24.400

Proseminar in philosophy I

Fall 2003

Tarski's Definition of Truth

Let *L* be a first-order language with identity, where the logical vocabulary is '∀', '&', '~', together with an infinite supply of variables 'x_1', 'x_2',… The non-logical vocabulary of *L* comprises 'Bernhard', 'Clare', 'Jason', 'Johanna', 'Selim', and a two-place predicate 'L'. The quantifiers of *L* are stipulated to range over MIT graduate students, '=' is interpreted as expressing the identity relation, 'Bernhard', 'Clare',… as referring to Bernhard, Clare,… respectively, and 'L' as expressing the loving relation.

A materially adequate and formally correct definition of 'true sentence of *L*' is as follows:

Definition of the denotation of a term relative to a sequence

(i) The ith variable of *L* denotes an object **o** relative to a sequence S iff **o** is the ith member of S. (So, relative to <Adina, Adam, Bernhard, Clare, Adam, Asta, …>, 'x_4' denotes Clare.)

(ii) A name ν of *L* denotes **o** relative to S iff ν=' Bernhard' and **o**=Bernhard, or ν='Clare' and **o**=Clare, or ν='Jason' and **o**=Jason, or ν='Johanna' and **o**=Johanna, or ν='Selim' and **o**=Selim.

Definition of the truth of a formula relative to a sequence (or of a sequence satisfying a formula)

(i) An atomic formula $\ulcorner(\alpha = \beta)\urcorner$ is true relative to a sequence S iff the object denoted by the term α relative to S is identical to the object denoted by the term β relative to S.
[Alternatively: a sequence S *satisfies* an atomic formula $\ulcorner(\alpha = \beta)\urcorner$ iff the object denoted by the term α relative to S is identical to the object denoted by the term β relative to S.]

(ii) An atomic formula $\ulcorner(L\alpha\beta)\urcorner$ is true relative to a sequence S iff the object denoted by the term α relative to S loves the object denoted by the term β relative to S.

(iii) A formula $\ulcorner(\sim\phi)\urcorner$ is true relative to S iff ϕ is not true relative to S.

(iv) A formula $\ulcorner(\phi\&\psi)\urcorner$ is true relative to S iff ϕ is true relative to S and ψ is true relative to S.

(v) A formula $\ulcorner(\forall\nu\phi)\urcorner$, where ν is the ith variable, is true relative to S iff ϕ is true relative to every sequence S* that differs from S at most in the ith place.

Then, for all sentences *x*, *x* is a true sentence of *L* iff *x* is true relative to every sequence of graduate students (or: is satisfied by every sequence).

So far we only have an "inductive definition" of 'denotation relative to a sequence' and 'true relative to a sequence'. That is, we have fixed the application of these expressions without supplying other expressions to which they are equivalent. But this may be remedied as follows:

α denotes **o** relative to S iff there is a set D such that $\langle\alpha, \mathbf{o}, S\rangle \in D$, and the members of D are exactly the triples $\langle\gamma, \mathbf{o}\#, S\#\rangle$ such that either (i) γ is the ith variable and **o**# is the ith member of S#; or (ii) γ is a name and either γ='Bernhard' and **o**#=Bernhard, or γ='Clare' and **o**#=Clare, or γ='Jason' and **o**#=Jason, or γ='Johanna' and **o**#=Johanna, or γ='Selim' and **o**#=Selim.

ϕ is true relative to S iff there is a set T such that $\langle\phi, S\rangle \in T$, and for all formulas x and sequences S#, the members of T are exactly the pairs $\langle x, S\#\rangle$ such that either (i) x is $\ulcorner(\alpha = \beta)\urcorner$ and the object denoted by the term α relative to S# is identical to the object denoted by the term β relative to S#; or (ii) x is $\ulcorner(L\alpha\beta)\urcorner$ and the object denoted by the term α relative to S# loves the object denoted by the term β relative to S#; or (iii) x is $\ulcorner(\sim\phi)\urcorner$ and $\langle\phi, S\#\rangle \notin T$; or (iv) x is $\ulcorner(\pi\&\psi)\urcorner$ and $\langle\pi, S\#\rangle \in T$ and $\langle\psi, S\#\rangle \in T$; or (v) x is $\ulcorner(\forall\nu\psi)\urcorner$, where n is the ith variable, and $\langle\psi, S^*\rangle \in T$, where S* differs from S at most in the ith place.

Then the explicit definition of 'true sentence of *L*' is:
For all sentences *x*, *x* is a true sentence of *L* iff there is a set T_L such that $x \in T_L$ and for all sentences *y* of *L*, $y \in T_L$ iff there is a set T such that for all sequences S′ , $\langle y, S'\rangle \in T$, and the members of T are exactly the pairs $\langle x, S\rangle$ such that either

(i) x is $\ulcorner(\alpha = \beta)\urcorner$ and there is a set D such that:

for all terms γ, objects **o**#, and sequences S#, the members of D are exactly the triples $\langle\gamma, \mathbf{o}\#, S\#\rangle$ such that either (a) γ is the ith variable and **o**# is the ith member of S#; or (b) γ is a name and either γ='Bernhard' and **o**#=Bernhard, or γ='Clare' and **o**#=Clare, or γ='Jason' and **o**#=Jason, or γ='Johanna' and **o**#=Johanna, or γ='Selim' and **o**#=Selim.

and there are objects **o′**, **o″** such that $\langle\alpha, \mathbf{o}', S\rangle \in D$ and $\langle\beta, \mathbf{o}'', S\rangle \in D$, and **o′** is identical to **o″**;

or:
(ii) x is $\ulcorner(L\alpha\beta)\urcorner$ and there is a set D as above and there are objects $\mathbf{o}'$, $\mathbf{o}''$ such that $<\alpha, \mathbf{o}', S> \in D$ and $<\beta, \mathbf{o}'', S> \in D$, and $\mathbf{o}'$ loves $\mathbf{o}''$;

or:
(iii) x is $\ulcorner(\sim\phi)\urcorner$ and $<\phi, S> \notin T$;

or:
(iv) x is $\ulcorner(\pi\&\psi)\urcorner$ and $<\pi, S> \in T$ and $<\psi, S> \in T$;

or:
(v) x is $\ulcorner(\forall\nu\psi)\urcorner$, where ν is the ith variable, and $<\psi, S^*> \in T$, where S^* differs from S at most in the ith place.