will be discussed and one source of error due to viscous dissipation at the blade tips will be estimated.

The Euler turbine equation described by Kerrebrock [19] is based on the conservation of fluid energy and angular momentum. Figure 4.4 shows a control volume adapted to the geometry of the macro-compressor. Applying the conservation of energy across the control volume:

\[
P_{imp} = \dot{m}(T_{t,o} - T_{t,i})C_p, \tag{4.13}
\]

where

\[P_{imp}\] = rate of energy addition by impeller to flow,

\[T_{t,o}\] = total temperature exiting control volume,
\( T_{i,o} \) = total temperature entering control volume.

The conservation of angular momentum is described by

\[
\tau_{\text{imp}} + \tau_{\text{casing}} = \dot{m}(U_{r,o} \sin \beta_o - U_{r,i} \sin \beta_i),
\]

(4.14)

where

\( \tau_{\text{imp}} \) = torque force acting on gas by the impeller,

\( \tau_{\text{casing}} \) = torque force acting on rotating gas by the stationary impeller casing, of opposite sign to \( \tau_{\text{imp}} \),

\( \beta_i, \beta_o \) = flow angle at the inlet and exit of the control volume.

For the case of the macro-compressor, flow enters radially and the last term in (4.14) is zero. Equations (4.13) and (4.14) may be related using

\[
P_{\text{imp}} = \omega \tau_{\text{imp}}.
\]

(4.15)

Combining (4.13), (4.14) and (4.15) gives the equation for the work addition across the impeller:

\[
\left( \frac{T_{r,o}}{T_{r,i}} - 1 \right) = \frac{\omega}{C_p T_{r,i}} \left( U_{r,o} \sin \beta_o - \frac{\tau_{\text{casing}}}{\dot{m}} \right).
\]

(4.16)

For most conventional axial compressors the amount of turning put into the flow by each stage and the fraction of the total blade span near to the casing is small, so the last term in (4.16) is usually neglected. With the last term neglected, the Euler equation becomes a very convenient means of measuring compressor stage efficiency. For the case of a radial compressor, however, the circumferential velocity component is large and the casing affects a much larger fraction of the blade span due to the relative shortness of the blades. The net effect is that the stationary casing, through viscous dissipation on the rotating air, removes a substantial amount of turning from the flow. The effect of the casing on the flow should increase as Reynolds numbers decrease, due to the increased effect of viscosity. The Euler equation thus becomes a less accurate measure of efficiency than was originally anticipated because the torque forces acting on the casing are difficult to measure experimentally.
Viscous Loss at the Blade Tips

One loss source that is due to $\tau_{casing}$ is the shear work done between the tips of the blades and the casing wall. The shear work done between two small areas in relative motion is given by

$$dP_{shear} = \tau UdA,$$

where $\tau$ is the fluid shear stress, $dA$ is the surface area and $U$ is the relative velocity of the two surfaces. For a radial system, the above equation must be integrated from the leading edge to the trailing edge of a blade:

$$P_{blade} = \frac{\omega^2 \mu}{\Delta} \int_{r_{min}}^{r_{max}} (\theta_p(r) - \theta_s(r)) r^3 dr,$$

where

$r_{\text{min}}$ = leading edge radius of blade
$r_{\text{max}}$ = trailing edge radius of blade
$\omega$ = Angular velocity of impeller = $424.5 \times 2\pi$,
$\Delta = airgap = 3.556 \times 10^{-4} m$,
$\theta_p, \theta_s$ = angular distribution of pressure and suction blade surfaces.

The static temperature of the air and its viscosity increase as it passes through the impeller passages. To simplify the calculation, the viscosity will be assumed to be constant at the value corresponding to the temperature measured at the exit of the impeller (using Eq 4.5),

$$\mu = \text{Viscosity of air at 405K (measured)} = 2.2 \times 10^{-5} N \cdot \text{sec} / m^2.$$

Using this maximum viscosity will result in an upper bound estimate of the blade tip loss.

The result of this calculation is that about 15 Watts of power per blade is lost through dissipation at the blade tips, for a total of 90 Watts for six blades (in the macro-compressor). This source of loss accounts for about 10% of the total impeller power at 100% speed (as determined by the spindown method, described in the following section). Repeating this calculation at 42% impeller speed gives a tip loss of 15.9 Watts, which is about 17% of the total impeller power.

In addition to the blade tip loss, there will be additional loss over the remaining area of the casing. Neglecting the radial velocity component of the gas flow, it is roughly estimated to be about $1/20$ of that at the blade tips due to the much larger clearance between the endwalls (compared to that at the blade tips). These sources of loss will decrease the actual efficiency below that predicted by the Euler equation that neglects the last casing
torque term. However, the uncertainty in the calculations performed above is large and it is concluded that the Euler equation will not produce accurate efficiency estimates for the compressor in its current configuration. It should be noted that the difficulty with the Euler equation is removed for a compressor with a rotating shroud. In this case, the last term of (4.16) will be removed, and all of the turning put into the air by the impeller will remain in the gas flow.

4.2.3 Efficiency Measure 3: Shaft Efficiency

Another means of determining the shaft work is to measure the deceleration of the impeller when it is allowed to coast or 'spindown', unpowered by the drive motor, so that the power is provided by that stored in the rotating system. The energy stored in the rotating system is given by

$$E = \frac{1}{2} I \omega^2, \quad (4.17)$$

where

$I$=total rotational inertia of system=0.1881 kg.m$^2$ (estimated in Appendix A),
$\omega$= angular velocity of the impeller.

If the angular velocity is a continuous function of time, $\omega=\omega(t)$, the above equation may be differentiated with respect to time:

$$P_{shaft} = \frac{dE}{dt} = I \omega \dot{\omega}. \quad (4.18)$$

The shaft power consumed as calculated in Eqn. (4.18) originates from two sources: mechanical dissipation in the drive system and the rate of aerodynamic work on the gas. Mechanical dissipation results from the various seals and bearings that support the rotating system. It may be estimated by performing a spindown test with the rig at much below operating pressure, when the gas work is close to zero.

The aerodynamic power is the quantity that must be isolated to give an efficiency measure. The total of the mechanical dissipation and aerodynamic power is determined by performing the same spindown test when the rig is at design pressure. Assuming that the bearing friction is unchanged by the operating pressure, the aerodynamic power can then be
separated from the total by subtracting the power found from the vacuum spindown test. This gas power may then be used with measurements of mass flow, pressure ratio and inlet temperature to give an estimate of the compressor efficiency. The temperature ratio across the impeller is given by

\[ \frac{T_{t,o}}{T_{t,i}} - 1 = \frac{P_{gas}}{\dot{m}CpT_{t,i}}, \]  

(4.19)

where

\[ P_{gas} = P_{total} - P_{bearings} = I\omega(\dot{\omega}_{press} - \dot{\omega}_{vac}), \]

\[ T_{t,o} = \text{total temperature at the impeller exit}, \]

\[ T_{t,i} = \text{total temperature at the impeller inlet}, \]

\[ \dot{m} = \text{compressor mass flow}. \]

The efficiency may be found by substitution into Equation (4.9):

\[ \eta = \frac{(\pi^{(\gamma-1)/\gamma} - 1) \cdot \dot{m}CpT_{t,i}}{I\omega(\dot{\omega}_{press} - \dot{\omega}_{vac})}. \]

(4.20)

4.2.3.1 Measurement of the Impeller Speed

Measurements of the impeller's rotation rate are made with a Hall-effect proximity probe, which is installed in the side of the motor-mount. Its head is positioned about 0.5mm away from the side of a flexible aluminum coupling that connects the motor to the shaft of the impeller. The probe functions by producing an electric signal once per shaft revolution as a steel setscrew in the coupling passes over the sensing head of the probe. The signal is relayed to a Hewlett Packard 5316A Universal Counter equipped with a GPIB port, which determines the frequency of the pulses and is serviced by the computer.

During a spindown test, the impeller is brought to a speed about 10 Hz higher than that being studied. Once the impeller speed is stable, the motor power is cut and the computer instructed to acquire speed information from the counter at a rate of seven readings per second. Once the speed has fallen to about 10 Hz below the point being studied, the motor power is restored and the impeller brought back up to speed. The total time taken for the impeller to drop through 20Hz is about one minute under 100% design conditions.

The actual spindown data for the compressor in its vaned diffuser configuration and at 100% speed are shown in Figure 4.5. Data from six separate spindown tests during a
single experiment run are shown, each beginning at an impeller speed of 433 Hz at time zero, so that the slopes may be visually compared. The shaft power calculated from this data is shown in Table 4.2. The data series labeled 1 and 2, which match each other so closely that it is difficult to visually discern them, were both recorded with the rig evacuated to below 0.1 torr. These lines give information about the bearing dissipation. Note from Table 4.2 that the bearing power indicated by these two lines differ by only 0.7 watts, or 0.1% of the full value, indicating very good reproducibility. The series labeled 3 was obtained with the inlet pressure set to 5 torr, or 50% design inlet pressure. Lines 4 and 5 where both measured with the inlet pressure set to 10.1 torr. Line 6 was measured with the inlet pressure set to 20.2 torr.

To evaluate the slopes of such data series, least squares fits with various powers of rotational speed were applied to the data. It was found, however that the type of fit made little difference due to the near linearity of the data over this relatively small range of impeller speeds, and a simple quadratic fit was ultimately chosen in order to simplify calculations. An indication of the goodness of fit to the data can be obtained by computing the mean squared residuals of each series of points, shown in Table 4.2

<table>
<thead>
<tr>
<th>Line Number</th>
<th>Condition</th>
<th>Power at 100% speed, W</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vacuum</td>
<td>922.3</td>
<td>4.2677e-05</td>
</tr>
<tr>
<td>2</td>
<td>Vacuum</td>
<td>921.6</td>
<td>1.6538e-04</td>
</tr>
<tr>
<td>3</td>
<td>50 % design Inlet Pressure</td>
<td>1398.2</td>
<td>7.1174e-05</td>
</tr>
<tr>
<td>4</td>
<td>100 % design Inlet Pressure</td>
<td>1908.0</td>
<td>4.6667e-05</td>
</tr>
<tr>
<td>5</td>
<td>100 % design Inlet Pressure</td>
<td>1933.0</td>
<td>1.4357e-04</td>
</tr>
<tr>
<td>6</td>
<td>200 % design Inlet Pressure</td>
<td>2640.8</td>
<td>4.5183e-05</td>
</tr>
</tbody>
</table>

Table 4.2: Results of 100% speed spindown test with rig in vaned configuration

where

$$
\sigma^2 = \frac{\sum_{i=1}^{n} (\omega_i - \hat{\omega}_i)^2}{n-2},
$$

$\omega$=measured shaft speed, Hz,

$\hat{\omega}$=shaft speed from best-fit curve, Hz,

n=number of data points acquired (400).

The small residual values indicate low noise levels in the spindown data and a very good fit by the least squares curve.
4.2.3.2 Mechanical Dissipation

Unlike most conventional turbomachinery, a large fraction of the shaft work produced by the motor is lost to mechanical losses in the rotating system. Table 4.2 shows that for the compressor operating at 100% design speed and inlet pressure, the bearings are responsible for about half of the total shaft work (compare lines 1 and 2 with lines 4 and 5). For this reason, the estimates of efficiency derived from the spindown method are very sensitive to changes in bearing dissipation between vacuum and pressure spindown runs. In this section, an attempt is made to evaluate the causes and importance of changes in bearing dissipation that may lead to errors in spindown efficiency measurements.

The essential features of the macro-compressor bearing configuration are shown in Figure 2.2. The shaft and impeller are supported on four Fafnir 2MMV99107-WN-CR-DUL angular contact ball bearings. Both of the bearings in the forward pair (1 and 2) are oriented to sustain an aft thrust on their inner races. They are soft-mounted to the rig structure in the radial direction by means of rubber O-rings. The bearings in the aft pair (3 and 4) are each oriented to sustain a forward thrust on their inner races and are rigidly mounted to the rig structure. To ensure that both pairs of bearings never become axially unloaded during operation, which would cause ball clippage, a spring was installed that supplies a forward axial preload of 580 N (130 lbs) to the outer races of the forward pair of bearings. This spring force is transmitted to the aft pair of bearings through the impeller shaft and acts to supply them with a preload as well.

It is expected that any change in the axial thrust load of the shaft during rig operation is almost entirely borne by bearings 3 and 4 because the rotating seals and the forward pair or bearings are much less rigidly mounted than the aft pair and because the motor is axially isolated from the impeller shaft. In addition, because bearings 1 and 2 are mounted closer to the mass of the impeller, it is expected (and has been observed experimentally using accelerometers) that this pair sustained most of the dynamic radial loads produced by the spinning impeller.

With the rig operating at 100% design speed and inlet pressure during a pressure spindown test, the static pressure on the front of the impeller varies from inlet static at the center region of the disk to the impeller exit static pressure at the rim (static pressure ratio=1.84 at 100% speed). However, since the cavity behind the impeller is directly connected to the exit region of the impeller, and the radial pressure gradient on the back of the impeller is expected to be negligible, the entire back surface of the disk is exposed to the impeller exit static pressure. The difference in average static pressure across the impeller disk produces an additional forward thrust force on the shaft, increasing the axial load
sustained by bearings 3 and 4. The overall result of this effect is that the friction in the aft pair of bearing is somewhat different between vacuum and pressure spindown conditions, leading to some error in the efficiency measurement.

The importance of this effect may be evaluated by performing a thrust balance analysis on the impeller at 100% speed and inlet pressure. Assuming that the air has the tangential velocity of the impeller as it flows radially outwards, the radial pressure gradient on the front of the impeller is

\[
\frac{dp}{dr} = \rho \omega^2 r. \tag{4.21}
\]

If it is further assumed that the flow in the impeller passages is isentropic, then

\[
\rho \rho_i = \left( \frac{p}{p_i} \right)^{1/\gamma}, \tag{4.22}
\]

where subscript (i) signifies conditions at the impeller inlet.

Equations 4.21 and 4.22 may be combined and numerically integrated radially outwards from the blade leading edges to the exit radius of the impeller to give the aft thrust force over this area. The net force acting on the impeller during 100% speed and inlet pressure conditions is given by (where the areas are shown in Fig. 2.2)

\[
F_{\text{press}} = A_a p_a + A_b p_b - A_c \overline{p}_c - A_d p_d,
\]

where

- \(A_a p_a\) = force acting across the rotating seal
  \[= \pi \times (0.0384m)^2 \times (1.103 \times 10^5 Pa - 1346 Pa) = 504.7 \text{ N (113.5 lbs.)},\]

- \(A_b p_b\) = force acting on the back of the impeller, outboard of the seal
  \[= \pi \times \left( (0.1501m)^2 - (0.0384m)^2 \right) \times 1.84 \times 1346 Pa = 163.8 \text{ N (36.8 lbs.)},\]

- \(A_c \overline{p}_c\) = net force acting over the bladed region of the impeller
  \[= 104.1 \text{ Pa (23.4 lbs.)} \text{ (evaluated from 4.17 and 4.18)},\]

- \(A_d p_d\) = force acting on the inner hub of the front of the impeller
  \[= \pi \times (0.0737m)^2 \times 1346 Pa = 23 \text{ N (5.2 lbs.)},\]

so that

\[
F_{\text{press}} = 541.4 \text{ N (121.7 lbs.)}.
\]

Performing the same analysis with an inlet pressure of 0.1 torr, corresponding to vacuum spindown conditions:
\[ F_{\text{vac}} = 511 \text{ N} + 1.6 \text{ N} - 1 \text{ N} - 0.2 \text{ N}, \]
\[ F_{\text{vac}} = 511.4 \text{ N} \ (115 \text{ lbs.}). \]

This analysis shows that the axial force sustained by bearings 3 and 4 will decrease by approximately 541.4 - 511.4 = 30 N (6.7 lbs.) between 10.1 torr and 0.1 torr inlet conditions. The fractional decrease in loading between the two inlet conditions is found by dividing this 30 N force by the total axial force applied to the aft bearings under 10.1 torr inlet conditions (due to the sum of air pressure and the preload spring forces):

\[
\text{fractional decrease in aft bearing thrust} = \frac{30}{(541 + 578)} = 2.7\%. 
\]

As explained earlier, the other bearings in the system are not influenced by the change in axial thrust. If it is assumed that no more than half of the total bearing dissipation is due to aft pair, and that bearing friction is proportional to the axial load (nearly true for bearings 3 and 4, which sustain small radial loads), then it is expected that the decrease in bearing dissipation between pressure and vacuum conditions is about 2.7%/2 = 1.4%. The vacuum spindown results are corrected for this error, and an uncertainty of ±1.4% in bearing friction is assumed. When a similar analysis was performed for operation at 42% speed, the difference between bearing loading under pressure and vacuum conditions was found to be negligible due to the small static pressure ratio at this speed.

4.2.3.3 Viscous Work on Sides and Back of Impeller

The spindown method will include all sources of gas dissipation on the impeller, including its rim and back. This project has adopted the convention of attributing losses on the back and in the bearing gap at the rim of the impeller, to the bearings. To avoid double booking these sources of viscous dissipation, they may be accounted for and subtracted from the gas power result before calculating an efficiency.

Fig. 2.3 shows two gaps at the rim of the impeller. For each of these gaps the dissipation will be given by

\[ P_p = \tau AU, \]

(4.23)

where

\[ \tau = \frac{\mu U}{\Delta_{ap}}. \]

For the gap near the front of the impeller

\[ \mu = 2.0 \times 10^{-5} \text{ N \cdot sec/m}^2 \ (\text{at } 350 \text{ K}), \]
\[ \Delta_{ap} = 0.00094m, \]
\[ A = \pi \times 0.3m \times 0.00572m, \]
\[ U=400m/s, \]

So that

\[ P_{gap}(1)=18.4 \text{ W}. \]

For the rear gap

\[ \mu = 2.0 \times 10^{-5} N \cdot \text{sec/m}^2, \]
\[ \Delta_{ap} = 0.00559m, \]
\[ A = \pi \times 0.3m \times 0.0469m, \]
\[ U=400m/s, \]

\[ P_{gap}(2)=25.3 \text{ W}. \]

The dissipation on the back of the impeller, neglecting the bolts joining the impeller to the shaft, is found by integrating (Eqn. 4.23) over the area of the disk:

\[ P_{back}=3.6 \text{ W}. \]

So dissipation on the sides and back of the impeller during operation at 100% speed will account for about 5% of the total aerodynamic power done by the impeller. At 42% impeller speed, windage power is estimated to be 8.3 W which about 9% total aerodynamic power.

4.2.3.4 Errors in the Measurement of Efficiency using the Spindown Method

The sensitivity of the spindown efficiency to experimental measurements is found by differentiating Eqn. (4.20) with respect to \( \pi, \dot{m}, T_{i,j}, I \) and \( \dot{\omega}_{\text{vac}} \), the results are shown in Table 4.3. It should be noted that the values used below represent fractions of the absolute value of the efficiency, and not efficiency points.

<table>
<thead>
<tr>
<th>Measured Quantity</th>
<th>% change in ( \eta ) due to +1% change in measured quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure ratio, ( \pi )</td>
<td>+1.6%</td>
</tr>
<tr>
<td>Mass flow, ( \dot{m} )</td>
<td>+1.0%</td>
</tr>
<tr>
<td>Inlet total temp, ( T_{i,j} )</td>
<td>+1.0%</td>
</tr>
<tr>
<td>Rotational inertia, ( I )</td>
<td>-1.0%</td>
</tr>
<tr>
<td>Decel rate (vac), ( \dot{\omega}_{\text{vac}} )</td>
<td>+1.0%</td>
</tr>
</tbody>
</table>

Table 4.3: Sensitivities in the calculation of spindown efficiency
Pressure Ratio

The uncertainty in the pressure ratio was determined to be \( \pm 0.2\% \). It introduces an uncertainty in the efficiency given by

\[
\delta \eta = \pm 0.2\% \quad (42\%, \ 100\% \ speed).
\]

Mass Flow

The uncertainty in the mass flow rate was discussed in Section 4.1.2. The numerical values near the chosen design points are calculated to be

\[
\delta \dot{m} = \pm 5\% \quad (42\% \ speed),
\]
\[
\delta \dot{m} = \pm 1.5\% \quad (100\% \ speed).
\]

The uncertainty in the efficiency is given by

\[
\delta \eta = \pm 5\% \quad (42\% \ speed),
\]
\[
\delta \eta = \pm 1.5\% \quad (100\% \ speed).
\]

Inlet Temperature

The uncertainty in the inlet temperature was determined to be \( \pm 1\% \). It introduces an uncertainty in the efficiency given by

\[
\delta \eta = \pm 1\% \quad (42\%, \ 100\% \ speed).
\]

Rotational Inertia

The inertia of the rotating system is computed in Appendix A. The uncertainty in the inertia of the rotating system is dominated by the uncertainty in the value for the motor, which was estimated only roughly. A worst case value for the uncertainty may be estimated by the fraction of the total inertia contributed by the motor, which is about 2\% (see Appendix A). This introduces an uncertainty in efficiency of

\[
\delta \eta = \pm 2\% \quad (42\%, \ 100\% \ speed).
\]

Bearing Friction

It was found in Section (4.2.3.2) that the bearing friction (which is proportional to \( \dot{\omega}_{vac} \)) will decrease by about 1.4\% between pressure and vacuum spindown tests, with an
uncertainty of ± 1.4% (at 100% impeller speed). The efficiency is increased by 1.4% to remove the systematic error. The uncertainty introduced is
\[ \delta \eta = \pm 1.4\% \quad (100\% \text{ speed}). \]

**Aerodynamic Power**

The uncertainty on the aerodynamic power is assumed to be given by the level of non-reproducability seen when two pairs of spindown tests are performed under the same conditions. This was found to be about 3% at both 42% and 100% speed, so that
\[ \delta \eta = \pm 3\% \quad (42\%, \ 100\% \text{ speed}). \]

**Propagation of Experimental Errors**

The random errors computed in this section are combined by taking their RMS value:
\[ \delta \eta_{tot} = (\delta \eta_1^2 + \delta \eta_2^2 + \cdots + \delta \eta_3^2)^{1/2}. \]

The result of this analysis is that
\[ \delta \eta = \pm 6.2\% \quad (42\% \text{ speed}), \]
\[ \delta \eta = \pm 4.3\% \quad (100\% \text{ speed}). \]

**Summary of Uncertainties**

Table 4.4 summarizes the uncertainties in the primary performance parameters that were determined in this chapter. The values shown were computed for conditions near the operating point where most observations were made.

<table>
<thead>
<tr>
<th></th>
<th>42% impeller speed</th>
<th>100% impeller speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass flow</td>
<td>±5.0%</td>
<td>±1.5%</td>
</tr>
<tr>
<td>pressure ratios</td>
<td>±0.21%</td>
<td>±0.21%</td>
</tr>
<tr>
<td>spindown efficiency</td>
<td>±6.2%</td>
<td>±4.3%</td>
</tr>
</tbody>
</table>

*Table 4.4: Uncertainties in the primary performance parameters*
Figure 4.1. Normalized inlet duct velocity profiles

Figure 4.2. Flow factor vs. inlet duct centerline Reynolds number
Figure 4.3. Uncertainty in the corrected mass flow vs corrected mass flow for 42% and 100% impeller speed.

Figure 4.4. Impeller control volume
Figure 4.5. Data from 100% speed spindown tests, compressor in vaned configuration.
Chapter 5

Experimental Results

This chapter presents the experimental findings for the macro-compressor in both its vaned and vaneless diffuser configurations. Measurements were taken at impeller tip speeds of 400 m/s and 170 m/s, corresponding to 100% and 42% of design respectively.

5.1 Speedlines

This section will present the variation of the inlet-total to plenum-static pressure ratio vs mass flow (called a speedline) at both impeller speeds and diffuser designs investigated. Figure 5.1 shows the 100% speed, vaned and vaneless diffuser results with measurement uncertainties, along with the 2-D predictions of the MISES code with vaned diffuser. Figure 5.2 shows the corresponding results at 42% speed. In both of these plots the mass flow has been normalized by that occurring at the 2-D code's 100% speed design point, which is 0.11g/s. It should be noted that for the vaned configuration at both 100% and 42% speeds, the static pressure measured in the plenum is very close to the centerline total pressure at the exit of the diffuser (see Table 5.3) so that it is appropriate to compare the vaned experiment results with the predictions of the 2-D code (which gives essentially centerline results). The 42% 2-D code speedline results are limited to normalized mass flows above 0.3 because of the difficulty in achieving numerical convergence at operating conditions that are far from design. Fig 5.1 also includes a single point predicted by the 3-D code at 100% speed for a vaned configuration. The two labeled points on each experimental speedline were selected for study of the efficiency and diffuser performance, which are discussed in the following sections.

Discussion

Figure 5.1 shows that for 100% impeller speed and normalized mass flows under 1.0, the total-to-static pressure ratio for a given mass flow was higher for the vaned configuration, than for the vaneless diffuser. Figure 5.2 similarly shows that for 42% speed and normalized mass flow below 0.25, higher pressure ratios were achieved with the vaned configuration. This result is due to the superior performance of the vaned over the
vaneless diffuser when the compressor is operating with relatively low mass flow (corresponding to relatively high pressure ratios). The diffuser performance at the labeled points is discussed in Section 5.3. It should be noted that at both impeller speeds and for relatively high mass flow rates (above 1.0 for 100% speed and above 0.25 for 42% speed), the performance of the vaneless configuration is superior to that of the vaned. This may be due to the increased importance of blockage caused by the vanes at higher mass flow rates.

A comparison of the 2-D code speedline with the experimental vaned configuration results indicate that at the design pressure ratio of 2.1, the macro-compressor had about 25% less mass flow than was predicted by the 2-D code. This result is expected because the 2-D code does not take into account the blockage caused by boundary layer growth on the endwalls. The performance of the experimental vaned configuration was significantly better than that predicted by the 3-D code (Dawes), achieving about 30% more mass flow. This difference may be because impeller inlet blockage caused by separation at the right angle corner in the inlet is not as important as was found in the 3-D calculation (the importance of inlet separation on the compressor performance will be tested in future experiments with an ideal smooth inlet). Another reason may be that the performance of the macro-compressor was enhanced by the removal of heat from the flow to the cooler rig structure, while the 3-D calculation was performed assuming adiabatic conditions. This temperature effect is discussed below.

Rig Temperature Effects

It should be noted that the data in Figure 5.1 were collected near the beginning of an experiment when the rig was close to room temperature. Of practical importance to the operation of the micro motor-compressor is the performance when the device is at operating temperature, which is expected to be above that corresponding to adiabatic conditions due to heat addition from its motor. Although the macro-compressor was not designed to allow the control of its wall temperature, trends can be obtained by observing changes in performance as the rig heats up over several hours during a run. To give a qualitative idea of the importance of wall temperature, two additional points have been included in Fig.5.1. The point indicated by the ‘o’ is the new location of the labeled, vaned-diffuser operating point, taken towards the end of the experiment when the rig was warm (but the throttle setting was the same). This indicates that over the course of the run, the pressure ratio decreased from 2.05 to 2.03, while the normalized mass flow decreased from 0.78 to 0.73. No quantitative information can be given about the change in rig temperature as no wall thermocouples were installed when the vaned tests were performed, however, the value of $\frac{T_s}{T_w}$ is expected to be about 1.25 as was later determined in the vaneless run.
The labeled vaneless diffuser operating point in Fig. 5.1 showed a larger change over the course of the experiment. As indicated by the ‘+’, the pressure ratio decreased from 1.98 to 1.92, while the normalized mass flow decreased from 0.70 to 0.63. Figure 5.3 shows the measured impeller exit gas temperature and the diffuser wall temperature over the course of the 100% speed vaneless run. In this figure the time period in which vaneless speedline measurements were made is bound by the two vertical solid lines. The first dashed vertical line indicates the time when the labeled vaneless operating point was measured, while the second indicates the time when the ‘warm rig’ point corresponding to the same throttle setting was measured. As shown in Fig. 5.3, the vaneless diffuser wall temperature increased from 38°C to 65°C over the time period when these two points were collected. However, the maximum diffuser temperature observed during the experiment was well below the measured impeller exit gas temperature of 132°C, indicating that heat was still being removed from the flow when the ‘warm rig’ observation was made.

One reason for the decrease in mass flow over time may be that the throttle setting changes as the density of the air passing through it decreases with increased temperature, while the area of the throttle remains constant. If this were the only effect of increased rig and air temperature, a corresponding increase in the pressure ratio would also be expected. However, the observation that both mass flow and pressure ratio decrease indicates that the compressor characteristics are indeed changed by higher wall temperature, shifting the speedline downwards and to the left.

It is cautioned that all experimental measurements taken during this project were made with heat removal from the flow. This may produce performance results that differ from the micro motor-compressor, where heat transfer is expected to be in the opposite direction.

### 5.2 Impeller Performance

Table 5.1 shows the performance parameters for the impeller at 42% and 100% design speed. The values of impeller power (which were scaled down by a factor of 75 to make them relevant to the micro device) and efficiency are shaft values obtained from spindown tests. They have been corrected by removing the scaled windage loss developed at the sides and back of the impeller, which were estimated in Section (4.2.3.3) to be 0.11 W and 0.63 W at 42% and 100% impeller speeds respectively. Neglecting this windage correction will result in efficiency values about 1.5% lower than those shown in Table 5.1, for each impeller speed and diffuser design tested.
2-D CFD results by Jacobson [1] and 3-D CFD results by Mehra [14], which include the interaction of the inlet with the impeller, have been included in Table 5.1 for comparison. The most significant difference between the results of the experiment and CFD is the impeller efficiency, which was measured to be about 30% lower than that predicted by the 3-D code and about 47% lower than the 2-D code.

**Blade Tip Losses**

One source of impeller loss in the macro-facility that was not taken into account in the CFD calculations (which assumed a rotating shroud instead of a stationary casing) is the viscous loss between the blade tips and the casing wall. This loss was estimated in Section (4.2.2) to be an important loss mechanism in the impeller, accounting for about 10% and 17% of the shaft power at 100% and 42% impeller speeds respectively. Tip loss is estimated to be responsible for a drop in efficiency of 4.5% at 100% speed and a drop of 5% in efficiency at 42% speed.

<table>
<thead>
<tr>
<th></th>
<th>Total Pressure Ratio $p_2/p_1$, (non dim.)</th>
<th>Mass Flow Rate $\dot{m}$, (scaled)</th>
<th>Power, Watts $P$, (scaled)</th>
<th>Efficiency $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>42% speed, vaned</td>
<td>1.21±0.2%</td>
<td>0.172±0.01</td>
<td>1.15±0.04</td>
<td>0.27±0.016</td>
</tr>
<tr>
<td>42% speed, vaneless</td>
<td>1.21±0.2%</td>
<td>0.169±0.01</td>
<td>1.23±0.04</td>
<td>0.25±0.016</td>
</tr>
<tr>
<td>100% speed, vaned</td>
<td>2.23±0.2%</td>
<td>0.731±0.01</td>
<td>12.8±0.4</td>
<td>0.48±0.02</td>
</tr>
<tr>
<td>100% speed, vaneless</td>
<td>2.26±0.2%</td>
<td>0.622±0.01</td>
<td>12.5±0.4</td>
<td>0.43±0.02</td>
</tr>
<tr>
<td>100%, M'SES 2D</td>
<td>2.19</td>
<td>1.0</td>
<td>8.77</td>
<td>0.953</td>
</tr>
<tr>
<td>100%, Dawes 3D</td>
<td>2.44</td>
<td>0.62</td>
<td>8.34</td>
<td>0.744</td>
</tr>
</tbody>
</table>

Table 5.1: Impeller performance parameters of the macro-compressor compared to CFD

**Alternative Measurements of the Impeller Efficiency**

Table 5.2 shows the impeller efficiencies indicated by the Euler turbine equation and rake temperature measurements for the vaneless configuration at 42% and 100% speed. The rake efficiencies shown in the uncorrected column have not been corrected for the effects of thermocouple error or heat loss into the rig. If the thermocouple correction factor determined in Section 3.6.3 is used, the 100% speed rake efficiency is calculated to be about 0.53 (though this still neglects heat loss into the rig).
As discussed in Section 4.2.2, the difference between the Euler and the shaft efficiencies from spindown measurements arise mainly because of the viscous shear work done at the impeller casing.

<table>
<thead>
<tr>
<th></th>
<th>Euler</th>
<th>Rake, uncorrected</th>
<th>Rake, corrected</th>
<th>Shaft (spindown)</th>
</tr>
</thead>
<tbody>
<tr>
<td>42% speed</td>
<td>0.986</td>
<td>0.53</td>
<td>—</td>
<td>0.25±0.016</td>
</tr>
<tr>
<td>100% speed</td>
<td>0.870</td>
<td>0.65</td>
<td>0.53</td>
<td>0.43±0.02</td>
</tr>
</tbody>
</table>

*Table 5.2: Impeller isentropic efficiency measurement comparison*

The efficiencies indicated by rake measurement, listed in Table 5.2, are intermediate between the Euler and shaft values. These results are difficult to interpret because a large amount of dissipation may have occurred near to the shroud wall where the heat generated would be quickly conducted into the cooler metal. However, unlike the Euler results, the rake efficiency values showed the same trend with impeller speed as did the shaft results, indicating that rake measurements may at least be useful in giving information about changes in efficiency.

It should be noted that neither the Euler nor the rake measurement techniques are expected to give accurate indications of the impeller efficiency because of the large systematic errors inherent in each, so they should be used for comparison purposes only. The spindown technique was shown in Section (4.2.3.2) to be largely unaffected by systematic error and it is believed to be a better measure of the shaft efficiency.

**Impeller Exit Flow Geometry**

Figure 5.4 shows the measured angle distributions across the flow path for 42% and 100% impeller speeds. In this figure, the distance from the impeller-side wall has been normalized by the flow-path height so that '1' on the x-axis indicates the location of the shroud wall. Shown in Figures 5.5 and 5.6 respectively, are the radial and circumferential components of the impeller exit velocity, plotted against the normalized flow-path height. The measurement uncertainty has been included in each of the three plots, with the exception of data points closest to the shroud walls, where the uncertainty is unknown (although expected to be larger that of the closest data point in which an uncertainty is given) because of the possible interaction of the wall instrument port with the Cobra head probe that was used to take the measurement.
Fig. 5.4 shows that there was a variation in impeller exit flow angle of at least 15° across the flow path. This large variation was not predicted by the 3-D code, which assumed a rotating shroud on both sides of the flow path, and is an indication of the strong influence that the stationary impeller shroud (casing) had on flow through the impeller. The negative angles near the shroud suggest that flow in this region may be radially inwards. As was pointed out by Tan [18], such a result (which is also illustrated in Fig 5.5) is typical of radial compressors with stationary casings.

5.3 Diffuser Performance

Table 5.3 compares the stage pressures of the macro-compressor in its vaned and vaneless configuration, all normalized by the inlet total pressure (computed by the sum of the inlet static and dynamic pressure). All total pressure measurements are centerline values.

<table>
<thead>
<tr>
<th></th>
<th>100% Speed</th>
<th></th>
<th>42% Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vaned</td>
<td>Vaneless</td>
<td>MISES</td>
</tr>
<tr>
<td>$p_2/p_{1t}$</td>
<td>2.23</td>
<td>2.26</td>
<td>2.19</td>
</tr>
<tr>
<td>$p_2/p_{1t}$</td>
<td>1.84</td>
<td>1.81</td>
<td>1.81</td>
</tr>
<tr>
<td>$p_3/p_{1t}$</td>
<td>2.05</td>
<td>1.99</td>
<td>2.10</td>
</tr>
<tr>
<td>$p_3/p_{1t}$</td>
<td>2.04</td>
<td>1.88</td>
<td>2.02</td>
</tr>
<tr>
<td>$p_4/p_{1t}$</td>
<td>2.03</td>
<td>1.92</td>
<td>.</td>
</tr>
</tbody>
</table>

Table 5.3: Stage pressure ratios for the macro-compressor, all measurements ±0.2%

In Table 5.3

$\ p_{1t}$ = inlet total pressure,
$\ p_2$ = impeller exit total pressure,
$\ p_3$ = impeller exit static pressure,
$\ p_3$ = diffuser exit total pressure,
$\ p_4$ = diffuser exit static pressure,
$\ p_4$ = plenum static pressure.
The overall effectiveness of the diffuser in converting the dynamic pressure at the impeller exit to static pressure in the plenum may be expressed in terms of a pressure coefficient:

\[ C_p = \frac{P_4 - P_2}{P_{2i} - P_2}. \]

Evaluating this coefficient for the four experimental cases in Table 5.3 gives

\[ C_p = 0.48 \pm 0.02 \quad (100\%, \text{ vaned diffuser}), \]
\[ C_p = 0.24 \pm 0.015 \quad (100\%, \text{ vaneless diffuser}), \]
\[ C_p = 0.46 \pm 0.10 \quad (42\%, \text{ vaned diffuser}), \]
\[ C_p = 0.13 \pm 0.06 \quad (42\%, \text{ vaneless diffuser}). \]

These results show that for both impeller speeds, the vaned diffuser is about twice as effective as the vaneless diffuser at recovering the impeller exit dynamic pressure.

**Over-all System Efficiency**

The over-all efficiency (at the points tested, from spindown measurements) of the macro-compressor configured with each diffuser design and at both impeller speeds is calculated to be

- 100\%, vaned \hspace{1cm} 0.42 \pm 0.02,
- 100\% \hspace{0.5cm} \text{vaneless} \hspace{1cm} 0.33 \pm 0.02,
- 42\% \hspace{0.5cm} \text{vaned} \hspace{1cm} 0.23 \pm 0.016,
- 42\% \hspace{0.5cm} \text{vaneless} \hspace{1cm} 0.19 \pm 0.016.

**Discussion**

The value of \( C_p \) for the macro-compressor/vaned diffuser at 100\% impeller speed compares well with the predictions of 2-D (MISES) CFD (\( C_p=0.55 \)) and 2-D (Dawes) CFD (\( C_p=0.44 \)) but is significantly higher than the prediction of 3-D CFD (\( C_p=0.20 \)). The performance of the vaneless diffuser agrees well with the predictions of 3-D CFD (\( C_p=0.145 \)). There are at least two possible reasons for the superior performance of the macro compressor's vaned diffuser over the prediction of 3-D CFD. First, the unsteadiness
in the flow through the macro-compressor may help to keep boundary layers in the diffuser attached, improving its performance. Second, cooling of the flow by the rig may also improve performance by stabilizing the boundary layers.

The relatively good performance of the diffusers compared to CFD predictions is encouraging in light of the skewed diffuser inlet angle distribution shown in Figure 5.4, because CFD work had predicted that the vaned diffuser’s performance would be sensitive to inlet flow angle. It is anticipated that the diffuser performance may be improved by reducing the variation of diffuser inlet angle by for example, including a rotating shroud in the impeller design.

**Performance Comparison with Conventional Turbomachinery**

It is of interest to compare the performance of the macro-rig to that of conventional centrifugal turbomachinery. Truck diesel engine turbochargers with vaneless diffusers are the closest in size and mass flow capacity of commonly used turbomachines to the micro-compressor. In a compilation of data by Japikse [20], the peak isentropic total-to-static efficiencies of turbochargers with mass flows of 100-200 g/s and pressure ratios of 1.5 to 2.5, range from about 77% to 83%. Compared to these devices, the peak efficiency of 42% for the macro-compressor is very poor. However, it should be remembered that the mass flow of the micro-compressor is three orders of magnitude smaller than these machines and it is difficult to determine how the current design compares to what is achievable at this scale.

The performance of the vaned and vaneless diffusers in the macro-compressor may also be compared to that in conventional systems. In a compilation by Cumpsty [21], vaned diffuser Cp values ranging from 0.53 to 0.73 are listed for centrifugal compressors with mass flows of 1-2 kg/sec and pressure ratios of 4.5:1 to 10:1 (there appears to be little correlation between pressure ratio and diffuser performance). The Cp of 0.48 measured for the macro-rig’s vaned diffuser is comparable to values of conventional devices. The performance of vaneless diffusers is shown by Cumpsty to vary inversly with impeller exit flow angle. A Cp of about 0.2 for a conventional scale vaneless diffuser is predicted for the relatively large average exit flow angles of 85° observed in the macro-rig. This compares well to the 100% speed vaneless value of 0.24 observed for the macro-rig.

In summary, it appears that the performances of both diffusers used in the macro-rig are similar to those in conventional sized devices, while the overall efficiency is far lower than in larger systems.
5.4 Performance at Off-Design Reynolds Numbers

The micro-compressor operates in a low Reynolds number regime where the effects of viscosity are expected to play an important role in the fluid dynamics of the device. To investigate the trends of compressor performance with Reynolds number, which would vary with the size of the actual device, the macro-compressor was operated with inlet pressures below and above that corresponding to the 1/75 atm. 100% scale pressure.

Total-to-Static Pressure Ratio

Figure 5.7 shows the dependence of the total-to-static pressure ratio on Re at 100% impeller speed. This plot shows that the pressure ratio is nearly independent of Re for the vaned configuration in the range tested. For the vaneless configuration, the pressure ratio drops relatively quickly for Re less than 200%. It is expected that the impeller performance is influenced little by the type of diffuser installed downstream, so it is deduced that the difference in pressure ratio Re dependence is due to a difference in the sensitivity of the diffuser performance to Re. One explanation for these observations is that flow in the vaned diffuser is less prone to loss producing separation because the vanes help to control the adverse pressure gradient. It follows that the vaned diffuser’s performance is less affected by the thicker boundary layers encountered at lower Re that tend to make the flow more prone to separation.

This theory is also supported by the findings of pressure ratio sensitivity to rig temperature, discussed in Section 5.1. It was shown in Figure 5.1 that the total-to-static pressure ratio of the vaneless configuration decreased more over the course of the experiment than the vaned configuration. This might be also be explained by the difference in sensitivity of the two diffusers to the destabilizing effects of increased wall temperature on the boundary layers.

Efficiency

The 100% speed!, total-to-static efficiency for the vaned and vaneless configuration was measured at off-design inlet Reynolds numbers using both Euler and Spindown methods. Figure 5.8 shows the results of this study.

These results are difficult to interpret because the tests at 100% Reynolds number were performed several hours after the 50% and 200% Re test were done, which may have introduced a significant rig temperature effect into the results. In particular, Fig. 5.8 seems
to suggest that the 100% Re spindown shaft efficiency was about the same or slightly lower than the 50% Re values. However, the rig temperature (as measured at the diffuser) was about 28°C higher during the 100% Re test, which may have removed any efficiency increase due to higher Re. The 50% and 200% Re tests were performed within 20 minutes of each other so the rig temperature was nearly the same, and a significant increase in the spindown efficiency was observed. This result may be due to the diminished importance of viscous dissipation in the impeller and diffuser as Re was increased.

The Euler efficiencies, which are thought to over predict the true efficiencies, are nearly constant with Reynolds number.
Figure 5.1: Speedlines at 100% impeller speed for the vaned and vaneless configuration.

Figure 5.2: Speedlines at 42% impeller speed for the vaned and vaneless configuration.
Figure 5.3. Diffuser wall and impeller exit gas temperature vs. time during the 100% speed, vaneless diffuser experiment.

Figure 5.4. Impeller exit angle distribution, 42% and 100% design speed and vaneless diffuser.
Figure 5.5: Impeller exit radial velocity distribution, 42% and 100% design speed and vaneless diffuser.

Figure 5.6: Impeller exit circumferential velocity distribution, 42% and 100% design speed.
Figure 5.7: The dependence of total-to-static pressure ratio on inlet Reynolds number.

Figure 5.8: The dependence of 100% speed total-to-static efficiency measurements on inlet Re. Spindown measurements have been corrected for windage loss on the back of the impeller.
Chapter 6

Summary and Conclusions

6.1 Summary of the Research

This project made an experimental assessment of the performance of a super-scale model of a 2:1 design compression ratio motor driven micro-compressor. Tests were performed with both a vaned and a vaneless diffuser configuration. Each configuration was tested at both 100% and 42.4% of the design tip speed of 400 m/s.

Speedlines were measured for each diffuser configuration and impeller speed investigated. It was found that the total-to-static pressure ratios developed were in general agreement with the CFD predictions for the vaned configuration. For a given pressure ratio with the vaned configuration, it was found that the measured design-point-normalized mass flow of 0.774 was between the predictions of 2-D (MISES) (1.0) and 3-D (Dawes) CFD (0.62) codes.

A comparison of the vaned and vaneless configurations revealed that at both impeller speeds, the vaned diffuser performed substantially better than the vaneless design ($C_p=0.48$ vs. 0.24 for 100% impeller speed). Although the vaned diffuser performed favorably compared to CFD prediction (0.55 for 2-D, 0.2 for 3-D CFD), a large span-wise variation in impeller exit angle was discovered at both impeller speeds, indicating that the vaned diffuser was operating off of its design angle over most of its span.

The primary technique for determining the adiabatic efficiency of the compressor relied on the measurements of the deceleration rate of the impeller while coasting. It was found that the overall efficiency at 100% speed was 0.42 with the vaned and 0.32 with the vaneless configuration. At 42% speed the overall efficiencies for the vaned and vaneless configurations were about 0.23 and 0.19 respectively. These values are significantly lower than any CFD estimate made. It is theorized that these low efficiencies were in large part due to the presence of a stationary shroud covering the impeller that was not taken into account in the CFD analysis. This theory was substantiated by the observation of a large spanwise impeller exit flow angle variation. It was concluded that impeller shroud had a very large influence on the fluid dynamics of the impeller, and it should not be neglected in future CFD work. The 3-D calculations indicated that blockage and flow distortion caused by the right angle inlet might also be an important source of loss. The importance of loss at the inlet will be determined in future experiments with the original right angle inlet replaced by a smooth design. In addition to the spindown method, the efficiency was evaluated
using the Euler turbine equation and direct temperature measurements. The Euler equation was shown to be unsuitable for this application due to the removal of turning from the flow by the impeller casing. Temperature measurements were shown to be subject to large errors due to difficulties in achieving adequate heat transfer to the measuring thermocouple under low-pressure conditions, and to the loss of heat into the cooler rig structure.

6.2 Recommendations for Future Work

The following recommendations are based on the author's experience with the assembly and operation of the test facility and reduction of the experimental data.

1. The cobra head and diffuser exit pitot probes should be replaced with instruments with larger impact holes and internal passages. This will improve the instruments' time response and increase the Re of their impact holes thereby increasing the accuracy of pressure measurements.

2. An attempt should be made to replace as much of the plastic tubing in the pressure line system as possible with metal. This will minimize the effects of outgassing, which complicate the accurate measurement of pressures.

3. A vacuum pump with a larger capacity should be acquired. This will reduce the time needed to evacuate the rig in preparation for a vacuum spindown test.

4. A smooth impeller inlet should be tested to determine the importance of inlet losses on the performance of the compressor.

5. The effects of an increased gap between the blade tips and the impeller casing should be investigated. This may allow the reduction of viscous loss in the gap while not appreciably changing its sealing ability.

6. An impeller design including a rotating shroud should be constructed. It is hoped that this may help improve the impeller efficiency by removing the flow distortion and viscous loss caused by the stationary casing.

7. In light of the evidence for the great importance of the impeller shroud in the impeller fluid dynamics, CFD calculations that take it into account should be performed.
Appendix A

Estimation of the Impeller System’s Rotational Inertial

A.1 Impeller Disk

The model of the impeller that was used for this calculation is shown in Figure A.1. For the sake of simplicity, all dimensions are given in inches, but were converted to meters for the calculation. The density of the titanium alloy used in the impeller is

\[ \rho_{\text{Ti64}} = 4430 \ \text{kgm}^{-3} \]

Estimating the rotational inertial of each segment of the impeller:

\[ I_A = \frac{m_A}{2} \left( R_{A(\text{max})}^2 + R_{A(\text{min})}^2 \right) \]

\[ I_A = 0.10948 \ \text{kgm}^2 \]

\[ I_B = \frac{m_B}{2} \left( R_{B(\text{max})}^2 + R_{B(\text{min})}^2 \right) \]

\[ I_B = 0.05151 \ \text{kgm}^2 \]

\[ I_C = \frac{m_C}{2} R_C^2 \]

\[ I_C = 0.00418 \ \text{kgm}^2 \]

\[ I_D = m_D \bar{R}_D^2 \]

\[ I_D = 0.00097 \ \text{kgm}^2 \]

\[ I_E = m_E \bar{R}_E^2 \]

\[ I_E = 0.00010 \ \text{kgm}^2 \]

The rotational inertia of the blades was calculated from

\[ I_F = 2\pi h_F \rho_{\text{Ti64}} \int_{r_{\text{min}}}^{r_{\text{max}}} \left( \theta_p(r) - \theta_s(r) \right) r^4 \, dr, \]
\( \theta_p(r) \) and \( \theta_s(r) \) are the angular distributions of the pressure and suction surfaces, respectively. These distributions were fit with a cubic spline from specified points on the original blueprint. Numerical integration gives

\[
I_p = 0.01482 \text{ kgm}^2. \quad \text{(all six blades)}
\]

Summing over all sections of the impeller disk:

\[
I_{\text{imp}} = 0.1811 \text{ kgm}^2.
\]

### A.2 Impeller Shaft

A schematic of the impeller shaft is shown in Figure A.2. The density of the steel alloy is given by

\[
\rho_{Fe} = 7830 \text{ kgm}^{-3}.
\]

The rotational inertia of the shaft is calculated from

\[
I_{\text{shaft}} = \sum_i \frac{m_i}{2} R_i^2,
\]

This was evaluated to give

\[
I_{\text{shaft}} = 0.00342 \text{ kgm}^2.
\]

### A.2 Motor

The motor’s rotational inertia is expected to be small compared to the other structures in the rotating system. For the sake of simplicity, it will be modeled as a cylinder of copper of radius 1.8” and length 4”, which corresponds roughly to the size of the motor’s rotor:

\[
I_{\text{motor}} = 0.0036 \text{ kgm}^2.
\]

The complete rotational inertia for the system is found by summing over all of the component parts:

\[
I_{\text{tot}} = 0.1881 \text{ kgm}^2.
\]
Figure A.1. Impeller disk cross section

Figure A.2. Impeller shaft cross section
Appendix B

Speed Line Data Points

The speed line data points that were plotted in Figures 5.1 and 5.2 are listed below.

<table>
<thead>
<tr>
<th>Point No.</th>
<th>$p_r/p_{hi}$</th>
<th>$\frac{m}{m(\text{design})}$</th>
</tr>
</thead>
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<td>1.094</td>
<td>0.334</td>
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<tr>
<td>2</td>
<td>1.096</td>
<td>0.334</td>
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<tr>
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<td>0.249</td>
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<tr>
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<td>0.223</td>
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<td>0.203</td>
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<td>0.173</td>
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<tr>
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Table B.1. Speed line data points for 42% speed vaned run.

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<th>$\frac{m}{m(\text{design})}$</th>
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Table B.2. Speed line data points for 100% speed vaned run.
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<th>$\frac{m}{m(\text{design})}$</th>
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<tr>
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<td>1.166</td>
<td>0.105</td>
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Table B.3. Speed line data points for 42% speed vaneless run.

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<th>Point No.</th>
<th>$p_d/p_{1t}$</th>
<th>$\frac{m}{m(\text{design})}$</th>
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Table B.4. Speed line data points for 100% speed vaneless run.
References


