

24.400
Proseminar in philosophy I

Fall 2003

Is Number a property of external things?

Distinguish: numbers are properties of “agglomerations”; numbers are properties (as opposed to objects); arithmetic is a physical science; arithmetic is empirical; arithmetic is contingent. The objection to be discussed only concerns the first.

The Property view:

An ascription of number, for instance ‘The leaves on the tree are 1000’, ascribes a property – *being 1000* – to a *composite object*, the aggregate or “agglomeration” of leaves.

(Mill adds that the property is “the characteristic manner in which the agglomeration is made up of, and may be separated into, parts”.)

This is not at all unnatural:

In language, numbers [numerals] most commonly appear in adjectival form and attributive constructions in the same sort of way as the words ‘hard’ or ‘heavy’ or ‘red’, which have for their meanings properties of external things. (§21)

However:

[I]t is quite true that, while I am not in a position, simply by thinking of it differently, to alter the colour or hardness of a thing in the slightest, I am able to think of the Iliad either as one poem, or as 24 Books, or as some large number of verses.

And:

If I give someone a stone with the words: Find the weight of this, I have given him precisely the object he is to investigate. But if I place a pile of playing cards in his hands with the words: Find the Number of these, this does not tell him whether I wish to know the number of cards, or of complete packs of cards...To have given him the pile in his hands is not yet to have given him completely the object he is to investigate; I must add some further word – cards, or packs, or points. (§22)

This might suggest that Frege is pointing to a contrast between sentences like:

(1) These are four

And:

(2) This is yellow

(3) This is hard

The contrast is this. Fixing the referent of 'these' to be (as the Property theorist thinks) such-and-such an agglomeration is not to fix the proposition expressed by an utterance of (1), but fixing the referent of 'this' is to fix the proposition expressed by an utterance of (2) or (3). In this sense, one can alter the number of things by "thinking of them differently". And if so – the argument might continue – the explanation is that *being four* – unlike *being yellow* or *being hard* – is not a property. For, if *being four* were a property (of some agglomerations), presumably fixing the referent of the demonstrative to be such-and-such an agglomeration *would* fix the proposition expressed by an utterance of (1).

However, this argument is not very convincing, because the contrast is dubious. Fixing the referent of 'this' *isn't* to fix the proposition expressed by an

utterance of (2) or (3). (Imagine (2) uttered while demonstrating a watermelon, and (3) uttered while demonstrating a chunk of gold. Plausibly the referent of the demonstrative in (2) is the watermelon, and the referent of the demonstrative in (3) is the chunk. The details could be filled out so that the proposition expressed by (2) is true iff the interior flesh of the watermelon is yellow; alternatively, iff the skin of the watermelon is yellow. Similarly – and perhaps more convincingly – with (3).)

Although color and hardness are not entirely happy as contrasting examples, Frege does have an excellent point. Agglomeration $a =$ agglomeration b if a and b have the same parts: the agglomeration of leaves = a certain agglomeration of cells, the agglomeration of two packs = a certain agglomeration of 104 cards. On the Property view,

(4) These packs are two

ascribes a certain property to the agglomeration (allegedly) picked out by 'these packs'. (Perhaps 'are two' is like 'is hard', expressing different properties in different contexts, but never mind if so.) Further, that agglomeration of (two) packs is a certain agglomeration of (104) cards. Since the agglomeration picked out by 'these packs' is a certain agglomeration of cards, according to the property theorist the following sentence is true (relative to a context C in which the referent of each demonstrative is the same pile of 104 cards):

(5) These packs = these cards

And (relative to C) (4) and (5) seem to imply

(6) These cards are two

which is false.

Contrast: This is hard/This is Frege's favorite paperweight/Therefore, Frege's favorite paperweight is hard.